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Preface

In issuing its forty-third annual volume of TRANSACTIONS, the American Institute of Electrical Engineers has published in chronological order the papers and discussions presented at the four conventions and two regional meetings held under its auspices during the year, 1924. The articles have all been printed in the JOURNAL either in full or in abridged form; they are published here in entirety with the discussions for each special group immediately following. Owing to lack of space several articles of importance but of somewhat transitory interest have been omitted. These will be found listed on page 1369. The reports of the Technical Committees at the Annual Convention are included; also the Board of Director's annual report for the fiscal year ending April 30, 1924, and lists of the several officers, committeemen, Section and Branch officers for the corresponding period, appear toward the end of the volume. The index contained herein is a greatly improved feature, the subjects being classified under general headings chosen with regard to the information contained in the papers. In many instances the subjects are also cross-referenced.

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Superpower Transmission

Economies and Limitations of the Transmission System of Extraordinary Length

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Review of the Subject.—*This paper is a study of the transmission of very large blocks of power for extraordinary distances and has for its purpose the bringing out of the major operating characteristics of such a system, the characteristics which it possesses which are different from those of shorter transmissions and the duties imposed upon generators, transformers, synchronous condensers, switches, etc., primarily as affecting their design. The paper defines a superpower transmission line as a line of great length in which the charging kilovolt-amperes per mile of length is of the same order of magnitude as the reactive kilovolt-amperes developed by the full-load line current passing through the reactance of the line and in which the resistance is small relative to the reactance.*

Such a line is adapted for economical transmission only for a fairly definite amount of load and any great increase or decrease below this point leads to poor economy or unstability. Since, however, the load appropriate to a given line depends upon the voltage, an appropriate line can be laid out for any reasonable amount of power to be transmitted.

In order to secure a definite set of conditions to serve as a specification for determining the performance of generators, transformers etc., all applying consistently to the same system, a typical hypothethical transmission has been assumed, namely, a delivery of 400,000 kw. over a distance of 500 miles over four circuits 220,000-volt at each end.

The characteristics of this line are worked out showing the efficiency, condenser capacity required, data on circuit breaker arrangements, protective relays connection to receiving network, provision for spare parts, switching of units and ratings of various apparatus.

It is shown that such a system is very sensitive to the receiving end voltage; when this voltage drops there will be a tendency for the generators to run away if the system is not properly laid out.

In addition a discussion is given of the effects of various prescribed values for the terminal voltage between 220,000 and 245,000 to show the effect of increasing the voltage 10 per cent, of maintaining the generating end 10 per cent higher than the receiving end and also of stabilizing the middle point with synchronous condensers.

The typical hypothetical case chosen shows one layout, it being recognized that other layouts may be chosen. This particular layout is operated without any high-tension switching of live lines. The layout is intended to relieve the duty on circuit breakers and as a matter of fact no breaker can be called upon to interrupt a short circuit of more than ¾ of a million kilovolt-amperes; this is a very favorable condition and is secured without materially limiting the equalization of the load. The layout connects with the assumed distribution net at a considerable number of points and no large portion of the total delivered power can be concentrated at any one point; this serves to secure a very intimate connection between the network and the transmission, and at the same time prevents any one breakdown, however, complete, from materially disturbing the major portion of the transmission system.

INTRODUCTORY

THE art of electric power transmission is just entering upon another stage of development, and this a most important one. The period characterized by the general adoption of the 100,000-volt line has been most fruitful. The coming period with lines of voltages in the neighborhood of 225,000 volts and of very great length will be marked by important changes from the old apparatus and the old practise. Considering electrical problems, the 100,000-volt line, to the usual designer, is a problem in drop, to keep the voltage at the substation within reasonable range of the generator voltage with varying load, to secure a good efficiency, and to watch the charging current to see that the light load conditions do not cause an unduly high potential or overload a generator. There is also the problem of insulation, but this is constituted largely of selecting a good make of insulator and watching the manufacturer, also the maintenance of the insulators in good condition during operation.

For the superline of the coming period, the problem of the designer is much more complex. The matter of power factor instead of being merely one of the underlying factors in drop calculations, becomes the all-important feature of the line, serving as the only feasible means of controlling voltage and efficiency. It must be completely under control of the operator. Instead of establishing the voltage at one end of the line as a means of controlling the voltage at the other end, the voltage in the superline must be controlled at both ends and care taken to keep track of the voltage at the center of the line to see that it does not rise too high. Instead of adding reactance to keep down the heavy current in short circuits, as in the 100,000-volt system, the designer will find that with the superline the short-circuit current may not greatly exceed full-load current and that the securing of enough current over the line to insure the holding of the machines on the two ends of the line in synchronism at times of disturbance becomes a serious problem. Other features bring forward novel conditions.

It is the purpose of this paper to discuss the nature of the super transmission system, bringing out some of the peculiarities characteristic of it and to offer some numerical data to give some measures of its economies and limitations. In order to give concreteness to the discussion, a typical but hypothetical example is chosen and the layout worked out in enough detail to develop the novel problems involved and suggest solutions. A sufficient number of calculations are made to determine

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the performance of the system under all conditions normal and abnormal.

Capacity

Unlike the ordinary 100,000-volt transmission line, chosen for efficiency or regulation, the superline has a substantially definite kilowatt capacity which cannot be practically exceeded. In this use of the term, a "Superline" is to be taken as a long line in which the kilovolt-ampere of charging current is numerically of the same order as the kilovolt-ampere developed by the load current in the reactance of the line, and in which the resistance is small in comparison with the reactance.

The dominating role of this relationship of charging kilovolt-ampere to line reactance kilovolt-ampere, set forth in a paper of my own in 1909, has been well expounded by Mr. Harold Goodwin, Jr.[1]

Very briefly its significance may be stated as follows: Since the resistance component of line drop is proportional only to the resistance, which can be made as small as desired, while the reactive component is proportional to the reactance of the line, which is substantially fixed beyond the control of the designer, the kilowatt capacity of a line cannot be made to exceed the fixed limitation imposed by the reactance unless some additional factor be introduced. The reactance considered alone would greatly reduce the capacity of superlines below that actually available. It would largely eliminate any advantage in the reduction of resistance beyond a certain point.

This situation may be avoided, however, by proper control of the line charging current; which fact may be explained as follows: For any definite section of line the charging current represents a certain kilovolt-ampere value, having a phase 90 deg. in advance of the voltage ($\frac{1}{2}\,C\,V^2$). Similarly in that section of line there is a kilovolt-ampere value due to the current passing through the line reactance ($\frac{1}{2}\,I^2\,X$) which is 90 deg. behind the line current in phase. If now the power factor is unity and the current and voltage are in phase with each other, the phases of these two kilovolt-ampere values will be exactly opposite and if equal they will neutralize each other as far as the rest of the system is concerned. This will leave the resistance as the only quantity causing line drop. That is, by the proper correlating of power factor, line voltage, and load current, the voltage drop may be made to be that due to resistance only. To put it another way. In an excited open-circuited high-voltage line the charging current causes a rise of potential along the line toward the open end. On the other hand, in a loaded line without charging current, the line reactance causes a drop in voltage toward the loaded end. If these two tendencies be made numerically equal by properly choosing voltage and load and be made opposite by establishing unity power factor, the tendency to rise and the tendency to drop will neutralize, leaving the resistance to determine the actual line drop.

1. Qualitative Analysis of Transmission Lines, A. I. E. E. JOURNAL, January 1923.

It goes without saying that in an actual line, some departure must be made from these ideal conditions, and it is the function of the designer of the superpower line to so control these departures as to secure the best compromise between efficiency, regulation, operating quality, etc. and cost.

For carrying large loads obviously a high voltage is necessary, but for any given load a particular voltage is most suitable. In this particular relation frequency plays little part, since the neutralization described above is not affected by change of frequency. Neither does the length enter as a factor theoretically, except to the extent that the consequences of a departure from ideal conditions will have a smaller effect with a short line and for the effect of ohmic resistance. Similarly frequency is of great importance when the ideal neutralization is not secured, as occurs with light load short circuits, etc.

Having chosen the load to be carried and the proper voltage, or having chosen the voltage and the proper load corresponding, the resistance can be chosen to determine the energy loss or efficiency. The designer does not, however, have a free hand to vary the resistance, as it is the resistance that is one of the potent factors tending to upset the balance of reactance and capacity, and is thus closely linked with power factor. The formula given by Goodwin, viz., kv-a. = $e^2/.04$ Where "e" is the line kilovolts gives a good idea of the equalizing capacity corresponding to various voltages.

In my paper of 1909 above referred to, I have suggested certain methods of increasing the load capacity of a super line without increasing the voltage, such as using divided conductors. While I see no reason why this method is not perfectly feasible, so far no occasion seems to have arisen in which there is not some other method less novel which would serve to secure the necessary capacity. This method is not a part of the subject matter of the present paper and will not be further touched on.

Regulation

In the superpower line, voltage regulation is naturally a most important function and fortunately, thanks to the competent means available, may be very satisfactorily controlled.

In the first place it will be necessary to have control of the voltage at both ends of the line independently. Since synchronous type apparatus is used on both ends and since the voltage drop is extremely sensitive to power factor, no single setting of the field current on the synchronous machines on either end of the line would serve to secure satisfactory voltage at that point.

While not theoretically necessary, usually the most satisfactory regime for regulation will be to maintain voltage constant automatically at each end at whatever voltage may be most suitable at that end. The

voltage, if necessary, may even be somewhat higher at the receiving end without serious additional expense. To arrive at the nature of voltage regulation in a superline, we may assume that the line is carrying full load with the capacity kilovolt-amperes and the reactance kilovolt-amperes equalized. If now a small leading current be made to flow from the generating to the receiving end, this leading current passing through the line reactance will cause a tendency for a rise in potential toward the receiving end. By choosing the amount of this leading current the amount of rise can be controlled. The effect of the resistance is of course, such as to cause a drop due to this leading current, but since in the superline the reactance is much greater than the resistance, and since the resistance component of the leading current voltage effect will be out of phase with the leading current, it will be substantially negligible; conversely with a lagging current.

To put this in another way. If we establish a voltage at each end of the line corresponding to the condition corresponding to the leading current just described, the line currents must flow as described to produce these voltages. Since any constant potential synchronous machine has a definite voltage and will supply, within its capacity, any current to its circuit required to maintain its voltage, such machines are entirely suitable for superpower line regulation especially in connection with field regulators.

Connected to the receiving end of such a line, a synchronous machine will tend to supply whatever current the transmission line takes at that voltage and at the same time to supply the load current required by the system connected to the line. The synchronous condenser actually takes, however, only the difference between these currents, which may be large or small and positive or negative as the case may be. In case the power factor of the load is low and the power factor required by the line close to unity, there will be a large out-of-phase kilovolt-amperes to be supplied by the synchronous machine at the receiving end and this is the most salient fact about the use of the synchronous machines, usually synchronous condensers.

Thus it is clear that any fixed voltage at either end may be easily maintained and for any load up to full load if proper synchronous condensers are available. This voltage will be held fixed by automatic regulators.

Equation of Typical Line

To illustrate numerically where these fundamental principals lead, I have assumed a superline of rather an ambitious capacity, but still an entirely feasible one for our present day knowledge, and have made the necessary calculations.

This line is 500 miles long, this being taken as an exceptionally long line and is insulated for 250,000 volts, 60 cycles. Its conductor has an outside diameter of 1 3/32 in. and its resistance is 0.015 ohms per thousand feet of cable, or the equivalent of 700,000 cm. The mean conductor spacing is 15.8 feet[2].

Performance of Line

Fig. 1 shows the efficiencies of this line for loads from about 10,000 kw. delivered up to 150,000 kw. on the following four assumptions as to terminal voltage conditions:

(A) that a voltage of 220,000 is maintained at each end.

(B) that a voltage of 220,000 is maintained at each end and that 220,000 volts is also maintained at the middle.

(C) that voltage of 220,000, is maintained steady at the receiving end and of 245,000 volts at the generator end.

(D) that the voltage is maintained constant at both ends at 245,000 volts.

Fig. 1 also shows the power factor at both ends for each of the four assumptions and the voltage at the middle of the line for (A) and (C).

These four cases are chosen to illustrate the relative effect on efficiencies and power factors of these variations of voltage.

As a matter of interest the efficiency of one half the length of this line, giving a transmission of 250 miles is added in Fig. 1 with 220,000 volts maintained on both ends.

Method (*A*) shows the same voltage at both ends, viz. 220,000 with no limitations on the voltage at the middle of the line, which will rise above the ends on light loads. Method (*B*) the same voltage at both ends and in addition the voltage at the middle stabilized at the same value. This can be accomplished by placing a step-down station at the center of the line with synchronous machines of some sort provided with automatic means of fixing the voltage.

The effect of this middle station as far as voltage conditions go is to divide the line into two lines, each of half the total length. Since the voltage at the middle of the line is maintained constant either half of the line may be considered as a line independent of the other half, except that the energy delivered by the receiving end of the first line must equal the energy delivered to the second line (plus the energy loss in the synchronous

2. As a matter of information it may be noted that the "Equations" of this line are

$$V \sin \varphi = .532\, S \sin (\theta + 5° 4') + 606.5\, Q \operatorname{Sin} 84° 58'$$
$$I \operatorname{Sin} \psi = .532\, Q \operatorname{Sin} 5° 4' - 0.00119\, S \operatorname{Sin} (\theta - 88° 56')$$

Where
V = Generator voltage
S = Load voltage
I = Generator current
Q = Load current
θ = Angle of advance of S over Q
$\operatorname{Cos} \theta$ = Power factor of load
φ = Phase of V
ψ = Phase of I
$\operatorname{Cos} (\varphi - \psi)$ = Power factor of generator current

From these equations any desired set of terminal conditions may be calculated.

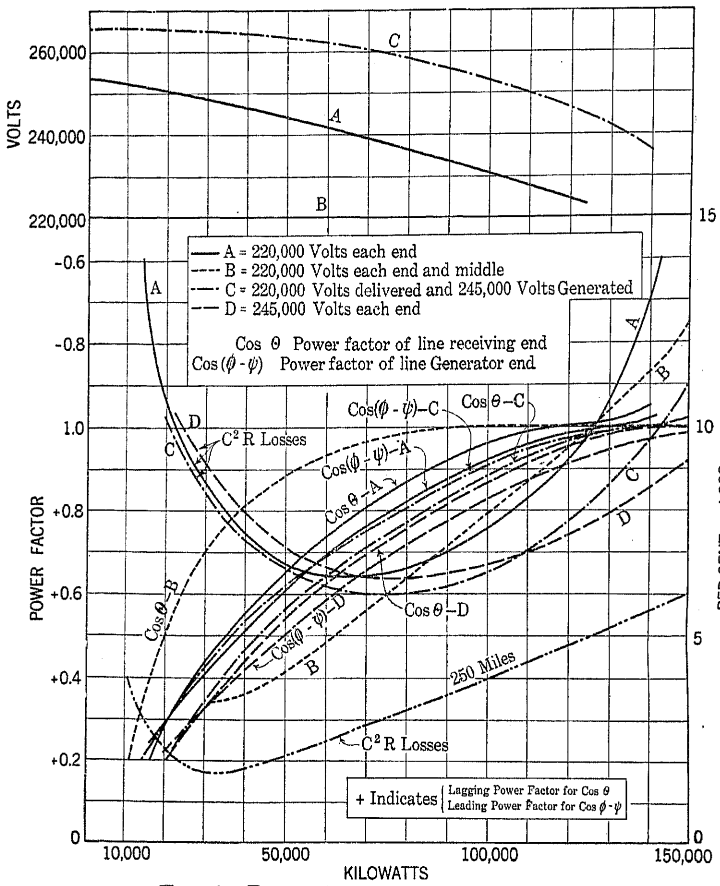


Fig. 1—Power Factor and Line Loss

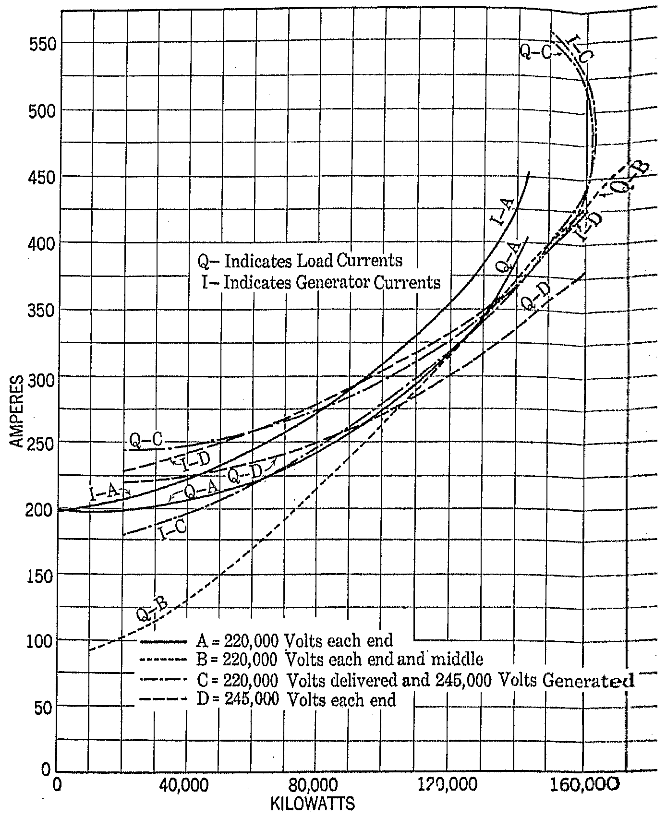


Fig. 2—Generator Currents

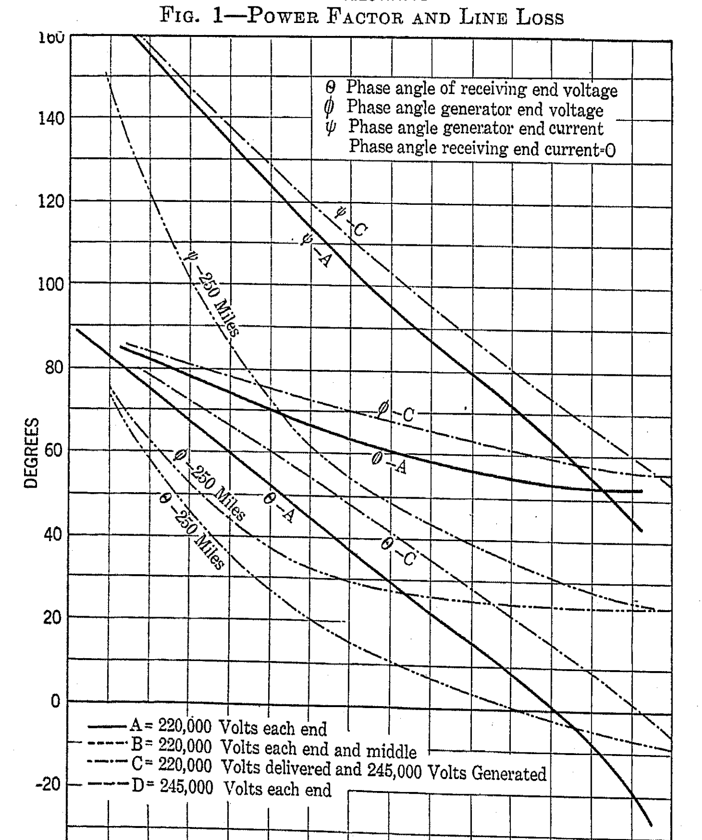


Fig. 3—Phase Position of Voltages and Currents Phase of Load Current Assumed as O

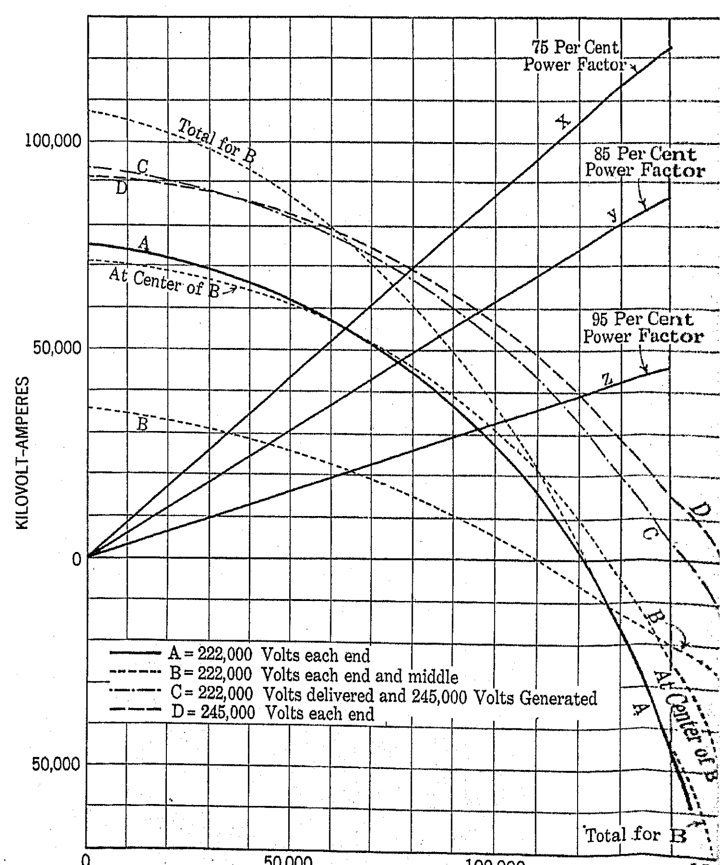


Fig. 4—Out-of-Phase Kilovolt-Amperes for Various Loads, Receiving End

machine) unless some load is taken off or some power supplied at the middle point. It is of course necessary that the synchronous machine at the center supply all the out-of-phase kilovolt-ampere required to maintain the receiving end of the first half at 220,000 volts and all that is required to maintain the sending end of the second half at 220,000. The kilovolts required for these two purposes may be of the same sign or opposite sign according to the load and the power factor and the resultant may be leading or lagging. This is the constant voltage transmission recently recommended by Baum.

Method (C) is to show the effect of a 10 per cent slope of potential in the direction of the load.

Method (D) is to show the effect of a 10 per cent increase in voltage over Method (A), the voltage being 245,000 volts at both ends.

With regard to the power factor curves of Fig. 1 it should be remembered that for any particular value of energy delivered the voltage assumed can be attained only with the load power factors shown.

To further illustrate the relations of these quantities Fig. 2 is added showing the currents at the two ends of the line with varying loads and Fig. 3 to show the relative phase positions of the voltages and currents at the ends of the line, the load current phase being taken as 0.

Since the delivery of any desired load at a definite voltage requires that the power must be delivered at a definite power factor, the synchronous condensers at the end of the line must be prepared automatically to deliver as much leading or lagging kilovolt-amperes as may be required and if they cannot supply this amount of energy the voltage will change appropriately.

As a practical matter of cost, the kilovolt-amperes required to maintain this prescribed power factor is a most important matter. These kilovolt-ampere quantities for controlling the line are shown in the set of curves in Fig. 4.

Three curves are added, marked X, Y and Z, showing the out-of-phase kilovolt-ampere component of the *load* the curve X being for a load power factor of 75 per cent, Y for 85 per cent and Z for 95 per cent. The condenser must supply both of these out of phase quantities.

Some comments may profitably be made on the curves of Figs. 1, 2 and 3. The efficiency curves no longer have the old familiar form. Good efficiencies are obtained only over a short range. The use of a middle point condenser station changes the form of the efficiency curve as seen in Fig. 1 in curve B. While the line losses are less with the middle point station at high and low loads, they are greater over the operating range and this without including transformer and condenser losses.

As would be supposed the maximum output occurs with power factors pretty near to unity.

The voltage at the middle of the line runs 15 per cent high at no-load with both ends at the same voltage, but only 10 per cent high when the sending voltage is 10 per cent higher than the receiving voltage.

The phases of the voltages and the corresponding phase positions of the machine armatures vary over an astonishing range, some 135 deg. for different conditions.

Special Conditions

The following special conditions on this line are of interest.

If the receiving end of this line be open-circuited and the generated voltage maintained at 220,000 volts, the open end will rise to 413,000 volts and the charging current will be 302 amperes, case 4, Table "A".

If the receiving end be short-circuited and 220,000 volts maintained at the generator, the generator current will be 223 amperes and the current at the receiving end will be 420 amperes. The power at the generator will be 7470 kw. at a power factor of 0.176 lag, case 8, Table "A".

Should the synchronous condenser pass out of step with the voltage at each end 220,000 volts, the maximum possible current that would be developed would be about 640 amperes approximately, the same on both ends. At the moment of maximum current the power factor would be 0.182 lag at the generator, giving a drag of 44,000 kw. with a negative load of 12,700 kw. at the receiving end with a power factor of 0.053 lead.

This condition is transitory but existing momentarily as the synchronous machines pass the condition of maximum current, case 7, Table "A".

The minimum line current with the voltage of 220,000 at both ends will be 196 amperes as shown in Fig. 2, with a load delivered of 10,000 kw.

The maximum power that can be delivered by the generator theoretically with its voltage kept at 220,000 as distinguished from the maximum current, is 166,000 kw. and will occur with the generator current about 441 amperes with a power factor of 0.986 lagging and the load delivered would be about 143,000 kw., the load current about 405 amperes at a power factor of 0.925 lead, if the load voltage is kept at 220,000 volts, case 3, Table A.

If in this last case the load voltage drops to 110,000 the maximum power that can be delivered by the generator will be about 94,000 kw. and will occur when the generator current is approximately 332 amperes and power factor 0.745 lagging and the energy at the load end will be about 75,000 kw. with current of 433 amperes and a power factor of 0.90 leading; case 6, Table A.

These conditions and several other special conditions are fully tabulated for convenience in Table "A".

Important as are the above features of the superline, the operating characteristics are more so and involve some most interesting discussions. Various aspects will be considered in turn.

TABLE A—500-MILE, 60-CYCLE TRANSMISSION

Case Number	Kw. on Generator High-Tension Side Transformers	Generator Power Factor "—" =leading	Generator Current, amperes I	Relative Phase Positions LoadCurrent = 0			Voltage, kv. Generator End	Voltage, kv-Load End	Kw. in Load-High Tension side	Load Power Factor, taken in Line "—" =leading	Current of Load, amperes	Line Energy Loss, %.	
				Generator Voltage φ	Generator Current ψ	Load Voltage θ							
500 Miles—Voltage Constant at Both Ends. 220,000 Volts													
1	108,400	−.922	308	56° 30′	79°	15° 30′	220	220	100,000	+.962	272.5	7.64%	Normal full load
2	132,200	−.984	354	53° 20′	63° 30′	0 50′	220	220	120,000	+.999	316	9.27%	Maximum load, to be take quired in emergency
3	166,000	+.986	441	52° 30′	43°	22° 27′	220	220	143,000	−.925	405	..	Maximum possible deli power, approximate
4	8,060	−.070	302	5° 4′	91° 4′	0	220	413	0	..	0	..	Line open at load end
5	1,660	−.0233	195.3	90° 30′	179° 10′	90°	220	220	0	..	198	..	No load at receiving end
6	94,080	+.745	332	69° 45′	41° 51′	−25° 4′	220	110	75,000	−.903	433	..	Maximum power possibl half voltage load end, a mate
7	44,400	+.182	640	80° 50′	1° 0′	−93°	220	220	−12,700	−.053	640	..	Maximum possible c approximate
8	7,470	+.176	223	84° 58′	5° 4′	..	220	0	..	..	420	..	Short circuit on receiving en
9	70,800	−.722	253.3	64° 30′	107° 30′	39°	220	220	66,667	+.784	225	6.4 %	Two thirds of normal full loa
500 Miles—Voltage Constant at Both Ends and at the Middle. 220,000 Volts													
10	300,000	+.82	960	..	..	..	220		220,000	..		27%	Maximum possible deliver power, voltage fixed at 2 at middle of line
11	108,640	−.973	..	..	..	..	220	220	100,000	+.998	..	7.96%	Full load with voltage fi middle point by conden 220 kv.
12	63,400	−.878	..	..	..	..	220	220	60,000	+.931	..	4.83%	6/10 Full load with voltag at middle point by conden 220 kv.
13	132,640	−.992	..	..	..	..	220	220	120,000	−.999	..	9.55%	Maximum load taken as re in emergency, witn voltag at middle point by conden 220 kv.
500 Miles—Voltage Constant at Both Ends. 245,000 Volts at Gènerator End, 220,000 Volts at Load End													
14	10,700	−.907	279	63° 0′	87° 0′	27° 40′	245	220	100,000	+.885	296	6.5 %	Normal full load
15	205,000	..	480	58°	37°	−22°	245	220	166,000	..	463	..	Maximum possible del power, approximate
16	104,000	+.812	302	70° 45′	35°	− 5° 4′	245	110	83,660	−.996	443	..	Maximum power possible half voltage, load end, ap mate
17	62,480	+.554	266	77° 24′	21°	− 5° 4′	245	55	43,400	−.996	458	..	Maximum power possible quarter voltage, load en proximate
18	48,800	−.625	184	102° 40′	231° 20′	120°	245	220	−52,000	+.50	274	6.16%	Example — half load deli backward over line
250 Miles—Voltage Maintained Constant at Both Ends. 220,000 Volts													
19	104,000	−.97	282	24° 45′	38° 50′	3°	220	220	100,000	+.998	262	4.0	Normal full load
20	126,000	−.99	336	23° 15′	31°	− 2° 40′	220	220	120,000	−.999	316	4.77	Maximum load taken as re in emergency
21	130,800	+.688	523	..	..	..	210	110	111,760	1.0	588	..	Maximum possible power livered—half voltage at approximate
22	70,000	+.352	681	..	..	..	210	55	62,060	−.866	755	..	Maximum possible power livered—one quarter volta load, approximate

STARTING

When one end of a super transmission line is open and the other end excited, the voltage at the open end rises very high; in our case to 413,000 volts from 220,000 volts. This is an impracticable operating condition and the open-circuiting of either end of the line cannot be permitted. Fortunately the maximum current that can be gotten steadily by any combination of conditions is not greatly in excess of full-load current at the receiving end so that for the duration of a starting period or an emergency, such as a short circuit or falling out of step, no damage would be done to apparatus permanently connected to the line, even if capacity be considerably less than the full curren the line, while an excessive rise of potential woul prevented.

If it be then assumed that the synchronous appar at the receiving end of the line be permanently nected to the line the problem of starting may be c for in several ways.

(a) The generator may be excited and started from rest and the synchronous condenser will fall i step as the generator comes up to speed.

(b) The generator may be brought up to speed v

the field open and then the exciting of the field will start the synchronous motor, if properly designed, which will then pull quickly into step.

(c) If means for revolving the synchronous condenser be provided, both this machine and the generator while still connected may be brought up to speed and they lock into step when their speeds become equal.

It is not the purpose of this paper, however, to discuss the details of starting, but merely to point out how the starting may be practically accomplished.

Synchronizing

Synchronizing may be accomplished in the low tension at either end. As a matter of fact, as a first approximation this superline is in some ways the equivalent of a very high reactance connected between the synchronous machines at the two ends of the line, so great in value that to get full power over the line it is necessary for the two synchronous machines to swing out of step by a very wide angle, thus developing enough voltage to support full-load current.

Considering "*A*" in Fig. 3, the difference in phase of the voltage of the two synchronous machines at the two ends of the line changes from about 5 deg. at a load of 8500 kw. to about 70 deg. with a load of 140,000 kw. That is, if a line with synchronous condensers carrying no load is connected at one end to a similar line loaded, there will be a very wide phase difference between the voltages at the unconnected ends of two lines. When these are connected, however, their phases will come together and the load will divide between them. However, while under the assumed conditions the line conditions can adjust themselves without disturbance on synchronizing at the second point, the condenser must adjust its position more slowly and may cause some heavy current interchange with other condensers but it will very likely be possible to accomplish synchronizing in a manner to avoid such a violent change in the phase position of the condenser. This is a matter of interest rather than a serious difficulty of operation. However, syncroscopes used in the ordinary way would not indicate at all correctly, as to the proper moment for closing the synchronizing switches. Of course, if one end of the two lines is already connected, the other may be also connected without use of a synchroscope, provided it be proper to make this connection at all.

Synchronizing a lone line with an operating substation works differently. If the line be idling the phase of the voltage at the receiving end will be nearly in phase with that at the generator end, as there is little load passing over the line. If it then be synchronized with the load system already operating there will be no immediate change, for the phase of the generator must advance relatively by many degrees before it can take much load. This will occur in a few cycles if the governor of the generator prime mover be set to take such load. It must also drop behind many degrees to take power from the main system. This adjustment of position will occur very quickly in actual practise.

Presumably this connecting of an idle line to the substation is the proper condition for synchronizing. The governor of the generator prime mover can be set for higher speed to cause it to take load when the synchronizing has been accomplished. If one line thus synchronized to a substation is then to be paralleled on the generator end with other already synchronized lines, it should be so loaded (the voltage being maintained constant automatically) that the generators are all in phase before closing the synchronizing switch at the generator end.

While synchronizing is not likely to cause much difficulty, the matter of pull-out torque and holding in synchronism through electrical trouble is much more difficult. If a generator feeds a superpower line synchronized at the receiving end with a load system and a sudden increase of load comes the load system will tend to slow down momentarily. To escape losing synchronism the generator must slow down also, but to accomplish this slowing down, more load must be passed over the transmission line. If now the generator be already loaded as far as the line will permit *by efficiency and voltage* considerations, as for example to 140,000 kw. it will be found by examining the curves of Figs. 1 and 2 and case 3 Table "A" that very little more load can be got over the line. If, however, a certain margin exists so that the normal full load over the line may be increased by perhaps 50 per cent when called upon by a load increase, then a sudden increase of load and slowing down of the load system will cause an increase of power delivered over the line and the generator will slow down, if properly designed, especially if it be a waterwheel with the usual small overload capacity and if its governor have a wide range of speed variation with load.

Again, if a local short circuit occur in the load system near the receiving end of the line, the local voltage will drop, which greatly limits the power that can pass over the line as shown in cases 6, 15 and 16, Table "A" and the generator will immediately tend to pull out of step if the rest of the system *tends to slow down.* Obviously this is a more serious matter and at first sight a most difficult one to handle.

Cases 21 and 22 of Table "A" show that a line of half the length is considerably better and that the maximum power that can theoretically be delivered is greater than for the full length of line with the same terminal voltages. A further treatment of this subject will be found below:

Typical System Diagram

The determination of the best arrangement for the use of breakers on the superpower line and its connected load network, when taken in connection with automatic relay protection for short circuits, will be found to be a very complex matter. The purposes of this paper,

which is largely illustrative, will be best served by assuming a layout which is well suited to the usual conditions and discussing its action under various normal and abnormal conditions. Such an arrangement is shown in the one line diagram in Fig. 5.

It will be assumed that the line voltage is normally maintained automatically at 220,000 volts 60 cycles, at each end by regulators working on the fields of the generators and condensers. The amount of synchronous capacity required for various cases is shown in Fig. 4.

The four circuits, which may well be carried by two two-circuit tower lines, make a well rounded super transmission installation. One circuit may be taken out for repairs and the other three lines be made to carry the total or nearly the whole power.

This overloading of the three remaining lines calls for a very large capacity of synchronous condensers. By reference to Figs. 1, 2 and 4, it will be seen that the demands on the synchronous condensers will be less excessive if at the same time the voltage at the generating end be raised even by such a small amount as 5 per cent and this should be permissible for temporary operation. In this case account is being taken of the requirements of the load as well as the line. This resultant will be the difference between the *X*, *Y* and *Z*, and the *A*, *B*, *C* and *D*, curves.

Synchronous Condensers

We may take as an illustration 400,000 kw. as the total normal load on the transmission with 220,000 volts at each end. This means 100,000 kw. delivered per circuit. From the curves, case 1, Table "A" it will be seen the normal generator power factor is 0.92 leading. An approximate capacity of 115,000 kv-a. is required, but to cover somewhat less favorable contingencies and some overload, 130,000 kv-a. may be taken at a power factor of 90 per cent. If the prime movers are waterwheels they will presumably be given a *maximum* continuous rating corresponding to about 140,000 kw. per circuit.

As to the condensers, we must always satisfy the condition when the line is disconnected from the load circuit, which calls for out-of-phase kilovolt-amperes of about 75,000 kv-a. lag (see Fig. 4) on the line. This means 78,000 kv-a. lag on the condenser terminals. Condenser capacity to meet this no-load condition must be furnished, whatever loading may be assumed for full load. The effect of raising the voltage of the sending end of the line is seen from case *C*, Fig. 4, viz., to increase the no-load kilovolt-amperes required. The *full load* kilovolt-ampere requirements, however, will be less with the higher generator voltage.

Strangely the major part of the charging current is fed from the lower voltage end. This is because the voltage naturally rises along an unloaded line away from the generator. By referring to case 4, Table "A", it is seen that if the receiving voltage were raised to 413,000 volts, this end would supply no charging current at all.

In order, however, to take account of the effect of transformers and condensers we may assume that if the load at the receiving low-tension bus bars is 100,000 kw. at 85 per cent power factor the load will be 105,000 kw. in the high tension, allowing for transformer and condenser losses. The high-tension out-of-phase component of the *load* will be 71,500 kv-a. lagging, assuming the transformer has 8 per cent reactance e. m. f. The line delivering 105,000 kv-a. requires 22,500 of lagging kv-a., giving 49,000 kv-a. lead to be supplied by the condenser in the high tension. Therefore, the kv-a. required from the condenser in its low-tension circuit is 51,500 kv-a. lead, assuming a 5 per cent reactance in the transformer tertiary winding. This is well within the capacity required for the light line running, *viz.*, 75,000 kv-a., the difference being available as overload capacity.

If one line be down and each of the remaining lines deliver 133,000 kw. low tension, the high-tension, out-of-phase kv-a. of the *load* (85 per cent power factor) on the 220,000-volt busbars is 94,000 lag. The out-of-phase high tension required by the line is 47,000 leading, giving a total condenser requirement of 141,000 kv-a. lead on the high-tension side and 5 per cent in excess on the low-tension side or 148,000 kv-a. lead. The amount is far in excess of that required for light line and this is not a desirable operating condition.

Various expedients may be used to reduce this requirement of 148,000 kv-a. For example:

(1) If one line is taken out, the total load required of the system may usually be somewhat reduced, say to 120,000 kw. per circuit, which will naturally reduce the requirements from the condensers.

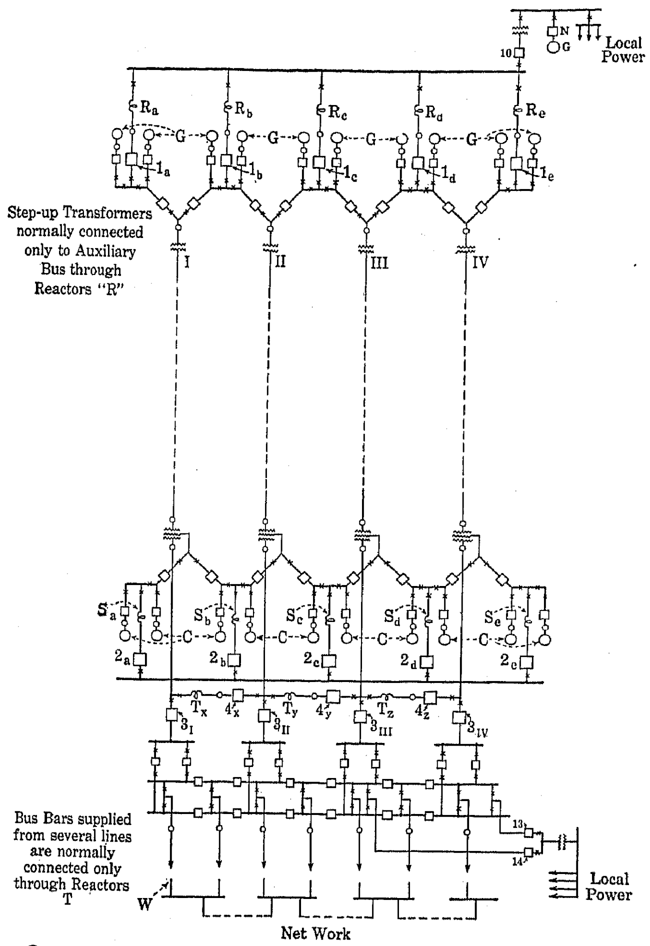
(2) If the generator voltage be raised 10 per cent to 245,000 volts, the requirements of the condensers will be reduced to 95,000 kv-a. lead which is somewhat more than the no-load requirement.

(3) If the power factor of the load at the step-down bus bars be raised to 95 per cent, the kv-a. out of phase of the load is 55,000 high tension, giving with the 47,000 required by the line, $102,000 \times 105 = 107,200$ lead.

(4) Since when one line is down its condensers are not in use, these may be added to give additional capacity to the three good lines. This will bring the available condenser capacity for each of the three operating lines to 133 per cent of normal, or 100,000 kv-a. if normal be taken as 75,000 kv-a.

(5) Normally spare capacity will be provided for condensers, and if it be assumed that this may be used when one line is out and the spare be 20 per cent of the total as in the system of Fig. 5, there will be capacity of two extra condensers for the three circuits which will provide the requisite capacity for 133,000 kw. delivered, counting somewhat on overloading.

However, it would no doubt be more practicable



Ⓖ —Main generator and prime mover, 65,000 kw-95 % P. F. units.
Ⓒ —Synchronous condensers, 75,000 kv-a.
[1] —Automatic oil circuit breaker.
□ —Oil switch, non automatic.
× —Disconnect switches, air break.
○ —Relays for breakers—actions as described below.

—Reactors—at generator station—to pass 400,000 kv-a. on full voltage at condenser end—to pass 300,000 kv-a.
—Step up transformers—130,000 kv-a. Internal reactance as low as practicable, say 8 per cent.
—Step down transformers, 110,000 kv-a. on secondary, and 75,000 kv-a. on tertiary, reactance between primary and secondary say 8 per cent. Between primary and tertiary—very low—say 5 per cent and between tertiary and secondary very high say 12 to 15 per cent.
↓ —Feeders to network.

Fig. 5—One Line Diagram of Typical System Superpower Transmission

In generator leads, relay kills the field of its own generator but only while short lasts; inverse time limit 0.2 seconds on 750,000 kv-a.,—0.5 seconds on 400,000 kv-a.

In lead to breaker [1], relay overload instantaneous, minimum setting 160,000 kv-a.

In high-tension of step up transformers, relay instantaneous to ring a bell as long as power factor is below 50 per cent, lagging.

In primary of step down transformer, (or a combination of currents representing tertiary and secondary windings to give the same result) relay to open breakers [2]. [3] and [4], instantaneous, minimum setting 175,000 kv-a. Also opening [2], [3] and [4] when power factor drops below 50 per cent leading (as motor), in 3 seconds.

In leads to step-down transformer secondary, relay opening [3] in 0.4 seconds.

In leads to synchronous condensers or to transformer, relay opening [2] inverse time limit 0.2 seconds on 400,000 kv-a., and minimum operating value 100,000 kv-a.

In leads to switch [4]—relay definite time limit 0.75 seconds—minimum operating kv-a., 125,000.

In feeders, relay inverse time limit but clearing any short as high as 100,000 kw. in less than 0.25 seconds.

Standard protection for short circuits for generators and local station power.

to count on operating only 120,000 kw. on each of the three remaining circuits when one goes out, thus requiring 95,000 kv-a. lead from each synchronous condenser, this being based on a network load power factor of 85 per cent. Even with this arrangement some of the idle condenser capacity should be used for continuous running at 120,000 kw. generator.

Apparently a very effective arrangement could be secured by providing to have the generator voltage automatically run lower on light load and higher on heavy load. A change of 5 per cent would be an important help and one of 10 per cent would greatly reduce the required condenser capacity.

Thus it is seen that for economic conditions involving the transmission of very heavy loads, capacity in synchronous apparatus may be required numerically in the neighborhood of the amount of the load transmitted. On the other hand a number of simple expedients may be used to materially curtail these requirements. It should be clearly understood that the values so far given are not intended to indicate that these are practicable for an actual installation. They are offered as a basis for discussion. No doubt more consideration will be given to these subjects before such an installation will be made.

It should be noted that to maintain constant voltage on the high tension at full load or with the overload condition, that voltages on the condenser circuits must be regulated to be a little above or a little below the line voltage according to whether the passing of power causes a rise or a fall of voltage in the transformers. For this line and normal full load there will be a drop through the step down transformer and the condenser voltage should be at a lower value than the ratio of turns of the transformer and the line voltage would indicate. This refers to the ratio of turns of primary and tertiary windings. If the load be increased for example, to the point where three lines will carry the load of four or under the conditions when a sudden disturbance calls for a still larger flow of current the condenser will be forced to supply a leading current and the drop in the step-down transformers will become a rise, so that the condenser terminal voltage must be higher than the ratio of turns would call for. The synchronous condenser must be adapted to sustain this condition. Since the primary and tertiary windings are closely related magnetically, these voltage variations will be small.

Similarly at the generating end the voltage regulator must be laid out to correct for the voltage drop or rise of the step-up transformer. Space will not permit a fuller discussion of this point at this time.

As a matter of good design, it is likely that it will be more favorable to provide most of the power factor correction as far as it is required to raise the power factor of the load by synchronous apparatus in the neighborhood of the load itself as this will improve the regulation and efficiency performance of the network as well as adapting the load to the requirements

of the line. It will also reduce the kv-a. load on the step down transformers. For this purpose any available existing synchronous apparatus would serve. The part of the corrective kv-a. that must be varied during changes in operation should be under immediate control and should be supplied by condensers at the step-down station.

For spares add a fifth unit to generators and condensers.

The original assumption of the loading of the line may now be modified to give a favorable layout as follows:

Normal full-load 400,000 kw., 100,000 per circuit on the distribution bus bars (requiring 420,000 kw., 105,000 per circuit on the high-tension side at power factor of 98 per cent lagging). Condenser kv-a. required per circuit 24,000 kv-a. lagging including requirement of step-down transformer, assuming *load* corrected in the network to *unity power factor* at the high-tension side.

Kilovolt-ampere required for no-load condition 79,000 lag per circuit on condenser terminals (only occasionally and for brief periods).

Maximum steady load provided for on three lines 360,000 total, 120,000 per circuit on distribution bus bar (380,000 kw., 127,000 per circuit on high-tension side at power factor 0.995 per cent leading) calling for 11,400 kv-a. per circuit leading on condenser terminals assuming the load corrected in the network to unity power factor at the high-tension side.

Capacity of condensers provided 75,000 kv-a. per circuit. Note that when the 79,000 kv-a. above is required on no-load that the spare condenser can be called upon should this condition continue indefinitely. The extra condenser capacity on the over load condition is highly desirable as a margin for operating reasons. Sufficient corrective capacity must be provided in the network (or at the condensers) to bring the load power factor to unity. The amount of this is shown for various power load factors in Fig. 4.

Efficiency of line, normal 92 per cent, loss 8 per cent; add transformer loss 2×1.5 per cent and condenser losses 4 per cent of 75,000 kw. total 14 per cent loss.

Step-up transformer capacity 130,000 kv-a. per circuit, step-down transformers capacity for load winding 110,000 kv-a., for tertiary winding 75,000 kv-a. This apparatus will carry the overload corresponding to one line out as above, until additional capacity can be added, but will be sensitive to voltage drop.

Generator capacity 130,000 kw. per circuit at 95 per cent power factor.

It should again be noted that this layout is proposed for discussion as a maximum and optimum system and no doubt a more conservative design would be adopted in any particular installation.

Switching

Considering the layout of Fig. 5 more particularly from the point of switching, the most conspicuous characteristic is the omission of all breakers or automatic oil switches in the high-tension circuits. This will be seen to have several important advantages and few disadvantages.

(a) It protects the line from accidental open-circuiting at either end and the very high voltage resulting at the open point.

(b) It eliminates a number of high-tension insulation points and greatly simplifies the high-tension layout.

(c) It eliminates a large item of cost and maintenance. Such a system as in Fig. 5 laid out in accordance with the usual practise would call for from 25 to 30 high-tension breakers at a cost of perhaps over a half a million dollars.

(d) The principle disadvantage of the omission of high-tension switches is the limitation of flexibility; for example, if a line is down in the layout of Fig. 5, its transformers are also down and similarly with the line if its transformer is down. If high-tension disconnect switches are installed any desired reconnection of lines and transformers may readily be obtained, but only at the expense of shutting down another line to make the change in the case of some operations. This use of disconnects in this particular situation offers several very interesting possibilities which space will not permit discussing here.

It may well be argued that for such a system as Fig. 5, feeding into a large network with other generating stations of large aggregate capacity, the simplicity of this layout is of great advantage. Here there is the one function of carrying a large block of power, 24 hours a day with no occasion for changing of connections except in case of some accident and for the regular inspection of generators and condensers. This inspection is provided for by the spare units shown which can be switched in and out one at a time without disturbing load. In case of emergency or to make the infrequent inspections of transformers and lines (that is—such line inspection and testing as cannot be done alive), three lines can be made to carry nearly the load of four. Ordinarily, however, when one line is to be shut down it will be feasible to arrange for a reduction of the load by 10 or 15 per cent on the transmission system. With careful line inspection and maintenance it may be confidently expected that trouble in the major elements of the system will be very rare. This favorable result assumes a very careful design, with the most painstaking scrutiny of details, but not an elaborate or excessively conservative design. The important thing is not to have an extreme factor of safety, but to insure an adequate factor of safety at *all* points. In making this statement as to the rarity of major trouble, reservation must be made to cover the possibility of some unusual and unexpected local disturbance. Such a condition might be malicious interference or one of the rare locally occurring fog belts which coat insulators or (possibly but very improbably) extreme lightning severity, etc.

At this point I would like to state that the practise of working on live lines at 100,000 volts and higher is worthy of most serious consideration on these super lines. While at first thought this practise seems dangerous, a more careful analysis does not bear out this appearance; neither does the experience with such work. I think there is little question that live line testing of high-tension insulators is feasible and reasonably safe. It has already been carried out very extensively. The question then arises whether very simple line operations are not also feasible. The best example of such a simple operation is the changing of an insulator string and this is the most useful operation in practise. A little consideration will show what a wonderful advantage the possibility of changing defective insulators without taking load off the line would be, especially in a line of the length of the one under discussion. In my judgment this advantage is so great as to warrant very considerable exertions. Some superintendents who do not care to do this work with their own crews may obtain more or less the same result by contracting to have their defective insulators changed periodically by outside expert crews as has already been done in many cases. While there will naturally be a great inertia of conservatism holding back the adoption of live line work, such as insulator changing, it is difficult to see how it can be logically ruled out in the end, when the live line testing is already pretty well accepted.

Automatic Relay Protection

Most of the features of the relay protection of Fig. 5 involve the standard use of standard apparatus but there are a number of novel features. The most satisfactory way of bringing out the intended operation will be the analyzing of the effect of various possible accidents. Single-phase grounds and short circuits will have the same effect on these lines since the high-tension neutral is assumed substantially dead grounded at a sufficient number of points. Assume all the generators except the right hand pair are operating, each pair on its own line and all breakers [1] closed, also all condensers except the left hand pair operating, each pair connected to its own transformer and all switches [2] closed; also all breakers [4] closed but distribution bus bars all operated in four groups.

A—Short Circuit on Generator End of Line I

The relays in the leads of the generators feeding the short circuit kill the generator fields, operating in 0.6 seconds. The breaker [1-*a*] will open instantaneously opening about 300,000 arc kv-a.; the other breakers [1] do not open as the current will be too small. The short circuit is then killed as far as the generating end is concerned. Meanwhile the only effect at the receiving end is to develop a current of 223 amperes and to change the power factor of the line to say 18 per cent leading (Table A, Case 8) (current would be lagging if condenser is considered a generator) and to drop the delivery of power that had been passing over this line. Through the action of a relay designed for this purpose, this low leading power factor in the step down transformer primary will open the breaker [3-I] in 3 seconds and at the same time [2 *b*] and [4 *x*]. This disconnects the line entirely from the system but leaves both generator and condensers connected and running. If this renews the short circuit the generator fields will be restored and the condensers fall into step again or, if so designed, once opened, the generator field circuit may remain open in which case the condenser will come to rest and restarting will be necessary. The delay of three seconds is to give the short circuit a chance to clear. No harm is done, since the short circuit will not pull more than about 7500 kw. to 15,000 kw., (Table "A", case 8) and less than full-load current. When the operators see that one line is out they should immediately connect in the spare generators and condensers. If the bad line can then be put in again this may be done at once, but otherwise the full load or a suitable portion can be taken over the three good lines. In case of the dropping out of line *I*, the three waterwheels still on the line will open up their governors and may take load up to the extent of their capacity—say 20 per cent excess making, 60 per cent excess in all of the loss from the bad line and the rest of the load will be taken by the generators in the network. If the latter have more sensitive governors they may take still more of the load. The steady distribution of the load between line circuits can be controlled at will by hand adjustment of the speed of the generator governors.

When the opening of [3-I] cuts off certain feeders to the network, the corresponding load will presumably be supplied around through the network by other routes over the good circuits.

B—Short circuit on primary leads of step up transformers

Substantially the same actions as in Case *"A"*, but currents will be 50 per cent heavier.

C—Short circuit on auxiliary bus, generator station

All breakers [1] go out instantaneously and before the relay in the generator leads can open the fields, since these are inverse time limit and the short circuit is limited by the reactances. No other effect will be produced as the line units can all run perfectly well without the auxiliary connection. A short circuit on the auxiliary bus should be very rare.

D—Short circuit at condenser end of line I

Breaker [3-I] opens instantaneously from relay on primary of step-down transformer and [2 *b*] and [4 *x*] are open by the same relay. Nothing happens at the generator end until the attendant cuts off this line at his leisure. The generator end current will be only 223 amperes (Table "A", case 8) and the current at the load end only 420 amperes. It might be well to have the low generator power factor 0.176 lag, ring a bell to call attention to the condition.

E—Short circuit at the middle of the line 1

This will pull 640 amperes at the generator end and cut off the generator fields, which will open [1-*a*] in 0.5 seconds clearing this end. It will act the same as *A* at the receiving end, [3-1], [2-*b*] and [4-*x*] going out by the same relay.

F—Short circuit on left hand operating condenser leads

Corresponding breaker [2] opens in 0.2 seconds leaving line to go down as in *A*– [3-*I*] opens in 0.4 seconds and [4-*x*] in 0.75 seconds.

G—Short circuit on condenser auxiliary bus

Breakers [2] go out in 0.2 seconds clearing the trouble without disturbing the operation, provided synchronism can be held. If reactances are used with switches [2], conditions will be much easier.

H—Short circuit on distribution feeders

[3 *x*] opens in 0.4 seconds on overload leaving the power from line 1 to pass through [4 *x*] to the other feeders and into the network where it would find its way to the original destination. The time limit of 0.4 seconds is to permit a feeder breaker to clear the individual feeder before [3 *x*] opens. If [3-I] fails to clear the circuit [4-*x*] opens in 0.75 seconds and line I will be disconnected.

The protection of the local station power circuits involves no particularly difficult problems and need not be discussed here.

Synchronizing Power in Typical System

As already pointed out, the characteristics of the superline for efficient full-load transmission of power are so extremely favorable on account of the neutralizing of the reactance effect by the capacity effect, that for other conditions when this neutralization does not occur to the same favorable extent, as for example with overloads, very light load or low voltage, transmission becomes very unfavorable. The very light load condition has been considered above. The overload condition, as an overload problem, is the least serious and easiest to meet, but the low voltage or short-circuit condition is serious and difficult from the point of view of holding synchronism.

Before discussing this matter with regard to the system of Fig. 5, I will consider a single superline circuit such as one of the four here shown. It will transmit considerably more power than full-load power if the voltage at both ends be kept at normal even as high as 145,000 kw. (See Table "A", case 3). If, however, this voltage drop below normal, even if at the receiving end only, the amount of power that can be delivered is greatly reduced as seen in Table "A", case 6. For example, in this case the actual power delivered with half voltage at the receiving end is less than the full load, viz., only 75,000 and this condition gives nearly the maximum deliverable power for this voltage. Since, however, there is little likelihood that the failure to deliver *power* under conditions of abnormal voltages (due presumably to short circuits) will be *per se* of importance, since such conditions can exist for a period of perhaps only one or two seconds, we have only to consider the danger of loss of synchronism or the opening of breakers.

Several cases occur which have to be considered independently.

First, when a short circuit occurs at the receiving end of the system on the step-down side. In this case there is a tendency for the receiving end voltage to drop, which will tend to limit the amount of power delivered by a line. But the full input to the generator continues to be delivered by the prime mover, and the only thing that prevents its speeding up to the light load speed of the governor (considering now one circuit by itself) is such power as may be taken by the line. This is the most serious condition to be met in maintaining synchronism and will be critical when the short circuit is such as to tend to cause the network machines to *slow down;* for example, when the short circuit is through a feeder line with very little armature reaction drop in the generator voltage. Of course, the overspeed governor on the generator prime mover will prevent any dangerous increase of speed, but it will be useless for preserving synchronism.

It follows from the above that for this case the critical factor will be the load developed by the generator, not the load delivered by the line and this generated power may be considerably greater than the delivered power. For example, the drag on the generator in Table "A", case 6, is 94,000 kw. compared with 75,000 kw.

But short circuits are of many varieties. If of such a sort that the actual kw. developed in the short is less than at full load, on account of low power factor in the short circuit, the generators in the network and the main generators, at opposite ends of the line will tend to *speed up* and the load that can be delivered over the line in one direction or the other may be sufficient to hold them in step, even if less in amount than full load power. In this case the governors of the prime movers will have to ultimately control the speeds and the more nearly their speed curves are alike the better the chance of staying in step. The range through which synchronism would be retained would be, on the one hand that condition in which the receiving end prime movers speed up more rapidly than the main generators and are only restrained from exceeding the speed of the latter by the load sent backward over the line helping the prime mover to speed up the main generators and on the other hand the condition in which the main prime movers are just able to pull up the speed of the receiving end machines until equilibrium is reached. This is presumably a very wide range and successful operation through this range is not limited to line voltage conditions in which full load may be passed over the line.

In the case of a single line and a condition such that the prime movers in the net work at the receiving end tend to slow down, synchronism cannot be maintained if the receiving voltage on the line drops to the point

where full load cannot be delivered over the line. In this case it must be noted, however, that the critical voltage is the high-tension line voltage and not the step-down voltage. This is a very important distinction for the drop in the step-down transformer will be considerable on a short circuit especially if the design of the step-down transformer is such that the magnetic interlinking of the tertiary with the primary is much closer than that of the tertiary with the secondary, for the short-circuit kv-a. supplied by the condensers must go through this step-down transformer before reaching the short circuit and this tends to sustain the line voltage. As a matter of fact, if the condensers were connected to taps in the grounded end of the primary windings some gain would result in close interlinkage. A heavy flywheel capacity in the condensers is presumably very helpful in maintaining stable conditions.

is, the restoring torque will develop at least *partially* with the *progressive* separation of the phase positions of the several prime movers generators. The form of curve connecting power transmitted with phase displacement (Fig. 6) shows a condition very favorable for suppressing the pendulum action without the machine getting out of step. Of course, proper *damping* devices will also be of *great* assistance.

Enough has been said to make it clear that for any given case the resultant action is very complex, involving the relative flywheel effects of the different units, their different short-circuit characteristics, their governor speed curves and the damping factor. It may very likely be that in an actual installation the most favorable adjustment of conditions may be secured by providing adjustment in such factors as flywheel effect, short circuit kv-a., etc., and making a

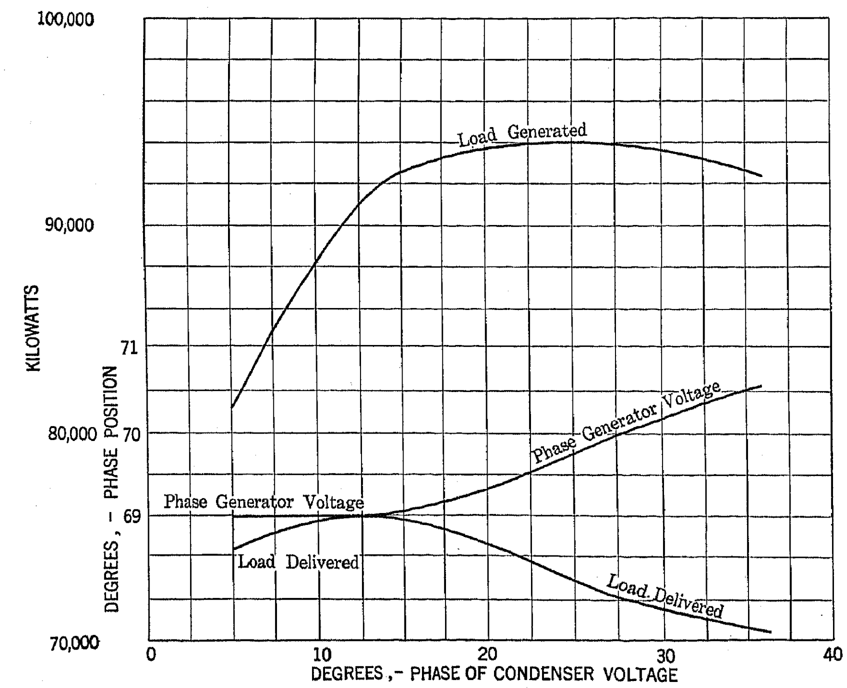


FIG. 6—MAXIMUM POWER TRANSMISSION

final determination in the field. These are matters for the study of specialists and cannot be profitably carried further at this point.

This discussion so far applies to a single circuit or a group of circuits connected directly in parallel. The action of the system of Fig. 5, however, is in effect quite otherwise. There are several critical points of difference.

As has been pointed out by others, the tendency for the various parts of the system to fall apart is theoretically markedly increased by the pendulum effect between two machines starting in equilibrium under normal conditions and subject to a sudden changing in conditions due to a short or even load change due to switching. A new equilibrium position must be assumed and in swinging to this new position there will be a tendency to over run the position of equilibrium and if this position of equilibrium be the maximum torque position also, the temporary over run may cause a falling out of synchronism. The seriousness of this can be determined only after further data are available, for the percentage increase of torque required for maintaining synchronism may not be great and the change will probably not be very sudden, that

First: The four lines while dividing the load freely between one another, are yet sufficiently separate electrically so that a short circuit on one or on the step-down end of one will not greatly disturb the voltage on the others. The high-tension circuits are not directly connected, but the step-down transformers secondaries are connected together through choke coils having a suitable reactance to permit equalization of the loads.

A similar interconnection exists at the generator end.

Second: The synchronous condensers are in parallel and are thus able to hold strongly together at times of disturbance in the network. This serves to hold all four high-tension circuits together. If found desirable to prevent too much disturbance of a condenser by trouble on another line, reactances can be connected between each condenser and the auxiliary bus.

Third: The main generators are connected in parallel through reactances. This serves to tie these generators all in parallel so that no one generator is likely to drop out of synchronism.

Fourth: The feeders on the step down side of the four transmission circuits are so connected through the reactances that a dead short circuit on one will not greatly disturb the voltage of the others, which will, therefore, continue firmly in synchronism through any kind of disturbance, however severe.

Thus this combination of circuits, in addition to providing a simple and practical layout for segregating and disconnecting any circuit suffering a short circuit, will hold itself firmly in synchronism and in synchronism with the network in spite of any breakdown that may occur.

For example should a dead short circuit occur three miles out on the left hand local feeder, *viz.*, at *W*, Fig. 5, current would be discharged into it through the feeder from line *I*, from the condenser bus through the tertiary winding of the transformer and through breaker [4 *x*]. While the voltage would be dropped materially on the local bus fed by line *I*, depending on the reactance of the bad feeder, the high-tension voltage on the line *I* would be dropped to a much less extent and the voltage on the condenser bus very little. The current passed through switch 4 would probably not exceed 200,000 kv-a. which would be supplied partly from the network and partly from the three other lines and would leave the voltage on the three good lines practically undisturbed. Meanwhile the relays would cut off the fault.

This system, to use an analogy, compared to the usual system, is like a locomotive which has an equalized, spring supported frame, compared to the farmer's hay wagon without springs. Every stone in the road causes a shock to the whole wagon while it would be passed over by the locomotive almost unnoticed.

Summary

To give a more concrete idea, numerically, of the effect of abnormal conditions in producing heavy short circuits and to indicate the duty of the oil breakers, the following approximate values are given, assuming a load of 90 per cent power factor at low-tension busses.

	Kw.	Kv-a.
Normal full load per circuit, low-tension bus	100,000	111,000
Maximum synchronous converter load per circuit	3,000	75,000
Normal current in reactors (4) representing interchange of load, estimated	10,000	12,000
Short circuit in high tension at receiving end of high-tension circuit, arc kv-a. from generator, at point of short circuit, 420 amperes		160,000
Kv-a. corresponding at generator, 223 amperes		85,000
Kv-a. through receiving transformer from condensers and network about		1,250,000
of this say 200,000 comes through reactors (*T*) and 350,000 from network. Of the 200,000, part comes from network and part from three good lines, the balance of 750,000 from the condensers and tertiary windings		
Short circuit in high tension at sending end of the line—		
From the generators arc kv-a.		600,000
From the line, amperes 420		160,000
From the network and condensers at receiving end corresponding to 223 amperes high tension		85,000
Short circuit at center of line, from each end 735 amperes		280,000
From generators and also from network corresponding, 642 amperes each		244,000
Short circuit on generator auxiliary bus, each generator 350,000 all		1,400,000
Short circuit on condenser auxiliary bus, each condenser 350,000		
Each tertiary winding 350,000 part from line and part from network—total		2,800,000

Note: If reactances be installed in series with the breakers (2), this value can easily cut in half or less, but little would be gained as no one breaker has to handle more than 700,000 arc kv-a.

The essential features of the system of Fig. 5 have now all been touched upon and the performance of the whole system seems to be very favorable. Of course, many variations can be made and changes introduced to meet special cases, but as long as the essential characteristics are maintained, the good performance should still be retained. The more fundamental features are the maintaining substantially separate of the main units as far as short circuits are concerned, the interconnection for purposes of load equalization, the means of maintaining a high voltage at both ends of the line on the high-tension side even when short-circuits occur, the automatic control of voltage and power factor for purposes of regulation and the proper choice of governor characteristics and flywheel effects for the various units.

Intermediate Condenser Station

An intermediate station at the middle of the line with synchronous condensers materially improves certain features of the line performance as for example, synchronizing power at time of disturbance and variation of voltage along the line. Furthermore, a much smaller synchronous condenser capacity is required at the receiving end. This advantage is more than balanced, however, by the large condenser capacity required at the center of the line.

Considering the layout of Fig. 5 as distinguished from a single transmission circuit, apparently, on the whole, little or nothing would be gained in efficiency or operating performance by the intermediate condenser station. The no-load voltage at the center would be reduced 15 per cent which is a material point, the line efficiency over the working range would be no better and at least 30,000 kv-a. of condensers additional total would be required, representing a loss and expense. The synchronizing power would be materially greater, perhaps 50 per cent greater. On the other hand the extra cost, especially when spare apparatus and interconnections are considered, and the very serious operating handicap of a third station with a third set of operators to be coordinated and the problem of relay protection, make it clear that very considerable sacrifices would be justified in avoiding the intermediate station and retain the simpler layout of Fig. 5.

It is interesting to note from case 18, Table "A", that power can be transmitted backward over the line with 220,000 volts at the condenser end and 245,000 volts at the generator end and at a better efficiency than in the normal direction.

Conclusion

The net upshot of this layout of Fig. 5 is a system delivering regularly 400,000 kw. (perhaps reduced to 360,000 for brief intervals, for work on a transmission circuit) over a distance of 500 miles, with losses including line, transformers and condensers of about 15 per cent, with flexibility such that any unit may be shut down at will without disturbing operation. This is a system in which disturbance caused by short circuits will apparently be less than with the usual transmission system and a system well adapted for automatic relay protection.

On the basis of power generated at $20.00 per kw. year on the switchboard and saleable at the receiving step-down bus bars in quantity at $50.00 per kw. year, an expenditure of something between $75,000,000 and $90,000,000 would seem to be justified, assuming 12 per cent to be sufficient to cover all fixed charges, or $75,000 to $90,000 a mile for a double circuit tower line including cost of condensers and auxiliaries.

It appears from the above discussion that the superline, while it has great possibilities of astonishing efficiency and an extraordinary range of transmission, has also some very distinct limitations and some real problems of its own that will warrant much study and discussion.

Discussion

For discussion of this paper see page 71.

Some Theoretical Considerations
of Power Transmission

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Review of the Subject.—*The following points are developed in the paper:*

1. A proof of the circle diagram maintaining therein the idea of the angle between the generator and receiver voltages.

2. It is shown how the characteristics of the synchronous condenser limit the maximum power that can be transmitted over a line.

3. The effect of character of load on stability.

4. Comparison of 500-mile straightaway line with 500-mile line with condenser at mid-point.

THE power circle diagram of transmission line performance has proved of great value to the line designer and to the analyst. This diagram was first presented by R. A. Philip[1] before the Institute. The circle gives the relation between real and reactive powers, which must be maintained at the receiver to obtain the assumed voltage conditions at the generator and receiver. Mr. R. A. Philip restricted its application to short lines in which the capacity could be neglected, while H. B. Dwight[2] extended its use to the general case of any line. Later H. B. Dwight[3] in a paper presented before the Institute and R. D. Evans and H. K. Sels[4] in a series of articles in the *Electric Journal* developed the diagram still further and indicated simple means whereby the impedances of the transformers may be included. The latter also developed simplified formulas for the calculation and plotting of losses and efficiencies. It is the purpose of the present paper to develop further the uses and properties of the circle diagram.

Let us consider for the present the diagram for the line itself. Let

r = Resistance of line in ohms per mile
x = Reactance of line in ohms per mile
g = Conductance of line in mhos per mile (very small and usually neglected)
b = Susceptance of line in mhos per mile
l = Length of line in miles

then $Z = l\,(r + j\,x)$
$Y = l\,(g + j\,b)$

The familiar steady state line equations may be written

$$\begin{aligned} \breve{E}_s &= A\,\breve{E}_r + B\,\breve{I}_r \\ \breve{I}_s &= A\,\breve{I}_r + C\,\breve{E}_r \end{aligned} \qquad (1)$$

Where $\breve{E}_s$ = Voltage (phase-to-neutral) at generator end
$\breve{I}_s$ = Current per line at generator
$\breve{E}_r$ = Voltage (phase-to-neutral) at receiving end
$\breve{I}_r$ = Current per line at receiving end

1. TRANSACTIONS A. I. E. E., Vol. 30, 1911, pp. 596-636.
2. *Electric Journal,* August 1914, Vol. XI, p. 487.
3. TRANSACTIONS A. I. E. E., Vol. 41, 1922, pp. 781-784.
4. *Electric Journal,* July, Aug. and Dec. 1921 and Feb. 1922.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

$$A = \cosh\sqrt{Z\,Y} = 1 + \frac{Y\,Z}{2} + \frac{(Y\,Z)^2}{24} + \frac{(Y\,Z)^3}{720} + \ldots$$

$$B = \sqrt{Z/Y}\,\sinh\sqrt{Z\,Y} = Z\left[1 + \frac{Y\,Z}{6} + \frac{(Y\,Z)^2}{120} + \frac{(Y\,Z)^3}{5040} + \ldots\right]$$

$$C = \sqrt{Y/Z}\,\sinh\sqrt{Z\,Y} = Y\left[1 + \frac{Y\,Z}{6} + \frac{(Y\,Z)^2}{120} + \frac{(Y\,Z)^3}{5040} + \ldots\right]$$

Mr. R. D. Evans and Mr. H. K. Sels in a companion paper show how the transformer constants may be included in the above constants.

Equations (1) may be written in the form:

$$\left.\begin{aligned} \breve{I}_s &= (\alpha + j\,\beta)\,\breve{E}_s - (\gamma + j\,\delta)\,\breve{E}_r \\ \breve{I}_r &= (\gamma + j\,\delta)\,\breve{E}_s - (\alpha + j\,\beta)\,\breve{E}_r \end{aligned}\right\} \qquad (2)$$

where

$$\left.\begin{aligned} (\alpha + j\,\beta) &= A/B \\ (\gamma + j\,\delta) &= 1/B \end{aligned}\right\} \qquad (3)$$

PROOF OF THE CIRCLE DIAGRAM[5]

From this point a proof of the circle diagram is readily derived. The conjugate equations of (2) may be written:

$$\begin{aligned} \hat{I}_s &= (\alpha - j\,\beta)\,\hat{E}_s - (\gamma - j\,\delta)\,\hat{E}_r \\ \hat{I}_r &= (\gamma - j\,\delta)\,\hat{E}_s - (\alpha - j\,\beta)\,\hat{E}_r \end{aligned}$$

Multiplying $\hat{I}_s$ by $\breve{E}_s$ and $\hat{I}_r$ by $\breve{E}_r$ we obtain the power at generator and receiver respectively.

$$\begin{aligned} P_s + j\,Q_s &= \breve{E}_s\,\hat{I}_s = (\alpha - j\,\beta)\,\breve{E}_s\,\hat{E}_s - (\gamma - j\,\delta)\,\hat{E}_r\,\breve{E}_s \\ P_r + j\,Q_r &= \breve{E}_r\,\hat{I}_r = (\gamma - j\,\delta)\,\hat{E}_s\,\breve{E}_r - (\alpha - j\,\beta)\,\hat{E}_r\,\breve{E}_r \end{aligned}$$

but $\breve{E}_s\,\hat{E}_s = E_s^2$ and $\breve{E}_r\,\hat{E}_r = E_r^2$.

If we let $\breve{E}_r$ be the datum line and $\breve{E}_s = E_s\,\epsilon^{j\,\theta}$ then

5. An alternate simpler proof not involving conjugate imaginary quantities is given in the companion paper by Evans and Sels. The idea of the angular displacement between generator and receiver voltages, however, is lost.

$\hat{E}_r \check{E}_s = E_r E_s \epsilon^{j\theta}$ and $\check{E}_r \hat{E}_s = E_r E_s \epsilon^{-j\theta}$. Substituting in above

$$P_s + j Q_s = (\alpha - j \beta) E_s^2 - (\gamma - j \delta) E_s E_r \epsilon^{j\theta} \quad (4)$$

$$P_r + j Q_r = (\gamma - j \delta) E_s E_r \epsilon^{-j\theta} - (\alpha - j \beta) E_r^2 \quad (5)$$

With E_s and E_r constant, equation (4) is the equation of a circle with its center at $(\alpha - j \beta) E_s^2$ and radius equal to $\sqrt{\gamma^2 + \delta^2}\, E_s E_r$. For different values of E_r with E_s constant the equation may be represented by a family of concentric circles. These are the so-called "supply circles."

For $\theta = 0$

$$P_s + j Q_s = (\alpha - j \beta) E_s^2 - (\gamma - j \delta) E_s E_r$$

Operating by $\epsilon^{j\theta}$ merely rotates the vector $-(\gamma - j \delta) E_s E_r$ through an angle θ in a positive (counterclockwise) direction. Similarly $P_r + j Q_r$ may be

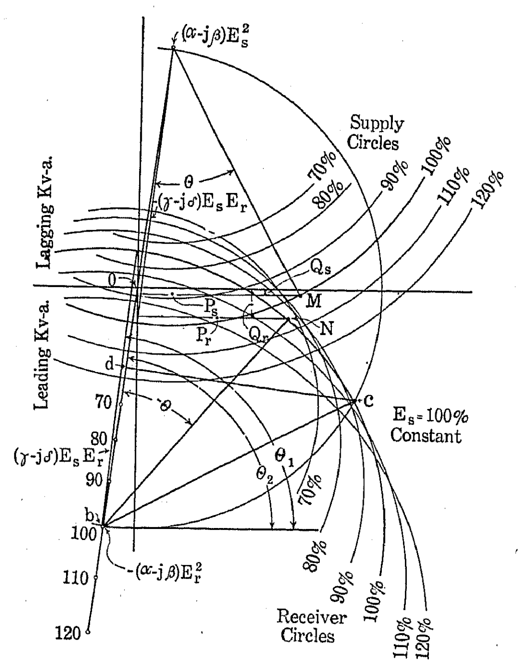

FIG. 1

Showing family of supply and receiver circles, relation between $P_r + j Q_r$ $P_s + j Q_r$ for any angle θ between E_s and E_r, and graphical determination of envelope.

represented by a circle locus, the center of which is given by $-(\alpha - j \beta) E_r^2$ which changes with each value of E_r and whose radius is $\sqrt{\gamma^2 + \delta^2}\, E_s E_r$. The vector $(\gamma - j \delta) E_s E_r \epsilon^{-j\theta}$ makes the angle $-\theta$ with $(\gamma - j \delta) E_s E_r$ as datum. For different values of E_r the equation may be represented by a family of non-concentric circles. These are the so-called "receiver circles."

Fig. 1 shows such a family of "supply" and "receiver" circles. These circles indicate the relation between inductive and real power which must be maintained to obtain the voltage conditions assumed for each circle, *viz.*, E_s constant and giving E_r the values indicated in each circle. It will be noted in this proof that the idea of the angle between the generator and receiver voltages has not been lost. Therefore, for any given value of $P_r + j Q_r$ the value of $P_s + j Q_s$ at the sending end may be obtained by using the same scalar values of voltage and laying off the same angles from their respective datum lines. For example suppose we are operating at the point N on the 100 per cent receiver circle. By laying off the same angle θ on the 100 per cent supply circle as that corresponding to $-\theta$ on the receiver circle the point M is obtained. This gives the value of true and reactive power at the generator end necessary to maintain the given conditions at the receiver end. $P_s - P_r$ represents the true line loss. This method does not offer very great accuracy, but for most work should be close enough.

The circle diagram could be made more complete by plotting thereon loci of constant angle between generator and receiver circles. It can be shown that these loci are parabolas. However, the angle loci are relatively unimportant and of academic interest only.

GRAPHICAL DETERMINATION OF ENVELOPE

In the analysis of transmission line problems, it is sometimes convenient to know the envelope of the receiver circles. One of its applications is developed in the companion paper by Evans and Sels. Equation (5) may be rewritten as follows:

$$(P_r + \alpha E_r^2 + j (Q_r - \beta E_r^2) = (\gamma - j \delta) E_s E_r \epsilon^{-j\theta} \quad (6)$$

Its conjugate is

$$(P_r + \alpha E_r^2) - j (Q_r - \beta E_r^2) = (\gamma + j \delta) E_s E_r \epsilon^{j\theta} \quad (7)$$

The product of the two gives

$$(P_r + \alpha E_r^2)^2 + (Q_r - \beta E_r^2)^2 = (\gamma^2 + \delta^2) E_s^2 E_r^2 \quad (8)$$

It is shown in works on the differential calculus that the envelope of such a function having E_r as the parameter determining consecutive loci, is the locus of the equation obtained by differentiating (8) with respect to the parameter E_r and eliminating E_r from the resulting equation and equation (8). This truth may not at first be evident. That it is so may be shown in a crude way as follows: Consider two power circles determined by the parameters E_r and $(E_r + \Delta E_r)$ respectively. As ΔE_r approaches zero the intersection of the two circles approaches and finally coincides with the envelope as a limit. E_r and $E_r + \Delta E_r$ substituted in equation (8) furnishes two equations from which the point of intersection could be calculated, two equations, two unknowns, *viz.*, P_r and Q_r. Subtracting the two equations and dividing by ΔE_r furnishes a third dependent equation. This third equation when ΔE_r approaches zero forms the differential of equation (8). It is evident then that the point of intersection or the envelope may be obtained by the simultaneous solution of one of the first two equations, *i. e.*, equation (8) and the third equation, *i. e.*, the differential. Since the envelope must be independent of the parameter E_r and since the simultaneous solution of equation (8) and its differential must be the envelope,

the latter operation may be accomplished by eliminating E_r from equation (8) and its differential.

Differentiating (8) with respect to E_r

$$2\,\alpha\,(P_r + \alpha\,E_r^2) - 2\,\beta\,(Q_r - \beta\,E_r^2) = (\gamma^2 + \delta^2)\,E_s^2 \qquad (9)$$

Let

$$(\gamma - j\,\delta) = \sqrt{\gamma^2 + \delta^2}\,\epsilon^{j\,\theta_1} = \sqrt{\gamma^2 + \delta^2}\,(\cos\theta_1 + j\sin\theta_1)$$

$$(\alpha - j\,\beta) = \sqrt{\alpha^2 + \beta^2}\,\epsilon^{j\,\theta_2} = \sqrt{\alpha^2 + \beta^2}\,(\cos\theta_2 + j\sin\theta_2)$$

Substituting in equation (6) and equating reals and imaginaries, we obtain

$$P_r + \alpha\,E_r^2 = \sqrt{\gamma^2 + \delta^2}\,E_s\,E_r\cos(\theta_1 - \theta)$$

$$Q_r - \beta\,E_r^2 = \sqrt{\gamma^2 + \delta^2}\,E_s\,E_r\sin(\theta_1 - \theta)$$

Substituting these values in equation (9)

$$2\sqrt{\gamma^2 + \delta^2}\,E_r\,[\,\alpha\cos(\theta_1 - \theta) - \beta\sin(\theta_1 - \theta)\,] = (\gamma^2 + \delta^2)\,E_s$$

Now since

$$\frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} = \cos\theta_2 \text{ and } \frac{\beta}{\sqrt{\alpha^2 + \beta^2}} = -\sin\theta_2$$

then $2\,E_r^2\sqrt{\alpha^2 + \beta^2}\cos(\theta_1 - \theta_2 - \theta) = \sqrt{\gamma^2 + \delta^2}\,E_s\,E_r$ (10)

This relation must be true for every point on the envelope. Now from Fig. 1 it will be seen that $(\theta_1 - \theta_2 - \theta)$ is the angle between $b\,N$ and the line of center $\overline{o\,b}$, $E_r^2\sqrt{\alpha^2 + \beta^2}$ is the length $o\,b$ and $\sqrt{\gamma^2 + \delta^2}\,E_s\,E_r$ is the radius of the power circle. How can we determine the particular position of N such that equation (10) will also hold true? With O as center draw a circle through b. The diameter of the circle will be $2\,E_r^2\sqrt{\alpha^2 + \beta^2}$ and $b\,c$ is $2\,E_r^2\sqrt{\alpha^2 + \beta^2}\cos(\theta_1 - \theta_2 - \theta)$. The point c, therefore, satisfies equation (10) and lies on the envelope. To determine the envelope locus lay off $o\,c$ equal to $o\,b$ for the different receiver power circles.

Point where Envelope Cuts Line of Centers

It is sometimes convenient to know the point at which the envelope cuts the line of centers. This point is determined when $\theta_1 - \theta_2 - \theta = O$.

From equation (10) we then have

$$E_r = \frac{\sqrt{\gamma^2 + \delta^2}}{2\sqrt{\alpha^2 + \beta^2}}\,E_s$$

The distance of the point of tangency from the origin is the radius of the $(P_r + j\,Q_r)$ circle or $\sqrt{\alpha^2 + \beta^2}\,E_r^2$ which is equal to

$$\frac{\gamma^2 + \delta^2}{4\sqrt{\alpha^2 + \beta^2}}\,E_s^2$$

This is also the smallest circle which touches the envelope.

Equation of the Envelope

Let us take the point where the envelope cuts the line of centers as origin (e in Fig. 1) and the line of centers as the X axis. The abscissa x of any point c on the envelope is then

$$x = c\,o + o\,b - d\,b$$

$$= \frac{\gamma^2 + \delta^2}{4\sqrt{\alpha^2 + \beta^2}}\,E_s^2 + \sqrt{\alpha^2 + \beta^2}\,E_r^2 - \frac{\gamma^2 + \delta^2}{2\sqrt{\alpha^2 + \beta^2}}\,E_s^2$$

$$= \sqrt{\alpha^2 + \beta^2}\,E_r^2 - \frac{\gamma^2 + \delta^2}{4\sqrt{\alpha^2 + \beta^2}}\,E_s^2$$

The ordinate $y = d\,c$

$$= \sqrt{\gamma^2 + \delta^2}\,E_s\,E_r\sin(\theta_1 - \theta_2 - \theta)$$

$$y^2 = (\gamma^2 + \delta^2)\,E_s^2\,E_r^2\sin^2(\theta_1 - \theta_2 - \theta)$$

$$= (\gamma^2 + \delta^2)\,E_s^2\,E_r^2\left[1 - \frac{\gamma^2 + \delta^2}{4\,(\alpha^2 + \beta^2)}\,\frac{E_s^2}{E_r^2}\right]$$

$$= \frac{(\gamma^2 + \delta^2)\,E_s^2}{\sqrt{\alpha^2 + \beta^2}}\left[\sqrt{\alpha^2 + \beta^2}\,E_r^2 - \frac{\gamma^2 + \delta^2}{4\,(\alpha^2 + \beta^2)}\,E_s^2\right] = \frac{(\gamma^2 + \delta^2)\,E_s^2}{\sqrt{\alpha^2 + \beta^2}}\,x$$

This equation is one of a parabola referred to its apex as origin.

Maximum Power Delivered by Line

The maximum power that may be drawn over a transmission line depends upon the line constants, generator and receiver voltage and also upon the characteristics of the synchronous condenser. With a condenser of infinite capacity the maximum power is determined when $\dfrac{d\,Q_r}{d\,P_r}$ is equal to infinity. From the geometry of the circle diagram this value is seen to be

$$\sqrt{\gamma^2 + \delta^2}\,E_s\,E_r - \alpha\,E_r^2$$

However, with a condenser of finite size the characteristics of the condenser must be taken into consideration.

Maximum Power Delivered by Line with Finite Condenser

Let us consider a definite case. Fig. 2 shows the receiver circle diagram of a single 220-kv., 3-phase, 250-mile line, 600,000-cir. mil copper conductor with 22 ft. triangular spacing. The condenser characteristics which we will choose for this line are represented by the full lines in Fig. 3, terminal voltage being plotted against leading and lagging kilovolt-amperes for constant field current. The transformer reactance is included in the condenser characteristics. Assuming for the time being a dead resistance load, the loci of constant field current may be plotted on the circle diagram as shown in Fig. 2. For the case in which all or part of the load consists

of inductive apparatus, the variation of reactive power with voltage must be considered in plotting the loci of constant field current on the circle diagrams. Care must be exercised in reading such a graph. The loci of constant excitation are not double valued functions of voltage, as might appear at first, since some of the curves cut the same voltage circle twice. The inter-

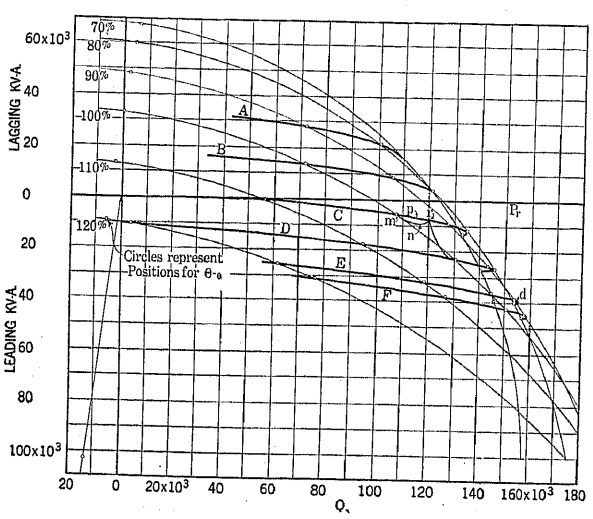

Fig. 2—250-Mile Line with 35,000 Kv-a. Leading and Lagging Condenser

sections of the upper portions of the curves with voltage circles refer to the higher values of voltage while the intersections of the lower portions refer to the lower values of voltage. The maximum power that may be drawn over the line for a given field excitation is determined when $\frac{d P_r}{d Q_r}$ is zero. Another relation[6]

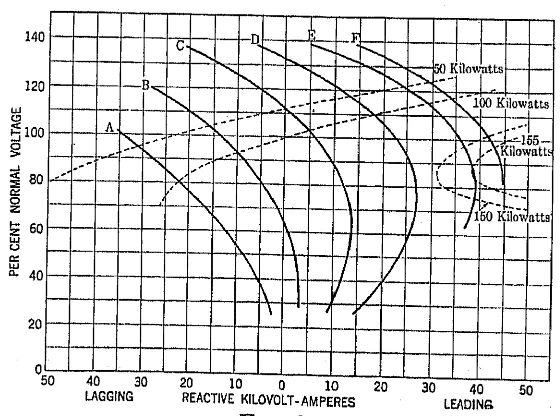

Fig. 3

Full lines represent condenser characteristics with constant excitation. Dotted lines represent reactive kv-a. relation for constant power as taken from circle diagram.

which must be satisfied at the maximum power is that $\frac{\partial Q_r}{\partial E_r}$ for the circle diagram (constant real power) be simultaneously equal to $\frac{\partial Q}{\partial E_r}$, where Q is the combined reactive kilovolt-amperes of the condenser and the load. The truth of this relation is shown graphically in Fig. 3, by plotting the values of reactive power against voltage for constant true power (dotted lines). As can be seen the maximum power which permits of a solution for a fixed excitation occurs for that value of power the curve of which is tangent to the excitation curve.

Stability with Load of Constant Resistance

The physical significance of what occurs when an increment of load is applied may be best illustrated by

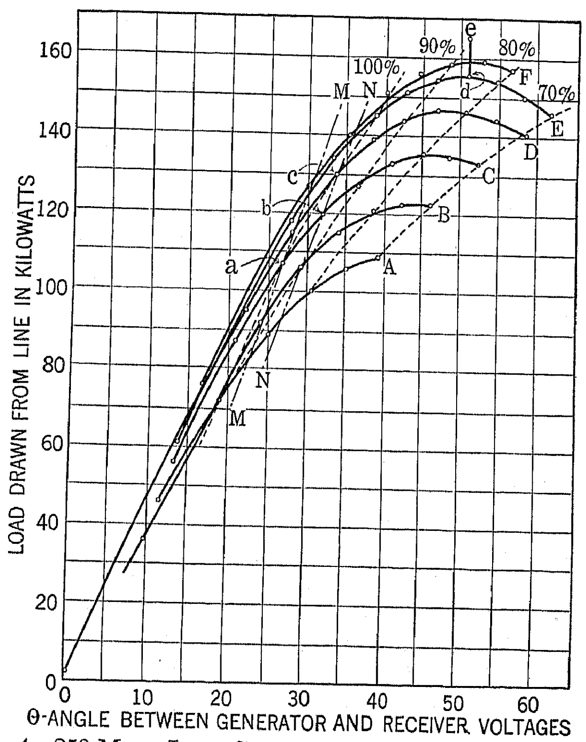

Fig. 4—250-Mile Line Characteristics with 35,000 Kv-a. Condenser

Full line—constant excitation.
Dotted line—constant voltage.
Dot-dash—constant load resistance.

still another curve. Fig. 4 shows the synchronous condenser curves of constant field current plotted with angle between generator and receiver voltage as abscissas and power drawn from the line as ordinates. These values were obtained from the constant field current loci in the circle diagram. Fig. 4 also shows loci of constant voltage and constant load resistance for the transmission line. The former represent the relation between the power drawn from the line and the angle between the generator and receiver voltage for constant voltage and the latter express the power taken by the load as a function of the angle between the generator and receiver voltage for constant load resistance. For our present discussion let us assume the generator voltage fixed in magnitude and phase posi-

6. The companion paper by Evans and Bergvall discusses this relation in greater detail.

tion, *i. e.*, a generator of zero impedance and armature reaction.

Suppose we are operating at the point a on Fig. 4, the intersection of the curve of constant field current C, of constant voltage 100 per cent normal and of constant resistance $M - M$. Now reduce the load resistance to that corresponding to the locus $N - N$ still maintaining the same field current. The operating point tends to move to the new point b, but in doing so the angle between the generator and receiver voltages increases; *i. e.*, with fixed generator voltage the terminal voltage of the condenser lags behind its original position a. Due to its inertia the rotor is unable to follow the terminal voltage instantly. This condition requires the condenser to function as a generator and supply part of the increase in load during the period in which the rotor is slowing down to its new position approximately in phase with the terminal voltage.

The division of load between the condenser and line accompanying a decrease in load resistance (constant field current) is explained as follows. With a dead resistance load such as we are considering all of the change in reactive power even during the transition period must be supplied by the condenser. This latter quantity is approximately equal to

$$\frac{E_r^2 - E_n E_r \cos \phi}{X}$$

Where E_r = Terminal voltage
E_n = Rotor voltage
X = Leakage reactance of condenser
ϕ = Angle between E_r and E_n

For steady state and also for the small angular swings during the transition period $\cos \phi$ will be very nearly equal to unity. For our present discussion we may then assume the reactive power to be a function of X and the scalar value of E_r. This shows that the terminal voltage during the transition period must vary along a locus similar to those of constant excitation shown on Fig. 2. These new loci differ from those of constant excitation only in the fact that the latter use leakage reactance and the former use synchronous impedance. For zero leakage reactance the loci coincide with the voltage circles. The true loci lie between the loci of constant voltage and the loci of constant excitation in which the synchronous impedance were used. In this way the relation between voltage and angular position of the terminal voltage which must be maintained during the early part of the transition period is determined. After the field in the condenser has built up, the synchronous impedance must be used. To state these facts slightly differently. Suppose we are operating at the point m in Fig. 2. With a condenser of zero impedance the operating point would be constrained to lie on locus of constant voltage $m\,p$ even during the transition period. With a condenser of zero armature reactance, *i. e.*, the leakage reactance equal to the synchronous reactance the operating point would be constrained to lie on the locus of constant field current during the transition period. But with a practical condenser, as the load resistance is changed with constant excitation the operating point will be constrained to lie on the curve $m\,n\,r$. The exact curve is a complex function of time, varying as the effective reactance changes from the leakage valve to the synchronous value and also varying with the speed the rotor changes its angular position. Immediately after the load resistance is changed the operating point will jump to an intermediate point which can be definitely determined. In the determination of this point the leakage reactance shall be used.

Now coming back to Fig. 4. Suppose we are operating at the point a and the load resistance is decreased. The operating point will be constrained to move along a curve lying between the locus of constant voltage and the locus of constant excitation. With a condenser impedance other than zero the receiver voltage drops with increasing angular displacement between the generator and receiver voltage as we move along the constraint. For every value of angular displacement there is a definite value of voltage. The power taken by the load can then be determined by the relation E_r^2/R where R is equal to the new load resistance. Fig. 5A shows an enlarged view of the section of Fig. 4 in the vicinity of point a Curve $a\,d\,b$ is the receiver voltage constraint and $e\,g\,f$ is the power taken by the load. The vertical distance between $e\,f$ and $a\,d\,b$ represents the excess of the power required by the load over that supplied by the line.

Due to its inertia the rotor cannot follow the terminal voltage instantly as it moves from a. Neglecting the effect of damper windings the power supplied by the condenser for an angle ϕ lag of terminal voltage behind the rotor is equal to $\frac{E_n E_r}{X} \sin \phi$. Draw $a\,g\,h$ such that the vertical distance between it and $a\,d\,b$ is equal to the above quantity where ϕ is measured from a. The angle to which the terminal voltage will jump is then determined by the intersection g. At this point the excess of the power required by the load over that supplied by the line is equal to that supplied by the condenser.

In supplying this energy the rotor of the condenser begins to lag behind its original position. This requires the curve $a\,g\,h$ to move to the right taking the positions shown by the dotted lines. In doing so the intersection moves from g downward along $e\,f$, the power supplied by the rotor steadily decreasing until the point b is reached. The rotor may overshoot but oscillates about b as a limiting position.

The locus $a\,d\,b$ is drawn assuming the field had built up and developed the full synchronous reactance. The dot-dash line shows the locus when the field has not had time to develop. Under certain conditions of low-frequency oscillations in the field the operating point

might even encircle the point b. With an extremely heavy rotor the operating point will move slowly from point d and with a light rotor correspondingly faster. In the above discussion the effect of the damper windings was neglected. As soon as the receiver voltage leaves a relative motion between the terminal voltage and the rotor takes places in such a manner as to convert the condenser into an induction generator. The damper windings and induced currents in the field coils and field structure determine the rate at which the operating point moves from a. We thus have the double effect of synchronous generator and induction generator action in retarding the rotor as it moves to its position b. Beyond the point b both effects reverse,

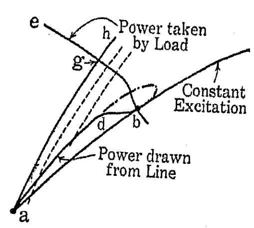

Fig. 5A

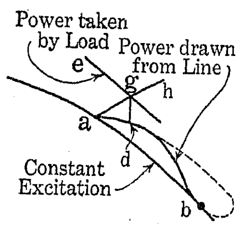

Fig. 5B

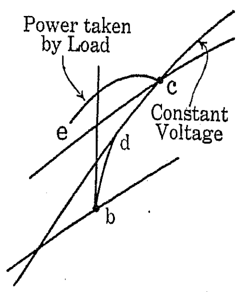

Fig. 5C

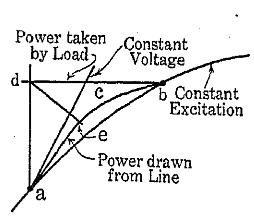

Fig. 5D

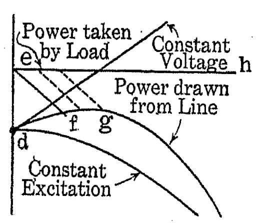

Fig. 5E

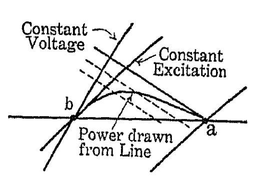

Fig. 5F

thus tending to bring the rotor back to b. There appears to be no reason in this case for suspecting a condition of instability as far as the condenser is concerned.

Referring to Fig. 2, the question sometimes arises as to the stability with a dead resistance load of operating at a point on the lower side of a locus of constant excitation. Fig. 5B shows a diagram similar to that of 5A except that it represents a point on the under side of the constant excitation locus. There appears to be no tendency toward instability. This condition alone, is then insufficient to determine the limit of instability with a dead resistance load. The true limit of instability appears to be at that point where $\frac{dP}{d\theta}$ for constant excitation is equal to $\frac{dP}{d\theta}$ for constant load resistance. If this angle is exceeded by decreasing the load resistance further the amount of power supplied by the rotor increases as the rotor lags and finally pulls out of step.

Now it will be noted that in decreasing the load resistance with constant excitation the terminal voltage drops. This brings the voltage regulator of the condenser into action. Let us then consider the distribution of power as the load resistance is kept constant and the field current is increased. Let the operating point be b Fig. 4 and increase the excitation from that corresponding to C to that corresponding to D. Fig. 5C shows an enlarged view of that part of the section. Inasmuch as it is the field of the condenser itself which produces the change, the operating point will not jump to an intermediate point immediately. The process is more gradual, the point moving from b toward c by the line $b\,d\,c$. The exact position of $b\,d\,c$ depends upon the inertia of the rotor, the magnetic circuit, etc. The power absorbed by the load is $e\,c$. This line is conditioned by the variations in voltage imposed by the locus $b\,d\,e$. The rotor in retarding supplies the differences in power designated by the vertical distance between $b\,d\,c$ and $e\,c$.

Stability with Inductive Load

The foregoing considered the conditions pertaining to a dead load. With an induction motor load which is essentially constant power independent of voltage, other factors must be considered. Taking proper cognizance of the variation of its reactive load with voltage, a set of curves similar to those of Fig. 4 can be constructed. However, for our discussion let us use the same curves.

Let us assume again that we are operating at the point a Fig. 4. Let us increase the load from that corresponding to a to that corresponding to b. Fig. 5D shows the transition period. The line $d\,b$ represents the power taken by the load. The line $a\,c$ is determined by the reactive kilovolt ampere relations that must be satisfied, (leakage reactance used instead of synchronous reactance and variation of lagging kilovolt-ampere taken by induction motor load as voltage changes). The initial operating point is constrained to lie upon this line. The vertical distance between any point on $a\,c$ and the line $d\,b$ represents the excess of the power required by the load over that drawn for the line. Draw $d\,e$ such that the vertical distance between $d\,e$ and $d\,b$ represents the power supplied by the rotor as the receiver voltage lags behind the rotor. The intersection of $a\,c$ and $d\,e$ therefor, represents the point to which the operating point will jump (neglecting damping action) as the load is increased. After this point is reached the operating point moves toward b. The exact path taken depends

upon the inertia of the rotor, the magnetic circuit, etc. There is no apparent instability.

But suppose we are operating on one of the excitation curves where the slope is zero (Fig. 4), say at the point d on excitation curve E. Suppose the load is increased to the value corresponding to that of e. Fig. 5E represents the transition diagram. The line $d\,f$ again represents the locus of constant leakage reactance, determining the initial point f to which the operating point jumps. As the rotor lags the line $e\,f$ moves to the right as shown by the dotted lines. After the point g is reached the power supplied by the retardation of rotor, *i. e.*, the vertical distance between $e\,h$ and $d\,g$ begins to increase; the affect becomes cumulative, the more the motor lags the greater the amount of power it supplies and the faster it retards until pull-out occurs. Under these conditions we may then say that stable operation results provided the excitation curve Fig. 4 has a positive slope. Referring to Fig. 2 stable operation results up to the point where $\frac{d\,P_r}{d\,Q_r}$ for constant excitation becomes equal to zero.

Changing the excitation maintaining the load constant produces a transition diagram similar to Fig. 5F This condition is likewise stable until the slope of the particular excitation curve Fig. 4 is zero.

An induction motor load has another effect. It has been observed in low-voltage transmission circuits in which the resistance is about equal to the reactance that hunting of the condenser at the receiver end takes place. It has been further observed that the presence of an unloaded induction motor tended to stabilize the line. This result may be explained as follows: In the foregoing, as instantaneous changes in load or field current occurred the receiver voltage changed instantly to a new position. However, with an induction motor, as soon as the vector of terminal voltage moves from its original position, it represents an increase or decrease in actual vector velocity. This in turn increases or decreases the slip. As a result the rotor of the induction motor momentarily takes care of the increase or decrease of the load. This occurs as soon as the voltage begins to change position. The induction motor then acts as a cushion in absorbing the shocks incident to change of load, excitation, etc. It decreases hunting but does not otherwise affect the power limit.

Considerations Involved in Extremely Long Lines

The question sometimes arises as to the stability of a condenser located at the end of an extremely long line. To investigate this point let us consider a 220-kv., 60-cycle, 500-mile line. Fig. 6 shows the circle diagram of such a line with the loci of constant excitation superimposed thereon. Fig. 7 shows data for the 500-mile line similar to that shown in Fig. 4 for the 250-mile line. Applying the same type of analysis to Fig. 7 as has been applied to Fig. 4 we may arrive at the same conclusion, *viz.*, for a dead resistance load there appears to be no apparent stability even beyond the point of maximum power for fixed excitation, but for an induction motor load the maximum load occurs when the differential of real power with respect to θ for any voltage is equal to zero. This latter condition also

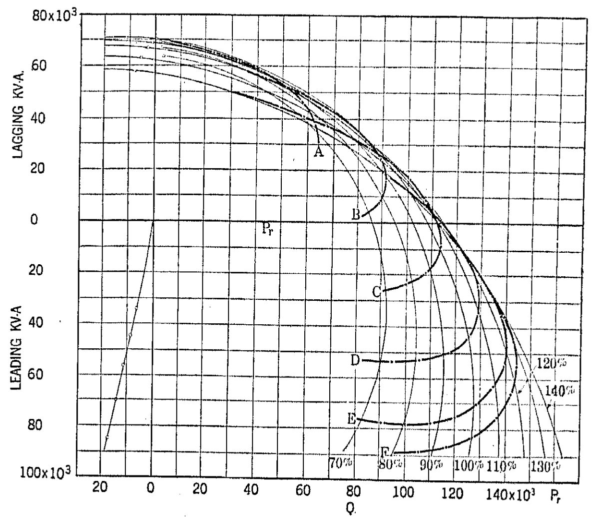

Fig. 6—500-Mile Line with 70,000 Kv-a. Leading and Lagging Condenser

requires that $\frac{d\,P_r}{d\,Q_r}$ for the locus of constant excitation be zero, *i. e.*, when the slope on Fig. 6 is vertical.

The value of plotting the condenser characteristics as on Fig. 6 may be illustrated. A condenser capable of supplying 70,000 kv-a. lagging and 70,000 kv-a. leading

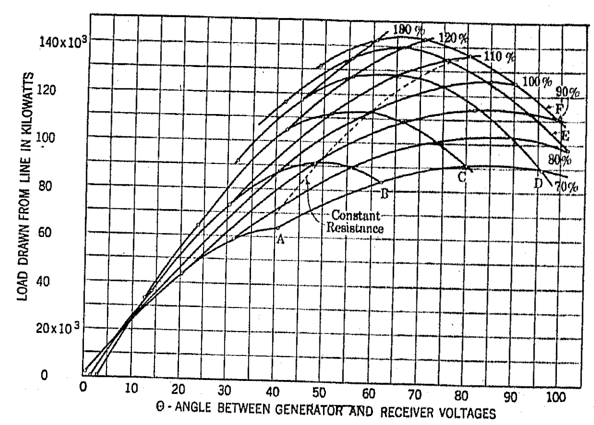

Fig. 7—500-Mile Line Characteristics with 70,000 Kv-a. Condenser

Letters refer to constant excitation. Figures refer to per cent of normal voltage.

at 100 per cent voltage was selected. From the circle diagram alone one would suppose that with 70,000 kv-a. leading capacity at 100 per cent voltage about 126,500 kw. could be drawn from the line. The actual limit, however, as determined by the slope of the constant excitation locus is 110,000 kw.

500-Mile Line with Intermediate Condenser Station

In recent years much thought and analysis have been given to the question of transmission of large blocks of power over great distances. Mr. Frank Baum,[7] in particular, has made a very comprehensive survey of our national resources, power markets, etc. His general solution of the superpower problem suggests placing synchronous condenser stations at intermediate points along the line to supply the charging kilovolt-ampere of the line, maintain constant voltage, and increase the power limit of the line. The first step in the logical development of such a system necessitates the analysis of, say, the 500-mile problem with a condenser station at the midpoint. The circle diagram for the receiving end section assuming constant 100 per cent potential at the midpoint would be the same as Fig. 2. With the given condenser, the maximum power limit at 100 per cent voltage is 151,000 kw. and for reduced voltage the power limit may be extended to 159,000 kw. The questionable element lies in the combined line and condenser characteristics at the

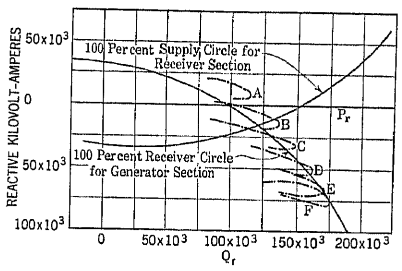

Fig. 8

Constant field current loci of condenser at mid-point of 500-mile line, plotted against power transmitted at mid-point and reactive power drawn from generator section, 100 per cent voltage at generator and at receiver.

midpoint. Fig. 8 shows the loci of constant field current for a 70,000-kv-a. leading and lagging condenser. These loci represent the relation between real and reactive power drawn from the generator section at the midpoint for constant excitation of the condensers, assuming constant normal voltage at the generator and receiver ends. The maximum power limit in this case occurs on the 100 per cent voltage circle at 175,000 kw. This value is more than sufficient to supply the losses in the receiver section. We could probably use a much smaller condenser. In practise this capacity would be built in several units. We can safely say then that the maximum power limit of the 500-mile with a condenser at the midpoint is 159,000 kw. The rated power of such a line allowing for switching power surges, let us estimate at about 120,000 kw. The actual reactive power required at this load is zero at the midpoint and about 15,000 kv-a. leading at the receiver.

Let us compare this line with a straight 500-mile line. Examination of Fig. 6 will show that little can be gained by supplying leading reactive power at the receiver. Let us assume then the power limit to be 115,000 kw. at zero reactive power. The rated power on the same basis as above, *viz.*, 75 per cent of maximum power would be about 86,000 kw. The active reactive power at this load at the receiver is 29,000 kv-a.

What have we gained? The rated capacity of the line has been increased from 86,000 kw. to 120,000 kw. an increase of 40 per cent. At the same time the condenser capacity required at these loads has been decreased from 29,000 kv-a. to 15,000 kv-a.

What has it cost? The no-load synchronous condenser capacity is practically unchanged. For the simple line the no load reactive power is sufficient to regulate up to the maximum power limit assumed, but the "compound" line requires 35,000 leading kv-a. at the receiver for load conditions in addition to the 35,000 kv-a. lagging at no load. The midpoint also requires about 70,000 kv-a. leading capacity under load conditions in addition to 70,000 kv-a. lagging under no load. But the greatest disadvantage is the housing of the condensers at the midpoint with all the necessary switching facilities. This last point should, however, be greatly discounted by the greater reliability of service obtained. It is quite improbable that a 500-mile line would be constructed without providing appropriate switching facilities at the midpoint to provide for emergency conditions. Mr. Frank G. Baum has pointed out other advantages of the intermediate condenser station.

Considering the pros and cons it is apparent that the increased line rating and reliability of service is well worth the additional condenser station equipment. For longer lines the need for intermediate condenser stations is still more pronounced. The exact number of stations in longer lines rests upon the economy of the situation,—the amount of power that can be sold, the cost of power at the power-house bus bars and similar questions affecting the problem.

Conclusion

The writers have attempted to bring out the following points:

1. A proof of the circle diagram has been given maintaining therein the idea of the angle between the generator and receiver voltages.
2. A graphical means for the determination of the envelope of the receiver power circles and an analytical proof showing the envelope to be a parabola with its axis as the line of centers of the circles.
3. It has been shown how the characteristics of the synchronous condenser limit the maximum power that can be transmitted over a line.
4. The effect of character of load on stability has been discussed.
5. The characteristics of a 500-mile line have been discussed and compared with the characteristics of the same line with a condenser station at the midpoint. This comparison resulted very favorably for the latter combination.

Discussion

For discussion of this paper see page 71.

7. Atlas of U. S. A., *Electric Power Industry*.

Power Transmission

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Introduction

THIS paper briefly reviews the development of power transmission, pointing out the power limitations of long distance, high capacity transmission systems. The paper serves as a general introduction to a group of papers, giving the results of analytical and experimental investigations of the power limits and stability of transmission systems. This group of papers which is to be presented at the A. I. E. E. Midwinter Convention includes, in addition to this paper by Mr. Hanker, papers by R. D. Evans and H. K. Sels; R. D. Evans and R. C. Bergvall; and C. L. Fortescue and C. F. Wagner.

The industrial supremacy of the United States can be traced directly to the general utilization of power in manufacture, transportation and agriculture. In their infancy, prime sources of power, both hydraulic and steam, were utilized directly, and consequently were relatively small in capacity and the use was restricted as to area. Today we can begin to appreciate the remarkable influence our electricity supply systems have had on the industrial expansion and can visualize to some degree the necessity of a national system that would remove any artificial barriers that arise in the path of full and free industrial development.

A similar condition existed in the development of the earlier power systems and the growth from the small isolated stations can be traced through several distinct stages. Initially, the area served was limited by the low voltage available for distribution and as a result only a small capacity station could be utilized. At that time the possibilities of voltage transformation were not recognized and, consequently, transmission and distribution were restricted to very nominal voltages. The advantage of the alternating-current system was soon appreciated, and as a result of the larger area it was possible to supply, the capacity of generating stations increased and the necessity for frequent stations of small capacity eliminated. The load on the early stations was almost entirely lighting and it was not until the advantages of the new power for industrial use became generally recognized that electricity supply systems became an important factor in the industrial expansion. As a result of the more general utilization of electric power through the development of higher voltages for transmission and distribution, small companies operating in adjacent districts were merged and the advantage of generation in larger units secured. In addition, there was the advantage accruing from the ability to supply diversified industries with less capital investment than by individual plants. From this condition, the second stage developed, and we had what may be termed group operation.

We are now at the beginning of the third stage, where the advantages of group operation have been fully realized and the desirability of the expansion of this plan is appreciated. Adjacent groups have interconnected, and with the full development, the economies of regional or national operation will be realized. A scheme of operation that will utilize the hydraulic resources of the country to the fullest extent will undoubtedly show the greatest economy. This cannot be accomplished by independent operation of different power units as it is impracticable to economically develop a number of projects due to the inability to utilize the power in a way that would justify the expense of development. With a well organized and practicable scheme of regional or national development it will be possible to work out a program that will gradually coordinate the available hydraulic resources with steam electric stations, utilizing various types of fuel to the fullest advantage.

The development of the art of generation and transmission soon attracted attention to hydroelectric possibilities located at points removed from the load centers that could utilize them to advantage. Studies of these propositions soon developed that there were definite limitations to the amount of power that could be transmitted over a single circuit without resorting to special conductor arrangements or intermediate regulating stations. Beyond these limitations the voltage conditions were found to be unstable and impossible of control. This situation led to careful analytical studies of fundamentals, so that today there is a fuller appreciation of the limitations of the system as a whole. Previous studies that have been published have dealt largely with individual units, as, for instance, a transmission line, without taking into full consideration the effect of the characteristics of the generating station, the transforming substation and the synchronous condenser equipment that would be required for voltage control. It was during the study of the transmission that was the basis of the general power system covered by the survey made of the districts contiguous to the northeast Atlantic sea-coast that improvements in the system layout were first given consideration by the engineers who have prepared the group of papers on this important problem.

The analytical studies made at that time indicated that the limitations to the amount of load that could be transmitted over a circuit could be increased materially by locating regulating stations at intermediate points along the line. These regulating stations would be

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

essentially the same as terminal stations now in use on transmission systems where synchronous condenser equipment is utilized for voltage regulation. In a similar way the synchronous condensers would be utilized to compensate for the reactive component of the line. As these stations would not be prime sources of power, there was some discussion as to their operating stability. It was fully recognized that a similar system with generating stations located at corresponding points was entirely satisfactory, but with this type of station the prime movers would have drooping speed characteristics and furnish a stabilizing element to the frequency.

With the proposed method of operation utilizing intermediate regulating stations, it was recognized that a number of conditions existed that could not be analyzed completely, as there was no operating experience or test data that were applicable. There was a certain amount of information available from analyses of tests made a number of years ago, from which it was possible to predetermine with a fair degree of accuracy the characteristics of the tie lines necessary to maintain parallel operation of the generating stations. These data apparently indicated a limitation to the amount of power that could be transmitted that would render the scheme under consideration prohibitive. It was realized that the success of the suggested layout was dependent on the stability of the regulating stations, both as to frequency and voltage. For that reason it was felt that a comprehensive experimental investigation should parallel the analytical studies. This was considered essential, due to the numerous assumptions it was necessary to make of the operating characteristics.

The group of papers here presented discuss the subject in all its phases, and the results of the mathematical analyses have been fully checked by the tests made on the experimental lines discussed in the paper presented by Messrs. Evans and Bergvall.

In the papers on "Power Limitations of Transmission Systems" by Messrs. Evans and Sels, the analytical studies leading to an understanding of power limitations are fully outlined. A number of very interesting and valuable conclusions developed through the analytical studies have been fully confirmed by the test results that are given in the companion paper. Analytical studies of the problem, both general and specific, are given in the paper by Messrs. Fortescue and Wagner. All these studies cover important phases of the problem and, supported as they are by experimental data, will be extremely valuable to engineers investigating power projects where the amount of power to be transmitted exceeds present practise. The results have such an important bearing on the economic layout of transmission systems that the importance of the conclusions cannot be over-estimated.

Discussion

For discussion of this paper see page 71.

Power Limitations of Transmission Systems

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Review of the Subject—*Several independent studies have been made recently to determine the economies of a large, uniform power system. The two studies of more general interest were those conducted by the Department of the Interior, under the direction of W. S. Murray, for the Superpower Zone, and by F. G. Baum for the United States. Both of these investigations are available in published form.*

During the progress of the Superpower Survey, one of the longest transmission lines proposed was that extending 350 miles from the Niagara Falls Development to New York City. Under emergency conditions on this line, the power limit for the maximum amount of power was approached by two twin-circuit tower lines with three circuits carrying the emergency load. The maximum power limit would have been exceeded if two single-circuit tower lines had been employed, even though the transmission voltages and the total copper cross-section were the same as with the two twin-circuit tower lines. Similarly, several long, high-voltage lines will be required in a nation-wide system, especially through the middle western region as shown by Mr. Baum's report. The tendency to extensive transmission systems has emphasized the necessity of considering the factors which will limit the amount of power that can be transmitted any distance with the highest practical transmission voltage. On account of the transmission line characteristics, the power limits will be greater when the system is regulated by synchronous apparatus than when no such apparatus is used so that two power limits will be considered in this paper; first, for an unregulated system; and second, for a regulated system. However, while we are primarily interested in high-voltage systems in this paper, it should be kept in mind that these same methods of calculation may be applied to lower voltages in determining the power limitations of station tie lines.

It is commonly accepted that different types of networks have certain power limitations. For example, a very simple case quite generally known is that of a simple resistance circuit in which the power delivered is a maximum when the resistance of the load is equal to that of the line. Another familiar case is that of the electric arc furnace where the maximum power occurs when the resistance of the furnace arc is equal to the reactance of the electric furnace leads. The general phenomenon of maximum power limit in circuits of fixed reactance and variable resistance or load has been recognized and its theory worked out for numerous cases, such as short transmission lines, rotating machines, and transformers.

A power transmission system may be regarded as a special type of network. Ordinarily it consists of long, high-tension transmission lines and apparatus connecting generating stations with distant load centers which may be either at the terminus or at intermediate points on the high-tension lines. In large systems, the high-tension lines may form a network similar to an ordinary local distribution system.

Where synchronous condensers are not installed, the problem of the maximum amount of power which may be delivered through the system is similar to the simple resistance and reactance cases cited above in that additional load or shunt impedance simply alters the load and voltage in accordance with the relative impedances of the system.

The employment of synchronous condensers at the load centers or along the transmission lines to alter the power factor and maintain the voltage at the load materially increases the maximum amount of power that may be delivered over a given transmission network. The theoretical maximum amount of power, however, cannot be obtained under operating conditions because the synchronous equipment at the receiver drops out of step with the supply. Also, fluctuations in load will produce unstable conditions, which may accumulate sufficiently to cause the momentary swings in load to exceed the power limit, resulting in the receiver falling out of step with the supply at a lower load than it would under steady conditions. This is usually characterized as "hunting out of step."

In order to investigate the power limitations of a transmission system, it was necessary to rearrange and extend the present methods of transmission line calculations to make them more convenient for the study of the practical limit of maximum power. The method which has been found best adapted for this purpose is a development of the power circle diagram combined with the characteristic curves of the synchronous machines used to regulate the system. This power diagram has been made applicable to all the types of transmission systems by including the transmission line, step-up and step-down transformers, series and parallel circuits, so that the most complex transmission system may be represented by a single equivalent set of transmission constants. A general discussion of the methods of calculation, the maximum power limits, the practical operating limit and illustrated examples is given in Part I of the paper, and the analytical development upon which the discussion is based is given in Part II.

Part I

THE POWER DIAGRAM FOR TRANSMISSION SYSTEMS

SEVERAL methods of computing the performance of transmission lines have been developed and published from time to time. A review of numerous methods previously proposed by different authors, with examples illustrating each method of solution and other useful data, has been written by William Nesbit. The methods described employ current and voltage or corresponding vector quantities throughout. However, it has been found more convenient in this study to represent the supply and receiver voltages and receiver loads of a transmission system by means of power circle diagrams, such as illustrated in Fig. 1. A circle diagram may be defined as a graph of all receiver or supply loads that may be transmitted, assuming definite supply and receiver voltages. Various forms of circle diagrams have been presented, notably by R. A. Philip in 1911, followed by H. B. Dwight in 1913. A full analytical treatment of this diagram as applied to a complete transmission system is given in Part II of this paper.

In order to plot the circle diagram, it is necessary to determine only three constants, l, m and n which, when combined with the supply and receiver voltages selected, determine the centers and radii of the circles respectively for the assumed voltage conditions. It may be pointed out that the constant l is determined

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largely by the resistance of the circuit; the constant m by the reactance of the circuit; and the constant n by the impedance of the circuit.

Referring to Fig. 1, the circle diagram shows at once:

(1) The synchronous condenser capacity required at no load, as indicated by the point x.

(2) The maximum load at unity power factor that may be delivered for the particular voltages assumed, as indicated by the point y.

(3) The maximum power that may be delivered for the particular voltages assumed, as indicated by the point z.

The diagram readily permits the determination of the synchronous condenser capacity required for any particular load condition at the receiver; for example, in Fig. 1, circle a, the receiver load represented by the point U corresponding to 120,000 kw. real power and 74,000 kv-a. reactive power (corresponding to 85 per cent power factor), will require a synchronous conden-

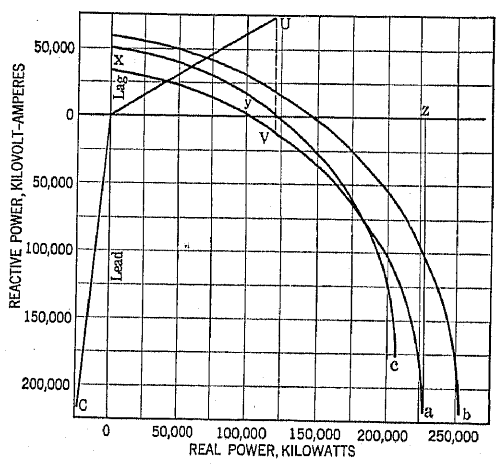


FIG. 1—TYPICAL CIRCLE DIAGRAM OF A TRANSMISSION LINE

This circle diagram is drawn for a 250-mile, 3-phase, 60-cycle transmission line with 21 ft. equivalent spacing.

	Supply Voltage E_s	Receiver Voltage E_r
Circle a..................	100%	100%
Circle b..................	110%	100%
Circle c..................	100%	90%

220 kv. = 100 per cent Voltage

The point z indicates the maximum power limit for 220 kv. at both supply and receiver, which corresponds to the vertical tangent of the receiver circle a.

ser capacity represented by the line $U\,V$, or 85,000 kv-a.

Not only does the circle diagram permit representation of all receiver loads that may be transmitted for certain fixed supply and receiver voltage conditions, but it is readily adapted to represent changes in supply or receiver voltages. If the supply voltage only is varied, the radius of the receiver circle is varied in direct proportion, and the center of the circle is unchanged. In Fig. 1, the circle b represents the conditions with the supply voltage 10 per cent above that shown for circle a. If the receiver voltage only is varied, the radius of the receiver circle is varied in direct proportion and in addition, the center of the circle moves on the line $O\,C$ in proportion to the square of the receiver voltage. In Fig. 1, the circle c represents the conditions with the receiver voltage 90 per cent of that shown for circle a.

For the majority of cases, the transmission supply voltage may be assumed constant, and this leaves only the receiver voltage and receiver load as variables. By plotting the power circles for receiver voltages from

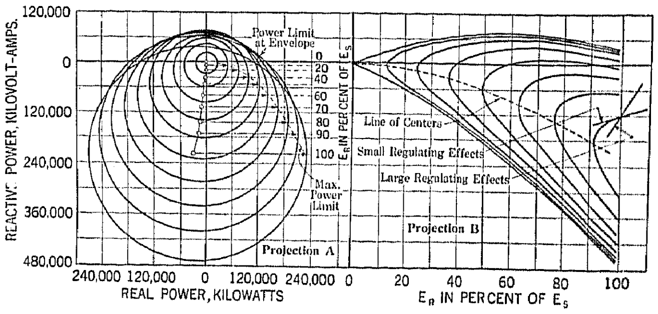


FIG. 2—VOLTAGE POWER DIAGRAM

Projection "A" shows the voltage power surface as plotted against real power and reactive power. Projection "B" represents the voltage power surface as plotted against reactive power and receiver voltage expressed in terms of supply voltage. These diagrams are based on a 250-mile, 3-phase, 60-cycle transmission line with 750,000 cir. mil copper conductor, with 21 ft. equivalent spacing. In projection "B", two lines are drawn which indicate the power limits for different characteristics of synchronous machines with respect to their regulating effect.

zero to more than 100 per cent of the supply voltage, it is possible to obtain a very complete picture showing how changes in receiver load affect the receiver voltage and vice versa. In Fig. 2, projection A, a family of such receiver power circles is plotted for various receiver

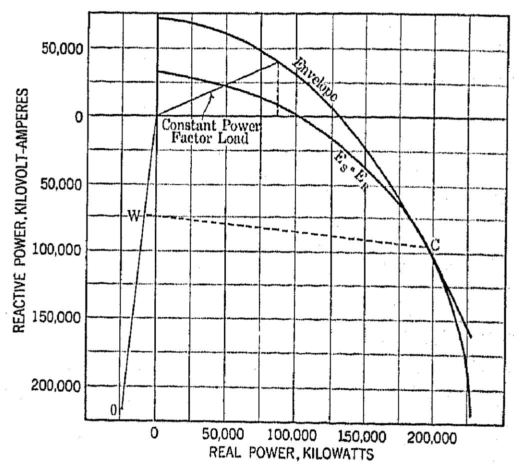


FIG. 3—THE ENVELOPE OF THE RECEIVER POWER CIRCUITS

This figure is based on the transmission circuit described in Figs. 1 and 2, with 220 kv. at the supply end. The circle drawn represents a condition with 220 kv. at the receiver. The power limit at the envelope and the maximum power limit for this transmission system are indicated.

voltages in steps equal to 10 per cent of the supply voltage. It will be observed that a curve can be drawn which will be tangent to the receiver power circles.[1] This curve is important in the study of maximum power

1. An independent investigation has also been made by J. Ossanna, *Electrotechnische Zeitschrift*, August 11, 1922.

limits, and will be designated as the "envelope" throughout this paper. It has not been shown on Fig. 2 as it would tend to obscure the circle diagrams. In Fig. 3, the envelope for the same transmission line as that plotted in Fig. 2 is shown together with the circle diagram for the receiver voltage equal to 100 per cent of the supply voltage.

Delivered power at a definite power factor is represented by a point in Fig. 2. If a point representing delivered power be selected within the envelope two power circles can be drawn intersecting at this point. This means that for any point within the envelope, there are two receiver voltages which will satisfy the receiver load and supply voltage conditions. For a point on the envelope, only one receiver voltage will satisfy the conditions, while for a point outside the envelope, there is no receiver voltage which will satisfy the conditions and consequently a point outside the envelope represents an imaginary operating condition. Therefore, the point at which any particular circle becomes tangent to the envelope is one type of power limit which will be described in greater detail in a separate section.

To understand the true significance of the power circle diagrams, it is desirable to conceive of them as a three-dimensional figure, that is, considering the supply voltage constant, the general equation representing the kilovolt-amperes delivered involves just three variables: The receiver power, the receiver reactive power, and the receiver voltage, which may be plotted along the X, Y and Z axes, respectively. By virtue of its construction, the surface of the figure represents all possible voltage-power conditions with a constant supply voltage. Since this surface has many interesting properties which are worthy of investigation, Fig. 2 has been plotted to show two projections, A and B, of the voltage-power surface; projection A shows the projection on the X-Y plane of inter-sections of the surface with planes parallel to the X-Y plane passing through points representing receiver voltages of 10 per cent, 20 per cent and so on of the supply voltage. Projection B shows the projection on the Y-Z plane of intersections of the surface with planes parallel to Y-Z plane passing through points representing 0 kw., 30,000 kw., 60,000 kw. and so on.

As previously mentioned, varying the supply voltage simply changes the radius of the receiver circle. Unless mentioned otherwise in the following discussion throughout the paper, the supply voltage will be considered constant.

Theoretical Maximum Power Limit

Heretofore, it has been the general practise of transmission engineers to consider only one circle diagram or one set of voltage conditions in their calculations, without relating the solution generally to other circle diagrams or calculations representing other voltage conditions. This interesting relationship is very important and gives the voltage power surface referred to above, which is plotted in Fig. 2. The study of these relationships is facilitated by the graphical conception of a surface which may be represented by an actual model.

The receiver load at any constant power factor may be considered as an impedance connected across the circuit, and if this impedance is varied from infinity to zero, the receiver load increases to a maximum and then decreases to zero. This is shown graphically by the line for constant power factor load in Fig. 3 with the maximum amount of power determined by the envelope.

If synchronous apparatus is available at the receiver for maintaining the voltage and controlling the power factor, the power limit may be increased. With infinite capacity in synchronous apparatus available, the power limit is that determined by the vertical tangent to the receiver circle for the voltages assumed.

Study of the voltage-power surface has developed some very interesting features which are associated with the performance characteristics of the synchronous apparatus used to regulate the system. These features will be further discussed in a subsequent section. For the time being the maximum limits may be considered as falling into two classes:

First: The power limit defined by the envelope for any given power factor of load and resulting voltage conditions. The load will include all kinds of electrical equipment, but if it includes any rotating equipment, this will fall out of step if the voltage drops sufficiently, or reduce load by a drop in speed if sufficient torque is still available when the power limit is passed.

Second: The power limit defined by the vertical tangent to the circle diagram. In addition to the load, synchronous apparatus is required to regulate the voltage-power conditions up to the maximum power point when the receiver will pull out of synchronism with the supply end.

For purposes of illustration, these two power limits have been identified on Fig. 2, the power limit for any given power factor being marked the "Power Limit at the Envelope," and the power limit described by a vertical tangent, the "Maximum Power Limit."

Practical Maximum Power Limit

The usual high-voltage transmission problem involves a regulated system with synchronous machines of limited capacity and commercial design. The Practical Maximum Power Limit, therefore, lies between the limits for an unregulated and a regulated system with infinite condenser capacity, that is, between the power limit at the envelope and a vertical tangent to the receiver circle. The characteristics of the synchronous machines which affect the power limit of a transmission system include not only the capacity in kilovolt-ampere, but also the regulating effect. By "regulating effect" is meant the change in reactive kilovolt-ampere for a definite change in voltage, the greater the change in reactive kilovolt-ampere, the

greater the regulating effect. The relation of the characteristics of synchronous machines to the power limits of a transmission system is indicated by the curves of regulating effects of loads sketched on Fig. 2.

In the discussion so far, no reference has been made to the practical limits of voltage regulation and power loss of transmission. In short transmission lines, the limits of voltage drop require synchronous condenser regulation, and even then the economic limits of power loss are usually reached before the envelope or maximum power limit is reached. However, in a long transmission line, especially above 350 miles, the chracteristics of the transmission system may be such as to make the envelope and maximum power limits within the practical limits of voltage regulation and power loss. In general, then, the discussion of the envelope and maximum power limit should be considered with a large transmission system, or in connection with the synchronizing stability of short lines between power stations where large loads are momentarily thrown on the tie lines so that they exceed the power limits and the stations fall out of synchronism. In this connection, it may be noted that a short circuit on the secondary system may be considered as a shunt loading on the transmission system and the corresponding circle diagram obtained on this basis.

Effect of Transmission Design on the Power Limits

The methods for increasing the Practical Maximum Power Limits of transmission systems will next be considered. These methods may be divided into two classes.

(1) Those which increase the power limits as determined by the envelope and the vertical tangent to the receiver power circles, and

(2) Those which permit stable operation at points closer to the maximum power limit.

The first class depends upon the modification of the circuit constants of the transmission line or increasing the transmission voltage, so that the power limits are actually increased. The second class is dependent upon the characteristics of the rotating machinery in the load, which will be discussed in a companion paper.

The relation between the circuit constants of the transmission line which will give the maximum power limits, has been analyzed in Part II of this paper. The results of this analysis show that for long transmission lines the l constant should be small, and the n constant large; and for short transmission lines, that the resistance should be made as small as practicable, and that the reactance be equal to $\sqrt{3}$ times the resistance. It becomes necessary, therefore, to distinguish between the cases where the reactance of the transmission line is large or small in comparison with the resistance. In general, the reactance of a transmission circuit will decrease very slowly with increase in conductor size, whereas the conductivity increases directly. Consequently, for large capacity, high-voltage circuits, the reactance becomes several times the resistance, and this case will be considered first.

In general, the power limits of a transmission system are increased by the use of larger conductors or increased voltage. However, at any given voltage, an indefinite increase in conductor size will not increase the power limits proportionately, because of the slight effects on the l and n constants, or in other words, when the resistance becomes negligible in comparison with the reactance, and the reactance is only slightly reduced. Curves B and C of Fig. 4 show the two power limits for a group of 3-phase, 60-cycle transmission lines, 250 miles in length, operated with 220 kv. at both supply and receiver ends. It will be noted that the increase in size of copper conductor from 600,000

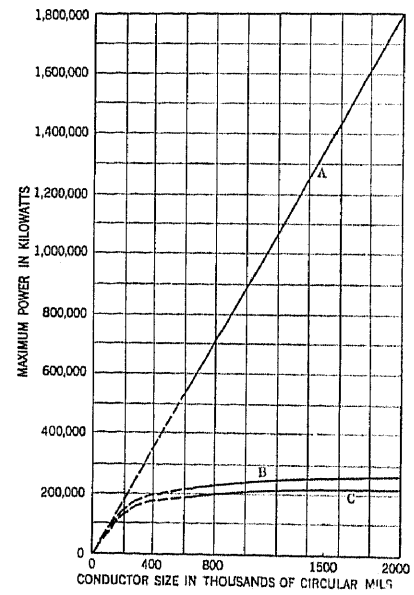


Fig. 4—Relation of Power Limits to Conductor Size

This diagram is based on transmission lines, 250 miles in length, with 3-phase, 60-cycle supply, and 21 ft. equivalent spacing, with 220 kv. maintained at the supply. Curve "A" shows the maximum power limit with unrestricted receiver voltage and condenser capacity, Curves "B" and "C" show the maximum power limit, and power limit at the envelope respectively for the condition with 220 kv. maintained at both supply and receiver. The difference between curves "A" and "B" or "C" indicates the limitation in the transmission of power due to the reactance of the transmission circuit with 60-cycle supply.

cir. mils to 2,000,000 cir. mils only increases the maximum power limit about 20 per cent. Therefore, it is apparent that the conductor size should not be increased indefinitely, except to economically reduce the power loss consistent with the increased cost of line. In a high-voltage transmission system, the corona effect may require a conductor size above which no appreciable gain in power limits will be obtained.

Curve A of Fig. 4, has been plotted as a matter of interest to show the maximum power that can be transmitted with unlimited receiver voltage and synchronous condenser capacity. This curve is given by equation 35 in Part II. By reducing this equation to the short line case, equation 36, it will be seen that

under these conditions of operation, the maximum power is only limited by the resistance of the line as in the case of direct current, the reactance of the line being offset by the leading power factor of the load and high receiver voltage. With small conductor sizes, the resistance is the determining factor of the power transmitted, so that all limits approach curve A, but with increased conductor size and limited receiver voltage the power transmitted is determined by the reactance, so that for large conductor sizes, there is a very large difference from curve A.

One method for increasing the power limit of a single circuit involves the use of a divided conductor. The arrangement would be similar to that proposed by Percy H. Thomas in a discussion on the "Critical Load"[2] of a transmission line. The increase in the power limit that may be accomplished by this method is determined by the limitation in the separation of the individual conductors due to the formation of corona. Wherever practicable, the maximum number of circuits should be employed. For example, double-circuit tower lines instead of single-circuit tower lines, should be used, thus greatly increasing the power limits.

Conductor spacing has an important effect on the power limits; the closer the spacing, the greater the power limits. However, variation in the spacing of conductors is not a practical method of increasing the power limit, for the reason that the closest spacing consistent with the avoidance of interruption to service and possible operation at higher voltages in the future, will always be employed.

The case in which the reactance of the transmission conductor is small in comparison with the resistance rarely occurs with 60-cycle power supply, except possibly with low-voltage, small capacity circuits, or an underground cable system. With 25-cycle power supply, however, this condition is likely to occur with moderate capacity systems. The conditions for obtaining the maximum amount of power in this case require that the resistance be made as low as practicable, and that the reactance be increased to equal $\sqrt{3}$ times the resistance. Hence, in this particular case, increased spacing of the transmission conductors or the use of reactors with underground cable would result in increasing the power limits of the circuits.

The ratio of the resistance to the reactance of a transmission circuit has an interesting effect on the power limits. If the load on a transmission circuit operating under constant voltage conditions is increased from no load, the receiver loads follow along the circle diagram, passing through the point of tangency with the envelope, and through the point of maximum power. If the reactance of the transmission circuit is several times the resistance, the power limit at the envelope is reached before the maximum power limit; on the other hand, if the reactance is small in comparison with the resistance, the maximum pow limit is reached first. Consequently, for transmissi circuits having a low ratio of reactance to resistan the special regulating characteristics of synchrono machines become unimportant.

The frequency of the power supply has an importa effect on power limits, particularly for the transmissi of large amounts of power where high voltage ar large-capacity circuits will be employed. For suc conditions, the 60-cycle reactance is many times tl resistance, and the use of a lower frequency, for exan ple, 25 cycles, would reduce the reactance and increas the power limit by more closely approaching tl theoretical conditions for transmitting the maximu amount of power with a given conductor. The max mum power limit for a 250-mile transmission line wit 636,000 cir. mil aluminum conductors is shown i Fig. 5, with curve B showing the power limit for 6 cycles, and curve D the corresponding power limit for 25 cycles. It will be noted that the power limit is increased at the lower frequency from 35 per cent to 65 per cent, depending upon the distance.

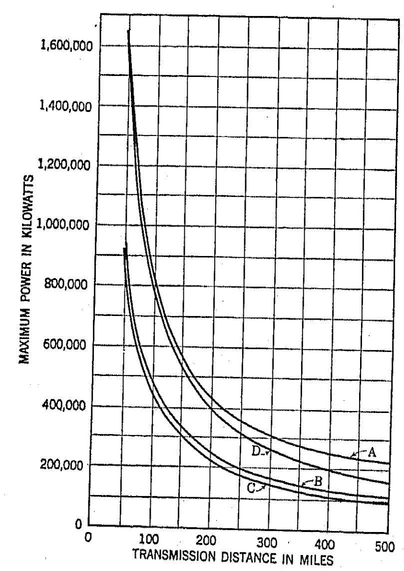


FIG. 5—RELATION OF POWER LIMITS TO LENGTH OF TRANSMISSION LINE

These diagrams are based on a transmission circuit employing 636,000 cir. mil aluminum conductor with 21 ft. equivalent spacing, and with 220 kv. maintained at the supply end. Curve "A" shows the maximum power limit with 60-cycle supply with unrestricted receiver voltage and condenser capacity. Curve "B" shows the maximum power limit with 60-cycle supply and 220 kv. maintained at the receiver. Curve "C" shows the power limit at the envelope with 60-cycle supply and 220 kv. maintained at the receiver. Curve "D" corresponds to curve "B", except that the power supply is 25 cycles.

The effect of considering the reactance of transformers has not been mentioned previously. It is evident from the preceding discussion that the addition of the reactance of transformers might increase the power limit for certain conditions, particularly on moderate-capacity, 25-cycle systems. On the other hand, for high-capacity, high-voltage systems, such as the 220-kv. systems which have been proposed, the

2. Percy H. Thomas, A. I. E. E. TRANSACTIONS, 1910.

addition of the transformer reactance will decrease the power limits appreciably.

Within the last few years, the idea of a loaded transmission line has been developed. By this means, it is possible to obtain equivalent transmission constants which are more favorable for the transmission of power than those that could be obtained by modification in transmission line conductor spacing, arrangement or size. With a loaded transmission line, it is proposed to install at intervals a "lumped" impedance connected either in series or in parallel with sections of the "smooth" transmission line. For the condition in which the reactance of a short transmission line is less than $\sqrt{3}$ times the resistance, the power limit will be increased by adding series reactors connected at intervals in the transmission line, as proposed by R. A. Philip.[3] The method of series loading with inductance, however, is of very limited application, because with the usual transmission problem, the reactance is already in excess of the desired amount.

The use of shunt loading with synchronous machines was proposed in a paper before the A. I. E. E. in 1921, by F. G. Baum. This method of loading for increasing the power limits is, in effect, one of variable shunt capacity. The effect of loading a transmission line with intermediate synchronous condensers which are controlled so as to maintain constant voltage at these points, is to increase the power limits of the system as a whole to the power limit for the individual sections. In other words, a 500-mile transmission line with one intermediate loading point, in effect increases the amount of power that may be transmitted to that of a 250-mile section. In Fig. 5, curve B shows that the maximum power limit with 220 kv. at the supply and receiver ends of a 3-phase, 60-cycle, 636,000 cir. mil aluminum conductor transmission line is about 121,000 kw. for the 500-mile line, and about 180,000 kw. for the 250-mile line. Consequently, a loaded transmission line, 500 miles in length, would be capable of delivering 180,000 kw., less a small amount for losses in the second 250-miles section of the line. In order to bring out quantitatively the present limits of high-voltage transmission, four problems will be worked out which will also show the advantage of loading the transmission circuit.

Examples of High-Voltage Transmission

From an engineering standpoint, it is practically impossible to predict the limits which high-voltage transmission will reach. However, from a commercial viewpoint, 220 kv. and certainly not more than 330 kv. will meet our requirements for some time to come, as good operating practise will limit the amount of power which should be entrusted to a single circuit. As the power systems grow, these limits will be raised, but this will be a process of considerable development.

Assuming a tentative standard of 220 kv. as the present highest transmission voltage, it is interesting to note the characteristics of different lengths of lines. It has been pointed out in Fig. 5 that the power limit is rapidly reduced by increased distance so that considerably more power can be carried by making up a long line of several sections with synchronous condenser loading at intermediate points.

Table I shows the transmission constants for 250, 500 and 750 mile lines, and Tables II, III and IV show

3. R. A. Philip, "Economic Limitations," A. I. E. E. Transactions, 1911. This proposal was based on short lines as the effect of capacity was neglected.

TABLE I
TRANSMISSION CALCULATIONS
750,000 cir. mil Copper Conductor, 21 ft.
Equivalent Spacing, 60 Cycles

Distance, miles	250	500	750
Resistance per mile	0.0782	0.0782	0.0782
Reactance per mile	0.8	0.8	0.8
Susceptance per mile	0.00000542	0.00000542	0.00000542
Z	$19.55 + j200$	$39.1 + j400$	$58.65 + j600$
Y	$0 + j0.001355$	$0 + j0.00271$	$0 + j0.004065$
$A = A_o = D_o$	$0.8675 + j0.01265$	$0.5048 + j0.04391$	$0.007227 + j0.07631$
$B = B_o$	$17.82 + j191.2$	$26.08 + j332.1$	$19.02 + j385$
$l = l'$	0.000485	0.00025	0.0001986
$m = m'$	0.004493	0.0015	0.000008958
$n = n'$	0.005208	0.003	0.002594

TABLE II
TABULATION OF CIRCLE DIAGRAM CONSTANTS FOR 250-MILE LINE

E_S = 220,000 volts m = 0.004493
l = 0.000485 n = 0.005208

E_R % of E_S	A_R	B_R	C_R	A_S	B_S	C_S
100	23,400	217,400	252,100	23,400	217,400	252,100
90	19,000	176,100	226,900	"	"	226,900
80	15,000	139,200	210,700	"	"	201,700
70	11,500	106,500	176,500	"	"	176,500
60	8,450	78,280	151,300	"	"	151,300
50	5,870	54,360	126,000	"	"	126,000
40	3,760	34,790	100,800	"	"	100,800
30	2,110	19,570	75,630	"	"	75,630
20	939	8,698	50,420	"	"	50,420
10	234	2,174	25,210	"	"	25,210
0	0	0	0	"	"	0

TABLE III
TABULATION OF CIRCLE DIAGRAM CONSTANTS FOR 500-MILE LINE

E_S = 220,000 volts m = 0.0015
l = 0.00025 n = 0.003

E_R % of E_S	A_R	B_R	C_R	A_S	B_S	C_S
100	12,100	72,610	145,300	12,100	72,610	145,300
90	9,800	58,820	130,750	"	"	130,750
80	7,740	46,470	116,200	"	"	116,200

TABLE IV
TABULATION OF CIRCLE DIAGRAM CONSTANTS FOR 750-MILE LINE

E_S = 220,000 volts m = 0.000008958
l = 0.0001986 n = 0.002594

E_R % of E_S	A_R	B_R	C_R	A_S	B_S	C_S
100	9,615	433.6	125,600	9,615	433.6	125,600
90	7,786	351.2	113,000	"	"	113,000
80	6,153	277.5	100,500	"	"	100,500

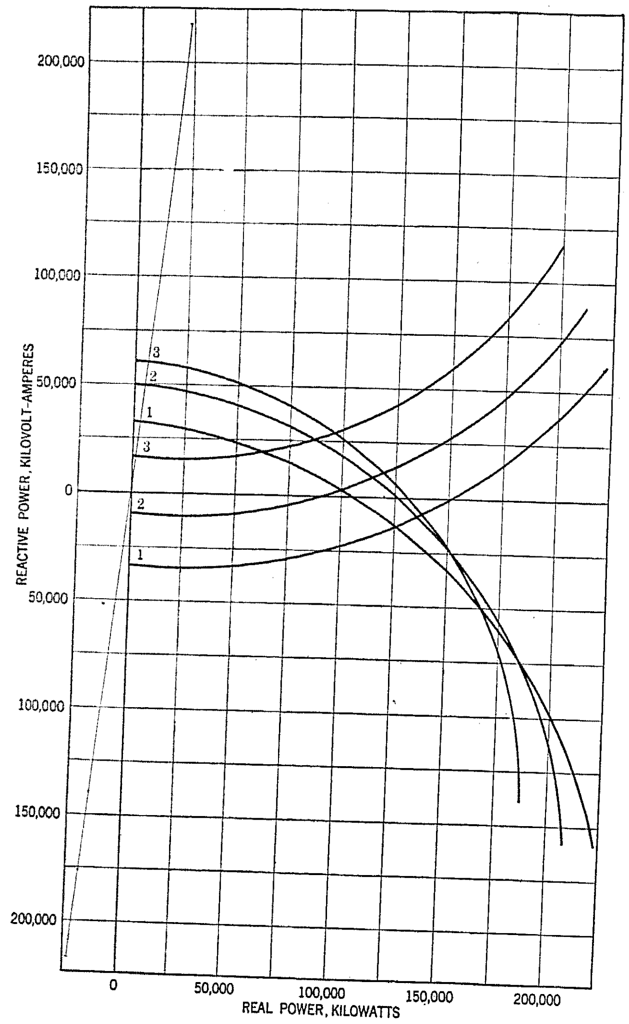


Fig. 6—Circle Diagram for 250-Mile Transmission Line

This diagram is based on 750,000 cir. mil copper conductors, 21 ft. equivalent spacing, with 220-kv., 3-phase, 60-cycle supply. Circle 1 represents the condition for the receiver voltage equal to the supply voltage. Circles 2 and 3 represent the conditions with the receiver voltage equal to 90 and 80 per cent of the supply voltage respectively. The receiver power circles have their centers below the "X" axis, while the supply circles have their centers above the "X" axis.

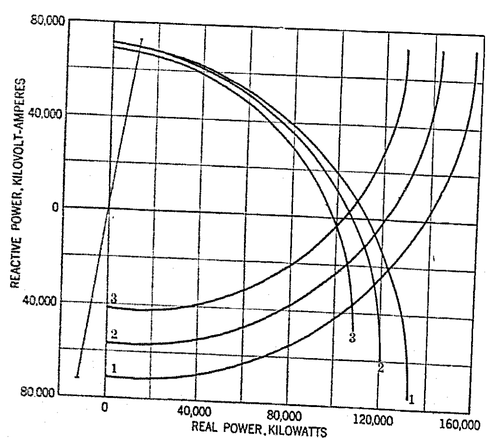


Fig. 7—Circle Diagram for 500-Mile Transmission Line

This diagram is similar to Fig. 6, except for the increase in transmission distance. Curve 1 represents the receiver circle for a voltage equal to the supply voltage of 220 kv. Circles 2 and 3 represent the conditions with receiver voltages 90 and 80 per cent of the supply voltage respectively.

the circle diagram constants for these lines respectivel All of the previous figures except Figs. 4 and 5 ha been plotted from the constants in Table II. Tl circle diagrams for each line have been plotted in Fig 6, 7 and 8 for three-voltage conditions, so as to indicat the general shape of the diagram with respect to tl voltage power surface. The supply power circle di grams corresponding to the receiver power circle di grams are also plotted in Figs. 6, 7 and 8.

Fig. 6 of the 250-mile line corresponds to the usu diagram for a short line indicating the amount of lin charging kilovolt-ampere which must be taken care o The diagram for the 500-mile line, Fig. 7, shows a larg amount of charging kilovolt-ampere which complet

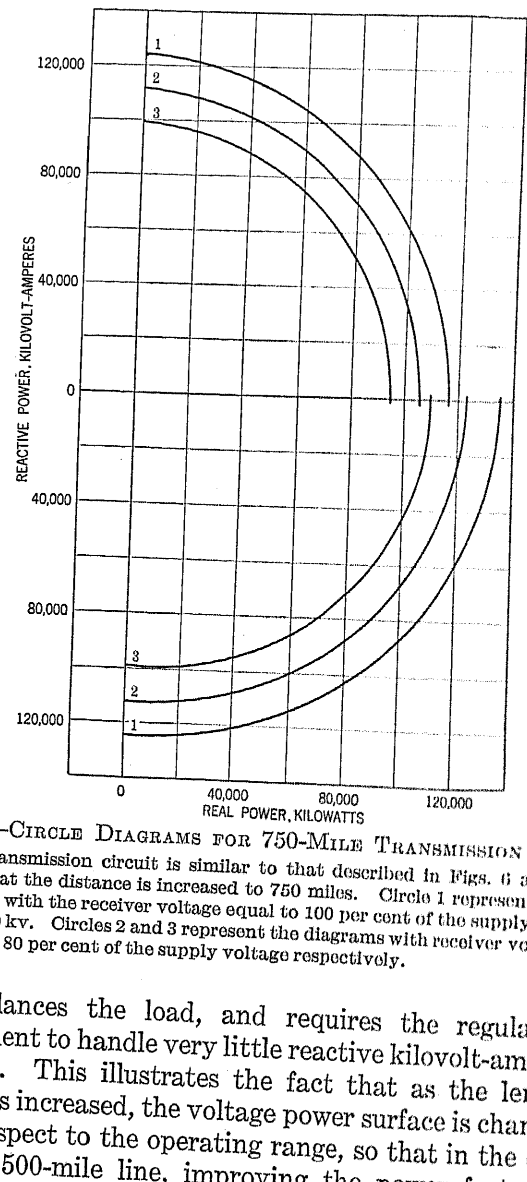


Fig. 8—Circle Diagrams for 750-Mile Transmission Line

The transmission circuit is similar to that described in Figs. 6 and 7, except that the distance is increased to 750 miles. Circle 1 represents the condition with the receiver voltage equal to 100 per cent of the supply voltage of 220 kv. Circles 2 and 3 represent the diagrams with receiver voltages of 90 and 80 per cent of the supply voltage respectively.

overbalances the load, and requires the regulating equipment to handle very little reactive kilovolt-ampere leading. This illustrates the fact that as the length of line is increased, the voltage power surface is changed with respect to the operating range, so that in the case of the 500-mile line, improving the power factor to leading decreases the voltage at all loads and the operating range is at lagging power factor. This imposes a heavy requirement on the characteristics of the regulating equipment, since it must be designed for heavy lagging loads, and hence high reactance when,

in order to obtain maximum stability, the machine should have minimum reactance.

In Fig. 8 for the 750-mile line, which is approximately one-quarter wave length for 60 cycles, the receiver must operate at lagging power factor, and the supply at leading power factor for all loads. This gives a higher voltage throughout the middle of the line at all loads, and the poor operating power factor serves to make the system unsatisfactory aside from all possibility of resonant conditions with either supply or receiver open-circuited or unstable operation of synchronous machines.

From a general consideration of the characteristics of the line, it is apparent that the longest transmission section of a line should not exceed 250 miles, and shorter sections would be preferable to obtain a higher average power factor, and hence lower losses at all loads, and a considerable increase in the power limits and general stability of the system. Based on the economic number of loading points, it is estimated conservatively that 135,000 to 150,000 kw. per circuit can be transmitted from any existing power supply to a load center with equipment commercially available.

Conclusion

To recapitulate on the points of general interest in transmission system design, there are several items to be considered:

(1) Methods of calculation have been developed in the power circle diagram which accurately determines the maximum power limits under various circuit and electrical conditions.

(2) Two types of power limits have been discussed with their relation to the load and regulating equipment.

(3) Methods of increasing the power limits have been discussed in transmission design.

(4) On the basis of economy and stability of operation, it is recommended that a single section of line be less than 250 miles, and longer lines loaded at intermediate stations with synchronous condensers so that no section exceeds 250 miles.

In concluding, it is important to note the general application of the methods developed to any transmission layout and to the determination of the load capacity or synchronizing capacity between stations in studying the performance of a system.

Part II

Theoretical Discussion

General Circuit Constants

In any transmission system[4] the relation between supply and receiver voltages and currents may be expressed in the following form:

$$\left.\begin{aligned} \dot{E}_s &= A_o \dot{E}_r + B_o \dot{I}_r \\ \dot{I}_s &= C_o \dot{E}_r + D_o \dot{I}_r \\ \dot{E}_r &= D_o \dot{E}_s - B_o \dot{I}_s \\ \dot{I}_r &= -C_o \dot{E}_s + A_o \dot{I}_s \end{aligned}\right\} \qquad \textbf{(1-4)}^5$$

the nomenclature for which is fully explained under caption "Nomenclature." These equations are in the familiar form in which A, B and C constants are employed, but the D_o constant has been added to provide for the general case. The use of A_o, B_o, C_o and D_o constants is required when the entire transmission system including transmission line and transformers is considered. The D_o constants has a value different from A_o for all cases where the transmission system is unsymmetrical about its center.

The general circuit constants may be derived from the consideration of the two elementary networks, the series impedance Z and the shunt admittance Y, and their combinations in series or parallel. The value of the general circuit constant as applied to these elementary networks is given in Table V.

TABLE V
GENERAL CIRCUIT CONSTANTS

Constant	Series Impedance	Shunt Admittance
A_0	1	1
B_0	Z	0
C_0	0	Y
D_0	1	1

Two of these elementary networks may be combined in series with constants, voltages and currents as indicated in Fig. 9. The equations are as follows:

$$\left.\begin{aligned} \dot{E}_1 &= A_1 \dot{E}_r + B_1 \dot{I}_r \\ \dot{I}_1 &= C_1 \dot{E}_r + D_1 \dot{I}_r \\ \dot{E}_s &= A_2 \dot{E}_1 + B_2 \dot{I}_1 \\ \dot{I}_s &= C_2 \dot{E}_1 + D_2 \dot{I}_1 \end{aligned}\right\} \qquad \textbf{(5-8)}$$

By eliminating $\dot{E}_1$ and $\dot{I}_1$ from equations (**5**) to (**8**) a solution in the form of equations (**1**) and (**2**) may be obtained where:

$$\left.\begin{aligned} A_o &= A_1 A_2 + B_2 C_1 \\ B_o &= B_2 D_1 + A_2 B_1 \\ C_o &= A_1 C_2 + C_1 D_2 \\ D_o &= B_1 C_2 + D_1 D_2 \end{aligned}\right\} \qquad \textbf{(9-12)}$$

It will be noted that for the general case of two elementary networks in series the constants A_o and D_o are not equal in the combined network though they are both equal to unity for each elementary network. Consideration of equations (**9**) to (**12**) show that an indefinite number of networks may be added in series

4. The term transmission system, as employed in this paper, not only includes the transmission line itself but the remainder of the system as well; the transmission system is equivalent to a general network where each branch has constant impedance characteristics.

5. These equations may be interpreted as follows: In any transmission system, the supply voltages varies with receiver voltage and current. A_o and B_o being proportionality constants dependent upon the particular system involved. Similarly the supply current varies with receiver voltage and current, C_o and D_o being proportionality constants dependent upon the particular system involved.

and the form of equations (1) and (2) will not be changed. The constants of the smooth transmission line may be obtained with this method by combining an infinite number of elementary sections of series impedances and shunt admittances. It will also be noted that equations (9) to (12) cover the series combinations of two general networks and the process may be extended to cover all cases. Hence the use of A_o, B_o, C_o and D_o constants is applicable to all networks made up of elementary networks in series.

Elementary networks may similarly be combined in parallel. The equation for the case with constants,

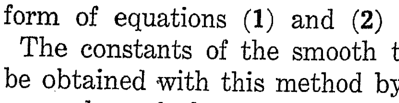


FIG. 9—TWO NETWORKS IN SERIES

voltage and currents as indicated in Fig. 10, are as follows:

$$\dot{E}_s = A_1 \dot{E}_r + B_1 \dot{I}_r' = A_2 \dot{E}_r + B_2 \dot{I}_r'' \quad (13)$$

$$\dot{I}_s = C_1 \dot{E}_r + D_1 \dot{I}_r' \quad (14)$$

$$\dot{I}_s'' = C_2 \dot{E}_r + D_2 \dot{I}_r'' \quad (15)$$

$$\dot{I}_s' + \dot{I}_s'' = \dot{I}_s \quad (16)$$

$$\dot{I}_r' + \dot{I}_r'' = \dot{I}_r \quad (17)$$

By eliminating $\dot{I}_r'$, $\dot{I}_r''$, $\dot{I}_s'$ and $\dot{I}_s''$ from equations (13) to (17), the equations may be written in the form of equations (1) and (2) with:

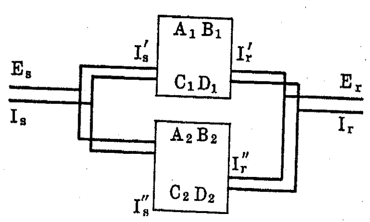


FIG. 10—TWO NETWORKS IN PARALLEL

$$\left.\begin{aligned}
A_o &= \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} \\
B_o &= \frac{B_1 B_2}{B_1 + B_2} \\
C_o &= C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2} \\
D_o &= \frac{B_1 D_2 + B_2 D_1}{B_1 + B_2}
\end{aligned}\right\} \quad (18\text{-}21)$$

It will be noted that for the general case of two elementary networks in parallel that the A_o and D_o constants are equal, and further that if two general networks are paralleled, the A_o and D_o constants are unequal if the same constants are unequal in either branch, and the equations are similar to equations (1) and (2).

By the combination of series impedances and shunt admittances any network may be formed. Hence the use of A_o, B_o, C_o and D_o constants covers the general case of a transmission system.

It may be stated from an observation of equations (10), (11), (19) and (20) that the B_o and C_o constants are identical, whether calculated from the receiver end or supply end. On the other hand constants A_o and D_o when calculated from the receiver end are equal to the D_o and A_o constants respectively, when calculated from the supply end.

The preceding discussion has shown the applicability of general circuit constants to the solution of transmission problems not only for the smooth transmission line itself but also for the transmission line with transformers, the loaded line and transmission lines in parallel. For certain important cases, the solutions of the general circuit constants are given in Table VI.

DEVELOPMENT OF THE POWER DIAGRAM

For the analysis of the power limitations of transmission systems, the use of the power or circle diagram has been found very desirable. The circle diagram may be derived from the equations for the general case of transmission systems employing general circuit constants in the following manner; equation (1) and its conjugate may be written as follows:

$$\dot{E}_s = A_o \dot{E}_r + B_o \dot{I}_r \quad (1)$$

$$\bar{\dot{E}}_s = \bar{A}_o \bar{\dot{E}}_r + \bar{B}_o \bar{\dot{I}}_r \quad (22)$$

After multiplying these two equations together and simplifying the product by recalling that

$\dot{E}_s \bar{\dot{E}}_s = E_s^2$ and $\dot{E}_r \bar{\dot{E}}_r = E_r^2$ and $\dot{E}_r \bar{\dot{I}}_r = P_r + jQ_r$

the following equation may be written

$$E_s^2 = A_o \bar{A}_o E_r^2 + (A_o \bar{B}_o + \bar{A}_o B_o) P_r + j Q_r (A_o \bar{B}_o - \bar{A}_o B_o) + B_o \bar{B}_o \left(\frac{P_r^2 + Q_r^2}{E_r^2} \right) \quad (23)$$

The above expression may be written in the form of a circle as follows:

$$(P_r + l E_r^2)^2 + (Q_r + m E_r^2)^2 = n^2 E_r^2 E_s^2 \quad (24)$$

Where

$$l = \frac{A_o \bar{B}_o + \bar{A}_o B_o}{2 B_o \bar{B}_o} \quad (25)$$

$$m = j \frac{A_o \bar{B}_o - \bar{A}_o B_o}{2 B_o \bar{B}} \quad (26)$$

$$n = \frac{1}{\sqrt{B_o \bar{B}_o}} \quad (27)$$

Equation (24) is expressed in terms of line to neutral voltage and apparent power in each phase. For the usual case of three-phase transmission it is more convenient to use line voltage and total apparent power.

Item	Circuit Constants			
	A_0	B_0	C_0	D_0
(a)	1	Z	O	1
(b)	1	O	Y	1
(c)	$1+\frac{T\,Y}{2}$	$T\left(1+\frac{T\,Y}{4}\right)$	Y	$1+\frac{T\,Y}{2}$
(d)	$A=\cosh\sqrt{Z\,Y}=\left(1+\frac{Z\,Y}{1.2}+\frac{Z^2\,Y^2}{1.2.3.4}+\ldots\right)$	$B=\sqrt{Z/Y}\sinh\sqrt{Z\,Y}=Z\left(1+\frac{Z\,Y}{2.3}+\frac{Z^2\,Y^2}{2.3.4.5}+\ldots\right)$	$C=\sqrt{Y/Z}\sinh\sqrt{Z\,Y}=Y\left(1+\frac{Z\,Y}{2.3}+\frac{Z^2\,Y^2}{2.3.4.5}+\ldots\right)$	Same as A_0
(e)	A	$B+A\,Z_r$	C	$A+C\,Z_r$
(f)	$A+C\,Z_s$	$B+A\,Z_s$	C	A
(g)	$A+C\,Z_s$	$B+A\,(Z_s+Z_r)+C\,Z_s\,Z_r$	C	$A+C\,Z_r$
(h)	$A\left(1+\frac{T_r\,Y_r}{2}\right)+B\,Y_r$	$B\left(1+\frac{T_r\,Y_r}{2}\right)+A\,T_r\left(1+\frac{T_r\,Y_r}{4}\right)$	$C\left(1+\frac{T_r\,Y_r}{2}\right)+A\,Y_r$	$A\left(1+\frac{T_r\,Y_r}{2}\right)+C\,T_r\left(1+\frac{T_r\,Y_r}{4}\right)$
(i)	$A\left(1+\frac{T_s\,Y_s}{2}\right)+C\,T_s\left(1+\frac{T_s\,Y_s}{4}\right)$	$B\left(1+\frac{T_s\,Y_s}{2}\right)+A\,T_s\left(1+\frac{T_s\,Y_s}{4}\right)$	$C\left(1+\frac{T_s\,Y_s}{2}\right)+A\,Y_s$	$A\left(1+\frac{T_s\,Y_s}{2}\right)+B\,Y_s$

(j)

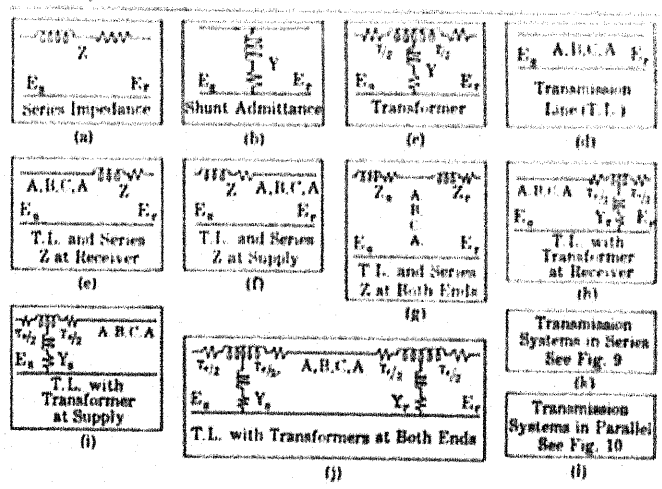
$$A_0=A\left[\left(1+\frac{T_r\,Y_r}{2}\right)\left(1+\frac{T_s\,Y_s}{2}\right)+T_s\,Y_r\left(1+\frac{T_s\,Y_s}{4}\right)\right]+B\,Y_r\left(1+\frac{T_s\,Y_s}{2}\right)+C\,T_s\left(1+\frac{T_r\,Y_r}{2}\right)\left(1+\frac{T_s\,Y_s}{4}\right)$$

$$B_0=B\left(1+\frac{T_r\,Y_r}{2}\right)\left(1+\frac{T_s\,Y_s}{2}\right)+A\left[T_r\left(1+\frac{T_r\,Y_r}{4}\right)\left(1+\frac{T_s\,Y_s}{2}\right)+T_s\left(1+\frac{T_r\,Y_r}{2}\right)\left(1+\frac{T_s\,Y_s}{4}\right)\right]+C\,T_s\,T_r\left(1+\frac{T_r\,Y_r}{4}\right)\left(1+\frac{T_s\,Y_s}{4}\right)$$

$$C_0=C\left(1+\frac{T_r\,Y_r}{2}\right)\left(1+\frac{T_s\,Y_s}{2}\right)+A\left[Y_r\left(1+\frac{T_s\,Y_s}{2}\right)+Y_s\left(1+\frac{T_r\,Y_r}{2}\right)\right]+B\,Y_r\,Y_s$$

$$D_0=A\left[\left(1+\frac{T_r\,Y_r}{2}\right)\left(1+\frac{T_s\,Y_s}{2}\right)+T_r\,Y_s\left(1+\frac{T_r\,Y_r}{4}\right)\right]+B\,Y_s\left(1+\frac{T_r\,Y_r}{2}\right)+C\,T_r\left(1+\frac{T_r\,Y_r}{4}\right)\left(1+\frac{T_s\,Y_s}{2}\right)$$

Item	A_0	B_0	C_0	D_0
(k)	$A_0=A_1\,A_2+C_1\,B_2$	$B_0=B_1\,A_2+D_1\,B_2$	$C_0=A_1\,C_2+C_1\,D_2$	$D_0=B_1\,C_2+D_1\,D_2$
(l)	$A_0=\frac{A_1\,B_2+B_1\,A_2}{B_1+B_2}$	$B_0=\frac{B_1\,B_2}{B_1+B_2}$	$C_0=C_1+C_2+\frac{(A_1-A_2)(D_2-D_1)}{B_1+B_2}$	$D_0=\frac{B_1\,D_2+D_1\,B_2}{B_1+B_2}$



General Equations

$E_s = A_0\,E_r + B_0\,I_r$

$I_s = C_0\,E_r + D_0\,I_r$

$E_r = D_0\,E_s - B_0\,I_s$

$I_r = -\,C_0\,E_s + A_0\,I_s$

Nomenclature

E_s = Supply Voltage to neutral

E_r = Receiver Voltage to neutral

I_s = Supply Current per line

I_r = Receiver Current per line

Z = Total Impedance

Y = Total Admittance

T = Transformer impedance

s and r subscripts refer to supply and receiver ends

For this condition equation (24) becomes:

$$(P_R + l\, E_R^2)^2 + (Q_R + m\, E_R^2)^2 = n^2\, E_R^2\, E_S^2 \qquad (28)$$

the equation of the Receiver Power Circle Diagram.

The circle diagram is plotted with real power along the X axis and with reactive power along the Y axis. A point in the first quadrant represents $P_R + j\, Q_R$, a positive load at lagging power factor. Some prefer to plot the power diagram with positive values of $P_R + j\, Q_R$ representing leading power factor.[6] The A. I. E. E. Standardization Rules do not cover the method of plotting power diagrams. However, if the voltage vector is chosen as reference then positive values of $P + j\, Q$ should represent lagging power factor. Also it would appear that the following expressions are mutually consistent when positive numerical values represent lagging power factor:

$R + j\, X$ for impedance

$I_1 - j\, I_2$ for current

$P + j\, Q$ for power

For the receiver power circle the center is located at $-\, l\, E_R^2,\ -\, m\, E_R^2$ and the radius is equal to $n\, E_R\, E_S$.

The scale for the circle diagram from Equation (28) is expressed in volt amperes which is usually changed to kilovolt-amperes for plotting.

The corresponding circle diagram for the load at the supply end and for definite supply and receiver voltages may be derived in a similar manner from equation (3). The equation for the supply power circle diagram is as follows:

$$(P_S - l'\, E_S^2)^2 + (Q_S - m'\, E_S^2)^2 = (n'\, E_S\, E_R)^2 \qquad (29)$$

which represents a circle with center at $l'\, E_S^2,\ m'\, E_S^2$ and with radius equal to $n'\, E_R\, E_S$.

Where

$$l' = \frac{D_o\, \overline{B}_o + \overline{D}_o\, B_o}{2\, B_o\, \overline{B}_o} \qquad (30)$$

$$m' = j\frac{D_o\overline{B}_o - \overline{D}_o B_o}{2\, B_o\, \overline{B}_o} \qquad (31)$$

$$n' = \frac{1}{\sqrt{B_o\, \overline{B}_o}} \qquad (32)$$

It should be noted that the constants given by equations (25-27) are the same as those given above by equations (30-32) except that the A_o and D_o constants are used respectively.

Formulas for Power Limits

The formulas for determining the power limits for different conditions can be derived from the equation of the Receiver Power Circle Diagram. The relation between the receiver apparent power, supply and receiver voltage is given by equation (28). If unlimited condenser capacity at the receiver is considered available then for maximum power the condenser capacity Q_R may be chosen as equal to $(-\, m\, E_R^2)$ and equation (28) may be reduced to the following form—

$$P_R = n\, E_R\, E_S - l\, E_R^2 \qquad (33)$$

This equation shows that the receiver power is unlimited if the supply voltage is not restricted in value. Hence the formulas for power limits are based on a fixed supply voltage. By differentiating equation (33) with respect to P_R and E_R and considering E_S as a constant the value of E_R for maximum power is obtained which is

$$E_R = \frac{n}{2\, l}\, E_S \qquad (34)$$

Hence the theoretical maximum power that may be delivered over a given transmission system is given by the following equation—

$$P_R = \frac{n^2}{4\, l}\, E_S^2 \qquad (35)$$

which for short lines reduces to the familiar form of

$$P_R = \frac{E_S^2}{4\, R} \qquad (36)$$

The formulas for the theoretical maximum power derived above assumes a fixed supply voltage and a receiver voltage unrestricted in value. In the practical case, however, the receiver voltage must be restricted to a value approximately equal to the supply voltage. If the receiver voltage is made equal to the supply voltage then the expression for maximum power under this condition may be derived from equation (33) with the following result:

$$P_R = (n - l)\, E_S^2 \qquad (37)$$

which for short lines becomes

$$P_R = \frac{\sqrt{R^2 + X^2} - R}{R^2 + X^2}\, E_S^2 \qquad (38)$$

The formulas given above have been derived on the assumption that condenser capacity was available to deliver the maximum power over the transmission line for the different voltage conditions. It is sometimes desirable to determine the maximum power that may be delivered over a circuit without condensers. The maximum power for this condition is obtained graphically by the intersection of a line corresponding to the power factor of the receiver load and the envelope of the receiver circles as shown in Fig. 3. The equation of the envelope is obtained by taking the partial derivative of equation (28) with E_R as the parameter and eliminating E_R between the new equation (39) and the original equation (28).

$$2\, P_R\, l + 2\, l^2\, E_R^2 + 2\, Q_R\, m + 2\, m^2\, E_R^2 = n^2\, E_S^2 \qquad (39)$$

The equation of the envelope of the receiver circles is as follows:

$$4\, m^2\, P_R^2 + 4\, l^2\, Q_R^2 - 8\, l\, m\, P_R\, Q_R + 4\, l\, n^2\, E_S^2\, P_R + 4\, m\, n^2\, E_S^2\, Q_R - n^4\, E_S^4 = 0 \qquad (40)$$

6. See Discussion by J. R. Dunbar and H. B. Dwight, A. I. E. E. Journal December 1922, page 1027.

The maximum load at any power factor with $Q_R = \theta P_R$ may be obtained by eliminating Q_R from equation (**40**) and solving for P_R which gives

$$P_R = \frac{\sqrt{(l^2 + m^2)(1 + \theta^2)} - (l + \theta m)}{2(m - \theta l)^2} n^2 E_S^2 \quad (\mathbf{41})$$

The maximum load at unity power factor that may be delivered over a given line with fixed supply voltage is obtained by letting $\theta = 0$ in equation (**41**) which gives

$$P_R = \frac{\sqrt{l^2 + m^2} - l}{2 m^2} n^2 E_S^2 \quad (\mathbf{42})$$

which for short lines reduces to

$$P_R = \frac{\sqrt{R^2 + X^2} - R}{2 X^2} E_S^2 \quad (\mathbf{43})$$

The envelope also determines the power limit for a regulated transmission system employing synchronous apparatus of negligibly small capacity at the receiver. The power limit at the envelope is the point common to the particular circle corresponding to the supply and receiver voltages and the envelope of the receiver circles. This point is obtained from the simultaneous equations (**28**) and (**40**). In solving these equations it was found convenient to use the center of the receiver circle as origin and to rotate the axes using the following substitution:

$$P_R = \frac{W l}{\sqrt{l^2 + m^2}} + \frac{Z m}{\sqrt{l^2 + m^2}} - l E_R^2 \quad (\mathbf{44})$$

$$Q_R = \frac{W m}{\sqrt{l^2 + m^2}} - \frac{Z l}{\sqrt{l^2 + m^2}} - m E_R^2 \quad (\mathbf{45})$$

The solution of the equation to determine the power limit at the point on the envelope and a particular circle in Fig. 3, gives the following value for

$$W = \frac{n^2 E_S^2}{2\sqrt{l^2 + m^2}} \quad (\mathbf{46})$$

The corresponding value of P_R is as follows:

$$P_R = - l E_R^2 + \frac{n^2 l E_S^2 + m\sqrt{4 n^2 E_S^2 E_R^2 (l^2 + m^2) - n^4 E_S^4}}{2(l^2 + m^2)} \quad (\mathbf{47})$$

Although the above expression is rather complicated, there is fortunately a simple graphical construction for obtaining the value of P_R. This construction is based on equation (**46**), and it is necessary only to measure from the center of the receiver circle a distance

$$O - W = \frac{n^2 E_S^2}{2\sqrt{l^2 + m^2}} \quad \text{and}$$

to erect a perpendicular $W C$. The intersection of the perpendicular with the particular voltage circle gives the power limit at the envelope as shown in Fig. 3.

In order to emphasize the significance of the several formulas for Power Limits it is convenient to summarize them as follows:—

Equation 35. Maximum power that can be transmitted with unlimited receiver voltage and condenser capacity. (Regulated system).

Equation 37. Maximum power that can be transmitted with unlimited condenser capacity but with the receiver voltage equal to the supply voltage. (Regulated System)

Equation 41. Maximum power that can be transmitted at any given power factor with the resulting receiver voltage condition at the envelope (Unregulated System).

Equation 47. Maximum power that can be transmitted at the envelope with any given supply and receiver voltage condition.

Synchronous condensers may be required to maintain the receiver voltage assumed.

Effect of Variation in Circuit Constants on Power Limits

The formulas for the Power Limits have been derived and the effect of changes in circuit constants on these limits will next be considered. Equation (**35**) shows that with unrestricted receiver voltage that (1) for long transmission lines the values of l should be made small and the value of n should be made large, (2) for short lines the resistance should be made as small as possible in order to transmit the maximum amount of power.

For the condition with equal supply and receiver voltage but with unrestricted condenser capacity, the equation (**37**) shows that n should be made large and l should be made small. For this same condition the solution of equation (**37**) with (**1**) the resistance R assumed fixed, the value of X should equal $\sqrt{3}\,R$ and (**2**) with the reactance X assumed fixed, the value of R should equal $(= X/\sqrt{3})$ and for the practical case R should have as small a positive value as possible.

The effect of variation in the value of the circuit constants on the envelope will next be considered. For the condition with unrestricted receiver voltage and condenser capacity it may be shown that the maximum value of the envelope coincides with the power limit and the formulas for this condition are the same as derived for power limit, namely, equation (**35**) and (**36**). Hence for this condition the value of n should be made large and the value of l should be made small, which for short lines reduces to the requirement that the resistance should be made small.

For the condition with equal supply and receiver voltage the maximum load from the standpoint of the envelope is given by equation (**47**) with $E_R = E_S$. For short lines this condition reduces to the following expression

$$P_R = \frac{\sqrt{3}\,X - R}{2(R^2 + X^2)} E_S^2 \quad (\mathbf{48})$$

For the practical cases of positive values of X and R the value of P_R is a maximum with R as small as possible and with $X = \sqrt{3}\,R$.

Nomenclature

The following nomenclature has been used in this article:

E_S = Supply line voltage
E_s = Supply voltage to neutral
E_R = Receiver line voltage
E_r = Receiver voltage to neutral
I_s = Supply current per line
I_r = Receiver current per line
P_R = Three-phase power in watts
P_r = Phase to neutral power in watts
Q_R = Three-phase reactive volt-amperes
Q_r = Phase to neutral reactive volt-amperes
R = Circuit resistance per phase
X = Circuit reactance per phase
Z = Circuit impedance per phase
Y = Circuit admittance per phase
A_o, B_o, C_o, D_o = General circuit constants for fundamental equations. See Table 6 for different networks
l, m, n = Receiver circle diagram circuit constants
l', m', n' = Supply circle diagram circuit constants
A_R, B_R, C_R = Receiver circle diagram constants
A_S, B_S, C_S = Supply circle diagram constants

Discussion

For discussion of this paper see page 71.

Experimental Analysis of Stability and Power Limitations

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Review of the Subject.—*A method of determining the power and voltage stability limit of a transmission system taking into account the characteristics of the synchronous condenser and load in conjunction with the characteristics of the line is described.*

The power limit of a straight 500-mile transmission line is calculated by this method. The power limit is also calculated for the same line with a synchronous condenser at the mid-point which divides the line into two sections. The addition of the synchronous condenser at the mid-point increased the power limit 42 per cent.

Tests were made on a 625 kv-a. transmission system operated at 2300 volts to determine experimentally the power and voltage stability limit of a transmission line with a synchronous condenser at the mid-point. The tests check closely with the calculated values.

Tests on hunting caused by prime mover pulsations, line characteristics and voltage regulator adjustment were made.

It was found that the damper windings on synchronous condensers do not have much effect in reducing hunting, particularly at resonant points. There was practically no difference between the standard high-resistance windings used for starting purposes and low resistance copper damper windings.

On a high-resistance and low-reactance transmission line hunting was found to occur at high condenser field excitations. The addition of reactance stabilized the system. On high-voltage transmission lines the reactance is high as compared to resistance and no difficulty from hunting due to line characteristics is to be expected.

A loose adjustment of the voltage regulator dash pot was found to set up hunting.

Short circuits were applied through reactance to a system carrying load to determine the severity and duration of a short circuit required to cause the system to pull out of synchronism.

The time element between the application of the short circuit and pull out was found to be due to the time required to reduce the voltage to a point where the power limit of the system was exceeded at the reduced voltage.

INTRODUCTION

DUE to the rapid growth in the use of electrical energy economic conditions have made it necessary to consider the transmission of large blocks of power over great distances. Theoretical investigations of these transmission problems have indicated that there is a definite limit to the amount of power which can be transmitted over a given line.

On high-voltage transmission lines this limit is determined principally by the reactance, which is large as compared to the resistance, on account of the wide conductor spacing. Previous investigations have mainly considered the limitations of the transmission line alone. In an actual transmission system the nature of the load and the size and characteristics of the synchronous condensers must be taken into account in determining the maximum amount of power which can be transmitted without hunting, loss of synchronism, or voltage instability taking place.

The maximum amount of power which can be transmitted over a given line may be increased by "loading the line" with synchronous condensers at intermediate points. The use of synchronous condensers in this manner was given consideration for the transmission lines from the St. Lawrence and Niagara developments of the Superpower Survey. This subject has been presented before the Institute by F. G. Baum in a paper published in the August 1921 A. I. E. E. JOURNAL.

The present paper covers an investigation, supplemented by actual tests, of the factors affecting the stability of transmission systems. The tests were made on two artificial "*T*" transmission lines which were built up of resistors, reactors, and static condensers, and operated normally at 2380 volts. Power was obtained from a 625-kv-a. generator and two 425-kv-a. synchronous condensers were available, thus permitting the "loaded line" operation to be tested. Standard voltage regulators of the vibrating type were used with the generator and each of the synchronous condensers. The tests were carried out on a large scale so that results comparable with those in actual operation could be obtained.

DETERMINATION OF THE POWER LIMITATIONS OF A TRANSMISSION SYSTEM

Use of circle diagrams. Power Limit of Transmission Line Alone.

An actual transmission line with its step-up and step-down transformers is quite a complicated network, since it involves series impedances and exciting admittances of the transformers in addition to the distributed resistance, inductance, and capacity of the line itself. The characteristics of this complicated network can conveniently be represented by means of circle diagrams. An exact method of combining the constants of the line with those of the transformers to obtain the general circuit constants of the entire network is described in the companion paper entitled "Power Limitations of Transmission Systems." From the general circuit constants thus derived the mathematically exact circle diagrams can be obtained. The method of calculating circle diagrams and their characteristics has been covered in other papers[1] and there-

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

1. R. A. Philip, Economic Limitations to Aggregation of Power Systems, 1911, A. I. E. E.
H. B. Dwight's book on "Constant Voltage Transmission"
R. D. Evans & H. K. Sels, *Elec. Journal* for July, August, Dec. 1921 and February 1922.

fore only a brief review of their characteristics will be given.

Consider the conditions at the receiver end of the transmission system shown in Fig. 1, with the generator voltage regulator set to maintain a constant voltage of 100 per cent at the supply end of the line. If the voltage regulator for the synchronous condenser is set to maintain the same voltage (100 per cent) at the receiver end of the line independent of the load, then as the kilowatts drawn over the line are gradually increased

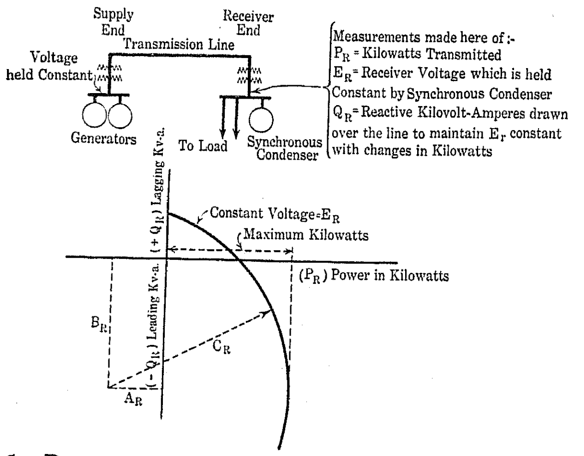

FIG. 1—RECEIVER VOLTAGE CIRCLE FOR THE LINE SHOWN ABOVE

the reactive kilovolt-amperes drawn over the line must be varied to maintain the receiver voltage constant. The relation between kilowatts and reactive kilovolt-amperes for a constant receiver voltage is a circle whose center has the coordinates A_R and B_R and whose radius is C_R.

The maximum amount of power which can be delivered over the line with 100 per cent voltage maintained at both ends of the line is indicated on the circle diagram. It should be noted that the line is unable to deliver more than the maximum kilowatts shown even though normal voltage is maintained at both the supply and receiver ends of the line. Power is transmitted over the line due to a difference in phase between the supply and receiver voltages. The power limit of the line itself as determined from the receiver circle diagram corresponds to the point at which a further increase in the phase displacement between the ends of the line results in a decrease in the amount of power which can be delivered.

With the supply voltage still maintained at 100 per cent and the synchronous condenser voltage regulator set to maintain 90 per cent voltage at the receiver end of the line the relation between kilowatts and reactive kilovolt-amperes at the receiver is still a circle, the coordinates of its center being $\left(\frac{90}{100}\right)^2 = 81$ per cent of the A_R and B_R for the 100 per cent receiver circle. The radius is reduced to 90 per cent of the C_R for the 100 per cent circle. In this manner a family of circles showing the relation between kilowatts and reactive kilovolt-amperes at the receiver for different receiver voltages with a constant supply voltage can be drawn. Fig. 16 shows the 90—100—110 per cent receiver circles for one of the line conditions tested and Fig. 2 shows a large number of receiver circles for a typical 250-mile transmission line. The relation between kilowatts and reactive kilovolt-amperes at the supply end of the line is also a circle. The supply circle for 100 per cent receiver voltage is also shown on Fig. 2.

The maximum amount of power at various constant receiver voltages, which the line (with transformers) *considered by itself* can deliver, may be determined from the receiver circle diagrams. The power limit of the entire system, taking into account the characteristics of the apparatus connected to the line will, in general, be less than the power limit of the line alone.

Power Limit of Entire System. Stability of Receiver Voltage. The necessity for considering the characteristics of the synchronous condensers and load as well as the characteristics of the line in determining the maximum amount of power which a system can deliver has already

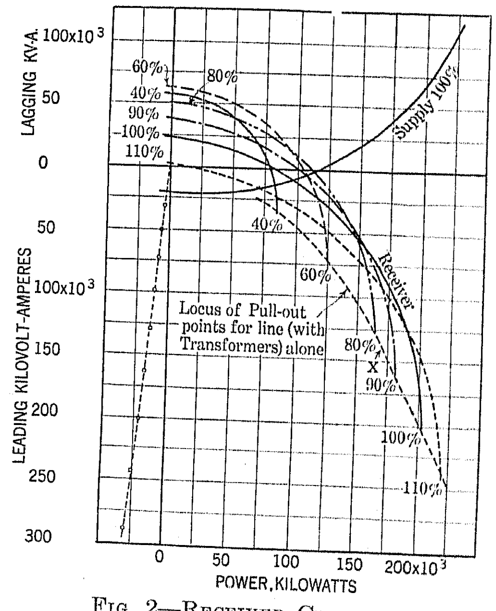

FIG. 2—RECEIVER CIRCLES

250-mile transmission line with stepdown transformers, 750,000 cir. mil copper, 19 ft. equivalent spacing, 150,000 kv-a. transformer bank. Supply voltage constant at 220 kv.

Receiver voltages expressed in per cent of 220 kv.

been pointed out. The power limitations and stability of the receiver voltage are so closely related that it is best to consider them at the same time. When a constant amount of power is being delivered at the receiver, the usual conception is that an increase in lagging kilovolt-amperes drawn over the line will decrease the receiver voltage, or conversely, an increase in leading kilovolt-amperes will increase the receiver voltage. On most transmission lines this is not true beyond a certain load.

Fig. 2 shows the receiver circle diagram for a 250 mile transmission line with step-down transformers included as a part of the line. The voltage has been assumed to be held constant at 220 kv. at the high-tension side of the step-up transformers. The various circles show the relations between kilowatts and reactive kilovolt-amperes for different voltages at the receiver end expressed as a percentage of 220 kv.

It is apparent that, with a constant kilowatt load being transmitted, the receiver voltage (with constant supply voltage) depends on the reactive kilovolt-amperes drawn over the line. For a particular load this relation can be determined from the receiver circles in Fig. 2, for example, at 175,000 kw. the 90 per cent circle requires 105,000 kv-a. leading, the 100 per cent circle requires 90;000 kv-a. leading and the 110 per cent circle requires 100,000 kv-a. leading. The power limit of the line alone when transmitting 175,000 kw. occurs at point x when the receiver voltage has been reduced to 86 per cent. The reactive load then is 145,000 kv-a. leading. The curves drawn in full on Fig. 3 were derived from Fig. 2 in the foregoing manner and show the relation between receiver voltage and reactive kilovolt-amperes at a number of constant loads.

Referring to Fig. 3, when a 175,000 kw. are being delivered at 100 per cent voltage 90,000 leading kilo-volt-ampere must be drawn over the line to maintain this voltage. It appears that at 90,000 kilovolt-amperes leading a considerable fluctuation in receiver voltage could take place, namely, from 98 per cent to 102 per cent. Below 100 per cent voltage an *increase in leading kilovolt-amperes* produces a *decrease* in receiver voltage, which is exactly the reverse of the usual action. *If* an increase in excitation of the synchronous condensers were always accompanied by an increase in the leading kilovolt-ampere drawn over the transmission line, the operation of the voltage regulator set to maintain 100 per cent voltage, would become unstable at 175,000 kw., because with a drop in voltage the regulator would increase the condenser excitation, causing more leading kilovolt-amperes to be delivered and still further decreasing the line voltage. However, voltage instability in general does not occur at this point because, as will be shown later, an *increase* in excitation of a synchronous condenser is not always accompanied by an increase in the leading kilovolt-amperes delivered by the condenser, or if operating in the lagging kilovolt-ampere range of the condenser an increase in excitation does not always reduce lagging kilovolt-amperes drawn over the line as will be shown later. A 500-mile line would be inoperative if this were true.

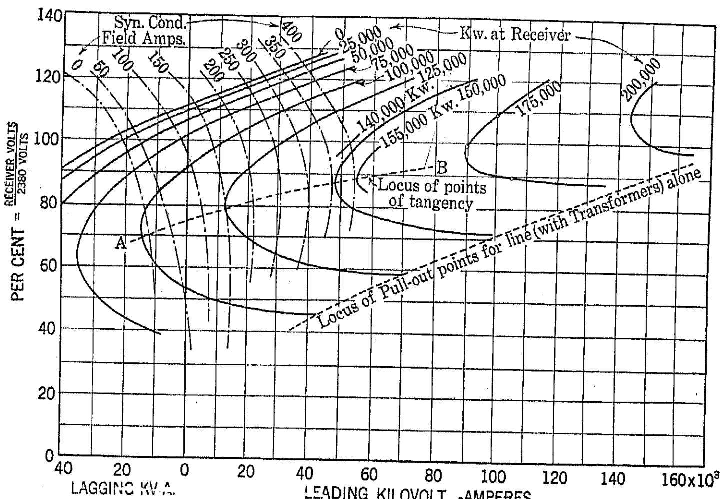

Fig. 3—Transmission Line and Synchronous Condenser Characteristics

250-mile line with stepdown transformers and two 25,000 kv-a. lead and 10,500 kv-a. lag. Synchronous condensers.

——— Curves show variation in receiver voltage with reactive kv-a. at the receiver for constant kw. loads. Curves obtained from Fig. 2.

-·-·-·- Curves show variation in synchronous condenser voltage with reactive kv-a. For constant field currents. Condenser voltage and receiver voltages of the line are identical.

Curve *A B* is the locus of points of tangency between line and synchronous condenser curves. This is the actual pull-out point locus for the system. It is impossible to operate below *A B*.

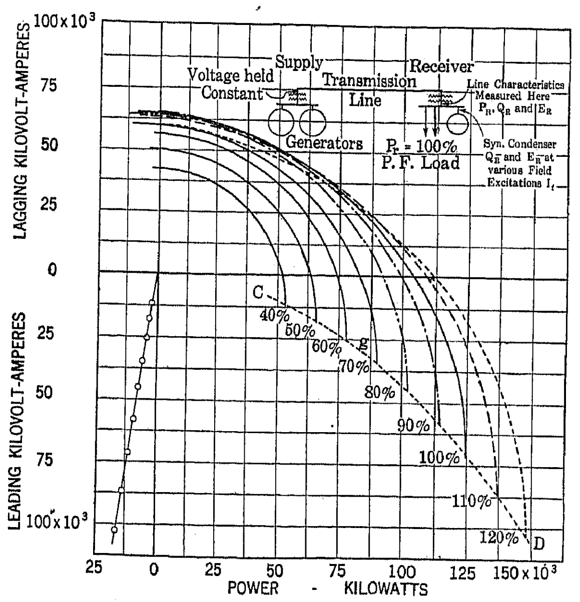

Fig. 4—Receiver Circles

500-mile transmission line. with stepdown transformers; 750,000 cir. mil copper, 19 ft. equivalent spacing; 90,000 kv-a. transformer bank; supply voltage constant at 220 kv.; receiver voltages expressed in per cent of 220 kv.

From the circle diagrams for the receiver end of a 500-mile transmission line shown by Fig. 4 it may be noted that a load of 100,000 kw. an *increase* in the lagging in the kilovolt-amperes, or what amounts to the same thing a *decrease* in the leading kilovolt-amperes, will actually raise the receiver voltage. The curves shown in full lines in Fig. 5 were obtained from the receiver circles in Fig. 4 by plotting the relation between receiver voltage and reactive kilovolt-amperes at a number of constant kilowatt loads in the same manner as previously described in detail for the 250-mile line. When delivering 50,000 kw. at 100 per cent voltage 50,000 lagging kilovolt-amperes are required. At this

point with the lagging kilovolt-amperes *remaining very nearly constant* at 50,000 kilovolt-amperes a variation of the receiver voltage from 94 per cent to 102 per cent could take place. If voltage instability occurred at this point as might be expected from a cursory consideration of the synchronous condenser characteristics, only 50,000 kw. could be delivered over the line. Actually there will be no difficulty in operating at this point because with other factors such as the excitation of the synchronous condensers remaining constant the fluctuation in voltage is always accompanied by a variation in reactive kilovolt-amperes and this variation tends to prevent the change in voltage.

To determine the point at which voltage instability will occur it is necessary to devise some means of plotting the variation in receiver voltage with a change in synchronous condenser excitation at different kilowatt loads. The fundamental condition for successful voltage regulator operation is that an increase in the

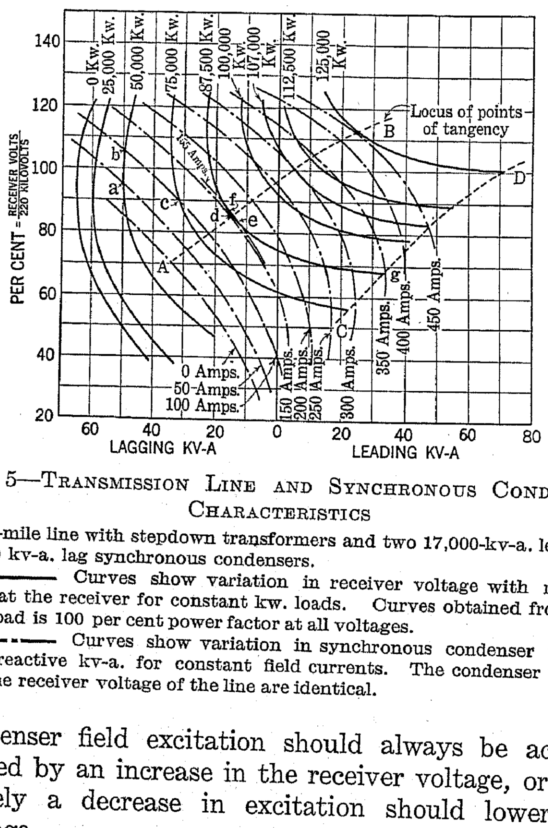

FIG. 5—TRANSMISSION LINE AND SYNCHRONOUS CONDENSER CHARACTERISTICS

500-mile line with stepdown transformers and two 17,000-kv-a. lead and 30,000 kv-a. lag synchronous condensers.

——— Curves show variation in receiver voltage with reactive kv-a. at the receiver for constant kw. loads. Curves obtained from Fig. 4. Load is 100 per cent power factor at all voltages.

–·–·– Curves show variation in synchronous condenser voltage with reactive kv-a. for constant field currents. The condenser voltage and the receiver voltage of the line are identical.

condenser field excitation should always be accompanied by an increase in the receiver voltage, or conversely a decrease in excitation should lower the voltage.

In investigating the relation between receiver voltage and condenser excitation a load, whose kilowatts remain constant and at 100 per cent power factor independent of the voltage, will be used for convenience. Synchronous motor load simulates such a load in that the kilowatts are independent of the voltage since the load remains constant as long as the frequency (speed) does not change. The power factor will of course vary with the voltage provided the field excitation is constant. The hypothetical 100 per cent power factor load is only used for convenience and modifications to take care of practical cases will be introduced after the theory is developed.

Since the load power factor is always constant at 100 per cent, all of the reactive kilovolt-amperes required to maintain the line voltage comes from the synchronous condenser. Referring to the sketch on Fig. 4, when constant voltage is maintained at the supply end of the line the conditions at the receiver end of the line are completely determined provided the following quantities are known:

P_R—The kw. transmitted over the line.

E_R—The receiver voltage.

Q_R—The reactive kilovolt-amperes drawn over the line to maintain the voltage.

Likewise the synchronous condenser point of operation is determined by the following variables:

I_F—The field current.

E_R—The terminal voltage.

Q_R—The reactive kilovolt-amperes supplied by the condenser.

The reactive kilovolt-amperes drawn over the line and the reactive kilovolt-amperes delivered by the condenser are equal since the load has been assumed to remain at 100 per cent power factor. The terminal voltage of the condenser is also identical with the receiver voltage of the line. This suggests plotting for the line a family of curves showing the variation in receiver voltage with reactive kilovolt-amperes at a number of kilowatt loads and superposing on them another family of curves which show the terminal voltage of the synchronous condenser plotted against the reactive kilovolt-amperes delivered by the condenser at a number of different field currents. Such a diagram could then be used to determine the change in receiver voltage and reactive kilovolt-amperes drawn over the line when the field current of the condenser is varied while the kilowatts transmitted remain constant. Similarly, a study of variations in the receiver voltage and reactive kilovolt-amperes for a constant value of field current as the power drawn over the line is changed will be of value.

The curves in Fig. 5 drawn in full lines show the relation between the receiver voltage and reactive kilovolt-amperes at the end of the 500-mile transmission line in Fig. 4. The chain dotted curves show the relation between synchronous condenser voltage, which is the same as the line receiver voltage, and the reactive kilovolt-amperes delivered by the condenser at a number of different field currents. These curves are for two synchronous condensers operating in parallel, each having a rating at 100 per cent voltage of approximately 30,000 kv-a. lagging at the minimum excitation and 17,000 kv-a. leading at the maximum excitation. The method of obtaining these curves from the no-load and zero per cent power factor saturation curves of a synchronous condenser will be described subsequently.

When supplying a constant load of 50,000 kw. the receiver voltage will be 95 per cent when the syn-

chronous condenser excitation is 50 amperes. This corresponds to point *a* on Fig. 5. With the load remaining constant at 50,000 kw. and the excitation of the synchronous condenser increased to 100 amperes, the receiver voltage will rise to point *b* or 106 per cent. Notice during this change in voltage the lagging kilovolt-amperes remained practically constant at 50,000 kv-a. Voltage instability does not occur even though a large change in voltage can occur with the lagging kilovolt-amperes constant, because a *considerable change in excitation* of the synchronous condenser is required to produce the change in voltage.

To investigate the variation in voltage with variation of load in kilowatts when the excitation of the synchronous condenser is constant, consider the operation at point "*b*," which corresponds to 50,000 kw. at 106 per cent voltage with 100 amperes of synchronous condenser excitation. Keeping the synchronous condenser excitation constant at 100 amperes and gradually increasing the load from 50,000 to 75,000 kw. point *c* is reached by following the 100 ampere field current curve as the load is increased. During this change in load the receiver voltage dropped from 106 per cent to 90 per cent.

Starting at point *c* and keeping the load constant at 75,000 kw. the voltage will rise from 90 per cent to 104 per cent if the excitation is increased from 100 amperes to 150 amperes. It should be noted particularly that the lagging kilovolt-amperes has been increased from 31,500 kv-a. to 34,400 kv-a., that is, *an increase in the lagging kilvolt-amperes drawn over the line actually increased the receiver voltage.* By keeping the load constant and increasing the excitation to 250 amperes the voltage will rise to 122 per cent. In Table *A* the variation in receiver voltage and reactive kilovolt-amperes drawn over the line as the synchronous condenser excitation is changed from 100 to 250 amperes with a constant load of 75,000 kw. is tabulated from the points taken on the 75,000-kw. curve.

TABLE A

Kw. Load	Syn. Cond. Field Amps.	*Receiver Voltage	Lagging Kv-a.	Lagging Current
75,000	100	90%	31,500	91.6 amps.
75,000	150	104%	34,400	86.5 "
75,000	200	114.5%	34,400	78.6 "
75,000	250	122%	33,300	71.6 "

*100 per cent voltage = 220 kv.

The lagging component of current drawn over the line, which was calculated from the receiver voltage and lagging kilovolt-amperes at the different points, is also tabulated. By increasing the receiver voltage from 90 per cent to 104 per cent, the lagging kilovolt-amperes drawn over the line are increased from 31,500 to 34,000 kv-a. while the lagging current drawn over the line decreased from 91.6 amperes to 86.5 amperes. The decrease in lagging current drawn over the line as the synchronous condenser excitation is increased accounts for the rise in line voltage even though the lagging kilovolt-amperes goes up. The lagging kilovolt-amperes increase because the line voltage is going up faster than the lagging current is going down.

Assume that the load is constant at 87,500 kw. and the condenser field excitation is 200 amperes, thus giving a receiver voltage of 105.5 per cent. If the power remains constant and the field excitation of the condenser is gradually decreased to 155 amperes point *d* will be reached and the voltage will be 85.5 per cent. At this point the line and synchronous condenser characteristic curves are tangent, and the voltage will be unstable because, with a constant condenser excitation of 155 amperes and a practically constant load of 87,500 kw. the voltage and the reactive kilovolt-amperes would fluctuate in the region from *e* to *f*.

If it were possible to get beyond the point of tangency between the two families of curves an increase in synchronous condenser excitation would actually decrease the receiver voltage, for example, an increase in the excitation from 155 to 200 amperes would reduce the voltage from 84.5 per cent to 73.5 per cent, with the load remaining constant at 87,500 kw. A still further increase in the excitation to 350 amperes would reduce the receiver voltage to 68 per cent as is indicated at point *g*, which corresponds to the pull-out point of the line alone for 68 per cent voltage. By referring to Fig. 4 it can be seen that point *g* corresponds to the maximum power point of the 68 per cent voltage circle.

If a voltage regulator set to maintain 85.5 per cent receiver voltage at all loads were in use, the regulator would operate satisfactorily at all loads up to 87,500 kw. At loads higher than this with the voltage maintained at 85.5 per cent, an increase in condenser excitation would decrease the voltage, which in turn would cause the regulator to still further increase the excitation and the action would be cumulative until the voltage dropped sufficiently for pull-out to occur.

It is even impossible in practise to get beyond point *d* with hand operation of the synchronous condenser field because, in order to get below *d* (the load remaining constant at 87,500 kw.), the synchronous condenser excitation must be decreased to 155 amperes and then increased. If the rheostat steps were such that the field current would be reduced to slightly less than 155 amperes, for example to 150 amperes, pull-out of the synchronous condenser and load would take place because the synchronous condenser with 150 amperes excitation can not deliver sufficient reactive kilovolt-amperes required for the transmission of 87,500 kw. at any voltage.

This discussion has dealt with the pull-out point of the entire system for a receiver voltage of 85.5 per cent. At a higher receiver voltage more power could be transmitted without this point being reached. The line *A B* (Fig. 5) is the locus of points of tangency between the family of line characteristic curves and

synchronous condenser characteristic curves, and is therefore the locus of pull-out points of the entire system for different receiver voltages.

When operating *with 100 per cent receiver voltage* 107,000 kw. is the maximum power that can be transmitted because the 107,000-kw. curve becomes tangent to the 300-ampere excitation curve of the synchronous condenser. In actual operation pull-out could be expected to occur slightly before this point of tangency. For example, when operating at 100,000 kw., and 100 per cent voltage, if the load suddenly increased to 107,000 kw., the voltage regulator would not have time to make a corresponding increase in the excitation. This gives the effect of an increase in load at a constant field excitation so that the voltage would drop down enough to reach the point of tangency and cause pull-out to take place before the regulator had time to get into action.

The locus of the pull-out points *of the line alone* is along the line $C\,D$, which corresponds to the pull-out point determined from the various receiver voltage circles shown in Fig. 4.

The voltage regulator is not directly affected by the reactive kilovolt-amperes. It merely operates to increase the condenser excitation when the receiver voltage should be increased, or to decrease the excitation when the receiver voltage should be decreased. Fig. 5 shows the change into reactive kilovolt-ampere during the regulating operation, and Fig. 6, which is derived therefrom by eliminating the reactive kilovolt-amperes, shows the variation in receiver voltage with

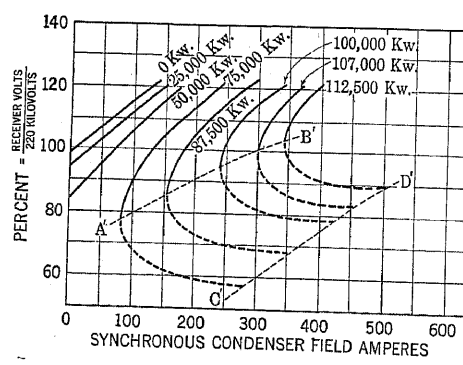

FIG. 6

These curves were obtained from Fig. 5. Curve $A'\,B'$ is the locus of points at which voltage instability and pull out will occur. It corresponds to $A-B$ on Fig. 5. Curve $C'\,D'$ is the locus of the maximum power points of the line alone as determined from the various receiver voltage circles. Corresponds to $C-D$ on Figs. 4 and 5.

condenser excitation at a number of constant kilowatt loads.

The line A'—B' in Fig. 6 corresponds to the line $A\,B$ on Fig. 5 where voltage instability and pull-out of the system occur. The dotted portions of the curve cover the range between the lines $A\,B$ and $C\,D$ in Fig. 5 where an increase in excitation would decrease the receiver voltage, provided it were possible to operate in this range. The line C'—D' in Fig. 6 is the locus of the maximum power points of the transmission line (with transformers, considered *alone* and is identical with the locus of maximum power points determined from the receiver diagrams and shown as the line $C\,D$ in Fig. 4.

The transmission line and synchronous condenser characteristic curves for a 250-mile line controlled by two synchronous condensers having a total capacity of 50,000 kv-a. leading and 21,000 kv-a. lagging at 100 per cent voltage are shown by Fig. 3. These curves are based on a hypothetical load which remains constant at 100 per cent power factor independent of the

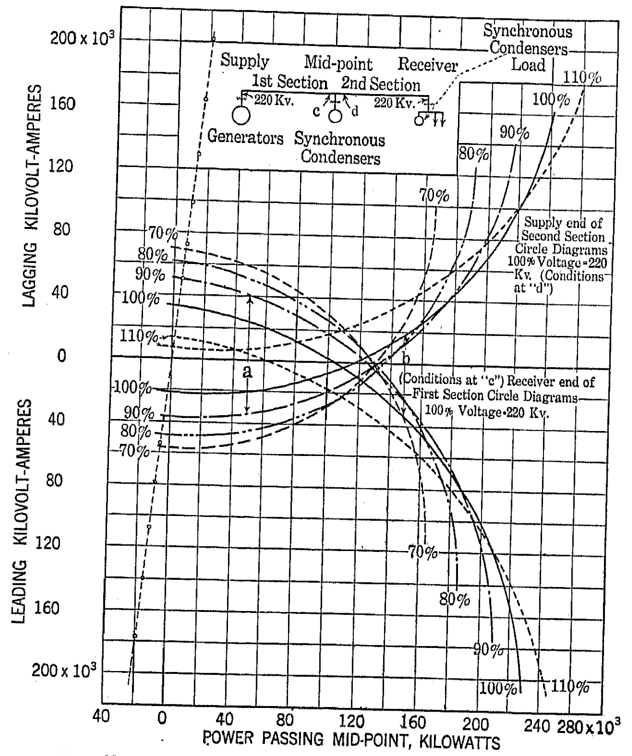

FIG. 7—MID-POINT OF 500-MILE LINE

220 kv. maintained at supply end of first section and receiver end of second section.

voltage. The locus of the points of tangency between the two families of curves is shown by the dotted line $A\,B$ and was obtained in the same manner as for the 500-mile transmission line previously investigated.

With the voltage maintained constant at 100 per cent the maximum load that can be drawn over the line is 140,000 kw. because the maximum synchronous condenser excitation is reached at this load. Pull-out will not occur at this point, but additional load can only be drawn over the line by leaving the condenser excitation constant at 400 amperes and allowing the voltage to drop as the load is increased. When the load reaches 155,000 kw. the voltage will be reduced to 89 per cent which corresponds to the point of tangency between the two families of curves and the system will pull out of synchronism.

The synchronous condenser characteristics happened to be such that the point of tangency occurs almost exactly where an increase in leading kilovolt-amperes drawn over the line would decrease the line voltage.

This is merely a coincidence and the point where the line curves turn back on themselves can not in general be considered as a point where instability will occur.

Power Limit of Loaded Line. By "loading" the 500-mile transmission line covered by Fig. 4 with synchronous condensers at the mid-point, the maximum amount of power which can be delivered to the receiver end will be increased. The sketch shown on Fig. 7 gives the schematic diagram of the system. By installing synchronous condensers at the mid-point, the 500-mile line is divided into 250-mile sections. The voltage is assumed to be maintained constant on the high-tension side of the step-up transformer at the supply end, thus making the two sections slightly different because transformers are not included with the first section while the step-down transformers are included with the second section since the voltage is held constant on the low-voltage side.

As the load at the receiver end is gradually increased some means must be devised for determining the amount of reactive kilovolt-amperes which the condenser at the middle of the line must supply to maintain any desired value of voltage at the mid-point. This can be done by considering the receiver end of the first section and the supply end of the second section independently; the only tie between them being that their voltages are always equal and if the losses in the synchronous condensers are neglected the power delivered at the receiver end of the first section is the same as the power supplied to the second section. The reactive kilovolt-amperes will be different and the synchronous condenser must supply the difference in the reactive kilovolt-amperes required for each section.

In determining the reactive kilovolt-amperes which must be supplied by the synchronous condenser at the mid-point under different conditions of voltage and load, the supply and receiver voltages at each end of the 500-mile line will be considered as being maintained constant at 220 kv. as is indicated on the sketch in Fig. 7. The losses in the synchronous condenser will be neglected.

Assume that it is desired to maintain 100 per cent voltage at the mid-point under all loads. The reactive kilovolt-amperes which the synchronous condenser must supply at any kilowatt load may be obtained in the following manner. The 100 per cent receiver circle for the first section (*i. e.* at *c* on sketch in Fig. 7) shows the reactive kilovolt-amperes which must be drawn over this section to maintain 100 per cent voltage as the power *passing the mid-point* is increased. The 100 per cent supply voltage circle (*i. e.* at *d*) for the second section shows the variation in reactive kilovolt-amperes taken by the second section with a change in power passing the mid-point.

At any particular kilowatt load passing the middle of the line the synchronous condenser must furnish the difference in reactive kilovolt-amperes between the 100 per cent receiver and supply circles in order to maintain 100 per cent voltage. As an example it is shown at *b* on Fig. 7 that the condenser must deliver 53,000 kv-a. leading to maintain 100 per cent voltage at 150,000 kw.

To obtain a perspective of the entire situation at the mid-point, consideration must be given to the conditions where mid-point voltages are above and below 100 per cent, with the voltage at both ends of the 500-mile line still constant at 220 kv. (100 per cent).

As an example the conditions at 90 per cent voltage at the mid-point can be determined in the same manner as for 100 per cent voltage by drawing the 90 per cent receiver and supply circles for the first and second sections respectively. As is shown at *a* on Fig. 7 the synchronous condenser must supply 75,000 kv-a. lagging to maintain 90 per cent voltage when 50,000 kw. are being transmitted past the middle of the line.

By drawing a considerable number of circles and obtaining a large number of points in the manner just described, the curves drawn in full on Fig. 8 were

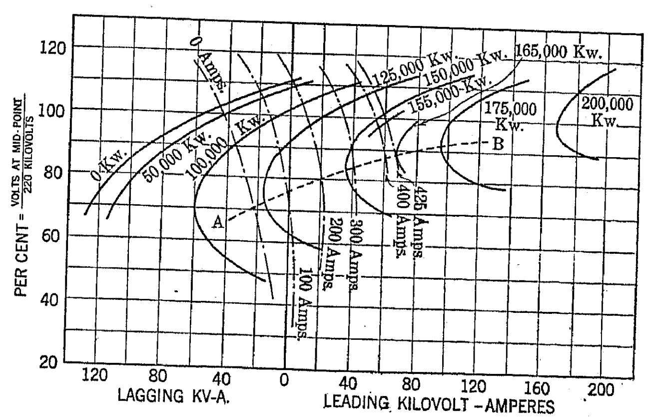

FIG. 8—MID-POINT OF 500-MILE LINE

Constant voltage = 220 kv. maintained at each end of the line.

——— Curves show relation between volts at mid-point and reactive kv-a. which must be supplied by the mid-point synchronous condenser for different amounts of power passing the center of the line. Obtained from Fig. 7.

–·–·– Curves show relation between synchronous condenser voltage and reactive kv-a. delivered (at high voltage side of transformer bank) for different values of field current.

Curve *A B* is locus of points of tangency between line and synchronous condenser curves. It is impossible to operate below *A B*.

obtained. These curves show the variation in voltage at the mid-point for different amounts of reactive kilovolt-amperes delivered by the synchronous condenser at a number of different kilowatt loads passing the middle of the line. The kilowatts actually delivered at the receiver end of the 500-mile line can be obtained by subtracting the losses in the second section from the kilowatts passing the middle of the line.

The chain dotted curves show the characteristics of the synchronous condensers used at the mid-point. These curves are for two synchronous condensers each of 30,000 kv-a. leading and 20,000 kv-a. lagging capacity operating in parallel.

The line *A B* (Fig. 8) is drawn through the points of tangency between the two families of curves and is the locus of points at which voltage instability and

pull-out of the synchronous condenser at the mid-point will occur.

With 100 per cent voltage maintained at the mid-point the limit of the synchronous condenser excitation is reached at 155,000 kw. corresponding to 425 amperes of excitation. The pull-out will not occur at this point and load may be increased to 165,000 kw. by keeping the field current constant at 425 amperes and allowing the voltage at the middle of the line to drop. At 165,000 kw. the voltage at the mid-point will have dropped to 89 per cent at which the line and condenser curves become tangent and voltage instability and pull-out will occur. The load at the receiver end of the 500-mile line at the time of pull-out is 152,000 kw. as obtained by subtracting the losses in the second section from the load of 165,000 kw. at the middle of the line.

Loading the 500-mile line with 60,000 kv-a. in synchronous condensers at the mid-point, therefore increases the power limits of the system from 107,000 kw. to 152,000 kw. an increase of 42 per cent.

From the foregoing it may be noticed that the power limit of the loaded line is to a large extent determined by the characteristics of the synchronous condensers at the mid-point. If a sufficiently large synchronous condenser were used, the power limit of the system would closely approach that of the first section.

In this investigation the voltages at both ends of the line were assumed to be constant. Actually the voltage regulators at the generator and receiver synchronous condenser cannot change the voltage instantaneously and some variation in voltage will occur. This does not appreciably effect the pull-out point as can be seen from the tests.

Method of Investigating Different Types of Load. The preceding discussion was based on a kilowatt load which remained constant and at 100 per cent power factor independent of the voltage. Synchronous motors simulate such a load in that a change in frequency (speed) is required to alter the kilowatt load. A variation in voltage merely changes the reactive kilovolt-amperes taken by the synchronous motors. Within certain limits, the same may be said of induction motor load.

In investigating the pull-out point of a system whose entire load consists of synchronous motors with a synchronous condenser for regulating the voltage, no change need be made in the line characteristic curves previously used for showing the relation between receiver volts and reactive kilovolt-amperes at constant kilowatt loads. Instead of superposing on these curves only the synchronous condenser characteristics, a new set of curves showing the variation in reactive kilovolt-amperes of the entire load (synchronous condenser and motors) with voltage at different synchronous condenser field excitations should be used.

A reactor or static condenser connected in parallel with a synchronous condenser can be handled in the same manner by plotting the variation in reactive kilovolt-amperes of the combination with voltage at a number of condenser field currents.

Reactors, static condensers, or resistors at the load end of a transmission line can also be combined to form a part of the transmission line by use of the general circuit constants. New circle diagrams can then be calculated for the transmission line with a part of its load, considered as a unit, and the characteristics considered in conjunction with those of the synchronous condenser. This method has the disadvantage that a new set of circle diagrams would be required for every change in load.

Another method of handling a purely resistance load with the receiver voltage controlled by a synchronous condenser can best be illustrated by considering a resistance load on the 250-mile transmission line shown by Fig. 2. With an amount of resistance which would draw 100,000 kw. at 100 per cent voltage, the synchronous condenser would have to supply 10,000 kv-a. leading as determined from the 100 per cent circle. With the same resistance remaining in use, if the voltage were reduced to 90 per cent, the load taken by the resistor will drop to 81,000 kw., and at this load the synchronous condenser must supply change to 12,500 kv-a., lagging to maintain 90 per cent voltage. By continuing this process for other voltages a curve giving the variation in line voltage with reactive kilovolt-amperes for this *particular value of resistance* can be obtained. A family of such curves for different amounts of resistance at the load may be calculated. These curves can be used for investigating the stability of the system by superposing on them the characteristic curves of the synchronous condenser in the same manner as for the constant power (kw.) curves drawn in full on Fig. 3.

Stability Tests on an Artificial Transmission System

An artificial transmission system on a sufficiently large scale so that results could be obtained comparable with those expected in actual operation was used for making tests on the power limitations, voltage stability, hunting, and effect of short circuits in causing the system to pull out of step.

Transmission System. Two artificial lines of the T type constructed of reactance coils, resistance grids, and static condensers were used. The lines were operated normally at 2380 volts. Power was obtained from a 625-kv-a. generator driven by a direct-current motor, which made the proportion between the capacity of the power plant and the load approximate that of an actual power system.

Two 425-kv-a. synchronous condensers were available thus permitting the two transmission lines to be operated either alone or in series with an intermediate condenser station for actually testing a "loaded" line. Voltage regulators for use with the generator and each of the synchronous condensers were provided.

Two 200-kw. synchronous motor-generator sets were

used as loads. The direct-current generators of these sets were connected to resistance racks to provide a dead load instead of loading back on the direct-current shop system. The fields of the direct-current generators were separately excited so that the load could be varied gradually by raising or lowering the d-c. voltage.

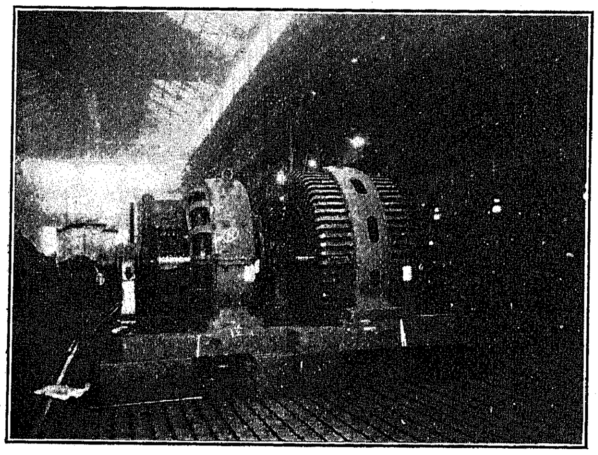

Fig. 9

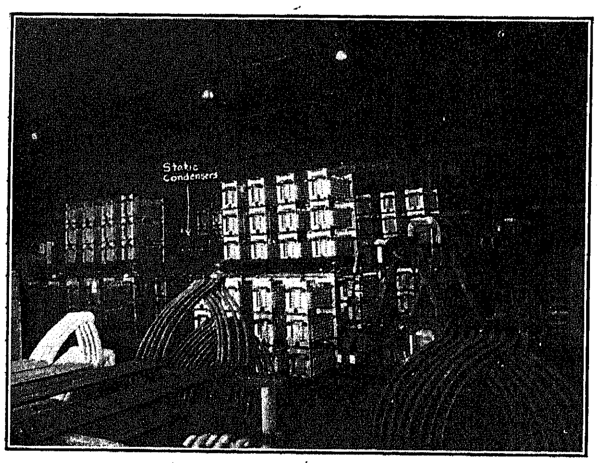

Fig. 10

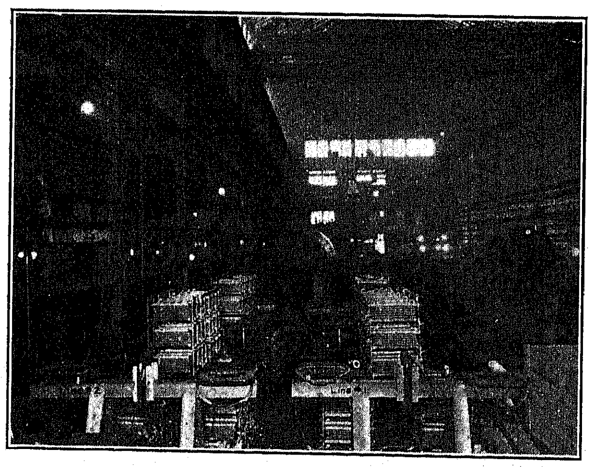

Fig. 11

The motor-generator set used as a source of power is shown by Fig. 9, and Fig. 10 and 11 show views of the two transmission lines. Fig. 12 is a photograph of one of the static condenser banks. One of the synchronous condensers with its belted exciter is shown in Fig. 13.

The photographs show how the reactance coils were grouped to minimize the mutual effects. From measurements made during operation, it was ascertained that the mutual effects were negligible. The reactance and effective resistance of the coils at 60 cycles were obtained from measurements. It happened that the $I^2 R$ loss and the eddy current loss at normal temperatures were about equal thus making the effective resistance of the coils practically constant over the temperature range at which they were operated during the test.

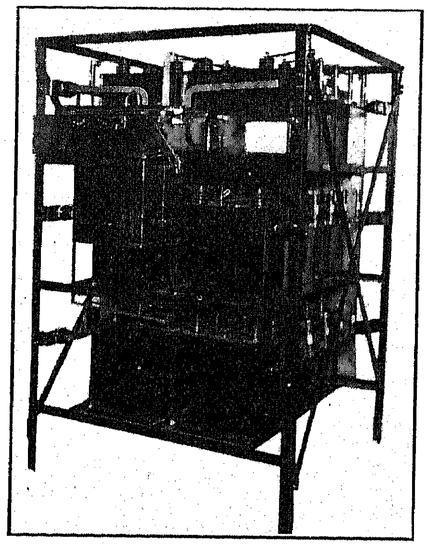

Fig. 12

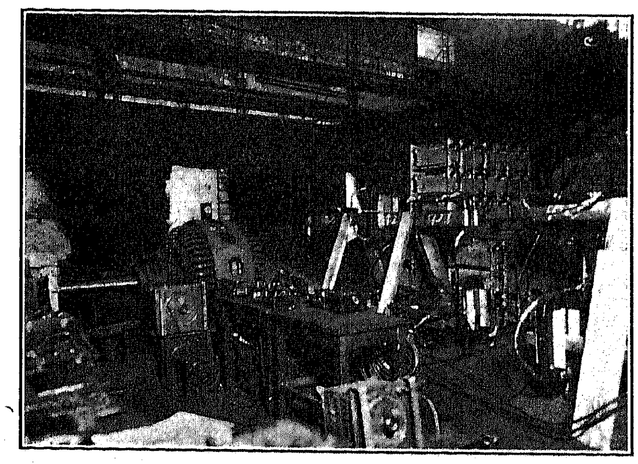

Fig. 12a

The resistance grids were arranged so that the amount of the resistance in the circuits could be varied. The resistance of the grids changed considerably with the temperature and in order to obtain the actual resistance the voltage drop across the grids was taken at each reading. In actual tests sufficient time was

allowed for a steady temperature state to be reached before any readings were taken.

The static condensers used with each transmission line had a capacity of 171 kv-a. at 2380 volts as determined from measurements. The condensers were

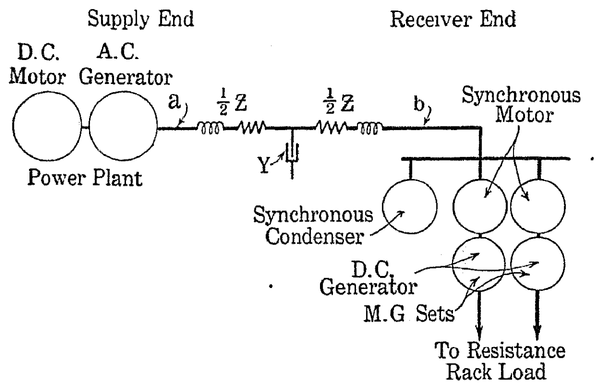

FIG. 13—"T" TRANSMISSION LINE WITH SYNCHRONOUS MOTOR LOAD

Line is three-phase.
Z = Impedance to neutral.
Y = Admittance to neutral of static condensers.
Measurements made at "a" and "B".

connected in star and arranged so that a smaller number of units could be used if desired. The admittance to neutral of the entire bank was $Y = +j\ 0.0302$ mhos.

Based on the generating capacity of 625 kv-a. the

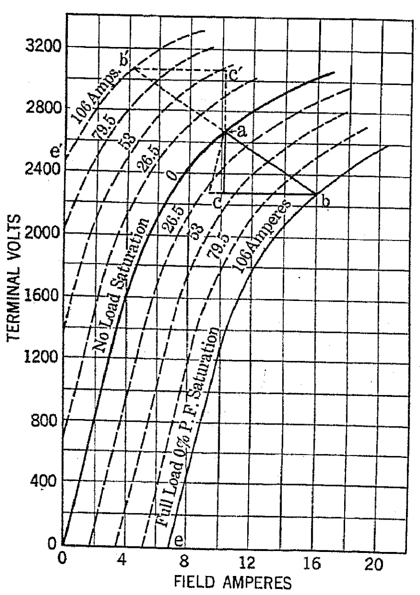

FIG. 14—SATURATION CURVES OF THE SYNCHRONOUS CONDENSERS USED IN MAKING TESTS

425 kv-a.; 0 per cent power factor; saturation curves obtained from tests. The other curves drawn with equal spacing on lines parallel to bb'.

following range of line constants was available in each transmission line:

Resistance 0 per cent to 72 per cent
Reactance 0 per cent to 134 per cent

For comparative purposes the constants based on 150,000 kv-a. transmitted over 250 miles of 220 kv. transmission line are as follows:

Resistance 7 per cent
Reactance 70 per cent

In analyzing the tests the constant field current characteristic curves of synchronous condenser No. 1 and No. 2, which are identical, are required. These curves may be obtained from the no load and zero per cent power factor saturation curves shown in Fig. 14 in the following manner. The curve $e\ b$ is usually spoken of as the zero per cent power factor saturation curve at full-load lagging current, or looking at it from the other point of view the condenser is at its maximum excitation and delivering full-load current leading. Draw in the triangle $a\ b\ c$ of which $a\ c$ is the reactive drop at 106 amperes and $c\ b$ is the field current necessary to overcome the armature reaction at 106 amperes. By inverting this triangle to the position $a\ b'\ c'$ the 106

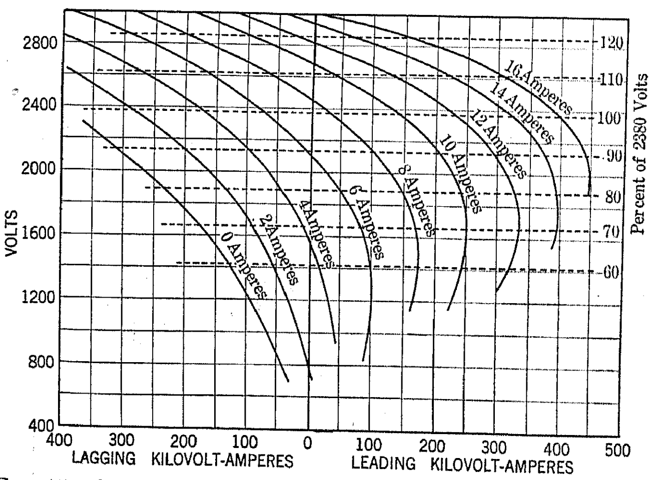

FIG. 15—CONSTANT FIELD CURRENT CURVES OF SYNCHRONOUS CONDENSERS USED IN TESTS

Curves show reactive kv-a. delivered by synchronous condenser when field current is held constant and terminal voltage is varied. These curves calculated from the saturation curves on Fig. 14.

ampere curve $e'\ b'$ for the condenser operating under-excited or delivering lagging kilovolt-amperes may be obtained. The saturation curves for ¼, ½, and ¾ full-load currents at zero per cent leading and lagging power factor may then be drawn by spacing them at equal intervals along lines parallel to $b\ b'$.

At a constant field current of 4 amperes the condenser will deliver 26.5 amperes leading at 950 volts which corresponds to 44 kv-a. leading. At 1600 volts the current is zero and the reactive kv-a. is therefore zero. At 2140 volts the current is 26.5 amperes lagging or the condenser is delivering 98 kv-a. lagging. The curves shown in Fig. 15 give the relation between condenser voltage and reactive kilovolt-amperes delivered, at a number of field currents.

Measurements of the power and reactive kilovolt-amperes being transmitted over the line were made at a and b in Fig. 13. The losses in the synchronous condenser are therefore included with the kilowatts transmitted

and the reactive kilovolt-ampere measurements include that supplied by the synchronous motors as well as the synchronous condenser.

Test No. A—1. In this test the power limit was determined for a transmission system having line characteristics similar to those of an actual 250-mile transmission line. The schematic diagram of the connections used in the test is shown in Fig. 13. To simulate a high-voltage transmission line the resistance was made small compared with the reactance.

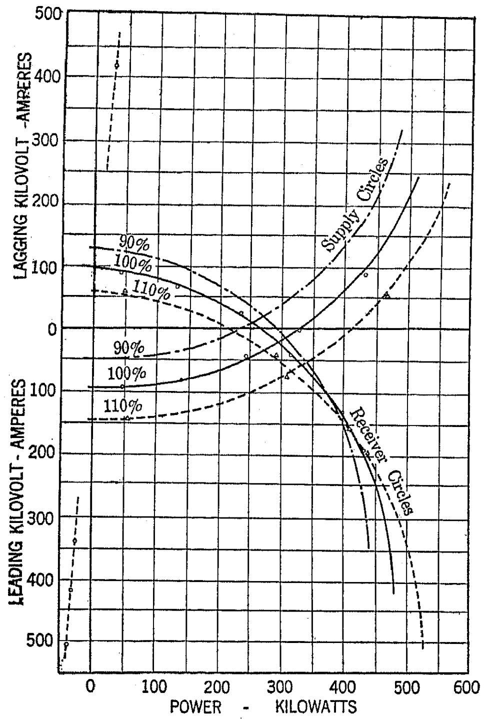

FIG. 16—TEST LINE SIMILAR TO AN ACTUAL 250-MILE LINE
Refer to Fig. 13 for connections.
$Z = 0.80 + j\,12.12$ ohms.
$Y = +j\,0.032$ mhos.
Supply voltage maintained constant at 2380 volts = 100 per cent.
Circles show relation between power and reactive kv-a. for 90 per cent, 100 per cent, 110 per cent receiver voltages.

Referring to Fig. 13 the impedance and admittance of the line to neutral were as follows:

$Z = 0.80 + j\,12.12$ ohms.

$Y = +j\,0.0302$ mhos.

The receiver and supply circle diagrams of the line calculated from these constants are shown in Fig. 16. In Fig. 17 the curves drawn in full show the reactive kilovolt-amperes which must be drawn over the line to maintain various receiver voltages at a number of kilowatt loads transmitted over the line. These curves were derived from Fig. 16. Superposed on these curves are the chain dotted curves which show the relation between voltage and reactive kilovolt-amperes delivered by the *load and synchronous condenser* at a number of condenser field currents.

The voltage at the receiver end of the line was controlled by the synchronous condenser voltage regulator. The two 200 kw. synchronous motor generator sets were used as load.

Readings were taken to check points on the calculated circle diagrams (Fig. 16) and to determine the pull-out points:

SUPPLY END
(Measurements made at "*a*")

Reading No.	Volts	Line Amps.	Kw.	Reactive Kv-a.
1	2380 (100%)	25.2	47	− 94
2	" (100%)	39.0	140	− 83
3	" (100%)	60.0	244	− 44
4	" (100%)	80.5	330	− 4
5	" (100%)	107.0	430	+ 83
6	" (100%)	No readings taken.		
7	" (100%)	" " "		

RECEIVER END
(Measurements made at "*b*")

Reading No.	Volts	Line Amps.	Kw.	Reactive Kv-a.
1	2380 (100%)	23.6	43	+ 89
2	" (100%)	35.5	134	+ 65
3	" (100%)	57.5	236	+ 22
4	" (100%)	76.5	317	− 45
5	" (100%)	103.0	400	−134
6	" (100%)	. .	432	−194
7	" (100%)	Pulled out at 450 kw. which was the average of a number of tests made under the same conditions. In these tests the pull-out point ranged from 445 to 455 kw.		

The above points have been plotted on the calculated 100 per cent receiver and supply circle diagrams in Fig. 16.

Until the load was increased to slightly more than 432 kw. at the receiver, all conditions were very stable

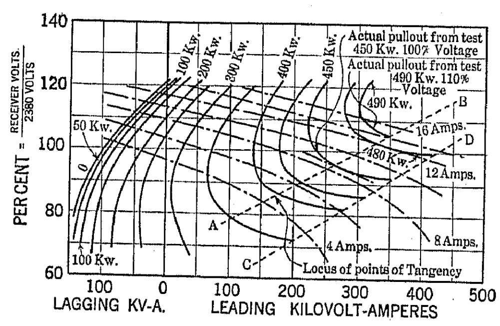

FIG. 17

——— Curves show relation between receiver volts and reactive kv-a. for different loads. These curves obtained from receiver circles on Fig. 16.

–·–·– Curves show relation between load voltages and reactive kv-a. delivered by *load and synchronous condenser* for different condenser field currents. Reactive kv-a. supplied by M. G. sets based on field excitation used in test and a constant load of 450 kw. with variable voltage.

and accurate meter readings could be made. Near the point where pull-out occurred the kilowatts and voltages were very constant but the reactive kilovolt-amperes fluctuated considerably as would be expected, because the line and synchronous condenser characteristic curves were approaching tangency as shown in Fig. 17.

When the load was gradually increased to about 450 kw. the synchronous machines at the load suddenly pulled out of step and slowed down rapidly. While slowing down the synchronous condenser and the motor generator sets remained in step with each other. When

pull-out occurred the polyphase wattmeter used for reading the kilowatts at the receiver end of the line dropped back without taking an upward swing. The voltmeter appeared to drift back and forth for an instant before dropping back. The instrument reading leading kilovolt-amperes made a sudden upward swing.

The oscillogram shown in Fig. 18 was taken at the pull-out point. The load was brought as near as possible to the pull-out point, and the oscillograph started. The load was then suddenly increased to slightly more than 450 kw. causing pull-out to occur.

The following series of points were taken on the 110 per cent receiver circles.

SUPPLY END
(Measurements made at "*a*")

Reading No.	Volts	Line Amps.	Kw.	Reactive Kv-a.
1	2380	38	56	−145
2	"	76	310	− 78
3	"	106	440	+ 26
4	"	113	468	+ 52
5	"	..	520	Pull Out

RECEIVER END
(Measurements made at "*b*")

Reading No.	Volts	Line Amps.	Kw.	Reactive Kv-a.
1	2620 (110%)	17	50	+ 59
2	" (110%)	64.5	292	− 43
3	" (110%)	97.5	411	−156
4	" (110%)	105.0	439	−198
5	" (110%)	..	490	Pulled Out

The above points have been plotted on the calculated 110 per cent receiver and supply circles shown in Fig. 16.

Referring to Fig. 17 it may be noted that pull-out occurred slightly before reaching the point of tangency

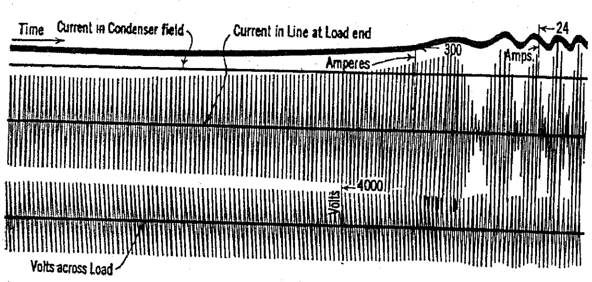

FIG. 18—PULL-OUT AT POWER LIMIT

between the line and synchronous condenser characteristic curves, both for 100 per cent and for 110 per cent receiver voltage. This can be accounted for by slight voltage fluctuations, since the operation of the voltage regulator is not instantaneous.

During the preceding part of the test when the receiver and supply voltages both were 100 per cent (2380 volts), an investigation of hunting due to faulty adjustment of the voltage regulator at the generator was made. The dash pot on the voltage regulator was loosened so that the regulator was not "dead beat" and the resulting voltage fluctuations caused hunting to take place. The load consisted of synchronous motors, which took a constant kilowatt load at all voltages; therefore, as the voltage changed the power component of current also changed. The power component of current drawn through the reactance of the line produced a shaft in phase between the supply and receiver voltages.

The synchronous motors hunted in attempting to follow the continual shift in phase of the receiver voltage. With proper damping of the voltage regulator there was no difficulty from hunting.

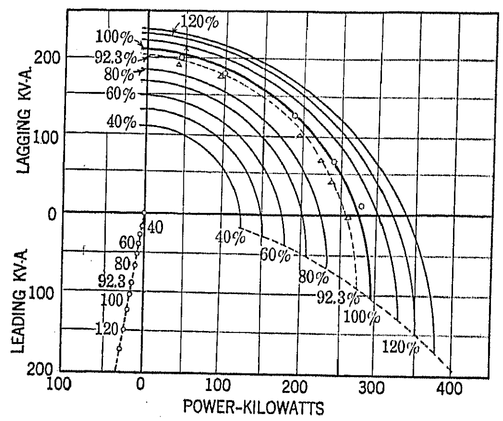

FIG. 19—RECEIVER CIRCLE DIAGRAM OF TEST LINE SIMILAR TO AN ACTUAL 500-MILE LINE

Two lines same as the one covered by the circle diagram on Fig. 16 connected in series. Connections identical with Fig. 22, except synchronous condenser No. 1 was disconnected.

$Z = Z = 0.80 + j\,12.12$ ohms. $Y_1 = Y_2 = j\,0.0302$ mhos.

See Fig. 20 for supply circles.

Supply voltage kept constant at 2380 volts equals 100 per cent. Circles show relation between power and reactive kv-a. at receiver for various receiver voltages.

Test No. A—2. This test was made on a transmission system having line characteristics similar to those of an actual 500-mile line, as may be seen by comparing Figs. 4 and 19.

The test line was built up by connecting in series two sections, each identical with the transmission line used in Test *A—1*. Referring to Fig. 22:

$$Z_1 = Z_2 = 0.80 + j\,12.12 \text{ ohms}$$

$$Y_1 = Y_2 = +\,j\,0.0302 \text{ mhos.}$$

During this test the synchronous condenser No. 1 shown in Fig. 22 at the middle of the line was disconnected.

The receiver circle diagrams calculated from these constants are shown in Fig. 19 and the supply circle diagrams in Fig. 20. The curves drawn in full in Fig. 21 show the relation between voltage and reactive kilovolt-amperes at the receiver end of the line for a number of different kilowatt loads transmitted. These curves were derived from the receiver circles in Fig. 19. It should be noted that with the power transmitted remaining constant, an *increase* in lagging kilovolt-amperes drawn over the line will *increase* the receiver voltage.

By referring to Fig. 31 it may be noted that an

increase in excitation of the synchronous condenser will always be accompanied by a rise in the receiver voltage when operating above the line $A-B$, which is the locus of the points of tangency between the two families of curves.

The voltage regulator at the supply end was set to maintain 2380 volts and the following points were taken with 100 per cent (2380) receiver voltage:

SUPPLY END
(Measurements made at "*a*")

Reading No.	Volts	Kw.	Reactive Kv-a.	Volts at Mid-Point *i. e.* at "*c*"
1	2380 (100%)	52	−216	..
2	" (100%)	112	−201	2860 (120%)
3	" (100%)	200	−154	2760 (116%)
4	" (100%)	260	− 94	2640 (111%)
5	" (100%)	300	− 48	2520 (106%)
6	" (100%)	320*	..	2360 (99%)

*Pull-out point.

RECEIVER END
(Measurements made at "*e*")

Reading No.	Volts	Kw.	Reactive Kv-a.	Volts at Mid-Point *i. e.* at "*c*"
1	2380 (100%)	46	+198	Stable
2	" (100%)	102	+180	"
3	" (100%)	192	+128	"
4	" (100%)	243	+ 69	"
5	" (100%)	280	+ 14	"
6	" (100%)	302	..	Pulled Out

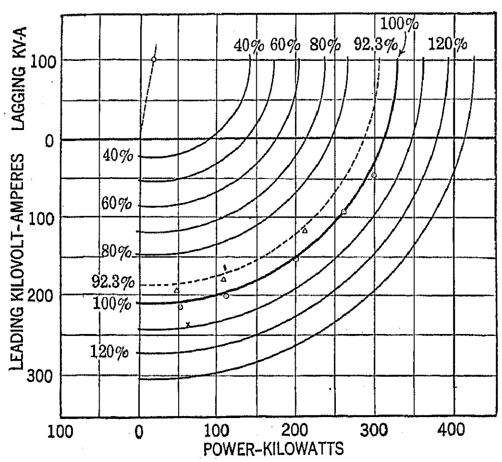

Fig. 20—Supply Circle Diagram of Test Line Similar to an Actual 500-Mile Line

For line constants, see the receiver circle diagram on Fig. 19. The supply voltage is constant at 2380 volts = 100 per cent. The circles show relation between power and reactive kv-a. at the supply end for various receiver voltages.

Within the above range of load the voltage regulator functioned perfectly, even though an increase in receiver voltage required an increase in the lagging kilovolt-amperes drawn over the line, because an increase in condenser excitation always produced a rise in receiver voltage. This can be determined by referring to Fig. 21.

The test points have been plotted on the circle diagrams in Figs. 19 and 20. Pull-out occurred at 302 kw., which corresponds quite closely with the maximum power point of the line alone as determined from the 100 per cent circle in Fig. 19. By referring to Fig. 21, it may be noted that the characteristic curves of the line and synchronous apparatus become tangent close to the maximum power point of the line alone, as determined from the circle diagrams.

Another series of tests was made to take points on the 92.3 per cent circle and one point was taken on the 110 per cent circle to show that an increase in lagging kilovolt-ampere drawn over the line actually increased the voltage, as would be expected from the circle diagram.

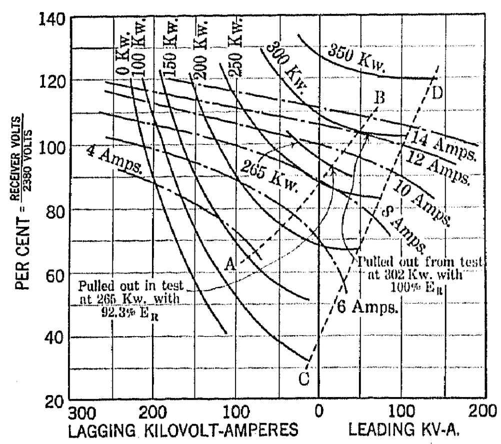

Fig. 21—Test Line Similar to an Actual 500-Mile Line

Connections same as Fig. 22, except synchronous condenser No. 1 disconnected; circle diagram and constants of the line shown on Fig. 19. ———— Curves show relation between receiver volts and reactive kv-a. for different loads. These curves obtained from receiver circles on Fig. 19. - — - — Curves show relation between load voltage and reactive kv-a. delivered by *load and synchronous condenser* for different condenser field currents. Reactive kv-a. supplied by *M. G.* sets based on field excitation used in test and a constant load of 300 kw. with variable voltage.

SUPPLY END

Reading No.	Volts	Kw.	Reactive Kv-a.	Volts at Mid-Point *i. e.* at "*c*"
1	2380	48	−194	2790 (117%)
2	2380	108	−180	2760 (116%)
3	2380	212	−118	2580 (108%)
4	2380	No readings taken		
5	2380	No readings taken		
6	2380	No readings taken		
7	2380	270	Pulled Out	
8*	2380	62	−239	3040 (128%)

*Point on 110 per cent receiver circle.

RECEIVER END

Reading No.	Volts	Kw.	Reactive Kv-a.	Remarks
1	2200 (92.3%)	40	+191	
2	2200 (92.3%)	96	+176	
3	2200 (92.3%)	198	+103	
4	2200 (92.3%)	226	+ 72	No readings at supply
5	2200 (92.3%)	240	+ 47	No readings at supply
6	2200 (92.3%)	260	0	No readings at supply
7	2200 (92.3%)	265	Pulled Out	
8	2620 (110%)	52	+208	

These points have been plotted on the receiver and supply circle diagrams shown in Figs. 19 and 20. The

pull-out point of the line alone with 92.3 per cent voltage corresponds very closely to the point of tangency between the two families of curves shown in Fig. 21.

During the above tests the load was suddenly varied by changing the amount of resistance used as load for the motor generator sets. The system stabilized after

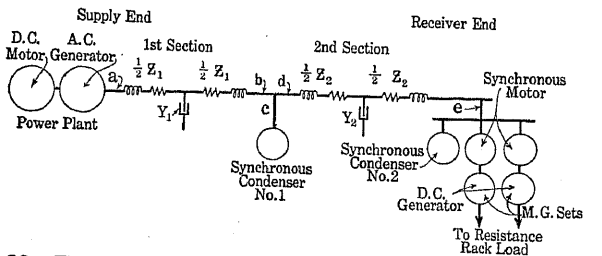

FIG. 22—TRANSMISSION LINE WITH SYNCHRONOUS CONDENSER AT MID-POINT

Line is three-phase.
Z_1 = Impedance to neutral of 1st section.
Z_2 = Impedance to neutral of 2nd section.
Y_1 = Admittance to neutral of static condenser on 1st section.
Y_2 = Admittance to neutral of static condenser on 2nd section.
Measurements made at *a-b-c-d-e*.

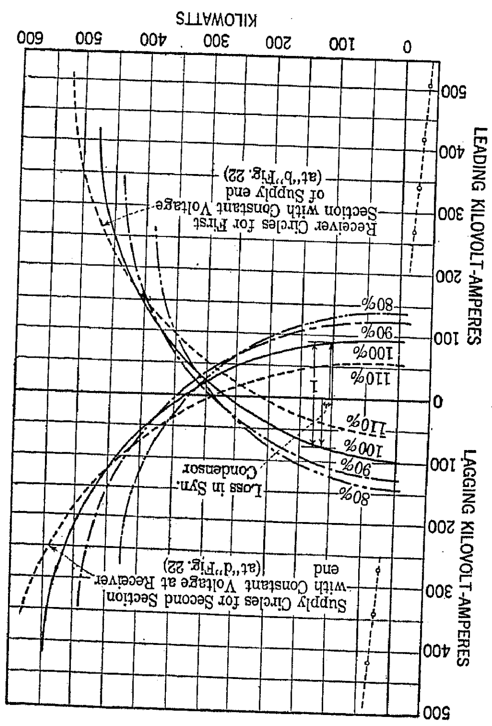

FIG. 23—MID-POINT OF TEST LINE WHICH IS SIMILAR TO AN ACTUAL 500-MILE LINE

2380 volts = 100 per cent maintained at supply end of first section and receiver end of second section. Each section identical with Fig. 16. Referring to Fig. 22 the line constants to neutral are: $Z_1 = Z_2 = 0.80 + j\,12.12$ ohms.
$Y_1 + Y_2 = j\,0.0302$ mhos.

1 = lagging kv-a. which must be supplied at mid-point to maintain 100 per cent voltage when the load at the receiver end of first section is 117 kw. To obtain the lead at the end of the line subtract losses in synchronous condenser and the second section. By continuing this process curves showing relations between volts and reactive kv-a. at different loads as in Fig. 24 were obtained.

a few oscillations and pull-out did not occur unless sufficient change in load was made to swing below the Line "A—B" in Fig. 21.

Test No. A—3. This test was made to check the operation of a loaded transmission line. Conditions were identical with those in Test *A—2* except that synchronous condenser No. 1 was connected at the mid-point of the line, as is shown in Fig. 22. The voltage regulators were set to maintain 100 per cent voltage (2380 volts) at the mid-point of the line, as well as at the supply and receiver ends.

The curves drawn in full lines in Fig. 24 show the reactive kilovolt-amperes which must be supplied by the synchronous condenser at the mid-point to maintain any desired voltage, at a number of different loads. The kilowatts shown refer to the power delivered at the end of the first section and measured at point *b* in Fig. 22. The corresponding amounts of power delivered at the receiver end of the line (at *e*), may be obtained by subtracting the losses in synchronous condenser No. 1 and in the second section. These curves were obtained from Fig. 23 in the same manner that the curves for the 500 mile line shown by Fig. 8 were obtained from Fig. 7, except that the losses in synchronous condenser No. 1 were included in the manner indicated in Fig. 23. This method may be extended to take into account loads at the mid-point.

The load was gradually increased, and the following readings taken:

SUPPLY END OF 1ST SECTION
(At "*a*" Fig. 22)

Reading No.	Volts	Line Amps.	Kw.	Reactive Kv-a.
1	2380 (100%)	26.5	64	−90
2	2380 (100%)	35	120	−83
3	2380 (100%)	50	196	−62
4	2380 (100%)	61	248	−38
5	2380 (100%)	77.5	320	..
6	2380 (100%)		430	Pulled Out

MID-POINT OF LINE
(See Fig. 22)

Readings No.	End 1st Sect. (at "*b*")			Syn. Cond. No. 1 (at "*c*")			Begin. 2nd Sect ("*d*")	
	Volts	*Kw.	Reactive Kv-a.	*Field Amps.	Kw.	*React. Kv-a.	Kw.	React. Kv-a.
1	2380 (100%)	62	+87	3.9	16	+175	45	−90
2	2380 (100%)	117	+75	4.4	15	+158	102	−85
3	2380 (100%)	190	+47	5.1	15	+116	171	−70
4	2380 (100%)	237	+23	5.8	15	+ 75	224	−52
5	2380 (100%)	303	−21	7.3	15	0	286	−23
6	2380 (100%)	410	Pulled out					

*May be used to check calculated curves in Fig. 24.

RECEIVER END OF 2ND SECTION
(At "*e*" Fig. 22)

Reading No.	Volts	Line Amps.	Kw.	Reactive Kv-a.
1	2380 (100%)	22	45	+82
2	2380 (100%)	30.5	99	+72
3	2380 (100%)	41.6	170	+47
4	2380 (100%)	52.5	219	+25
5	2380 (100%)	67.5	278	−11
(Extra)	2380 (100%)	..	302	−55 Meters all were steady
6	2380 (100%)	..	384	Pulled out

Considering the two sections independently the kilowatts and reactive kilovolt-amperes at their supply and receiver ends may be checked against the 100 per cent circles shown in Fig. 23, which are the same as those in Fig. 16 used with Test *A—1*.

For loads above 330 kw. at the receiver end of the line (at *e* Fig. 22), which corresponds to 370 kw. at *b*, the reactive kilovolt-amperes fluctuated to a limited extent although the voltage and power remained steady. At a load of 340 kw. at *e* (380 kw. at *b*), the system showed signs of becoming unstable, although pull-out did not actually occur until the load was increased to 384 kw. (410 kw. at *b'*). This point was checked by several trials.

The addition of the synchronous condenser at the mid-point of the line used in Test *A—2* increased the load that could be delivered over the line from 304 kw. to 384 kw., an increase of 26 per cent.

An investigation at a number of different loads was made of the effect of suddenly throwing on about 25 per cent additional load. The figures given refer to kilowatts at the receiver end of the line, and for convenience in referring to Fig. 24, the corresponding kilowatts at *b* are given in brackets.

Trial No.	Kw. at "e"			Remarks
	On	Off	On	
1	293 (321)	243 (263)	293 (321)	Stabilized rapidly
2	320 (360)	283 (308)	320 (360)	Voltage oscillated ± 10 per cent at first, but settled down after a few oscillations.
3	344 (384)	300 (330)		Pulled out when load was thrown back.

Test No. A—4. This test is identical with Test *A—3*, except that one motor generator set was moved from the receiver end of the line to the mid-point of the line and operated in parallel with synchronous condenser No. 1. The loads were adjusted so that about one-half of the load was delivered at the mid-point, and the other half transmitted to the end of the line.

When pull-out occurred, 440 kw. (measured at *b*) were being transmitted over the first section. This is very close to the pull-out point of the first section when tested alone, which was 450 kw. in Test *A—1*. Up to 400 kw. (at *b*), the system would stabilize quickly after a sudden change in load.

In Test *A—3* with the synchronous condenser No. 1 alone at the mid-point, pull-out occurred at 410 kw. at *b*, while in this test with the motor-generator set operating in parallel with the condenser, pull-out occurred at 440 kw. This was because the additional synchronous capacity at the mid-point changed the slope of the chain dotted curves in Fig. 24 in such a manner as to increase the power limit; in other words, it gave the effect of a larger synchronous condenser at the mid-point.

Test No. B—1. The preceding series of tests was made on a transmission line having low resistance, high reactance and capacity to simulate a high-voltage transmission line.

This test was made on a transmission line having an impedance to neutral of:

$Z = 3.84 + j\ 8.08$ ohms

$Y = 0$

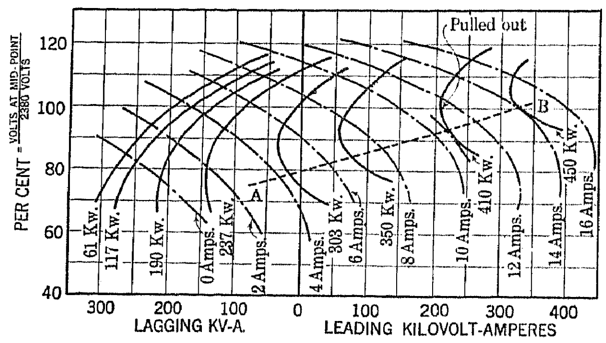

Fig. 24—Mid-Point of Test Line Similar to an Actual 500-Mile Line

Constant voltage = 2380 volts = 100 per cent *E* maintained at receiver and supply ends of the line.

———— Curves show relation between volts at mid-point and reactive kv-a. which must be supplied by the synchronous condenser for kw. measured at "*B*" on Fig. 22. The load actually delivered at receiver end of the line can be obtained by substracting the losses in the synchronous condenser and 2nd section.

- — - — Curves show relation between synchronous condenser voltage and reactive kv-a. delivered for particular values of field current. Data for curves giving relation between volts and reactive kv-a. at mid-point obtained from Fig. 23.

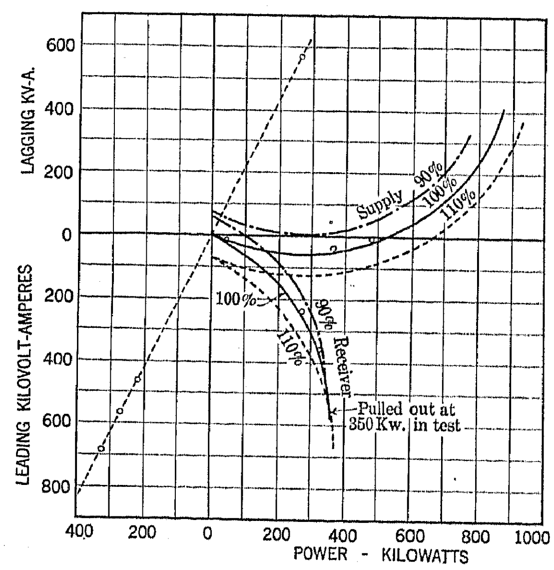

Fig. 25—Test Line Receiver and Supply Circles

Refer to Fig. 13 for connections.

$Z = 3.84 + j\ 8.08.$

$Y = 0$

Supply voltage maintained constant at 2380 volts = 100 per cent.

Circles show relation between power and reactive kv-a. for 90 per cent, 100 per cent, 110 per cent receiver voltages.

This relation between resistance and reactance is typical of a low-voltage transmission line. The circle diagrams calculated from the above constants are shown in Fig. 25. Fig. 26 shows the characteristic curves of the line and synchronous load superposed upon each other.

At 100 per cent voltage, the points of tangency between the two families of curves are almost identical with the power limits of the line considered by itself.

The connections and apparatus employed are shown in Fig. 13. The voltage regulators were set to maintain constant voltage at both the receiver and supply ends of the line. The load was gradually increased with the following results:

SUPPLY END
(At "*a*" Fig. 13)

Reading No.	Volts	Line Amperes	Kw.	Reactive Kv-a.
1	2380 (100%)	..	48	−17
2	2380 (100%)	88	364	−37
3	2380 (100%)	117.5	480	− 7
4	2380 (100%)	..	560	Pulled out

RECEIVER END
(At "*b*" Fig. 13)

Reading No.	Volts	Kw.	Reactive Kv-a.
1	2380 (100%)	46	− 19
2	2380 (100%)	274	−238
3	2380 (100%)	326	−360
4	2380 (100%)	350	Pulled out.

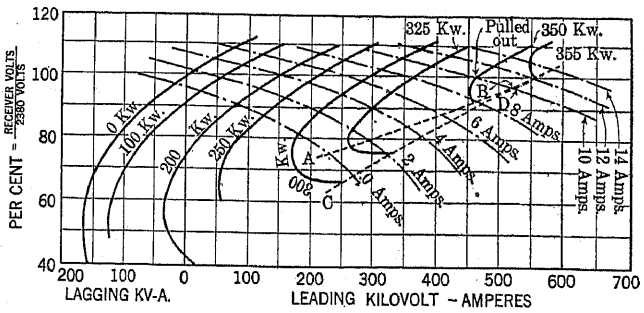

FIG. 26—RECEIVER END OF LINE IN TEST B-1

2380 volts = 100 per cent voltages.

———Curves show relation between receiver volts and reactive kv-a. at different loads. These curves obtained from receiver circles on Fig. 25. The supply voltage was constant at 2380 volts = 100 percent.

- — - —Curves show relation between load voltages and reactive kv-a. delivered by *load and synchronous condenser* for different condenser field currents. Reactive kv-a. supplied by *M. G.* sets based on field excitations used in test and a constant load of 350 kw. with variable voltage.

The test results have been plotted in Fig. 25. The points checked the calculated circle diagrams very closely, particularly the higher readings, because the circle diagrams were based on the resistance of the grids at the higher loads.

Pull-out occurred very sharply at 350 kw. without much sign of instability below this load, as would be expected by referring to Fig. 26, since the two families of curves approach tangency abruptly very close to the maximum power point of the line.

Test No. B—2. This test was made to determine whether an individual part of the system could be pulled out of synchronism without causing the entire system to pull out.

The same transmission line and apparatus were used as in Test *B*—1. The entire load was carried by motor generator set No. 2.

M. G. set No. 1 was running idle to assist the synchronous condenser.

The load transmitted was kept constant at 200 kw. The receiver voltage was gradually reduced by decreasing the excitation of the synchronous machines at the load. At 50 per cent voltage M. G. set No. 2, which was carrying the load, pulled out of step and slowed down, while M. G. set No. 1 and the synchronous condenser remained in synchronism with the generator.

The system as a whole did not pull out of step because its power limit was not exceeded when 200 kw. at 50 per cent voltage were being transmitted. The pull-out point of M. G. set No. 2 considered by itself was exceeded and it pulled out of step.

Test No. B—3. Two transmission lines each identical with the line used in Test *B—1*, were connected in series as shown on Fig. 22, except that synchronous condenser No. 1 was disconnected. The line constants were:

$$Z_1 = Z_2 = 3.84 + j\,8.08 \text{ ohms}$$
$$Y_1 = Y_2 = 0$$

The total impedance of the line was $7.68 + j\,16.16$ ohms.

A test run was made with the voltage held constant at 2380 volts at both ends of the line. Pull-out took place at 190 kw. which corresponded closely to the pull-out point of the line alone as determined from the calculated circle diagram. The characteristic curves of the line and synchronous apparatus become tangent to each other practically at the pull-out point of the line. These courses are not shown.

Test No. B—4. To determine the power limit of a "loaded" transmission line synchronous condenser No. 1 was connected at the middle of the line used in test *B—3*. The connections were then exactly in accordance with Fig. 22.

The load was gradually increased with the voltage regulators set to maintain constant voltage at *a-b-e*, and the following readings taken:

SUPPLY END OF 1ST SECTION
(At "*a*" Fig. 22)

Reading No.	Volts	Line Amps.	Kw.	Reactive Kv-a.
1	2380 (100%)	14	61	−10
1	" (100%)	50	208	−31
3	" (100%)	82	345	−32
4	" (100%)	134	550	..

MID-POINT OF LINE
(See Fig. 22)

	End of 1st Sect. (at "*b*")			Syn. Cond. No. 1 (at "*c*")			Begin. 2nd Sect. (at '*d*")	
Reading	Volts	Kw.	React. Kv-a.	Field Amps.	Kw.	React. Kv-a.	Kw.	React. Kv-a.
1	2380 (100%)	61	− 17	7.0	14	+ 8	47	−25
2	" (100%)	174	− 94	7.9	15	− 45	158	−49
3	" (100%)	264	−200	10.0	17	−144	250	−53
4	" (100%)	350	−427	15.5	20	−380	330	−47

RECEIVER END OF 2ND SECTION
(At "*e*" Fig. 22)

Reading No.	Volts	Line Amps.	Kw.	Reactive Kv-a.
1	2380 (100%)	14	48	− 34
2	" (100%)	41	148	− 92
3	" (100%)	62	208	−156
4	" (100%)	84	268	−229

Immediately after reading No. 4 was taken the system pulled out of step of its own accord without any further increase in power having been made. This indicated that reading No. 4 was very close to the pull-out point. The system was very stable when this reading was taken and all the meters could be read accurately.

The conditions at the middle of the line were as shown in Fig. 27. The curves were derived in the same manner as the corresponding curves used with Test *A-3*. The family of receiver and supply circles from which these curves were derived are not shown. From Fig. 27 it can be seen that the transmission line and the condenser characteristic curves become tangent to each other practically at the pull-out point for the first section, that is, slightly above 350 kw. as measured at point *b* (Fig. 22). Since the curves became tangent very suddenly it was to be expected that the voltage

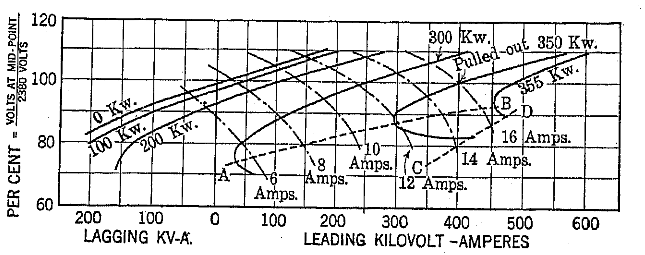

FIG. 27—MID-POINT OF TEST LINE USED IN TEST B-4

Refer to Fig. 22 for connections. $Z_1 = Z_2 = 3.84 + j\,8.08$ ohms. $Y_1 = Y_2 = 0$.

Constant voltage = 2380 volts = 100 per cent maintained at receiver and supply ends of the line (*i. e.* at "*A*" Fig. 22).

———— Curves show reactive kv-a. which the synchronous condenser at the mid-point must supply, to maintain various voltages at a number of different loads in kw. measured at receiver end of 1st section (at "B" on Fig. 22). The load actually delivered at the receiver end of the line (at "*E*" Fig. 22) can be obtained by subtracting the losses in the synchronous condenser and second section.

- — - — Curves show relation between synchronous condenser voltage and reactive kv-a. delivered for particular values of field currents (from Fig. 15). Note the large increase in reactive kv-a. between 350 and 355 kw.

regulator would operate satisfactorily almost up to the point where pull-out occurred.

At reading No. 4 with 134 amperes in the first section and 84 amperes in the second section, the per cent resistance and reactance voltages in each section were as follows:

1st Section $I R + j I X = 37$ per cent $+ j$ 79 per cent

2nd section $I R + j I X = 22$ per cent $+ j$ 49 per cent

There was no tendency toward hunting even with such high percentage resistance and reactance drops. The system stabilized after a few oscillations following a sudden change in load, provided the maximum swing did not exceed the pull-out point of the system.

As determined from Test *B—1*, 350 kw. was the maximum power which could be transmitted over the first section. In Test *B—3*, two such lines were connected in series without a synchronous condenser at the mid-point, and pull-out occurred at 190 kw. With the circuits arranged as in Test *B—3*, but with a synchronous condenser at the mid-point, 268 kw. could be transmitted. The addition of the synchronous condenser at the mid-point raised the power limit 40 per cent.

Test No. B—5. This test was made to determine the effect of damper windings on the stability of the system, particularly at times when sudden changes in load occur. The conditions were identical with Test *B—4*, except that the damper windings were removed from synchronous condenser No. 1 located at the mid-point, and from one of the motor generator sets.

The load was gradually increased and points were taken on the circle diagrams for the first and second sections in the same manner as in Test *B—4*. Pull-out occurred when the load transmitted over the first section (measured at *b*) was 356 kw., which checks closely with 350 kw. obtained in the preceding tests.

The load on the motor generator set without damper windings was changed suddenly. The tabulation below shows the fluctuation in load at the end of the second section. For convenience the corresponding loads at "*b*" (Fig 22) are given in brackets.

Trial No.	Kw. at "*c*" Fig. 22			Remarks
	On	Off	On	
1	232 (293)	160 (192)	232 (293)	Oscillations of considerable amplitude but short duration.
2	240 (301)	170 (205)		Pulled out at once. Evidently oscillated beyond "*A-B*" on Fig. 27.

The above tests show that the damper windings on synchronous condensers do not contribute materially to the stability of the system either during normal operation or with sudden changes in load.

Test No. B—6. This test was made to investigate the effect of damper windings in reducing hunting produced by forced oscillations on a system, such as would be caused by a reciprocating engine or a faulty prime mover governor. The forced oscillations were produced by cutting a block or resistance in and out of the circuit supplying the d-c. motor driving the generator used as a source of power. The severity of the hunting was gaged by the fluctuation in power as indicated by the polyphase wattmeters.

The transmission line and connected equipment were the same as used in Test *B—1*. The synchronous condenser was equipped with copper damper windings and the motor generator sets with standard brass damper windings. The voltage regulators of the generator and synchronous condensers were set to maintain 2380 volts each of the line. The load at the receiver was kept constant at 180 kw. and the frequency of the applied pulsations varied. The accompanying table shows the fluctuations in power at the receiver end of the line as indicated by the polyphase wattmeter:

LOAD AT RECEIVER END ABOUT 180 KW.

Impulses per Minute	Kw. Swing at Receiver
60	16 = 164 to 180
70	20 = 168 to 188
75	4 = 168 to 172
80	36 = 164 to 200
85	120 = 120 to 240
90	28 = 176 to 204
100	4 = 168 to 172

The hunting covered a range of 120 kw. at 85 impulses per minute which corresponds to the natural frequency. At 10 impulses per minute above or below this point the hunting was small. From these tests it can be seen that the resonant point is sharply defined and that no difficulty need be anticipated from hunting due to forced pulsations unless they are quite close to the natural frequency.

The damper windings were then removed from the synchronous condenser and one of the motor generator sets, and the test repeated. Except in the immediate vicinity of the resonant point no great change in the amount of hunting from that shown above, took place. In the vicinity of the resonant point the hunting was somewhat more severe than with damper windings. Even with damper windings the hunting at the resonant point rendered the system inoperative for practical purposes.

A similar group of tests was made with the two sections in series and a synchronous condenser at the mid-point, as in tests *B—4* and *B—5*. The results obtained were the same as in the test described above except that hunting took place at a frequency of 55 impulses per minute.

Test No. C—14. These tests were made to investigate hunting caused by the inherent characteristics of the transmission line. No external periodic pulsations were used to start hunting as was done in the preceding tests. When rotary converters first came into use considerable difficulty was experienced with hunting and investigations made at that time indicated that hunting usually occurred when the line resistance exceeded 25 per cent.

Synchronous condenser No. 1 (425 kv-a.) with copper damper windings was placed at the end of a transmission line and the line resistance and reactance were varied to investigate the effect on hunting. Power was obtained from the 2300-volt shop system to eliminate the effect of generator reactance. Hunting would not take place below the field excitations given in the table.

Condition No.	Line Impedance to neutral in ohms	Syn. Cond. excitation when hunting began
(a)	$6.66 + j\,0$	7.0
(b)	$6.66 + j\,2.02$	12.5
(c)	$6.66 + j\,4.04$	No hunting at maximum excitation = 18 amps.
(d)	$4.8 + j\,0$	9.0
(e)	$3.33 + j\,0$	No hunting at maximum excitation = 18 amperes

On the high-resistance line in (a) hunting began approximately when the excitation of the synchronous condenser was increased above that to give normal voltage on open circuit. Refer to Fig. 14 for the saturation curve.

Reactance added to the high resistance line reduced the tendency toward hunting and in (c) it was not possible to cause hunting.

When the damper windings were removed from the synchronous condenser hunting started at 4.0 amperes with the line condition at (a), instead of at 7.0 amperes with copper dampers. This shows that damper windings have some tendency to reduce hunting.

Another test was made with the set up shown in Fig. 13 and a line impedance $Z = 6.66 + j\,0$ as in (a). Power was obtained from the 625-kv-a. motor generator set. When the excitation of all synchronous machines was brought up to the value corresponding to normal voltage at no-load, vigorous hunting would occur. When hunting occurred the periodic pulsations of the synchronous machines could be heard plainly.

When the excitation was removed from any one of

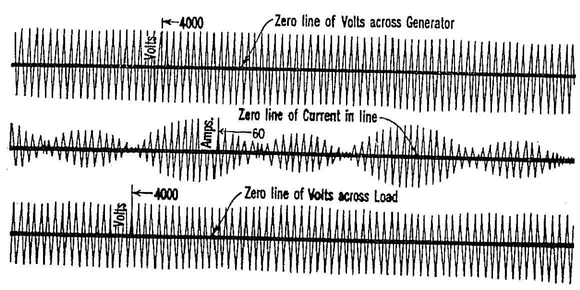

FIG. 28—HUNTING WITH HIGH RESISTANCE LINE

the synchronous machines hunting could not be started by over exciting the other two machines.

The oscillogram in Fig. 28 shows the pulsations in line current when hunting was taking place.

The foregoing tests show that a line having a large resistance as compared to reactance is likely to cause synchronous machinery to hunt. The addition of reactance to a resistance line tends to stabilize the system. On a high-resistance line, hunting is more likely to occur at high field excitation than at low field excitation of synchronous apparatus.

Test No. D—1. A number of tests were made to determine the severity of short circuits that could be applied to a system carrying load without causing pull-out to occur. The short circuit was applied through reactors to produce the effect on the main transmission system of a short circuit on a distribution circuit.

A single section of transmission line identical with that in Test B—*1* was used. The line constants were $Z = 3.84 + j\,8.08$ and the circle diagram is shown in Fig. 25. Power was obtained from the 625 kv-a. generator and the load consisted of a synchronous condenser with copper damper windings and the two

motor generator sets, all connected in accordance with Fig. 13.

When 256 kw. were being transmitted over the line (about 75 per cent of the power limit), a three-phase short circuit was applied at the receiver end, as is shown in Fig. 29. The line was short circuited through reactors having an impedance of $0.27 + j\,4.04$ to neutral. The system pulled out of step about $1\frac{1}{2}$ seconds, as measured by a stop watch, after the short circuit was applied. The oscillogram in Fig. 30 shows the reactions taking place at the time of the short circuit. The voltage regulators increased the condenser

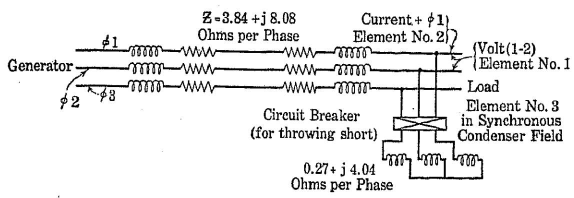

Fig. 29—Short-Circuit Test D-1

Three-phase short circuit in Fig. 30 shown. For the single-phase short circuit in Fig. 31 the oscillograph elements were connected as shown and a single-phase short circuit of $0.27 + j\,4.04$ ohms applied across phases 1 and 2.

field current to the maximum value determined by the setting of the voltage limiting rheostat in an effort to maintain the voltage.

The conditions under which pull-out will occur may be calculated by the general method previously described for determining the power limits of a transmission system, the circle diagrams of the transmission line being modified to include the reactance in the applied short circuit as a part of the line. The characteristic curves of the line (with shunt reactance)

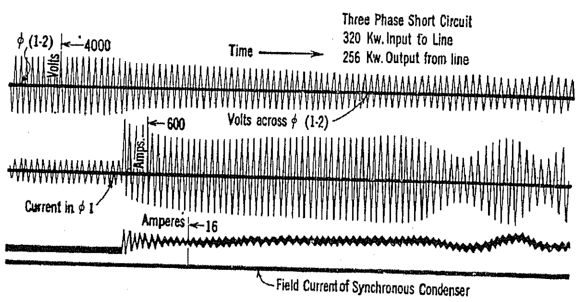

Fig. 30

and the synchronous apparatus at the load can then be obtained and superposed. If the synchronous condenser is controlled by hand the field excitation will remain constant and the maximum amount of power, which can be transmitted without pull-out taking place, can be determined from the power limit corresponding to this field excitation. If the voltage regulator is in the use it will boost the synchronous condenser excitation to the maximum value determined by the voltage-limiting rheostat and thereby increases the amount of power which may be transmitted without pull-out being caused by the short circuit.

As shown by the oscillogram in Fig. 30 pull-out did not occur until the voltage had been reduced sufficiently for the power limit of the system to be exceeded. The voltage at the receiver does not drop instantaneously because of the time required to demagnetize the synchronous condenser.

This test was repeated by applying the short circuit for only one second. The system remained in synchronism because the receiver voltage did not have time to fall until the power limit was exceeded.

Calculations made in the manner outlined above indicate that pull-out will take place with a 256-kw. load when the voltage has been reduced to about 50 per cent of normal, which checks quite closely with the oscillogram.

Fig. 31 shows the effect of a single-phase short circuit through an impedance of $0.27 + j\,4.04$ ohms applied between phases 1 and 2 at the receiver end. The other conditions were identical with those in the foregoing test. Pull-out took place in about 1-1/2 seconds as determined by a stop watch. From the oscillogram it appears that with a phase-to-phase short

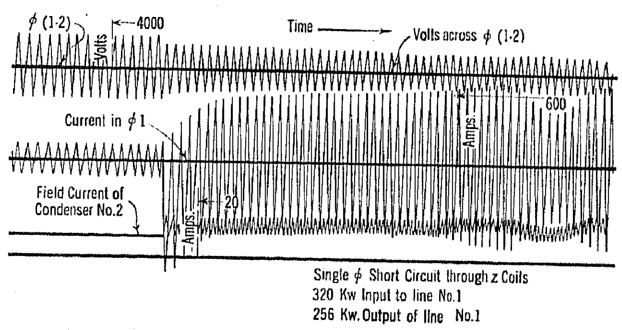

Fig. 31

circuit pull-out occurs when the voltage across the short-circuited phase has been reduced to about the same value as required to cause pull-out with a three-phase short circuit. The double frequency pulsations in the sychronous condenser field current due to the single-phase load can be seen on the oscillogram.

Conclusion

It is of fundamental importance to maintain stable operating conditions on power systems particularly for long distance transmission and interconnected power networks, not only for normal operating conditions, but also at times of short circuits on secondary systems. The method for determining the limits of stable operation presented in the paper and substantiated by tests also have application to the problem of interconnection and station tie-lines.

Power Limit of Transmission System. The method of determining the power limits of a transmission system described in this paper takes into consideration the characteristics of the load and synchronous condensers as well as those of the line with transformers.

The importance of considering the nature of the

load and characteristics of the sychronous condensers in combination with those of the transmission line is illustrated by the conditions existing on the 500-mile line.

It is reassuring to know that in the majority of the cases investigated, the power limit of the entire transmission system, when equipped with present commercial apparatus, approaches very closely the power limit of the transmission line considered by itself. As transmission distances are increased and high voltages employed, it becomes more important to consider the effect of load and synchronous condenser characteristics in combination with those of the transmission line.

Power Limits of Loaded Line. The theoretical investigations and tests show that the amount of power transmitted over a circuit may be appreciably increased by "loading the line" with synchronous condensers at intermediate points. Particular attention was given to the case where the synchronous condenser was located at the mid-point of the line. When synchronous condensers are applied at the mid-point, the power limit of the loaded line approaches that of the first section. The investigation shows that with the loaded line the power limit is to a large extent determined by the capacity of the condenser at the mid-point.

Hunting. Hunting of synchronous apparatus may be due to periodic pulsations applied to the system, or to the transmission line characteristics, when the resistance is large as compared to the reactance. In high-voltage transmission neither of these factors is of great importance. In this connection it is to be pointed out that improper adjustment of the voltage regulators may cause hunting.

The tests show that very little reduction in hunting results from operating with low-resistance copper damper windings as compared to operating without damper windings. The standard high-resistance brass damper windings commonly used for starting purposes are therefore adequate.

Pull-out Caused by Short Circuit. On a loaded line where power is distributed from loading points, there should be sufficient reactance between the transmission and distribution lines to prevent a short circuit on the distribution system from causing the entire transmission system to pull out of step.

Short circuits on a power system cause it to pull out of step as soon as the power limit at the reduced voltage is exceeded. The tests indicate that the time interval between the occurrence of a short circuit and pull-out is determined largely by the time required to reduce the line voltage.

Discussion

For discussion of this paper see page 71.

The Limitations of Output of a Power System
Involving Long Transmission Lines

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***Review of the Subject.**—The conditions of stability of a system are discussed and it is pointed out that while the various transmission line diagrams as used at present implicitly assume the terminal voltage at the two ends to be constant the degree of voltage regulation as determined by load conditions is an important factor in the determination of the limit of output. A type of combined diagram is proposed whereby this factor and other characteristics of the load may be included. The effect of the inherent regulation of synchronous condensers is taken up particularly with respect to compound transmission lines.*

A numerical example of a 300-mile line is considered and various characteristic curves are drawn. The relation between the maximum output and the capacity of condensers installed at the mid-point shows the benefit obtained by increasing the condenser capacity—within certain economic limits.

Mathematical analyses are presented to cover a number of different conditions.

General Considerations and Theory

THE transmission of electric power over the long distances now considered practicable presents to the engineer a wide variety of problems for solution, some of which must be primarily economic, while others will be more strictly confined within the limits of the art. Even these purely technical problems are of radically different kinds; for instance, the necessity of the use of higher voltages results in questions of insulation and corona protection, the design of switching equipment, transformers, etc. The particular problem here considered, however, is of the capability of electric circuits of high relative impedance to deliver large quantities of power. This limitation due to impedance, becomes more prominent as the length of the lines increases and it is pertinent that methods of calculation be devised to represent as nearly as practicable the actual conditions under which a line may be expected to operate. This is the object of the present paper.

If the power demanded from a transmission system be in excess of its capability to supply, the connected apparatus at the receiving end will drop out of step and the system become inoperative. This limitation may be called the limit of output, the limit of stability or the point of pull-out. Methods of calculation of this condition based on transmission line constants and assumed voltages, or the diagrams usually substituted for these calculations, would give true results if the transmission line were the only link for the supply of power, but the electrical circuit of a transmission system comprises generators, transformers, transmission lines, synchronous condensers and the load circuit, and the limit of output will be determined by the combined effect of all of these. It may be said, therefore, that while the various transmission line diagrams as ordinarily employed give correct results as far as the definite inter-relations of power, voltage and current are concerned, the limits of output are only conventional values greater than the actual limits, and should be used only for comparative results in the case of important lines. It is possible, however, to combine the characteristics of the component parts of a power system to form a single diagram for the whole system. To make such a diagram complete becomes a much too complicated process, but it is believed that by including some of the more important factors, particularly the characteristics of synchronous condensers and of the load, a fair approximation may be made with only a justifiable amount of extra labor.

The methods presented in this paper are particularly applicable in determining the limit of output for "compound transmission lines"[1] by which is meant those lines employing synchronous condensers at intermediate stations for the purpose of voltage regulation. This type of line, recommended by Frank G. Baum,[2] promises to become of increasing importance due to the present tendency toward the development of the more distant sources of waterpower and the inter-connection of separated systems.

Before making a detailed study of the limits of output of power systems a general conception of stability of operation may be outlined. The principles involved are not advanced entirely as assumptions nor as hypotheses, but by presenting the phenomenon in this way the purpose of the various diagrams may be rendered clearer.

The characteristics of a transmission line as an electric circuit are closely similar to those of a synchronous machine, the main differences being the effect of magnetic saturation in the latter, and the much greater magnitude of the distributed capacity effect in the former. It will be found, in fact, that the various transmission line diagrams may be derived directly from the classic synchronous motor diagrams of Blondel. That the limit of stability of a line is physically re-

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

1. The term "compound transmission line" has been introduced as conveying most nearly the conception of this type of line as given in the following discussion.

2. Voltage Regulation and Insulation for Large Power, Long Distance Transmission Systems, Frank G. Baum, JOURNAL A. I. E. E., August, 1921.

lated to the pull-out of a synchronous motor is shown by the sudden and complete stalling of connected apparatus when this point is exceeded. When a synchronous motor is loaded, its rotor drops back in phase position by an angle governed by the synchronous impedance of the machine, the condition of stability being represented by the fact that an increase in angle results in an increase of torque. When the limit of stability is passed, the torque decreases with an increase in the phase angle so that this represents an impossible condition of operation. The point of transition between the two conditions occurs when the phase angle is approximately 90 electrical degrees, or more strictly, $\theta = \tan^{-1} x/r$ which may be referred to as the angle of pull-out. Similarly, the conventional limit of stable operation of a transmission line is reached when the phase angle between the sending and receiving ends has reached the corresponding value ($\tan^{-1} x/r$). This angle it will be noted is a "line constant" in both cases and is entirely independent of the values of voltage assumed.

While the angle of pull-out may be used as the fundamental expression of stability some of its accompanying relations are found to form a more convenient means of investigation in more complicated circuits. Thus in studying a simple reactive tie line with constant voltages at either end, it is found that the voltage as measured at any intermediate point will drop as the phase angle between the two extremities increases. When the pull-out angle is reached the rate of voltage change with load must be infinite. Expressing this in mathematical terms; when $\theta = \tan^{-1} x/r$, then $\dfrac{\delta \text{ voltage}}{\delta \text{ load}} = -\infty$ at any intermediate point. This has assumed that the terminal voltages do not change with load; if this condition does not exist the rate of voltage decrease will be greater over the whole line and pull-out will occur at an angle less than $\tan^{-1} x/r$ and at a correspondingly smaller load. In other words, the output limit is determined not only by the actual voltage values at the line terminals at the instant of pull-out, but also by the degree of their regulation. With this idea in mind the difference between the conventional and actual output limitations may be more definitely explained. Transmission line diagrams assume constant voltages at the sending and receiving ends so that the output limit or maximum power so derived will represent only this ideal condition. Actually, the voltage regulation is not perfect at either end of the line, and the conventional value will not be realized.

This naturally brings up the question of the effect of automatic voltage regulators as used on generators and synchronous condensers to maintain constant voltage at both ends of the line. The response of voltage to the action of regulator contacts is much too sluggish when compared with changes of phase displacement due to load to allow the vibrating regulator to be considered in a different sense from a rheostatic regulator operating with occasional adjustment and therefore their use will not modify appreciably the analysis of stability as given above. If a vibrating regulator of a much quicker degree of response were devised an entirely different state of stability might be reached. This may be considered as follows,— when the actual limit of stability is reached a condenser at the end of the line will commence to drift out of step at a rate determined by the excess load, and the voltage will drop correspondingly. A sudden increase in field excitation materially increasing the voltage would bring the rotor and voltage phase angle back again tending to cause an overshoot in the forward direction and consequent high voltage. A reduction of field current will result in the initial condition being regained, followed by the repetition of the cycle. This represents a state of artificial stability in which, with a quick enough response, it would be possible to reach the conventional output limit, or the equivalent of perfect voltage regulation would be obtained. This theory of artificial stability, although perhaps not a practical possibility, is here outlined mainly for the purpose of preventing any misconception regarding the capabilities and limitations of the commercial vibrating regulator in connection with the present subject. Moreover, if the state of artificial stability were attainable it is very doubtful that it would be desirable to depend on this apparatus as the main link in maintaining the operation of the system. In deriving the limit of output for power systems it may be considered that the condensers are operating under a definite value of excitation for each value of load, this value being adjusted by the regulator as the load conditions change.

The general conception of stability may be considered in connection with the theory of compound lines. It has been stated that if the voltages at two points in a circuit are perfectly regulated the limit of stability occurs when the phase angle between the two points equals $\tan^{-1} x/r$, this being independent of the remainder of the circuit. Applying this to the compound line it will be seen that if the voltage regulation at every condenser station be considered perfect the limit of stability is that of the individual sections of line. Adding sections of line will not reduce the output except by the losses in the sections themselves. Actually, of course, the inherent regulation characteristics of the condensers used are finite and the actual output limit of a compound line must be somewhere between the value of the "weakest" section as referred to above and that of the complete line neglecting the intermediate condensers. To accurately evaluate such a line the actual regulation characteristics of the condensers used must be combined with the line diagram.

The synchronous condenser has inherently very desirable characteristics from the standpoint of voltage regulation; it forms a kind of reservoir of magnetization by delivering magnetizing current to the line in in-

creased amounts as the voltage falls. The rough mathemtaical expression for the characteristic of a condenser with constant excitation is as follows:

$$I_o = \frac{E_c - E_r}{X_c}$$

I_o = magnetizing current.

E_c = open-circuit voltage of condenser.

E_r = applied terminal voltage.

X_c = synchronous impedance of condenser.

If a condenser be small, the regulation will be poor and its effect towards increasing the capabilities of the compound line will be limited. Considering, on the other hand, the hypothetical case of a condenser of infinite capacity, X_c will equal zero and the inherent regulation must be perfect, and in the case of the compound line the limit of stability would be truly that of the "weakest" section alone. This comparison demonstrates clearly the importance attached to the choice of condensers for such applications.

In the above connection it may be noted that static condensers are inherently unsuitable for the regulation of long lines. Their inherent characteristics may be expressed thus,

$$I_o' = E_r / X_c'$$

which signifies that the magnetizing current decreases with the voltage, the effect on regulation being therefore negative. This same argument applies to the distributed capacity of the line itself.

The subject of stability might be examined from one other angle. In a paper before this Institute[3] it has been inferred that if the load on a transmission line could be maintained at a certain critical value, the distance to which the power might be transmitted is limited only by the line losses. On examining the examples given from the standpoint of stability it was found that the conventional limit of stability occurred at the distance of one quarter wave length. It may be stated then that even for the restricted conditions considered the use of synchronous condensers at intervals of a few hundred miles would be a necessity, not to change the voltage conditions, for they might be merely floating on the line without carrying reactive current, but to reduce the relation $\frac{\delta \text{ voltage}}{\delta \text{ load}}$ along the line and thus maintain the stability of the system.

LINE DIAGRAM

In dealing with long distance transmission lines it has become the recognized practise to treat the effects of distributed capacity by means of hyperbolic functions. The conventional vector equations may be written.[4]

$$E_s = E_r \cosh \sqrt{Z\,Y} + I_r\, Z \frac{\sinh \sqrt{Z\,Y}}{\sqrt{Z\,Y}} \qquad (1)$$

$$I_s = E_r\, Y \frac{\sinh \sqrt{Z\,Y}}{\sqrt{Z\,Y}} + I_r \cosh \sqrt{Z\,Y} \qquad (2)$$

E_s and E_r are respectively the voltages at the sending and receiving ends, and I_s and I_r the corresponding values of current. Z is the total line impedance, and Y the admittance. All quantities are complex.

These expressions may be abbreviated to the form[5]

$$E_s = A\, E_r + B\, I_r \qquad (1a)$$

$$I_s = C\, E_r + A\, I_r \qquad (2a)$$

and rearranging—

$$E_r = A\, E_s - B\, I_s \qquad (3a)$$

$$I_r = -\, C\, E_s + A\, I_s \qquad (4a)$$

where

$$A = \cosh \sqrt{Z\,Y}$$

$$B = Z \frac{\sinh \sqrt{Z\,Y}}{\sqrt{Z\,Y}}$$

$$C = Y \frac{\sinh \sqrt{Z\,Y}}{\sqrt{Z\,Y}}.$$

The relation of the above formulas to those for the simpler short lines may be noted by assuming the distributed capacity Y negligible. Under this condition $\cosh \sqrt{Z\,Y}$ and $\frac{\sinh \sqrt{Z\,Y}}{\sqrt{Z\,Y}}$ both become equal to unity. Then $I_s = I_r$ and $E_s = E_r + I_r\, Z$ which is the familiar expression for the simple circuit.

A simple voltage vector diagram may be drawn, very convenient for the purpose in hand, directly from equation (1) — or (1a). This diagram has already been published several times and in some cases expanded in an elaborate manner.[6]

For convenience in graphical construction and algebraic analysis the line constants may be written in form

$$A = A_1 + j\, A_2$$

$$B = B_1 + j\, B_2$$

$$C = C_1 + j\, C_2$$

3. "Qualitative Analysis of Transmission Lines," H. Goodwin, Jr. JOURNAL A. I. E. E. January, 1923.

4 See Chap. 15 "Transmission Line Formulas," H. B. Dwight. D. Van Nostrand & Co., etc., for the derivation of these expressions.

5. For the development of these expressions to include unsymmetrical lines and combined lines and transformers, see companion paper "Power Limitation of Transmission Systems," R. D. Evans and H. K. Sels.

6. A Graphic Method for the Exact Solutions of Transmission Lines," Holladay, JOURNAL A. I. E. E., Nov. 1922; also "Calculs, diagrammes et régulations des lignes de transport d'énergie à longue distance." Thielemans, Revue Générale d'Electricité, September 25, 1920 and following: etc.

while as a scalar quantity—

$$B = \sqrt{B_1^2 + B_2^2}$$

Similarly I_r and I_s may be resolved into their power and wattless components

$$I_r = I_{rw} + j\,I_{ro}$$

$$I_s = I_{sw} + j\,I_{so}$$

I_{rw} and I_{sw} being in phase with E_r and E_s respectively.

The general construction of the voltage diagram using E_r as a datum of reference is shown in Fig. 1.

From the geometrical construction it is evident that with E_r assumed constant and considering only the power component of current, ($I_{ro} = 0$), the locus of E_s must lie on the line $T\,S$, while for any given power component of current, varying the wattless component will merely move the locus of E_s along a line perpendicular to $T\,S$,—the line $G\,H$, for instance. With these relations in mind it may be seen that the lines $T\,S$ and $T\,K$ may be utilized as the axes of the two components of current, or if preferred, of kilowatts and reactive kv-a. The former is utilized mainly in the following discussion.

Considering further the geometrical relations of Fig. 1 it is obvious that E_s' is the minimum value of voltage at the sending end which will sustain a load at the receiving end corresponding to the point S, and conversely that with the voltage E_s' this load represents the maximum value, or remembering that the voltages are taken as constant this will represent the conventional limit of stable operation of the transmission line. E_s' is parallel with $T\,S$ so that this point will occur when the phase angle between the voltages at the two ends of the line is equal to $\tan^{-1} B_2/B_1$ which is the equivalent of $\tan^{-1} x/r$ of either the simple circuit or of the synchronous machine. This angle as mentioned before is purely a line constant.

From Fig. 1 can be obtained E_r, E_s, I_r, the power factor at the receiving end, and the voltage phase angle θ. With a slight complication I_s and its phase angle with E_s may be included as well. Thus, from equation (3a)

$$B\,I_s = A\,E_s - E_r \tag{5}$$

In Fig. 2 $O\,L = (A_1 + j\,A_2)\,E_s$ and $L\,V = (B_1 + j\,B_2)\,I_s$. By using the same reasoning as for Fig. 1, I_{sw} is found to be along the axis $L\,W$ and I_{so} at right angles to it. The current values can be measured on the same scale as I_{rw} and I_{ro}. If kw. and reactive kv-a. are desired, the scale will differ from the corresponding one at the receiver end by the ratio E_s/E_r. The value of power-factor at the two ends of the line may be taken as $\cos\phi_r$ and $\cos\phi_s$ respectively. The line efficiency may be obtained from the quantities given by a short numerical calculation.

Although usually more convenient, it is not necessary to resort to graphical methods to determine certain conditions of operation. Certain expressions which are particularly convenient as occasional checks on graphical results are given below. Their derivation will be found in the Appendix.

For the conventional point of pull-out, $\theta = \tan^{-1} x/r$

$$I_{rw}\ \text{(pull-out)} = E_s/B - M\,E_r \tag{6}$$

$$P_r\ \text{(pull-out)} = E_r\,(E_s/B - M\,E_r) \tag{7}$$

where P_r is power at the receiving end in watts per phase assuming E_r to be the voltage to neutral.

$$M = \frac{A_1\,B_1 + A_2\,B_2}{B^2}\ \text{(a line constant)} \tag{8}$$

These quantities are all scalar. With the same conventional assumptions of perfectly constant terminal voltages it is possible to derive additional relations. In the case of a direct-current circuit it will be remembered that the power flow into any part of the circuit is a maximum when the voltage across this part is equal to one-half the total voltage or $E_r = E_s/2$. For a transmission line the equivalent condition is found to be represented by the expression

$$E_r = \frac{E_s}{2\,M\,B} \tag{9}$$

for which condition

$$I_{rw}\ \text{(maximum pull-out)} = \frac{E_s}{2\,B} \tag{10}$$

$$P_r\ \text{(maximum pull-out)} = \left(\frac{E_s}{2\,B}\right)^2 1/M \tag{11}$$

In investigating the significance of these equations it will be found that the limit of output is a maximum in the case of a normal 60-cycle line when the receiver voltage is considerably higher than the sending, although it is true that on account of the large condenser capacities required for such voltage relations this will not represent an economical condition of operation. For lines of 25 cycles and less it will probably be found that the maximum output of the line will be obtained when E_s is raised appreciably higher than E_r. In the case of the higher frequency lines more accruate assumptions as regard voltage regulation will naturally tend to change these relations somewhat, particularly when the condenser is such a large factor in the result. When lines are operated so that E_r is greater than the critical value $\frac{E_s}{2\,M\,B}$ it is found, as expressed in some of the following diagrams, that the actual and conventional limits of stability coincide; that is, the line actually pulls out with the angular displacement equal to $\tan^{-1} B_2/B_1$

The reactive component of current at the receiver end, I_{ro} may also be expressed in mathematical form.

$$I_{ro} = E_r\,K \pm \sqrt{(E_s/B)^2 - (P_r/E_r + E_r\,M)^2} \tag{12}$$

K being an additional line constant of the value

$$K = \frac{A_1\,B_2 - A_2\,B_1}{B^2} \tag{13}$$

The quadratic equation (**12**) will have two real solutions, a singular solution, and imaginary solutions. These may be interpreted from Fig. 1 as follows,—the smaller real solution (that of minus sign) will represent the condition of stable operation, ($\theta < \tan^{-1} B_2/B_1$), and the greater solution (that with plus sign) will represent the unstable zone where $\theta > \tan^{-1} B_2/B_1$. The singular solution corresponds to the point of pull-out, $\theta = \tan^{-1} B_2/B_1$. The imaginary solutions are for conditions where the conventional pull-out load is exceeded. For the singular solution equation (**12**) may be expressed.

$$I_{ro} = K\,E_r \qquad \textbf{(14)}$$

which shows that the value of reactive current at the conventional point of pull-out is a direct function of the receiver voltage and independent of the other conditions of operation.

Equations (**12**) and (**14**) represent only the reactive current in the line itself; to obtain the total condenser current there must be added to I_{ro} the magnetizing current required by the load as estimated at this same point. A synchronous condenser must be chosen on the basis of the algebraic sum of the two.

The System Diagram

The preceding diagram may be properly applied only to the transmission line. To define the limiting conditions of operation of a transmission system the characteristics of the line must be combined with those of the rest of the circuit. In this respect, the line transformers represent no particular difficulty, their concentrated

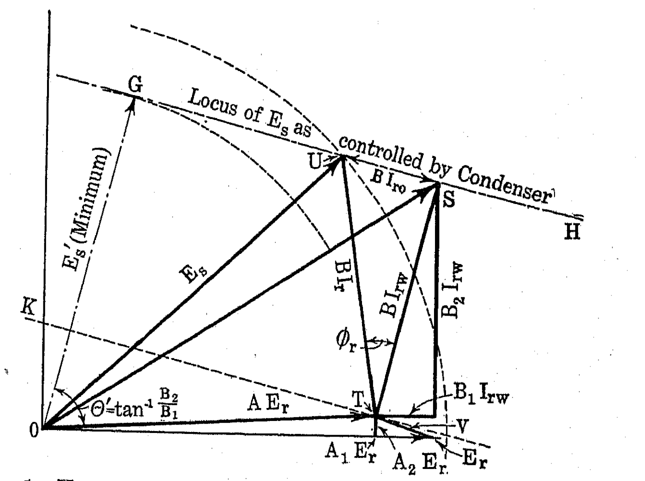

Fig. 1—Voltage Vector Diagram of a Transmission Line

impedances may be included with that of the line and a set of line constants derived by the method already referred to. The diagram of Fig. 1 will then represent the line with transformers. Due to the fact that many of the assumptions made in transmission line calculations must necessarily be rather broad approximations it may not be entirely necessary to take into account the magnetizing current of the transformers. If the necessary data for this is available, however, it should be combined with the characteristics of the adjacent synchronous condenser. The magnetizing current and the condenser currents result from shunt admittances at the same point of the circuit so that the above suggestion is technically correct. The magnetizing current decreases at a considerably faster rate than the transformer voltage so that it will add very slightly to the stabilizing effect of the condenser station. In the case of the transformers at the sending end the main effect of the magnetizing current is a very slight increase in the

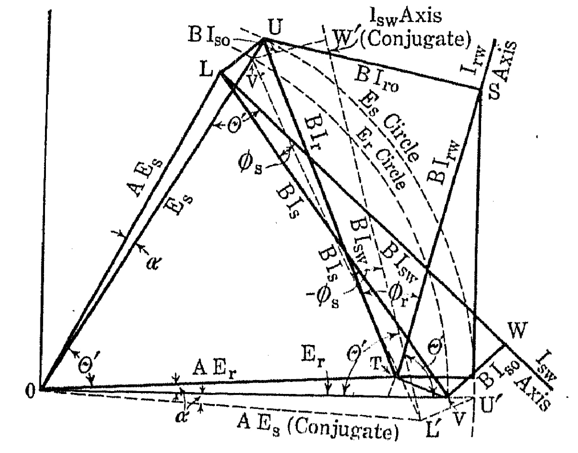

Fig. 2—Diagram Including Current Relations at Sending End

field current of the generators and may be otherwise neglected.

In the following discussion the effect of the inherent voltage regulation of the main generators has been neglected. This is justified on the grounds that the error is slight while the labor involved would increase greatly. Two methods, almost equally laborious may be used in taking this factor into account; one is the ordinary method for the compound line, while the other is an extension of the method used for transformers. The synchronous impedance of the generator may be added to that of the transformer at the sending end and a diagram used for the line transformers and generator. The "sending voltage," *i. e.*, the internal voltage of the generator, becomes a variable and a separate diagram must be worked out for each value of excitation while in addition these diagrams must be correlated with the conditions at the actual sending terminal of the line. Both of the above methods become somewhat complicated.

The effect of the load on the limits of a transmission line is important and its characteristics should be approximated with a fair degree of accuracy. For this purpose two general classes of loads may be considered; first where the power required is independent of voltage, and second, where the power required decreases with voltage. The rotating machine where the output is determined by frequency is typical of the first class, while a lighting or heating load where the power varies with the square of the voltage is typical of the second. The actual load of a system is a combination of a large number of units of both of these classes, with possibly

other classes less clearly defined in addition. The rotating load is usually of preponderant value although the component of lighting load will produce some tendency for a decrease with voltage which may be estimated when the relative proportions have been determined.

Due to the importance of the constant power type of load its characteristics may be studied in some detail. It may be noted then, in Fig. 1, that if the power com-

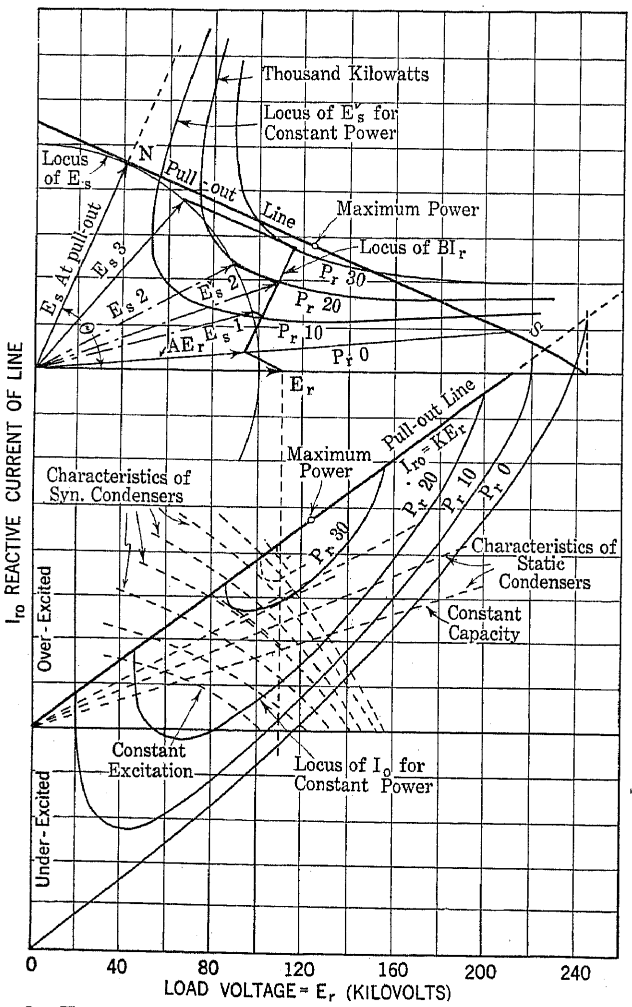

Fig. 3—Transmission Line Diagram. Diagram of Voltage Vectors

Constants $A_1 = 0.85$
$A_2 = 0.075$
$B_1 = 168$
$B = 183$

Fig. 3A—Reactive Current Curves Derived from Voltage Diagrams

ponent of current I_{rw} be varied inversely with E_r the locus of the point S, $(O S = A E_r + B I_{rw})$, will be a hyperbola with the lines $O T$ and $O G$ as asympotes. The hyperbolas for various values of P_r are shown in Fig. 3, the various vectors corresponding with those of Fig. 1. The reactive components of current are indicated by the intervals measured parallel to the line $N S$ and between the hyperbolas and the circular locus of E_s. The zone of stable operation considered on the conventional basis is then represented by the triangle $N O S$, $N S$ being designated as the "pull-out line." One hyperbola may be drawn which will just become tangent to the pull-out line; this represents the condition of maximum output expressed by equation (**11**). It may be observed that in consequence of the locus E_s becoming tangent to the line $N S$ the rate of change of reactive current, $\frac{\delta I_{ro}}{\delta E_r}$, is infinite for the condition of pull-out.

Although the conditions of stability may be readily visualized from Fig. 3, it will be found much more convenient for actual calculations to replot the data as in Fig. 3A. These curves show the relation between power, voltage, and reactive current at the receiving end. The characteristic curves are slightly similar in form to the V-curves of a synchronous motor although plotted on a somewhat different basis. The curves represent the requirements of reactive current for the different conditions of operation, so if the regulation curves of the synchronous condenser, representing the reactive current capable of being furnished under various conditions of voltage and excitation, be plotted with reference to the same axes, the intersections of the two sets of curves gives resultant points of operation of the combination. An additional assumption is made that the load is of unity power-factor.

Assuming the condenser to be operating with constant excitation the limit of output occurs where the two sets of curves would become mutually tangent. Thus, it is noted that the $P r = 30$ curve becomes tangent to one of the condenser regulation curves at 90 kilovolts. According to the theory already advanced, if the condenser were furnished with a vibrating regulator set to maintain 90 kilovolts on the line the limit of output would be identical with the above, even assuming the load to be free from fluctuations. The conventional limit, where the various P_r curves cross the pull-out line would indicate this load to be stable until the voltage decreased to 86 kilovolts. These curves show definitely how the actual limitations of a line depend upon the condenser characteristics, and furthermore that the static condenser does not have the proper inherent characteristics for transmission line regulation. These relations may be better appreciated by referring to Fig. 4 which is derived from Fig. 3A. Here a comparison is presented between the conventional limit, the limit with a condenser of finite characteristics, and the limit with a static condenser for line regulation. It will be noted that the various limits converge to the critical point where $E_r = \frac{E_s}{2 M B}$ (**9**). At this point and above, pull-out must occur where $\theta = \tan^{-1} B_2/B_1$ so that it may be deduced that in the case of low-frequency lines where the normal voltage relations will approach those of equation (**9**) the conventional line diagram will give reasonably accurate results for steady

loads, although such will not be the case for lines of higher frequency as is shown in the later examples.

In combining the line and condenser characteristics in Fig. 3A the reactive component of the load has been neglected; Fig. 5, however, shows the combination of

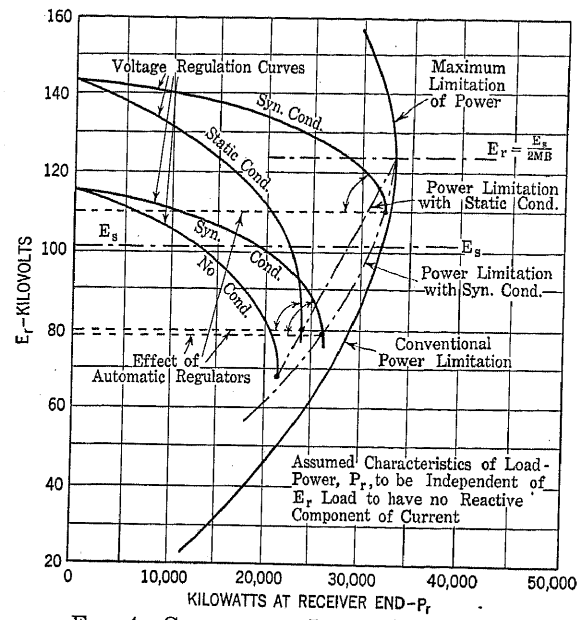

Fig. 4—Comparative Limits of Output

the line characteristics with those of a synchronous motor at constant excitation. The intersections of corresponding curves giving the points of operation, the limiting condition being as before the point of mutual tangency of the two sets of curves. To indicate the effect of a synchronous condenser on the above com-

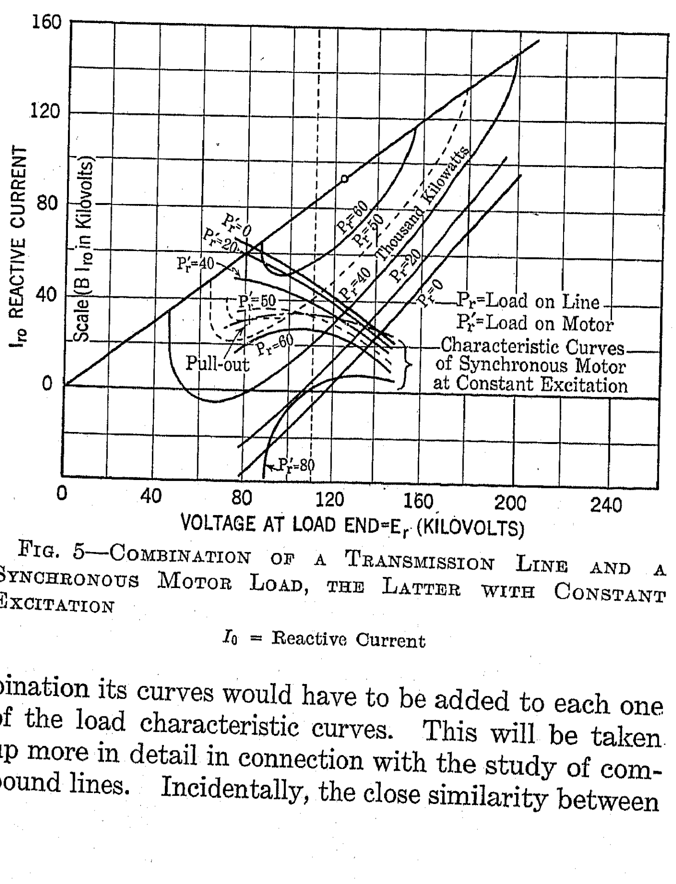

Fig. 5—Combination of a Transmission Line and a Synchronous Motor Load, the Latter with Constant Excitation

I_0 = Reactive Current

bination its curves would have to be added to each one of the load characteristic curves. This will be taken up more in detail in connection with the study of compound lines. Incidentally, the close similarity between the characteristics of the synchronous motor and those of the transmission line may be noted in Fig. 5.

To pass now from the constant power type of load, Fig. 6 has been drawn to represent a load of constant current, a state somewhere between a motor load and a lighting load. Fig. 6A shows that the actual limit of output will always coincide with the conventional value just as in the case of the constant power load when E_r is greater than $\frac{E_s}{2\,M\,B}$. The above condition, it should be noted, will not hold necessarily in the case

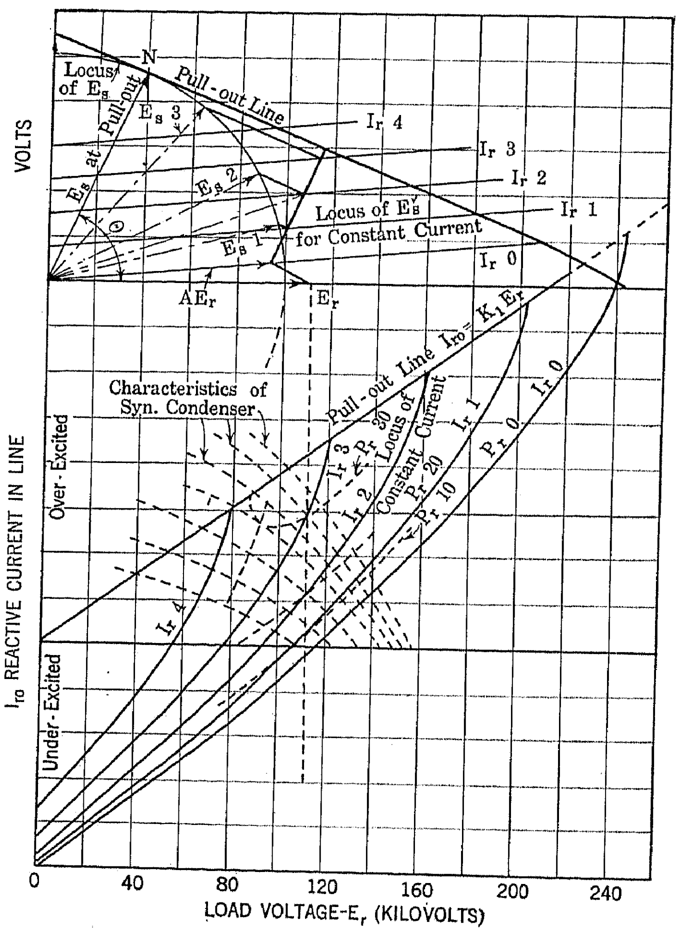

Fig. 6—Transmission Line Diagram. Diagram of Voltage Vectors

Constants A_1 = 0.85
A_2 = 0.075
B_1 = 73.1
B_2 = 168
B = 183

Fig. 6A—Reactive Current Diagram Derived from Fig. 6

of a compound line for the inherent characteristics of the load are masked to some extent by the effect of the condenser at the receiving end. In general, such a load is very favorable from the standpoint of stability.

The regulation curves of the synchronous condenser depend upon the rated capacity of the machine and upon the various design features involved. The curves may be obtained in the ordinary way from the load saturation curves at zero power factor[7] with the excep-

7. See Section 4394 of A. I. E. E. Standards, 1922, etc.

tion that the condensers will always be connected to the transmission lines through transformers, the impedance of which will modify their characteristics as considered with respect to the line. This effect is equivalent to an increase in the armature reactance of the condenser, so that corrected load saturation curves may be drawn by increasing the reactance component of voltage drop by the reactance drop in the transformer. This method was employed in obtaining the data for Fig. 7. When the relation between magnetizing current and voltage is known for the transformer this current can be subtracted from the corrected condenser current at corresponding voltages, the result being a complete regulation curve for the condenser and transformer.

It may be remarked that the increased impedance due to the transformer has a material effect on the regulating action of the condenser and that it should be reduced as low as is practicable. The operating experience obtained with such transformers of high impedance fully verifies this conclusion.

In connection with the operation of automatic voltage regulation one further limitation may be mentioned. The operation of the regulator is only possible up to the point where the full exciting voltage is impressed across the condenser field winding, at which point the regulator contacts become blocked in the closed position. If the requirements from the condenser are still increased the machine operates with constant maximum excitation, the voltage dropping as the load increases. On account of economic considerations involved in the application of condensers this point of maximum current will usually be reached considerably before the limit of output. Therefore pull-out should occur with constant excitation on the condenser.

Compound Transmission Lines

The compound transmission line can probably be studied best by means of an actual example where the quantitative factors are directly involved. Consider then a 300-mile transmission line with a condenser station at the mid-point. Actually the condenser would be somewhat more beneficial if placed slightly closer to the sending end.

The following are the data on the assumed line:

Complete line

Length of line, 300 miles.

Conductors, 500,000 cir. mil aluminum cable.

Spacing—15 ft. effective spacing 18.9 ft.

Voltage between conductors 150,000 volts (86,600 volts to neutral).

Derived constants for 300 mile line.

$A = 0.807 + j\,0.043$	$A_1 = 0.807$
$B = 48.7 + j\,229$	$A_2 = 0.043$
$C = (-\,0.0228 + j\,1.52)\,10^{-3}$	$B_1 = 48.7$
$K = 3.33 \times 10^{-3}$	$B_2 = 229$
$M = 0.913 \times 10^{-3}$	$B = 234$

Derived costants for each 150 mile section.

$A = 0.9507 + j\,.0112$	$A_1 = 0.9507$
$B = 27 + j\,120.6$	$A_2 = 0.0112$
$C = (-\,0.0036 + j\,0.80)\,10^{-3}$	$B_1 = 27$
$K = 7.51 \times 10^{-3}$	$B_2 = 120.6$
$M = 1.78 \times 10^{-3}$	$B = 123.5$

For the purposes of the present discussion the impedance of the line transformers has been neglected, mainly so that the two line sections may be considered to be identical. When this factor is included the above constants will merely suffer some modification. For accurate line calculation it should be included.

The characteristics of a 25,000-kv-a. synchronous condenser combined with a transformer of 8 per cent reactances is shown in Fig. 7. The effect of the magnetizing current of the transformers has been neglected, although if desired this can be taken into account by the method already referred to. Although condensers of various ratings are introduced into the discussion the regulation characteristics of all are assumed to vary

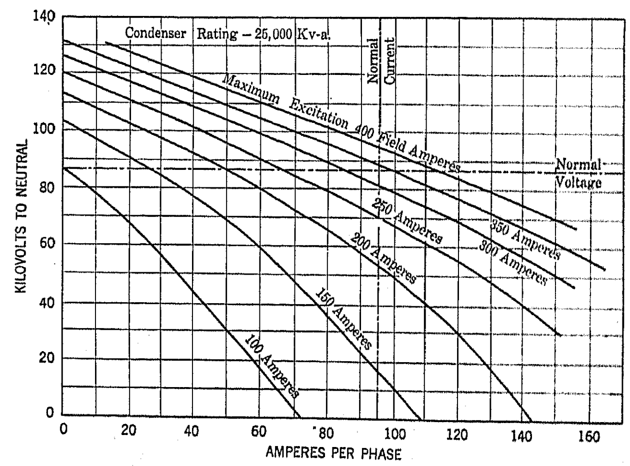

Fig. 7—Voltage Regulation Curves of Synchronous Condenser and 8 Per Cent Reactance Transformer

with these curves proportional to the rating assumed for the condenser.

For the first set of data discussed the voltages at both the sending and receiving ends of the line have been assumed constant. For the receiver voltage this means that it is unnecessary to make any assumptions regarding the change of load with voltage. The limit of output may then be obtained from equation (7), or

$$P_r = 86{,}600\,(86{,}600/234 - 0.913 \times 86.6) \qquad \textbf{(15)}$$
$$= 25{,}200 \text{ kw. per phase or } 75{,}600 \text{ kw. total.}$$

This is the conventional limit of the 300-mile line without intervening condensers. When a condenser of finite capacity is placed at the middle point of the line, a type of diagram similar to Fig. 5 must be employed. Assuming various values of voltage at the middle condenser A, from a diagram similar to Fig. 1 there may be worked out for each value of load at the receiver end, the power and reactive current at A. Also with a diagram such as Fig. 3 the conditions of voltage, power and reactance current may be worked out for the sending

section of the line. Due to the discrepancy between the power at the point A and the point B these two sets of data may, for conveneince be plotted up together in the form of Fig. 8. Here the full line and dot-dash curves represent the receiver section of the line and the dotted curves represent the sending section. With a load at the receiver end corresponding to $o\ m$ the line loss in the receiver section will be $n\ p$. The reactive current furnished to the station A will be $r\ p$ while that required for the sending section will be $-\ r\ q$, the net result being that there is an excess of magnetizing current at A of the amount $p\ q$. These data may then

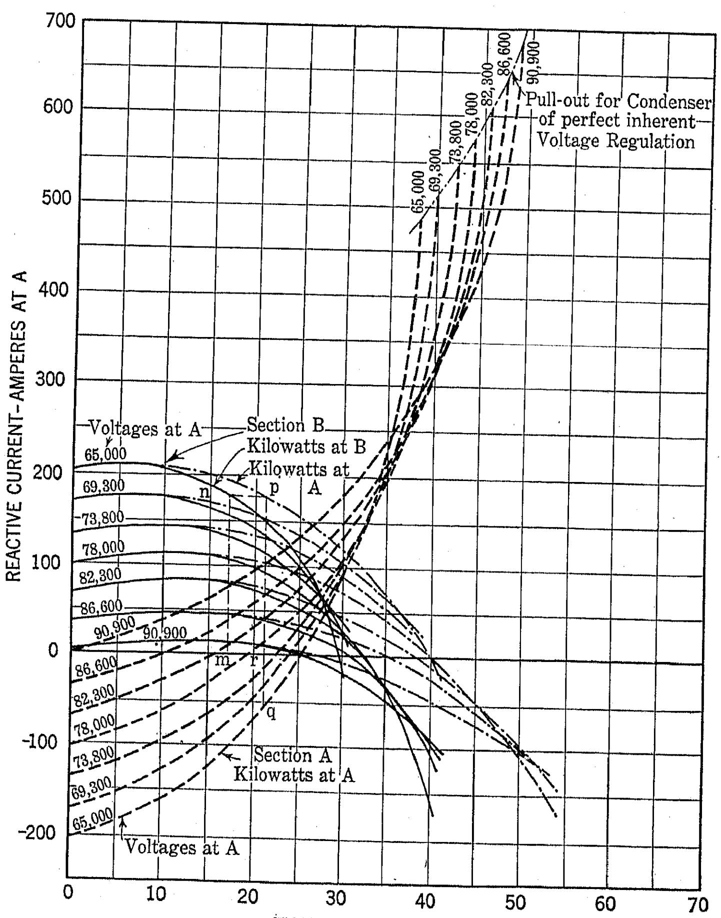

FIG. 8—CONDITIONS AT MID-SECTION OF LINE FOR DETERMINING CONDENSER CAPACITY REQUIRED

be plotted as shown in Fig. 9, the full lines representing the characteristics at the point A for the whole line. The intersections of these curves with the condenser regulation curves give the points of operation of the combination of line and condensers. By employing different condenser curves the effect of the choice of condenser is readily apparent. Fig. 10 gives the voltage regulation curves at the station A assuming condensers of various sizes to be employed. The point at which the voltage regulators become blocked is shown as well as the load and voltage at which pull-out occurs.

In connection with Fig. 10 it may be interesting to note that although the 175,000-kv-a. condenser is sufficiently great in capacity to allow the regulator to operate completely up to the pull-out line at 86,600

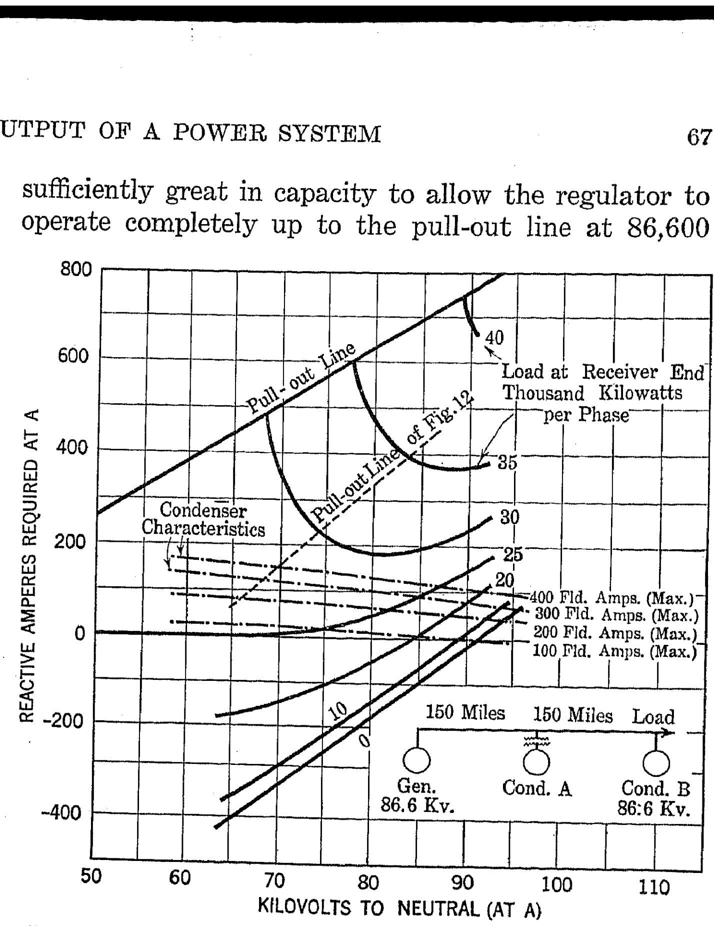

FIG. 9—RELATIONS BETWEEN CURRENT AND VOLTAGE OF THE MID-POINT OF A 300-MILE TRANSMISSION LINE RECEIVER VOLTAGE CONSTANT

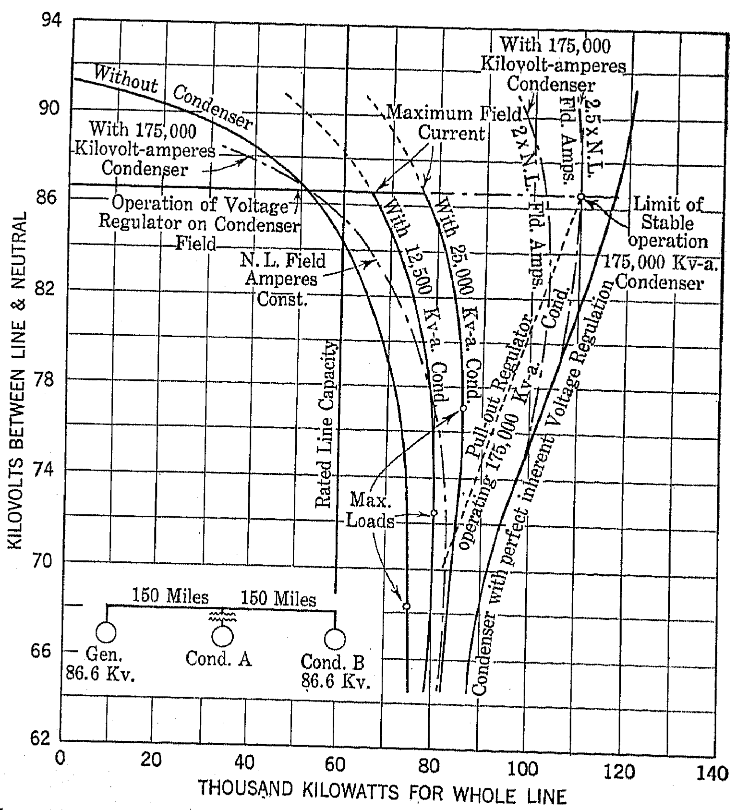

FIG. 10—CHARACTERISTICS OF VOLTAGE AND LOAD AT MIDDLE POINT OF 300-MILE TRANSMISSION LINE

volts, the inherent characteristics of the machine result in the limit of stability occurring at 110,000 kw. instead of the conventional 117,000 kw. Fig. 10 is of the same type as Fig. 4.

Fig. 11 has been plotted to show the relationship existing between the limit of output of a compound line and the size of the condenser used. It will be noted that to transmit the maximum amount of power over a line must become very uneconomical even if examined no farther than from the standpoint of the capital cost of synchronous condensers.

A final and rather elaborate study was made of this compound transmission line on the assumption that the condenser at the point B was finite in its characteristics. Two condensers, (a total of 50,000 kv-a.), each with characteristics represented by Fig. 7 were placed at this point while the load was assumed to be independent of voltage and at a constant power-factor of 90 per cent. The amount of labor resulting from this change was several times greater than before, the line being equivalent of a compound line of three sections. It is believed, however, that it may be possible to reduce this labor very considerably by using kinematic devices to represent the line characteristics. Fig. 12 is the diagram, resulting from this study, it is of the same general character as Fig. 9 although the limits are still further reduced. The above assumptions made concerning the load characteristics are only roughly approximate. If, for instance, the load is made up of 70 per cent rotating load and 30 per cent lighting load the increase of current with the decrease of voltage will be roughly half as great as in the case of constant power, which would tend to increase the stability. The assumption of the power factor being independent of load is probably not greatly in error. In a highly diversified load, where both synchronous and induction motors may be operated at various fractions of their ratings their effect on power factor with a change in voltage may be largely equalized. Where closer approximations of the load may be made the hyperbolic loci may be modified to correspond, an instance of which is given in Fig. 6.

The results of Fig. 12 have been included in Fig. 11 so that the error made in assuming a constant voltage at the receiving end is indicated as the difference between the two curves. With a 25,000-kv-a. condenser at the station A in each case the limit of output is reduced from 86,000 kw. to 75,000 kw.

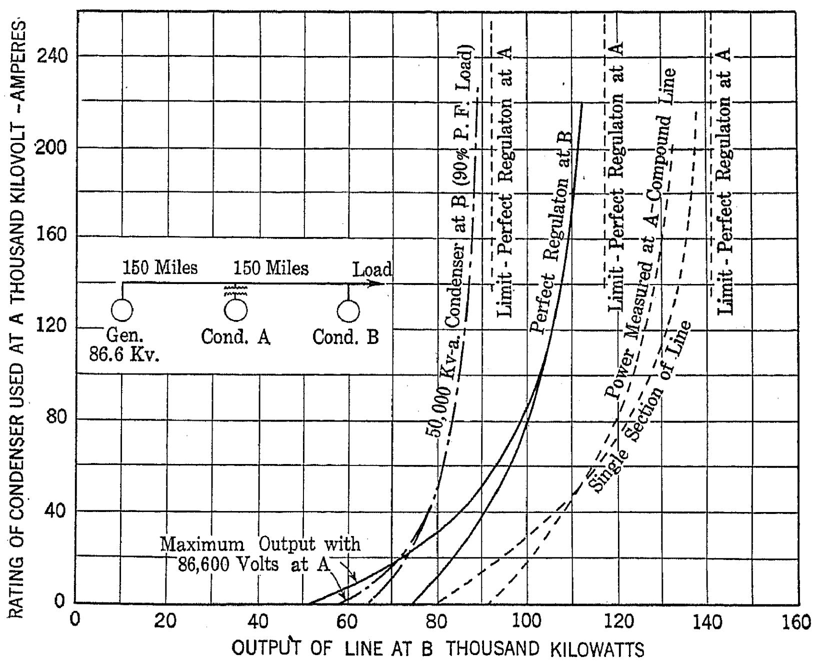

Fig. 11—Limits of Output of a Compound Transmission Line as Determined by the Condenser Characteristics

Summary

In the foregoing pages a method for determining some of the limitations of transmission systems has been developed in accordance with the conception of operation outlined, representing as nearly as practicable, it is believed, the actual conditions under which such a system will operate. In using this method it is essential that the characteristics of the apparatus connected to the transmission line, such as synchronous condensers and transformers, shall be accurately known as well as those of the line itself, and that reasonable approximations should be made regarding the characteristics of the load even when it is diversified. While this may often involve tedious complications the greatly increased accuracy of transmission line computations will much more than justify its use, if not making it almost a necessity in the case of compound lines.

The comparison of the various types of calculations given in Fig. 11 shows the importance of the more complete methods for these lines and how the maximum rating of a line may be increased by adding to the condenser capacity at the various stations. Beyond some point, however, the condenser capacity must increase rapidly for an increase of load. Such curves will furnish new data for the economic study of transmission systems.

A conclusion has been drawn that static condensers are not inherently suitable for line regulation even though their cost might compare favorably with that of the synchronous machine. The vibrating regulator in connection with the field circuit has been indicated is too sluggish to introduce a state of artificial stability

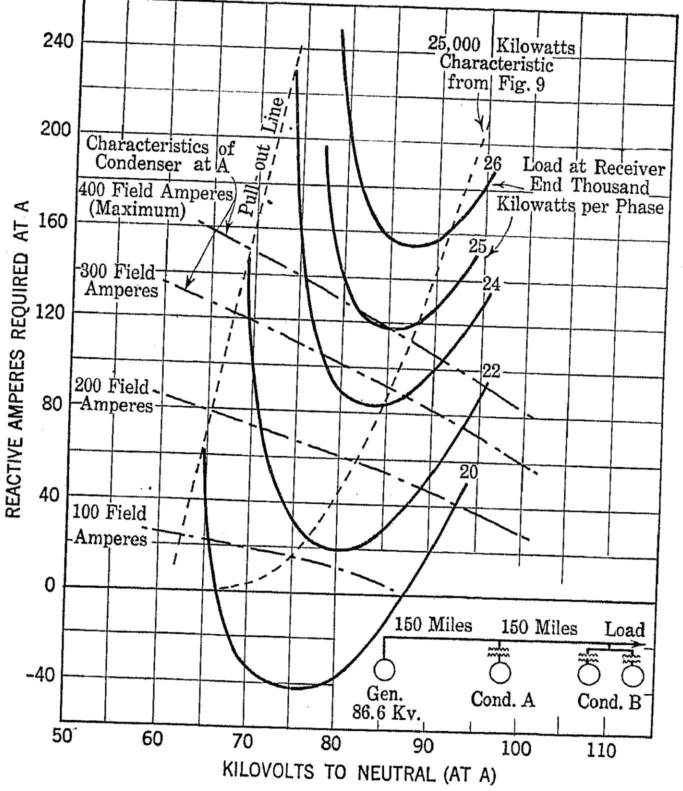

FIG. 12—RELATIONS BETWEEN REACTIVE CURRENT AND VOLTAGE AT THE MID-POINT OF A 300-MILE LINE WITH TWO 25,000 KV-A. CONDENSERS AT THE RECEIVER END

Load 90 per cent power factor

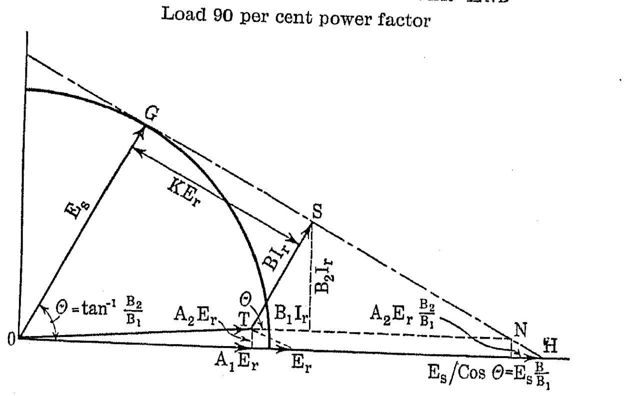

FIG. 13—VOLTAGE AND CURRENT RELATIONS FOR MAXIMUM POWER

extending beyond that produced by constant excitation even on steady loads.

The effects of fluctuating loads and load surges have not been considered. A load change of the type of a short circuit may be studied on the assumption that the field current of the condensers does not change during the period of load adjustment, but for fluctuating loads it will be practically impossible to determine the rate of increase of excitation with respect to the rate of increase of load.

Appendix

DETERMINATION OF I_s.

The construction referred to for determining the current at the sending end, I_s, has the disadvantage that a new axis of I_{sw}, ($L\,W$ in Fig. 2) must be drawn for each value of phase angle θ even although E_s is of constant magnitude. By transposing a part of Fig. 2 so that E_s falls on E_r and E_r on E_s, Q' will correspond to Q and V' to V. The triangle $L'\,V'\,W'$ corresponds to triangle $L\,V\,W$ so that $L'\,W' = B\,I_{sw}$ and $W'\,V' = B\,I_{so}$. By this modification of construction the I_{sw} axis becomed fixed for each magnitude of E_s. When obtaining series of values at different loads this will simplify greatly the graphical constructions. It should be noted, however, that the inversion of the diagram will also invert the angles of lag and lead with reference to I_{so}.

In the line diagram the values of current are expressed explicitly as voltages, $B\,I_{rw}$ for instance represents a voltage; by constructing a scale in the ratio of $1/B$ to the voltage scale whereby all currents may be read directly.

MATHEMATICAL DERIVATIONS

A number of mathematical relations may be derived as follows: Let Fig. 13 represent a transmission line operating at the conventional point of pull-out. As in Fig. 1

$$E_s^v = (A_1 + j\,A_2)\,E_r + (B_1 + j\,B_2)\,I_r \qquad (16)$$

Then as P_r is a maximum when $\theta = \tan^{-1} B_2/B_1$

$$T\,N = E_s\,B/B_1 - (A_1\,E_r + A_2\,E_r\,B_2/B_1) \qquad (17)$$

$$B\,I_{rw} = \left\{ \frac{B\,E_s - (A_1\,B_1 + A_2\,B_2)\,E_r}{B_1} \right\} \cos\theta$$

$$= \frac{B\,E_s - (A_1\,B_1 + A_2\,B_2)\,E_r}{B} \qquad (18)$$

I_{rw} (pull-out)

$$= E_s/B - \frac{(A_1\,B_1 + A_2\,B_2)\,E_r}{B^2} = E_s/B - M\,E_r \quad (6)$$

$$(P_r \text{ (pull-out)} = E_r\,I_{rw} = E_r\,(E_s/B - M\,E_r) \quad (7)$$

The value of P_r expressed by (7) will be a maximum when:

$$\frac{\delta}{\delta\,E_r} \{E_r\,(E_s/B - M\,E_r)\} = 0 \qquad (19)$$

$$E_s/B - 2\,M\,E_r = 0$$

or

$$E_r = \frac{E_s}{2\,M\,B} \qquad (9)$$

by combining (6) and (9)

$$I_{rw} \text{ (Maximum pull-out)} = \frac{E_s}{2\,B} \qquad (10)$$

the corresponding value of power is naturally

$$P_r \text{ (Maximum pull-out)} = \frac{E_r\,E_s}{2\,B} = \left(\frac{E_s}{2\,B}\right)^2 1/M \quad (11)$$

It is frequently desired to be able to determine the condenser capacity required under certain conditions of operation. To determine the reactive current in the line, but not that of the load, the following method may be used. From Fig. 1, the following relations may be derived.

$$\begin{aligned} E_s^v &= (A_1 + j\,A_2)\,E_r + (B_1 + j\,B_2)\,I_{rw} \\ &= (A_1\,E_r + B_1\,I_{rw}) + j\,(A_2\,E_r + B_2\,I_{rw}) \qquad (20) \\ E_s &= E_s^v + j\,(B_1 + j\,B_2)\,I_{ro} \\ E_s^2 &= (A_1\,E_r + B_1\,I_{rw} - B_2\,I_{ro})^2 \\ &\quad + (A_2\,E_r + B_2\,I_{rw} + B_1\,I_{ro})^2 \qquad (21) \end{aligned}$$

by substituting P_r/E_r for I_{rw} and solving for I_{ro} this becomes

$$I_{ro} = \frac{E_r\,(A_1\,B_2 - A_2\,B_1)}{B^2}$$

$$\pm \sqrt{(E_s/B)^2 - \left\{ P_r/E_r + E_r\,\frac{(A_1\,B_1 + A_2\,B_2)}{B^2} \right\}^2}$$

$$= E_r\,K \pm \sqrt{(E_s/B)^2 - (P_r/E_r + E_r\,M)^2} \qquad (12)$$

By eliminating the quantity under the radical

$$(E_s/B)^2 - (P_r/E_r + E_r\,M)^2 = 0 \qquad (22)$$

or $\qquad E_s/B = P_r/E_r + E_r\,M$

and

$$P_r = E_r\,(E_s/B - M\,E_r) \qquad (7)$$

Therefore, it is verified that the quantity under the

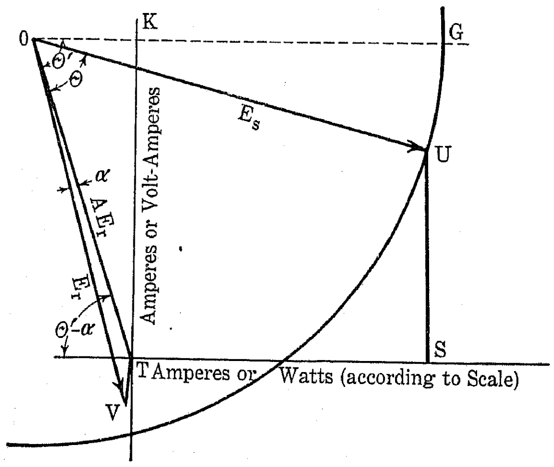

FIG. 14—POWER-CIRCLE DIAGRAM FOR RECEIVING END

radical sign in **(12)** disappears at the conventional point of pull-out, and that at this point

$$I_{ro} = K\,E_r \qquad (14)$$

POWER CIRCLE DIAGRAM.

It is of interest to show that the power circle diagram[8] is essentially the same as the diagram used in this paper.

Fig. 14, a typical power circle diagram is merely Fig. 2 rotated around the point T and using the line $T\,S$ as a longitudinal axis. The lettering is maintained to conform to that of Fig. 1. By using the scale ratio $1/B$, *i. e.* $O\,T = \dfrac{A\,E_r}{B}$ and $O\,U = E_s/B$ then $T\,S = I_{rw}$ and $S\,U = I_{ro}$, whereupon $T\,S$ and $T\,K$ become axes of in-phase and out-of-phase components of current respectively. By using the scale ratio of $\dfrac{3\,E_r}{1000\,B}$, *i. e.*

$$O\,U = \frac{E_s\,3\,E_r}{1000\,B},$$

the scales of the axes become kw. and reactive kv-a. respectively, which is the usual form of the power inch diagram. $T\,S$ is then the power output of the line and $U\,S$ the reactive kv-a. at the receiver end.

The upper part of Fig. 15 is identical with Fig. 14. The lower part is the part of Fig. 2 used to determine the conditions at the sending end of the line. The closed figure $O'\,T\,W\,V'$ of Fig. 15 corresponds to

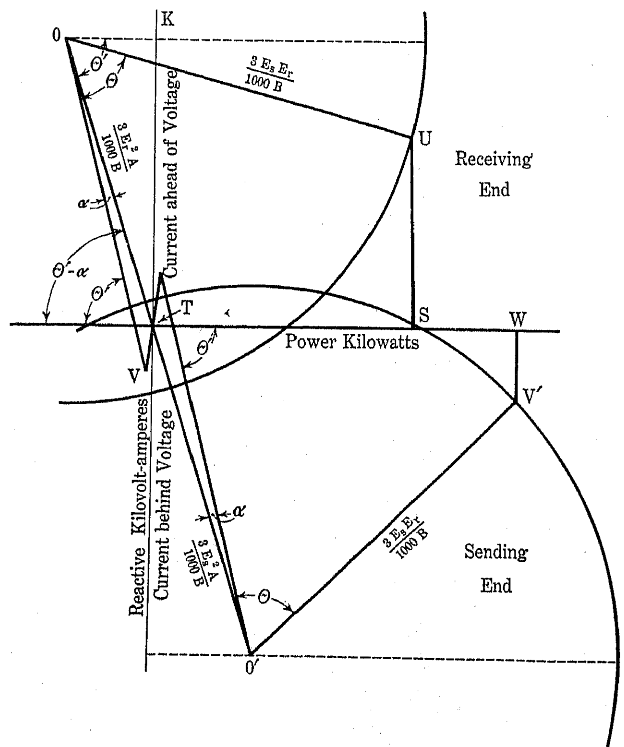

FIG. 15—POWER-CIRCLE DIAGRAM FOR BOTH ENDS OF TRANSMISSION LINE

$O\,L\,W\,V$ of Fig. 2. The corresponding scale ratio for this part of the diagram would be $1/B$ for ampere relations, or $\dfrac{3\,E_s}{1000\,B}$ for kw. and kv-a. With this latter ratio $T\,W$ will represent the power input to the line and $W\,V$ the reactive kv-a. at the sending end. The connecting link between the two parts of the diagram is that the angle $T\,O\,U$ must equal the angle $T\,O'\,V'$.

The line efficiency is $\dfrac{T_s}{T_w}$. To determine the angle $\theta' - \alpha$ for the purposes of construction.

$$\theta' = \tan^{-1} B_2/B_1$$
$$\alpha = \tan^{-1} A_2/A_1.$$

8. Economic Limitations to Aggregation of Power Systems, R. A. Philip, TRANS. A. I. E. E. 1911.

Constant Voltage Transmission, H. B. Dwight.

Transmission Line Circuit Constants and Resonance, etc., R. D. Evans and H. K. Sels, *Elec. Journal*, July, Aug. Dec. 1921, Feb. 1922.

Discussion

ECONOMICS AND LIMITATIONS OF THE SUPER TRANSMISSION SYSTEM
(THOMAS)

SOME THEORETICAL CONSIDERATION OF POWER TRANSMISSION
(FORTESCUE AND WAGNER)

POWER TRANSMISSION
(HANKER)

POWER LIMITATIONS OF TRANSMISSION SYSTEMS
(EVANS AND SELS)

EXPERIMENTAL ANALYSIS OF THE STABILITY AND POWER LIMITATIONS OF TRANSMISSION SYSTEMS
(EVANS AND BERGVALL)

LIMITATIONS OF OUTPUT OF A POWER SYSTEM INVOLVING LONG TRANSMISSION LINES
(SHAND)

F. G. Baum: The papers that have been presented so far are based on calculations made using the hyperbolic functions. On any transmission line of considerable length in which we apply voltage at one end, we immediately get some charging current near the station. That charging current tends to raise that voltage, and as we go along the line, each increment of the current tends to raise that voltage, and we have the rising characteristic of that transmission line.

However, if we are going to have a general power system, it is necessary, as far as I can see, that we have a practically constant transmission line.

If we are going to have constant voltage transmission, then you can see at once that the charging kv-a. per mile of line or hundred miles of line will be the same. That is, it is not a quantity then that is gradually changing as you proceed out from the station to the line, but every mile of line is a repetition of every other mile, so we have a straight line of relation of the charging line, if we maintain constant potential. That makes the attack of the problem very much simpler and you can use a simpler calculating device. The paper by Mr. Shand uses the old-time method of vector equations, which is correct so long as the charging kv-a. per unit length of line is constant, and that is the assumption we are starting out with.

Edward L. Moreland: The four papers which have been presented are interesting to anyone who is working on the problem of long-distance transmission—by long distance I mean distances of from 250 miles up to 500 miles or more.

Our office has for some time been working on a specific problem of this kind,—one of our clients having asked us about a year and a half ago to study for them the electrical and mechanical feasibility of transmitting a large block of power from Canada, a distance of approximately 500 miles. We made a preliminary report in October, 1922, that such a project is feasible, and have continued our investigations of the details since that time. Mr. Booth of our office has borne the brunt of this work. Professor Bush of the Massachusetts Institute of Technology has also aided in these studies and has worked with Mr. Booth in the development of the methods of analysis which we have applied to the problem.

In the course of our studies, we have also conferred from time to time with others who might be interested in problems of this kind, particularly with the authors of some of these papers and with the engineers in the research department of the General Electric Company, but we have apparently carried our analyses further than the authors of these papers.

There is a common point of weakness in the Thomas paper, the Evans and Bergvall paper, and the Fortescue and Wagner paper, namely, that all three make their analyses on steady-state conditions, and base their conclusions as to stability of operation and limitations of power on these analyses. The limitations are, however, not imposed wholly by steady-state conditions but also by transient conditions, and consequently analyses of steady-state conditions alone do not give a proper basis for conclusions. Before accurate conclusions can be drawn, analyses must be made taking into account the transient conditions produced by sudden changes in the load, including the effects of kinetic energy of rotating equipment connected to the system and the transient effects induced in the fields of synchronous apparatus connected to the line. Fortescue and Wagner discuss the transients in a general way but make no effort to calculate the effects, and their conclusions are based on the steady-state analysis.

Steady-state analysis is interesting only as a step in the complete analysis, but does not furnish a sound basis for drawing conclusions as to stability of the line or of power limitations.

Mr. Thomas bases his design upon steady-state analysis. This takes no account of the behavior of the synchronous condensers, or of the behavior of the system during load changes. A consideration of these matters soon shows that a 500-mile unsectionalized line is unsuitable for 100,000 kw. per circuit at 220 kv. normal terminal voltage. In lines of this length steady-state analysis is not sufficient, for the instability of the system is made apparent only by transient analysis involving the electrical and mechanical constants of the connected apparatus. Mr. Thomas uses a very large conductor, but our analysis has shown that stability under switching is not greatly affected by the size of conductor, within reasonable limits. The reason for these difficulties of stability lies in the fact that 220 kv. is inherently a low voltage for 500-mile transmission at 60 cycles per second.

Messrs. Evans and Bergvall point out the limitation that at a certain load the action of the regulators becomes indeterminate and produces instability. This again is based on an analysis which considers the performance of the system to be a succession of steady states. It, hence, applies at normal operating receiver voltage, but does not apply during fluctuations; for during fluctuations various factors, such as the kinetic energy of the rotating masses and the transients in condenser fields, come into play which are not considered at all in steady-state analysis. Their analysis, therefore, shows one limit to steady operation, but it gives no information about stability during switching. There is hence here no basis for general conclusions regarding the operation of a system where sudden load changes may occur due to switching. The tests made by Messrs. Evans and Bergvall are also limited to conditions of gradually applied load, for a motor-generator set cannot transfer its load to the system until it swings back in phase. This is clearly indicated by the oscillograph record shown in one of their charts. The behavior for a suddenly applied load is very different, as will be shown.

The chart given herewith (Fig. 1) shows the power curves for a 250-mile, single-section, 220-kv., 60-cycle line, with four 20,000-kv-a. condensers at the receiving end. The locus of steady-state Tirrill instability is also shown. Considering steady-state conditions alone, the chart shows that at 100 per cent normal voltage the line can deliver 190,000 kw. This would mean that if the line were delivering 100,000 kw., the load could be increased by 90,000 kw. without pulling the line out of step. This would be true if the load were gradually increased so that the regulators had time to function and keep the voltage constant.

If, however, the load is suddenly increased, the conditions are very different due to the transient effects. The heavy wave line indicates the cycle of voltage conditions through which the

system will pass if the load is suddenly increased from 100,000 kw. to 140,000 kw. The complete cycle has not been plotted but the oscillations will center on the 140,000-kw. line and ultimately come back to it. It will be observed that the first swing carried the system into the region of apparent Tirrill instability and safely out again; this is again due to transient effects. If, however, the load is suddenly increased from 100,000 kw. to 150,000 kw., the voltage will continue to drop on the first swing and will not recover, but the line will break out of step.

The significance of this chart is that it shows that steady-state analysis alone would have made it appear that 90,000 kw. could be successfully switched onto a line already carrying 100,000 kw.; whereas, in fact, less than 50,000 kw. could be suddenly switched onto it without breaking the system apart. This clearly shows that steady-state analysis alone does not give results sufficiently accurate to warrant using such analyses as a basis for drawing conclusions.

Discussions presented by Mr. Booth and Professor Bush give some specific illustrations of complete analyses under transient conditions.

The studies made by the authors are of great interest and the data from their tests form a valuable contribution to the knowledge available on this subject, but the conclusions drawn

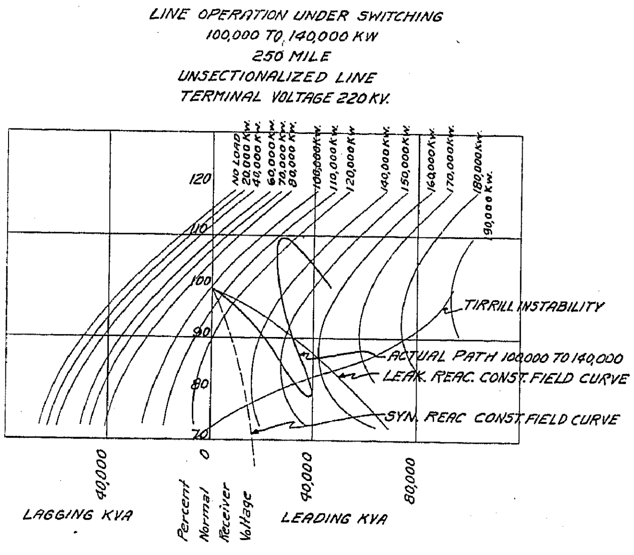

FIG. 1

should be qualified by the statement that various factors which radically affect the results have not been considered in their analyses.

PART I

R. D. Booth: Evans & Bergvall have analyzed the action of the Tirrill regulators on synchronous apparatus of the system for various steady-state voltages and found that instability occurred in the steady state at the points of tangency of the load and the field-current characteristics. However they apparently are applying this criterion to transient conditions. Mr. Moreland has, we believe, shown the fallacy of their method.

In particular we cannot agree that they have presented any substantiation for their general conclusions regarding the sectionalized line.

They have examined the load at which the regulators on the mid-point condensers become unstable in operation, and have assumed that this sets the maximum load limit for the system. Incidentally in this analysis they have assumed the receiver voltage to remain normal, and hence the only point of their curve of regulator instability which has physical significance is that corresponding to normal voltage. They have tested these conclusions by tests made with loads slowly applied through a motor-generator set. When a load is thrown on to the generator of the motor-generator set as in their tests, it does not become transferred to the system until the set falls back in phase. This occurs slowly enough so that their assumption of a succession of steady states is fairly close to being correct for this condition. But all of this leaves out of consideration what may happen in the system when a load increment is very suddenly added, as for example by the cutting out of circuit of a generating unit at the receiver end. In other words, they have examined one load limit for the system, but have drawn general conclusions which assume this to be the only limit which need be considered.

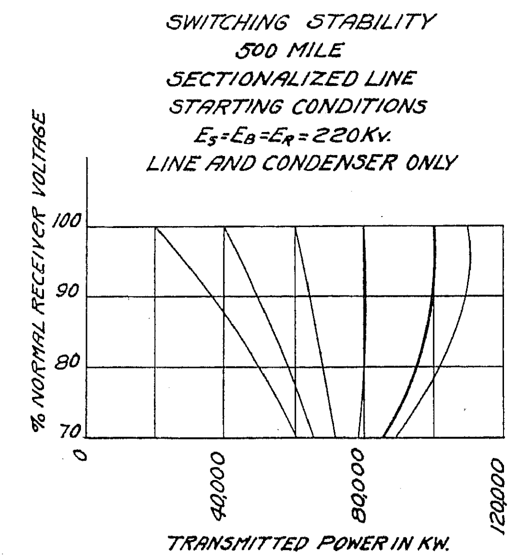

FIG. 2

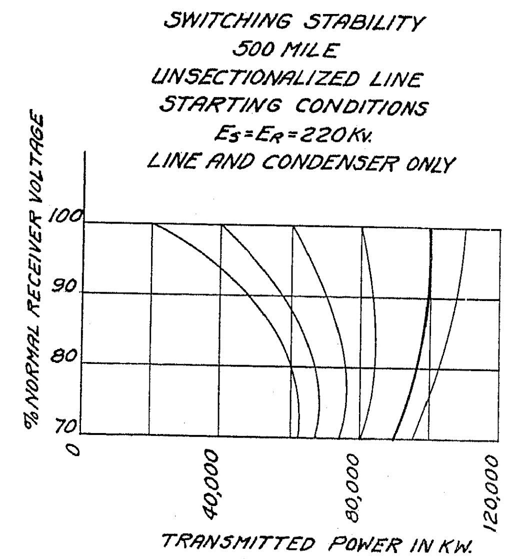

FIG. 3

In the early part of our analysis, we made for simplicity the assumption on which this paper is based, *i. e.*, neglecting the inertia of the condenser and the changes in the condenser fields. But we have examined on this bassis, in addition to regulator action in steady state, the effect of sudden switching. Even on this basis alone our conclusions as to the load limit of the sectionalized line would be very different from theirs.

We show in Fig. 2 and Fig. 3 the variation in receiver potential of the 500-mile line upon a sudden increase in load, Fig. 3 being for the unsectionalized line and Fig. 2 for the same line, with a mid-point condenser added. These curves were computed by

neglecting the inertia of the condenser as was done for the curves in the paper, but the changes of both terminal and receiver voltages in the sectionalized case were taken into account. The curves show the variation of receiver voltage upon suddenly increasing the load above that at normal voltage. The curves are here drawn for a load which is independent of the voltage, since this is the type of load considered in the Evans & Bergvall paper.

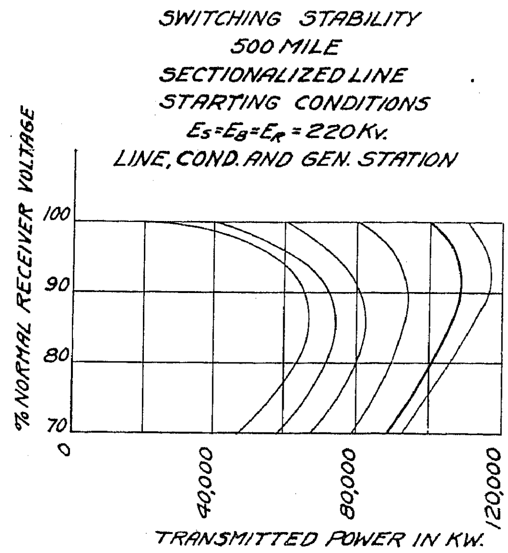

FIG. 4

It will be noted that the two curves of Fig. 2 and Fig. 3 above are not far different from one another. In each case they indicate that if the line were initially carrying 100,000 kw. a sudden increase in load of only 2 per cent, if applied before the regulators could act, would break the system apart. The curves are based, as are those of the paper under discussion, on the steady-state

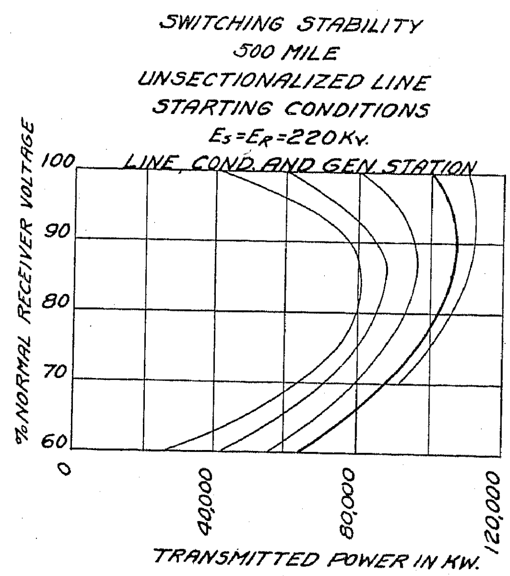

FIG. 5

characteristics of the condensers, that is, on their synchronous reactance, although, as noted, these do not strictly apply during such a transient condition.

In the statement of the methods followed in our analysis, you will note also that our studies include the coincident variation of receiver and mid-point potential for a sudden load increment, while the curves of the paper for the sectionalized line consider only the mid-point potential. It is the receiver-end potential that in this case actually sets the limit to load as far as switching is concerned, and for the sectionalized line it sets this limit far below the value found by the authors for difficulty with regulator action. Hence the premises of their paper, if applied to sudden load switching, indicate that adding a mid-point condenser with the mid-potential normally held at the same value as at the ends does not increase the capacity of the line appreciably for the type of load here considered. This is widely different from the conclusion of their paper that 40 per cent increase in capacity is obtained in this manner, and clearly indicates the need for

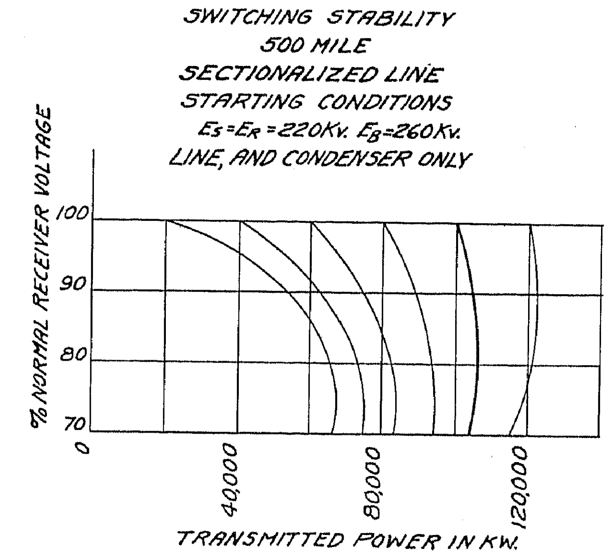

FIG. 6

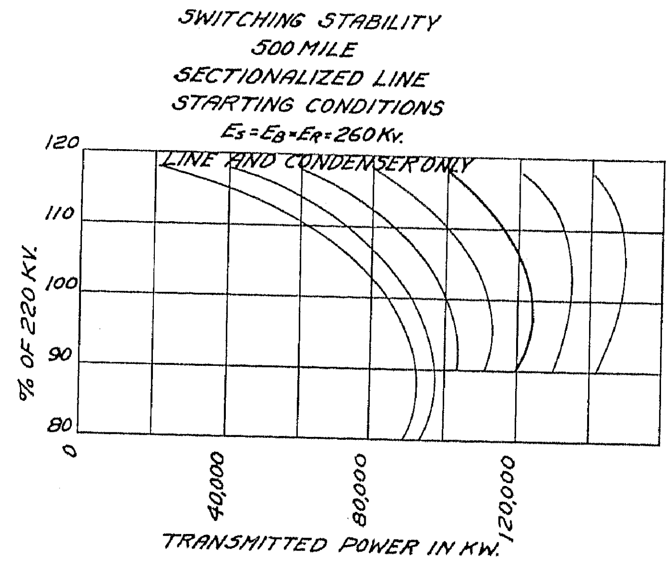

FIG. 7

complete study taking into account the factors here neglected. We have added, Fig. 4, a similar curve with the regulator at the mid-point set to maintain normally 260 kv., and Fig. 5 for line voltage of 260 kv. throughout. The installed capacity of synchronous condensers is, of course, different for these cases. This indicates that there is some benefit by this voltage increase. The stability of the line for a load which varies with the voltage may be readily obtained by plotting the characteristics of the load on the diagrams here presented.

The stability of a transmission system is of course, affected by all rotary apparatus connected to it, and therefore we have

shown the effect of the synchronous apparatus in a large generating station connected at the receiver end of the transmission line.

It must be emphasized again, however, that the analysis of the paper and of the part of our work presented above, neglects certain factors which greatly modify the action during switching.

PART II

Note on Method of Computing Stability of the Two-Section Line

(Kinetic Energy and Tirrell Transients Neglected)

The method of computing stability curves for the line with a mid-point condenser will now be outlined. This is based on the assumption of no inertia in the condenser, so that the phase of the condenser is assumed to follow that of the terminal voltage instantly. We have also used the steady-state characteristics based upon synchronous impedance. This involves the assumption of neglecting the transients produced in the condenser field current by terminal voltage variation during switching. The characteristics based upon leakage reactance may be used

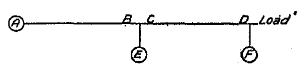

Fig. 8

instead, if desired, but this involves the assumption of neglecting the dying-out of field current transients. It is more conservative to use the synchronous reactance, so we have adhered to this assumption in our approximate analyses.

We have certain initial conditions on the system, and hence know the initial voltages, condenser fields, etc. We wish to examine the variation in voltage which will ensue if the load is suddenly increased from its initial value.

The method of analysis consists in general of finding a set of values which satisfies simultaneously the conditions at the mid point and the receiver end.

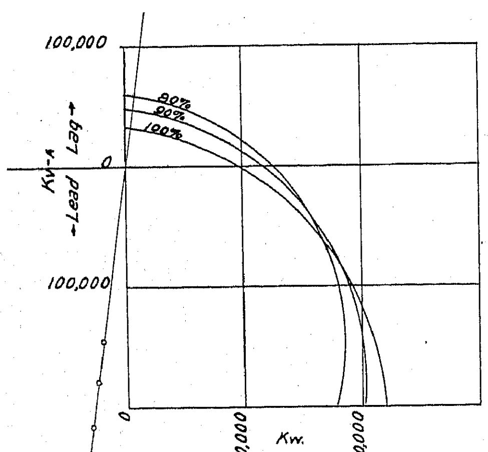

Fig. 9

The system we shall consider is as shown in Fig. 8. For the mid point we have the condition that the power at B drawn over the line must equal the input at C to the second section except for condenser losses. Also we have the fact that the difference in quadrature kv-a. at these two points must equal the kv-a. of the mid condenser. To satisfy these we will use the circle diagrams for the two sections of line, and the characteristic curves of the condensers at mid point.

Fig. 9 is the diagram for the first section based on a constant voltage at the generating station, and giving the relation between kw. and kv-a. at B for various values of mid-point voltage.

Fig. 10, drawn for the second section, is based on normal receiver voltage and similarly gives the relation between kw. and kv-a. at C for various values of mid-point voltage.

Fig. 11 gives the mid-point condenser characteristics, that is the variation of kv-a. output with terminal voltage for various values of condenser field current. The length (a) is the mid-condenser output for normal voltage at mid point and receiver,

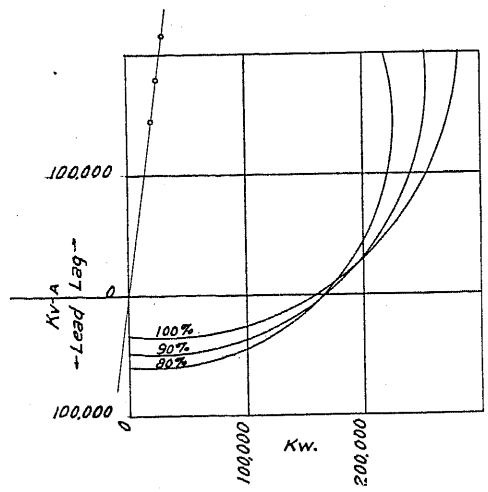

Fig. 10

and is found in the usual manner. The lengths (b), (c), (d) give the condenser output for this same field setting, and for various values of voltage at B.

Now superpose Fig. 10 on Fig. 9 as shown in Fig. 12 so that the scales coincide and pick off the values of kw. corresponding to a vertical distance between similarly labelled curves of the corresponding values (a), (b), (c), (d). Another way is to displace the curves vertically by the distances (a), (b), (c), etc.,

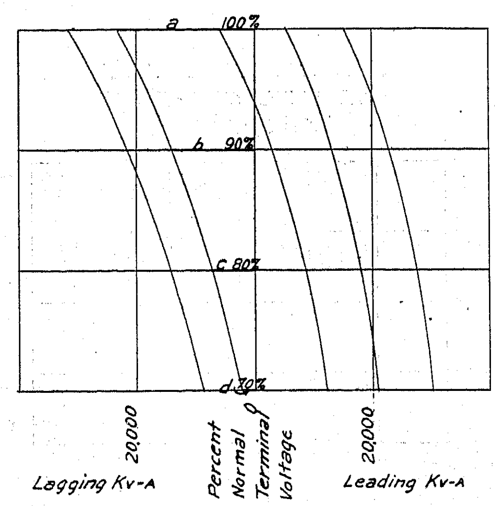

Fig. 11

in turn, as shown in Fig. 13 and read curve intersections. If the condenser output is lagging the curves will overlap. If it is desired to take into account condenser losses, the curves should simultaneously be displaced horizontally by the kw. of condenser losses corresponding to each successive value of mid-point voltage.

Now, convert the values of mid-point power to receiver end power, by means of a circle diagram. Plot the resulting values of mid-point voltage and receiver-end kw. as in Fig. 14 and label

the resulting curve 100 per cent E_r, corresponding to the normal receiver voltage which was assumed. It will be evident that points on this curve satisfy all conditions at the mid point for the assumed value of receiver voltage.

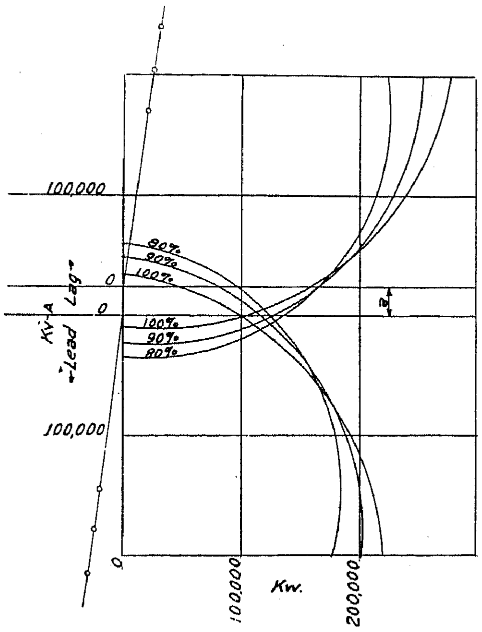

FIG. 12

Now, assume a new value of receiver voltage, for example 90 per cent of normal, use the circle diagram for the second section corresponding to this receiver voltage, and repeat this process. Plot the results as the curve 90 per cent E_r, in Fig. 14. Repeat for other values of E_r.

Turn now to conditions at the receiving end. We have here the conditions that the quadrature kv-a. of the line at D must equal the sum of the quadrature kv-a. of the condenser and the load. To satisfy these, use the circle diagram for the second

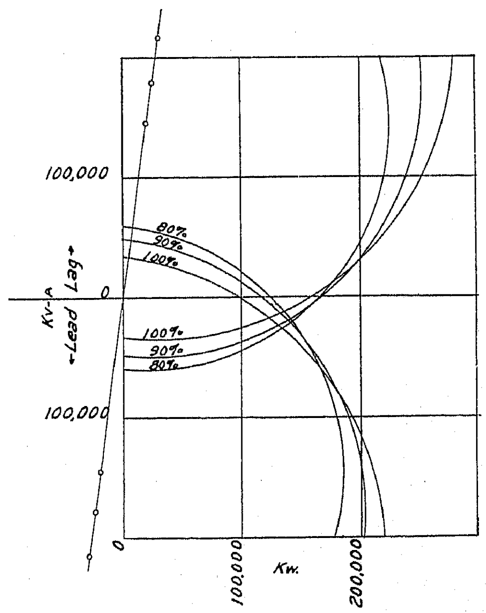

FIG. 13

section based on normal receiver voltage, Fig. 19, which shows the kw. and kv-a. over the line for various values of mid-point voltage. We make also a chart, Fig. 15, for the receiving end condenser, similar to Fig. 11, but based on a different capacity. Also we have the characteristic of the load, that is the variation in power factor of the load for different values of receiver end voltage. We can in addition take into account the characteristics of any generating station that may be connected

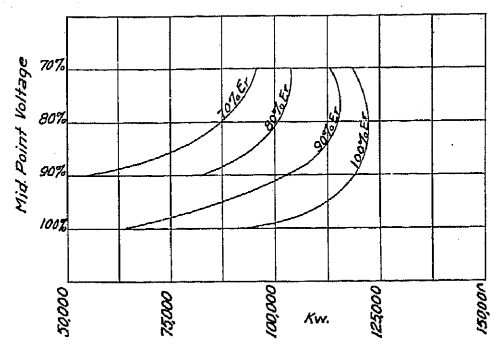

FIG. 14

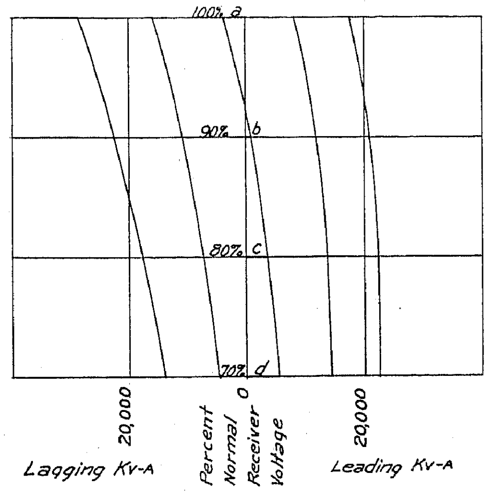

FIG. 15

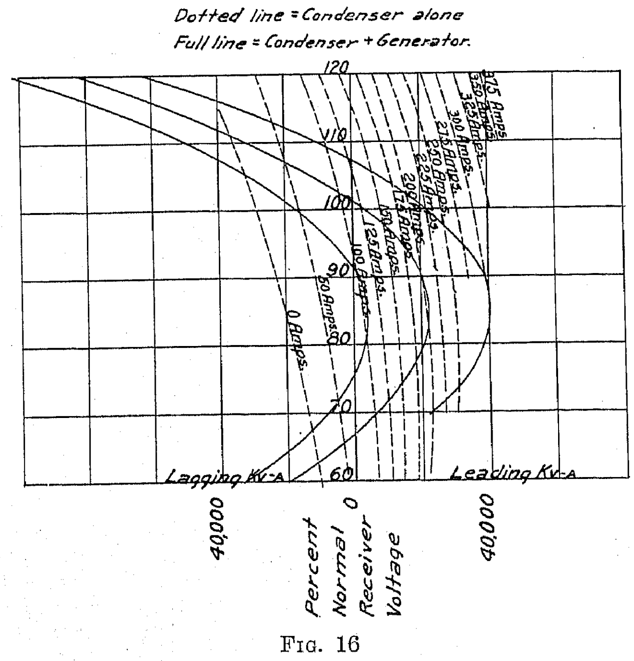

FIG. 16

to the receiver end of the line by adding them to the characteristics of the condenser station.

An operating point for the receiver end for each mid-point voltage will be at a certain load at which the kv-a. of the line is

equal to the kv-a. of the condenser plus the load. These points are shown on the above Figs. 17 and 18. For any constant E_r chart, we will have but one condenser value to consider, this may be taken in connection with the load at that particular receiver voltage.

From the intersections on each of these charts we obtain a curve of operating points which would fulfill all the operating requirements of the receiving end for the particular receiver

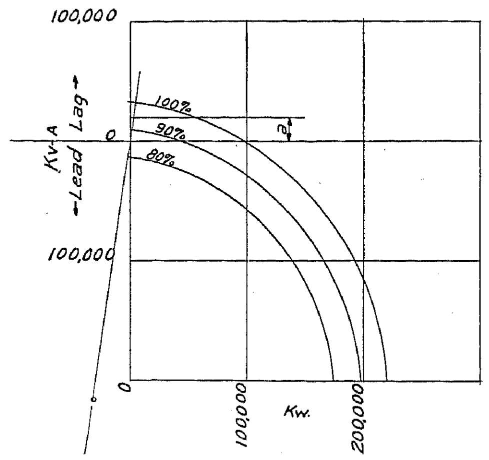

FIG. 17

voltage considered. We can then plot with voltage at the mid point against load at receiver end.

Calculations made with a number of these receiver charts for different receiver voltages will give a series of these curves for the different values of voltage at the receiver end, as shown in Fig. 19.

We now have two graphs plotted against identical functions, one of which shows the operating characteristics of the first

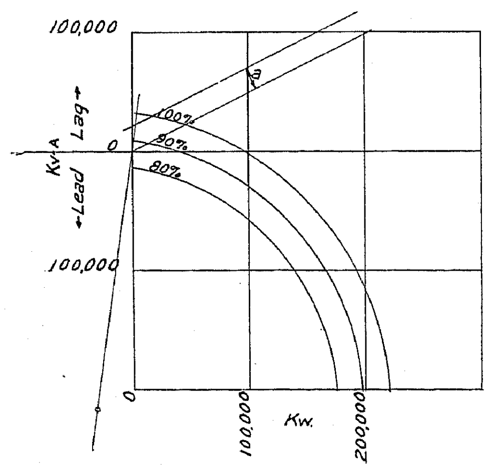

FIG. 18

section, and the other the characteristics of the second section; therefore by superimposing these as in Fig. 20, we can find the conditions which satisfy both sections. The intersections of the curves correspondingly labelled give values of receiving-end voltage and receiver-end power which satisfy all the requirements of the system. These points hence are operating points for the particular set of initial conditions considered. A group of these curves for various starting conditions shows the complete behavior of the line under sudden switching conditions with the single limitation that we have neglected inertia and condenser field changes in the computations. (Fig. 21.)

Upon this set of stability curves we can readily show the action of the line when loads of various resistance are applied. In Fig. 21 we have shown a set of stability curves with load lines representing fixed load of 85,000 kw. and another of fixed resistance of ohms.

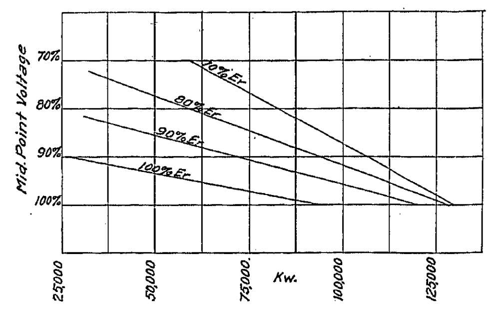

FIG. 19

With a line operating at 80,000 kw. and switching upon it a load of such character as to make the total equal to a resistance load as shown by line b we should have fallen immediately to point p and then as the regulator operated increase along line b to the operating point Q.

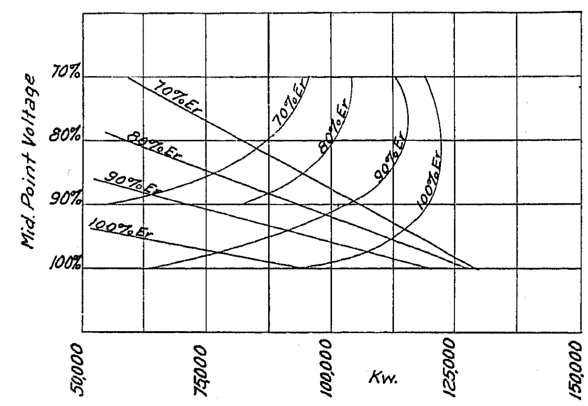

FIG. 20

It will be readily apparent that the above method can be extended to cover the case when load is drawn from or supplied to the mid point of the system. This is done by combining the station or load characteristics with those of the condenser at that

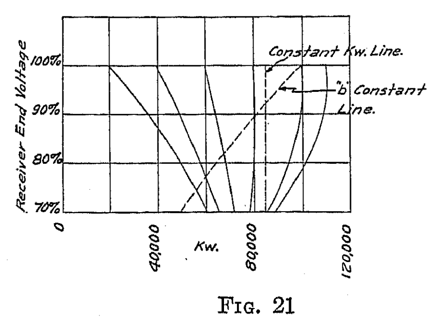

FIG. 21

point. Thus we have a method of treating a long line supplied with some power at a point along the line.

This proceedure undoubtedly appears laborious. It is really not, however, when the curves for lines and machines are once made available. Two computers can readily determine a complete sheet of stability curves in a day.

V. Bush: The criticism that has been made of Messrs. Evans and Bergvall's paper applies to the paper by Fortescue and Wagner. This paper discusses at some length the effect of kinetic energy in rotating masses, but so far as results from computations are concerned, the conclusions are based only on steady-state conditions and neglect the stored energy in rotating masses. Yet definite conclusions are again drawn in regard into consideration in the computations. We have not been willing in our work to accept conclusions obtained by neglecting condenser overswing, etc. Hence we have analyzed the behavior of a line during switching taking into account the kinetic energy of rotating masses, the transients produced in condenser fields, and the action of regulator and exciter during the transient period. We have completed this analysis for a 250-mile single-

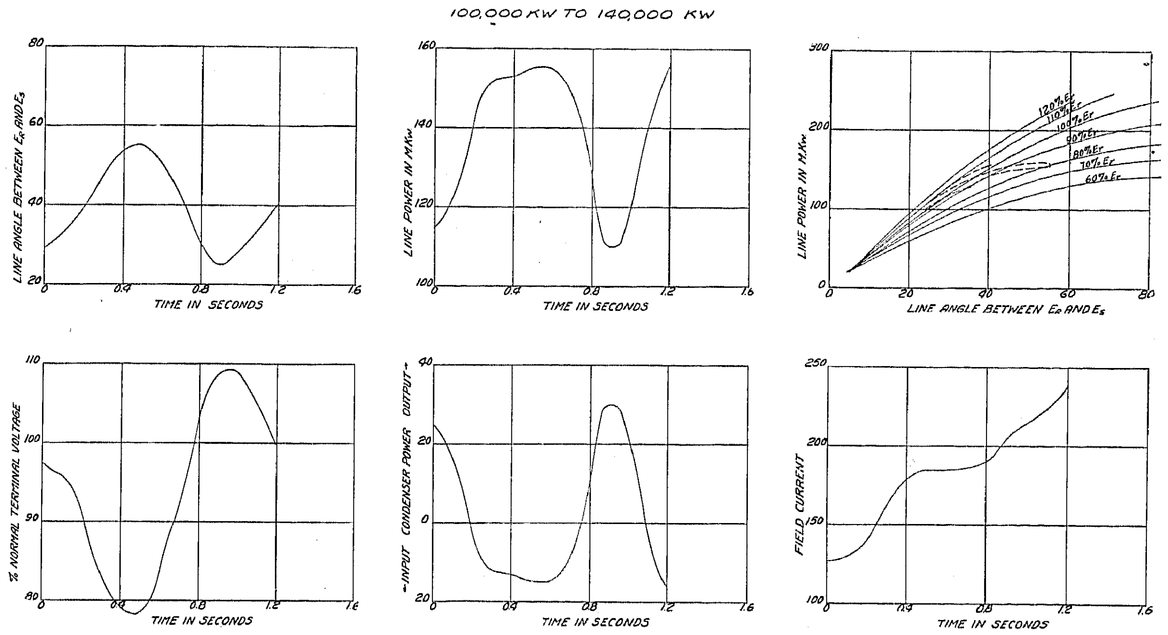

FIG. 22

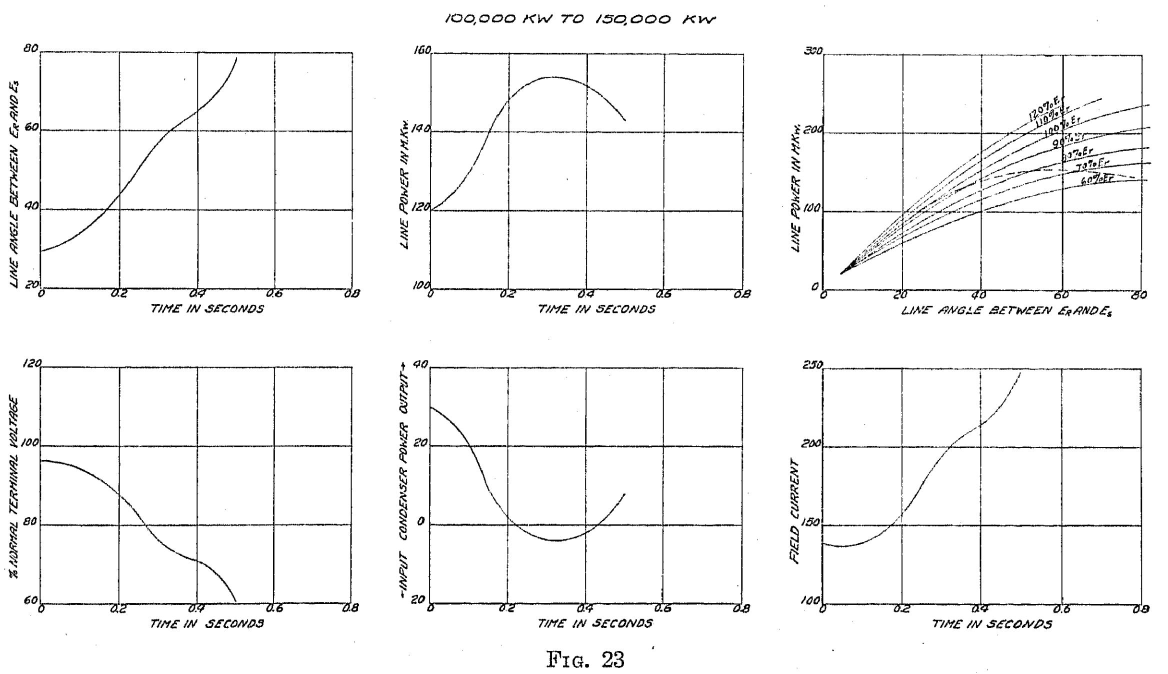

FIG. 23

to the increase in capacity of the line expected to result from adding a mid-point condenser. There is a qualitative discussion of the effect of some of the neglected factors and an arbitrary allowance of 25 per cent is made to take care of these, but this cannot be regarded as more than a guess. The authors evidently recognized that the factors neglected may be important, but they attempt no computations in which they are involved. Yet they proceed to draw general conclusions. General conclusions should not be drawn until the factors here neglected are taken

section line and now have in process the analysis for a 500-mile double-section line. The analysis of the 250-mile line was made as it forms the first step in the analysis of a two-section line. In Fig. 22 is shown the effect produced on a 250-mile line by suddenly increasing the load from 100,000 to 140,000 kw., the load in this case being assumed to have unity power factor. The variation in terminal voltage, phase angles, condenser field current and power from the condenser are shown. The field-current variation is controlled both by the terminal voltage

variation and the effect of the regulator and exciter. In this case the voltage was restored and the system did not pull out of step. In Fig. 23 the load was suddenly increased from 100,000 to 150,000 kw. and the system broke apart. Similarly in Fig. 24 the load charge was from 80,000 to 13,000 kw. and the system held together; while in Fig. 25 the charge was from 80,000 to 140,000 kw. and the system broke apart. In each of these cases the amount of load in kilowatt was assumed to be independent dict about the same load-switching limit as obtained by the complete analysis. This is, however, a coincidence. It happens that in this case the overswing of the condensers and the condenser field variation produced by voltage change and by the regulator, just about offset each other in effect. There is no basis, however, for any conclusion that this will happen generally; in fact, there is no such coincidence in certain other cases we have examined. We have presented here the curves applying to the

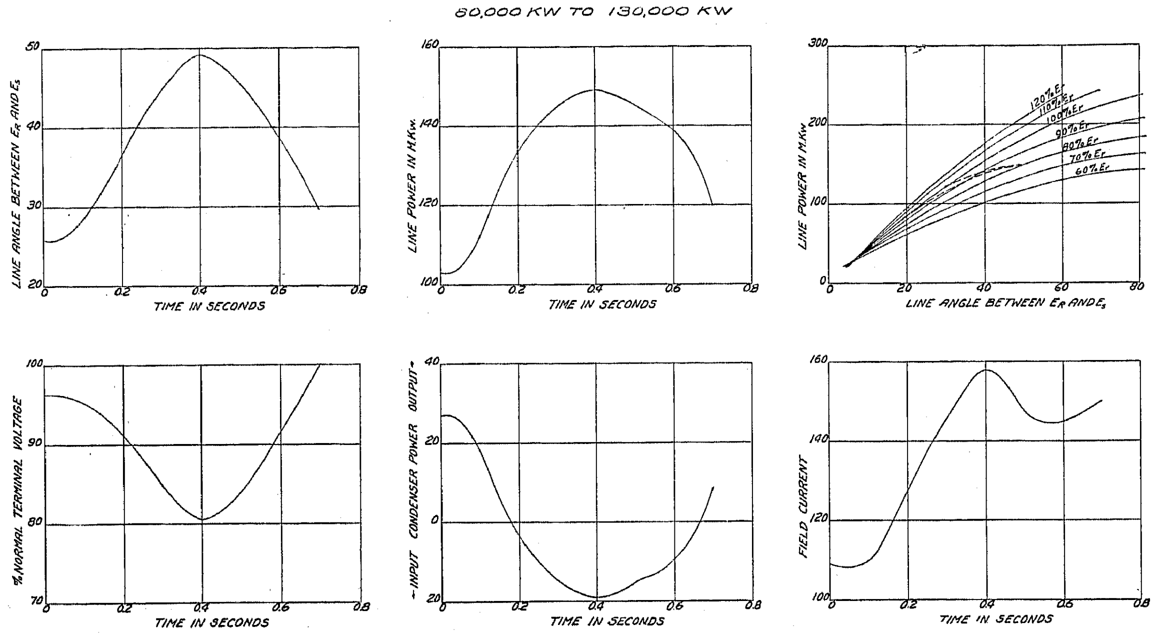

Fig. 24

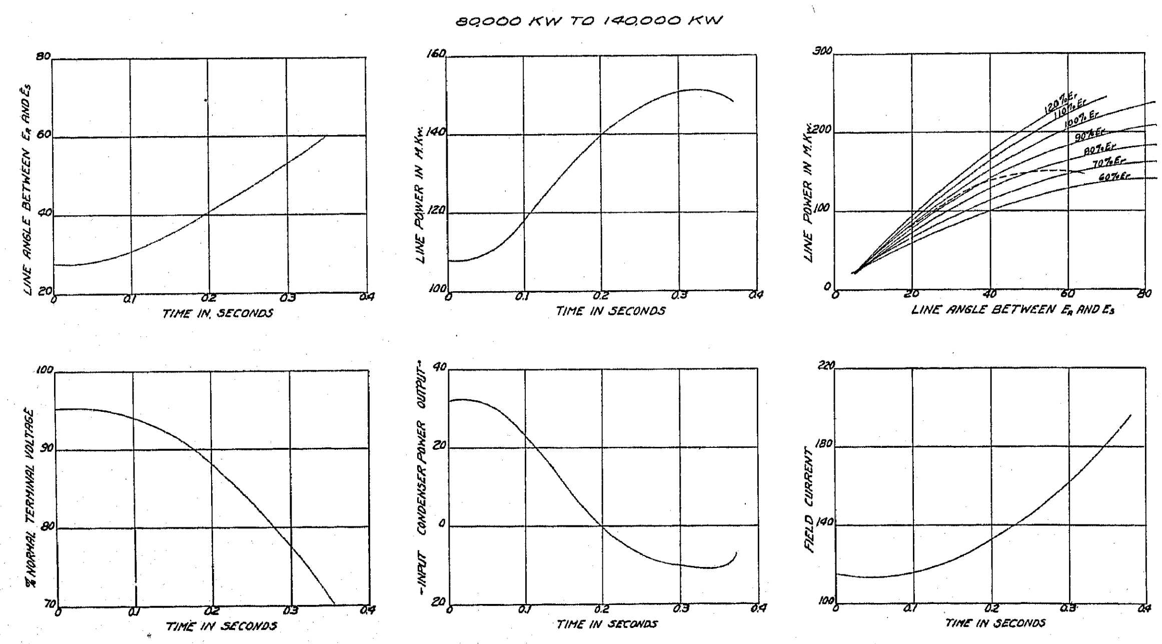

Fig. 25

of the voltage, since this is the type of load most readily considered in computation. Loads of a different character may be treated by the same analysis. The method of computing these curves and of taking into account the factors neglected in the papers presented here today, is too complicated to be explained at this time, but will be given in a subsequent paper.

In Fig. 26 are shown the stability curves of this same line neglecting the inertia of the condensers and changes in condenser field current during switching. It will be noted that they predict line and condensers only, as this is the part of the system considered as a unit in these papers. The stability of this unit will, of course, be affected by all rotary apparatus connected. Mr. Booth, in his discussion showed the effect of a generating station as determined in that approximate analysis and shown in Fig. 7.

H. Goodwin, Jr.: Under the caption "Growth of High-Voltage Networks Introduces Control Problems" the *Electrical World* on September 8, 1923, outlined very ably some problems that are confronting those interested in such development and

closed with the sentence: "Some genius has an opportunity to obtain a better solution to the general problems on such systems."

After outlining a comparatively simple network the editor asked these questions: How can the voltage regulation at each substation be controlled? How can the load division between generating stations be controlled? What are the limits of stable operation of the system if certain short circuits occur? Later the present devices and methods are referred to as unsatisfactory for "the new type of system where an immense amount of energy is to be handled and reliability of service is paramount."

In the October 20, 1923 issue of the same paper there appeared

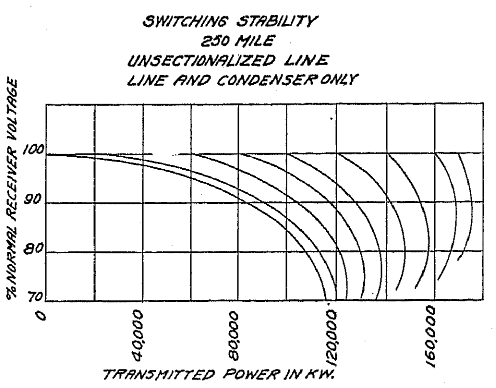

FIG. 26

a letter referring to the editorial and claiming that most of the points had already been answered.

But I and many others cannot agree with the correspondent and welcome most heartily the papers presented here today and congratulate their authors on the opportunity they have had to make the investigations and further on the simple terms to which they have reduced the problem in presentation.

Mr. Thomas' paper is a most excellent introduction to the subject (but not limited to introducing) and I should like to go into it and accentuate its many good points. But I shall have to pass on with recommending it for the study which only will give a full appreciation of its comprehensiveness. Issue might

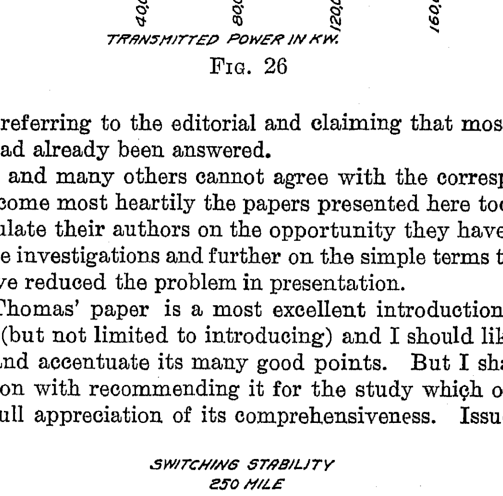

FIG. 27

be taken with his suggestion of a 400,000-kw. substation. It would, in general, be better to split it in two or three. His detail proposal of generating and substation connections at first seems out of place, but on more careful investigation it is evident that a close relation exists between the general problem and these detail connections. The exact set-up has a considerable bearing on the operation of such a system as he is considering.

It is most interesting to compare the integral operation he gives, for his system under short-circuit conditions with that proposed for a superpower system in a report of considerable authority on that subject. The latter says: "If a short circuit occurs the system will be automatically separated at selected points into several systems that are complete in themselves, in order to limit the energy interrupted to amounts that can readily be handled."

The other papers deal with the circle diagram and advocate its use. May I suggest two other things almost as potent as the circle diagram for elucidating the details of action of a transmission line: 1st.: Following Dr. Kennelly in the use of polar coordinates for vectors wherever possible instead of the rectilinear form "$a + j\,b$." 2nd.: The use of "Qualitative Analysis of Transmission Lines" as a means of forming a mental picture on which to hang and by which to understand the detail mathematics. Mr. Thomas's paper is very good in the picture it gives of the system in addition to its mathematics.

On the first page of Messrs. Evans & Bergvall's paper, reference is made to the method of calculating circle diagrams previously given by Messrs. Evans & Sels in the *Electrical Journal.* This says in one place:[1] "When such a diagram has been finished, the most desirable conditions of operation can be readily selected, and the rigid mathematical solution may be applied to the particular case with any further degree of accuracy that may be desired." Later:[1] "The loss formula may be put in the form of a circle and plotted in connection with the regular circle diagram. This seems desirable to do, as the losses *neglected* when using the general circuit constants are practically constant for different loads, so that the most economical point of operating the line can be determined from the diagram." (Italics by the speaker). Yet do we find any "rigid" check of any of the results taken from the circle diagrams? And some results are almost hair-splitting.

Perhaps the paper by Messrs. Fortescue and Wagner, giving a more vigorous proof is supposed to serve instead. But why then refer to the old articles? Also, Messrs. Fortescue and Wagner do not refer to the "Loss Circle." Can it not also be proved by rigid mathematics? These questions are not post mortems. The old articles by Messrs. Evans and Sels have been made a part of the present group by reference. The circle diagram has had a very checkered career in the past. If it is now to be used as the foundation for a great forward step in the art, as in these papers today, it must be cleared of all doubt and set forth in new clean clothes. Messrs. Fortescue and Wagner do this in part. How about the rest particularly the "Loss Circle"? Has anything been "neglected" in the use of the circle diagram in these papers?

There is apparently duplication between several of the papers. This may have the advantage of giving different points of view of the same subject. But on the whole it seems that it would have been preferable, since the authors are so closely associated, to have put all related matter together in the best form and devoted the remaining space to some phases that have not been covered, *e. g.*:

A fuller development of the problems involved with comparatively short ties of large capacity systems such as: Newark & Philadelphia; Baltimore & Washington; East Central & West Central Pennsylvania. These are of almost immediate importance.

(Messrs. Evans & Bergvall give a most encouraging point in this connection on page 15. It appears that a number of stations in parallel along a line will help to hold each other in step at times of short circuit.)

The design of synchronous condensers to meet the requirements found, instead of assuming present design as the limit of operating conditions.

In the discussion of a 750-mile line, Messrs. Evans & Sels paper there does not appear to be proof of the statement, "the receiver must operate at lagging power factors," etc. and this is apparently at variance with Messrs. Fortescue and Wagner, in regard to a 500-mile line requiring 15,000 kv-a. *leading* at the receiver. In the next paragraph the reasons for limiting to 250 miles are not at all clear.

1. ***Electrical World*, Decembe. 1921, p. 530 and p. 533.**

In the same paper the notation used in the equations is not standard or defined. It is assumed that the special symbols indicate vectors and conjugate vectors.

We meet the suggestion that "the condenser capacity Q_R may be chosen as equal to $(- m \, E_R{}^2)$." On looking for the value of m in equation (26) we find it is imaginary. Referring to Pender's Handbook page 240 we read:

"A real quantity cannot be equal to an imaginary quantity."

But even if these differences can be adjusted more detail of references are necessary to clarify the process of deduction and prove its correctness. It is not clear that the balance of equation (26) will not have an angle in it in addition to the j.

"Infinite" condenser capacity is referred to in some places while "unlimited" is used in others. Infinity is a rather dangerous toy and if the discussion could be confined to "unlimited" some will follow the deductions with greater ease and assurance.

Mr. Shand draws inferences from my paper of last year "Qualitative Analysis of Transmission Lines" and then proceeds to find fault with them. At the time the paper was presented, two of the other authors of today's papers "by inference" developed things to discuss adversely that were not in the paper. The one inference to be drawn from that paper and the whole point is that an understanding of the fundamental physics and most simple mathematics of the transmission line and the "critical load" derived therefrom are of the greatest benefit in understanding the action of a line and such detail mathematics as have been presented today.

The paper of last year showed a simple method of determining the critical load of a line and the characteristic action of loads above and below this on present commercial lines. It was then shown, by example, that if the critical load was put on an extra long line in a certain way, there would be no reactive energy drawn from synchronous condensers even if they were placed along the line. The circle diagram was then used in adverse discussion. It is most interesting to see its use here confirm the points made in last year's paper: In Mr. Shand's paper, read of operation at critical load, that the intermediate condensers "might be merely floating on the line without carrying reactive current." In Messrs. Fortescue and Wagner's paper, read the author's judgment that while with large amounts of synchronous-condenser capacity distributed on a 500-mile line, larger loads would be possible, the practical rated load would be the critical load. "The actual reactive power required at this load is zero at the midpoint and about 15,000 kv-a. leading at the receiver." In the discussion last year the circle diagram was used to prove that the most efficient operating conditions would be with reactive kv-a.

Your attention is also drawn to the many circle diagrams and the general intersection of the circles at the critical load and a slightly leading power factor which is to be expected from qualitative analysis.

Now these things are not said to continue or develop a controversy but to point out that in today's papers a difficult subject has been ably dealt with. The subject is so difficult and involved with so many practical considerations, that it is apparently impossible of general mathematical solution. The only means at hand is the working out of specific problems and generalizing from them. Generalizing is a difficult and dangerous thing without a guide. The best guide I know is qualitative Analysis, enabling one to relate the results of a specific problem to the fundamental physical conception and thereby be able better to judge the effect of change of any of the conditions or constants.

A. E. Silver: One point in Mr. Thomas' paper of particular interest is that of the avoidance of all switching on the 220-kv. side as shown in the circuit diagram, Fig. 5. This scheme is ingenious and embodies many advantageous features. However, I believe it doubtful if there can be anticipated many actual situations, when analyzed on the basis of probable conditions, in which high-tension switching can be advantageously avoided. Any 220-kv. transmission development that may at present appear to offer some probability of materialising in this country, would, I fear, prove unsuited if laid out without at least providing for later high-tension switching.

Especially at the receiving end of such a system it may be expected that interconnection between large load centers will occur hand-in-hand with, and in some cases precede, the construction of long transmission circuits of this kind coming in from distant water powers or fuel fields. It also may be expected that circuits for such interconnection must be of capacity commensurate with that of the long 220-kv. transmission circuits, otherwise full advantage cannot be taken of such factors as load diversity, reserve capacity in existing plants and load equalization over generally parallel circuits. Should several long 220-kv. lines emanate from one center of generation it is probable that, in the beginning at least, they would diverge at the receiving end so that one or perhaps a pair of circuits would deliver at each of several separated major load centers. The reserve against outage of any such 220-kv. circuit would in considerable part be provided over the interconnecting circuits between these receiving stations. Such interchange through the medium of the network on the low-voltage side of the 220-kv. transformers would, in general, appear inadequate and uneconomical because of the relatively large increase required in the transformers and lower voltage line capacity.

In other words, regardless of how simple the beginning may be, I believe the prevailing situations in laying out such a 220-kv. system will require looking ahead to a process of extension that will continue until it has developed and merged into a 220-kv. network embracing several or many major load centers and extending over a wide power consuming area. It would seem impracticable to develop such a network simply and economically and with the needed flexibility if based on other than 220-kv. switching.

At the generating end of such a group of long 220-kv. transmission circuits it occasionally may occur that the supply will be derived from a single large development so located relative to other power developments as to preclude the practicability of tying together at 220-kv. It is believed, however, that the more frequent condition will be the supplying of such a group of circuits from several moderately large plants separated by appreciable distances, as for example, a series of developments along a river. Usually, therefore, it is believed that 220-kv. switching at the generating end will be better suited for flexibility and load equalization.

As Mr. Thomas points out one definite disadvantage of the scheme of eliminating 220-kv. switching which places the transformers as an integral part of the line is the fact that the overload capacity of transmission circuits is entirely dissimilar to that of transformers. The overload characteristic of a transformer is similar to that of a generator, that is, a definite limit relatively close to normal-capacity rating beyond which the apparatus cannot be loaded without damage from heating. On the other hand, the transmission circuit has no such limitation as concerns its own safety, its practical load limit being set by voltage-regulating equipment. Economical design may permit a load, on shorter lengths of line, as great even as several hundred per cent of the load normally carried on a circuit.

It is thus evident that for utilizing the overload capacity of any of the circuits during times of maintenance or emergency, the needed flexibility can most readily be obtained by paralleling and switching on the 220-kv. side.

A system designed for 220-kv. paralleling and switching, I believe to be capable of greater simplicity than a similar system laid out for equivalent load equalization and emergency transfer of power entirely through the network on the low-tension side of the 220-kv. transformers.

It must be recognized, of course, that adoption of 220-kv.

bussing and switching for an interconnected network places the problems of circuit and apparatus arrangement, design and performance in a field of magnitude well advanced over that of the more or less conventional practise of today for lower voltages. It means that persistent attention must be given to developing and perfecting simple and dependable circuit and switching arrangements and apparatus including selective relaying and switch operation. Of particular importance is the problem of suitably dealing with short-circuit conditions. Also there will be required simple and effective devices for 220-kv. synchronizing and metering and for performing any other functions which would logically be required in the major bus of a station where the lower voltage switching facilities have been minimized. There seems no good reason to doubt that by persistent effort these devices and methods will be made available as needed.

It is, of course, not the intent of Mr. Thomas' paper to cover the situation of 220-kv. networks which I have tried to describe but rather the case of a single large generating center directly connected by high-tension lines to a single large receiving station. I am sure we would all appreciate an extension of his studies to include this companion problem of an interconnected network of large generating and receiving centers.

Mr. H. R. Summerhayes: The papers show that the transmission engineer is in the position of a navigator approaching an unfamiliar coast who pauses to take soundings, consult his charts and get bearings on a lighthouse or some established guide to navigation.

This subject of long distance transmission is of especial importance now because of the financial and economic pressure. A 500-mile line under certain conditions of cost of water power and price of coal, must transmit in the neighborhood of 100,000 kw. to justify the construction of the line from a financial and economic standpoint.

It would be easy probably to transmit 50,000 kw. over a 500-mile line, but when it comes to 100,000 kw. we are approaching the limit, as far as practical operation is concerned. A 250-mi. line is of the order of fifty or sixty per cent reactance when you are considering one hundred thousand kw. A five hundred million is over a hundred per cent reactance and it becomes a rather thin tie between generating stations of that size and larger. I think that several of the speakers discussing the papers have called attention to the fact that the transient condition should be considered and some engineers whose work I am familiar with have given special attention to these points, that is, to the sudden addition of load and what happens on the line when an increment of load is suddenly added.

To me the most significant part of these studied is the effect of the high reactance in the reduction of voltage.

Now, speaking of the charts of the navigator, the engineers have made some very creditable and thorough studies and we have been introduced to curves and whole families of curves, not simple ones but a lot of them so that one begins to think we are not only approaching the limits of transmission, but the limits of understanding the human mind. We meet so many of these curves and try to understand them that it becomes difficult.

However, this voltage question is large, and one can't put the whole subject in a nutshell, but it does seem to me that one conception of it is this: that you have to have voltage enough to transmit the power. If you suddenly increase the load so that the voltage momentarily goes down and there isn't voltage enough left to transmit the increased power, then before your voltage regulator can get busy your station will fall out of synchronism and the determination of that point of lost stability is the aim of a great deal of this work.

Edith Clarke: Messrs. Evans and Sels have explained the calculation of transmission lines by means of circle diagrams in a manner which is clear and easy to follow. I read their treatment of the subject in the *Electrical Journal* and have employed it in transmission line calculations. It is an accurate and rapid method. A complete circle diagram can be made in less than an hour and all information in regard to the transmission line is then available.

I have been accustomed however, to presenting the results of transmission line calculations in a different manner, and as this method shows clearly the variation of the receiver voltage as the load is increased or decreased, the generator voltage remaining constant, I have prepared a chart to illustrate it. (Fig. 28).

This chart illustrates a method of plotting results of calculation. The calculations themselves can be made by any method whatever. In this particular case, the calculations were made on the transmission line calculator described in the *General Electric Review* of June 1923. Such curves can be run off in a few minutes by using the calculator. They could be made as well from values taken from the circle diagram. The circle diagram could also be made from these curves.

Fig. 28 shows two families of curves, one for a 100-mile line, the operating characteristics of which are familiar to you, and the other for a 500-mile line which is the subject under discussion.

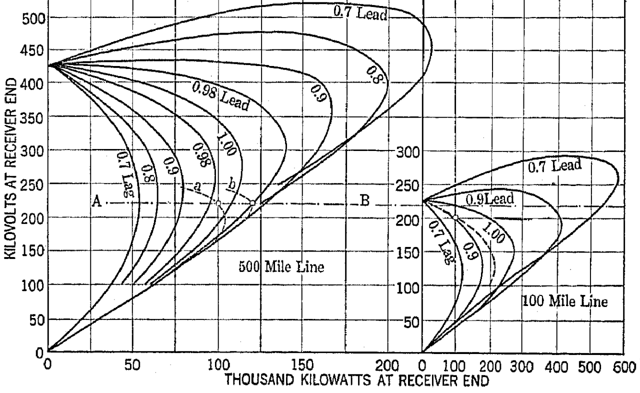

FIG. 28

You will notice in these curves kilowatts at receiver end against kilovolts at receiver end, generator voltage remaining constant for various power factors at receiver end. For each power factor at the receiver end there will be a different curve. The same conductor is used for both families of curves, the same frequency (60 cycles) and the same generator voltage (220 kv.). They have the same general shape. They are like the regulation curves of a shunt generator. With no load at the receiver end on the 100-mile line the voltage of the receiver is 225 kv., while on the 500-mile line it is 425 kv. due to the large charging current.

Let us look at the 100-mile line first. It is customary to operate such a line with the receiver voltage lower than the generator voltage and with a power factor of unity or less. If we follow the unity power factor curve, starting off with no load at the receiver and adding power, the receiver voltage continues to drop until we come to the maximum power for this power factor. This point is far below the usual operating voltage. It will be noted that as the power factor increases towards leading, the voltage at which the maximum power occurs becomes higher. Maximum power for a given line insulation is obtained by having the voltage at both ends the same, *i. e.*, with kilovolts at receiver equal to kilovolts at the generator. For the 100-mile line with kilovolts at the receiver equal to 220 kv., for maximum power we would have a power factor of about 0.7 leading and could deliver over 500,000 kw. Notice however, that we are now operating on the underside of the curve for 0.7 power factor leading and the case is comparable with the 500-mile line.

Let us pass to the 500-mile line. As we go out along the line *A*, *B*, Fig. 28, (220 kv. receiver equals 220 kv. generator), a more leading power factor shows an increase in kilowatts until we come to the limit of the line at this voltage. For the line chosen the limit is about 130,000 kw. and the power factor is 0.9 leading

while 0.8 power factor leading gives less power and 0.7 less still. This same thing has been shown on the circle diagram. With the receiver voltage equal to the generator voltage, for all constant receiver power factors we are below the maximum power points. Therefore, the 500-mile line would be unstable for a shaft load of constant power factor. Fortunately however, an induction or synchronous load does not have a constant power factor but a power factor which becomes more leading as the voltage drops and more lagging as the voltage rises. The constant power factor curves, therefore, are not the curves of receiver power against receiver voltage for induction and synchronous loads. You will notice two dotted curves. These are for induction motor loads in parallel with synchronous condensers with constant excitation on the condensers. The circles indicate the points of normal voltage operation. In Curve *a* we are delivering 100,000 kw. with normal voltage of 220 kv. An increase in load is accompanied by a drop in voltage and an improvement in power factor. Beyond a certain point, however, a drop of voltage means a more leading power factor but no more load. In Curve *b* we are delivering 120,000 kw. with normal voltage of 220 kv. We cannot get any more power with a drop of voltage although we get a more leading power factor. There is no margin. Now look at the dotted curve in the 100-mile line graph. We have the same load and the same condenser as in the dotted Curve *a* on the 500-mile line graph. We have also constant synchronous condenser excitation. A drop of voltage is accompanied by a more leading power factor and also more load, so that we can pass from 100,000 kw. to 200,000 kw. and beyond before we reach the maximum power point. The 100-mile line is stable for the load chosen, but the load is far from the maximum power the line will deliver. The 500-mile line looks dangerous for we are very near the maximum power of the line at 220 kv.

We can increase the maximum power of the 500-mile line by raising the voltage, by reducing resistance or reactance or by going to a lower frequency. A higher voltage would increase the maximum power as the square of the voltage. Decreasing the resistance when it is small in comparison with the reactance makes but little difference in the maximum power. Decreasing the reactance in the manner proposed by Mr. Percy Thomas, in a paper before the Institute in 1910, by employing split conductors, would increase the maximum power by 50 or 60 per cent. Decreasing the frequency to 25 cycles would make the 500-mile line equivalent to a 60-cycle line whose length is 500 X 25/60 = 208 miles, but whose resistance is inversely as the frequency or 60/25 as great. With 25 cycles, more than 150,000 kw. can be delivered over a 500-mile line with stability assuming constant generator terminal voltage.

The use of a synchronous condenser in the middle of the 500-mile line as suggested by the authors, will increase the maximum power that can be delivered over the line. Assuming 60,000 kv-a. in condensers at the mid point and constant voltages of 220 kv. at the generator and receiver ends but constant field on the condenser at the mid point, I find we could deliver approximately 150,000 kw. which agrees with the 152,000 kw. mentioned in Messrs. Evans and Bergvall's paper. In arriving at the 107,000 kw. mentioned in their paper for the rating of the line without condenser in the middle, constant field was assumed on the receiving end condensers but not constant field on the generators as voltage was assumed constant at the terminals of the step-up transformers. In arriving at the 150,000-kw. rating of the line with condenser at the mid point, constant field was assumed only on the condensers at the mid point. Both of these ratings would be very materially reduced if constant field were assumed at all points. Mr. Doherty discusses this phase at some length and gives some figures as to what this maximum power rating may become.

The curves in Fig. 29 are plotted as the others were, with kw. at the receiver against kv. at the receiver, (or per cent normal receiver voltage) and with constant generator voltage. I have assumed the condition shown on the graph. The upper set of curves is for the 500-mile line without condensers in the middle. The solid curves are for a constant generator voltage of 220 kv. at the terminals of the step-up transformer. A 70,000-kv-a. condenser was assumed and a load power factor of unity constant with voltage. The losses in the transformers and condensers have been neglected. The condenser field was adjusted to give the proper power factor for getting the assumed load over the line at normal receiver voltage of 220 kv. The field was then held constant on the condenser and the graph of kw. against kv. plotted. The circles indicate the points of normal voltage operation. When delivering 100,000 kw. at normal voltage we are

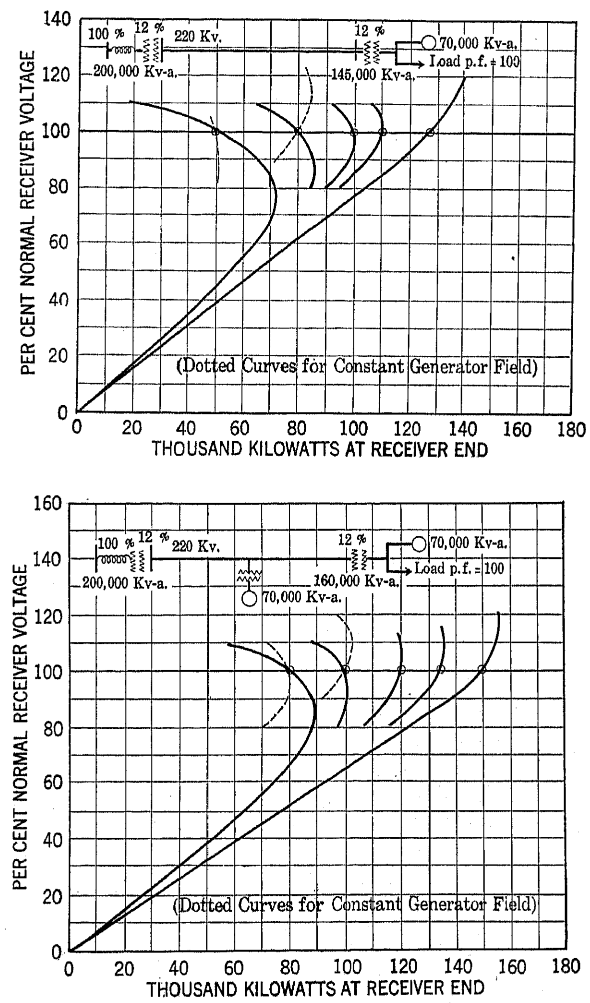

Fig. 29

above the maximum power point and when delivering 110,000 kw. we are below it. The maximum power possible over this line with constant generator voltage equal to 220 kv. and with the condenser capacity and load power factor assumed, is therefore, about 107,000 kw. as has been stated. The dotted curves are for constant field on the generator. The maximum power in this case is approximately 70,000 kw.

The lower graph of Fig. 29 is for the 500-mile line with condenser at the mid point. The solid curves are for a constant generator voltage of 220 kv. at the terminals of the step-up transformer. The receiver voltage has been allowed to vary with the addition of load as well as the mid-point voltage. Messrs. Evans and Bergvall held the receiver voltage constant and allowed the voltage at the mid point of the line only to vary with a change in load. With a 70,000-kv-a. condenser at the receiver and a 70,000-kv-a. condenser at the mid point and constant excita-

tion on these condensers, the lower graph of Fig. 29 shows how the receiver voltage varies with the load. The circles indicate the points of normal voltage operation. When delivering 100,000 kw. at normal voltage we are on the upper side of the curve, but if 2000 kw. were added suddenly, before the regulator could increase the field on the condenser the voltage would fall, and if the load were a shaft load, it would fade away. When delivering 120,000 kw. at normal voltage we are already on the underside of the curve and for a shaft load we are unstable at this point. The maximum power, therefore, with the condenser capacity assumed and constant generator terminal voltage is less than 120,000 kw., probably about 118,000 kw. The dotted curves are for constant field on the generator. The maximum power in this case, is approximately 90,000 kw. It is possible by increasing the amount of condenser capacity to get more power over this line with a condenser in the middle. However, calculations made with a 150,000-kv-a. condenser of the usual type at the end and at the middle of the line, show that it is not possible to get 150,000 kw. over the line with stability assuming constant generator terminal voltage. If the generator field instead of the generator terminal voltage is assumed constant, the case would be worse. An increase in the amount of power with stability can be obtained if a special highly saturated synchronous condenser is used at the middle of the line.

If it is necessary to transmit 150,000 kw. per circuit 500 miles, I feel sure that it will be done, if not with the equipment at present commercially available, then by means of specially designed apparatus.

R. E. Doherty: While I would not detract one iota from the many points of real value in these papers, I nevertheless believe that a wrong impression may possibly be obtained from them as to the maximum power limit of such long lines, and I therefore wish to submit for your consideration the maximum limits which, in the present state of engineering knowledge and experience, I consider to be justified.

After a careful and extended investigation, and considering the present experience and the extent to which the proposed colossal project of a 500-mile line extends beyond the bounds of experience, my conclusion is that the estimates of the maximum power given in these papers are too high. Take the case of a 500-mile straightaway transmission; where the authors proposed about 110,000 kw., we calculate about 70,000 kw. in the case of the sectionalized line with a 70,000-kv-a. condenser in the middle. Where they propose 150,000 kw. our estimate is 90,000 kw. These figures represent the maximum power which can be transmitted over the line at 220 kv. with the synchronous apparatus assumed. All assumptions are the same as those stated by the authors, excepting the generators. I have assumed a 90,000-kv-a. generator with the usual degree of saturation, and with 60 per cent synchronous reactance: which is equivalent, so far as the above results are concerned, to a 90,000-kv-a. generator with 45 per cent synchronous reactance, in which saturation is negligible. The usual value of synchronous reactance is about 100 per cent. The authors assume, in the calculations, that it is zero, when they assume constant voltage, *i. e.* $\frac{d\,E}{d\,P} = 0$.

Do not misunderstand. This difference in maximum power does not come from differences in premises, methods of calculation, or character of the synchronous apparatus considered. It is altogether a question of assumptions. Starting with the authors' assumptions, and with the same theory, long since generally accepted, we naturally arrive at the same conclusions; but the difference in assumptions is just the difference between 110,000 kw. and 70,000 kw. in the case of straightaway transmission, and between 150,000 kw. and 90,000 kw. in the case of a sectionalized line with a condenser at the mid-point.

The difference in assumptions is this: in the case of 500-mile straightaway, the authors assume a constant voltage at the generator terminals, or perhaps at the high side of the transformer; that is, they neglect the fact that the synchronous generators impose limitations upon maximum power in the same manner, and to a comparable extent, as the synchronous apparatus at the receiving end. That is, the difference in this case is altogether that they have neglected the effect of the generators, and we have included it.

In the sectionalized line with the condenser at the mid-pont, Messrs. Evans and Bergvall have not only neglected the limitations due to the synchronous generators, as all of the authors have done in the straightaway line, but have also even neglected the limitations imposed by the synchronous condenser at the receiving end—which limitations they had considered of sufficinent importance to include in calculating the straightaway transmission. In other words, in the sectionalized 500-mile line, the only limitation considered, outside of the line itself, is that imposed by the condenser at the middle of the line. That is, the difference between our conclusions regarding the sectionalized line is that the authors have neglected the limitations of the apparatus both at the sending, and at the receiving, end, and we have not.

Therefore, in comparing these conclusions, I wish to make it clear that the difference in results arises, not from any difference of opinion as to the fundamental theory of the transmission line, of the apparatus involved, or from essential difference of method, but only from a difference of opinion as to the importance of taking into account in the calculations the limitations imposed by the certain groups of the apparatus involved.

What about these assumptions? Which of them is justified? I submit that the colossal magnitude and importance of the long-distance lines now under consideration, and also the extent to which, in this study, we are projecting beyond the limits of experience, both demand that we adhere to the following principle: That we must show the system to be *inherently* stable—that is to say, we must neither gamble that a voltage regulator will be able to insert a supporting prop under an otherwise falling system, nor *depend* for stability during load transients, upon possible, momentary, favorable conditions due to momentum and field transients. These may add up in the right direction, but engineers had better keep them up their sleeves, just as they have done in the past in most other apparatus applications, particularly in generating and transmission equipments. Moreover, it should not be forgotten that the values here discussed are the ultimate maxima of power that can be transmitted at normal voltage with the apparatus considered. These are factors which must be carefully accounted for in deciding what asumptions are justified, and what are not.

The point of view which I am here outlining is presented in a masterly way in the first few pages of Mr. Shand's paper—which pages I should commend to your very careful study. After discussing generator characteristics and voltage regulators, he says, "This theory of artificial stability, although perhaps not a practical possibility, is here outlined mainly for the purpose of preventing any misconception regarding the capabilities and limitations of the commercial vibrating regulator in connection with the present subject. Moreover, if the state of artificial stability were attainable it is very doubtful that it would be desirable to depend on this apparatus as the main link in maintaining the operation of the system. In deriving the limit of output for power system it may be considered that the condensers are operating under a definite value of excitation for each value of load, this value being adjusted by the regulator as the load conditions change." Then, somewhat as a shock, he follows up with this statement: "In the following discussion the effect of the inherent voltage regulation of the main generators has been neglected. This is justified on the grounds that the error is slight while the labor involved would increase greatly." What do Evans and Bergvall say? "In this investigation the voltages at both ends of the line were assumed to be constant. Actually the voltage regulators at the generator and receiver

synchronous condenser cannot change the voltage instantaneously and some variation in voltage will occur. This does not appreciably affect the pull-out point as can be seen from the tests."

Now I think it is perfectly clear from the foregoing that there is complete agreement in fundamental theory, and in our conceptions of the general character of the behavior of all apparatus involved. But when we come to the point of making actual calculations—then, and not until then, we reach the point of disagreement. It is merely a difference of opinion as to whether the limiting effects of synchronous apparatus, which we both acknowledge, are significant or not; I am certain that they are. And inasmuch as we do agree in theory, it is only necessary for the authors to make the calculation including these effects, even if it is greater labor, and they will undoubtedly find substantially the same results. As a matter of fact, Mr. Shand has already calculated for a 300-mile line, the effect of the limited condensers at the receiver—still neglecting, however, the limitations of the generator. Why not do it for a 500-mile line, take the final step to make a complete job of it, and include the limitations of the generator. That would be more convincing than to assume that the factory test settled it. The difference is on the wrong side to let the test settle it.

I hope I have made it clear to you where the difference in results arises. The doctors do not disagree regarding diagnosis, nor as to the treatment, but merely regarding the *extent* of the treatment. And I trust that further study and calculation will bring agreement there.

T. A. Worcester: As has been brought out in some of the discussion this afternoon, the practical load limit of a 500-mile, 60-cycle line is not more than 100,000 kw., probably is in the neighborhood of 80,000 kw. If this is the case, it is well that we check up to see whether such a line would be economically practicable. The cost per kw. year of a 60-cycle transmission system to deliver 80,000 kw. per circuit is about $29. This figure includes the transmission line, step-up and step-down transformers and synchronous condensers, and to it must be added the cost of generation of power in the hydro station. It is not to be expected that power can be generated at anything less than $15 per kw-year and would likely be $20 or more. To these figures must be added the cost of transmission losses which will be approximately 25 per cent. The cost for $15 power when delivered over the 500-mile line will then be $48 per kw-year and of $20 power $54 per kw-year.

On the basis of mils per kw-hr. for 7200-hour load these figures reduce to 6.6 mils for $15 power and 7.5 mils for $20 power. To these figures must be added an amount necessary to take the power from the low side of the receiving station of the hydro system to a point where a competing steam plant might be placed, and it is very doubtful if the hydro power on this basis can compete with steam power unless the price of coal increases materially above the present-day value.

As the 80,000 kw. per circuit 60-cycle line is questionable from the economic point of view, the problem immediately arises as to how to increase the capacity of the line and lower its cost per kw.

Higher voltages have been considered but there is a strong indication that the increase in carrying capacity is about equalled by the increase in cost.

Another scheme of increasing the line carrying capacity is to divide the conductors as suggested by Mr. Percy Thomas. This will increase the capacity of the line approximately 50 per cent and may be the answer to the problem. Operating engineers, however, have shied at the difficulties involved in operating a split-conductor line as they think they have enough troubles with one wire on an insulator string.

Another alternative which involves the use only of standard equipment and methods is to use a lower frequency. A 25-cycle 500-mile line is equivalent to a 60-cycle 200-mile line and no one would worry about such a line. With slightly leading power factor 150,000 kw. can be carried over one circuit with ample margin for a sudden increase in load and there is no question of stability in the same sense as with the 60-cycle line with equivalent load. Twenty-five cycles has been avoided as the main load in the industrial districts is 60 cycles and unquestionably the growth is in that direction and should by all means be encouraged. It is necessary then to convert the 25-cycle power to 60 cycles and charge the cost of transmission with frequency changers and their losses. Assuming the losses to be 10 per cent, the useful power delivered will be 135,000 kw. per circuit instead of 150,000 just mentioned. On this basis I have figured the cost of delivering 60-cycle power into the metropolitan district and find that it can be done for 6 mils for $15 hydro power and 7 mils for $20 power. These figures include the cost of one transformation after going through the frequency changers and if it is necessary to make an additional transformation the comparison is not quite so favorable to the long line. However, they are on the same basis as the figures given above for the 60-cycle transmission and they show about 10 per cent lower cost. This percentage difference is somewhat disappointing as the margin is not sufficient to assure one that the hydro power can compete with steam. It is, however, sufficient to warrant a very close analysis for any particular undertaking, as some slight difference in assumption might throw the balance heavily on one side of the other. For instance, if any large part of the power could be used at 25 cycles, the cost and losses in the frequency changers would be reduced, with a consequent advantage for the hydro system.

Frederick E. Terman: The influence of circuit constants on the performance of the transmission line as reflected in the circle-diagram coefficients can be clearly shown by expressing these coefficients in their simplest hyperbolic form, and expanding the result into a power series. Dropping the unimportant terms, and judiciously combining the others leads to a very simple expression.

Expressed in the simplest form, the conjugate expressions of the Evans and Sels diagram coefficients become, in the case of the transmission line alone:

$$1 = \text{Real part of } (A/B) = \text{Re}\,\frac{\coth + \theta}{Z_o}$$

$$m = \text{Imag. part of } (A/B) = \text{Imag}\,\frac{\coth \theta}{Z_o}$$

$$n = \text{Modulus } (1/B) = \text{Mod}\,\frac{\operatorname{cosech} \theta}{Z_o}$$

Taking R, X, and Y as the total resistance, reactance, and admittance of the line, and remembering that a term of the character $X\,Y$ is dependent only upon the frequency and length, but is independent of the physical arrangements due to the reciprocal relation of capacity and inductance of open air lines, the following results are obtained:

$$1 = \frac{k}{(R^2 + X^2)^{\frac{1}{2}}}$$

$$m = \frac{R}{R^2 + X^2}$$

$$n = \frac{X}{R^2 + X^2} - k''\,Y$$

$k = \{ 1 + 1.133 \times 10^{-9} f^2 L^2 s \,(2 + 1.133 \times 10^{-9} f^2 L^2 s) \}^{\frac{1}{2}}$
$s = (0.1667 + 0.0221 \times 10^{-9} f^2 L^2)$
f = frequency
L = length of line in miles
$k'' = (0.333 + 0.0252 \times 10^{-9} f^2 L^2)$

For lines not exceeding 500 miles these results are accurate to within 1 per cent for open-air lines at commercial frequencies, (not over 60 cycles), and furnish an easy and yet sufficiently accurate method for the determination of the circle constants. In the ordinary line R/X is small, so we see that changing R does practically nothing to the circle diagram except alter m in pro-

portion, while a change in X affects all of the coefficients. Furthermore, it is seen that Y is only a secondary influence since the term $k'' Y$ is not of great size.

In general the characteristic one would like are: (1) A large $(n - m)$, which gives a high power limit; and (2) a small $m/1$ which gives a line requiring only a minimum of variation in synchronous capacity from no-load to full load transmission. These requirements show the desirability of first a low reactance, and next a low resistance, although the change in characteristics is not great after the conductor section increases beyond a reasonable size.

Prof. Bush has called my attention to the fact that R, X, and Y can be expressed as functions of conductor spacing and diameter, making it possible to draw a set of curves showing the relation between the physical and electrical constants of the line. This has been done in Fig. 30 for a copper conductor, and using the data for R, X, and Y taken from the Westinghouse Reprint 82.

Before the present form of the Evans and Sels circle diagram becomes too well established, it would be well to call attention to the fact that plotting leading kv-a. in the lagging position leads to several conflicts with the customary conventions.

This point becomes evident by an examination of Fig. 31, which is in general similar to the diagrams of Thielman and Holladay[2]. In the original form this diagram is suited only for constant receiver potential since it depends upon the fact that the power

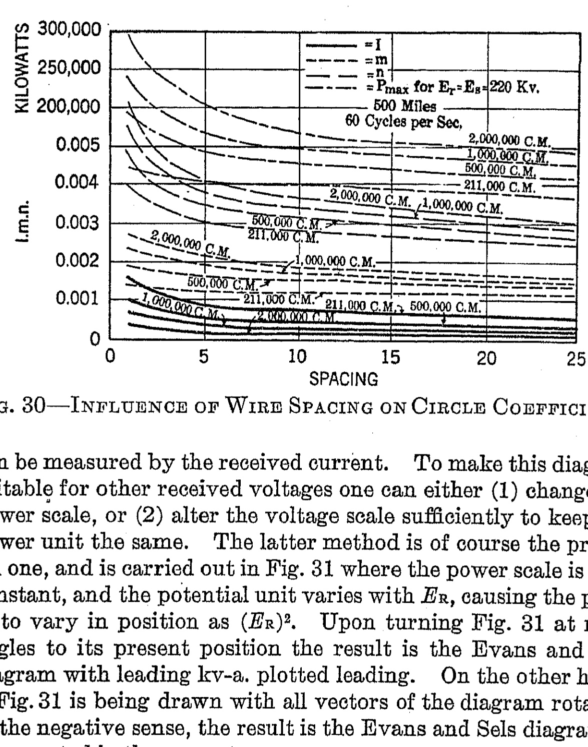

FIG. 30—INFLUENCE OF WIRE SPACING ON CIRCLE COEFFICIENTS

can be measured by the received current. To make this diagram suitable for other received voltages one can either (1) change the power scale, or (2) alter the voltage scale sufficiently to keep the power unit the same. The latter method is of course the practical one, and is carried out in Fig. 31 where the power scale is kept constant, and the potential unit varies with E_R, causing the point A to vary in position as $(E_R)^2$. Upon turning Fig. 31 at right angles to its present position the result is the Evans and Sels diagram with leading kv-a. plotted leading. On the other hand, if Fig. 31 is being drawn with all vectors of the diagram rotating in the negative sense, the result is the Evans and Sels diagram as is presented in the present paper.

This inconsistency was justified in an earlier paper[3] of the present writers by the fact that inductive impedanceis $r + j x$, but analysis shows this to be an argument on the other side. If the equation given by Evans and Sels for the circles in terms of power is divided by E_R^2 an interesting, and for some purposes, very valuable circle diagram in terms of current results, but with the $+ Q$ taken as lagging, then lagging current must be plotted leading. Again dividing by E_R^2 gives the circle diagram in terms of admittance, where with the convention adopted by the writers, $g + j b$ represents inductive admittance, clearly not in agreement with inductive impedance as $r + j x$.

Putting the center of the circles in the second quadrant as in Fig. 31, and plotting leading kv-a. leading has the great advantage of avoiding all of these inconsistencies, and conforming to the common custom.[4]

C. A. Nickle: In the following, a method is developed for calculating the maximum power under steady state for a system comprising generators, transmission line, synchronous condensers and receiving circuit. This method has been developed on the assumption that saturation in the generator does not exist to any significant degree, although it may exist to any known degree in the condensers. Saturation in the generators would increase the maximum power over the values thus calculated.

The general equations of a transmission line having distributed inductance, capacity, resistance, and leakage are:

$$E = E_R \cos\beta 1 \cosh\alpha 1 + j E_R \sin\beta 1 \sinh\alpha 1$$

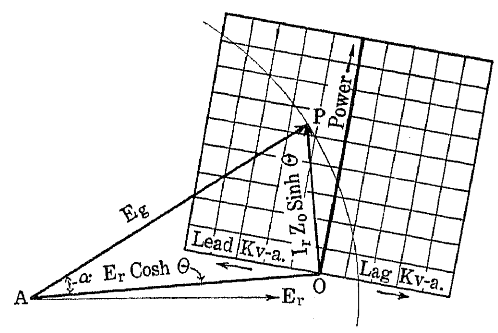

FIG. 31—VECTOR BASIS OF EVANS AND SELS CIRCLE DIAGRAM

$$+ I_R \sqrt{\frac{Z}{Y}} \cos\beta 1 \sinh\alpha 1 + j I_R \sqrt{\frac{Z}{Y}} \sin\beta 1 \cosh\alpha 1 \quad (1)$$

$$I = I_R \cos\beta 1 \cosh\alpha 1 + j I_R \sin\beta 1 \sinh\alpha 1$$

$$+ E_R \sqrt{\frac{Y}{Z}} \cos\beta 1 \sinh\alpha 1 + j E_R \sqrt{\frac{Y}{Z}} \sin\beta 1 \cosh\alpha 1 \quad (2)$$

where

E_R = receiver voltage line to neutral
I_R = receiver current per phase
Z = line impedance per phase per mile
Y = line admittance to neutral per phase per mile
α = attenuation constant $= \sqrt{\tfrac{1}{2}(z y + r g - b x)}$
β = propogation constant $= \sqrt{\tfrac{1}{2}(z y - r g + b x)}$
1 = distance to any point on the line measured from the receiver end.
E = voltage to neutral 1 mile from receiver.
I = current 1 mile from the receiver.

For the 500-mile line under discussion the line constants expressed in per cent (as a fraction) on the basis of 100,000 kv-a. at 22 kv. are:

$$\sqrt{\frac{Z}{Y}} = 0.793 - j\,0.0501$$

$$\sqrt{\frac{Y}{Z}} = 1.26 + j\,0.0796$$

$$\alpha = 0.000131$$
$$\beta = 0.00207$$
$$\alpha 1 = 0.0655$$
$$\beta 1 = 1.035$$
$$\cosh\alpha l = 1.002$$
$$\sinh\alpha l = 0.0655$$
$$\cos\beta l = 0.515$$
$$\sin\beta l = 0.86$$

2. Thielman, *Rev. Gen. d' El.* 1920-1921.
Holladay, A. I. E. E. 1922.
3. Evan & Sels in the "*Electric Journal*" 1921.

4. J. R. Dunbar in discussion on page 789 of 1922 A. I. E. E. TRANSACTIONS, on the vector representation of reactive power.

Substituting these constants in the general equations, we get the voltage and current expressed as a fraction of normal voltage and current, as follows:

$$\dot{E} = 0.516\,\dot{E}_R + j\,0.0563\,\dot{E}_R + 0.0699\,\dot{I}_R + j\,0.6813\,\dot{I}_R \quad (3)$$

$$\dot{I} = 0.516\,\dot{I}_R + j\,0.0563\,\dot{I}_R - 0.0261\,\dot{E}_R + j\,10.089\,\dot{E}_R \quad (4)$$

The receiver current expressed in terms of receiver power and receiver reactive kv-a. is

$$\dot{I}_R = \frac{P}{E_R} + j\,\frac{Pj}{E_R} \quad (5)$$

where

P = power per phase at receiver.

Pj = reactive kv-a. per phase at receiver.

E_R = receiver voltage per phase used as a reference vector.

Substituting (5) in (3) and (4)

$$\dot{E} = 0.516\,E_R + 0.0699\,\frac{P}{E_R} - 0.6813\,\frac{Pj}{E_R} + j\left(0.0563\,E_R + 0.6813\,\frac{P}{E_R} + 0.0699\,\frac{Pj}{E_R}\right) \quad (6)$$

$$\dot{I} = -\,0.0261\,E_R + 0.516\,\frac{P}{E_R} - 0.0563\,\frac{Pj}{E_R} + j\left(1.089\,E_R + 0.0563\,\frac{P}{E_R} + 0.516\,\frac{Pj}{E_R}\right) \quad (7)$$

The nominal e. m. f. of the generator is

$$\dot{E}_n = \dot{E} + j\,x_0\,\dot{I} \quad (8)$$

where x_0 is the synchronous reactance of the generator.

Substituting (6) and (7) in (8)

$$\dot{E}_n = \Bigg[(0.516 - 1.089\,x_0)\,E_R + \left(0.0699 - 0.0563\,x_0\right)\frac{P}{E_R} - \left(0.6813 + 0.0516 x_0\right)\frac{Pj}{E_R} \Bigg] + j\Bigg[(0.0563 - 0.0261\,x_0)\,E_R \left(0.6813 + 516\,x_o\right)\frac{P}{E_R} + \left(0.0699 - 0.563\,x_o\right)\frac{Pj}{E_R} \Bigg] \quad (9)$$

Expressing in absolute values, dividing through by E_R^2, completing the squares, and rearranging,

$$\left[\frac{P}{E_R^2} + \left(\frac{0\,0745 - 0.0939\,x_0 + 0.0478\,x_0^2}{0.4688 + 0.6941\,x_0 + 0.2692\,x_0^2}\right)\right]^2 + \left[\frac{Pj}{E_R^2} + \left(\frac{-\,0.3476 + 0.4706\,x_0 + 0.560\,x_0^2}{0.4688 + 0.6941\,x_0 + 0.2692\,x_0^2}\right)\right]^2 = \frac{E_n^2}{E_R^2}\left[\frac{1}{0.4688 + 0.6941\,x_0 + 0.2692\,x_0^2}\right] \quad (10)$$

Equation (10) is the equation of a family of circles having a parameter $\dfrac{E_n}{E_R}$

and coordinates $\dfrac{Pj}{E_R^2}$ and $\dfrac{P}{E_R^2}$. Thus the center of these circles is located at

$$\frac{Pj}{E_R^2} = -\left[\frac{-\,0.3476 + 0.4706 x_0 + 0.560\,x_0^2}{0.4688 + 0.6941\,x_0 + 0.2692\,x_0^2}\right] \quad (11)$$

$$\frac{P}{E_R^2} = -\left[\frac{0.0745 - 0.0939 x_0 + 0.0478\,x_0^2}{0.4688 + 0.6941 x_0 + 0.2692\,x_0^2}\right] \quad (12)$$

The radii are given by

$$R = \frac{E_n}{E_R}\sqrt{\frac{1}{0.4688 + 0.6941\,x_0 + 0.2692\,x_0^2}} \quad (13)$$

For any given generator, x_0 is known, and one set of concentric circles will give the performance of the line.

The condition $x_0 = 0$ represents an infinite generator, and a set of circles for this condition may be used for determining the voltage at the generator terminals. Fig. 32 shows a family of circles for $x_0 = 0$ and Fig. 33 for $x_0 = 0.50$, which corresponds approximately to a 200,000 kv-a. generator. The method of obtaining the maximum power which can be transmitted over a 500-mile straightaway line from these curves is as follows:

For any given receiver power, the reactive kv-a. in the line at the receiver, required to hold normal voltage at both the receiving and sending ends, is obtained from the set of curves for $x_0 = 0$. For these conditions $E_R = 1.0$ and $\dfrac{E_n}{E_R} = 1.0$.

Having found the reactive kv-a. required at the receiver, the excitation on the receiving condenser can be obtained from the condenser characteristics. A typical set of these characteristics is shown in Fig. 34.

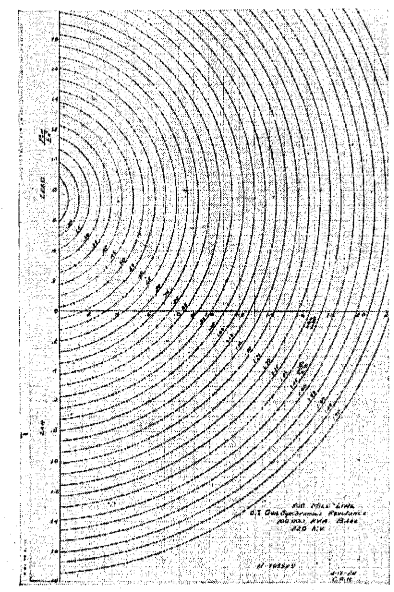

Fig. 32—500-Mile Line. 0 Per Cent Gen. Synchronous Reactance. 100,000 Kv-a. Base. 220 Kv.

Thus for any receiver power setting, the receiver reactive kv-a. and receiver power are known, and from the curves for $x_0 = 0.50$, the approximate excitation on the generator may be read directly.

The variation of receiver voltage with receiver power is then obtained under the assumption that the generator and condenser excitation remain fixed. To obtain this variation, a new receiver voltage is assumed. For this new receiver voltage, a new value of $\dfrac{Pj}{E_R^2}$ is obtained from the condenser characteristics and the new value of $\dfrac{E_n}{E_R}$ is known. Hence from the curves for $x_0 = 0.50$ the new value of $\dfrac{P}{E_R^2}$ is obtained.

Repeating this process for several assumed values of E_R, a curve may be plotted with P_R as abscissae and E_R as ordinates. The maximum power for these conditions occurs at the point where $\dfrac{d\,P}{d\,E_R} = 0$ and the maximum power of the line under

steady conditions occurs for the settings which give $\frac{d\,P}{d\,E_R} = 0$ for $E_R = 1.0$.

Fig. 35 shows a set of such curves obtained for a 500-mile line under the following assumptions:

75,000-kv-a. condenser at receiver.
Generator synchronous reactance $x_0 = 0.50$
Generator synchronous reactance $x_0 = 0$
Unity power factor motor load.

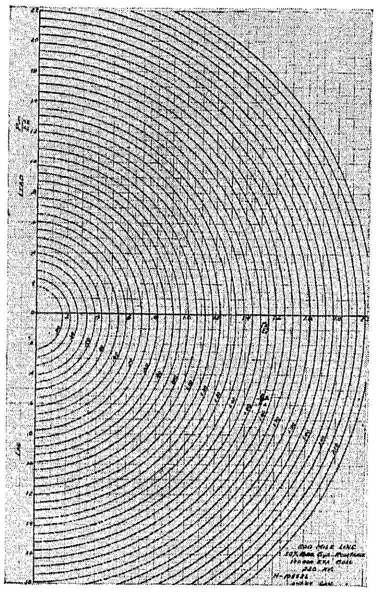

FIG. 33—500-MILE LINE. 50 PER CENT GEN. SYNCHRONOUS REACTANCE. 100,000 KV-A. BASE. 220 KV.

The solid curves are for $x_0 = 0.50$ and the dotted curves for $x_0 = 0$. For $x_0 = 0$, $\frac{d\,P}{d\,E_R}$ is zero for $E_R = 1.0$ at approximately 117,000 kw. and, for $x_0 = 0.50$, at approximately 67,000 kw. With the finite generator assumed the maximum power is thus 57 per cent of that which could be obtained with an infinite generator.

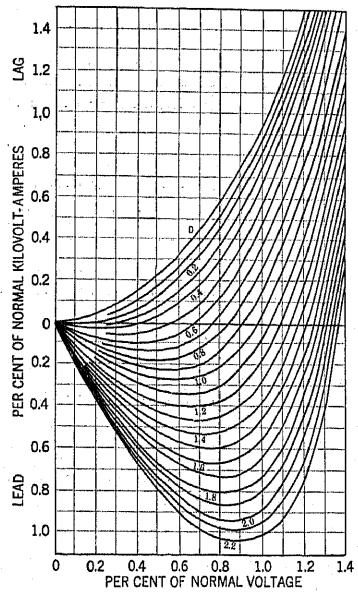

FIG. 34

SECTIONALIZED LINE

The process of obtaining these curves for a system having a condenser located at the middle of the line is considerably more difficult. For this case, separate computations are necessary for each 250-mile section. Curves, similar to those drawn for the 500-mile line, are obtained for a 250 mile line, and in addition, circles giving line loss and reactive kv-a. at the sending end are drawn for the condition $x_0 = 0$. These are shown in Figs. 36 and 37 and are derived as follows:

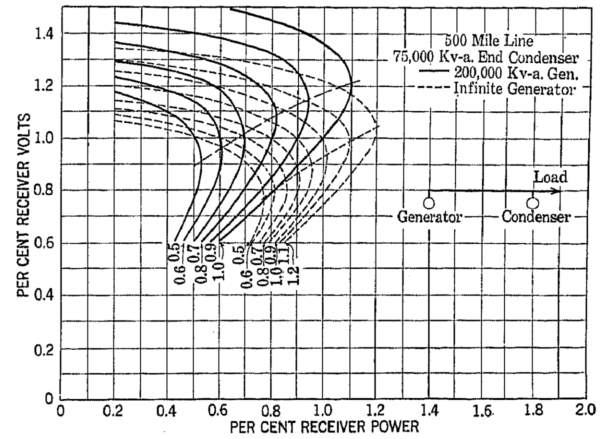

FIG. 35

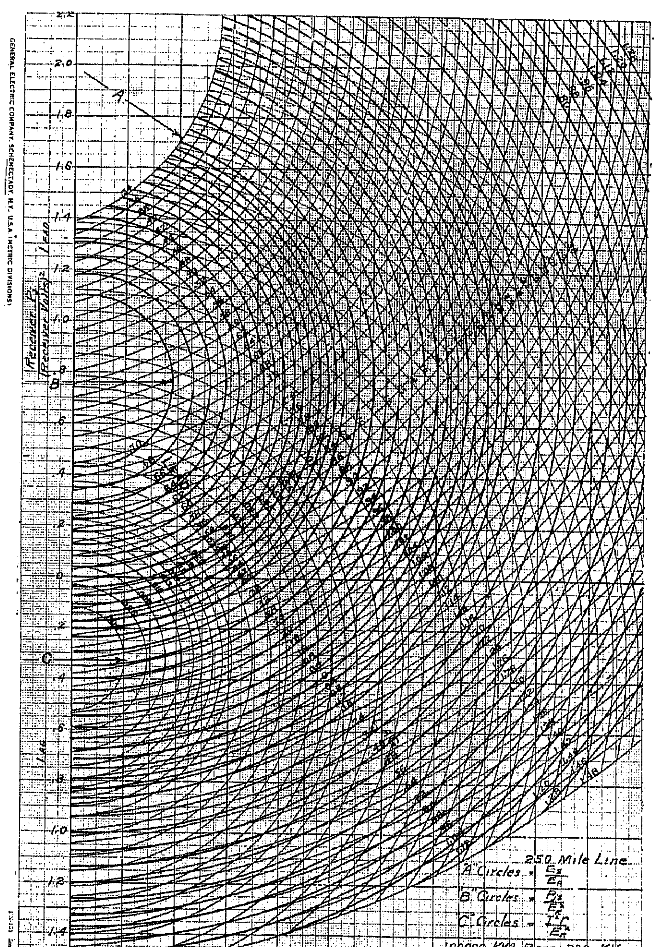

FIG. 36

For the 250 mile line, the line constants are

$$\sqrt{\frac{Z}{Y}} = 0.793 - j\,0.0501$$

$$\sqrt{\frac{Y}{Z}} = 1.26 + j\,0.0796$$

$$\begin{aligned}
\alpha &= 0.000131 \\
\beta &= 0.00207 \\
\alpha\, l &= 0.0327 \\
\beta\, l &= 0.517 \\
\cosh \alpha\, l &= 1.001 \\
\sinh \alpha\, l &= 0.0327 \\
\cos \beta\, l &= 0.87 \\
\sin \beta\, l &= 0.493
\end{aligned}$$

Substituting these constants and equation (5) in the general equations (1) and (2) we get

$$\dot{E} = 0.87\, E_R + 0.0473 \frac{P}{E_R} - 0.389 \frac{Pj}{E_R}$$

$$+ j \left(0.0161\, E_R + 0.389 \frac{P}{E_R} + 0.0473 \frac{Pj}{E_R} \right) \tag{14}$$

$$\dot{I} = -\,0.00345\, E_R + 0.87 \frac{P}{E_R} - 0.0161 \frac{Pj}{E_R}$$

$$+ j \left(0.620\, E_R + 0.0161 \frac{P}{E_R} + 0.87 \frac{Pj}{E_R} \right) \tag{15}$$

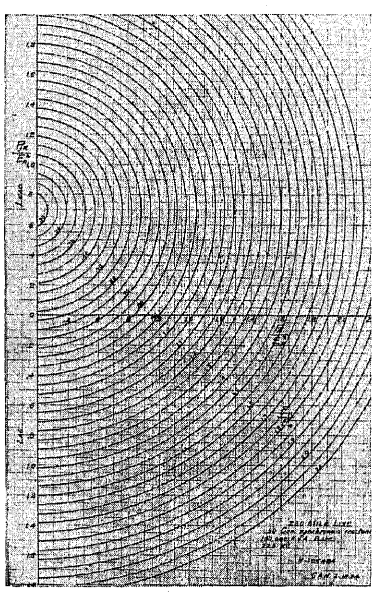

FIG. 37—250 MILE LINE 0.50 GEM SYNCHRONOUS REACTANCE, 100,000 KV-A BASE 220 KV.

Telescoping (14) and (15) for power at the sending end.

$$P_s = \dot{E}\, \dot{I}$$

$$= 0.007\, E_R^2 + P + 0.0306\, Pj + 0.0474 \frac{P^2}{E_R^2} + 0.0474 \frac{Pj}{E_R^2} \tag{16}$$

The line loss is the difference between the power at the sending end and the power at the receiving end or

$$P_L = 0.007\, E_R^2 + 0.0306\, Pj + 0.0474 \frac{P^2}{E_R^2} + 0.0474 \frac{Pj^2}{E_R^2} \tag{17}$$

Dividing through by E_R^2, completing the squares and rearranging,

$$\left(\frac{P}{E_R^2} \right)^2 + \left(\frac{Pj}{E_R^2} + 0.343 \right)^2 = \frac{21.1\, P_L}{E_R^2} - 0.0430 \tag{18}$$

This is the equation of a family of circles of radii,

$$R = \frac{21.1\, P_L}{E_R^2} - 0.0430 \tag{19}$$

and the center at

$$\frac{Pj}{E_R^2} = -\,0.343 \tag{20}$$

$$\frac{P}{E_R^2} = 0 \tag{21}$$

The reactive kv-a. at the sending end is the magnitude of the cross product of (14) and (15). Thus

$$P_{j_s} = 0.540\, E_R^2 + 0.0306\, P + 0.516\, Pj - 0.337 \frac{P^2}{E_R^2} - 0.337 \frac{Pj^2}{E_R^2} \tag{22}$$

Dividing through by E_R^2, completing the squares and rearranging,

$$\left(\frac{P}{E_R^2} - 0.0454 \right)^2 + \left(\frac{Pj}{E_R^2} - 0.765 \right)^2 = 2.193 - 2.97 \frac{P_{j_s}}{E_R^2} \tag{23}$$

This is the equation of a family of circles of radii

$$R = \sqrt{2.193 - 2.97 \frac{P_{j_s}}{E_R^2}} \tag{24}$$

and the center at

$$\frac{Pj}{E_R^2} = 0.765 \tag{25}$$

$$\frac{P}{E_R^2} = 0.0454 \tag{26}$$

The nominal e. m. f. of the generator is

$$\dot{E}_n = \dot{E} + j\, x_0 \dot{I} \tag{8}$$

Substituting (14) and (15) in (8)

$$\dot{E}_n = \Big[(0.87 - 0.62\, x_0)\, E_R + (0.0473 - 0.0161\, x_0) \frac{P}{E_R}$$

$$- (0.389 + 0.87\, x_0) \frac{Pj}{E_R} \Big]$$

$$+ j \Big[(0.0161 - 0.00345\, x_0)\, E_R + (0.389 + 0.87\, x_0) \frac{P}{E_R}$$

$$+ (0.0473 - 0.0161\, x_0) \frac{Pj}{E_R} \Big] \tag{27}$$

Expressing in absolute values, dividing through by E_R^2, completing the squares, and rearranging,

$$\left[\frac{P}{E_R^2} + \left(\frac{0.0478 - 0.0306\, x_0 + 0.007\, x_0^2}{0.1537 + 0.676\, x_0 + 0.757\, x_0^2} \right) \right]^2$$

$$+ \left[\frac{Pj}{E_R^2} + \left(\frac{-\,0.338 - 0.516\, x_0 + 0.54\, x_0^2}{0.1537 + 0.676\, x_0 + 0.757\, x_0^2} \right) \right]^2$$

$$= \frac{E_n^2}{E_R^2} \left[\frac{1}{0.1537 + 0.676\, x_0 + 0.757\, x_0^2} \right] \tag{28}$$

This is the equation of a family of circles of radii.

$$R = \frac{E_n}{E_R} \sqrt{\frac{1}{0.1537 + 0.676\, x_0 + 0.757\, x_0^2}} \tag{29}$$

and the center at

$$\frac{Pj}{E_R^2} = -\left(\frac{-\,0.338 - 0.516\, x_0 + 0.54\, x_0^2}{0.1537 + 0.676\, x_0 + 0.757\, x_0^2} \right) \tag{30}$$

$$\frac{P}{E_R^2} = -\left(\frac{0.0478 - 0.0306\, x_0 + 0.007\, x_0^2}{0.1537 + 0.676\, x_0 + 0.757\, x_0^2} \right) \tag{31}$$

In Fig. 36, circles are drawn giving reactive kv-a. at the sending end, voltage at the sending end, and line loss.

In Fig. 37, generator nominal voltage circles are given for a generator synchronous reactance $x_0 = 0.50$.

The sectionalized 500-mile line may be represented by two 250-mile sections in series as shown in Fig. 38.

The reactive kv-a. at D required to give normal voltage at C and D can be obtained from Fig. 36. From this reactive kv-a. the excitation required on the receiving condenser C_2 can be

obtained from the condenser characteristics, Fig. 34. The power and reactive kv-a. at C may also be obtained from Fig. 36. The power at B is the same as the power at C excepting the losses in the condenser C_1. If the condenser losses are significant, they should be added to the power at C to obtain the power at B. The power at B, thus obtained, is the receiver power for the section A B and from Fig. 36 we can get the reactive kv-a. required at B to hold normal voltage at A and B. The difference between the reactive kv-a. required at B and the reactive kv-a. at C, due to the load at D, gives the reactive kv-a. which the condenser C_1 must furnish.

The excitation on the condenser C_1 is then found from the condenser characteristics, Fig. 34. From the values of power and reactive kv-a. at B, the generator excitation can be obtained from Fig. 37.

Next, assume a new value of power at D. Then assume three or four different values of voltage at D, with the assumed new value of power. For each of the assumed voltage at D, the power and reactive kv-a. at D are known since the excitation on the condenser C_2 is assumed to remain fixed. From these values of power and reactive kv-a. the voltage, power, and reactive kv-a. at C can be obtained from Fig. 36. Then from the voltage at C and the known excitation on the condenser C_1 the reactive kv-a. furnished by this condenser is found from the condenser characteristics, Fig. 34. The reactive kv-a. at B is then the sum of the kv-a. at C and the condenser kv-a.

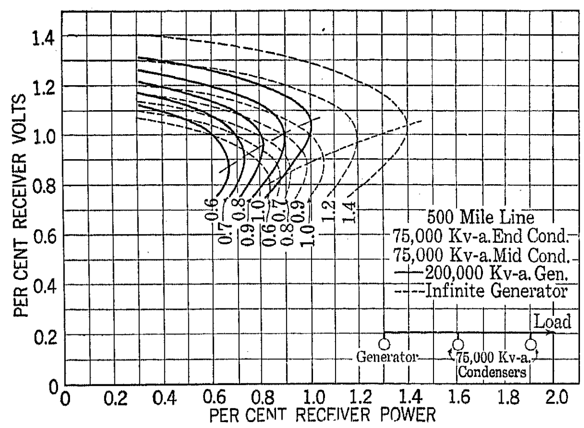

FIG. 38

Thus, for each assumed voltage at D, the power conditions at B are found, and, since these are the receiver conditions for the section A B, the excitation required on the generator G to give these conditions is found from Fig. 37.

The values of generator nominal voltage thus obtained are plotted against the assumed values of receiver voltage at D and that point on the curve, where the generator nominal voltage is the same as the value previously obtained, gives the correct value of receiver volts which correspond to the power assumed at D. This process is repeated for different assumed values of receiver power at D and a curve is plotted with receiver power as abscissae and receiver volts as ordinates. Similar curves are drawn for different power settings, and that power setting which gives

$\frac{d P}{d E_R} = 0$ for $E_R = 1.0$ is the maximum power that can be transmitted over the line at $E_s = 1.0$.

In Fig. 38 curves are shown for a sectionalized 500-mile line for the following assumptions;

75,000 kv-a. condenser at receiver.
75,000 kv-a. condenser at mid-point.
Generator synchronous reactance $x_0 = 0.50$.
Generator synchronous reactance $x_0 = 0$
Unity power factor load at receiver.

The maximum power for an infinite generator is thus 130,000 kw. and for the finite generator, 90,000 kw.

E. A. Smith: In referring to the paper on Superpower Transmission, I have noticed that Mr. Thomas has brought up some interesting points, such as I had recently figured on and by following Mr. Harold Goodwin, Jr. in his paper on Qualitative Analysis of Transmission Lines, a thorough analytical study can be made covering the general principles of very high tension transmission problems.

I doubt very much if the average engineer can solve and foresee the many difficulties encountered with 250 and 500-mile transmission systems of the 220,000 or higher voltage type.

The capacity of the transmission lines as well as the voltage regulation required for each particular system has to be taken into consideration to solve the principle factors from the performances on the systems as outlined under all conditions to operate efficiently. The different loads on a transmission system as well as the heating effects and losses have to be expounded in a way to cover the starting and stopping of the necessary apparatus at the distributing centers.

The costs of a system have to be studied in two ways, namely, construction and operation and by putting a system into operation, the best economical results are required to produce the means to cover these two principles. The period of construction being only of a short duration, it must be overlooked after completion, if the entire system is on a satisfactory operating basis.

Owing to the many theoretical assumptions for high voltages of this kind, a series of complications will arise in time, causing many difficult problems to appear on these systems at enormous expense and to keep the receiving end supplied, studies will have to be undertaken to analyze thoroughly all the principles involved in generation and transmission to bring out such points of interest which will help eliminate all unnecessary troubles and expenses.

I think Mr. Thomas is quite right in his assumption of installing a stepdown station in the center of the transmission system, by placing synchronous condensers therein, and providing for automatic means of fixing the potential which at the same time would divide the system entirely into two separate sections or lines. This would make the system more expensive as far as construction is concerned, but in the end would pay for itself.

I feel that the transmission system at 220,000 volts will not be able to stand up under all maximum load and weather conditions for any great length of time as the qualities of the conductors are not up to the point of satisfaction to withstand the continuous heat factors.

As a matter of fact the load at the receiving ends must be supplied at a fixed voltage and power factor and the synchronous condensers must be arranged to operate automatically to deliver the leading or lagging kv-a. as required, otherwise if this is not provided, the voltage will change accordingly to load conditions and losses. It will be seen by referring to Figs. 1, 2, 3 and 4, the curves will also change accordingly, owing to the fluctuating conditions of the load, voltage and losses which take place on long transmission lines, whereas on short-distance lines the losses would be considerably reduced and better operation would result. From this point it will be seen, that by introducing a central stepdown station at the middle of the transmission system, the line losses would be reduced at different loads.

According to calculations it is noted that with a maximum output of the generators, the power factor will be nearly unity, but due to many changes in the load, it will be unable to retain this value and the losses will be greater.

Assuming that all the curves in Figs. 1, 2, 3 and 4 and the values in Table A are correct, we have to compare them with actual practice, although, they appear to be fair values in connection with this superpower transmission when operating under no-load and full-load conditions.

E. A. Smith: In reference to Messrs. Evans and Sels paper although the curves represent certain conditions for a transmission system, the details are more or less based on theoretical assumptions.

It is seen that the curves are plotted according to the con-

ditions of the load and voltage of the system and depend mostly upon the resistance, reactance and impedance of the circuits described. By following the diagrams as shown it is readily seen that the synchronous condenser capacity required for any load at the receiver end, can be determined for approximate values at different power factors, but if the supply voltage varies between wide limits the radius of the receiver circle changes, which offsets the assumed fixed values and results in greater transmission losses for the length of the line.

Although the receiver load must retain a fixed power factor and is also assumed to act as an impedance across the circuit, it must have synchronous apparatus available for maintaining the voltage and controlling the power factor at the receiver end, to increase the power limit.

For any long transmission system, the most suitable arrangement would be, a synchronous condenser station at the center of the system in which the synchronous apparatus would tend to help regulate the voltage up to the maximum power limit.

As this system will include all kinds of electrical equipment as well as rotating apparatus, the system will be subjected to voltage fluctuations at different periods of the day (due to load conditions) and so will cause the rotating apparatus to fall out of step when the voltage drops below its fixed minimum value.

Professor V. Karapetoff: For the last year or so, we have been constructing at Cornell University a mechanical device to imitate the performance of a long transmission line. This device consists of pivoted weights and of springs, each weight taking the place of the inductance of a small section of the line and each spring representing the capacitance of that section. We are practically ready now to assemble this apparatus and then we shall have to provide a device for communicating to it sinusoidal vibrations to imitate the generator, and a load consisting perhaps of a friction disk, a spring, and a flywheel. Such a mechanical device is subject to the same equations of motion as a transmission line, under any desired transient conditions. Therefore, no mathematics is necessary: All you have to do is to impose a desired mechanical condition upon that model and to observe what happens. The natural frequency of vibration of each element is several seconds so that it is easy to follow a transient along such a line with a naked eye.

The authors of the papers use the method of complex quantities, and I should be the last one to deprecate it or to speak against this method, having used it so much myself. However, we now have to make another step, namely, in the direction of the so-called vector analysis. In the JOURNAL for last December I have a little article on the application of vector analysis to the circle diagrams of certain types of a-c. machines, principally commutator motors, and I urge the use of vector analysis, especially of the scalar product and of the vector product of two vectors, in the study of transmission lines under steady conditions.

In view of the importance which the circle diagram of power has acquired of late, it seems advisable to reduce its proof to the simplest possible terms, in order to make its use more general. The authors use the familiar method of complex quantities; in the case of power this necessitates a multiplication of a voltage by a current which is conjugate of the actual current. In other words, they use the relationship $P + j\,Q = (e + j\,e')\,(i - j\,i') = (e\,i + e'\,i') + j\,(e'\,i - e\,i')$ (11)

While this method is perfectly legitimate and leads to correct results[5], it is also of interest to solve the same problem using the principles of *Vector Analysis*. This branch of mathematics has not been used much as yet in electrical engineering, although its use has been found of great advantage in many branches of physics and mechanics. Since the theory of electrical engineering is largely based on these sciences, it is only a question of time when Vector Analysis will find its place in engineering investigations.[6]

Referring to Fig. 39, let vectors of voltages and currents be drawn in the $X\,Y$ plane. Let unit vectors parallel to the X and Y axis be denoted by $\mathbf{x}$ and $\mathbf{y}$ respectively. Then a current vector and a voltage vector may be written in the following form:

$$\mathbf{E} = \mathbf{x}\,e + \mathbf{y}\,e' \qquad (12)$$

$$\mathbf{I} = \mathbf{x}\,i + \mathbf{y}\,i' \qquad (13)$$

Vector quantities are denoted by bold-face letters, scalar quantities by italics.[7]

In Vector Analysis two kinds of products of two vectors $\mathbf{M}$ and $\mathbf{N}$ are distinguished: the *scalar product* (or the dot product)

$$\mathbf{M} \cdot \mathbf{N} = M\,N \operatorname{Cos} \theta \qquad (14)$$

and the *vector product* (or the cross product)

$$\mathbf{M} \times \mathbf{N} = \mathbf{q}\,M\,N \operatorname{Sin} \theta \qquad (15)$$

in these expressions θ is the angle between the vectors $\mathbf{M}$ and $\mathbf{N}$, and $\mathbf{q}$ is a unit vector in a direction normal to the $X\,Y$ plane (see figure)

From these definitions we may write

$$\mathbf{x} \cdot \mathbf{x} = \mathbf{y} \cdot \mathbf{y} = 1;\ \mathbf{x} \cdot \mathbf{y} = 0 \qquad (16)$$

$$\mathbf{x} \times \mathbf{x} = \mathbf{y} \times \mathbf{y} = 0;\ \mathbf{x} \times \mathbf{y} = -\,\mathbf{y} \times \mathbf{x} = \mathbf{q} \qquad (17)$$

These expressions follow from eqs. (14) and (15), by putting $\theta = 0$ or $\theta = 90°$, and $M = N = 1$. Forming a scalar product of eqs. (12) and (13), we get

$$\mathbf{I} \cdot \mathbf{E} = \mathbf{x} \cdot \mathbf{x}\,i\,e + \mathbf{y} \cdot \mathbf{y}\,i'\,e' + \mathbf{x} \cdot \mathbf{y}\,(e\,i' + i\,e') \qquad (18)$$

Using eqs. (16), we find that

$$P = \mathbf{I} \cdot \mathbf{E} \qquad (19)$$

In other words, the real power is equal to the scalar product of the current and voltage vectors. This also follows directly from the expression $P = I\,E \operatorname{Cos} \phi$, which is identical with the definition (14).

Similarly, forming a vector product (cross product) of eqs. (12) and (13), we obtain:

$$\mathbf{I} \times \mathbf{E} = \mathbf{q}\,(i\,e' - i'\,e) \qquad (20)$$

In other words, the reactive power

$$\mathbf{Q} = \mathbf{I} \times \mathbf{E} \qquad (21)$$

Thus $\mathbf{Q}$ may be represented by a vector normal to the $X\,Y$ plane. Since for any combination of values of $\mathbf{E}$ and $\mathbf{I}$, $\mathbf{Q}$ is always normal to the $X\,Y$ plane, this result is not objectionable on the score that reactive power cannot be a vector. It is a vector which always is in the same direction, or which for a certain range of values of angle ϕ may be negative, but which is always normal to the plane in which $\mathbf{E}$ and $\mathbf{I}$ are drawn.

It is convenient to consider P and Q as orthogonal components of a vector, say $\mathbf{U}$, so that

$$\mathbf{U} = \mathbf{p}\,P + \mathbf{q}\,Q \qquad (22)$$

where $\mathbf{p}$ and $\mathbf{q}$ are unit vectors at right angles to each other. But $\mathbf{q}$ is perpendicular to the $X\,Y$ plane; hence $\mathbf{p}$ lies in that plane, as shown in the Fig. 39. The exact position of the P axis with respect to the X axis is immaterial; in fact the $P\,Q$ diagram can be drawn entirely separate from the $X\,Y$ diagram. Thus, from the foregoing theory we have for the supply power

$$\mathbf{U}_s = \mathbf{p}\,(\mathbf{I}_s \cdot \mathbf{E}_s) + \mathbf{I}_s \times \mathbf{E}_s \qquad (23)$$

Eliminating $\mathbf{I}_r$ from eqs. (1) in the paper, we get

$$B\,\mathbf{I}_s = A\,\mathbf{E}_s - \mathbf{E}_r \qquad (24)$$

which is equivalent to the first of the eqs. (2). Multiply eq. (24) separately by $\cdot\,\mathbf{E}_s$ and by $\times\,\mathbf{E}_s$ and substitute the results in eq. (23). This will give

$$B\,\mathbf{U}_s = \mathbf{p}\,(A\,\mathbf{E}_s \cdot \mathbf{E}_s - \mathbf{E}_r \cdot \mathbf{E}_s) + (A\,\mathbf{E}_s \times \mathbf{E}_s - \mathbf{E}_r \times \mathbf{E}_s) \qquad (25)$$

But, according to eqs. (14) and (15),

$$\mathbf{E}_s \,.\, \mathbf{E}_s = E_s^2;\ \mathbf{E}_s \times \mathbf{E}_s = 0 \qquad (26)$$

Hence $\quad B\,\mathbf{U}_s = \mathbf{p}\,A\,E_s^2 - (\mathbf{p}\,\mathbf{E}_r \cdot \mathbf{E}_s + \mathbf{E}_r \times \mathbf{E}_s) \qquad (27)$

The last equation may be simplified as follows:

$$\mathbf{p}\,\mathbf{E}_r \cdot \mathbf{E}_s + \mathbf{E} \times \mathbf{E}_s = (E_r\,E_s)\,(\mathbf{p} \operatorname{Cos} \theta + \mathbf{q} \operatorname{Sin} \theta) = (E_r\,E_s)\,\mathbf{r}_\theta \qquad (28)$$

where $\mathbf{r}_\theta$ is a unit vector in the $P\,Q$ plane at an angle θ to the P

5. For a deduction of the terms on the right-hand side of this equation, see for example, V. Karapetoff, "The Electric Circuit," pp. 91 and 92.

6. As a beginning in this direction, the present writer has recently published an article entitled "The Use of the Scalar Product of Vectors in Locus Diagrams of Electrical Machinery," this JOURNAL, 1923, Vol. 842, p. 181.

7. For an elementary exposition of the general principles of the subject see, for example, J. G. Coffin's "Vector Analysis" (Wiley).

axis, and θ is the phase angle between $\mathbf{E}_r$ and $\mathbf{E}_s$. Thus, eq. (27) becomes

$$\mathbf{U}_s = \mathbf{p}\,(A/B)\,E_s^2 - (E_r\,E_s/B)\,\mathbf{r}_\theta \qquad (29)$$

Let E_s be kept constant; then the term $\mathbf{d} = \mathbf{p}\,(A/B)\,E_s^2$ is also constant. (A/B) is a complex quantity, which may be written in the form:

$$A/B = c\,\epsilon^{j\sigma}$$

Consequently, $\mathbf{d}$ is a vector of length $c\,E_s^2$, at an angle σ to the p axis.

The complex quantity B may be written in the form

$$B = b\,\epsilon^{j\tau} \qquad (31)$$

so that

$$(E_r\,E_s/B)\,\mathbf{r}_\theta = (E_r\,E_s/b)\,\mathbf{r}_{\theta-\tau} \qquad (32)$$

which is a vector of length $E_r\,E_s/b$, at an angle $\theta - \tau$ to the p axis. If E_r is also kept constant, this term represents a circle of radius

$$R = E_r\,E_s/b \qquad (33)$$

The above expressions for $\mathbf{d}$ and R are identical with the values given in the paper for the position of the circle and its radius.

While the above deduction may seem long, its length is due to the general introduction on vector analysis. The specific problem to be solved begins with eq. (23) and ends with eq. (29).

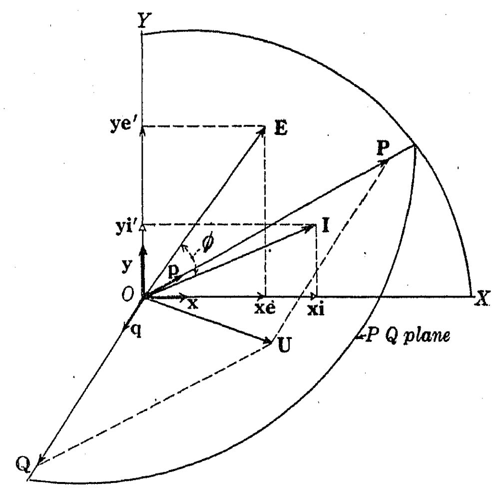

FIG. 39

which is the desired equation of the circle when E_s and E_r are constant.

By assuming E_r to be variable and differentiating eq. (29) *vectorically* (that is, with respect to both $\mathbf{r}$ and θ), a new vector equation is obtained. Eliminating E_r between this equation and eq. (29), gives a vector equation of the envelope. It is not necessary to introduce the components $\alpha, \beta, \gamma, \delta$, and the awkward square roots of the sums of the squares.

C. M. Longbottom (by letter): I propose to discuss one or two aspects of long-distance transmission which were not brought out in the papers. Reference has been made by Messrs. Evans and Bergvall to the amount of reactive kv-a. required at the mid-point of the line when a synchronous condenser is placed there to maintain constant voltage at this point. It seems very pertinent that one should carry this discussion a little further in order to examine the conditions arising from such subdivision of the line and what is more important, to determine how many subdivisions should be made in any case for the most economical arrangement of equipment.

With this object in view, I have examined four particular cases of a 500-mile transmission line operating at 220 kv. It was assumed that transformer banks of 200,000 kv-a. were situated at either end of the line. While banks of this rating would be too big in the case of an "unloaded" line as in case (1) outlined below, banks of this rating would be too small for case (4) but as this is an average value of the maximum load delivered it is not considered necessary to take different transformers in each case. This fact will not materially effect the conclusions to be drawn from this analysis.

The following four cases were considered; (1) a 500-mile transmission line with transformer banks at each end, ample synchronous-condenser capacity at the receiver and ample generator capacity at the supply end. (2) The same line as in (1) but with a synchronous condenser stationed at the mid-point thus dividing the line into two equal portions and forming what we will term "nodal" points at the generator, the mid-point and the receiver end. (3) Similar to (1) but with four nodal points equally spaced or, in other words, two intermediate condenser stations. (4) As in (1) but with five nodal points equally spaced. It will be assumed that the voltage at nodal points is kept constant at 220 kv., that the synchronous condenser

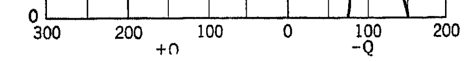

FIG. 40

losses will be neglected and that the generator characteristics will also be neglected.

Figs. 40 and 41 represent the four cases outlined. In Fig. 40 is represented the variation in the net resultant reactive kv-a. injected into the system with variations in loads for a 500-mile line with two nodal points, that is, at the receiver and supply ends. By net reactive kv-a. is meant the algebraic sum of the reactive kv-a. components delivered to the line at each nodal point. Thus in case (1) there will be delivered to the line approximately 150,000 kv-a. lagging, part of this being supplied by the generator and part by the condenser at the receiver end. At any other load the net resultant kv-a. can be read off from the cruve.

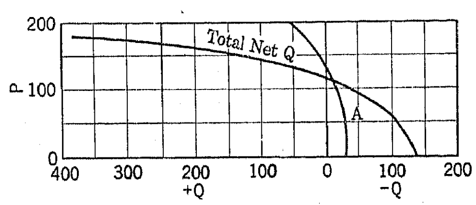

FIG. 41

Curve A, in all cases, refers to the generator characteristics and is actually the circle diagram for the supply end of the system. In Fig. 40 at no load it will be seen that there is nearly 80,000 kv-a. charging current to be supplied by the generator. This reactive kv-a. seriously effects the operation of the system since it tends to over-excite the generators so that the excitation of these machines must be very substantially reduced and this in itself presents a problem of no little difficulty. Reference to Figs. 41, 42 and 43 will show that in general the more nodal points, that is, synchronous condenser stations distributed along the system the less the generator excitation needs to be interfered with. It will be seen from Fig. 43 that with five nodal points the generator power factor differs very little from unity under any condition of load. It follows, therefore, that the best use is made of the generators when a large number of condenser stations are used. In other words, the rating of the generating equipment is higher under these conditions.

A question which is equally important with that of the generator is the amount of synchronous-condenser equipment that must be installed under the various conditions. This represents a capital cost which must be taken into account in the same way

as the cost of the conductors. In general, it might be said that a synchronous condenser placed at any point along such a line is less effective in regulating the voltage than one placed at the receiving end, due to the fact that the inductance between that station and the generator is less than that for the whole line.

The amount of reactive kv-a. supplied at the nodal points has no direct relation to the amount of plant required since leading kv-a. may be supplied at one point and lagging kv-a. at another as is shown in the accompanying tables. It must be pointed out that the total net kv-a. shown in Figs. 41 to 43 gives no indication of the power factor on the system. The curve is of more theoretical interest than practical. Fig. 44 shows this net reactive kv-a. for the four cases considered plotted for different values of kilowatts transmitted. As a matter of interest, it

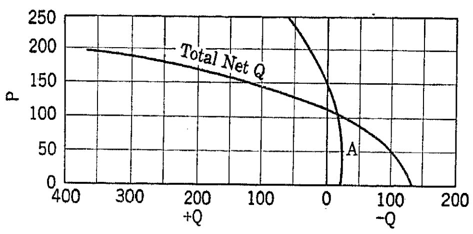

FIG. 42

might be pointed out that the load curve for zero reactive kv-a. represents approximately the conditions mentioned by Mr. Percy H. Thomas on Page 2 of his paper. If resistance were neglected there would be no reactive kv-a. injected into the line for this particular line for this particular load. Although Fig. 40 shows that there is no net reactive kv-a. injected at 125,000 kw. nevertheless Table 1 shows that the generator is actually supplying 24,000 kv-a. and the receiver condenser is drawing 24,000 kv-a. The apparent discrepancy in Fig. 44 from this purely theoretical consideration, is due to several causes among which are; first, the neglect of resistance in the theoretical case; secondly, the rise in voltage along the line in the practical case; and thirdly, possible inaccuracies in calculation as no great accuracy was aimed at in deriving these figures.

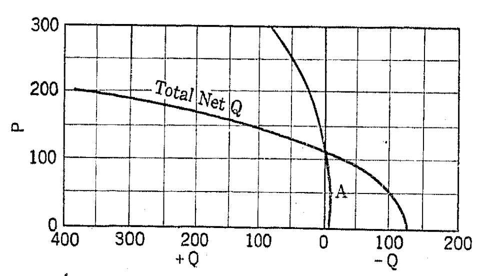

FIG. 43

Tables I to IV show the amount of plant in synchronous condensers required for any particular number of nodal points. In general it might be said that the more nodal points give the greater amount of synchronous-condenser plant required but at the same time the load which may be transmitted over the line is increased. Fig. 45 is a purely theoretical analysis of the four cases considered. The maximum load referred to is the maximum possible load which would be transmitted over the line regardless of any questions of stability. This corresponds to the power limit of the line as originally mentioned by Mr. H. B. Dwight and others some time ago. And in general will be the limiting load of the first section of the line, that is, the generator end. This maximum load has been used to derive the curve showing the ratio between the total net reactive kv-a. and maximum power.

In summarizing the above discussion the following conclusions may be drawn:

1. The effect upon the generator excitation of increasing the number of nodal points in a long transmission line is to increase the stability of the generator at light load and, general to improve the power factor and hence increase the station capacity.

2. An increase in the number of nodal points leads in general to an increase in the synchronous-condenser equipment required.

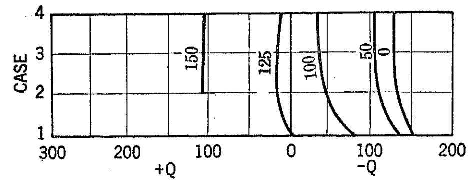

FIG. 44

At the same time the output of the line is materially increased.

I have purposely avoided discussing questions of stability as the characteristics of the load have such a direct bearing on this subject that each particular case must be considered on its merits. It is for this reason that I have chosen transformer banks of 200,000 kv-a., although actual stability limitations would call for transformer banks of approximately half this rating.

I wish to raise one point in connection with part 2 of the paper by Evans and Sels in their discussion of parallel transmission lines. It sometimes happens that two transmission lines of different voltage are working in parallel and it is necessary to determine the line constants for the combined line. This may be done in the following manner. Use one voltage as a reference voltage and determine the line constants, A_1, B_1, C_1 and D_1 of this line and also the line constants A_2, B_2, C_2 and D_2 of the other line. It is necessary now to transfer the second set of constants into new constants referred to the reference voltage. The A constant will remain the same, the B constant must be multiplied by the square of the ratio of the voltages, and the C constant must be multiplied by the reciprocal of that ratio squared, the D constant remaining the same. Equations **18** to **21** may then be applied.

The method of determining the division of load between the lines may be had by reference to Fig. 1 of the paper by C. L. Fortescue and S. C. F. Wagner, since whichever path is followed the voltage vector must turn through the same angle for equal voltage so that by plotting the circle diagram for both lines

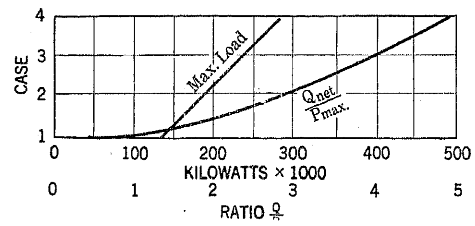

FIG. 45

separately and for the combined lines we can determine the angle θ for any given load on the combined line and by drawing this same angle on the separate line diagram, determine the amount of load on each line.

TABLE I

Kv-a. in thousands required at salient points for various loads delivered. Case I.

Load	Kv-a.$_S$	Kv-a.$_R$	Total Kv-a.$_{net}$
125	−24	+24	0
100	−48	−25	− 73
75	−63	−48	−111
50	−73	−64	−137
0	−78	−77	−155

TABLE II

Kv-a. in thousands required at salient points for various loads delivered. Case 2.

Load	Kv-a.$_{R}$	Kv-a.$_{M}$	Kv-a.$_{S}$	Total Kv-a.
188	+103	+250	+152	+505
150	+ 45	+ 61	+ 26	+132
100	+ 3	− 22	− 15	− 34
80	− 20	− 63	− 25	−108
0	− 30	− 75	− 30	−135

TABLE III

Kv-a. in thousands required at salient points for various loads delivered. Case 3.

Load	Kv-a.$_{R}$	Kv-a.$_{M_2}$	Kv-a.$_{M_1}$	Kv-a.$_{S}$	Total Kv-a.
200	+77	+106	+132	+61	+376
150	+35	+ 37	+ 41	+14	+127
100	+ 7	− 13	− 15	−12	− 33
50	−10	− 38	− 40	−20	−108
0	−20	− 45	− 48	−20	−133

TABLE IV.

Kv-a. in thousands required at salient points for various loads delivered. Case 4

Load	Kv-a.$_{R}$	Kv-a.$_{M_3}$	Kv-a.$_{M_2}$	Kv-a.$_{M_1}$	Kv-a.$_{S}$	Totalkv-a.
250	+109	+147	+143	+225	+120	+744
200	+ 65	+ 72	+ 66	+101	+ 48	+352
150	+ 34	+ 20	+ 15	+ 32	+ 12	+113
100	+ 12	− 12	− 17	− 7	− 5	− 29
50	− 2	− 30	− 35	− 27	− 12	−106
0	− 11	− 35	− 40	− 34	− 10	−130

L. A. Herdt (by letter): I read with great interest the paper by Messrs. Fortescue and Wagner on "Some theoretical consideration of Power Transmission" in the February 1924 issue of the JOURNAL of the A. I. E. E.

I noted (and they are not the only writers that do so) that the term *Synchronous Condenser* is used—Leading and Lagging Condenser Fig. 2. Applied to such machines this term is a misnomer, giving the impression that the chief function of the machine is to supply leading current while this represents but one-half of its use in service. The other and fully as important function is to supply *lagging* current.

The use of the term *Synchronous Reactor* for such machines was suggested by the writer (see the *Electric Journal* September, 1915—Constant Voltage Operation of a High-Voltage Transmission System) in view of the common use of the term reactance as positive to represent inductive reactance and negative to represent condensive reactance.

A synchronous reactor is a machine having the property of reactance positive or negative, that is inductive or condensive.

A. E. Kennelly (by letter): The paper by Mr. Thomas is valuable in presenting the conditions that may be expected to occur during the steady-state, over a power-transmission line of unusual and hitherto unattained length, operated three phase at 220 kilovolts.

On three-phase power-transmission lines of the lengths that have hitherto come into service, the hyperbolic-formula corrections for the effects of distributed capacitance at the fundamental frequency of 60 cycles per second, have been small. In most cases, these corrections might be ignored in first-approximation calculations, and the lines treated on the basis of capacitance placed either in one lump at the middle of the line (for the nominal T), or in two equal lumps at the ends of the line (for the nominal π).

In the case, however, of Mr. Thomas' line, 500 miles long (804.7 km), the nominal π of any one of the three line conductors has an appreciable error at 60 cycles per second, and has to be corrected by some process which takes the uniform distribution of capacitance into account. This is done in the paper by means of the equations at the foot of its third page, and which have been shown by Mr. Thomas in an earlier paper to be suitable for the purpose. An alternative method, preferred by the writer, is to form the equivalent[8] π of one line, such that the electrical behavior of this π, according to Ohm's Law, gives the same voltages, currents and powers at its terminals under the assigned conditions of load, as the actual line with its distributed constants.

In the accompanying Fig. 46, $A\ B\ G\ G$ represents the nominal π of one of the conductors considered in Mr. Thomas' interesting paper. The line has $R = 39.6$ ohms resistance in the conductor, and also $X = 370$ ohms reactance, at the working frequency of 60 cycles per second, assuming that there are no harmonics present in the voltage or in the current. The line leakance is ignored $(G = O)$, and the total dielectric susceptance is taken as $j2.882$ millimhos, divided into two terminal condensers of $j1.441$ millimhos each.

The angle θ of the line is $\sqrt{Z\,Y} = 1.035 \angle 86°.56'.44''$ hyperbolic radians, and the correction factors, by reference to the charts prepared for that purpose,[9] are $0.832 \angle 1°.12'$ for the architrave of the π, and $1.10 \nwarrow 0°.37'$ for each pillar. Applying these factors, we obtain the equivalent π, $A'\ B'\ G'\ G'$ in the lower part of the Fig. 46. The line behaves as though it had 27.0 ohms resistance instead of 39.6, a reduction of 31.8 per

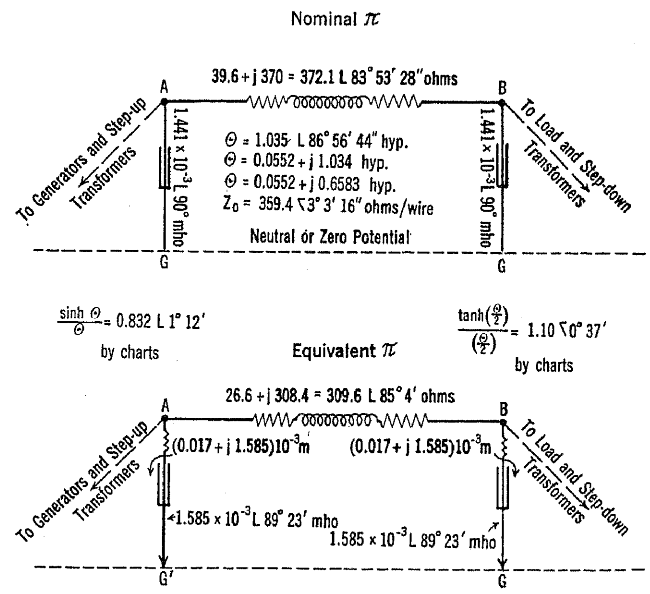

FIG. 46

Nominal and Equivalent π of one of the three-line conductors considered in the Thomas superpower transmission paper:

L = 804.7 km. = 500 mi. $f = 60\sim$ $D = 15.8' = 4.816m$ $d = 0.0278$ m.
r = 0.04921 ω/w. km. = 0.07920 ω/w. mile $R = 39.6$ ω/wire
l = 1.220×10^{-3} h/w. km = 1.963×10^{-3} h/w. mi; $j\,x = j\,0.46$ ω/w. km = $j\,0.74$ ω/w. mi; $j\,x = j\,370$ ω/wire.
L = 0.9815 h/wire; $g = 0$ $G = 0$. $c = 0.950 \times 10^{-8}$ f/w. km. = 1.529×10^{-8} f/w. mi.
C = 7.645×10^{-6} f. $j\,b = 3.581 \times 10^{-6}$ m/w. km. = $j\,5.763 \times 10^{-5}$ n/w. mi. $j\,B = j\,2.882 \times 10^{-3}$ n/wire.

cent, and $j\,312.4$ ohms reactance instead of $j\,370$, a reduction of 15.5 per cent. The effect of distributed capacity upon the π of the line $A\ B\ G\ G$, is to reduce its apparent conductor resistance by nearly one third. The admittance of each pillar is, however, increased ten per cent, to 1.585 millimhos, with a small leak element added of 0.017 millimho. If now the load to neutral be attached at B' and the generator apparatus at A', the equivalent π enables the terminal voltages, currents and

8. "Artificial Lines for Continuous Currents in the Steady State" A. E. Kennelly, *Am. Ac. of Arts & Sciences*, Vol. 44, No. 4. Aug. 1908, page 97.

9. "Electrical Characteristics of Transmission Lines" by Wm. Nesbit, Westinghouse El. & Mfg. Co., Publication, Pittsburgh. Feb. 1922.

powers to be determined by the ordinary vector Ohm's law calculation.

It is evident that the hyperbolic correction factors for a line of this length operated at 60 cycles, are by no means negligible.

F. R. Sharpe: The mathematical results found by Fortescue and Wagner may be expressed in the following simple form.

(A) Consider the normals in Fig. 47 drawn at any point P of a parabola to meet the axis in N. A circle with its center at N and of radius $P\,N$ will touch the parabola at P and at the symmetrical point P'. The envelope of all such circles is clearly the parabola. The smallest circle has the radius $2\,p$ and is tangent to the parabola at its vertex V.

If we denote $F\,N$ by h the radius is

$$P\,N = \sqrt{P\,M^2 + M\,N^2} = \sqrt{4\,p\,V\,M + (2\,p)^2} = \sqrt{4\,p\,h}$$

Hence the envelope of the circles

$$(x - h)^2 + y^2 = 4\,p\,h \tag{1}$$

is the parabola $y^2 = 4\,p\,(x + p)$, the origin being the focus F.

In the case of the receiver circles the vectorial form of the equation (1) is

$$R = \frac{P_r + j\,Q\,v}{\alpha - j\,\beta} = E_r^2 - \frac{\gamma - j\,\delta}{\alpha - j\,\beta}\,E_s\,E_{re}{}^{-j\,\theta}$$

$$= E_r^2 - \sqrt{\frac{\gamma^2 + \delta^2}{\alpha^2 + \beta^2}}\,E_s\,E_{re}{}^{-j\,\theta'} \tag{2}$$

where $\theta' = \theta + \theta_1 - \theta_2$

so that $h = E_r^2$ and $4\,p = \dfrac{\gamma^2 + \delta^2}{\alpha^2 + \beta^2}\,E_s^2$

(B) We may also derive the same results by vector differentiation of (2): The vector to a point of intersection Q of (2) with the neighboring circle is given by (2) and by

$$R = (E_r + \delta\,E_r)^2 - \sqrt{4\,p}\,(E_r + \delta\,E_r)\,e^{-j\,(\theta' + \delta\,\theta')}$$

and is therefore determined by the vector equation

$$0 = 2\,E_r\,\delta\,E_r - \sqrt{4\,p}\,\delta\,E_r\,e^{-j\,\theta'} + \sqrt{4\,p}\,E_r\,e^{-j\,\theta'}\,j\,\delta\,\theta'$$

as in the case of ordinary differentiation. Hence taking the real and imaginary parts we have

$$(2\,E_r - \sqrt{4\,p}\cos\theta')\,\delta\,E_r - \sqrt{4\,p}\,E_r\sin\theta'\,\delta\,\theta' = 0$$

and

$$\sin\theta'\,\delta\,E_r - E_r\cos\theta'\,\delta\,\theta' = 0$$

Hence

$$\frac{2\,E_r - \sqrt{4\,p}\cos\theta'}{\sin\theta'} = \frac{\sqrt{4\,p}\sin\theta'}{\cos\theta'}$$

that is

$$E_r\cos\theta' = \sqrt{p}$$

$$E_r = \sqrt{p}\ \mathrm{sec.}\ \theta' \tag{3}$$

Hence the envelope from (2) is

$$R = p\sec^2\theta' - 2\,p\sec\theta'\,e^{-j\,\theta'}$$

$$= p\tan^2\theta' - p + 2\,p\tan\theta'\,j = x + j\,y$$

Hence $y = 2\,p\tan\theta'$; $x = p\tan^2\theta' - p$; or eliminating $\tan\theta'$, $y^2 = 4\,p\,(x + p)$.

(C) The differentiation performed graphically gives at once the result

$$\cos(\theta' + \delta\,\theta') = \frac{\sqrt{4\,p}\,\delta\,E_r}{2\,E_r\,\delta\,E_r} \text{ approximately}$$

because

$$\cos Q\,N'\,N = \frac{Q\,N' - Q\,N}{N\,N'}$$

and taking the limit, as $\delta\,E_r$ and $\delta\,\theta'$ approach 0,

$$\cos\theta' = \frac{\sqrt{4\,p}}{2\,E_r} \text{ as in (3).}$$

F. W. Peek, Jr: All of these papers refer to a specific problem,—the transmission of a large amount of power 500 miles at 220 kilovolts, sixty cycles. It happens that a 500-mile line is not economical unless power of the order of 100,000 kw. per circuit can be delivered over it. This is a rather large amount of power for 500-mile transmission at 220 kv. under the conditions fixed by the problem. Since the load must be increased to the limit for economic reasons the question of stability is an important factor. A higher voltage or lower frequency would permit greater power per circuit without approaching the instability point. The difficulties, therefore, arise to a considerable extent from the fixed conditions of the problems rather than to limitations of high-voltage transmission.

It is not the actual physical length of the line that causes limitations. Limitations are caused by certain factors that depend upon physical length, mainly inductance, but to some extent capacity and resistance. If the inductance is reduced the line has the characteristics of one of short length. Some time ago Mr. Percy Thomas suggested how this might be done by "split conductors." Split conductors are also advantageous from the corona standpoint. At 25 cycles this line would have characteristics similar to present 220-kv. lines 200 miles in length.

In considering the effects of transient load changes on stability it is not only necessary to consider how quickly the generator

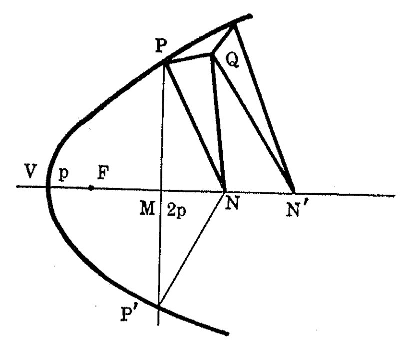

FIG. 47

can respond but also what the increments of load are and how rapidly they can be changed. The size of the system is an important factor.

It is interesting that it is not necessary to discuss insulation limitations. As the transmission voltages and distances of transmission increase the size of apparatus units increases and becomes an important problem. This introduces the question of size of material and difficulties of transporting built-up units.

Chairman Baum: I have been working, as some of you know, with quite a large transmission system for the last twenty-five years. As I say, the stability of that system has been increasing all the time and the last addition of the synchronous condensers has increased the stability so markedly that if you talk to the operators, they will tell you right away how easy it is to operate and how difficult it is to shake that system apart.

I don't think the load transients should apply to your regulation, back to include your generator reactance.

I think Mr. Thomas is too pessimistic about his system, but the only criticism I would make is this: he thinks the switching is difficult. Therefore, he eliminates the high-tension switching. As a matter of fact, the high-tension switching has been of no trouble on our system, and is one of the most satisfactory thing-we have. We laid down the rule twenty years ago that the highs tension switching must be the same as low-tension switching, and I don't believe you can have a successful transmission system without that condition being fulfilled.

If you put that condition down, that you must have switching in and out, no matter what the time of day or what the conditions are, then you must have that line cut up into sections, you can't help that, and that means of course adding some cost. People say the cost of condensers and the cost of switching is

too great. When you get through, the cost per kilowatt transmitted is the real thing you are after, and the cost per kilowatt at two hundred twenty thousand is about one-half the cost per kilo watt at one hundred ten thousand and that is the real answer to your transmission problem.

R. C. Bergvall: Our papers show a method of determining the maximum amount of power which can be transmitted with a load gradually applied. Mr. Moreland has incorrectly inferred that we propose to *rate* a transmission system as being capable of carrying this amount of power under normal operating conditions. The maximum amount of power which can be transmitted under steady state conditions can only be used as a basis for forming an engineering judgment as to the amount of power which should be transmitted over a given system. The point at which a transmission line should be worked will depend on local factors such as continuity of service requirements, a discussion of these factors being obviously outside of the scope of this paper.

During transient conditions there are so many variables, which would have to be taken into account, including the action of machine governors, which has not been considered by Mr. Moreland, that the reliability of any transient mathematical analysis is open to a great deal of question. The analysis carried out by Mr. Moreland and his associates which show that the *rate of change of load* instead of the amount of suddenly applied load is the important factor, is not borne out by practical operating experience or by our tests. There are numerous cases of two transmission lines operating in parallel where a 100 per cent increase in load results on one of the lines whenever the second line is tripped out and pull-out does not result. In one case where the load thus added to the second line approached the maximum power limit, the system occasionally pulled out of step after two or three seconds, thus indicating that the process is a slow one.

Mr. Moreland has stated that the loads in our tests were gradually applied, basing his conclusion on the oscillogram shown by Fig. 18. This oscillogram does not refer to the suddenly applied load tests as may be ascertained by reading the test data. This oscillogram shows the pull out taking place as the load is *gradually* increased. The suddenly applied load tests were made by intermittently shunting out a portion of the resistance used as load. The inductance of the armature of the direct-current generator of the motor-generator set is very low as compared to the resistance in the load so that the resulting time constant is negligible. Calculations taking into account the inertia of the motor-generator set show that the load variation would be applied to the system in from 1/20 to 1/10 of a second. The results of this test do not agree with the conclusions arrived at by Mr. Moreland and his associates, as to the effect of suddenly applied loads.

The amount of load which can be suddenly applied without causing pull-out to occur can be approximately determined by considering that the synchronous machine excitation remains constant, due to the inductance of its field, resulting in a drop in the receiver voltage. As an example, refer to Fig. 5 in our paper. Assume that the line is carrying 80,000 kw. at 100 per cent voltage, which corresponds to a synchronous condenser excitation of 155 amperes. If the load is suddenly increased to 87,500 kw. and the excitation of the synchronous condenser remains constant, the voltage would drop to point *D* or 58 per cent voltage, resulting in pull out. Mr. Moreland assumes that we would continue out at 100 per cent voltage and could therefore, stand a swing to 107,000 kw. instead of 87,500 kw. without pull out taking place.

The assumption of constant synchronous condenser excitation is, if anything, pessimistic because the armature reactance during transient conditions is in such a direction as to assist the synchronous condenser voltage regulator in increasing the field current. Mr. Evans, in his discussion of Mr. Thomas' paper, presented our ideas of a method for determining the rating of a transmission line and the results are in substantial agreement with those of the other speakers, whereas Mr. Moreland's discussion would make it appear that our results would be widely different from theirs.

Our calculations were based on a constant voltage at the generator end of the line as stated in the paper. This, of course, would only be approached for a very large generating station as compared to the transmission line connected to it. The generator used in our tests was somewhat larger than the maximum load of our transmission line and our tests indicate that for this condition the effect of generator synchronous reactance was small. However, in most practical cases it will be necessary to take into account the generator synchronous-reactance by merely adding it to the reactance of the line and considering the internal voltage of the generator instead of the supply voltage of the transmission line. Our method of calculation may readily be extended to meet such condition.

R. D. Evans: Mr. Thomas, in his paper on "Superpower Transmission" has sought to fix a rating of a 500-mile transmission line. Mr. Thomas states that the maximum permissible load on the line is 143,000 kw., and he proposes to operate the

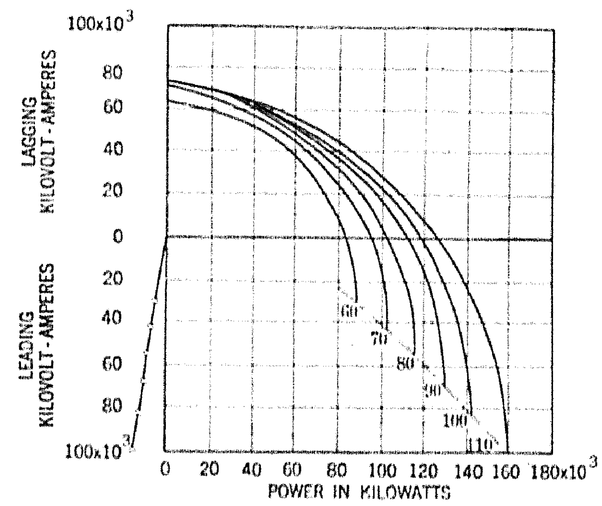

Fig. 48—Circle Diagram for 500-Mile Transmission Line

line at 120,000 kw. continuously when one of the transmission lines is out of service. We do not find that Mr. Thomas made any allowance for the characteristics of synchronous condensers and load which would reduce the maximum permissible output of the line to an amount not exceeding 100,000 kw., and, in our opinion, this amount should be considerably lower to allow a reasonable factor of safety.

We have made calculations, taking into account the characteristics of the synchronous condenser and load, using the transmission line, synchronous-condenser capacity, et cetera, as given in Mr. Thomas' paper. The performance of the transmission line is shown in the circle diagram given in Fig. 4, Condition *A* of Mr. Thomas' paper. A family of power circles derived therefrom is reproduced in Fig. 48. The synchronous-condenser capacity is based on 75,000 kv-a. in condensers per line with spares, the condensers having equal leading and lagging kv-a. range. Under emergency conditions, with one transmission line out of service, the condenser capacity per circuit with all the spare units in operation, amounts to 125,000 kv-a. per line, as indicated in Mr. Thomas' Fig. 5.

Mr. Thomas assumes the power factor of the load as 85 per cent. The characteristics of the load for changes in voltage are not indicated, but a reasonable assumption would be that 70 per cent of the load consists of rotating machines of essentially

constant power characteristics. The remainder of the load is assumed as a lighting or resistance load which varies approximately as the square of the voltage. It has been assumed that the reactive kv-a. of the load remains substantially constant, independent of the voltage. Referring to Fig. 49 the curves in solid lines show how the reactive kv-a. required by the line varies with voltage for loads corresponding to the amounts indicated, at 100 per cent voltage; that is, these curves take into account the decrease in power due to the decrease in resistance load with lower voltage. The dotted-line curves show how the reactive kv-a. of the load and synchronous condensers varies with voltage at various constant condenser field excitations. The line *A B* is the locus of points of tangency between the two families of curves. The system will pull out of step when the conditions of operation fall below the line *A B*.

In the actual operation of transmission lines, even in large systems, there are inevitably many sudden fluctuations of load due to short circuits and switching operations either on the main lines or on auxiliary lines or systems tied in therewith. When the load on a transmission line is suddenly increased, the inertia of the rotating apparatus will momentarily supply part of the

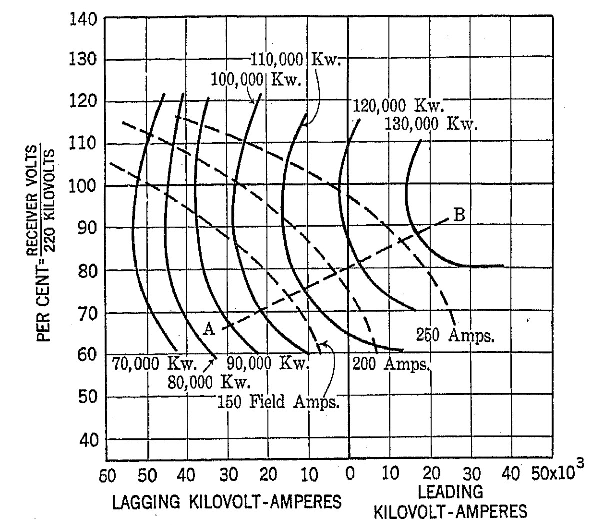

FIG. 49—500-MILE TRANSMISSION LINE WITH 125,000-KV-A. IN SYNCHRONOUS CONDENSERS

additional load requirements. Even allowing for the load increment momentarily taken by the rotating apparatus, a 15 per cent sudden increase in power which the line must supply is a small alowance to make for sudden increase of load due to loss of a transmission line or other disturbing condition. Applying this test to the curves shown in the Fig. 49, it will be noted that if the transmission line is operated at a load of about 101,000 kw., corresponding to a condenser excitation of 200 amperes, pull-out will occur with about 15 per cent increase of load.

Mr. Thomas has tacitly assumed that the line is stable up to the maximum power capacity of 143,000 kw. He has taken no account of the characteristics of the load and synchronous condensers, or of the fact that the machine excitation will not be varied as rapidly as the load can change.

The above analysis indicates a maximum permissible load of 100,000 kw. beyond which there is no certainty of securing stable operation and which must be reduced to allow for the necessary margin of safety. Assuming a 20 per cent margin, the capacity of the line would be reduced to about 80,000 kw.

It is to be pointed out that assumptions have been made which are favorable to the transmission of a large amount of power. For example, our calculations of the transmission-line constants for the particular conductor described by Mr. Thomas show a power limit of approximately 10 per cent below that stated by him and assumed in the preceding discussion. The condenser capacity assumed is very liberal, taking all the spare units in operation. The assumption that 30 per cent of the total load is lighting and resistance load varying with the square of the voltage is also liberal. Mr. Baum's survey shows that lighting load constitutes about 15 per cent of the total. The effect of generator characteristics in limiting the power output of the line has been neglected. The final assumption made in the preceding discussion was the amount of increase in load that should be provided for. The value selected appears quite small when it is pointed out that with three lines in service, the loss of one transmission line would increase the load on the remaining circuits about 50 per cent.

Our analysis indicates that it is impossible to give a rating to the circuit of 120,000 kw. for continuous operation under emergency condition with one line out of service. In our opinion, the maximum permissible amount of power would be of the order of 70,000 to 80,000 kw.

The importance of a consideration of the characteristics of the synchronous condenser and load is emphasized by the fact that the transmission line in Mr. Thomas' paper should not be expected to carry more than about 80,000 kw.

E. B. Shand: My paper on the subject of the maximum output of transmission systems has been the result of a study made several years ago on the principles of stability of synchronous generators both under the condition of constant excitation, and also when the terminal voltage is maintained constant by means of a vibrating regulator. This study gave rise to the distinctions of inherent and artificial stability as defined in the paper. No thought of applying these principles to transmission systems had occurred to me until the proposal of F. G. Baum in 1921 to use synchronous condensers to regulate the voltage at intermediate points of lines suggested itself as permitting a broad extension of the same principles.

On account of the absence of any experimental data regarding the relative speeds of regulator action and of inertia changes in regulated machines, the writer considered it probable that the maximum steady load with a vibrating regulator would be somewhat in excess of the limit determined by inherent conditions; but on account of sudden load increases, where it was appreciated that the regulator action would be too sluggish, he proposed the maximum load as determined by the excitation values of the machines preceding the load increase as the criterion of stability, that is, inherent condition of stability.

With this conception of the problem in mind, he evolved a method for the determination of the limitation of inherent stability of a system utilizing the actual characteristics of the synchronous condenser and of the load. As far as his information extends he believes that this was something entirely new in the study of transmission systems.

At the close of this work the writer came in contact with the power-circle diagram as developed by Messrs. Evans, Sels and Fortescue, and an actual system was investigated using this as a basis. Further study, however, convinced him that the voltage vector diagrams expressed the conditions of stability with greater significance so that this type of diagram was used in the presentation of the paper.

The tests described in the paper of Messrs. Evans and Bergvall, which were made some time later, were of special interest from the writer's standpoint, for they demonstrated definitely that with the vibrating regulator as now developed and with the relative magnitudes of machine inertia as used on the tests, the possible range of artificial stability was negligible. It has been noted that the criterion of stability had already been proposed on the basis of inherent stability, therefore, the result of these tests laid additional emphasis on the importance of this limitation. The co-operated endeavors of the authors of the various transmission papers, both before and after these tests,

cleared up many other questions regarding the operation of transmission systems and their limitations. This is believed to be evident from the papers themselves.

Some of the above discussion, although somewhat recapitulatory, has been included particularly with respect to the discussion by a number of the members,—Messrs. Moreland, Booth, Bush, Doherty, etc., regarding the steady-state stability and surge stability. From additional and more careful reading of this discussion, it seems that a part of their criticism at least is due to a misapprehension or perhaps a mis-application of the data given in the papers. Although the papers referred to take no definite account of transient effects such as inertia, etc., in their calculations, the writer most certainly agrees that in the case of a sudden increase of load the limit of stability will depend mainly on the excitation values of the machines considered prior to the increase of load and not the values required to maintain the proper voltage after the increase. The writer had never comprehended that this point was in question.

Referring now particularly to my own paper, the sole question discussed was that of maximum output, and neither (a) the maximum permissible load, nor (b) the maximum rated load

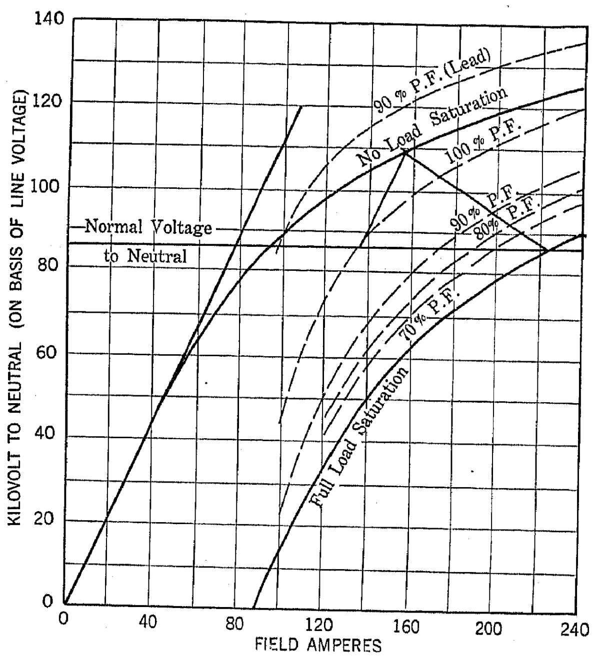

FIG. 50—CHARACTERISTIC CURVES OF AN A-C. GENERATOR CONNECTED TO A TRANSMISSION LINE

was considered, the purpose of the paper being rather to explain the one phenomenon discussed. The results of examples serve, therefore, as criteria of design, that is, they represent a definite limitation determined by the system itself and not by a combination of variable conditions of operation. The rated capacity on the other hand, is a quantity which may be chosen to give satisfactory results over a more or less restricted range which represents a balance between capital cost and reliability. It may be pointed out, therefore, that a single margin, or factor of safety, might be as judicially chosen as the amount of sudden load increase to be used in determining a new limitation to which a new factor of safety must be added to obtain either a safe maximum load or the rated capacity. Of the two methods of procedure above outlined the writer prefers the former, although to determine a suitable margin it has been his practise to determine the maximum limit of stability under reduced excitation, for instance, that corresponding to normal load on the line, which will, it is believed, check roughly with the methods referred to in the discussion. Fig. 10 of my paper represents the method referred to for the special case of maximum excitation on the synchronous condensers. It will be observed that pull-out occurs at reduced voltages.

Messrs. Moreland and Bush have strongly recommended the inclusion of inertia and field transient effects in calculations. Until these gentlemen have given us the details of their methods it is impossible to discuss them fully. While it would undoubtedly be desirable to include these factors, the writer feels that these members have over-estimated the importance of these factors, and under-estimated the difficulties of accurately taking them into account, particularly in the case of the more complicated types of systems.

Mr. Doherty has censured the writer for his neglect of the effect of generator reactance in the examples of his paper and I have already admitted culpability in the matter. It is only fair to state, however, that if Mr. Doherty had continued his quotation from my paper it would have been evident that the matter was not passed entirely by. Since the presentation of the paper,

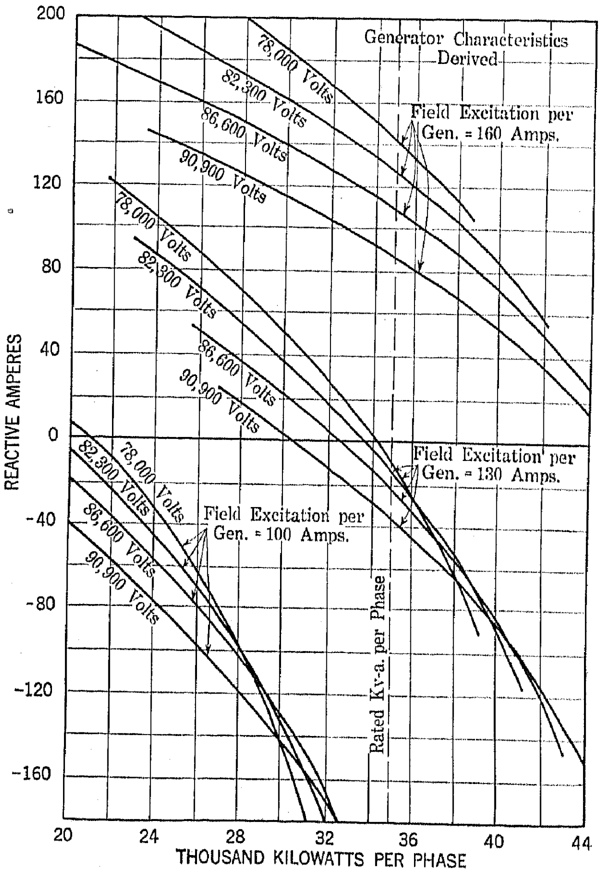

FIG. 51

however, I have extended the diagrams to which he referred, to include the effect of generator reactance. This was accomplished by the first of the two methods referred to in my paper, because the second will prove inaccurate unless great care is used in the choice of synchronous impedance to represent a given machine.

Fig. 50 represents in the conventional manner, the electrical characteristics of the generators considered. The total full-load rating of all the generators combined was assumed to be 35,000 k-va. per phase or 105,000 k-va. total. Fig. 51 gives these same characteristics replotted in accordance with the conventions used in the paper. Three separate values of field excitation are represented. The characteristics of the same compound line of Fig. 12 of my paper, represented by Fig. 52 are derived from an extension of the general method employed for compound transmission lines. All condensers are assumed to be operating under maximum excitation, which is constant regardless of the

voltage conditions at the various points regulated. As a matter of fact, the voltage at the mid-point comes out slightly above normal value which tends to increase the effect of generator impedance slightly. It also causes the generator excitation to be somewhat less than it would otherwise be.

The superposition of the generator characteristics on those of

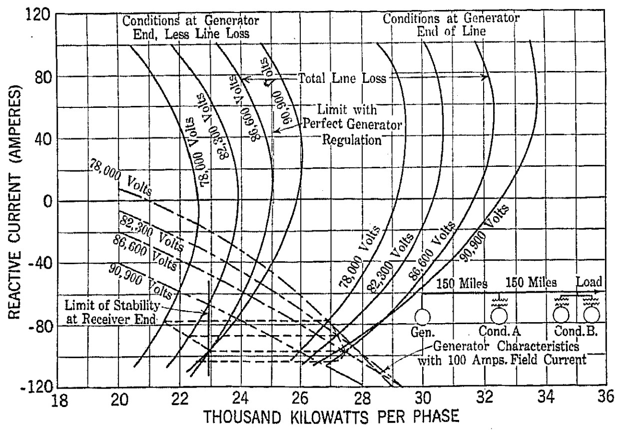

FIG. 52

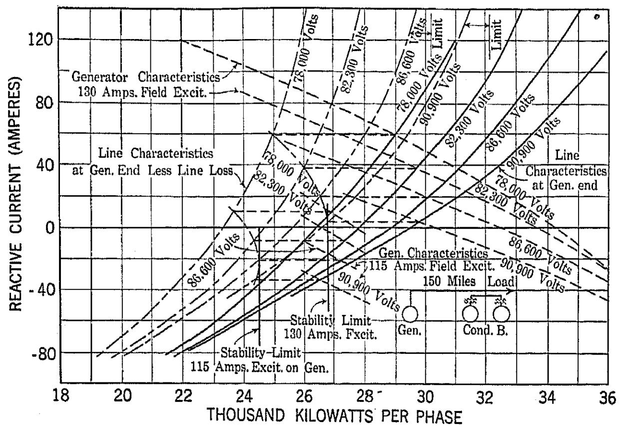

FIG. 53

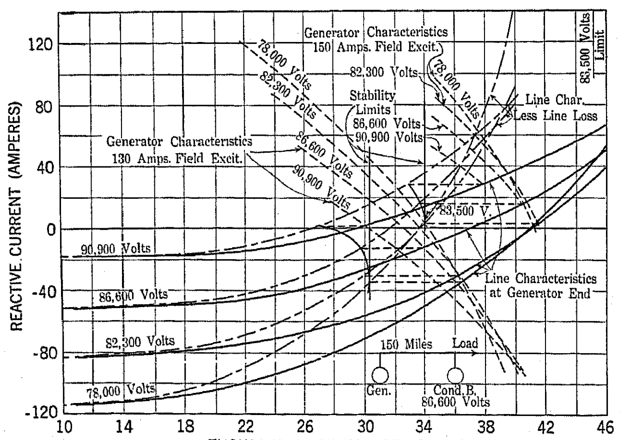

FIG. 54

the line allows the operating condition of the combination to be determined. With a generator field of 100 amperes, the point of maximum output at the receiver end occurs at approximately 86,600 volts at the generator terminals. The load value itself is 22,900 kw. per phase with a power input of 27,400 kw. per phase. The corresponding output with perfect generator regulation is 25,050 kw. which is the same value as that given in the paper. The effect of the generator impedance in this case is a reduction of output of 2150 kw. per phase or by 8.5 per cent. Considering that the generators are underloaded, this value will be slightly smaller than what might normally be expected, but in any event the effect should not be greater than 10 per cent.

Fig. 53 has been plotted to indicate the corresponding effect on a simple line of one half the length of that used above, the rated condenser capacity at the receiver end being 50,000 kv-a. as before. In this case the generator impedance reduces the maximum output from 30,200 kw. to 24,500 kw. which is a reduction of 19 per cent.

A third case, Fig. 54, assumes the conditions to be the same as in the last case except that the receiver end is perfectly regulated. The effect of generator impedance will then be approximately 33 per cent.

This successive increase in the effect of generator impedance is quite to be expected as each change is equivalent to a reduction in line impedance while the generator impedance remains practically constant.

The general conclusion from this investigation is that for transmission lines of the length of those considered in the papers and discussion the effect of generator impedance should be of the general magnitude of 10 per cent reduction of output and that this will naturally increase as the line is reduced in length.

Mr. Terman has brought up in his discussion a point of technicality regarding the conventions in plotting power-circle diagrams. While this is perhaps of minor importance it may be noted that Figs. 14 and 15 in the appendix of my paper comply with his suggestions.

C. L. Fortescue: I thought I had read my own paper over very carefully, but it appears from the remarks made by Mr. Moreland and Mr. Booth and Dr. Bush that I haven't. However, in rebuttal of their accusations I want to point out that it is an entirely different thing to make a hypothetical load in a paper as a basis for comparison and to advocate it as a proper rating.

Now, Mr. Moreland intimates that we advocated a certain rating which was an impractical rating on a transmission line. We have not done so. We have merely taken a hypothetical rating to form a basis of comparison between two different systems: one, a system of 500 miles straightaway, and the other one, 500 miles in length loaded at a middle point. That is quite an appropriate thing to do.

As regards the question of rating, I have always taken this point of view: I say, let's figure up the stability under the worst possible conditions and call these emergency conditions. Let's take a good margin of safety and then under normal operating conditions we shall be good and safe. That is the position I take.

Now, I am an optimist, and I saw a passage in a recent book by Havelock Ellis, in which he says, "The nice balance between optimism and pessimism is the best thing." I think Mr. Moreland and his associates have gone to the other extreme. They are indulging in a great deal of pessimism. They have done a lot of work and hard digging, but it is all theoretical work, and they haven't shown the way they obtained their results. They have shown us a number of curves but how did they get these curves? I hope that Mr. Moreland and his associates will sometime tell us how these curves were obtained.

We, as an actual matter of fact, made tests on a system that imitated on a small scale a 500-mile straightaway line; also a 500-mile line loaded at its middle point, and with this system we had ordinary generators of fair size. We took generators of approximately 500-kw. capacity which at that time we considered would give a fair basis of comparison with those used on large transmission lines.

Now, we supposed it fair to assume that the inertia conditions in those generators and synchronous condensers and also their

synchronous reactance were about on the same scale as these quantities for the generators and synchronous reactors used on an actual 500-mile, 220-kv. line. Under the test conditions we were able to throw off and on blocks of power, 25 per cent of the maximum power which was obtained over that line without throwing the synchronous condensers out of step. These are actual results. I feel a whole lot more optimistic about the problem than Mr. Moreland and his associates.

As regards Mr. Doherty's discussion, I think he has given us a splendid one. He has pointed out that there are other factors that enter into the problem which should be considered. I tried to present the fact that while we, so as to simplify the problem, used the usual convention of ignoring certain factors that we considered did not enter into the problem sufficiently to affect the results, properly speaking, we should have also considered these factors. In other words, I think it is important to get as complete a solution of the problem as possible, and as I pointed out, by the analytical method you can go as far as you please, provided you have sufficient perseverance.

Professor Karapetoff says that the doctors do not agree. He is wrong. The doctors do agree. The only point of difference is the extent to which we should consider all the factors that enter into this problem.

I don't disagree with Mr. Doherty and Mr. Summerhayes in the essentials of the problem. I think we ought to take all the factors into account. We ought to take them into account to see how much they influence the problem, and we shall do so as soon as we can get time.

Percy H. Thomas: All the papers except the first, and most of the discussion treat of the conditions of theoretical stability of a hypothetical *single-circuit* transmission line of *extraordinary length* taken as the *sole supply of power* for a system of distribution containing a mixed load such as is supplied by the usual public utility system.

All are agreed that such a line is capable of carrying certain large blocks of power with 220 kv. maintained at each end, provided the power factor of the passing power at each end has a suitable value. This value of power factor will be different for each value of power transmitted and the line will not transmit this amount of power with the 220 kv. at each end at any other power factor. There is no difference of opinion about this.

Now the line itself has no control over the power factor of the load which it is expected to carry, so that some means must be provided to keep the power factor of the load at the right value for the particular load passing at the moment. Furthermore, the generator must be able to accept the load at 220 kv. at the power factor at which it is brought to the generator by the line.

If either of these conditions is not met, the particular required amount of power will not be transmitted. This power of adjusting the power factor to the proper value to suit the momentary conditions is naturally the function of synchronous apparatus, the generator at one end and a synchronous condenser or another generator at the other end, which *tends* to accomplish this result automatically. So the ability of the system as a whole to carry the load depends directly on the performance of the generator and the synchronous condensers.

Now it so happens that the designers of such apparatus have for sometime been doing their best to build machines that will deliver a minimum amount of arc kv-a. on short circuit, relying on automatic voltage regulators to maintain voltage with varying load. This tendency in design is emphasized since it makes a cheaper machine. But this characteristic of a small short-circuit current is directly opposed to the ability of a generator or condenser to control power factor as required by the *long line.* Furthermore, the action of voltage regulators in readjusting voltages of the machines is too slow (as these systems now operate) to satisfy the line requirements and a second or two of hyatus occurs. Naturally, therefore, when the authors of the papers attempted to determine the performance of the long-line system as a whole, using the electrical performance of commercial machines designed to limit short circuit, the latter were found to be inadequate so that the system as a whole would refuse to transmit the full amount of power expected, at least until regulators had time to act.

These principles and methods of analysis and mathematical and graphical treatment therefore have been beautifully and thoroughly worked out in these papers and have been materially extended by the discussion. The report of tests on a model system is an addition of great value. But after all, the importance of the limitation developed in the transmission system may easily be over emphasized, and wrong impression is perhaps created by the unqualified statements frequently here made that the capacity of a 500-mile line is only so and so much, with or without the midpoint condenser station, etc. In such statements the qualification should have been added, that it is assumed that present commercial designs of machines developed for another service are used, and that a single circuit, or its equivalent is used, without other means of connecting the supply with the load network. These limitations are of the greatest importance as will be seen in a moment.

Returning now to my own paper, I would like to point out that the object of this paper was to cover in a broad way all the pertinent characteristics of an isolated 500-mile transmission feeding an extended network, taken as a whole. In such a broad consideration it would not be feasible to discuss *in detail* all the new features of design, of which there are many, besides the performance of synchronous apparatus.

In studying this subject I soon discovered the weakness of a single-circuit, heavily loaded long transmission line, which is vulnerable not only when supplied with inadequate condensers but also when *for any reason* the voltage at either one or both ends falls seriously. It will do no good to have a perfect condenser if a short circuit on the line will cause the voltage to drop and the load to be lost and the generator thus be left light-loaded to pull out of step. A broader remedy than a perfect condenser is required.

The layout described in my paper and shown as a one-line diagram in Fig. 5, is proposed as one scheme which is not subject to the limitations of the machines brought out in the papers, at least to anything like the same extent as a single circuit and which further provides for many other likely contingencies not considered by the others,—and naturally so, as they would be outside the scope of their papers. I am sorry that no consideration seems to have been given to the method of meeting the situation that I have proposed. Mr. Goodwin has rightly stated that the great detail in the connection to the network at the receiving end of the line shown in Fig. 5, is introduced because it is an essential part of the layout as a whole.

The features of the layout which give it relative immunity to the effects of limited condensers and accidental voltage disturbances are the following:

1st—Each circuit, of which there are four, with its transformers is an independent unit, electrically speaking, and electrically connected to a separate part of the general network, so that a short on one circuit will not cause a serious drop of voltage on the others and hence will not prevent them from carrying their normal load and a certain amount of overload, for a very considerable margin of overload is allowed in the system of my paper. This overload capacity together with the generators in the *receiving network* will be able to do the duty of the affected line.

2nd—After all, the effect of a drop in voltage or of an increase in load which are taken as fatal to synchronism in the limited single-circuit transmission considered in the other papers, unless very severe, is not to cause the circuit to drop all the load from the affected line, but perhaps to drop 10 or 15 or 20 per cent of the load, which can easily be taken up by the other circuits in Fig. 5, provided they all stay in synchronism.

3rd—If we assume large generating capacity in the network, this will go far to relieve the transmission line of its burdens. For example, suppose a big generator or two drops out in the network throwing an additional load on the rest of the system, which is the worst case that is assumed. This will not primarily cause a great drop in voltage per se (if unaccompanied by a short circuit) and the added load will be taken up by the other network generators. It is sufficient in preserving synchronism for the transmission line to carry the same load as before. In any case, any showing down of the system as a whole, will be slow on account of the very small excess load and the large flywheel effect and the voltage regulators will have time to act to restore any drop in voltage.

4th—The condensers of Fig. 5 cannot fall out of step readily because they are all on the same bus bar—similarly with the auxiliary bus at the generator station. This keeps all generators in synchronous operation through any disturbance on any one line.

5th—Four generators or four condensers are available to support any one circuit in case of need, instead of only one condenser per circuit as is considered for single-circuit operation.

Finally, it may be pointed out that much may be expected from efforts to design machines to give a favorable performance for *long-line service.* A comparatively small improvement in the behavior of the machines will greatly better the performance of the system and largely eliminate the machines as a limitation.

I would like to say that I entirely agree with much that Mr. Silver says to the effect that in most actual situations, long transmissions without intermediate stations will not occur, but in this paper I have set myself this particular problem—I have considered the broad network in a second paper, presented at the Birmingham Convention entitled "New Type of High-Tenion Network." In this Birmingham paper I have made free use of high-tension switching.

With regard to high-tension switching, the essential thing in the layout of Fig. 5, is not the elimination of high-tension breakers so much as keeping the high-tension circuits electrically independent. Until some other means is shown of meeting high-tension short circuits without loss of synchronism, this high-tension separation stands as the most feasible proposal for a *heavily loaded isolated long line.*

Mr. Moreland, Mr. Booth and Mr. Bush have greatly emphasized what might be called the pendulum effect of the revolving parts of machines resulting from sudden changes in load. It is certainly true that the effects of such action must be considered. As these gentlemen do not tell by what method their curves are derived, no intelligent analysis of them can be made. If they are based on theoretically perfect pendulum action it is not likely that they represent the actual practical case on a large scale. If they take into account all the pertinent factors in the case, I do not see how they can have any general application, since these factors will vary greatly in different cases and are many of them subject to design.

They state that I have ignored such pendulum-action transients in my paper, but I would point out that this subject is treated on page 13 and in Fig. 6. It is true that it is not gone into at great length, but the system of my Fig. 5 is such as to largely eliminate any noticeable effect from this phenomenon.

I will close with calling attention to the fact that all the limitations of condensers, generators and pendulum action are of an exceedingly transient nature perhaps of the order of one or two seconds and that all that would be required to eliminate their effect is to speed up the automatic voltage regulator and exciter system to meet the actual speed of change in phase position of rotors, sufficiently so that machines do not have time to pull out of step. This is by no means a hopeless remedy. In this I assume that no sudden increases of load can occur in any big system able to cause more than a very moderate per cent increase of the load on any one generator, when divided pro rata over the system.

H. K. Sels: In regard to Mr. Goodwin's comments on our reference to the articles brought out in the *Electrical Journal* some time ago, our reference is not made to bring in the loss diagram in our present paper but merely the matter of general calculations of power diagrams. The two statements that he brings up were, first, the application of the mathematical solution for purposes of checking the power diagram and second, the "losses neglected" in the loss diagram. The point that we meant to bring out in the first statement was that analytical methods would only be applied when the scale on the power diagram came out, say, 5000 kw. in dealing with loads of 10,000 kw. In this case you would have to go to analytical methods to determine what you were doing. As none of the examples we have ever selected come in this class, we have never shown a parallel mathematical solution but considered the diagram sufficiently accurate. In regard to the loss diagram there are no losses neglected in the derivation of the loss formula or diagram. What we did have in mind when we said "the losses neglected when using the general circuit constants are practically constant," were the ordinary assumptions every transmission engineer makes for transformer exciting current, line leakage or corona losses. In other words one rarely includes these factors in a practical problem and furthermore being practically constant, "the most economical point of operating the line can be determined from the diagram" without including them.

Mr. Goodwin also refers to Equation 26 as being imaginary. He will see when he multiplies out the vector quantities, he gets another j term which makes a j^2 or -1, giving the m term a real value.

In regard to the remarks of Mr. Moreland, Mr. Booth and Mr. Bush on throwing sudden loads on lines producing heavy transient conditions, I wonder where that sudden load will come from. About the only way that you will get it would be to have one line drop out due to short circuit and the others assume that sudden load. It would never be obtained in the normal load curve of a system.

I want to agree heartily with Mr. Silver's remarks on high-tension switches. That is, these super-power systems are going to grow by connecting up nearby load centers, which may be done before you connect these to some outside power supply, so that you again come back to short sections. That means that sudden loads on long lines will not be obtained when these short sections drop out.

In regard to the statements of Mr. Moreland and Mr. Peek about getting increased power limits by raising the voltage, I want to point out that raising the voltage does not change the general shape of what we call the voltage power surface. When we use up the additional power that an increase in voltage gives us, we still have the problem of the general shape of this voltage power surface.

In regard to Mr. Worcester's remarks on the 25-cycle system with frequency changers as against the 60-cycle system, I might say that calculations made on 350-mile lines show them about equal, and I would suspect that his conclusions of slight reduction in cost would be correct for a 500-mile line.

H. K. Sels (Communicated after adjournment): In the discussion of the 750-mile line, Fig. 8 of our paper is sufficient proof that "the receiver must operate at lagging power factor." Mr Goodwin's statement that this is at variance with the 500-mile line is correct, but the comparison is wrong and no statement was made that the 500-mile line did not operate at leading power factor. A comparison of the independent diagrams for 500 miles shows them to be in entire accord with each other.

Mr. Terman raised the question of how the power diagram should be plotted. Apparently we are in agreement on this point

as the derivation of the power equation is on the basis that the following quantities represent lagging power factor.

$R + j\,x$ for impedance
$I_1 - j\,I_2$ " current
$P + j\,Q$ " power

which gives:

$$\frac{Q}{E} = - I_{-2}$$

Therefore, dividing the power equation by E_R^2 gives a current equation where lagging current is plotted lagging, yet lagging power is plotted leading.

Fundamentally, there is no basis for plotting power quantities and voltage and current quantities on the same diagram so that if mathematics rule the case, they should be plotted according to our analysis or if customary convention rules, lagging quantities should be plotted lagging. Our position at this time is to have everyone recognize this distinction so that the Standards Committee can define the plotting of power quantities according to the proper rule acceptable to everyone.

Considerable discussion has centered about the amount of power that can be transmitted 250 and 500 miles. All of this shows remarkable agreement in results and as Mr. Doherty points out in giving his figures, any differences are largely a matter of the conditions assumed. Some of these, I believe, are over-conservative and therefore some of the limitations are a matter of personal opinion. With all conditions considered, I am confident that given the problem to transmit 150,000 kw. per circuit 500 miles we will do it with equipment now commercially available if it is economical to do so at all.

Messrs. Hanker, Fortescue, Wagner, Evans, Bergvall and Shand (communicated after adjournment): The discussions which have been presented on these papers on "Power Transmission" have been very interesting and important. Due to the number and length of the discussions, it will not be possible to comment fully on the individual discussions. The papers were presented as a group, and a general reply will be made jointly by the several authors and supplemented by answers to questions raised on individual papers.

The discussions by Mr. Moreland, Mr. Booth and Dr. Bush will be considered first. Mr. Moreland, in connection with the Canadian power development, is concerned with the problem of determining the highest permissible loading of the transmission line under consideration. He apparently has confused this problem with the related one presented in the group of papers, namely, that of determining the maximum load that could be carried on a given transmission system with definite assumptions as to voltage, line and load characteristics. This attitude is particularly surprising to us in view of the visit to East Pittsburgh by one of his associates to secure information on stability, at which time we presented our methods of calculation and test data and pointed out that the maximum load was considerably in excess of the probable rating of a line. Furthermore, at that time we discussed the effects of electrical and mechanical transients of rotating machines and also we fully appreciated that the complete solution of the stability problem could not be obtained until all of these effects had been evaluated.

The maximum load on a transmission system must necessarily be obtained under steady state conditions, that is, by increasing the load in negligibly small increments. A definite method of solving this problem is presented in the group of papers. The complete solution to the problem of determining the highest permissible load of a transmission system from the standpoint of stability, was not given in the papers, because the problem is extremely complicated and the type of solution will vary with individual cases. The problem is complicated because the solution depends upon transients produced by changes in voltage, excitation, relative angular position of rotating parts, and governor and regulator characteristics. In the very nature of things, a simple solution cannot be obtained, and therefore, no attempt at this time was made to present a complete solution of the problem which Mr. Moreland infers was the object of the papers. The object was to present a general discussion supplemented by actual test data on stability because, in our opinion, the importance of transmission stability was not generally recognized.

The authors, however, did not avoid the problem of determing the permissible loading in view of the transient characteristics, but have indicated a method of calculation and submitted the results of tests. This method for analyzing transient conditions is based on the application of a suitably chosen static stability criterion. This static condition is determined by assuming:

(1) That the changes in the fields of synchronous machines produced by changed load or circuit conditions have been completed, and,

(2) That the machine excitations have not been changed. The static stability limit is easy to determine and has been proven both analytically and experimentally, as developed in the papers.

Mr. Moreland criticized the papers because static stability methods were employed to investigate transient stability, but he failed to read the papers to see how this static stability method was applied by the authors. To illustrate, Mr. Moreland submits a chart, Fig. 1, and implies that the methods given by the authors would indicate that for the conditions assumed, the load could be increased from 100,000 kw. to 190,000 kw., without loss of stability. The authors' method would have given the permissible increase as 38,000 kw. Mr. Booth and Dr. Bush in their discussion give the permissible load for this condition as being more than 40,000 kw., and less than 50,000 kw.

It appears, therefore, that the methods employed by Mr. Moreland and his associates and those employed by the authors give closely the same results, though their methods are based on "transient stability analysis," whereas the method employed by the authors is based on a static stability criterion. Consequently in analyzing Mr. Moreland's discussion, there can be found no support for his contention that the conclusions drawn by the authors are invalid, because of the methods employed to investigate the transient conditions.

Dr. Bush has presented a number of curves giving the results of computations of stability conditions for sudden increases in load. The method that takes into account accurately all the transients involved for changed load conditions (including prime mover governor, not mentioned by Dr. Bush) must necessarily be very complicated and require many assumptions. In the verbal discussion, Mr. Fortescue requested Dr. Bush to submit the basis for his calculations, which has not been given. Consequently, judgment of the value of Dr. Bush's method must be reserved until such time as adequate explanations are made.

It is perhaps desirable to point out some of the reasons why a static stability criterion should give good results for the transient conditions. This method in effect gives the limit toward which the analysis of the transient conditions must approach. Mr. Moreland apparently believes that the static stability method will fail because of the suddenness of changes introduced by switching operations. It is to be pointed out that part of the momentary demands are supplied by the reduction in kinetic energy of the load, condensers and generators at supply and receiver. Furthermore, for loads less than the static stability limit, there are many forces which tend to oppose the forces that would cause the system to pull apart. These forces include the kinetic energy of rotating masses, the transients in the exciter circuits, and the operation of regulators and exciters. During the transient condition, the flux in the main machines does not change instantly, and thus produces in effect an appreciable increase in the leading kv-a. available to hold the system in step. All these actions tend to prevent the instantaneous pulling out of step due to changed circuit or load conditions. It has been

observed on long distance transmission lines where instability occurs that the action of pulling out of step is a relatively slow process, requiring several seconds. The method of static stability analysis used by the authors therefore appears to have a reasonable basis, in assuming that flux in the main machines has been entirely changed in accordance with the new load or circuit conditions; and is pessimistic in assuming that the regulator and exciter systems have not had opporunity to act.

Mr. Booth's discussion on the intermediate synchronous condenser station is pertinent. In particular, he criticises the basis for considering the effectiveness of the intermediate condenser as presented in some of the papers, because the coincident variation in receiver and mid-point potentials is not considered. He feels that the assumptions made do not correspond with the usual operating condition. We do not understand that Mr. Booth questions the calculations submitted on the assumptions stated. Mr. Booth's point is pertinent, and if he had studied Mr. Shand's paper, he would have found that the particular point he raised was investigated at considerable length.

Mr. Booth's discussion leads to a conclusion with which the authors, however, must thoroughly disagree, namely, that the addition of an intermediate condenser station does not increase both the theoretical maximum output and the permissible output for various operating conditions. The calculations given in Mr. Shand's paper for the intermediate condenser stations show an important increase in the maximum load. The tests made in the shop prove very definitely that under favorable conditions, very large increases in the maximum load of a system can be obtained by the addition of an intermediate condenser station. In order to obtain stability, it is necessary, in general, to maintain the voltage adequately. In order to compensate for reduced voltage, the synchronous apparatus connected to the system should be of such design that a reduced voltage will cause an appreciable decrease in the lagging kv-a. or an appreciable increase in the leading kv-a. delivered. Mr. Booth's discussion, however, does bring out this fact that if the intermediate condenser station is of small capacity or high impedance, the increased output may not be important. This difficulty may be avoided by using synchronous condenser of larger capacity and of lower synchronous impedance.

The discussions by Messrs. Summerhayes, Doherty and Worcester and Miss Clarke are very interesting and important, particularly the discussion by Mr. Doherty. The importance of considering generator characteristics was recognized by the authors, but Mr. Doherty is justified in criticising the omission of calculations and data presenting figures on the effect of generator characteristics. His discussion was very clear and convincing, and in a sense rounds out the presentation of the subject by the authors.

The effect of generator characteristics, while important and at all times requiring consideration, is not nearly so great in limiting the power output as one might expect from the comparison of machine synchronous reactance with the reactance of the transmission line. This is due to the fact that the characteristics of the generator are considerably different from that of the line. With a decrease in voltage, a generator will supply an increased amount of reactive power, whereas a transmission line at reduced voltage requires an increased amount of reactive power due to both the increase of the $X\ I_2$ of the line for constant power, and to the decreased charging kv-a. Mr. Shand has carried out his calculations for the problem considered in this paper, and has found that the genertor characteristics limit the maximum output about 8 per cent. The tests given in the Evans and Bergvall paper show that the effect of generator characteristics for the particular system tested had a negligible effect in limiting the output because the test results checked very closely with calculations neglecting generator effects.

The simplified expressions for l, m, and n given in the discussion by Frederick E. Terman are similar to expressions used by one of the authors for some time. He has found them very convenient in studies involving the line alone. Mr. Terman also raises the question as to the correct manner of plotting leading and lagging kv-a. Following the analytical treatment given on page one of the paper by Fortescue and Wagner, one must logically arrive at the method of plotting employed. The basis for the equation:

$$P_s + j\ Q_s = \breve{E}_s\ \hat{I}_s$$

is given in a paper by Mr. Foretscue on "The Measurement of Power in Polyphase Circuits" presented at the Midwinter Convention in 1923. By reference to this paper it can be seen that this method of plotting has a firm analytical basis.

We have read C. A. Nichol's discussion with a great deal of interest. The manner of including the effect of the generator in limiting the power is relatively simple when the constant value of synchronous reactance is used. In work which we have done subsequent to the writing of the papers we have found that it is not sufficiently accurate to use the so-called "synchronous reactance." We have preferred rather to use more accurate regulation charts for the generators and have evolved an approximate method of taking the generator characteristics into consideration, which are quite accurate.

The kinematic device for imitating the performance of long transmission lines which Professor V. Karapetoff is constructing at Cornell University, like his other devices, will undoubtedly prove to be highly instructive. While delighting in the use of mathematics, Professor Karapetoff derives still greater pleasure in demonstrating a phenomena in an entirely unmathematical manner. It would be highly entertaining to see a surge applied to one point of a line and observe its propagation along the line.

Mr. Longbottom has presented an interesting study of effect of the number of intermediate synchronous condenser stations on the performance of a transmission line. His figures show very clearly that under heavy load conditions the total exciting kv-a. required of all the synchronous machines is reduced when the number of condenser stations is increased.

E. B. Shand (Communicated after adjournment): In connection with the theory of transmission systems, there are one or two points I would like to amplify. First, in the paper by Messrs. Fortescue and Wagner, a mathematical analysis is given of the envelope of a certain family of power circles, while in the paper of Messrs. Evans and Sels, the same envelope is referred to as defining one type of power limit. The relation of this envelope to the system may be conceived in somewhat the following way.

When power is transmitted over a reactive circuit, such as a transmission line, the receiver voltage being controlled by varying the reactive current in the circuit, the ordinary relation is that a decrease of receiver voltage is accompanied by a decrease of reactive current (a leading current with respect to the line being considered positive). If the assumption be made that the load is independent of voltage, the power component of current must become greater at an increasing rate as the voltage is lowered, so that a point is eventually reached where the increasing reactive effect of the load current neutralizes the natural tendency for the magnetizing requirements to drop with voltage. For each value of constant power load there will be a value of receiver voltage at which the magnetizing requirements at the receiver end will be a minimum. Letting I_0 represent the magnetizing current at the receiving end, and Q the corresponding kv-a., two critical points may be expressed mathematically as follows:

$$\text{(a)} \quad \frac{S\ I_0}{S\ E_r} = 0 \text{ (For constant power)}$$

$$\text{(b)} \quad \frac{S\ (I_0 \times E_r)}{S\ E_r} = \frac{S\ \ Q}{S\ E_r} = 0 \text{ (For constant power)}$$

The envelope referred to is the locus of the condition represented by (b).

The above expressions represent the actual power limitations for two very special conditions of load and of regulating apparatus. Equation (b), represents the limit where the load has the characteristics of constant power, and zero or constant reactive kv-a. irrespective of voltage. The condenser would also have to deliver constant kv-a. irrespective of voltage. Equation (a) represents a similar condition, except that the reactive current characteristics are constant, rather than the reactive kv-a.

The assumption of a load of constant power and constant reactive kv-a. may often be approximated in actual practise, but the conditions for the regulating apparatus will differ considerably from this assumption. If a static condenser be considered, the reactive current will increase with the voltage, and the kv-a. with the square of the voltage. For a synchronous condenser, the current will increase with a drop in voltage, and within the operating range the reactive kv-a. will also increase as the voltage drops. With a load of the assumed characteristics and a synchronous condenser, the "envelope limit" would be slightly conservative.

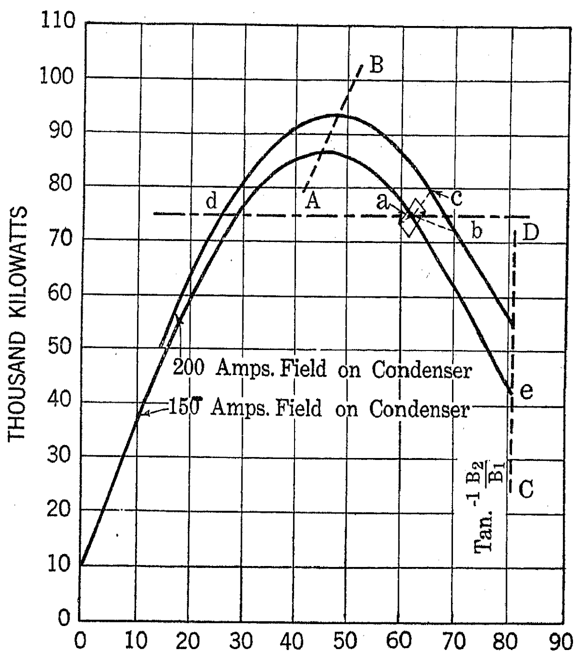

Fig. 55

When compound lines are considered, the reactive kv-a., required by the second section at the mid-point, may be expected to increase rapidly with a decrease of voltage and may more than neutralize the characteristics of the synchronous condenser. Therefore, the kv-a. available for the first section of the line may decrease with the voltage. This is illustrated by the 300-mile compound line, discussed in my paper. The "envelope limit" of this line is approximately 108,000 kw. From Fig. 11 of my paper the actual limit of output, with a size of condenser economically justified, would not be greater than 80,000 or 90,000 kw.

In the paper of Messrs. Evans and Bergvall it is mentioned that in making tests with a line purely of resistance, hunting was observable with practically normal excitation on the terminal condenser and without load on the line. The tendency to hunt, moreover, increased with the excitation. This is logical. A line without reactance becomes equivalent to a d-c. circuit and power can be transmitted only by a drop in voltage and not by a difference of phase angle. Therefore, by increasing the condenser excitation, the receiver voltage would approach the sending voltage, at which value it would be impossible to transmit any power except for the stabilizing effect of the condenser impedance. In this case the latter was comparatively small. These extreme line characteristics are not found in practise, although trouble is occasionally experienced on short high-resistance lines with over-excited synchronous motors at the end.

One further point in the paper of Messrs. Evans and Bergvall, will probably admit additional explanation. In referring to Fig. 5 of this paper the authors state: "If it were possible to get beyond the point of tangency between the two families of curves, an increase in synchronous-condenser excitation would actually decrease the receiver voltage," and further that the cumulative action of the regulator and reduction of voltage would eventually produce pull-out. According to the hypothesis made, *i. e.*, that the operation between the limiting lines A B and C D be stable, the above conclusions are perfectly correct, as may be deduced from the curves; the statement, however, without further analysis conflicts with the conclusions of my own paper regarding the condition of artificial stability, which, although not of great practical importance, is of interest on account of the principle involved.

Neglecting other phenomena which may be taking place concurrently, an increase in the field current of the condenser furnishes additional magnetization to the connected circuit and will increase its voltage. Also, an increase of field-current will increase the amount of power tending to flow over the line, rather than increase the tendency of the line to fall out of step. An exception to this latter statement will occur in a line of high relative resistance, that is, when the operating voltage is lower than the critical voltage $\frac{E_s}{2\,M\,B}$ This is very unusual.

These relations are probably more clearly seen in Fig. 55 in which the data of Figs. 4 and 5 of Messrs. Evans and Bergvall's paper, has been plotted on the basis of phase angles. The lines A B and C D correspond to similar lines in Fig. 5. The zone between these two lines represents the conditions under consideration.

Assuming a load of 75,000 kw. with a field excitation of 150 amperes, and at the point a the system will tend to drift out of step at e as is indicated by the slope of the curve of constant excitation. If the field is being increased so that its effect with respect to change of phase angle may be represented by the line a b, the power available will from that point be less than the requirement, and although pull-out will be retarded somewhat it will not be prevented. The voltage will also be higher than if the field had not been increased. If the rate of field increase were sufficiently rapid to be represented by the line a c, the power available would be greater than the load requirements and the masses would accelerate, stability finally being reached at the point d, which is on the stable slope of the curve.

From this discussion, it is apparent that the action of an automatic regulator in the zone considered will depend upon its resultant speed with respect to changes of angular displacement of the condenser rotor. With this being sufficiently great, the action of the regulator at the point a might be represented by the closed polygon about a, the rotor and the receiver voltage being kept in constant oscillation. This is the condition of artificial stability referred to in my paper.

Fortieth Anniversary Celebration of the A.I.E.E.

Addresses in Philadelphia by Three Charter Members with a Résumé of Electrical Engineering Progress

The 40th anniversary of the Institute was fittingly celebrated during the Philadelphia Midwinter Convention at a meeting held Monday evening, February 4th, at which President Harris J. Ryan presided. Addresses were delivered by Elmer A. Sperry, T. Commerford Martin and Elihu Thomson, all charter members, and by John J. Carty, past-president of the Institute.

President Ryan presided, and in opening the meeting said:

"The hour is one in which we all rejoice to have come to Philadelphia because it was the home city of the first American electrician, the man of whom we all know, and who enabled us to understand very clearly and for the first time in this world what is lightning. From what he did for us in that way, we have had many an inspiration of the most valuable sort to carry us forward to know many of the other things and what they are and why they are.

"Through all this came the inspiration on the part of a group of men forty years ago to give us this, our American Institute of Electrical Engineers. We have assembled to do honor to those men tonight, to remember the beautiful and valuable organization that they gave us through their vision and their enthusiasm as young men. We are glad to know and to be together to rejoice over the considerable number that have lived through the entire generation, the lifetime, the time of one generation of active business at all events.

"The first man I am going to call up who joined with others to give us our Institute in 1884 is the youngest charter member who had at this same time to come the greatest distance for the organization meeting. He came all the way from Chicago to New York. Mr. Elmer A. Sperry."

Address of Mr. Sperry

I feel that the Institute is very young; it is really the youngest member of our technical bodies. You remember the Civils were formed in '52; then the Miners came along some time in '77; then the Mechanicals were organized about 1880, and then this body came into existence in 1884.

These things, of course, don't happen by themselves. The pioneer is doubly in his own way, always having a hard time when he starts, or he imagines he does, then when he comes along later to tell how it did all happen to happen, he gets in the way again, and the youngsters with the forward look and on their toes as the members of this Institute always have been, say, "Well, make it brief." So I will try to make my part of it brief.

There were two groups that had their young enthusiasm and were willing to make some sacrifice to see if we could not have an American Institute of Electrical Engineers. In the Western Group—I was living in Chicago—we had about eight people. Our keynote was that England had had the Institution of Electrical Engineers some four or five years. They had had, it is true, that marvelous body, the Society of Civil Engineers, but we in America had matched that in 1852, as I stated before, but they were definitely ahead of us, and it was a going concern, and the papers that they heard and the scientific matter that they put forth at that time were an envy to us all here. In the Western group we felt as though something should be done. We cast about and found that there was an equally enthusiastic group in the East, but we in the West were all young fellows.

In the East it was quite different. We found a man who was then forty-six years old, who was really the leader, and do you know that we have the great and unusual honor of having him here tonight? It was due to Nathaniel S. Keith, sitting yonder, that the matter was thoroughly stirred up in the East, and finally a petition was circulated, which never got West, by the way, containing the names of those who were willing and anxious to see such a society formed.

That body got together on April 15th in the rooms of the Civils. I have one or two pictures which I have had taken showing how that building looks today and about how it looked when we went up the steps that night, May 13, just one month later.

The meeting in April succeeded in getting ten more names on this list of "Come, let us have one," and the time of the inaugural meeting was then settled upon as the next month, Tuesday, May 13, and then we felt as though we had in sight our Institute, our dream. There were five of us in Chicago who purposed to attend the New York meeting. Two dropped out and finally two more, and it was wished on me to come down and do my part in swinging my arms and seeing to it that the Institute was put on the map.

So I came, arriving in New York as I remember, at ten o'clock in the morning and proceeding directly to

the Astor Library and working there all day. I don't suppose any of you know that the Astor Library never had any artificial illumination. The British museum had found that the products of gas lights ruined the books, so they couldn't have gas, and Edison was confined to the downtown quarters at that time, so they had no artificial illumination and I was turned out at early candle light and proceeded to the rooms of the Civils, away uptown, 127 East 23d Street, to attend the inaugural meeting.

Getting my dinner early in that way, I happened to be about the first, if not the very first, to arrive in the rooms. As I remember, I waited quite a little time and the second man to appear was Dr. Keith, who had come to see his child born. I complained to Dr. Keith, and so far as I can remember, asked him how it came about that his people down here in the effete East talked so much and did so little.

He calmed me down and told me to wait, that the people in the East possibly would be a little late, but they would be right there, which they were. Some forty of us finally gathered there in the rooms.

One little incident in the first meeting was that we had Mr. Edison there, and that was a long time before we knew it was impossible to get him to speak. So T. Commerford Martin, and some of the others concocted a scheme to head him off from going upstairs, and we would rush him from below, and when he got halfway up, we would all cry, "Speech." That was before we knew he would rather climb under the table, which he did at the World's Electrical Congress at Chicago in '93, rather than make a speech. But we totally failed to elicit anything but a bow from Thomas A., although he was the greatest electrician of the time. That was the word that was used, as I will quote presently, and you will notice how different our nomenclature is at the present time from what it was then.

He had not long previous to that time sold to the telegraphic interests his quadruplex and it was noised around that he had received four hundred thousand dollars for it, and that was a stupendous urge to the young imagination. He was himself a young man at that time, as of course, you know.

To indicate the relative importance of factors of electrical activity of the time, I only have to say that we elected Norvin Green as our first President. He was the President of the Western Union Telegraph Company. That was one of the most important factors of electrical engineering then, and a cross-section of the activities of the time indicates that the telegraph had been achieving rapid progress by doing the unheard of thing at that time of getting four messages over a single line at once. Dr. Jewett of the Western Electric Company, here present, would smile at this from the present-day standpoint as his company are now putting a great many telephone and telegraph messages simultaneously over one wire.

In those years the membership was to be as follows: (this is from the *Electrical Review*). Matters in the electrical field were extremely active in those years and history was in the making. Here were three live journals reporting this meeting. The description of who could become members and who could not is rather amusing.

"Members are those who may justly be called electricians because of professional employment and general recognition as such. All others are to be associate members, persons who are connected with electrical matters directly or indirectly, but not ranked as electricians. Thus the president of an electrical company might or might not be entitled to full membership, but there would be no question of the professional electrician, employed by such a company."

Our light at that time was nearly all arc lights. Edison, just a year and a half before in September, had set into operation the Pearl Street Station, and divided electricity, which our English savants said couldn't be done, but he just used horse sense. He said, "The gas mains go down the street and are tapped for individual residences. Let's do the same thing with electricity," and he did it. He put it over, and that is one of the reasons we are here tonight, because the scheme was so wonderfully successful. He had started the Edison Company then only shortly before. The Brush Electric Company was our oldest company, dealing entirely with arc-lights. Streets were illuminated and municipal companies had been formed, but just a little before that time there had been a junction formed between our redoubtable Elihu Thomson, whom you will hear later tonight, and the great C. A. Coffin, whose dream was the General Electric Company. Few people live to see their dreams come into full fruition as Dr. Thomson and Mr. Coffin have. All honor to them both!

It was about this time that Edison saw fit to curtail the great long legs of his generator and find that spindling legs were out of style, that they could be shorter and plumper and still do the same work, and have a higher efficiency.

But Edison in those days refused to have anything to do with alternating current. He rather spurned the idea that a great system could be induced to change its mind as to the impressed potential direction, especially as frequently as one hundred twentieth of a second. The facts are that in those days we had frequencies much higher than this. It was left to Elihu Thomson and others to bring their mathematical conception and intuition to bear and determine, "Yes, we will make the whole network change its mind every one hundred and twentieth of a second, because by so doing we can make marvelous gains in the territory served by a given distribution."

It was the initiation and bringing forward of the alternating current system that was one of the elements of the great expansion of the Thomson-Houston Com-

pany. I was working in Chicago and gradually had acquired my share of the trade in arc lighting in the West. We had a number of cities, Sperry towns we called them, but later the spread of the alternating current gave us hard competition with its ability to distribute much smaller light units.

The Hochhausen Company was in operation at this time and Edward Weston's Company also. When I reached Chicago I found one plant of the Weston operating in the old Palmer House. The Brush Company had been in the field just previous to the others and they had a number of towns both in the East and West lighted by arcs. Soon the Vanderpool Company and others came forward with systems.

One of the great troubles that we immediately discovered when we commenced to put in arc illumination with these municipal plants in the towns, was the terrific interference with the telephone. The telephones in those times, as you remember, were one wire with common ground. Nobody was making any money. They were poor, struggling concerns in little and big towns, as was our own company, and to think of having two wires for one conversation at that time was impossible. It was less than a year until a convention was called together of all the arc-light producers in America, to see what on earth we could do to get rid of the very great interference with the telephone company.

The town council would make us put our poles on one side, and the telephone people theirs on the other side of the street, and yet whenever we started our plants, and especially when the Thomson-Houston Company with its sinful three coils came in, the induction was something fierce and every telephone went bad.

We claimed that the telephone wasn't much good anyhow as it couldn't be used when it stormed nor over any long distance in any event, and they on their side claimed that our lights hissed and were too strong and didn't amount to anything either. There we were.

At the time of this convention, which was the actual formation of the National Electric Light Association, (allow me a word; I told this tonight at dinner, but it is really worth repeating), an envoy from the great Theodore N. Vail rose up in the meeting and said, "I represent Mr. Vail of Boston. He sent his regrets that he couldn't be present in person."

Then he took out of his pocket a little document which showed the greatness of Theodore N. Vail even at that early date. The document went on to say, "It is a thing that we will overcome, this interference of the service one with the other. All we have to do is be patient with one another. Be patient and you will find us ready to do everything we can to help the cause along."

Think of the beautiful cooperative spirit! There is in this room, and is to speak from this platform, the boy that solved that problem within that year, General J. J. Carty. He found such wonderful effects by using two wires for one conversation without ground return that it spread like wildfire. Before the year was over, our telephone companies, though poor, adopted the system and our troubles were over. So it was the telephone company after all that did it. We swung our arms and found much fault, but did very little, and it was through the wonderful initiative of Vail and his great organization that that thing was straightened out.

Now, it is a strange thing that this last winter the meeting of our own Society had as its keynote the very self-same thing, telephonic interference by power lines, and the cycle seems to be about forty years. I should like to be on earth and know what comes around forty years hence. I have made up these remarks from my memory of the impressions I was receiving in those early times when this Society was in the process of formation. I thank you.

Mr. Sperry then showed several views of the first home and the first secretary of the Institute.

President Ryan: The next speaker, who on the occasion referred to by Mr. Sperry could not induce Mr. Edison to speak, did succeed in making him speak. We members of the American Institute of Electrical Engineers as it exists today, have all a great inheritance from him. He did a wonderful work in the early days of the Institute in helping the organization to get down to a plan of running its meetings, conducting discussions, in reporting the same, and in broadcasting the results of these activities. He is one of the Past Presidents of the Institute, one of the early Presidents, Mr. T. Commerford Martin.

Address of Mr. Martin

Fortunately it is not like deciphering palimpsests to get down to the fundamental data concerning the early history of the American Institute of Electrical Engineers. The inmost rings of its growth are still very close to the bark. While not extant in lavish profusion, the printed record may still be found on many library shelves, and stray beams of human memory still flicker over the archives.

The creation of the Institute coincided happily with two other notable occurrences in electrical development on this Continent. It was a corollary of both; while all three events may be definitely traced and attributed to that ancient splendid exemplar of scientific leadership, the Institute so well named after out great pioneer, the Immortal Franklin. Dating from the Centennial Exposition of 1876 with its memorable exhibits such as the Bell telephone, the Edison telegraphic inventions, and the Wallace-Farmer arcs, there had followed a period of electrotechnical advance unparalleled before or since. The Franklin Institute in 1883—with the same prescience that has just led it to bring over J. J. Thomson to lecture on the electron—felt that the times were highly propitious for the First American

Electrical Exhibition, to excell those of London and Paris. The Franklin Institute might well have waited for the full decade after 1876 to elapse; but it is not in the nature of Philadelphia to wait when it wants anything whole heartedly. Moreover, the British Association for the Advancement of Science was meeting in Montreal in 1884 and would bring a whole swarm of European scientists and physicists across the Atlantic. These could be lured Southward from Canada to the projected Electrical Congress the same year, timed to synchronize with the Exhibition; while they would not easily retrace their steps later, if the opportunity to enroll them were pretermitted for a year or more.

Hence, with two such compelling factors as to 1884, the Exhibition and the Congress, it was indeed, inevitable that the foundation of the A. I. E. E. should occur in that year. Possibly there was some propaganda traceable to Philadelphia and the Institute, but it was, if so, wholly desirable and necessary, everybody felt, that not only to greet fraternally the visitors from abroad, but for the sake of the art itself, there should now be a national electrical engineering society, before any time was lost. Crystallizing with energy and enthusiasm, all the ideas and impulses finding vague indefinite expression, Dr. Nathaniel S. Keith published in April, 1884, his memorable circular reprinted on the very first page of Vol. I of the Transactions of that year. That circular, prepared and issued by himself and colleagues, among whom the present writer is proud to be included, was submitted to the leaders in the electrical field in various parts of the country, with separate name sheets that could afterwards be assembled and thus permitted simultaneous canvassing even in quarters that in those highly-strung days were avowedly hostile and antagonistic to each other, but all of which recognized the common need of a central technical authority without bias. It was convincing also to be able to point to the American Society of Civil Engineers founded in 1852; the American Institute of Mining Engineers, in 1871; the American Society of Mechanical Engineers, in 1880, "which have been so prosperous and of such great advantage to their members." Besides that, none of these bodies made any provisions for "electricals" as such, and even if Sir William Thomson said that "electricals" were "nine-tenths mechanical," that would perhaps not always be true; and it needed more than that to thwart the sound normal American instinct to organize another Society whenever a decent chance offers.

All went well. The Roster lists were carried upstairs and downstairs and into the magnate's chamber, and failed not once of the signature sought. Advice and help came freely from the Franklin Institute, never out of touch, and at last on a dirty, miserably wet night, Tuesday, April 15, 1884, the first meeting was held for organization, with an excellent attendance. Preliminary work had been done carefully, and the stage was quickly set for Tuesday, May 13, when the Committee on Organization reported not only a complete slate of proposed officers, but a brief Constitution. Both were unanimously adopted and can be found fully set forth in Volume I of the A. I. E. E. TRANSACTIONS now an exceedingly scarce document, but still available bound up usually with the three subsequent Volumes. The first Constitution, admirably brief, was adequate for the times. It has been frequently enlarged and modified, but Constitutional amendments are not always and wholly admirable.

The rendezvous for all those meetings in 1884 and others, for a year or two later, was the stately old home of the Civil Engineers, 127 East 23rd Street, where its courteous and efficient Secretary John Bogart, later State Engineer and consulting engineer for Niagara Power development extended a most cordial welcome. Then and later, until the splendid gift of Dr. Carnegie came and the United Engineering Building was erected on West 39th Street, the various Societies occupied others like it—fine old typical New York dwellings. Indeed, for the Electricals, a serious effort was soon put forth to secure the charming old home of Morse on West Twenty-second street, just off Fifth Avenue; and to make it possible, the writer offered to occupy with his family one or two of its upper floors for a rental that would guarantee the investment. But negotiations with the owners, the Bourne family, fell through on the score of price. The Twenty-third street region was then at a high crest of values, and was marked for commercial devastation. The Civil Engineer rooms were, moreover, admirable for their purposes, and there were no strings to the welcome except an injunction from Bogart to keep down the gas bills—talk after 11 wasn't worth it.

The Institute was fortunate in its first President, Dr. Norvin Green, a towering, swarthy Southerner, who had long before ridden the circuits as a rural doctor with pills and plasters in his saddle bags, and who out of the ruck of provincial presidencies of subordinate telegraph companies had lifted himself to well won supremacy as head of the Western Union Telegraph Company, with headquarters in New York. He was leadership personified, and out of scant leisure, he gave the best that was in him to the affairs of the tiny Institute, in whose opportunity, necessity and mission he had a profound belief. He was always available for Executive and other Committee meetings, with prompt suggestions for diplomatic and practical disposal of problems. But he was much more than a formal President. His warm Southern blood, like his sympathies, ran swiftly, and he loved to temper debate with anecdote. Pat to the occasion or incident, rollicking good stories would come in delightful succession from their endless store of a born raconteur; and even if it was maybe "at long interval," his board meetings might well be compared with those of Lincoln and his cabinet. It was, indeed, easy for Dr. Green to set an example on attendance, for he lived in cosy apartments quite nearby,

where hospitality was more than the proffer of a dry cracker.

Equally fortunate was the Institute in its first Secretary, Dr. Nathaniel S. Keith, who still lives, unquenched, and can tell his own story, but who deserves the tribute of his colleague and successor. A master of his own mining profession, he was a skilled and accomplished electrical engineer, which then meant more than plain electrician. He was studiously familiar with foreign electrical literature, from which he translated one of the first and one of the best German treatises on dynamo electric machinery, an art then obscure and occult. He was a brilliant technical journalist, and entirely competent in the politic arts and graces of secretaryship. All he lacked just then was the gift of continuity, for no sooner had he seen his hopes and ideals realized in the foundation of the Institution than he went to the furthest West, to stay for years, to grow up with it and electricity, and become the first American to build any kind of electrical machinery on the Pacific Coast. He left as a timid successor and probationer the present writer, his equal only in faith as to that future of the A. I. E. E. which they had forecast with buoyant optimism.

Other honored names might well be mentioned here and dwelt on, of those who at the very start gave their highest endeavor to the fostering and upbuilding of the Institute—in Philadelphia, Carl Hering, E. J. Houston, W. D. Marks; in New York, C. O. Mailloux, Joseph Wetzler, "Steve" Field, nephew of Cyrus, Francis Jones, George Hamilton—in the East, Thomas D. Lockwood; in the West, Elmer T. Sperry—but it must suffice to signalize Franklin L. Pope, the second President, and a Vice-President for 1874-5. Not long before, he and Edison together had in telegraphic journals announced themselves to the world as "electrical engineers;" and diligent search has failed to encover any earlier attribution of that honored title for the new profession and the new practitioner for whose legitimate working spheres no limits have ever yet been found in the electrical era *now* beginning. Pope's brief Presidential valedictory, in May 1887, was characteristic of his broad outlook, and not less typical of his simplicity and modesty in omitting any reference to the highly useful work he had done as organizer and administrator. A scholar and student, author of technical textbooks, he came from that wonderful group of well-trained, well equipped telegraphers whose scientific class room was the noisy- key-clattering office of a commercial company but had there learned the secrets whose mastery carried them to leadership in all our later fields of marvellous electrical achievement. Ponderous in movement, slow in speech, deliberate in action, pontifical in dignity, Pope was far more the conventional Englishman than he was the born Yankee—until some cause like the Institute moved him to unsuspected depths, and he became transfigured with the vision glorious. Never was the chair of the Institute more worthily filled by an "electrical engineer" than by him who first, in unwitting prophecy, gave himself the accolade of that appelation.

Reference has been made to the hospitable halls of the "Civils" as the home of the Electricals, and rallying place they were for a few years. But fortunately, the Institute officers decided early that an annual meeting would not suffice, so the young Society made a practice of getting together at frequent intervals for papers and discussions—in New York, of course, as only there could a good membership attendance then be registered. The writer's report as Secretary—the first ever made to the Institute, dated May, 1885—stated proudly that it then had already 279 members and associates; but they were "in all parts of the country." It has been interesting to see resumed in the last year or two the local New York meetings that marked the initial stage of Institute work, although to insure a good attendance and be comprehensive, topics are now chosen that will interest local members of all the four national engineering societies. The writer has enjoyed lately several excellent meetings of this character. This offsets also the centrifugal tendency which has scattered such members so widely over the huge metropolitan territory, in which one can now hardly ever go home to dinner and get back for the meeting. Forty years ago, most members lived in close proximity to the meeting hall. It was thus found desirable and feasible to get together every few months in "special meetings" when, no matter what the topic, a very large percentage of the local members turned out. Especially was this the case when the meeting was held at some restaurant, with preliminary dinner, and it is curious to note as a sign of local changed times and conditions that some of those meetings were held as far down town even as Broad Street, or else Broadway South of City Hall. One very far northern rallying point was a famous restaurant cafe, Martinelli's, on Fifth Avenue, south of Madison Square. In his Presidential address, Mr. Pope said of the "Special" meetings in his seat that they "have proved to be most successful and enjoyable reunions and have certainly done much to strengthen the Institute."

But all that fades into relative insignificance before the great functions with which in Philadelphia in 1884 the Institute was so closely associated and to promote which it had, on its creation, joined hands most promptly with the Franklin Institute. The first of these was the Electrical Congress of 1884, the first held on the American Continent. It was a most successful affair, marked by large attendance from abroad—notably such men as Sir William Thomson and W. H. Preece—by brilliant and animated debates on the definition and establishment of fundamental electrical units. Even at this remove one can recall vividly the masterly presentation by Prof. Harry Rowland of his great work in the determination of the Ohm. The good results of this first American Congress

made it easy even in the panic year of 1893, to bring together, again with distinguished visitors from abroad, notably Von Helmholtz, the Electrical Congress in Chicago. This again was accompanied by a splendid electrical exhibit as part of the Columbian World's Fair.

But as the Institute is tenting and camping tonight on the old battlefield of 1884 here in Philadelphia, it may forgive—even expect—a casual peep at its own modest little performance of September and October that year. The Exhibition Building was the spacious old West Philadelphia depot of the Pennsylvania Railroad Company. There the Franklin Institute had generously placed two large rooms at the disposal of Dr. Keith. They were fitted up comfortably by him and were open to members and guests from September 2 until October 11. As a great many members were actively connected with the numerous fine exhibits of electrical and steam apparatus, those Institute headquarters were immensely useful and popular. Everybody met there every day. On October 7 and 8, the first convention ever held by the Institute was conducted in the rooms and at the fine old Continental Hotel—the Bellevue-Stratford of its day as to splendor and comfort—but fortunately not as to prices. The report of that 1884 Philadelphia Convention constitutes the bulk of Volume I of A. I. E. E. TRANSACTIONS—traversing such diversified arts as to wireless, underground wires, multiplex telegraphy, earth return, carbon filaments and the incandescent lamp, batteries and electrochemistry, electric roads and subways, the Edison vacuum bulb effect from whose study such extraordinary results in electricity and physics have followed, and some of the first explorations of the dynamo. It was and is all in harmonious precedence and prophecy of such a program as that of this week for the Twelfth Midwinter Convention.

The Exhibition, the Congress and the Convention were soon followed by a most valuable series of Jury Reports issued by the Franklin Institute, on the Exhibits, and to the preparation of which a large number of A. I. E. E. members contributed in various important ways, notably such experts as Prof. E. J. Houston and the indefatigable original "man from Missouri," Dr. Carl Hering. Nor was that all in the way of electrical literature springing from the events of 1884. Dr. Houston, confirmed and inveterate pedagogue, issued during the Exhibition an admirable little series of electrical leaflets. Their matter and manner were cleverly mimicked and plagiarized in a rival series issued by the Mystic Order of Kazoos. Frank and open confession may here and now be made that the cosy rooms of the Institute were just as convenient for their surreptitious writing and publication, as for the preparation of Volume I of the TRANSACTIONS of 1884, then undertaken by the writer with valuable Keith collaboration; soon to be followed by Volumes 2, 3 and 4, from the hands of Secretary Ralph Pope, elected in 1885, author of the excellent first A. I. E. E. YEAR BOOK. Some of the quips and jokes of the priceless Kazoo leaflets were part of electrical haus spreche for many years thereafter. Almost up to the moment when before the Institute a score of years ago, Steinmetz enunciated his epochal "Law of hysteresis," one could hear once or twice a week at least, as comment on alleged advances in the dynamo electric arts, a laughing quotation of the similar Kazoos' apothegm that "Dynamos may be painted any color without increasing their efficiency."

Yes, there is still some vital savor, some drop of red blood, some human appeal in all the dim and dusty records of humble beginnings on which rests with superimposing crush the colossal electrical structure of today. As President in 1887, the writer was proud to assert that the total capitalization of all American electrical industry was about $375,000,000. This week the Institute awards its Edison Gold Medal to the member whom it has recognized as chief engineering exponent of just one of its industries—that of electric light and power—which alone has now attained an investment of Six Billion Dollars, and employs thousands of its member electrical engineers in most varied capacities.

Best of all, perhaps, above and beyond the archaic technical record of those early dingy volumes of "TRANSACTIONS" is the undying evidence they embody that even then while the older men were prophecying estactically as to a world to be wholly electrified in the near future the young men were transforming wild dreams into sober realities as arts and industries, careers and utilities. A great national society that had the courage to take for its third President an untried young man of 30, still calls youth to its banners, must forever renew its own life from the well spring of youth, and will forever with higher aims, higher ambitions, higher aspirations, higher service, interpret the engineering science that leaves no department of human affairs without benefit of its healing touch, and asseverates truthfully and reverentially; "Behold, I make all things new!"

PRESIDENT RYAN: The next speaker is an eminent American who is responsible among other things for having caused General Carty to put those twists into the telephone that serve us so well today. He did many things of wonderful value aside from that. He gave us the first electric meter that the customer could read, and with which to satisfy himself in regard to his bill.

We have had three great international electrical conventions in our country, two of which, as I remember, were referred to by Mr. Martin; the first being the one in Philadelphia under the auspices of the Franklin Institute; the next one in Chicago, in 1893, and the third and last one in St. Louis in 1904.

This gentleman carries the fine distinction for us in this country of having been elected unanimously

President of each of the last two international electrical congresses.

I take very great pleasure in calling him up to speak likewise of the early days of the Institute. Mr. Martin has spoken of the wonderful things that are to be found in the archives of that day. I must speak of the gentleman who will thus quickly be called up as a prophet, for if you do not realize that he is truly one of our great prophets, turn to the TRANSACTIONS in the fall of 1889. He was the President of the American Institute of Electrical Engineers in that year, and while in London was inspired to speak in a prophetic fashion. It is truly wonderful to read what our TRANSACTIONS say at that time.

ADDRESS OF DR. ELIHU THOMSON

In commemorating, on this occasion, the completion of the fourth decade of our Institute, it is difficult for us early pioneers to realize that much of our present membership has no knowledge, from actual contact, of the beginnings of our Institute, from which have been built up the enormous volume of electrical engineering today. Incidentally, it has given birth to the profession which our Society represents, the profession second to none in the wide world.

As its President, as has been mentioned by our own President now, in the fifth year of the existence of the Institute, it was my duty to represent the Institute at a large gathering and dinner at the Guild Hall in London, at which about three hundred of the most prominent engineers of London met, and a visiting body of Americans, mostly Civil, Mechanical and Mining, with a sprinkling only of those who had begun their activities on the electric side of engineering. I felt almost as an interloper, a trespasser on the older domain, but did my best to dignify the new profession in response to a toast to our infant society.

I want to interpose here that just before I was called upon to speak, naturally a youngster among a great many much older men, I received a message. The speaker who represented the American Society of Mechanical Engineers couldn't be heard ten feet away. I was close to him myself, and I couldn't hear what he was saying. He read too meekly altogether, and there was such a roar in the audience that a speaker could not expect to be heard even if talking much louder than he did.

At that moment I received a little scrap of paper from Professor R. H. Thurston of Cornell. He said, "Thomson, it is your turn next. For God's sake, get them back. Get them back, for God's sake."

I nodded to him that I would. Now, it was a big job for any one to do, but I yelled as I never yelled in my life before. I took in my lungs full, and I let it out with almost every word, and the actual fact was that I did get them back.

They quieted down, and I think most of them heard everything I had to say. I have forgotten what it was all about, but I was bound to make this little engineering institute count for something, even among that great body of men, and I was free in my predictions of what was to come in the electrical engineering field, naturally, because I believed they were to be true, and whenever I have made such predictions and found out afterwards that they actually happened, I found that several hundred per cent more had happened than I could possibly predict, so I was justified.

Lord Kelvin, then Sir William Thomson, was there. We had met once before at Philadelphia at the Electrical Congress in 1884, held in connection with the Franklin Institute Electrical Exhibition in the fall of that year. I have liked to regard him as a representative of the theoretical and practical side of electrical engineering. He it was that first understood the meaning of capacity and inductance in submarine cables, which led to the laying of the first Atlantic Cable in 1866 after the failure in 1858, due largely to imperfect knowledge of others.

His insight at that early day was most exceptional. Not since the days of Franklin's kite experiment here in Philadelphia, had there been any electrical engineering undertaking of any magnitude.

Franklin's lightning rod was a great thing and still is the foundation of our protective means for lightning.

Except for a lighthouse lamp here and there run by alternating current, a single generator to a single arc-light, there was not until about the middle of the decade between 1870 and 1880 any marked progress in what might properly be called electrical engineering outside of land and cable telegraphy.

There was no field for one who felt that his tendencies led him in the direction of large electrical application—he must create that field. At the centennial exhibition of 1876 here in Philadelphia, there were only two exhibits of dynamos in action; the Gramme, unique as it was, and the Wallace-Farmer; the latter long ago obsolete because too inefficient. Both of them were a development of the ideas of Paccinotti of the previous decade. There was also a Gramme machine exhibited by Prof. W. A. Anthony—later a President of the Institute—and run at times. I am informed by Dr. Bedell of Cornell that it is still intact being at present used as a motor.

Here and there was an example of the earlier shuttle wound armature Ladd and Wilde machines, and the only one of which I knew as existing in this country was at the University of Pennsylvania in the collection of philosophical apparatus. I knew Professor Robert E. Rogers, who was professor of natural philosophy at the University. When I first saw this early type of dynamo operated in the years just after 1870, I remember standing before it spellbound. Something of the significance of thus turning mechanical power into current for the future overcame me, though the

machine itself was crude and inefficient. The earlier types of permanent magnet machines of Saxton and others made no such impression. The magneto electric had given way to the self-excited field machine of unlimited power and capabilities seen by a sort of prevision. This principle of self-excitation came to be known as the "reaction principle" and was discovered in 1866.

The Gramme exhibit at the centennial in 1876 was made more impressive by the use of such a machine as an electric motor in addition to running single arc-lights, one to a machine.

It is well known that the telephone of Bell was first shown in operation to a body of scientific men among whom was Sir William Thomson, at the centennial in Philadelphia.

A curious accident which may be known to some of you, but not to others, I may as well relate. Sir William borrowed from Bell a couple of instruments, of the magneto telephone type. The two instruments were electro-magnetic, the one to talk into and the other to listen from. He took them abroad with him. He tried to show these two instruments to the Royal Society in England, and he failed in getting the instruments to work. Some little accident had happened to the support of a diaphragm and prevented the operation. That little accident saved the Bell patents in England. If that instrument had worked there would have been no possibility to have had valid Bell patents in England. A patent there depends upon the prior non-introduction publicly of the invention, and as Sir William would have been the one to introduce it publicly and not Bell, Bell couldn't have had any patent.

That was one of the accidents of fate, only it happened in that case favorably to Mr. Bell.

There was another thing that happened in those days, not so favorable to Mr. Bell. It was said that when the German Patent Office was asked to grant a patent on Bell's telephone, they said, "Oh, no, it is too valuable an invention to be patented," and they refused it. In other words, the more valuable a thing a man gets up, from the German standpoint, the more definitely must he be refused protection.

From 1874 on, while living in Philadelphia, I was constructing small machines and using them as motors and at the close of 1876 had finished a machine which required about a horse power to drive it, and which would work a small arc-light. I had wound the field shunt and series as a compound winding and used it as such, subsequently finding that a British patent to S. A. Varley had already shown such a winding in 1876. However, I made use of this machine of mine, in a course of five lectures in electricity given at the Franklin Institute early in 1877, the object of which lectures I had designed to be the demonstration of the fact that electricity from any source was indeed the same.

In 1877 the Franklin Institute appointed a committee to investigate the properties of arc-lights and machines for running them, one machine to one light, of course. Professor Houston and I served on the Committee for the electrical measurements, the first, I think, published. This was in 1878.

Then followed a more rapid advance. The Avenue de Opera was lighted during 1878, exposition year, by Jablochkoff candles, later found too expensive. Brush brought out out early in 1879 his series system of arc-lights, closely followed by Thomson-Houston in the same year.

I may say as a matter of personal note that about four blocks west of where we are now, the first T.-H. dynamo was built early in 1879.

Its armature had on it the first three-phase windings, used with the three segment commutator, of course. The patent application showed the winding of today connected to collector rings for a-c. current.

The machine was the very first of its type though of about five hundred volts and ten amperes or about five kw. No model had been made, but so confident was I of the merit of the construction for constant-current work, that I was willing to risk success. It worked perfectly from the start and lighted a bakery, near where it was built, through the summer of 1879, the room temperature on account of the large bake ovens reaching on a hot night 140 deg. fahr. Why didn't it kill us? It did nearly kill two of us, but I was the one not affected.

I am going to give a piece of advice. Houston had to lie off from heat prostration. Thomas H. McCollin, interested in the matter, had to go off from heat prostration, but I didn't, and the reason was before I got hot enough to feel very uncomfortable I drank quantities of ice water. They said, "You will kill yourself."

"No, I won't. I am keeping my temperature down," and it worked.

In 1880 our newborn enterprise was taken to New Britain, Connecticut, and later to Lynn, forming the nucleus of the great works there.

Another little personal note I may interject here. When I got to New Britain, a manufacturing town in the interior of the State of Connecticut, I found hardware, tacks, nails, hinges, and everything of that kind were made there. They did not know anything about electrical apparatus, and didn't appreciate what we were trying to do. We got to Lynn three years later. There was a Weston plater in one of the shops in New Britain. Probably some of you know the old type of multiple field plater.

I was the electrician (not the electrical engineer) of our Company, The American Electric, at that time, and I was asked to come and see that machine. I said, "Send it over." They put it on a wagon and sent it over. We found it was a machine of fairly large size used for working plating baths in the hardware operations. I turned it over and took a monkey-wrench

and tightened a screw on the bottom. One of the nuts that tightened up the connection was loose, and of course, with such a low voltage as plating voltage, the current couldn't cross that kind of a joint. I gained some prestige from that.

Late in 1879 Edison had settled on carbon as the material for a burner or incandescent lamp filament and the famous Menlo Park exhibition of such lamps running in parallel from a dynamo took place at the close of the year.

Years elapsed before the prodigious work of producing the much-needed appliances in such a new art as incandescent lighting was accomplished. The Pearl Street Station in New York, with its direct connected "Jumbo" dynamos and underground distributing mains, was the outcome in 1882.

The following year the Brockton Edison station was opened as the first example of city distribution by three-wire circuits. The pace of development was from now on rapid, but we have now about reached the time of the organization in 1884 of our Institute, and later that of the first electrical exhibition in the fall of 1884, in Philadelphia, the Franklin Institute exhibition where were shown in one building all the notable advances made in the years preceding.

I will not attempt to catalogue them, only to say that this event was, as it were, a most fruitful and interesting landmark in the electrical progress in our country.

Edison in 1875 had thought he had a new force, etheric force. Prof. Houston and I thought it was electrical and set about proving it. The whole account of it was published in the Franklin Institute *Journal* at the time. At the conclusion of the experiments, I said, "Let's do this thing on a big scale." So we set up a large Ruhmkorff coil on the lecture table attaching one terminal by a wire several feet long to a large tin vessel mounted on a tall glass jar, and the other coil terminal was connected under the table to ground through a water pipe, there being a spark gap between the coil terminals. When we did that we could go all over the building and outside and explore by a pencil point, which by the way, Edison had used as a detector. We went to the observatory, five floors away, and we could get from doorknobs a signal that the machine was in operation on the first floor.

This was at the old Boys Central High School, North Broad Street, about a mile away from here. The building is still there.

Was there in this anything suggestive of wireless? Yes, there was. In the old Boys Central High School, the small set of experiments made in 1875 and 1876, pointed that way. They have been detailed elsewhere.

Was there anything of transformer work known in those days? Yes. In the Franklin Institute lecture room in February, 1879, were run two transformers (induction coils) with their fine coils of many turns in parallel from the collecting ring end of a dynamo which had been constructed to be self-exciting and yield a-c. currents.

There were no incandescent lamps for the coarse short wire secondaries, so we had to be content with semi-incandescent lamps with carbon, and coils of iron wire heated red hot. This is probably the first time such a combination was used, pointing the way to transformer work of today.

In New Britain one of our first jobs (I am trying to give you a picture of the electrical engineering of the time) was to find out what, if any, was the loss in an armature core of a dynamo. Some had said there wasn't any loss in the iron, and others had said there was a lot of loss in the iron. Some had said that if you use insulated wire you wouldn't get any loss, or very little, if you use bright iron wires for winding the cores you will have losses, and this was true.

We didn't have sheet metal. I will tell you later about that.

So we set up a dynamometer, built the armature cores of iron wire and ran them in an excited field to determine how much loss there really was in the iron, and we found the iron varied and the loss was in some cases much greater than in others, and we selected our iron wire accordingly but we always found there was a loss outside of windage air loss, but it was very difficult to determine how much it was. It was not so serious a loss as compared with the resistance loss in the copper. So we had to go along in that way.

I want to say that sometimes we had to go to the Patent Office in those days to tell the Patent Office what we were about. They didn't understand. I tried to tell them something about the reactance or inductance coil, how it worked unlike a resistance. They said, "You can't tell us that."

So I had to make an elaborate description. Think what the trip to Washington cost us in those days! You left Philadelphia and went to Baltimore. Through the streets of Baltimore was a line of mule teams, and they took the railroad coaches and pulled them through the heart of Baltimore. That was in 1880. Those whose memories can carry them back to that time will bear me out. It wasn't an easy job to get to Washington then.

Current control in those days depended on regulation by hand, and it was only later that we got into automatic regulation.

Now, I want to say just a word before I stop as to the handicaps we were working under in the days just before the organization of this Institute.

Wire: How and where could we get wire? In Philadelphia there was a bonnet wire factory. As you know bonnets in those early days were framed of wire, braided or wound with cotton. That bonnet wire factory did on the side a little business of winding cotton on copper. That is the way we had to get the wire for dynamos we were constructing. We had difficulty as late as '84 or '85 in getting the fine wire used in the shunt circuit in arc-lights, which should have the same resistance throughout. We should have coils of the same resistance; we gaged the wire at one end of a hank, and it

was either smaller than at the other end, or larger than at the other end. During the time of pulling the thousands of feet of wire through the wire drawing die of steel, the die wore, and the wire got bigger and bigger all the time. That was one of the difficulties that we had.

How could we make coils that would measure up to any definite resistance? Well, I had to take the wire makers into my confidence and say, "We won't buy that wire unless you jewel draw it. You will have to draw it through jewel dies. That is the way it will have to be done."

They set up the jewel dies. They advertised that they had now the process of producing wire the same at one end as at the other, and they never gave me any credit for it.

It was the same with the insulations. The insulations at first were poor, and our line wire was poor; even bare wire was used in those days to distribute arc-lighting. All of us who go back far enough know that the loops that went into stores and the wires on the poles were bare copper, and machines might have forty arc-lights or more in series.

I know one case where they tried to couple up in 1882 seven forty arc-light machines of the Brush type in Cincinnati on bare wire alone. It is a wonder they didn't set the whole place on fire, but the fact of the matter was, they broke down all the machines themselves so they couldn't set fires. The circuit was off.

We had no slate and porcelain. Mica was used for stove doors, and that was about all the use there was; and it was ourselves that introduced mica and pasted it together for the insulation in our arc-light machines.

Permanent Magnets: We had to study and manufacture magnets for our meters and study the whole case of production of permanent magnets and how to season them so that they would be permanent. Otherwise the meter was no good. Those were the things we had to do. Nothing had been done for us. Remember also that the steel we had at first for magnets was only tool steel. We hadn't any special magnet steel such as we have now.

In 1878 there were no telephones. You couldn't call up anybody by 'phone.

There were no typewriters. We had to do all our writing by longhand, and have it put in a copy book. The typewriter came later. There was an awful lot of work done in those days that people don't have to do nowadays.

There were no suitable steam engines in those days, and that applies directly to the time of the start of the Institute. The engines of those early days, except for some special instances where they were made for it, were not good enough to drive electric machinery. You know the old slide valve engine found everywhere. We would put electric lights on those old engines, and every time there was a fluctuation in speed the lights went up and down.

It was the development of the high-speed engine that made it possible to drive the Jumbo dynamo at a proper speed. In the years before everything was belted instead of direct connection. All the machines were belted to the engine power and subject to the vagaries of old-fashioned governors. One of the earliest cases of direct connection was the famous Gordon alternator in England.

I could go on and tell you more and more about the difficulties, but better things came gradually, sheet metal, sheet iron. We have been even criticized at times by being asked, "Why didn't you use sheet iron in those early machines?"

We couldn't get any sheet iron except stove pipe iron. That wasn't any good for magnetic effects. We would have been glad to get punch presses and use sheet iron, we couldn't get either. More than that, the punch presses in those days were used to punch out pie plates and similar objects only. There was not much done. The punch press room now is the wonderful institution of any great electrical works. We had to design our machines to fit the tools we had in the shop. If we did not have a big enough planer to take a machine, we had to design a machine so the planer would take it. That will explain some of the designs that we had to use. We didn't have any steel castings in those days. There weren't any steel foundries that were producing steel castings. It was only in 1886 that we established a steel casting foundry in Lynn and mainly because we could not get such castings made for us.

I will weary you if I keep on, but I wanted to give you a sort of an idea of the fact that things were not in those days as they are today.

President Ryan: The first year as instructor in college, I was called upon to proctor in an examination in Greek life. It was a new experience. I looked over the list of questions and found that there was but one in regard to which I could conjecture anything. However, I had no responsibilities in the matter in that regard. It didn't matter whether I could conjecture anything or not in regard to the examination. I had not the questions to write or the papers to read. The question that appealed to me was this one: "In the understanding of the wise men of ancient Greece, what was the limit in population of a city?"

When the papers began to come in, I looked them over. Four or five sufficed. It was an easy question to answer, evidently. The answers were all in agreement, and I came to the conclusion I knew the answer. The answer was this interesting one: "No city in the understanding of the wise men of ancient Greece in population could grow beyond a point whereat the people might gather around the city's advocate and hear his voice and learn the truth."

I wonder what a wise man of ancient Greece would think today to come into our country and find that through the wonderful work of the next speaker and the organization that he built up, there is no limit to

the size of a city in population. For in 1915 he extended his lines clear across the continent and from that day to this, at any time if we care to, it is physically possible for us as a nation to sit down at home, or nearly so, and listen to the voice of the President of our great country.

I take great pleasure in calling up as the last speaker in this series, a past President of the American Institute of Electrical Engineers, General Carty.

Address of General Carty

Among the four national engineering societies, composed of Civil, Mining, Mechanical and Electrical Engineers, the Society of the Electricals is the youngest. Of the other societies mentioned, it may be said that the art which they represent existed before they were formed, but of the Electricals it can be said, with pardonable pride, that its members created their own art.

The Centennial Exposition, held here at Philadelphia in 1876, marked the beginning of a new era in electrical development. Prior to that time the use of dynamos had practically been limited to such purposes as electroplating, and electric motors were a curiosity—a toy, not for children, but only for college professors to play with. The principal use of electricity was in the different forms of signaling, of which the telegraph was the most important.

At the Centennial Exposition in Philadelphia, one who was to become a member of the American Institute, and also one of its Presidents, Dr. Alexander Graham Bell, made known to the world his invention of the telephone. This gave a wonderful stimulus to the advancement of electrical science.

Soon followed the invention of the phonograph and the astonishing development of the dynamo and the electric light, which we so largely owe to another who was to become one of the members of our Institute yet to be formed—Mr. Thomas A. Edison.

Working here at Philadelphia about this time, there was another whose name was destined to be memorable in the annals of our Institute—Dr. Elihu Thomson. His fundamental work in laying firmly the foundations of the art of electrical engineering will serve as an inspiration to our members during the years to come.

Between the years 1876 and 1884, such marked progress had been made by the electrical pioneers, the future founders of our Society, that the importance of forming a national electrical engineering society was recognized, so that in 1884 the electrical pioneers of the period joined in the formation of an organization which should be a worthy exponent of electrical progress throughout America.

Prior to the work which was done by these pioneers, many of whom are still alive and active members of our society, there was no art of electrical engineering, and there was no university conferring the degree of Electrical Engineer. Electrical engineering was not even recognized among the professions. Those who devoted themselves to the practical affairs of electricity were known as electricians—a term now generally applied only to electrical artisans.

It was our society and the work of our members which have secured for electrical engineering its present position among the foremost of the learned professions.

Since the formation of the American Institute of Electrical Engineers, progress in electrical engineering has been a record of rapid and continuous and marvelous achievement.

The story of this achievement is to be found in the history of the American Institute of Electrical Engineers and in the careers of its members. So numerous and so important are the contributions of our members that adequately to describe them would require the writing of the history of the art of electrical engineering.

While our colleagues of the Civil Engineers were celebrating the joining of the Atlantic Ocean and the Pacific Ocean by the Panama Canal, the members of the American Institute of Electrical Engineers were celebrating the joining of the Atlantic Ocean and the Pacific Ocean by electric wires spanning the continent and carrying the human voice from coast to coast.

It is among our membership that we find the men, who, taking the feeble telephone which Bell exhibited at the Centennial Exposition in 1876, have built upon it the entire art of telephony, and have constructed telephone lines and installed telephone stations so that it is now possible to transmit the human voice from San Francisco to all of the states in the Union, connecting with wire systems reaching more than fifteen million stations and aggregating more than forty million miles of wire, representing an investment of more than two billion dollars, and carrying more than nineteen billion messages in a year.

In our membership are to be found men who have developed the wonderful art of electric power transmission, whereby the melting snows of our mountains are transmuted into power and light and heat and current for transmitting speech, for lighting the homes of our nation, for propelling railroad trains and ministering in countless ways to the happiness and welfare of our people.

It is a great satisfaction to our members to observe these important contributions which they have made to the comfort and beauty of the homes and the halls of our country.

In the progress of society from the primitive conditions found in former times to the present high state of civilization, communication and transportation have been the two great forces in the building up of the civilized state. It was said before the founding of our government that west of the Alleghanies there never could be a people which could form a part of a nation

on the Atlantic Coast. This was because of the impossibility of communications and transportation, which are essential to a community of interest and common action.

This indeed was true at the time, and would still be true were it not for the work of engineers. Communication north and south along the Atlantic Coast was accomplished well enough at the time by coasting vessels plying between the numerous harbors which there abound, but it was not until the advent of railroads, which were the work of engineers, both civil and mechanical, that the growth of our country westward as a permanent part of our nation, was assured, and it was not until the great State of California was connected by rail with the East, that our Union was complete.

Coextensive with the development of the railroads came the telegraph, supplying communities with the great essential, quick communication. Thus, with the railroad, the telegraph and the mail, the essentials of growth for the time being were provided, but as our progress has become more complex, further development of transportation and communication was required.

Our members have provided these in the telephone and in the electric railroad. The telegraph and the steam railroad connect places; the telephone and trolley reach directly to homes and to offices. They connect not only places, but they connect people. In many other ways that are little thought of, our membership has contributed to the upbuilding of our nation.

The American Institute of Electrical Engineers stands for the highest achievement in electrical engineering and for the most distinguished attainments among its members.

From the art of electrical engineering as its exists in the world today, take away the contributions of the members of the American Institute of Electrical Engineers, and that which would be left would make a sorry showing by comparison.

The art of telephony would disappear and all of those wonderfully coordinated activities of both peace and war, depending upon that means of communication, would instantly be paralyzed.

In electric lighting and power and current distribution, the contributions of our members have been so fundamental, so important and so numerous that it is impossible to picture the chaos which would result if, by some black magic, their wonderful work should be undone.

By its papers and meetings and discussions, by the spread of knowledge through its published JOURNAL and TRANSACTIONS, and above all, by the high ideals of its members and by the unsurpassed character of their achievements, the Institute has taken a foremost place among the forces making for the welfare and unity of mankind.

It is with feelings of pride and satisfaction that today we celebrate the fortieth anniversary of the founding of the American Institute of Electrical Engineers.

We are holding this celebration in the Golden Age of Electricity—the age of electrical communications, of electrical transportation, the age of light and power. What words these are to conjure by! Light to see, and power to move and to do—to do that which no man has done before. What inspiration they send forth, urging our members to still higher achievement!

Gaseous Ionization in Built-up Insulation—II

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Review of the Subject:—*It has been suggested frequently that the failure of high-voltage armature bars might be due to deterioration caused by gaseous ionization in entrapped air spaces. In an earlier paper a series of tests on a number of 6600-voltage mica folium armature bars made up with different degrees of mica content was described. The variations of the dielectric losses with voltage and with temperature were studied and by means of the application of pressure it was shown to what extent losses due to internal ionization were present. The influence of these ionization losses on the life of the bars was also studied.*

The tests of the foregoing paper are continued in the present paper, extending to a wider range of type of armature insulation. The general results are as follows:

1. The absolute values of loss due to internal ionization in well constructed armature bars are small compared with dielectric losses of other types.

2. The losses due to internal ionization do, however, cause a progressive deterioration of the insulation. This is shown by a gradual increase in the loss and power factor of the insulation.

3. It is indicated that the principal function of mica in this type of insulation is in the reduction of the conductivity of the insulation and the withstanding of the action of internal ionization. The indications are that full mica folium content can be safely reduced only by use of the best grade of mica folium and the best conditions of application without variation. Several bars having relatively low mica content have compared favorably with those having the maximum mica content, indicating reliable insulation over long periods. It appears that high mica content is necessary in order to maintain a high factor of safety to cover the variations in factory processes.

4. In the drying out period this type of insulation is subject to great danger from relatively brief periods of application of voltage. During these periods the losses are high, internal temperature is raised, with further increase of loss, leading to breakdown. The greatest increase of loss in this danger period is in the range 25 deg. to 50 deg. cent.

5. The drying out period at 125 deg. cent. and during which normal voltage may not be applied to the insulation for more than a minute or two, varied in the specimens studied from fifteen days to three months.

In a foregoing paper, under the above title, a series of tests has been described, investigating the presence of gaseous ionization, or corona, in various types of armature insulation, with particular reference to its magnitude and its influence on the life of insulation. It was shown that the magnitude of the power loss due to ionization is relatively small, as compared with the dielectric loss due to other causes. It was shown that under sustained voltage and temperature the loss in those bars which initially had the highest ionization loss, increased gradually, indicating a progressive deterioration of the insulation due to these causes. These life tests were made on bars No. 33, 46 and 49; the two former having standard 6600-volt mica folium insulation, and the last a slightly smaller proportion of mica.

In the present tests the study of the progressive changes in the internal loss is continued, the principal purpose in view being to investigate the influence of varying degrees of mica content in two groups of bars, one having paper, and the other treated cloth, as the supporting binder for the mica folium. A description of the make-up of the various bars is given in the foregoing paper.

The general outline of the present tests is indicated by the following list of the bars tested. The list includes the bars upon which report has already been made as indicated. The remaining bars have been selected as having decreasing amounts of mica, and as having the greatest percentage corona loss in their respective groups. However, in two groups of paper bars no bar showed a corona loss higher than 2 per cent.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

Thus as regards the influence of corona, No. 39 is the most conspicuous new case in the paper group.

TABLE I.

Paper Insulation	Cloth Insulation
No. 33—full mica folium (92 per cent) 14 per cent corona loss—already studied	No. 46—full mica folium (no cloth) 92 per cent, 3 per cent corona—already studied
No. 36—70 per cent mica folium 2 per cent corona	No. 49—73 per cent mica folium 37¼ per cent corona—already studied
No. 39—50 per cent mica folium 14 per cent corona	No. 54—42 per cent folium 28 per per cent corona
No. 41—28 per cent mica folium 2 per cent corona	No. 55—22 per cent mica folium 32 per cent corona
	No. 60—0 per cent mica folium 14 per cent corona

Bars 36, 39 and 41 were placed in one oven and Nos. 54, 55 and 60 in a second duplicate oven. These ovens were of the Fries type and were electrically controlled up to about 135 deg. cent. maintaining temperature constant to about 2 deg. in the upper range. In the tests the temperature was raised by steps lasting several days each, to a maximum of 125 deg. cent. The ovens were equipped with high-voltage bushings and 10-kv. was applied for 16 out of every 24 hours continuously, to the central conductors on the sample bars. Other leading-in bushings afforded connections to the central test electrodes and to the guard ring electrodes on each bar. At stated intervals the voltage was interrupted and measurements made of dielectric loss and charging current at 7.5 kv. and also of insulation resistance as based on application of continuous voltage for one minute. The alternating frequency was 60 cycles. Other conditions surrounding the methods of measurement are described in the earlier paper.

Experimental Results

The general results of the tests may be seen in Fig. 1, which gives the record of continuous observation on the

bars mentioned, subjected to continuous temperature over a period of ten months. The continuous application of voltage was possible over a period of six months only, on bars Nos. 36, 39 and 41, and not at all on Nos. 54, 55 and 60 for reasons described below.

Before beginning the temperature run the losses in all of the bars were measured at atmospheric temperature, and the values compared with those taken one year previously, when the measurements, described in the earlier report, were taken. The values of loss in bars Nos. 36, 39 and 41 were found to be practically unchanged indicating no alteration on standing idle at temperatures in the neighborhood of 25 deg. over a period of one year. On the other hand, the bars of the cloth group showed increases in loss as follows: No. 54—15 per cent; No. 55—6 per cent; No. 60 (no values permitting the application of voltage over reasonable periods of time. The continuous application of voltage was possible only after 3½ months, on the bars having the larger percentage of mica, and on two of the bars low in mica it was never possible, as described below. During the initial period, therefore, measurements were made only on bars Nos. 36 and 54 as pilot bars for their respective groups.

Internal Losses

Referring to Fig. 1 the sharp increase in dielectric loss following the increase in temperature is at once evident by the rise between 24 deg. to 42 deg.; the losses in each of the bars 36 and 54 are practically doubled. There is little change in the loss at this temperature and on increasing to 63 deg. there is a further increase

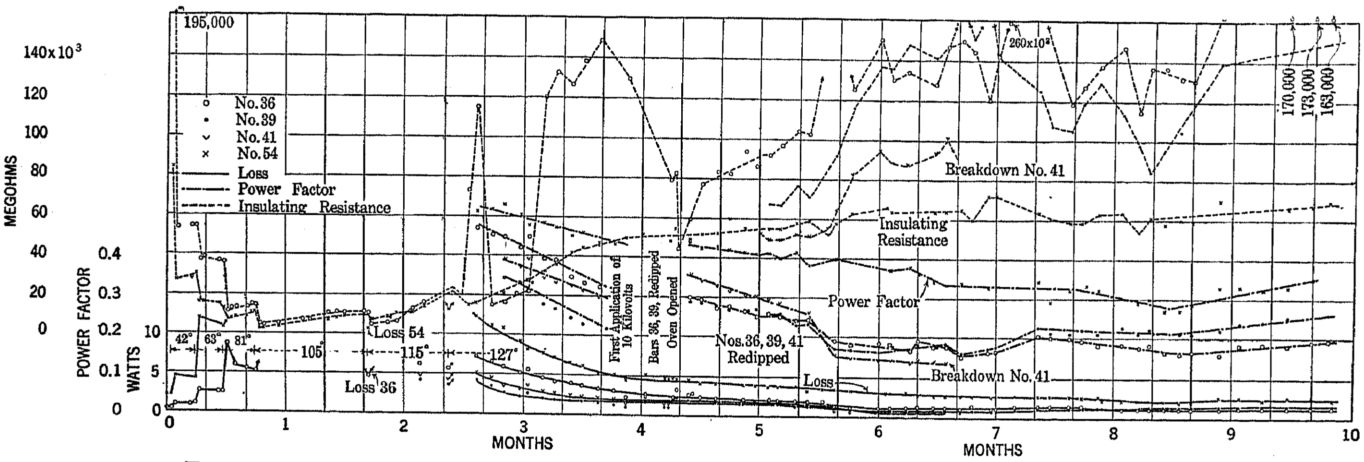

Fig. 1—Losses in Armature Insulation at High Temperature and High Voltage

mica)—40 per cent; thus green bars having cloth as a supporting binder show a tendency to increase their internal losses with age. Absorption of moisture suggests itself as the most probable cause of this increase.

After assembling in the ovens the bars were first measured at atmospheric temperature 24 deg. cent. The temperature was then raised to 42 deg. for 6 days, then to 63 deg. for 7 days, then to 81 deg. for 8 days, to 105 deg. for 29 days, to 115 deg. for 20 days, and finally to 125 deg., at which value it was held continuously. The duration of the run at each temperature was determined solely by the behavior of the insulation as regards its internal loss. Every elevation of temperature was followed by a marked increase in the internal loss, the value of the loss thereafter decreasing at various rates. During the later increases of temperature the losses increased so sharply as to make it impossible to apply voltage continuously, as breakdown would follow after very short intervals. In fact, above 81 deg. it was found inadvisable to apply the measuring voltage of 7½ kilovolts over the period of 8 minutes, necessary for the measurements, for fear that breakdown would occur. It was only after two months, and after the insulation had been subjected to 115 deg. for sometime, that the losses had fallen to of loss by about 2½ times. At sustained 63 deg. there is a tendency to a decrease of loss. On raising the temperature to 81 deg. the voltage cannot safely be applied to No. 54, and No. 36 shows an 18-fold increase of loss over its initial value. At 81 deg. however the loss falls off fairly rapidly, although it increases again sharply when the temperature is raised to 105 deg. From this point onward for a period of nearly two months, it was not possible to make complete loss measurements owing to the high values of loss. The conditions of the bar could be told by a brief application of voltage in its effect on the wattmeter reading. A slowly increasing wattmeter indicates internal heating of the bar, and the danger of the continuous application of voltage. Isolated readings are shown toward the end of the second month. A regular program of loss measurements was begun after about two and a half months, when the bars had been raised to 125 deg. cent. and maintained at that temperature for several days. During this early period regular readings of insulation resistance were possible, and these also afforded some information as to the condition of the insulation. Comment in this connection is made below.

Bars Nos. 55 and 60 after their initial reading at atmospheric temperature would at no time stand 7½

kv. for a period sufficiently long to obtain the loss measurement. They showed a rapid increase in wattmeter readings and when subjected to voltage on January 26th, after nearly three months, No. 55 punctured under this test. In the earlier report several curves are given showing the increase of internal loss under the continuous application of potential. In Fig. 2 herewith is shown this effect as applied to bar No. 56 of the same group as No. 55 taken at atmospheric temperature. For the same reasons, although the regular loss measurements on the remaining bars began at two and one-half months, it was not possible to apply 10 kv. continuously to them until well on to the end of the third month. For example, No. 54 early in the third month showed an increase of 25 per cent in its internal loss in a period of 20 minutes.

It is noteworthy that in the present tests this initial period of high loss and unstable condition, often referred to as the drying out period, extended over a period of four months, whereas in the tests of the earlier report, having to do with insulation having high mica content, this period was limited to only about 15 days. While the losses were somewhat high after that period they were still sufficiently low for measurement, and as to cause no steady rise due to internal heating. In fact, comparing the results of the present tests with those on the bars having higher mica content, we find that the initial rises of loss with increasing temperature are less for the high mica bars and of shorter duration. After one month all three high mica bars show less than 3 watts each at 110 deg., while after two and one-half months the low mica bars are at from 3 to 12 watts each at the same temperature. The low mica bars dried out much more slowly and approached constant values at six months as against two and one-half months for those high in mica. Note, however, that the final constant minimum values of low mica bars (0.6 to 1.25 watts) are only slightly higher than those high in mica (0.6 to 1 watt).

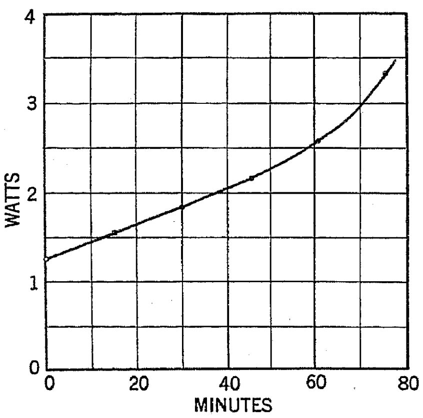

Fig. 2—Loss-Time, Bar 56, 7.5 kv.

The conclusion from these facts is that in some way the presence of mica affects not so much the ultimate values of the dielectric losses themselves, but the way in which they change when subjected to temperature. Observations on the values of dielectric absorption for this type of insulation give indications that the major proportion of the dielectric loss is due to absorption. Consequently, the variation in behavior between high mica and low mica insulation with time, will probably be found to lie largely in the variation of dielectric absorption.

The regular decrease in values of loss under continuous high temperature, as indicated in Fig. 1, begins toward the end of the second month. Toward the end of the third month 10 kv. was applied to the bars for 16 out of each 24 hours, together with the sustained temperature of 125 deg. cent. These conditions were continued until the end of the test. Occasional irregularities in these curves were caused by temporary fluctuations in temperature, and on two occasions it was necessary to open the ovens and re-dip one or more bars in insulating compound. This insulating compound was necessary to prevent surface discharge between the exposed end of the central conductor and the measuring electrodes. It will be observed, however, that even at these points of irregularity in the curves the relative values of the losses in the several bars remain approximately the same.

Considering the results plotted in Fig. 1 we may note as follows:

Of the six bars investigated only three, Nos. 36, 39 and 54, survived the test of ten months of high temperature and continuous application of voltage.

Bar No. 36, 70 per cent mica folium and 2 per cent corona, shows throughout a very low value of loss. In the earlier stages this loss was slightly higher than those of No. 39 and 41 although the three had much the same value of loss in the sixth month, when No. 41 broke down. After seven months No. 36 had the lowest loss and lowest power factor and remained in this position until the end of the run, showing, therefore, best prospect for long life. No. 36 started with the lowest initial corona loss and its good showing at the end of the test is in accord with the results of the tests described in the earlier paper, in which the bar showing least initial corona loss, showed in the final stage lowest total loss and best prospect of long life.

Bar No. 39, 50 per cent mica folium, 14 per cent initial corona, shows throughout a very low value of loss. During the seventh month, however, its loss gradually rises above that of No. 36, its insulation resistance falling at the same time. This increase in loss is attributed to the presence of internal ionization in No. 39, and its gradual influence in causing deterioration of insulation. The effect is also seen in the curves of power factor.

Bar No. 41, 28 per cent mica folium, 2 per cent initial corona. This bar is especially interesting because notwithstanding its low mica content it showed from the beginning a very low value of loss, this value being comparable with that of Nos. 36 and 39 after four

months. This low value of loss was maintained for six and one-half months, when unfortunately the insulation punctured. The breakdown occurred to a guard ring electrode, and so the approach to the breakdown condition was not observed on the instruments. The final low value of loss of this bar is the same as that obtained in bar No. 46 of the first report, which had full mica folium content, and which showed the best performance in the earlier tests. The low loss in No. 41 is significant as showing that low mica content may under some conditions give as good characteristics as high mica content. It appears that No. 41 must be regarded as an exceptional case in which a relatively low mica content, well preserved, after careful application, results in good electrical characteristics which, however, can hardly be considered as typical in view of the history of other low mica bars. In fact, as stated, this bar broke down after six months.

Bar No. 54, 42 per cent mica folium, 28 per cent corona, is the only bar in the cloth group surviving to the final period. It shows a relatively high loss which is to be attributed rather to its low resistance (*i. e.* high absorption) than to its high initial value of corona. There is little, if any, evidence of increase of loss in the final stages. However, it is to be noted that on account of its high total loss it was not possible to submit this bar to continuous high voltage. Therefore, it offers no conclusion as to the influence of internal ionization on its life.

Bars Nos. 55 and 60 never reached a stage at which voltage could be continuously applied. Both of these bars had high loss at atmospheric temperature, preventing power measurement for sometime after their temperature was raised. No. 55 broke down at the end of two months, at 115 deg. cent. during an attempt to measure its power loss. At the start at atmospheric temperature the resistance of No. 55 was 6500 megohms, and that of No. 60 340 megohms, showing the very great influence of the mica in No. 55. Two months later at 115 deg. No. 60 showed 16.5 megohms which, however, rose under sustained temperature to 47 megohms in another month, and 236 megohms in four months. These bars were then removed from the oven to make room for the others. No. 60 is interesting in several ways. In spite of its very low insulation resistance when cold it had the relatively low value of loss of 5.35 watts. Dielectric absorption in No. 60 is completely masked by its high conductivity. The value of insulation resistance given above is determined from a one minute reading at continuous potential, but this value remains practically unchanged over one-half hour or more. The value of resistance so computed, however, accounts for only 16 per cent of the total dielectric losses. Although the absorption is masked it is present, as is shown in another section of this report, and we have here clear evidence of the importance of dielectric absorption as distinct from conductivity in causing the losses in dielectrics.

INSULATION RESISTANCE

The curves of insulation resistance in Fig. 1 are plotted from observations with continuous potential in the neighborhood of 200 volts, and current in a high-sensitivity galvanometer. The galvanometer reading was taken at exactly one minute after the application of voltage. The insulation was then short-circuited one minute and a galvanometer reading with reversed voltage at the end of another minute again taken. The mean of these two readings was used in computing the resistance. As is well known, the value so obtained does not represent the true ohmic resistance but merely gives one point on the absorption curve. In many cases and particularly in some of the bars under observation, a steady reading of the galvanometer only results after a long period of time, say one hour or more. It often happens that the one minute reading is many times greater than the final approximate steady value. However, except under conditions in which the shape of the absorption curve is materially altered, the proportionate relation between the one minute and final readings remains constant under a variety of conditions. While, therefore, the curves in Fig. 1 do not give, as suggested, the final values of resistance, they do represent in their mutual relation the relative changes in the final resistance of the several bars. They probably represent more exactly the relative values of the absorption, since the one minute reading is in most cases well up on the initial steep portion of the absorption curve. In taking this point of view it should be noted that high values of insulation resistance indicate low values of dielectric absorption and vice versa. It is highly probable that the major portion of dielectric losses are due to absorption, and it is for this reason that there is a fairly uniform relation between the so called insulation resistance curves and the loss curves. The relation, however, is total loss and absorption, rather than loss and final ohmic resistance. Referring then to the insulation resistance curves, it will be seen that at atmospheric temperature they indicate a very high value of resistance and that this resistance decreases very sharply when passing from 24 deg. cent. to 42 deg. cent. The decrease here is much greater than in any other of the successive temperature steps leading up to 125 deg. cent. It is especially significant that absorption should be relatively low in the green state of insulation, at room temperature, and be greatly increased by a slight elevation of temperature. This moderate elevation of temperature leads to a condition of high absorption, low resistance, and high loss extending over a period of two or three months. If this change of condition is due to the presence of moisture, which is the only influence which suggests itself, would it not be possible to assemble and apply this insulation at 40 deg. or 50 deg. cent. and so avoid the low dielectric strength of the weak period, and also its long duration? Under the influence of higher temperatures, the insulation resistance after a time begins to rise again, gradu-

ally increasing in value and being accompanied always by corresponding decrease in the values of loss. The readings of the galvanometer in these regions of high resistance become quite erratic and apparently the condition of the insulation as regards resistance and absorption is in a continual process of change. For example, the readings immediately following the removal of the alternating test voltage (10 kv.) from bar 36 will be quite different from those taken 15 minutes later. The first application of high alternating test voltage evidently causes some shake up in the structure of the insulation, and it is followed by a change in the value of the insulation resistance. The opening of the oven for a brief period may also be followed by changes in this quantity. Expansion of the sides of the bars, particularly those containing mica, following long periods at high temperature, lead to thicker air layers, a high resistance, and also probably to unstable mechanical structure. The insulation resistance curves, therefore, in their regular progress are to be regarded for the light they throw on the variation in dielectric absorption. In their upper erratic ranges, however, they give evidence of little else than unstable internal condition. See Appendix A.

Influence of Compression

Since internal ionization takes place in entrapped air layers, compression of the insulation should reduce the volume of these layers, or increase the density of the entrapped gas, and so reduce the loss due to ionization. At the same time pressure will reduce the total thickness of the insulation, and thus increase the voltage gradient and the losses in the solid parts. In the earlier paper it was shown that in far the greater number of cases the reduction of the ionization greatly offsets any increased dielectric losses, and that an increase of loss on compression appeared in only two bars out of 30 investigated, and in each case the increase was very small.

Under sustained high temperature the insulation on the flat sides of armature bars tends to expand forming a curved, instead of a flat surface. This obviously causes increase in the thickness or number of the enclosed air spaces, resulting in a decrease in loss in the low-voltage range, and a greatly increased loss if the voltage is above the ionizing value for the entrapped air or gas. This expansion also causes a loosening up of the structural rigidity of the insulation, probably causing the shifting of mica, and so further increasing the likelihood of failure. The expansion of the insulation in the present tests is undoubtedly greater than could occur in the slot of a generator armature, but even in the latter case the assembly clearance provides space for some expansion. It is, therefore, important to know whether a solid structure under compression, reducing the internal gaseous ionization, and maintaining fixed position of all interior layers, would result in longer life for the insulation.

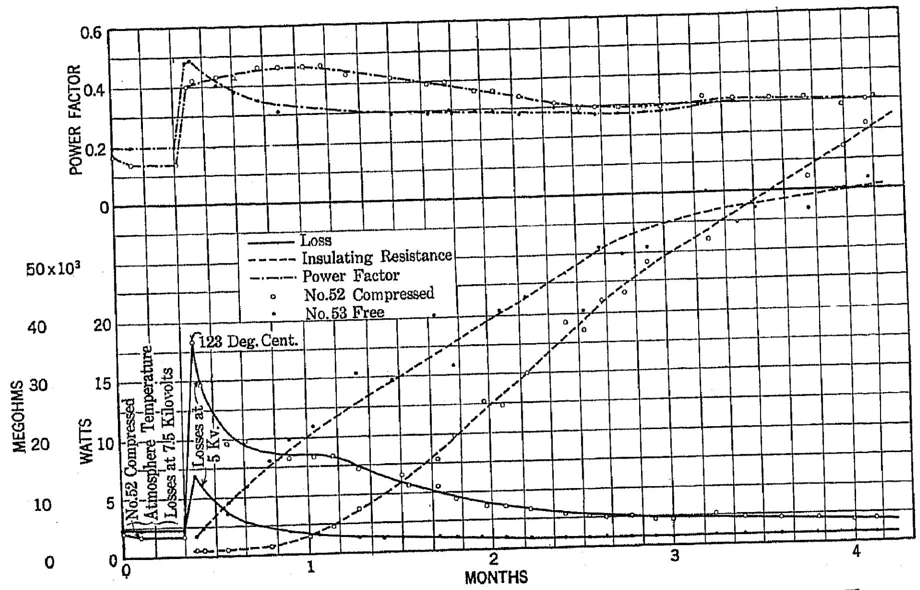

Fig. 3—Losses in Armature Insulation Influence of Compression and Temperature

For test of this question bars No. 52 and No. 53 both of the same structure (treated cloth and 42 per cent mica folium) were used. The insulation under the electrodes of No. 52 was tightly compressed with steel plates and screw clamps, that on No. 53 being left free. Each was dipped in insulating compound to prevent surface leakage and the two set up in one of the constant temperature ovens, at 125 deg. cent., replacing No. 55 and No. 60, withdrawn for reasons described above. The continuous record of this test is shown in Fig. 3. The loss measurements at atmospheric temperature were made at 7.5 kv. Those at higher temperatures were made at 5 kv. instead of 7.5 kv., since in the early stages the losses at 7.5 kv. were so high as to endanger the insulation.

Referring to Fig. 3, the initial readings were taken at atmospheric temperature and with no compression on either bar. The readings of the second day were

taken with No. 52 firmly compressed as described and No. 53 free, both at atmospheric temperature. An 8.5 per cent reduction of the losses in No. 52 is evident. This reduction is due to the decrease in the loss due to internal gaseous ionization. These readings were repeated a few days later and the bars were then dipped and placed in the oven and their temperature raised to 123 deg. cent. This rise in temperature is accompanied by the usual sharp increase in dielectric loss, and increase of absorption, *i. e.*, decrease in insulation resistance. It is especially interesting to note that the increase in the loss of bar No. 52 under compression is very much greater than that of the free bar No. 53, in spite of the initial lower values of the former. This immediately suggests that the compression of the insulation greatly retards the drying out process. This bears out the conclusion reached in the foregoing section of this report, that the greater part of the reduction in insulation resistance, or the increase in absorption, takes place during the first stage of temperature rise, and also takes place quite rapidly. Whatever the process of drying out may be, such as the driving out of water, it is apparent that it is seriously retarded by the compression of insulation. The insulation resistance curves also clearly indicate this condition. Immediately following the sharp decrease in resistance attendant upon high temperature, the resistance of No. 53 begins to rise rapidly while that of No. 52 remains practically stationary over several days and begins to rise quite slowly. The further history of the bars over the period of four months shows that No. 53 maintains its lower value of loss but that of No. 52 is gradually approaching it. It is to be borne in mind that the values of loss shown by these curves are taken at 5 kilovolts, at which voltage the losses of internal ionization are very low. It would appear that No. 52 should ultimately fall below No. 53 since it was lower at the start. The tendency of the insulation resistance curves indicates this more definitely than do the loss curves. It will be noted that in the final stages the resistance of No. 53 is increasing less and less rapidly, while the rate of increase of the resistance of No. 52 is rising more rapidly and, therefore, tending to higher values than that of No. 53. Apparently, however, considerable further time would be necessary to bring the two bars to their final constant values of loss. Unfortunately the work was of necessity interrupted before it was possible to apply continuously, the normal operating voltage on these bars, and to continue observations on them so as to make a relative study of their life histories. It would appear that a continuation of tests of this character should give important information as to the influence of ionization and compression on the life of this type of insulation. See Appendix B.

Conclusions

From the results of the life tests as shown in Fig. 1 we conclude as follows:

1. The conclusion in the earlier paper that internal ionization increases the dielectric loss and tends to shorten the life of built up insulation is supported by the results of the present tests. In each group in which comparison is possible the longest life is associated with low corona loss. In the two groups, highest and next to highest mica, the bars having high initial corona loss, at the end of the test, show evidence of gradual deterioration as compared with steady conditions in the bars initially showing low internal ionization.

2. Considering in general the influence of the amount of mica content: In the earlier test the best showing was made by a full mica (92 per cent) bar (No. 46). In the present test the best showing is made by a second group bar, No. 36 (70 per cent mica folium), and No. 39 (50 per cent mica folium) a third group bar is also still in good condition at the end. Moreover, No. 41 (28 per cent mica folium) made a run of six months with very low losses. These, however, are the only ones in which a relatively low content of mica folium gives a performance approximately that of full mica folium content. Note for example that No. 54 (42 per cent mica folium) and No. 55 (22 per cent mica folium, were not in condition after ten months drying out to stand the constant application of voltage. Throughout the course of both series of tests there have been a number of instances of high internal loss and heating and several cases of breakdown. These cases for the most part have been found in bars of the third group, or below, that is, those having less than 70 per cent mica folium content.

3. In general the indications are that the principal function of mica is in the reduction of the conductivity of the insulation, and in withstanding the action of internal ionization. While definite conclusions are scarcely permissible from the small number of bars tested, the indications are that the full mica folium content can be safely reduced only by insuring the best grade of mica folium, and best conditions of application, as regards uniformity and compact structure, these conditions being maintained to a high degree in continued factory production. Several bars having relatively low mica content have compared favorably with those having the maximum mica content. They are relatively few, however, in the total number studied. It appears that high mica content is necessary in order to maintain a high factor of safety to cover the variations in factory process. On the other hand, it has been shown that under best conditions considerably less than the full mica content may yield reliable insulation.

4. In the present tests the drying out period is much longer than that of the earlier tests, and the increase of loss with increase of temperature, much greater. The average mica content was higher in the earlier test, although in one case bars of the same group showed the difference noted. The greatest step in the

increase in loss and decrease in insulation resistance occurs in the change from 25 deg. to 50 deg. cent.

5. An uncompleted study of the relative performance of compressed and free insulation, indicated that the former passes through a more dangerous drying out period, but ultimately approaches equality with the free bar. From then on it should show superiority owing to the higher internal ionization loss in the free bar. This test should be continued.

Appendix A

Insulation Resistance

Irregularities in the one minute galvanometer readings for insulation resistance have been attributed, in the foregoing, principally to a continuous process of

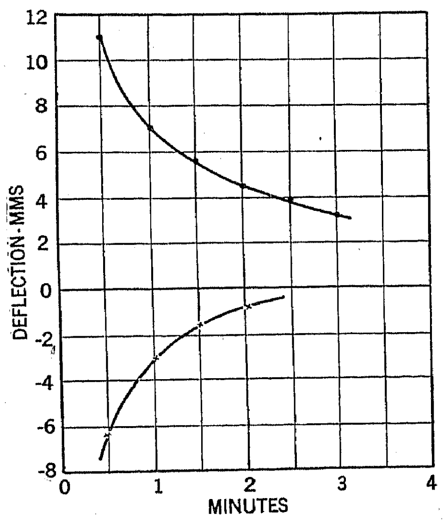

Fig. 4—Residual Charge After Alternating Voltage

alteration of the structure of the insulation under sustained high temperature and high voltage. The matter has been studied further, and apparently the principle influence in causing the variation in the galvanometer readings is the residual or absorbed charge in the insulation, resulting from the sustained alternating voltage which was interrupted only for the brief intervals required for loss and insulation resistance measurements. A series of continuous charging current observations was taken on bar No. 56 immediately following the interruption of the 10-kv. alternating voltage, 30 seconds being required for the change of connections. Right and left galvanometer readings one minute after the application of the continuous voltage, and with one minute short circuit in between, showed great irregularities. The values of the readings in either direction, and the relative values between right and left readings, varied widely. Thus in two successive observations the right and left readings were 6 and 8, and 12.5 and 6 mm. respectively, at 285 volts d-c.

A series of discharge curves was then taken. As promptly as possible after the removal of the 10-kv. alternating, bar No. 36 was connected for discharge through the circuit used for insulation measurements (see former report) the resulting deflections were right and left without regularity, and the curve showing the change of deflection with time has the characteristic form of the discharge of dielectric absorption. Fig. 4 shows two of these curves one of right and the other of left hand deflection. These curves, surviving after a 30-sec. interval necessary after interruption of the alternating voltage, but of irregular initial value and polarity, show clearly the residual absorption due to the foregoing alternating voltage. In the course of the life tests the insulation resistance measurements were taken sometimes immediately after interruption of the alternating voltage, and sometimes after the power measurement, and without regulation of the time interval following the removal of the alternating voltage. Hence it seems clear that the variation in the values of insulation resistance is in large measure due to this cause.

Appendix B

Influence of Compression

It is significant that towards the end of the run of Fig. 3, bar No. 52, under compression, is tending to a lower power factor, and a higher insulation resistance than No. 53, and that its loss is still falling while that of No. 53 is about constant. If continued with the application of sustained high voltage this test probably

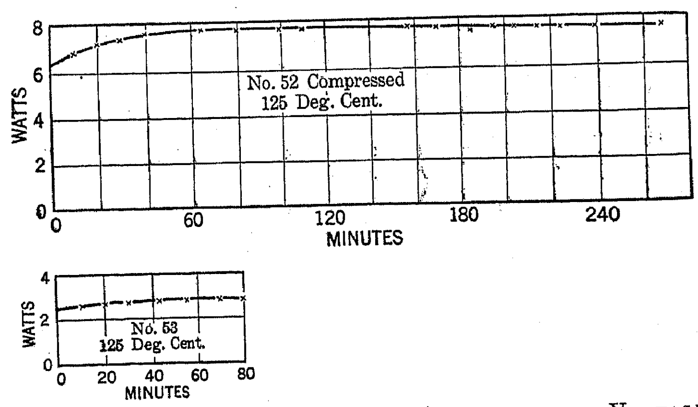

Fig. 5—Influence of Continued Application of Voltage on Dielectric Loss

would have shown next a decreasing resistance and an increasing loss for No. 53 due to its internal ionization. The original ionization loss in bar No. 53 was about 25 per cent, that or No. 52 at the beginning of the present test being considerably less, and reduced to a minimum by heavy compression.

Fig. 5 shows the results of a continuous run for a few hours at 10 kv. on both these bars to determine whether they were ready for the continued application of alternating voltage. It is seen that there is no increase in loss in either case after the first hour and thus that the internal losses are dissipated as fast as generated. The initial greater rise in loss in No. 52 is due to the elevation of internal temperature due to its greater losses. It is unfortunate that this run was of necessity interrupted at a time when the bars were just ready for studying the influence of voltage and internal ionization.

Discussion

F. M. Farmer: I just want to mention one thing you may be interested in. Dr. Whitehead has referred to the critical point in the curve between the dielectric losses and voltage. It is proposed to use this characteristic in a practical way in connection with paper-insulated cables. As you all know, a paper-insulated cable is an impregnated laminated structure, and we have always felt that an important element in the satisfactory operation of such a cable is thoroughness of impregnation. The question is, How are we going to tell when the insulation is thoroughly impregnated, that is, free from moisture and voids?

It has been proposed that we use this critical point. (Illustrating on the blackboard). A little easier and more practical method is to use power factor, instead of voltage. Normally the curve between power factor and voltage is horizontal with the abscissa, up to this point which Dr. Whitehead has called the critical point. At that point the line will depart. This point is generally referred to as the point where ionization begins.

Now, if ionization is a matter of variation in impregnation, we get some kind of a measure of impregnation by the slope of this curve, particularly the slope of this line beyond the critical point,—in other words, the change in power factor with a given change of voltage.

It is proposed that such a test could be applied to every reel of high-tension cable as a test of thoroughness of impregnation. It is a test that can be made fairly easily, and is a perfectly practical one, but the question still in the minds of a good many of us is just how important is it? We need quantitative information as to just how significant a given change in the slope of this line is as an indication of the quality of the insulation.

Dr. Whitehead's experiments seems to show ionization produces a destructive effect. But we need more experimental evidence before we can draw definite conclusions because there seem to be a good many cases where high voltage cables have been in service for many years at a voltage considerably above this critical point or ionization voltage without giving any trouble whatever.

Everett S. Lee: Professor Whitehead in giving his paper has attempted to make a statement regarding practically all of the variations which have come about, and in most cases I think we can say that we would agree with him. In other cases, other statements might be made that would equally fit the case, particularly in connection with the insulation resistance. Although I noted that in his remarks he cautioned us about making too much use of it, I just want to dwell on that point for one moment because it does seem that in the results which he obtained, the insulation-resistance curve, as shown in Fig. 1 did seem to show in an inverse way, about the same thing as the loss curve showed.

Now, if that is true, it is just one more phase in the insulation game which makes us pause and consider, because of the fact that the losses were obtained at 60 cycles, at a potential of about 5 kv., whereas, the insulation resistance was obtained with dire electromotive force of about 200 volts, the time of application being about one minute.

In other words, if we can connect up quantities obtained under such widely varying conditions, it is very interesting indeed, and the only way we can really determine it is to examine closely all of the available data to see if that is true. I know in a great deal of our experience we have come to the conclusion that we could not tie up these two factors, and although it has been done in Professor Whitehead's paper to some extent, I think the statement that he made in his paper bears the conclusion out that so far anyway we do not have a good direct relation between the two quantities of insulation resistance, taken as we take it, and dielectric energy loss obtained, as we take it.

I was impressed, upon reading the paper, that on making the measurements of loss, it was almost impossible to obtain them, because of the fact that the loss increased considerably even in a few moments, causing an increase of temperature which we know is cumulative, higher temperature resulting in higher loss, and even going so far as causing two of the samples to break down.

It is quite significant to note that one sample which broke down had practically the lowest loss of all, and there again we find a condition here which we meet in practise, That is, we have material which has a very low loss and yet we find its dielectric strength may also be low.

The general law, therefore, from our experience seems to be, that if we have relatively high loss we may expect low dielectric strength, whereas, if we have low loss we will not necessarily get high dielectric strength. There seems to be a point of low loss beyond which it is not necessary to go, to have high dielectric strength.

Further, in connection with the paper and also in connection with what Mr. Farmer has said, I would call to your attention the use of the property of power factor. If we express our results of tests upon insulation in terms of loss, in order that we may compare the results obtained by one observer with those obtained by others, we must know the exact physical dimensions of the samples, and it is very important because the loss measurement gives the total loss in the sample, and that will be determined by the size of the sample, the manner in which it is built up, the manner in which the voltage is applied, the kind of electrodes, etc.

Now, we have a property which, although varying to some extent, eliminates quite a few of those factors, and that, I believe, is the property of power factor, and if we could use power factor in our results, I believe it would come nearer to being a term which when one observer expressed his results as such and such a power factor, it could be used by other observers in comparing their results.

Now, if that is true (and I hope we may have other discussions on that), I believe it would be well worth while for us to consider that, because as I understand it, the insulation section of the engineering division of the National Research Council is interested in having available as far as possible the results of different observers, and if we can agree on some term in which we can express our results, so that they will be more nearly comparable, that of itself will be a step in advance.

W. A. Del Mar: There are two elements in research work which I think are not always recognized. The first element is the gathering of information, *i. e.*, the building up of knowledge in experimental work.

The other element is the inspirational, where the subconscious mind brings these data into juxtaposition and produces a new idea. Real progress in research cannot be made without these two elements.

Thus far, insulation research has been mostly of the former type, but the insulation problem is not like many of our problems, a matter of discovering some equations or series of equations which will give an answer to our questions. We are concerned with something that is very complicated, involving the effect of small quantities of air, small quantities of moisture, and the motions of exceedingly small particles of matter. We also have chemical problems. When there is ionization, nitrous oxide and ozone are formed, both of them exceedingly active chemically, and Professor Whitehead has shown us how those materials affect the insulation; but there is another effect that ionization produces. It changes mild surface leakage into streamer discharges thus setting up local surges of an exceedingly high frequency, and often of a violent character. It may be that most, or possibly all cases of laminated insulation failure are due to those local surges. It seems to me doubtful whether the actual voltage that you measure on the voltmeter is the real voltage that causes the disruption at a given point in the insulation. I think in every case there is a local gradient which is not measured on the instruments.

In the matter of cable performance, this matter of ionization is, as Mr. Farmer pointed out, a very live question. There are cables, to my knowledge, which are working with considerable ionization and working satisfactorily. There are others which are failing and apparently the failure is due to the presence of air. We don't understand the reason why in some cases the air seems to cause trouble and in other cases it does not. There is a big field for research work of a very interesting and important character. Mr. Whitehead's researches on ionization show a thorough realization of the vital point of attack in the insulation problem.

P. L. Alger: In looking over Mr. Whitehead's paper, I thought it would be interesting to bring out the relation between the state of the art as indicated by his results, and the parallel arts of cable insulation and condenser insulation. It is customary, I believe, in the manufacture of cables to operate at a voltage stress of perhaps fifty or sixty volts per mil and to have a power factor of five per cent or less. In condensers made of oil and paper it is usual to operate at several hundred volts per mil and to have power factors of one-half of one per cent, whereas in Mr. Whitehead's paper the experiments show a stress of perhaps forty volts per mil under normal conditions and a power factor of perhaps two-tenths. Of course, there are differences in the manufactures of the different types of apparatus which partly account for these differences in stress and power factor—for example in cables the insulation is protected from air by the outside sheath.

In armature coils the insulation is exposed to air, and therefore moisture and oxidation both give trouble. But I believe the major part of the differences is due to the fact that the binder used in the armature coil insulation is ordinarily some kind of shellac or other material, which is not physically stable at normal temperatures and which gives off volatile gases or boils at temperatures of fifty deg. cent. or more.

For this reason I feel that such a power factor-voltage curve as Mr. Farmer has drawn is not entirely reliable unless the temperature around the cable is lowered as the voltage is raised so as to preserve the same temperature inside the insulation throughout the tests. In this way only will a true curve of power factor against voltage that is independent of the temperature be obtained. I feel that the rise of power factor with temperature shown by Mr. Whitehead is due to the boiling away of some of the sticker or compounding material used, and consequently, that the best way to advance in this armature insulation art is to study the stability of the materials used under high temperatures. No electrical tests at all seem essential until a material is found which is satisfactorily stable and which will not give off gases until after temperatures of perhaps one hundred degrees are attained.

Alexander Nyman: I was particularly struck by the statement of Mr. Whitehead that on applying the pressure to insulation, the losses due to corona effect were reduced.

In building static condensers, particularly of the mica type, this effect is very pronounced. We find that in making condensers we have to apply the pressure whether mica or paper is used as insulator.

We have made samples of condensers where it was possible to observe the corona effect, visually, and we found in these condensers that even with the highest pressure applied, although the corona effect was reduced, it was not eliminated at high voltages, except by a special construction.

The only way to minimize the corona effect is to fill up all the smallest crevices in the insulation, with some other insulating compound, wax or varnish or anything that will eliminate air entirely. It is therefore necessary not only to apply the pressure, but also to evacuate the particular piece of apparatus and then impregnate it with the necessary material.

I would very much like to know what pressure Mr. Whitehead applied to the insulation in order to eliminate the losses altogether. The reason for the corona losses disappearing on this curve is, I believe, that the dielectric losses are large, and when you apply the pressure although the corona losses are not eliminated, they are so small that they become negligible compared to dielectric losses. In static condensers where the insulation is pure mica, the dielectric losses are small and the corona losses become important even at very high pressures.

In paper condensers, the effect is exactly the same as in mica condensers. You must not only apply the pressure but also fill in all the minutest spaces with some impregnating compound. There are different types of paper used in static condensers. For instance, I believe that some condensers have been manufactured with paper which has been impregnated before applying the condensers. I believe this is a mistake.

In general it will be found that with that kind of paper, although you can eliminate most of the air spaces, you can't eliminate all of them and there will still be losses. It is only a paper that can be impregnated throughout by some suitable compound that would give reduced losses.

I believe this will possibly shed some light on cable construction as well. I heartily agree with the statements of one of the speakers on the use of power factor in determining the quality of an insulation as far as dielectric losses are concerned.

Herman Halperin: Dr. Whitehead has stated that the $I^2 R$ loss in insulation is a small part of the dielectric loss and I understood Mr. Lee to say that the insulation resistance did not give any indication of the dielectric loss.

It has been found for the impregnated paper insulated cables bought for the Commonwealth Edison Company that as long as the manufacturer was using about the same kind of impregnating compound and paper, a curve of dielectric loss versus insulation resistance, similar to the one below, would be obtained.

I wish to agree with the other speakers in regard to the value of plotting the power factor of the charging current instead of dielectric loss, against voltage. As a matter of comparison between the various kinds of built-up insulation on cables, it might be well to use the maximum dielectric stress in the cable for the abscissa instead of the voltage, since it appears that the ionization in the cable starts at some definite dielectric stress for a given insulation.

I also want to emphasize the remarks made by one of the speakers as to the great value of any investigations along the lines of dertermining the effect of this ionization loss on the life of the insulation.

H. L. Curtis: One of the most interesting points in Dr. Whitehead's paper was his statement that he had come to the conclusion that there is no definite relationship between insulation resistance and dielectric loss. That is quite generally agreed to by most physicists. But dielectric loss is probably connected in some way with dielectric absorption as Professor Whitehead has said, though the exact way, the mathematical formula by which it is connected, we do not yet know.

It is a very difficult question as to how to measure insulation resistance. Much of the work that is reported as measurement of insulation resistance is merely a measurement of some type of absorption. The last speaker did not say how he measured his insulation resistance but I am of the opinion that he did not measure what we may call the true resistance, but in some way he was measuring an absorption phenomenon, and he is showing by that exactly what Dr. Whitehead brought out, namely, there is a connection between dielectric absorption and power factor.

J. C. Lincoln: The point brought out by Dr. Whitehead that mica was not injured by the effects of ionization was new to me. The question I want to ask is this: I imagine that the reason for the destruction of fibrous material is the production of ozone and nitrous oxides from the ionization of the gas.

I would like to know if that is the case, and also if these same materials, ozone and nitrous oxide, will affect such materails as rubber. I imagine they will, but are there other materials besides mica which are not affected by the production of ionization?

J. B. Whitehead: Several of the speakers beginning with Mr. Farmer have mentioned the power-factor curve as being perhaps the best method for determining the point at which losses begin. In the first section of my paper, presented at Swampscott, a number of power factor curves are given, and from the beginning we attempted to use the power factor curves as a means for detecting the point at which internal ionization begins.

In no single case did the power factor curve serve us in that connection. The power factor curves seem to run in a very erratic way, and show no marked breaks at the point at which the ionization sets in as determined by our own methods.

There was a definite break in the logarithmic curves and we have attributed this break to the beginning of ionization, particularly as pressure reduces the break in the curve.

It does not take a great deal of ionization to cause trouble. In the extreme cases we found the losses from ionization, those portions of the loss we could squeeze out of the sample, were relatively small. I have been wondering whether the ionization losses don't begin far down on the curve, but are still sufficiently small as not to reflect themselves very prominently. May we not have loss and still not yet reach a point where the cable is actually beginning to overheat, where the absorption itself and the major part of the dielectric loss which I take to be due to absorption, is not yet setting in in such a marked way that the cable itself is becoming unstable.

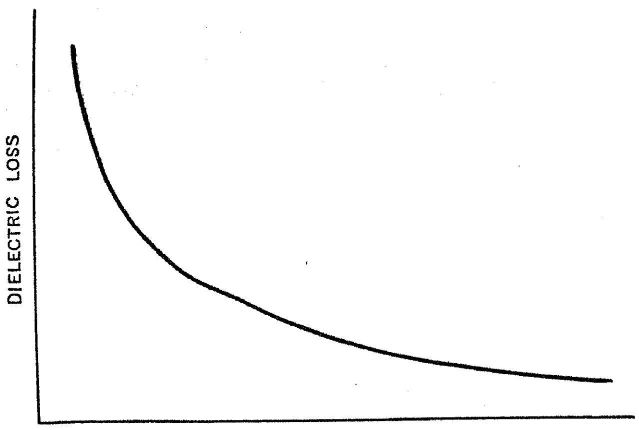

FIG. 1

If you carry any of this insulation sufficiently high in voltage you get to a point where the loss is cumulative, not due to ionization, but due to the increasing temperature caused by the dielectric loss directly due to dielectric absorption.

It seems to me there is one other reason why we should not rely too much on power factor as a quantity for defining the ionization point. Power factor, after all, involves two quantities, the dielectric costant, of course, and such losses as are in the dielectric. Why try to tie those two things together when they certainly may vary, each of them independently of the other.

As regards the destructive effects: I have another paper which was published I think in the December number of the JOURNAL which is a by-product of the work described in the present paper. The title of it is "The Influence of Gaseous Ionization on Fibrous Materials and Mica." The results are qualitative, but do show that gaseous ionization in very small amounts is quite sufficient to cause the most serious kind of deterioration of fibrous insulation of any kind within a few hours. Papers lose their calendered surfaces, lose their tensile strength. The whole mechanical structure of fibrous materials goes to pieces very soon, within the course of a few hours. We tried a number and none of them would stand up against ionization.

I think we also concluded very definitely that the most active agent was ozone. Nitrous oxides do not exist in air, in their own state, in any quantity, at least, at temperatures below several hundred degrees, so that it is fairly evident as shown in the paper referred to that the active agent is ozone. It is a slow oxidation, not rapid by any means. The results of this deterioration do not show up as ordinary combustion, as the blackening of the insulation. The deterioration seems to penetrate into the material in paths by some form of slow oxidation.

Mr. Lee pointed out in one case, at least, an armature bar having a very low dielectric loss was one of those to break down. That is perfectly true, but in the paper it is pointed out that the problem in applying mica in insulation of this character is largely one of uniformity and of being able to obtain a continuous product of high dielectric strength and good insulating properties uniformly through a course of continued factory production. So occasionally we unquestionably do find a bar well wrapped, in which the insulation is tight, and the internal ionization is low and which nevertheless fails. However, I think we reviewed a quite sufficiently large number of these bars to show that the one case of that kind that we found was simply fortuituous.

Mr. Del Mar has pointed out the importance of approaching this whole subject of insulation from a higher standpoint than we have done in the past and I simply want to call attention to the fact that the Engineering Committee of the National Research Council which is made up of our own research members, is attempting to take this higher viewpoint of the large mass of data that has been accumulated in this field of insulation. We are looking for assistance in carrying on that work, and if there are any here who themselves are interested, or know of others who are interested in the field of insulation, and are willing to take part in our present efforts of coordinating the results of the past, I would greatly appreciate it if they would communicate with me. We can give somebody a good, interesting and important job.

Mr. Alger, I think it was, pointed out his belief that the increase in the power factor is due to the boiling away of the material, the material of the binder. I don't know that I can make any comment on this except to point out that we have traced in this paper a very direct relation between the ionization loss itself and the life of the bars. We find that a bar which has low ionization loss, when it is tightly wrapped, and in which it is impossible to squeeze out any ionization loss is the bar which goes on and shows the longest life and vice versa. Unquestionably the bars with the greatest amount of ionization loss in them immediately show a tendency to run down or break down.

Mr. Dawes' comments were really made principally on the other paper I have referred to, which was published in the JOURNAL and which we are not discussing here.

As regards the pressure: One of the speakers, Mr. Nyman, has asked what the pressure was that was used here, and raised the question as to whether all of the ionization loss was squeezed out. I quite agree with him that it is not all squeezed out, but we were able to squeeze it down to a point where the logarithmic loss is very straight. It did not seem to be worth while to attempt to measure the actual pressures that were used in this case. It was quite evident that this ionization loss must be very variable; it must be present in varying amounts, even in different parts of the same sample, and so the difficulty that would be encountered in measuring pressures, while one is also applying high voltage, led us to simply try to obtain approximately the same pressure for all these samples. That we did by constructing a special form of clamp in which the screws of this clamp were applied uniformly all over the surface and tightened up approximately to the same amount.

Mr. Halperin raises the question as to the $I^2 R$ square loss. $I^2 R$ loss as computed from the insulation resistance of the sample is unquestionably a very small part of the total loss. In that connection we can only consider the resistance as determined from the final value of the leakage current through the sample. If you apply continuous voltage, leave it long enough for the current to maintain a steady value, use that current

as the basis of calculation of resistance, and then compute loss on the basis of that resistance, it will be found to be an extremely small part of the total observed loss.

As regards my comments in connection with insulation resistance, I mean to point out only that it seemed to me that it would be better if we used dielectric absorption itself as a means and a method of describing the material, rather than to speak of the insulation resistance. There is undoubtedly a very definite relation all the way through an experiment of this kind, between the value of the direct current which will be observed after one minute, and the alternating current dielectric losses which will be observed. In other words, as the resistance curves go up, indicating increased resistance, the loss curves come down, but the same thing would be found for a continuous curve measured at any other interval, whether one or two minutes, or one or two hours, and the connection is between absorption and loss and not between resistance and loss. I think it must be conceded unless there is some feature that hasn't come to my attention, that it is perfectly obvious that the great portion of loss of dielectric is due to the dielectric absorption and to nothing else.

Overdamped Condenser Oscillations

BY CHARLES P. STEINMETZ*
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Review of the Subject.—*In the classical equations of the discharge of a condenser of capacity C into an external circuit of resistance r and inductance L, it is found that the discharge is oscillatory if* $r < \sqrt{\frac{4L}{C}}$, *and is impulsive if* $r > \sqrt{\frac{4L}{C}}$.

As the perfect condenser can never be realized in practise, it is the purpose of this paper to show the effect of the condenser leakage on the discharge wave.

The imperfect condenser is represented by the perfect condenser C shunted by the conductance g. This condenser discharges into the circuit L and r as above. The mathematical discussion shows that no matter what the relation between r and $\sqrt{\frac{4L}{C}}$, *there may always exist some value of g for which the discharge is oscillatory. This is the case when*

$$(r/L - g/C) < \sqrt{\frac{4}{LC}}$$

$$i, e., r < \sqrt{\frac{4L}{C}} + \frac{gL}{C}$$

It is to be borne in mind that the above holds true only when r, L, C, and g are constant. If the resistance is that of a third class conductor, the discharge will always be oscillatory.

An apparent paradox is found in the statement that the current may be more than 90 deg. out of phase with the voltage. A study of the derived equations, remembering that the current under consideration is only one of the two components forming the total discharge current $\left(C\frac{de}{dt}\right)$ *of the condenser (the leakage current ge being the other), will show that this is true. A study of the oscillograms will further confirm the theory.*

I. General and Experimental

THE classical equations of the condenser discharge through an inductive circuit show that the discharge is oscillatory, if the resistance of the discharge circuit is less than a certain critical value $2\sqrt{L/C}$; and in this case the damping of the wave is goemetric, that is, the quotient of two successive half waves is constant. The discharge is impulsive, if the resistance of the discharge circuit is greater than the critical value.

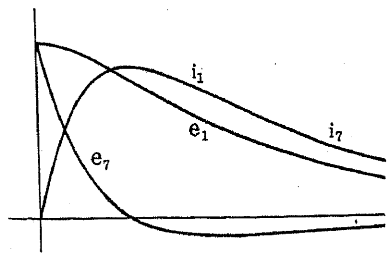

Fig. 1

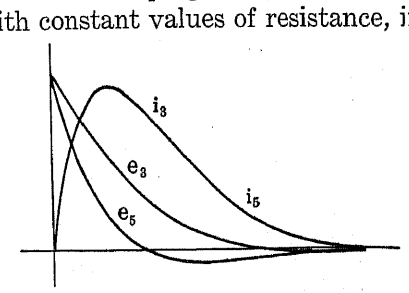

Fig. 2

This, however, applies only if the circuit factors: resistance, inductance and capacity, are actually constant. It does not apply if the resistance is that of a third class conductor, that is, if the potential drop across the resistance decreases with increase of current. In this case the discharge is always oscillatory, but the damping is either arithmetic, that is the difference of two successive half waves is constant, or it is a combination of arithmetic damping and geometric cumulation.[1]

Even with constant values of resistance, inductance, and capacity, the classical condenser discharge equations hold only for the *ideal condenser*, that is, a condenser in which no energy losses occur. *If energy losses occur in the condenser*—as is usually the case—*the discharge remains oscillatory even for values of the discharge resistance greater than the critical value* $2\sqrt{L/C}$, and the discharge oscillates the more, the higher the

*Deceased.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

1. See A. I. E. E.

energy losses in the condenser. No matter how high the resistance of the discharge circuit, there always exists a value of energy loss in the condenser (which depends on the value of the discharge resistance) at which no impulsive discharge is possible, but the discharge is always oscillatory. This occurs when the

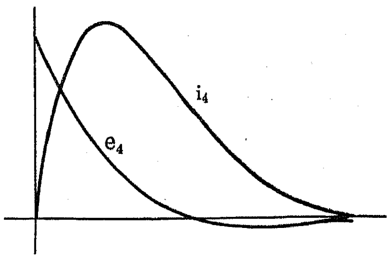

Fig. 3

damping of the wave due to energy losses in the condenser, equals that due to the discharge resistance. The damping is then geometric.

The frequency of the oscillation, and the limit up

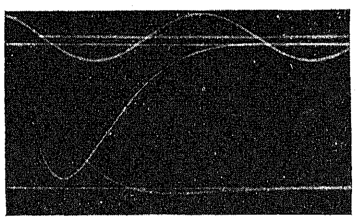

Fig. 4

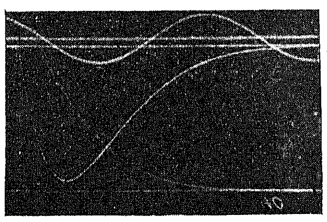

Fig. 5

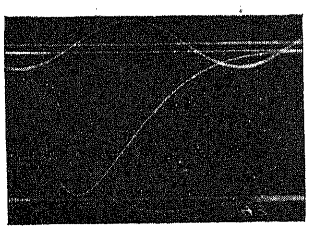

Fig. 6

to which the discharge is oscillatory, depends on the *difference* between the power dissipation in the discharge resistance, and that in the condenser losses, while the attenuation or damping of the discharge wave depends on the *sum* of the losses. As a result of the combined losses, such discharge waves may have rates of attenuation far greater than possible in the classical equations of the ideal condenser. They may therefore be called *Overdamped Oscillations*.

In such overdamped discharge oscillations of the imperfect condenser, only a fraction of the first half wave may be of appreciable magnitude. That is, in

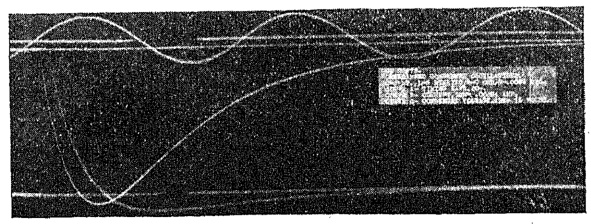

Fig. 7

the current wave, the maximum (which with the alternating wave occurs at 90 deg., and with a wave of low damping near 90 deg.) may occur at 20 deg. to 30 deg. and still earlier; the wave from then is very much steeper than the wave tail, similar to the case in the non-oscillatory impulse; the wave shape however,

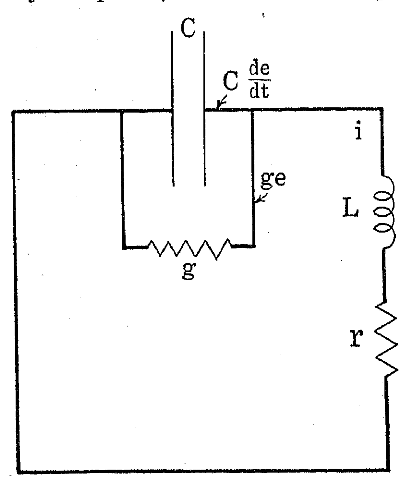

Fig. 8

differs from the latter, and the current and voltage wave pass through zero at a definite point, while in the non-oscillatory impulse they never reach zero, but gradually fade out.

As illustrations are shown in Figs. 1 to 3 the calculated current and voltage waves and in Figs. 4 to 7

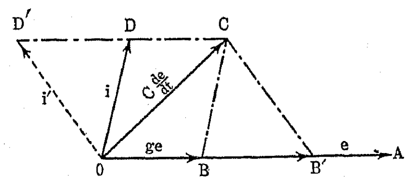

Fig. 9

the observed oscillograms of a series of such overdamped condenser discharge waves, for a condenser of $C = 5\ \mu$f. capacity discharging through an inductance of 5 henrys. This gives the critical resistance $r_0 = 2\sqrt{L/C} = 2000$ ohms.

As (1) is shown the discharge, given by the classical equations for zero condenser loss and a discharge resistance of $r = 2400$ ohms, that is, an impulsive or unidirectional discharge.

In (2) the discharge resistance has been reduced to 2100 ohms (which with a loss free condenser would still give an impulsive discharge) but such a power dissipation in the condenser has been assumed, as to give the same damping constant as (1). The discharge then has become oscillatory.

(3) to (7) give the discharges for successively lower discharge resistances. $r = 1800$, 1200, 600, 300 and 0 ohms, but with successively increased power dissipation in the condenser, so as to give the same total resultant dissipation constant. As seen, in (7), for zero discharge resistance, the discharge has again become impulsive, due to the strong damping by the condenser losses, although the shape of the voltage impulse is very different from that in (1). The frequency of the discharge increases with increasing condenser losses, up to (4), the case at which the condenser losses equal the losses in the external circuit and the condenser losses neutralize the effect, on the frequency, of the discharge resistance, and the frequency is the same as that of an undamped wave.

(1) to (7) thus give forms of waves, of the same condenser discharging through the same inductance with the same damping, that is, the same attenuation constant, but the losses, that is, the power dissipation which causes the damping, shifting from all in the external circuit and none in the condenser, in (1) to all in the condenser and none in the external circuit, in (7). As seen, the current waves in (7), (6) and (5) are the same as in (1), (2) and (3); but the voltage waves are materially different.

As a further illustration, the current and voltage discharge waves of the same condenser are calculated, $C = 5\ \mu$f. over the same inductance $L = 5$ henrys, and a constant discharge resistance $r = 2400$ ohms, and with amounts of power dissipation in the condenser, varying from zero in (1), up to a value in (5), which doubles the dissipation constant of the discharge.

II. Mathematical

Let:

C = Capacity of the condenser

L = Inductance of the discharge circuit

r = Resistance of the discharge circuit (including the effective resistance representing the energy losses in the inductance).

If energy losses occur in the condenser, an energy component occurs in the condenser current, which can be represented by an effective shunted resistance or shunted conductance. Thus let:

g = Effective shunted conductance representing energy losses in the condenser.

We thus have, in the general case:

C = Coefficient of energy storage by the voltage, in the capacity of the condenser

L = Coefficient of energy storage by the current, in the inductance of the external circuit

g = Coefficient of power dissipation by the voltage in the effective shunted conductance of the condenser

r = Coefficient of power dissipation by the current, in the (effective) resistance of the external circuit

Let:

e = Voltage at condenser terminals, with $e = e_0$ a initial value at time $t = 0$

i = Current in the external circuit, with $i = 0$ a initial value.

It is then:

In the external circuit:

$$e = L \frac{d\,i}{d\,t} + r\,i \qquad (1$$

In the condenser:

$$-\,i = C \frac{d\,e}{d\,t} + g\,e \qquad (2$$

Substituting (1) and its differential into (2) give the differential equation of current:

$$\frac{d^2 i}{d\,t^2} + 2\,u \frac{d\,i}{d\,t} + \frac{1 + r\,g}{L\,C}\,i = 0 \qquad ($$

where:

$$u = 1/2\,(r/L + g/C) \qquad ($$

is the attenuation constant, and

$$m = 1/2\,(r/L - g/C) \qquad ($$

is the distortion constant.[2]

Equation (3) is integrated in the usual manner terms of the form:

$$i = A\,\epsilon^{-ct}$$

which, in the case that c becomes imaginary, combine a term:

$$i = B\,\epsilon^{-ct} \sin q\,t$$

and by the substitution of the terminal conditions, giv

A. Impulsive Discharge

$$m^2 > \frac{1}{L\,C}$$

$$i = \frac{e_0}{2\,s\,L}\left(\epsilon^{-c_1 t} - \epsilon^{-c_2 t}\right)$$

$$e = e_0\left(\frac{m+s}{2\,s}\,\epsilon^{-c_1 t} - \frac{m-s}{2\,s}\,\epsilon^{-c_2 t}\right)$$

where:

$$\left.\begin{aligned} c_1 &= u - s \\ c_2 &= u + s \end{aligned}\right\}$$

2. See A. I. E. E., "The General Equation of the Ele Circuit," 1907. Also: "Theory and Calculation of Tran Phenomena, Section IV," page 462, 509.

$$s = \sqrt{m^2 = \frac{1}{L\,C}} \qquad (10)$$

The current is a maximum at the time:

$$t = \frac{\log c_2 - \log c_1}{2\,s \log \epsilon} \qquad (11)$$

B. CRITICAL DISCHARGE

$$m^2 = \frac{1}{L\,C} \qquad (12)$$

$$i = e_0/L\,t\,\epsilon^{-ut} \qquad (13)$$

$$e = e_0\,\epsilon^{-ut}\,(1 + m\,t) \qquad (14)$$

The current is a maximum at the time:

$$t = \frac{1}{u} \qquad (15)$$

C. OSCILLATORY DISCHARGE

$$m^2 < \frac{1}{L\,C} \qquad (16)$$

$$i = \frac{e_0}{q\,L}\,\epsilon^{-ut} \sin q\,t \qquad (17)$$

$$e = e_0\,\epsilon^{-ut}\,(\cos q\,t + m/q \sin q\,t) \qquad (18)$$

where:

$$q = \sqrt{\frac{1}{L\,C} - m^2} \qquad (19)$$

The current is a maximum at the time:

$$\tan q\,t = q/u \qquad (20)$$

As seen, with increasing energy loss in the condenser, m decreases to zero and then becomes negative, and the phase relation between voltage and current thus changes from less than quadrature to quadrature to more than quadrature.

Although at first glance this statement may seem impossible, and may be impossible for steady conditions, we believe that this, like many other seeming paradoxes in the study of transients, is perfectly true.

Referring to the diagrams, in Fig. 8 the capacity c, shunted by the conductance g, represents the imperfect condenser.

$c \dfrac{d\,e}{d\,t}$ = current supplied by condenser

$g\,e$ = current through g (through imperfect condenser)

i = current through L and r (external circuit)

e = condenser voltage

(1) The current $g\,e$, being through the pure resistance g, must be in phase with the voltage e.

(2) The vector sum of the currents through the external circuit (L and r) and through g must be equal to the current supplied by the condenser.

In Fig. 9

$A\,O$ represents the condenser voltage e

$B\,O$ represents the current $g\,e$ in phase with e

$C\,O$ represents the condenser current $C \dfrac{d\,e}{d\,t}$, out of phase with e by the angle $C\,O\,A$

$D\,O$ represents the external current i, out of phase with e by the angle $D\,O\,A$

Now, change the circuit constants (g, L, and r) so that $C \dfrac{d\,e}{d\,t}$ has the same relation to e as before, but $g\,e$ has taken the position $O\,B'$. Then, as the vectorial sum of $i/g\,e$ must be equal to $C \dfrac{d\,e}{d\,t}$, then i must take the position $O\,D'$ which is displaced from c by the angle $D'\,O\,A$ obviously greater than 90 deg.

A study of oscillograms for cases 5, 6 and 7 of the first instance given in the paper will corroborate the above analysis.

D. NOTES

The energy dissipation in the system is given by the same equations in all three cases, as:

$$r\,(I^2) + g\,(E^2) = r \int_0^\infty i^2\,d\,t + g \int_0^\infty e^2\,d\,t$$

$$= \frac{r\,e_0^2\,C}{4\,L\,u\,(l + r\,g)} + \frac{g\,e_0^2\,(l + u\,r\,C)}{4\,u\,(l + r\,g)} \qquad (21)$$

The current slope is a maximum for:

$$\left|\frac{d\,i}{d\,t}\right|_{t=0} = e_0/L$$

The voltage slope is a maximum for:

$$\frac{d^2\,e}{d\,t^2} = 0$$

at:

$$\tan q\,t = \frac{q^2 - u^2}{2\,q\,u}$$

$$\frac{d\,e}{d\,t} = -\,e_0\,\epsilon^{-ut}\,(u \cos q\,t + q \sin q\,t)$$

In the classical equations of the condenser discharge, the term $\dfrac{r}{2\,L}$ takes the place of both of the terms, u and m, of the general equations.

III. Instances

As an instance, let: e_0 = 10,000 volts.

C = 5 μf. = 5×10^{-6} = capacity of condenser

L = 5 henrys = inductance of discharge circuit, thus:

$z_0 = \sqrt{L/C}$ = 1000 ohms = surge impedance, and

$r = 2\,z_0 = 2000$ ohms = critical resistance, at which the discharge of the ideal condenser changes from oscillatory to impulsive.

$$\frac{1}{L\,C} = 200 = \text{frequency constant.}$$

Choosing then the discharge resistance r, and the shunted conductance of energy loss in the condenser g, so as to give a constant attenuation, that is, at constant total attenuation u, the energy losses are divided between the external circuit and the condenser, in different proportions.

$$u = 1/2\,(r/L + g/C) = 240$$

Seven cases are given in Figs. 1 to 3 and oscillograms Figs. 4 to 7, for the constants:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
No. Figs.	1	..	2	3	2	..	1
	..	..	4	5	6	..	7
$r =$	2400	2100	1800	1200	600	300	0 ohms
$g =$	0	0.3	0.6	1.2	1.8	2.1	2.4×10^{-3} mhos
thus $m =$	240	180	120	0	−120	−180	−240
$q = \sqrt{\frac{1}{L\,C} - m^2} =$	..	87.2	160	200	160	87.2	..
$s = \sqrt{m^2 - \frac{1}{L\,C}} =$	132.6	..	..	..	..	..	132.6

Current	Terminal Voltage at Condenser
(1) $i = 7.55\,(\epsilon^{-107.4t} - \epsilon^{-372.6t})$	$e = 10000\,(1.41\,\epsilon^{-107.4t} - .41\,\epsilon^{-372.6t})$
(2) $i = 23\,\epsilon^{-240t}\sin 87.2t$	$e = 10000\,\epsilon^{-240t}(\cos 87.2t + 2.07 \sin 87.2t)$
(3) $i = 12.5\,\epsilon^{-240t}\sin 160t$	$e = 10000\,\epsilon^{-240t}(\cos 160t + .75 \sin 160t)$
(4) $i = 10\,\epsilon^{-240t}\sin 200t$	$e = 10000\,\epsilon^{-240t}\cos 200t$
(5) $i = 12.5\,\epsilon^{-240t}\sin 160t$	$e = 10000\,\epsilon^{-240t}(\cos 160t - .75 \sin 160t)$
(6) $i = 23\,\epsilon^{-240t}\sin 87.2t$	$e = 10000\,\epsilon^{-240t}(\cos 87.2t - 2.07 \sin 87.2t)$
(7) $i = 7.55\,(\epsilon^{-107.4t} - \epsilon^{-372.6t})$	$e = 10000\,(-.41\,\epsilon^{-107.4t} + 1.41\,\epsilon^{-372.6t})$

In the oscillograms, as nearly as possible the same constants were chosen, and as seen, the oscillograms are identical in wave shape with the calculations, (except for a slight inductive effect at the beginning of the voltage wave). For convenience, the current is shown in reverse direction, and as time measure, a 60-cycle wave is given.

As a second instance, consider:

The same e_0, C, L, z_0, etc. but a constant value of the discharge resistance:

$$r = 2400 \text{ ohms}$$

and various values of shunted conductance g, from zero to a value equal in energy dissipation to r. That is:

	No.	(1)	(2)	(3)	(4)	(5)
	$g =$	0	.3	.6	1.2	2.4×10^{-3} mhos
thus	$u =$	240	270	300	360	480
	$m =$	240	210	180	120	0
	$q =$	..	..	87.2	160	200
	$s =$	132.6	64.2	..	..	..

Current	Voltage at Condenser Terminals
(1) $i = 7.55\,(\epsilon^{-107.4t} - \epsilon^{-372.6t})$	$e = 10000\,(1.41\,\epsilon^{-107.4t}\epsilon .41\,\epsilon^{-372.6t})$
(2) $i = 15.6\,(\epsilon^{-205.8t} - \epsilon^{-334.2t})$	$e = 10000\,(2.14\,\epsilon^{-205.8t} - 1.14\,\epsilon^{-334.2t})$
(3) $i = 23\,\epsilon^{-300t}\sin 87.2t$	$e = 10000\,\epsilon^{-300t}(\cos 87.2t + 2.07 \sin 87.2t)$
(4) $i = 12.5\,\epsilon^{-360t}\sin 160t$	$e = 10000\,\epsilon^{-360t}(\cos 160t + .75 \sin 160t)$
(5) $i = 10\,\epsilon^{-480t}\sin 200$	$e = 10000\,\epsilon^{-480t}\cos 200t$

Discussion

Prof. V. Karapetoff: I should like to draw your attention to a simple mechanical model which is represented mathematically by the same differential equations as the electric circuit discussed in the paper. Think of a horizontal helical spring attached at one end to an immovable wall and at the other end to a mass supported on frictionless rollers. Let there also be a tank with viscous fluid, and let a vane attached to the mass be immersed in it. Without this viscous fluid we would have a simple oscillating system; by starting it from a position which is not one of equilibrium, and letting it go, we would produce oscillations which theoretically would go on forever. By adding a vane and a viscous liquid we get damped oscillations; mathematical equations for this system are exactly the same as the equations for a circuit containing R, L and C, but no leakage.

Now, to imitate the circuit which Dr. Steinmetz considers, we shall remove the solid wall and replace it by a yielding reaction which we can imagine to be a cylinder, with a piston, a piston rod to which the above spring is attached, and a by-pass between the two sides of the piston. Let the cylinder be filled with a viscous fluid. The liquid circulating through the by-pass will constitute a yielding support for the spring.

If the support is immovable, there is only one differential equation, but by adding this yielding support, you add a second equation, namely, that the velocity of motion of the mass consists of the velocity due to the compression of the spring plus that due to the conductance of the by-pass. This gives Steinmetz' second equation. This model enables us to study and to visualize, so to say, this electric circuit. If you cannot see from the equations why the circuit becomes more oscillatory by the addition of that by-pass, your engineering intuition might tell you that this is so from an inspection of the model.

If for purely qualitative analysis, this model is too complicated, you can use a simple pendulum. Suspend a simple pendulum in a viscous fluid and you get the first equation in the paper. Add a yielding support, and kind of imperfect support, and you get the second equation of the paper. To record the phenomenon under discussion put a soft brush on the pendulum, dip it in ink, and let the brush make a record on a piece of paper moving at a uniform speed past the pendulum.

Free Convection of Heat in Gases and Liquids-II

BY CHESTER W. RICE

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Review of the Subject.—*The present paper is an extension of my earlier work which was found necessary in order to account for the observed convection for both large and small temperature differences.*

General expressions are developed, by the method of dimensions, which enable us to calculate the convection from any system of similar bodies when the three undetermined constants K, m, and n have been experimentally determined.

The available experimental data for horizontal and vertical cylinders, spheres and plane surfaces have been analyzed and the results expressed in the form of simple equations.

* * * * *

Introduction

In the discussion of my recent paper on convection,[1] Montsinger called my attention to the fact that the free convection from large vertical plane surfaces varies as the 5/4 power of the temperature difference, whereas my equations made the watts convection vary approximately as the first power of the temperature difference for small temperature rises.

A careful study was therefore made of the available data on free convection from plane surfaces. The previous theory assumed that the film thickness was independent of the temperature difference and merely depended upon the ambient properties of the fluid. The watts convection should therefore be directly proportional to $\Delta \varphi$. In Fig. 3 the available data on free convection from vertical plane surfaces have been plotted against $\Delta \varphi$ and also against Δt on log: log paper. A 45 deg. line has been drawn through the w_c vs. $\Delta \varphi$ points as required by the previous theory. Above 100 deg. cent. temperature difference the agreement is satisfactory but for the lower temperature differences the points depart systematically from the 45 deg. line by more than the probable errors. On the other hand it will be seen that the data are well represented by the 5/4 power of the temperature difference.

We are therefore forced to conclude that at least for large surfaces and small temperature rises the film thickness is a function of the temperature difference. Langmuir[2] has pointed out that for extremely small temperature differences the film thickness should increase without limit. It may be of interest to know that when the previous work was being written a more general dimensional expression for the film thickness which included the additional variables of temperature difference and temperature coefficient of density change was investigated and abandoned because the simpler expression appeared to satisfy the data then under investigation.

In view of the lack of agreement between theory and experiment for low temperature differences it seemed desirable to re-examine the high and low temperature data on the basis of the most general expression for the film thickness so as to obtain if possible a single expression which would be universally applicable for both large and small temperature differences. In the following paper it will be shown that the desired universal expression exists.

General Expression for Effective Film Thickness in Free Convection

Assume that the film thickness will depend on the temperature difference, the coefficient of density change per degree temperature change, the acceleration of gravity and the density and viscosity. Thus the driving force of the convection currents will be proportional to the difference in density produced by the expansion times the acceleration of gravity and it is the viscosity which opposes this driving force. The motion will obviously depend upon the shape of the hot body. The film thickness may also depend upon the rate at which heat is transmitted through the film and the specific heat. The addition of these variables also allows us to include the effects of heat carried by the slow mass motion within the film. We will therefore assume that the ratio of film thickness to the size of the body is given by the following expression.

$$B/D = K\,(\alpha\, g)^x\, \Delta\, t^y\, D^z\, \mu^p\, \rho^n\, k^m\, c_p^{\,r} \tag{1}$$

where B = Film thickness (L)

D = A linear dimension (L)

K = A numerical constant

α = Coefficient of density change per deg. cent. (θ^{-1})

g = Acceleration of gravity $(L\,T^{-2})$

Δt = Temperature difference deg. cent. (θ)

μ = Viscosity of fluid $(L^{-1}\,M\,T^{-1})$

ρ = Density of fluid $(M\,L^{-3})$

k = Heat conductionly of fluid $(H\,L^{-1}\,T^{-1}\,\theta^{-1})$

c_p = Specific heat per gram at constant pressure $(H\,M^{-1}\,\theta^{-1})$

Substituting the dimensions of the various factors in equation (1) we obtain

$$O = (L\,T^{-2}\,\theta^{-1})^x\,(\theta)^y\,(L)^z\,(L^{-1}\,M\,T^{-1})^p\,(M\,L^{-3})^n\,(H\,L^{-1}\,T^{-1}\,\theta^{-1})^m\,(H\,M^{-1}\,\theta^{-1})^r \tag{2}$$

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1. Paper presented before A. I. E. E. at Swampscott, June 27, 1923.

2. Irving Langmuir, *Trans.* Amer. Electro-Chem. Soc., Vol. XXIII, page 318, April 1913.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924. An abstract of this paper was presented at the Cincinnati meeting of the Am. Phy. Soc. Dec. 1923. See also discussion Jour. A. I. E. E., Dec. 1923. A companion paper on Forced Convection of Heat in Gases and Liquids-II will be presented at the April meeting of the Am. Chemical Society.

From which we obtain the following system of simultaneous equations:

$$\left.\begin{array}{ll} \text{By Length} & O = x + z - p - 3\,n - m \\ \text{Mass} & O = p + n - r \\ \text{Time} & O = -\,2\,x - p - m \\ \text{Heat} & O = m + r \\ \text{Temperature} & O = -\,x + y - m - r \end{array}\right\} \qquad (3)$$

The solution of the simultaneous equations (3) gives the following relations:

$$\left.\begin{array}{l} x = n/2 \\ y = n/2 \\ z = \dfrac{3\,n}{2} \\ p = -\,n - m \\ r = -\,m \end{array}\right\} \qquad (4)$$

From physical considerations we know that the film thickness should increase with the viscosity of the fluid. We therefore change the sign of n and substitute the relations (4) in equation (1) and obtain

$$B/D = K\left[\frac{\nu}{(\alpha\, g\, \Delta\, t)^{1/2}\, D^{3/2}}\right]^{n}\left(\frac{k}{\mu\, c_p}\right)^{m} \qquad (5)$$

where $\nu = \mu/\rho$ = Kinematic viscosity of fluid.

Free Convection from Horizontal Cylinders

In order to correlate the data in different fluids and at different temperature differences, it was necessary to determine the temperature at which ν, α, k, μ, c_p, should be taken. A study of the data in gases and liquids showed the best agreement by taking the value of the variables at the average of the surface and ambient temperatures.

In Table I the available data have been recalculated from Tables VII and X of the free convection section of the previous paper.[1] For perfect gases the factor $k/\mu\, c_p$ in equation (5), is practically independent of temperature and pressure and does not vary greatly from one gas to another.[3] For gases this factor was therefore omitted and the calculated values of B/D were plotted against $\nu_{avg.}/[(\alpha\, g\, \Delta\, t)^{1/2}\, D^{3/2}]$ on log:log paper. The points all fell on a smooth curve as demanded by the theory. An exponent $n = \frac{1}{2}$ was selected as a close approximation and the corresponding values of K_1 have been calculated in Table I, column XIII. Inspection of column XIII under liquids, shows a different value of K_1, from that obtained for the gases and also a variation of the constant among the different liquids. It is therefore apparent that the factor $k/\mu\, c_p$ in equation (5) cannot be neglected. By trial it is found that an exponent of $m = \frac{1}{4}$ brings the data for gases and liquids into agreement. In Fig. 1 we have plotted $(B/D)\,(\mu\, c_p/k)^{1/4}$ vs. $\nu_{avg.}/[(\alpha\, g\, \Delta\, t)^{1/2}\, D^{3/2}]$ on log:log paper. It will be seen that all of the points fall on a single curve within the probable experimental errors. There seems to be a tendency for the slope of the curve to gradually increase for large values of $(B/D)\,(\mu\, c_p/k)^{1/4}$. The single straight line which best represents the points, has a slope of $n = 0.54$. The value of K found by averaging all of the points, assuming $n = 0.54$, is $K = 2.60$. The line of slope $n = \frac{1}{2}$ has been drawn in such a manner as to give most weight to the last 17 points, since the accuracy of determining $(B/D)\,(\mu\, c_p/k)^{1/4}$ is greater for thin films, assuming equal accuracy in determining the watts convection. Thus for $n = \frac{1}{2}$ we take $K = 2.12$.

The values of $k/\mu\, c_p$ for gases in column XIV were obtained by dividing the value for $k/\mu\, c_v$ given by Jeans[3] by the values of γ as function of pressure from Landolt Bornstein Tabellen. The values assumed for the liquids will be found in Table II of the next section.

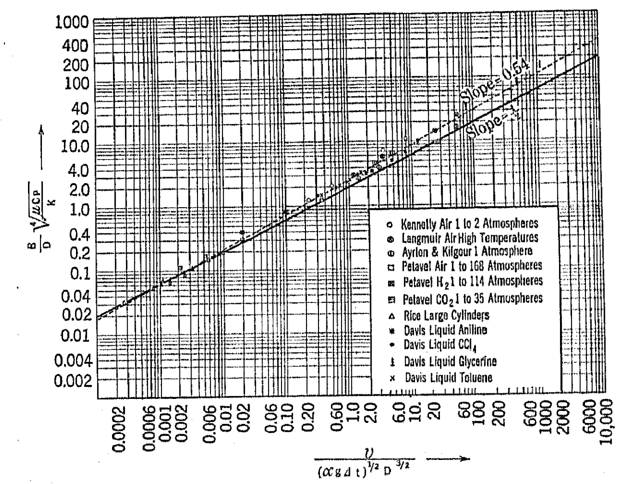

Fig. 1

Calculation of Heat Conductivity of Liquids from Free Convection Tests

The assumptions of the previous paper[1] lead to equations from which the heat conductivity of fluids could be calculated from convection tests when the viscosity and density were known for the ambient temperature alone. The more general expression (5) now found to be necessary, to completely satisfy the available data, requires a knowledge of the viscosity, density and temperature coefficient of density change, as functions of temperature, as well as an approximate knowledge of the heat conductivity and specific heat, before calculations of the heat conductivity can be made. It is therefore now apparent that free convection tests do not constitute a primary method of obtaining heat conductivities. It will be observed, however, that the heat conductivity enters the equation for the film thickness to the $\frac{1}{4}$ power only, so that the method is still of interest where an approximate value of the heat conductivity already exists. In Table II we have recalculated the heat conductivities on the present basis from the data given in Tables IX and XI of the previous paper.[1] The film thickness

3. J. H. Jeans, The Dynamical Theory of Gases. Camb. Univ. Press 2nd edition 1916, p. 317.

TABLE I

SUMMARY OF COLLECTED DATA ON FREE CONVECTION FROM HORIZONTAL CYLINDERS IN GASES AND LIQUIDS

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII
Source and Gas	Cyl. Diam. cm.	Abs. Gas Pres. atm.	Amb. Temp °C	Cyl. Temp °C	Conv. Watts per cm. Length	$\Delta\varphi$ Watts per cm.	b cm.	B cm.	B/D	Avg. Temp. ν c. g. s.	Avg. Temp. $\frac{\nu}{(\alpha g \Delta t)^{1/2} D^{3/2}}$ c. g. s.	$\frac{X}{XII}$ K_1 $n=1/2$	$\frac{k}{\mu c_p}$	$\sqrt[4]{\frac{k}{\mu c_p}}$	$(B/D)\times\sqrt[4]{\frac{\mu c_p}{k}}$	$\frac{XVI}{XII^{1/2}}$ K $n=1/2$	$\frac{XVI}{XII^{.54}}$ K $n=.54$
Kennelly	.0114	.987	20°	185°	.098	.050	.279	.1338	11.70	.235	8.03	4.10	1.35	1.08	10.9	3.85	3.54
Air	.0114	2.22	"	"	.112	"	.188	.088	7.73	.105	3.59	4.07	"	"	7.17	3.77	3.59
⊙	.0691	.987	"	"	.186	"	.374	.1525	2.21	.235	.528	3.04	"	"	2.05	2.81	2.89
	.0691	2.22	"	"	.238	"	.258	.0945	1.37	.105	.236	2.82	"	"	1.27	2.61	2.77
Langmuir	.00404	1.0	27°	1027	.84	.537	.225	.1105	27.4	.84	54.5	3.70	"	"	25.4	3.43	2.94
Air	.00691	"	"	"	.97	"	.224	.1085	15.7	"	24.9	3.14	"	"	14.6	2.92	2.58
⊗	.0251	"	"	"	1.30	"	.338	.156	6.22	"	3.52	3.33	"	"	5.77	3.09	2.93
	.051	"	"	"	1.71	"	.368	.159	3.12	"	1.22	2.84	"	"	2.89	2.63	2.60
Ayrton	.0031	1.0	10.5	299	.164	.094	.1134	.0551	17.75	.302	54.3	2.41	"	"	16.5	2.24	1.91
and	.0051	"	13.0	301	.192	"	.1110	.0530	10.40	.306	26.7	2.01	"	"	9.65	1.87	1.64
Kilgour	.0152	"	12.0	300	.220	"	.224	.1044	6.88	.304	4.97	3.09	"	"	6.38	2.86	1.89
Air	.0206	"	12.3	300	.264	"	.194	.0865	4.20	.304	3.10	2.39	"	"	3.89	2.21	2.12
Φ	.0236	"	15.9	304	.250	"	.250	.1132	4.80	.312	2.69	2.92	"	"	4.45	2.72	2.61
	.0356	"	14.5	303	.307	"	.244	.1042	2.93	.310	1.43	2.44	"	"	2.72	2.27	2.24
Petavel	.1106	168.4	16	916	14.82	.450	.1335	.0115	.104	.00433	.00206	2.29	.775	.94	.111	2.45	3.13
Air	"	65.4	"	"	9.72	"	.148	.0187	.169	.01115	.00530	2.32	1.00	1.00	.169	2.32	2.88
⊡	"	.969	"	"	2.18	"	.404	.1467	1.33	.754	.359	2.22	1.35	1.08	1.23	2.05	2.14
Petavel	.1106	114.	16	916	30.82	2.8	.1955	.0425	.384	.0430	.0205	2.68	.83	.955	.403	2.82	3.37
Hydrogen	"	23.3	"	"	17.52	"	.3010	.0952	.862	.210	.100	2.72	1.21	1.05	.822	2.60	2.85
⊠	"	.971	"	"	8.27	"	.932	.411	3.72	5.05	2.40	2.40	1.34	1.08	3.44	2.22	2.14
Petavel	.1106	34.8	16	916	9.77	.405	.1432	.0163	.147	.0127	.00605	1.89	.990	1.00	.147	1.89	2.23
CO_2	"	10.2	"	"	5.14	"	.1815	.0355	.321	.0435	.0207	2.23	1.13	1.03	.311	2.18	2.52
|三|	"	.99	"	"	2.30	"	.3340	.112	1.01	.448	.213	2.18	1.17	1.04	.972	2.10	2.24
	4.28	1.0	14.0	142	1.66	.037	4.92	.32	.075	.205	.00108	2.28	1.35	1.08	.0695	2.11	2.78
	"	.35	"	"	1.08	"	5.32	.52	.122	.585	.00307	2.20	"	"	.113	2.03	2.58
Rice	"	.11	"	"	.672	"	6.05	.885	.207	1.86	.00977	2.10	"	"	.192	1.95	2.32
Air	5.56	1.0	28.8	81.8	.687	.014	6.32	.38	.068	.182	.00101	2.14	"	"	.0630	1.98	2.62
△	"	1.0	103.2	150.4	.697	.015	6.37	.405	.073	.260	.00144	1.92	"	"	.0676	1.78	2.31
	11.35	1.0	12.3	97.4	2.08	.022	12.15	.40	.035	.182	.000272	2.12	"	"	.0324	1.96	2.72
	"	.35	"	"	1.29	"	12.65	.65	.057	.520	.000778	2.05	"	"	.0528	1.89	2.54
	"	.11	"	"	.81	"	13.45	1.05	.093	1.65	.00247	1.87	"	"	.0860	1.73	2.19
										Avg. Last 11 Points..... 2.1			Avg. Last 17 Points..... 2.12				
Free Convection in Liquids																	
						$k\Delta t$					Avg. of Amb. and Wire Temp.						
Davis	.0155	1.0	14.0°	19.0	.0257	.00855	.125	.0545	3.52	.0488	12.62	.993	.0159	.355	9.93	2.80	2.53
Aniline	"	"	"	24.0	.0590	.0171	.0957	.0401	2.59	.0450	8.12	.910	.0172	.363	7.15	2.51	2.30
C_6H_7N	"	"	"	34.0	.136	.0342	.0754	.0300	1.94	.0376	4.94	.875	.0206	.380	5.11	2.30	2.16
*	"	"	"	64.0	.406	.0855	.0580	.0213	1.38	.0241	1.94	.990	.0321	.423	3.26	2.35	2.28
Davis	.0155	1.0	13°	18.0	.0219	.00530	.0708	.0277	1.79	.00660	1.38	1.53	.122	.592	3.03	2.59	2.55
CCl_4	"	"	"	23.0	.0490	.0106	.0605	.0225	1.45	.00638	.938	1.49	.128	.600	2.42	2.50	2.50
●	"	"	"	33.0	.108	.0212	.0530	.0188	1.21	.00589	.632	1.53	.138	.610	1.98	2.51	2.55
	"	"	"	63.0	.312	.0530	.0450	.0148	.955	.00478	.319	1.69	.168	.640	1.49	2.64	2.77
Davis	.0155	1.0	19°	24.0	.0224	.0140	.782	.388	25.1	6.10	2030.	.56	.000150	.111	226	5.03	3.76
Glycerine	"	"	"	29.0	.0498	.0280	.528	.256	16.5	4.80	1140.	.49	.000191	.118	140	4.15	3.11
$C_3H_8O_3$	"	"	"	39.0	.113	.0560	.347	.166	10.7	3.05	503.	.48	.000300	.132	81.2	3.63	2.82
‡	"	"	"	69.0	.339	.140	.209	.097	6.27	.770	77.7	.76	.00119	.186	33.7	4.10	3.24
Davis	.0155	1.0	15.5°	20.5	.0253	.00642	.0760	.0303	1.95	.00690	1.53	1.58	.129	.600	3.25	2.63	2.58
Toluene	"	"	"	25.5	.0570	.0129	.0638	.0242	1.56	.00672	1.07	1.51	.132	.604	2.59	2.51	2.50
C_7H_8	"	"	"	35.5	.129	.0257	.0541	.0193	1.25	.00634	.708	1.48	.141	.613	2.04	2.43	2.46
×	"	"	"	65.5	.381	.0642	.0448	.0147	.945	.00535	.373	1.54	.166	.640	1.48	2.42	2.52
														Avg. All Points..... 2.60			

has been determined from equation (5), in which $K = 2.60$; $n = 0.54$ and $m = \frac{1}{4}$. The available data on viscosity as function of temperature appear to be satisfactory except for glycerine and olive oil. For glycerine the existing data have been extrapolated to higher temperatures by plotting the fluidity $1/\mu$ against temperature on semi-log paper, since, under these conditions, the points lie almost on a straight line. For olive oil a pure guess has been made, as follows: Osborne Reynold's data[4] on fluidity for a certain olive oil have been plotted on semi-log paper and a parallel line drawn through the fluidity given by

4. Osborne Reynolds, *Phil. Trans.*, Roy. Soc., Vol. 177, Part I p. 170, 1886

TABLE II

CALCULATION OF HEAT CONDUCTIVITIES FROM FREE CONVECTION TESTS ON HORIZONTAL WIRES

Liquid Observer Assumed properties	Amb. Temp. t_a °C	Wire Temp. t °C	$t - t_a$ °C	W_c/L Watts per cm.	$(\rho/\mu)_{avg.}$ c. g. s.	2 B cm.	$\varphi - \varphi_a$	$\Delta \varphi$ Watts per cm.	Δt °C	$k_{avg.}$ $\frac{\Delta \varphi}{\Delta t}$	$t_{avg.}$ °C
Toluene ⊡	15.5	20.5	5	.025	145.	.0598	.0063	. .	. .	. .	. .
$C_7 H_8$	"	25.5	10	.057	149.	.0494	.0130	.0067	5	.00134	23.
Davis	"	30.5	15	.092	154.	.0439	.0198	.0068	"	.00136	28.
Platinum Wire	"	35.5	20	.129	158.	.0403	.0263	.0065	"	.00130	33.
.0155 cm. Diam.	"	40.5	25	.169	163.	.0376	.0332	.0069	"	.00138	38.
α = .00109	"	45.5	30	.210	168.	.0355	.0398	.0066	"	.00132	43.
ρ = .866	"	50.5	35	.253	172.	.0340	.0466	.0068	"	.00136	48.
k = .000307	"	55.5	40	.295	177.	.0324	.0530	.0064	"	.00128	53.
c = .42	"	60.5	45	.338	182.	.0312	.0593	.0063	"	.00126	58.
	"	65.5	50	.381	187.	.0299	.0651	.0058	"	.00116	63.
Glycerine ⊙	19	24.	5	.022	.165	.54	.0125	. .	. .	. .	. .
$C_3 H_8 O_3$	"	29.	10	.050	.206	.424	.0266	.0141	5	.00282	26.5
Davis	"	34.	15	.081	.255	.356	.0410	.0144	"	.00288	31.5
Platinum Wire	"	39.	20	.114	.315	.310	.0555	.0145	"	.00290	36.5
.0155 cm. Diam.	"	44.	25	.149	.385	.275	.0695	.0140	"	.00280	41.5
α = .00050	"	49.	30	.189	.480	.246	.0838	.0143	"	.00286	46.5
ρ = 1.26	"	54.	35	.224	.585	.223	.0975	.0137	"	.00274	51.5
k = .00067	"	59.	40	.262	.720	.202	.1100	.0125	"	.00250	56.5
c = .58	"	64.	45	.300	.870	.186	.1225	.0125	"	.00250	61.5
	"	69.	50	.340	1.32	.160	.1320	.0095	"	.00190	66.5
CCl_4 ×	13	18.	5	.022	151.	.0568	.00538	. .	. .	. .	. .
Davis	"	28.	15	.077	163.	.0412	.0159	.00510	5	.00102	25.5
Platinum Wire	"	33.	20	.108	170.	.0378	.02125	.0054	"	.00108	30.5
.0155 cm. Diam.	"	38.	25	.140	175.	.0368	.0271	.0058	"	.00116	35.5
α = .00124	"	43.	30	.174	182.	.0331	.0316	.0045	"	.00090	40.5
ρ = 1.582	"	48.	35	.208	187.	.0317	.0369	.0053	"	.00106	45.5
k = .0002	"	53.	40	.242	194.	.0301	.0416	.0047	"	.00094	50.5
c = .203	"	58.	45	.277	200.	.0290	.0464	.0048	"	.00096	55.5
	"	63.	50	.312	207.	.0278	.0509	.0045	"	.00090	60.5
Aniline ●	14	19.	5	.027	20.5	.111	.00905	. .	. .	. .	. .
$C_6 H_7 N$	"	24.	10	.060	22.5	.0896	.01825	.00920	5	.00184	21.5
Davis	"	29.	15	.096	24.7	.0781	.0275	.0092	"	.00184	26.5
Platinum Wire	"	34.	20	.136	27.0	.0707	.0371	.0096	"	.00192	31.5
.0155 cm. Diam.	"	39.	25	.179	29.0	.0650	.0470	.0099	"	.00198	36.5
α = .000855	"	44.	30	.223	31.5	.0608	.0566	.0096	"	.00192	41.5
ρ = 1.023	"	49.	35	.268	34.0	.0568	.0657	.0091	"	.00182	46.5
k = .000408	"	54.	40	.314	36.5	.0535	.0748	.0091	"	.00182	51.5
c = .514	"	59.	45	.360	39.0	.0510	.0833	.0085	"	.00170	56.5
	"	64.	50	.406	41.5	.0490	.0923	.0090	"	.00180	61.5
Olive Oil ⊗	18	23.	5	.016	.80	.310	.00776	. .	. .	. .	. .
Davis	"	33.	15	.060	.85	.227	.02635	.0099	5	.00198	30.5
Platinum Wire	"	38.	20	.083	.87	.208	.0352	.0088	"	.00176	35.5
.0155 cm. Diam.	"	43.	25	.110	.90	.194	.0457	.0105	"	.00210	40.5
α = .00070	"	48.	30	.136	.93	.183	.0551	.0094	"	.00188	45.5
ρ = .916	"	53.	35	.165	.96	.175	.0662	.0111	"	.00222	50.5
k = .000392	"	58.	40	.194	.99	.164	.0758	.0096	"	.00192	55.5
c = .47	"	63.	45	.223	1.02	.161	.0864	.0106	"	.00212	60.5
	"	68.	50	.252	1.05	.155	.0965	.0101	"	.00202	65.5
						.256					
No.12 Transil Oil △	24.8	35.	10	.045	7.0.	.1435	.01355	. .	. .	. .	. .
Rice	"	55.	30	.175	10.0.	.0960	.04370	.0152	10	.00152	50.
Silver Wire	"	65.	40	.250	11.5.	.0855	.05870	.0150	"	.00150	60.
.0254 cm. Diam.	"	75.	50	.325	13.5.	.0766	.0717	.0130	"	.00130	70.
α = .000813	"	85.	60	.405	16.0.	.0700	.0855	.0138	"	.00138	80.
ρ = .83	"	95.	70	.485	18.5.	.0644	.0978	.0123	"	.00123	90.
k = .000355	"	105.	80	.570	21.5.	.0592	.1095	.0117	"	.00117	100.
c = .50	"	115.	90	.665	25.0.	.0550	.1225	.0130	"	.00130	110.
	"	125.	100	.760	28.5.	.0512	.1340	.0115	"	.00115	120.
	"	135.	110	.865	32.0.	.0486	.1480	.0140	"	.00140	130.
	"	145.	120	.970	36.0.	.0456	.1585	.0105	"	.00105	140.

Davis[5] for olive oil at the assumed temperature of 18 deg. cent.

In Fig. 2 the calculated heat conductivities have been plotted against temperature, from which we see that the temperature coefficients are now negative for all the liquids except for olive oil, whereas, the method of the previous paper [1] gave positive temperature coefficients. There are very little data in the literature on this susbject and of this some investigators find negative and others positive values for the coefficient for the same liquid. It is interesting to note that

5. A. H. Davis, *Phil. Mag.*, Vol. 44, p. 929, 1922

Bridgman's[6] recent determinations show negative temperature coefficients of conductivity for all of the fifteen liquids which he has studied. In Table III I have, therefore, compared the present calculated heat conductivities with the data by those investigators who have found negative coefficients where available. The value calculated for olive oil cannot be given much weight, due to the uncertainty of the viscosity value.

TABLE III
COMPARISON OF HEAT CONDUCTIVITIES AS CALCULATED FROM FREE CONVECTION TESTS WITH THOSE OBTAINED BY DIRECT EXPERIMENT

Liquid	Temp °C	k Cal. Sec.$^{-1}$ Cm.$^{-1}$ °C^{-1}	Ratio Exp./Calc.	Temp Coef. α	Source
Toluene, C_7H_8.......	0	0.000349		−.0014	Goldschmidt
" "	0	0.000358	0.97	−.0029	Calculated
Carbontetrachloride; CCl_4..............	0	0.000266		....	Goldschmidt
Carbontetrachloride; CCl_4..............	0	0.000287	0.93	−.0038	Calculated
Aniline; C_6H_7N......	0	0.000434		....	Goldschmidt
" "	0	0.000478	0.91	−.0018	Calculated
Glycerine $C_3H_8O_3$....	25°	0.000682		−.0044	Lees
" "	25°	0.000715	0.95	−.0038	Calculated
Petroleum Oil........	13°	0.000355		+.011	Graetz
No. 12 Transil Oil.....	13°	0.000392	0.91	−.0026	Calculated
Olive Oil.............	6.°6	0.000392		....	H. F. Weber
" ".............	6.°6	0.000394	0.99	+.0055	Calculated

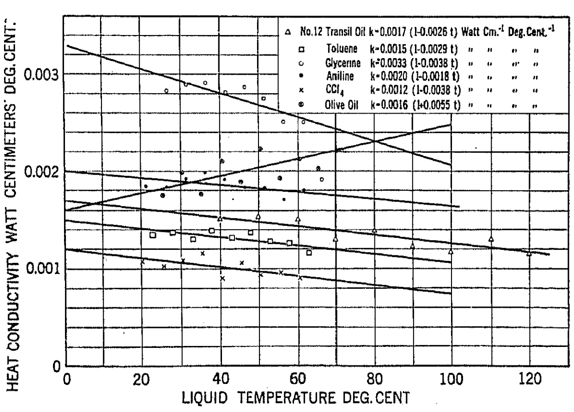

FIG. 2

FREE CONVECTION FROM LONG VERTICAL CYLINDERS

For this case we have the data by Griffiths and Davis[7] on a series of cylinders 17.43 cm. diam. and of various lengths up to 263.5 cm. In these experiments we are dealing with thin films and moderate temperature differences and may therefore write

$$W_c = (A/B)\, k_{avg.}\, \Delta\, t \text{ watts convection} \qquad (6)$$

Since the tests are made in air we may omit the second factor in equation (5) and express the film thickness as

$$B = K_1 D \;\{\nu/[(\alpha\, g\, \Delta\, t)^{1/2}\, D^{3/2}]\}^n \text{ cm.} \qquad (7)$$

When we plot the watts convection per unit length against the temperature rise on log:log paper for the four longest cylinders we find that all points fall on a single curve as should be the case, when the length is large compared with the diameter. Actually the shortest of the four cylinders is only a little over three times the diameter. From the slope of the curve we find that the watts convection per cm. length is approximately proportional to the 4/3 power of the temperature difference. If we put $n = 2/3$ in equation (7) and substitute the value of B in equation (6) and take ν at the average of the surface and ambient temperature we obtain.

$$W_c = \frac{\pi\, D\, L\, (\alpha\, g)^{1/3}}{K_1}\, (k/\nu^{2/3})_{avg.}\, \Delta\, t^{4/3} \text{ watts} \qquad (8)$$

This equation is seen to be of the proper form since the factor $k/\nu^{2/3}$ only varies by about 4 per cent in the temperature range covered by the experiments. In the last column of Table IV the values of K_1 have been calculated from the data on the longest cylinder. Data in liquids are needed in order to determine the exponent m in equation (5).

FREE CONVECTION FROM SPHERES

In the first paper[1] Compan's[8] data on the free and forced convection from spheres were used to determine the convection laws. In the course of the present work I have re-examined Compan's data and compared them with the work of Péclet[9] and M'Farlane[10] with the result that Compan's experiments do not agree at all with the others. It seems probable that the trouble with Compan's work is due to the use of a small sphere covered with a thick coat of lamp black. For small spheres the ratio of film thickness to diameter is greater than for larger spheres, but the actual film thickness will be smaller, and therefore a relatively thin coat of poorly conducting lamp black may easily double the effective film thickness on small objects. The total emissivity for a lamp black surface calculated from Compan's data gives $E = 0.43$ at 200 deg. cent. as compared with the now commonly accepted value of approximately 0.9. We are thus led to conclude that Compan's thermocouple did not give the surface temperature. In the present work his data have therefore been discarded.

Péclet worked with three hollow copper spheres, 7.46, 10.58 and 15.38 cm. in radius, filled with water. These were placed in a water-jacketed constant-temperature cylindrical enclosure 100 cm. high and 80 cm. in diameter. The water in the sphere and jacket of the enclosure was kept in constant agitation and the surface temperatures assumed to be the same as that

6. P. W. Bridgman, *Phys. Rev.*, Vol. 21, abstract No. 9, p. 704, June, 1923

7. Ezer Griffiths and A. H. Davis. The Transmission of Heat by Radiation and Convection. Food Investigation Board. Special Report No. 9 Department of Scientific and Industrial Research Published by His Majestys' Stationary Office, London.

8. Paul Compan, *Annales, de Chemie et de Physique*, VII Serie Vol. 26, pp. 488-574, 1902.

9. E. Peclet, Traite de la Chaleur, Third Edition. Victor Maison et Fils, Paris, 1860, Vol. III, pp. 418-481.

10. Donald M'Farlane, *Proc. Roy. Soc.* London, Vol. 20, p. 90, 1871 and 1872.

of the water. He eliminated the radiation from his observations in the following manner: He determined the total rate of heat loss under given temperature conditions for a lamp-black-covered copper sphere and later under the same conditions, except that the sphere surface was highly polished. The difference of these results gives the difference in radiation for lamp black and polished copper surfaces, since the convection is independent of the nature of the surface. He next obtained the ratio of radiation from a lamp black surface to that of a polished copper surface by measurements with a thermopile. Having thus determined the difference in radiation and the ratio of the two radiations he solved for the absolute value of each. The lamp black surfaces were obtained by first covering the object with paper and then depositing lamp black on the paper from a flame. To see whether the presence of the paper influenced the convection he determined the total rate of cooling with 1, 2, 3 and 4 sheets of paper applied to the object without finding any noticeable change. For spheres of this size in air the film thickness is of the order of ½ cm. and since the conductivity of paper is approximately 4 times that of air only a small proportion of the total temperature drop will be consumed by the paper.

In this manner Péclet obtained the following equation for the free convection per square cm. of surface for his spheres.

$$w_c = 0.000064\,(1.778 + 26./D)\,\Delta\, t^{1.233} \text{ watts per cm.}^2 \quad \mathbf{(9)}$$

where D = sphere diam. in cm.
In his experiments $\Delta\, t$ varied from about 25 deg. cent. to 70 deg. cent.

In the above experiments we are dealing with thin films and small temperature differences and therefore obtain the following relations from equations (**6**) and (**7**).

$$\text{If } n = 1/3;\ w_c \propto \Delta\, t^{7/6}/D^{1/2} \text{ watts per cm.}^2 \quad \mathbf{(10)}$$

$$n = 1/2;\ w_c \propto \Delta\, t^{5/4}/D^{1/4} \text{ watts per cm.}^2 \quad \mathbf{(11)}$$

Péclet's data give approximately,

$$w_c \propto \Delta\, t^{1.233}/D^{0.412} \text{ watts per cm.}^2 \quad \mathbf{(12)}$$

Thus from temperature we would select equation (**11**) while by size we find a closer agreement with equation (**10**).

M'Farlane[10] measured the rate of cooling of a 4-cm. diam. polished and lamp black covered copper sphere placed in a water-jacketed enclosure at atmospheric pressure for temperature differences up to 60 deg. cent. The size of the enclosure is not stated.

In correcting M'Farlane's data for radiation we have assumed $E = 0.9$ for the lamp black surface and $E = 0.15$ for his polished copper. The total emissivity for polished copper calculated from the relation given by Foote[11] is $E = 0.0116$ at O deg. cent. and $E = 0.0161$ at 100 deg. cent. A study of the measurements of total emissivity of various metal surfaces by Randolph and Overholzer[12] as function of time and temperature makes it seem likely that some degree of oxidation had taken place before M'Farlane's tests were started. We have therefore assumed an average value for the temperature range of $E = 0.15$. On this basis the convection from the polished copper surface is approximately 15 per cent greater than for his lamp black surface. The lamp black surface is probably subject to some correction for the temperature drop through the deposit.

On the above basis we find that for M'Farlane's sphere the convection varies approximately as the 1.12 power of the temperature difference and is therefore in better accord with equation (**10**) than (**11**).

TABLE IV
FREE CONVECTION FROM VERTICAL CYLINDER IN AIR AT ATMOSPHERIC PRESSURE. DATA BY GRIFFITHS AND DAVIS
Cylinder Diameter = 17.43 cm.; Length = 263.5 cm.
Ambient Temperature Assumed 20°C

Amb. Temp. °C$_2$	Surf. Temp. °C	Temp. Diff. $\Delta\, t$	Watts Conv. per cm. Length W_c/L	$\frac{10^6\, k}{\nu^{2/3}}$ Avg. Temp.	$\Delta\, t^{4/3}$	$\frac{8.38\, k \Delta t^{4/3}}{\nu^{2/3}}$	K_1 $n = 2/3$
20	32.0	12.7	.252	863.	29.5	2.13	8.45
"	36.3	16.3	.308	861.	41.5	3.00	9.75
"	41.9	21.9	.505	858.	61.0	4.39	8.70
"	42.0	22.0	.55	858.	61.5	4.43	8.05
"	44.2	24.2	.574	857.	70.	5.03	8.77
"	70.3	50.3	1.36	845.	184.	13.0	9.56
"	71.9	51.9	1.52	844.	192.	13.6	8.95
"	90.3	70.3	2.52	836.	290.	2.03	8.05
"	94.2	74.2	2.80	835.	310.	21.7	7.75
"	106.0	86.0	2.96	829.	380.	26.4	8.93
"	106.6	86.6	3.16	829.	380.	26.4	8.35
"	107.9	87.9	3.21	828.	390.	27.1	8.45
						Avg. =	8.65

In Table V we have given the values of K_1 as calculated from equation (**6**) and (**7**) on the two alternative assumptions. The extensive work of Dulong and Petit[13] on the cooling of large spherical thermometer bulbs accords best with assumption $n = ½$. Until further data are available it is probably safest to calculate the free convection from spheres using $n = ½$ and $K = 2.0$ in equation (**5**). Data on the convection of spheres in liquids are needed to determine the exponent m.

FREE CONVECTION FROM VERTICAL PLANE SURFACES

The available data on convection from vertical plane surfaces do not belong to any definite system of similar plane figures, *i. e.*, similar rectangles, triangles, etc., or circular disks of varying diameters, and therefore, strictly speaking, cannot be correlated.

An interesting type of vertical plane surface consists of a long horizontal ribbon whose thickness is small com-

11. Paul D. Foote, *Bull. Bur. Stds.* Vol. 11, p. 607, 1915.

12. C. P. Randolph and M. J. Overholzer, *Phys. Rev.*, Vol. 2, p. 144, 1913.

13. Dulong and Petit Ann de Chemie et de Phys. 2e tom. VII pp. 225 and 337, 1817. See abstract in Preston's Theory of Heat.

TABLE V

FREE CONVECTION FROM POLISHED COPPER SPHERES IN AIR AT ATMOSPHERIC PRESSURE

Observer	Sphere Diam. Cm.	Amb. Temp. °C	Sphere Temp. °C	Temp. Diff. Δt	Watts Conv. W_c	$10^6\, k\, \nu^{1/2}$ Avg. Temp.	$\Delta t^{5/4}$	K_1 $n = 1/2$	$\Delta t^{7/6}$	K_1 $n = 1/3$
Péclet............	15	14	34	20	6.43	635	42.5	2.09	32.8	0.532
	15	"	74	60	24.8	631	167	2.11	118	0.503
	20	"	34	20	10.0	635	42.5	2.21	32.8	0.528
	20	"	74	60	38.7	631	167	2.23	118	0.500
	30	"	34	20	19.3	635	42.5	2.30	32.8	0.500
	30	"	74	60	74.5	631	167	2.33	118	0.475
M'Farlane........	4	14	24	10	.349	636	17.8	1.59	14.6	0.600
	"	"	34	20	.762	635	42.5	1.73	32.8	0.618
	"	"	44	30	1.20	634	70.0	1.80	52.8	0.635
	"	"	54	40	1.66	633	101	1.88	73.5	0.642
	"	"	64	50	2.11	632	133	1.94	95.0	0.655
	"	"	74	60	2.53	631	167	2.04	118	0.680
							Average K_1	= 2.02..................		0.574

pared with the vertical height. As a rough approximation we will assume that the available data are applicable to this case and determine the constant K_1 and exponent n of equation (**7**). For narrow ribbons the shape factor will require evaluation as the flow between confocal elliptical cylinders, but for thin films the shape factor takes the simple form, $S = A/B$. For thin films and moderate temperature differences we will determine the convection from equations (**6**) and (**7**). For our ideal ribbon surface the horizontal length is large compared with the vertical height and therefore the convection per unit length will be independent of the length. We will therefore associate D in equation (**7**) with the vertical height H.

Langmuir [2] studied the convection from one side of a circular disk 19.1 cm. diameter under various surface conditions. We have selected the values of total loss obtained for the polished silver and calorized copper surfaces since the radiation corrections are fairly definite and rather small. The vertical height H has been taken as the square root of the area.

We have taken the data by Griffiths and Davis[7] for the total loss from one side of a polished aluminum surface 127 cm. by 127 cm. Here we have taken $H = 127$ cm.

Doherty[14] studied the heat loss from a double-sided lamp-black-coated hot plate 25.4 cm. high, 38.1 cm. long and 1.27 cm. thick.

Montsinger[15] studied the loss from a double-sided plate 80 cm. high, 33.2 cm. wide, and 2.7 cm. thick. His original data on the polished nickel and black surfaces have been used.

Griffiths and Davis[7] have studied the heat loss from a series of vertical cylinders of the same diameter and various heights. We have included their data on this series for the four shortest cylinders, since for these the circumference is fairly large compared with the vertical height and therefore may be considered to fit in with the very rough correlation here being attempted.

Griffiths and Davis[7] have assumed that their data on the convection from this series of vertical cylinders of different heights accurately determines the law of variation of convection with respect to height for vertical plane surfaces. This assumption is undoubtedly an oversight, since when the height is great compared with the diameter we are dealing with the problem of convection from long vertical cylinders, in which case, the convection per unit area, or per unit vertical length, should be independent of the total height, and I can see no reason for believing that the conditions are at all analogous with those which apply to the problem of a long horizontal ribbon of negligible thickness and definite vertical height.

The values of *total emissivity* used in deducing the radiation correction for the polished surfaces have been calculated as functions of temperature from the following relation derived by Foote.[11]

$$E_{total} = 0.5736 \sqrt{r\,T} - 0.1769\, r\, T \qquad (\mathbf{13})$$

where T = Absolute temperature of radiating metal surface

r = Volume resistivity in ohm cm. at the temperature T

We have determined r from the usual relation, $r = r_0 (1 + \alpha\, t)$. Equation (**13**) is applicable to unoxidized polished metal surfaces below 1500 deg. cent., or to the spectral region where resonance phenomena do not exist. Foote[11] shows that the experimental values of *total emissivity* for platinum from 0 deg. cent. to 1700 deg. cent. are well represented by the equation.

The radiation corrections calculated on this basis are probably somewhat too small because of the marked effects due to oxidation.[12] In this connection it should be pointed out that where accurate work on convection is attempted provision should be made for a simultaneous determination of the total emissivity of the actual surface. In this manner the uncertainties in the value of emissivity as functions of time and temperature and previous history will be eliminated. Tests in inert gases such as H_2 and N_2, etc. are also recommended.

14. Data kindly supplied by R. E. Doherty. These and additional data will probably soon be published in the A. I. E. E.

15. Data kindly supplied by V. M. Montsinger. A summary of this data is given by Montsinger in his discussion of paper reference (1).

In Fig. 3 we have plotted the watts convection per square cm. of surface against temperature difference on log:log paper. From this we see that the convection per unit area is proportional to the 5/4 power of the temperature difference. Above 100 deg. cent. rise the radiation may be considerably off due to oxidation. If we plot in a similar manner, the convection given by Griffiths and Davis[7] for the four shortest cylinders, excluding the points below 10 deg. cent. rise, we obtain a family of four approximately parallel curves in which the convection per unit area is again approximately proportional to the 5/4 power of the temperature difference. We will therefore *assume* that the watts convection per unit area for our ideal ribbon surface, under the conditions of thin films and moderate temperature differences, will vary as the 5/4 power of the temperature difference. This assumption requires us to take $n = \frac{1}{2}$ in equation (**7**) when combined with equation (**6**). We thus obtain for ideal gases

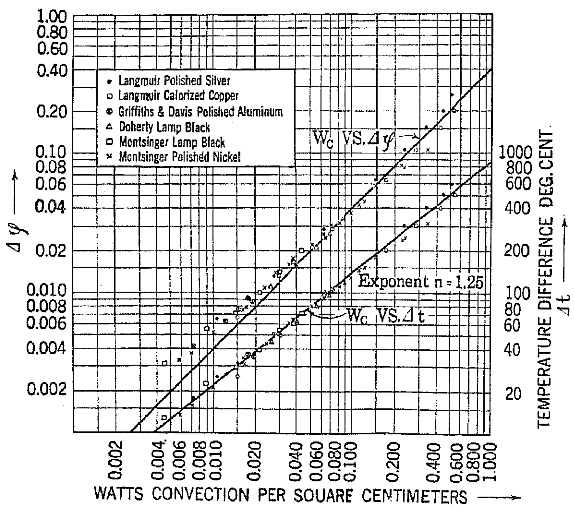

Fig. 3

$$w_c = 1/K_1\,(\alpha\, g/H)^{1/4}\,(k/\nu^{1/2})_{avg.}\,\Delta\, t^{5/4} \text{ watts cm.}^{-2} \quad (\mathbf{14})$$

There are no data available from which the exponent m in equation (**5**) can be evaluated but if we assume $m = \frac{1}{4}$ as was found to be the case for horizontal cylinders, where the exponent n was equal to $\frac{1}{2}$, we obtain

$$w_c = (1/K)\,(c_p\,\mu/k)^{1/4}{}_{avg.}\,(\alpha\, g/H)^{1/4}\,(k/\nu^{1/2})_{avg.}\,\Delta\, t^{5/4} \text{ watts cm.}^{-2} \quad (\mathbf{15})$$

where

K = Numerical constant for all fluids

c_p = Specific heat at constant pressure for the average of surface and ambient temperatures, joule gram^{-1} deg. cent.$^{-1}$

μ = Viscosity of fluid for the avg. temp. cm.$^{-1}$ gram sec.$^{-1}$

k = Heat conductively of fluid for avg. temp. watt cm.$^{-1}$ deg. cent.$^{-1}$

α = Coefficient of density change per deg. cent. temperature change. For ideal gases $\alpha = 1/273$

g = Acceleration of gravity = 980 cm. sec.$^{-2}$

H = Vertical height of long horizontal plane in cm.

ν = μ/ρ = Kinematic viscosity of fluid at the average temperature cm.^{2} sec.$^{-1}$

$\Delta\, t$ = Temperature difference between surface and ambient fluid deg. cent.

Equation (**15**) in a more compact form becomes

$$w_c = (1/K)\,(c_p\, g\, k^3\, \alpha/\mu\, H)^{1/4}\,\rho^{1/2}\,\Delta\, t^{5/4} \text{ watts cm.}^{-2} \quad (\mathbf{16})$$

It is now interesting to observe that this is practically the equation derived by Lorenz[16] for this case, to which Langmuir[2] has called attention. In our notation Lorenz equation is

$$w_c = 0.548\,(c_p\, g\, k^3/\mu\, H\, T)^{1/4}\,\rho^{1/2}\,\Delta\, t^{5/4} \text{ watt cm.}^{-2} \quad (\mathbf{17})$$

where T = Absolute ambient temperature deg. K. Here the fluid properties were taken for the ambient temperature while in our equation the average values are used. Our equation contains the coefficient of density change per deg. cent. instead of the reciprocal of absolute ambient temperature T.

In Table VI the values of K_1 have been calculated from equation (**14**). As expected the values of the constant varies from one surface to the next since the data do not belong to any definite system of similar surfaces. The average value of the constant K_1 is 1.46. If we assume equation (**15**) we obtain $K = 1.35$ which is not greatly different from the value $K = 1.82$ calculated by Lorenz.

Free Convection from Large Surfaces

A large surface is here defined as one in which the film thickness is small compared with the characteristic dimension of the body. We will also limit the discussion to moderate temperature differences and assume the relation

$$\Delta\,\varphi = k_{avg.}\,\Delta\, t \text{ watts per cm.} \quad (\mathbf{18})$$

The total watts convection is then given by

$$W_c = (A/B)\,k_{avg.}\,\Delta\, t \text{ watts} \quad (\mathbf{19})$$

where

A = Area of surface in cm.^{2}

B = Effective film thickness in cm.

k = Heat conductivity for the average of surface and ambient temperature in watt cm.$^{-1}$ deg. cent.$^{-1}$

$\Delta\, t$ = Temperature difference in deg. cent.

The effective film thickness is given by

$$B = K\,D\,(k/\mu\, c_p)^m\,\{\nu/[(\alpha\, g\,\Delta\, t)^{1/2}\, D^{3/2}]\}^n \text{ cm.} \quad (\mathbf{20})$$

where

K = Experimental constant depending on the system of similar bodies under consideration

D = Characteristic linear dimension of body in cm.

k = Heat conductivity at average temperature watt cm.$^{-1}$ deg. cent.$^{-1}$

μ = Viscosity of fluid at average temperature cm.$^{-1}$ gram sec.$^{-1}$

c_p = Specific heat at constant pressure for average temperature joule gram^{-1} deg. cent.$^{-1}$

m = Experimental exponent

16. L. Lorenz, *Annder Physik*, Vol. 13, p. 586, 1881.

TABLE VI

SUMMARY OF COLLECTED DATA ON FREE CONVECTION FROM VERTICAL PLANE SURFACES IN AIR AT ATMOSPHERIC PRESSURE

Source and Surface	Exposed Area cm.^{2}	Amp. Temp. t_a °C	Surface Temp. t_s °C	Temp. Diff. Δt °C	Total Watts per cm.^{2} w_t	Watts Rad. Black Body per cm.^{2}	Total Emissivity E	Watts Rad. per cm.^{2} w_R	Watts Conv. per cm.^{2} w_c	Watts per cm. $\Delta \varphi$	$\Delta t^{5/4}$	K_1 assuming $n = 1/2$
		27	52.	25.	.011	.018	.0140	.00025	.0107	.0065	56.	2.19
Langmuir	Disc.	"	77.	50.	.028	.040	.0150	.00060	.0274	.013	133.	2.04
Polished	19.1 cm. Diam.	"	127.	100.	.065	.104	.0175	.00182	.0632	.028	315.	2.09
Silver	286 sq. cm.	"	227.	200.	.156	.322	.0220	.00710	.149	.064	750.	2.12
●	$H = \sqrt{286} = 17$ cm.	"	327.	300.	.260	.717	.0265	.0190	.241	.104	1250.	2.18
		"	427.	400.	.385	1.37	.0310	.0425	.342	.151	1800.	2.21
		"	527.	500.	.530	2.37	.0325	.0770	.453	.200	2350.	2.18
		"	627.	600.	.670	3.82	.0395	.151	.519	.259	2970.	2.40
		27.	52.	25.	.018	.018	.175	.0031	.0149	.0065	56.	1.58
Langmuir	Disc.	"	77.	50.	.037	.040	.177	.0071	.0299	.013	133.	1.87
Calorized	19.1 cm. Diam.	"	127.	100.	.090	.104	.178	.0185	.0710	.028	315.	1.86
Copper	286 sq. cm.	"	227.	200.	.233	.322	.180	.0580	.175	.064	750.	1.80
⊙	H = 17 cm.	"	327.	300.	.420	.717	.182	.131	.289	.104	1250.	1.76
		"	427.	400.	.680	1.37	.185	.254	.426	.151	1800.	1.78
		"	527.	500.	.99	2.37	.188	.445	.545	.200	2350.	1.81
Griffiths	Plate	20.	37.5	17.5	.0073	.00755	.0187	.000143	.00179	.0041	36.5	1.29
and Davis	127 cm. × 127 cm.	"	46.1	26.1	.0127	.0168	.0193	.000324	.0123	.0062	60.	1.24
Polished	16,100 sq. cm.	"	51.0	31.0	.0153	.0206	.0196	.000404	.0149	.0075	75.	1.28
Aluminum		"	55.2	35.2	.0192	.0238	.0198	.000470	.0187	.0086	87.	1.18
⊗		"	56.3	36.3	.0188	.0263	.0200	.000495	.0183	.0090	90.	1.25
Doherty	Plate	16.4	60.8	44.4	.0532	.0305	.90	.0275	.0257	.0110	117.	1.73
Lamp	2,100 sq. cm.	17.0	78.4	61.4	.0798	.0445	"	.0400	.0398	.0157	173.	1.66
Black	25.4 cm. High	18.1	97.8	79.7	.116	.0667	"	.0600	.0555	.0213	240.	1.64
◬	38.1 cm. Long	18.9	116.6	97.7	.152	.0922	"	.0830	.0690	.0270	310.	1.71
	1.27 cm. Thick											
		30.0	42.6	12.6	.0125	.00893	.90	.00803	.0045	.0031	24.3	1.54
Montsinger	Plate	32.7	55.2	22.5	.0234	.0161	"	.0145	.0089	.0055	50.	1.61
Lamp	5,920 sq. cm.	25.6	55.2	29.6	.0333	.0207	"	.0186	.0147	.0073	70.	1.36
Black	80 cm. High	28.5	67.2	38.7	.0477	.0293	"	.0264	.0213	.0100	98.0	1.32
⊡	33.2 cm. Wide	31.0	82.5	51.5	.0675	.0418	"	.0377	.0298	.0137	140.	1.34
	2.7 cm. Thick	29.7	90.6	60.9	.0845	.0513	"	.0462	.0383	.0166	172.	1.29
		29.7	101.8	72.1	.102	.0647	"	.0582	.0440	.0200	206.	1.34
		23.2	36.9	13.7	.00608	.00888	.036	.00032	.00576	.0033	27.0	1.34
		28.5	43.9	15.4	.00741	.0103	.037	.00038	.00703	.0037	31.0	1.26
		30.0	51.0	21.0	.0107	.0147	.038	.00056	.0101	.0051	45.5	1.29
		29.5	63.1	33.6	.0175	.0247	.040	.00099	.0165	.0086	82.0	1.42
		33.5	83.6	50.1	.0298	.0419	.043	.00180	.0280	.0135	135.0	1.38
		28.2	92.4	64.2	.0409	.0537	.044	.00236	.0385	.0172	183.	1.36
		26.3	106.6	80.3	.0552	.0735	.046	.00338	.0518	.0220	240.	1.32
		23.8	115.4	91.6	.0675	.0843	.047	.00396	.0635	.0252	286.	1.29
		25.0	117.8	92.8	.0675	.0875	.048	.00420	.0633	.0258	290.	1.31
Montsinger	Plate	27.0	128.8	101.8	.0757	.103	.049	.00503	.0707	.0289	325.	1.32
Polished	6,000 sq. cm.	24.3	134.4	110.1	.0877	.112	.050	.00558	.0821	.0313	360.	1.25
Nickel	as above with less	25.6	144.2	118.6	.0952	.127	.051	.00645	.0887	.0343	400.	1.29
×	Edge and Leads	25.2	152.4	127.2	.106	.141	.052	.00732	.099	.0371	430.	1.24
	covered	28.3	171.8	143.5	.124	.176	.055	.00970	.114	.0430	500.	1.25
		22.6	175.1	152.5	.134	.178	.056	.00996	.124	.0454	540.	1.24
		28.2	89	60.8	.0377	.0508	.043	.02218	.0355	.0162	172.	1.38
		25.3	56.6	31.3	.0177	.0226	.039	.00088	.0168	.0078	76.	1.29
		24.8	84.9	60.1	.0383	.0488	.043	.0021	.0362	.0158	168.	1.33
		24.7	67.4	42.7	.0248	.0318	.040	.00127	.0235	.0108	112.	1.36
		28.7	338.5	309.8	.405	.745	.078	.0581	.347	.106	1300.	1.07
		30.7	272	241.3	.263	.446	.069	.0315	.232	.081	960.	1.18
		26.2	205	178.8	.162	.252	.060	.0151	.147	.058	660.	1.29
		28.5	270.5	242.0	.273	.452	.069	.0312	.242	.079	970.	1.15
		20.	30.	10.	..	.00605	.90	.00545	.0113	.00220	18.3	.95
Griffiths	Short Cyl.	"	45.	25.	..	.0162	"	.0146	.0339	.00570	56.5	.97
and	4.65 cm. High	"	70.	50.	..	.0368	"	.0331	.0797	.0124	134.	.98
Davis	17.43 cm. Diam.	"	120.	100.	..	.0932	"	.0838	.184	.0274	315.	1.00
											Avg......	.975
		20.	30.	10.	..	.00605	.90	.00545	.00838	.00220	18.3	1.11
Griffiths	Short Cyl.	"	45.	25.	..	.0162	"	.0146	.0260	.00570	56.5	1.11
and	8.0 cm. High	"	70.	50.	..	.0368	"	.0331	.0599	.0124	134.	1.14
Davis	17.43 cm. Diam	"	120.	100.	..	.0932	"	.0838	.141	.0274	315.	1.14
											Avg.....	1.13

TABLE VI—*continued*

SUMMARY OF COLLECTED DATA ON FREE CONVECTION FROM VERTICAL PLANE SURFACES IN AIR AT ATMOSPHERIC PRESSURE

Source and Surface	Exposed Area cm.²	Amp. Temp. t_a °C	Surface Temp. t_s °C	Temp. Diff. Δt °C	Total Watts per cm.² w_t	Watts Rad. Black Body per cm.²	Total Emissivity E	Watts Rad. per cm.² w_R	Watts Conv. per cm.² w_c	Watts per cm. $\Delta\varphi$	$\Delta t^{5/4}$	K_1 assuming $n = 1/2$
Griffiths and Davis	Short Cyl. 15.2 cm. High 17.43 cm. Diam.	20.	30.	10.	..	.0605	.90	.00545	.00583	.00220	18.3	1.35
		"	45.	25.	..	.0162	"	.0146	.0180	.00570	56.5	1.35
		"	70.	50.	..	.0368	"	.0331	.0428	.0124	134.	1.35
		"	120.	100.	..	.0932	"	.0838	.101	.0274	315.	1.34
											Avg....	..1.35
Griffiths and Davis	Short Cyl. 28.8 cm. High 17.43 cm. Diam.	20.	30.	10.	..	.0605	.90	.00545	.00424	.00220	18.3	1.57
		"	45.	25.	..	.0162	"	.0146	.0134	.00570	56.5	1.55
		"	70.	50.	..	.0368	"	.0331	.0319	.0124	134.	1.55
		"	120.	100.	..	.0932	"	.0838	.0763	.0274	315.	1.52
											Avg....	..1.55
									Average for all Surfaces.......			1.46

ν = μ/ρ = Kinematic viscosity of fluid for average temperature cm.² sec.⁻¹

α = Coefficient of density charge per deg. cent. for average temperature in deg. cent.⁻¹

g = Acceleration of gravity = 980 cm. sec.²

Δt = Difference between surface and ambient temperature deg. cent.

n = Experimental exponent for the system of similar bodies under consideration

The coefficient α is the coefficient in the familiar equation:

$$\rho = \rho_0 (1 + \alpha t) \text{ gram. cm.}^{-3} \text{ at temperature } t \text{ deg. cent.} \quad (21)$$

where

ρ_0 = density of fluid at 0 deg. cent.

t = Fluid temp. deg. cent.

α = 1/273 for the case of ideal gases and the values for liquids are available in Physical Tables.

If we substitute equation (**20**) in (**19**) we obtain as the general expression for thin films and moderate temperature differences.

$$W_c = (A\,k\,\Delta t/K\,D)\,[\,(\alpha\,g\,\Delta t)^{1/2}\,D^{3/2}/\nu]^{\,n}\,(\mu\,c_p/k)^{\,m} \text{ watts} \quad (22)$$

Where no data are available on convection in liquids we are unable to determine the exponent m. Fortunately for ideal gases, this factor does not vary greatly from one gas to another and may therefore be neglected without seriously affecting the results. On this assumption we obtain for ideal gases

$$W_c = (A\,k\,\Delta t/K_1\,D)\,[\,(\alpha\,g\,\Delta t)^{1/2}\,D^{3/2}/\nu]^n \text{ watts} \quad (23)$$

Equation (**22**) is seen to be similar to the form of relation deduced from certain theoretical considerations by Davis[17] which in our notation becomes

$$W_c = (A\,k\,\Delta t/K\,D)\,[S^2\,g\,D^3\,\alpha\,\Delta t/k^2]^m\,(S\,\nu/k)^r \quad (24)$$

where $S = \rho\,c_p$ = specific heat per unit volume joule cm.⁻³ deg. cent.⁻¹

For *ideal gases* we have from the kinetic theory k proportional to $\mu\,c_p$. If we make these substitutions in equation (**24**) we obtain our equation (**22**). Thus for ideal gases the two methods of attack give the same result but for liquids or imperfect gases the results will be different.

We obtain from the previous paper[1] the following approximation for *air* at *atmospheric pressure*. In the heat conductivity equation we have here assumed the value of specific heat at 400 deg. k. instead of 300 deg. k. so as to obtain a better average value.

$$\nu = \mu/\rho = 7.05 \times 10^{-6}\,T^{1.754} \text{ cm.}^2 \text{ sec.}^{-1} \quad (25)$$

$$k = 3.46 \times 10^{-6}\,T^{.754} \text{ watts per cm. per deg. cent.} \quad (26)$$

We also have for *air*

$$\mu\,c_p/k = 0.74 \text{ numeric} \quad (27)$$

Large Horizontal Cylinder in Gases and Liquids. For this case we substitute D = Cyl. diam. in cm.;

$K = 2.12$; $n = 1/2$ and $m = 1/4$ in equation (**22**) and obtain

$$W_c = 0.472\,A\,[\,(\mu\,c_p/k)\,(\alpha\,g/D)\,]^{1/4}\,(1/\nu)^{1/2}\,k\,\Delta t^{5/4} \text{ watts} \quad (28)$$

For *air* we obtain the following relation by making use of the approximate equations (**25**), (**26**) and (**27**).

$$W_c = 0.000785\,A\,(1/D)^{1/4}\,p^{1/2}\,(1/T_{avg.})^{0.123}\,\Delta t^{5/4} \text{ watts} \quad (29)$$

where

A = $\pi\,D\,L$ = Area in square cm.

D = Cylinder diameter in cm.

p = Absolute air pressure in atmospheres

$T_{avg.}$ = Average of cylinder and ambient absolute temperature deg. K

Δt = Temperature difference deg. cent.

As an example let us apply this equation to calculate the convection from the large cylinder 11.35 cm. diameter and 152.5 cm. long for the atmospheric pressure point.

$$\overline{W}_c = 0.000785 \times \pi \times 11.35 \times 152.5\,(1/11.35)^{1/4} \times 1^{1/2} \times (1/327.9)^{0.123} \times (85.1)^{5/4} = 295 \text{ watts.}$$

The observed value given in the table for this length is 317 watts. Thus, the experimental value is 7 per cent high, which is not far from the probable error.

17. A. H. Davis, *Phil. Mag.*, Vol. 44, p. 938, 1922.

We see from equation (**29**) that for a constant temperature difference the convection will vary approximately inversely as the $\frac{1}{8}$ power of the average absolute temperature. Thus the convection per deg. cent. temperature difference will decrease slowly with rising ambient temperature. In the previous paper[1] an oversight was made in section V under *Effect of Ambient Temperature*. Here the experiments were tested by calculating the film thickness for the high and low ambient temperatures from the observed convection. These calculations showed that the film thickness increased with increasing ambient temperature as was required by the theory, and therefore, the agreement was considered satisfactory. Now the heat conductivity increases with the temperature but not quite so fast as the film thickness increases and, therefore, the convection per deg. cent. should decrease with increasing ambient temperature whereas, the experiments showed an increased convection per deg. cent. for the high ambient temperature. Thus the *Oven Tests* are contrary to the previous as well as the present theory. It is probable that extraneous convection currents were the cause of the trouble.

Long Vertical Cylinder in Ideal Gases. For this case we have only the data by Griffiths and Davis on a single large cylinder. We have, therefore, had to rely solely on the variation of convection with temperature to determine the exponent n and constant K_1 of equation (**23**). The values obtained in Table IV are $n = 2/3$ and $K_1 = 8.65$ and substituting in (**23**) gives

$$W_c = 0.116\, A\, (\alpha\, g)^{1/3}\, (1/\nu)^{2/3}\, k\, \Delta\, t^{4/3} \text{ watts} \qquad (\mathbf{30})$$

For *air* we obtain

$$W_c = 0.00166\, A p^{2/3}\, (1/T_{avg.})^{.415}\, \Delta\, t^{4/3} \text{ watts} \qquad (\mathbf{31})$$

From this equation we observe that for this system of similar figures the convection per unit area is independent of the size.

Large Spheres in Ideal Gases. Here the available data are meager and unsatisfactory. Until further data are available we will take $n = 1/2$ and $K_1 = 2.0$ in equation (**23**) and obtain

$$W_c = 0.5\, A\, (\alpha\, g/D)^{1/4}\, (1/\nu)^{1/2}\, k\, \Delta\, t^{5/4} \text{ watts} \qquad (\mathbf{32})$$

For *air* we obtain

$$W_c = 0.00088\, A\, (1/D)^{1/4}\, p^{1/2}\, (1/T_{avg.})^{0.123}\, \Delta\, t^{5/4} \text{ watts} \qquad (\mathbf{33})$$

A comparison of this equation with the corresponding equation (**29**) for a horizontal cylinder shows that for the same diameter the convection per unit area is approximately the same in both cases.

Large Vertical Plane Surfaces in Ideal Gases. Here we have taken the available data on mongrel types of vertical plane surfaces and assumed that they can be roughly correlated by a vertical plane surface whose horizontal length is large compared with the vertical height and of small thickness. In this manner we obtained $n = \frac{1}{2}$ and $K_1 = 1.46$. If we make these substitutions in equation (**23**) and let $D = H$, the vertical height of our ribbon surface in cm. we obtain,

$$W_c = 0.685\, A\, (\alpha\, g/H)^{1/4}\, (1/\nu)^{1/2}\, k\, \Delta\, t^{5/4} \text{ watts} \qquad (\mathbf{34})$$

For *air* we then obtain

$$W_c = 0.00121\, A\, (1/H)^{1/4}\, p^{1/2}\, (1/T_{avg.})^{0.123}\, \Delta\, t^{5/4} \text{ watts} \qquad (\mathbf{35})$$

It is well to emphasize the fact that these equations can only be used as approximations in engineering calculations. In working up the available data for Table VI it was found that for each type of surface the convection per unit area could be represented by an equation containing a single value of the constant and the 5/4 power of the temperature difference. Under these conditions the method of dimensions shows us that the convection per unit area must also vary inversely as the $\frac{1}{4}$ power of the size factor for strictly similar surfaces. There is of course, no inherent reason why the exponent should remain the same over a great range in either size or temperature but the fact that for horizontal cylinders the exponent changes only slightly over the tremendous range of sizes and temperatures covered by the various experiments makes it appear probable that the same will hold for other systems of similar figures such as spheres similar rectangular, circular disks, similar short vertical cylinders, etc. The values of the constants K_1 for the different systems which have the same exponent have been gathered together below for comparison.

1. Long horizontal cylinders $n = 1/2; K_1 = 2.10$
2. Spheres $n = 1/2; K_1 = 2.02$
3. Langmuir, 19.1-cm. disk inserted in one end of short insulating cylinder 24 cm. diam. by 12.5 cm. deep. Here the characteristic dimension has been taken as the square root of the hot area $n = 1/2; K_1 = 2.00$
4. Griffiths and Davis rectangular plate 127 cm. × 127 cm., back and edged covered by 7.8 cm. thick covering $n = 1/2; K_1 = 1.25$
5. Doherty rectangular plate 25.4 cm. high, 38.1 cm. long, 1.27 cm. thick $n = 1/2; K_1 = 1.68$
6. Montsinger rectangular plate 80 cm. high by 33.2 cm. wide and 2.7 cm. thick with tapered edges and certain edge coverings $n = 1/2; K_1 = 1.32$
7. Griffiths and Davis short vertical cylinder 17.43 cm. diam. by 4.65 cm. high with 7.62 cm. high caps on top and bottom $n = 1/2; K_1 = 0.975$
8. Griffiths and Davis short vertical cylinder 17.43 cm. diam. by 8.0 cm. high, lower cap 7.62 cm. high, upper cap 15.24 cm. high $n = 1/2; K_1 = 1.13$

9. Griffiths and Davis short vertical cylinder 17.43 cm. diam. by 15.2 cm. high upper and lower caps each 7.62 high $n = 1/2; K_1 = 1.35$
10. Griffiths and Davis short vertical cylinder 17.43 cm. diam. by 28.8 cm. high upper and lower caps each 7.62 cm. high $n = 1/2; K_1 = 1.55$

Avg. of 10 values, $K_1 = 1.54$

Each of the above 10 surfaces belongs to a different system of similar surfaces in which the convection per unit area varies approximately as the 5/4 power of the temperature difference but each system requires a different value for the constant.

Strictly speaking, therefore, it is not possible to determine the convection from a transformer tank from experiments on a rectangular plate or from the convection data obtained on a long ribbon surface of the same vertical height. Accurate calculations of convection can only be expected when the exponent n and constant K_1 is known for one, of a similar line of tanks of various sizes. Corrugated cylindrical tanks will be similar, when for each size, the ratio of diameter to height, depth of corrugations to height, size of flange to height, etc., are the same.

Elliptical or rectangular tanks each belong to a separate line of similar tanks. Montsinger's[18] tests on convection from transformer tanks bring out the kinds of variations to be expected for three different types of tank.

The fact that we cannot expect accurate calculations of convection from large surfaces of various types from a single equation does not mean that it is not desirable to take an equation such as (35) and use it as an approximation for engineering calculations, where a closer approximation is not available. For such work, Montsinger has proposed the following equation for air.

$$W_c = 0.000217\, A p^{1/2} \Delta t^{5/4} \text{ watts} \qquad (36)$$

Here the convection per unit area is taken independent of the height of surface.

For surfaces over 100 cm. high the approximate equation suggested by Griffiths and Davis for air at atmospheric pressure gives

$$W_c = 0.000205\, A\, \Delta t^{5/4} \text{ watts} \qquad (37)$$

For a surface 100 cm. high and an average of surface and ambient temperature of 350 deg. our equation (**35**) gives

$$W_c = 0.000186\, A p^{1/2} \Delta t^{5/4} \text{ watts} \qquad (38)$$

The constant in equation (**36**) was obtained by Montsinger from tests on the vertical rectangular surface 80 cm. high by 33.2 cm. wide. The constant in equation (**37**) was obtained by Giffiths and Davis from the long vertical cylinder 17.43 cm. diameter and 273.5 cm. high by forcing the convection to follow the 5/4 power of the temperature difference. The constant in equation (**38**) was obtained from averaging the column of figures in Table VI. By giving different weights to the various experiments we could obtain a considerably larger or smaller value of the constant. In this connection, tests on a series of *similar surfaces* such as large cylindrical tanks would be of considerable interest and value. Highly polished surfaces should be used so as to reduce the uncertainties involved in the radiation corrections.

18. V. M. Montsinger, TRANS. A. I. E. E., Vol. XXXV, Part I, p. 606, 1916

SUMMARY AND CONCLUSIONS

(1) The more general expression for the effective film thickness, here developed, is seen to be superior to the simpler expression of the previous paper since the resulting equations account accurately for the convection from large and small bodies at both *high* and *low* temperature differences.

(2) The general expression for the effective film thickness obtained by the method of dimensions for free convection is

$$B = K\, D\, (k/\mu\, c_p)^m \{\nu/[(\alpha\, g\, \Delta t)^{1/2} D^{3/2}]\}^n \text{ cm.} \qquad (39)$$

where K = Experimental constant depending on the system of similar bodies under consideration
D = Characteristic linear dimension of body in cm.
k = Heat conductivity at average temperature watt cm.$^{-1}$ deg. cent.$^{-1}$
μ = Viscosity of fluid at avg. temp. cm.$^{-1}$ gram. sec.$^{-1}$
c_p = Specific heat at const. pressure for avg. temp. joule gram.$^{-1}$ deg. cent.$^{-1}$
m = Experimental exponent
$\nu = \mu/\rho$ = Kinematic viscosity of fluid for average temperature cm.^{2} sec.$^{-1}$
α = Coefficient of density change per deg. cent. for average temperature in deg. cent.$^{-1}$, for ideal gases $\alpha = 1/273$
g = Acceleration of gravity = 980 cm. sec^2
Δt = Difference between surface and ambient temperature deg. cent.
n = Experimental exponent for the system of similar bodies under consideration.

(3) When dealing with *ideal gases* the first factor in equation (**39**) can be neglected since it does not vary greatly from one gas to another. Under these conditions we write the effective film thickness

$$B = K_1 D \{\nu/[(\alpha\, g\, \Delta t)^{1/2} D^{3/2}]\}^n \text{ cm.} \qquad (40)$$

(4) The free convection from *a long horizontal cylinder* in *gases* and *liquids* is given by

$$W_c = 2\,\pi\, L\, \Delta\, \varphi/\log_\epsilon [(2\,B + D)/D] \text{ watts} \qquad (41)$$

where L = Length of cylinder in cm.
$\Delta\varphi$ = Thermal conduction in watts per cm.
D = Diameter of cylinder in cm.
B = Film thickness in cm. from eq. (**39**)

A sufficiently close approximation is usually obtained by taking $n = \frac{1}{2}$; $K = 2.12$ and $m = \frac{1}{4}$ in equation (**39**).

(5) The free convection from *a long vertical cylinder* in *gases* is given by equation (**41**) and the film thickness by equation (**40**). The meager available data give

$n = 2/3$ and $K_1 = 8.65$

(6) The free convection from a *sphere* in *gases* is given by

$$W_c = 2\,\pi\,\Delta\,\varphi/\,[1/D - 1/(2\,B + D)] \text{ watts} \quad (42)$$

and the film thickness by equation (**40**) in which

$n = \frac{1}{2}$ and $K_1 = 2.02$

(7) For *large surfaces* the body size will usually be small compared with the film thickness and the shape conductance will then take the simple form $S = A/B$. For moderate temperature differences we may also take $\Delta\,\varphi = k_{avg.}\,\Delta\,t$. Under these conditions our general expression for free convection becomes

$$W_c = (A\,k\,\Delta\,t/K\,D)\,(\mu\,c_p/k)^m\,[(\alpha\,g\,\Delta\,t)^{1/2}\,D^{3/2}/\nu]^n \text{ watts} \quad (43)$$

For *ideal gases* we may omit the first factor without great error and obtain

$$W_c = (A\,k\,\Delta\,t/K_1\,D)\,[(\alpha\,g\,\Delta\,t)^{1/2}\,D^{3/2}/\nu]^n \text{ watts} \quad (44)$$

(8) For *a large long horizontal cylinder* in *gases* and *liquids* we take $K = 2.12$; $n = 1/2$ and $m = \frac{1}{4}$ in equation (**43**).

For the convection in *air* we obtain

$$W_c = 0.000785\,A\,(1/D)^{1/4}\,p^{1/2}\,(1/T_{avg.})^{0.123}\,\Delta\,t^{5/4} \text{ watts} \quad (45)$$

where $A = \pi\,D\,L$ = Area in square cm.

D = Cylinder diam. in cm.

L = Length of long cylinder in cm.

p = Absolute air pressure in atmospheres

$T_{avg.}$ = Average of cylinder and ambient absolute temperature deg. K

$\Delta\,t$ = Temperature difference deg. cent.

(9) For *a large long vertical cylinder* in *gases* we take

$n = 2/3$ and $K = 8.65$ in equation (**44**)

For *air* we obtain the following approximation

$$W_c = 0.00166\,A\,p^{2/3}\,(1/T_{avg.})^{0.415}\,\Delta\,t^{4/3} \text{ watts} \quad (46)$$

(10) For *large spheres* in *gases* we take

$n = \frac{1}{2}$ and $K_1 = 2.0$ in equation (**44**)

For *air* we then obtain the following approximation

$$W_c = 0.00088\,A\,(1/D)^{1/4}\,p^{1/2}\,(1/T_{avg.})^{0.123}\,\Delta\,t^{5/4} \text{ watts} \quad (47)$$

(11) For a *long thin vertical plane* surface (ribbon surface) in *gases* we may take as an approximation $n = \frac{1}{2}$ and $K_1 = 1.46$ in equation (**44**). We also take D equal to the vertical height H in cm. For *air* we then obtain

$$W_c = 0.00121\,A\,(1/H)^{1/4}\,p^{1/2}\,(1/T_{avg.})^{0.123}\,\Delta\,t^{5/4} \text{ watts} \quad (48)$$

(12) The more general theory of the present paper shows that *free convection* tests *can not be used* as a *primary method* of *obtaining* heat *conductivities*. The method however, is still of interest where an approximate value of the heat conductivity already exists. Below the heat conductivities have been recalculated by this method.

No. 12

Transil Oil	$k = 0.0017\,(1 - 0.0026\,t)$	watt cm.$^{-1}$	°C.$^{-1}$	
Toluene	$k = 0.0015\,(1 - 0.0029\,t)$	"	"	"
Glycerine	$k = 0.0033\,(1 - 0.0038\,t)$	"	"	"
Aniline	$k = 0.0020\,(1 - 0.0018\,t)$	"	"	"
CCl_4	$k = 0.0012\,(1 - 0.0038\,t)$	"	"	"
Olive Oil	$k = 0.0016\,(1 + 0.0055\,t)$	"	"	"

The value for olive oil should not be given much weight, due to the large uncertainty concerning the viscosity. The present method gives, excepting olive oil, negative temperature coefficients, whereas, the method of the previous paper gave positive values.

In conclusion, the writer wishes to express his thanks to Mr. V. M. Montsinger for his valuable criticism of the first paper which brought out the limitations of the previous theory and stimulated the present work. The writer is also indebted to Mr. R. E. Doherty for the use of his unpublished data on convection from plane surfaces.

Discussion

V. M. Montsinger: About ten years ago or something over, Dr. Langmuir published his first paper on the film theory, in which he showed the remarkable agreement between the test results and calculations, at high temperatures, principally from 100 up to 1000 degrees. I could never get satisfactory results at temperatures below 100 degrees, which is a very important field for electric apparatus, especially transformers, generators and motors, and in fact, Dr. Langmuir himself agreed with me that the film theory did not check exactly at these low temperatures, and there was some factor that was not exactly known. Now, fortunately, Mr. Rice has shown us the method to use and has introduced a factor that takes care of the discrepancies.

For the first time I feel that we really understand the phenomena of free convection in air for both small and large temperature differences. The solution of this has been made possible principally by the use of the apparently little used and little understood "Method of Dimensions."

It will be worth while to say a few words as to what the Method of Dimensions is and how it is used. Method of dimensions simply means that each factor such as air density, viscosity, conductivity, temperature rise, etc., that affects the rate of transfer of heat from a heated surface to the air is expressed in terms of the dimensions of the body like L for the length and L^3 for the volume or T for time or θ for temperature rise. For instance, density is represented by mass M, divided by the volume L^3. Conductivity is represented by the heat, H, divided by the factors of length, L, time T, and temperature drop θ. In Mr. Rice's equation (1), each factor to start out with has its particular exponent of say, x, y, z, or n, etc. In equation (2) the dimensional equivalents are substituted for the factors themselves such as density, viscosity, etc., but the equation retains its original exponents. By equating the value of the exponents for length, mass, time, heat, and temperature and solving the simultaneous equations any exponent may be expressed in terms of some particular one, like, say n or m.

Having obtained the relationship between the letters representing the exponents, the exponents of each factor in equation (1) can now be expressed in the same terms as for instance; n or $\frac{3\,n}{2}$. It happens however, that all but one can be expressed in terms of n, the remaining one being m.

The numerical value for anyone of these exponents must, of course, be obtained by experimental test. But after we have determined the value of, say n, for one factor like the variation in loss with air density, we also have the solution for the correct exponents for all the other factors because they are also expressed in terms of n. In other words if we find that loss is proportional to the square root of the air density in which case n equals ½, we know how the loss is affected by a change in each of the other factors such as viscosity, temperature rise, or dimensions of the object. Thus by taking advantage of what the method of dimensions can tell us we greatly reduce the number of experiments which have to be tried in order to determine the law for the system of similar bodies under consideration. This is where the Method of Dimensions comes in and serves as a very useful tool.

In addition to the value of the correct exponent for temperature rise, that is, whether the loss is proportional to temperature rise raised to the 5/4 or some other power, there has been a considerable difference of opinion as to the correct value of the exponent for air density. In other words, does the loss vary as the density raised to the ½ or 2/3 power or some other power? The value of exponential values claimed by different investigators has been from about 2/5 to 2/3 or ¾. I have never made any accurate laboratory tests to determine this but from my experience in testing transformer tanks having different shapes of surfaces at different altitudes I have found that the value of ½ checks best the observed values.

In regard to the convection loss varying as the temperature rise raised to the 5/4 power I have been quite certain for several years that this exponential value is about correct.

As mentioned before since the Method of Dimensions shows a definite relationship between the exponents for the different factors if we are certain of one we know also what the others should be. I note with a great deal of interest that according to the Method of Dimensions if the 5/4 power is correct exponent for temperature rise it requires that the exponent must be ½ for air density. I believe therefore that Mr. Rice has proven beyond a reasonable doubt that loss by free convection in air should vary as the square root of the barometric pressure especially for large vertical surfaces.

As to the effect of height of plane on loss by convection my experience has been that for heights over three or four feet the affect of height is very small. I had occasion some two or three years ago to test the efficiency of different shapes of corrugations having a uniform (unity) vertical surface temperature gradient. These tests showed that the convection loss for a height of 72 in. was only about 3 per cent less than the loss for the same corrugations 36 in. in height. In other words the convection decreased roughly 1 per cent for each foot increase in height over 3 ft. In fact in practise I have never made any decrease in loss per unit area of corrugations (where most of the heat is carried away by convection) for heights over three or four feet.

Griffeths and Davis referred to in the paper found that the loss by convection per unit area was approximately the same for heights ranging from about 23 in. to 104 in. although they found that the value of the exponent increased slightly over 5/4 for the higher surfaces. However for heights under about one foot they found that height had a very decided effect on the loss by convection. This is as it should be according to the factor H ¼ in the method of dimensions.

C. W. Rice: There is just one thing which I would like to emphasize in closing the discussion and that is the need of more data on the free convection from various families of similar figures such as spheres, short cylinders, etc.

The Magnetic Properties of the Ternary Alloys Fe-Si-C

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Review of the Subject.—*The variations in the magnetic properties of iron and iron-alloys, even of supposedly constant composition, has been puzzling to the users and investigators of ferro-magnetic materials ever since the introduction of such materials for electrical apparatus. The author started to investigate this problem over ten years ago at the University of Illinois, and has continued it at the Westinghouse Research Laboratory since 1916, concentrating on iron and iron-silicon alloys. While the results obtained do not eliminate 100 per cent of the difficulties, they go a long way in that direction.*

It has been found that carbon is largely responsible for the variations, because of the fact that amounts so small as previously to be regarded as traces—less than 0.01 per cent—remain dissolved in the iron in the solid state, even after slow cooling, and have a tremendous influence on the magnetic properties. Of much less effect is carbon precipitated as pearlite, free cementite and graphite, the effect being in the order named. If the effect of dissolved carbon be represented by 100, the effect of carbon as pearlite is 16.5, of carbon as Fe_3C 2.25, and of carbon as graphite nearly nil. The form assumed by carbon—aside from the carbon in solution—depends largely on the silicon content, and can best be explained by referring to Fig. 29.

Besides carbon, it has been found that the grain size has a large and definite influence on the magnetic properties due to the accompanying inter-crystalline amorphous cement that may be regarded as an impurity similar to other inter-crystalline impurities.

The detrimental effect of sulphur, phosphorus and manganese on pure iron is in the order named, while phosphorous has a beneficial effect on high silicon alloys.

The evidence obtained is to the effect that the increased reluctivity, coercive force, or hysteresis loss due to carbon and other impurities that are precipitated combined with iron—including in this class the inter-crystalline cement—is caused by the inherent corresponding property of these precipitated impurities.

Regarding the tremendous effect of carbon in solution, it is suggested that this is due to the entering of carbon into the more or less stable equilibrium arrangement of the ferro-magnetic structure, upsetting this equilibrium arrangement.

Introduction

THE superiority of silicon-steel over ordinary iron for magnetic purposes was discovered by Hadfield in 1899.[1]

The steel—containing 4 per cent silicon—was first used commercially in England in 1903[2] and immediately proved a decided success.

The first patent in the United States[3] on silicon steel was granted Hadfield on December 1, 1903, but it was not until 1906 that the first transformer was built in this country using the new steel. Since that time silicon steel has been used almost exclusively for transformers, both here and abroad, with an enormous saving in energy, and it is still without a competitor, in spite of the many investigations[4] made for the purpose of discovering one. However, the 4 per cent silicon steel of today is much better, magnetically, than Hadfield's original steel, due to improvements in manufacturing processes, in raw materials and in methods of heat treatment. Roughly speaking, the hysteresis loss is only one-half and the maximum permeability is at least twice that of Hadfield's steel, but these improvements were largely accomplished prior to 1910. During the last ten years the average improvement—as far as can be gathered—has been approximately 10 per cent, and this improvement has been due more to minor improvements in the heat-treatment than to refinements in composition.

1. Barrett, Brown & Hadfield; *Sci. Trans.* Royal Dublin Soc. VII, Ser. 2, part 4, Jan. 1900.
Journal Inst. Elec. Engrs. Vol. 31, p. 674, (1902).
2. Hadfield: History of the Metallurgy of Iron and Steel, *Proc.* Inst. Mech. Engrs., Feb. 8, 1915, p. 332.
Yensen: The Development of Magnetic Materials, *Elect. Journal* 18, p. 93, March 1921.
3. U. S. Patent No. 745,829.
4. See: TRANS. A. I. E. E., 34, p. 2455 et seq. Oct. 1915, *Historical Review*. Bull's 72 and 83. Eng. Exp. Stn. Univ. of Ill., Historical Reviews.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

Resumé of Previous Investigations

The effect of the ordinary impurities on the magnetic properties of iron and iron alloys has not to any great extent formed the main object of systematic scientific investigations. The most probable reason for this is that the factors influencing the magnetic properties are too numerous and were not sufficiently well known to enable the investigators to eliminate the undesirable variables. The objects have rather been to determine the magnetic properties of alloys of iron with elements that will, or may, bestow decidedly new and useful properties upon the iron, and to disregard the effect of such incidental impurities as usually occur in commercially pure iron of the best grades. Hadfield[5], for example, used Swedish charcoal iron as the base for his investigations and prepared his alloys under commercial conditions. Burgess and Aston[6] on the other hand attempted to get away from this uncertainty and were the first to use electrolytic iron as the base for their alloys. Unfortunately, however, the prep-

5. Barrett, Brown & Hadfield: Sci. *Trans.* Royal Dublin Soc. VII. Ser. 2, Part. 4, Jan. 1900.
Journal, Inst. Elect. Engrs. 31, p. 674, (1902).
6. *Trans.* Am. Electrochem, Soc. XV. p. 369, 1909.
Chem. & Met. Engr. Jan., Feb., Mar., Apr., 1910.

TABLE I
PROGRESS MADE IN RECENT YEARS

Year	Investigator	Kind of Material used	Maximum Permeability	Coercive Force gilberts per cm.		Hysteresis Loss ergs per cc. per ~*	
				For $B_{max} = 10000$	For $B_{max} = 15000$	For $B_{max} = 10000$	For $B_{max} = 15000$
1900	Hadfield	Sw. Char. Iron	4,000	0.920	1.00	abt. 2700	abt. 5500
1900	Hadfield	2½ per cent Si-Iron	5,100	0.72	0.79	" 2200	" 4700
1901	Gumlich & Schmidt	Wrought Iron	8,350		0.60		
1903	Baker	4.9 per cent Si-Iron			1.20		" 6200
1910	Terry	Electrolytic Iron	11,000				
1912	Gumlich & Goerens	0.4 per cent Si-Sheets	11,600		0.54		
1912	Gumlich & Goerens	4.0 per cent Si-Sheets	9,400				
1912	Paglianti	1.75 per cent Si-Iron		0.60	0.75	1650	3500
1914	Yensen	Pure Vacuum Iron	19,000		0.29	813	1640
1915	Yensen	0.15 per cent Si-Vacuum Iron	66,500	0.09	0.16	286	916
1915	Yensen	3.40 per cent Si-Vacuum Iron	63,300	0.08	0.15	280	1025

*1000 ergs/cub. cm./cycle = 0.80 watts/kg. at 60 cycles for 4% Si-Steel (Sp. Gr. 7.5) and = 0.76 watts/kg. for pure Fe (Sp. Gr. 7.9).

arations were made under conditions (a Hoskins carbon plate furnace) that reintroduced carbon in varying amounts into their alloys and they consequently lost the advantage they had in using a pure base. When the writer started his investigations in 1912 he took advantage of the results of Burgess & Aston and prepared his alloys in a vacuum furnace in such a way that carbon was not reintroduced to any appreciable extent, with the result[7] that the magnetic properties obtained, both for pure iron and for iron-silicon alloys, were far superior to those obtained by previous investigators. This improvement is shown in Table I, reproduced from previous publications.[8] The results for pure iron were later confirmed by Gumlich, using Fischer electrolytic iron and vacuum treatment.[9]

In these cases, the efforts were concentrated on eliminating and on keeping away the impurities, but in spite of these efforts it was impossible to obtain consistent results and to duplicate results. Not having control over some of the most important factors, and—what was worse—not even knowing what they were, it was evidently useless to attempt to determine the effect of impurities that might produce changes far less than the changes caused by the uncontrollable impurities. Aside from the usual impurities, the question of grain size enters as an important factor affecting the magnetic properties. Ruder has found[10], in the case of silicon steel that the larger the grain size the better the magnetic properties; he also showed how the grain size could be increased, by cold deformation followed by high temperature annealing, without, however, being able definitely to control it.

Preliminary Work

When, in 1916, the writer transferred his activities to the Research Laboratory of the Westinghouse Company and there tried to duplicate the results previously obtained at the University of Illinois, difficulties were encountered. The results for pure iron checked fairly well but not so for the iron-silicon alloys (Fig. 1). The magnetic properties were generally very much inferior and the more so the higher the silicon content; furthermore, the results varied depending upon the grade of silicon used in spite of the fact that chemical analysis of the alloys revealed no consistent differences. After a great deal of experimentation on 4 per cent *Fe-Si* alloys, it was found that annealing under oxidizing conditions at a temperature of 950-1100 deg. greatly improved the magnetic properties, while annealing in vacuum, hydrogen or nitrogen had no such beneficial effect (Figs. 2, 3, 4). By applying this method of heat treatment to the original series of *Fe-Si* alloys it was found that all of the alloys were susceptible to the treatment. Some responded very quickly (an hour or two at 1100 deg.) while others were much more resistant (requiring 8 to 10 hours) but in all cases, the maximum permeability reached 20,000-35,000 and the hysteresis loss 400-550 ergs per cu. cm. per cycle for $B = 10,000$ gauss (Figs. 5 and 6). Fur-

7. Univ. of Ill. Engr. Exp. Sta. Bull's 72, 77, 83, 95, 1914-17.

8. Bull. No. 83, p. 44, Eng. Exp. Stn. Univ. of Ill. and Trans. A. I. E. E. 34, p. 2601 (1915). It may be mentioned that the last two values of maximum permeability probably are too high, as mentioned in the above papers, due to errors in the Burrows method of testing, when large compensating currents have to be used, so that the true maximum permeability probably is in the neighborhood of 40,000 instead of 60,000.

9. Wissensehaftliche Abhandlungen der *Physikal. Tech. Reichsanstalt*, Berlin, Vol. IV, No. 3, 1918, & Vol. V, No. 2, 1922. *ETZ* 36, pp. 675-77, 691-94, 1915.
Phys. Zeitschr. 19, pp. 434-36, 1918.
ETZ, Nos. 26 and 27, 1919.
Stahl u. Eisen, 41, p. 1249-54, 1921.
For a rod sample of electrolytic iron cut from a cathode sheet and annealed in a vacuum furnace at 1000 deg. cent. Gumlich obtained the very low coercive force of $H_c = 0.11_5$ gilberts/cm. after subjecting it to a magnetizing force of $H = 150$, a value that has not been obtained for fused electrolytic iron, the lowest value being $H_c = 0.15$ (See Table 4). However, a value of $H_c = 0.10$ for $B = 15{,}000$ was obtained by the writer in 1915 for ring samples of 3.0 per cent Fe-Si alloys. (See: University of Illinois Eng. Exp. Sta. Bulletin No. 83, p. 67 Table 13, or Trans. A. I. E. E., Vol. 34, p. 2664, Table 10).

10. A. I. M. E. Trans. 1913, p. 2805.

thermore, it was found that commercial 2 and 4 per cent silicon steels could be improved to nearly the same extent as the laboratory prepared alloys (Fig. 7).

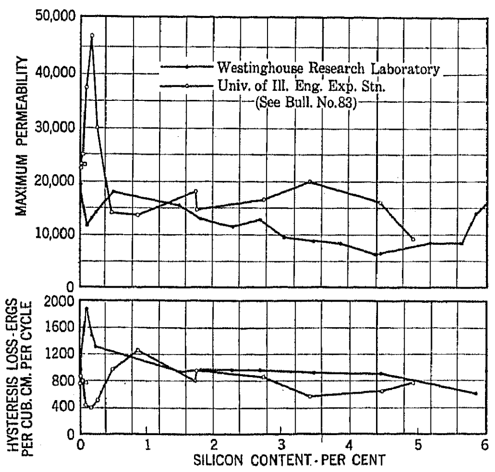

Fig. 1—Magnetic Properties of Iron-Silicon Alloys Annealed at 900 Deg. Cent. in Vacuo

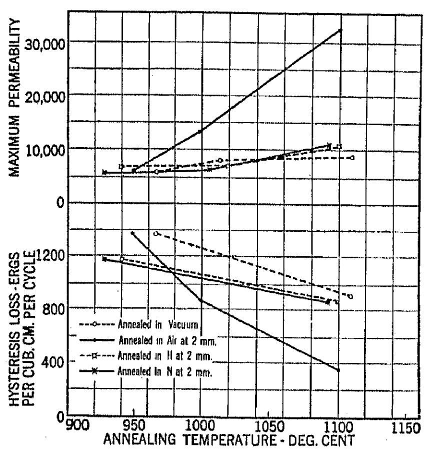

Fig. 2—Magnetic Properties of 4 Per Cent Iron-Silicon Alloys. Effect of Annealing in Vacuum, N, H or Air

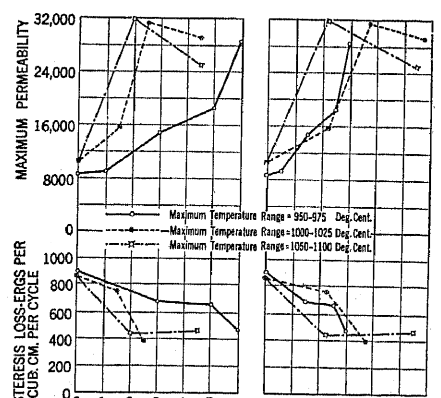

Fig. 3—Magnetic Properties of 4 Per Cent Iron-Silicon Alloys. Effect of Annealing of various Temperatures on Current of Air at 2 mm. Hg.

Analysis of the gases given off during the annealing confirmed the suspicion that carbon was being eliminated and quantitative tests revealed some relationship between the magnetic properties and the amount of carbon eliminated (Figs. 8, 9 and 10). It was concluded that elimination of carbon was the cause of the improvements, although direct evidence in the way of chemical analysis of the annealed test pieces was not yet obtainable for the reason that the methods available were not sufficiently accurate for this purpose, bearing in mind that 0.05 per cent means high carbon. On account of the great importance attributed to car-

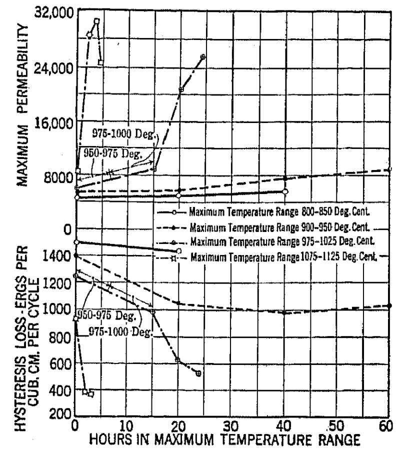

Fig. 4—Magnetic Properties of 4 Per Cent Iron-Silicon Alloys. Effect of Annealing in Current of Air at 10 Hg. at various Temperatures

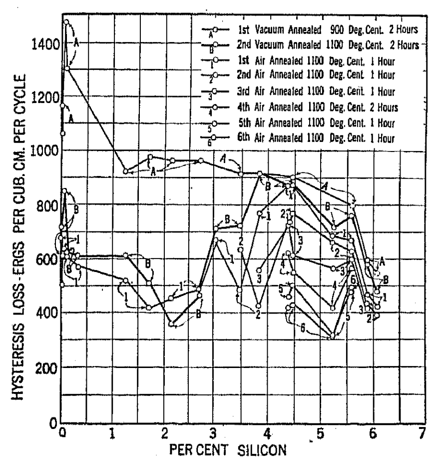

Fig. 5—Maximum Permeabilities of Iron-Silicon Alloys. Effect of Vacuum and Air Treatments

bon a new method was developed for carbon analysis[11] whereby the sample to be analyzed could be heated first to 600 deg. in vacuum and then to 1100 deg. in oxygen, and the resulting CO_2 frozen out in a liquid air trap and measured by the pressure exerted when evaporated into a known evacuated volume (Fig. 11). An accuracy of $\pm$ 0.0001 per cent was obtainable by this method and this furnished the means for getting the desired data.

11. Carbon in Iron, *Trans.* Am. Electrochem. Soc. 37, p. 227, 1920.

A number of samples of 2 per cent and 4 per cent silicon steels variously decarbonized, were tested and analyzed for carbon. The results completely verified the expectations, the curves for hysteresis loss vs. carbon content having the equation (see Fig. 12).

$$W_h = 5370 \times C^{0.425}$$

for 4 per cent Si steel, and

$$W_h = 9650 \times C^{0.493}$$

for 2 per cent Si steel.

These equations forming the first approximation to

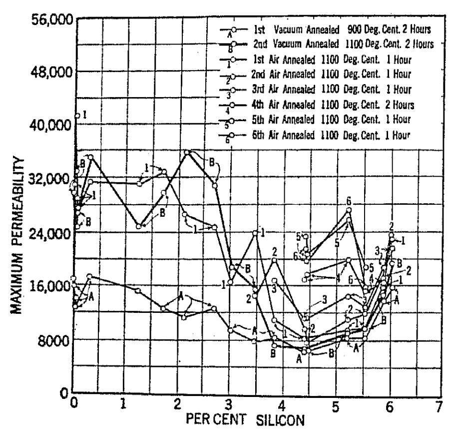

Fig. 6—Hysteresis Losses of Iron-Silicon Alloys. Effect of Vacuum and Air Treatments

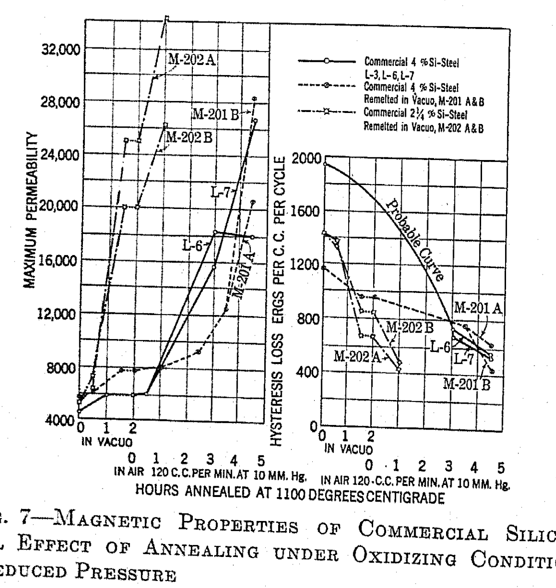

Fig. 7—Magnetic Properties of Commercial Silicon-Steel Effect of Annealing under Oxidizing Conditions at Reduced Pressure

the true relationship between the magnetic properties and carbon, would, if correct for all carbon contents, lead to the startling conclusion that zero carbon should correspond to zero hysteresis loss and that carbon is the only factor affecting the magnetic properties, as none of the other impurities, nor the structural characteristics, were considered in plotting the above data. Before drawing such sweeping conclusions, however, it was deemed advisable to get more reliable data, to use test samples in the form of rings, and to determine carefully the effect of impurities like sulphur, phosphorus and manganese.

In order to cover the range of probable silicon contents it was decided to investigate the four series: 0, 2, 4, and 6 per cent silicon. The effect of sulphur, phosphorus and manganese was investigated for the 0 and 4 per cent alloys only.

A description of the method of magnetic testing and of the general procedure will be found in the appendices to this paper.

FIRST SERIES: 0 PER CENT SILICON

The effect of carbon, sulphur, phosphorus, manganese and grain size on the magnetic properties of pure iron.

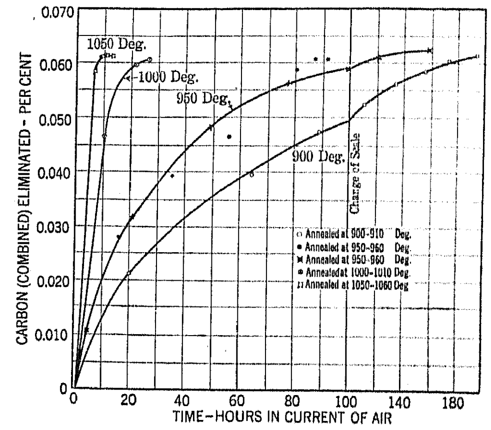

Fig. 8—Elimination of Carbon from 4 Per Cent Silicon Steel by Annealing in Current of Air at Various Temperatures

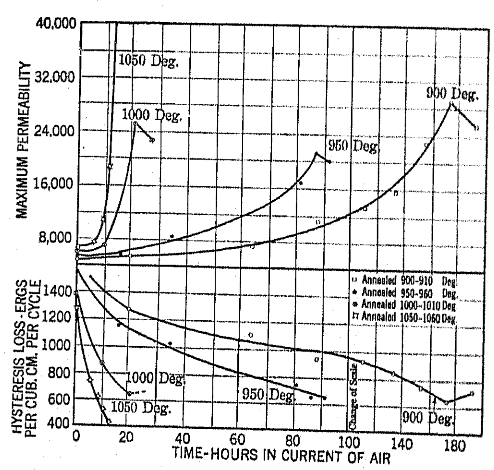

Fig. 9—Magnetic Properties of 4 Per Cent Silicon Steel and the Effect of Annealing under Oxidizing Conditions at Various Temperatures

A large number of samples were prepared both with and without additions of carbon. Analyses of ingots made from electrolytic iron with no other ingredients added are as follows:

C —0.005—0.020 per cent
S —0.001—0.026 per cent
Si —0.020—0.030 per cent
Mn —0.001—0.002 per cent
P —0.002—0.005 per cent

The variations in sulphur content have been found to

be due to the degree of care with which the electrolyte was washed off the electrolytic iron before charging it into the furnace and the variations in carbon to the degree to which the graphite parts of the furnace had been baked out at the time the ingot was made. From

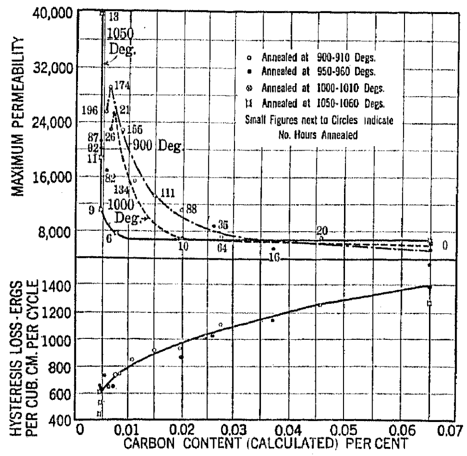

FIG. 10—MAGNETIC PROPERTIES VS. CARBON CONTENT. 4 PER CENT SILICON STEEL (N. 1 AW)

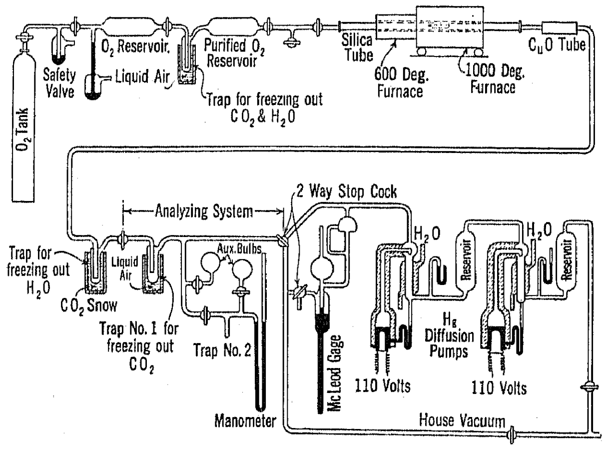

FIG. 11—APPARATUS FOR CARBON DETERMINATION

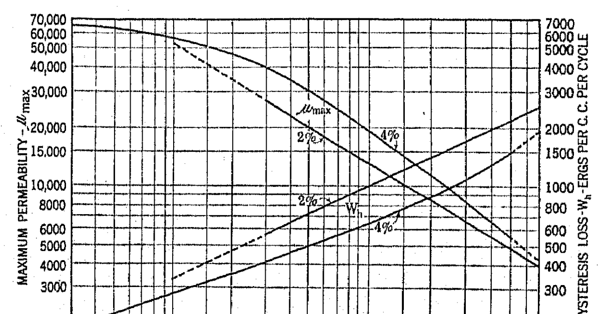

FIG. 12—MAGNETIC PROPERTIES VS. CARBON CONTENT, 2 PER CENT AND 4 PER CENT COMMERCIAL SILICON STEEL

this, it is apparent that it is practically impossible to prepare samples entirely free from all incidental impurities and furthermore, it is also very difficult to prepare a set of samples in which the incidental impurities are constant[12]. On these accounts, it became necessary at first to obtain an approximate relationship between carbon and the magnetic properties by ignoring the incidental impurities (0.01—0.026 per cent S, 0.002—0.005 per cent P, and 0.001—0.002 per cent Mn), then to use this relationship to determine the effect of sulphur, using different samples, containing up to 0.10 per cent S, as this element appeared to be the second most important, and then to use these two approximate relationships to determine the effects of phosphorus and manganese, again using two other sets of samples, one containing up to 0.14 per cent P and the other up to 0.60 per cent Mn. Having done this, the effect of carbon was re-determined making use of the approximate effect of S, P and Mn, just found. This second approximation for the effect of carbon was then used to re-determine the effects of S, P and Mn. This process was repeated until the last determination checked the previous one. To go through this process for all the various magnetic

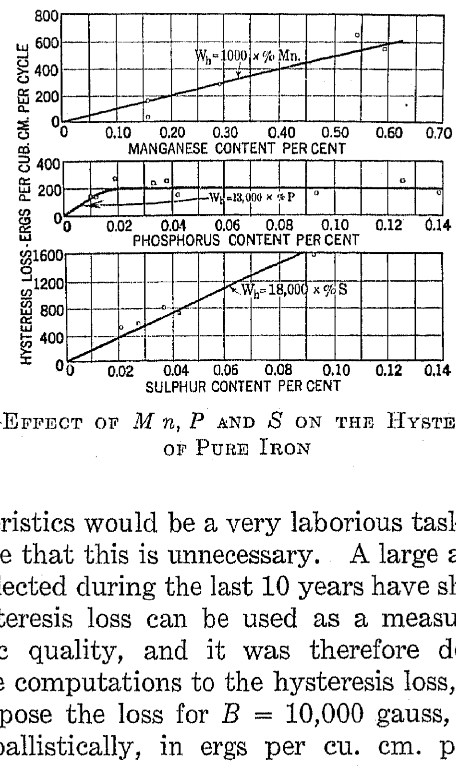

FIG. 13—EFFECT OF Mn, P AND S ON THE HYSTERESIS LOSS OF PURE IRON

characteristics would be a very laborious task and it is fortunate that this is unnecessary. A large amount of data collected during the last 10 years have shown that the hysteresis loss can be used as a measure of the magnetic quality, and it was therefore decided to limit the computations to the hysteresis loss, using for this purpose the loss for $B = 10,000$ gauss, as determined ballistically, in ergs per cu. cm. per cycle. However, additional data in regard to permeability and reluctivity are given in Appendix 3 with no corrections made for incidental impurities. The original data are given in Tables IV-VII. inclusive in Appendix 5.

The final calculations in regard to hysteresis loss have been plotted in Figs. 13, 14 and 15. From these

12. Since the completion, about two years ago, of the experimental work upon which this paper is based, high frequency induction furnaces have been installed, and this, in conjunction with a more careful preparation of the electrolytic iron, has to a large extent eliminated these variables, so that we now can prepare ingots with 0.005 per cent C and 0.004 per cent S, the variations being only ± 0.001 per cent.

curves it will be seen that manganese has a very small effect,

$$W_h = 1000 \times Mn$$

For such small amounts of Mn as occur incidentally, namely 0.001—0.002 per cent, the effect of Mn is to raise the hysteresis loss only 1—2 ergs. However, as the Mn content is increased the effect on the hysteresis loss increases in proportion so that for 0.5 per cent Mn the increase in the loss is 500 ergs.

Phosphorus acts very differently from manganese. For low amounts (up to 0.015 per cent) the hysteresis loss increases in accordance with the formula:

$$W_h = 13{,}000 \times P,$$

but from this point on (0.015 per cent) the effect remains constant (200 ergs) even up to 0.14 per cent. For the incidental amounts of P (0.003—0.005 per cent) the effect is to increase the loss by 25–65 ergs. This effect of phosphorus indicates solubility of P up to 0.015 per cent, followed by precipitation of P as a non-magnetic compound of Fe for larger amounts.

Sulphur again acts like manganese, only to a much greater degree.

$$W_h = 18{,}000 \times S$$

Up to the limit of the present investigation, namely 0.1 per cent S, sulphur thus is very effective in increasing the hysteresis loss, even in the incidental amounts of <0.026 per cent, for which case the increase in loss is 470 ergs.

The previous cases have been investigated partly for the purpose of determining what corrections are necessary due to the incidental impurities in order to determine the effect of carbon. On account of the slight effects of P and Mn these latter are disregarded and the effect of S only is taken into account. Figs. 14 and 15 show the effect of carbon. For carbon contents below 0.006 per cent, and disregarding for the present the effect of grain structure, carbon should increase the hysteresis loss in accordance with the formula:

$$W_h = 220{,}000 \times C$$

Between 0.006 and 0.010 per cent C, the slope is very much less, dropping to 2250 for the upper range. For higher carbon contents, 0.1 to 1.0 per cent, the rate is again higher, being 16,700, but does not begin to approach the rate for very low carbon contents.

A great deal of consideration has been given to explaining this peculiar effect of carbon. As the curve developed, and as it became evident what its shape would be, it was suggested that it be due to different forms in which carbon might exist in the iron. Pearlite being the familiar form, it was natural to suggest that this was the form in which carbon existed in the region 0.1 to 1.0 per cent. It was also thought since the effect of carbon was much less in the region 0.01 to 0.10 per cent that carbon here might be in the form of $Fe_3\,C$ unassociated with ferrite. Finally, it was believed on account of the tremendous effect of carbon in the region below 0.01 per cent that carbon here must be in solution in the iron. At that time the knee of the curve had not been definitely determined, but it was known to be in the neighborhood of 0.01 per cent. In the meantime Mr. N. B. Pilling had been investigating this problem metallographically and it was gratifying to find that his results substantially confirmed the original assumptions. According to his results the knee of the curve was in the neighborhood of 0.005 per cent C and subsequent results obtained magnetically agreed fairly well with this figure. Here is another striking proof that the various physical and structural characteristics go hand in hand. The gradual change in the slope of the curve (Fig. 14) from 220,000 to 2250 indicates that there is no sharp dividing point between the region in which carbon is in solution and the region in which it is precipitated as $Fe_3\,C$. The probability is that carbon continues to go into solution to some extent above 0.006 per cent, perhaps depending upon the annealing conditions. The curves for minimum reluctivity (Fig. 31 Appendix 3) and for electrical resistance (Fig. 37 Appendix 4) indicate that carbon continues to go into solution to some extent up to 0.02 per cent.

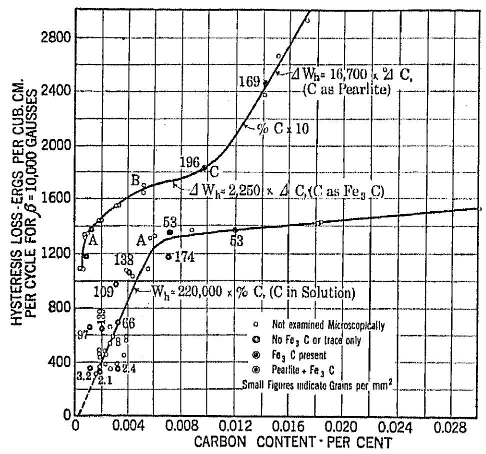

FIG. 14—EFFECT OF CARBON ON THE HYSTERESIS LOSS OF PURE IRON (EFFECT OF GRAIN SIZE NOT ELIMINATED)

A few of the above alloys were examined under the microscope, and photo micrographs obtained (see Fig. 16) using the ordinary etching reagent to bring out pearlite, namely, picric acid, and a magnification of 100 dia. In order to reveal small amounts of $Fe_3\,C$ (cementite) Mr. Pilling developed a new reagent, consisting of a dilute solution of nitric acid and methyl alcohol in nitrobenzol (min. & met. 5, p 31, 1924.) This reagent does not attack the ferrite and so does not reveal the grain boundaries; consequently the $Fe_3\,C$ shows up much more distinctly and there is not the chance of confusion present as is the case when picric acid is used. After the above photo micrographs were taken, the specimens were re-polished, etched with nitrobenzol and examined under a magnification of 1000 diameters, the result of which is given in Table II and in the captions of Fig. 16. In this illustration the alloys are arranged in order of C contents. In the first three the C contents are 0.001,

0.001 and 0.0018 per cent, respectively, and the microanalysis shows no carbide; in the three following the carbon contents are 0.0031, 0.0031, 0.0030 and the microanalysis shows a trace of carbide, *i. e.*, it is doubtful whether there is any carbide present. It is only when we come to the third group where the carbon contents are 0.0071 and 0.012 per cent that the microanalysis shows definite quantities of Fe_3C; and finally when in the fourth group the carbon contents are 0.10 and 0.142 per cent carbon appears as pearlite in addition to free Fe_3C.

Another interesting point to be noted is that the ratio of the slope of the curve (Fig. 14) between C and D (16,700) to that between B and C (2250), 16,700/2250 or 7: 1, agrees very closely with the ratio of the slope of the carbon containing constituent, vs. carbon content for the two cases, pearlite between C and D, and Fe_3C between B and C, namely 7.35.[13] Consequently by plotting carbon-containing constituent as abscissa, the curve B-C-D would be a straight line. In other words, the hysteresis loss for pure iron saturated with C

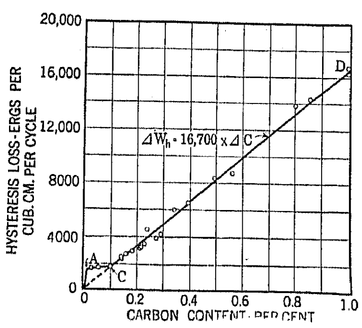

FIG. 15—EFFECT OF CARBON ON THE HYSTERESIS LOSS OF PURE IRON (EFFECT OF GRAIN SIZE NOT ELIMINATED)

in solution is increased in direct proportion to the amount of the iron associated with the precipitated carbon—or the carbon-containing constitutent.

For carbon contents above 0.01 per cent the points deviate but slightly from the smooth curve, while for carbon contents below the knee of the curve there is considerable deviation. Part of this is undoubtedly due to experimental errors both in the carbon determinations and in the magnetic testing. A variation of $\pm$ 0.001 per cent would nearly account for the deviation, but the analyses were carefully checked, and it is felt that the errors due to this source are less than the above figures. The same statement can be made with regard to the magnetic testing. Another variable was therefore looked for and in view of the great importance attributed to grain size by some investigators an approximate determination of the average grain size was made of some of the samples.[14]

In Fig. 17 the points for very low carbon contents (0.001—0.004 per cent) have been plotted.[15] The mean value of the carbon content for the samples with large grains is 0.0020 per cent and that for samples with small grains 0.0026 per cent. By considering the carbon content as 0.0023 per cent the variation is only $\pm$0.0003 per cent and can consequently be regarded as a constant for the present purpose, which is to determine

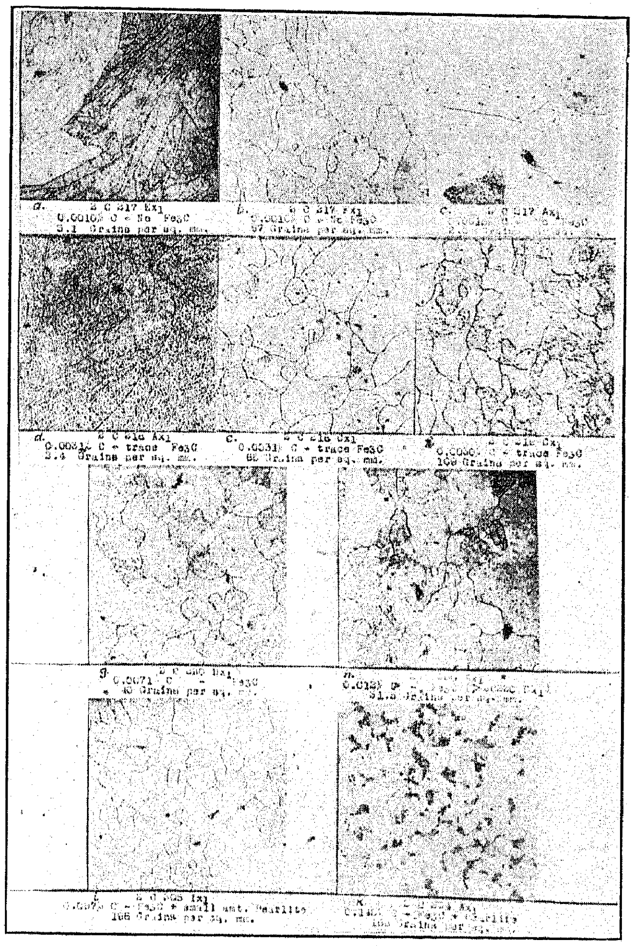

FIG. 16—FE-C ALLOYS. ETCHED WITH PICRIC ACID. 50 DIAM. CARBON DETERMINATION BY VACCUM-LIQUID AIR METHOD. FORM OF CARBON DETERMINED BY MEANS OF NITRO-BENZOL ETCHING REAGENT AND 1000 DIAM. MAGNIFICATION

13. Fe_3C contains 6.67 per cent $C \therefore 1$ part C makes 15 parts Fe_3C. Pearlite contains approx. 0.90 per cent $C \therefore 1$ part C makes 111 parts pearlite.

The ratio $\frac{\text{pearlite}}{Fe_3C}$ (for the same C content) = 111/15 = 7.35 (by weight or volume).

14. The grain size was determined by actual grain count on the ground glass screen of the camera attached to the microscope. On account of the great difficulty of obtaining definite grain boundaries as well as on account of the non-uniformity of the grains, accuracy is, in some cases, not as great as might be desired.

15. It will be noted from Fig. 17 that three out of five of the samples having small grains had been annealed in nitrogen, while all of the samples having large grains had been annealed in vacuum, which would indicate that nitrogen may have some influence on the grain growth—possibly through the agency of minute quantities of iron-nitride, although nitrogen is not supposed to react with iron at any temperature according to Tchijevski (Rev. de Met. 11 E pp. 617-18, 1914).

the effect of grain size upon the hysteresis loss. The carbon content being constant its effect must necessarily be constant, and as the effect of the grain size must be zero when the grain size is infinitely large, (or the number of grains per unit area is zero), the relationship between grain size and hysteresis loss for a constant carbon content may be expressed as

$$W_h = a N^b + K,$$

where N = number of grains per sq. mm. and K = the effect of carbon.

By giving the values of grain size weight in accordance

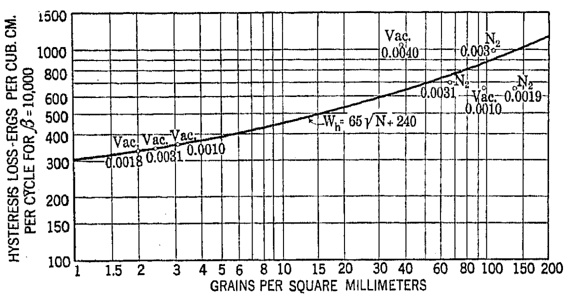

FIG. 17—HYSTERESIS LOSS VS. GRAIN SIZE FOR CARBON CONTENTS OF 0.001-0.004 PER CENT

Vac = Annealed in vacuo N_2 = Annealed in Nitrogen Small figures show carbon content.

with the approximate accuracy with which they could be determined, the weighted mean value of grain size for the upper group of points is 85 grains per sq. mm. corresponding to a hysteresis loss of 810 ergs. The equation for the curve as drawn through this point and the lower points then becomes:

$$W_h = 65\sqrt{N} + 240$$

As this equation holds for a carbon content of 0.0023 per cent this means that 240 ergs is the loss due to 0.0023 per cent C. Hence in general, the effect of carbon in solution on the hysteresis loss can be written:

$W_h = 240/0.0023 \times C = 104{,}000 \times C$ instead of the previously obtained value of $220{,}000 \times$ per cent C.

The next question is whether the above relationship between grain size and hysteresis loss holds when carbon is thrown out of solution. This question will be taken up later under the 4 per cent silicon alloys, where it is shown that for carbon contents between 0.02 and 0.06 per cent, in which range carbon is precipitated as Fe_3C, the effect of grain size can be expressed by the equation:

$$W_h = 3N$$

(see Fig. 23). As there is no reason to suppose that this effect is different in the case of pure iron containing appreciable amounts of Fe_3C, than in the case of 4 per cent silicon alloys, the above equation will be used in order to eliminate the effect of grain size on the hysteresis loss of iron with medium and high carbon contents (*i. e.* with carbon contents of 0.02 per cent and above).

Using these data in regard to grain size and deducting the effect of grain size from the values of hysteresis loss given in Table II, using for this purpose the equations just derived, namely:

$W_h = 65 \times \sqrt{N}$ for carbon in solution, and

$W_h = 3 \times N$ for carbon in excess of 0.02 per cent, the net loss due to carbon is tabulated in the last column of Table II, and plotted in Fig. 18.[16]

This curve then represents the hypothetical hysteresis loss of pure iron-carbon alloys having no inter-crystalline cement, except as far as the carbon containing constituent is concerned, and no other inter-crystalline or intra-crystalline impurities. The equation of this curve is

$$W_h = \underset{C \text{ in sol.}}{104{,}000 \times C} + \underset{Fe_3C}{2250(C - 0.008)} + \underset{\text{pearlite}}{16{,}700(C - 0.09)}$$

upper limits $C = 0.008$ $C = 0.09$ $C = 0.90$

The complete equation for the effect of carbon and grain size on the hysteresis loss of pure iron can consequently be written:[17]

1. For carbon in solution:

$W_h = 65\sqrt{N} + 104{,}000 \times C$ for $C < 0.008$

2. For precipitated carbon:

$W_h = 3N + 830 + 2250(C - 0.08) + 16{,}700(C - 0.09)$

Upper limits $C = 0.09$ $C = 0.90$

3. For small amounts of the common impurities:

$W_h = 65\sqrt{N} + 104{,}000 \times C + 18{,}000 \times S + 1000 \times Mn + 13{,}000 \times P$

Upper limits $C = 0.008$ $S = 0.10$ $Mn = 1.0$ $P = 0.015$

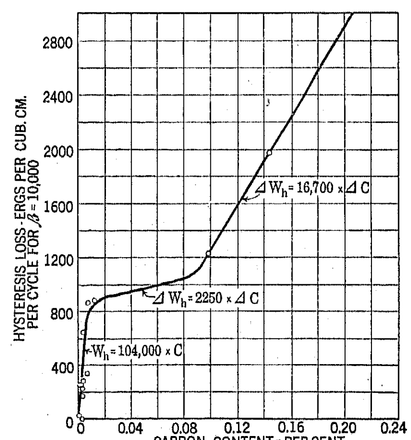

FIG. 18—EFFECT OF CARBON ON THE HYSTERESIS LOSS OF PURE IRON; EFFECT OF ALL OTHER IMPURITIES HAVING BEEN ELIMINATED

Of all the factors influencing the magnetic properties the dissolved carbon is thus seen to be by far the most important. For higher carbon contents (0.5—1.0 per cent), this effect has long been made use of for permanent magnets, by quenching the alloys from a temperature at which the carbon is in solution and thereby preventing it from being precipitated. Other ele-

16. In drawing this curve, use was made of the general shape of the curves of Figs. 14 and 15.

17. N = no. grains per sq. mm. C, S, Mn and P are the percentages of these elements respectively. The upper limits given are the values of the elements (in per cent) above which the constants either do not hold or have not been investigated.

TABLE II—HYSTERESIS LOSS OF PURE *Fe*. EFFECT OF CARBON AND GRAIN SIZE

Specimen Heat Treatment					Composition (see Tab. 1 and 2)		Hysteresis Loss ergs per cub. cm. per cycle for $B = 10000$			Micro-analysis		Hysteresis Loss due to Grain Size ergs per cc. per cycle	Net Hysteresis loss due to Carbon
No.	Kind Test Pce	Anneal Temp. °C.	Atm	Anneal. Period hrs.	C %	S %	Total as per Tab. 4 and 5	Portion due to S (see Fig. 13)	Net Loss due to Carbon and Grain Size	Grains per sq. mm.	Form of Carbon		
2- 206*I*	Ring	976	Vac	2	.0025	.018	873	324	549				
" *F*	"	"	"	"	.0028	"	914	324	590				
2- 207*I*	"	"	"	"	.0021	.018	800	324	476				
2- 251*I*	"	"	"	"	.0026	.010	530	180	350				
" *F*	"	"	"	"	.0017	"	647	180	467				
2- 213*K*	"	900	Vac Char-coal	"	.0018	.012	660	216	444				
2- 213*L*	"	1000	Vac Char-coal	"	.0038	.012	810	216	594				
2- 213*M*	"	1100	Vac Char-coal	"	.0087	.012	1,595	216	1,379				
2- 219*L*	"	900	Vac	"	.0035	.020	825	360	465				
2- 220*L*	"	"	"	"	.0018	.020	850	360	490				
2- 221*L*	"	"	"	"	.0025	.019	1,000	342	658				
2*C*205*I*	"	920	"	"	.097	.009	1,990	162	1,828	196	Small am't. Pearl-ite + Fe_3C	590	1240
2*C*206*I*	"	936	"	"	.174		3,120	180	2,940				
2*C*207*I*	"	"	"	"	.213		3,320	180	3,140				
2*C*208*I*	"	930	"	"	.205		3,260	180	3,080				
2*C*209*I*	"	"	"	"	.283		4,250	180	4,070				
2*C*210*I*	"	"	"	"	.268		4,050	180	3,870				
2*C*211*a*	"	900	N_2	"	.235		4,800	270	4,620				
" *b*	"	"	Vac	"	.151		2,940	270	2,670				
2*C*212*a*	"	"	N_2	"	.339		6,280	270	6,010				
" *b*	"	"	Vac	"	.227		3,620	270	3,350				
2*C*213*a*	"	"	N_2	"	.485		8,600	270	8,330				
" *b*	"	"	Vac	"	.387		6,830	270	6,560				
2*C*214*b*	"	"	Vac	"	.559		8,950	270	8,680				
2*C*215*a*	"	"	N_2	"	.850		14,600	270	14,330				
" *b*	"	"	Vac	"	.791		14,200	270	13,930				
2*C*216*a*	"	"	"	"	.985		16,400	270	16,130				
" *b*	"	"	"	"	.981		17,000	270	16,730				
2*C*217*A*	"	"	"	"	.0018	.015	602	270	332	2.0	No Fe_3C	91	241
" *B*	"	"	"	"	"	"	637	270	367				
" *C*	"	"	300mm. N_2	"	.0019	"	925	270	655	139	trace Fe_3C	780	0
" *D*	"	"	700mm. N_2	"	.0038	"	835	270	565				
" *E*	"	900-1000	Vac	"	.0010	"	630	270	360	3.1	No Fe_3C	110	250
" *F*	"	900	"	"	.0010	"	926	270	656	97	No Fe_3C	645	11
2*C*218*A*	"	"	"	"	.0031	.026	810	470	340	2.4	trace Fe_3C	120	220
" *B*	"	"	"	"	"	"	840	470	370				
" *C*	"	"	300mm. N_2	"	.0031	"	1,170	470	700	66	trace Fe_3C	520	180
" *D*	"	"	700mm. N_2	"	.0054	"	1,560	470	1,090				
" *E*	"	"	Vac	"	.0014	"	790	470	320				
" *F*	"	"	"	"	.0044	"	1,505	470	1,035				
2*C*219*A*	"	"	"	"	.0022	.021	763	380	383				
" *B*	"	"	"	"	"	"	858	380	478				
" *C*	"	"	300mm. N_2	"	.0030	"	1,370	380	990	109	trace Fe_3C	710	280
" *D*	"	"	700mm. N_2	"	.0060	"	1,720	380	1,340				
" *E*	"	"	Vac	"	.0056	"	1,690	380	1,310				
" *F*	"	"	"	"	.0040	"	1,440	380	1,060	38	No Fe_3C	420	640
2*C*220*A*	"	"	"	"	.0032	.016	840	290	550				
" *B*	"	"	"	"	"	"	850	290	560				
" *C*	"	"	300mm. N_2	"	.0038	"	1,370	290	1,080				
" *D*	"	"	700mm. N_2	"	.0071	"	1,660	290	1,370	53	Fe_3C	490	880
" *E*	"	"	Vac	"	.0120	"	1,670	290	1,380	53	Fe_3C > in *E*	490	890
" *F*	"	"	"	"	.0070	"	1,470	290	1,180	174	trace Fe_3C	850	330
2*C*221*A*	"	"	"	"	.0303	.018	1,870	320	1,550				
" *B*	"	"	"	"	"	"	1,870	320	1,550				
2*C*222*A*	"	"	"	"	.0182	.021	1,825	380	1,445				
" *B*	"	"	"	"	"	"	1,825	380	1,445				
2*C*223*A*	"	"	"	"	.0505	.015	1,970	270	1,700				
" *B*	"	"	"	"	"	"	1,910	270	1,640				
2*C*224*A*	"	"	"	"	.1420	.017	2,780	305	2,475	168	Pearlite	504	1971
" *B*	"	"	"	"	"	"	2,690	305	2,385				

ments like tungsten and molybdenum assist in keeping carbon in solution and are consequently used extensively in the manufacture of permanent magnets. It has hitherto been assumed, however, that the dissolved carbon is precipitated as pearlite by slow cooling from 900 deg. or above. The present results, however, confirmed by the analyses of Mr. Pilling, show that this is not the case, at least not under the conditions of the present investigation, where cooling from 900 deg. or above was done at the slow rate of 30 deg. cent. per hour. Under these conditions 0.006 to 0.008 per cent C remains in solution (martensite) 0.09 per cent is precipitated as Fe_3C (cementite) and any carbon in excess of this is precipitated as pearlite. Were it possible to precipitate all the carbon, we should expect the magnetic properties to be very greatly improved, even though the total carbon content were as high as 0.1 per cent.

Various more or less vague suggestions have been made to explain this tremendous effect of carbon on the magnetic properties. One is that the ferro-magnetic properties are caused by a certain more or less stable equilibrium arrangement of the electrons in the atomic structure and that the carbon entering into solution affects this structure by distorting the space lattice and upsets the equilibrium.[18]

Gumlich and Steinhaus[19] in discussing the effect of dissolved carbon on the coercive force, H_c, give the following figures:

0.01 per cent C as pearlite increases H_c by 0.07 gauss
0.01 per cent C as martensite " H_c by 0.80 gauss

These results were obtained for medium and high carbon contents, in the case of martensite by quenching and in the case of pearlite by slow cooling.[20]

For the sake of comparison the data in regard to coercive force vs. carbon content from the present data have been plotted in Fig. 19, giving the following results:[21]

For $0 < C < 0.006$	$H_c = 80 \times$ per cent C *i. e.* 0.01 per cent C produces a coercive force of 0.8 gilberts per cm.
For $C > 0.09$	$\Delta H_c = 5.4 \times$ per cent ΔC *i. e.* 0.01 per cent C produces a coercive force of 0.054 gilberts per cm.
For $0.006 < C < .09$	$\Delta H_c = 0.8 \times$ per cent ΔC *i. e.* 0.01 per cent C produces a coercive force of 0.008 gilberts per cm.

In other words, the effect on the coercive force of carbon in the three successive regions is as 100:1:6.8 respectively, or practically the same ratios as in the

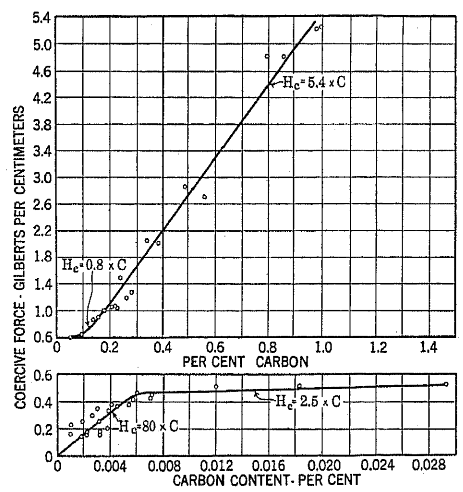

FIG. 19—COERCIVE FORCE VS. CARBON CONTENT FE-C ALLOYS. (EFFECT OF GRAIN SIZE NOT ELIMINATED)

18. Since writing the manuscript for this paper a very interesting contribution to the subject of Solid Solutions has been made by Dr. Walter Rosenhain: 'Solid Solutions' issued with Mining and Metallurgy, TRANS. A. I. M. E. 1923, Reprint No. 1250-N. In this paper Dr. Rosenhain explains the 'Substitution' theory of Solid Solutions, according to which atoms of various metals are capable of displacing each other in the space lattice of the crystal structure. The amount of solubility and the effect the solute has on the solvent will largely depend upon the relative properties of the atoms of the solute and the solvent. Such substitution would in any case result in some change in the forces between the adjacent atoms of the solvent, and some sort of distortion of the lattice is bound to follow, the more so the greater the difference between the two atoms. Most of the well known phenomena of solid solutions are satisfactorily explained by this theory. In regard to carbon and phosphorus, Dr. Rosenhain believes that their atoms are so small compared with those of iron that they can find room in the lattice interstices, but with effects similar to those of 'substitution' atoms. It is readily seen that this theory harmonizes very well with the suggestion given above as to the action of dissolved carbon on the magnetic properties of iron. Dr. Rosenhain purposely avoids discussing the beneficial effect of silicon (and aluminum) on the hysteresis loss of iron, and merely remarks that this effect may be due to a neutralizing effect on the distortions caused by the impurities present. In view of the results given in this paper it would seem, however, that the beneficial effect of silicon is independent of the impurities and is due to the causes stated later on in this paper.

19. "Variations in the B-H curves of Low Hysteresis Materials" *Phys. Tech. Reichsanstalt.*

ETZ 36 pp. 675-77, 691-94, 1915 (Received after the close of the war).

20. Means were not available for analyzing very low carbon contents directly. This was done indirectly, however, by extrapolating the curve between dissolved carbon and H_c to zero, assuming it to be a straight line. Carbon contents below 0.01 per cent were then determined by quenching the previously annealed and tested test pieces in order to get all the carbon into solution, then redetermining H_c and finally from the increase in H_c and the above extrapolated curve obtaining the carbon content. This was then assumed to be the carbon content of the original test pieces. This method is obviously open to the objection that it does not take into account the effect of strain on the magnetic properties caused by the quenching. If it were not for this effect, H_c should be the same before and after quenching for $C < 0.008$ per cent because C is then in solution even after slow cooling. In other words, the increase in H_c by quenching is attributed to strain, and not to a change in the form of carbon, as assumed by Gumlich.

21. In this case the effect of sulphur on the coercive force has been taken into account by deducting from the measured coercive force 4.4 x per cent S, but the effect of grain size has been included in order to compare these results with those of Gumlich & Steinhaus.

case of the hysteresis loss (98:1:7.4). Comparing the above results with those of Gumlich and Steinhaus, it is seen that our coefficient for carbon contents of less than 0.006 per cent check theirs for dissolved carbon (martensite) exactly ($H_c = 80 \times$ per cent C), thus confirming again our previous conclusion that for carbon contents below 0.006—0.008 per cent, carbon is in solution.

In the case of pearlitic carbon, however, the agreement is not so good. Gumlich and Steinhaus give $H_c = 7.0 \times$ per cent C while the present data give $H_c = 5.4 \times$ per cent C. This disagreement is readily understood, if,—as is probable—they assumed that all the carbon is precipitated as pearlite by cooling from 900 deg. or above. On this assumption, by drawing a straight or slightly curved line from the origin through the points obtained for H_c corresponding to carbon contents of 0.1 to 1.0 per cent a coefficient of 7.0 can readily be obtained but based on the present results this is obviously not correct.

In view of the results obtained for hysteresis loss, the magnetic properties depend to a large extent upon the grain size and as the latter does not depend altogether on the carbon content, the grain size should be introduced into the equation for the coercive force in order to reconcile it with the experimental facts. If this be done in the same way as it was done above in the case of the hysteresis loss the following equation is obtained:

$$H_c = 0.025 \sqrt{N} + 40 \times C$$

for carbon in solution and corresponding equations for precipitated carbon.

The electrical resistance of the *Fe-C*, *Fe-Mn*, *Fe-S* and *Fe-P* alloys as tabulated in Tables 4-7 in Appendix 5 have been plotted in Fig. 37 in Appendix 4. From this it will be seen that carbon in small amounts, (less than 0.02 per cent) increases the resistance by $82.5 \times$ per cent C, whereas carbon in larger amounts increases the resistance by only $4.5 \times$ per cent C, that is, in the ratio of nearly 20:1. All evidence, therefore, is to the effect that small amounts of carbon remain in solution in iron at ordinary temperature, in spite of very slow cooling from above the A_3 transition point.

Maximum permeability and minimum reluctivity curves will be found in Appendix 3 (Figs. 30 and 31 respectively.) It will be seen that the reluctivity curves are of the same general shape as the hysteresis loss curve indicating that carbon is all in solution up to 0.006 per cent and continues to go into solution to some extent up to 0.02 per cent. It is interesting to compare the slopes of the curves for the hysteresis loss, coercive force, and minimum reluctivity in the three different regions (effect of grain-size not eliminated):

	Carbon in Sol.		Carbon as Fe_3C		Carbon as pearlite
Hysteresis Loss..	98	:	1	:	7.4
Coercive Force	100	:	1	:	6.76
Min. Reluctivity	94.5	:	1	:	6.78

As these ratios are very nearly the same it may be concluded that the above properties all are identical functions of the carbon content and of the state of the carbon.

Second Series, 4 Per Cent Silicon

The effect of carbon, sulphur, phosphorus, manganese and grain size on the magnetic properties of 4 per cent iron-silicon alloys.

In case of the 4 per cent silicon alloys the same procedure was followed as in the case of the simple iron-carbon alloys, *i. e.* the effects of sulphur, phosphorus and manganese were determined, so that the effect of carbon could be more accurately ascertained. Three series of alloys were prepared. The first series, in which Goldschmidt-Thermit Co. 91 per cent ferro-silicon was used, was furthermore divided into two groups: Group "a" in which electrolytic iron was used as a base and containing all the alloys used for obtaining the effects of sulphur, phosphorus and manganese, and group "b"

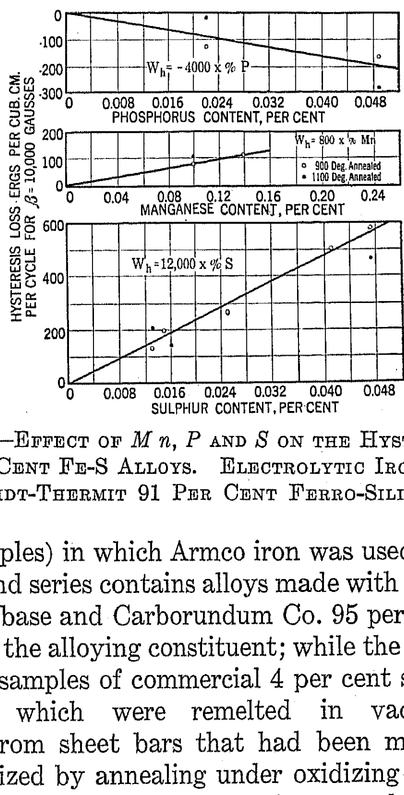

Fig. 20—Effect of Mn, P and S on the Hysteresis Loss of 4 Per Cent Fe-S Alloys. Electrolytic Iron Base and Goldschmidt-Thermit 91 Per Cent Ferro-Silicon

(two samples) in which Armco iron was used as a base. The second series contains alloys made with electrolytic iron as a base and Carborundum Co. 95 per cent ferro-silicon as the alloying constituent; while the third series contains samples of commercial 4 per cent silicon steel two of which were remelted in vacuum and six cut from sheet bars that had been more or less decarbonized by annealing under oxidizing conditions.

The effects of sulphur, phosphorus and manganese are shown graphically in Fig. 20 and can be expressed by the equation,

$$W_h = 12{,}000 \times S + 800 \times Mn - 4000 \times P$$

In other words sulphur and manganese affect the magnetic properties of 4 per cent *Fe-Si* alloys in the same way as—although to a slightly smaller degree than—they affect the magnetic properties of unalloyed iron. Phosphorus, on the other hand, improves the properties of the 4 per cent *Fe-Si* alloys while it has the opposite effect on unalloyed iron.

After eliminating the effects of the above impurities the effect of carbon is shown graphically in Figs. 21

and 22. Fig. 21 for 3.48 – 3.70 per cent *Si* and Fig. 22 for 3.70 – 4.30 per cent *Si*, disregarding for the present the effect of grain size. For carbon contents below 0.007 per cent carbon is in solution and the relationship of hysteresis loss to carbon content can be expressed by the equation:

$$W_h = 110{,}000 \times C$$

Carbon probably continues to go into solution to some extent up to 0.02 per cent but is largely precipitated as Fe_3C up to 0.10 per cent, while for carbon contents in excess of 0.10 per cent carbon is precipitated not as pearlite as in the case of unalloyed iron, but as graphite, with a correspondingly slight effect. These conclusions are confirmed by the minimum reluctivity curves shown in Fig. 32 in Appendix 3.

Another difference between these two materials is that relating to grain size. Under the conditions of the present investigation the grain size of unalloyed iron with low carbon (< 0.006 per cent) may vary from large

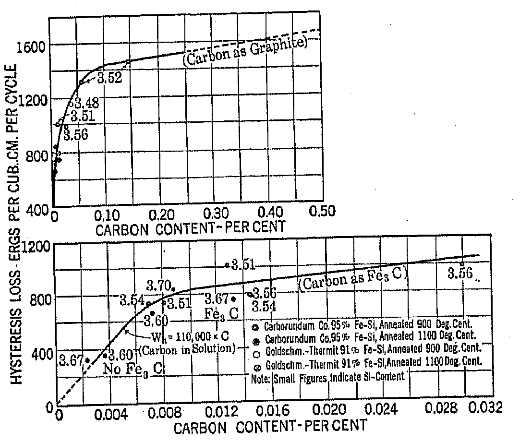

Fig. 21—Hysteresis Loss vs. Carbon Content. 3.5 Per Cent Fe-Si Alloys (3.48 – 3.70 Per Cent Si). Effect of Grain Size not Eliminated

to very small (2-100 grains per sq. mm.) and for medium carbon (0.01-0.10 per cent) the grain size is very small (see Fig. 14). For 4 per cent *Si*, on the other hand, the grain size is large both for low and medium carbon contents. In order to determine the effect of grain size on the properties of 4 per cent *Si* alloys it was therefore necessary to produce alloys with small grains and to treat them in such a way that extensive grain growth could not take place. Two separate alloys had been so treated. Samples had been rolled at 600, 800 and 1000 deg. in order to test out the effect of various rolling temperatures, and separate samples had been annealed at 900 and 1100 deg. tested magnetically and inspected microscopically. Three other samples, 26, 31 and 14 mils, respectively, were also tested. In order to get data in regard to alloys with medium carbon content and small grains it was necessary to make use of samples of commercial 4 per cent *Si* steel, because with the methods available at the laboratory the rolling operation would decarbonize the samples. Finally, four samples of decarbonized steel were included. The data have been plotted in Fig. 23.[21] The samples arrange themselves in two distinct groups, one including those having carbon contents of less than 0.005 per cent *C*, and the other composed of those with carbon contents of 0.020-0.060 per cent *C*. The equation for the curve drawn through the centers of density of the points for low carbon contents is so nearly the same as in the case of unalloyed iron that the two can be regarded as indentical, namely,

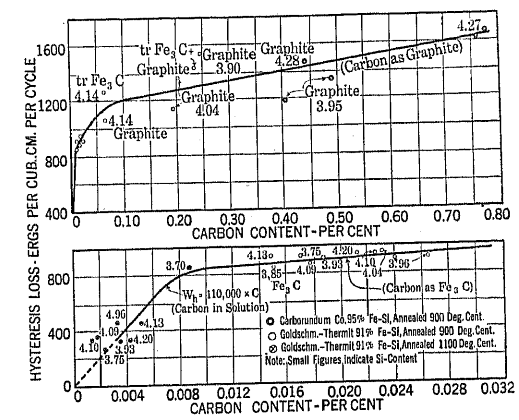

Fig. 22—Hysteresis Loss vs. Carbon Content. 4 Per Cent Fe-Si Alloys (3.70 – 4.30 per Cent Si). Effect of Grain Size not Eliminated

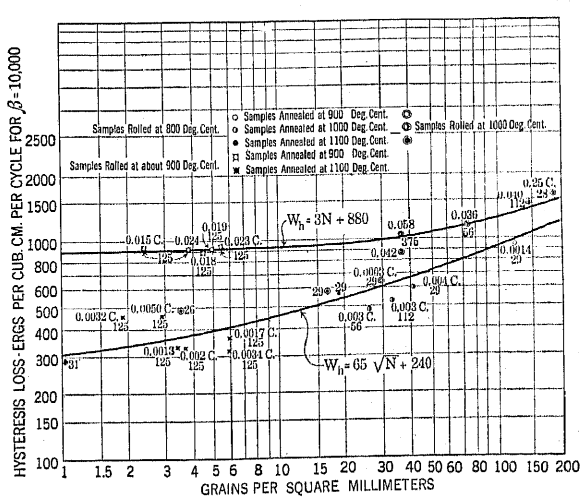

Fig. 23—Hysteresis Loss vs. Grain Size. 4 Per Cent Fe-Si Alloys small Figures show Carbon Content and Thickness of Samples (Mils)

$$W_h = 65\sqrt{N} + 240$$

where the first term, $65\sqrt{N}$, represents the effect of grain size and the second term, 240, represents the effect of carbon, which in this case averages 0.0020 to 0.0025

21. In order not to introduce unnecessary variables the results for the 14 mils samples were not plotted, because of the excessive oxidation these were subjected to during the rolling operation. Furthermore, the results for the samples rolled at 600 deg. were not plotted. Although no apparent reason is yet known for disregarding these samples the rolling temperature is so far below the standard that it is believed preferable to leave them out for the present purpose.

per cent. The hysteresis loss for 0 per cent silicon and for 4 per cent silicon having the same carbon content (less than 0.005 per cent) is consequently the same and depends primarily upon the grain size. For higher carbon contents (0.020-0.060 per cent) where carbon in excess of about 0.010 per cent is precipitated as $Fe_3\,C$ the effect of grain size is not nearly so large. As the effect of a small variation of carbon in this case is already known to be very small (see Fig. 22) the carbon content and its effect on the hysteresis loss may for the present purpose be regarded as constant, and the relationship between grain size and hysteresis loss for this carbon content can be expressed by the equation

$$W_h = 3\,N + 880$$

where the first term, $3\,N$, represents the effect of the grain size, and the second term, 880, represents the effect of the carbon which in this case averages 0.03 per cent.

Deducting now the effect of grain size in accordance with the above equations, from the loss due to the com-

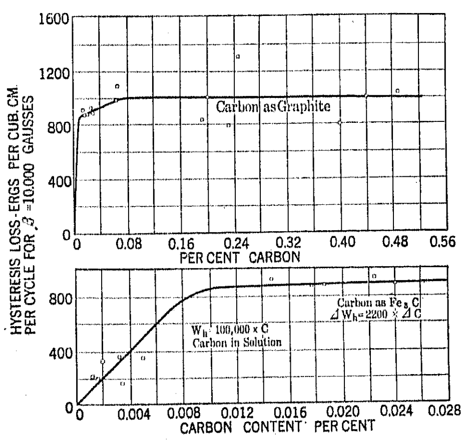

Fig. 24—Effect of Carbon on the Hysteresis Loss 4 Per Cent Iron-Silicon Alloys; Effect of All other Impurities having been Eliminated

bined effect of grain size and carbon the net loss due to carbon is obtained. The result is plotted in Fig. 24. The equation for the compound curve thus obtained is,

$$W_h = 100{,}000 \times C + 2200\ (C—0.0085) + 0\ (C—0.08)$$

Upper limits $C = 0.0085$ $C = 0.08$ $C = 0.50$
(C in solution) (C as $Fe_3\,C$) (C as graphite)

Comparing this curve with that for 0 per cent Si, it will be seen that the two are practically identical up to a carbon content of about 0.080 per cent. From this point on carbon is precipitated as pearlite for 0 per cent Si, with the result that the loss increases by 16,700 $\times$ C, while for 4 per cent Si carbon is precipitated as graphite with practically no additional increase in the hysteresis loss.[22]

What at first, therefore, appeared as a difference in the effect of carbon on the two materials (unalloyed iron and 4 per cent Fe-Si alloys) has resolved itself into a difference caused by the action of silicon: Firstly, by its action on the grain size, namely, that *without* silicon the normal grain size is small and it is difficult to get large grains, while *with* silicon (4 per cent) the normal grain size is large due to the absence of the A_3 transformation point, and small grains can be had only by severe mechanical working; secondly, by the action of silicon on the form of carbon, namely, by precipitating it as graphite instead of as pearlite for carbon contents in excess of about 0.08 per cent. The conclusion can, therefore, be drawn that the effect of carbon is the same on unalloyed iron and on 4 per cent Fe-Si alloys and depends only upon the form in which it occurs.

To summarize, the effect of carbon and grain size on 4 per cent Fe-Si alloys can be expressed by the following equations:

1. For dissolved carbon

$$W_h = 65\sqrt{N} + 100{,}000 \times C \qquad \text{for } C < 0.0085$$

2. For precipitated carbon

$$W_h = 3\,N + 850 + 2200\ (C—0.0085) + 0\ (C—0.08)$$

Upper limits $C = 0.08$ $C = 0.50$

For small amounts of impurities:

$$W_h = 65\sqrt{N} + 100{,}000 \times C + 12{,}000 \times S + 800 \times Mn - 4{,}000 \times P$$

Upper limits $C = 0.0085$ $S = 0.05$ $Mn = 0.20$ $P = 0.05$

Calculation of Hysteresis Loss

In order to ascertain further the validity of the above formulas, a number of samples of commercial 4 per cent Si steels treated in various ways so as to give a variety of compositions and structures were analyzed chemically and microscopically. The first set of samples marked "A" in Table III were remelted in vacuum, forged and machined into test rings (⅜ in. cm. thick). Two of these were annealed at 900 deg. and two rings at 1100 deg. in a current of air at reduced pressure resulting in carbon contents of 0.03 per cent and 0.003 per cent, respectively. The second set, marked "B", consists of samples made from regular sheet bars, that had been more or less decarbonized by annealing them in iron oxide at 1000 deg. cent. at reduced pressures. The machined rings were annealed at 1000 deg. in vacuum for 20 hours, the final carbon contents varying from 0.002 to 0.023 per cent. The third set, marked "C", consists of samples made from sheets rolled to various gages, 0.35—9.5 mm. (14—375 mils,) by the commercial rolling mill. The samples were prepared by punching rings 4.5 cm. × 3.2 cm. from the rolled sheets, except in the case of the first sample that was machined from the sheet bar. The samples were annealed at 1000 deg. for two hours in vacuum. The carbon contents in this case varied from 0.013 to 0.058 per cent. All the samples were cooled slowly after being annealed and were then tested and analyzed by sampling the test pieces themselves. The results, as far as hysteresis loss is concerned, are given in Table III. The actual losses are

22. The variation from the smooth curve for high carbon contents is probably due to variation in the relative amounts of carbon precipitated as $Fe_3\,C$ and as graphite.

TABLE III

CALCULATION OF HYSTERESIS LOSS BY MEANS OF COMPOSITION & MICROSTRUCTURE. 4% SI-STEEL

Specimen No.	Gage mils	Heat Treatment: Ann'l. Temp °C	Ann'l. Period hrs.	Atm.	Composition: Si %	C %	S %	P %	M_n %	Grain-Size Grains per mm.^{2}	Hysteresis Loss ergs per cub. cm. per cycle for B = 10000 — See Figs. 20-23 and Equations: Due to C	Due to S	Due to P	Due to M_n	Due to Grain Size	Total Loss: Calc.	Act.	Difference: ergs	%
A. 4% $S\,i$-Steel Remelted in Vacuo																			
M-203A	125	900	4	12 mm. Hg. 120cc. air/mm.	3.98	.0270	.024	.018	.110	est.4	895	290	−70	90	10	1215	1245	+ 30	+ 2
204A	"	"	"		4.01	.0287	.020	.019	.120	" 4	900	240	−80	95	10	1150	1180	+ 30	+ 3
203B	"	1100	2		3.98	.0033	.024	.018	.110	" 2	330	290	−70	90	90	730	680	− 50	− 7
204B	"	"	"		4.01	.0030	.020	.019	.120	" 2	300	240	−80	95	90	635	635	0	0
B. 4% $S\,i$-Steel Decarbonized by annealing in $F\,e_3\,O_4$ at 1000°C. Not Rolled																			
M-14-10	375	1000	20	Vac	4.09	.0049	.027	.018	.125	0.12	490	320	−70	100	20	860	775	− 85	−10
30	"	"	"	"	3.95	.0028	.026	.022	.130	est.0.10	280	310	−90	100	20	620	620	0	0
40	"	"	"	"	3.85	.0033	.036	.021	.130	" 0.10	330	430	−80	100	20	800	877	+ 77	+10
50	"	"	"	"	3.87	.0022	.027	.016	.135	" 0.10	220	320	−60	110	20	610	675	+ 65	+11
60	"	"	"	"	4.06	.0229	.028	.016	.137	" 1.0	890	340	−60	110	3	1283	1242	− 41	− 3
C. 4% $S\,i$-Steel. Not decarbonized. Rolled to various Gages in Commercial Mill.																			
M-14-00	375	1000	2	Vac	3.97	.058	.029	.015	.120	est. 37	950	350	−60	95	110	1445	1420	− 25	− 2
01	112	"	"	"	"	.040	"	"	"	act. 145	930	"	"	"	435	1750	1790	+ 40	+ 2
02	56	"	"	"	"	.036	"	"	"	" 75	920	"	"	"	225	1530	1530	0	0
03	28	"	"	"	"	.025	"	"	"	" 189	895	"	"	"	570	1850	1940	+ 90	+ 5
05	14	"	"	"	"	.031	"	"	"	" 225	910	"	"	"	675	1970	1870	−100	− 5

given in the column next to the one giving the calculated losses and the difference given in ergs and per cent in the last two columns. While the maximum errors are $\pm$ 10 per cent it will be noted that the algebraic mean errors for the three sets are − 0.5 per cent, + 1.6 per cent and 0 per cent, respectively, and the mean error for all the sets is only + 0.43 per cent, showing that by taking the mean for a few samples, the individual errors due to analysis are eliminated and very accurate results are obtained.

Third and Fourth Series: 2 Per Cent Silicon and 5-6 Per Cent Silicon

Note: These two series were not investigated as fully as the previous two series, thus the effects of S, P and Mn on the magnetic properties were not investigated separately but assumed to be the same as for 0 and 4 per cent silicon, respectively. Furthermore, only a few samples of the 2 per cent series were inspected for grain structure and none of the fourth series. While the final calculations, therefore, are based on a number of assumptions, a close examination will show that the errors involved should be well within $\pm$ 5 per cent. These two series are included in the paper for the sake of correlating all the information available at the present time.

Third Series: 2 Per Cent Silicon

The effect of carbon on the magnetic properties of 2 per cent iron-silicon alloys.

Two sets of alloys were made, one using Carborundum Co. 95 per cent ferro-silicon and the other using Goldschmidt-Thermit Co. 91 per cent ferro-silicon. No investigation of the effect of S, Mn and P was made for 2 per cent Si, but based on the results obtained for 0 and 4 per cent Si, the error in using the constants for 0 per cent Si cannot be appreciable. The effect of P and Mn—being present in only minute quantities (Mn—0.01 per cent, P—0.003 per cent) is therefore disregarded and the effect of sulphur is assumed to be

$$W_h = 18{,}000 \times S.$$

On this assumption the hysteresis loss for pure Fe-Si-C alloys containing 1.75—2.25 per cent Si has been calculated and plotted in Fig. 25.

On account of the large deviation from the curve in

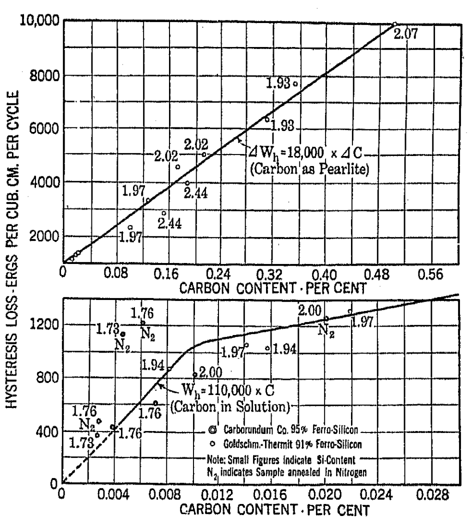

Fig. 25—Hysteresis Loss vs. Carbon Content for 2 Per Cent Fe-Si Alloys (1.73 − 2.44 per Cent Si). Effect of Grain Size not Eliminated

the case of two of the rings (carbon contents 0.0044 per cent and 0.0060 per cent, respectively) the grain structure was suspected, and two rings having nearly the same carbon content (0.0044 and 0.0026 per cent) but one having a high loss and one a low loss were inspected under the microscope and photographed (see Fig. 26). The result confirmed the suspicion, one ring having 48 grains per sq. mm., while the other had less than 0.5

grains per sq. mm. or 100 times as large grains as the former. Applying the same formulas as derived for 0 per cent and for 4 per cent Si, namely,

$W_h = 65 \times \sqrt{N}$ for 0—0.008 per cent C

the effect of the grain size on the hysteresis loss is 10 times as great in the one case as in the other, being 450 ergs to 45 ergs, which is sufficient to account for the difference in the total hysteresis loss. No cementite is visible in either case, showing that carbon is in solution.

A few more samples were analyzed for grain size and the effect on the hysteresis loss eliminated in accordance with the equations deducted for 0 and 4 per cent silicon.

The result has been plotted in Fig. 27. Based on the

FIG. 26A—2 G 201 $L \times 100$, C – 0.0026 PER CENT. $W_{10} = 576$. $N < 0.5$ GRAINS PER MM.²

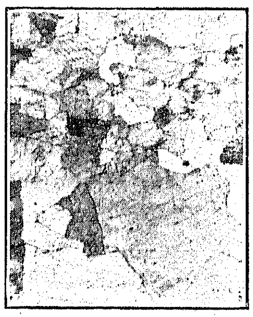

FIG. 26B—2 G 202 $L \times 100$, C – 0.0044 PER CENT. $W_{10} = 1216$. $N = 48$ GRAINS PER MM.²

combined magnetic and microscopic results the conclusion is that for carbon contents of less than 0.0085 per cent carbon is in solution, as in the case of 0 and 4 per cent Si, the effect of carbon being expressed by the equation,

$$W_h = 105{,}000 \times C \quad \text{for } C < 0.0085$$

Carbon in excess of 0.0085 per cent is precipitated as pearlite. This is in contrast to the cases for 0 per cent and 4 per cent Si in both of which cases carbon in the region 0.008—0.09 per cent is precipitated as Fe_3C unassociated with iron. As a consequence, the total effect of precipitated carbon is greater for 2 per cent Si than for 0 and 4 per cent Si, being expressed by the equation:

$$W_h = 105{,}000 \times C + 16{,}120\ (C—0.0085)$$

Upper limits	$C = 0.0085$	$C = 0.50$
	C in solution	C as pearlite

but the rate of increase is practically the same as for pearlitic carbon for 0 per cent silicon, (16,120 vs. 16,700). On the other hand, due to the silicon, the normal grain size is so much larger for 2 per cent silicon alloys with low carbon contents that the actual hysteresis loss is less than for 0 per cent Si up to a carbon content of about 0.04 per cent.

Minimum reluctivity curves for the 2 per cent Si alloys will be found in Appendix 3 Fig. 33, indicating that carbon may go into solution to some extent up to 0.02 per cent as in the case of 0 and 4 per cent Si.

FOURTH SERIES: 5-6 PER CENT SILICON

The effect of carbon on the magnetic properties of 5-6 per cent iron-silicon alloys.

The effect of S, P and Mn was not determined separately for these alloys but was assumed to be the

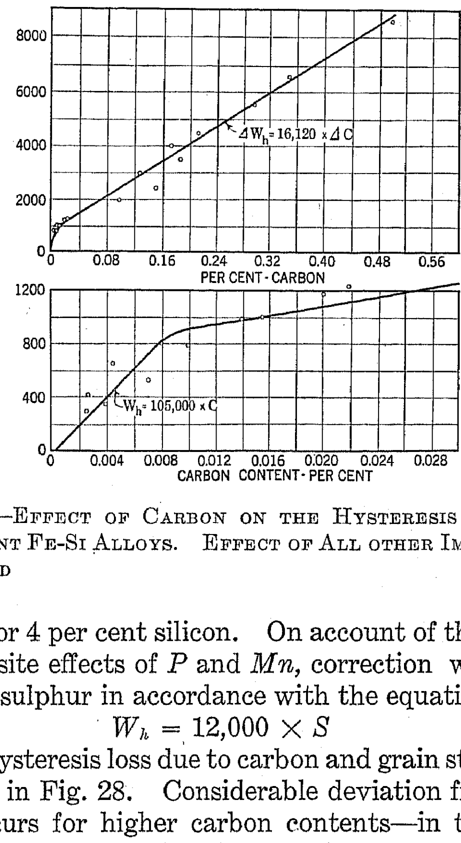

FIG. 27—EFFECT OF CARBON ON THE HYSTERESIS LOSS OF 2 PER CENT FE-SI ALLOYS. EFFECT OF ALL OTHER IMPURITIES ELIMINATED

same as for 4 per cent silicon. On account of the slight and opposite effects of P and Mn, correction was only made for sulphur in accordance with the equation:

$$W_h = 12{,}000 \times S$$

The net hysteresis loss due to carbon and grain structure is plotted in Fig. 28. Considerable deviation from the curve occurs for higher carbon contents—in the case of the samples made with Goldschmidt-Thermitt ferro-silicon—probably due to variation in the grain structure. Based on the results obtained for the 4 per cent Si alloys in regard to grain size the hysteresis loss has been reduced in accordance with the equation:

$$W_h = 3\,N$$

(where N = no. grains per sq. mm.) resulting in the horizontal curve in Fig. 28 for carbon contents in excess of 0.007 per cent. From these results it can be concluded that for Fe-Si alloys containing 4.7 per cent Si and above, carbon is in solution up to 0.007 per cent

and carbon in excess of this amount is precipitated as graphite. The effect on the hysteresis loss is

$$W_h = 100{,}000 \times C + 0\ (C - 0.007)$$

Upper limits $C = 0.007$ $C = 0.50$
C in solution Graphite

i. e., there is no further increase due to carbon percipitated as graphite. The reason for the increase in the *total* hysteresis loss is probably to be found in the decrease in the grain size due to the disturbance caused by the graphite with the consequent increase in the amount of the intercrystalline material. One might say, of course, that the primary cause of this increase is carbon and write the equation.

$$W_h = 100{,}000 \times C + 700\ (C - 0.007)$$

Upper limits $C = 0.007$ $C = 0.50$

but as it is possible to vary the grain size even with a high carbon (graphite) content it would be preferable to write the equations:

$$W_h = 65\sqrt{N} + 100{,}000 \times C \qquad \text{for } C < 0.007$$

and

$$W_h = 3\,N + 700 \qquad \text{for } C > 0.007$$

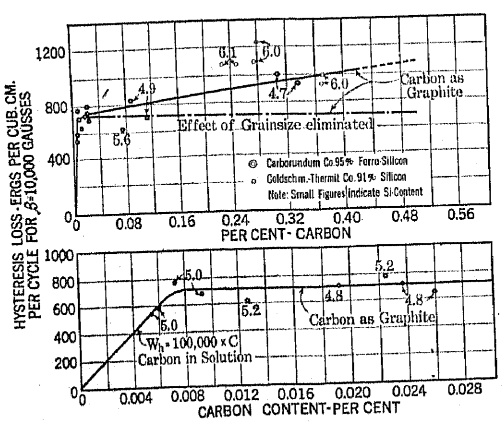

Fig. 28—Hysteresis Loss vs. Carbon Content 5-6 Per Cent Alloys

Minimum reluctivity curves for the 5 and 6 per cent alloys will be found in Appendix 3 Fig. 34.

The Question of Grain Size and Intercrystalline Impurities

The question naturally arises: Why should the grain size affect the magnetic properties? There must be something that increases as the grain size decreases, and upon which the hysteresis loss partly depends, or, in other words, that causes an increase in the hysteresis loss over and above that due directly to carbon and other impurities. As far as is known at present there is only one material that increases, as the grain size decreases, and this is the amorphous inter-crystalline cement, so called. It is, therefore, logical to conclude that there is a definite relation between the amount of this material and the hysteresis loss. It has been shown that the hysteresis loss per unit volume for alloys with carbon in solution is proportional to $\sqrt{N}$. It can also be shown that the amount of the inter-crystalline material per unit volume is proportional to $\sqrt{N}$[23]. It follows, therefore, that the hysteresis loss is directly proportional to the amount of the inter-crystalline cement, and that the latter acts just like any other inter-crystalline impurity, such as $Fe_3\,C$, pearlite, $Fe\,S$, etc.

Another question that remains to be answered is in regard to the mechanism by means of which these inter-crystalline impurities increase the hysteresis loss. Does the loss occur in the impurities themselves—or do they increase the loss in the ferrite grains by disturbing the uniformity of the flux distribution? Or, do they act merely as inert material, like graphite or air, having relatively speaking, no capacity for conducting a magnetic flux? In favor of the first suggestion speaks the fact that pure pearlite, consisting of $Fe_3\,C$ and ferrite, has a maximum permeability of 1000 and a hysteresis loss of about 15,000 ergs., 13,500 of which is due to $Fe_3\,C$ as pearlite (the remainder being due to carbon in solution). This is nearly 15 times the loss for iron saturated with carbon in solution (0.006—0.008 per cent C). Now, pearlite consists of about 13.5 per cent $Fe_3\,C$ and 86.5 per cent ferrite, and it is difficult to conceive how a reduction in the amount of active ferrite of only 13.5 per cent can possibly increase the flux density in any part to such an extent as to increase the hysteresis loss to 15 times the original value. On the other hand, how can we account for the difference in the effect on the hysteresis loss of $Fe_3\,C$ as free cementite and of $Fe_3\,C$ associated with ferrite as pearlite, the ratio of the former to the latter being as 1 to 7. Furthermore the permeability of $Fe_3\,C$, even at 1000 gauss (it saturates at less than 3000 gauss)[24], is so extremely low compared with that of ferrite with 0.008 per cent C at 10,000 gauss that the amount of flux passing through the $Fe_3\,C$ would be very small, unless the $Fe_3\,C$ formed a more or less continuous barrier across the flux path.

The only conclusion we can arrive at is, therefore, that there must be a hysteresis loss in the intercrystalline impurities, including in this term the amorphous inter-crystalline cement, and that the more or less continuous films formed by these impurities force the flux to pass through them, and furthermore, that more flux passes through the cementite when it occurs in the finely divided form of pearlite than when it occurs unassociated with the ferrite. It follows from this that the hysteresis loss of $Fe_3\,C$ must be approximately

$$13{,}500/0.135 = 100{,}000 \text{ ergs per cu. cm. per cycle.}$$

23. Assume that the grains are cubes and that the number of grains per $m\,m^2$, N, is the same in every direction. Let the thickness of the cement be "t" $m\,m$. No. grains per $m\,m^3$ = $N^{3/2}$ and each grain has a volume of $1/N^{3/2}\ m\,m^3$. Each grain is bounded by 6 faces of $1/N\ m\,m^2$.

Assign one-half the thickness of the intercrystalline cement to each grain, then the volume of cement per grain is one-half x 6 x $t \times 1/N = 3\,t/N\ m\,m^3$. Vol. of cement per cu. mm. = $3\,t/N\ N^{3/2} = 3\,t\,\sqrt{N}\ m\,m^3$ or vol. of cement per unit volume = $3\,t\,\sqrt{N}$ units.

24. Honda & Murakauri: Spec. Magnetism of Cementite. Tohuku University. *Sci. Repts.* 6 p. 23-29, 1917.

If Fe_3C saturates at a flux density of less than 3000 gauss, as stated by Honda, the flux density in the Fe_3C with 10,000 gauss in the pearlite (obtained by a magnetizing force of 15 gilberts) would be considerably less than 3000 gauss. Supposing, for the sake of calculation, that it is 2500 gauss in the Fe_3C, corresponding to a loss of 100,000 ergs. Applying Steinmetz' coefficient 1.6, the hypothetical loss corresponding to 10,000 gauss would be

$W_{10} = W_{2.5}(10/2.5)^{1.6} = 100{,}000 \times 4^{1.6} = 913{,}000$ ergs.

Now, by regarding Fe_3C as 6.67 per cent carbon dissolved in iron, and applying the formula for dissolved carbon, namely,

$$W_h = 100{,}000 \times C$$

the loss for pure Fe_3C for $B = 10{,}000$ gauss should be

$$W_{10} = 100{,}000 \times 6.67 = 667{,}000 \text{ ergs.}$$

While these two results differ considerably (25 per cent) it is interesting to note that by these two different methods of approach, we can arrive at figures for the loss of Fe_3C that are of the same order of magnitude, and that the hysteresis loss of Fe_3C would approach 1,000,000 ergs for a flux density of 10,000 gauss if such a flux density could be reached.

Summary and Conclusion

1. The effect of sulphur, phosphorus and manganese on the magnetic properties, as represented by the hysteresis loss (ergs per cu. cm. per cycle for $B = 10{,}000$ gauss) can be expressed by the equations:

For 0 per cent *Si* alloys,

$W_h = 18{,}000 \times S + 1000 \times Mn + 13{,}000 \times P + 0(P - 0.015)$

Upper limits $S = 0.10$ $Mn = 1.0$ $P = 0.015$ $P = 0.14$

For 4 per cent *Si* alloys,

$$W_h = 12{,}000 \times S + 800 \times Mn - 4000 \times P$$

Upper limits $S = 0.50$ $Mn = 0.16$ $P = 0.05$

2a. Carbon in amounts up to 0.007—0.008 per cent exist in solid solution (martensite) in iron and in all iron-silicon alloys, even after long annealing and very slow cooling, and has an exceedingly detrimental effect on the magnetic properties:

$$W_h = 100{,}000 \times C$$

b. Carbon goes into solution to some extent up to 0.02 per cent, the extent probably depending upon the annealing conditions.

3. In amounts greater than 0.007—0.008 per cent carbon occurs in one or more of three forms (1) graphite with practically no effect, (2) Fe_3C (cementite) with the effect $\Delta W_h = 2250 \times \Delta C$ or (3) pearlite with the effect $\Delta W_h = 16{,}500 \times \Delta C$. The effect of the latter two is in the ratio of 1:7.4 or in the same ratio as the per cent of carbon containing constituents of the two materials for the same actual carbon content.

4. With 0 per cent *Si*, as carbon increases, it changes from solid solution through Fe_3C to pearlite; in 2 per cent *Si* alloys, the change is directly from solid solution to pearlite; in 4 per cent *Si* alloys the change is from solid solution through Fe_3C to graphite, and in 5-6 per cent *Si* alloys the change is directly from solid solution to graphite. These changes are shown graphically in Fig. 29.

5. In addition to the ordinary impurities, it has been found that the magnetic properties depend to a large extent upon the grain structure, in particular upon the inter-crystalline amorphous cement. Denoting the number of grains per sq. mm. by N, it has been shown that for low carbon contents (for carbon in solution) the effect of grain size on the hysteresis loss may be expressed approximately by the formula,

$$W_h = 65\sqrt{N},$$

whereas for precipitated carbon the effect may be expressed more nearly by the formula,

$$W_h = 3\,N$$

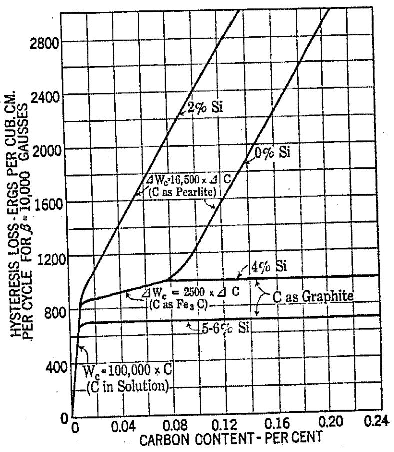

Fig. 29—Effect of Carbon on the Hysteresis Loss of Iron-Silicon Alloys. All other Impurities and Effect of Grain Size Eliminated

6. The hysteresis loss of any *Fe-Si-C* alloy, containing the ordinary impurities, *S*, *P* and *Mn* in small amounts, as for instance commercial electrical steels, can be calculated with a good degree of accuracy by means of the relationships given, using for this purpose one set of constants for 0 and 2 per cent silicon and a second set for 4-6 per cent silicon.

Thus, for carbon in solution, the hysteresis loss is,[25]

For 0—2 per cent *Si*:

	$W_h = 65\sqrt{N} +$	$100{,}000 \times C$	$+ 18{,}000 \times S$	$+ 1000 \times Mn$	$+ 13{,}000 \times P$
Upper Limits		$C = 0.006$	$S = 0.10$	$Mn = 0.60$	$P = 0.015$

For 4—6 per cent *Si*:

	$W_h = 65\sqrt{N} +$	$100{,}000 \times C$	$+ 12{,}000 \times S$	$+ 800 \times Mn$	$-4000 \times P$
Upper Limits		$C = 0.008$	$S = 0.05$	$Mn = 0.16$	$P = 0.05$

25. N = No. grains per sq. mm.

C, S, Mn and P are the percentages of these elements, respectively. The upper limits given are the values of the elements (in per cent) above which the constants either do not hold or have not been investigated.

For precipitated carbon the equations vary more with the silicon content, thus:

	Form of Precipitated Carbon			
	Fe_3C	Pearlite	Graphite	
For 0 per cent Si:				
$W_h = 3N + 800$	$+ 2250 (C - 0.008)$	$+ 16{,}500 (C - 0.09)$		$+ 18{,}000 S + 1000 Mn + 13{,}000 P$
Upper limits........	$C = 0.09$	$C = 0.90$		$S = 0.10 \quad Mn = 0.60 \quad P = 0.015$
For 2 per cent Si:				
$W_h = 3N + 800$		$+ 16{,}500 (C - 0.008)$		$+ 18{,}000 S + 1000 Mn + 13{,}000 P$
Upper limits........		$C = 0.50$		$S = 0.10 \quad Mn = 0.60 \quad P = 0.015$
For 4 per cent Si:				
$W_h = 3N + 800$	$+ 2250 (C - 0.008)$		$+ 0 (C - 0.08)$	$+ 12{,}000 S + 800 Mn - 400 P$
Upper limits........	$C = 0.08$		$C = 1.0$	$S = 0.05 \quad Mn = 0.16 \quad P = 0.05$
For 6 per cent Si:				
$W_h = 3N + 700$			$+ 0 (C - 0.007)$	$+ 12{,}000 S + 800 Mn - 4000 P$
Upper limits			$C = 1.0$	$S = 0.05 \quad Mn = 0.16 \quad P = 0.05$

7. The evidence obtained is to the effect that the increased hysteresis loss due to carbon and other impurities that are precipitated combined with iron is caused by the inherent hysteresis loss of these precipitated impurities.

8. As it is found that the hysteresis loss (for carbon in solution) is proportional to the amount of intercrystalline amorphous material, it is concluded that the latter can be regarded as belonging to the same class as the precipitated impurities, and that the increased hysteresis loss due to decreased grain size consequently is caused by the inherent hysteresis loss of this amorphous material.

9. Regarding the tremendous effect of carbon in solution it is suggested that this is due to the entering of carbon, into the more or less stable equilibrium arrangement of the ferro-magnetic structure, and upsetting this equilibrium arrangement.

10. An attempt has been made in this investigation to separate the various factors influencing the magnetic properties. On account of the large number of factors involved and the large number of separate steps required to obtain the data for the various sets of samples, the conclusion may in some individual case be based on what may seem insufficient evidence. However, if the results are viewed as a whole it will be seen that evidence in one part of the investigation can be used to strengthen evidence in other parts that may seem rather scant if taken by itself. The writer does not wish that the results be considered as final in regard to the various factors affecting the magnetic properties, but it is believed that they are qualitatively correct and that they have opened a new path in the unexplored region of magnetism that may be of service in the further exploration of the same.

Acknowledgements

The alloys were prepared partly by Mr. E. F. Long, partly by Mr. A. A. Frey and partly by the writer; the annealing has been attended to largely by Mr. D. C. Mayne; the carbon analysis has been done partly by Mr. A. L. Shields and partly by Mr. D. C. Mayne.

The magnetic testing has been done under the direction of Mr. Thomas Spooner; the chemical analysis, other than the carbon analysis, under the direction of Mr. C. J. Rodman; and the micro-analysis under the direction of Mr. N. B. Pilling.

All of the above work has been done in the Research Laboratory under conditions insuring great accuracy and the results are therefore believed to be as reliable as is possible with the time and money available. The writer feels greatly indebted to all the above colleagues and takes this occasion to express his appreciation for their valuable cooperation.

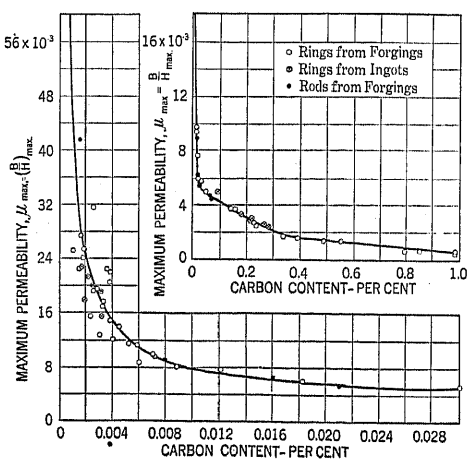

Fig. 30—Maximum Permeability vs. Carbon Content 0 Per Cent Si. No Correction made for Incidental Impurities of Grain Size

Appendix I

Magnetic Testing

In the previous work reported by the writer test pieces was used in the form of round or rectangular bars and these were tested by the Burrow's compensated double-bar and yoke method. For low permeabilities and for uniform bars this method is very accurate, but with high permeabilities and in case of bars of non-uniform permeability, it became evident that the results were far from reliable.[26] In order to get definite data

26. See Eng. Exp. Sta. Univ. of Ill., Bull. No. 83 & 95, 1915 & 1917.

in regard to this method, Mr. Thomas Spooner had test pieces prepared that could be tested both as bars by the Burrow's method and as rings, the ring method being regarded as the ultimate method of reference. His results[27] show that the Burrow's method falls down for maximum permeabilities above 10,000 and becomes increasingly inaccurate as the permeabilities increase. It was, therefore, decided for the purpose of the present investigation where accuracy was a prime factor, to rely altogether on the ring method.[28] While the results obtained with bars in the early part of the investigation were tabulated in a few cases, only the results obtained with rings have been plotted and used as a basis for drawing conclusions.

Appendix II

Apparatus, Materials, Test Pieces and General Procedure

The alloys were all prepared in an Arsem type vacuum furnace by melting together under a pressure of about 2 mm. $H\,g$, electrolytic iron, ferro-silicon, and Acheson graphite in the desired proportions, making 2-kg. ingots.

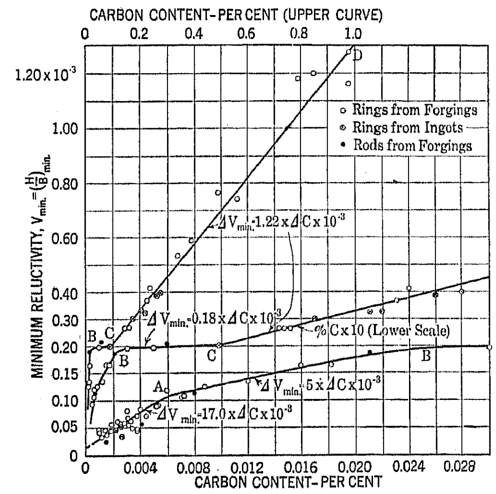

Fig. 31—Minimum Reluctivity ($= 1/\mu_{max}$) vs. Carbon-Content for 0 Per Cent Si. Effect of Incidental Impurities and of Grain Size not Eliminated

Analyses of the electrolytic iron gave the following impurities:

S—0.001—0.005 per cent
P—0.003—0.005
Mn—0.001
C—0.010
Cu—Trace
Si—0.01—0.02
Al—0.01—0.02

Total 0.035—0.061

27. *Elect. Journ.* 18, p. 351, 1921.
28. See T. Spooner: "Rapid Testing of Magnetic Materials," *El. World*, 74, pp. 4-6, July 5, 1919.

Two grades of ferro-silicon were used; namely, Carborundum Co. grade "B" and Goldschmidt-Thermit Co. 91 per cent analyses of which gave the following results:

	Carb. Co. Grade B.	Goldschmidt-Thermit 91 per cent
Si.........	94.89 per cent	91.00 per cent
Al.........	2.06	1.21
C..........	0.20—0.40*	Trace*
S..........	Trace	0.017
Mn.........	Trace	0.04
Fe..........	2.31	6.67
Ti..........	0.12	0.12
Cr..........	Trace	Trace
Ca..........	0.02	
Total........	99.60	99.06

*Not Reliable

The ingots, after being allowed to cool in the furnace,

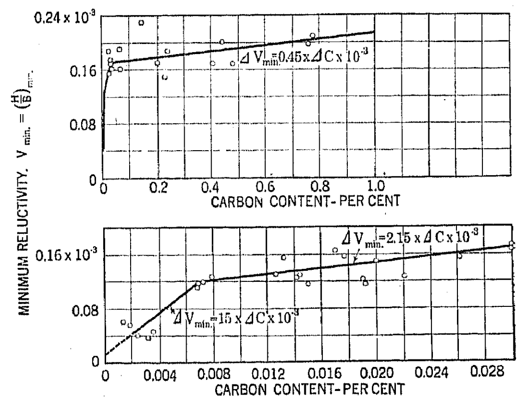

Fig. 32—Minimum Reluctivity ($= 1/\mu_{max}$) vs. Carbon Content for 4 Per Cent Si. Effect of Incidental Impurities and of Grain Size not Eliminated

were forged into bars 3.18 cm. (1¼ in.) dia. from which the test rings were machined 2.46 cm. (31/32 in.) × 1.83 cm. (23/32 in.) × 1 cm. (⅜ in.) or 2 cm. (¾ in.) long. In a few cases rings of different dimensions were used and in the early part of the investigation rods were used but the conclusions are based on results obtained with rings only. The shavings from the machining of the rings were used for chemical analysis for S, Mn and P. The samples were annealed at various temperatures in a closed silica tube under a pressure of < 1 *m m. H g.* In some cases a stream of air 120 cc. per min. at 12 mm. pressure was used in order to reduce the carbon content. The annealed rings were tested magnetically, the B-H curve and the hysteresis loop for $B = 10{,}000$ being obtained in every case. One half of each of the tested rings was then cut up into fine shavings and analyzed for C by the liquid-air-vacuum method and in a large number of cases the other half was examined microscopically to determine the grain size and other structural characteristics.

Appendix III

Permeability and Reluctivity

In the body of this paper only the hysteresis loss has been considered, because the hysteresis loss can be used as a criterion of magnetic quality. As it is desirable, however, for certain purposes to have actual figures in regard to permeabiilty the following curves have been prepared. The maximum permeability

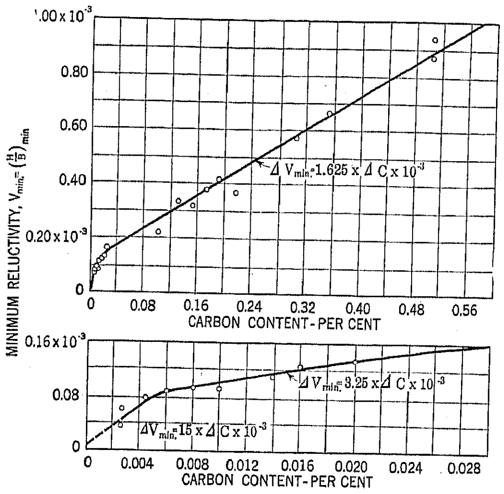

Fig. 33—Minimum Reluctivity ($= 1/\mu_{max}$) vs. Carbon Content for 2 Per Cent Si. Effect of Incidental Impurities and of Grain Size not Eliminated

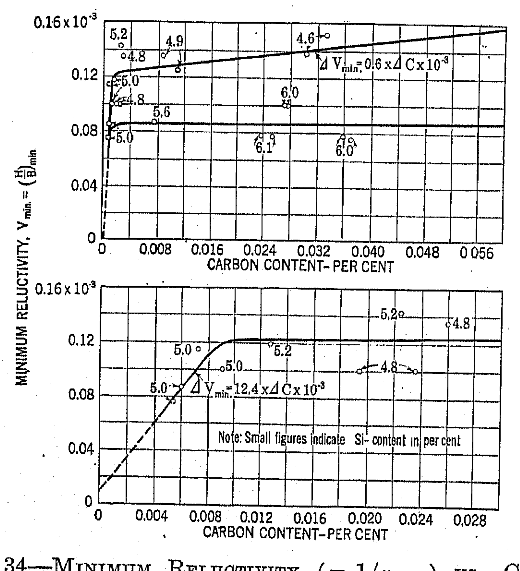

Fig. 34—Minimum Reluctivity ($= 1/\mu_{max}$) vs. Carbon Content for 5-6 Per Cent Si. Effect of Incidental Impurities and of Grain Size not Eliminated

of the Fe-C alloys (0% Si) is plotted in Fig. 30. No attempt has been made in this case to correct for the incidental impurities. The general form of the curve for low carbon contents is seen to be that of a rectangular hyperbola, *i. e.* $\mu_{max} \times C =$ constant. Therefore, by plotting minimum reluctivity, ν_{min} ($= 1/\mu_{max}$), instead of μ_{max} a straight line will be obtained: $1/\mu_{max} = \nu_{min} = C \times$ const. In Fig. 31, ν_{min} has been plotted against per cent C and it will be seen that the shape of this curve is almost identical with that of the curve for hysteresis loss (Figs. 14 and 15). As a matter of fact the ratio of the slopes of the various parts of the curve, OA:BC:CD is 94.5: 1:6.78 or nearly the same as for the hysteresis loss (98:1:7.40) and for the coercive force (100:1:6.76), Fig. 19.

It must therefore be concluded that these three quantities: minimum reluctivity, hysteresis loss, and coercive

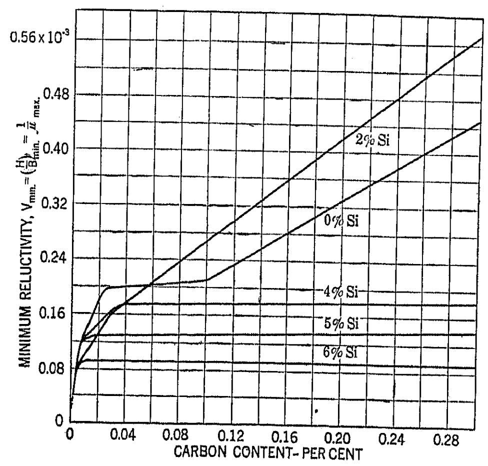

Fig. 35—Minimum Reluctivity ($= 1/\mu_{max}$) vs. Carbon Content for Fe-Si Alloys. No Correction made on Account of Incidental Impurities and Grain Size

force are all identical functions of the carbon content and of the state in which carbon exists.

In Figs. 33, 32 and 34 the minimum reluctivity data have been plotted for 2 per cent, 4 per cent and 5-6 per cent Si respectively, and in Fig. 35 all the curves thus obtained have been redrawn for comparison. While there are certain minor differences between these

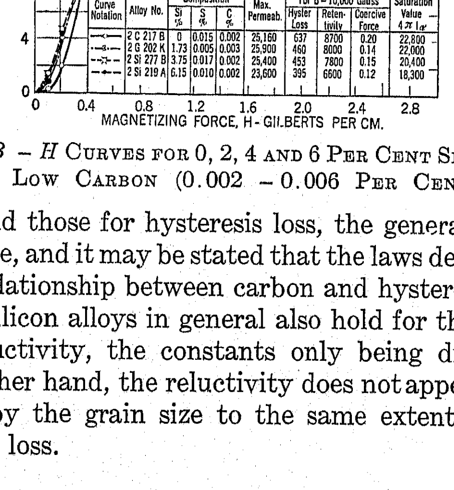

Fig. 36—$B - H$ Curves for 0, 2, 4 and 6 Per Cent Si Alloys. All Low Carbon (0.002 – 0.006 Per Cent)

curves and those for hysteresis loss, the general shape is the same, and it may be stated that the laws developed for the relationship between carbon and hysteresis loss for iron-silicon alloys in general also hold for the minimum reluctivity, the constants only being different. On the other hand, the reluctivity does not appear to be affected by the grain size to the same extent as the hysteresis loss.

In Fig. 36 will be found representative *B-H* curves and other data for 1, 2, 4 and 6 per cent *Fe-Si* alloys with low carbon contents (0.002-0.003 per cent), uncorrected for incidental impurities and for grainsize. The maximum permeability is practically the same for all of them, 25,000, but the high induction permeability and saturation value decreases in proportion to the silicon content. The hysteresis loss and coercive force are seen to be higher for the 0 per cent *Si* than for the other alloys on account of the smaller grainsize. Data un regard to the electrical resistance will be found in Appendix 4.

Appendix IV

Electrical Resistance

1. Fe-C, Fe-Mn, Fe-S & Fe-P alloys

The data tabulated in Appendix VI in regard to electrical resistance have been plotted in Fig. 37, giving the following results:

1. Pure *Fe* has a resistance of 9.6 michroms per cu. cm. at 20 deg. cent.
2. With addition of *Mn*, $\rho = 9.6 + 7 \times Mn$.
3. With addition of *S*, $\rho = 9.6 + 12 \times S$.
4. With addition of *P*, $\rho = 9.6 + 60 \times P + 3.5 \times (P—0.015)$.
   Upper limits.................. $P = 0.015$ per cent $P = 0.12$ per cent
5. With addition of *C*, $\rho = 9.6 + 82.5 \times C + 4.5 \times (C—0.02)$
   Upper limits.................. $C = 0.02$ per cent $C = 0.85$ per cent
   C in Sol. *C* as Fe_3C and pearlite

For iron containing small amounts ofll a of these impurities the electrical resistance will therefore be:

$$\rho = 9.6 + 7\,Mn + 12\,S + 60\,P + 82.5\,C$$

Upper limits $Mn = 1.0$ $S = 0.1$ $P = 0.015$ $C = 0.02$

Thus, for iron containing 0.01 per cent each of *Mn*, *S*, *P* and *C* the resistance will be:

$$\rho = 9.6 + 0.07 + 0.12 + 0.60 + 0.83 = 11.22 \text{ microhms per cu. cm.}$$

In regard to the effect of *C* the curve obtained from Gumlich's formula[29]

$$\rho = 10.5 + 3 \times C + 2\,x\,C^2$$

has been drawn in Fig. 37. This formula was deduced from results obtained by means of materials prepared by two commercial firms (Phoenix and Lindenburg) and contained only a few hundredths of a per cent of *S* and *P* but "relatively high percentages of *Si* (up to 2.4 per cent) and *Mn* (up to 0.52 per cent), the latter being "frequently disturbingly noticeable in the measurements", and "could only partly be taken into consideration."

It is, therefore, believed that our values are the more reliable, our alloys containing only about 0.015 per cent *S*, 0.02 per cent *S i* 0.0015 per cent *Mn* and 0.003 per cent *P*, and that the discrepency is due to the necessity in Gumlich's case of making large and uncertain reductions because of impurities. Another point to be noted is that Gumlich's formula is based on samples, the lowest carbon content of which is 0.07 per cent, and he therefore was unable to detect the important part of the curve below 0.02 per cent *C*.

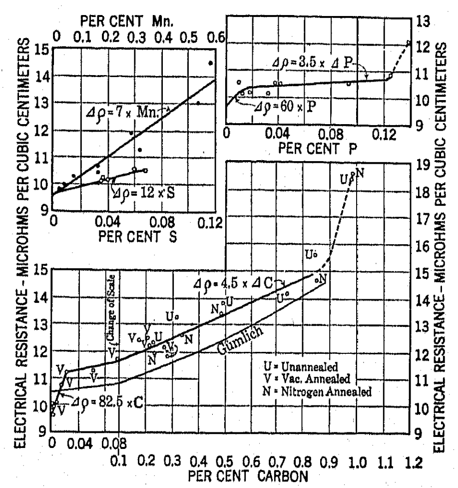

Fig. 37—Electrical Resistance of Fe-C, Fe-Mn, Fe-S and Fe-P Alloys

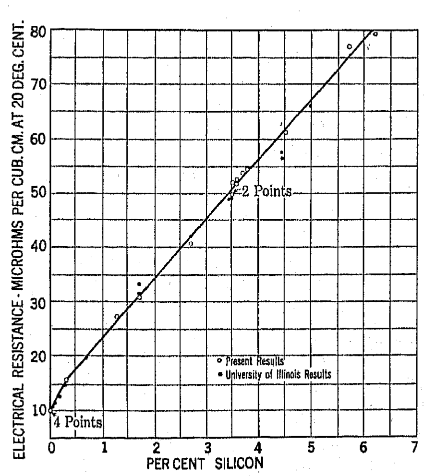

Fig. 38—Electrical Resistance of Iron-Silicon Alloys

Whether the sudden rise above 0.85 per cent (the eutectoid point) is real or due to an experimental error can not be stated, because 1.0 per cent *C* was the limit of the carbon content. This same statement must be made in regard to the sudden rise in the curves for *Fe-P* alloys for $P > 0.12$ per cent. With these excep-

29. *E. T. Z.* July 3, 1919 pp. 325-8.

tions these curves for electrical resistance in general check those for hysteresis loss as shown in Figs. 13, 14 and 15, and thus serve the useful purpose of confirming the conclusions in regard to the relationship between carbon and iron.

2. Fe-Si Alloys

The results obtained for 0–6 per cent *Si* are shown in Fig. 38.

This curve can be expressed in the form of an equation:

$$\rho = 9.6 + 18.4 \times Si + 11.1 \times (Si - 0.35)$$

Upper limits $Si = 0.35$ per cent $Si = 6$ per cent

that is, the curve is composed of two straight lines with a slight bend at $Si = 0.35$ per cent[30].

30. The solid points were obtained by the writer at the University of Illinois in 1915. (See Bull. No. 83 Eng. Exp. Sta.).

Gumlich[31] gives the following equation for the resistance of *Fe-Si* alloys up to 4 per cent *Si*.

$$\rho = 9.9 + 12 \times Si$$

but accompanies this by a statement, that this can not be regarded as accurate, because his curve is slightly concave toward the x-axes. His curve checks the writer's results fairly closely. Thus for 1, 2, 4 and 6 per cent *Si* the following results are obtained.

Per cent *Si*	Gumlich Equation	Gumlich Curve	Writer Equation	Writer Curve
0	9.9	9.9	9.6	9.6
1	21.9	23.0	23.3	23.0
2	33.9	36.0	34.4	34.2
4	57.9	57.5	56.6	56.5
6	81.9	75.0	78 8	78.7

31. *E. T. Z.* July 10, 1919 pp. 334.

Appendix V.

TABLES GIVING ORIGINAL DATA

TABLE IV—MAGNETIC AND ELECTRICAL PROPERTIES OF VACUUM FUSED ELECTROLYTIC IRON. VARIOUS HEAT TREATMENTS

Specimen		Composition before Heat Treatment		Heat Treatment			Composition after Heat Treatment		Elect. Resist. microhms per cub. cm.	Maximum Premeability	Density B in kilo-gausses for H =						Saturat. Value $4\pi I_S$ gausses	For B = 10000 gausses		
No.	Kind Test Piece	S %	Com'd C %	Temp. °C	Cond. Bedding	Cond. Atm.	S %	Com'd C %			0.2	0.4	1	4	20	100		Hysteresis Loss ergs/cc/~	Retentivity gausses	Coercive Force gilberts /cm.
2-202	Rods	..	.016	905	M_gO	Vac	..	$.004_1$	9.91	19,200	0.5	6.4	13.4	15.1	16.2	17.9	22,800	991	9500	0.32
2-203	"	..	..	"	"	"	..	..	9.91	17,200	0.3	4.0	13.4	15.1	16.2	18.0	22,700	1168	9500	0.36
"	"	..	..	1142	"	"	..	..	..	30,000	0.2	11.6	13.9	14.8	15.7	17.5	23,000	713	9700	0.24
"	"	..	..	1093	"	Air 10mm.	.027	$.001_2$	..	31,000	0.4	12.4	14.1	14.8	15.8	17.6	23,200	680	9500	0.22
2-204	"	..	..	905	"	Vac	..	..	10.02	12,500	0.1	0.7	11.7	15.2	16.3	18.2	22,700	..	..	..
"	"	..	..	950	"	Air 10mm.	..	..	10.02	16,400	0.4	5.7	12.4	15.0	16.1	17.9	20,500	955	9200	0.32
2-206	Ring Ingot	..	.006	976	"	Vac	..	$.002_5$	..	20,000	0.8	7.8	13.4	15.5	16.0	17.8	..	873	8200	0.28
"	Ring Forg.	.016	$.007_5$	976	"	"	.018	$.002_8$	..	19,150	0.4	6.0	13.4	15.3	16.1	18.0	..	914	8200	0.30
"	Rods	.016	..	950	"	Air 10mm.	.018	..	9.93	9,000	0.2	1.7	9.2	15.0	16.3	18.1	22,500	1470	7000	0.47
"	"	..	..	900	"	Vac	..	..	9.93	13,500	0.5	4.0	11.4	15.6	16.5	18.3	23,000	1090	8200	0.35
2-207	Ring Ingot	..	$.010_8$	976	"	"	..	$.002_1$	..	21,200	1.0	8.0	13.1	15.0	16.0	17.8	..	800	7800	0.25
2-210	Rods	..	.003	920	"	"	..	..	10.25	16,000	0.2	1.6	13.0	15.9	16.9	18.6	22,400	1060	9000	0.32
"	"	..	..	1142	"	"	..	..	..	33,300	2.8	11.8	14.5	15.5	16.4	18.2	22,950	610	9350	0.20
"	"	..	..	1093	"	Air 10mm.	.014	$.001_5$	..	41,500	6.7	12.9	14.6	15.4	16.4	18.3	23,050	505	9400	0.17
2-251	Ring Ingot	..	$.002_5$	976	"	Vac	..	$.002_6$	..	31,700	5.4	11.1	13.2	14.4	15.6	17.9	..	530	8200	0.17
"	Ring Forg.	.010	$.001_1$	"	"	"	..	$.001_7$	..	27,300	2.4	10.2	12.7	13.9	15.0	17.2	..	647	8400	0.21
"	Rods	.010	$.002_5$	950	"	Oxid	..	..	10.15	16,500	1.4	6.6	12.1	14.5	15.8	17.7	21,600	850	8800	0.26
2-213K	Ring Forg.	.012	$.004_3$	923	"	Vac	..	..	..	20,000	0.9	7.5	13.2	15.2	16.1	18.0	..	789	8900	0.26
"	"	..	..	900	Wood Charc.	" 1 hr.	..	$.001_8$	..	24,000	2.2	9.5	13.6	14.8	15.7	17.4	..	660	9000	0.20
" L	"	.012	$.004_3$	1009	M_g O	Vac	..	..	..	24,900	1.5	9.9	14.1	15.5	16.4	18.2	..	735	9200	0.23
"	"	..	..	1000	Wood Charc.	" 1 hr.	..	$.003_8$	..	20,200	0.8	7.8	13.9	15.4	16.3	17.9	..	810	8900	0.26
" M	"	.012	$.004_3$	1110	M_g O	Vac	..	..	..	20,050	0.6	7.7	13.6	15.3	16.3	18.2	..	904	9520	0.27
"	"	..	..	1100	Wood Charc.	" 1 hr.	..	$.008_7$	..	8,000	0.2	1.5	8.0	13.7	16.0	17.8	..	1595	7600	0.46
" N	"	.012	$.004_3$	910	M_g O	Air 10mm.	..	..	..	25,000	3.9	9.7	13.3	14.6	15.9	18.0	..	517	8700	0.15
"	"	..	..	1200	Wood Charc.	Vac 1 hr.	..	Saturated	..	..	..	..	..	..	..	..	..	..	..	..*
2-219L	"	.020	..	900	None	Vac	..	$.003_5$	..	21,200	..	8.1	14.1	15.8	17.0	19.0	..	825	9100	0.26
2-220L	"	.020	..	"	"	"	..	$.001_8$	..	6,200	..	2.1	5.1	10.1	15.5	18.5	..	850	3400	0.34†
2-221L	"	.019	..	"	"	"	..	$.002_5$	..	19,200	..	6.4	13.8	15.3	16.7	19.0	..	1000	9200	0.33

*Ring partly fused.

†Small Crack in Ring.

TABLE V.

MAGNETIC PROPERTIES OF IRON-CARBON ALLOYS ELECTROL. IRON AND ACHESON GRAPHITE, MELTED IN VACUA

Specimen		Carbon (Graphite) Added %	Composition by Chem. Anal.		Car-bon Lost %	Heat Treatment			Carbon Cont. after Anneal. Comb'd. %	Electr. Resist. microhms per cc.	Maximum Permeability	Density, B, in kilogausses for H = (gilberts per cm.)							Satura. Value $4 \pi I_s$ kilo-gausses	For B = 10000 gauss			Micro-Anal.	
						An-neal Temp °C	Cond.																	
No.	Kind Test Piece		S %	C comb'd. %			Bed-ding Mate-rial	Atm.				0.2	0.4	1	4	20	100	400		Hyst. Loss ergs per cc. per ~	Re-ten-tivity gausses	Coer-cive Force gil-berts per cm.	Grains per sq.mm.	Form of Carbon
2C201	Rods	0.05	0.019	0.014	0.04	910	M_gO	Vac	0.008	10.28	9,000	0.1	0.9	9.0	15.6	17.4	19.0	21.6	22.80	1,700	8100	0.58		
202	"	0.10	0.013	0.020	0.08	"	"	"	0.016	10.81	6,150	0.2	0.8	5.9	14.3	16.8	18.6	21.4	22.50	1,960	7100	0.68		
203	"	0.15	0.010	0.030	0.12	"	"	"	0.021	11.43	5,400	0.2	0.6	4.8	13.4	16.3	18.2	21.1	22.65	2,070	7600	0.67		
204	"	0.20	0.012	0.076	0.12	920	"	"	0.061	11.47	4,670	0.1	0.5	3.8	12.2	15.9	18.0	21.0	22.60	2,150	6550	0.76		
205	"	0.25	..	..	..	"	"	"	..	11.87	4,670	0.2	0.7	4.2	11.9	15.7	17.8	20.8	22.50	1,980	6200	0.70		
"	Ing. Ring	"	0.009	0.094	0.16	"	"	"	0.097	..	5,000	0.2	0.8	4.7	12.2	16.1	18.2	21.7	..	1,990	6400	0.70	196	$F e_3 C$ + small Am't. Pearlite
206	Rods	0.30	..	0.176	0.12	936	"	"	..	12.58	4,470	0.2	0.5	3.8	11.0	15.0	17.5	20.7	22.30	2,480	7350	0.76		
"	Ing. Ring	"	..	0.167	0.13	"	"	"	0.174	..	3,300	0.2	0.4	2.5	10.0	14.8	17.5	20.7	..	3,120	6900	1.00		
207	Rods	0.35	..	0.206	0.14	"	"	"	..	12.29	3,300	0.1	0.4	2.5	9.7	14.6	17.2	20.4	22.05	2,940	6700	0.98		
"	Ing. Ring	"	..	0.209	0.14	"	"	"	0.213	..	3,100	0.1	0.4	2.4	9.3	14.7	17.2	20.5	..	3,320	6600	1.05		
208	Rods	0.40	..	0.205	0.20	930	"	"	..	12.55	3,330	0.2	0.5	2.7	9.9	15.0	17.4	20.6	22.40	2,850	6500	0.94		
"	Ing. Ring	"	..	0.206	0.19	"	"	"	0.205	..	3,000	0.1	0.3	2.4	9.3	14.7	17.4	20.5	..	3,260	6700	1.05		
209	Rods	0.45	..	0.282	0.17	"	"	"	..	11.82	2,870	0.2	0.5	2.2	9.1	14.4	17.1	20.2	22.20	3,620	7000	1.12		
"	Ing. Ring	"	..	0.276	0.17	"	"	"	0.283	..	2,500	0.1	0.3	1.8	8.2	13.9	16.7	19.9	..	4,250	7000	1.25		
210	Rods	0.50	0.010	0.284	0.22	"	"	"	..	12.20	2,870	0.1	0.4	2.2	9.0	14.4	17.2	20.3	22.10	3,590	6900	1.12		
"	Ing. Ring	"	..	0.283	0.22	"	"	"	0.268	..	2,600	0.1	0.3	1.8	8.7	14.6	17.5	20.9	..	4,050	7050	1.20		
211a		0.40	..	0.224	0.18	900	"	N_2	0.235	12.09*	2,450	..	0.2	0.8	8.9	14.4	16.8	..	..	4,800	7400	1.50		
" b		"	..	"	"	"	"	Vac	0.151	12.40†	3,766	0.1	0.5	3.1	10.9	15.3	17.6	..	..	2,940	7400	0.90		
212a		0.60	..	0.315	0.28	"	"	N_2	0.339	12.70*	1,850	..	0.1	0.6	7.3	13.6	14.9	..	..	6,280	7300	2.05		
" b		"	..	"	"	"	"	Vac	0.227	13.40†	2,800	0.1	0.5	2.5	9.2	14.4	17.0	..	..	3,620	7300	1.03		
213a		0.80	..	0.492	0.31	"	"	N_2	0.485	13.50*	1,310	..	0.1	0.4	5.1	12.5	14.2	..	..	8,600	7500	2.85		
" b		"	..	"	"	"	"	Vac	0.387	13.90†	1,700	..	0.3	1.4	6.6	13.5	16.1	..	..	6,830	7600	2.02		
214a		1.00	..	0.742	0.26	"	"	N_2	..	13.80*	743	..	..	0.2	1.3	11.6	14.6	..	..	16,400	7600	5.35		
" b		"	..	"	"	"	"	Vac	0.559	14.20†	1,340	..	0.3	1.4	5.3	14.2	17.5	..	..	8,950	7500	2.70		
215a		1.20	..	0.846	0.35	"	"	N_2	0.850	14.70*	833	..	..	0.3	1.9	11.8	13.8	..	..	14,600	7700	4.80		
" b		"	..	"	"	"	"	Vac	0.791	15.65†	845	..	0.1	0.3	2.1	12.2	15.8	..	..	14,200	7900	4.80		
216a		1.50	..	1.016	0.48	"	"	"	0.985	18.70‡	860	..	0.1	0.2	1.6	10.7	14.2	..	..	16,400	8000	5.30		
" b		"	..	"	"	"	"	"	0.981	18.50†	780	..	0.1	0.2	1.6	10.6	14.0	..	..	17,000	8400	5.30		
217A		0.02	0.015	0.004	0.02	"	None	"	0.0018	..	25,160	1.8	9.4	14.3	15.7	16.7	18.7	300 21.5	..	602	8600	0.20	2	No $F e_3 C$
B		"	"	"	"	"	"	"	"	..	25,160	2.0	9.4	14.3	15.7	16.7	18.6	21.6	..	637	8700	0.20		
C		"	"	"	"	"	"	N_2	0.0019	..	17,840	0.1	5.8	13.4	15.1	16.1	18.2	21.0	..	925	8800	0.31	139	Trace $F e_3 C$
D		"	"	"	"	"	"	"	0.0038	..	22,000	0.1	8.9	13.7	15.2	16.2	18.3	20.9	..	835	9200	0.26		
E		"	"	"	"	900-1000	"	Vac	0.0010	..	24,600	0.5	9.6	15.8	16.6	17.7	19.6	22.0	..	630	8600	0.23	3	No $F e_3 C$
F		"	"	"	"	900	"	"	0.0010	..	22,250	0.1	8.9	14.1	16.0	17.3	19.5	22.4	..	926	8950	0.30	97	No $F e_3 C$
218A	Rings from forgings	0.04	0.026	0.012	0.03	"	"	"	0.0031	..	18,500	0.8	7.1	13.5	15.5	16.5	18.5	21.5	..	810	8800	0.26	2.4	Trace $F e_3 C$
B		"	"	"	"	"	"	"	"	..	18,500	0.7	6.9	13.6	15.6	16.5	18.6	21.5	..	840	8800	0.28		
C		"	"	"	"	"	"	N_2	0.0031	..	15,130	0.1	2.1	12.7	15.1	16.3	18.3	20.9	..	1,170	8800	0.38	66	Trace $F e_3 C$
D		"	"	"	"	"	"	"	0.0054	..	11,250	0.1	0.5	10.7	14.9	16.3	18.4	21.0	..	1,560	9000	0.50		
E		"	"	"	"	"	"	Vac	0.0014	..	22,200	0.1	6.9	14.6	16.3	17.5	19.5	22.2	..	790	8800	0.28		
F		"	"	"	"	"	"	"	0.0044	..	13,900	0.1	0.7	11.6	15.7	17.0	19.6	22.4	..	1,505	8840	0.48		
219A		0.06	0.021	0.008	0.05	"	"	"	0.0022	..	15,500	1.0	6.1	12.8	15.6	16.6	18.7	21.6	..	763	8000	0.25		
B		"	"	"	"	"	"	"	"	..	15,500	1.0	5.9	12.7	15.6	16.7	18.7	21.4	..	858	8000	0.27		
C		"	"	"	"	"	"	N_2	0.0030	..	12,500	0.1	1.0	11.4	15.1	16.3	18.5	21.0	..	1,370	8800	0.45	109	Trace $F e_3 C$
D		"	"	"	"	"	"	"	0.0060	..	8,400	0.1	0.3	8.5	14.7	16.3	18.6	21.4	..	1,720	8600	0.58		
E		"	"	"	"	"	"	Vac	0.0056	..	11,250	0.1	0.4	10.7	15.6	17.3	19.4	22.0	..	1,690	8800	0.50		
F		"	"	"	"	"	"	"	0.0040	..	12,000	0.1	0.9	11.6	15.6	17.0	19.4	22.1	..	1,440	8950	0.48	38	No $F e_3 C$
220A		0.08	0.016	0.015	0.06	"	"	Vac	0.0032	..	16,950	0.7	5.6	13.7	15.8	16.8	18.9	21.7	..	840	8400	0.28		
B		"	"	"	"	"	"	"	"	..	17,350	0.7	5.9	13.7	15.8	16.8	18.8	21.7	..	850	8900	0.28		
C		"	"	"	"	"	"	N_2	0.0038	..	14,280	0.1	2.1	11.9	14.8	16.1	18.3	20.8	..	1,370	9000	0.42		
D		"	"	"	"	"	"	"	0.0071	..	9,450	0.1	0.6	9.4	14.5	16.1	18.2	20.8	..	1,660	8800	0.52	53	$F e_3 C$
E		"	"	"	"	"	"	Vac	0.0120	..	7,600	0.1	0.4	7.5	14.9	17.2	19.4	21.8	..	1,670	7600	0.58	53	$F e_3 C$ > for E
F		"	"	"	"	"	"	"	0.0070	..	9,750	0.1	0.7	9.6	15.0	16.8	19.0	21.8	..	1,470	8300	0.49	174	Trace $F e_3 C$
221A		0.10	0.018	0.065	0.04	"	"	"	0.0303	A	5,840	0.3	1.1	5.9	12.8	16.5	18.7	21.6	..	1,870	7000	0.60		
										.. B	5,870	0.3	1.1	5.9	12.8	16.5	18.7	21.6	..	1,870	7000	0.60		
222A		0.15	0.021	0.049	0.10	"	"	"	0.0182	A	6,000	0.2	0.8	6.0	12.9	16.4	18.6	21.5	..	1,825	7500	0.60		
										.. B	6,000	0.2	0.8	6.0	12.9	16.4	18.7	21.5	..	1,825	7500	0.60		
223A		0.20	0.015	0.110	0.09	"	"	"	0.0505	A	5,040	0.2	0.9	4.9	12.9	16.2	18.5	21.4	..	1,970	6800	0.67		
										.. B	5,040	0.2	0.9	4.9	12.9	16.3	18.6	21.3	..	1,910	6800	0.66		
224A		0.30	0.017	0.177	0.12	"	"	"	0.142	A	3,800	0.2	0.7	3.5	11.0	15.6	18.3	20.9	..	2,780	6800	0.85	168	Pearlite
										.. B	3,800	0.2	0.7	3.5	11.0	15.5	18.3	21.0	..	2,690	6800	0.84		

*Annealed Rods.
†Unannealed Rods.
‡Rod Annealed in N_2.
Ing. Ring = Ring machined from Ingot.

TABLE VI

MAGNETIC PROPERTIES OF IRON-MANGANESE ALLOYS. ELECTROLYTIC IRON USED AS BASE

Specimen No.	Kind Test Piece	Addition: Element	Addition: %	Chem. Analysis of Forging: C %	Chem. Analysis of Forging: Mn %	Chem. Analysis of Forging: S %	Chem. Analysis of Forging: P %	Heat Treatment: Temp. °C	Heat Treatment: Condition	Carbon Content after Heat Tr. Comb'd. C %	Electr. Resist. microhms per cub. cm.	Max. Permeability	Density, in kilogausses for H = .2	.4	1	4 or 5	20	100	200 300 or 400	Satura. Value $4 \pi I_S$ gausses	For B = 10000: Hyst. Loss ergs per cc/~	Retentivity gausses	Coercive Force gilberts per cm.
2*Mn*201	Rods	*Mn*	.05		.036			950	Vac		10.19	10,600	.5	2.7	9.8	4 15.1	16.5	18.2	400 21.2	22,850	1050	7300	0.35
202	"	"	.10		.047			"	"		10.40	10,000	.3	2.1	9.5	15.0	16.3	18.1	21.1	22,800	1200	7300	0.43
203	"	"	.15		.074			"	"		10.70	9,400	.3	2.0	9.0	14.9	16.3	18.0	21.2	22,800	1120	7100	0.40
204	"	"						918	"		10.81	13,150	.4	2.4	11.8	15.7	16.9	18.7	21.3	22,600	1105	8100	0.39
"	Ing Ring	"	.20		.16	.014		950	"	0.0021		11,700	.5	4.2	10.4	14.5	15.9	18.2	300 20.9		869	7000	0.29
205	Rods	"	.30		.16	.017		918	"		11.04	10,000	.4	2.2	9.9	15.4	16.7	18.4	400 21.4	22,800	1070	7200	0.38
"	Ing. Ring	"						950	"	0.0030		12,800	.5	4.9	10.6	14.5	15.8	17.9	300 20.5		997	7000	0.31
206	Rods	"	.40		.32	.016		918	"		11.55	10,200	.4	2.0	10.2	15.3	16.5	18.4	400 21.3	22,700	1185	7500	0.41
"	Ing. Ring	"						950	"	0.0016		15,400	1.7	6.3	11.6	15.2	16.5	18.6	300 21.3	Spec Gravity	664	8400	0.21
207	Ring Punch	"	.53		.29	.006		"	"			12,000	.7	3.3	11.0	5 15.0	16.0	17.9	200 20.2	7.84	1040	7830	0.34
"	"										12.24*	11,920	.6	2.8	10.5	14.7	15.7	17.6	19.9	7.85	1090	7680	0.36
"	Ring	"						"	"	0.0018		16,100	1.9	6.4	11.4	14.9	16.1	18.1	20.5	7.85	802	8000	0.24
208	Ring Punch	"	.74		.54	.027		"	"			12,420	.6	3.0	11.6	15.1	16.0	17.8	20.0	7.80	1120	8190	0.36
"	"	"						"	"		13.42*	12,200	.5	2.7	11.4	15.1	16.1	18.0	20.3	7.80	1104	8100	0.37
"	Ring	"						"	"	0.0013		10,930	.4	1.4	10.9	15.2	16.1	17.9	20.2	7.89	1460	8600	0.48
209	Ring Punch		1.05		.59	.028		"	"			13,000	.4	2.2	11.8	15.1	16.0	17.8	20.0	7.90	1207	8700	0.39
"	Ring	"						"	"		15.04*	13,000	.5	2.2	11.8	15.0	16.0	17.8	20.0	7.89	1173	8480	0.39
"	"	"						"	"	0.0020		10,920	.4	1.3	10.9	15.1	16.1	17.9	20.1	7.90	1480	8600	0.49

*Annealed Rods ¼ in. dia.

Spec. Grav. shown in same Col. as Sat. Valve

Ing. Ring = Ring machined from Ingot

Ring. Punch = Ring punched from ⅛ in. plate

Ring = Ring machined from Forging

Rod = Rod machined from Forging

TABLE VII

MAGNETIC PROPERTIES OF IRON-SULPHUR AND IRON-PHOSPHORUS ALLOYS. ELECTROLYTIC IRON USED AS BASE.

Specimen		Addition		Chem. Analysis of Forging				Heat treatment		Carbon Content after Heat Tr. Comb'd. C %	Electr. Resist. microhms per cub. cm.	Max. Permeability	Density, in kilogausses, for H =							Satur. Value $4\pi I_s$ gausses	For B = 10000		
No.	Kind Test Piece	Element	%	C %	Mn %	S %	P %	Temp. °C	Condition				.2	.4	1	4 or 5	20	100	300 or 400		Hyst. Loss ergs per cub. cm. per or~	Retentivity gausses	Coercive Force gilberts per cm.
2 *S* 201	Ring	*S*	.005			.020		923	Vac	0.0014		20,000	0.6	7.5	13.6	4 15.2	16.2	18.1	300 20.4		820	9000	0.27
202	"	"	[illegible]			.020		"	"	0.0017		17,500	0.6	5.2	13.3	15.2	16.2	18.1	300 20.6		926	8900	0.30
"	Rod	"	"			.035		"	"		10.30	14,500	0.1	1.4	12.9	15.5	16.5	18.2	400 21.1	22,750	1130	9100	0.37
203	Ring	"	.020			.042		"	"	0.0009		16,200	0.2	4.3	12.8	14.7	15.8	17.8	300 20.4		1009	9200	0.32
"	Rod	"	"			.036		"	"		10.40	14,500	0.1	1.4	12.9	15.5	16.5	18.2	400 21.1	22,750	1130	9100	0.37
204	"	"	.030			.060		"	"		10.65	10,000	0.1	0.4	9.5	15.8	17.0	18.7	400 21.4	22,700	1775	9200	0.58
205	Ring	"	.040			.036		"	"	0.0005		14,300	0.2	2.7	12.5	14.9	15.9	18.0	300 20.6	(7.90)	1110	9200	0.35
"	Rod	"	"	.009		.068		"	"		10.55	6,000	..	0.4	4.2	15.3	17.0	18.8	400 21.3	22,300	2440	9000	0.79
206	Ring	"	.050			.092		"	"	0.0020		8,600	0.05	0.3	7.8	14.3	15.9	18.0	300 20.4	(7.89)	1985	9300	0.63
"	Rod	"	"			.040		"	"		10.30	11,200	0.05	1.2	11.1	16.2	17.3	18.9	400 21.6	22,450	1420	9300	0.47
2 *P* 201	Ring	*P*	.005			.021	.010	924	Vac	0.0014		18,700	0.5	7.1	12.9	15.2	16.1	18.2	300 20.8	(7.89)	842	9000	0.27
"	Rod							"	"		10.80	16,300	0.2	3.1	13.3	15.3	16.3	18.0	400 20.8	21,900	960	8900	0.32
202	Ring	"	.010			.020	.012	"	"	0.0014		20,000	0.5	8.0	13.7	15.1	15.9	18.0	300 20.4	(7.90)	806	9200	0.26
"	Rod							"	"		10.35	21,700	0.6	7.5	14.6	16.2	17.1	18.7	400 21.5	22,700	820	9000	0.28
203	Ring	"	.020			.016	.019	"	"	0.0008	Rod 10.35	18,700	0.4	6.3	13.1	15.2	16.2	18.2	300 20.6	(7.89)	881	9200	0.29
204	Ring	"	.030			.023	.033	"	"	0.0015		17,500	0.5	5.2	13.4	15.2	16.2	18.1	300 20.6	(7.89)	980	9200	0.32
"	Rod							"	"		10.45	20,000	0.3	4.4	14.4	16.3	17.2	18.8	400 21.5	22,400	905	9200	0.31
205	Ring	"	.040			.020	.037	"	"	0.0011		18,800	0.4	6.1	13.4	15.1	16.2	18.1	300 20.6	(7.90)	922	9200	0.31
"	Rod							"	"		10.80	18,300	0.2	2.1	14.2	16.2	17.1	18.7	400 20.9	22,400	1037	9100	0.35
206	Ring	"	.050			.020	.041	"	"	0.0016	Rod 10.85	19,600	0.6	7.3	12.4	14.3	15.7	17.6	300 20.4	(7.88)	863	9200	0.28
207	Ring							922	"			15,320	0.4	4.2	12.6	5 15.5	16.4	18.2	300 20.4	(7.95)	1070	9100	0.32
"	Punchings	"	.070			.036		"	"		Rod 11.00	17,100	0.6	5.6	13.2	15.7	16.6	18.3	300 20.5	(8.19)	985	9100	0.29
"	Ring						.094	"	"	0.0006		15,000	0.3	3.8	12.2	15.4	16.3	18.1	300 20.1	(7.93)	1116	9180	0.35
208	Ring							"	"			15,850	0.6	5.0	12.9	15.7	16.6	18.3	300 20.5	(8.10)	1010	9000	0.31
"	Punchings	"	.100			.029		"	"		Rod 11.34	18,400	0.7	7.1	13.3	15.7	16.8	18.6	300 20.9	(8.15)	833	9100	0.25
"	Ring						.125	"	"	0.0017		15,880	0.3	3.2	13.0	15.4	16.2	17.9	300 20.0	(7.91)	1163	9190	0.37
209	Ring							"	"			14,260	0.3	2.3	12.8	15.7	16.9	18.7	300 21.0	(8.08)	1200	9040	0.38
"	Punchings	"	.150			.028		"	"		Rod 12.55	16,960	0.4	4.1	13.6	16.0	16.9	18.7	300 20.7	(8.08)	1200	9020	0.38
"	Ring						.139	"	"	0.0016		17,000	0.3	4.0	13.0	15.2	16.1	18.3	300 20.1	(7.95)	1034	9100	0.33

Spec. Grav. shown in ( ) under Sat. Value.

TABLE VIII

MAGNETIC PROPERTIES VS. CHEMICAL COMPOSITION 4%. Fe-Si ALLOYS & COMMERCE 4% SI-STEEL. RING TEST PIECES

Specimen No.	Elements Added Si %	Elements Added C %	Elements Added S %	Elements Added P %	Elements Added M_n %	Chem. Anal. of Forging Si %	Chem. Anal. of Forging C %	Chem. Anal. of Forging S %	Chem. Anal. of Forging P %	Chem. Anal. of Forging M_n %	Heat Treatment Temp °C	Heat Treatment Condition	Chem. Anal. after Heat Treatment C %	Chem. Anal. after Heat Treatment S %	Max. Permeability μ_{max}	B for μ_{max} gausses	Density, B, in kilogausses for H = .2	1	4	20	300	For B = 10000 gauss: Hyst. Loss ergs/cc. per ~	Retentivity gausses	Coercive Force gilberts per cm.	Micro-Anal.: Grains per sq. mm.	Form of Carbon
First Series: Goldschmidt-Thermit Co. 91% Ferro-Silicon used. (a) Electrolytic Iron Base.																										
2 Si 277	4	0	0	0	0	3.75	.027	.017	.003	.010	900	Partial Vac. of 12mm. Hg. – 120 cc. Air per min. passed through. 900° Anneal – 4 hrs. 1100° Anneal – 2 hrs.	.017	.015	6,120	4600	.3	5.8	12.3	15.3	20.6	1100	4800	.42		
											1100		.002	.016	25,400	5600	4.7	12.4	14.2	15.4	20.4	453	7800	.15		
2 Si 278	4	0	0	0	0	4.09	.028	.013	.003	.020	900		.018	..	6,400	4800	.3	6.1	12.4	15.4	20.4	1040	4800	.44	4.5	
											1100		$.001_7$	..	17,700	6200	3.4	12.3	14.3	15.4	20.2	517	7400	.17	5.9	
2 Si 280	4	0	.05	0	0	4.16	.023	.046	.003	.010	900		.020	..	3,080	4000	.1	2.9	8.9	14.2	20.1	1800	4000	.71		
											1100		$.005_6$	..	5,000	4400	.2	5.0	11.4	14.7	20.1	884	3200	.34	2.5	
2 Si 281	4	0	0	.02	0	3.96	.015	.015	.022	.010	900		.024	..	7,470	5000	.3	7.2	12.7	15.4	20.1	1005	5200	.34	3.9	
											1100		$.003_2$	..	18,750	7500	3.2	12.0	14.1	15.3	20.3	552	7000	.19	1.9	
2 Si 282	4	0	0	.04	0	4.13	.022	.014	.049	.010	900		.015	..	7,100	5400	.4	7.5	13.8	16.6	20.4	905	4800	.34	2.4	
											1100		.005	..	26,000	5200	5.2	12.8	12.2	15.4	20.5	432	7200	.14	2.9	
2 Si 283	4	0	0	0	.10	4.10	.010	.011	.016	.100	900		.023	..	6,000	5500	.2	6.1	12.9	15.4	20.1	1090	4400	.49	5.5	
											1100		$.001_3$	..	24,000	4800	5.0	11.9	14.0	15.0	19.8	480	7000	.15	3.4	
2 Si 284	4	0	0	0	.20	3.93	.005	.023	.003	.140	900		.019	..	5,820	6400	.2	5.8	12.9	15.5	20.2	1280	5200	.50	5.0	
											1100		$.003_4$	..	15,800	6000	1.2	11.1	14.0	15.5	20.4	687	7200	.23	5.9	
2 Si 285	4	0	.05	.02	.15	3.77	.011	.047	.025	.110	900		.015	..	5,370	6400	.2	5.4	13.1	15.8	20.7	1450	5400	.58	5.2	
											1100		$.002_2$	..	20,400	7350	.3	12.5	14.5	15.6	20.3	760	8600	.26	5.2	
2 Si 286	4	0	.05	0	0	4.18	.021	.025	.002	.010	900		.018	..	6,420	5600	.2	6.2	12.9	15.6	20.6	1175	5200	.45	2.1	
											1100		$.003_7$	..	17,200	6900	1.0	12.0	13.9	15.3	20.3	674	7600	.22	3.3	
2 Si 287	4	0	.10	0	0	4.07	.026	.041	.002	.010	900		.021	.041	5,500	5500	.1	5.5	11.7	15.1	20.2	1420	5400	.52		
											1100		.005	.040	13,300	8000	.1	11.5	14.0	15.3	20.2	1040	8800	.34	4.4	
2 Si 290	4	0	0	0	0	3.85	.029	.008	..	..	900	2hrs N_2	.015	..	8,700	5400	.5	7.9	12.5	14.8	20.0	950	5500	.33		Fe_3C
2 Si 291	4	0	0	0	0	4.04	.029	.008	..	..	900	" "	.023	..	8,100	6000	.4	7.6	12.5	14.7	20.0	1060	6100	.40		
2 Si 294	4	0	0	0	0	3.70	.013	.009	..	..	1050	" Vac	$.008_6$	..	5,700	4000	1.3	5.4	11.2	14.7	19.6	945	3800	.40		
1 G 216	4	.10	0	0	0	3.48	..	.011	..	..	"	" Part Vac	.003	..	16,500	6600	1.3	10.3	12.5	14.4	19.3	660	7200	.21		
											900	" "	.038	..	6,400	4800	.3	5.6	12.3	15.5	20.8	1295	4050	.46		
1 G 217	4	.20	0	0	0	4.14	..	.008	..	..	"	4 " "	.034	..	5,450	6000	.2	5.5	12.5	15.6	20.9	1310	5400	.42		
											"	2 " "	.065	..	6,500	6500	.4	6.5	13.3	15.4	20.6	1167	5100	.45	29	Graphite
1 G 218	4	.30	0	0	0	4.04	..	.011	..	..	"	4 " "	.062	..	6,800	6020	.3	6.0	13.2	15.6	20.6	1370	5200	.45	61	trace Fe_3C
											"	2 " "	.200	..	5,810	6800	.3	5.7	12.4	14.9	20.3	1495	5860	.57	128	Graphite
											"	4 " "	.190	..	1,560	5200	.2	1.5	6.3	14.5	20.2	1270	1200	.50	101	Graphite Ring Cracked
1 G 219	4	.50	0	0	0	3.90	..	.008	..	..	"	2 " "	.244	..	5,500	6600	.3	5.4	12.4	15.0	20.2	1640	5870	.61	74	Graphite
											"	4 " "	.229	..	7,000	5550	.3	5.3	12.4	15.2	20.3	1520	5660	.60	226	tr. Fe_3C + Gr.
b. Armco Iron Base																										
3 Si 203	4	0	0	0	0	3.96	..	.030	.005	.050	900	4 hrs. 12mm. Hg. Vac. 120cc. Air per Min.	.027	..	6,500	5800	.3	6.4	13.0	15.6	20.6	1305	5600	.48		
3 Si 204	4	0	0	0	0	4.20	..	.023	.005	.030	"	"	.021	..	7,300	5800	.2	7.1	13.3	15.5	20.6	1230	5600	.40		
											1100	2 "	$.004_2$	..	15,300	7200	1.1	11.7	14.1	15.2	20.1	605	7200	.22		
Second Series: Carborundum Co. 95% Ferro-Silicon used. Electrolytic Iron Base																										
2 Si 295	4	0	0	0	0	3.60	.011	.010	..	..	1050	2hrs. Vac	$.007_1$	..	8,400	4200	.7	7.5	12.3	14.9	19.8	790	4400	.30		
2 Si 296	4	0	0	0	0	3.67	..	.006	..	..	"	" Part Vac	$.003_5$	..	22,800	8000	2.8	12.5	13.6	14.8	19.6	485	7000	.16		No Fe_3C
											1050	" Vac	$.013_1$	..	6,450	4000	.5	5.9	11.5	14.9	19.7	820	3800	.33		Fe_3C
2 G 205	3.85	.05	0	0	0	3.51	.013	.005	tr	..	"	" Part Vac	$.002_2$	..	7,000	1400	1.4	4.0	8.5	14.4	19.2	390	1800	.16	Small	Crack
											936	" Vac	.013	..	7,880	5200	.5	7.0	11.9	13.8	19.5	1064	5300	.36		
2 G 206	3.85	.10	0	0	0	3.54	.015	.005	tr	.	900	" N_2	.008	..	8,220	4600	1.0	7.6	12.3	14.2	20.3	805	4600	.25		
											936	" Vac	.014	..	8,030	4900	.7	7.4	12.2	14.5	19.8	840	4900	.31		
2 G 207	3.85	.20	0	0	0	3.56	.048	.005	tr	..	900	" N_2	.007	..	8,700	5400	1.0	8.1	12.5	14.5	20.4	794	4900	.25		
											936	" Vac	.030	..	5,960	4000	.5	5.6	11.2	14.1	19.9	1043	3800	.35		
2 G 208	3.85	.30	0	0	0	3.52	.117	.006	tr	..	900	" N_2	.014	..	8,570	5400	1.0	7.9	12.7	14.4	20.4	844	4900	.26		
											936	" Vac	.145	..	4,420	6000	.3	4.2	11.9	14.9	19.6	1535	4950	.64		
2 G 216	4	.80	0	0	0	4.26	.467	..	..	..	900	" N_2	.057	..	5,340	4800	.4	5.3	11.8	15.1	20.0	1375	5000	.44		
2 G 217	4	1.00	0	0	0	3.95	.585	..	..	..	"	" "	.440	..	5,000	5000	.3	5.0	11.2	14.6	19.6	1610	5600	.59	144	Graphite
											"	" "	.405	..	6,000	6000	.3	6.0	12.6	15.2	19.8	1320	5800	.50	154	"
2 G 218	4	1.20	0	0	0	4.27	.817	..	..	..	"	" "	.485	..	6,000	6000	.3	5.9	12.2	15.0	19.8	1475	5800	.52	111	"
											"	" "	.760	..	5,070	6600	.3	5.0	11.9	14.5	19.2	1770	6200	.63		
											"	" "	.775	..	4,820	6000	.3	4.8	11.6	14.5	19.3	1820	6000	.63		
Third Series: 4% Commercial Silicon Steel (No. 1 A W). Variously Treated.																									Remarks	
M- 203	0	0	0	0	0	3.98	..	.024	.018	.110	900	4hrs. 12mm. Hg. Vac. 120cc. Air per Min.	.027	..	6,500	6500	.3	6.5	13.4	15.7	20.5	1245	5400	.46	Melted in Vacum	
											1100	2 "	$.003_5$	..	18,000	7200	.12	11.9	14.1	15.1	19.8	680	8100	.22		
M- 204	0	0	0	0	0	4.01	..	.020	.019	.120	900	4 "	.029	..	6,600	5400	.3	6.5	13.0	15.6	20.7	1180	5400	.43		
											1100	2 "	$.003_5$	..	18,200	6000	.13	12.0	14.2	15.3	20.0	635	7800	.21		
M-14-00	0	0	0	0	0	3.97	.058	.029	.015	.120	1000	20 " Vac	.042	..	6,750	6000	.3	6.6	11.1	15.3	19.5	1205	5400	.48	Rings machined from Sheet bars that had been decarbonized.	
M-14-10	0	0	0	0	0	4.09	.008	.027	.018	.125	"	20 " "	$.005_2$	..	13,650	8100	.3	10.6	13.7	15.0	19.7	775	6600	.27		
M-14-30	0	0	0	0	0	3.95	.007	.026	.022	.130	"	20 " "	$.002_3$	..	21,900	9200	2.1	12.9	14.3	15.7	19.8	620	8200	.20		
M-14-40	0	0	0	0	0	3.85	.006	.036	.021	.130	"	20 " "	$.003_3$	..	15,100	8000	.8	11.8	14.0	15.2	19.7	877	7900	.30		
M-14-50	0	0	0	0	0	3.87	.003	.027	.016	.135	"	20 " "	$.002_2$	..	19,150	6900	1.5	12.5	14.2	15.3	19.8	675	7800	.22		
M:14-60	0	0	0	0	0	4.06	.018	.028	.016	.137	"	20 " "	.023	..	7,170	5800	.4	7.1	13.2	15.3	19.7	1242	5800	.46		

TABLE IX

MAGNETIC PROPERTIES OF 2% Fe-Si ALLOYS. RING TESTPIECES

Specimen No.	Elements Added: Si %	Elements Added: C %	Chem. Analysis of Forging: Si %	Chem. Analysis of Forging: C %	Chem. Analysis of Forging: S %	Heat Treatment: Temp °C	Heat Treatment: Atm.	Heat Treatment: Anneal Period hrs.	C-Cont after Annealing %	Magnetic Properties: Max. Permeability	Density for max. μ	Density in kilo-gausses for H = .2	1	4	20	300	For B = 10000 gauss: Hyst. Loss ergs per cc. per ~	Retentivity gauss	Coer. Force gilberts per cm.	Net* Hyst. Loss due to Carbon & Grain Size
									First Series: Carborundum Co. 95% Ferro-Silicon											
2G201 K	1.86	.05	1.76	.013	Assumed 0.005%	900°	Vac	3	.0037	18,750	6000	2.7	11.3	13.0	14.4	21.2	. 513	6400	.170	423
" L						"	N_2	2	.0026	16,600	5800	2.6	11.4	13.3	14.8	21.1	576	6400	.175	486[1]
2G202 K	1.86	.10	1.73	.013		"	Vac	3	.0025	25,900	7000	4.1	13.1	14.3	15.6	21.3	460	8000	.140	370
" L						"	N_2	2	.0044	13,100	9200	0.5	11.5	14.2	15.4	20.2	1,216	8900	.380	1126[2]
2G203 K	1.86	.20	1.76	.013		"	Vac	3	.0070	22,000	8000	1.6	12.9	14.0	15.4	21.2	700	8800	.210	610
" L						"	N_2	2	.0060	11,900	8800	0.3	10.8	13.6	14.9	20.3	1,300	8900	.380	1210
2G204 K	1.86	.30	2.00	.046		"	Vac	3	.0100	11,650	7000	0.4	10.1	14.2	15.9	21.0	915	7200	.300	825
" L						"	N_2	2	.0200	7,450	7000	0.4	7.4	13.4	15.8	20.9	1,335	6700	.430	1245[3]
									Second Series: Goldschmidt-Thermit. 91% Ferro-Silicon											
1G201 Ax_1	2.0	.35	2.44	.220	.009	900	Vac	4	.1890	2,400	6000	0.1	1.6	9.2	15.1	20.6	4,130	6800	1.35	3970
1G202 "	"	.40	2.02	.200	.007	"	"	"	.2150	2,720	6800	0.1	0.8	9.5	15.0	20.2	5,200	7900	1.65	5070
1G203 "	"	.50	1.93	.325	.002	"	"	"	.3530	1,500	6000	0.1	0.5	6.0	13.9	20.1	7,700	7500	2.30	7660
1G204 "	"	.70	2.07	.530	.016	"	"	"	.5030	1,150	6900	0.1	0.3	3.8	13.4	19.4	10,250	7500	3.40	9960
1G211 "	"	.15	1.97	.138	.010	"	"	"	.1300	3,000	7200	0.1	1.6	10.3	15.5	20.2	3,470	6700	1.20	3290
1G214 "	"	.10	1.97	.052	.012	"	"	"	.0218	5,830	6000	0.2	5.7	12.7	15.9	20.7	1,520	5900	0.52	1300
1G221 "	"	.11	1.94	..	.013	"	"	"	.0157	8,100	6000	0.2	7.8	13.5	15.8	20.8	1,270	7100	0.40	1040
1G201 Ax_2	"	.35	2.44	.220	.009	"	"	8	.152	3,150	6300	0.1	2.4	10.4	15.3	20.6	3,000	6300	1.05	2840
1G202 "	"	.40	2.02	.200	.007	"	"	"	.1760	2,600	7000	0.1	1.2	9.6	14.8	20.4	4,700	8000	1.45	4570
1G203 "	"	.50	1.93	.325	.002	"	"	"	.3090	1,750	7000	0.1	0.8	7.6	14.0	20.2	6,400	7300	1.90	6360
1G204 "	"	.70	2.07	.530	.016	"	"	"	.5030	1,060	5300	0.1	0.4	4.1	13.2	19.4	10,200	7600	3.30	9910
1G211 "	"	.15	1.97	.138	.010	"	"	"	.1010	4,550	6000	0.1	4.1	11.8	15.7	20.6	2,390	6700	0.75	2210[4]
1G214 "	"	.10	1.97	.052	.012	"	"	"	.0140	9,100	7000	0.2	8.5	13.7	15.7	20.8	1,270	7600	0.41	1050
2G221 "	"	.11	1.94	..	.013	"	"	"	.0080	10,900	7000	0.2	9.9	14.2	16.0	21.1	1,100	7800	0.34	870

*Deducting 18,000 × % S from the total hysteresis loss.

1. >0.5 Grains per mm.^{2} No Fe_3C.
2. 48 Grains per mm.^{2} No. Fe_3C.
3. 18 Grains per mm.^{2} Fe_3C + Pearlite
4. $\left\{\begin{array}{l}\text{18 Grains per mm.}^2\\ \text{1640 Grains per mm.}^2\end{array}\right\}$ Pearlite.

TABLE X

MAGNETIC PROPERTIES OF 5-6% Fe-Si ALLOYS. RING TEST PIECES

Specimen No.	Elements Added: Si %	Elements Added: C %	Chem. Analysis of Forging: Si %	Chem. Analysis of Forging: C %	Chem. Analysis of Forging: S %	Heat Treatment: Temp °C	Heat Treatment: Atm.	Heat Treatment: Anneal Period hrs.	C-Cont after Annealing %	Magnetic Properties: Max. Permeability	Density for max. μ	Density in kilo-gausses for H = .2	1	4	20	300	For B = 10000: Hyst. Loss ergs per cc. per ~	Retentivity gauss	Coer. Force gilberts per cm.	Net* Hyst. Loss due to Carbon & Grain Size
									First Series: Carborundum Co. 95% Ferro-Silicon											
Kx_1					Assumed 0.006%	936	Vac	2	.0225	7,060	4,800	.5	6.6	11.7	13.4	19.0	862	3650	.31	790
2G210 Kx_2	5.4	.10	5.18	.018		900	Vac	4	.0189	2,130	4,000-10,000	.3	2.0	8.5	13.5	19.8	455	600	.25	383†
Lx_1						900	N_2	2	.0128	8,400	4,200	.9	7.7	11.9	13.5	19.3	688	3800	.26	616
Kx_1						936	Vac	2	.0260	7,430	5,200	.6	7.1	11.8	13.6	19.4	732	3700	.27	660
2G211 Kx_2	5.4	.20	4.84	.026		900	Vac	4	.0237	10,000	5,000	.2	9.0	12.0	13.7	19.7	790	5200	.28	718
Lx_1						900	N_2	2	.0194	10,000	5,000	1.2	8.2	11.6	13.4	19.4	785	4500	.29	713
2G212 Kx_1	5.4	.30	5.64	.085		936	Vac	2	.0770	11,700	5,000	1.0	9.3	12.9	14.6	19.5	670	4750	.20	600
Lx_1						900	N_2	2	.0660	3,000	600	.3	1.5	5.0	13.4	18.7	397	600	.22	325†
Kx_1						936	Vac	2	.0072	8,770	5,000	.7	8.0	12.8	13.7	19.4	828	4800	.30	756
2G213 Kx_2	5.4	.05	5.02	.0086		900	Vac	4	.0060	11,500	4,600	.3	8.8	12.9	14.8	20.0	650	5300	.22	580
Lx_1						900	N_2	2	.0092	10,000	5,000	.8	8.3	12.2	14.0	19.3	760	5000	.27	688
Lx_2						900	Vac	4	.0055	13,300	4,800	.3	8.9	12.3	14.0	19.8	600	5400	.17	530
2G214 Ax_1	4.0	.40	4.88	.129		900	Vac	4	.1110	8,000	4,000	.5	7.1	12.8	14.7	20.2	813	4400	.31	693
Bx_1						900	Vac	4	.0860	7,330	4,400	.5	6.9	13.0	15.0	20.1	925	4600	.33	805
2G215 Ax_1	4.0	.60	4.65	.372		900	Vac	4	.3080	7,200	7,200	.4	7.0	12.9	15.0	20.0	1100	5600	.42	980
Bx_1						900	Vac	4	.3350	6,600	6,000	.4	6.5	12.8	15.1	19.8	1040	5200	.44	920
									Second Series: Goldschmidt-Thermit. 91% Ferro-Silicon											
1G209 Ax_1	6.0	.25	6.39	.120	.002	900	Vac	4	.1450	4,100	4,100	.6	4.1	8.8	13.2	18.4	565	2330	.22	541†
1G210 Ax_1	6.0	.35	6.11	.255	.010	900	Vac	4	.2480	12,900	8,000	.2	10.5	12.7	13.9	18.4	1192	9100	.32	1070
Ax_2						900	Vac	8	.2290	12,700	8,000	.1	10.2	12.4	13.7	18.3	1200	9210	.35	1080
1G213 Ax_1	6.0	.50	6.02	.314	.015	900	Vac	4	.2770	10,000	8,000	.2	9.6	13.1	14.4	18.6	1400	8800	.44	1220
Ax_2						900	Vac	8	.2770	10,000	8,000	.2	9.6	12.8	14.2	18.9	1370	8800	.42	1090
1G222 Ax_1	6.0	.50	5.96		.005	900	Vac	4	.3660	13,330	8,000	.3	10.3	12.6	14.1	18.8	1050	8700	.32	990
Ax_2						900	Vac	8	.360	13,000	7,400	.4	10.7	13.7	15.5	20.8	970	7900	.31	910

*Deducting 12,000 × % S from the total Hysteresis Loss. †Ring Broken.

Discussion

Thomas Spooner: In conversation recently with an engineer of a large public service company, I asked what the value would be to his company of reduced core losses in transformers. Without any hesitation, showing that he had previously given this matter careful consideration, he told me that core losses were worth to his company approximately $800 per kilowatt for distributing transformers and from $300 to $400 for power transformers.

Taking these figures and applying them to certain standard distributing transformers, by simple calculation, I found that for a 5 kv-a. transformer, a reduction of twenty per cent in core loss would make it possible for a customer to pay approximately eight cents a pound more for the core material. For a 200-kv-a. transformer, this figure was approximately twenty cents a pound. This is on a basis of no re-design of the transformers.

The improved electrical sheet, made by the methods outlined by Mr. Yensen, would of course cost considerably more to manufacture. I think though there is no question but what progressive customers who know how to get the most for a dollar, would be very willing to pay the increased cost of this material in order to obtain transformers of improved quality.

M. G. Newman: The most interesting and vital point that is brought out is the effect of carbon. If there is carbon even in very small quantities, as Mr. Yensen points out, it is very detrimental, and silicon, while really it does not affect the magnetic properties of the material itself, does do something to the carbon there.

Most of our magnetic sheet steel, however, is used in alternating-current machinery in very thin laminations, and silicon does help out there, of course, in increasing the electrical resistance and decreasing the eddy-current losses. That brings up one question I would like to ask Mr. Yensen. Will it be possible to roll this material into sheets and obtain the same curves he has presented to us, and if not, what will be the effect on the material when rolled into sheets where it is finally used?

J. B. Whitehead: We know that the basic laws of the electrostatic field, and those of the magnetic field all take their beginnings from very much the same character of fundamental physical principles; but we also know that the parallel which exists between electrostatic phenomena and magnetic phenomena cannot be carried very far.

The increase in electrostatic flux density which follows the introduction of a dielectric, is called "electrostatic induction" just as the increase in flux density in magnetic material following the introduction of iron is called "magnetic induction." However our parallel cannot be carried any further. We find the dielectric constant of the material is really a constant so far as we can determine all the way up to breakdown, whereas the permeability of magnetic materials as we know departs very markedly indeed from a constant value, very early in the increase of magnetic induction. So while we have always hoped that some day we would be able to tie together in some fundamental way the behavior of magnetic materials, and the behavior of dielectric materials, and in a way which could be traced back to our fundamental relations, where they are of the same character, we have never yet I think come so near to it, as we seem to be doing in some of the results that are obtained by Mr. Yensen. I refer particularly to the initial portions of these curves in which he speaks of the carbon as being in solution with the iron, and his showing there that the losses that occur are the direct result of the combination, in the form of solution of these two materials.

Now, wherein does the parallel lie? I think it lies in this: That so far as the losses in dielectric are concerned, about the only safe starting point we have is the suggestion of Maxwell, that dielectric losses are really to be attributed to the mixtures of different materials. We are still uncertain as to whether we can say absolutely that dielectric absorption is present at all in pure materials, but the suggestion of Maxwell, and those who have followed after him in the study of dielectric hysteresis, is that the loss itself is largely due to the mixture of two different materials. Mr. Yensen now shows that magnetic hysteresis loss is also due to a combination of materials.

T. D. Yensen: In regard to the question raised by the first speaker about phosphorous: Of course, this curve does not mean that if you start with the material of zero hysteresis loss, that is, with all the impurities eliminated and of infinite grain size, and then add phosphorous to it, that a material of negative hysteresis loss will result. It means that in connection with other impurities and in connection with material having a finite grain size, phosphorous seems to be beneficial to the magnetic properties of four per cent silicon steel. As a matter of fact, this effect is so very small anyhow, as compared with the effect of sulphur and carbon, that it does not make very much difference whether the curve is drawn as shown or horizontal. In other words, the effect of phosphorous on *Fe-Si* alloys can be regarded as negligible.

In regard to Mr. Newman's question as to whether these same relationships hold for sheets, I can say that as far as we know they do. There is one factor that we have not yet considered, that we have not studied systematically, and that is, the effect of oxidation. In heavy materials, in big samples, the effect of oxidation is not very great, but the thinner the material becomes, the greater will be the effect of surface oxidation, and great precautions must be taken to prevent excessive oxidation when this material is rolled into sheets.

As far as actual results are concerned, I might add this: by taking special precautions we have been able to get material in sheet form with a loss considerably less than one watt per kilogram for 10,000 gausses at sixty cycles. Those of you who are familiar with magnetic materials will realize that this is quite an advance, because the ordinary materials that you have been using in transformers up to the present time have a loss of one and a half watts per kilogram or more. Better than this has been obtained during the last few years, but that is an average figure, so you will see that if you can reduce the loss to less than one watt per kilogram, you will have obtained an improvement of over fifty per cent. Mr. Spooner mentioned the fact that if we could decrease the loss by twenty per cent, we could afford to pay twenty cents a pound for the material, and by decreasing it fifty per cent, we might be able to place iron-silicon alloys in the same class as iron, nickel and other semi-precious materials.

Haakon Styri (by letter): I find two points which would be of great interest to have explained; one is the probability of having the second term of the hysteresis loss corresponding to free cementite continued to higher carbon contents when the steel has been prepared in such manner as to produce granular pearlite instead of lamellar pearlite. Such condition of the steel can readily be obtained by quenching at high temperature, followed by proper drawing.

The other question which to me particularly is of great interest, but which would probably mean a large amount of work, is to find by the same kind of magnetic analysis, how the saturation point for solid solution of carbon varies with temperature. I would believe that by increasing temperatures the amount of carbon going into solution, should increase.

If variation of magnetic properties with temperature could be determined, this would certainly throw some light on whether it was proper to consider the carbon in solution either as cementite or as free carbon, and whether the influence of carbon on the magnetic properties might be due to dissociated ionized carbon. It can certainly not be possible that carbon could have such influence on the magnetic properties if only mechanically dissolved.

T. D. Yensen (by letter): Replying to Dr. Styri, my conclusion in regard to the slopes of the hysteresis loss vs. carbon for 0% Si between A and C and C and D (Fig. 15) was that the hysteresis loss in both cases is due to the inherent hysteresis loss of Fe_3C, but that the actual flux passing through the Fe_3C is much greater in the case of pearlite (for $C > 0.1\%$) than in the case of free Fe_3C $(0.01 < C < 0.1)$. It is my opinion, therefore, that the actual hysteresis loss will, in any given case, depend upon the relative distribution of Fe_3C and ferrite. If granular pearlite gives a distribution whereby the magnetic flux will tend to avoid the Fe_3C more than in the case of lamellar pearlite, then the slope of the $C - D$ part of the curve should become less. However, as the concentration of Fe_3C becomes greater I should venture to say that the slope should increase and approach the slope $C - D$ shown in Fig. 15. To test this out experimentally would be somewhat difficult, as we should have to introduce another variable due to the heat-treatment, apart from the effect on the Fe_3C, and this might screen the effect due to the difference between granular and lamellar pearlite.

The second question brought out by Dr Styri, namely; whether the saturation point for carbon in solution (0.006 — 0.008 for 20 deg. cent.) increases with increase in temperature, is a very interesting one, and I hope to be able to investigate this point at some future date. It would be natural to assume that the solubility should increase with the temperature; on the other hand, we know that Fe_3C all goes into solution at the A_1 transformation point which may or may not harmonize with a gradually rising solubility curve between 20 deg. and the A_1 point.

Whether carbon in solution (martensitic) exists as Fe_3C or as free carbon apparently is a question not yet answered to everybody's satisfaction. Reference might be made to arguments of Jeffries (*Chem. & Met. Eng.*, Vol. 24, p. 1057, 1921 and Vol. 26, p. 250, 1922 which are in favor of the free carbon hypothesis. Westgren at first opposed this view, but later admitted that Jeffries' conclusion probably is correct (*Jour. Iron & Steel Inst.*, Vol. 95, p. 241, 1922, No. 1.)

The tremendous effect of carbon in solution is, to my mind readily explained either by the substitution theory, whereby one atom of a foreign element is capable of replacing one of the solvent atoms and thereby distorting the entire original space lattice, or on the assumption that the carbon atoms enter the interstitial spaces between the Fe and Si atoms. (See Walter Rosenhain: Solid Solutions, A. I. M. E. preprint No. 1250 N, June 1923). Dr. Styri has just called my attention to a paper by Dr. Westgren (*Jernkontorets Annaler* 1923) where he proves that the latter view is the correct one.

E. Gumlich (by letter): I was pleased to note that Mr. Yensen, by means of much better basic materials, perfected analytical methods and plentiful support, has succeeded in going much further than I could in my own investigations. Twenty years ago when most of my test pieces were prepared, the standards in regard to purity of iron were much lower than now, when we have available for the preparation of alloys an almost pure base, *viz.* electrolytic iron; it is therefore the more gratifying that our results, at least in general, coincide. For instance, the fact, observed by Mr. Yensen, that in the region of C-content between 0.006 per cent and 0.09 per cent, free iron carbide, Fe_3C, affects the magnetic properties and electrical resistance much less than Fe_3C in solution or in the form of pearlite. This is in agreement with the observation made by me in many cases, that the effect of carbon on the electrical resistance as well as the coercive force and the saturation value is much less pronounced for alloys with C-content in excess of 0.9 to 1 per cent, than for alloys with less than this amount of carbon, whether the material is slowly cooled or quenched; in other words, whether the carbide grains, precipitated in hypereutectoid alloys, is embedded in a pearlitic or a martensitic base.

The corresponding diagrams can be found in full only in my complete article in Wissenschaftlichen Albandlungen der Ph. Tech Reichsanstalt, Vol. IV, Part 3, and only partially in an abstract published in E. T. Z., 1919.

I consider very important the fact, found by Mr. Yensen and perfectly well established, that carbon to the extent of at least 0.006 per cent, remains in solution even after very slow cooling and, therefore, has a correspondingly large effect. This phenomenon was naturally overlooked by me, because of my much less pure material and less exact method of C determination. At that time these factors were of no practical importance, while today, when the manufacture of entirely pure iron and the investigation of its magnetic and electric properties is our goal, these factors come to assume greater and greater importance. (In Fig. 14, there seems to be a mistake in that the C-content plotted as abscissas only refers to the lower, not to the upper curve; for the latter the abscissa values should be multiplied by about 20; also in the table entitled "Form of Precipitated Carbon" in the section for 4 per cent Si there seems to be a misprint; it should be"—4000 P" instead of "—400 P.")

Of particular interest to me was Mr. Yensen's demonstration in regard to the relationship between the grain size and the magnetic properties, which has often been mentioned, but never definitely established.

On the other hand, according to our measurements I cannot consider the data on saturation values for pure iron to be correct; using field intensities at our disposal, (6-7000 gauss), we have always obtained by various methods $4\pi I = 21{,}600$, while the values given in the paper are 23,000 and over. We cannot believe that the comparatively small amount of impurities in our electrolytic iron can noticeably affect the saturation value. It is, therefore, probable that the method used in obtaining the saturation value (not described in the paper) may be responsible for the discrepancy.

Finally, quoting the motto "Qui tacet, consentire videtur," I should like to correct one point in the article, which concerns me personally. In the first paragraph of the introduction it is stated that the discovery of the advantage of silicon steel as compared with ordinary iron was made by Sir Hadfield. Strictly speaking this is not incorrect, since Messrs. Barrett, Brown and Hadfield found that magnetic properties of many Si and Al alloys were better than their basic materials (their article in the *Sc. Trans.* Roy Dublin Soc. VII, 1900); but absolute values for these new alloys were in no way exceptional and did not reach those for rolled iron or good ingot iron. This is clear from the fact that the first sample of Hadfield's Si-iron, which was sent by Fr. Krupp Co. to the Reichsanstalt for magnetic test, had a coercive force of 1.45 and a hysteresis loss of 12,300; at the same time the best grades of iron at that time had a coercive force of between 0.6 and 1 gauss and a hysteresis loss of between 4900 and 10,000, and the director of the Krupp Co. told me personally, that his factory had no interest in this new material at all. This interest was only aroused after it had occurred to me to utilize the high resistivity of the Si-alloys for lowering the eddy-current loss in transformer and generator sheets and after I had communicated this idea personally not only to representatives of metallurgical works and rolling mills, but also to ETZ in 1901 and 1902. Thus it happened, that at the end of 1903, when Sir Hadfield obtained his first patent in England, in Germany the manufacture of silicon sheet steel, started by my activities, was in full swing and could not be restricted by foreign patents. Whether Sir Hadfield, independently of me, came to the idea of using Si-alloys in sheet form for construction of transformers is unknown to me; anyhow, I can insist upon my priority, because the first publication by Barrett, Brown and Hadfield, by which I had been moved to my activities, gives no reference to the possibility of utilizing the high resistivity. The best proof of the correctness of my contention can be found in the fact that I could open my address in the Faraday Society of London in 1912, under the presidency of Sir Hadfield, with the following words:

"When in 1900 Messrs. Barrett, Brown and Hadfield published the results of their interesting researches on the magnetic properties and the electric resistance of iron-aluminum alloys and iron-silicon alloys, I had the idea to utilize the high specific resistance of the silicon alloys for the diminution of the eddy currents in transformer and dynamo sheet metal. The P. T. R. therefore requested some prominent German firms to produce transformer sheets out of silicon alloys, and began itself to make experiments with the new material. It resulted in the course of these experiments that more had been attained than had been expected; for not only were the eddy currents weakened in accordance with the higher specific resistance, but also the hysteresis loss was often smaller, and the permeability in low fields was higher than in the usual dynamo iron. Thus the so-called "legierte Blech" could not fail almost entirely to replace the usual material within a short time, in spite of the initial difficulty of production and of the much higher price, and German electrical engineers are much indebted to Messrs. Barrett, Brown and Hadfield for the researches which supplied the foundation for this great improvement in transformer material."

This presentation of the matter has not been corrected by Sir Hadfield; I therefore consider myself justified, in claiming the credit for being the first to have conceived and realized the idea of the application of silicon sheet steel to transformer construction.

T. D. Yensen (Communicated): Replying to Dr. Gumlich's communication, I note that he is in substantial agreement with my results in regard to the relative effect on the magnetic properties of C in the form of free Fe_3C, pearlite, and martensite. He states in his discussion that "Carbon in excess of 0.9—1.0 per cent has a much smaller effect on the electrical resistance, the coercive force, and the saturation value, than carbon contents below this value, whether the samples have been slowly cooled or quenched, *i. e.*, whether the carbide grains in these hypereutectoid alloys are embedded in a pearlitic or martensitic base." I believe this statement, to make the facts clear, should be modified by specifying that the quenching be done from a temperature sufficiently low to permit the free hypereutectoid Fe_3C to be precipitated, *i. e.*, from just above the A_1 point. If the quenching is done from a higher temperature, then obviously carbon is not precipitated at all (or only partially) but remains in solution with the corresponding effect on the physical properties. This is well illustrated in Fig. 8 of Dr. Gumlich'spaper in *ETZ* of July 3, 1919, page 328. By quenching from 900 and 1000 deg. the curves have no breaks in them at $C = 1.0$ per cent because the quenching temperatures lie above the A_3 point for all the alloys. In the case of the 1.8 per cent C alloy quenched from 900 deg. there may be a slight amount of precipitated Fe_3C but most of the carbon will remain in solution. By quenching at 800 deg. however, the carbon above about 1.2 per cent will have a chance to be precipitated as Fe_3C and he consequently obtains the break shown. It is interesting to note that the slope of the upper part of the curve ($C > 1.0$ per cent) for the 800 deg. quenching in Fig. 8 is 2.5 or identical with the slope obtained by me for the coercive force in the region $0.01 < C < 0.1$, *i.e.*, where C occurs as free Fe_3C. This consistency is most gratifying, because it shows that we have found relationship that can be depended upon. It also answers a question asked by Dr. Haakon Styri as to whether the relationship given for the region $0.01 < C < 0.1$ holds for higher C contents whenever carbon occurs as free Fe_3C. I believe the evidence given in Fig. 8 of Dr. Gumlich's paper is sufficient to answer this in the affirmative.

Dr. Gumlich's remark in regard to Fig. 14 "that the abscissa values only hold for the lower curve, not for the upper" is correct. This is taken care of, however, in the chart as it will be noted that the upper curve is marked "Percent $C \times 10$." There is, however, a misprint in the table entitled "Form of Precipitated Carbon" in the equation for 4 per cent Si. The constant for P should be "—4000" instead of "—400." Another mistake has been made in the abscissa values for the upper part of Fig. 34. These should all be multiplied by 10; *i. e.*, they should be 0.08 to 0.56.

In regard to the saturation values, it is possible but not probable that Dr. Gumlich's value is correct. Our values were obtained by plotting $\rho = \dfrac{H}{B - H}$ vs. H for values of H up to 500 gilberts per cm. for the rod samples. As the curves were apparently straight lines for $H = 300$ to 500 we used the reciprocal of the slopes for this region as the saturation value. This is a method extensively used in the past and has been regarded as reliable. (See J. D. Ball "The Reluctivity of Silicon Steel as a Linear Function of the Magnetizing Force" *Gen. Elec. Review* 15, page 750, 1913, and "Some Notes on Magnetization Curves" *Gen. Elec. Review*, Jan. 1915) but it is possible to get values that are far off if the plotting is done carelessly. My colleague, Mr. Thomas Spooner, has recently conferred with the Bureau of Standards in Washington in regard to this matter, and finds that the saturation value for pure iron reported by the Bureau varies from 21,200 to 21,900, the most reliable value being regarded as 21,600, which checks Dr. Gumlich's value. On the other hand, Mr. Spooner has obtained actual values of $B - H$ for some of our rod samples that exceed 21,600. For example, for $H = 600$ he obtained in one case 21,890 after making all proper corrections for air space. The calculated saturation value in this case was 23,200. These results were obtained with an apparatus that checks the Bureau of Standards results within one per cent. To be doubly certain of our values, we recently sent the rods to the Bureau for checking our results for high values of H. The values obtained by the Bureau are appoximately one per cent *higher* than ours. As far as we know now, therefore, the saturation values reported in the paper are correct within a few per cent. They also check the value obtained by Dr. E. H. Williams at the Univ. of Ill. for vacuum fused electrolytic iron in the form of elipsoids (See *Gen. Elect. Rev.* 18, p. 881, Sept. 1915), the value obtained being 22,800.

As far as the values for the ring samples are concerned the writer regrets to have to report that all values of B are 3.5 per cent too high, due to an error just discovered, in the formula for calculating the cross section.

Finally, as regards the credit for the benefits that have resulted from the use of silicon steel in transformers, I think Dr. Gumlich states the matter very clearly in the introduction to his lecture before the Farraday Society in 1912 when he ends by saying:"———and the German electrical engineers are much indebted to Messrs. Barrett, Brown and Hadfield for the researches which supplied the foundation for this great improvement in transformer material." Dr. Gumlich, undoubtedly deserves great credit for having seen and understood the importance of the results obtained by his English colleagues, and for having pushed the application thereof in Germany and thereby in the rest of the world, and I think Sir Hadfield would be the last man to deny that this credit is due him. On the other hand, the world is indebted to Sir Hadfield and his colleagues for having laid down the foundation upon which the present magnificent structure has been built.

E. Gumlich (Communicated): Mr. Yensen is of course perfectly correct, when he states that the effect of free iron carbide in a base of martensite will be observed only when the quenching temperature is sufficiently low, or the carbon content sufficiently high, so that the carbon at the temperature in question will not all go into solution. While this is quite obvious to experts in this particular field, it will perhaps make things clearer if the statement is made as follows:

The effect of carbon, etc., is much less for alloys with more than 0.9 per cent—1 per cent carbon than for alloys with less than this amount, no matter whether we deal with slowly cooled alloys (in which the precipitated free carbide is embedded in a pearlitic

base) or with alloys that have been quenched from a sufficiently low temperature (so that the precipitated free carbide is embedded in a martensitic base).

E. A. Smith (by letter): Within the last 10 years, many investigations have been carried on to improve alloys, for use in transformers and generators. Some of the authors covered various phases of iron and steel alloys, yet, many difficulties were still to be encountered due to the hysteresis effects and only lately the improvements have progressed fairly well. The papers that I am referring to, have been published by the German Electro-Technical Societies, The Faraday Society, Imperial Academy of Sciences of Germany and Austria, The German Iron and Steel Institute and the German Society of Engineers. I have also compared Mr. Yensen's Bulletins in 1915 and 1916 published by the University of Illinois Experiment Station, with those of the above Institutions covering ironsilicon alloys and found them all to check closely in their respective tests.

In my estimation, the present paper of Mr. Yensen seems to be fairly accurate with the magnetical properties of alloys as described and further investigations will result in many improvements to the construction of electrical apparatus.

In most metals and alloys more or less impurities exist and the grain size affects the magnetic properties and the hysteresis losses considerably, therefore, a special annealing process is being worked out at the present time by one of the manufacturing companies in Germany.

The best results can be obtained by eliminating the carbon and other impurities which appear to affect the magnetic flux much more than the grain structure. The maximum permeability of the individual alloys cannot be changed, but the resistance and reluctance can be altered, by the removal of the impurities. The numerical coefficients for the effect of the carbon on the hysteresis losses can be calculated if the structural and composition characteristics possess fixed constants or factors.

While the analysis of different alloys shows varying factors in their physical states it proves from actual conditions that many of the factors can be improved upon by the introduction of changes in their grain structures and compositions.

Alkali Vapor Detector Tubes

By HUGH A. BROWN and CHAS. T. KNIPP

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Review of the Subject:—*This paper describes unusual and very desirable results that are obtained on priming tungsten filament three-electrode vacuum tubes with an alloy of potassium and sodium when such tubes are used as detectors of radio frequency currents in receiving circuits.*

The method of preparing the tubes and of introducing the alloy is described in some detail.

Data obtained for these alkali vapor tubes are shown in the form of curves. They show the static characteristics of the tube, the operation characteristics when used as detectors, also the effect of filament temperature increase upon their performance.

The results obtained when the tubes are used as detectors include—lower optimum plate voltage (5 to 10 volts), less critical adjustment of plate voltage, steady action, considerably greater sensitiveness to weak signals than for gas content tubes, and no distortion in detector action.

CONTENTS

I. Introduction

ABOUT two years ago the authors of this paper completed an experimental investigation on the effect of various residual gases in three-electrode vacuum tubes upon the characteristics of the tubes and upon their performance as detectors and demodulators. It was found that the introduction of certain gases improved to some extent the sensitivity of the tubes as detectors.[1] Argon at a pressure of 0.005 mm. of mercury gave the best results. The most important result of this earlier investigation was the effect of gases upon the critical characteristics of detector tubes. The data relating to these are summarized in the form of curves in Fig. 1. The curves show how critical the adjustment of plate voltage becomes for air, nitrogen, neon, etc., when the pressure within the tube is increased. Helium, having a higher ionizing potential than the gases just mentioned, causes a detector tube to function best at a corresponding higher plate voltage. It should be noted also that mercury vapor, due to its lower ionizing potential, causes a tube to function best at correspondingly lower plate voltages. However, at the same time the allowable per cent variation of plate voltage for mercury vapor with no change in audibility was found to be greater than for the other gases except helium. Furthermore, if the pressure of the residual gas is increased the plate voltage required for the best audibility of signal response, which will be called the "operating voltage," was found to vary as shown in the curve of Fig. 2. Hence it was concluded that if a gaseous medium possessing an extremely low ionizing potential could be provided the tube would function as a detector on a minimum plate voltage.

The characteristic of mercury vapor shown in Fig. 1 indicates that a metallic vapor possessing a very low ionizing potential would not only operate on low plate potentials but also would not be critical as regards plate voltage adjustments. Vapors of certain alkali metals, and also of certain alloys of these metals, have very low ionizing potentials. Notable among these is the alloy of potassium and sodium, the vapor of which

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

1. *Phys. Rev.*, 19, (2), No. 3, 1922.

has an ionizing potential of 4 volts or less. Accordingly, a small amount of this material was introduced into several vacuum tubes by methods described presently. Curve *E* of Fig. 1 is typical of the results obtained. Curves similar to this one will be shown later. In addition to this other results were obtained, which were entirely unexpected, and will be described in due course.

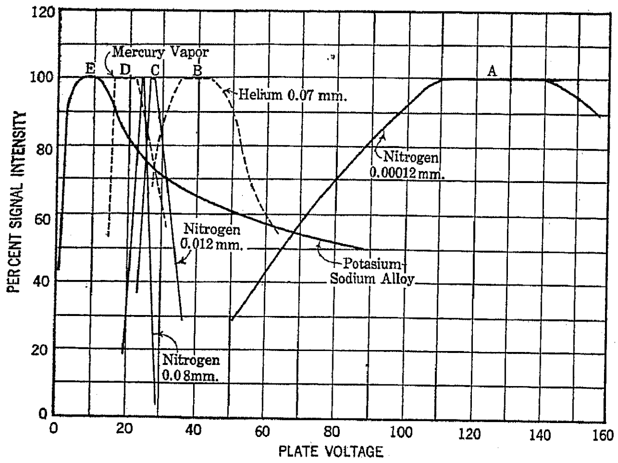

Fig. 1

II. Method of Priming Tubes

Standard types of detector and amplifier tubes, and also special forms made in this laboratory, were primed with a molecular alloy of potassium and sodium. This alloy which at ordinary temperatures is a fluid resembles mercury in appearance though its density is less than one. The potassium-sodium alloy was contained in an evacuated glass supply tube *A*, Fig. 3. Two or more

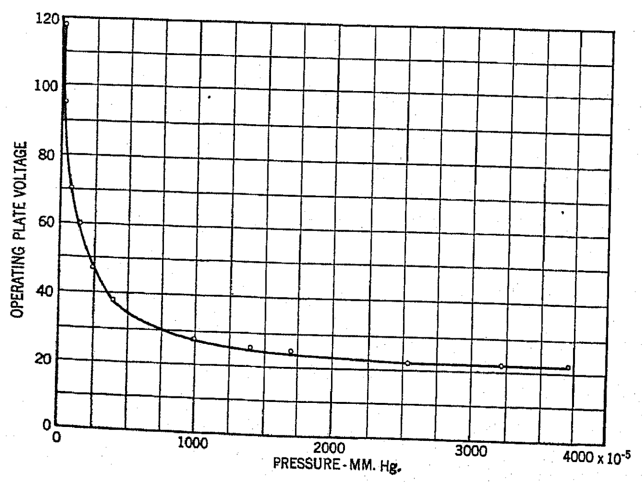

Fig. 2

three-electrode tubes were fused in a vertical position onto a horizontal manifold as shown in the figure. This manifold was connected through a *T*-tube, one branch of which, *B*, led to the evacuating system, and the other to the *K-Na* alloy supply tube *A*. The evacuating system consisted of a charcoal tube in liquid air, a phosphorous pentoxide tube, a mercury vapor trap also immersed in liquid air, a McLeod gage, a mercury condensation pump, and a rotary supporting oil pump. The supply tube, *A*, had a number of branch outlets, each branch being drawn down into a slender tube *D*, and also provided with a constriction, for the introduction of the alloy into the manifold and for subsequent sealing off. Connection between the outlet tube and manifold was made by heavy walled rubber tubing lubricated with rubber cement and tightly wired down, as shown in the figure.

When the pressure was of the order of 10^{-4} mm. of mercury the tip at *D*, which was previously nicked with a file, was broken off by bending sharply the rubber tubing. The evacuation was continued and in order to completely as possible outgas the tubes the filaments were kept incandescent, and also 150 volts were applied to the plates. This outgassing process was continued until the electron tubes (having been previously con-

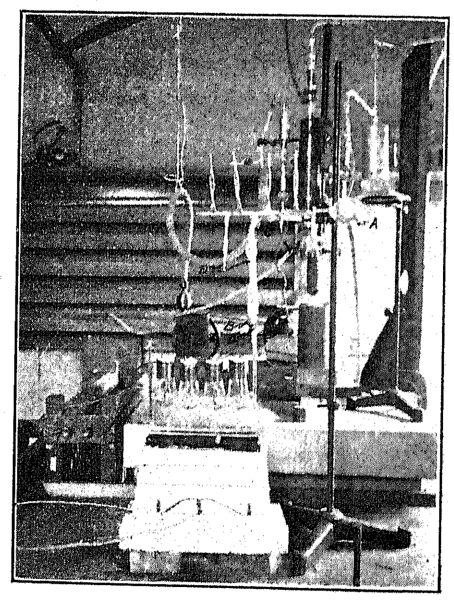

Fig. 3

nected to receiving circuits) functioned best as detectors with 60 to 80 volts on the plates. This test indicated that the tubes were fairly well outgassed. The supply-tube *A* was now tilted up and a little of the alloy was allowed to run down into the manifold, after which tube *A* was sealed off at *C*. To guard against possible leaks, due to the rubber connecting tube, another seal off was made at *E*.

The oil bath with its electric heater attached was now raised so that the horizontal manifold was immersed in the oil. The bath was gradually heated, with the pumps going, and ultimately the temperature was pushed to about 230 deg. cent., at which temperature the alloy was fully vaporized. The vapor passing through the capillary tubes connecting the manifold to the electron tubes was condensed on the colder walls of the latter forming a thin film, at first of varied purplish hues but shortly becoming silvery white, when

viewed on the inside as the deposited film became thicker. Heat coming to the bulbs by reason of being mounted vertically over the oil bath was intercepted by strips of cardboard and by turning on an electric fan. The filaments were kept incandescent at nearly normal filament current during this distillation process,

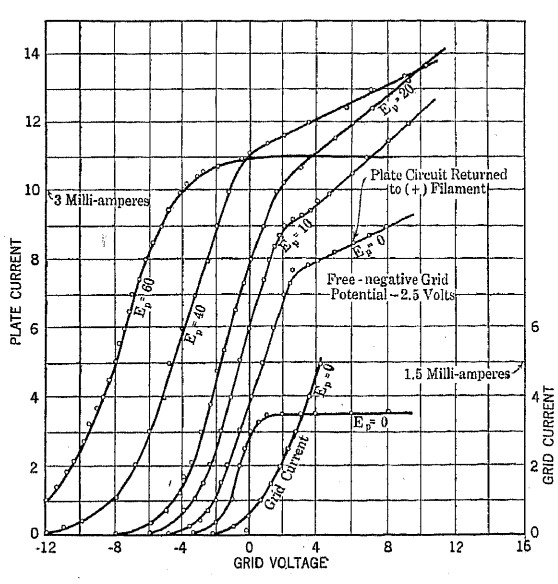

FIG. 4

thus preventing a deposit of the alloy on the inner metal parts of the tube, as well as aiding in the process of outgassing. After the inner walls of the bulb were well covered with the alloy the oil bath was removed, and the tubes sealed off. The pressure throughout the

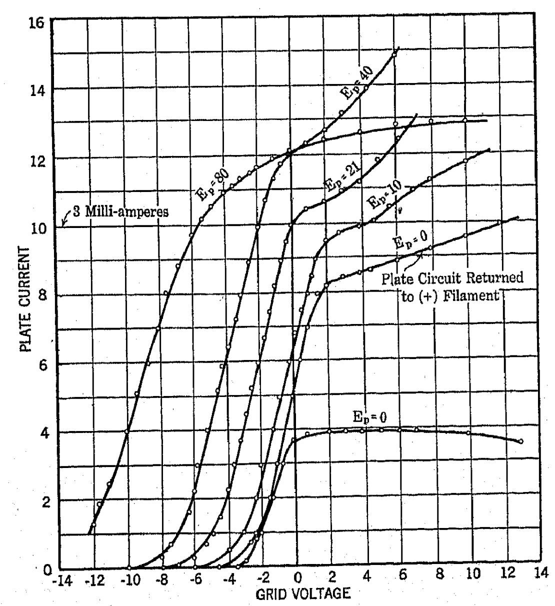

FIG. 5—K-NA VAPOR TUBE NO. 7

final stages of priming was easily maintained at approximately 0.00004 mm. mercury. The presence of the alloy within the tube will "clean up" the remaining traces of air and hence the vacuum improves with time.

It is extremely important to clean all glass and rubber tubing with "aqua regia" and distilled water, otherwise the alloy will stick to the walls of the tubing. The writers preferred to construct new manifolds rather than attempt to clean the used ones, also to use fresh rubber tubing each time. In order to facilitate distillation into the electron bulbs the sealing off strictures should be fairly large—about 1 to 2 mm. inside diameter. Any of the alloy condensed in the stricture should be evaporated gently before attempting to seal off, as excessive heat will burn the alloy giving it a brown color.

The potassium-sodium alloy was prepared by putting potassium and sodium into a glass tube in proportion to their atomic weights, *i. e.*, 39 to 23, the tube was then quickly closed by fusing and the pumps started. Heat was then slowly applied, but not in sufficient quantities to melt the metals until the gage showed a vacuum of the order of 0.001 mm. mercury. The heating was

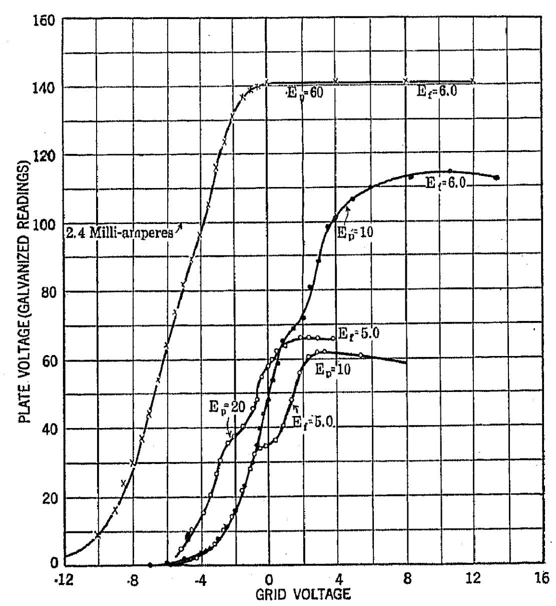

FIG. 6—K-NA VAPOR TUBE NO. 20, POORLY EVACUATED

then continued until the metals were melted and thoroughly mixed, after which the flame was removed, the connection to the pumps sealed off, and when cool the mixture was poured through small funnel-like strictures into the previously attached glass supply tube *A*, which in turn was sealed off, thus providing a store of potassium-sodium alloy for future use. The alloy must never be exposed to the air.

III. CHARACTERISTIC CURVES

All data and curves shown, with one exception, were obtained with standard Radiotron UV 201 amplifier tubes primed with potassium-sodium alloy vapor. This type was used because the stock tubes as purchased came with the electrodes carefully freed of gas and hence they required a minimum amount of time in subsequently out gassing. Note is made on all curves, the data for which were obtained for other types of tubes. Figs. 4 and 5 show conventional charac-

teristic curves for different tubes containing potassium-sodium alloy vapor. The vapor pressure in each tube corresponds to the temperature of the coolest portions of the walls. The most remarkable curves are those obtained for zero plate voltage when the plate circuit return was connected to the negative filament terminal, thus making the plate actually negative to a portion of the filament, yet plate currents as high as one milli-ampere flowed from plate across the vacuum to the filament. This action is the same as though a positive external potential was applied to the plate. These currents at zero plate voltage were 50 to 100 times as great as those obtained for the conventional vacuum

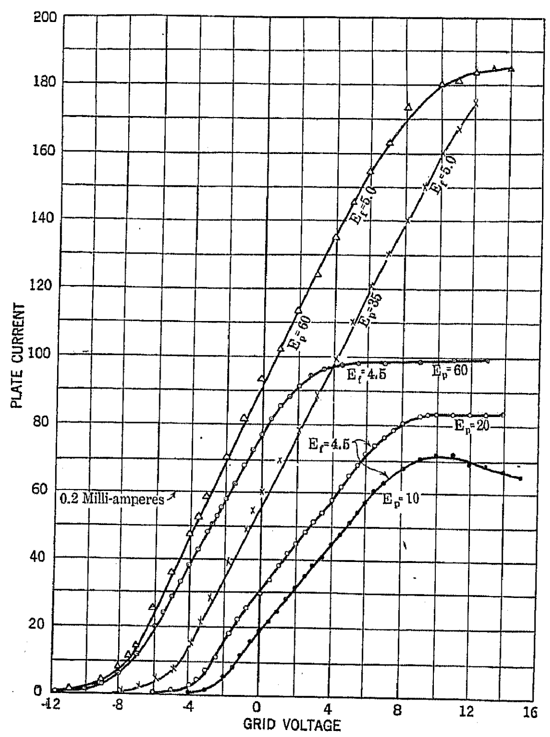

FIG. 7—K-NA VAPOR TUBE NO. 22

or gas content tubes now in use. It should also be noted that the curves are smooth and straight for negative grid voltages, when plotted against plate voltages, and that when a certain plate voltage is exceeded the curves approach a saturation point at negative grid potentials.

If the tube contains a large amount of gas the characteristic humps of gas content tubes are present, however, if the filament temperature is substantially increased these humps disappear and the saturation points are not reached until the grid voltages approach positive values. These results are shown in Figs. 6 and 7. With the filament voltage increased from 4.5 volts to 5 volts the curve for 60 volts on the plate remains straight up to 7 volts positive grid potential. In Fig. 8 this same tube was used with the filament voltage increased to 5.5 volts, and with 100 volts plate potential. This indicates that increased filament temperatures make the alkali vapor more active and effective and also that vacuum tubes containing alkali vapor at high vapor pressures possess the characteristics of high vacuum amplifier tubes. At the same time the considerable plate currents that are obtained at low plate voltages, and the steep slope of these low-voltage curves, indicates that the tubes will function as detectors at low plate potentials. This point will be dwelt on later. It was found that to obtain tubes with smooth characteristic curves at normal filament tem-

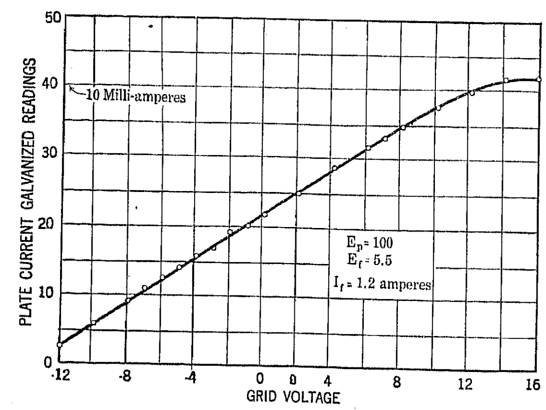

FIG. 8—K-NA VAPOR TUBE NO. 22

peratures, and lower, the evacuation must be made as complete as possible, just as in the preparation of high vacuum amplifier tubes.

Grid current curves and emission curves for sample alloy vapor tubes are shown in Figs. 9 and 10, respectively. The high degree of assymetry of the grid-current curves at about 2 volts negative grid potential should be

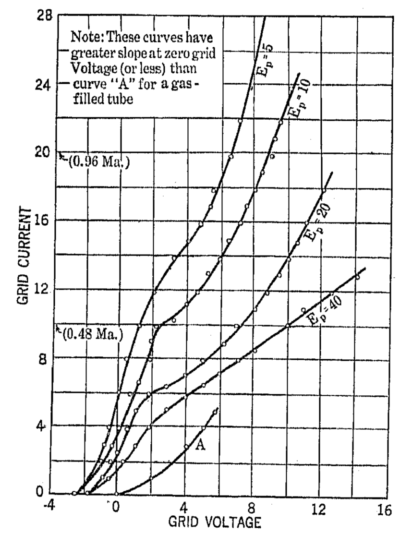

FIG. 9—GRID CURRENT, GRID VOLTAGE CHARACTERISTICS, K-NA VAPOR TUBE NO. 1

noted. Fig. 11 shows plate voltage-grid voltage curves at constant plate currents for three of these tubes. Curves A and B are typical for gas content tubes and for amplifier tubes and are given for comparison. The slope of these curves is a measure of the amplification constant of the tube. The slope for the alkali vapor tubes is steeper than that for the vacuum amplifier tubes, and the curves of the former are parallel straight lines.

The amplification constants of several tubes were measured for varying plate voltages by an audio-frequency a-c. method described on page 203 of Van der Bijl's "The Thermionic Tube." The results are shown

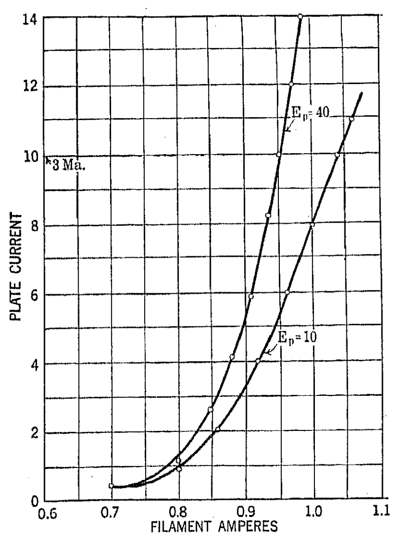

FIG. 10—K-NA VAPOR TUBE NO. 7
$E_g = 0$

in Fig. 12. It is not certain whether or not the high values obtained at low plate voltages were due to extra sensitiveness of the tubes, or to some discrepancy in the method when working at such low plate voltages.

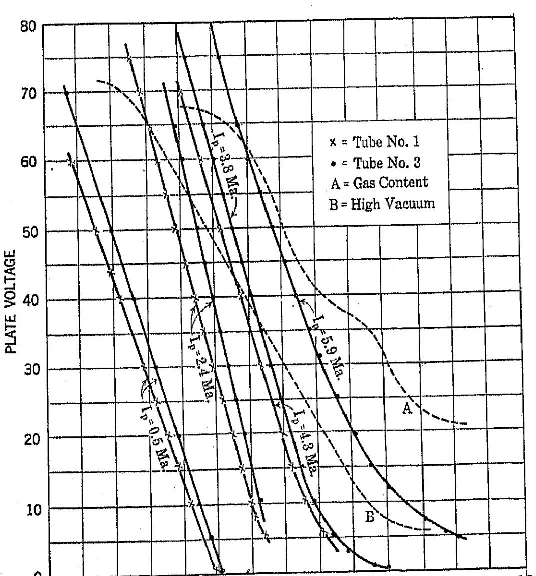

FIG. 11—K-NA VAPOR
x = TUBE NO. 1
• = TUBE NO. 3
A = GAS CONTENT
B = HIGH VACUUM

In the measurements the impressed 1000-cycle e. m. f. and the negative grid potential were varied, but no marked change in results occurred. It should be noted that the amplification constant falls off as the plate voltage is increased, and in Curve A, Fig. 13, it falls to zero at about 140 volts. However, increasing the filament current has the effect of keeping the curve from falling to zero, at least not until a considerably higher plate voltage is reached. In Curve B it is seen that the value of μ does not fall below 9.0 at 225 volts plate potential. In Fig. 14 are plotted values of mutual conductance for varying plate voltages, showing that the variation of this constant is much more gradual than it is for the case of gas content detectors.

IV. DETECTOR AND AMPLIFIER PERFORMANCE

One of the most important features of a detector tube is the degree of critical adjustment of plate voltage and filament current. It is known that low vacuum tubes, containing nitrogen, air, etc., (see Fig. 1) are quite critical. The alkali vapor tubes have been found to be much less critical in spite of the low plate voltages necessary for best efficiency, hence careful tests were made on several tubes used as detectors in laboratory receiv-

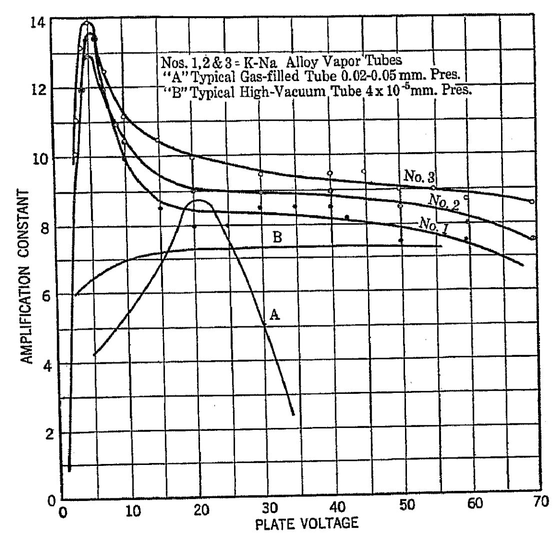

FIG. 12—VARIATION OF AMPLIFICATION CONSTANT WITH PLATE VOLTAGE

ing circuits, receiving modulated undamped waves from a small loop transmitter. Measurements on the intensity of response in the telephones were made by matching the intensity of the received signal with another signal which could be attenuated in definite proportions from which per cent comparative audibilities could be calculated.[2] The results are shown in Fig. 15. Curves for low and high-vacuum tubes are shown for comparison. Not only are the alkali vapor tubes much less critical than the conventional gas content tube, but the tests showed several of the former to be more than three times more sensitive than are the latter on weak signals, *i. e.*, three times as loud signal response. It is regretted that space does not permit a detailed discussion of the method of making the tests and the precautions[3] that are necessary.

2. Van der Bijl's "The Thermionic Tube," pp. 337 and 347.
3. See paper by the authors, *Proc. I. R. E.*, Vol. 10, No. 6, pp. 460-3.

Another very interesting effect of filament temperature was here noted.

The value of μ, the amplification constant, and also the audibility, vary with the filament current in a manner similar to that of a high vacuum amplifier tube. These quantities increase from zero to full value rapidly as the filament current increases from 75 per

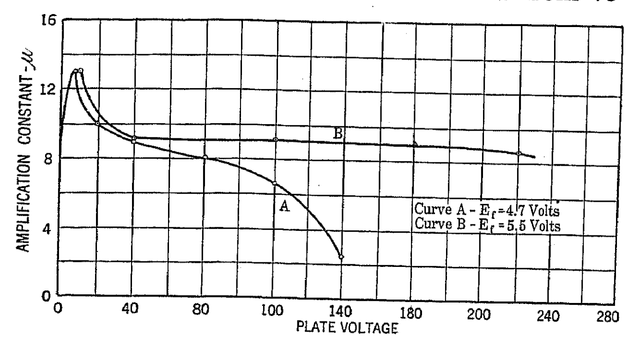

Fig. 13—K-Na Vapor Tube No. 23

cent to 90 per cent normal rated value, and they remain practically constant as the filament current increases to full value and on to 10 per cent above normal. Above this value the amplification constant falls off gradually.

If a tube contains considerable gas in addition to the alkali vapor the performance curve will be that of curve

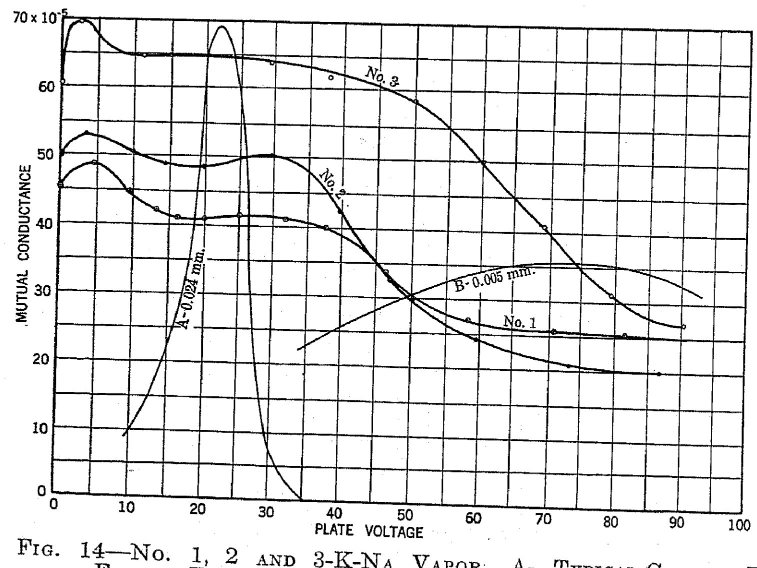

Fig. 14—No. 1, 2 and 3-K-Na Vapor. A—Typical Gas-Filled Tube; B—Typical Vacuum Amplifier

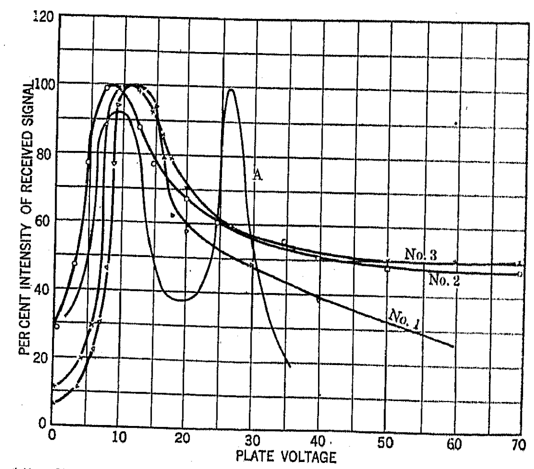

Fig. 15—Detector Performance, K-Na Vapor Tubes

A, Fig. 15, having a second peak in the region of the ionizing potential of the residual gas. If now, the filament current is increased, say, from 0.97 ampere to 1.02 amperes, the second peak will entirely disappear, and the curve will be similar to one of those shown in the figure, say for tube No. 1. These results, in addition to the mentioned effect of increased filament temperature on the amplification constant at high voltages, and the results showing effect on the characteristic curves, indicate that the heat of the filament makes the alkali vapor more effective in producing desirable characteristics that overcome the well known effects of the residual gas in tubes.

The quantitative results obtained indicate that potassium-sodium vapor tubes should be efficient and practical detectors. Actual use of these tubes has proved such to be the case to a surprising degree. The best plate voltage to use is about 8 or 10 volts. With this voltage users in this vicinity (Urbana-Champaign, Illinois) report excellent results. When using only the positive filament drop, with no additional "B" battery, the writers have often received broadcasting stations in Kansas City, Atlanta, Schenectady and Pittsburgh, using a variometer type of regenerative receiver with an antenna 12 ft. above the ground and 40 ft. long, and with *no* amplifier. The reception from the above stations was fairly loud and very distinct. Again, using an antenna 40 ft. high, and with *no* external "B" battery, broadcasting stations in Los Angeles, Cal., were received fairly loud. The above stations could even be heard faintly with the plate circuit return connected to the negative filament lead, and with no "B" battery. With these conditions the tubes were also used as beat receivers of the autodyne type receiving the high-power stations on the coasts, when using the negative filament drop as plate potential. In addition to their high degree of sensitiveness the tubes have exhibited a remarkable degree of selectivity when functioning as detectors, especially in zero beat reception. They also give reception absolutely free from distortion which is not the case of many gas content tubes. In listening to piano music, for instance, every note in the runs was received perfectly lear and distinct.

When used in receiving circuits the tubes do not function efficiently until the filament has produced enough heat to render the alkali vapor active. This requires from 20 seconds to 1 minute. In most tubes the filament current can be maintained 10 to 20 per cent lower

than for gas content tubes. Some of the alkali vapor tubes prepared in the laboratory have been used intermittently for 8 months, and no deterioration of the filaments has been noted. It is known that potassium and sodium can form alloys with tungsten, but it is very improbable that, owing to the filament temperatures, any alkali metal remains in contact with the filament. Another peculiar effect studied qualitatively is the ability of the tube to function most efficiently as a detector at a certain definite wave length. The wavelength of the transmitted signal was varied and the receiver tuned to receive it. The maximum audibility occurred at about 650 meters. Vacuum amplifier tubes did not show this effect when used in the same receiver.

V. Effect of Variation in Spacing of Electrodes

The foregoing data and discussion apply to the standard Radiotron U. V. 201 tubes, and also to a few tubes of different makes having nearly the same electrode spacings. To determine the effect of various spacings upon tubes primed with potassium-sodium alloy vapor was also undertaken. For tubes with slightly greater or slightly smaller spacings than those previously studied there was found no marked difference in the characteristics. When the spacings were increased to 6 mm. and 8 mm. from filament to grid, and to plate respectively, the amplification constant was found to pass through a maximum value for a plate voltage of 50 volts as the latter was varied. When the tube was used as a detector the audibility of the received signal was also maximum at this plate voltage, and passed through a maximum value for a filament current of about 50 per cent of the normal rated value as the latter was varied. Some of the results are shown in Fig. 16, and are typical for several tubes tested. Some of these tubes were extremely sensitive detectors under the above conditions, low filament current and high plate voltage and in the testing circuits gave approximately 10 times louder response than did those of normal spacing, but their action seemed erratic and unsteady. The response at zero plate voltage was very weak. The characteristic curves of the tubes having large spacings proved to be similar in shape to those shown in this paper. The results of these tests indicate that in order to obtain the low plate voltage characteristics the spacings of the electrodes must be small.

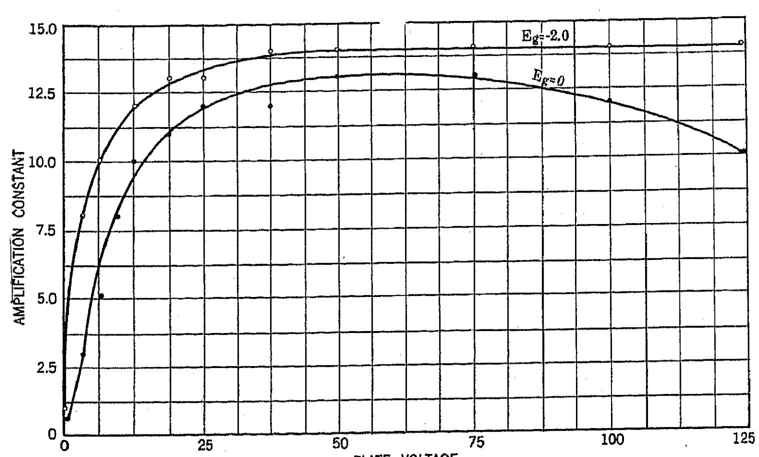

Fig. 16—K-Na Alloy Vapor Tube with Large Electrode Spacings, 50 Per Cent of Arted Filament Current

VI. Oxide Coated Filament Tubes

The effect of potassium-sodium alloy in oxide coated filament tubes is very peculiar. Several of the "dry cell" types of vacuum tubes were filled with the alloy and proved to have extremely high plate resistance with normal filament current flowing. The measured *d-c.* plate resistance was upwards of a million ohms. The tubes were insensitive as detectors below filament currents that were 100 per cent above normal. This shows that for low temperatures the alkali vapor actually opposes the emissive power of the cathode, and it is only when the latter becomes so hot as to set the alkali vapor into very violent agitation that the plate current flows. Other effects such as contact potential, photo sensitivity, and ionizing action, probably combine in a complicated way to produce the results obtained. It should be mentioned that the Western Electric type VTI tubes primed with potassium-sodium alloy functioned fairly well as detectors at low plate voltages and normal filament current due to the considerable heat produced by the oxide coated filament in this type of tube. The application of heat externally improves their action. Characteristic curves obtained for such a tube are shown in Fig. 17. Due to the large increase in plate current when the voltage was raised from 10 to 20 it was necessary to lower the

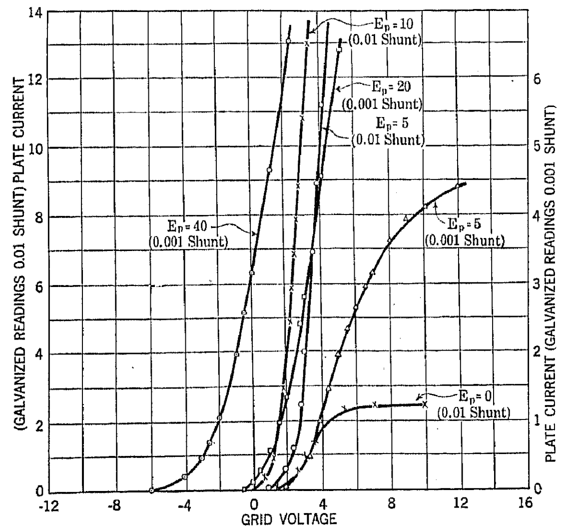

Fig. 17—Western Electric V. T. I. Tube, Containing K-Na Alloy Vapor, Normal Rated Filament Current

VII. Summary

Three electrode tubes containing potassium-sodium alloy vapor, and probably other alkali metal vapors, were found to be very sensitive detectors in radio

receiving circuits at low plate voltages. This sensitivity is due to the low ionizing potentials of these vapors, and probably also to other characteristics of such vapors, as photo-sensitivity, contact potential, etc. These tubes are much less critical than are gas content detectors. They are steady in their action, needing no frequent adjustments of filament current or plate voltages. The electrode spacings must be small to obtain the low plate voltage characteristics. Curves and measurements show excellent amplifier characteristics especially at higher filament temperatures. value of the shunt across the galvanometer that read plate currents, hence the two sets of coordinates. These curves are extremely steep and that they are shifted to the right of the zero grid voltage line as compared to the tungsten filament tubes is significant. The effect of residual gas on the detector and amplifier characteristics is nullified by raising the filament temperature from 10 to 15 per cent. For sensitive detector operation the filament temperature necessary to make the alkali vapor active is a little lower than the rated value for that type of tube.

Credit should be given Dr. Jakob Kunz who recommended the alkali metal vapors in response to an inquiry as to what gases possessed the lowest ionizing potentials. The writers are also obliged to Mr. Orlando Whelan for valuable assistance.

Discussion

G. D. Robinson: Will the authors please state what decrease or increase of filament life is found to accompnay the use of the alkali vapor?

Referring to Mr. Knipp's discussion of Figs. 4 and 5, where he states that plate current flows with the "plate actually negative to a portion of the filament," it appears that the plate is actually negative with respect to all active parts of the filament.

Alexander Nyman: The first curve, I believe, shows the sensitivity of the detector. As a rule, it is a very difficult thing to measure the sensitivity, and I would like to know how it was measured in this case. I believe several curves show amplification constants. That is another doubtful thing in the tubes. There are two ways of measuring amplification constants; one is the amplification constant on open circuit, and the second, is with full current on the plate circuit. The easiest way to get the second is by simply having a family of plate-current and grid-voltage curves, and computing the ratio of voltages from these series of curves. I would like to know which particular amplification constant is used.

With regard to the application of this tube, it seems that if it can meet all the qualifications of a detector and amplifier, it ought to make quite a good commercial product.

There is one factor that hasn't been discussed and that is the stability of this tube as an oscillator. In other words, if you have a regenerative circuit, and you are adjusting for the most sensitive conditions, unless the tube is a fairly stable oscillator, the signal will swing from non-oscillatory conditions to an oscillatory condition and distort the signal.

Hugh A. Brown: Replying to Mr. Robinson's question, no accurate data on the effect of the alloy on the life of the filament have been obtained. However, some of the tubes have been in use intermittently for nearly two years and the filament resistance has changed very little. It seems as if the presence of the alloy is not nearly so serious a factor on the filament life as is the presence of the argon and helium in the conventional gas content detector tubes. This is probably due to the fact that the alkali vapor tubes function at about 10 per cent lower filament current than do the former. It is known that potassium and sodium can form an alloy with tungsten under proper conditions, but it may be true that the alloy of these two metals is not so active. This is true in the case of glass, the separate metals will crack glass in time, but the alloy seems to have no effect.

Replying to Mr. Nyman's questions, the curve he refers to was obtained principally for the purpose of showing how critical the tubes are with varying plate voltages. The method of testing is described very briefly on page 4, of the paper. A more detailed description of the measuring apparatus can be found in the I. R. E. paper by the authors referred to on page 5. The tubes were used as "plain" detectors on a weak signal which showed 5 to 7 "times audibility" on a standard *J* tube used as a detector. The amplification constants were measured by an a-c. method described in Van der Bijl's book, "The Thermionic Tube," and which method the authors understand is used by the Western Electric Company in rating amplifier tubes. It is a "no load" amplification constant, but plate current flows during the measurement. The methods of measuring both "no load" and "full load" amplification constants are described in Morecroft's "Radio Communciation." The tubes are exceedingly stable oscillators, especially in the weakly excited condition and are thus excellent for "zero beat" reception.

Transient Performance of Electric Elevators

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Review of the Subject.—*The investigation of transient conditions is vitally important in elevator engineering, both electrical and mechanical. In no other method of human transportation are traveled distances so short and speeds and loads so frequently varied. Passengers must be hoisted and lowered safely and expeditiously. Landings must be accurately made. The elevator is consequently a machine that is very sensitive to transient effects and the entire apparatus must therefore be designed on the basis of transient analysis.*

Pure mathematical determination of elevator transients is evidently both difficult and laborious. In some cases it is impossible because the functions are unknown. Graphic methods of analysis lead to a much clearer understanding of the reasons for transient conditions, thus considerably aiding development.

Interdependent mechanical and electrical transients of a hoist system are first developed in the following. Then separate mechanical and electrical transients are considered. Finally, temperatures of the electrical apparatus, resulting from elevator operation in service are determined.

The basic method of transient determination used in this paper is to plot one or more curves representing integrals which each contain a single variable and which, when integrated, will give the desired solution. These integrals are evaluated between limits by measurement of the areas defined by these curves.

Methods of determining elevator transients of speeds, distances, currents, voltages, powers, forces and fluxes, are developed in this paper by means of this system of graphic integration.

* * * * *

TRANSIENT PERFORMANCE AS A MEASURE OF ELEVATOR QUALITY

TO determine the quality of elevators, including general efficiency and ability to supply service, it is necessary to study their transient performance. Predetermination of elevator transients is often desirable, and it is the purpose of this paper to describe comparatively simple means for obtaining transients from data that are available from the usual tests of electric elevator apparatus.

For maximum transportation, it is not only essential that an elevator have a full speed as high as can be advantageously utilized, taking into account the height of the building, the service required and the apparatus available; but it is also necessary to accelerate and retard as quickly as practical, and always to make and maintain accurate landings.

Quantity and quality of transportation are both essential considerations, particularly where elevator service is intensive, which is almost always the case in high buildings. The practical height for a building is dependent upon the service that can be obtained from available elevator equipment. The hatchway and hall space required for elevators capable of giving the required service, is the principal factor in determining how high a building is commercially practical for a given plot. Elevator equipment that will give maximum service is invariably justified for high buildings.

Changes in torque, with accompanying changes in speed, acceleration, and retardation, must proceed properly in order to secure satisfactory elevator operation. Shocks must be avoided for safety considerations and to prevent objectionable sensations of passengers. Constancy of acceleration and retardation, with all loads within the allowable capacity moving in either direction, is very desirable. When acceleration and retardation change, as they must at starting, at coming to constant speed, at the beginning of retardation, and at stopping, the changes must be gradual, tending toward constancy of rate of change of acceleration and retardation. These considerations are necessary even though the elasticity of the human body allows these changes to be made in fractions of a second.

Transient conditions near the stop are important as affecting the accuracy of the stopping point. Accuracy, expediency, and smoothness in making landings have a considerable effect on both quantity and quality of transportation.

The fitness of an elevator for providing a required amount of transportation is dependent upon the rapidity, smoothness and constancy of its transient performance, and the story of this performance is definitely written by the curves of transient speeds, forces, powers, currents, voltages and temperatures resulting from its operation.

ANALYSIS OF TRANSIENT PERFORMANCE

A detailed technical analysis of the transient performance of elevators is useful for preliminary investigations, for definite determination of satisfactory operation, for locating the causes of unsatisfactory operation, for checking performance stated to be the result of tests, and for definite comparisons of different apparatus.

It is important to study transient performance both by means of direct tests, usually made with curve drawing instruments, and by development of theory to the extent that the necessary values for plotting performance curves can be calculated from basic data. Most of the necessary data can be obtained from the characteristic test records of the electrical apparatus employed.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

Theoretical Methods of Predetermining Performance

By comparatively simple methods, it is possible to predetermine performance with sufficient accuracy for practical purposes. This paper describes methods of determining transient performance which avoid seriously involved mathematics and which enable calculations to be made, when the equations of functions are unknown, when functions are discontinuous, and when both conditions exist. An example of a solution readily made by the method described, but which is impossible on a strict mathematical basis, is that of the determination of variation of flux with time through a coil having a magnetic core. The saturation curve is known, but its equation is unknown. Even if an approximate equation is assumed to represent the saturation curve, complete mathematical treatment is seriously involved and the results obtained are too inaccurate to be of practical value.

Complete treatment of all the problems of transient electric elevator performance has not been attempted here, but it is believed that sufficient cases have received consideration to enable the process to be extended to solve any practical transient problems of electric elevators.

General Equations

In general, we have the differential equation:

$$\frac{d\,v}{d\,t} = f/M \tag{1}$$

From which for a hoist system

$$f_1\,d\,t = M\,d\,v \pm F\,d\,t \pm f_2\,d\,t \tag{2}$$

f_1 = the variable driving force
v = the variable velocity of the load
f_2 = the friction force, which may be variable
M = the constant equivalent mass of the system
F = the constant net load hoisted or lowered.

M, which is the equivalent mass of the moving system reduced to speed v, must be determined.

The process of calculating M is laborious and likely to be inaccurate, so that it is best to determine it by test. Various test methods for obtaining this value are described, and finally that giving the most accurate results.

To Determine the Kinetic Energy and Equivalent Mass of a System Including Rotating and Translating Masses

1. The times in which the system stops against friction from various velocities are known, and the power necessary to overcome friction at a single velocity is also known.

Plot the curve of time t to stop from velocity v. See Fig. 1.

Then if P_1 = the known friction power at speed V_1
and F_1 = friction force at speed V_1

$$F_1\,V_1 = P_1$$

and $$F_1 = P_1/V_1$$

Referring to the curve, we have

$$\frac{d\,v}{d\,t} = \tan\alpha = f/M$$

and $$\frac{d\,V_1}{d\,T_1} = P_1/V_1\,.\,1/M = \tan\alpha_1$$

in which $\tan\alpha_1$ may be obtained by drawing the tangent to the curve at point T_1, V_1.

Therefore $$M = P_1/V_1\,.\,\frac{1}{\tan\alpha_1}$$

Thus, by determining the tangent and substituting the power and velocity values, the equivalent mass M may be determined. Consequently the kinetic energy at any speed v is $1/2\,M\,v^2$.

This method is subject to error both in drawing the tangent and in determining the friction power at a single velocity.

Note: In calculating transients it is necessary to know the friction at various velocities, but it is often difficult to measure the friction power at various velocities. With the speed-time curve and the friction power at one velocity, known, as above, the friction power-time curve may be determined as follows.

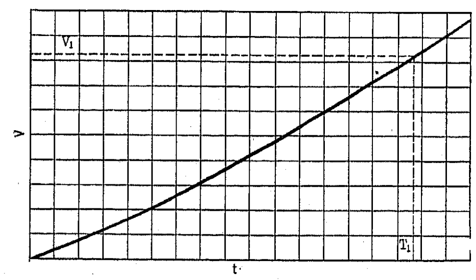

Fig. 1

We have $\tan\alpha = 1/M\,.\,p/v$

Therefore $p = M\,v\tan\alpha$

Substituting $M = P_1/V_1\,.\,\dfrac{1}{\tan\alpha_1}$

We have $p = \dfrac{P_1}{V_1\tan\alpha_1}\,v\tan\alpha$

By drawing tangents to various points on the curve (Fig. 1) and substituting the corresponding values of v, we can obtain the corresponding values of p and thus draw the Friction Power-Velocity Curve.

Any errors that there may be in the determination of the friction power P_1 and the $\tan\alpha_1$ still exist, and the curve so constructed is subject to these errors.

2. The Friction-Power at various velocities and the time required to come to rest from a single velocity are known

From equation (1)

$$t = M\int 1/f\,d\,v$$

This integral cannot be directly integrated since f is a mathematically unknown function of v.

If we draw a curve (Fig. 3), plotting various values of $1/f$ against corresponding values of v, we have Evidently the area of this curve is $\int 1/f\, d\, v = \times B$

It is therefore only necessary to take the area up to any velocity with a planimeter, to determine the time corresponding to this velocity.

$$\int_0^{V_1} 1/f_1\, d\, v = B_1$$

$$T_1 = M\, B_1 \text{ and } M = T_1/B_1$$

and $T_2 = M\, B_2$ etc. and $M = T_2/B_2$

Taking the area (as B_1) corresponding to a velocity (in this case V_1) and taking T_1 corresponding to V_1, we determine M.

This method of drawing a curve in such a manner that it defines an area which represents a certain integral, then measuring the area by means of a planimeter, is a simple method of solving differential equations. It avoids both the mathematical solution of the integral and the determination of constants of integration.

This solution is subject to any error there may be in the measurement of the time required to stop from a single velocity.

3. If the times required for the machine to come to rest against friction, from various velocities, and the friction-powers at various velocities are known.

a. Run the balanced hoist system at various velocities and, cutting off power, allow it to stop opposed only by friction, noting the time required to stop from each velocity.

b. Plot the speed-time curve, Fig. 4.

c. Drive the system at various speeds, noting the friction power required at each speed.

d. Plot the friction-power-velocity curve, as shown in Fig. 2.

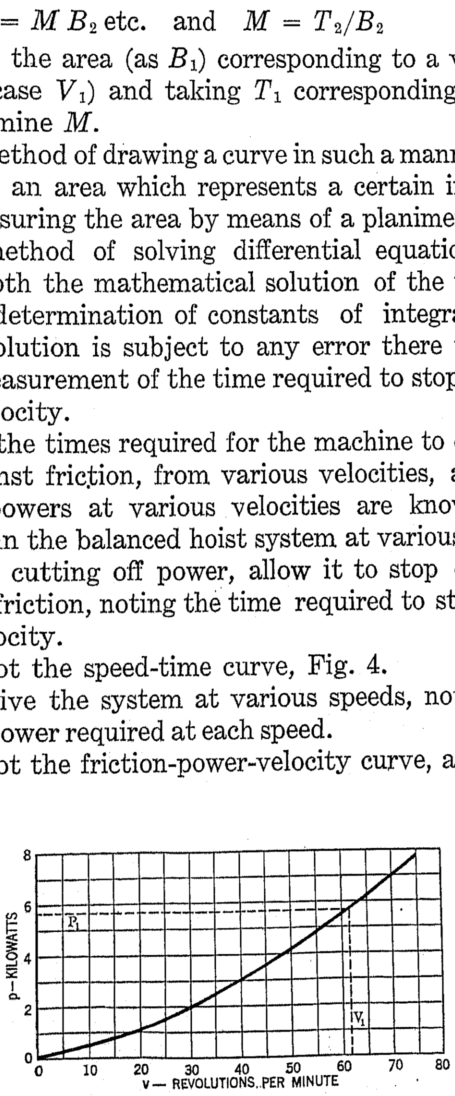

FIG. 2

e. Plot a friction power-time curve, taking the power and time from Figs. 2 and 4, for the same values of velocity.

The area of this curve, which may be obtained by a planimeter, is

$$f\, v\, d\, t = \int p\, d\, t = 1/2\, M\, v^2 = A$$

Taking area A_1 for a velocity V_1

$$M = \frac{2\, A_1}{V_1^2}$$

Note that it is necessary to use proper multipliers in determining actual values from the above.

This is an accurate method of obtaining equivalent mass and kinetic energy, since all test points are averaged in the result. It is only necessary to employ an ammeter, voltmeter, stop-watch and tachometer, instruments usually available and which may be used without difficulty, to obtain the value of the equivalent mass M of the hoist system.

Mathematical Treatment

Proceeding now to the consideration of elevator transients, having shown a simple and accurate method

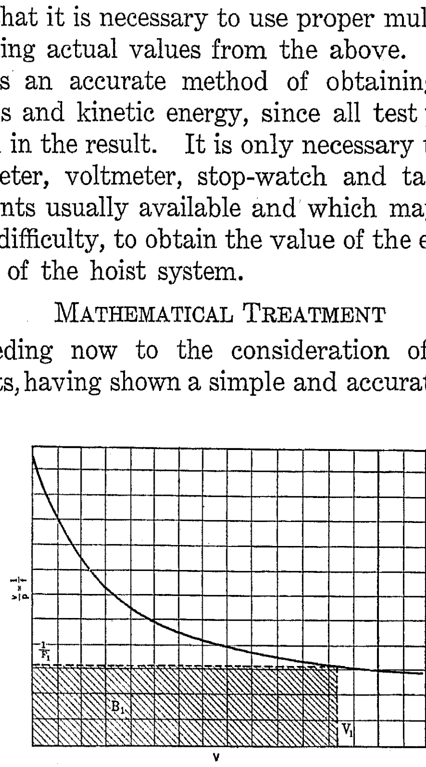

FIG. 3

for obtaining the value M to be used in the equations, we may first investigate pure mathematical treatment. A simple case will serve to demonstrate that pure mathematical method is complicated and often impossible.

Taking a very simple example, we have a direct-current motor having a constant field, operated on constant e. m. f. A constant field has been assumed to simplify the problem. Such a condition is, however, seldom, if ever, met with in elevator practise. To further simplify this case, we shall neglect the effect of armature reaction, which is to reduce torque by weakening the field. The effects of core losses and armature circuit inductions are also neglected. The friction force is assumed constant.

Elevator Speed-Time Curve

Direct-Current Motor with Constant Field.

E = Constant line voltage

R = Constant resistance of armature circuit, including external and armature resistance

e = the variable counter e. m. f.

i = the variable current in the armature.

We have $e\, i\, d\, t = M\, v\, d\, v \pm F\, v\, d\, t + F_1\, v\, d\, t$

In this equation the plus sign is used with F when the load is being driven by the elevator machine and the minus sign when the load is driving the machine.

t = the variable time

v = the variable velocity

M = the constant equivalent mass of the moving system at speed v

F = the constant hoist load, positive or negative
F_1 = the friction load (assumed constant)

Assuming with constant field that

$e = K v$ where K is a constant.

Neglecting the induction of the armature circuit, we have

$$i = \frac{E - K v}{R}$$

Substituting these values in the differential equation, we have

$$d t = M R \frac{d v}{(E K - F R - F_1 R) - K^2 v}$$

Integrating and substituting the constants of integration, we have the equation of the speed-time curve:

$$t = \frac{M R}{K^2} \log \epsilon \frac{E K \mp F R - F_1 R}{E K \mp F R - F_1 R - K^2 v}$$

also $$v = \frac{E K \mp F R - F_1 R}{K^2} \left(1 - \epsilon^{-\frac{K^2}{MR} \cdot t}\right)$$

To obtain the equation of the current-time curve, we have

$$e = E - R i$$

$$v = \frac{E - R i}{K}$$

$$d v = - R/K \, d i$$

From which we have, by substituting in the differential equation:

$$d t = - \frac{M R}{K} \cdot \frac{d i}{K i \mp F - F_1}$$

Integrating, we have the equation of the current-time curve

$$t = \frac{M R}{K^2} \log \epsilon \frac{F_1 \pm F - K I}{F_1 \pm F - K i}$$

where $I = E/R$, the initial current admitted to the armature.

Note again that it is necessary, in determining actual values, to insert the proper conversion multipliers to make the energy equation read in the same units on both sides.

In plotting the speed-time and current-time curves, the resistance of the armature circuit changes with every step of resistance, so it is necessary to change the value of R in the equation for every controller step.

This mathematical solution is made possible by assuming, for the sake of simplicity, conditions that are seldom, if ever, encountered in elevator practise. In some cases the armature reaction must be taken into account, and in almost all cases there is a variable field.

With the usual variable field it is impossible to get algebraic equations for the transients, because the equation of the saturation curve is unknown. If an equation is assumed to represent the saturation curve, the result, in the case of elevator motor fields, is too inaccurate to be of practical value. For low saturations, the flux will be approximately directly proportional to the magnetizing force and the equations

$$i = E/R \left(1 - \epsilon^{-\frac{R}{L} t}\right) \text{ for an increasing field}$$

and $i = I \epsilon^{-\frac{R}{L} t}$ for decreasing field may be used. Since high saturations are used in the accelerating and retarding periods of elevators, the actual saturation curve must be taken into account. Secondary currents must also be considered.

In his work on "Transient Phenomena," Dr. Steinmetz has treated the cases of combined mutual and self-induction where the magnetism is assumed directly proportional to the magnetizing force and also self-induction alone, using Froehlich's formula for the saturation curve. He notes that the equations obtained are only approximately correct for low saturations.

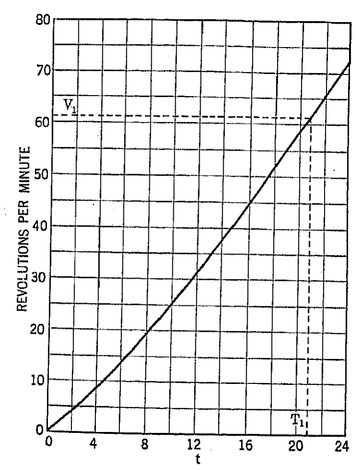

FIG. 4

By a simple graphical method, the flux and current in the case of highly saturated variable fields, such as are generally used with elevator motors, may be taken into account with sufficient accuracy for all practical purposes. By the graphical method described in this paper, we have treated the cases of self-induction and also mutual induction, neglecting primary and secondary leakage, in both cases taking the actual saturation curve into account. Mutual induction, including primary and secondary leakage, is an involved problem, but it is possible by combining the methods we employ, with the Steinmetz method, to obtain a step-by-step solution of actual fluxes and currents, taking the actual saturation curves into account to any degree of saturation.

Before considering the field problem, we shall take up the determination of speed-time and distance-time elevator curves when the speed-torque curve of the motor and the speed-friction curves of the elevator are known.

Graphical Treatment

It is evident, if all the variables are to be taken into account, that it is practically impossible to determine electric elevator transients by pure mathematical methods.

If we attempt to deduce the equation of the speed-torque curve of a motor, we must take into account a large number of variables. In some cases theory has not been sufficiently developed to enable us to consider the variables mathematically. Even if theory were available that would make it possible for us to determine the equation of the speed-torque curve of a motor, taking all the variables into account, it is evident that the resulting equation would be too cumbersome to be of any practical value in determining actual transients.

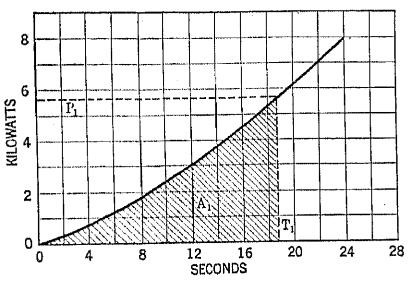

Fig. 5

The speed-torque curve of an electric motor can always be obtained from simple tests and the necessary test data are usually available from standard tests made to determine the motor characteristics. This curve takes all the variables into account, but its equation is unknown; that is, the torque is a mathematically unknown function of the speed.

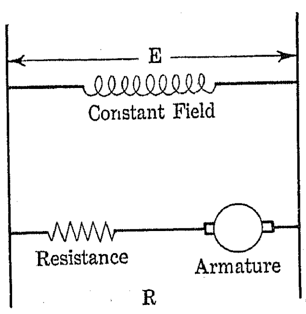

Fig. 6

Graphically, however, this function is known, since it is exactly expressed by the speed-torque curve.

In determining elevator transients it is evidently necessary to take the correct speed-torque curve into account. As this is only known graphically, some graphical method must be employed for using it in calculations of elevator transients. The method described in the following is easily applied to practical problems, and employing no approximations and being based on the actual speed-torque curve, gives correct transient values. The accuracy is only limited by the accuracy of the instruments used in making the speed-torque tests, and the accuracy of their observation.

To Determine the Speed Time Curve when the Force Causing Speed Change Varies with Velocity.

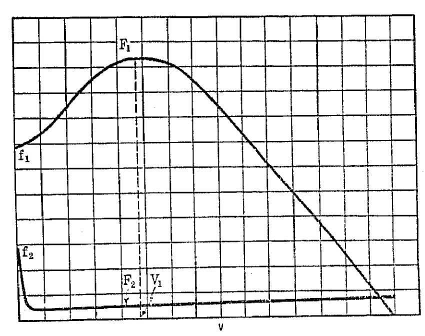

Fig. 7

Given: The Curves (Fig. 7) representing the speed-torque characteristic of the motor as determined by the usual tests and the change in friction force of the elevator with velocity.

Obtaining f_1 and f_2 from the test curves above plot, the curve (Fig. 8).

From the differential equation (2), we have

$$t = M \int \frac{1}{f_1 - f_2 \mp F} \, d v$$

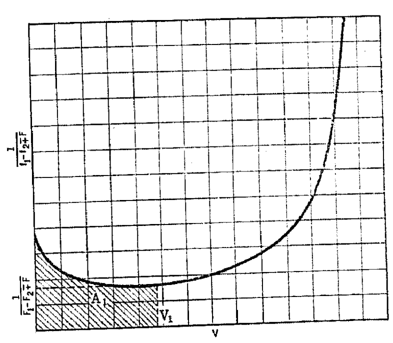

Fig. 8

Therefore $\quad T_1 = M A_1$

Since A_1 (which is the area up to point V_1)

$$= \int_0^{V_1} \frac{1}{f_1 - f_2 \mp F} \, . \, d v$$

Also $\quad T_2 = M A_2$

$T_3 = M A_3$, etc.

To find the time corresponding to a speed V_1, take the area of the curve up to V_1 (shown shaded) using a planimeter. Multiply this area by M (the equivalent mass) and the result is the time T_1 corresponding to velocity V_1.

In this case we have two mathematically unknown

functions, the speed-torque curve and the speed-friction curve. By the very simple graphical method using a planimeter, we are able to determine the speed-time curve of a hoist system with an accuracy only limited by the accuracy of the instruments used and the average accuracy of observation.

To Determine the Distance-Time Curve of an Elevator

If the speed-time curve (Fig. 9) has been determined,

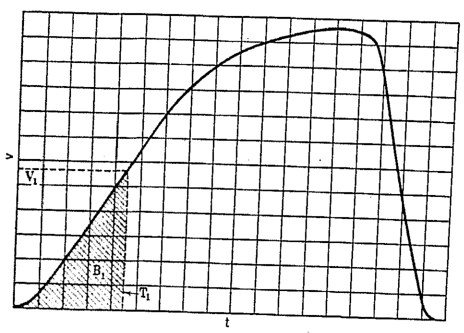

FIG. 9

the distance-time curve may be obtained by integrating this curve by means of a planimeter.

We have $\dfrac{d\,s}{d\,t} = v$

and $s = \int v\,d\,t$

At time T_1 $S_1 = B_1$ where B_1 = shaded area

At time T_2 $S_2 = B_2$ etc.

To Determine the Speed-Distance Curve of an Elevator

We have, as before $\dfrac{d\,v}{d\,t} = \dfrac{f_1 - f_2 \mp F}{M}$

Also $\dfrac{d\,s}{d\,t} = v$

Therefore $d\,s = M \dfrac{v\,d\,v}{f_1 - f_2 \mp F}$

Then $s = M \int \dfrac{v\,d\,v}{f_1 - f_2 \mp F}$

Plot the curve (Fig. 10)

Then

$$S_1 = M\,C_1$$

$$S_2 = M\,C_2 \text{ etc.}$$

Take the area to V_1 by planimeter, and this area multiplied by M is the distance traveled to speed V_1.

To Determine the Speed-Time Curve when the Force Causing Speed Change Varies with Distance

The curve plotted in Fig. 11 is known:

f = force causing speed change

s = distance traveled while speed is changing

$$\int_0^{S_1} f\,d\,s = \text{work done} \Big/_0^{S_1} = \text{Area } D_1$$

This work is converted into kinetic energy.

If V_1 is the velocity at distance S_1. Then at B_1 we have

$$D_1 = 1/2\,M\,V_1^2$$

$$V_1 = \sqrt{\frac{2\,D_1}{M}}$$

$$V_2 = \sqrt{\frac{2\,D_2}{M}}$$

We have $\dfrac{d\,s}{d\,t} = v$

Therefore $d\,t = \dfrac{d\,s}{v}$

and $t = \int 1/v\,d\,s$

If we plot curve (Fig. 12)

Obtaining values for $1/V_1$ from $V_1 = \sqrt{\dfrac{2\,D_1}{M}}$

and for $1/V_2$ from $V_2 = \sqrt{\dfrac{2\,D_2}{M}}$ etc.

We have $T_1 = B_1$ for distance S_1

$T_2 = B_2$ for distance S_2 etc.

Areas B_1 and B_2 are obtained by the planimeter. This solution is applicable to elevator buffer problems.

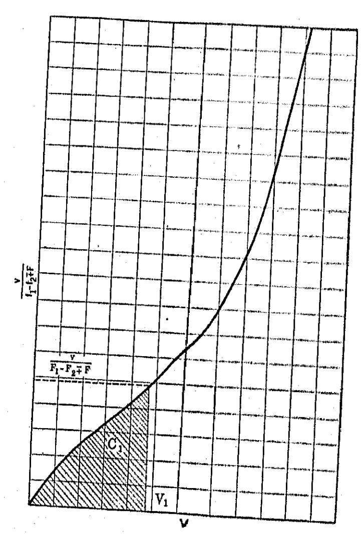

FIG. 10

Having obtained the speed-distance curve, tangents may be drawn to various points on it, determining

$$\frac{d\,v}{d\,s} = \tan \alpha.$$

Then $v \dfrac{d\,v}{d\,s}$ = the retardation, and by plotting the curve of retardation against distance, the maximum retardation of the buffer may be obtained.

The force causing change of speed is the algebraic sum of the forces acting to accelerate and retard the mass of the system.

Elevator Buffers

When an elevator buffer operates, it is necessary to know the retardation and the stresses in the mechanical structures, under all conditions. The human body can only withstand retardations below a certain maximum. The mechanical structures can only withstand limited stresses.

Some elevator buffers apply a retarding force varying only with distance. The following example gives the solution for transient retardation when a traction elevator runs into a buffer having a retarding force varying in a graphically known relation to distance.

The sheave is turning so as to cause the elevator car to descend, at the time that the buffer is struck, and continues to turn in the same direction, the ropes slipping on the sheave.

f_s = Buffer retarding force
s = Distance buffer is compressed
g = Acceleration of gravity
a = Acceleration or retardation

$$T_2 = W_2\,(1 + a/g) - f_s$$

$$T_1 = T_2\,(\epsilon^{\mu\alpha})$$

$$T_1 - T_2 = [W_2\,(1 - a/g) - f_s\,(\epsilon^{\mu\alpha} - 1)\,]$$

When driving sheave is rotating and ropes are slipping over it.

$$W_1 - W_2 + f\,s - (T_1 - T_2) = M\,a$$

$$a = \frac{W_1 - W_2\,\epsilon^{\mu\alpha} - f\,s}{M = W_2/g\,(\epsilon^{\mu\alpha} + 1)} = K_1 - K_2 f\,s$$

$$\frac{d\,v}{d\,t} = \frac{v\,d\,v}{d\,s} = K_1 - K_2 f_s$$

$$v\,d\,v = (K_1 - K_2 f_s)\,d\,s$$

$$1/2\,v^2 = \int (K_1 - K_2 f_s)\,d\,s$$

Plot Curve of s against $K_1 - K_2 f_s$ (Fig. 14).

$$\tfrac{1}{2} v^2 = A$$

$$V_1 = 2\,A_1$$

$$V_2 = 2\,A_2 \text{ etc.}$$

Plot curve between v and s (Fig. 15).

$$\frac{d\,v}{d\,s} = \tan\alpha$$

Plot curve of $v \tan\alpha$ against s to determine the retardation at any point.

Speed-Time and Distance-Time Curves of Elevators

The methods described enable these transients to be easily determined under all conditions of operation, no matter what kind of control is used. If acceleration and retardation are regulated by speed, the speed-torque curve is drawn from the characteristic curve data. Fig. 16 is a curve of this type, the steps occurring where the controller changes at certain velocities.

The curve of $\dfrac{1}{f - f_1 \mp F}$ against v, which is drawn from the above, may be directly integrated to determine corresponding time and speed.

If time control of acceleration and retardation is employed, the speed-time curve is determined up to the time when the controller changes and then the speed-torque curve corresponding to the controller change is used to determine the corresponding portion of the speed-time curve.

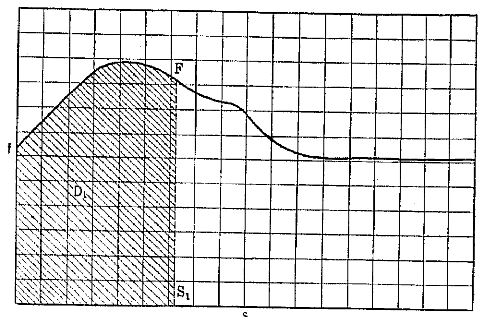

Fig. 11

If acceleration and retardation are controlled by distance, the controller change point is determined from the distance-time curve, shifting to the new speed-torque curve at the specified distance.

Speed-time curves may be determined with any kind of control of acceleration or retardation, for example, current or voltage control, by the comparatively simple methods described.

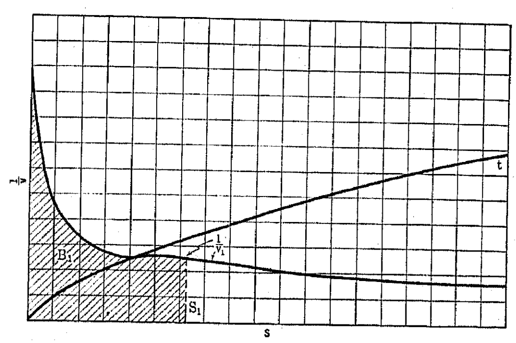

Fig. 12

Varying Fluxes

When fields vary, as they frequently do in elevator operations, the corresponding flux must be taken into account, in determining the torque at any instant. Fields are operated at high, as well as at low saturations, so that it is necessary to use the correct saturation curve in making calculations. By the method employed here, it is comparatively easy to determine fluxes and field currents, including the effect of secondaries, which are usually present in direct-current elevator fields. Since

the correct saturation curve, the equation of which is mathematically unknown, is taken into account by this graphical method, the results are correct for all saturations.

In addition to determining the transient flux of motor fields, it is also important to determine the transient flux of brake and controller magnets used with elevators, and the same method is applicable to these cases. Magnet time constants have an important effect on operation and it is essential to be able to definitely determine this effect.

Rise of Flux and Current with Time in an Inductive System Consisting of a Coil with a Core of Magnetic Material, when a Constant Direct-Current Electromotive Force is Applied, Assuming no Secondary Current

The saturation curve of current and corresponding flux is first plotted.

Let φ = the variable flux
and i = the variable magnetizing current
N = the number of turns in the coil
R = the resistance of the coil
E = the constant impressed d-c. electromotive force
t = variable time

We have first the electromotive force equation:

$$E = R\,i + N\frac{d\,\varphi}{d\,t}$$

According to the saturation curve

$$\varphi = f\,(i)$$

Then
$$d\,\varphi = f'\,(i)\,d\,i \quad \frac{d\,\varphi}{d\,i} = f'\,(i) = \tan\alpha$$

$$d\,\varphi = \tan\alpha\,d\,i$$

$$t = N\frac{d\,\varphi}{E - R\,i} = N\frac{\tan\alpha}{E - R\,i}\cdot d\,i$$

If now we take a number of points of the saturation curve and draw tangents to these points, these tangents will be $\tan\alpha$.

If we now plot a curve of $\dfrac{\tan\alpha}{E - R\,i}$ against i, and take the area of this curve up to the value of i, we obtain t from the above equation. Assuming as an example a direct-current motor field of six poles having 1200 turns per pole, the total field resistance being 36 ohms, we have

$$t = 7200/10^8\,\frac{\tan\alpha\,d\,i}{E - R\,i}$$

The saturation curve is shown as Fig. 16. Tangents have been drawn to points on this curve and plotted against i, in Fig. 17.

The curve $\dfrac{\tan\alpha}{E - R\,i}$ plotted against i, is shown as Fig. 18. Taking areas of this curve, with planimeter the curve Fig. 19 has been drawn, plotting t against i. In this case the flux is simultaneous with the current. (See Fig. 20.)

Rise of Flux and Current with Time in an Inductive System Consisting of a Primary Coil Connected to a Constant Direct-Current Electromotive Force, and a Secondary Assuming the Same Flux Passing through Primary and Secondary

We have previously developed the current and flux time curves without secondary currents, and as the next step, we are developing the current time and flux time curves with short-circuited secondary, including the leakage both of primary and secondary magnetism.

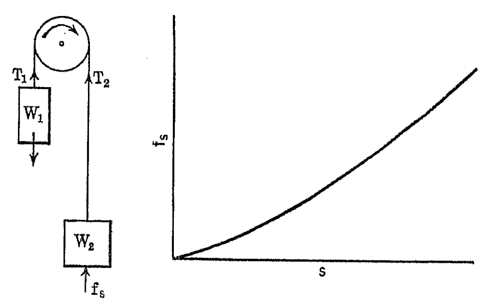

Fig. 13

In this case
Let R_1 = Primary resistance
R_2 = Secondary resistance
i_1 = Primary current
i_2 = Secondary current
N_1 = Turns in the primary coil
N_2 = Turns in the secondary coil

Electromotive Force Equations:
$$\begin{cases} E = R_1\,i_1 + N_1\dfrac{d\,\varphi}{d\,t} \\[2ex] O = R_2\,i_2 + N_2\dfrac{d\,\varphi}{d\,t} \end{cases}$$

$$i_1 = 1/R_1\left(E - N_1\frac{d\,\varphi}{d\,t}\right)$$

$$i_2 = -\,1/R_2\left(N_2\frac{d\,\varphi}{d\,t}\right)$$

$$i_0 = i_1 + N_2/N_1\,i_2 = 1/R_1\left(E - N_1\frac{d\,\varphi}{d\,t}\right) - 1/R_2\left(N_2\frac{d\,\varphi}{d\,t}\right)N_2/N_1$$

$$i_0 = E/R_1 - \frac{d\,\varphi}{d\,t}\left(N_1/R_1 + \frac{N_2^2}{N_1 R_2}\right)$$

$$d\,t = \left(N_1 + \frac{N_2^2 R_1}{N_1 R_2}\right)\frac{\tan\alpha\, d\, i_0}{E - i_0 R_1}$$

$$\text{and } t = \left(N_1 + \frac{N_2^2 R_1}{N_1 R_2}\right)\int \frac{\tan\alpha\, d\, i_0}{E - i_0 R_1}$$

if $R_1 = R_2 = R$ and $N_1 = N_2$

$$d\,t = 2N\frac{\tan\alpha\, d\, i_0}{E - Ri_0}$$

$$t = 2\,N\int\frac{\tan\alpha\, d\, i_0}{E - R\, i_0}$$

Therefore, in this case, if the secondary has the same copper section as the primary, the time for a given flux

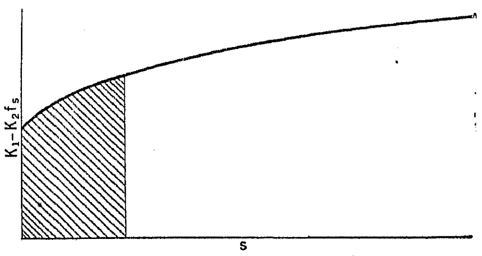

FIG. 14

is doubled. If the secondary has half the copper section of the primary, the time for a given flux is increased 50 per cent.

i_0 in this case represents the magnetizing current which is the combined primary and secondary currents.

We first plot the curve of i_0 and time, obtained as above. See Fig. 21.

Referring then to the saturation curve, we obtain the fluxes corresponding to the various values of i_0.

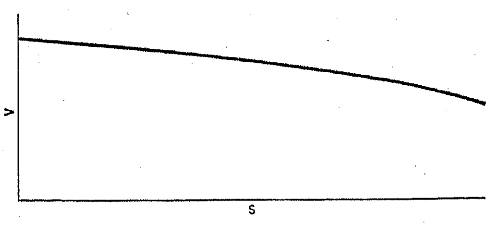

FIG. 15

The flux time curve can then be plotted. See Fig. 22.

If primary and secondary currents are wanted, we have

$$i_1 = 1/R_1\left(E - N_1\frac{d\,\varphi}{d\,t}\right)$$

and

$$i_2 = -\,1/R_2\left(N_2\frac{d\,\varphi}{d\,t}\right)N_2/N_1$$

and

$$i_0 = i_1 + N_2/_1N\, i_2$$

Drawing tangents to the flux-time curve, we obtain the values of $\frac{d\,\varphi}{d\,t}$ corresponding to various values of i_0. See Fig. 17.

Substituting these values in the above equations, we obtain the primary and secondary current-time curves. See Fig. 21.

Fig. 23 shows primary, secondary and magnetizing currents when there is mutual induction with primary and secondary magnetic leakage.

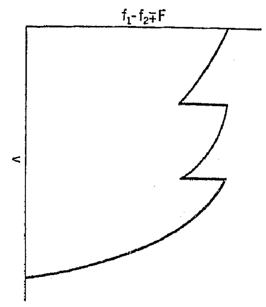

FIG. 15A

Temperature Resulting from Cyclic Operation of Elevator Motors

It is often desirable to determine the maximum temperatures attained by elevator motors in cyclic operation as in actual service. The heat generated in the running period, with a given load, is a function of velocity, and the temperature rise depends on the dissipation of heat, which is dependent on two factors; first, the difference in temperature between the radia-

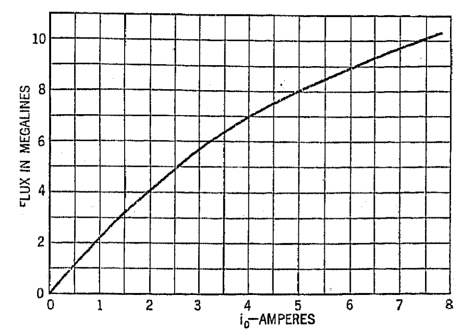

FIG. 16

ting surface and the surrounding air, and second, the ventilation, which is a function of velocity and temperature.

In this case, the motor, if suitable for the work, must have its maximum temperatures limited to proper values, and must therefore finally reach a temperature where the heat dissipated in a cycle equals the heat generated.

When the working cycles are of short duration, as they always are with elevators, it is evident that the elevator motor will not change its temperature materially during the cycle, when heated to the final balanced

condition. It is therefore practically correct to assume that the temperature of the motor is constant during the final cycle.

The following method of calculating final temperatures of elevator motors in service is based upon the above substantially correct assumption, which considerably simplifies the problem.

To Determine the Heating of the Driving Motor Resulting from Cyclic Operation

Let T_1 = final temperature rise at $t = \infty$

" T = temperature of the motor after time (t)

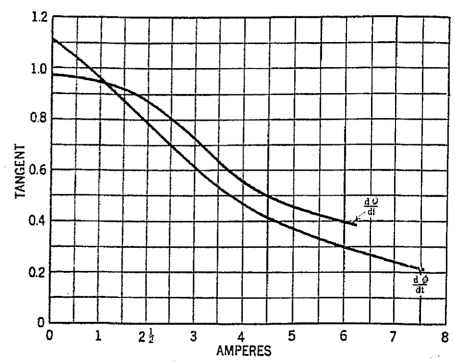

Fig. 17

" R = radiation of the motor per degree rise in temperature

" M = mass of the motor

" L = specific heat of the motor

" W = watts loss of the motor

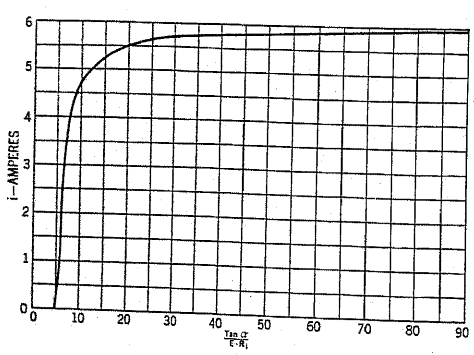

Fig. 18

Then,

$$\int_0^t W\,d\,t = \int_0^t R\,T\,d\,t + M\,L\,T \qquad (1)$$

$\int W\,d\,t$ = energy required to heat the motor to T temperature

$\int R\,T\,d\,t$ = energy radiated by motor

$M\,L\,T$ = Absorption of heat by motor

It is evident that if the motor is run until final temperature is reached, or in other words, if the above equation is integrated between O and ∞ the term $M\,L\,T$ will be comparatively small and may be neglected. Then the above expression reduces to:

$$\int W\,d\,t = T_1 \int R\,d\,t$$

or

$$T_1 = \frac{\int W\,d\,t}{\int R\,d\,t} \qquad (2)$$

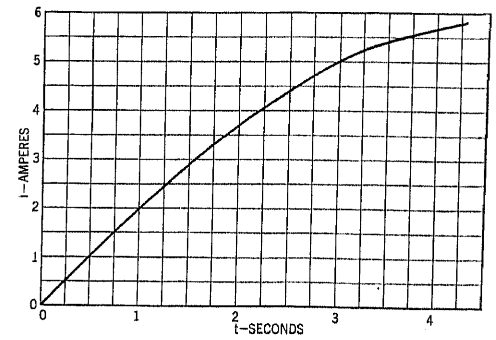

Fig. 19

Below are derived formulas for both numerator and denominator of expression (2) in terms of known values of the motor.

$$\int R\,d\,t = \int a\,[c_1 + c_2 f\,(n)\,]\,d\,t$$

Where a = radiating surface of the motor in square inches

" $f\,(n)$ = some known function of n

" C_1 and C_2 are constants depending upon the construction of the motor

" n = speed of motor in revolutions per second

$$\int R\,d\,t = \int a\,c_1\,d\,t + \int a\,c_2 f\,(n)\,d\,t$$

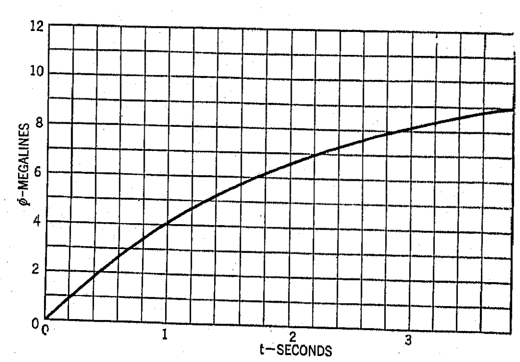

Fig. 20

But

$$d\,t = \frac{M\,K}{Q}\,d\,n$$

Where M is the equivalent mass, K is a constant and Q is the net torque of the motor, causing speed change.

Then

$$\int R\,d\,t = \int_{n_1}^{n_2} \frac{a\,c_1\,M\,K}{Q}\,d\,n + \int_{n_1}^{n_2} \frac{a\,c_2\,M\,K\,f\,(n)\,d\,n}{Q}$$

Or $$\int R\,d\,t = a\,c_1 M K \int_{n_1}^{n_2} 1/Q\,d\,n$$

$$+ a\,c_2 M K \int_{n_1}^{n_2} \frac{f(n)}{Q}\,d\,n$$

Or $$\int R\,d\,t = M K a\,[c_1 H + c_2 F] \qquad (3)$$

Where $$H = \int_{n_1}^{n_2} 1/Q\,d\,n \text{ and}$$

$$F = \int_{n_1}^{n_2} \frac{f(n)}{Q}\,d\,n$$

In order to obtain H plot a curve between n and $1/Q$ which are values obtained from test. (See Fig. 24).

The area H (See Fig. 24) between speed n_1 and n_2 is the integral.

$$\int_{n_1}^{n_2} 1/Q\,d\,n$$

In order to determine F plot the ratio $\frac{f(n)}{Q}$ against the speed n. Such a curve is shown in Fig. 25.

The area F (See Fig. 25) between speeds n_1 and n_2 is the integral.

$$\int_{n_1}^{n_2} \frac{f(n)}{Q}\,d\,n$$

Now substituting the value of $d\,t$ in the numerator:

We have $$\int W\,d\,t = \int_{n_1}^{n_2} W \frac{M K}{Q}\,d\,n$$

Or $$\int W\,d\,t = M K \int_{n_1}^{n_2} W/Q\,d\,n$$

Or $$\int W\,d\,t = M K G \qquad (4)$$

Where $$G = \int_{n_1}^{n_2} W/Q\,d\,n$$

In order to obtain G plot the ratio of W/Q against speed n. (See Fig. 26).

The area G (See Fig. 24) between speeds n_1 and n_2 is the integral.

$$\int_{n_1}^{n_2} W/Q\,d\,n$$

Substituting now the known values of $\int W\,d\,t$ and $\int R\,d\,t$ in equation (2).

We have $$T_1 = \frac{G}{a\,(c_1 H + c_2 F)} \qquad (5)$$

c_1 and c_2 can be determined as follows:

Take constant load heat tests on motor at two distinct speeds, n_1 and n_2 until final temperatures T_4 and T_2 respectively are obtained.

Determine the losses W_4 and W_2 on each test.

Substituting these values in the following formula taken from equation (2) for constant values of W and R.

Thus: $$T_1 = \frac{W}{a\,[c_1 + c_2 f(n)\,]}$$

We get $$c_1 = \frac{T_4 W_2 f(n_2) - T_2 W_4 f(n_1)}{a\,T_2 T_4\,[f(n_2) - f(n_1)\,]}$$

and $$c_2 = \frac{T_2 W_4 - T_4 W_2}{a\,T_2 T_4\,[f(n_2) - f(n_1)\,]}$$

Using test results, solve above formulas for numerical values of c_1 and c_2; substitute same in equation (5) together with values of H and F and G previously determined, and determine T_1, the final temperature rise.

Temperature Resulting from Cyclic Operation

In certain cases, where temperatures change rapidly during the final cycle, the assumption made in the previous example is not correct and the temperature change during the final cycle must be taken into account in determining the maximum temperature. Fuses and other high-temperature electrical devices are in this class.

Devices which change temperature rapidly are usually stationary, so that there is no speed element of cooling. The following example refers to stationary devices.

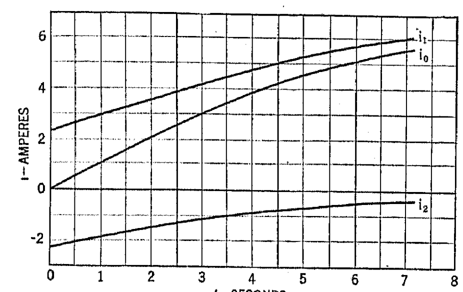

Fig. 21

Temperature Rise due to Input of Power and Dissipation of Heat

Let W = Power input

" R = Radiation constant of body

" A = Absorption constant of body

" T = Temperature of body above surrounding medium at any instant.

The general differential equation is

$$W\,d\,t = R\,T\,d\,t + A\,d\,T \qquad (1)$$

Separating the variables T and t, we get

$$(W - R\,T)\,d\,t = A\,d\,T$$

hence $$d\,t = \frac{A\,d\,T}{W - R\,T} \qquad (2)$$

W, the power input may be constant or variable. We will first consider the case where W is constant.

Case 1: W Constant.

With W constant, equation (2) may be integrated as follows:

$$d\,t = \frac{A\,d\,T}{W - R\,T}$$

$$t = A \int_{T_0}^{T} \frac{d\,T}{W - R\,T}$$

(T_0 = initial temperature)

Let $W - R\,T = Q$, then $d\,T = -\,1/R\,d\,Q$

Substituting, we get

$$t = A \int_{T_0}^{T} -\,1/R \frac{d\,Q}{Q} = -\,A/R \int_{T_0}^{T} \frac{d\,Q}{Q}$$

$$= -\,A/R \log_e Q \int_{T_0}^{T}$$

$$t = -\,A/R\,[\log_e (W - R\,T) - \log_e (W - R\,T_0)\,]$$

$$t = A/R \log_e \frac{W - R\,T}{W - R\,T_0} \qquad (3)$$

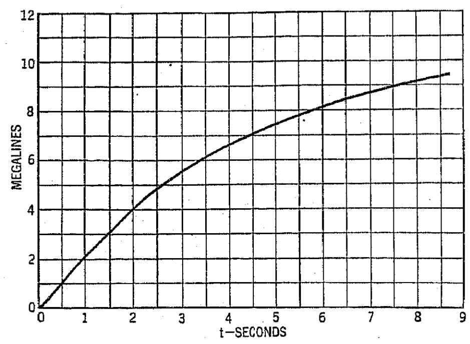

FIG. 22

$$\text{or} \quad -\,t\,R/A = \log_e \frac{W - R\,T}{W - R\,T_0}$$

$$e^{-tR/A} = \frac{W - R\,T}{W - R\,T_0}$$

$$(W - R\,T_0)\,e^{-tR/A} = W - R\,T$$

$$\text{hence} \quad T = \frac{W - (W - R\,T_0)\,e^{-tR/A}}{R}$$

$$= W/R - W/R\,e^{-tR/A} + T_0\,e^{-tR/A}$$

$$T = W/R\,(1 - e^{-tR/A}) + T_0\,e^{-tR/A} \qquad (4)$$

Case 2: W Variable—Expressable Graphically as a Function of t.

Let Fig. 27 represent the known graphical relation between W and t or $W = f\,(t)$.

Required: the relation between T and t, or $T = F\,(t)$. It is evident that this relation can only be known graphically.

Draw the known curve $W = f\,(t)$. Let the curve $T = f\,(t)$ represent the required graphic relation between T and t. Take any interval t, and divide it into n equal increments such that for each increment the input W is constant. In that case, the equations (1) to (4), which were derived for constant input, become applicable for each interval.

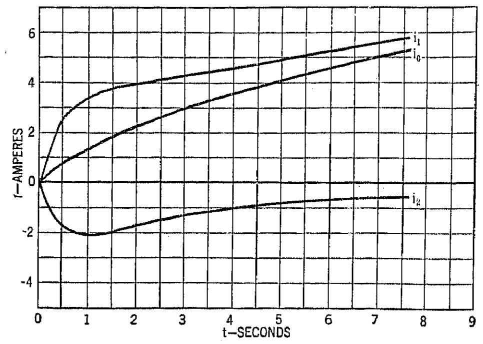

FIG. 23

Let T_0 = Initial temperature
W_0 = Initial rate of input
T_1 = Temperature after 1st interval t_1/n
W_1 = Rate of input after 1st interval t_1/n
T_2 = Temperature after 2nd interval t_1/n
W_2 = Rate of input after 2nd interval t_1/n
etc. (See Fig. 28).

From equation (4) we get

$$T_1 = W_1/R\,(1 - e^{-(t_1/n)\,(R/A)}) + T_0\,e^{-(t_1/n)\,(R/A)}$$
$$= W_1/R\,(1 - E) + T_0\,E$$

where

$$E = e^{-(t_1/n)\,(R/A)}$$

$$T_2 = W_2/R\,(1 - E) + T_1\,E = W_2/R\,(1 - E) + W_1/R\,(1 - E)\,E + T_0\,E^2$$

$$= \frac{1 - E}{R}\,(W_2 + W_1\,E) + T_0\,E^2$$

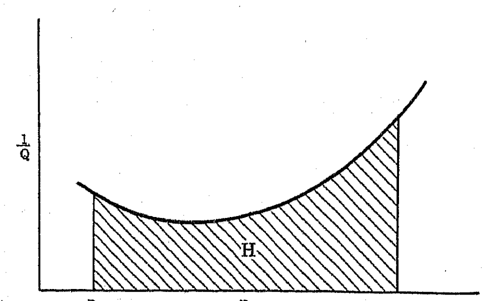

FIG. 24

$$T_3 = W_3/R\,(1 - E) + T_2\,E = \frac{1 - E}{R}\,(W_3 + W_2\,E + W_1\,E^2) + T_0\,E^3$$

accordingly,

$$T_n = \frac{1 - E}{R}\,(W_n + W_{n-1}\,E + W_{n-2}\,E^2 + \ldots\,W_1\,E^{n-1}) + T_0\,E^n \qquad (6)$$

If we let n approach infinity then the terms of the series within the parenthesis may be evaluated as follows:

The 1st term which let us call $y_0 \quad = W_n$

The nth term, $y_n = W_1 E^{n-1} = W_1 e^{-(t_1/n)(R/A)(n-1)}$
$= W_1 e^{-t_1 R/A}$

The $(\frac{1}{4} n)^{th}$ term, $y_{\frac{1}{4}n} = W_{n-\frac{1}{4}n} E^{\frac{1}{4}n} = W_{\frac{3}{4}n} e^{-\frac{1}{4} t_1 R/A}$

The $(\frac{1}{2} n)^{th}$ term, $y_{\frac{1}{2}n} = W_{n-\frac{1}{2}n} E^{\frac{1}{2}n} = W_{\frac{1}{2}n} e^{-\frac{1}{2} t_1 R/A}$

The $(\frac{3}{4} n)^{th}$ term, $y_{\frac{3}{4}n} = W_{n-\frac{3}{4}n} E^{\frac{3}{4}n} = W_n e^{-\frac{3}{4} t_1 R/A}$

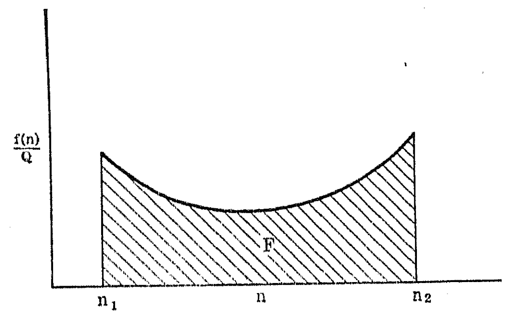

Fig. 25

Similarly, we can obtain the value of any (fraction of $n)^{th}$ term from the value of t_1 and the various values of W.

If the various terms as calculated above are laid out as y ordinates corresponding to the fractional values of t (see Fig. 29), then

$$\frac{\text{area } B}{t_1} = \text{average } y \text{ term}$$

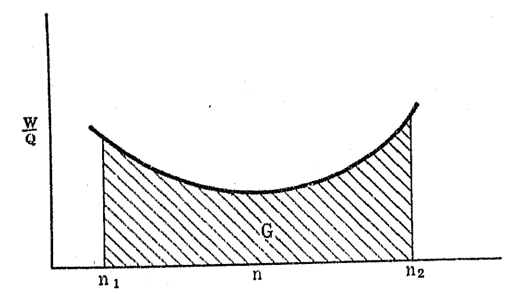

Fig. 26

The average term multiplied by the number of terms n will give the sum of the entire series in (6).

or $\qquad$ Sum of Series $= B\, n/t_1$

Substituting in (6), we get

$$T_n = 1/R\,(1 - E)\, B\, n/t_1 + T_0 E^n$$
$$= 1/R\,(1 - e^{-(t_1/n)(R/A)})\, B\, n/t + T_0\, e^{-(t_1/n)(R/A)n}$$

$$T_n = B/R\,\frac{(1 - e^{-(t_1/n)(R/A)})}{t_1/n} + T_0\, e^{-t_1 R/A} \qquad (7)$$

As n approaches infinity, the quantity

$$1 - \frac{e^{-(t_1/n)(R/A)}}{t_1/n} = \frac{1 - e^{-xR/A}}{x} = 0/0$$

$x = t_1/n = 0$

To evaluate this indeterminate expression, differentiate the numerator and denominator with respect to x:

$$\frac{d\,(1 - e^{-xR/A})}{d\,(x)} = \frac{R/A\, e^{-xR/A}\, d\,x}{d\,x} = R/A\, e^{-xR/A}$$
$$= R/A \text{ when } x \text{ approaches } 0$$

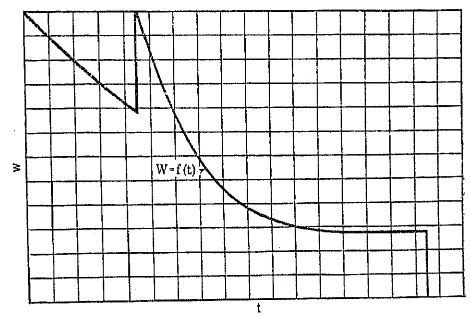

Fig. 27

Substituting in (7), we get

$$T_n = (B/R)\; R/A + T_0\, e^{-t_1 R/A}$$

$$T = B/A + \frac{T_0}{e^{t_1 R/A}} \qquad (8)$$

Combining equation (8) with equation (15), when R/A is known, but not A, we have

$$T_n = \frac{T_f\, R}{W_c}\; B/A\; \frac{T_0}{e^{t_1 R/A}}$$

$$T = B/W_c\, R/A\; T_f + \frac{T_0}{e^{t_1 R/A}} \qquad (8a)$$

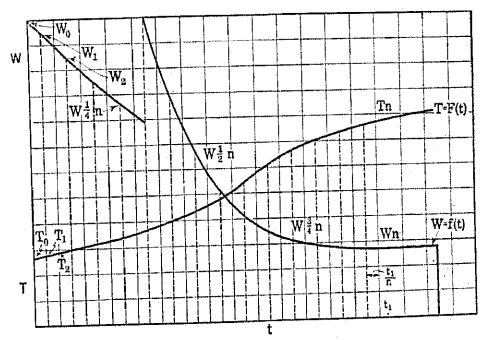

Fig. 28

Final Temperature

Assume a duty cycle of heating and cooling continued indefinitely as shown in Fig. 30. What be will the final temperature?

The final temperature will be reached when equilibrium is established. That is, when the temperature rise during the heating period t_1 is exactly equal to the drop in the temperature during the cooling period t_2 (See Fig. 31).

From equation (8) we get

$$T^1 = B/A + \frac{T_0}{e^{t_1 R/A}} \qquad (9)$$

T''' may be obtained from the differential equation of the cooling curve, as follows:

$$R\,T\,d\,t \;=\; -\,A\,d\,T \tag{10}$$

$$d\,t \;=\; -\,A/R\,\frac{d\,T}{T}$$

Integrating between the limits T' and T''', we get

$$t_2 \;=\; A/R \log_\epsilon T'''/T'$$

$$e^{-t_2 R/A} \;=\; T'''/T'$$

Hence $$T_0 = T''' = \frac{T'}{e^{t_2 R/A}} \tag{11}$$

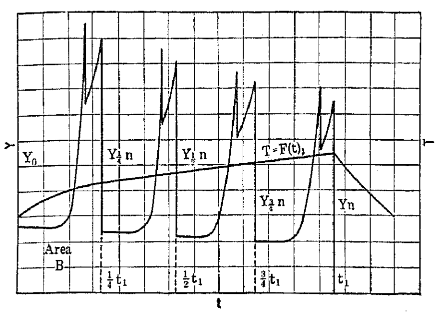

FIG. 29

From equations (**9**) and (**11**), we get

$$T_0 = B/A + \frac{T_0}{e^{t_1 R/A}}\;\frac{1}{e^{t_1 R/A}}$$

$$T_0\,e^{t_2 R/A} - \frac{1}{e^{t_1 R/A}} \;=\; B/A$$

Hence the final $$T_0 = B/A\,\frac{e^{t_1 R/A}}{(e^{(t_1+t_2)R/A} - 1)} \tag{12}$$

and the final $$T' = B/A\,\frac{e^{(t_1+t_2)R/A}}{(e^{(t_1+t_2)R/A} - 1)} \tag{13}$$

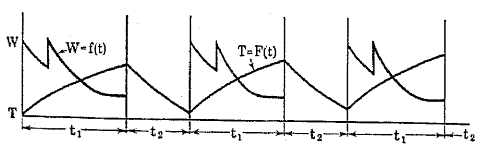

FIG. 30

If the heating cycle is repeated indefinitely *without* any intermediate cooling period, the t_2 in equations (**12**) and (**13**) equals 0.

Hence $$T_0 = T' = B/A\,\frac{e^{t_1 R/A}}{(e^{t_1 R/A} - 1)} \tag{14}$$

If the input is not in cycles but remains *constant* indefinitely then the final temperature is obtained by substituting in equation (**4**) $t = \infty$.

Hence $$T_f = W_c/R$$

or $$\frac{T_f\,R}{W_c} = 1 \tag{15}$$

Where W_c = the constant input to produce the final temperature T_f.

Note: In equations (**12**), (**13**) and (**14**) the value A may not be known. Instead only the ratio R/A may be known. In that case these equations may be combined with equation (**15**) as follows:

$$T_0 = \frac{T_f\,R}{W_c}\;B/A\;\frac{e^{t_1 R/A}}{(e^{(t_1+t_2)R/A} - 1)}$$

$$T_0 = B/W_c\,(R/A)\,\frac{e^{t_1 R/A}}{(e^{(t_1+t_2)R/A} - 1)}\,T_f \tag{16}$$

and $$T' = B/W_c\,(R/A)\,\frac{e^{(t_1+t_2)R/A}}{(e^{(t_1+t_2)R/A} - 1)}\,T_f \tag{17}$$

W_c may be in any units provided only that the same units are employed in obtaining B.

T_0 of equation (**16**) is the final temperature at the beginning of each cycle and T' (eq. **17**) is the final temperature at the end of each cycle. The maximum temperature, however, may occur between these points.

To get the maximum temperature, the entire curve $T = F(t)$ between the final T_0 and T' should be constructed as per Fig. 27.

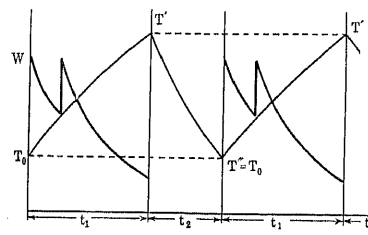

FIG. 31

The methods described in this paper have numerous practical applications. The effects of any kind of control and machine may be definitely predetermined. Different kinds of elevator apparatus may be definitely compared. Test results may be checked. Having determined the power-time curves for various distances, the power consumption of the elevator under all conditions may be accurately calculated. The complete performance of elevator apparatus in all kinds of service may be determined from basic data without special tests. Durability and dependability of apparatus may be forecast. The determination of transients is also of advantage in fixing maximum electrical and mechanical stresses and the variations of these stresses.

Since the transient curves of elevator performance tell the story of the qualifications of the apparatus for the service, any means of calculating them is of value, particularly when the calculations may be quickly and easily made, as by the methods demonstrated.

Discussion

Bassett Jones: It strikes me that the paper by Messrs. Lindquist and Yearsley presents a method of solving a very large class of differential equations of almost any order and complexity involving both continuous and discontinuous functions. The method should prove of inestimable value in mathematical engineering. As the authors have shown, the

method makes *the solution of even an unknown equation,* relating two or more variables, a practical possibility provided only that the actual physical relationship between these variables can be observed and graphically represented. It is not required that the mathematical expression for this relation be known, or if known in abstract form, it does not follow that the detail form of the expression is required.

The method makes it possible for the ordinary man who posesses the necessary instruments to solve real physical problems that lie beyond the reach of any purely analytical method, unless it be a most searching and complicated mathematical treatment. Unfortunately for rigorous mathematical methods, most of our real and pressing day-by-day engineering problems are of this type. If we adhere to the strict rules of the game, no solution is possible, for, as a very great man once said, "most differential equations take refuge in a definite integral and only rarely can these integrals be evaluated." By a very ingenious device, Mr. Lindquist has forced the integrals into the light of day where we can look at them, and measure them, and make an end of them. Heaviside did the same sort of thing by what he called algebraising the differentials. But if the equations are complicated, this method requires marked mathematical agility on the part of the worker.

Mr. Lindquist's method requires merely that the relation between the variables be measurable. To be of any real value, this relation must be measurable. Being measurable it can be properly represented in graphical form. The resulting mechanical integration of such unknown, and generally unknowable functions, will be quite as accurate as the basic data used.

The general method set forth in this paper opens a wide field for investigation that, mathematically speaking, up to the present time has remained either a closed book or subject only to the vaguest sort of approximations.

K. L. Hansen (by letter): Seven years ago the present writer presented a paper before the Institute, entitled "Analysis of Starting Characteristics of Direct-Current Motors" (TRANSACTIONS A. I. E. E., Vol. XXXVI). Although the problems of acceleration discussed in that paper did not refer specifically to elevators, they were nevertheless, in many respects, similar to those discussed by Messrs. Lindquist and Yearsley.

At that time controller engineers appeared to take little interest in analysis of the transient phenomena incidental to acceleration and retardation of electric motors. The feeling seemed to be that rough approximations based on experience and cut-and-try methods were preferable to mathematical analysis. In spite of this sentiment my belief was that interest in the mathematical predetermination of these transients would increase as the requirements became more exact. The paper under discussion indicates that this belief will be justified.

To illustrate the similarity of the problems discussed in the two papers, consider the following formula developed by the authors

$$t = \frac{M\,R}{K^2} \log \epsilon \frac{F_1 \pm F - K\,I}{F_1 \pm F - K\,i}$$

F_1 and F being constants $F_1 \pm F$ may be combined into one constant, T_c, which may be positive or negative. Also writing $\frac{E}{R}$ for its equivalent I. and transforming the equation slightly we have

$$\frac{t\,K^2}{M\,R} = \log \epsilon \frac{T_c - \dfrac{K\,E}{R}}{T_c - K\,i}$$

Expressing this in the exponential form

$$\frac{T_c - \dfrac{K\,E}{R}}{T_c - K\,i} = \epsilon^{\frac{K^2 t}{MR}}$$

and solving for i

$$i = \frac{T_c}{K} + \left(\frac{E}{R} - \frac{T_c}{K} \right) \epsilon^{-\frac{K^2 t}{MR}}$$

By inserting the proper conversion multipliers this formula is readily seen to be identical with formula (10) of my paper. Incidentally it may be remarked that when publishing a formula, which has already been derived in a previous paper, it is customary to make some references to the earlier publication.

However, while the problems discussed in the two papers are similar in many respects, the authors of the paper under discussion have mainly employed a different method of obtaining the solutions. The authors have arrived at useful and valuable results by an ingenious application of the well known method of graphical integration, but have apparently underestimated the limitations of this method and greatly overestimated its accuracy and usefulness as compared with straight analytical methods.

The classes of differential equations, which can be integrated by quadratures, as this method is called, are very limited indeed, being those of first order and first degree, in which the variables can be separated, and those of first order from which one of the variables is explicitly absent. Very few of the differential equations met with in engineering and physics belong in this group. A little reflection will show that the limitations of the method have, in fact, prevented the authors from arriving at the complete solution of the problem they proposed to solve.

To illustrate, consider the motor and elevator having the characteristic curves shown in Fig. 7 of the paper. Assume all conditions to remain unchanged, except that a considerable inductance is inserted in the motor armature circuit. As this in no wise alters the speed-torque characteristic of the motor or the friction curve of the elevator, the same speed-time curve would be arrived at by the authors' method. The fact is that this curve may be considerably modified by an appreciable change in inductance of the main circuit. Indeed, in elevator service such inductance is sometimes employed to limit the current peaks in transition from one controller step to the next, thereby effecting smoother acceleration.

The authors' statement that "the accuracy of their method is limited only by the accuracy of the instruments used in making the speed-torque tests, and the accuracy of their observation" appears therefore to be quite inaccurate. The limitation of the method itself in not being readily extended to take into account the electro-magnetic energy stored in the system is liable to introduce errors greatly exceeding the inaccuracies of observation. The limitation of the method is again illustrated by the necessity of going to a step-by-step calculation in order to take into account the primary and secondary leakage fluxes in the problem of mutual induction.

The authors state that it is comparatively easy to apply the method they describe to determine the speed-time curves when the field currents vary, but the method of procedure in that case is not indicated. It would be of interest to see the method applied, without resorting to step-by-step calculation, to a case of continuously varying field flux, as, for example, to the system of control described by Mr. Bouton in his paper, "Variable Voltage Control Systems as Applied to Electric Elevators."

Step-by-step calculations, like the method described in the paper, are found very useful in many cases when it is impossible to deduce analytical expressions for the variables involved. However, in the vast majority of cases, analytical solutions are preferable, even when certain simplifying assumptions are made in their derivation. That the authors have greatly overestimated the errors that are likely to result from such assumptions, pertaining to these problems, will become apparent by a careful study of the calculated and test curves in Figs. 6, 7, and 8 of my paper.

When the authors state that graphic methods lead to a much clearer understanding of the reasons for transient conditions, is

that statement to be interpreted to mean that they have a better understanding of these problems than one who has deduced the solutions analytically?

The expression "seriously involved" mathematics is obviously a relative term. One is likely to consider mathematics beyond one's own knowledge seriously involved. To some, the elementary calculus used in the paper undoubtedly seems like seriously involved mathematics; while to others, the application of the theory of functions to the solution of differential equations may be a simple matter. As seems to be the case with most mathematical papers, probably very few feel that the authors have used just the kind of mathematics to suit them.

E. W. Yearsley (by letter): The equation referred to by K. L. Hansen was published in the *Proceedings* of the Association of Iron and Steel Electrical Engineers in 1908 as a part of a paper on Electric Motor Drives, by E. W. Yearsley. In this paper the general treatment of systems of electric drives was outlined, both on analytical and practical bases. These equations, which are so simple as to be obvious to those who have even an elementary knowledge of physics and the calculus, were introduced in the present paper for the purpose of leading up to the graphical treatment and to call attention to the difficulties of the analytical method.

It is, of course, impossible to take into account unknown functions such, for example, as the equation of the magnetic or saturation curve in determining transients, unless either a graphical method is used or an empirical equation assumed, which approximates the function. This latter is necessary when purely analytical methods are used and the result is usually very inaccurate.

Dr. Steinmetz, in his work on Transient Phenomena, considers various magnetic problems analytically, and is compelled to resort either to the approximation of Froelich's formula or to confine himself to the basis of magnetism directly proportional to magnetizing force.

The writer has used both Froelich's and Kapp's formulas in analytical investigations of transients as long ago as 1903, but the results obtained were not sufficiently accurate. Those who have read the work of Dr. Steinmetz will appreciate the difficulties of a purely analytical treatment of magnetic transients. The purpose of the paper under discussion was to simplify the treatment of transient problems so that practical solutions could be obtained that would be of advantage in developing satisfactory apparatus for actual service.

To solve all the differential equations of electro-mechanical systems was not within our expectations, but we have obtained practical results by the methods outlined and have had no great difficulty in taking into account continually varying field fluxes.

We have been engaged in the development and investigation of transients of various sorts of electro-mechanical systems, particularly those applied to elevators, for a number of years and have investigated the operation of electric motors under many kinds of control, including the Ward-Leonard System, as described by Mr. Bouton in his paper "Variable Voltage Control Systems as Applied to Electric Motors." This system has recently come into more general favor, particularly on account of the demand for high-speed elevator equipment to be operated on alternating-current supply. It has been found comparatively easy to develop the transients of an elevator system employing this type of control.

Personally, I am interested in mathematics as a means of obtaining practical results. In this connection it does not make much difference what sort of mathematics suits various mathematicians. The means that are simplest are most attractive to those who are developing practical apparatus.

Variable Voltage Control Systems as Applied To Electric Elevators

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Review of the Subject.—*Low-speed electric elevators, using d-c. motors, came into use about 1890. Later, a-c. motors were employed, but on account of the difficulty of speed control could not be used for the high speeds necessary in tall buildings. Since the height of buildings is dependent upon the elevator system, and in many districts only a-c. is available, the need for high-speed equipments that can be operated from a-c. is evident. A solution to the problem is found in the variable voltage system of control.*

In this system each elevator motor is supplied by an individual generator driven by a motor operating from the a-c. or d-c. supply voltage. The generator's voltage, and hence the elevator motor's speed, are controlled by varying the field of the generator.

The apparatus consists of:

1. *An ordinary shunt-wound d-c. elevator motor.*
2. *A d-c. generator of special design for the elevator motor.*
3. *Control panel for the generator and elevator motor.*
4. *An a-c. or d-c. motor to drive the generator.*
5. *A starting device for the driving motor*

And if the supply is a-c.

6. *A direct-connected exciter for the field, brake and control circuits.*

The control panel (1) makes the proper connections for up and down motion of the car; (2) releases, or sets, the brake; (3) controls the speed; (4) discharges the generator field during retardation, and on stopping (5) demagnetizes the generator and (6) opens the circuit between the armature of the generator and the armature of the elevator motor.

From tests the following conclusions in favor of the variable voltage over the rheostatic control system are drawn:

1. ***Speed.***—*High-speed installations are now possible for any commercial a-c. voltage and frequency.*

2. ***Acceleration and Retardation.***—*The rate of acceleration increases gradually to about half speed, then decreases uniformly until full speed is reached. The time and power required for acceleration are less than with rheostatic control. The time remains practically the same for all loads. The higher rate of acceleration and retardation permits higher speeds, and the smoothness, besides reducing wear and tear on the machinery, makes riding entirely comfortable to passengers. The impression of falling which is often given, under rheostatic control, by a heavily loaded car when descending, is inherently avoided, since the car speed follows the generator voltage which at no time changes suddenly.*

3. ***Speed Control and Regulation.***—*The speed regulation remains flat at as low as one tenth full speed. Consequently it is easier to make an accurate landing, fewer false stops are made the car may be "inched" easily and quickly. In stopping, regenerative braking is set up which brings the car quickly but smoothly to a low speed before the friction brake is applied. Positive speed control enables the limit stops to be made in less time and shorter distances, and without over-travel.*

4. ***Efficiency and Economy of Power Consumption.***—*Less power is required in acceleration and retardation. This saving of power over the rheostatic control is greatest when the number of starts and stops is large. Power is returned to the line while making the limit stops. Power consumption is not increased in making small movements of the car or in running at low speed. During idle periods standby losses may be eliminated by shutting down the motor generator set.*

5. ***Maintenance.***—*Since the switching of major currents is eliminated and they are controlled indirectly, the number of arc rupturing contacts is reduced to a minimum the control as a whole is simpler, less adjustments are necessary and maintenance costs are lowered.*

6. ***Safety.***—*Inherent safety features make higher speeds possible. Limit stops are made accurately and positively. A second independent means is provided for stopping the car in emergencies. On failure of power a dynamic braking circuit closes and a field is maintained, on the elevator motor, making certain the stopping of the car. An overspeed contact on the motor generator set opens the safety circuit independently of the speed governor.*

THE application of electric power to elevators dates back to 1890. The early motors were direct-current machines and were applied to relatively low-speed elevators. Later a-c. motors came into use, both phase-wound and cage-wound secondaries being used. Higher-speed elevators were developed as building heights increased. The modern sky scraper would not be practical except through the use of high-speed elevators. The d-c. motor and its controller were perfected for high-speed work largely because of the difficulty in controlling the speed of the a-c. motor. Until quite recently high-speed elevators invariably required the use of d-c. power.

In the last three or four years a great deal of engineering effort has been put into the problem of developing a high-speed a-c. elevator equipment. These efforts have been very fruitful and a-c. power can now be applied to elevators at as high a speed as can d-c. One of the a-c. systems that has recently been perfected is known as variable voltage. It is this system that will be described in this paper. The development of this system has also greatly improved the operation of all classes of elevators to which it has been applied and because of its superior operation and high economy has frequently been used on d-c. power lines.

During the last few years the cost of buildings and the ground upon which they are built has increased so rapidly that effective use must be made of every available square foot of floor space. The space occupied by the elevators brings in no direct income and should be reduced to a minimum. Higher car speed and reduction of lost time due to better control offer considerable help in solving this problem. Variable voltage control has been used very successfully for elevators running at speeds as high as 700 feet per minute. The system with further refinements in some details will no doubt be used for still higher speeds in the future.

I. Requirements of Elevator Service

It is not the purpose of this paper to discuss all the factors that go to make up elevator service. Elevator

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service is, however, so affected by the kind of electrical equipment installed that in discussing any particular control system for elevator work, it will be necessary to consider its fundamental requirements. The requirements of elevator service that are affected by the electrical equipment and particularly the control may be enumerated as follows:

1. *Elevator Speeds.* The speed at which cars are run affects the service, and is very largely dependent upon the electrical equipment used and upon the control that can be obtained. Low-speed elevators require relatively simple equipment, while the maximum speed that can be used is limited by the control systems in use at the present time, and increases in speed are probably dependent upon further control developments.

2. *Acceleration and Retardation.* The rate of acceleration and retardation has a very marked effect upon elevator service, not only in so far as comfort to the passengers is concerned, but it also has a direct bearing upon the speed at which cars can be run. It is mostly dependent upon the control system and the ideal control system is one which will accelerate and retard the car at the highest possible rate, and still operate so smoothly as to subject the passengers to no discomfort.

3. *Speed Control and Regulation.* Elevator speeds in the past have increased directly as methods of increasing the speed range over which motors could be controlled have developed. No matter how high a car speed is used, the speed from which the landing can be made will remain more or less fixed. The regulation of the equipment affects elevator service because if the cars slow down too much under load the service is slowed up at the very time when maximum service is demanded.

4. *Efficiency and Power Consumption.* Electrical elevator equipment at the present time has reached a high stage of development and the matter of economy in operation probably receives more attention than in almost any other industry. Electrical elevator equipment to give good service must not only handle passengers as quickly and smoothly as possible, but it must also do it in an economical manner.

5. *Upkeep and Maintenance.* Upkeep and maintenance constitutes a considerable item in the cost of elevator service. It is, therefore, desirable that this item be kept as low as possible. It has an even more important bearing upon the continuity of service. Equipments that are easy to maintain and which are readily kept in good condition are much less subject to shutdown than those which require constant attention to keep them in operating condition.

6. *Safety.* Although this item appears last in the list it is far from being the least in importance. In fact safety is the first consideration in giving good elevator service. Any developments in control systems or apparatus which increase the service must at the same time be absolutely safe in operation, and in no case must the safety of passengers be jeopardized in order to save time or reduce the first cost of installation. Any control system having features which increase the safety with which passengers can be handled is worthy of serious consideration from that standpoint alone.

II. Description of the System

The variable voltage system consists of five main elements, a d-c. motor to drive the elevator, a d-c. generator for each elevator motor, an a-c. or d-c. motor to drive the generator, a control panel for the elevator motor and generator and a starting device for the motor driving the generator. If the supply is a-c., a direct-connected exciter is added. The auxiliary equipment, such as the car master switch, limit switches and magnet brake may be the same as those used for

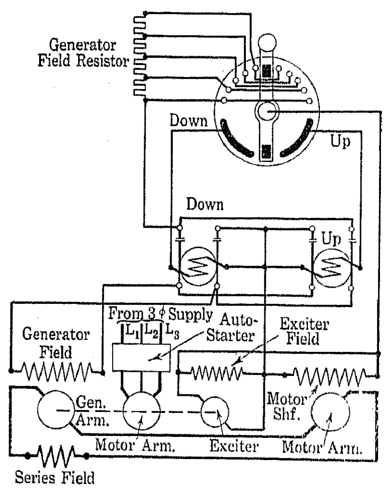


Fig. 1

Elementary diagram showing principal connections of variable voltage control system. The generator is driven by an a-c. motor and the set has a direct-connected exciter.

other well known systems of control. Fig. 1 is a schematic diagram showing how the apparatus is connected together.

1. *Elevator Motor.* The elevator motor requires no special features and is shunt wound which is the accepted elevator practise. No speed adjustment by field control is necessary although it is good practise to strengthen the field during the starting period for applications where high starting torque is required. A constant-speed motor for a given rating will have a lower weight and cost, and better electrical performance than the corresponding adjustable-speed motor.

2. *Generator.* The generator is one of the most important units of the system and is designed almost entirely from the standpoint of its control functions. As shown in the schematic diagram the motor and generator armatures are connected directly together electrically without the use of any resistors. The direction of rotation and the speed at which the elevator motor runs depend upon the direction and value of the generator voltage. The field magnet is excited from a separate source and the field excitation is governed by the controller.

The generator has commutating poles and sufficient commutator capacity to take care of the current peaks that must be handled during the accelerating period. High efficiency and good voltage regulation are incorporated as essential features. Fig. 2 shows the performance curves of a 25-kw. machine that are typical of what can be obtained. The efficiency reaches a maximum of 89 per cent and remains high with overloads so that acceleration is accomplished without excessive generator losses.

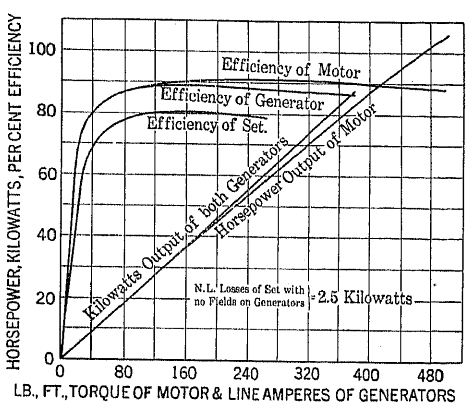


Fig. 2

Performance curves of a double generator motor generator set with an a-c. driving motor.

Fig. 3 shows a test on the rate of building up of a 25-kw. generator designed for variable voltage control service. Full excitation voltage was thrown on the field coils in one step. The machine builds up to 85 per cent of its final voltage in $1\frac{1}{4}$ seconds and in $2\frac{1}{2}$ seconds is generating full voltage. Curve 2 shows the effect of a short-circuited damper winding on the field poles. The damper winding has the effect of increasing the time constant of the field at the early stage of building up but does not materially affect the total time of

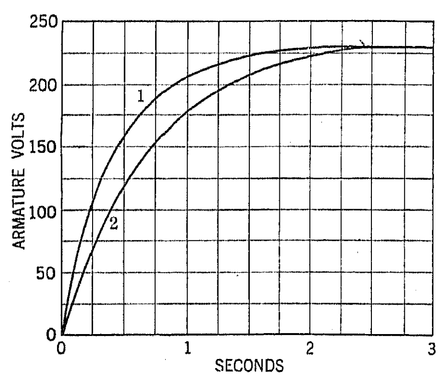


Fig. 3

Curves showing the rate of building up of generator voltage. The curves show the effect of a short-circuited damper winding on the field poles. The maximum rate of acceleration can be adjusted by changing the resistance of this damper winding.

building up. Practical use of this characteristic of the field coils can be made in giving the proper shape to the acceleration curve.

3. *Driving Motor of the Motor Generator Set.* The driving motor of the set may be either a-c. or d-c. depending upon the character of the power supply. Alternating-current motors are cage-wound and designed for low slip and high efficiency. Direct-current motors are shunt wound. It is of particular importance that the no-load or idling losses be kept as low as possible. The motor generator set whose performance curves are shown in Fig. 2 has a no-load loss of 2.5 kw. or 1.25 kw. per elevator.

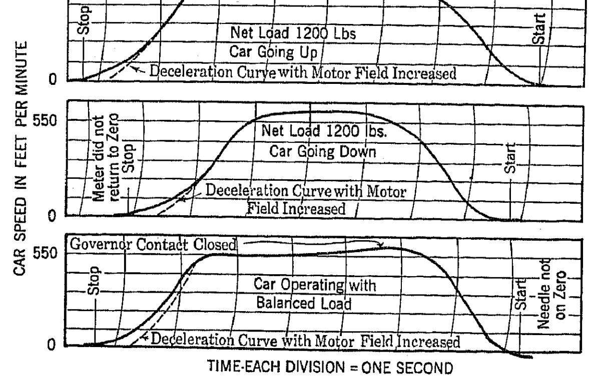


Fig. 4

Speed Time Curves for a Gearless Traction Elevator. Duty 2000 lbs. at 500 F. P. M. The dotted lines show the effect of increasing the motor field during retardation.

4. *Motor Generator Set Starter.* Since the motor generator sets are started infrequently and always start without load a relatively simple automatic starter

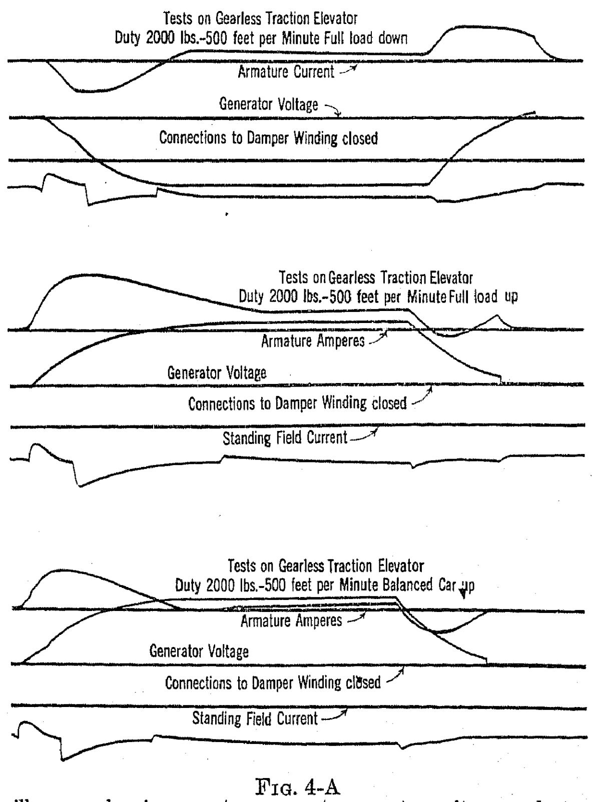


Fig. 4-A

Oscillogram showing armature current, generator voltage, and standing field current.

may be used. For d-c. motors a counter e. m. f. starter with two or three steps of light duty starting resistance is used. The only special feature required

is a counter e. m. f. relay interlocking with the elevator controller to prevent energizing the generator field and brake circuits when the motor generator set is not running or before it has come up to speed. For the a-c. sets a resistance type starter is used. Alternating-current sets have a direct-connected exciter and a switch controlled by the voltage of the exciter is used to short circuit a step of primary resistance when the motor has come up to speed. No interlocking relays are required since there will be no voltage to energize the d-c. circuits unless the motor generator set

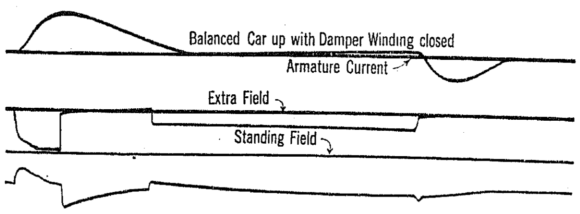


Fig. 5
Oscillogram showing armature current and current in extra and standing motor fields.

is running. Reverse phase protection is inherent in the combination without the use of relays because the exciter being a self-excited machine will not build up its voltage if the motor generator set is started in the wrong direction.

5. *Elevator Controller.* The elevator controller proper carries the contactors and relays to perform the following primary functions:

1. Connect the generator field to the excitation

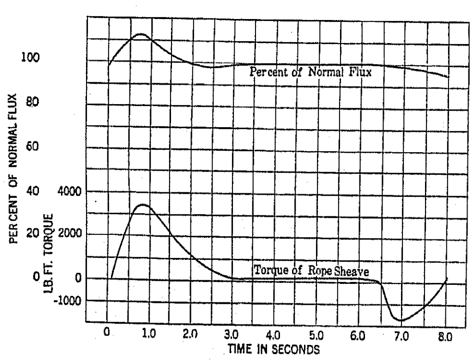


Fig. 5A
Curves showing motor field flux and torque.

voltage in the proper direction for up or down motion of the car.

2. Energize the brake coil. The brake coil is opened on both sides and is fed through the same contactors that energize the generator field.

3. Control the car speed by controlling the excitation of the generator field.

4. Provide the proper discharge circuits for the generator field during retardation.

5. Connect in an extra demagnetizing field on the generator in stopping.

6. Open the connection between the generator and elevator motor after the car has come to rest.

7. Control the setting of the brake by means of its own self induction and so obtain a smooth stop.

There is only one contactor carrying armature circuit current. This contactor is included as an additional safety measure and in normal operation opens only after the car has completely stopped and the brake has set. It also opens immediately when the safety devices, such as the car emergency switch, overspeed governor, etc., operate. In normal operation no current is ruptured by this contactor. The other contacts on the controller handle only field, brake and control circuit currents. When the armature circuit contactor opens its back contacts close a low-resistance dynamic-braking circuit for the elevator motor. This contactor has auxiliary contacts through which the brake coil

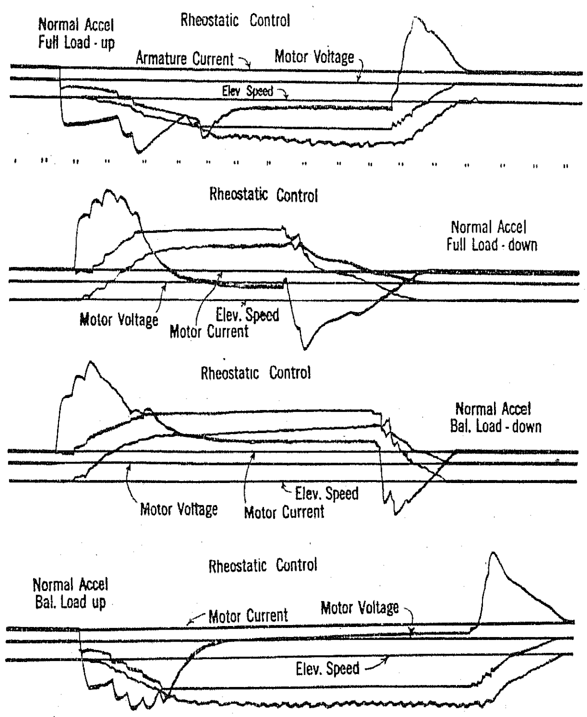


Fig. 6
Accelerating characteristics of a gearless traction elevator with rheostatic control.

current passes so that the brake cannot be released unless the armature circuit is closed.

III. Characteristics and Operation

1. *Acceleration and Retardation.* To obtain maximum service from an elevator car it should be accelerated and retarded at as high a rate as possible without discomfort to passengers. The uncomfortable feeling frequently experienced while riding in high-speed elevators is not due to a high rate of acceleration but to rapid changes in the rate of acceleration. Therefore the rate of acceleration should not change suddenly during the accelerating period.

When a fully loaded car is to be accelerated in the down direction the weight of the unbalanced load alone is sufficient to accelerate the car. Under a system of rheostatic control, if the motor is now connected to the line with a resistance in series with its armature, the

voltage impressed on the armature will be reduced only by a flow of positive current in the resistance current flowing to the motor from the line being regarded as positive; current returned from the motor, acting as a generator, to the line, as negative. This current if it passes through the motor armature would increase the accelerating torque to a value which would be very uncomfortable to passengers. It is therefore customary to connect resistances in parallel with the motor armature so that a voltage drop can be produced in the armature series resistance and retard the acceleration. This is very uneconomical and accomplishes the desired result imperfectly so that it is quite a common experience to ride in cars which give one the sensation of falling when descending heavily loaded.

The acceleration is considerably affected by the load and there is a considerable difference in the shape of the speed-time curves obtained with different loads. With full load up the car accelerates too slowly and with full load down the acceleration increases too rapidly for comfort. The rate of acceleration increases very rapidly when the motor is first connected to the line and when each step of resistance is short-circuited.

With variable voltage control it is possible to obtain more uniform accelerating and retarding characteristics under widely varying load conditions. The generator voltage builds up uniformly, following a fixed law and the car speed follows the generator voltage. The torque developed by the motor may be either positive or negative depending upon the load, but the speed in all cases depends upon the voltages generated by the machines.

If the car is to be stopped from high speed in a minimum of time, the maximum rate of retardation must be maintained during nearly the whole retarding period. When the controller handle is moved to the low-speed point resistance is connected in series and parallel with the generator field. The field shunt or discharge resistance can be adjusted if the generator field has the proper time constant, so as to give the car exactly the right rate of retardation. As the field dies down the generator voltage falls below that of the elevator motor and regenerative braking brings the car to a low speed. The generator voltage falls with absolute smoothness and there are no steps as in the case of rheostatic control. The application of dynamic braking in steps produces peaks in the rate of retardation that tend to cause the cables to slip so that the average rate of retardation must be kept well below the peaks. The absence of these peaks makes it possible to keep the average rate of retardation very high.

Electrical or regenerative braking is used to bring the car to a very low speed from which the final stop can be made with a friction brake. The transition from electrical to friction braking is perfectly smooth as a result of the characteristics of the electrical system and the method of controlling the friction brake.

The friction brake is a very important factor in making a smooth stop. It does not help matters much to have the retardation perfectly smooth if the final stop produced by the brake is rough or if the car slides a considerable distance after the brake sets. Electrical braking will reduce the speed of the car to a low value but except under certain load conditions will not bring it to a complete stop. It is, therefore, only necessary to apply the brake in such a manner that an abrupt change in the rate of retardation is not produced.

The distance that the car will slide when the friction brake sets is given by the formula:

$$d = (K_1 M_1 + K_2 M_2) \frac{V^2}{T_B + T_L} \qquad (1)$$

See appendix for symbols.

Equation 1 shows that the slide of the car when the brake sets varies as the square of the speed and inversely as the net torque exerted by the brake. The net torque developed by the brake is the algebraic sum of T_L and T_B. The torque of the load T_L assists the brake to stop when the load is positive or is being hoisted. It acts against the brake when the load is being lowered and increases the slide. The velocity of the car at the time when electrical braking becomes ineffective, however, has the greatest effect on the distance the car will slide through the brake since the slide varies as the square of the velocity. It is evident therefore that for minimum slide the car speed should be as low as possible before the friction brake is called upon to stop it.

To avoid a jar when the final stop is made there should be a smooth transition from the retardation produced by electrical braking to that developed by the friction brake. To accomplish this result it is necessary to apply the friction brake gradually so that full braking torque is not exerted instantly. The brake must be designed and adjusted so as to stop the car smoothly both when stopping from full speed and when stopping from creeping speed.

When stopping from full speed the brake should not set until the car has been slowed down to a low speed by electrical braking. When stopping from low speed the brake should set in a very short time after the control handle is thrown to the off position but the torque should build up gradually so as not to produce a jar. The most practical means of controlling the setting of a d-c. brake is to utilize the self induction of the brake coil in such a way that the flux is maintained in the magnet cores and retards or opposes the action of the brake spring in developing braking torque.

With rheostatic control in which the motor armature is disconnected from the line in stopping it is difficult to obtain good brake action and very careful adjustment must be maintained. If the time element of the brake is made long enough to make the action smooth with light loads the slide will be excessive with full load down and loads in the up direction may drop back several inches before the brake can take hold and bring the car to rest. With variable voltage control the armature

circuit is not opened immediately in stopping. When the controller handle is moved to the off position the generator shunt field is disconnected from the line and the generator voltage becomes practically zero. Since the line between the generator and motor armature is not broken, sufficient current will circulate to hold the car firmly under control while the brake sets. The control of the flux in the brake core is obtained with suitable contacts on the control panel and on the brake itself. After the brake has set and the car stopped, the contactor in the armature circuit is opened to eliminate circulating currents that might be produced by residual magnetism of the generator.

Tests were made on a gearless traction elevator rated 2000 lb. at 500 ft. per min., equipped with variable voltage control. The motor generator set consisted of an a-c. driving motor, a variable voltage generator, and a direct-connected exciter. The tests were made with the generator voltage adjusted to give a car speed of 550 ft. per min. with balanced load. The car was started by moving the master controller directly to the full-speed position in starting and stopped by moving directly to the off position. Fig. 4 shows the speed-time curves obtained on these tests. The curves were drawn by a graphic meter which recorded the voltage generated by a magnet driven from the shaft of the elevator motor.

Fig. 4A shows oscillograph records of armature current, generator voltage, and current in the standing field of the motor. The motor also has an extra or starting field which is connected to the line for starting. This increases the total field during the first part of the accelerating period, giving a high starting torque. During the last part of the accelerating period, the generator field is overexcited, in order to maintain a high rate of acceleration. The current in the standing field shows rapid changes in value, but due to its self inductance and to the mutual inductance between this field and the extra starting field the resultant change in flux is small and takes place quite slowly. Fig. 5 shows the currents in the two motor fields and Fig. 5A shows the resultant flux and motor torque. The action of these two fields is discussed more fully later.

The torque developed by the motor is proportional to the product of the armature current times the field flux. As shown by the curves the motor current and torque increase at a uniform rate during the first half of the accelerating period and decrease at a uniform rate during the last half. The rate of change of acceleration is proportional to the rate of change of motor torque and is determined by the tangent or slope of the curve. The speed time curves show that the rate of acceleration increases uniformly as the car starts and reaches a maximum at approximately one half speed. Above half speed the rate of acceleration decreases uniformly as the car comes up to speed. The time required to reach full speed is less than two and one half seconds and is practically the same for all loads.

The retardation curves have the same general characteristics as the acceleration curves. The time required to stop from full speed is very nearly the same for all loads. In practise this uniform retardation under all load conditions makes it possible for car operators to gage the stops very accurately in stopping from high speed and so reduce the number of false stops and time lost in making landings. A dotted portion of the retardation curves is added to show the effect upon retardation of increasing the total field by the addition of the extra field.

Fig. 6 shows the results of tests similar to those shown in Figs. 4 and 5 but made on a gearless traction elevator with rheostatic control. The curve of armature current shows notches as the steps of the starting resistance are short-circuited. These changes in the armature current correspond to changes in motor torque and in the rate of acceleration. The same notches are apparent as the dynamic braking resistance is short-circuited. These rapid changes in the rate of acceleration and retardation make it difficult to obtain smooth

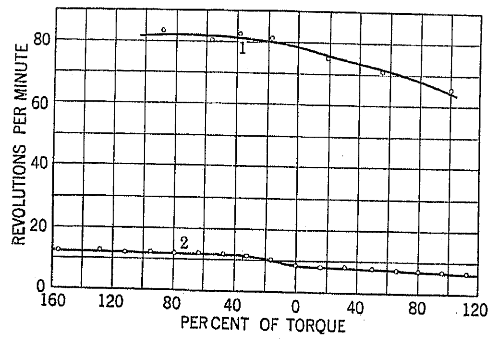


Fig. 7

Speed torque curves of gearless traction elevator motor at full speed and at low speed with variable voltage control.

and comfortable operation. The speed time curves are less uniform than those shown in Fig. 4 and show more variation in time for different loads.

Experience has proved that variable voltage control is smoother in operation than the older system and that passengers can be handled more comfortably and quickly.

An analysis of the test data shows the following reasons for these results:

1. The rate of acceleration increases uniformly as the car comes up to speed and is not subject to rapid changes such as are produced by increasing the armature current in steps.

2. The time required to reach full speed is reduced.

3. The accelerating time and retardation time is constant for all conditions of load.

2. *Regulation and Speed Control.* The speed at which elevator cars can be run depends very largely upon having a system of control that will give a positive low speed for making landings which does not fall off appreciably under load. Fig. 7 shows a test on an

equipment operating on reduced generator voltage. The speed is reduced to 1/10 of full speed at no-load and only falls slightly below this value when hoisting a load. With overhauling load the speed remains low. This flat regulation at speeds as low as 1/10 of full speed makes it possible for the operator to have complete control of the car under all loads.

In elevator work it is entirely practicable to operate the motor over this speed range, or greater, by control of the generator field. Fig. 3 shows the speed at which the generator fields can be changed. The generator field can be varied over such a wide range that very little field control on the motor is necessary. Motor field control,

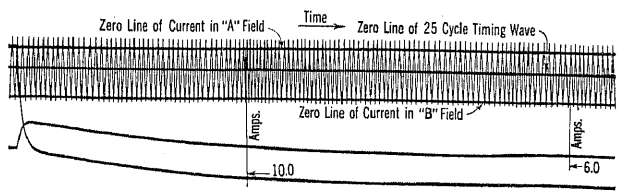


FIG. 8

Oscillograph records showing the rate of change of field currents of a low-speed elevator motor.

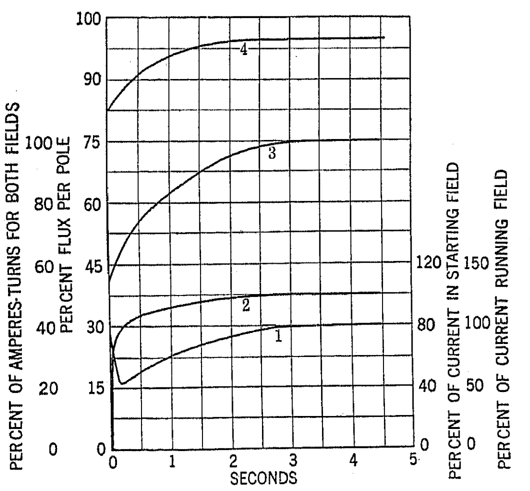


FIG. 9

Curves derived from the oscillograph tests shown in Fig. 8. The curves show the time required to change the motor speed by field control.

if used for any considerable speed range, requires a very large motor. With large low-speed machines it is not practicable to use a very great range of motor field control because of the slowness with which the field flux changes.

Fig. 8 shows oscillograph records of the field control of a gearless traction elevator motor. The motor has two field windings known as the running or B field and the starting or A field. With both fields energized the motor runs at 50 rev. per min. and with the starting field disconnected at 65 rev. per min. The curves in Fig. 8 shows the current in the running or B field with the motor at full speed and the current in the starting or A field when it is connected to the line to slow the motor down to 50 rev. per min. The current builds up quite rapidly in the A field but the current in the B field is reduced by the mutual inductance between the field coils.

Fig. 9 shows derived curves of the combined ampere turns of both field coils and the resultant field flux. Curve No. 4 shows that it requires approximately 3½ seconds for the field flux to reach its final value and

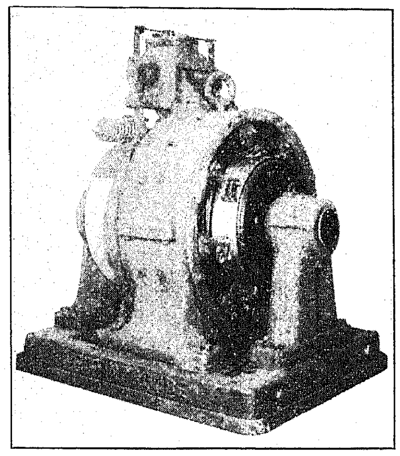


FIG. 10

Photograph of gearless traction elevator motor on which the tests shown in Fig. 8 were taken.

change the motor speed from 65 to 50 rev. per min. In practise an elevator must be retarded and stopped in from 1½ to 2 seconds so it is quite obvious that the full-speed range of this machine cannot be utilized. It is doubtful if a speed range greater than 10 or 15 per cent is practicable for this class of elevator motor. A machine of this type is illustrated in Fig. 10.

Armature series and shunt resistance gives inherently

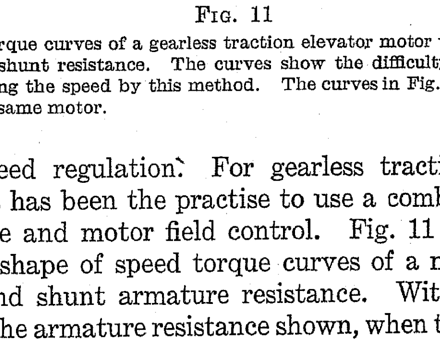


FIG. 11

Speed-torque curves of a gearless traction elevator motor with armature series and shunt resistance. The curves show the difficulty encountered in controlling the speed by this method. The curves in Fig. 7 and Fig. 11 are for the same motor.

poor speed regulation. For gearless traction equipments it has been the practise to use a combination of armature and motor field control. Fig. 11 shows the general shape of speed torque curves of a motor with series and shunt armature resistance. With a motor having the armature resistance shown, when the no-load

speed is reduced to 25 per cent of the full-speed value the motor torque is reduced to 70 per cent at stand still. This means that the motor will not have sufficient torque to hoist full load on this controller point, except by some automatic adjustment of the armature resistance. Considerable complication has been introduced in the control to make it possible to lift full load to the top floor. With overhauling loads it is difficult to make the speed low enough to obtain accurate stops. At best the speed range is quite limited and large currents are drawn from the line when operating on the low-speed points.

The method of speed control by varying the generator voltage shows considerably better speed regulation and a greater range than the other two methods. This is clearly shown by comparing the results shown in Fig. 7 and Fig. 11. In practise this better control results in the following advantages for the variable voltage system.

1. It is much easier for the operator to land accurately at the floors and the number of false stops are reduced.

2. The car may be "inched" to the floor level very accurately and quickly in case the operator does not stop accurately the first time.

3. The limit stops can be made in a minimum distance and without loss of time or over travel with all loads.

4. Elevators may be run at a higher speed than is possible with rheostatic control.

3. *Efficiency and Power Consumption.* The factors that affect the power consumption of an elevator are so many and varied that it is extremely difficult to predict what it will be for a new installation. The more important factors are as follows:

(a) Load and speed.

(b) Number of stops and starts per mile of travel.

(c) The number of miles per hour that the car makes while in service.

(d) The weight of the moving masses that must be accelerated at each start.

(e) Method of operation.

A large number of tests have been made in which the cars are operated on a fixed schedule with different loads and the power consumption measured. These tests are useful in making comparisons between different equipments but do not give accurate information for determining the power consumption in actual service where the cycle of operation may be quite different. Different operators handle the cars in different ways and this will have an effect on the power consumption. In actual operation considerable power and time may be wasted in the following ways:

(a) Acceleration and retardation for every start and stop.

(b) Inching the car to correct for inaccurate stops at the landings.

(c) Running at reduced speed when the cars are ahead of their schedule. (With rheostatic control).

(d) Making the limit stops.

(e) Long idle periods in which the standby loss in the motor field and motor generator sets uses up power. (With variable voltage control).

The power required to drive the elevator at full speed is usually considerably less than that used up in the items enumerated above. The distribution of the power losses will depend very greatly upon the system of control used.

Fig. 12 shows a typical cycle for a high-speed elevator with variable voltage control operating with the average

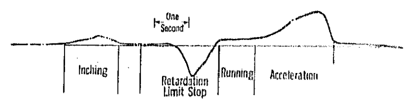


FIG. 12

Power cycle of high-speed elevator with variable voltage control tested with balanced load. The chart was made by a graphic wattmeter.

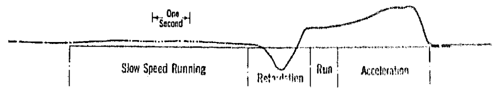


FIG. 13

Power cycle of high-speed elevator with variable voltage control tested with load. The chart was made by a graphic wattmeter.

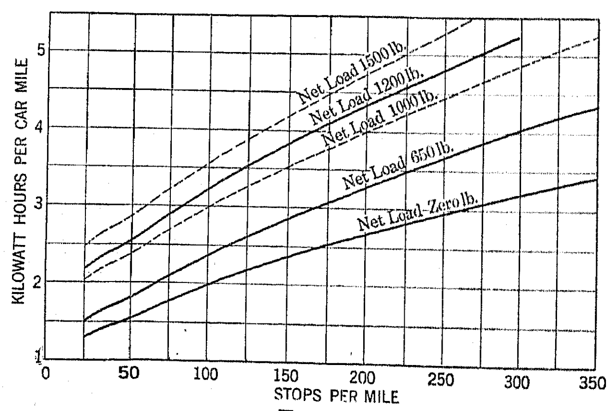


FIG. 14

Energy consumption tests of gearless traction elevator with variable voltage control. Duty 2000 pounds at 500 feet per minute.

or balanced load. The power input is greatest during the accelerating period, and falls off to a very low value as the car comes up to speed. During retardation the power becomes negative and is returned to the line. The effect of inching the car is shown directly following the retarding period. The maximum is only 25 per cent of that required during acceleration. This does not mean that the elevator motor does not develop sufficient torque to move the car promptly. On the contrary the current flowing in the generator and elevator motor circuit is high enough to develop full-load torque or greater but since the generator voltage is quite low the actual power input is also low. Fig. 13 shows a similar cycle but with a loaded car and shows a

period of low-speed running. In this case the power at low speed is ⅓ of that required to run at full speed.

A series of tests were made on a gearless traction elevator having a duty of 2000 pounds at 500 feet per minute and equipped with variable voltage control. Fig. 14 shows the energy consumption in kilowatt hours per car mile plotted against stops per car mile. With balanced car and 125 stops per mile the energy consumption is 2.2 kilowatt hours per mile. The energy consumption shown includes all the power taken from the line.

Fig. 15 shows a test made on an elevator having a capacity of 2500 pounds at 600 feet per minute with rheostatic control. Fig. 16 shows a similar test on the same motor with a smaller sheave and variable voltage control operating at 550 feet per minute. With balanced load and 50 stops per mile the energy consumption is 2.2 kilowatt hours for both equipments. At 150 stops per mile the energy consumption is 3.6 kilowatt hours per mile for the variable voltage equipment and 5.2 kilowatt hours for the rheostatic. While the two

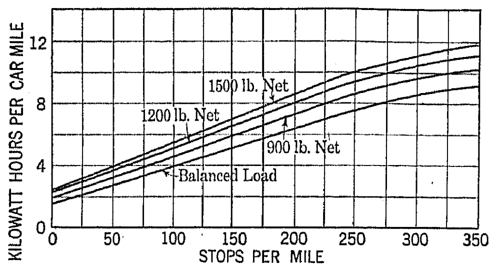


Fig. 15

Energy consumption tests of gearless traction elevator with rheostatic control. Duty 2500 pounds at 600 feet per minute.

motors are not working under exactly the same conditions the slope of the curves gives a fairly good comparison between the two systems of control.

From an analysis of the load curves and the test data we can draw the following conclusions:

1. The variable voltage system requires less power for acceleration and retardation because of the absence of rheostatic losses and because of the power returned to the line during retardation.

2. Very little power is taken from the line to make small movements of the car due to the low generator voltage.

3. Low-speed running does not increase the power consumption.

4. Power is returned to the line while making the limit stops.

5. The greatest gain in economy will be shown where the number of starts and stops is large.

6. Long rest periods will cause waste of power if the motor generator sets are left running. This waste can be eliminated by shutting down the motor generator set through a start and stop push button station which may be located at a point accessible to the starter or mounted in the car.

IV. Safety Features

Elevators have a number of devices which protect the passengers riding in the car. I shall discuss only those that are affected by the control system.

1. *Limit Stops.* An elevator control system should be so designed that the car will be slowed down and finally stopped as it approaches the terminal landings. In order to be sure that the car platform will come flush with the top and bottom landings under all conditions of load, the stop contact on the slow-down device must not open until the car has reached the floor level. It is preferable that the point of cut-off be a few inches beyond the floor level so that the operator will not form the habit of depending entirely upon the automatic device for making the stop. It is necessary then, when the car reaches the cut-off point, that it be slowed down to a low enough speed so as not to drift further than the distance provided for over travel. If the car runs by the over-travel limit switch it is not possible, with the usual connections, to back out and the assistance of an attendant is necessary to put the car into service again. To stop a fully loaded car within the usual over-travel distance has always been a considerable problem with rheostatic control because of the extremely poor regulation of the motor when operating at reduced speed with armature series and shunt resistance. The characteristics of the machine operating under these conditions have already been discussed. Refer to Fig. 11.

A switch is mounted on the car having contacts actuated by an arm and roller which engages a cam in the elevator shaft. As the car approaches the top and bottom landings these contacts operate to insert steps of resistance in the generator field, in the variable voltage system, which reduces the voltage. The car speed reduces with the reduction of voltage due to the regenerative braking that takes place until, as the landing is approached, the car is running at very low speed.

The inherent regulation of the variable voltage system makes it possible to slow down a fully loaded car in almost the same distance as an empty car. It is possible to stop the car at the terminal landings under all conditions of load within one inch of the same spot. The positive action of the automatic slowdown at the limits of travel greatly increases the safety of operation. The automatic slowdown and stop at the terminal landings is one of the elements in the system of safety devices provided for the elevator and it is important that it function properly.

2. *Emergency Stop.* When making normal stops with the regular controlling means such as the car master switch or the automatic limit stops, the generator field is reduced to zero which sets up regenerative braking in the elevator motor which slows it down. At the same time the friction brake sets which brings the car to rest. To provide against failure of the master switch contacts to function or a failure of the regular stopping cycle the car is equipped with the usual

emergency switch. This switch opens the armature circuit contactor which disconnects the elevator motor entirely from the generator and connects it to a low-resistance dynamic braking circuit. The coils to the brake contactors are opened and the brake circuit itself is also opened by auxiliary contacts on the armature contactor. This arrangement gives two entirely independent means of stopping the car and is an advantage over the older forms of control in which the emergency switch forms only an additional means of opening the same contactors that disconnect the motor in regular operation.

3. *Failure of Power*. The failure of the main power supply on a large system is a comparatively rare occurrence, but failure of power from some cause to elevator motors has been a rather frequent source of trouble. Failure of power to an elevator may be due to (1) failure of the main power supply to the building, (2) the opening of the main circuit breaker in the building, (3) opening of the circuit breaker or blowing a fuse in the line feeding the elevator. I shall analyze the result of a power failure under several conditions and discuss means for taking care of the condition.

When the motor is operating under positive load, that is when hoisting an unbalanced load, there is no particular danger from power failure as the car will stop of its own accord in a comparatively short distance. The condition of balanced or overhauling load presents a more serious problem. Let us consider first the case of overhauling load produced by a fully loaded car going down. Consider first the case of a d-c. motor controlled by armature series and shunt resistance and motor field control. With overhauling load the motor is operating as a generator and returning power to the line and is kept at constant speed by the supply voltage. The contactor magnets on the controller are energized from the supply voltage. If now the power source is cut off by an opening of the circuit breaker in the line to the motor, the reservoir into which the motor has been delivering its power will have been disconnected and the motor and car will overspeed. There will be no warning to the operator that the power has failed until the car has reached a considerable overspeed, probably high enough to trip the safety clamps under the car. The control magnets will be kept energized by the voltage generated in the armature of the elevator motor so that the dynamic braking circuits will not be set up or the friction brake disconnected until the overspeed governor driven by the car has opened its contacts. In other words, there is nothing to start the functioning of the control to stop the car until there has been an actual overspeeding of the car. After the control has functioned to stop the car the friction brake will be called upon to make most of the stop since the electrical braking will be rendered more or less ineffective due to the dying field of the elevator motor. Control schemes have been developed to connect the field of the motor to the armature so as to maintain the excitation. In some cases the motors have been equipped with special fields for this purpose. Such schemes are only partially successful and the principal difficulty is to initiate the functioning of the control before the car has reached considerable overspeed. In special cases the control circuits have been connected through auxiliary contacts on the circuit breaker.

Alternating-current squirrel-cage elevator motors that use a separate winding which must be excited from the supply line have no means of obtaining regenerative braking when power fails and the friction brake alone must stop the car.

Consider next the case of an elevator motor driven through an motor generator set with an a-c. driving motor and having either one or two generators. The motor generator set comprises a small direct-connected, self-excited generator to furnish d-c. excitation to the generator and motor fields, brake and control circuits. In case of overhauling load the elevator motor is acting as a generator, the generator end of the motor generator set is acting as a motor, the motor of the set acts as an induction generator and is running above synchronous speed. It should be kept in mind that the elevator motor is generating a voltage higher than its terminal voltage due to the drop in the windings. For the same reason the generator, acting now as a motor, is generating a counter voltage lower than its terminal voltage by the value of its internal drops.

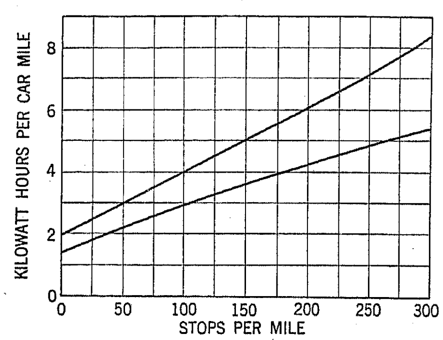


FIG. 16

Energy consumption tests of gearless traction elevator with variable voltage control. Duty 2500 pounds at 550 feet per minute.

When the circuit breaker opens the energy that is being developed by the descending car is expended in accelerating the motor generator set which speeds up until the generator voltage equals the voltage generated by the elevator motor, the internal drops reducing to a very low value. The result will be an overspeed on the set of 15 or 20 per cent above synchronous speed. The motor generator set is equipped with an overspeed device which opens a contact in the safety circuit of the elevator controller that sets the friction brake and sets up the emergency dynamic braking circuit. The motor generator set is now running free at overspeed and due to its inertia runs for several seconds before the direct-connected exciter

loses its voltage. The field of the elevator motor is connected directly to the exciter so that a strong field is maintained for dynamic braking long enough to bring the car completely to rest before the exciter loses its voltage. The same cycle is performed when the driving motor of the set is a d-c. machine. The only difference is in the connections of the field of the elevator motor which are such that the counter e. m. f. of the driving motor maintains the excitation on the elevator motor field. Tests have shown that the car is stopped promptly and without over travel when a power failure occurs while making the limit stops.

4. *Overspeed.* The cars are equipped with the usual overspeed governor which performs the usual functions of first slowing down the car by field control, opening the safety circuit and finally setting the wedge clamp safety under the car. The overspeed contact on the motor generator set acts as an additional factor of safety as it opens the safety circuit independently of the speed governor. It can be set to open its contacts before the governor trips and so stop the car entirely by means of the electrical and friction brakes, saving the inconvenience and loss of time in releasing the wedge clamp and resetting the governor before the car can be moved to the nearest landing.

IV. Maintenance

The upkeep of equipment is one of the factors that go to make up the cost of operating electric elevators. Large buildings usually employ someone who is competent to care for and maintain the elevator equipments. Such buildings usually have excellent elevator service and experience little trouble from shutdowns and delays. At the other extreme we find buildings in which no regular help is employed for this purpose and elevator maintenance consists of periodic inspection for which service contracts are often let.

Maintenance of the electrical equipment consists mainly of inspection, cleaning, making adjustments, oiling and installing renewal parts. The first three items involve labor costs only, while the last two items involve both labor and material. It should be the desire of every building owner to purchase equipment on which he can reduce the last three of these items to a minimum.

The art of designing rotating machinery has advanced to a point where maintenance is merely a matter of routine. Elevator machinery is usually installed in comparatively clean places so that insulation trouble is comparatively rare. The elevator cycle is such that machines which are designed to have the proper operating characteristics seldom reach, in actual service, the maximum temperature rise which they are guaranteed to stand. The use of commutating poles has made it possible to design high-speed d-c. machines that will commutate heavy-current peaks with practically no sparking. The present accepted standard of commutation is that the commutators shall improve their appearance, in actual operation. Bearings, oiling systems and general mechanical design have been developed to a high degree of perfection.

Elevator controllers have in recent years reached a high state of development but there is one inheren condition that has made the maintenance problem more difficult. I refer now to the rupturing of heavy currents in starting and stopping the motor and in performing other control functions such as changing speeds. A contactor in elevator service is rupturing heavy currents continuously and the arc produced when the circuit is opened is bound to burn away the contacts. Considerable improvement has been made in recent years in the method of rupturing arcs, such as the use of arc splitters, and improvements in the design of the magnetic blowout. The use of rolling contacts has considerably improved the life of the contacts themselves.

It is quite probable that these details are now pretty well standardized and that little improvement can be expected along this line. The next step is to eliminate the rupturing of major currents altogether and to control them indirectly. This is accomplished by switching only the field current of the generator, which, as its voltage changes, controls the elevator motor. The equipment can be so designed that the maximum current ruptured by the controller is not more than five amperes. From the standpoint of maintenance this results not only in an increased life of the contacts themselves but in reduced cost of renewal parts, since the contactors are much smaller in size than if they had to handle the motor current directly. The elimination of heavy arcs reduces the burning of other parts and the chances of damaging flashovers. The acceleration and retardation is governed by the characteristics of the generator field and does not depend to a great extent on adjustment of relays and contactors so that the controller requires very little attention to keep it in good operating condition.

Another item that has an effect upon the maintenance of elevators is the wear and tear on machinery incidental to severe service. The life of the cables and the wear on gears are both affected by the control equipment. High-speed geared elevators develop backlash in the gears and this is aggravated by the notching of the ordinary controller. Sharp peaks in the retardation torque have a tendency to cause slippage of the cables and reduce their life. Fig. 5 shows the smoothness of acceleration and retardation and the absence of abrupt changes in motor torque. This smoothness will result in longer life and less maintenance cost of the mechanical equipment.

V. Conclusion

In the foregoing part of this paper I have attempted to describe the variable voltage system of control as applied to electric elevators and to set forth the characteristics and operation obtained. In conclusion

I shall summarize its principal advantages for elevator work as compared with older systems.

1. Elevators running at the highest speeds may be driven from a-c. supply lines of any commercial voltage and frequency.

2. The acceleration and retardation are perfectly smooth and uniform.

3. A high rate of acceleration and retardation is obtained which permits the use of high car speeds and increased elevator capacities.

4. Positive speed control under all conditions of load makes it easy for operators to land the car accurately without loss of time.

5. High economy of power is obtained.

6. A number of inherent safety features make it particularly suitable for high car speeds.

7. Maintenance costs are reduced to a minimum and the reliability of service increased because of the simplicity and flexibility of the system and because no large currents are ruptured in operation.

Appendix

Symbols used in equation (1).

M_1 = Moment of car counter weights and other moving parts.

M_2 = Moment of motor armature.

V = Velocity of the car.

T_B = Torque developed by the friction brake.

T_L = Torque developed by the load.

K_1, K_2 = Constant.

Discussion

K. L. Hansen: Because of the fact that the inherent speed-torque characteristic of the induction motor makes it less suitable than the direct-current motor to applications where masses of considerable inertia have to be accelerated and retarded at frequent intervals, the application of alternating-current power to certain classes of elevators presented a formidable problem. It appears that the system of control described in Mr. Bouton's interesting paper was first thought of in connection with elevators as a solution to this problem.

However, it is evident from the paper that, even when considered purely from the standpoint of control, the system must have shown some very desirable characteristics in operation, as it has frequently been extended to cases where conversion from alternating to direct current is not the object, that is to elevators on d-c. power lines.

It has long been recognized, but has of late become much more forcibly impressed upon us, that in motor applications where the starts and stops are frequent, control of acceleration by means of resistors in the armature circuit is extremely wasteful of energy, and there is consequently a general tendency to devise more economical methods of control in such cases. Application of the system described in the paper to elevators where d-c. power is available is obviously in line with the general trend towards greater economy.

Another general trend in the evolution of control apparatus is to control transient conditions, such as acceleration and retardation of motors by continuous electromagnetic changes inherent in the machines, rather than by mechanical changes, such as switching operations in the main circuits. In this respect, the system described appears to be ideal, the switching operations of the main circuit having been reduced to a minimum. The advantages of having the acceleration controlled by a continuous change in the generator field flux are also clearly set forth in the paper as being smoothness of operation and reduced maintenance cost.

However, while the system is thus seen to possess some very marked desirable features, there are some drawbacks. The total machine capacity required is more than three times that required for the elevators themselves. This, of course, refers to capacity and not to physical dimensions, as the motor-generator set is undoubtedly relatively much smaller than the elevator motor, being operated at much higher speed. But even at best, the necessary additional machine capacity is undeniably a serious drawback. Furthermore, the losses in the additional machines, especially the standby losses, offset to a considerable extent the efficiency gained during acceleration and retardation.

It, therefore, becomes of interest to determine whether or not it is possible to devise a system which will materially mitigate these drawbacks and at the same time retain the most desirable features. I believe this is possible, and will briefly describe a proposed system and submit it for comments and criticism. The system is shown as applied to direct current only, the conversion being accomplished by other means when the supply is alternating current. It will first be described in combination with a three-wire supply line as the connections are then simplified somewhat.

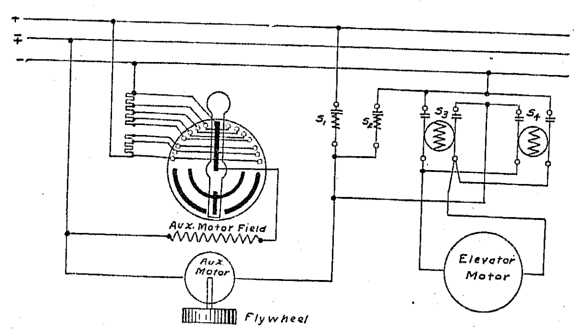


Fig. 1

For the sake of simplicity the diagrams show only the distinguishing features, omitting those common to other systems, such as the main disconnecting switches, the circuits energizing the magnet switches, the field circuit of the main motor, etc.

Referring to Fig. 1, it will be seen that the system employs an auxiliary motor, with a flywheel to increase the inertia of its revolving parts. In the standstill position switches S_1, S_3 and S_4 are open and switch S_2 is closed. The auxiliary motor is running at normal speed between neutral and negative line. The auxiliary motor starter, being of the usual type, has been omitted from the diagram.

To start the elevator S_2 is open and S_3 or S_4 (according to whether the motion is up or down) is closed, thus inserting the main motor in series with the auxiliary motor between neutral and negative line. Continued movement of the controller handle weakens the auxiliary motor field, and as the inertia prevents its armature from speeding up rapidly, sufficient current flows to accelerate the main motor armature.

When the auxiliary motor field has been reduced to zero the main motor is running between neutral and negative line at approximately half voltage and speed. Up to this point the auxiliary motor has absorbed energy, its e. m. f. being in opposition to the line voltage and its speed has therefore increased somewhat. Further movement of the controller handle reverses the auxiliary motor field and gradually strengthens it. The auxiliary motor now acts as generator, and by reducing its speed, gives up the energy which was previously stored. Its induced

e. m. f. is now in the same direction as the line voltage and therefore boosting this, so that the voltage impressed on the main motor continues to increase.

When the auxiliary motor field reaches full strength in the opposite direction its speed will have been reduced to approximately normal and its voltage added to the line voltage is approximately equal to the voltage between the outside main lines. The voltage of the main motor, being of approximate equality to the main line voltage, switch S_1 can be closed, making each machine run independently of the other, the main motor between the outside lines and the auxiliary motor between neutral and positive line. The switch S_1 is so connected and adjusted that it closes only when the controller handle is in running position and the main motor voltage bears a certain ratio to the line voltage.

To stop the elevator the operations are reversed by moving the controller to standstill position. During retardation the auxiliary motor voltage adjusts itself so that a regenerative current flows and a breaking torque is produced practically down to standstill.

It is obvious that the rate of acceleration and retardation follows a fixed law, precisely as in the system described by Mr. Bouton, if the controller is moved at once to the full running position in starting and to the off position in stopping. Equally, the rate of acceleration may be regulated by varying the resistance of a damper winding on the auxiliary motor field. Furthermore, the main current adjusts itself to retain approximately this rate under widely varying load conditions from large positive to negative or overhauling loads.

The acceleration and retardation curves and the speed-time curves when running at low speed can all be determined by mathematical analysis, but cannot be included in this brief discussion.

The current rating of the auxiliary motor depends on the relative amount of time consumed in acceleration and retardation and full speed running. At most it is equal to the current rating of the main motor, and as it is wound for one-half voltage its capacity is at most one-half that of the main motor. In case of d-c. power line the additional machine capacity is therefore less than one-fourth of that required with the control described in the paper. The losses, especially the standby losses, are, of course, correspondingly reduced.

Even when the supply is alternating current, the additional machine capacity and the losses can be materially reduced by using the auxiliary motor control and a synchronous converter for conversion from a-c. to d-c. current.

Fig. 2 shows the principal connections when the supply is a two-wire d-c. power line. In this case the main motor has two armature windings and two commutators connected in series during acceleration and retardation and in parallel when running at full speed. The switches S_1 and S_2 are double-pole. S_3' and S_4' always open and close simultaneously with switches S_3 and S_4 respectively. With these modifications the sequence of switching operations is the same as before and the operation is essentially the same as in the case of the three-wire system already discussed.

E. M. Claytor: Referring to Fig. 5A in Mr. Bouton's paper, it may appear that the maximum rate of acceleration would occur earlier than it actually does because the peak torque is reached in ¾ second after the start, while the maximum rate of acceleration does not come until one second after zero time. The rate of acceleration is proportional to the net torque available for accelerating the total equivalent mass having linear velocity. The curve in Fig. 5A is somewhat misleading in as much as it is labeled "Torque of rope sheave" which might be taken to mean net torque on the ropes for acceleration. As a matter of fact this curve is the total torque at 1 ft. radius (developed by the motor armature) divided by the radius of the driving sheave.

The net torque is what is left of the torque curve in Fig. 5A after the torques due to the brake, bearing friction, bending of cables, and sliding friction have been subtracted. This difference is represented by curve 2 in Fig. 3 shown herewith, and it is the true net torque used only for acceleration. The peak of this curve comes at the same time at which the maximum rate of acceleration occurs in Mr. Bouton's Fig. 4 for balanced load, *i. .,e* where the slope of that curve is steepest. Curve 1 shows the car speed as taken from balanced load curve in Fig. 4. Assuming that this acceleration curve is correct, we derive Curve 2, which is the net accelerating torque, in the following way.

Pick out some point, say at 1½ seconds, where we may assume that the brake and static friction have reduced to zero, and draw a tangent as accurately as possible to the acceleration Curve 1.

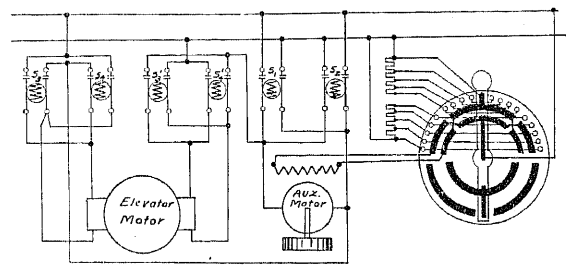


Fig. 2

Next take the total motor torque at 1½ seconds and from it subtract the running friction torque of 460 lb. as obtained from a total torque curve (not shown) at some point corresponding to 3 or 4 seconds. The difference will be 2200 lb. which is the net torque for the point on acceleration curve corresponding to 1½ seconds.

We now have the rate of acceleration, and the torque which produces that rate. The unknown quantity is the equivalent mass having linear velocity. This we obtain from the simple formula—

$$M = \frac{T \times 32.2}{a \times R}$$

Where, M = lb. mass having linear velocity.
T = torque in lb. at 1 ft. radius

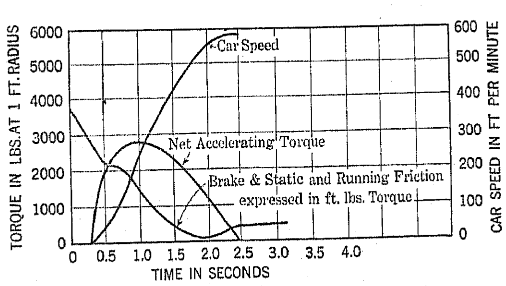


Fig. 3

R = radius of driving sheave in ft.
a = rate of acceleration in ft. per sec. per sec.
and 32.2 = gravity acceleration constant.

The equivalent mass in this case was 11,350 lb.

After determining the mass, we rearrange the same formula above as follows—

$$T = \frac{M \times a \times R}{32.2}$$

By substituting known values of M, by previous calculation, R, by measurement, and a, by drawn tangents to the graphic test curve, we can solve for T which is the net torque Curve 2 in the sketch.

Having determined that component of the total torque curve which is used only for accelerating the balanced car, we subtract that from the total torque curve and we have Curve 3 left. This latter curve represents all friction or anti-torque. We know from other oscillograph tests that the brake reduces its torque to zero in 0.6 second. This means that the static friction is about 2000 ft-lb. at the start and it does not reduce to zero until a point corresponding to $1\frac{1}{2}$ seconds. This indicates that the driving sheave and shaft have made more than a complete revolution before the static friction becomes zero.

The dip in the friction Curve 3 cannot be explained except by considering that oscillations or hunting occurs between accelerating torque and friction torque, or else the graphic meter had enough inertia in moving parts to overshoot the actual speed.

From the above analysis, we may draw the conclusion that

(a) An appreciable amount of power is wasted in friction during acceleration.

(b) A quick-acting brake on the pick-up is desirable.

(c) Anything done to reduce static friction will reduce the current and power peaks of the elevator motor.

W. F. Eames: In an effort to obtain what might be considered a perfect acceleration curve for an elevator, a number of curves have been analyzed, and although the results contain considerable speculation they are of interest in connection with the subject just discussed. It will be necessary first to define perfect acceleration and then to analyze the elements. Any remarks made in connection with acceleration apply in general equally well to retardation, as retardation may be considered as negative acceleration. Consequently we will consider only acceleration, and treat retardation as a special case.

Perfect acceleration as applied to elevators might be defined as that which will bring an elevator up to speed in a given time without discomfort to the passengers. Assuming that the speed of an elevator can be made to follow any curve in going from zero speed to full speed, some method must be used to select the best one. The problem of comfort to a passenger seems to be tied up psychologically in some measure with the idea of falling. That is, if the motion of the car is such as to suggest falling, a feeling of discomfort is produced. Falling is concerned with a high rate of acceleration especially a high rate suddenly applied, which corresponds to a high rate of change of acceleration. The continuous curve that represents values between zero and a maximum has necessarily an S shape and most of them can be represented by some combination of the powers of X in the general equation y = function of x.

Several curves have been plotted from equations of the form

Velocity $= K_1 t^2$ (Parabolic)

" $= K_1 t^2 - K_2 t^3$ (cubic)

" $= K_1 t^2 - K_2 t^4 + K_3 t^6$ (cosine)

and were all found to lie very close together, and to the eye showed no points where noticeable difference will be felt if an elevator were accelerated along any of them. Plotting the first and second derivaties the parabolic curve showed the highest accelerating rate and the lowest rate of change of acceleration. The cubic equation gave the opposite conditions and the cosine lay between.

Various observations indicate that the high rate of change of acceleration is more objectionable than a high rate of acceleration. If this is true the parabolic form is the most desirable of the three functions. It was also found that the variable-voltage elevator acceleration lies very close to the parabolic form at balanced car conditions and when lifting full load departs only slightly from it. Fig. 4A herewith shows the velocity curve for the variable voltage elevator rated at 2000 pounds at 550 feet per minute that has just been discussed by Mr. Bouton. The dotted curve shown in the same figure is a parabolic curve, that passes through the zero, mid-point, and full speed values of the velocity curve. Figs. 4B and 4C. shows the first and second derivatives, or the acceleration and rate of change of acceleration. The maximum acceleration is (7.5 ft./sec.)^2 and the maximum rate of change of acceleration is (6.4 feet/sec.)^3. As the operation of the car is very comfortable with the conditions shown, higher accelerations can be used than these shown and it is probable that with the variable-voltage system of control that a car can be accelerated to 650 to 700 ft./min. in $2\frac{1}{2}$ seconds without discomfort to the passengers. In this connection, however, it should be mentioned that as the rates of acceleration are increased other limiting factors appear in addition to the discomfort felt by the passengers. For example, slipping of the cables on the driving sheave occurs if the retardation rate is too great. This decreases the life of the cables. Also larger equipment is required when these high rates are used, than is required with the lower ones. This point becomes important especially when the elevator cab and other parts have a large inertia.

The above considerations apply equally well to retardation of the elevator. However, the rates used in retardation can be somewhat higher than those used in acceleration without causing discomfort. The reason for this is probably that a greater sense of security is felt because the car is slowing down. Although no extended experiments have been tried it has been found that, if a person shuts his eyes, much less discomfort is felt in an elevator using high rates of acceleration. If maximum rates are to be demanded in the future, it may become necessary to take steps

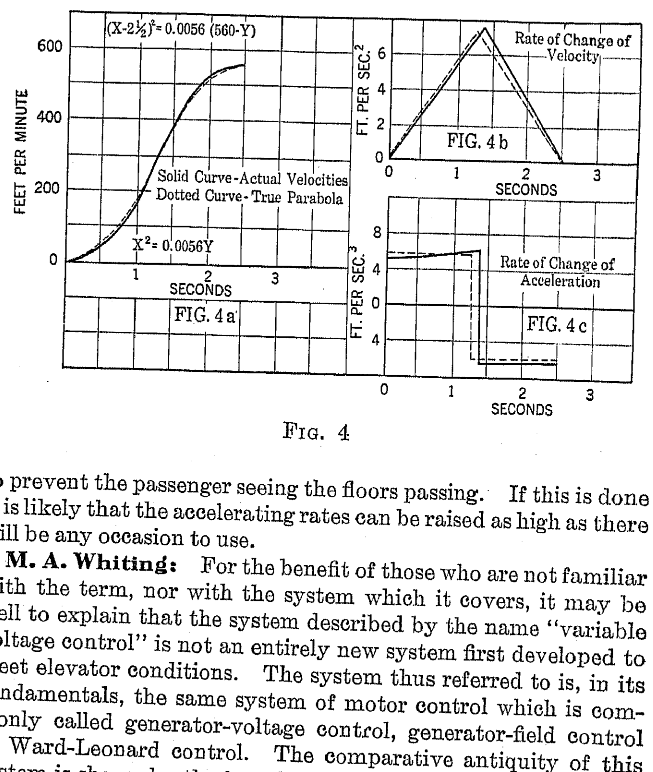


Fig. 4

to prevent the passenger seeing the floors passing. If this is done it is likely that the accelerating rates can be raised as high as there will be any occasion to use.

M. A. Whiting: For the benefit of those who are not familiar with the term, nor with the system which it covers, it may be well to explain that the system described by the name "variable voltage control" is not an entirely new system first developed to meet elevator conditions. The system thus referred to is, in its fundamentals, the same system of motor control which is commonly called generator-voltage control, generator-field control or Ward-Leonard control. The comparative antiquity of this system is shown by the fact that it helped sink the Spanish fleet off Santiago de Cuba in 1898. Merely as an indication of the extent to which this system is now used it may be mentioned that, over a period of 16 years, one manufacturer has built, or is now building, for steel-mill and mine-hoisting service alone, over 70 equipments totaling over 110,000 h. p. The corresponding totals for the other manufacturers covering these items and a complete list of other classes of service to which this system has been applied would also be impressive.

The new development presented in Mr. Bouton's paper is, therefore, not the fundamental system of "variable-voltage" or generator-voltage control itself but is rather the extensive application of this system to elevators. The problem, or the accomplishment, should not be belittled, however. Some of the previous practise in the application of generator voltage control

could be followed but some special problems were presented by the requirements of elevator service, and it has, therefore, been necessary to devise new practises.

One of the principal problems is that of speed regulation over the range of loads handled, which is discussed by the author under "Requirements of Elevator Service" and under "Characteristics and Operation." Attention is called particularly to the author's Fig. 7. The speed regulation shown on the low-speed controller point is much closer than is obtainable with any practicable rheostatic machine and provides a correspondingly

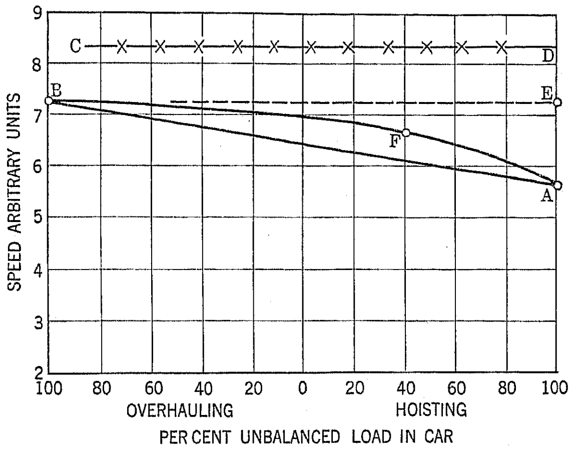


Fig. 5

greater ease of control in making landings. However, let us examine the author's Fig. 7 more closely. As the results are given in per cent of motor torque, an interpretation is required. It is reasonable to assume that 100 per cent positive torque corresponds to the hoisting of an elevator load not exceeding normal capacity; then when lowering the same elevator load, the motor torque will be not less than 60 per cent negative. The speed at 100 per cent load is about 6½ rev. per min. and at 60 per cent negative load is about 11½ rev. per min., or about a 75 per cent rise in speed from full load "up," to the corresponding load condition "down." If this speed regulation curve could be made flat at all loads, this would be found very useful in attaining a still greater ease of control for making landings.

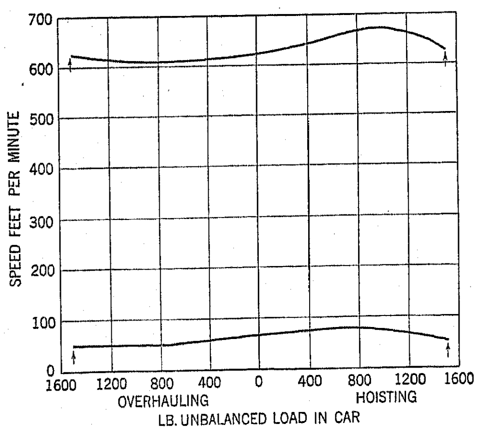


Fig. 6

On the full-speed point, the regulation shown by the author's Fig. 7 is from 64 rev. per min. at 100 per cent load to 82 rev. per min. at 60 per cent overhauling load, or a rise of 28 per cent. Let us consider further just what such a speed regulation at full speed means in the operation of an elevator.

Fig. 5 herewith shows a speed regulation curve for a hypothetical gearless elevator plotted to per cent unbalanced load in the car. The speed regulation (28 per cent rise from full load hoisting to full load overhauling) is assumed to be the same as that in the author's Fig. 7. The speed curve may be considered to be bow-shaped, as in the author's Fig. 7, but I believe that a nearly straight inclined line from *A* to *B* is equally typical, particularly if an extra contact on the speed governor is not used to operate within this speed range. Line *C-D* represents the speed at which the overspeed governor will set the safety clamps. It is commonly considered that the margin of speed, *B-C*, between maximum overhauling speed and the final emergency operation of the governor, should be not less than 15 per cent in terms of speed *B*. Let us assume also that the elevator rides comfortably and can be controlled without difficulty when overhauling at speed *B*. Now if a speed as high as *B* is suitable in all respects when thus lowering maximum load, a speed as high as *B* is also equally suitable when operating at balanced load or when hoisting full load; in other words, if speed *B* is right, a speed curve *B-E* is right.

The advantage of a flat speed-regulation curve, as *B-E*, will be greatest on express elevators during the rush hour going in. If the load goes up at speed *A*, and the empty car comes down (against the overbalance of the counterweight) at speed *F*, the average speed up and down will be only 85 per cent of the permissible speed *B*. Allowing about 30 per cent standing time and 70 per cent running time for rush-hour express operation, the loss of service per elevator due to the poorer regulation, compared with perfect regulation, is about 11 per cent.

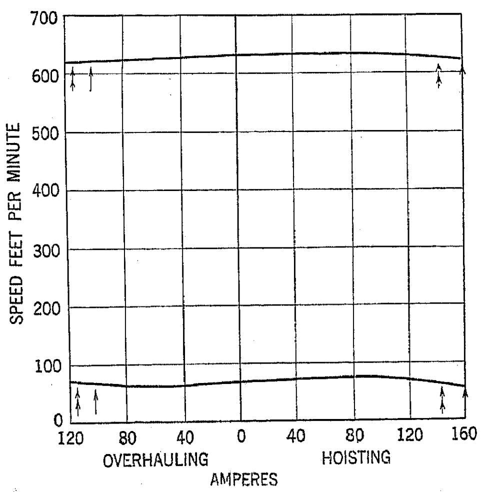


Fig. 7

In order to provide the maximum service possible from an equipment of a specified maximum overhauling speed, and also in order to obtain further improvements over previous equipments in ease of handling the elevator, an improvement in generator-voltage control has been developed by which regulations are obtained which approach closely the ideal which I have just described. I intend to prepare a technical paper in the near future covering this development, but some of the results are presented herewith.

Fig. 6 herewith shows results obtained on the first commercial installation embodying this development, the elevator being rated 600 ft. per min., 2500 lb. live load and counterbalanced for 40 per cent of the live load (maximum unbalance 1500 lb.). On the first or lowest speed point, the speeds at maximum load hoisting and maximum load overhauling are equal at 50 ft. per min., and the prevailing speed over the entire range of overhauling loads is actually *lower*, as an average, than over the range of hoisting loads.

On this same equipment, in Fig. 6, the full speeds at maximum load hoisting, at balance and at maximum load overhauling are

equal at 625 ft. per minute, and the overhauling speeds are lower, on the average, than the hoisting speeds.

Further developments of this system have been made and embodied in equipments of more recent manufacture. A shipment of three such equipments was made after only routine factory tests on the individual parts of the equipment; the machines were placed in successful commercial service promptly, without difficulty and without personal assistance from headquarters.

At the first opportunity, extensive factory tests were made on an equipment in accordance with these further developments. Numerous proportions and adjustments were studied, and a typical set of adjustments gave results as in Fig. 7 herewith. Since predictions may differ as to the mechanical losses in an elevator installation beyond the traction-motor armature, the results in Fig. 7 are given in amperes, as taken. If the mechanical efficiency is assumed as 80 per cent, the rated load of the elevator is represented by 160 amperes hoisting and 102 amperes overhauling. Or, on an assumed mechanical efficiency of 90 per cent, the corresponding loads will be 142 amperes hoisting and 115 amperes overhauling. On the former basis, when hoisting full load on the first speed point, the speed is 55 ft. per min. and when lowering full load, the speed is 67 ft. per min., an increase of only 12 ft. per min. at full load overhauling. On the basis of the higher mechanical efficiency, this regulation is even closer. If, on this regulation curve a horizontal straight line is drawn at 65 ft. per min., the speed at nearly all positive loads is slightly *above* and at nearly all negative loads is slightly *below* this average line. For all practical purposes, therefore, the speed regulation on the first speed point is flat over the entire range of loads.

The regulation on the full-speed point for the equipment in Fig. 7 (assuming 80 per cent mechanical efficiency, which is the less favorable case) is from 615 ft. per min. at full load hoisting to 620 ft. per min. at full load overhauling. The maximum speed, 630 ft. per min., occurs at balanced load and various heavier loads going up. The significant speed regulation is therefore 1 per cent, from full load up to full load down.

After this equipment (as in Fig. 7) is installed, results of its operation in actual service will be presented in the paper which I intend to prepare. In that connection, the system of control will be described and explained.

J. J. Matson: In Figs. 14 and 16 Mr. Bouton shows results of tests to determine elevator power consumption with various loads and stops per car mile. As the curves are obtained from tests the power consumption undoubtedly contains the motor-generator losses only during the period of test. Assuming this is true, the results greatly favor the variable-voltage control system. By this, I mean that if the power consumed, the stops made, the miles traveled and the average elevator load for one day were measured and the kilowatt hours per car mile calculated, the result obtained would be higher than is given in Mr. Bouton's curves. The reason is apparent if one stops to consider the actual cycle of operation for an elevator which consists of some time running, and some time standing (Includes time required for receiving and discharging passengers). During the standing time, which may be as high as 50 per cent of the total, the motor-generator set will surely be running and taking power. This power would, of course, be shown in the all-day run but when a test was run for power consumption, the running-idle losses of the motor-generator set would not be included as the elevator would be operated continuously until sufficient readings were taken to obtain the power-consumption values. The best way to compare power consumption for variable-voltage and rheostatic controlled elevators is on an all-day basis. Even under such conditions, the variable-voltage control shows the lowest power consumption, the saving increasing as the stops per mile increase.

A great deal of importance has been attached to elevator power consumption by all interested parties. In reality this is not borne out by a careful consideration of all factors. For example, assume a 20-story building having a hatch 7 ft x 6 ft. and the floor space renting for about $3.00 per sq. ft.; the elevator to cost $18,000; the actual charges then become on a yearly basis:

Rentable floor space occupied by elevator............	$2520
Interest depreciation, insurance, etc., (15%)...........	2700
Operator's salary..................................	750
Maintenance, etc..................................	300
Spare parts, re-roping, etc..........................	350
Power bill (20 miles per day, 2.5 kw-hr. per car mile, 300 days per year and 2c. power................	300
Total....................................	$6920

Thus, the power bill is the small part of the total yearly elevator bill, approximately 4.0 per cent.

Fig. 11 shows the regulation on a rheostatic-controlled elevator motor. The regulation between 100 per cent motor torque and 64 per cent generator torque (this assumes a mechanical efficiency of 80 per cent) is approximately 63 to 83 rev. per min. or 31.8 per cent. Tests on an elevator which has been installed about 18 months, show the speed regulation (without centrifugal speed governor) to be as follows: 100 per cent motor torque 64.5 r. p. m. Zero torque 62.0 r. p. m., and 64 per cent generator torque 68.5 r. p. m. This is regulation of 6.2 per cent. It is interesting to note that full-speed acceleration was obtained in about 3¼ seconds and with perfect comfort to passengers.

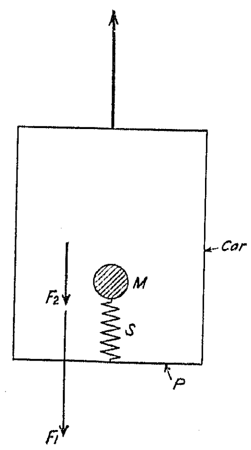


FIG. 8—THE GRAVITATION FIELD IN AN ACCELERATING ELEVATOR CAR

Bassett Jones: I want to add something to Mr. Eames' discussion of Mr. Bouton's paper. The fact that the most suitable time-velocity characteristic of a passenger elevator is a reversed parabola may be deduced directly from the mechanics of the passenger's body. This I have already discussed in "The Time-Velocity Characteristics of High Speed Passenger Elevators," *General Electric Review*, P. 111, February 1924.

In Fig. 8 shown herewith the mass, M, represents the mass of the passenger's body. The spring, S, represents the elastic elements of his bodily frame. If the car be standing still or traveling at any constant velocity, the force F_1 acting on M and causing a compression of S is the gravitational field of the earth. If the car accelerates on the up motion or retards on the down motion, a force, $F_2 = M\,a$, is added to F_1 and the spring is further compressed. If the car accelerates on the down motion or retards on the up motion, the force F_2 is deducted from F_1 and the spring expands.

Obviously if this force, F_2, is applied or removed suddenly as by constant acceleration, the resulting shock to the passenger's body will be uncomfortable. Therefore, it should change from zero to a maximum, and from a maximum to zero in some

gradual manner, and it seems reasonable to suppose that if this change be constant, the best conditions will result.

This requires that the *rate of change in acceleration* (or retardation) *be a constant*, that is,

$$\frac{d\,a}{d\,t} = p,$$

where p may be called the *physiological constant.* Therefore, if S be distance,

$$\frac{d^3\,S}{d\,t^3} = p \qquad (1)$$

From this, the velocity, V, in ft. per sec. at any time, t, is

$$V = 1/2\,p\,t^2 + C_1\,t,$$

and $C_1 = V_m/t_m - 1/4\,p\,t_m,$

or $V = 1/2\,p\,t^2 + (V_m/t_m - 1/4\,p\,t_m)\,t,$

where V_m is the maximum velocity in ft. per sec. attained in time, t_m, given in seconds.

Evidently the only possible case is when

$$V = 1/2\,p\,t^2, \qquad (2)$$

giving $(V_m/t_m - 1/4\,p\,t_m) = 0 \qquad (3)$

Therefore the time-velocity characteristic is a parabola. Also maximum acceleration reached is

$$a_m = p\,t_m/2.$$

Equation (2) holds between $V = 0$, $t = 0$ and $V = V_m/2$, $t = t_m/2$. From $V = V_m/2$, $t = t_m/2$ to $V = V_m$, $t = t_m$ the curve is reversed. A single equation for the entire curve may be developed, but is not a practical necessity.

From (3) it is obvious that

$$V_m = 1/4\,p\,t_m^2,\; p = 1/4\,V_m/t_m^2,\; t_m = 2\,(V_m/p)^{1/2} \quad (4)$$

Therefore if any two conditions, V_m, t_m or p, are given, the remaining condition is fixed. A few such ideal time-velocity characteristics are given in Fig. 9. The whole matter being more completely discussed in the article mentioned above.

From a practical standpoint it is not essential that the parabolic form be maintained except during the initial and final parts of the acceleration. Between these, the acceleration may be constant, or nearly so. Probably the time-velocity curve should approximate very closely to a parabola during the first and last 0.5 second of the acceleration period.

Obviously, for any given value of V_m the smaller is p the longer is t_m, and the round trip time for a given traffic will be increased.

Consequently, where comfort is a matter of moment, as in family hotels, hospitals and the like, a value of p smaller than can be properly employed, for instance, in office building equipment, must be used and the rapidity of service correspondingly sacrificed.

The next question is, what shall be the relative values of p in two such cases?

Having established as above, the manner in which the kinetic energy of motion must be communicated to the passenger's body, it is necessary to put a limit on the total amount that can be safely communicated in a given time without over stressing the elastic elements of the passenger's body. Given a certain time of acceleration, the impressed kinetic energy varies as the square of the velocity attained in this time, or as p^2. Mechanically speaking, the reverse of this case is precisely the same as determining the capacity of oil buffers for a given retardation time. If the oil buffer is to bring the car to rest in the same time, irrespective of the velocity, then, if the velocity of the loaded car be doubled, the capacity of the oil buffer must be quadrupled.

So, probably, if in two cases $p = 10$ and $p = 20$ for the same value of t_m, the discomfort of the passenger due to muscular stress, may be assumed to be as 100 is to 400 in the two cases. In the latter case he will experience four times the discomfort he experiences in the first case.

If, the ratio of comfort in the two cases is to be as 2:1, then, also the corresponding values of p are 20 and 14.

Assume that $V_m = 11.66$ ft. per sec. (700 ft. per min.) then from (4), and the values of p (20 and 14) given above, $t_m = 1.53$ and 1. 3. The latter of these is 19 6 per cent greater than the former.

Of course much of the above is theoretical in the sense that it is based on an assumed simplicity in the mechanics of the passenger's body that it does possess. Also, it requires a form of torque-time characteristic of the hoisting-engine motor that, so far as I know, can only be obtained by special adjustment and for one particular set of values of V_m and p for anyone motor and control, and for some one value of the load.

However, this discussion is intended merely to draw attention to the real problems faced in attempting to attain high velocities

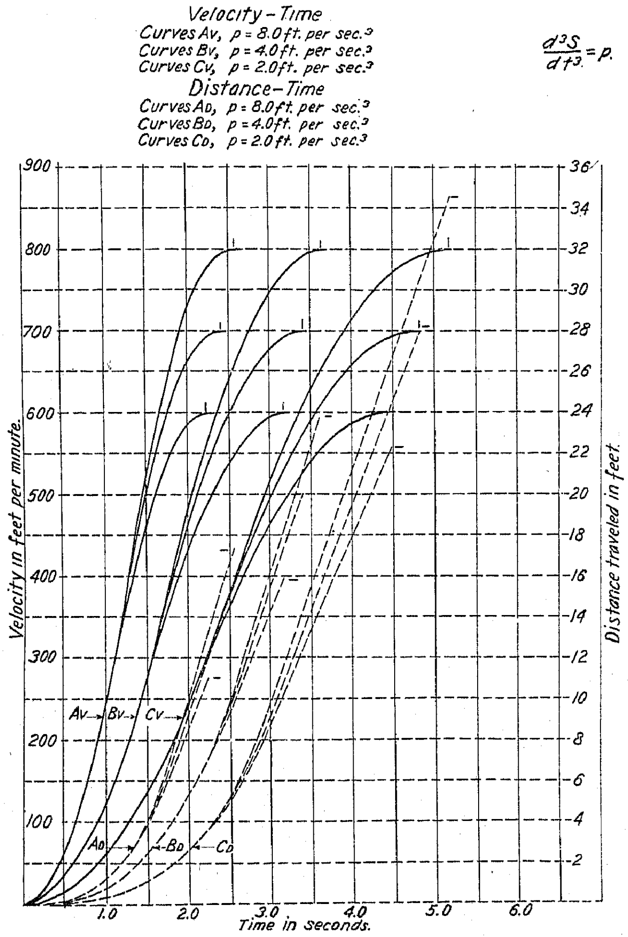


Fig. 9

and rapid acceleration in elevators. Definite solutions must wait on the necessary physiological tests required to set up the values of p.

A. A. Gazda: Looking back over the experience of the past three years in the installation and operation of upwards of 150 elevators controlled by the variable-voltage system, the one outstanding feature is its adaptibility to a broad range of applications. On one hand the advantage of smooth control for high-speed machines is quite apparent but on the other hand the feature of definite and positive speed points has been utilized not only in slower-speed passenger elevators but also in high-grade freight installations. For automatic or push-button controlled elevators particularly, we have found it of considerable advantage to have a system at hand whereby two or more definite speeds can be set at convenient values and thus insure accurate

stops even under changing load conditions. Hitherto, the nominal operating speed of automatic elevators has been limited by the design of induction motors with certain sub-synchronous speeds and in the case of direct-current machines by the lack of smooth transition from one speed to another. Introducing a motor-generator set (and the variable-voltage system) as outlined in Mr. Bouton's paper has solved both the problem of definite speed points and smooth transition. It is only necessary to locate the desired values on the generator-field rheostat and depend upon the time element of the magnetic circuit to make the swings smoothly. In accomplishing this, permanent damping windings must be provided. Also final adjustments may readily be made by placing discharge resistors across the shunt fields. If the time constants are too great the elevator operation will be sluggish. On the other hand short time constants throw severe peak loads on the commutators of both the generator and the elevator motor and if these peaks are frequent or of sufficient intensity the commutators will not stand up. This statement is made not to bring out disadvantages of the system but rather as a factor that must not be overlooked.

Another important field of application for this system lies in freight elevators which must be held level with the floor landing automatically. As shown in Fig. 7 of Mr. Bouton's paper a stable low speed can be maintained throughout the torque range and thus insure accurate stops. In such applications the problem really lies in changing the motor torque quickly on account of static friction. This has been accomplished by the proper choice of the generator series field and also by the use of momentary contact switches in the main control scheme. Our experience has demonstrated that this problem can be solved and there are now several installations of this type in successful operation.

In high-speed elevator work we find that the continued successful operation of the cars depends largely upon the care taken in maintaining the original adjustments particularly on the controller. Our experience with variable voltage installations has demonstrated that the original control adjustments are of a permanent character which do not change as contacts wear. Even as originally installed by average elevator constructors, the system is almost fool-proof and we have often been agreeably surprised by the smooth acceleration and deceleration obtained by inexperienced men. This ease of installation and operation is a factor of considerable importance from the standpoint of the elevator manufacturers particularly when it is so difficult to find competent mechanics.

W. L. Atkinson: Field experience with the system of control described by Mr. Bouton has indicated that we have here a means of obtaining operating characteristics in an elevator, not possessed by any other known means of control. While the comparative economic advantage for any given number of stops per car mile can be very readily analyzed and definitely stated, the important feature of smoothness of operation through the accelerating and decelerating periods, contributed by this method can only be demonstrated by actual experience with elevators in the field.

The company with which I am connected, has installed and in operation fifty-two elevators equipped with this system of control, operating at 400, 500, 600 and 700 ft. per minute, and in every installation a distinctive and characteristic smoothness is noticeable throughout the starting and slowing down periods. In the highest developed type of rheostatic control, making use of all the refinements which experience has shown to be of advantage we cannot attain a smoothness of operation with rheostatic control comparable with that quite ordinarily possible with the variable voltage scheme here described.

While we have been limited heretofore, to a maximum speed of about 600 ft. per minute, or in exceptional cases to 700 ft., we are now enabled by this means, we are quite sure, to go to speeds much beyond this limit without occasioning discomfort to passengers, and with the certainty of making definite stops under all conditions of loading.

Those of you who have worked on the traffic problem involved in moving the occupants of a densely populated tall modern office building in or out of the building, over the usual peake of say 20 to 40 minutes, will readily appreciate the value of any system of elevator control that offers a practical means of cutting down the time of a round trip or cycle of operation.

With modern elevator machinery there is no difficulty encountered in operating at any speed we may wish, once that uniform speed has been reached. The limitation has rather been imposed by the means available for bringing the car up to speed or slowing it down to a final stop, quickly and without discomfort to the passengers.

In a recent trial on an elevator in regular service equipped with the variable-voltage system of control, and having a nominal speed of 600 ft. per min., the car was operated at 850 ft. per min. without the occupants of the car noticing or being aware of the fact that it was not operating at the usual speed. From such data and experience in the field we are entirely confident in predicting successful operation of passenger elevators up to a speed of 1000 ft. per min.

Laurence D. Jones: Mr. Bouton's paper describes the application of the well known system of generator voltage control to the operation of electric elevators. When this system of control is used with any application where rapid rates of motor acceleration and retardation are required, these rates are in most cases dependent upon the rates of building up and dying down of the generator field.

As pointed out by the author, this fact offers one of the principal advantages of applying such a system of control to elevator service because it insures very smooth acceleration and retardation.

The inherent rates of change of generator voltage are dependent on the time constant of the field circuit which, in turn, depends upon the proportion of resistance and inductance in the circuit. It is possible to change the time constant of a field by changing the proportions of resistance and inductance. The usual method is to increase or decrease the resistance as the inductance is not so easily changed. The resistance may be increased by placing a resistor in the circuit and may be decreased by placing a short-circuited damping winding around the poles.

From the usual theory of building up of current in a circuit having resistance and inductance, the time required to reach a constant value depends upon the proportion of L to R. Bearing this fact in mind we find it rather difficult to understand how Mr. Bouton obtained the results shown in Fig. 3 of his paper which indicates that the use of a short-circuited damping winding has changed the rate of building up, but has not affected the time required for the voltage to reach a constant value. It is possible that the author has used some special means for producing the effect shown. If so, a description of how this was accomplished would prove of interest.

Similar tests which have been made under the same conditions as those described in Mr. Bouton's paper show a very material difference in the time required for voltage to reach constant value, with the field damped and undamped. In this case the time was 4.4 seconds with the damping winding in use and 2.0 seconds without this winding. Figs. 10 and 11 shown with this discussion are oscillograph records of the building up of field current and armature voltage of a generator having its field undamped and also damped. Fig. 12 is a comparison of the voltage curves for the two cases. The general conclusion from these tests is that the use of a damping winding is of value as a means of changing the time constant of the field so as to obtain the desired time for accelerating to full speed but has very little effect upon the shape of the curve.

From Part III of the paper one is likely to gain the impression that the traction motor will have acceleration and retardation

curves of the same shape as the building-up or dying-down curves of the generator voltage and that these curves are independent of the load. That this is not the fact is quite evident if we compare the shape of the acceleration curves in Fig. 4 with the curves in Fig. 3. The principal reason for the disagreement is that the shape of the speed curves is modified to some extent by voltage drop in the armature circuit. With further reference to Fig. 4, I would like to ask Mr. Bouton why a longer time is required for the motor to accelerate a loaded car going down to a

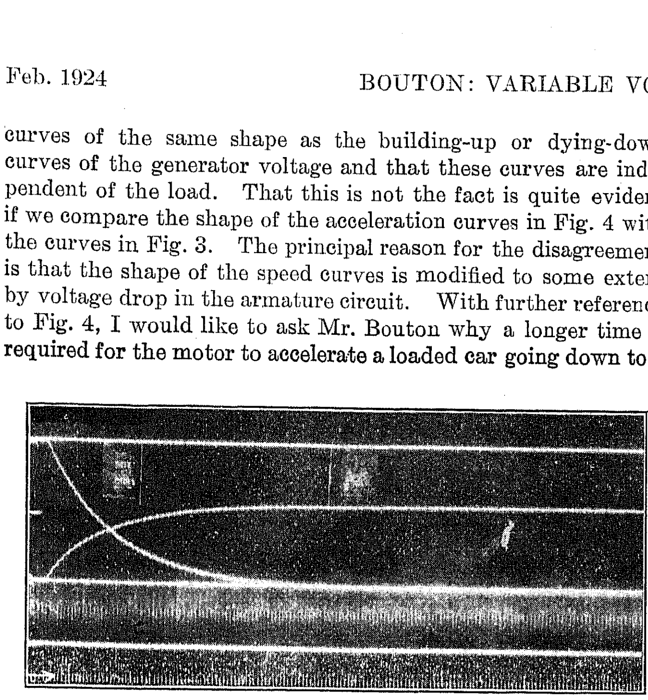


FIG. 10—OSCILLOGRAM SHOWING BUILDING UP OF GENERATOR FIELD CURRENT AND ARMATURE VOLTAGE WITHOUT DAMPING WINDING ON FIELD

Curve A—Armature voltage, 230 volts maximum.
Curve B—Field current.
Curve C—Timing wave. (40 cycles.)

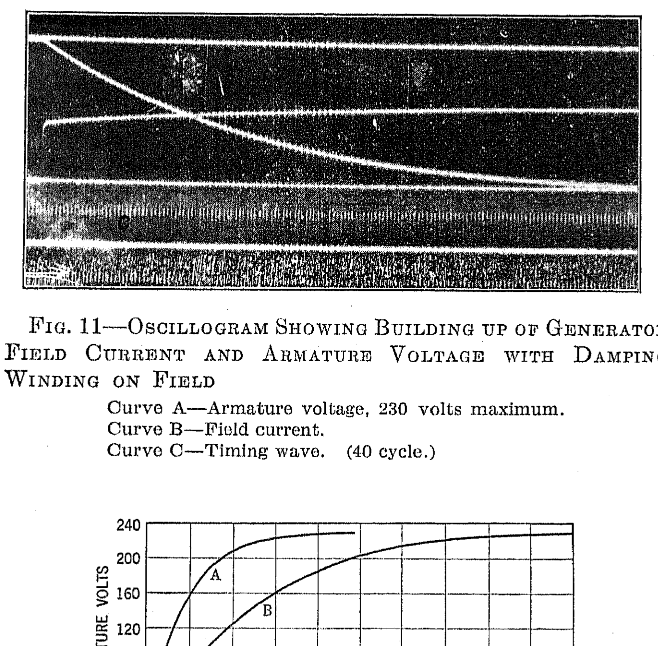


FIG. 11—OSCILLOGRAM SHOWING BUILDING UP OF GENERATOR FIELD CURRENT AND ARMATURE VOLTAGE WITH DAMPING WINDING ON FIELD

Curve A—Armature voltage, 230 volts maximum.
Curve B—Field current.
Curve C—Timing wave. (40 cycle.)

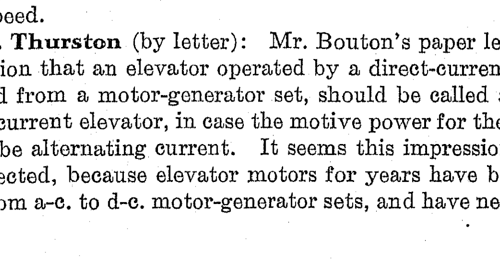


FIG. 12

given speed than is required to accelerate a balanced load to the same speed.

E. B. Thurston (by letter): Mr. Bouton's paper leaves the impression that an elevator operated by a direct-current motor, supplied from a motor-generator set, should be called an alternating-current elevator, in case the motive power for the generator set be alternating current. It seems this impression should be corrected, because elevator motors for years have been supplied from a-c. to d-c. motor-generator sets, and have never been considered as alternating-current elevators. (Indeed were the variable-voltage generator driven by water or steam power the elevator would be called a hydraulic or steam elevator with the same line of reasoning.)

Fundamentally the only elevator that can be truly called an alternating-current elevator is one in which the elevator machine is driven directly by an alternating-current motor.

Comparing the two systems—variable voltage and the straight alternating-current, as defined in the preceding paragraph—it is seen that the variable voltage as given in the paper consists of six main elements, while the straight alternating current consists of but two, the elevator motor and its controller.

Although the paper under consideration refers to the alternating-current elevator equipment, it is noted that none of the test information contained therein gives any consideration to an alternating-current power supply, but deals only with equipment requiring direct current. If the paper leaves the impression that the curves apply to equipment having an alternating-current supply it is misleading, for the amount of energy returned to the line during slow-down will certainly be somewhat different from an a-c. to d-c. motor-generator set. Another very important consideration is the fact that the power factor will be very low during average elevator operating conditions, and idle and slow-down periods, which will be 90 to 95 per cent of the total time in service.

The paper compares only two types of control—variable-voltage and rheostatic, and these with direct current supplied, whereas it is a fact that the standard shunt-control direct-current elevator unit, consisting of the two main elements of motor and controller, shows in regular service an energy consumption still lower than either of these.

The tests given in the paper are interesting, but actually of little value even for comparison with other types of control.

Energy-consumption tests, to be of value, must show what the particular types of control under comparison are doing in actual service, where a corresponding number of hours per day, miles per day, stops per mile and kilowatt hours per car mile over a number of months are recorded. This recorded information will represent quite accurately average conditions, and will be of great value to engineers in general.

The different types of elevator control might more fittingly be compared by giving consideration to an increased number of items as follows:

Economy as to first cost.
Economy as to maintenance.
Economy as to energy consumption.
Economy as to space.
Safety.
Simplicity.
Number of main elements.
Total number of parts.
Continuity of service.
Speed regulation.
Accelerating and decelerating characteristics.

As engineers we instinctively aim towards ultimate simplicity of equipment with maximum safety and refinement of operation combined with a minimum kilowatt-hour consumption for regular operating conditions.

It is, therefore, self-evident that a straight alternating-current elevator with only two main elements, which will give any desired car speed with positive control, smooth, quiet, rapid acceleration and deceleration, and a low overall operating cost as well as all of the safety requirements, is a goal towards which we, as electrical engineers, should turn our interest and attention.

E. W. Seeger (by letter): Although the acceleration of an elevator equipped with variable-voltage control is exceedingly smooth, it is possible to get practically the same results with other types of control. In decelerating and stopping, however, the variable-voltage system has a decided advantage because of the

possibility of obtaining a sustained braking effect and a stable slow speed. The quick acceleration and retardation possible with this type of control will probably lead to the more general use of high-speed elevators for local service.

It is always difficult to compare data on the power consumption of elevators because so many factors, other than the electrical equipment, affect the result. The exact method of making the test plotted in Fig. 14 is not stated. If the load were placed on the car and the elevator then started and stopped the required number of times, the test would not show results which would be obtained in actual service, but would favor the variable-voltage equipment.

Another factor which will affect power consumption is provision for night service in office buildings. Although it is possible to start the set each time the elevator is required, it is doubtful if this practise will be followed and probably one set will be allowed to run.

After acceleration is completed and the motor is running at full speed, the variable-voltage system, due to the losses in the motor-generator set, is probably the least efficient of any of the accepted methods of control.

Even after taking the above factors into consideration, the variable-voltage system will undoubtedly show a saving on any installation where a gearless traction machine is justified. This is particularly true with "green" operators because the power required for inching is less.

With slower-speed elevators driven by geared machines, it is more difficult to justify the use of the variable-voltage system. The rheostatic losses are, in general, a much smaller percentage of the total power, landings are easier to make, and the service is less frequent. It follows that the maintenance expense is also smaller.

There is, therefore, a very definite field for the two speed a-c. motor on moderate-speed geared elevators. The acceleration and deceleration obtained with this type of equipment are as satisfactory as obtained on most d-c. installations. The average a-c. installation, however, is comparatively noisy, and for this reason the variable-voltage system has been installed in some cases where quiet operation is essential.

In the majority of variable-voltage equipments which have been installed, the car operating switch has been used, not only to control the contactors on the board, but also to vary the generator field resistance directly.

The generator field resistance is usually placed on the car, and inasmuch as a number of resistance steps are used, there is considerable wiring on the car which is comparatively inaccessible. The writer believes that a more satisfactory arrangement is to have the car-switch-control multi-point relays placed on the control panel. With this arrangement, only one set of resistance is required and it can be placed on the control panel rather than on the car, where space is at a premium.

B. M. Jones (by letter): It has been my good fortune to do some work on a variable-voltage elevator built by Mr. Bouton's company, and it was a pleasure to see the ease with which the elevator was controlled and the smoothness of its operation.

The writer has also done considerable work on some of the early two-speed a-c. elevators using squirrel-cage motors having two speeds, covering a range of approximately three to one. This means that the a-c. motor has two primary windings. For starting the motor it is thrown on the low-speed winding and as the car accelerates the motor is automatically transferred to the high-speed winding through resistance which smooths out the acceleration. The star point of the windings is brought out of the frame of the motor and the resistance connected in these leads before the starting connection is made. Magnetic contactors short circuit this resistance in steps to accelerate the car. When switching from high to low speed, the resistance is thrown in the high-speed winding before the latter is thrown on the line, thus smoothing out the change in speed from low speed to high.

In connection with these two-speed elevator motors, as well as with any elevator motor, the number of stops per minute and the duration of the stops have a great effect upon the heating of the motor, as does the class of building in which the elevator operates. For example there are some department stores five or seven stories in height in which during non-rush periods the car only pauses at each floor, and, seeing no passengers, the operator proceeds on to the next floor. This constitutes a full stop and start, but with very little rest period. This is very severe duty on the motor and is considerably different from that in a tall office building where the elevators runs three, four, five, or, in some cases, ten, floors without a stop, thus operating the motor a great portion of its operating time at a high speed and consequently deriving the benefit of its ventilating effect.

The question of first cost of an elevator installation is a quite important one and the method of drive influences this cost somewhat. These points were not touched upon in this paper. It is also very interesting to know the power consumption of elevators per car mile, and the comparative power consumptions of variable-voltage d-c. elevators and two-speed a-c. elevators, as well as the amounts of maintenance required on these two schemes.

The question of speed of elevators is very interesting, especially for a tall office building where the ability to handle the peak load in a space of half an hour two or three times a day is very important. However, the ability to operate an elevator at such a speed that the operator can make the landing at the floor accurately without a great amount of juggling up and down, to which passengers object strenuously, is very important and should be kept in mind by the elevator designers as well as the motor and control designers when laying out elevators for a tall office building.

E. M. Bouton: Mr. Whiting has shown some very interesting results obtained with variable voltage control, by paying particular attention to the details that affect the regulation of the motor. Unfortunately, he has not described the method by which he obtained the results.

In general, the regulation of the elevator motor can be improved by increasing the compounding of the generator or by using control contacts which manipulate the generator or motor fields in response to changes in motor load or speed. The generator compounding may be accomplished by putting series turns upon its own field poles or by putting these series turns upon an auxiliary generator which excites or affects the excitation of the main generator. If the generator is compounded so as to have a flat speed torque curve at high speed, it will probably have a rising curve at slow speed. This is because the generator fields are more nearly saturated at full speed and a given number of series turns is more effective at the lower saturation. It is entirely practicable to compound the machine in this way, except that if carried too far it has a tendency to make the equipment unstable.

I believe that a little further discussion of Fig. 7 and Fig. 11 would be helpful. Curve 1 in Fig. 11 shows the inherent regulation of a gearless machine at full speed without the corrective effect of any speed governor contacts when connected to a constant potential of 230 volts. The regulation at low speed with armature series and shunt resistance will be as shown by Curve 2.

The test illustrated in Fig. 7 consisted in taking the same machine and driving it with a variable voltage generator. The generator compounding was so adjusted as to give approximately the same regulation at high speed as was obtained on the constant voltage supply. The inherent regulation at low speed is then shown by Curve 2. A comparison of curves 2 in Fig. 7 and Fig. 11 shows the advantage that variable voltage control has over rheostatic control in making landings when adjusted for the same regulation at full speed.

It was not the purpose of my paper to show the maximum improvement over rheostatic control that could be obtained, but rather to show that the variable voltage system has certain inherent characteristics that make it superior to other systems. It is entirely possible, as is shown in Mr. Whiting's discussion, to make the regulation at full speed as good as is desired. I wish to state, however, that additional complications are involved if extremely close regulation is desired.

The tests shown in Figs. 14 and 16 include the motor-generator losses only during the period of test. For this reason, in actual service the all day power consumption would be somewhat higher. For example, if the elevator runs 20 mi. in a 10-hr. day, the no load losses will be, (at 1.25 kw. per elevator)—$1.25 \times 10 = 12.5$ kw-hr. total for the day. This loss must be distributed over the 20 mi. traveled and will be $12.5 \div 20 = 0.625$ (625/1000) kw-hr. per car mile for the idling losses. It would not be right, however, to add all of this to the test value shown in Fig. 14 since time was actually consumed in making the test. It would probably be fair to add ½ of this value or approximately 0.3 (3/10) kw-hr. per mile to the test value to obtain the actual power consumption of an elevator operating on the schedule outlined above. For night service the motor-generator sets can be shut down so that the losses do not occur except while the car is running.

I wish to point out also that the test results shown in Fig. 15 for rheostatic control do not show all the rheostatic losses that will be present in actual operation. The test does not include any periods of slow speed running prior to the stops nor does it include "inching" the car to make a landing while in actual service, these operations occur very frequently. If the motor is controlled over a wide range by armature series and shunt resistance as is the practise with gearless machines, the elevator will frequently consume more power while running at low speed than at high speed. For this reason the test values probably favor the rheostatic system more than the variable voltage system when compared with actual service conditions.

There seems to be quite a tendency among engineers when comparing variable voltage control with other systems to pay considerable attention to whether or not there will be a saving in power. As a matter of fact, in the majority of cases the other advantages of variable control that I have described, such as smoothness of operation, accuracy of control, saving in time, etc., are much more important considerations. I think Mr. Matson's analysis of the cost of operating elevators brings out quite clearly that the power consumed is a small item in the cost of operation.

L. D. Jones has questioned the results shown in Fig. 3 which show that the rate of building up of the generator fields is changed by the use of a damper winding, but the total time of building up is not materially affected. The inductance of the field circuit is made up of two factors, the self inductance of the main field winding and the mutual inductance of the short-circuited damper winding. If the inductance were actually constant, the field coil would have a true time constant and the field flux would build up according to the well-known logarithmic law. However, the inductance is not constant but depends upon the permeability of the magnetic circuit and upon the amount of magnet leakage between the main winding and the damper winding. Now keeping in mind that the field poles are being worked at a lower point on the saturation curve when the field starts to build up than when it has built up to higher values, and also that the increasing magneto force of the main winding will cause a redistribution of the leakage flux, it is not unreasonable to expect a change in the shape of the curve when the damper winding is added. For other proportions of magnetic circuit and proportions of windings I would readily believe that different results would be obtained, as Mr. Jones states that he obtained from his tests. The results I have shown in Fig. 3 were obtained on test with an oscillograph.

The speed time curves obtained in Fig. 4 were taken by driving a magneto by the elevator motor and recording with a graphic voltmeter the voltages of the magneto. The voltage of the magneto was calibrated in terms of car speed with an ordinary tachometer. The method is not absolutely accurate and these inaccuracies no doubt account for the discrepancies in the curves that Mr. Jones mentions. I think Mr. Claytor's analysis of the curves explains why the motor speed does not follow exactly the building up of the generator voltage during the early part of the accelerating period. During the rest of the accelerating period and during retardation up to the point where the brake sets, I believe the speed and the generator voltage follow each other very closely.

Mr. Hansen has described an interesting system whereby he uses a voltage of rotation in an auxiliary machine to first oppose the impressed line voltage and then add to it to accelerate the main motor to a higher speed. By this means the energy which in a rheostatic system is dissipated in resistance is here stored in a flywheel and then returned as energy used in acceleration. It would seem that this system would be quite useful for constant speed work where acceleration and stopping make up the principal part of the cycle. It has the one drawback for elevator work, that speeds lower than full speed are not available. If the controller handle were left on a slow speed point the main motor would not run at this low speed except momentarily, because the energy input to the auxiliary motor would accelerate the flywheel and the main motor would accelerate due to the reduction in counter e. m. f. of the auxiliary motor. A slow speed which is constant and independent of load is one of the important requirements of elevator service and this requirement is one of the reasons why the variable voltage system has proved so successful in this field since it provides not only for smooth acceleration but also for speed control.

The discussion by Mr. Eames and by Mr. Bassett Jones illustrate the extent to which elevator engineers have analyzed the passenger himself when providing a conveyance for transporting him, in a vertical direction. I have no doubt that in the future more complete physiological as well as psychological tests will be made to determine the effect of acceleration upon the passenger, but I believe that a more exact solution of the problem depends equally as much upon the development of more accurate methods of testing the elevator to determine how closely it fulfills the required conditions.

A Novel Alternating-Current Voltmeter

BY LEON T. WILSON

Formerly Connecticut Company Research, Fellow, Yale University

Review of the Subject.—*This paper describes an improved thermo-voltmeter, which may be used at all frequencies up to and including 1,000,000 cycles. This meter retains the usual high sensitivity of thermovoltmeters so that it requires a very small current—for full scale deflection about 2 milliamperes.*

At present it is made in ranges from 1 to 20 volts inclusive. Higher voltage ranges can be made but probably at the expense of lowering somewhat the upper limit of frequency at which the instrument is still accurate.

* * * * *

History of Development

THE voltmeter to be described was first devised in 1918 to meet the requirements of a particular problem[1]. As then used, the instrument was primarily a laboratory one but its usefulness in that problem was so well demonstrated that it was deemed advisable to put the instrument into a commercial portable form.

Accordingly two preliminary models were designed and built. One of these was successfully used by two seniors in Electrical Engineering at Mass. Institute of Technology, to measure the resistances of some condensers and inductance coils at high frequencies. This work was part of a thesis problem and to the writer's knowledge has never been published.

The other model was submitted to representatives of the Weston Electrical Instrument Co. for their consideration. Following some preliminary measurements, this company undertook to change certain details in the design of the preliminary model to meet their own standards and manufacturing methods.

Largely through the efforts of Mr. Caxton Brown, and Mr. W. N. Goodwin, Jr., the development of the present model has been accomplished.

Principle of Operation

In principle the voltmeter falls in a class of instruments commonly known as thermo-voltmeters. In their usual form these instruments have been on the market for years and therefore will not be described. As ordinarily made these voltmeters may be used at all frequencies up to about two or three thousand cycles.

In the new voltmeter this upper limit in frequency has been increased to approximately one million cycles. This material increase has been accomplished by means of special shields within the meter case and a special disposition of the component parts of the meter.

How the improvement was brought about will be discussed in some detail.

Theory of the New Features

Probably the theory of the new features may be presented best by first showing why the ordinary thermo-voltmeter fails at frequencies above three thousand cycles and then by taking the reader through the same steps that were taken by the writer in his development of the meter to the final form which overcame the various difficulties encountered.

The following analysis therefore will be confined to the specific circuit in which the voltmeter was first developed and used.

That circuit is shown in Fig. 1. An ordinary thermo-voltmeter is shown connected to measure the potential drop across the resistance $A\,B$. The thermocouple is shown external of the meter case but the analysis is essentially the same as if it were enclosed. In such a circuit at high frequencies it has been found that not only does the voltmeter fail to register the true drop across $A\,B$ but it will actually give a large reading with no e. m. f. across its terminals, that is, with the resistance $A\,B$ short-circuited and again with either one

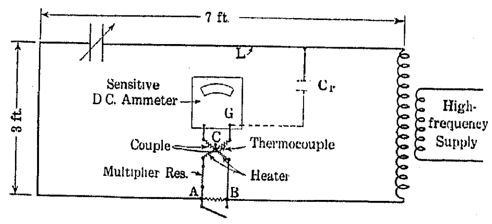


FIG. 1

of the voltmeter leads disconnected. In each case the reading was observed to increase as the meter was moved nearer the side of the circuit marked L.

Although this phenomenon appears mysterious at first sight, it is readily explained. It is due to extraneous or stray displacement currents. The principle one is that to the side of the circuit marked L because a greater difference of potential exists between that side of the circuit and the meter than between any other part of the circuit and the meter. For the present this is the only displacement current that will be considered.

Let us now trace out the path of this stray current. If $A\,B$ is short-circuited the current will divide between the paths $B\,C$ and $A\,C$. If only one lead is connected it will flow either from A to C or from B to C depending on which lead is disconnected. From C it goes to G which is that terminal to which are connected most of the metallic parts of the meter, such as the magnet, needle shield, and the like. From C it flows to L through the capacity existing between these metallic

1. *Proceedings* I. R. E., Vol. 9, page 56, 1921.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

parts and the line L. Although small this capacity is very real and cannot be neglected at high frequencies. It is represented by the fictitious condenser C_1 of equivalent capacity.

It will be seen that the stray current has, on its path, gone through the heater or at least through part of the heater. The current thus causes the meter to register and the observed phenomenon is explained. There still remains to be explained the fact that the observed reading increases as the meter is moved nearer the line L. This is due to the increase in the capacity C_1 accompanying the change in the position of the meter, which lowers the impedance of the path of the stray current and increases its value. The increased current produces a greater reading which is in accordance with the observations.

The phenomena we have just discussed begin to show at frequencies of about two or three thousand cycles and explain some of the reasons why an ordinary thermo-voltmeter begins to fail at such frequencies.

Now let us take the next step that was taken in the

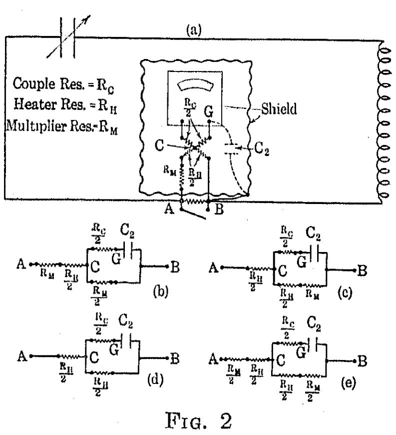


FIG. 2

development. We shall enclose the ordinary thermo-voltmeter in a metallic shield, as indicated schematically in Fig. 2a, and connect it to the point B and see what happens.

As before there will be a stray current from B to L but it does not now flow through the heater. Therefore when $A\ B$ is short circuited the meter is observed to read zero as it should. However a new difficulty is encountered. It is found that in measuring the drop across $A\ B$, if the leads to the voltmeter are reversed a different reading is obtained, not a common experience with an alternating-current instrument. The difference in the two readings is observed to be greater the larger the instrument series resistance.[2]

To explain this curious phenomenon we must first observe that there is a capacity existing between G and the shield, represented by the equivalent condenser C_2. This capacity must be taken into account in an analysis of the circuit through the voltmeter. When the leads of the voltmeter have the polarity shown in Fig. 2a the circuit through the instrument may be represented by Fig. 2b. When the leads are reversed the same circuit is represented by Fig. 2c. In both figures it is assumed that the heater and couple make contact at their respective electrical centers. Actual measurements of the resistances between terminals of the thermocouple showed this assumption to be justified.

Let us now compare Figs. 2b and 2c. It is at once obvious that although the same potential be applied between the terminals A and B in each circuit, the current through the heater in each case will not be the same. This explains why the meter gives a different reading when its leads are reversed.

It will be remembered that this difference in the two readings was found to be greater the larger the instrument resistance or stating it another way, the smaller this resistance the smaller the difference. To find an explanation for this observation, let us reduce the instrument series resistance to zero and see whether the difference in readings disappears. For this to be true both circuits must reduce to an identity. They do, for they both reduce to the circuit shown in Fig. 2d. Therefore with no series resistance the meter should have its reading unchanged when its leads are reversed. This conclusion was checked experimentally.

Why do the circuits 2*b* and 2*c* reduce to an identity without a resistor and not with one? Perhaps the answer to this question will show how to employ properly a resistor. A further study of the circuits shows that without a resistor there are equal resistances on each side of the contact between the heater and the couple, while with a resistor there is more resistance on the side in which the resistor is connected. This observation at once suggests splitting the multiplier and connecting half on each lead, so as to make the resistance on each side the same. Now let us determine whether or not the circuits 2*b* and 2*c* will reduce to an identity under these conditions. They do, for they both reduce to the circuit shown in Fig. 2e.

Two difficulties have now been overcome. The single shield makes the meter read zero when it should. The divided multiplier makes the meter read the same on the reversal of its leads.

However a closer study of Fig. 2*e*, shows one more difficulty to overcome. That difficulty is, that the reading of the meter is not independent of the frequency of the applied e.m.f., due to the fact that the current through the heater obviously depends on the impedance of the capacity C_2 which is not a constant but changes with the frequency.

In solving this problem we come to the final fundamental step in the development of the meter to its present form. That step consists in splitting the shield, as we did the multiplier, into two equal parts. One is connected to one terminal of the voltmeter; the other, to the other terminal, as is shown in Fig. 3a.

An analysis of the circuit through the instrument shows that it may be represented by Fig. 3b.

2. This series resistance corresponds to that marked "multiplier" in the drawings.

An inspection of this circuit shows its striking resemblance to an a-c. Wheatstone bridge. It is obviously balanced when C_3 equals C_4 and when the two resistances are equal to each other. That is, there is no alternating current flowing through $\frac{Rc}{2}$. When so bal-

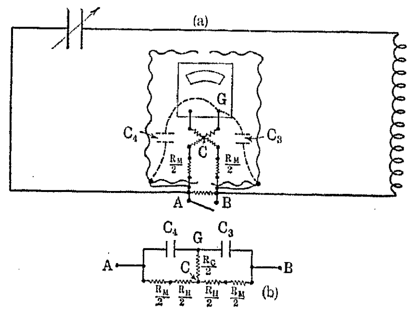


Fig. 3

anced, the current through the heater resistance depends only on the potential difference between A and B and not on the frequency, provided of course that the resistances have negligible inductance or distributed capacity.

In practice C_3 is made equal to C_4 by a symmetrical

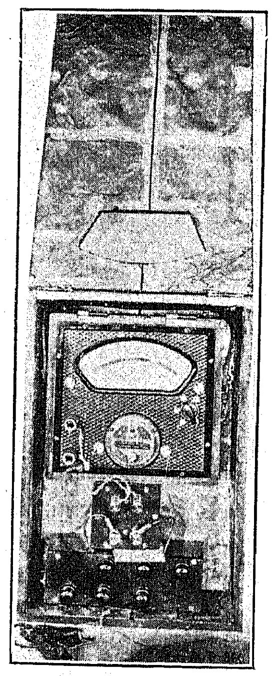


Fig. 4

placing of the meter with respect to the two halves of the shield, while the resistances are balanced by adjustment.

In this form the meter has met effectively the various requirements imposed on a high-frequency voltmeter.

The original laboratory form of the voltmeter is shown in Fig. 4.

Fig. 5 shows one of the two preliminary models in which the shielding, thermocouple, and multiplier resistance are all incorporated within the meter case.

Voltage Range

The Weston model is at present made in various voltage ranges from 1 to 20 inclusive. In these ranges the meter can be employed at all frequencies up to and including one million cycles with an error of less than 1 per cent of full scale value. Instruments of higher ranges can be built, of course, but it is doubtful whether they can be employed at so high a frequency with equal accuracy, on account of certain factors. For example, one of these factors is the distributed capacity of the multiplier resistance, which becomes more important the higher the voltage or the frequency.

Current Taken by the Voltmeter

On referring again to Fig. 3b it will be seen that the voltmeter draws two distinct currents; one through the resistances and in phase with the voltage; the other through the capacities C_3 and C_4 and 90 degrees out of phase with and leading the voltage. The first current is practically the same for all voltage ranges since the instrument resistance is proportionately greater the higher the range. The second current however is not

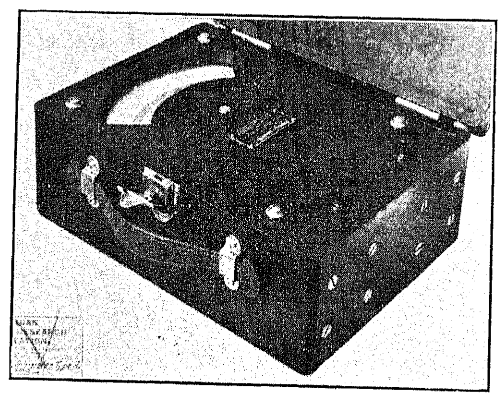


Fig. 5

the same for all voltage ranges. It increases with the applied voltage and also with the frequency of that voltage. This second current is small however and at the lower frequencies can be neglected entirely. At frequencies approaching the upper limit and particularly at the higher voltages this second current becomes appreciable and may equal or even exceed in magnitude the first current. That first current is only about two milliamperes so in any case the total current drawn by the voltmeter over its useful range is not excessive.

Precautions in the Use of the Meter

In addition to the usual well known precautions necessary in high frequency measurements, it is recommended that the voltmeter be kept well away from other parts of the circuit in which it is employed, in particular those parts which are at a widely different potential from the points to which the voltmeter is connected.

Also it is recommended that in general the meter be as symmetrically placed with respect to the various parts of the circuit in which it is employed as is feasible.

However, very good results have been obtained without taking any more than the usual care required in such measurements.

Discussion

R. S. Glasgow: The alternating-current voltmeter described by Mr. Wilson will be quite valuable in a large number of radio-frequency measurements. Particularly in the investigation of radio-frequency amplifier circuits is there a need of such an instrument. However, the type just described has too low a resistance to be employed in the majority of such circuits, as the radio-frequency current in the output circuit of the average vacuum tube amplifier is seldom more than a few milliamperes.

In order to overcome this objection to thermo-voltmeters I have employed a vacuum tube as an electrostatic voltmeter, using the circuit shown in Fig. 1, which is identical with a standard detector circuit. When a radio-frequency voltage, E, is applied to the terminals a b, the mean potential of the grid becomes more negative, causing a reduction in the average value of the plate current, I_p. The device may be calibrated by passing known radio-frequency currents through a known value of resistance, a typical calibration curve being shown in Fig. 2. Where considerable accuracy is desired a family of such curves should be obtained for various frequencies since the slope will vary slightly with the frequency of E. Since the resistance between the terminals a b is of the order of five to ten megohms, while the capacity can be reduced to less than 50 micromicrofarads, it is seen that the device can be bridged across almost any type of circuit without producing an appreciable change in the voltages and currents already present in the circuit. The terminal b should always be connected to that part of the circuit to be measured which is nearest ground potential.

By employing one of the several dry-cell types of tubes now available, the batteries, meters and tube may all be mounted in a shielded container and thus made portable. The accuracy of the device is of course lower than that of the volt-meter described by Mr. Wilson. Its calibration should be checked from time to time which is a common need with the majority of electrostatic voltmeters for low voltages. It will, however, enable the measurement of voltages across portions of a circuit wherein the amount of energy is too small to actuate the instrument described in Mr. Wilson's paper. The voltage range is limited only by the characteristics of the tube chosen and the plate voltage employed.

G. D. Robinson: It should be noticed that the indications of Mr. Wilson's meter depend upon the wave form as well as the effective voltage. A calibration made with one wave form may not hold for another.

Leon T. Wilson: The vacuum-tube voltmeter described by Mr. Glasgow in his discussion of my paper is not new. It has been in use in numerous laboratories for several years.

However Mr. Glasgow has rendered a service in bringing this useful device again to the attention of the electrical engineering profession.

The vacuum-tube voltmeter with its characteristic of a high input-impedance has a certain field of usefulness not covered by my voltmeter. On the other hand, as Mr. Glasgow mentions, it has the disadvantage of requiring frequent calibration. Also its calibration is dependent on the frequency, and therefore its calibration is materially affected by the wave shape of the voltage applied. In working with vacuum-tube voltmeters, it has been my experience that a great amount of care must be taken to insure reasonably accurate measurements. However, when such care is taken this type of voltmeter is a very useful instrument and well deserves the attention of engineers.

Mr. Robinson in his discussion states that "the indications of Mr. Wilson's meter depend upon the wave form as well as the effective voltage." The fact that the calibration of this voltmeter is practically independent of the frequency from low frequencies, say 25 cycles, to its upper limit, approximately 1,000,000 cycles, seems to me to be sufficient evidence to show that its indications, at least for all practical purposes, do *not* depend upon the wave form.

Since the presentation of my paper W. N. Goodwin, Jr. has kindly offered me the use of the results of his recent tests on the Weston model. The results follow:

The charging current, that is, that current taken by the voltmeter which leads the voltage by 90 degrees, was found to be one and one-half milliamperes for an applied e. m. f. of twenty volts and a frequency of one and one-half million cycles.

The error at full-scale deflection (20 volts) was found to be negligible at 600,000 cycles, 1/10 of one per cent at 1,000,000 cycles, and 4/10 of one per cent at 1,500,000 cycles.

Oscillographic Study of the Current and Voltage in a Permeameter Circuit.

BY W. B. KOUWENHOVEN and T. L. BERRY, JR.

Member, A. I. E. E.

Associate Professor, Electrical Engineering, Johns Hopkins University

Review of the Subject.—*The purpose of the investigation was to study the form of the voltage-time and current-time curves, existing in a permeameter circuit, and to reduce the time required for the reversal of the magnetizing current. The permeameters used in the investigation were of the U-shaped yoke type. Oscillograms were taken of the current and of the induced voltage during the opening and also during the reversal of the magnetizing current.*

The permeameter, with which the investigation was started, was fitted with brass end pieces to support the magnetizing coil. The oscillograms showed that the flux change lagged behind the magnetizing current. In fact, the secondary e. m. f. continued for about one second after the current change was completed. The cause of this lag was found to be due to eddy currents set up in the short-circuited paths provided by the brass end pieces. After these were removed oscillograms showed that the lag in flux behind the magnetizing current was negligible. This brought out clearly the fact that short-circuited paths in which eddy currents may be induced should be avoided in permeameter construction.

Two new permeameters were then constructed of the same type; one of these was made with a solid core of silicon steel and the other with a laminated core of the same material. Tests of these showed that use of the laminated core materially reduced the time required for the reversal of the current.

Several different types of switches were used for opening and reversing the magnetizing current. The oscillograms showed clearly that a quick-break snap switch operating under oil is superior to other types of switches.

* * * * *

THE purpose of the investigation was to study the form of the voltage-time and current-time curves in a permeameter, and to reduce the time occupied by the current reversal to a minimum.

One of the reasons for undertaking the investigation was the fact that a permeameter of the U-shaped yoke type, that had been constructed at the Johns Hopkins University, had never given satisfactory results. This instrument read too low in flux density, B, for a given magnetizing force, H. Another reason was that a search through the literature disclosed no publications showing the form of the secondary e. m. f. curve in a permeameter.

Permeameters

The permeameter, mentioned above, was of the *U*-shape type. The magnetizing coil consisted of 1935 turns of No. 16 B and S wire wound on a cast-iron core 5.08 cm. square. The ends of the spool supporting the coil were of 1/8 in. (about 0.3 cm.) sheet brass. The total length of the magnetic circuit including the specimen was approximately 65 cm.

During the investigation two other permeameters were constructed. The cores of these were of high-grade silicon tranformer steel. One was fitted with a solid core of silicon steel 6 cm. by 6 cm. square and the other with a core of 0.012 (0.0305 cm.) thick laminated silicon steel of the same dimensions. The cross-sectional area of the solid core was 36 cm. square and that of laminated core, allowing the customary 10 per cent stacking factor, was 32.4 cm. square. The ends of the spools supporting the windings in these two permeameters were of bakelite. These two permeameters were wound with the same number of turns, and were also of the yoke type.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

Circuit

The diagram of the connections used in the test are shown in Fig. 1. The current supply was taken from a storage battery, and the value of the magnetizing current measured by the ammeter A. A number of different types of switches were used for reversing the current and opening the circuit. In some cases automatic control was used for operating the switch and the oscillograph shutter simultaneously. In some of the tests condensers were placed in parallel with the primary winding of the permeameter, and in others, extra inductance was placed in the circuit. These conditions are described below.

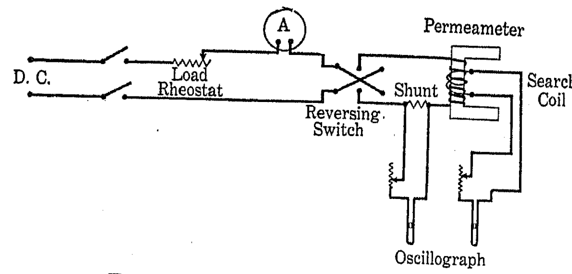


Fig. 1—Diagram of Connections

The shunt used in the magnetizing current circuit was non-inductive. The search coil consisted of 28 turns of No. 18 wire wound over the magnetizing coil of the permeameter. The e. m. f. induced in this search coil was measured by the oscillograph and the shape of this voltage curve shows the form of the e. m. f. that is induced in the secondary circuit of the permeameter.

Tests

Tests were made both of the opening and of the reversal of the magnetizing current.

Opening the Current

Two oscillograms were taken using a carbon break magnetic blow-out circuit breaker to open the magnetizing current of the permeameter with the cast-iron core. The circuit breaker was automatically tripped in this test. The results are shown in oscillograms 1 and 2, which are reproduced in Fig. 2. The magnetizing current was 7.5 amperes in both cases and in

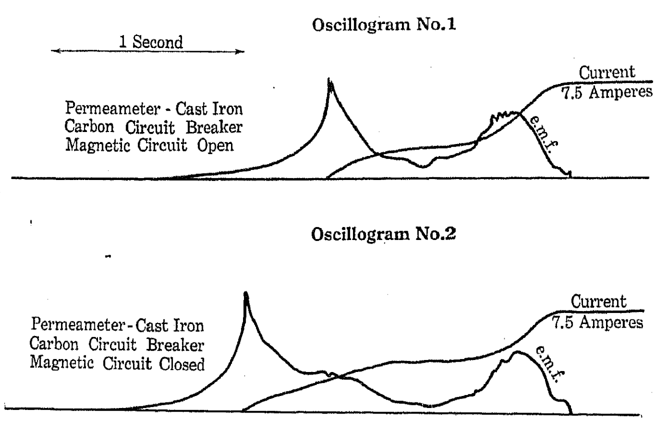


Fig. 2

oscillogram 1 the magnetic circuit was open, while in 2 it was closed with a soft iron specimen. As would be expected the time necessary to interrupt the current was greater in 2 than in 1. Oscillogram 1 shows that it required 1.25 seconds to reduce the current to zero and in 2 it took 1.68 seconds to accomplish the same thing. A study of the oscillograms show that the secondary e. m. f. has two pronounced peaks; one when

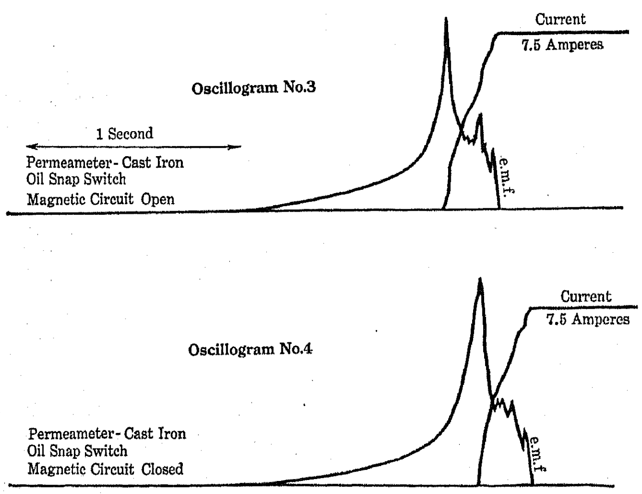


Fig. 3

the current starts to fall, and the other when it dies to zero; also that the e. m. f. in the secondary continues for some time after the magnetizing current has been reduced to zero. The explanation of this is given below.

Ballistic galvanometer readings were taken simultaneously with oscillograms 1 and 2, and were very erratic, as would be expected from the shape of the secondary voltage curve.

It was clearly evident that the circuit breaker did not give a quick clean break, and a 20-ampere snap switch immersed under oil was tried. The results with this switch are shown in Fig. 3, oscillograms 3 and 4. In 3 the magnetic circuit was open and in 4 it was closed. The time of interrupting the circuit was nearly the same in the two cases, 3 requiring 0.23 seconds and 4, 0.24 seconds. In oscillograms 3 and 4 there is only one pronounced peak of secondary e. m. f., but the secondary voltage rises to this peak in an irregular way. A study of these oscillograms shows also that the secondary e. m. f. continues to exist after the current has been interrupted. In fact, this continuation of

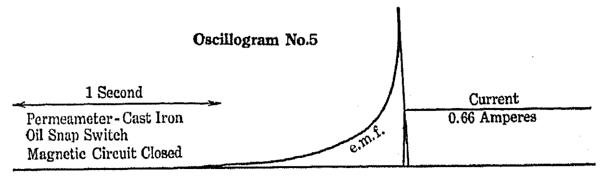


Fig. 4

e. m. f. after the cessation of the current lasts for about one second in all of the four oscillograms shown in Figs. 2 and 3.

In Fig. 4 oscillogram 5 is reproduced. This ocillogram was taken with a magnetizing current of 0.66 ampere through the solenoid of the cast-iron permeameter with a closed magnetic circuit and using the oil-break switch. It shows that the current is interrupted in about 0.02 seconds and also that the secondary e. m. f. continues for about one second after the magnetizing current ceases to flow. In other words, the secondary e. m. f. is maintained for about the same length of time irrespective of the current opened as found in oscillograms 1 to 4 inclusive.

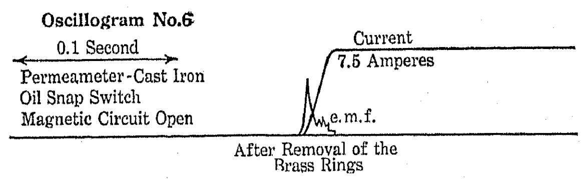


Fig. 5

These five oscillograms all show that the flux lags behind the magnetizing current that produces it. This lag was found to be due to the presence of eddy currents induced in the brass rings forming the ends of the spool supporting the magnetizing solenoid. This is conclusively proved by oscillogram 6, shown in Fig. 5. This oscillogram is taken with the cast-iron core permeameter with the brass rings removed. The oil-snap switch was used to break the magnetizing current and the magnetizing circuit was open. A study of the the oscillogram shows that the secondary voltage and current sink to zero practically simultaneously. The secondary e. m. f. builds up to a peak in an irregular

manner as before and then falls rapidly to zero. The total time that the secondary voltages continues is 0.02 seconds.

This series of six oscillograms brings out clearly the necessity of avoiding the presence of short-circuited paths in which eddy currents may be induced in permeameters. It also shows the superiority of the quick break switch operating under oil over the circuit breaker for opening the magnetizing current. The oil-break snap will open the circuit in less than 1/5 of the time required by the circuit breaker.

Two oscillograms were taken of opening the circuit under the same conditions except that in one the current was supplied by a 120-volt storage battery and in the other by a 10-volt battery. These oscillograms are very nearly identical. They show that it took a small fraction of a second longer, about one thousandth of a second as nearly as can be determined from comparing the oscillograms, to open 120-volt supply than the 10-volt supply.

Reversal of the Current

Before removing the brass ends from the cast-iron core permeameter, the problem of the reversal of the magnetizing current was investigated. In this work an ordinary double-pole, double-throw reversing switch, a special reversing switch that makes contact in the opposite direction before it opens the first circuit, and an oil-immersed snap reversing switch were tried. The special switch in its mid position short-circuits both the supply circuit and the permeameter winding. The oil-immersed reversing switch was a standard four-way snap switch such as is used in electric lighting circuits.

In the previous work the speed of a quick break under oil had been clearly demonstrated for opening the circuit in a very short space of time. It was, of course, realized that the problem involved in the reversal of the magnetizing current included the make as well as the break. It is, therefore, a question of the time constant of the circuit to a large degree. It was felt that this permeameter with the heavy damping, caused by the brass rings, was ideal to experiment with.

The results obtained with the three reversing switches are shown in Fig. 6. The current used was 0.66 amperes, and the magnetic circuit was closed. Oscillograms 7, 8 and 9 are for the double-pole, double-throw switch, the special switch, and the oil-reversing switch respectively. Oscillogram 7 shows that it takes considerable time to operate an ordinary double-pole, double-throw reversing switch of 180 deg. throw. The secondary e. m. f. sank to zero before contact was made in the opposite direction. Efforts to manually operate this switch at a high rate of speed were not successful. This type of switch is clearly unsuited for magnetic testing. Oscillograms 8 for the special switch, which has a throw of only about 15 degrees, shows clearly its superiority over the 180 deg. throw switch. The oil switch gave a much quicker break than the others, as expected. There is only one peak of secondary induced e. m. f. for both the special and the oil-reversing switches. All three oscillograms show the gradual building up of the current in the cast-iron core permeameter after its reversal. In all cases the secondary e. m. f. continues for several seconds before it sinks to a negligible value, for the special switch the time was 5 seconds and for the oil switch 3.9 seconds.

It is a well known fact that the theory of a ballistic galvanometer assumes that all of the charge passes through the galvanometer before it commences to deflect. Professor Laws has shown Laws-Electrical Measurements) that accurate results are possible with a ballistic galvanometer, provided the duration of the discharge is less than one twentieth of the galvanometer period.

It is apparent from oscillograms 7, 8, 9 and 10 that the largest factor influencing the time of reversal is the growth of the current. The use of a quick-break switch operating under oil will effectually solve the problem of opening the circuit in a very short interval of time.

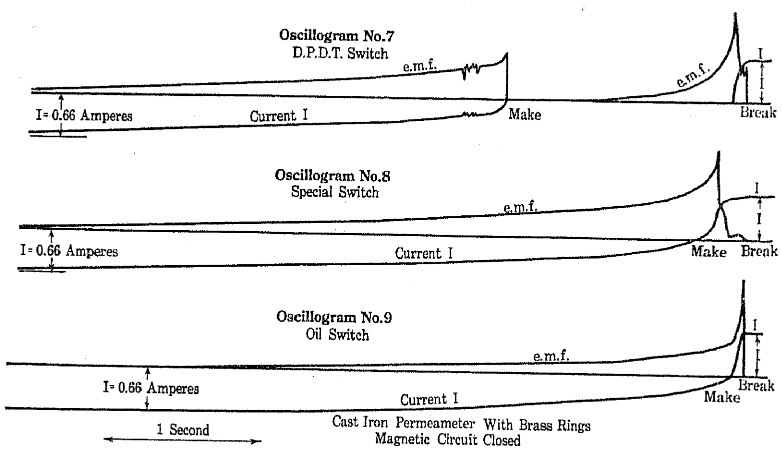


Fig. 6

Therefore, efforts were made to reduce the time constant of the circuit and to shorten the time required for the current to reestablish itself in the reverse direction. A number of schemes were tried.

Among these the effect of introducing a break in the iron of the magnetic circuit upon the time of reversal of the current was tried. Fig. 7, oscillogram 10, shows the effect produced by the introduction of two 1/16-in. gaps in the iron of the magnetic circuit. This oscillogram was taken with the special switch and 0.66 amperes of magnetizing current with the cast iron permeameter. In this case the secondary e. m. f. disappeared in 3.2 seconds as against 5 seconds without the gaps. With an open magnetic circuit the time fell to 1.8 seconds.

Another scheme of shortening the time of reversal of the current that was tried was to place condensers in parallel with the magnetizing solenoid of the permeameter, and neutralize the self induction of the winding.

Several arrangements were tried without success due to the production of current oscillotations in the circuit.

The method that met with the most success was to placing large inductances in series in the line between the source of supply and the special reversing switch, which does not open the initial circuit through the permeameter until it has closed the circuit in the opposite direction. As stated above, this special switch in its mid position short-circuits both the supply circuit and the permeameter winding. The low-voltage windings of a transformer were used for the inductance. Using this special reversing switch, the method reduced

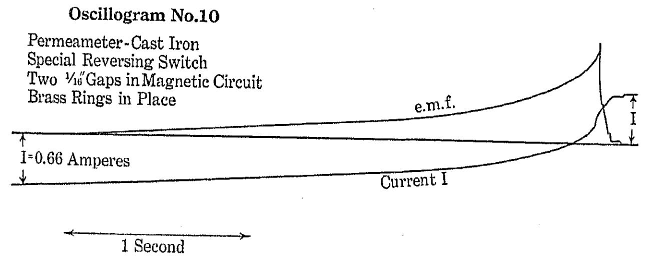


Fig. 7

the time of reversal about 40 per cent. The reason for this is that the inductance in the line tends to maintain the line current constant, and that the special reversing switch does not interrupt the line current, but simply reverses the current through the permeameter. The method of course failed to reduce the time of reversal when any type of switch was used that opened the first circuit; that is it opened the line current; before it closed the circuit in the opposite direction. This is due to the fact that in this case the extra inductance placed in the circuit is added to that of the permeameter windings and increases the time constant of the circuit.

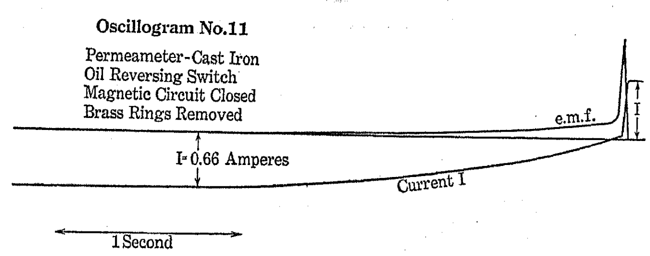


Fig. 8

In Fig. 8 oscillogram 11 shows the effect upon the time of reversal produced by removing the brass ends of the coils. Oscillogram 11 was taken with the magnetic circuit of the permeameter closed, and a magnetizing current of 0.66 amperes. The oil-reversing switch was used in taking this oscillogram. A study of this oscillogram shows that the secondary e. m. f. continues for 1.7 second, as against 3.9 seconds, shown in oscillogram 9, Fig. 6, taken under identical conditions, except as to the ends of the brass spool.

The investigation of the reversal of the current also brings out the importance of avoiding short-circuited paths in which eddy currents may be set up by the reversal of the magnetic flux.

Silicon Steel Permeameters

A number of tests were made with these two permeameters. Two of the oscillograms, 12 and 13, which were taken under the same conditions are reproduced in Fig. 9. Oscillogram 12 is for the solid silicon steel core permeameter and 13 for the laminated silicon steel core. A study of these shows that it required 0.23 seconds to reverse the current in the solid core permeameter compared to 0.08 seconds to reverse the same number of ampere turns in the laminated-core permeameter. The oil-reversing switch was used in both cases and the magnetic circuits were closed. In both cases the secondary e. m. f. dropped to a negligible value by the time the reversal of the current was completed. This test brings out the superiority of the laminated-core permeameter as far as the time of reversal of the magnetizing current is concerned.

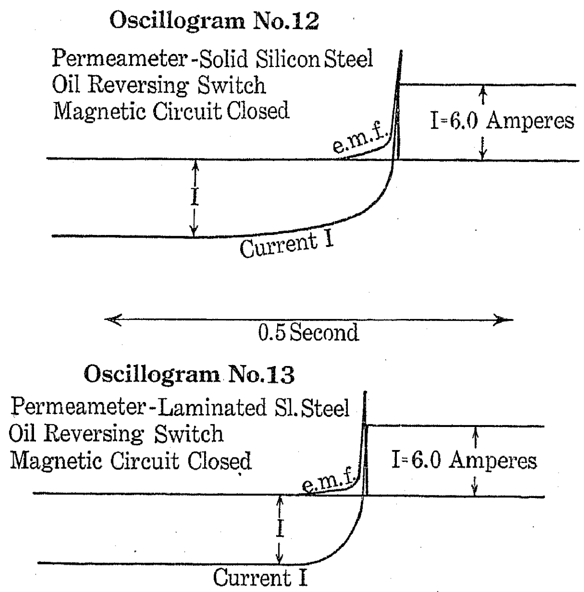


Fig. 9

Conclusions

The results of this investigation bring out clearly: *First,* the necessity of avoiding the presence of any unnecessary short-circuited path, which is cut by the flux of a permeameter. The frame upon which the coil for measuring the flux through the specimen is wound should be of such construction as to prevent the flow of eddy currents.

Second, that the use of a switch operating under oil will shorten the time of breaking the current.

Third, that the laminated-core type of permeameter will materially reduce the time required for the reversal of the current and thereby make the use of a short period ballistic galvanometer possible in magnetic testing. This will reduce the time required for a test.

The authors wish to thank Mr. H. C. Willis and Mr. W. C. Ball, postgraduate students in Electrical Engineering at The Johns Hopkins University, for their assistance in preparing this paper.

Discussion

S. L. Gokhale (by letter): In this discussion I intend to limit my remarks to two points, namely:

(1) The choice of the ballistic galvanometer for permeametry.

(2) The use of shunts to control the currents in the several magnetizing coils of a permeameter.

These remarks have reference to findings No. 1 and 3 in the 'conclusions' of the paper under discussion.

In the history of the ballistic galvanometer there was at one time the belief, that a ballistic galvanometer should have little or no damping if it is intended to function properly as a ballistic galvanometer. It is beginning to be recognized now, that the relation of ballistic function to freedom from damping is purely accidental, and that an overdamped galvanometer is not only as good, but in fact much better than the undamped galvanometer for purpose of ballistic measurement.

A ballistic galvanometer is a galvanometer for measuring an electric impulse by transforming it into a mechanical momentum in the first place, and subsequently into a deflection, in which form the impulse is ultimately measured. The impulse under measurement is either directly a current impulse $\int$ idt, or a votlage impulse $\int$ edt (although this also is converted into a current impulse before it affects the galvanometer). In the first case the total impulse is expressed as a charge or quantity of electricity: $\int \mathrm{idt} = Q$. In the second case the impulse which is generated by change of magnetic interlinkage is expressed in terms of equivalent flux change, $\int \mathrm{edt} = n \, \Delta \, \phi$. A ballistic galvanometer used for measuring $\int$ idt or Q may be called a quantometer; when used for measuring $\int$ edt or $\Delta \, \phi$ it may be called a flux meter. The relation of Q to θ or of $\Delta \, \phi$ to θ, (where θ represents the deflection), expressed generally as a graph, is the calibration of the galvanometer. (The term quantometer, used in the sense of fluxmeter as I have defined fluxmeter above, was first used by Mr. R. Bettie in the *Electrician* Dec. 25, 1902, p. 383).

The calibration of a ballistic galvanometer, may be obtained either empirically by a direct measurement of a known impulse, or indirectly by mathematical computation based on dynamic principles. In the latter case, the necessary data to be used as a basis of computation is obtained by measurements with a steady direct current, of known amount. The legitimate use of the dynamic formula involves in theory the fulfillment of two conditions, (1) long period, and (2) no damping. From this point of view a ballistic galvanometer may be defined as a galvanometer of long period and little or no damping. The first condition was easy to fulfill and was little thought of. The ballistic galvanometer thus came to be defined merely as a galvanometer of little or no damping. It is so defined even now in college textbooks, and dictionaries.

But times have changed; ballistic galvanometers for magnetic measurements are now generally calibrated directly by measurement of a known flux or interlinkage. The calibrating standard (which is frequently a mutual inductor) has its secondary coil in series with the test coil and galvanometer. Under these circumstances damping has no influence on the final results.

On the contrary, damping has some decided advantages, particularly when the damping is electromagnetic and follows the law.

$$r = - c. \, d\theta \, / \, dt$$

i. e., the resistance to motion due to damping is proportional to the angular velocity.

When an impulse is made up of two equal and opposite impulses with an interval of time between them, the total impulse being zero, the resultant deflection ought to be zero also. Such a case arises in the Bureau of Standards method of measurement by the compensated double-yoke permeameter. With an undamped galvanometer, it is impossible to say when the flux in the various test coils is balanced. Users of the compensated double-yoke permeameter are therefore compelled to use some type of over-damped galvanometer; the Grassot's fluxmeter seems to be the most popular (A. I. E. E. 1915 p. 2602, Fig. 2 B) although I prefer a regular ballistic galvanometer with a high internal resistance in the form of active copper.

The superiority of the overdamped galvanometer over the underdamped, may best be demonstrated by the following experiment:—

A ballistic galvanometer practically undamped when used with a high resistance *Rg.* in series with it, is connected as shown in Fig. 1. The reversing switch is operated with the key down. The impulsive emf. produces a ballistic deflection. The key is then released; under the influence of the critical damping shunt, the galvanometer returns to zero and comes to rest, without any

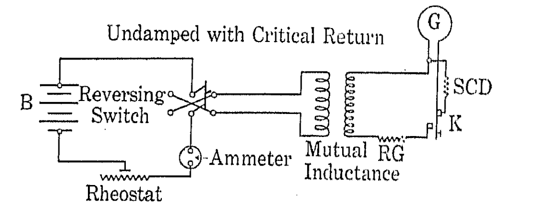


Fig. 1—Scheme of Wiring for Undamped and Overdamped Galvanometers

oscillations and with the least possible delay. The time taken up in the forward movement is ¼ of the complete free period. The time for return to zero is practically equal to the free period. During this time the reversing switch is generally brought back to the original normal position, so as to be ready for a second throw. The current is adjusted so as to produce a ballistic deflection of 100 mm.

The reversing switch is now operated forward and backward with the key down throughout the double operation. The final effect on the galvanometer ought to be zero; but the actual effect is positive or negative, large or small depending on the interval between the two reversals. It is never zero except by chance.

A low-resistance shunt *Sg* is now connected as shown in Fig. 2, reducing the resistance *Rg* until the deflection for the single reversal

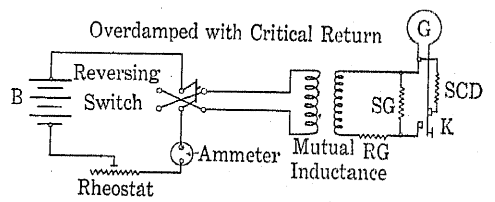


Fig. 2

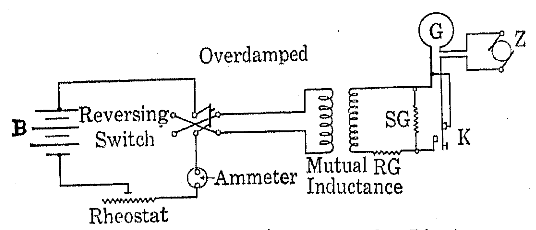
is 100 mm. as before. It will now be seen that the double reversal produces a double kick, the final result being a zero deflection as it ought to be. With a highly overdamped galvanometer the resultant deflection is zero even when the two reversals are separated by an interval of several seconds. Instead of the critical damping shunt, it is now the practice of our laboratory to use a zero-setting inductor Z (Fig. 3). Instead of waiting for the galvanometer to come to zero, the operator is then able to bring it to zero much more quickly and easily by turning the knob of the inductor.

As to the oscillographic study as a research by itself, Messrs. Kouwenhoven and Berry are entitled to credit for patience, and care with which the work seems to have been done; so to the practical value of the research, I expect the contribution to prove of value somewhere, but I fail to see its usefulness in perme-

ametry. For example, the authors have demonstrated the superiority of a laminated core over a solid core, in permitting the use of a short-period galvanometer (presumably undamped) in order to shorten the time spent in the test. This sort of solution ignores the nature of the problem.

The problem which confronts us in permeametry is not the choice of a test sample to suit our galvanometer, but the choice of a galvanometer to suit the sample. I had an occasion once, to measure the flux in the shaft of a turbo-generator. In such a case, a laminated sample for test is out of the question and one must choose his galvanometer to suit the material as it is. The research in question, was obviously undertaken with the behavior of undamped galvanometer in view. But as I said before, the times have changed. Twenty-two years ago Mr. R. Bettie demonstrated the possibilities of an overdamped galvanometer, and its immunity against errors due to slow or interrupted magnetic changes. (*Electrician*, Dec. 25, 1902, p. 383). In 1913 I had commenced an oscillographic study of the reversing switch, in connection with some difficulties I encountered in my experiments on the well known "Burrow's method," but the overdamped galvanometer seemed to be a complete solution of our difficulties and the oscillographic study was therefore discontinued.

It is true, that an overdamped galvanometer when deflected is very slow in coming back to its zero position; but one need not infer from this that such a galvanometer is slow for use, the fact being just the reverse. When a momentary impulse is applied, the overdamped galvanometer jumps almost instantly to the



Z = Zero Setting Inductor (Faraday Disc.)

FIG. 3

final position, where it stays long enough to take the reading conveniently; then it starts to creep back slowly, and would be several minutes in reaching zero if we had to depend entirely on its own free motion. It can however, be brought to zero very quickly by the zero setting inductor, or quicker still by reversing the first impulse when such reversal is possible and permissible, as it always is in permeametry. Consequently, the overdamped ballistic galvanometer is not only more accurate, but it is also easier to read and far more expeditious to use.

Messrs. Kouwenhoven and Berry have reached the conclusion that it is necessary to avoid a short-circuited path which is cut by the flux of a permeameter. This fact has long been known and has long been recognized as a serious obstacle in the way of simplification of modern permeametry. In Burrows' permeameter there are several magnetizing coils which call for independent adjustment of current. Dr. Burrows has recommended the separate-circuit system with separate reversing switches. At first we followed this plan, using a long-period ballistic galvanometer without damping; but the balancing of the circuit proved an impossible task. We therefore abandoned the separate circuits and substituted for it a system of divided circuits such as was partly used by Dr. Burrows. This was a considerable improvement, but not enough. About this time I discovered, (what was already well known to others), that an overdamped galvanometer was the best remedy for the trouble. At this time I learned also, that with such a galvanometer, I could use separate circuits just as easily as the multiple circuit. But both methods had their advantages as well as disadvantages. The separate circuits involves the use of a gang switch, which becomes a very complicated affair when the measurement is extended to hysteresis. The divided-circuit system does not need a complicated gang switch so far as permeability measurements are concerned, but it is not available for hysteresis

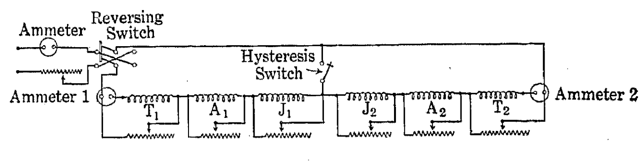


FIG. 4—SCHEME OF WIRING WITH FRACTIONIZING SHUNTS (UNCOMPENSATED)

measurements. A third conceivable system is the series system in which the several magnetizing coils are connected in series, the necessary independent control being obtained by the use of separate shunts for the several coils, see Fig. 4. Here we meet the condition referred to by Messrs. Kouwenhoven and Berry. Such a system, if it were permissible, would simplify the work considerably; but is it permissible? It is possible that the shunt would retard the change of flux in one coil much more than in

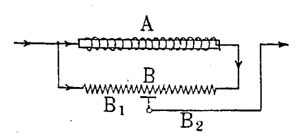


FIG. 5—FRACTIONIZING SHUNT (SELF-COMPENSATING)

A Magnetizing Coil
B Compensating Rheostat Shunt
B_1 Shunt Section
B_2 Compensating Section

the other. The difference of time might be so great as to make the balancing of the several circuits extremely difficult if not absolutely impossible. It is however possible to minimize the evil by the use of a self-compensating shunt, see Fig. 5. In this arrangement a part of the rheostat acts as a shunt and the other part as a self-compensating series resistance. The total resistance of the local circuit is therefore high and remains constant, being unaffected by the setting of the shunt. This scheme

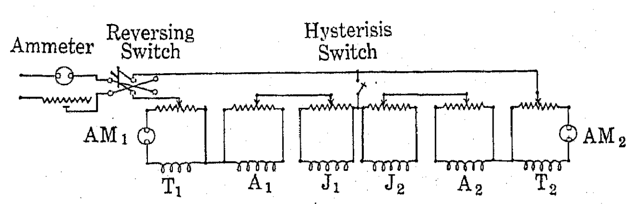


FIG. 6—SCHEME OF WIRING WITH FRACTIONIZING SHUNTS (SELF-COMPENSATING)

of wiring has been incorporated in the permeameter equipment of the General Engineering Laboratory of the General Electric Co. and has proved very simple and expeditious. (See Fig. 6.) This diagram represents schematically the complete magnetizing circuit, omitting the auxiliary apparatus. Section 1 is used for permeability tests up to H 25, the coils T_1, A_1 and J_1, being the three magnetizing coils characteristic of the Bureau of Standards method. Section 2 is an exact copy of Section 1, except that its polarity is reversed. Normally this section is cut out by means of the short circuiting switch "Hys.". When this switch is

opened, the coils T_2, A_2 and J_2 become operative and the resulting value of H is less than that produced by section (1) alone. This gives a point on the hysteresis loop. Section 3 (not shown in the diagram) contained more layers of coils T and A which raises the magnetizing force to 200 gilberts in steps of 25 gilberts or less as required. Tests for permeability are made by operation of the reversing switch alone, and for hysteresis, by the operation of the single switch "Hys". A complete discussion of the permeameter and the method of using it would be beyond the scope and purpose of this discussion. I have mentioned it here merely to show how by means of the self-compensated shunt, we avoided the difficulty referred to by Messrs. Kouwenhoven and Berry and at the same time eliminated the gang switch, which has always been since its invention, the dread of the operator who had the task of manipulating it.

W. B. Kouwenhoven: Mr. Gokhale's discussion of our paper and his description of the method he uses in operating the Burrow's Permeameter is of interest and value. Mr. Berry and I refer to an entirely different instrument, namely the Simplex Permeameter. This instrument is built with a U-shaped yoke usually of solid material. We found that better results could be more quickly obtained if the yoke were made of laminated material. Nothing was said regarding the use of laminated specimens. In our work an overdamped galvanometer was used when needed.

Mr. Gokhale states in his discussion that he fails to see the value of this paper, and in reply I will say, that, at the June Convention of the A. S. T. M., the engineers of two of our large electrical companies told me that they were now using laminated core simplex permeameters with excellent results.

Power Plant Auxiliaries and Their Relation to Heat Balance

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Review of the Subject.*—In the larger central station steam plants, efforts to increase the over-all economy and ease of operation were responsible for the use of motor-driven auxiliaries receiving their power from auxiliary turbogenerators, called house turbines, the exhaust from which is used for heating feed water. This scheme is reliable and economical. It is handicapped by the fact that over a considerable range of load some of the auxiliary power must be taken from the main bus or some of the energy generated must be fed to the main bus. This resulted in transfer motor-generators, etc., which were additional complications.*

Still greater economy is possible by bleeding steam from the low-pressure stages of the main unit to heat the feed water. Considerable heat that would otherwise be rejected with the condensing water is reclaimed, thereby allowing of a smaller condenser than would otherwise be required, the performance of the main turbine also being improved due to somewhat relieving the congestion of steam in the low-pressure stages.

To utilize fully the advantages of stage bleeding, the auxiliary power must be obtained from the main turbine. In order to insure an uninterrupted supply of auxiliary power to the essential drives it is suggested that an auxiliary generator be connected to and driven by the main turbine, thereby supplying the necessary reliability as long as the main turbine is available for load.

Due to the advantages of variable speed drive for circulating pumps, boiler feed pumps, etc., and the necessity of having direct current for excitation, a direct-current generator may be used to advantage. The use of the direct-connected auxiliary generator with stage heating, gives a maximum of flexibility, is especially reliable in that it entirely eliminates all small turbine and gear troubles, and permits of the use of the unit scheme of grouping and supplying auxiliaries, with what appears to be a maximum of economy at no apparent increase in cost.

Furthermore, the use of closed heaters with the fresh water storage located within the condenser hot well seems to offer an economical and highly satisfactory solution for the de-aeration of the feed water and to eliminate the possibility of the water picking up further air after leaving the condenser.

THE steam power station designer is confronted by the following basic requirements:

1. Maximum reliability is essential for the existence of the enterprise.

2. Economy demands that the station heat cycle approach the Carnot cycle as nearly as possible. This results in getting the maximum percentage of heat units fed into the furnace delivered to the station bus in the form of merchantable power.

3. Simplicity and flexibility of operation so that the calculated results may be obtained with minimum effort under wide variations in load, temperature and in the quality of fuel obtainable.

4. The proper balance must be maintained between initial outlay and cost of fuel so as to produce power at the minimum net cost per unit, including fixed charges as well as operating expense.

The all-steam-drive type of auxiliaries has been developed to a high state of reliability. It is economical at that station load which will utilize all of the exhaust steam. The economy decreases with the load, becoming very uneconomical at low loads. Where turbine drive is used, the economy of the auxiliary turbine falls off with reduction in speed. This is an important consideration, as the demand on practically all auxiliaries decreases with the load on the station. The cost of auxiliary steam and exhaust piping is quite a factor inasmuch as the surface exposed to radiation is large. With the advent of higher steam temperatures, the reliability of the auxiliary turbines will be decreased, while the first cost and maintenance cost will be greater.

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The all-electric-drive with power obtained from the station bus is simple to operate, but is not reliable as any disturbance on the transmission system may affect the station auxiliaries. Electric motors and controls have been developed to a high state of perfection and of themselves are very reliable, take up a minimum of space and require but little attention, the maintenance cost being lower than is the case with steam-driven auxiliaries.

The feed water may be heated by bleeding the main unit to open heaters which except in the case of specially designed turbines is not very economical unless bleeding takes place at several points so as to always bleed the steam at slightly above atmospheric pressure with variations in load. Special bleeder valves must be used in order to insure the safety of the unit in case of total loss of load.

Efforts to obtain the reliability of steam drive without the attending disadvantages, together with the advantages of electric drive without the questionable reliability of securing the current for motors from the main bus, resulted in the house turbine which has the additional economical advantage of lower water rate than individual steam auxiliaries, due to their relative size, which is further augmented when the house turbine is operated at partial vacuum. The house turbine gives a clean layout due to the absence of numerous steam and exhaust lines, is reliable and simple to control. The water rate is not so good as that of the main unit and the dual transformation of power from mechanical to electrical and back to mechanical is seldom more than 80 per cent efficient. Due to the advantages of variable speed drive which is most economically obtained by the use of d-c. motors, the house turbine must either drive a d-c. generator through a reduction gear with its attendant disadvantages or a rotary convertor must be used which still further reduces the efficiency of conversion.

Inasmuch as all efforts for economy point towards the utilization of low-temperature heat and towards the most economical source of auxiliary power, attention in both cases is directed to the main unit.

The main unit is inherently more economical than any of the smaller auxiliary turbines or the house turbine. In it the steam expands to the lowest pressure compatible with the available condensing water. Obviously, therefore, if steam is bled from a number of successive stages of the turbine and used to heat the feed water which is passed successively through surface heat interchangers, the power obtainable from a given amount of steam will be greatest when that steam is bled at the lowest pressure, and the power obtainable per lb. of steam will decrease as the pressure at which the steam is bled increases. Inasmuch as the condensate is usually cooler than the exhaust steam entering the condenser it would be most economical, theoretically, to bleed steam from each of the lower stages of the unit. Practical considerations will probably limit the bleeding to three or possibly four stages, the highest temperature stage being able to deliver relatively hot feed water at light loads. It must be remembered that steam bled from the main unit decreases the congestion in the low-pressure stages of the turbine, thereby improving the total performance of the unit somewhat and also decreases the duty on the condenser.

It is quite probable that still greater amounts of low head heat can be bled from the turbine prior to rejection to the condenser and used to preheat combustion air for the boiler furnaces. While bleeding steam to preheat air is somewhat novel and requires some further development of heat interchangers for this purpose, it shows considerable thermal gain. Present types of stokers using forced draft can handle air up to a total temperature of several hundred degrees Fahr. without any great difficulty. Complications would undoubtedly result from the attempted use of 300 deg. air on pulverized-coal-fired furnaces where air-cooled furnace walls are used. In the writer's opinion, however, the air-cooled powdered coal furnace is destined to be short lived, due, first, to the fact that it is obviously uneconomical to allow high head heat, which should be utilized directly for generating steam, to be used for preheating furnace air when there is so much low head heat readily available; secondly, the air-cooled furnace has not been entirely successful, the furnace maintenance undoubtedly being very high and the time the boiler is out for furnace repairs must be a considerable percentage of the service hours. The solution of this question would seem to lie in the development of a boiler and furnace so designed that the water heating surfaces absorb a large percentage of the heat which is now taken up by the furnace walls. This will allow of still higher furnace temperatures with the attending increase in efficiency.

It would, therefore, seem that the greatest opportunity for economy lies in the use of power from the main unit for auxiliary drive, this power being applied as directly as possible, together with the use of the maximum economical amount of low-temperature steam bled at several points from the main unit. The question now arises as to whether this scheme provides the essential flexibility and ease of operation.

By the use of motors it is possible to obtain compact drives, reduction gear troubles are eliminated and the mass of auxiliary steam, exhaust and water-cooling pipes is eliminated. Where d-c. motors with field control for speed variation are used there is no appreciable decrease in economy of the drive with decrease in speed. When properly designed ball bearings are used, practically no attention is required and the units may be started, stopped or the speed varied from remote points. The steel industry has investigated the ball-bearing motor very thoroughly and have made out a very strong case in its favor. Due to their location it is desirable to be able to control the speed of forced and induced draft fans and the stoker and clinker

grinder motors from the boiler room firing floor, and they are usually so located as to make it impracticable to give them relatively continuous attention without having special attendants at the several elevations where they are located. Boiler-feed and house-service pumps should also be capable of operating under the control of automatic regulators with a minimum of attention. Variable-speed d-c. motors lend themselves to this form of control just as readily as do steam turbines without the attendant change in economy with change in speed and without the attention and troubles incident to turbine packings, bearings and the possibility of damage due to overspeed on account of loss of load.

When the extra storage of boiler water is located in the hot well of the condenser and the makeup admitted directly to this hot well it is feasible to keep the water in the hot well at the temperature of the exhaust steam by raising the temperature of the hot well by the condensate from the lowest temperature feed heater, plus some additional steam bled from the last stage of the main turbine when necessary. This eliminates the use of de-aerating apparatus and makes possible the location of the make-up float valve so that it is readily accessible to the condenser attendant who can also look after the boiler feed pumps. The division of duty between the hot-well pumps and boiler-feed pumps can be so arranged as to allow minimum water pressure on the closed heaters compatible with reliable operation. The hot-well pump can be run at constant speed, the total speed variation being taken care of by the boiler-feed pump. The duty on the circulating pump can be varied to compensate for changes in load and of circulating water temperatures. This eliminates the necessity for the use of two pumps of reduced size, one being used for cold water or light loads, two for warmer water and high load; it being possible to obtain somewhat better performance by using one pump and varying the speed. Where two pumps are used for added reliability it is more economical to run both at reduced speed than to shut one down and speed up the other. Practically the same reliability can be secured by interconnecting the discharge lines on adjacent condensers and providing somewhat larger driving motors, thereby enabling one pump to serve two condensers in emergency with only a slight reduction in vacuum. By the use of a properly-designed motor the large low-speed circulating pump becomes the most reliable piece of auxiliary equipment.

The static or steam jet type of vacuum pump has proved very reliable in marine service and is becoming increasingly popular in central station service. When equipped with inter and after condensers, using condensate, and located respectively before and after the low-temperature bleeder heater, they provide a compact and reliable unit whose performance compares very favorably with a hydraulic or reciprocating air pump at a somewhat smaller first cost, and should show a lower maintenance cost and require less attention than either of the other types. The air and non-condensible vapors from the bleeder heaters can be discharged directly to the inter-condenser thereby lowering the duty on the primary jets, which should use the minimum of steam possible as it is desirable from an economical point of view to do nothing which will limit the amount of steam bled from the main unit to the low-temperature heater. Most of the operating difficulties experienced with steam jet air pumps have been due to poor performance on the part of the reducing valve supplying steam to the jets. This may be entirely eliminated by the use of a properly-designed orifice installed in parallel with either a hand or solenoid operated valve which may be opened either manually or automatically in case of a drop in steam pressure.

Since stage bleeding together with electric drive offers opportunity for higher efficiency with a maximum of flexibility at a cost as low or lower than with any other form of auxiliary drive, it now remains to provide an arrangement for obtaining power from the main units in such a manner as to guarantee continuity of auxiliary power supply eliminating all chance of trouble on the main bus being transmitted in any manner to the auxiliary bus.

All power plant auxiliaries fall into three general classes with respect to the necessity of their having a continuous power supply.

First, equipment which does not require a continuous power supply, such as coal and ash handling equipment, circulating water screens, sump pumps, machine tools, etc. This class of equipment can be eliminated from the present discussion as continuity of service is not essential.

Second, the auxiliaries which are needed only when the station is carrying load and which have to be shut down in case of an interruption which greatly reduces the plant load. To this class belong forced and induced draft fans while coal feed and clinker grinder drives might well be included as they are normally supplied from the same circuits as supply the fans. As it is essential to check the fires as soon as a general interruption occurs it is an advantage to have fan motors trip out due to low voltage. Where automatic starting panels are used, the service is automatically restored as the main bus voltage comes back. In actual practise following a general interruption the station load does not come back immediately after the restoration of full voltage it usually being at least ten minutes after the restoration of service before the load is normal. This allows ample time for furnace conditions to become normal. Where the nature of the interruption is such as to result in low frequency and somewhat lowered voltage, trouble may be experienced in holding the motors on the line due to the operation of the low-voltage relays. This can be readily overcome by arranging to throw the affected motor controls on the station control battery. The individual controllers can then be brought up to full speed setting until the

trouble is cleared, should this be necessary. Where the total number of boilers is comparatively few the contactors can be provided with a mechanical latch which will hold them in the full-speed position.

It is of course essential that the proper safeguards be used to minimize interruptions due to failure of the transformers, circuits and switching equipment between the main bus and the several motors.

The writer has operated a fairly large generating station for about five years where the boiler auxiliary drives were fed from the main bus. There have been in that time the usual number of system interruptions and

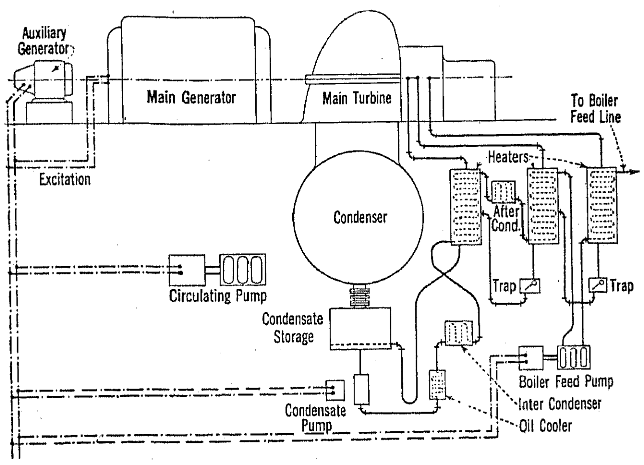

Fig. 1

disturbances without in a single instance causing any embarrassment due to inadequate power for the stoker fan and coal feed motors. It is the regular practise to have in service two banks of transformers either of which is of sufficient capacity to carry the essential service. The two transformer banks are supplied from either end of the main bus, each serving one-half of the auxiliary circuits and the other plant auxiliary load. The transformers are protected by necessary relays. The switches are so interlocked that if either high-tension transformer switch opens the corresponding secondary switch opens which in turn causes the switches supplying the non-essential auxiliary circuits to trip and parallels the two sections of the low-tension station service bus.

Third, the auxiliaries, which it is essential to keep in service in order to insure continuity of service and which it is essential to keep in service during times of interruption, are the exciters, boiler feed pumps and circulating pumps. Where the boiler-feed make-up storage is a part of the condenser hot well it is also essential that the hot-well pumps be kept in service.

The service for this third class of auxiliary drive can be most economically and advantageously supplied by a separate generator connected directly to and driven by each of the main turbine units.

Direct-connected exciters have been in quite general use for a number of years and have been so satisfactory that there seems to be no reason why the size of this piece of apparatus should not be increased sufficiently to take care of the additional load imposed by the so-called essential auxiliary drives. This auxiliary generator need be relatively small, in fact only about two and one-half per cent of the capacity of the main unit. This arrangement will require, as does the direct-connected exciter some auxiliary source of power for starting up. This can be taken care of by a motor generator, supplied from the house service transformers which if necessary can be equipped with a non-condensing turbine, exhausting to atmosphere, for starting up in case of a complete shutdown of considerable duration.

The attached sketch shows such an arrangement for a 25,000-kv-a. unit. The auxiliary generator is rated at 500 kw. 250 volts direct current. A 3000-gallon water storage will be provided in the condenser hot well. Three-stage bleeding will be used, the condensate from the high and intermediate temperature heaters will be trapped back to the next lower-temperature heater while the low-temperature heater condensate will be trapped back to the hot well where it will be used to heat the condensate and make up to the boiling point in order to drive off entrained air.

It will be noted that this scheme provides auxiliary power from the most efficient source, allows of the most economical scheme of feed heating without any complications. The auxiliary wiring is on the unit principle, the auxiliaries and generator being grouped together do not require long runs for low-voltage auxiliary feeders and there is no transformation loss in obtaining direct current for these drives. Careful analysis would seem to indicate that this scheme provides the necessary reliability without calling for any compromise that affects the economy of the general layout, at a first cost no greater, if not actually less than any of the arrangements in general use, with a maximum of flexibility of operation at varying loads, with a minimum of operating attendants.

Discussion

H. W. Brooks (by letter): Mr. Penniman's paper is of intense interest to many designers now working on *new* plant construction. There is, however, a vast field for application for heat-balancing devices in plants designed prior to the last three to five years as there are undoubtedly enormous losses taking place in many of these older plants owing to lack of proper heat balance. In private consulting practise last year the writer encountered a very interesting problem of this character in the plant of the Chicago, Aurora and Elgin Railroad Co. at Batavia, Ill., where a dual-drive exciter set of 300 kw. capacity was finally designed in cooperation with the manufacturer. The steam end consisted of a 200 lb. pressure, 150-deg. superheat, non-condensing, direct-connected turbine of the impulse type, while the electric drive end consisted of a three-phase, 25-cycle, 2300-volt, phase-wound, induction motor, both machines being direct-connected to the 300 kw. generator mounted between them and functioning at 750 rev. per min.

It will be noted from the following functions performed by this set that it covers all of the merits of the basic Clarke patents

on dual-drive heat-balancer sets and in addition covers automatically a number of new functions not heretofore performed automatically, and by so doing eliminates attention on the electrical end of the set previously required by either the operator on the engine-room floor or the switchboard operator. The complete functions of the set follow:

1. In case of failure of voltage on main a-c. generators motor cuts out and d-c. load is taken by the turbine.

2. In case of failure of frequency on main a-c. generators motor cuts out and d-c. load is carried by turbine.

3. In case motor is tripped out on account of under voltage it is restored to the line upon resumption of proper voltage.

4. In case motor is tripped out on account of low frequency it is restored to the line upon resumption of proper voltage.

5. In case of reversal of a-c. power caused by the induction motor, driven as an asynchronous generator, motor is tripped out and entire load carried by turbine and after turbine speed has been so reduced as to restore normal direction of flow on the alternating-current line motor is restored to the line.

6. In case of accident to the a-c. motor, motor trips and latches out and load is carried by turbine until such time as motor is repaired.

7. The various relays such as the frequency relay, selective watt relay, etc., are arranged with suitable interlockers in connection with the low-voltage relay equipment and the starting equipment so that each will operate independently.

8. Motor is capable of being thrown on the line either from zero speed or from full speed, which latter condition occurs when the set is started from the turbine end.

9. On reversal of d-c. power the d-c. breaker is open but need not necessarily be recloseable automatically upon restoration of normal direction of power flow. Should this occur, however, the steam and control must trip automatically to prevent running away of the machine in case low feed-water temperature should cause high-pressure valve at that time to be open.

10. Secondary motor control is entirely automatic, functioning at all times as the primary or breaker functions.

11. In general, in the event of any accident to the motor the turbine picks up the load and carries any part or all of it, and similarly in the event of an accident to the turbine the the motor must instantly pick up the load and carry any part or all of it. The combination is so arranged that the turbine will carry variable loads automatically normally, dependent upon the amount of steam necessary to pass through the turbine to keep the feed-water temperature up to 212 deg., the motor automatically pulling the balance. Load is automatically shifted from one method of drive to the other in such manner that the regulation of the direct current supplied will not be visibly affected.

12. Normally the turbine exhaust is automatically regulated in such manner as to pass only sufficient steam through the turbine to supply necessary exhaust steam heat to the incoming feed water so that no steam is lost by exhausting it to the atmosphere, the remaining load being carried by the motor.

13. Emergency features, however, are provided on the turbine as above mentioned so that if for any reason the electric supply to the motor should be cut off, the turbine would automatically open its valves and continue to drive the exciter without any interruption in the supply of excitation, even if by so doing considerably more exhaust steam would be supplied than would be necessary to raise the feed water to 212 deg., which means that in such abnormal circumstance exhaust steam would be wasted into the atmosphere.

Frank G. Boyce (by letter): I agree with the author that it is desirable to approach as near as possible all-electric drive for auxiliaries. However, I feel that it is good practise to have at least one boiler feed pump in the plant which is steam-driven, the thought in this being that continuous supply of water to the boilers is absolutely necessary in modern plants where the storage capacity in the boilers is reduced to a minimum and with the tendency towards higher ratings. It seems both economical and desirable to bleed the main turbines for stage heating of the feed water and preheating the combustion air for stokers.

I would suggest that instead of a direct-current generator being used mounted on the end of the main turbine shaft for source of auxiliary power, an alternating-current generator be used for this purpose and that the shaft be further extended to provide space for an exciter. The reasons why I would suggest this instead of the scheme suggested by the author are as follows:

1st, I am rather inclined to believe that the reliability of the unit would be somewhat reduced if the excitation were taken from the same generator which supplied power to the auxiliaries around the plant, so that troubles in the various motors and equipment would be transmitted to the field circuit and rotor, which might cause interruptions to service from the main unit.

2nd, I would suggest that the boiler feed pumps be driven by constant-speed alternating-current motors, except one emergency pump which should be steam-driven. This arrangement works very satisfactorily and instead of maintaining a constant differential pressure between the feed water and the boiler pressure, a constant feed-water pressure is held somewhat above the normal steam pressure. This will operate perfectly satisfactorily with modern types of boiler feed regulators. This, therefore, eliminates the necessity for variable-speed direct current motors on boiler feed pumps.

Of course, variable-speed drive for stokers is necessary and this could be obtained very nicely by the use of two rotary converters of sufficient capacity to furnish power for only the stoker motors. This makes a rather economical installation. However, it is also possible to obtain stokers driven by hydraulic pistons which eliminates the necessity for direct-current variable speed on the stoker drive.

The only drive now for which direct-current variable-speed motors seems necessary is condenser pumps. I suggest that a very economical arrangement is two pumps of different sizes installed on the condenser, one pump to be used at light load, a larger one at medium load, and both pumps to be used in case of very warm circulating water or heavy overloads.

I believe that the installation as mentioned, eliminating the direct-current variable-speed motor drives, will give as good satisfaction, will be more reliable, and the cost per kilowatt of investment will be less.

A. L. Penniman, Jr.: The discussion, by letter, of Frank G. Boyce admits the desirability of all-electric drive. The writer agrees that there should always be one boiler feed pump in a plant which is steam driven. This should be for purely emergency purposes and should take its suction from the service water supply. The turbine driving this pump may properly be allowed to exhaust to the atmosphere, as it will run only on very infrequent occasions.

Mr. Boyce's suggestion that an a-c. generator be direct connected to the main turbine shaft for auxiliary power is entirely feasible. It might be somewhat cheaper than a d-c. generator. A-c. motor drive for auxiliaries might also be slightly cheaper than d-c. motors. The combined efficiency of the generator and motors would, of course, be lower where alternating current is used with the motors running at reduced speed. In the writer's opinion the alternating current would be somewhat more reliable, due to the elimination of commutators and due to the fact that a short circuit could be much more readily cleared up than if direct current be used.

Variable speed motors driving the boiler feed pump are advisable though not necessary. On the closed system outlined in the paper the duty can be so divided between the hot-well pump and the boiler feed pump that the bulk of the work is done by the hot-well pump, leaving a relatively small amount for the boiler feed pump. The hot-well pump may then be driven at constant speed, the desired pressure being maintained by varying the speed of the boiler feed pump motor. It would seem that this

motor need be capable of carrying only 25 per cent of the power desired by the two pumps when both are operating at rated capacity.

The proof also includes a statement from the JOURNAL of Of February, 1924, having to do with the so-called Clarke Patents on dual-drive exciter sets in which, what are apparently the claims of the Clarke patent are stated as functions performed when the house generator is driven from the main turbine shaft. This, of course, is not a matter of fact, as the writer cannot conceive how, under normal conditions, the main generator can be driven by the main turbine and in turn supply the power for driving the auxiliary generator which is hooked on the same shaft. There can be no question that the auxiliary generator is driven by the main turbine. It is not called on to perform any of the functions performed by the dual-drive exciter.

The writer's scheme permits of taking the maximum advantage of stage bleeding from the main unit, and at the same time obtain a reliable source of power for auxiliaries by driving the auxiliary generator by the main turbine. Whether the generator supplies alternating current or direct current is largely a matter of personal preference of the designer and balancing the increase in first cost of direct current against its economy in operation.

Shaft Currents in Electric Machines

BY P. L. ALGER and H. W SAMSON

Member' A. I. E. E.

Both of the General Electric Company

Review of the Subject.—*This paper describes the causes of, and remedies for, the existence of "shaft currents" or "bearing currents" which sometimes flow across the rubbing surfaces of the bearings of electric machinery, thereby gradually damaging the shaft and bearings.*

Up to the present time the only cause of shaft currents that has attracted any particular attention has been the use of sectionalized stators, and the published discussions have been chiefly confined to synchronous alternators. Fleischman[1] and others have shown that sectionalizing causes shaft currents for the reason that the extra reluctance of the joints causes an unequal division of the flux between the clockwise and counter-clockwise paths in the yoke, thus giving a resultant flux linking the shaft.

Applying the same method of reasoning used in the case of sectionalizing to the general case of any machine with segmental punchings, the following facts are shown:

1. *A principal cause of shaft currents in revolving electric machines is the use of poles and segments in certain ratios.*

2. *The frequency of the shaft current due to joints in the stator yoke is an odd multiple of the frequency of the stator flux, the frequency of the shaft currents due to rotor joints is an odd multiple of the rotor frequency, and these frequency multiples are determined by the ratios of poles to segments.*

3. *Machines with 4, 8, 16, 24, 32, etc., poles are especially likely to have shaft currents, and machines with 6, 10, 14, 22, etc., poles are relatively immune.*

4. *By the proper choice of the number of segments for use with any machine, or by the use of segments with offset dovetails, or both, shaft currents can be effectively eliminated in most cases.*

The possibilities of shaft currents being caused by homopolar action as the result of magnetic flux flowing in the shaft, or by other means, are discussed, and it is concluded that such causes are seldom important. A possible useful application for the theory of shaft currents in the design of a high-current transformer is mentioned, and the possibility of obtaining multiple frequencies from a stationary transformer in this way is shown to be dependent upon the presence of magnetic saturation.

A table of combinations of poles and segments that will cause shaft currents is given, and a bibliography of the subject is appended.

1. Reference No. 8.

CONTENTS

INTRODUCTION

A common source of trouble in revolving electric machines is the presence of electric currents flowing across the rubbing surfaces of the bearings. These currents make their presence known by blackening the oil, pitting the bearing, and, in extreme cases, scoring the shaft.

Figs. 1 and 2 show photographs of damage done to a shaft and bearings by these currents. Other photographs are given by Adler[2], who states that currents greater than 1¼ amperes per square inch of bearing surface will damage the shaft, but that currents of lesser magnitude will harm the bearings only.

The usual type of shaft current flows in a circuit consisting of the shaft, the bearing pedestals or end shields, and the base. Interruption of this circuit by insulation under the pedestals, as shown in Fig. 3, is the most usual method of avoiding trouble from this source. In machines with end shield bearings, however, it is very inconvenient to insulate, and in no case does the use of insulated bearings afford any pleasure to either the manufacturer or the operator.

This paper has been written in order to present some supposedly novel ideas on the causes of shaft currents and methods of avoiding them. As the published information on this subject is rather scattered, and, being chiefly in German periodicals, is relatively inaccessible to American readers, the previously established principles of the subject are also explained.

First the three possible causes of shaft currents will be described, then the two causes of minor importance will be briefly discussed, next the major cause will be carefully examined under two headings, then means for

2. Reference No. 6.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

avoiding shaft currents will be explained, and finally a possible field of utility for shaft currents will be mentioned.

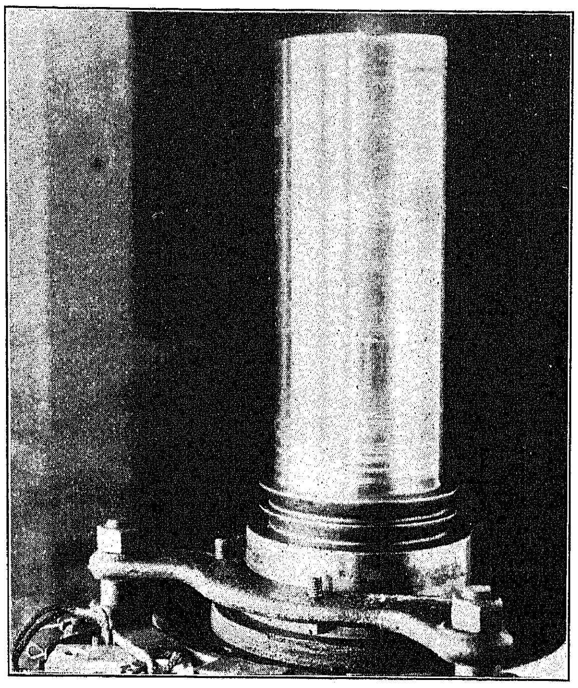

Fig. 1—Photograph Showing Effect of Shaft Currents on Shaft of 500-h. p. Induction Motor

Possible Causes of Shaft Currents

All shaft currents are due to the existence of an e. m. f. between shaft and bearing lining. There are three imaginable ways in which such an e. m. f. can be produced:

(a) By a direct or alternating flux flowing in the shaft.

(b) By a difference of potential between shaft and ground due to electrostatic effects, or to grounding of the rotor conductors to the core.

(c) By an alternating flux linking the shaft.

Of these, (c) is by far the most important, and the one which has occupied the greater share of the attention of previous writers. However, in order to leave a clear field for the study of (c) once it has been started, the minor causes (a) and (b) will be first considered.

Shaft Currents of Type (a), Due to Shaft Flux

If, for any reason, a magnetic flux flows in the magnetic circuit consisting of shaft, bearings, and base of a machine, a homopolar voltage will be induced in each bearing due to the revolving shaft cutting the radial lines of flux passing from shaft to bearing. The voltages so induced in the two bearings will exactly neutralize, if the flux passing through one bearing is equal to the flux returning from the other bearing to the shaft. Hence such shaft voltages will chiefly cause local currents within the bearings, and insulation of the bearing pedestal from the frame will be of little use, except in so far as it increases the magnetic reluctance of the flux path. The most convenient paths for such currents to flow in are from shaft to bearing through an oil ring placed at one end of the bearing and back from bearing to shaft through another oil ring placed at the other end of the bearing.

Whether the shaft flux is direct or alternating, a homopolar voltage will still be induced in the shaft, of the same frequency as the flux. Shaft fluxes will only appear as a result of a current linking the shaft. Only when an unsymmetrical construction of the windings is employed, such as sectionalized end rings for a squirrel cage, or a wave winding with only one bar per slot, will the multipolar machine be subject to this trouble. Homopolar machines, however, are likely to have a good deal of shaft flux.

An inspection of the developed two-circuit wave winding of Fig. 4 shows that the currents in the two circuits encircle the shaft in opposite directions, so that any inequality of these currents will give rise to a shaft flux.

A test was made on a large induction motor with such a winding on the rotor, to determine if appreciable shaft currents could be produced by shaft flux. One circuit of one phase of the two-circuit rotor wave winding was opened, and the motor was then operated under

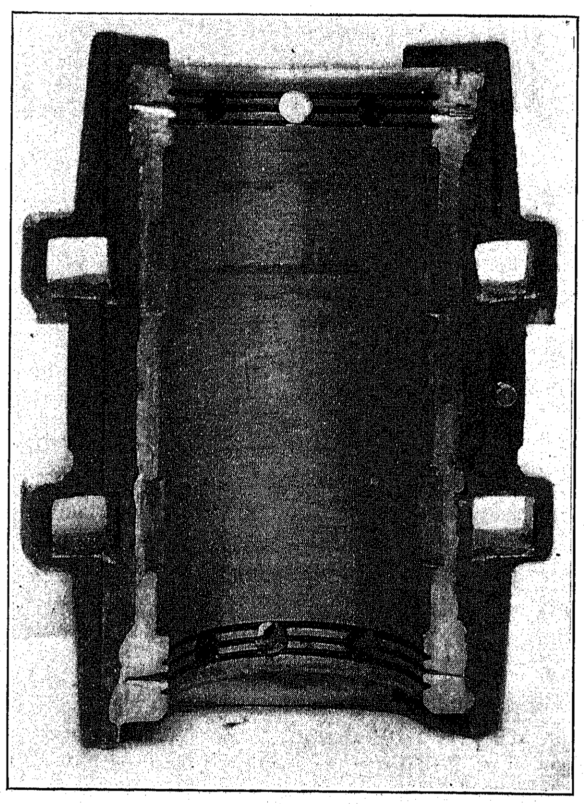

Fig 2—Photograph of Damaged Bearing of 500-h. p. Induction Motor

various starting and running conditions. Although under these conditions all the current of one phase of the rotor linked the shaft, it was found that only a few

millivolts were produced between shaft and bearings under the worst conditions. On disassembling the motor after these tests no signs of any shaft currents having been present could be detected.

There are other possible sources of shaft flux, such as uneven air gaps, and others listed by Buchanan[3]. No cases of shaft currents that could be proved due to these causes have come to the authors' attention, however, and it is their belief that the role of shaft fluxes in producing bearing trouble is a minor one. This belief is based on the experiment described above; and on the fact that insulation of the bearing pedestals is a generally accepted and succesful remedy for shaft currents, although such insulation would not materially reduce homopolar shaft currents due to shaft fluxes.

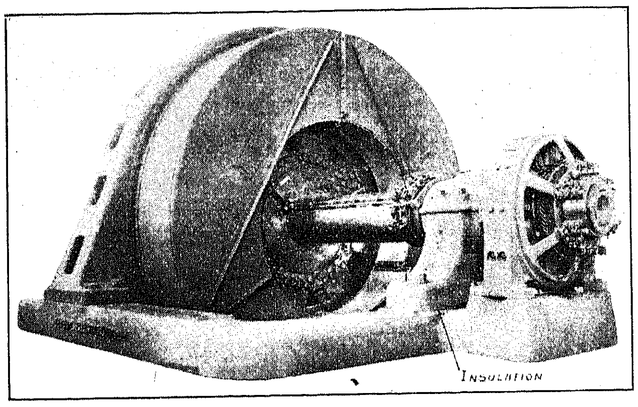

Fig. 3—Photograph of Insulated Pedestal Bearing

Two methods are available for avoiding trouble from shaft flux, if such trouble is feared. One is the use of non-magnetic bearing pedestals, or the equivalent. The other is the use of a coil linking the shaft through which current is passed in such a direction as to counterbalance the existing m. m. f. available for making shaft flux.

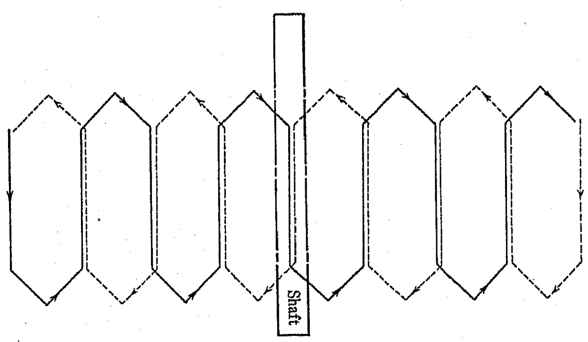

Fig. 4—Wave Winding Cause of Shaft Flux

Shaft Currents of Type (b), Due to a Potential Between Shaft and Ground

Electrostatic voltages between shaft and bearings may be set up by the friction of a belt or a pulley, or rubbing friction within the bearings themselves, or by reason of the potential of the rotor winding above ground. Electrical men are familiar with the sparks which may be drawn from a revolving leather belt on a dry day, and with the severe jolt that may be received from touching the frame of an electric motor placed on wooden blocks. Such shocks are evidence of the potential that may be built up by electrostatic effects. It is conceivable that for such reasons as these the rotor of an electric machine may be brought to a potential considerably above ground, and that when the potential reaches a certain value it may discharge through the oil film of the bearing. Constant repetition of such sparks might conceivably in time give the usual pitting effects of shaft currents.

If one part of the rotor winding is accidentally grounded to the rotor core, the whole rotor will be raised above ground potential to the potential of this point of the winding, and so an e. m. f. between shaft and ground will result. If, in addition, the rotor circuit is grounded elsewhere, a short circuit will occur through the bearings. An accident of this kind actually occurred in one instance, with the result that the shaft was badly scarred during the few moments that the power remained on.

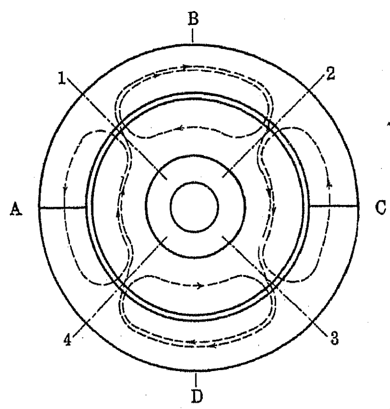

Fig. 5—Production of Shaft Voltages by Yoke Flux

No cases are known to the authors where shaft currents of this nature have given trouble except as the secondary results of accidents.

Shaft Currents of Type (c), Due to Alternating Voltages Induced in the Shaft

In every multipolar electric machine, the flux of each pole, after crossing the air gap, divides into two portions, one taking a clockwise and one a counterclockwise path through the yoke. If, for any reason, the clockwise flux, R, is not equal to the counterclockwise flux, L, the effect is the same as if their difference, $R-L$, flowed completely around the yoke. This circulating flux will link the shaft and, if it is alternating, will induce a voltage in the circuit composed of shaft, bearing pedestals, and base, causing a shaft current to flow. This type of shaft current is by far the most important,

3. Reference No 2.

and the one which has occupied the greater share of the attention of previous writers. It is characterized by the approximate equality between the shaft current at standstill, with the secondary open-circuited and full alternating voltage impressed on the primary, and the shaft current in normal operation. Also, this type of shaft current is approximately the same at no-load as at full-load. These characteristics serve to prove that it is not due to the load current, the end turn reactance, or the mechanical arrangement of the end shields, as has been variously suggested.

Consider for example, the classical case of a four-pole alternator with a stator built in two sections as shown in Fig. 5. At A and C the yoke flux passes through regions of much higher reluctance than at B and D. But pole 1 need not send its flux through A. On the contrary, it will send the major part of its flux through B to pole 2, allowing pole 3 to similarly send the larger part of its flux through D to pole 4. Thus the final distribution of flux gives a component linking the shaft, as shown by the sinuous curve of Fig. 5. As 1 will be a north pole at one instant and a south pole one half-cycle later, the flux linking the shaft will alternate at line frequency and will cause a shaft current of the same frequency.

This subject of shaft voltages caused by the use of sectionalized stators has been very fully discussed by previous writers, and reference to the articles listed in the bibliography will provide those interested with an over sufficiency of explanatory diagrams and discussions of the matter.

The general law which enables a prediction to be made as to whether or not any given sectionalizing of the stator will cause shaft currents with any given number of poles is:

(1) *Sectionalizing the stator will cause shaft currents if the ratio of twice the number of joints to the number of poles, expressed as a fraction reduced to its lowest terms, has an odd number for its numerator. The frequency of the shaft currents will be equal to this numerator times line frequency. If the numerator is an even number, no shaft currents will appear.*

For example, with 4 joints and 14 poles, the ratio reduces to 4/7, and as 4 is an even number, there are no shaft currents. With 2 joints and 8 poles, the ratio is 1/2, and line frequency shaft currents are set up. The foregoing rule applies only to machines with equally spaced and uniform joints. When the stator is divided into unequal sections special consideration must be given each particular case. In practically every case the joints between sections will vary enough to give some slight dissymmetry and, consequently, a small shaft current even though the numerator of the fraction twice joints over poles is an even number, but such accidental currents should not give serious trouble.

Sectionalizing the d-c. field of a synchronous or a direct-current machine will not cause shaft currents, since the flux linking the shaft will be unidirectional and constant.

Axial holes through the core for ventilation purposes, which are frequently used in high-speed machines, are another source of shaft currents, unless they are so located as to preserve perfect symmetry with respect to the poles. If the pattern of these holes is repeated every pole pitch, no shaft voltages will be produced; otherwise they will be. On the other hand, such axial holes may be so located as to partly neutralize the dissymmetries due to joints in the core, and so may be made to give beneficial effects.

The use of segmental punchings gives effects similar to sectionalizing. The joints in a segmental core are lap instead of butt joints, but they nevertheless have much higher reluctances than corresponding lengths of iron, and so they may cause marked variations between the reluctances of the parallel (clockwise and counterclockwise) paths in the yoke. Tests have indicated that at 8 kilolines per square cm., one lap joint has a reluctance equal to about 25 cm. of yoke, while at densities of 12 and 15 kilolines, respectively, the equivalent lengths of yoke path are roughly 20 and 10 cm.

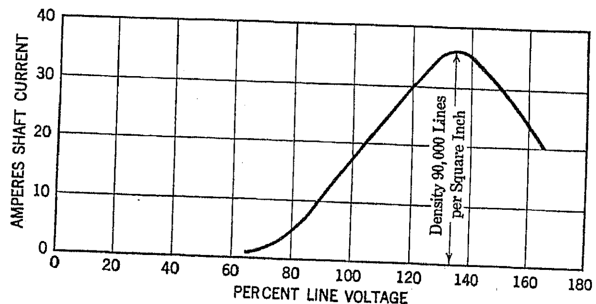

Fig. 6—Variation of Shaft Current with Voltage

It is the usual practise to provide each segmental punching with two symmetrically placed dovetails, and therefore to assemble a core with twice as many (lap) joints as there are segments. Thus the use of any given number of segments has the same qualitative effects as the use of twice as many sections. Therefore rule (1) previously stated also applies to the case of segmental construction. It may be restated as follows:

(2.) *The use of symmetrical segmental punchings will cause shaft currents if four times the segments over the poles, expressed as a fraction reduced to its lowest terms, has an odd number for its numerator; and the frequency of the shaft currents will be equal to this numerator times line frequency.*

For example, an eight-pole, six-segment, stator will have three times line-frequency shaft currents, and a 30-pole, 12-segment stator will have no shaft currents. The table of shaft current frequencies given in appendix, Fig. 15, will be found useful in determining what combinations of joints (two times segments) and poles are most favorable.

When the ratio of four times segments to poles is unity, all the poles share equally in the production of circulating flux and of shaft voltage. When this ratio is fractional, however, only a corresponding part of the poles contribute to the circulating flux, so that the greater the denominator of the fraction the lower will

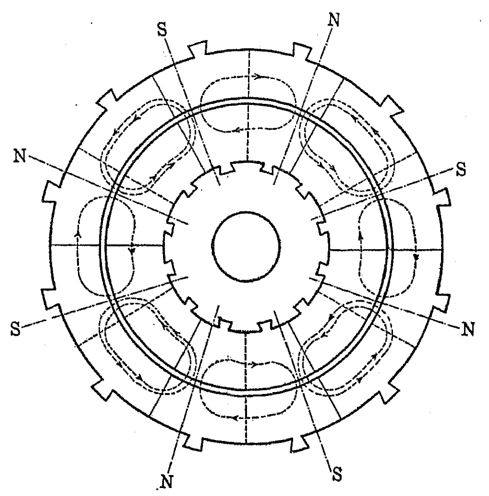

FIG. 7—PRODUCTION OF SHAFT CURRENTS IN AN 8-POLE 6-SEGMENT MOTOR

be the shaft voltage. Also, when the ratio is an odd integer greater than unity, the production of a shaft voltage is dependent on the presence of some degree of saturation in the yoke paths. For, referring to Fig. 7, which illustrates the flux paths in a motor having a ratio equal to 3 ($4 \times 6/8 = 3$), as adjacent joints in the

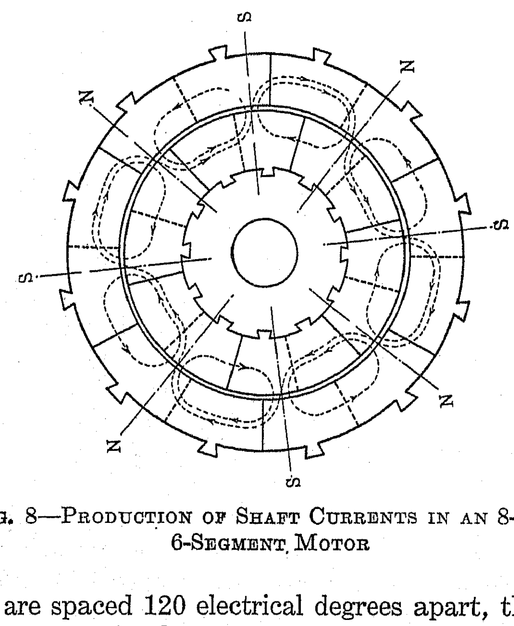

FIG. 8—PRODUCTION OF SHAFT CURRENTS IN AN 8-POLE 6-SEGMENT MOTOR

yoke are spaced 120 electrical degrees apart, the algebraic sum of the fluxes passing through 3 consecutive joints is zero, assuming a sinusoidal distribution. If, therefore, the ampere turns at each joint were proportional to the flux, the total ampere turns introduced by the joints into the clockwise flux path would be exactly equal to the ampere turns introduced into the counterclockwise flux path, and no tendency for a circulating flux to appear would exist. Actually, however, the ampere turns across each joint increase at a faster ratio than the flux, and so the two joints per pole in the clockwise flux path give less ampere turns than the single joint per pole in the counterclockwise path; a clockwise circulating flux being thus introduced.

In Fig. 6 a graph of a triple-frequency shaft current as a function of voltage is shown, which illustrates the effect of saturation. At very low densities the ampere turns across the joints are proportional to the flux, and at very high densities the entire yoke becomes so saturated that the reluctance of the entire flux path approaches that of air. In the first case the joint ampere turns in the clockwise and counterclockwise flux paths balance each other and in the second case the joint ampere turns become negligibly small in comparison with the ampere turns required for the

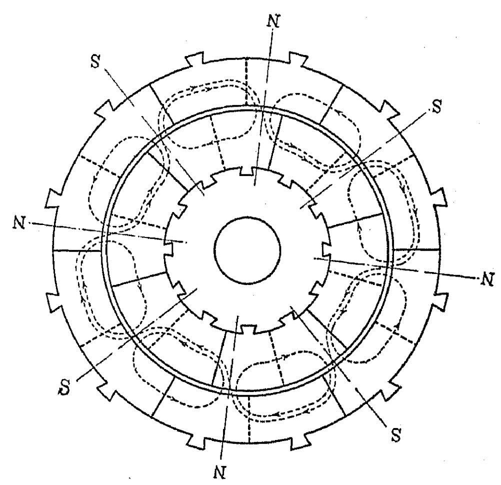

FIG. 9—PRODUCTION OF SHAFT CURRENTS IN AN 8-POLE 6-SEGMENT MOTOR

rest of the path; so that in both cases no circulating flux is produced. A very low value at reduced voltage is thus a characteristic of multiple-frequency shaft currents. Line-frequency shaft currents, on the other hand, are more nearly proportional to the voltage at low densities, and decrease less rapidly with saturation, as in these cases there are joints in only one of the flux paths.

When both rotor and stator of an induction motor are made with such a number of segments as to cause shaft currents, the resulting shaft voltage will be equal to the sum of the two shaft voltages that would be caused by the two sets of segments acting separately. The presence of rotor joints has very little influence on the effects of the stator joints, and vice versa. As the slip-frequency shaft voltages due to the rotor segments are small compared with the line-frequency voltages due to the stator segments, the rotor construction is of little importance in considering how to avoid shaft currents.

In order to obtain a clear idea of how a flux linking the shaft is set up in an induction motor with segmental rotor and stator, it is worth while to examine Figs. 7, 8 and 9. These show three positions of the rotor of an eight-pole motor with six segments in both rotor and stator. It is assumed that the slip is negligibly small, and so the center line of a north pole is in the same position on the rotor in every figure.

Consider first the stator alone. In Fig. 7 the flux is in such a position that there is an excess of flux in the clockwise yoke paths, as the reluctance of the eight

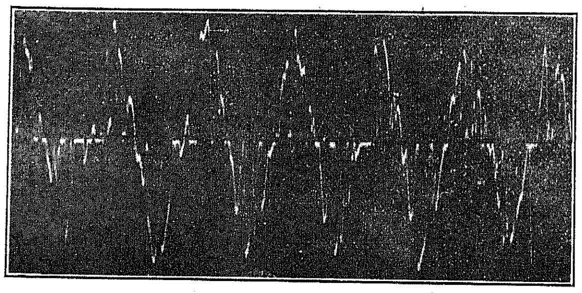

Fig. 10—Oscillogram of Shaft Voltage
Voltage across ends of shaft. Frequency 150 cycles; line frequency 50 cycles.

joints in series in the low-density regions of the core is less than the reluctance of the four joints in series in the regions of maximum core density. In Fig. 8 the flux has moved 1/24 of a revolution and now the counterclockwise flux predominates. In Fig. 9 1/12 of a revolution has been completed and the position is identical with Fig. 7. Thus the stator joints give a flux linking the shaft which completes 12 cycles per revolution, or gives three times line frequency.

Next consider the rotor alone. In all three figures the rotor joints give rise to a predominance of clockwise flux. If it is assumed that the air-gap distribution of

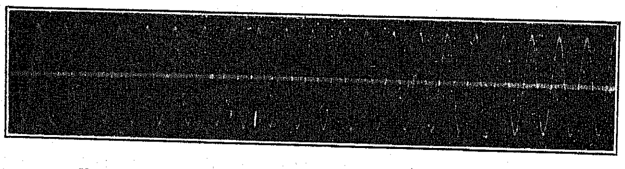

Fig. 11—Oscillogram of Shaft Current
Current across ends of shaft. Motor running light; frequency 180 cycles; line frequency 60 cycles.

flux is not affected by the unequal division of flux in the stator yoke, it is evident that the effects of the rotor joints are not dependent on the stator joints. At a later moment of the slip-frequency cycle the rotor joints will give a predominance of counterclockwise flux, and after one twelfth of a revolution of the flux with respect to the rotor the clockwise flux will again reach a maximum and a cycle of shaft-voltage variation due to the rotor segments will be completed.

Figs. 7, 8, and 9 give a net resultant clockwise flux in the first position, zero flux in the second, and a clockwise flux again in the third position. But one third of a slip cycle later the three positions will give counterclockwise, zero and counterclockwise fluxes. Therefore, the 180-cycle shaft voltage due to the stator segments is simply superposed on the 180 times per cent slip-cycle voltage due to the rotor segments and the magnitudes of each of the two voltages are the same as if they existed independently.

The shaft voltage shown in Fig. 10 was taken on a 16-pole motor with 12 segments in both rotor and stator. That shown in Fig. 11 was taken on an eight-pole motor with six stator segments and five rotor segments. In neither case is the shaft voltage due to the rotor segments (three times slip frequency in the first case and five times in the second case) large enough to be noticeable.

When an induction motor is at standstill, with open-circuit rotor, a shaft current due to the use of the same number of segments in both rotor and stator occurs at n times frequency instead of being composed of two different frequency voltages. But its magnitude then depends upon the rotor position, and it will vary through a complete cycle of values as the rotor is turned through an arc corresponding to one quarter of a segment. As the rotor accelerates the single-frequency shaft voltage breaks up into two components whose frequencies are at first near together and later separate more and more widely until, as full speed is reached, the shaft voltage consists of a large n times line-frequency component and a small n times slip-frequency component. Corresponding to these changes in the shaft voltage the shaft current will change its value in a seemingly erratic manner.

It is a well known fact that the slip of almost any induction motor may be counted by observing the beats of a millivoltmeter placed across the ends of the shaft. The discussion given above clearly shows that the beats of the millivoltmeter are due to a slip-frequency alternating flux encircling the shaft. And the widespread occurrence of this phenomenon shows that even very slight differences in the magnetic paths in the yoke will cause measurable shaft voltages. In many cases the ratio of rotor segments to poles is such as to give shaft voltages 3 or 5 times slip frequency instead of slip frequency itself. For this reason it is well to check the measured slip of an induction motor with a tachometer when the shaft voltage seems to give too high a frequency.

The two oscillograms of shaft currents shown on Figs. 10 and 11 pertain to induction motors with segmental stators having the ratio four times segments over poles equal to 3. In both these cases the oscillograms show the shaft voltages to be at three times line frequency thus verifying the rule (2). The pulsations in the voltage on Fig. 10 are ascribed to tooth-frequency variations in the core flux. It is interesting to note that any third harmonics in the core flux of these machines will give shaft voltages of the same triple frequency

that the fundamental flux does, as four times poles over segments for the 3rd harmonic is 1 instead of 3, as for the fundamentals.

In Figs. 12 and 13 are shown oscillograms of shaft voltage taken on two turbine generators with one-piece stator frames. Since in both cases the ratio of four times segments to poles is an even integer, the rules previously laid down do not explain these shaft currents, except on the assumption that they are due to inequalities in the core joints. Evidently the fundamentals of

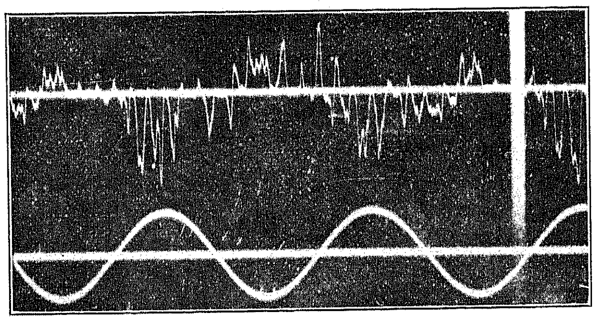

FIG. 12—OSCILLOGRAM OF SHAFT VOLTAGE
Upper curve, voltage shaft to frame. Lower curve, voltage wave no load.

these oscillograms are of line frequency, but the wave forms are extremely irregular. The reasons for these irregularities probably lie in the facts that the actual wave form of the alternator yoke flux is irregular, due to tooth pulsations, harmonics in the field flux, and saturation, and that the passage of this flux through so complicated a magnetic circuit as an annular steel core with irregular air gaps in it gives rise to a still more irregular m. m. f. wave form. If, however, all the

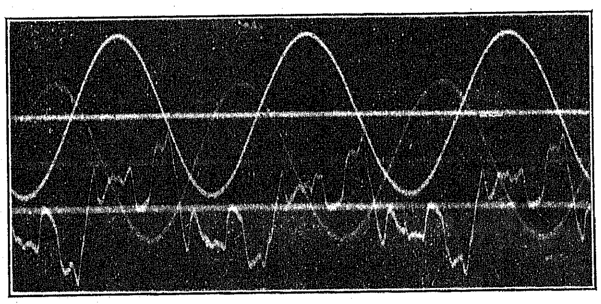

FIG. 13—OSCILLOGRAM OF SHAFT VOLTAGE
Upper curve, voltage, Phase 1-2. Middle curve, voltage, Phase 1-3. Lower curve, voltage, shaft to frame.

joints in the cores had been uniform, no shaft current should have appeared no matter how irregular the flux wave form.

It is the resultant m. m. f. acting around the periphery of the yoke that causes the circulating flux which links the shaft, and this resultant m. m. f., being the difference between two relatively large quantities, has a wave form that is more irregular the smaller its average value. The shaft voltages shown in Figs. 12 and 13 were actually quite small, so that their poor wave forms cannot be taken as an indication that the wave forms of the yoke fluxes were anything like as irregular as they are.

In well designed machines all the important fluxes and currents should have closely sinusoidal wave forms, and so the wave forms of any large shaft voltages should be reasonably sinusoidal. Thus the excellent wave form shown in Fig. 11 is not entirely a matter of chance, but an indication of good design. This motor has nearly closed rotor slots and a good ratio of rotor to stator teeth so that only very small tooth-frequency pulsations occur in the yoke fluxes. Also, the wave form of its exciting m. m. f. is very nearly sinusoidal, its gap permeance is very uniform, and none of the magnetic paths are too highly saturated. Consequently the wave form of its yoke m. m. f. should be very good, as the oscillogram actually indicates.

However, the irregularity of wave form of the three former shaft currents and the regularity of the latter are partly due to the differences in the voltage drops in the shaft circuit. In the turbine generator cases the voltage was taken from shaft to frame, so that the oscillograph current had to return through the other (inaccessible) bearing. In the induction motor cases the voltage was taken across the shaft through metallic brushes, so that the irregular voltage drop across the bearing was avoided. In taking Fig. 10 one brush was on the outside of the revolving shaft, making a rather irregular contact, while in taking Fig. 11 the contacts were made by axially-applied brushes on the two exposed ends of the shaft.

REMEDIES FOR SHAFT CURRENTS OF TYPE (C)

Of course the first essentials in avoiding shaft currents are to banish the combinations of poles and joints indicated to be objectionable by (1) and (2), or as tabulated in the appended table, and in every case to make the clearances between yoke joints as small and as uniform as possible.

Also, if the shaft and bearings are absolutely smooth and excessive speeds and bearing pressures are excluded, the oil films will act as effective insulators, and so damage to the bearings will be avoided. A large number of motors which are known to have triple-frequency voltages across the shaft of about one volt have been in successful operation for many years, so that small shaft currents are not necessarily dangerous. But any roughness on the shaft or excessive bearing pressure, will cause rubbing, which will allow the currents to flow, and these currents will soon pit the bearings and so aggravate conditions as to in time destroy the bearing surfaces.

The end play of the shaft will usually give intermittent metallic contact from one end of the shaft to the bearing lining, so that the fact that the oil films in the two bearings are electrically in series is of no importance in lessening the currents. The end play

explains the fact that frequently the voltage from shaft to ground will be zero on one end of the shaft and yet will be considerable on the other end. The presence of oil rings, which make metallic contact with the top of the shaft, affords a very convenient path for the shaft currents, and for this reason it will often be found that the first signs of bearing currents appear as scratches on the shaft under the oil rings. Thus permanent reliance can not always be placed on the oil film alone as an insulator.

The standard method of avoiding the effects of shaft currents is simply to insulate the bearing pedestals from the bed plate, or the bearing lining from the end shield. Such methods form the simplest way of getting around the difficulty, but they are difficult of adoption on machines with end shield bearings, and they are always a source of trouble and expense.

Several German patents have been taken out on other methods of avoiding these troubles. A common expedient is to place metallic brushes in contact with the ends of the shaft and connect them electrically to the frame, thus short-circuiting the bearings. But the contact drop of the brushes is so great that one-third to one-half the original current still flows through the bearings.[4] To improve this condition an A. E. G. patent[5] proposes to connect the primary of a transformer across the brushes, the secondary being wound around the core of the machine. By proper arrangements the voltage applied across the brushes may be made to approximately oppose and cancel the line frequency component of the shaft voltage.

Another patent[6] covers the case of a ring-wound coil encircling the stator yoke, this coil being fed with a line-frequency current of such a phase angle as to most nearly cancel the ampere turns set up by the joints. The current in this coil makes a circulating flux in the yoke which opposes the circulating flux set up by the dissymmetry of the magnetic circuit. This scheme affords a method of eliminating the line-frequency component of the shaft voltage but does not avoid the higher frequency currents shown in the oscillograms, Figs. 12 and 13.

A third patent[6] covers the use of an iron collar encircling the shaft inside the bearings, which collar is wound with a ring winding. Application of a suitable line-frequency voltage to this coil sets up an alternating flux whose phase relation to the circulating flux in the yoke is adjusted to give the best cancellation. This scheme also completely cures the fundamental frequency component of the shaft voltage but does not remove the higher frequency components. None of the patented schemes mentioned would benefit in any way the higher frequency shaft currents such as shown in Fig. 10 and Fig. 11 and in fact the application would exaggerate the shaft currents in these cases.

4. Reference No 6.
5. Reference No. 5.
6. Reference No. 5.

Liwschitz[6] has suggested the cutting of a notch or notches in the yoke in such a way as to add reluctance in the yoke paths that have lower than average reluctance and so bring all the reluctances up to the same level. The simple cutting of such notches in the finished yoke would be very objectionable on account of increased core losses. But a better result can be simply obtained by making butt joints in the punchings to balance the butt joints between sections in those cases where the desirable number of sections is such as to cause bearing currents with the given number of poles. For example, a two-section four-pole motor would now be built with butt joints between sections and lap joints elsewhere, and so would have shaft currents. By making two extra butt joints in the punchings halfway between the section joints, all poles would be made alike and so shaft currents would be eliminated. This assumes that all the four-butt joints could be made very closely alike.

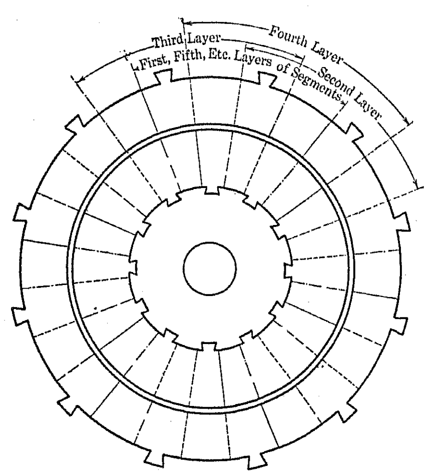

Fig. 14—Core Constructed with Offset Segments

Preferred Method of Avoiding Shaft Currents Due to Segments

The methods so far considered apply equally well to shaft currents due to sectional and segmental stators, but the methods involving the introduction of a bucking e. m. f. are of no use in the case of certain numbers of segments in both rotor and stator. If segments alone are the cause of trouble, as is most commonly the case, the best and altogether most desirable method of avoiding shaft currents is to use offset segments.

For, if the segments are laid out as in Fig. 14 instead of as in Figs. 7, 8 and 9 with the dovetail tags (or notches) placed at the ⅛ and ⅝ points instead of the ¼ and ¾ points as is usually the case, the core can be built up with four joints per segment instead of two, and ¾ of the full iron section at each joint instead of only ½. This will be accomplished by laying any single pair of layers of punchings with lap joints in the usual way and then laying the next pair of layers upside down, so that the new lap joints come midway between the first lap joints, as shown in Fig. 14. It is not impor-

tant that the dovetail tags come exactly at the ⅛ and ⅝ points, but it is essential that each tag come opposite a slot or a tooth, so that slots in the turned over punchings will coincide with those in the other punchings.

The usefulness of this scheme is somewhat limited by the fact that a blanking die with offset tags will seldom work with more than one number of slots per segment. If the tags are set at the ⅛ and ⅝ points, the number of slots per segment must be a multiple of 4, as 12, 16, 20 and 24. Thus one offset blanking die can be used

Number of Equally Spaced Joints

Number of Poles	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	18	20	22	24	28	30	32	36	40	42	44	48	50	52	56	60	64	66	72
2	-	3	-	5	-	7	-	9	-	11	-	13	-	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	1	3	-	5	3	7	-	9	5	11	-	13	7	15	-	9	-	11	-	-	15	-	-	-	21	-	-	25	-	-	-	-	33	-
6	-	1	-	5	-	7	-	3	-	11	-	13	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	1	3	1	5	3	7	-	9	5	11	3	13	7	15	-	9	5	11	-	7	15	-	9	-	21	11	-	25	13	-	15	-	33	-
10	-	3	-	1	-	7	-	9	-	11	-	13	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
12	1	1	-	5	1	7	-	3	5	11	-	13	7	5	-	3	-	11	-	-	5	-	-	-	7	-	-	25	-	-	-	-	11	-
14	-	3	-	5	-	1	-	9	-	11	-	13	-	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
16	1	3	1	5	3	7	1	9	5	11	3	13	7	15	-	9	5	11	3	7	15	-	9	5	21	11	-	25	13	7	15	-	33	9
18	-	1	-	5	-	7	-	1	-	11	-	13	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	1	3	-	1	3	7	-	9	1	11	-	13	7	3	-	9	-	11	-	-	3	-	-	-	21	-	-	5	-	-	-	-	33	-
22	-	3	-	5	-	7	-	9	-	1	-	13	-	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
24	1	1	1	5	1	7	-	3	5	11	1	13	7	5	-	3	5	11	-	7	5	-	3	-	7	11	-	25	13	-	5	-	11	-
26	-	3	-	5	-	7	-	9	-	11	-	1	-	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
28	1	3	-	5	3	1	-	9	5	11	-	13	1	15	-	9	-	11	-	-	15	-	-	-	3	-	-	25	-	-	-	-	33	-
30	-	1	-	1	-	7	-	3	-	11	-	13	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
32	1	3	1	5	3	7	1	9	5	11	3	13	7	15	1	9	5	11	3	7	15	-	9	5	21	11	3	25	13	7	15	-	33	9
34	-	3	-	5	-	7	-	9	-	11	-	13	-	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
36	1	1	-	5	1	7	-	1	5	11	-	13	7	5	-	1	-	11	-	-	5	-	-	-	7	-	-	25	-	-	-	-	11	-
38	-	3	-	5	-	7	-	9	-	11	-	13	-	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
40	1	3	1	1	3	7	-	9	1	11	3	13	7	3	-	9	1	11	-	7	3	-	9	-	21	11	-	5	13	-	3	-	33	-
42	-	1	-	5	-	1	-	3	-	11	-	13	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
44	1	3	-	5	3	7	-	9	5	1	-	13	7	15	-	9	-	1	-	-	15	-	-	-	21	-	-	25	-	-	-	-	3	-
46	-	3	-	5	-	7	-	9	-	11	-	13	-	15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
48	1	1	1	5	1	7	1	3	5	11	1	13	7	5	-	3	5	11	1	7	5	-	3	5	7	11	-	25	13	7	5	-	11	3
50	-	3	-	1	-	7	-	9	-	11	-	13	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
52	1	3	-	5	3	7	-	9	5	11	-	1	7	15	-	9	-	11	-	-	15	-	-	-	21	-	-	25	-	-	-	-	33	-
56	1	3	1	5	3	1	-	9	5	11	3	13	1	15	-	9	5	11	-	1	15	-	9	-	3	11	-	25	13	-	15	-	33	-
60	1	1	-	1	1	7	-	3	1	11	-	13	7	1	-	3	-	11	-	-	1	-	-	-	7	-	-	5	-	-	-	-	11	-
64	1	3	1	5	3	7	1	9	5	11	3	13	7	15	1	9	5	11	3	7	15	1	9	5	21	11	3	25	13	7	15	-	33	9
68	1	3	-	5	3	7	-	9	5	11	-	13	7	15	-	9	-	11	-	-	15	-	-	-	21	-	-	25	-	-	-	-	33	-
72	1	1	1	5	1	7	-	1	5	11	1	13	7	5	-	1	5	11	-	7	5	-	1	-	7	11	-	25	13	-	5	-	11	-
76	1	3	-	5	3	7	-	9	5	11	-	13	7	15	-	9	-	11	-	-	15	-	-	-	21	-	-	25	-	-	-	-	33	-
80	1	3	1	1	3	7	1	9	1	11	3	13	7	3	-	9	1	11	3	7	3	-	9	1	21	11	-	5	13	7	3	-	33	9
84	1	1	-	5	1	1	-	3	5	11	-	13	1	5	-	3	-	11	-	-	5	-	-	-	1	-	-	25	-	-	-	-	11	-
88	1	3	1	5	3	7	-	9	5	1	3	13	7	15	-	9	5	1	-	7	15	-	9	-	21	1	-	25	13	-	15	-	3	-
92	1	3	-	5	3	7	-	9	5	11	-	13	7	15	-	9	-	11	-	-	15	-	-	-	21	-	-	25	-	-	-	-	33	-
96	1	1	1	5	1	7	1	3	5	11	1	13	7	5	1	3	5	11	1	7	5	-	3	5	7	11	1	25	13	7	5	-	11	3
100	1	3	-	1	3	7	-	9	1	11	-	13	7	3	-	9	-	11	-	-	3	-	-	-	21	-	-	1	-	-	-	-	33	-
104	1	3	1	5	3	7	-	9	5	11	3	1	7	15	-	9	5	11	-	7	15	-	9	-	21	11	-	25	1	-	15	-	33	-
108	1	1	-	5	1	7	-	1	5	11	-	13	7	5	-	1	-	11	-	-	5	-	-	-	7	-	-	25	-	-	-	-	11	-
112	1	3	1	5	3	1	1	9	5	11	3	13	1	15	-	9	5	11	3	1	15	-	9	5	3	11	-	25	13	1	15	-	33	9
116	1	3	-	5	3	7	-	9	5	11	-	13	7	15	-	9	-	11	-	-	15	-	-	-	21	-	-	25	-	-	-	-	33	-
120	1	1	1	1	1	7	-	3	1	11	1	13	7	1	-	3	1	11	-	7	1	-	3	-	7	11	-	5	13	-	1	-	11	-
124	1	3	-	5	3	7	-	9	5	11	-	13	7	15	-	9	-	11	-	-	15	-	-	-	21	-	-	25	-	-	-	-	33	-

◇ Denotes combinations, seemingly the worst, where the above given ratio reduces to the odd integer 3. Shaft currents in each of these cases can be completely cured by offset segments.

○ Denotes all other cases given in this table that can be completely cured by offset segments.

COMBINATIONS OF POLES AND SEGMENTS PRODUCING SHAFT CURRENTS.

Fig. 15—Table of Shaft Current Frequencies

In any revolving electric machine, if the ratio of two times joints to poles expressed as a fraction reduced to its lowest terms has an odd number, N, for its numerator, shaft currents of N times line frequency will occur. By line frequency is meant the frequency of the magnetic flux in the jointed element of the machine. This table gives the values of N for various combinations of poles and joints. Combinations with which no shaft currents occur are indicated by dashes.

with 20 and 24 teeth per segment but not with 21 and 22. But the scheme can be used with any combination die. And, the use of this construction will probably reduce the yoke ampere turns and the core losses to a sufficient degree to make it worth considering for these reasons alone, regardless of the shaft current question. Transformer tests have shown that poorly made lap joints may increase the core loss by as much as 20 per cent, while at average densities each lap joint requires as many ampere turns as 6 or 8 inches of the full-section part of the yoke.

A crude alternative to the use of offset segments is to punch ventilating holes in the primary yoke in such a way as to create additional reluctances midway between the joints. These additional reluctances will offset the joint reluctances and so approximate the effect desired. This scheme of course increases the core loss and the yoke ampere turns, and so is not recommended.

Turning Shaft Currents to a Useful Purpose

By constructing both stator and rotor of symmetrical segmental punchings with butt joints and considerable clearances, it would be readily possible to make a large part of the total yoke flux link the shaft. In this way shaft voltages could be produced equal to a fair percentage of the volts per turn in the main winding, and relatively large currents could be drawn from the shaft at low voltages and with fairly good wave form. By making $2 \times \dfrac{\text{sections}}{\text{poles}} = 1$, 3 or 5, frequencies of 1, 3 or 5 times line frequency could readily be produced. Of course the multiple frequencies would only be due to saturation and so this arrangement would be very inefficient indeed if used as a power-frequency transformer.

Thus a nearly standard induction motor construction could be used to produce large low-voltage single-phase currents at any frequency $(2m - 1)$ times line frequency where m is any integer. Such a machine would have to deliver its current through brushes, and so would have some of the troubles of homopolar machines. Without proceeding to an extreme, an induction motor could be so constructed as to serve the dual purpose of rotating the shaft and also producing low-frequency currents. Of course, any such machine would have to have its bearings well insulated from the frame.

As previously noted, these shaft currents could be as well produced with the rotor stationary, and therefore the motor construction with bearings and air gap is entirely unnecessary, if the sole object is the production of these shaft currents. Such a machine would simply be a static transformer giving a secondary frequency 1, 3 or 5 times the primary frequency. If the frequency ratio is 1, the machine would be exactly equivalent to an ordinary transformer. If the secondary frequency is three or five times the primary frequency, due to proper choice of the segments, the machine is equivalent to a three or five-phase transformer of the usual type, with mesh-connected secondary.

These ideas on the utilization of shaft currents are rather fanciful, and it is not probable that they will be practically developed.

Conclusions

It is concluded that the most important cause of shaft currents is the presence of some dissymmetry in the magnetic circuits of the rotor or stator core, which causes more flux to flow in one of the two parallel (right and left hand) paths than in the other, and so gives a

resultant flux linking the shaft. The most prominent causes of such dissymmetries are sectionalized frames and segmental punchings. All cases where such divisions of the yoke will cause excessive bearing currents can be predicted by means of rules (1) and (2) given above. But the use of any sections or segments at all will give rise to some shaft voltages, due to the impossibility of making all joints exactly alike. These secondary shaft currents should not be harmful except in extremely high-speed machines like turbine alternators.

The most effective methods of remedying bearing currents, other than insulating, are to punch holes in the yoke in the regions of low reluctance, thus raising all parts of the yoke to the same level of reluctance, or to create a separate flux linking the shaft and opposed to the flux produced by dissymmetry or to make use of offset segmental punchings.

It is believed that the use of offset segmental punchings will effectively cure all ordinary cases of bearing currents in small induction motors, d-c. machines, converters and synchronous motors and, as offsetting the dovetails gives better core loss, lower yoke reluctance, and a more solid core construction, with very little expense or trouble (once the new dies are developed), it is recommended that this scheme be adopted whenever practicable.

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9. Punga, F. and Hess, W. Bearing Currents. (In German) *Elektrotechnik und Maschinenbau*, v. 25, pp. 615-18, Aug. 11, 1907. Theory and tests. Same, abstract translation: *Science Abstracts*, v. 10, pp. 421-22.

Shows that a sectionalized stator causes bearing currents and gives data on the ampere turns required to force flux across lap and butt joints in the punchings.

Discussion

W. F. Dawson: I can endorse one point which Mr. Alger made, namely that shaft currents are often associated with high saturation in the stationary armature cores of synchronous generators.

Some oscillograph records give the interesting information that the e. m. f. of shaft currents is greatly reduced as the generator voltage, and consequently the saturation of the armature core, is reduced.

My recollection is that we had approximately 5 volts potential at normal voltage and only 0.05 volts shaft potential with 50 per cent armature voltage.

We have built more than 2500 two-pole machines, of which less than 1 per cent have developed observable shaft currents, but on the other hand practically all of those that have given trouble should have been immune according to Mr. Alger's formula and the tabulation given as Fig. 15.

G. E. Luke: This is an interesting paper in that it describes a parasitic current or a loss which is peculiar in its effects. We are not interested in its effects on the performance or efficiency; we are interested in it solely due to its mechanical effects on the bearings. The bearing is a very simple part of the machine, but it is also a delicate part. A highly polished bearing will give a very low coefficient of friction. It is very important to reduce that to a low figure in high-speed machines. Any increase in the roughness of shaft will increase this coefficient and may cause bearing failure.

There is one point not mentioned by the authors and that is in connection with ball bearings or roller bearings. Of course, these bearings are ordinarily not used in large machines, but I have heard it said that the question of stray currents is a very serious limitation to their use in large machines because the ball bearing is especially sensitive to any imperfection on the surface of the bearing or race.

There is another point I would like to bring out, and that is: how did they measure the stray currents in the bearing, except by the usual method of putting a shunt around the bearing and using an ammeter, insulating the pedestal and shunting the insulation? Do the authors have any other methods which they use for measuring the currents in the bearing when they have an unsymmetrical magnetic field? The method of insulation in a pedestal machine is easy, but when we come to a bracket machine, it is not so easy on account of mechanical difficulties. Do they have any suggestions there as regards the elimination of the stray currents?

W. J. Foster: Last evening a designing engineer who has had a great deal of experience told me of a little telltale device that had been installed on some large generators of recent manufacture. It consisted in placing a low-voltage incandescent lamp across the insulation underneath the pedestal. The lamp stays lighted in varying degrees of intensity all the while, unless the insulation breaks down, and then it goes out. It is quite important that some such telltale device be installed, otherwise something may happen, such as I have seen on visiting plants, where after four or five years when a change in the management or a change in the operators has taken place, the insulation under the pedestal may be forgotten. In one case, iron steps had been placed on the base resting against the pedestal thus short-circuiting the insulation.

F. D. Newbury: As the authors of the paper bring out, the major cause of shaft currents has been known for a long time. I remember very distinctly a memorandum that Mr. Lamme circulated, I think it was in 1908, emphasizing the fact that the dissymmetry of the magnetic circuit was one of the principal causes of shaft current and cautioning against the use of certain combinations of poles and segments.

There is a possible inference from the paper that I hope will be avoided, that is the thought that insulation is, from this time on, to be frowned upon as an evidence of improper design. As Mr. Dawson brought out, there are cases in which results are difficult to predict, where, even with proper relations between poles and number of segments or between numbers of segments in the rotors and stators of induction motors, we do find shaft currents in spite of the proper relationship, and I think that all designers will continue to use bearing insulation as a precaution, even in those cases where shaft currents would not ordinarily be predicted.

A. M. Perry: On a trip through the Southeast about a year ago, I had the opportunity to observe a vertical-shaft water-wheel generator unit on which they had had considerable trouble with the bearings melting. They didn't look into the cause of the trouble, but they did put slip-rings on the lower end and the upper end of the shaft and connected them to a ground so that there was a very low-resistance connection between the shaft and what would correspond to the pedestal of the bearings. They told me that this method eliminated the trouble entirely.

W. J. Foster: In line with Mr. Newbury's remarks and Mr. Dawson's, that there may be cases of shaft currents where they would not be expected in accordance with the theory on which this paper is based, I would like to have the authors, in closing, make a statement as to whether they rule out all other causes.

Personally, from my own experience, I consider mechanical dissymmetry at the two ends of the shaft in certain cases as responsible,—for example, in a 12-pole vertical machine with a 6-arm bracket at the top and no corresponding bracket underneath; where shaft currents were much in evidence, it seemed to be perfectly natural to ascribe the reason to the stray flux cutting the arms synchron-

As I understand the paper, it does not necessarily shut out other sources of trouble besides the particular relations of the segments in the core.

E. A. Smith (by letter): Referring to the paper of Messrs. Alger and Sampson, I have noticed that a considerable number of the tests mentioned, have been treated in detail in some of the foreign societies' publications in which a special study is being undertaken to eliminate stray shaft currents. Particularly in Germany the electrical manufacturing concerns are trying a method of insulating the bearing pedestals from the base of the machines. They have so far not been able to insulate the end shield bearings but they are employing a non-magnetic babbit lining and non-magnetic pedestal.

No doubt in due time, by installing these non-magnetic babbit linings and by splitting the yokes as set forth in Messrs. Alger and Sampson's paper these stray shaft currents will be very much eliminated.

Most of this trouble has been found in synchronous and induction machines, where, uneven air gaps were found to exist, but in a few cases where new machines were installed, the trouble was due entirely to the flux traveling along the shaft to the bearings.

Tests were made in these cases and as far as could be observed, the only remedy would be for manufacturing concerns to investigate these conditions thoroughly and eliminate them as much as possible.

P. L. Alger: We believe that certain irregularities in the construction of the core are inevitable and therefore that any machine will have some shaft current, even though it is very well constructed, if you measure it with a delicate enough instrument. Therefore, it is not desirable to remove the insulation from very large and important machines, even though it may be believed that they will have no shaft currents. But we do believe that a great many machines do not need any insulation, although they are now provided with it as a matter of standard practice; and also we believe that certain machines which now, because of their classification, have no insulation, would be better provided with it.

I agree with Mr. Foster and Mr. Newbury, that it would be very undesirable to provoke a series of troubles with shaft currents by removing it, but I do think that operating men may profitably measure the shaft voltage in machines they have operating, and in those cases where it is not a vitally important, large, machine and where they find no shaft current by tests, they may with impunity take less care of the insulation. If enough experience is accumulated we may be able to make some progress in avoiding the troubles we now have.

The question was asked how we measured the shaft current. We do that ordinarily by merely placing an ammeter between the two ends of the shaft. In that way we measure a current in parallel with the path through the bearings. But we believe that when the oil film is still intact and not damaged by any cause, that the current flowing through the bearings themselves is negligible in comparison with that which will flow through a short-circuit path around the ends of the shaft. In those cases where we have had to insulate end-shield machines, we have done it by placing a band of insulation around the bearing itself, inside the bearing housing. In that way it is possible to make a satisfactory insulation, but it is very inconvenient to do, and if the bearing is removed it must be very carefully handled, so we do not wish to do that when it is possible to avoid it.

This question of shaft currents is particularly important in machines built for ship propulsion, where space is valuable and where the presence of salt water and of various other pieces of apparatus makes it very difficult to insulate, and also where end shields are almost always used to save space. We therefore believe it particularly desirable to apply this theory to machines intended for ship propulsion.

I will call your attention to the fact that in this paper as printed, the figures have been misarranged. Fig. 7 should be Fig. 8; Fig. 9 should be Fig. 7 and Fig. 8 should be Fig. 9. Furthermore, the true Fig. 9; that is, the one that is called Fig. 8 in the paper, has been turned through 90 degrees to the left, so that the top is now at the lefthand side. These figures are so much alike that it is hard to follow through the discussion without having the right titles.

Eddy Current Losses in Armature Conductors

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Review of the Subject.—*This paper is an extension to the author's paper in Vo. XXXIX Pages 997 to 1047 on Eddy Current Losses in Armature Conductors. In this present paper additional formulas are given for the cases where transposed coils are used and also methods given for quickly estimating the increased loss due to eddy currents.*

THIS paper is a supplement to the one presented by the author in the A. I. E. E. JOURNAL, Vol. XXXIX, Part I, for 1920. It is realized that the value of the formulas presented in this type of a paper depends upon their completeness and upon the simplicity of their application. Consequently, additional formulas have been added, covering types of windings neglected in the original paper, and an effort has been made to simplify the application of the formulas. An example has been worked through completely as an illustration.

In the original paper, presented in Vol. XXXIX of the A. I. E. E. TRANSACTIONS, a summary of the results were given between pages 1038 and 1045. This summary covered seven formulas for the calculations of the additional losses in the slot portion of a conductor for different groupings, and an additional formula for the loss outside of the slot portion. These seven formulas were designated A to G, inclusive, and occur on pages 1042 and 1043. They cover the usual winding combinations with coils wound "straight up" and "turned over," as illustrated on page 1012. The short circuits between strands were limited, however, in location. For short circuit occurring at the end of one-half turn, formula B applied. For a short circuit at the end of each full turn, formula D and F applied, and when the short circuit occurred only at the start and finish of the coil, then formulas C, E and G applied. With the above limitations, it is not possible to accurately determine the eddy current losses in a transposed multiple turn coil of the type using two coil sides per slot.

The data in this paper give the formulas necessary to cover this latter case. The nomenclature followed in the original paper will be used in this one, but additional terms will be employed where necessary. These additional terms will be explained at the time they are first used in the text. The formulas only will be given without going into their development which follows the outline previously laid down in detail.

The six formulas necessary to cover all the usual winding combinations with strands short-circuited at one-half turn or each full turn, or at the end of any integral number of full turns, are given below.

When conductors consist of solid bars and there are two coil sides per slot, formula A in Volume XXXIX applies.

When conductors are stranded and the strands are short-circuited at the end of each half turn, and there are two coil sides per slot, formula B in Volume XXXIX applies.

When the coil is wound continuously straight up and there is one coil per slot with a short circuit between strands at the end of ϕ turns, the formula is

$$\text{Ratio} = n^2 \left[\frac{m^2 - 1}{3} + m^2/4 \cos\beta - (m^2/4 + \theta^2/12 - 1/3) \right] M_s + [m^2/4 + \theta^2/12 - 1/3]\, M_c + O_c \quad \textbf{(III)}$$

When the coil is turned over, and there are two coil sides per slot, with a short circuit between strands at the end of turns, the formula is

$$R_t = n^2 \left[\frac{m^2 - 1}{3} + m^2/4 \cos\beta - (m^2/16 + \theta^2/12 - 1/3) \right] M_s + [m^2/16 + \theta^2/12 - 1/3]\, M_c + O_c \quad \textbf{(IV)}$$

$$R_b = n^2 \left[\frac{m^2 - 4}{12} - (m^2/16 + \theta^2/12 - 1/3) \right] M_s + [m^2/16 + \theta^2/12 - 1/3]\, M_c + O_c$$

When the coil is turned over, and there is one coil per slot, with a short circuit between strands at the end of turns, the formula is

$$R = n^2\, [m^2/4]\, M_s + \left[\frac{\theta^2 - 4}{12} \right] M_c + O_c \quad \textbf{(V)}$$

The formula for the loss ratio in the end turns for any of the winding combinations above is given by

$$R = O_c + \frac{n^2 - 1}{n^2}\, M/r^2\, (K) \quad \textbf{(VI)}$$

The new symbols employed have the following meanings: ϕ is used to designate the number of turns in the coil without a short circuit. In general, this would be the number of turns in the coil as formed in manufacture.

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*Deceased

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θ is used to designate the number of groups in a transposition. θ can be any integral factor of the slots per pole per phase. m is previously used as the equivalent number of conductors in a slot carrying equal currents. From the definitions above, $m = \phi\,\theta$ for one coil per slot windings, and $m = 2\,\phi\,\theta$ for two coil per slot windings. For example, a two-turn coil for a two-coil per slot winding, if transposed in five groups, would have a value of $m = 20$. A summary of the values given to the other symbols is shown on pages 1040 and 1041 of volume XXXIX.

In formula **VI**, r is used to indicate the ratio of a full turn to the embedded portion. K is used for the coefficient of M_c in that formula which covers the particular winding combination under investigation.

The six formulas give the loss ratios for any usual winding combination for the embedded and end portions of the coils. The most common winding in this country is completely covered by formulas **IV** and **VI.**

In investigating additional losses in armature coils caused by eddy currents, there are two things which it is desirable to know. The more important of the two is the amount of the excess where it has its greatest value. The average value for the entire coil is of secondary importance. The principal purpose of eddy current investigation is to determine the probable temperature rise of the coil. The insulation to ground, or the coil insulation for a 13,200-volt coil is 0.2 of an inch or more in thickness, where mica is used for insulating. The expected temperature drop through such a wall for an average heat dissipation of 0.6 of a watt per square inch would be in the neighborhood of 40 deg. cent. It is obvious, therefore, that any temperature gradient within the wrapper is of secondary importance, and that an average temperature within the wrapper can be assumed without appreciable error. As a consequence, the formulas have been shown indicating the loss ratio for all strands within a ground wrapper.

Formulas **IV** and **VI** cover the great majority of the windings in common use in this country. The application of these formulas appears complicated; in fact, however, it is a relatively simple matter to apply them, provided a preliminary approximation is employed. To illustrate, we will work out an example.

Let it be assumed that it is desired to limit the eddy loss factor in the top coil of a slot to 1.35. Let us assume further that the following conditions obtain. Our winding is to be of the two-coil per slot turned over type. The machine is assumed to be three-phase, four poles, sixty cycles, and to have seventy-two slots. The copper depth total in a slot can be 4.5 inches. The width of copper is 0.5 of an inch and the width of the slot is 1 inch. Assume that the ratio of the coil length to the embedded portion is 2.3. The coils are to be three turns per coil, and are to lie in slots 1 and 15.

The problem is to determine what winding combination will conform to our condition, and limit the loss factor in the top coil to 1.35 when the top coil carries currents in phase with those in the bottom coil. Also, to determine the loss factors for other parts of the coil, as follows: The factor for the top coil, when currents in the bottom coil are 60 degrees out of phase. The factor for the bottom coil and for the ends. And, finally, the factor for the whole winding as a unit.

The preliminary approximation referred to consists in first simplifying formula **IV** and then tabulating the results for different values of m and n. It was shown in the original paper that for small values of $\alpha\,h$ say less than 0.3, that $M_s = \frac{(\alpha\,h)^4}{3}$ and also $M_c = \left[1/n^2 + \frac{n^2-1}{n^2}\,1/r^2\right]\frac{(\alpha\,h)^4}{3}\,n^4$. See TRANSACTIONS, Vol. XXXIX, page 1038. Introducing these values and omitting O_s for the present, we find that formula 4 could be written with $(m\,n\,\alpha\,h)^4$ as a factor. Three new formulas will be written, and their values will give, when added to O_s the loss factor for the top coil side, for the bottom coil side, and for the end portion, respectively. One other change has been introduced in the formulas; that is, since $m\,n\,\alpha\,h$ is a factor, it can be given a definite value, say six and for any change, the incremental loss in excess of O_s will vary as the fourth power of the changed value to six. The simplified equations are

The incremental loss in excess of O_s for a top coil

$$\frac{36}{m^2\,n^2}\left[4 + 3\cos\beta + 1/4\,(3 + 1/\phi^2)\left(\frac{n^2-1}{r^2}\right) - 4/m^2\left(1 + \frac{n^2-1}{r^2}\right)\right] \quad (1)$$

The loss factor in excess of O_s for a bottom coil

$$\frac{36}{m^2\,n^2}\left[1 + 1/4\,(3 + 1/\phi^2)\left(\frac{n^2-1}{r^2}\right) - 4/m^2\left(1 + \frac{n^2-1}{r^2}\right)\right] \quad (2)$$

The loss factor in excess of O_s for the end portion

$$\frac{36}{m^2\,n^2}\left[1/4\,(3 + 1/\phi^2)\left(\frac{n^2-1}{r^2}\right) - 4/m^2\left(1 + \frac{n^2-1}{r^2}\right)\right] \quad (3)$$

In simplifying the equations (1), (2) and (3), above, for our first approximation, it will be noticed that for a value of ϕ equal to 1, the loss increment is a maximum. Also, it will be noticed that if we neglect that term in our equations, which contains m as a factor, that with these two changes the equations are very simply written. It can also be shown that for small values of $n\,\alpha\,h$ that O_s is essentially equal to unity.

With these changes, a table can be given, covering the incremental loss in the top coil under its maximum condition for loss. In this table, the assumption has been made that $r = 2$, and that $\cos \beta = 1$. The table is plotted for various values of m, and for n equal to 1, 2 and 3, respectively.

m	10	12	16	18	20	24	30	36	40	
factor	..	..	0.985	0.776	0.628	0.432	0.280	0.194	0.158	$n=1$
factor	0.690	0.481	0.270	0.222	0.174	0.121	0.078	0.054	0.043	$n=2$
factor	0.355	0.247	0.140	0.110	0.090	0.063	0.040	0.028	0.023	$n=3$

If we consider our problem, and assume that the copper temperature is known, say 100 deg. cent. total, then the calculation of α will be derived as in the previous paper, and would give a value of 1.83. The total copper depth was given as 4.5 inches, and this is equal to $m\,n\,h$. $4.5 \times 1.83 = 8.25$, and the values of the increment of the loss from the tables will be in the ratio of 3.58 for a coil 4.5 inches deep. Consequently, since the limit of the loss in the top coil was to be 1.35, the increment from the table will be 0.098, in order to keep the loss ratio to 1.35. In this connection, it is necessary to remember that there is a manufacturing limitation on the thickness of strands that can be handled; particularly, if mica tape is to be used to insulate the individual strands. A strand of 0.07 inches is about the smallest that can be successfully handled. From our table, therefore, it is evident that m must be 18, and n must be 3, in order to keep the increment approximately 0 098. This gives us a strand of a thickness of 0.0835 inches.

Having derived m and n from the approximation table, their values and the other factors given in our assumptions can be substituted in formulas (1), (2) and (3), for an accurate determination of the various loss factors. There are five factors to be determined: First, the loss in the top coil for $\cos \beta = 1$; Second, the loss in the top coil for $\cos \beta = 0.5$; Third, the loss in the bottom coil; Fourth, the loss in the end portion; and Fifth, the mean factor for the whole winding. These factors can all be determined by substitution in formulas (1), (2) and (3), except the last one, which is a factor of the ratio of the end windings to the embedded portion. The figures are given in the table below:

	Col. 1	Col. 2
Top coil $\cos \beta = 1$ Formula (1)........	0.1005	0.3600 a
" " $\cos \beta = .5$ " (1)........	0.0783	0.2810 b
Bot. coil " (2)........	0.0264	0.0946 c
End portion " (3)........	0.0141	0.0505 d
Factor for entire winding..............		0.1185

Column 2 is obtained from column 1 by multiplying by the factor 3.58. The loss increment for the whole winding can readily be obtained, as it is equal to $3\,a + 3\,b + 6\,c + 15.6\,d$ divided by 27.6. 15.6 is obtained from $(r - 1) \times 2 \times$ coils per pole and phase.

To the above figures in column 2 we have to add the value O_s. It is shown in the original paper how to obtain this value, but an approximation, when $m\,n\,\alpha\,h$ is equal to 6, is given by

$$O_c = 1 + \frac{130}{m^4\,n^2}\left[1 + \frac{N^2 - 1}{r^2}\right]$$

and for any other value of $m\,n\,\alpha\,h$ the increment of loss above unity is to be increased as the fourth power of the ratio of this value to 6. In our case, $O_s = 1.0025$.

It has been shown how relatively simple it is to predetermine the loss ratios in the different parts of a turned over two-coil per slot winding with transposition. It is recommended that any one who has occasion to use the approximation frequently will find it most convenient to plot his values on log-log paper. That is, on paper with logarithmic ruling for both ordinates and abscissas. If this method is followed, the table becomes a series of straight lines for different values of n.

One of the purposes in presenting a paper of this kind is to show that designing engineers are fully alive to the importance of theoretical investigations and the necessity of taking into consideration details in design which tend to improve the performance of their apparatus. This paper further illustrates how relatively simple it is to estimate accurately the effect of as complicated a phenomeon as the loss due to eddy currents in a stranded conductor.

Discussion

James Burke: With reference to Fig. 14 of the paper showing the offset segments, our company has used offset segments uniformly and continuously for 18 years in segmental punching machines and has not experienced any trouble from shaft currents.

P. L. Alger and R. F. Franklin: In view of the addition of Mr. Gilman's comprehensive papers to the already extensive literature on the eddy current losses in the conductors of large a-c. machines, it seems appropriate at this time to review the subject and describe the points of interest in the various articles from the point of view of a designing engineer. From this aspect, it is very desirable to be able to find in the shortest time and with the greatest accuracy and clearness what the eddy-current losses will be under any given conditions. Each of the articles that has appeared has features of particular value for the solution of certain problems, but most of them may be relatively inconvenient for use in the solution of any particular problem.

The first article to treat the subject in a comprehensive way was that by A. B. Field in the 1905 A. I. E. E. PROCEEDINGS. In this article, the physical phenomena of eddy currents are very neatly presented, accurate formulas starting with the classical differential equation are developed for most of the standard cases, and some interesting side-lights on the use of approximate formulas for the losses and on the calculation of reactance are given. This article may still be regarded as the best comprehensive review of the whole subject, considering synchronous machines, induction motors, and d-c. apparatus.

Mr. Gilman's article in the 1920 A. I. E. E. PROCEEDINGS treats the subject of large synchronous machines in much greater detail than Field's article. In it are derived accurate formulas starting with the classical differential equation for the eddy

losses in special cases not considered by Field, such as the carrying of a single strand through the whole phase belt before clipping it to the other strands of the same conductor. The alternator designer will find in this article formulas for all windings in use at the present time except those in which twisted conductors are employed. However, Mr. Gilman does not preserve the physical view of the phenomena while carrying out the solution of the equations, which is so necessary to a clear understanding of the effects of changes in conditions from those directly considered.

Mr. W. H. Taylor has published an extremely interesting article on the subject in the JOURNAL of the I. E. E. for April, 1920. He goes to the other extreme from Mr. Gilman by beginning with the physical viewpoint and building up the losses as a result of the summation of the effects of component eddy currents due to each conductor separately. In this way he derives formulas in the form of infinite series, in which only the first term is ordinarily used. However, the accuracy of these formulas is entirely sufficient in all cases of alternator windings in which the losses are not so excessive as to be unreasonable. Taylor treats all the standard cases of alternator windings and several curious cases in addition. For example, he shows that a coil that is turned over at the connection end but is wound straight up at the other end would have a very small loss, if such a coil were mechanically feasible to make. On the whole, I believe that Taylor's article is the best from the point of view of the designer of synchronous machines, as his formulas are the simplest (they require no curves or tables) and his presentation renders it easy to see the effects of whatever changes in the winding the designer may be able to imagine. Taylor also does not consider the twisted conductor of the Roebol type although he does treat the conductor that is turned over in the middle of the slot.

The twisted conductor has been considered at some length by German and French writers. For example, Fleishmann has shown that the circulating-current loss in a conductor will be completely eliminated if every strand in it has the same r. m. s. height in the slot. However, the use of such twisted conductors requires that they be clipped at each end, and so precludes the use of a machine-wound coil. For this reason, as well as because the labor in making them is greatly in excess of that for a normal conductor and the space factor of the slots is reduced by their use, they have not been adopted in this country. Therefore, most of the continental literature on the question is of relatively little interest to American designers.

The induction-motor designer is also interested in the question of eddy-current losses, since it is possible by exaggerating the eddy currents in the squirrel cage to obtain a greater starting torque per kv-a. than in the ordinary motor. Here, however, the losses are purposely made as large as possible, and so they fall outside the range of accuracy of Taylor's formulas. Also the feasible types of conductor are widely different from those used in alternators which are discussed by Taylor and Gilman. Finally, it is necessary in this case to consider the reactance as well as the resistance, and none of the articles so far mentioned gives any adequate method for calculating the reactance. For these reasons I believe the induction-motor designer had best put all his energy into the study of the article by Prof. W. V. Lyon in the 1922 A. I. E. E. PROCEEDINGS.

This article treats the whole subject of eddy currents by the vector method used by Kennelly and others in the solution of transmission-line problems. By this method the vector impedance of a conductor carrying alternating current is carried through the whole series of equations and found numerically, as a ratio to the d-c. resistance. This contrasts with the method followed by all the other writers in which the losses are separated from the reactive power at as early a stage as possible and thereafter they only are considered. While Lyon's article does not give numerical results directly usable by the designer, he does give methods and formulas which with a little further work can be put in a form to give accurate and complete values for the effective impedances of all kinds of squirrel-cage conductors. Thus this article is the most basic and comprehensive for use as the foundation of future work on the subject.

M. S. Vallarta: In a discussion on a paper by W. V. Lyon,[1] I presented certain results of an experimental investigation undertaken in the Research Division of the Electrical Engineering Department, Massachusetts Institute of Technology, the purpose of which was to test the correctness of the premises on which Field's[2] one-dimensional theory of skin-effect in armature conductors is built up. As all resistance-ratio formulas given in the paper under discussion are based on this theory, a reexamination of its underlying assumptions in the light of recent developments does not appear superfluous.

Perhaps the most fundamental and least evident of these assumptions is that the component of magnetic field strength parallel to the slot side vanishes everywhere within the cross-section of the conductor. In view of its importance, especial efforts were made in the course of the investigation already referred to in order to bring to light conclusive experimental evidence on this point, but in spite of them, the conclusion, stated in 1922, was: "No satisfactory proof of Field's assumption of no component of field strength parallel to the slot side has been found."[3] Unexplained discrepancies, ranging from 2 to

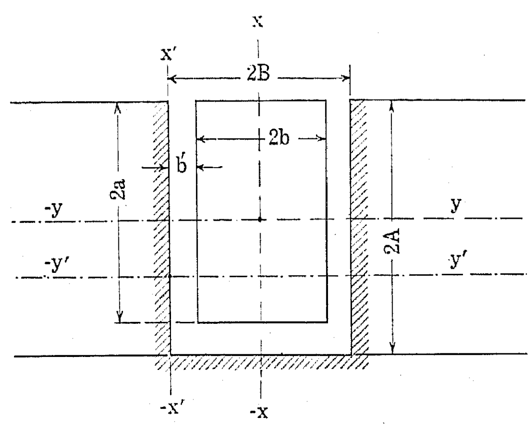

FIG. 1

5 per cent, were observed between calculated and measured resistance-ratios. Such discrepancies were believed to be due to the fact that the current density is not actually constant along planes parallel to the slot bottom, *i. e.*, that skin effect is not one-dimensional. The purpose of these lines is to show how far this two-dimensional skin effect may be expected to come into play and how it affects resistance-ratio formulas.

In a recent paper, Steidinger[4] has examined the problem for the case of a *single massive* conductor which does not completely fill the slot and given formulas for the Joule loss. We consider a single massive rectangular conductor of width $2\,b$ and height $2a$ embedded in a rectangular slot of width $2\,B$ and height $2\,A$ (Fig. 1). On its cross-section the current distribution is a function of x and y, that is, Field's assumption of one-dimensional skin effect, in which the current density is a function of x only (x = distance of current element from bottom of slot), is given up. The time variation of all electromagnetic quantities is assumed to be monoperiodic, as is done in Field's theory and elsewhere in the literature. We therefore write:

$$c = u\,(x, y) e^{j\omega t}$$

1. W. V. Lyon, "Heat Losses in Stranded Armature Conductors," TRANS. A. I. E. E., Vol. 41, p. 199, 1922.
2. TRANS. A. I. E. E., Vol. 24, p. 761, 1905.
3. TRANS. A. I. E. E., Vol. 41, p. 212, 1922.
4. *Archiv. fur Elektrotechnik*, Vol. 12, p. 149, 1923.

where c is the current density at a point $(x\ y)$ and ω is the circular frequency of this current. $u\ (x\ y)$ is determined by Maxwell's circuital equations:

$$\Delta \times H = \frac{4\pi}{c}\left(\sigma E + \frac{\epsilon}{4\pi}\frac{\partial E}{\partial t}\right),$$

$$\Delta \times E = -\frac{\mu}{c}\frac{\partial H}{\partial t}$$

with $\Delta . H = 0$ $\quad \epsilon \Delta . E = 4\pi\rho$

where H and E are the magnetic and electric field strengths, σ, ϵ, μ, the conductivity, permittivity and permeability, c is the velocity of light in free space, ρ the space electrical density, all expressed in absolute (Gaussian) units; and Δx, $\Delta .$ denote the operator's curl and divergence, in Gibbs' notation. Since skin effect is assumed to be two-dimensional, $\frac{\partial}{\partial 2}$ is an annihilator and the electric and magnetic field-strength components satisfy the equation—derived by curling from the two circuital equations:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = j\,\frac{4\pi\sigma\mu\omega}{c}\,u - \omega^2\frac{\epsilon\mu}{c^2}\,u$$

but, since in a metal the displacement current $\frac{\epsilon}{4\pi}\frac{\partial E}{\partial t}$ is negligible compared with the conduction current, this reduces to:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = j\,\alpha^2 u \text{ with } \alpha^2 = \frac{4\pi\sigma\mu\omega}{c}$$

Field's[5] equation is obtained from this, provided the conductor completely fills the slot, by simply noticing that for one-dimensional skin effect $\frac{\partial}{\partial y} = 0$ and that in electromagnetic units $c = 1$.

A particular solution of this last equation is:

$$u\ (x, y) = C \cosh\left[x\ \sqrt{n^2 + j\ \alpha^2} + \beta\right] \cos\ (n\ y + \gamma)$$

where C, n, β, γ are arbitrary constants, which may be complex numbers. By symmetry $u\,(+\,y) = u\,(-\,y)$, so $\gamma = 0$. n is determined by the condition equation obtained from the boundary conditions—continuity of the magnetic field—upon the plans $y = -\,b$ $y' = b'$

$$n \tan\ (n\ b) + \sqrt{n^2 + j\ (\alpha^2 + \beta^2)}\ \tan\left[b'\ \sqrt{n^2 + j\ (\alpha^2 + \beta^2)}\right] = 0$$

where b' is the distance from slot side to conductor, (see Fig. 1). This equation has an infinite number of complex roots n_k.

Likewise, the constant β is determined by the boundary conditions at the top and bottom. There is a difficulty here because the boundary conditions at the top of the conductor are not definitely known. In an alternator, they would depend on the relative position of the slot with respect to the pole face. If we write the solution in the form:

$$C = A\ e^{j\omega t} \sum_{k=1}^{\infty} C_k \cosh\left[x\ \sqrt{n_k^2 + j\ \alpha^2}\right] \cos\ (n_k\ y)$$

there is another difficulty because the normal solutions are not orthogonal and hence the coefficients C_k cannot be determined by Fourier's method. We shall not go into this here, but refer to Steidinger's paper (l. c. p. 152) for details. In what follows, the argument is limited to the fundamental harmonic ($k = 1$).

Assume that the conductor carries a total current $\sqrt{2}\ I\ e^{j\omega t}$. The average over a cycle of the total heat loss in the conductor is

$$\bar{Q} = \tfrac{1}{2} \int \rho / c /^2\ d\ v$$

ρ being the resistivity ($\rho = 1/\sigma$) and the integral being taken through the volume of the conductor. Integrating, and again referring to Steidinger's paper for details, we get:

$$Q = R\ I^2 \left[m_1\ a\ \frac{\sinh 2\ m_1\ a + \sin 2\ m_1\ a}{\cosh 2\ m_1\ a - \cos 2\ m_1\ a} \right]$$

$$\left[m_2\ b\ \frac{\sinh 2\ m_2\ b + \sin 2\ m_2\ b}{\cosh 2\ m_2\ b - \cos 2\ m_2\ b} \right] \qquad (1)$$

where R is the ohmic resistance, $m_1 = \alpha\ \sqrt{b/B}$ and $m_2 = \alpha\ \sqrt{b'/B}$. The resistance ratio K is $Q/R\ I^2$ by definition.

The function in parentheses is always greater than unity except for argument zero, when it becomes unity; it has been tabulated and plotted.[6] It is *only when the conductor completely fills the slot* ($b' = 0$) that the above equation reduces to Field's corresponding formula, which is derived also by Gilman, Lyon and others. It is only in this case that the resistance ratio, calculated by using one-dimensional skin-effect formulas, such as those given by Rogowski, Gilman, Lyon and others[7] can be expected to agree with facts. This conclusion is wholly in accordance with experimental evidence available to date.

In all practical cases, where the conductors do not completely fill the slot, the resistance ratio is *greater* than that calculated from one-dimensional formulas, such as given by Mr. Gilman in the paper under discussion. That the two-dimensional correction is usually negligible, but may easily become important is perhaps best shown by two illustrative examples.

As a first instance, take the test coil used in our M. I. T. investigation. Its dimensions, assuming a massive conductor, are, $2\ b = 1.418$ cm., $2\ a = 3.58$ cm. The slot dimensions are, $2\ B = 1.905$ cm., $2\ A = 7.62$ cm. The conductor is assumed to be laid flat on the slot bottom, symmetrically with respect to its sides. Then $b' = 0.24$ cm., and from the above formulas, $\alpha = 1.671$ cm.$^{-1}$ at 60 cycles for commercial copper and $m_2 = 0.861$ cm.$^{-1}$. The correction due to two-dimensional skin effect [2nd parenthesis in formula (1)] is 1.03 approximately, whereas the measured value was 1.02. Too much worth cannot be attached to this check, first, on account of the special construction of the coil which largely eliminated two-dimensional skin effect, second, in view of the uncertainty in the elimination of the iron loss, and third in view of the smallness of the correction.

As a second example, take the generator quoted in Gilman's paper under discussion. Here $2\ B = 1$ inch, $2\ b = 0.5$ inch. So $b' = 0.634$ cm. and $m_2 = 1.182$ cm.$^{-1}$. The two-dimensional correction is here about 1.12.

It should be carefully emphasized that the resistance-ratio formula (1) takes into account the fundamental harmonic only and that the two-dimensional skin-effect theory of n conductors per slot has not yet been worked out. In this sense, the above examples are only illustrative. Judging from available experimental evidence, two-dimensional skin effect seems to be more effective for conductors near the top than for those near the bottom. It follows that care and judgment should be exercised by the designing engineer in the use of Gilman's formula, particularly in the case of wide slots and highly insulated conductors.

R. B. Williamson: About 17 years ago the subject of eddy-current losses in armature conductors was brought before the Institute in a paper by A. B. Field. Previous to this it was known in a general way that generators sometimes developed high temperatures in the windings for no well explained reason. Such temperatures were not due to lack of sufficient copper, because the apparent current density in the copper was in many of these cases quite low. Mr. Field's paper showed that the excess heating was due to eddy currents and gave means for

5. This equation is also the starting point of Lyon's and Gilman's papers.

6. For instance by W. Rogowski in *Archiv fur Elektrotechnik*, Vol. 2, p. 97, 1913, also by W. V. Lyon, TRANS. A. I. E. E., Vol. 40, p. 1361, 1921 (Fig. 3). Notice that our m_1 is Gilman's "α" or $1/\sqrt{2}\ j$ Lyon s "α."

7. For instance Emde, *Elektrotechnik und Maschinenbau*, Vol. 40, p. 301, 1922. Also, Mayer, *Archiv. fur Elektrotechnik*, Vol. 12, p. 349, 1923.

estimating the amount of such excess loss. This paper, however, gave the methods of estimating the loss for solid conductors only, or for those infinitely laminated. In windings as actually made, the lamination can only be partial, and Mr. Gilman's previous paper presented before the Institute carried Mr. Field's work further and showed how to estimate the eddy current loss for different arrangements of winding and various degrees of stranding of the conductor. The present paper takes up some additional cases not covered by the previous one, and the two papers taken together form a very complete treatment of the whole subject. It is not possible to build successful large generators without taking this eddy-current effect carefully into account and arranging the winding accordingly. Contributions such as these by Mr. Gilman, while on the face appearing highly theoretical, are of the greatest practical importance to the designer and will doubtless form a valuable reference on this subject for many years to come.

S. L. Henderson: I think it has been pretty well brought out today that Mr. Field's article on eddy currents covered only two cases of conductors, namely, that of the solid conductor and that of the infinitely stranded conductor; obviously, only one practical case—the solid conductor.

I happened to be associated with Mr. Gilman at the time he developed an eddy-current formula and assisted him with the tests for checking the results of this formula, and while I am not entirely familiar with Mr. Taylor's article or with the discussion of Mr. Vallarta on whether the case comes under the one-dimensional or the two-dimensional theory, this much I do know: That the tests we made checked within one per cent of our calculation, and it is obvious, therefore, that in view of the other features of design which are not accurately determinable, any formula that will give results within one per cent is acceptable.

The comment was also made that Mr. Gilman's formula appeared involved and required considerable effort to apply. Of course, my point of view may be warped inasmuch as I have been using this formula probably for four or five years, but with the formula and the curves I think it is possible to check the eddy-current loss in any armature conductor within ten minutes; probably five minutes in the average case.

I think there was also a statement made that Field's article would cover most of the cases. This might be true on small machines, but where the size gets beyond, say, 1000 or 2000 kv-a., the conductors are stranded either twice or more times, and consequently it is necessary to be able to calculate the amount of eddy-current loss in the conductor, and it is only possible to do this with the aid of the formula as worked out by Mr. Gilman or possibly as stated by Mr. Taylor or Professor Lyon.

Tooth Pulsation in Rotating Machines

BY T. SPOONER

Member, A. I. E. E.

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Review of the Subject.—1. *An experimental method is presented of checking the magnitude of flux pulsations in the teeth of rotating machines where both members are slotted. The method consists in using metallic electrodes shaped like the teeth of a machine and an electrolyte or mercury to represent the air. Voltage is applied between the two members and the current through the tooth under consideration is measured. The magnitude of this current under different conditions is proportional to the magnitude of the flux which would flow under the analogous magnetic conditions.*

2. *The test results are compared with the pulsation amplitudes as calculated by two methods. The test results are in general slightly lower than the calculated but the agreement is fairly good.*

3. *It is believed that either of the above-mentioned methods can be used to calculate tooth pulsations without serious errors where saturation effects are not appreciable. These methods should be specially useful in determining which of two or more designs would be subject to the lesser pulsation losses.*

4. *The effect of saturation in the iron is determined experimentally by making the ratio of the mercury to the electrode resistance small. This ratio corresponds to the magnetic permeability. The effect of the tooth resistance on the amplitude of the pulsation is calculated by assuming three resistances in series, namely, the stator and rotor tooth resistances and the air gap resistance. The calculated and test values check reasonably well.*

5. *It is shown that the effect of saturation on pulsation amplitude for actual machines can not be calculated by adding directly the air gap and tooth reluctances due to the fact that the permeability of iron is not constant. In order to actually calculate the effect of saturation, it is necessary either to plot portions of the magnetization curve or to make use as we do of the incremental permeability values.*

6. *The method described of allowing for saturation is too complicated for ordinary design calculations but is useful in giving a clearer picture of just what these effects are and could be successfully used for special cases where the extra labor involved was warranted.*

7. *These methods do not apply when short-circuited windings are present in the slots. It is hoped at a later date to consider this aspect of the problem.*

8. *We believe it is possible by the use of the outlined methods below to calculate simple, reasonably reliable correction curves for saturation effects which may be used both for the case of uniform mean flux and for a sine distribution of the mean flux.*

INTRODUCTION

WHEN the rotor and stator of a rotating machine are both slotted, high frequency flux pulsations are produced in the teeth which give rise to hysteresis and eddy-current losses. These losses may amount to a considerable percentage of the total core losses of the machine. In order to predetermine the magnitude of the losses and to reduce them to a minimum by proper design it is first necessary to be able to calculate the magnitude of the tooth pulsations. These pulsations are the result of local changes in the air-gap reluctance and are reduced by saturation effects in the iron of the teeth; namely, when the reluctance of the teeth is appreciable with respect to the reluctance of the air gap the tooth pulsations are less. Also the presence of short-circuited windings in the slots such as in the case of rotors of squirrel-cage induction motors or the damper windings of salient-pole machines tend to reduce these pulsations. These latter effects, namely, of short-circuited windings, will not be considered in this paper but it is hoped that at a later date we may deal with them also.

There are various methods of estimating the magnitude of tooth pulsations but a discussion of these will be reserved until we have first described an experimental method which we have applied to this problem. It is a simple matter to measure the magnitude of tooth pulsations in an existing machine and it has been done in several ways. For instance, we may place an exploring coil around the tooth in question, excite the core with direct current, reverse the direct current and note the deflection of a flux-meter or ballistic galvanometer connected to the exploring coil. Or we may slowly revolve the rotor and note the amplitude of the deflection of the fluxmeter. Again we may connect the exploring coil to an oscillograph and from the record with the rotor revolving determine the amplitude of pulsation by the following method due to Hoseason.[2] If the ratio of the amplitude of the tooth-frequency voltage ripple to the fundamental-frequency voltage amplitude be divided by the ratio of the tooth frequency to the fundamental frequency we shall have the amplitude of tooth flux pulsation with reference to the fundamental flux.

Hoseason has used another method, namely, to connect his tooth exploring coil to an electrostatic voltmeter, apply normal alternating voltage to the stator and obtain the maximum and minimum voltage readings when the rotor is revolved very slowly. Another method is one which we have used with a fair degree of accuracy, namely, simply to connect the tooth exploring coil to an a-c. voltmeter of the dynamometer type, excite the stator with direct current (full current in one phase and one-half reversed in the other two for a three-phase machine), and calculate the tooth pulsations from the readings, thus:

$$E_T = E_F \sqrt{1 + (1/8 + 1/8\,n^2)\,P^2} \qquad (1)$$

E_T = total tooth voltage,

1. This is one of a number of articles being published by members of the Core Loss Committee of the National Research Council.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

2. Tooth Frequency Iron Losses in Slip Ring Induction Motors, D. B. Hoseason; *Electrician*, Sept. 7, 1923.

E_F = voltage as calculated from the fundamental frequency flux (no tooth pulsations)
n = the number of teeth per pair of poles,
P = flux pulsations based on the fundamental frequency flux.

If the frequency is sufficiently high it is necessary to correct the dynamometer readings for the inductance error of the instrument. The above formula was developed by Dr. J. Slepian and is based on the assumption of a sine wave fundamental-frequency and tooth-frequency flux distribution. Any of these methods are satisfactory for completed machines but do not give us a means of predetermining the tooth pulsations for new apparatus.

Experimental Method

For a number of important investigations involving the determination of magnetic or electrical flux distribution, an electrolytic method has been used in which the electrodes were shaped like the magnetic or conducting portions, and an electrolyte took the place of the air. The current is caused to flow from one electrode to another and will follow the same path in the electrolyte as a magnetic flux, for instance, would follow in the air when passing from one ferro-magnetic member to another. As previously used the electric field was explored in the electrolyte by means of a potentiometer or millivoltmeter arrangement and the equipotential lines plotted. The current or flux lines were of course obtained by plotting perpendicular to the equipotential lines. A similar method making use of templates cut from high-resistance sheet to simulate teeth and slots has been used very successfully by Douglas[3] and by Roeternik[4]. We believe this method was first suggested by Carter. As used by most investigators this method was capable of giving results corresponding to the condition of infinite permeability only. Roeternik simulated the condition of moderate permeabilities by using a comparatively low-resistance template for the iron portion and joined to this a high-resistance template for the air portion. These methods, while yielding quite accurate results, are very slow and tedious.

It would have been possible to attack the problem by using the method of Hele-Shaw[5], in which a viscous liquid is caused to flow through certain definite channels and the flow lines indicated by dyes introduced in the liquid in suitable manner. This is also a tedious process.

For our work we have made use of a modification of the electrolytic method but have greatly simplified the experimental process by automatically integrating the current for maximum and minimum reluctance conditions. The method is made clear by referring to Fig. 1.

In order to determine experimentally the magnitude of the tooth pulsations, use was made of the analogy between the flow of electric currents and the distribution of magnetic flux. A set each of copper and brass blocks were machined from solid material having the dimension shown in Fig. 2. The thickness was ½

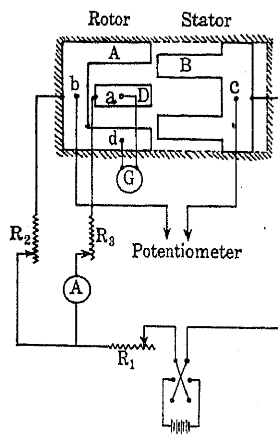

FIG. 1—DIAGRAM OF CONNECTIONS FOR TOOTH PULSATION TESTS

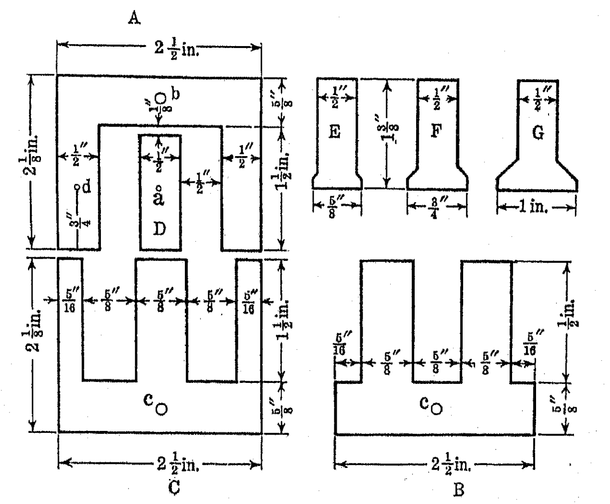

FIG. 2—COPPER AND BRASS BLOCKS FOR TOOTH PULSATION TESTS

inch (1.27 c m.). A tray was filled with paraffin wax with an opening left large enough to take a set of blocks arranged as shown in Fig. 1. The sides of the opening are shown by the dotted lines. Electrical connections were made to the blocks as indicated.

The principle of operation is as follows, tooth D being the one under test. It will be noted that block D is electrically separated from A in order that the current flowing into it may be measured. The openings between the blocks are filled with an electrolyte or mercury so that its level is just flush with the top of

3. The Reluctance of Some Irregular Magnetic Fields. John F. H. Douglas, TRANS. A. I. E. E., Vol. XXXIV. 1915, p. 1067.

4. Eine theoretische und experimentelle Untersuchung des Untenfeldes einer unbelasteten elektrischen Maschine. F. M. Roeternik: *Archiv fur Elektrotechnik,* Vol. 7, 1918-19, p. 292

5. Hydrodynamical and Electrical Investigations Regarding the Magnetic Flux Distribution in Toothed Core Armatures. H. S. Hele-Shaw, A. Hay, P. H. Powell. Inst. E. E. Vol. 34, 1904, p. 21.

the blocks. The blocks represent the iron teeth of a machine and the liquid represents the air. The ratio of the resistivity of the liquid to the resistivity of the block material gives the effective permeability. All we have to do then is to measure the current flowing into the block D for various conditions and this will be equivalent to the flux which will flow under the corresponding magnetic conditions, since the current corresponds to the flux and the voltage to the magnetomotive force. It will be seen that with the arrangement shown in Fig. 1, we have a maximum reluctance or resistance between tooth D and the teeth of the block B. Now if we remove block B and substitute block C (Fig. 2) we shall have the condition of minimum resistance. This change of blocks corresponds to a movement of the stator with respect to the rotor of one-half a slot pitch. The tests are made on the assumption of constant difference of magnetic potential between the stator and the rotor yokes and this corresponds to a constant electromotive force between points b and c.

The method of test was as follows. The block B was adjusted to the right or left (Fig. 1) to give a definite air gap between the stator and rotor blocks. The resistances R_2 and R_3 were adjusted until the galvanometer G showed no deflection. This means that tooth D and the teeth of A were at the same potential. Contact points a and d should really be at the left end of the teeth but it was thought better to put them as shown in order to allow the current in the tooth D to become uniform over the cross section. R_1 was next adjusted until the potentiometer connected to b and c gave the desired voltage which was kept constant for one set of readings. The ammeter A was then read. The battery was then reversed, the resistances adjusted and the ammeter again read. This reversal was for the purpose of eliminating thermoelectric effects. The air gap was then changed and another set of readings of the ammeter obtained. After the required range of air gaps was covered, block B was removed, block C substituted and the tests repeated. The percentage pulsation was then obtained by dividing the difference between the two ammeter readings for a given air gap by the maximum ammeter reading. The maximum instead of the average was used since in general hysteresis losses can more easily be calculated on a basis of maximum flux. After completing a set of tests with tooth D, it was removed and E, F and G substituted and the tests repeated. These latter teeth gave results for partly closed rotor slots. This gives a construction which of course is not met in commercial machines but for the purpose of checking our calculated results this does not matter.

If we use an electrolyte it will be seen that for all practical purposes its resistance is infinite with reference to the resistance of the copper or brass blocks. This will correspond then to the condition of moderate and low inductions in the teeth in which the permeability of the iron is many times that of air.

If, instead of an electrolyte, we use mercury with the copper blocks, for instance, we obtain a ratio of resistance of the copper to the mercury of approximately 55.5, which will be equivalent to a permeability of this value. By using brass blocks and mercury we obtain an effective permeability of 13.6.

With an electrolyte of course it was out of the question to use direct current so we went to a 500-cycle supply. In place of the galvanometer G, we used a telephone receiver and for the ammeter a thermo-electric instrument. Voltage was maintained constant by means of a Paul voltmeter placed outside of the ammeter A. Suitable corrections were made for the change in voltage due to the $I R$ drop in the ammeter. The resistance R_3 was eliminated.

TEST RESULTS

Preliminary Tests. Instead of using metallic electrodes a much greater range of permeability could be obtained by using a higher resistance material and various electrolytes. This would have the advantage also that only one set of electrodes would be required. Accordingly we first made a set of electrodes of cast tellurium (silicon was also considered but was abandoned due to the difficulty of machining and due to the difficulty of casting without blow holes). Tellurium has a resistivity of about 0.2 ohm-cm. which is the same order as that of high conductivity electrolytes. This would enable us to obtain effective permeabilities from the lowest values desired to the highest by simply varying the concentration of the electrolyte. Two difficulties were encountered. The first was the difficulty of making good electrical contacts with tellurium. This was overcome by coating the material where desired with copper by means of the Schoop spray. It was then possible to solder different parts together. The second and more serious difficulty was the fact that, no matter how carefully it was cleaned, the surface of the tellurium developed a much higher resistance than the average for the material. This means that the effective permeability could not be accurately determined.

After abandoning the tellurium, electrodes were made of a special graphite material supplied by Mr. G. M. Little and having a specific resistance of about one ohm. This was open to the same surface-resistance objection as the tellurium although to a much less degree. Both materials give good results with higher resistance electrolytes but were not suitable for a study of the effect of permeability changes.

Final Tests. With the copper and brass electrodes no serious difficulties were encountered. The tests under infinite permeability conditions were made with the copper electrodes and a strong copper sulfate solution. For the tests with the mercury bath the working surfaces of the electrodes were carefully

amalgamated. The top and bottom surfaces and the sides, except the working surfaces, were insulated with paraffin. Some precautions had to be taken against thermoelectric effects since the maximum currents in tooth D were of the order of 15 amperes and the maximum voltage drop between b and c only a fraction of a millivolt. Four air gaps were used for each set of blocks, namely, 0.04 in. (1.01 mm.), 0.081 in. (2.03 mm), 0.105 in. (3.8 mm.) and 0.180 in. (4.57 mm.).

Typical test results are given in Figs. 3-6 plotted between per cent pulsation based on the maximum

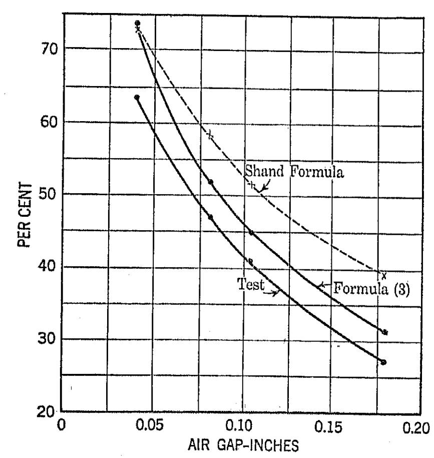

FIG. 3—TEST AND CALCULATED TOOTH PULSATIONS FOR TOOTH D AND INFINITE PERMEABILITY

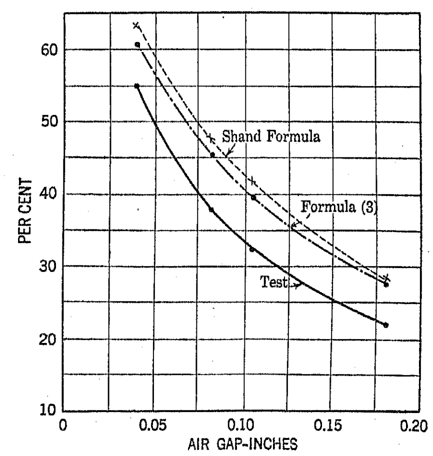

FIG. 4—TEST AND CALCULATED TOOTH PULSATIONS FOR TOOTH E AND INFINITE PERMEABILITY

current or flux and the air gap in inches. These curves are for infinite permeability. Fig. 7 shows the curve for tooth D and a permeability of 55.5, and Fig. 8 the corresponding curve for a permeability of 13.6. Fig. 9 gives a comparison between the test and calculated values for an air gap of .04 in., (1.01 mm.) and various permeabilities. Fig. 10 gives similar results for an air gap of .180 in. (4.57 mm.). Fig. 11 shows the relation between air gap and per cent pulsation of tooth D and the three permeability conditions.

METHOD OF CALCULATING PULSATIONS

The tooth pulsations could doubtless be predicted quite accurately by the well-known graphical method of plotting the flux flow lines. A good recent example of the use of this method is given by Lehmann,[6] for checking the accuracy of his air gap reluctance formulas for the case of slotted rotors and stators. This method, however, is quite laborious.

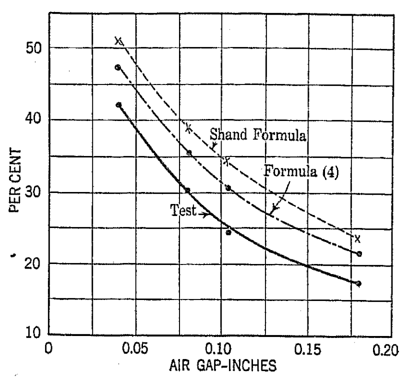

FIG. 5—TEST AND CALCULATED TOOTH PULSATIONS FOR TOOTH F AND INFINITE PERMEABILITY

A comparatively simple mathematical method which has frequently been used for air-gap reluctance calculations is to assume that the air-flux paths follow straight lines and arcs of circles. This method gives results when applied to the case of one smooth and one slotted member which check Carter's more exact calculations very well. Still[7] has recently used this method quite successfully in considering air gap

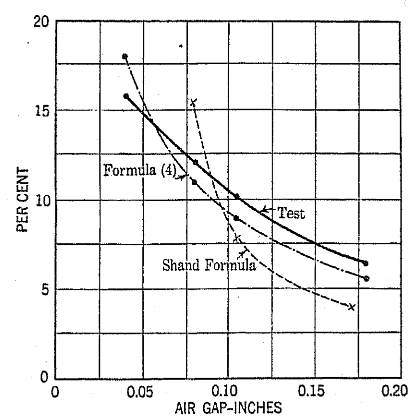

FIG. 6—TEST AND CALCULATED TOOTH PULSATIONS FOR TOOTH G AND INFINITE PERMEABILITY

reluctances when saturation effects are present. Mr. E. B. Shand has used this method for deriving curves according to certain definite assumptions by means of which he is able very quickly to estimate the mag-

6. Sur la Reluctancé de l'entrefer des Machines a Encoches Ouvertes dans le Stator et dans le Rotor. Th. Lehmann; *Rev. Gen. de l' Electricite*, Feb. 3, 1923, p. 165.

7. Flux Distribution in Air Gap and Teeth of Dynamos. Alfred Still; *Electrician*, Feb. 10-17, 1922.

nitude of the tooth pulsations for the case of infinite permeability. His data are not yet published.

Some time ago we devised formulas for calculating tooth pulsations making use of Carter's fringing constants which apparently work very well. Hoseason[8] has recently suggested the use of such a method but did not go into details as to how to apply it. Our method is as follows. Referring to Fig. 12, suppose we calculate the effective width (t_e) of the stator and rotor teeth by using Carter's fringing constants[9] and assume that all the flux crossing the air gap is confined to those widths and that it is of uniform density. Suppose that we also assume that the flux flows only over those portions of the effective tooth widths which happen to be opposite each other. Now by calculating the change in

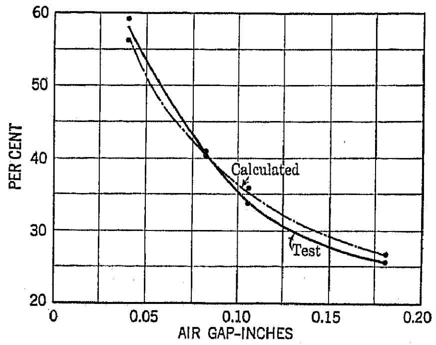

FIG. 7—TEST AND CALCULATED TOOTH PULSATIONS FOR TOOTH *D* AND A PERMEABILITY OF 55.5

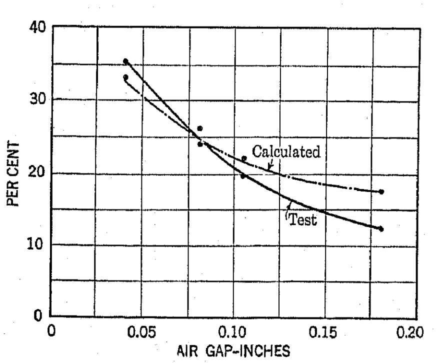

FIG. 8—TEST AND CALCULATED TOOTH PULSATIONS FOR TOOTH *D* AND A PERMEABILITY OF 13.6

tangential air gap widths opposite each other for the position of maximum and minimum reluctance, we can estimate the amount of the tooth pulsations. When Carter's coefficients for the stator and rotor teeth have already been calculated for other purposes, the effective width of the tooth may be obtained from the ratio $\lambda/g\ (= t_e)$ where λ is the tooth pitch and g is the Carter coefficient.

Referring now to Fig. 13 we have ~~the four~~ cases which are likely to occur in practise for induction motors. The stator tooth pulsations in per cent of the maximum flux can almost always be calculated by the formula (open stator slots).

$$P_s = \frac{\lambda_r - t_{er}}{t_{es}} 100 \qquad (2)$$

There are four cases for the rotor teeth depending on the relative dimensions. These four cases are covered by the following formulas:

Case 1 — Limits

$$P_{r1} = \frac{\lambda_s - t_{es}}{t_{er}} 100 \qquad (3)$$

P_{r1} cannot exceed 100

t_{es} is greater than t_{er}

Case 2

$$P_{r2} = \frac{\lambda_s - t_{er}}{t_{es}} 100 \qquad (4)$$

t_{es} is less than t_{er}

P_{r2} has a positive sign

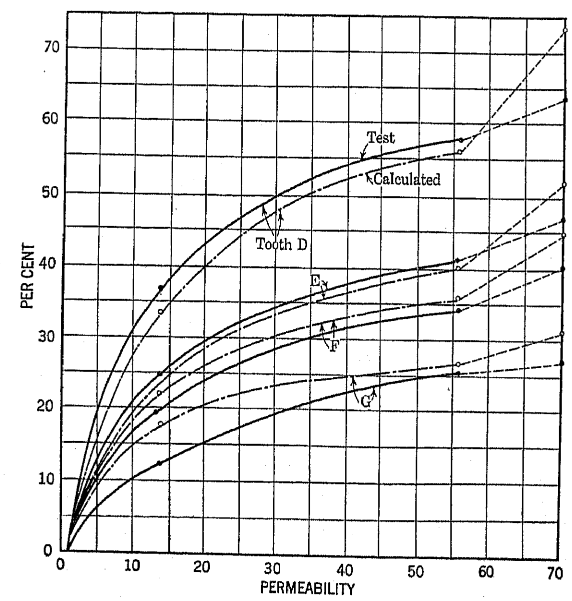

FIG. 9—TEST AND CALCULATED TOOTH PULSATIONS FOR VARIOUS TEETH AND PERMEABILITY AND AN AIR GAP OF 0.04 INCHES

Case 3

$$P_{r3} = \frac{t_{er} - \lambda_s}{t_{er} - \lambda_s + t_{es}} 100 \qquad (5)$$

t_{es} is less than $t_{er} - \lambda_s + t_{es}$

t_{er} is less than $2\,\lambda_s - t_{es}$

Case 4

$$P_{r4} = \frac{\lambda_s - t_{es}}{t_{er} - \lambda_s + t_{es}} 100 \qquad (6)$$

t_{er} is greater than $2\,\lambda_s - t_{es}$

t_{er} is less than $\lambda_s + t_{es}$

where,

P_s is the stator tooth pulsation in per cent of the maximum tooth flux

P_{r1}, P_{r2}, P_{r3} and P_{r4} are the rotor tooth pulsations for the four cases.

λ_s is the stator slot pitch

λ_r is the rotor slot pitch

t_{es} is the effective stator tooth width

t_{er} is the effective rotor tooth width.

Cases 1 and 2 are to be used when the rotor slot pit[illegible]ie [illegible]ully

8. Tooth Frequency Iron Losses in Slip Ring Induction Motors, D. B. Hoseason; *Electrician*, Sept. 7, 1923.

9. Electrical Machine Design. Alexander Gray; p. 44, fig. 40.

is less than the stator slot pitch and cases 3 and 4 when the reverse is true. The conditions for the four cases are shown graphically in Fig. 13. If the rotor slot pitch becomes greater than covered by case 4, a fifth and sixth case, etc., may be added if necessary but the above four cases will cover practically all commercial conditions. After the effective tooth widths are calculated it takes only a moment to determine which formula to use.

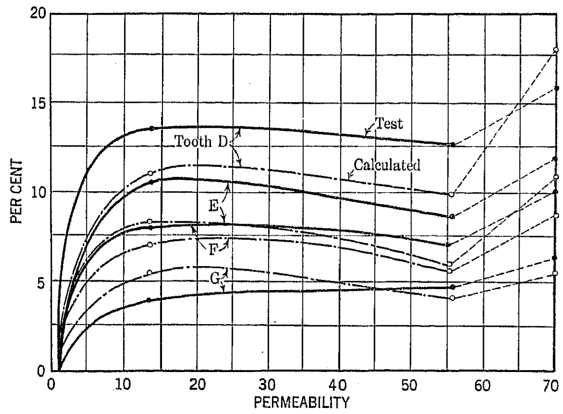

FIG. 10—TEST AND CALCULATED TOOTH PULSATIONS FOR VARIOUS TEETH AND PERMEABILITY AND AN AIR GAP OF 0.04 IN.

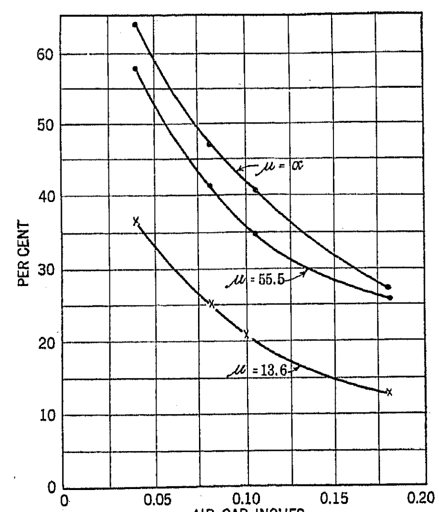

FIG. 11—TEST TOOTH PULSATIONS FOR VARIOUS AIR GAPS AND PERMEABILITIES FOR TOOTH *D*

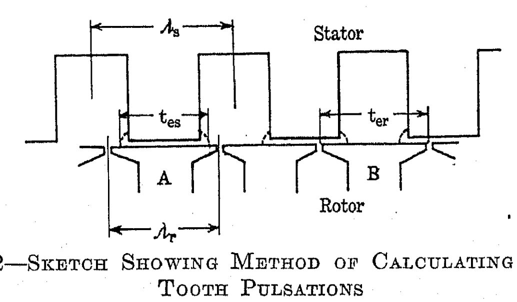

FIG. 12—SKETCH SHOWING METHOD OF CALCULATING ROTOR TOOTH PULSATIONS

These formulas do not give strictly correct results since they are based on assumptions which are not quite true. Their reliability will be discussed a little later. Fig. 14 shows the sort of pulsation curves that are obtained by their use. It will be noted that when passing from one formula to the next the curves are discontinuous. Approximately true results will probably be obtained by rounding off these curves at those positions. At the point of transition from one formula to the next, either formula will give the same result.

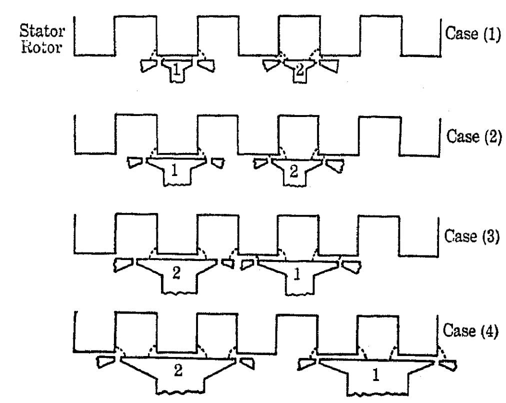

FIG. 13—SKETCH SHOWING METHOD OF DERIVING TOOTH PULSATION FORMULAS

The calculation of curves of this kind will yield a number of interesting points, some of which have been mentioned by Hoseason.[10] It should be noted that these formulas apply only to the case of infinite permeability.

Comparison between Calculated and Test Results. By

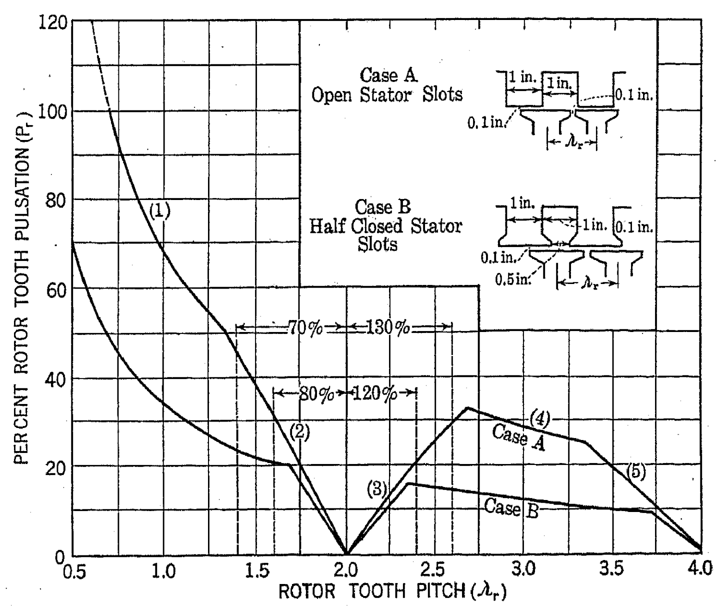

FIG. 14—SKETCH SHOWING EFFECT OF CHANGING ROTOR SLOT PITCH WITH REFERENCE TO STATOR SLOT PITCH

referring to Figs. 3-10, it will be seen how the test values compare with the pulsations as calculated by the above-mentioned methods. It will be noted that in general the calculated results are slightly higher but

10. Tooth Frequency Iron Losses in Slip Ring Induction Motors, D. B. Hoseason; *Electrician*, Sept. 7, 1923.

the shape of the curves checks very well. We feel quite confident that the formulas when properly applied may be used without any very large errors when the inductions are sufficiently low so that saturation effects are negligible.

Calculation of Saturation Effects. In order to calculate the effect of saturation on the pulsation values we made use of the following method, (see Fig. 15) which consists simply in adding the air-gap resistance to the tooth resistances. It was assumed that the specific resistance of the mercury was 1 and that the resistance of the electrode material was 1 divided by the permeability. It was also assumed that the vertical thickness was unity. The resistance of tooth D is then (Fig. 15, position 1).

$$R_1 = \frac{d_r}{t_r\, x\, \mu} \qquad (7)$$

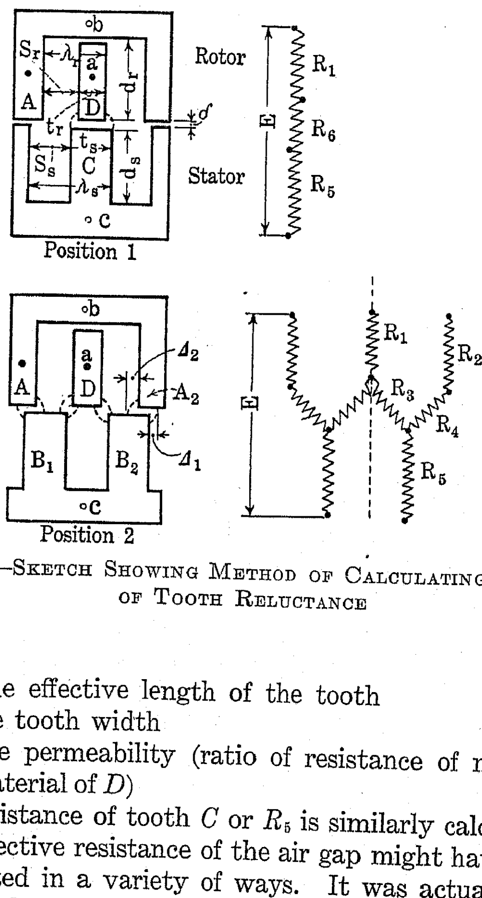

FIG. 15—SKETCH SHOWING METHOD OF CALCULATING EFFECT OF TOOTH RELUCTANCE

where,

d_r is the effective length of the tooth

t_r is the tooth width

μ is the permeability (ratio of resistance of mercury to material of D)

The resistance of tooth C or R_5 is similarly calculated. The effective resistance of the air gap might have been calculated in a variety of ways. It was actually calculated by using Carter's coefficients. The effective tangential width of the air gap since tooth D is narrower than tooth C (position 1) is then,

$$t_{er} \text{ or } \lambda_r/g_r$$

$$t_{er} = t_r + f_{sr} \text{ (see Gray p. 44)} \qquad (8)$$

λ_r is the slot pitch for $A - D$, and

g_r is the rotor tooth Carter coefficient.

The effective resistance of the air gap is then,

$$R_6 = \delta / t_{er} \qquad (9)$$

Position 2 for maximum reluctance is more complicated. The equivalent resistances are indicated at the right (Fig. 15). It will be seen that the right and left halves of the diagram are alike so we can use one-half only for the calculations.

R_1 is the same as above

R_2 is equal to R_1 for the particular case here considered

R_5 is the same as above

R_3 is obtained as follows, using the effective tooth width as calculated by Carter's method mentioned above.

$$R_3 = \frac{\delta \times 2}{t_{er} - \lambda_s + t_{es}} \qquad (10)$$

(see 11, case 1).

In order to calculate R_4 or the effective air gap resistance between A_2 and B_2, we have to determine the fringing for the two teeth and add this to the amount which the two teeth overlap, thus, (see Fig. 15, position 2).

$$\Delta_1 = \frac{t_{es} - t_s}{2} \qquad (11)$$

$$\Delta_2 = \frac{t_{er} - t_r}{2} \qquad (12)$$

$$\text{The overlap} = \frac{3\, S_s}{2} - \frac{3\, S_r}{2} = 0.187 \qquad (13)$$

$$R_4 = \frac{\delta}{\Delta_1 + \Delta_2 + 0.187} \qquad (14)$$

The equivalent resistance can now be calculated as indicated by Fig. 15. If now we assume that one volt is applied between the roots of the rotor and the roots of the stator teeth, then the maximum and minimum currents i_1 and i_1' in tooth D can be calculated as indicated and the pulsations would be proportional to these currents. In the case of teeth F and G where the resistance of the teeth has a larger influence on the results than in the case of D and E, the potential drop was taken between points a and c instead of c and b. Under these conditions the tooth resistances were taken only from the point a down (¾ in.) and a correction was made to take care of the resistance from point c to the roots of the B and C teeth.

The calculated curves of Figs. 7-10 were obtained by the above method.

Discussion of Results. Referring to Figs. 3-6, it will be seen that for infinite permeability the calculated results are in general somewhat higher than the test results and that the results as obtained by the use of Carter's coefficients are somewhat closer to the true results than those obtained by the Shand method. The differences, however, are not large and due to other uncertainties in calculating iron losses are probably quite accurate enough for estimating tooth pulsations. The reason that the calculated losses using Carter's coefficients are slightly high is probably due to the fact that uniform flux over certain distances is

assumed whereas in reality the flux does not terminate sharply but gradually shades off. It is surprising that the results check as well as they do.

When saturation effects come in, namely, with the copper and brass blocks and mercury, the checks between the calculated and test results are in general equally good. There are a few rather wide variations but these are due probably as much to testing errors as to inaccuracies in the method of calculation. At least the checks are close enough to show that the assumptions and methods are not in serious error. The testing for tooth G was specially difficult since the pulsations are small and an error of one per cent in reading the currents due to thermoelectric effects, an inaccuracy in setting the air gap or other causes would mean approximately a 10 per cent error in percentages. The apparatus was crude and the results could not be expected to check much better than they do.

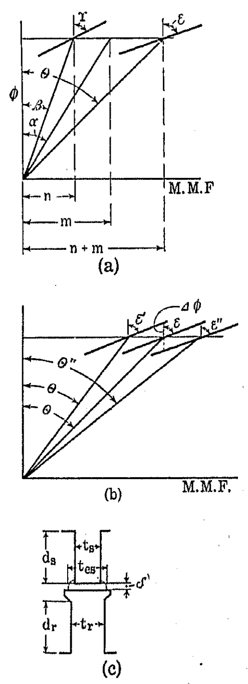

FIG. 16—SKETCHES SHOWING METHOD OF CALCULATING EFFECT OF SATURATION ON TOOTH PULSATIONS

Referring to Fig. 10 an actual increase in percentages of pulsation for the smaller permeability was noted both by test and calculation. This was due to the certain peculiarities of the testing and probably would not occur in any commercial machine.

Fig. 11 shows that for tooth D at least it is necessary for the permeability to be reduced to about 13 in order to reduce the pulsation percentage to one-half its value at infinite permeability. We shall discuss this further a little later.

Saturation Effects in Actual Machines. In order to calculate the saturation effects in the teeth of actual machines it would seem at first glance that it is simply necessary to add the reluctance of the teeth to the reluctance of the air-gap for the positions of maximum and minimum air-gap reluctance, calculate the total change in reluctance due to the change in air-gap reluctance for the tooth under consideration and then take this difference as a measure of the tooth flux pulsation. This procedure would be correct if the permeability of iron were constant. Since this is not the case, however, the method is far from valid. Again from permeability curves on commercial sheet iron it may be seen that it is necessary to go to inductions of over 120 kilolines per sq. in. before we reach a permeability as low as 100. According to the data just reported (see Fig. 11) a permeability of even as low as 50 does not very greatly affect the pulsation amplitude and yet we know from tests on actual machines that saturation effects begin to be noted at tooth inductions of about 70 kilolines per sq. in. where the permeability may be several thousand, and at 120 kilolines these saturation effects are quite large. The following analysis will make clear the reason why these saturation effects appear at such low flux densities and will also give a method of calculating them.

Referring to Fig. 16C let us assume two teeth opposite each other having the dimensions as indicated. These dimensions may be in any desired units provided the units are all the same; d_r is the effective length of the rotor tooth and is somewhat less than the actual length to take care of the wide tip which does not saturate: t_{es} is the effective tangential width of gap and is equal to t_s (actual stator tooth width) plus Carter's fringing constant times the stator slot width as above. Now referring to Fig. 16A the ordinate represents the total flux per tooth across the air gap and the abscissas the magnetomotive force necessary to force this flux through the various parts of the path from the root of a stator tooth to the root of a rotor tooth. The magnetomotive force m is that necessary to force the flux across the air gap from one tooth to the other and is equal to the flux times the air-gap reluctance R_δ where the air-gap reluctance equals the radial air-gap width δ divided by the effective tangential air-gap width t_{es}; n is the magnetomotive force necessary to force the flux through the stator and rotor teeth and is equal to $\phi \times R_f$, where R_f is the total tooth reluctance. The tooth reluctance must be calculated separately for a stator and rotor tooth and is equal to

$$\frac{d}{\mu \times t} \tag{15}$$

d is the effective length of tooth,
μ is the permeability corresponding to the maximum induction and t is the width of the tooth. The sum of the reluctance for the two teeth gives R_f.

Now referring to Fig. 16A, suppose we assume some arbitrary flux and plot a point corresponding to this flux and to the magnetomotive force $= \phi\, R_\delta$. Draw a line from this point through the origin and call the angle made by this line with the vertical α. Now the tangent of this angle is,

$$\frac{\phi \times R_\delta}{\phi} = R_\delta \tag{16}$$

and is proportional to the reluctance of the air gap. Similarly the tangent of the angle β is R_f and is proportional to the reluctance of the teeth. Suppose now we increase or decrease the flux ϕ. The line making the angle α (air-gap reluctance) will simply be extended. If, however, we consider the line making the angle β (tooth reluctance), when the flux is increased the line will not be extended but will follow a curve corresponding to the magnetization curve for the steel. In order to simplify this instead of plotting a magnetization curve, let us consider that the flux follows a straight

line, making the angle γ with the vertical. This angle γ is calculated in the same way as the angle β except that we use the incremental permeability μ_Δ[11] instead of the ordinary permeability. The incremental reluctivity $R_{f\Delta}$ is the sum of $\dfrac{d}{\mu_\Delta \times t_f}$ for the two teeth. This simply means that we are assuming that over the range of inductions covered by the tooth pulsations the magnetization curve is a straight line which is the line connecting the two tips of a minor hysteresis loop, having the same flux amplitude as the tooth pulsations and whse upper tip corresponds to the maximum tooth induction.

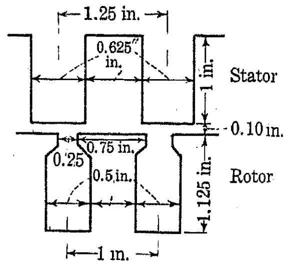

FIG. 17—SKETCH OF TEETH FOR ILLUSTRATING SATURATION EFFECT CALCULATIONS

Now if we combine the air gap and tooth reluctances we obtain a line making an angle θ with the vertical and the tangent of the angle will be $R_\delta + R_f$ and the tangent of the angle ϵ will be $R_\delta + R_{f\Delta}$.

Now referring to Fig. 16B the angle θ is the same as for 16A, namely, at the assumed flux the magnetomotive force equals $R_\delta + R_f$ where R_δ is the mean reluctivity of the air gap. If now we calculate the tooth pulsations by the appropriate formula (3)-(6) and make R_δ' one-half this per cent less than R_δ and R_δ'' the same amount greater we may calculate the tangents of the angles δ', δ'', ϵ', ϵ'', as shown by Fig. 16. Now if we draw a vertical line through the point of intersection of the line of θ and the assumed flux the length of this line $\Delta\,\phi$ (Fig. 16B) between the lines of ϵ', and ϵ'' will be the flux pulsation or will give the per cent pulsation numerically if the maximum flux is assumed to be 100.

In the above it is quite immaterial what the flux and magnetomotive force units are. In fact, it is perhaps convenient to make the tooth reluctance values equal simply to the tooth lengths in inches divided by the permeability and the tooth widths in inches. In order to make this analysis clearer a simple case will be assumed and the calculations gone through to illustrate the method.

Example. Refer now to Fig. 17.

$q_s = 6.25$ stator (S_s/δ)

$q_r = 2.5$ rotor (S_r/δ)

$t_{es} = 0.905$

$t_{er} = 0.916$

$$P_{r2} = \frac{1.25 - .916}{0.905} = 0.368 \text{ (see formula 4)}$$

Now from the derivation of formula 4 for P_{r2} we see that t_{es} represents the maximum effective contact area of the two teeth. Then the mean area equals:

$$0.905 - \frac{0.905 \times 0.368}{2} = 0.739$$

The above is on the assumption of infinite length of punchings axially.

This value 0.739 is t_{es} for Fig. 16C. Then,

$$R_\delta = 0.1/0.739 = 0.135$$

Now assume that the maximum tooth induction is 110 kilolines per sq. in. on a sine wave basis and let us assume no stator tooth pulsations. Also let us assume that due to saturation effects the rotor tooth pulsation is only one-half the calculated value of .368 or 0.184. This means that the true maximum rotor tooth induction will be

$$\frac{110}{1 - 0.184/2} = 121 \text{ kilolines,}$$

and the minimum will be 98 kilolines.

$$\text{Now } R_f = \frac{1}{295 \times 0.625} + \frac{1^*}{295 \times 0.5} = 0.0122$$

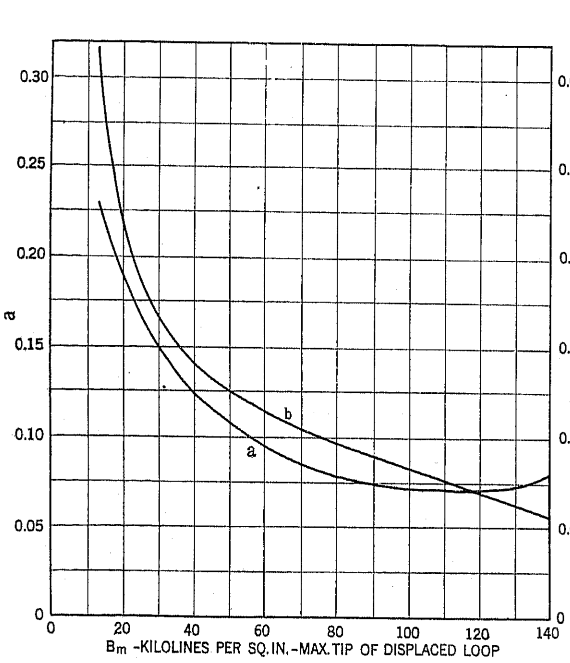

FIG. 18—CONSTANTS FOR INCREMENTAL PERMEABILITY CALCULATIONS

$\mu_\Delta = \mu_m\,(a + b \times \Delta\,B)$

(μ_m is the ordinary permeability corresponding to B_m)

The permeability value of 295 is an average value for *O H* electrical sheet at $B = 110$ kilolines.

$B_m = 121$

$\Delta\,B = 121 \times 0.184 = 22$

$\mu\,\Delta = 115\,(0.071) + 22 \times 0.0068) = 25.4$ (see Fig. 18)[13]

(115 is the μ corresponding to 121 kilolines)

11. Permeability. Thos. Spooner: A. I. E. E. JOURNAL, January 1923.

*12. Effective Length of rotor tooth taken as 1 inch.

13. Fig. 17 is derived from the data of Fig. 4, A. I. E. E. JOURNAL, p. 42, January 1923, but altered slightly at the high inductions to conform more closely to recent data and converted to English units.

$$R_{f\Delta} = \frac{1}{25.4 \times 0.625} + \frac{1}{25.4 \times 0.5} = 0.143$$

$$R_{\delta}' = 0.1/0.905 = 0.110$$

$$R_{\delta}'' = \frac{0.1}{0.905 \times 0.632} = 0.175$$

$$\begin{aligned} \text{Tan } \theta' &= 0.110 + 0.012 = 0.122 \\ \text{Tan } \theta'' &= 0.175 + 0.012 = 0.187 \\ \text{Tan } \epsilon' &= 0.110 + 0.142 = 0.252 \\ \text{Tan } \epsilon'' &= 0.175 + 0.142 = 0.317 \end{aligned}$$

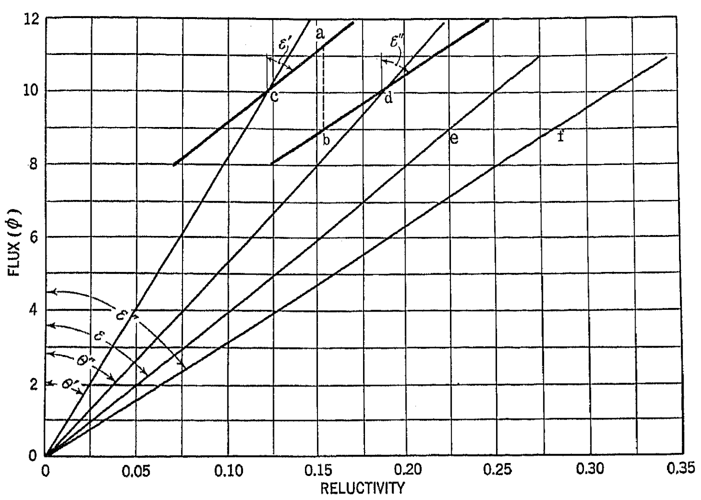

FIG. 19—DIAGRAM SHOWING METHOD OF CALCULATING EFFECT OF SATURATION FOR TEETH OF FIG. 18

$$P_r = \frac{11.25 - 8.95}{11.25} \times 100 = 20.4$$

Now referring to Fig. 19 for the vertical ϕ axis we may use any arbitrary scale we please but for convenience suppose we divide it into ten divisions. On the horizontal axis we have the reluctivity scale corresponding to the values as calculated above. On the line corresponding to the flux of 10 we plot the tangent θ' and θ'' and the tangent ϵ' and ϵ'' or reluctivity values, giving the lines $O - c$, $O - d$, $O - e$ and $O - f$, respectively. Now with a pair of triangles we transfer the lines $O - e$, $O - f$ so that they pass through the points c and d, respectively. Now at the midpoint between c and d, erect a perpendicular. The vertical distance between the points a and b gives the total flux pulsation in the teeth and divided by the maximum ϕ ordinate (point a) and multiplied by 100, gives the per cent flux pulsation or 20.4 as against 36.8 per cent if there had been no saturation effects. It will be noted that without taking into account the incremental permeability, the flux pulsation would be the difference between the line $O - c$, $O - d$ or nearly double the corrected value as just stated.[14]

This correction for saturation applies only for the teeth in the position of maximum induction. In the case of an induction motor where the flux varies approximately according to a sine wave, the saturation effects will be different for every flux value and this greatly complicates the correction. The actual use of this method is more difficult than indicated in the example due to the presence in commercial machines of tapered teeth, to appreciable leakage fluxes through the slots at high inductions and various other disturbing factors. We believe, however, that for careful machine design the method is valid and may be employed with confidence when the extra labor involved warrants its use. These pulsation calculations are correct only when short-circuited windings are not present. For the case of the squirrel-cage induction motor and the salient-pole machines with damper windings, corrections will be necessary for the counter-magnetomotive force of the short-circuited windings.

We have calculated saturation correction curves based on different relative tooth widths, lengths and air gaps for the case of induction motors. These curves have not been adequately checked experimentally and will not be included here. We hope, however, that they will prove to be reasonably accurate and if they are, will make a ready method of correcting for saturation effects in wound rotor induction motors.

14. If this calculated value had been much different from the assumed value of 18.4 per cent it might have been advisable to recalculate.

Discussion

For discussion of this paper see page 280.

Surface Iron Losses With Reference to Laminated Materials

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Review of the Subject.—*Surfaces or pole-face losses are assumed to be governed by ten factors and the influence of each on the resulting losses is considered. After reviewing the work of the previous investigators an account is given of some rather extensive tests recently completed by the writers. These tests cover results obtained on an experimental salient pole machine and an experimental 3-phase induction motor. Data are given for various types and thicknesses of commercial sheet.*

For a given material and machine the surface losses are assumed to be a function of the air-gap induction, ratio of slot width to single air gap, tooth frequency and slot pitch or width. The laws of variation of surface losses with respect to these variables are determined from experimental results and are shown to be in general exponential. The relation between hysteresis and eddy-current losses and the effect of variations in thickness and resistivity of laminations is considered in some detail.

Simple methods are given for applying these results to actual design problems which involve plotting the various functions on double log paper, a slide rule being then sufficient for all calculations.

The appendix gives a description of a graphical method of separating the various types of core losses which exist in a polyphase induction motor.

Introduction

Surface losses, or pole-face losses as they are sometimes called, are common to nearly all types of rotating machines in which at least one member is slotted. In many machines they are responsible for a very appreciable percentage of the total no-load losses. It is often very desirable to be able to calculate their magnitude with a reasonable degree of accuracy and it is sometimes well to know the best way in which to reduce them.

It is the purpose of this paper to analyze as accurately as possible from the data which we have available the various factors affecting surface losses and to combine them into easily usable design formulas.

We shall define surface losses as those hysteresis and eddy-current losses which occur just below the surface of a magnetized smooth-core laminated material which is adjacent to a slotted member having a relative motion with respect to the first member. One of the most familiar examples of pure surface losses is in the poles of salient-pole machines having no damper windings. When this first member is slotted as well as the second the losses of the former are increased due to the greater penetration of the pulsating flux and to the higher flux density. The losses due to high-frequency pulsations penetrating through the whole length of a tooth will be known as tooth-pulsation losses. A discussion of this latter type of loss is beyond the scope of this paper.

There have been published a number of formulas for calculating pole-face or surface losses. Some of them have been based on theoretical considerations alone and some on empirical formulas derived from test data on experimental or commercial machines. Still others are based on theoretical considerations but the constants are derived from actual tests on machines. Some of these formulas are difficult to use due to their complication; some are based on insufficient or inaccurate test data and some of the older ones applied to material which is no longer in commercial use. Most of them either have not been verified by other observers or if they have, the constants are known to be in doubt, or erroneous. Finally, so far as we know, the experimental data which have been published do not show the relative magnitude of the hysteresis and eddy-current components which leaves us in doubt as to the best choice of material with reference to thickness and magnetic quality to be used for pole shoes and for similar applications. For these reasons and others to be mentioned later it was therefore thought worth while to make a rather thorough investigation of this subject with the idea of putting it on a simpler and more rational basis having in mind especially that it should be possible to predict the performance of any laminated material under these conditions of use from a knowledge of its fundamental electrical and magnetic characteristics just as we can predict the core loss of a transformer from an Epstein test on samples of the material entering into its construction. The experimental work is limited to the case of open slots only.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

Factors Affecting Losses

In analyzing this problem we wish especially to acknowledge our debt to Adams and his collaborators for the aid received from their classic paper on "Pole Face Losses."[1] We have to a large extent followed the trail blazed by them and their predecessors. In considering this problem we have taken into account the following factors.

1. Air gap induction B_{AG}
2. Field form
3. Ratio of slot width to single air gap q
4. Tooth frequency f_t
5. Tooth pitch λ or slot width σ.
6. Resistivity of material ρ.
7. Thickness of individual laminations t.
8. Hysteresis coefficient r.

1. Adams, Lanier, Pope, Schooley. Trans. A. I. E. E., page 1133 (1909)

9. Insulation between laminations
10. Effect of punching.

Surface losses are a function of the flux density, the frequency of pulsation and the quantity of material involved as is the case for the simpler problem of the alternating-current transformer. The additional complications are due to four things: (1) the very appreciable departure of the pulsating wave form from that of a sine wave; (2) the rapid decrease in the amplitude of pulsations from the surface inward; (3) the fact that the high-frequency pulsating fluxes are superimposed on the fundamental-frequency flux variations (namely, we have a case of displaced unsymmetrical hysteresis loops which produce increased hysteresis losses over those caused by symmetrical hysteresis loops of the same amplitude of pulsation); (4) due to the high frequency of the tooth pulsations and to the thickness of material commonly used, skin effects become quite appreciable.

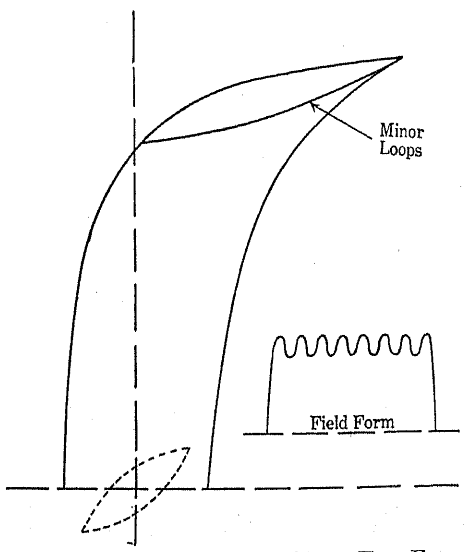

FIG. 1—HYSTERESIS LOOPS WITH FLAT TOP FIELD FORM

We have two general types of field form, *i. e.*, the flat top as given by a salient-pole machine with uniform air gap and the sine as usually approximated in the induction motor. With chamfered poles we may have almost any form in between. The flat-top field form produces a major hysteresis loop in the smooth-core member and near the surface there is superimposed a series of unsymmetrical hysteresis loops as shown by Fig. 1. These minor loops are all of approximately the same amplitude for a given depth of penetration and occur at the tips of the major loops. As we penetrate deeper away from the air-gap surface these minor loops decrease rapidly in amplitude. For a sine wave distribution of flux we have the following condition (see Fig. 2). Superimposed on the major loops are a series of minor loops changing in amplitude in proportion to the displacement from zero induction.

The flux density is determined chiefly by the average air-gap induction B_{AG}. Of course for a sine-wave flux distribution the maximum flux density is greater than for a flat top distribution for the same average air-gap induction. The frequency f_t or number of minor hysteresis loops is determined by the number of teeth which pass a given position per second. We have preferred to use tooth frequency rather than air-gap velocity as is used by some of the previous investigators since it seems to correspond better to our habits of thought when considering alternating-current iron losses. It is also more convenient when considering surface losses in connection with tooth pulsation losses as must be done in the case of induction motors, for instance. The magnitude of the flux pulsations in the air gap is chiefly determined by the ratio of slot width to radial air gap (equals q). The depth of penetration of the pulsating fluxes is chiefly a function of the slot pitch λ or perhaps more accurately the slot width σ.

In other words, the first five items of the factors affecting losses give us a measure of the three fundamental variables, induction, frequency and volume of material as previously outlined.

In considering the quality of the material which for the transformer is determined by the Epstein or some similar test we must consider items 6, 7 and 8. The sixth item, resistivity of the material ρ determines, together with item 7, the thickness of laminations t, the eddy-current loss to be expected, assuming perfect insulation between laminations. It must be remembered that these two factors also govern the skin effect as produced by the screening currents of Mr. Adams' previously-mentioned paper. These effects can by no means be neglected for commercial laminated poles. Item 8, the hysteresis coefficient governs the hysteresis loss. This in turn is affected by the screening currents and by the amplitude of the displacement of the minor high-frequency hysteresis loops from the normal position.[2,3,4] Items 9 and 10, the insulation between laminations and the effect of punching, are incidental to the process of manufacture and will be considered in detail later.

We shall make no attempt to derive theoretical formulas for surface losses but shall be content to combine the various factors by empirical means based on numerous tests to give a working formula. Due to the many variables, we believe that this is the only satisfactory mode of attack. We have, however, been guided by the able theoretical analyses of Rudenberg, Adams, Carter and others in separating and combining these various factors most effectively.

PREVIOUS WORK

We wish to discuss briefly the work of some of the most prominent of the previous investigators along this

2. The Effect of Displaced Magnetic Pulsations on the Hysteresis Loss of Sheet Steel. L. W. Chubb and Thos. Spooner, TRANS. A. I. E. E. (1915) page 2671.
3. The Unsymmetrical Hysteresis Loop, John D. Ball. TRANS. A. I. E. E. (1915) page 2693.
4. Tooth Frequency Losses in Rotating Machines, Thos. Spooner. A. I. E. E. JOURNAL 1921, page 751.

line, pointing out some of the limitations of the various analyses. Much of the pioneer work was done by Rudenberg[5] and Potier,[6] in establishing a sound fundamental theory on which to base and analyze the results of experimental work. Their original formulas, however, were for solid poles which is beyond the scope of this paper. Their theoretical formulas for the magnitude of pole-face losses in solid poles formed a basis for the laminated-pole formulas of Adams and others.

Later Wall and Smith[7] obtained some most interesting test results on pole-face losses for laminating material by an ingenious use of a special salient pole machine in which they measured the temperature of their poles after they had been subjected to surface loss conditions and then brought them to the same temperature with the machine unexcited by means of imbedded electrical heaters. The known heater input gave a measure of the surface losses. Their pole-face-loss formulas are given

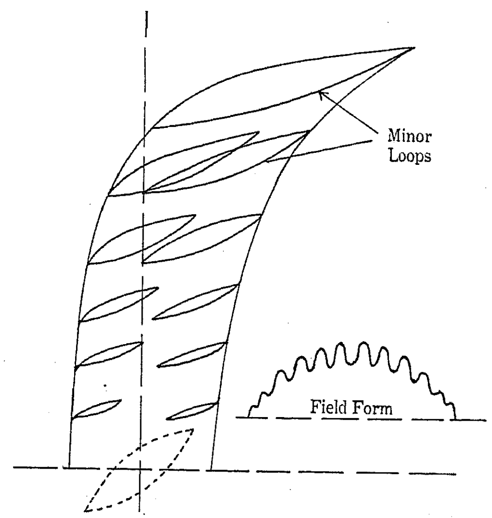

FIG. 2—HYSTERESIS LOOPS SINE-WAVE FIELD FORM

a little later in comparison with others. As pointed out by Adams their pole-face-losses were in error due to the fact that because of an unfortunate relation of pole-face width and slot pitch high-frequency pulsations penetrated through the whole stator core, thus giving incorrect results.

Adams has probably given us the most complete and accurate surface-loss formulas at present available. He derived these formulas based on the work of Rudenberg and Potier as previously mentioned and then checked his formulas by means of a large number of tests on a special salient pole machine driven only by electrica means and fitted successively with several rotors and sets of poles. The pole-face losses were segregated by testing with the same poles set for various air gaps. His actual working formulas which were based on his theoretical formulas are really empirical. These will be given later.

Mr. Lamme[8] derived a pole-face formula based on tests on a large number of commercial machines. His formula has the advantage that it is based on many tests on commercial machines rather than on a few tests on a single special machine and has the disadvantage that it is not based on all of the fundamental factors which affect surface losses and may therefore give inaccurate results on machines which depart much from the normal.

Mr. Hanssen[9] by taking the test results for numerous induction motors calculated the fundamental-frequency losses, subtracted these from the test values and obtained constants for empirical curves between watts per square inch of surface and a factor, f, where

$$f = \frac{B\,K\,S}{\delta\,P\,100}\text{, where} \qquad (1)$$

B = maximum density in air gap,
K = slot width,
S = total number of teeth,
δ = air gap,
P = total number of poles.

A different curve has to be used for each applied frequency. It is obvious that this formula gives simply mean values for the machines tested and includes both surface and tooth-pulsation losses, the surface losses being those obtained with a smooth-core rotor and the tooth-pulsation losses being the additional losses due to the presence of the rotor teeth.

Carter[10] has derived surface-loss formulas from theoretical considerations only. Due to the complexity of the problem, however, it is probably impossible, at present any way, to derive an accurate solution for surface losses from theoretical considerations alone and that this is so can be shown by a comparison of Carter's formulas with the known characteristic of surface losses as will be done later. Carter did one thing, however, which, so far as we know, had not been attempted by anyone else; namely, he has derived a formula for the hysteresis loss as distinct from the eddy-current loss. Previous theoretical formulas have been based on eddy-current losses alone.

Latour[11] has not, so far as we know, specifically attacked this problem of surface losses but he has recently derived some very valuable formulas for hysteresis and eddy-current losses for laminated material where skin effect is appreciable. His work was in connection with radio frequencies but we have checked

5. *Electrotechnische Zeitschrift*, Vol. 26, page 181. (1905).
6. *L'Industrie Electrique*, 1905, page 35.
7. Losses in Pole Shoes, Wall & Smith, I. E. E., Vol. 40, page 577 (1907).
8. Electrical Engineering Papers, Iron Losses in Direct-Current Machines, by B. G. Lamme, page 487.
9. Calculations of Iron Loss in Dynamo Electrical Machinery. I. E. Hanssen, TRANS., A. I. E. E., 1909.
10. Pole Face Losses, W. Carter, I. E. E. 1916, page 168.
11. Notes on Losses in Sheet Steel at Radio Frequencies, Marius Latour, Institute of Radio Engineers, Feb. 1919, page 61.

his formulas experimentally for frequencies of a few hundred and thousand cycles per second and have found them to agree qualitatively very closely with the test results. They are very valuable in explaining some of the test data which we shall later discuss.

Space does not permit of a resumé of the work of previous investigators so we shall have to be content to refer to their original articles and would specially recommend the discussion of damping and screening currents as given by Adams since he offers a very clear picture of the mechanism of surface losses. For laminated materials such as are used for commercial machines the damping currents may be neglected, namely, the depth of penetration of the pulsating flux is only slightly affected by tooth frequency and is determined almost completely by the dimensions of the slots, air gap, etc. The screening currents, however, crowd the flux toward the surface of the individual laminations so that looking at a cross section of the laminations the flux near the air-gap surface at any instant will be as shown by Fig. 3. This is the familiar skin effect and results in decreased eddy-current losses and corresponding increased hysteresis losses over those which would exist if the skin effect were inappreciable. Since the hysteresis loss is comparatively a small percentage of the total surface losses, usually the net result of skin effect is a smaller loss than would occur if it were not present.

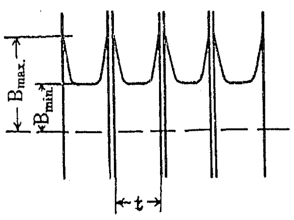

Fig. 3—Flux Distribution Due to Skin Effect in Plane at Right Angles to Plane of Laminations

The previously-mentioned investigators are not the only ones who have done valuable work along these lines but are the only ones whose work has been seriously considered in connection with our own investigation. These various formulas which are given below have all been reduced as far as possible to the same variables in order that they can be more readily compared.

Adams (theoretical)

$$S = 3.3 \times 10^{-7}\, a\, k^2\, t\, \rho^{-1/2}\, \mu^{-1/2}\, B^2 f_t^{1.5} \qquad (2)$$

Carter (theoretical)

$$S_e = \frac{\pi}{24}\, t^2\, \rho^{-1/2}\, \mu^{-1/2}\, (B^1)^2\, f_t^{1.5} \qquad (3a)$$

$$S_h = \frac{1}{3.2\,\pi}\, \rho^{1/2}\, \mu^{-1/2}\, (n + n^1 B^{1.9})\, (B^1)^{1.6} f_t^{0.5} \qquad (3b)$$

Adams (empirical)

$$S = K_1\, B^{2.3 \text{ to } 2.4} f_t^{1.55}\, q^{1.22 \text{ to } 1.5}\, \lambda^{1.05} \qquad (4)$$

The different exponents correspond to different thicknesses of laminations. The first exponents correspond to 0.014 in. material and the last to 0.06 in.

Wall and Smith (empirical)

$$S = K_2\, B^{2.1} f_t^{1.5}\, q^{3.5} \qquad (5)$$

Lamme (empirical)

$$S = \frac{K_3\, \sigma\, E^2}{C_f\, W_s\, g\, L} \sqrt{\frac{S_c}{R_s\, g}} \qquad (6)$$

(See page 8 for Hanssen's formula).

S = total surface loss per unit air-gap area.

S_e = the same except that it refers to eddy-current loss only.

S_h = the same except that it refers to hysteresis loss only.

a and k = constants depending on the relative slot, tooth and air gap dimensions.

K_1, K_2, K_3 = constants depending upon the thickness and magnetic and electrical quality of the material.

t = thickness of the individual laminations.

ρ = resistivity.

μ = mean permeability.

B = average induction over the pole face.

B^1 = amplitude of tooth pulsation (see Carter[10] for the relation of B and B^1).

f_t = tooth frequency in cycles per second.

λ = slot pitch.

r and r' are hysteresis coefficients; n' takes care of the increased loss due to the displacement of the hysteresis loop from its normal position.

$$q = \sigma / g = \frac{\text{slot width}}{\text{single air gap}}$$

e = generated voltage.

C_f = field-form constant.

W_s = armature windings in series.

L = width of poles.

V = peripheral velocity.

Scope of Experimental Work

Our experimental work was divided into two parts; (1) pole-face losses for salient pole machines; (2) surface losses for induction motors. The salient-pole losses were obtained on a special four-pole railway motor supplied with various sets of poles of different materials so arranged that the air gap could be varied at will. The induction-motor losses were obtained on a special induction motor provided with an open-slot stator and several rotors of the same material, but having different diameters.

It might be stated here that no tests were made on solid pole material due to the fact that such poles are practically never used in modern machines except in the case of turbo-generators and for interpoles. This is due chiefly to the fact that solid poles have much higher losses and that the construction of laminated poles is generally cheaper.

Test Apparatus

Railway Motors. The salient pole machine was mounted on ball bearings and direct-connected by a flexible coupling to a small variable speed d-c. motor. The latter had ordinary sleeve bearings. Two armatures were provided for the railway motor having the following general dimensions.

TABLE A

	Armature A.	Armature B.
Diameter	9 in.	9 in.
Length	7 in.	7 in.
Number of slots (open)	31	60
Slot width at air gap	0.370 in.	0.220 in.
Slot pitch at air gap	0.911 in.	0.471 in.
Slot depth	1.184 in.	0 8 in.

Neither armature had any vent ducts but armature *A* had three 5/8-in. band-wire slots 1/16 in. deep. Armature *B* was smooth. There were no windings on the armatures except certain exploring coils to be mentioned later. Several sets of poles were constructed for this machine made of the following materials.

0.125-in. Bessemer unenameled
0.0625-in. " enameled
0.0281-in. " unenameled
0.0281-in. " enameled
0.0172-in. 0.9 per cent silicon unenameled
0.0172-in. 0.9 per cent " enameled

These poles were shortened so that an adjustment of 1/2 in. could be made in the air gap by means of shims. The pole dimensions are given by Fig. 4.

Induction Motor. The induction-motor set consisted of a special ball-bearing induction motor direct-connected to a ball-bearing, adjustable-speed, d-c. motor. The latter was arranged to be driven by means of a storage battery. The general dimensions are as follows:

TABLE B.

Stator punchings	O.D. = 19in.	I.D. = 14 in.	
Length	= 6 in.		
Number of slots (open)	= 60 in.		
Slot width at air gap	= 0.346 in.		
Slot pitch at air gap	= 0.711 in.		
Slot depth	= 1.6 in.		
Rotor Punchings			
Diameter rotor No. 1	= 13 17/32 in.	Single air gap (mils)	= 31.5
" " No. 2	= 13 1/2 in.	" " " "	= 47
" " No. 3	= 13 7/16 in.	" " " "	= 78
" " No. 4	= 13 3/8 in.	" " " "	=109.5
" " No. 5	= 13 1/4 in.	" " " "	=172

The stator and rotor punchings were made of enameled 0.0172, 0.9 per cent silicon sheet steel. All of the punchings were obtained from the same lot of steel. Special pains were taken to remove the burs from the punchings before enameling. The three-phase stator windings were arranged in 12 sections and could be connected for 110, 220 and 440 volts, 60 cycles and 4 poles. Fractional-pitch windings were used so as to give an approximate sine-wave distribution of flux. The rotors were supplied with a few small longitudinal slots in the surface 1/16 in. square. Suitable exploring coils were placed in these slots and brought out to slip rings.

Methods of Test

Salient Pole Machine. The method of test consisted in placing a set of poles in the machine obtaining the friction and windage losses with no field current and then gradually increasing the field current and noting the increased input to the d-c. drive motor. These tests were repeated for various speeds and air gaps. By extrapolating the curves between loss and air gap to infinity air gap the magnitude of the pole-face losses was estimated. This was done for each set of poles as noted above. Of course, corrections were made for brush loss and $I^2 R$ loss in the windings of the driving motor. The armature inductions of the machine under test were determined by means of exploring coils on the armature covering approximately a pole pitch. The terminals of these coils were brought out to slip rings and the voltages measured by means of an a-c. voltmeter. It was assumed that for a given voltage on this exploring coil and a given frequency that the fundamental-frequency armature losses were a constant except as noted later. The total pole fluxes were determined ballistically by means of exploring coils surrounding the poles and connected to a fluxmeter. The air-gap inductions were obtained by dividing the total flux-per-pole by the gross area of the pole. Also readings were taken of the a-c. voltage generated in a coil wound on an armature tooth. This will be discussed more fully later.

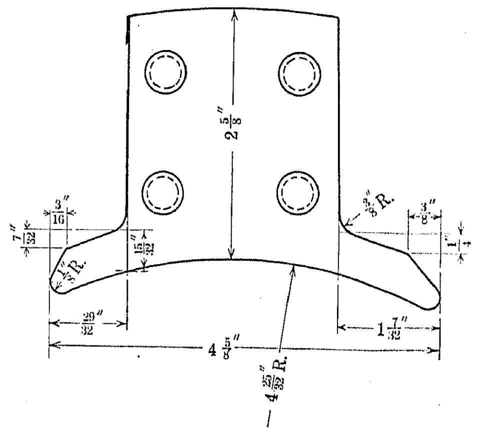

Fig. 4—Salient Pole Punchings

Pulsation Losses. In order to find out whether a pulsation of the total flux existed through the poles and frames of sufficient magnitude to cause any appreciable iron loss, which would appear as pole-face losses, the following special test was made: A single turn of wire was placed around a pole and connected to an oscillograph that had been previously calibrated. By this means it was found that for a 0.100-in. air gap and an

air gap induction of approximately 55 kilolines per sq. in. the pulsating fluxes were entirely negligible.

The method of test has the advantage over the method of direct electrical drive in that there are no $I R$ drops in the machine under test to be observed and no eddy-current losses in the copper. Also there are no flux distortions due to armature current. There is of course the disadvantage that we have greater windage and friction losses due to the extra armature and bearings and also the losses in the drive motor. Moreover, the particular drive motor we were using did not have ball bearings although the main motor under tests did. It would have been preferable if both machines had been fitted with ball bearings. However, the conditions of test were so well controlled that results could be duplicated very closely from day to day and differences of 2 watts from one time to another could readily be detected.

Induction Motor. Two methods of test were used for determining the induction motor losses.

1. Three-phase alternating current was applied to the stator, the rotor driven at various speeds below and above synchronism and the a-c. and d-c. inputs noted.

2. Direct current of a certain value was applied to one phase of the stator with one-half this current reversed in the other two phases. The rotor was then driven by the d-c. motor and the input to this motor noted for various speeds and stator excitations.

In both cases the input to the d-c. motor was observed for each speed without stator excitation. This consisted of friction, windage and losses in the d-c. drive motor. When the load was increased, the armature voltage of the d-c. drive motor was kept constant and the usual corrections made for the increased brush and $I^2 R$ losses.

The a-c. supply was obtained from a three-phase, 440-volt generator connected directly to the stator of the induction motor through suitable switching apparatus. The supply voltage was varied by altering the field of the a-c. generator. The a-c. generator was direct-connected to a d-c. drive motor and the a-c. supply frequency was adjusted by varying the field of this motor. For method 1, the a-c. input to the test motor was measured by means of a single-phase precision wattmeter introduced successively into the three phases. The potential circuit of the wattmeter was connected between the line and the motor neutral and was automatically shifted from one line to the next when its current coil was shifted. The a-c. input to the induction motor was equal to the sum of the three wattmeter readings. These meter readings included the losses in the voltmeter and shunt circuit of the wattmeter. These instrument losses together with the $I^2 R$ losses of the induction motor windings were subtracted from the sum of the wattmeter readings to give the net input.

For both methods of test the losses were based on average air-gap induction as determined by a pole pitch coil placed in the small slots on the surface of the rotor. This coil was connected through slip rings to a rotating commutator and d-c. voltmeter. For method 1, the rotor was held stationary in various positions and the a-c. stator voltage varied. The maximum readings of the d-c. voltmeter gave a measure of the average air-gap induction. The different positions of the armature were necessary due to an appreciable eccentricity between the rotor and the stator. For method 2, the rotor was revolved at 1800 rev. per min. and the average air-gap induction calculated as before from the readings of the d-c. voltmeter. For one rotor the a-c. values with d-c. excitation were checked by ballistic results obtained with the same rotor coil connected to a fluxmeter and the readings noted when the rotor was slowly revolved. As a matter of interest the d-c. field form was obtained ballistically by means of a coil placed in two rotor slots 0.2 in. apart. This coil was placed opposite a tooth, the d-c. exciting current reversed and the deflection of a ballistic galvanometer connected to the coil noted. This was done for each tooth giving the flux density opposite the tooth. Data were also obtained with the coil opposite the slots. Similar data were obtained by means of an oscillograph connected to a pole pitch coil or a radial coil with the stator excited by d-c. and the rotor revolved. These tests will be discussed in greater detail later.

In order to separate the various losses of method 1 (a-c. applied to the stator) an approximate method was developed as follows: A definite three-phase voltage was applied to the stator and the rotor revolved at various speeds below and above synchronism. The a-c. and d-c. inputs were noted, the results corrected for friction, windage, $I^2 R$ losses, etc. and then plotted. From the characteristics of the resulting curves plotted between input and rotor speed, the fundamental-frequency stator and rotor iron losses could be calculated and subtracted from the total input, thus leaving the surface losses. This method is described in detail in Appendix 1.

For method 2, (d-c. excitation) we have present no stator losses but only the rotor losses. After subtracting the friction and windage losses in the d-c. drive motor, the remaining losses as determined at 600, 1200 and 1800 rev. per min. for the five rotors having different diameters were plotted against air gap and the results extrapolated to infinity air gap. Assuming that the fundamental-frequency losses for all of the rotors were identical, the difference between these extrapolated values and the total rotor losses equalled the tooth-pulsation losses. This assumption is correct with a small adjustment for the different size of rotors except as the tooth pulsation losses may affect the fundamental-frequency losses and as the rotor losses may be affected by burs, difference in stacking pressure, etc. for the different rotors. There should be no appreciable

difference in materials as the rotors were all punched from the same lot of steel.

The a-c. method of test has the advantage that a separation of losses may be made with a single rotor and air gap. There is the disadvantage, however, that the testing methods are complicated; the method of calculation is somewhat tedious and the surface losses themselves are in general only a small portion of the total losses, thus requiring great accuracy of measurement. The d-c. method has the advantage of simplicity of measurement and calculation with the surface losses representing a considerable proportion of the total losses but has the disadvantage of requiring several rotors or some other means of varying the air gap and when several rotors are used the uncertainty as to the amount of the fundamental-frequency losses for the various rotors.

Test Results.

Salient Pole Machine. After obtaining the pole-face loss results by the method outlined above the losses were plotted on double-log paper against each of the four variables B_{AG}, f_t, q and λ, where

B_{AG} = air gap induction,
f_t = tooth frequency,
q = ratio of slot width to air gap,
λ = tooth pitch.

From the average slope of the curves the mean exponent for each variable was determined. In general, these functions gave approximately straight lines, indicating that the variations were truly exponential except for the values of q. As will be noted later the ratio of variation of q varies, decreasing as q increases. Of course, these various functions are not altogether independent of each other but for a first approximation they may be considered to be so. In fact, the dependence of one function upon another is surprisingly small. It is obviously impossible to reproduce even a small part of these curves here but the exponential values may be summarized as follows:

TABLE C.
Exponents

Material	B_{AG}	f_t	q	λ
0.0172 M. A.	2.2	1.7	1.7	1.1
0.0281 Bessemer	2.4	1.6	2.15	1.3
0.0625 "	2.6	1.6	2.2	1.3
0.125 "	3.1	1.6	2.3	1.3

Range of variables: B_{AG} = 25 -60 kilolines per sq. in.
f_t = 600-1800 cycles.
q = 2.2-4.1
λ = 0.471-0.911 inches.

From this information we may therefore write the surface loss equation for 0.0625 Bessemer, for instance, as follows:

$$W_s = K \times B_{ag}^{2.6} f_t^{1.6} q^{2.2} \lambda^{1.3} \tag{7}$$

This is quite similar to the formulas used by Adams. Such an equation will give a good estimate of pole-face losses over a considerable range of the several variables, but it is not in convenient form to use.

Derived Design Curves. We therefore suggest that the variables be plotted on double-log paper, thus greatly facilitating their use. For instance, see Fig. 7. In order to draw such a set of curves let us assume that the surface loss for the following conditions equals 1.2 watts per sq. in. of the surface:

B_{AG} = 49.5 kilolines per sq. in.
f_t = 500 cycles per second
q = 2.85
λ = 1.17

Now place a point at an ordinate of 1 and an abscissa of 49.5 and draw a straight line having a slope of 2.6. This gives the B_{AG} function. Similarly place a point at an ordinate of 1 and an abscissa, say, of 500 and draw a line having a slope of 1.6. This gives the frequency function. Similarly the q and λ functions may be drawn. Finally the known loss of 1.2 watts per sq. in. give the constant of the equation and we have

$$W_s = 1.2 \times K_{B_{AG}} \times K_f \times K_q \times K_\lambda, \tag{8}$$

where the K factors signify the respective ordinates for

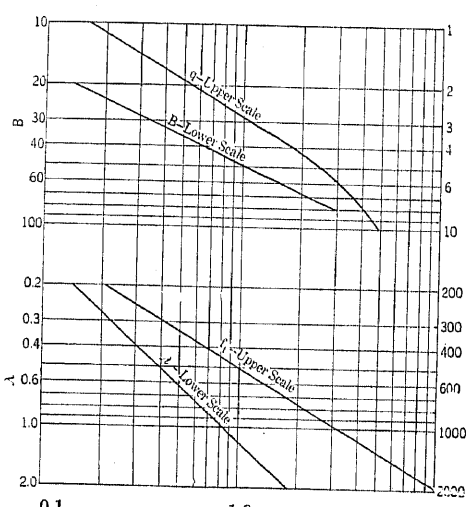

FIG. 5—SURFACE-LOSS CALCULATION CURVES FOR 0.0172 IN., 0.9 PER CENT SILICON STEEL
$W_s = 0.20\ K_{B_{AG}} \times K_{f_t} \times K_\lambda$

any desired value of B_{AG}, f_t, q and λ as read from the curves.

If desired the constant 1.2 may be eliminated by placing the reference point for one of the curves at an ordinate of 1.2 instead of 1. Similarly if the curves do not fit well on the sheet or overlap they may be shifted up or down at will and the constants changed accordingly. Or if one curve is shifted up and another shifted down an equal amount, no change in the constant need to be made.

Figs. 5–8 give the calculation curves for 0.0172 0.9 per cent silicon sheet, 0.0281, 0625 and 0.125 Bessemer sheet steel, respectively. These are all placed on the same basis so that for the reference conditions the constant of each equation is proportional to the actual surface losses for each class of material.

It will be noted that the q curves go to much higher values than were covered by the test results. These

curves were extrapolated from the induction-motor data and from test data by other observers. It is obvious that q can not have a constant exponent since as the air gap approaches zero, q approaches infinity while the pole-face losses approach a constant value. Therefore the exponent of q would actually begin to decrease if q were taken sufficiently large. We believe that the curves as shown are a very fair estimate of the way in which the pole-face losses actually vary with q.

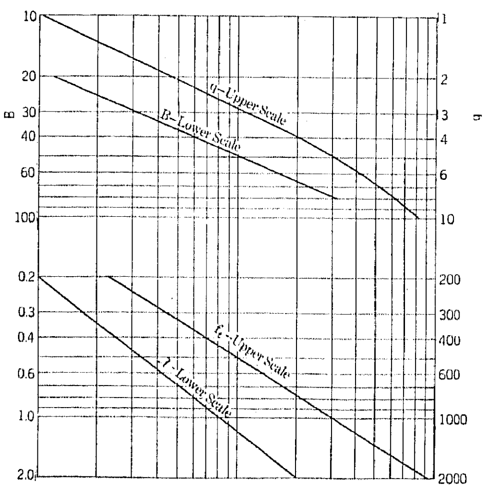

FIG. 6—SURFACE-LOSS CALCULATION CURVES FOR 0.0281-IN. BESSEMER STEEL

$W_s = 0.56\, K_{BAG} \times K_{f_t} \times K_\lambda$

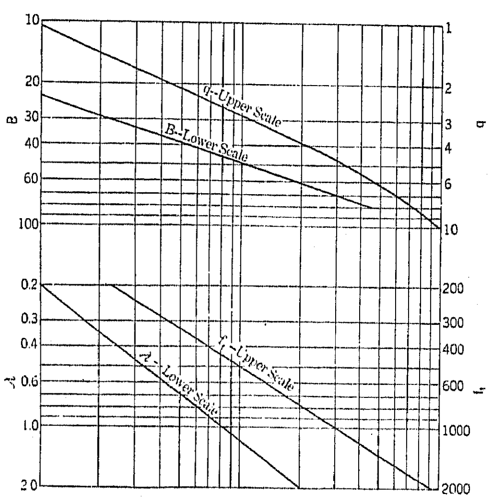

FIG. 7—SURFACE-LOSS CALCULATION CURVES FOR 0.0625-IN. BESSEMER STEEL

$W_s = 1.2\, K_{BAG} \times K_{f_t} \times K_\lambda$

Separation of Hysteresis and Eddy-Current Losses. It is obvious that if we wish to estimate the effect of changes in the magnetic quality of the materials entering into our poles, it is necessary to know the relation between the hysteresis and eddy-current coponents. In order to make an estimate of the relative values of these two components the pole-face losses for a given set of poles were reduced to watts/cycle and the results plotted against frequency. Fig. 9 gives a typical set of data. The intercepts on the vertical axis give a measure of the hysteresis components. In spite of the fact that in general there is present an appreciable skin effect causing the curves to be concave downward the extrapolation to zero frequency can be made fairly accurately.

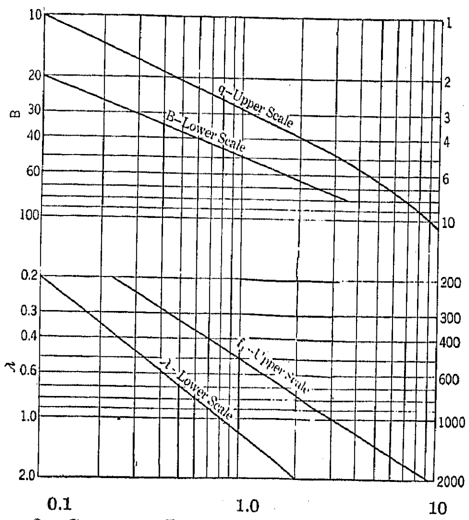

FIG. 8—SURFACE-LOSS CALCULATION CURVES FOR 0.125-IN. BESSEMER STEEL

$W_s = 2.6\, K_{BAG} \times K_{f_t} \times K_\lambda$

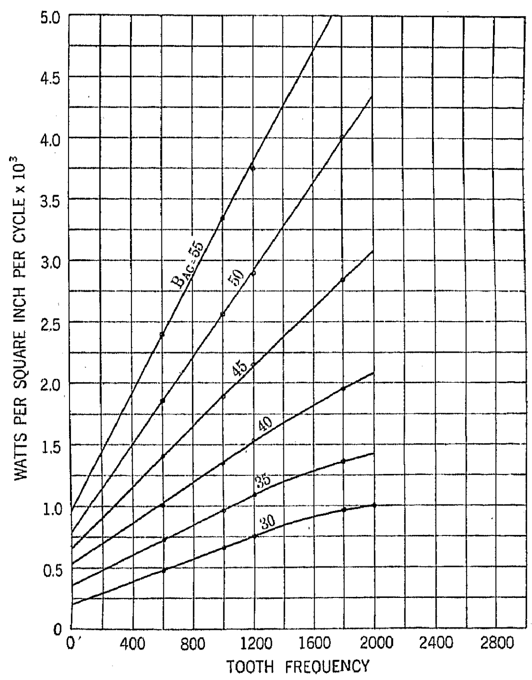

FIG. 9—SURFACE-LOSS IN WATTS-PER-CYCLE AGAINST FREQUENCY FOR 0.0625-IN. BESSEMER STEEL, SHOWING RELATION BETWEEN HYSTERESIS AND EDDY LOSSES

$q = 4.1$ in.

$\lambda = 0.47$ in.

The skin effect causes the hysteresis losses to increase somewhat faster than the first power of the frequency and the eddy-current losses to increase somewhat more slowly than the square of the frequency. This will be discussed in greater detail later.

From similar curves to those shown in Fig. 9 we have obtained the following average results for the relative hysteresis and eddy-current losses.

TABLE D.

Material			Total loss watts sq. /in.	Per cent Hysteresis	Per cent Eddy
0.0172 M. A.	$q = 4.2$	$\lambda = 0.471$	0.385	25	75
0.0281 Bess. (enameled)	$q = 3.7$	$\lambda = 0.912$	1.30	47	53
0.0281 " (unenameled)	$q = 3.7$	$\lambda = 0.912$	1.68	31	69
0.0625 " "	$q = 3.7$	$\lambda = 0.912$	3.7	35	65
0.0625 " "	$q = 3.0$	$\lambda = 0.471$	1.0	26	74
0.0625 " "	$q = 4.1$	$\lambda = 0.471$	1.9	35	65
0.125 " "	$q = 3.7$	$\lambda = 0.912$	8.1	20	80

In all cases $B_{AG} = 45$ and $f_t = 1000$.

Effect of Gage. In order to study the effect of gage, Fig. 10 has been plotted. This shows the relation between surface losses and thickness of individual laminations for various gages of Bessemer sheet steel and for various air-gap inductions. The points at zero thickness were obtained by the previously explained multiple-frequency method of separation, assuming that the zero thickness losses equal the zero frequency losses for the 0.0281 material. This is not strictly correct but the

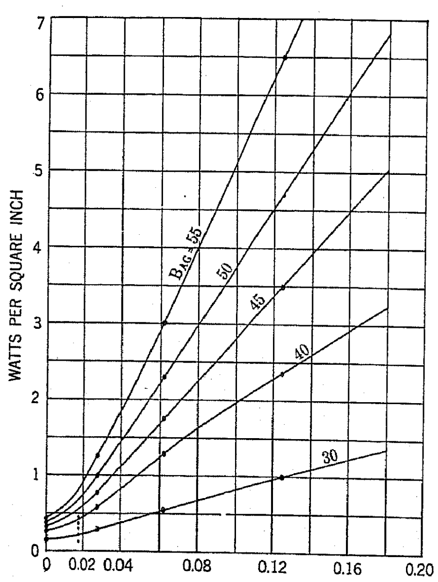

FIG. 10—RELATION BETWEEN SURFACE LOSSES AND THICKNESS OF LAMINATIONS FOR BESSEMER STEEL

$f_t = 620\sim$
$q = 3.7$ in.
$\lambda = 0.912$ in.

curves are accurate enough for most purposes and show an interesting relation about which little has hitherto been published. It should be noted that for quite thick material the rate of increase of pole-face losses with gage is approximately equal to the first power, while for thin gage material the rate of increase is approximately as the square of the thickness of the individual laminations. The vertical row of dots at a gage of 0.017 is for 0.0172 0.9 per cent silicon sheet steel. It will be noted that the 0.9 per cent silicon losses fall far below the Bessemer loss curves. This thin sheet has a considerably higher resistivity and considerably lower hysteresis loss than Bessemer steel which accounts for these differences.

Effect of Pole Chamfering. Fig. 11 shows some pole-face data for a set of standard poles and also for the same poles in which the air gap was 0.1 in. at the center and the tips machined off to give an air gap of approximately 0.2 in. It will be noted that pole-face losses are approximately the same for both the unchamfered and chamfered poles. In order to obtain these data certain

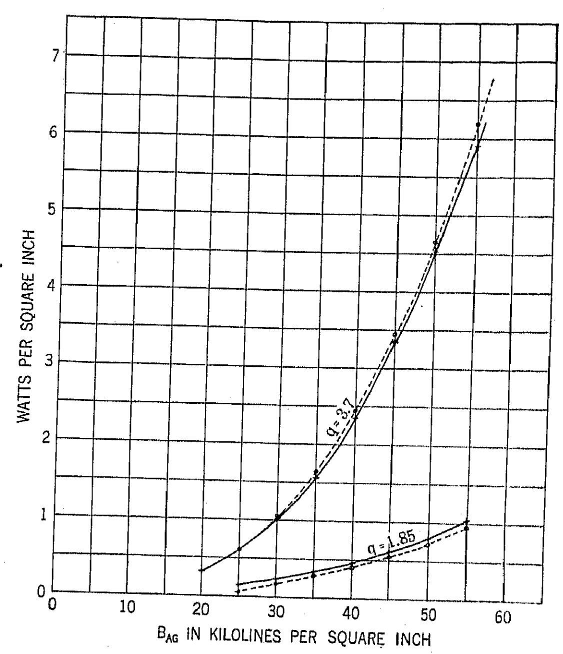

FIG. 11—EFFECT OF POLE CHAMFERING FOR 0.0625-IN. BESSEMER STEEL

$f_t = 930$
$\lambda = 0.912$ in.

Full lines before chamfering. Dotted lines after chamfering.

corrections had to be made in the armature losses. We shall not go into detail here in regard to these corrections but believe that they were made with a fair degree of accuracy.

Effect of Enameling. The net gain from enameling was very slight, ranging from 15 per cent in one case to zero in others. It may be noted from Table D, however, that enameling evidently results in decreased eddys and increased hysteresis.

Induction Motors. The method of handling the surface-loss test results as obtained by the test procedure outlined above was the same as for the salient-pole-machine data. The four functions B_{AG}, f_t, q and λ were plotted on double-log paper against surface losses and the average exponents determined with the following results for 0.0172 0.9 per cent silicon sheet steel.

TABLE E.

Function	Test Range	Exponent
B_{ag}	4-40	1.9
f_t	100-2700	1.55
q	2.01-11.	2.3-0.93
λ	0.711	1.1 (assumed)

Fig. 12 gives a typical set of surface-loss data plotted against q for the a-c. method of test.

Fig. 13 gives a typical set of rotor losses plotted against air gap as obtained by the d-c. method of test.

Fig. 14 gives a comparison between surface losses plotted against q as obtained by the a-c. and d-c. methods of test. There is also included a curve as calculated by Adams formula for 0.014 material.

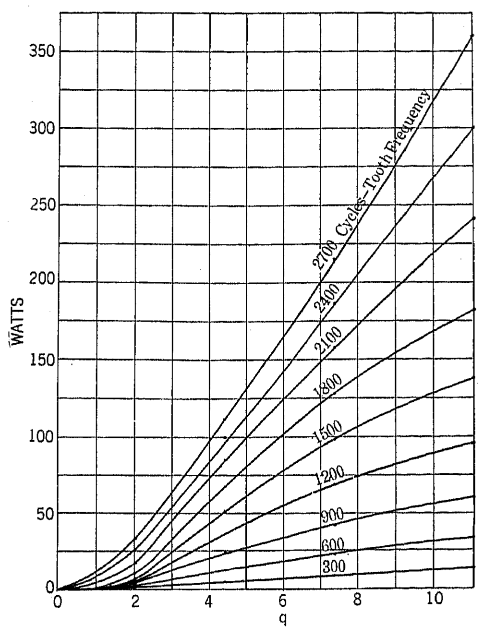

FIG. 12—INDUCTION MOTOR SURFACE LOSSES 60 CYCLES ON STATOR

B_{AG} = 16 kilolines per sq. in.

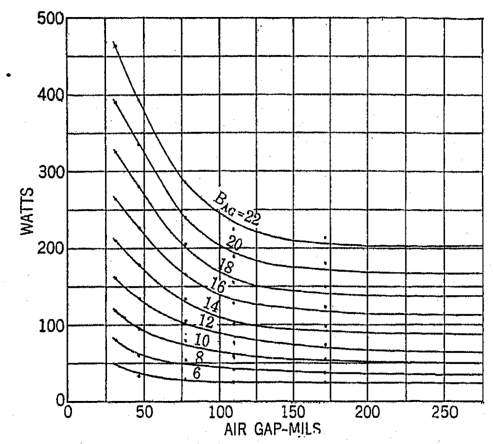

FIG. 13—TOTAL INDUCTION-MOTOR ROTOR LOSSES WITH DIRECT CURRENT ON STATOR

f_t = 1800 (= r. p. m.)

Fig. 15 gives a comparison of a-c. and d-c. surface losses for a 47-mil gap as plotted against average air-gap induction for the a-c. and d-c. methods of test.

Fig. 16 gives typical watts-per-cycle results plotted against tooth frequency for a 47-mil air gap. These curves make possible an estimate of the relative hysteresis and eddy-current losses.

Fig. 17 gives a calculation curve for surface losses for 0.0172 material under induction-motor conditions, namely, a sine-wave space distribution of flux.

Fig. 18 gives some calculated and observed data between q and per cent flux pulsation at the air gap. The observed values were obtained ballistically (circle points) as noted above by means of a narrow exploring coil placed in slots on the surface of the rotor located

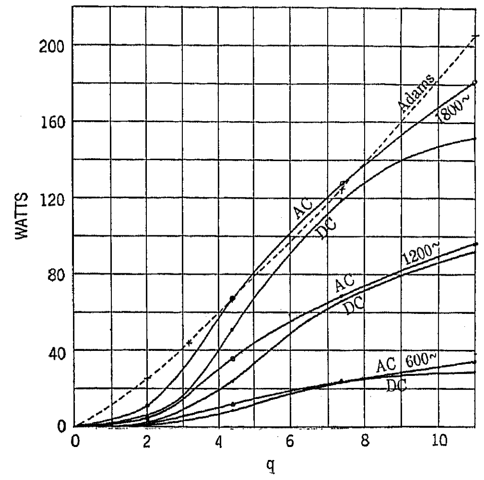

FIG. 14—COMPARISON OF INDUCTION-MOTOR SURFACE LOSSES WITH DIRECT CURRENT AND ALTERNATING-CURRENT STATOR EXCITATION AND WITH ADAMS FORMULA

B_{AG} = 16 kilolines per sq. in.

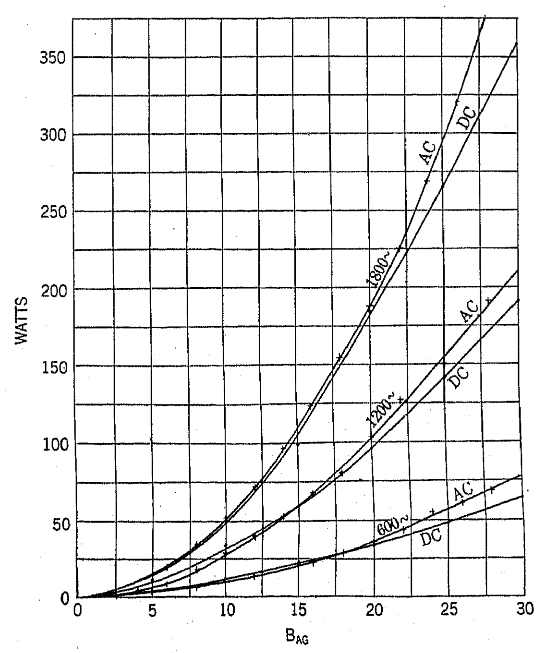

FIG. 15—COMPARISON OF INDUCTION-MOTOR SURFACE LOSSES WITH A-C AND D-C. STATOR EXCITATION

Air gap = 47 mils

0.2 in. apart. The points marked by crosses were obtained from measurements on oscillograms taken as previously described. The calculated curve marked by the dots was obtained from Carter's formula[10]. It will be noted that the oscillograms and calculated results check very closely indeed. The ballistically

observed pulsations are low, due to the fact that the exploring coils spanned an appreciable arc of the rotor surface and therefore did not show quite as deep dips in the flux wave as would have been shown by a very narrow coil.

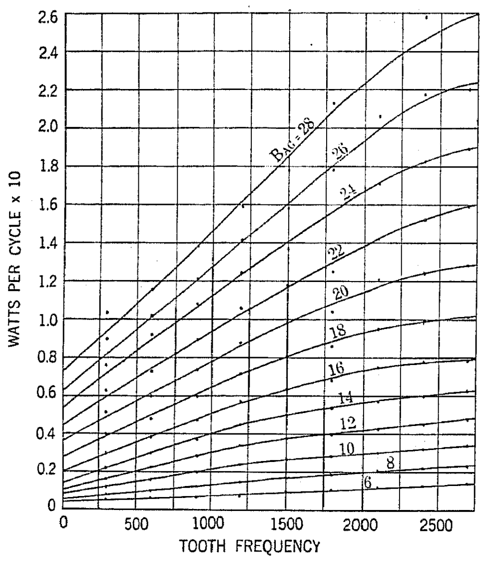

FIG. 16—SURFACE LOSSES IN WATTS-PER CYCLE AGAINST FREQUENCY FOR 0.0172-IN., 0.9 PER CENT SILICON STEEL SHOWING RELATION BETWEEN HYSTERESIS AND EDDY LOSSES

$q = 7.36$ in.
$\lambda = 0.711$

DISCUSSION OF RESULTS

Accuracy of Results. It is obvious that since these surface-loss data are obtained as the difference between two large quantities which may be several times the

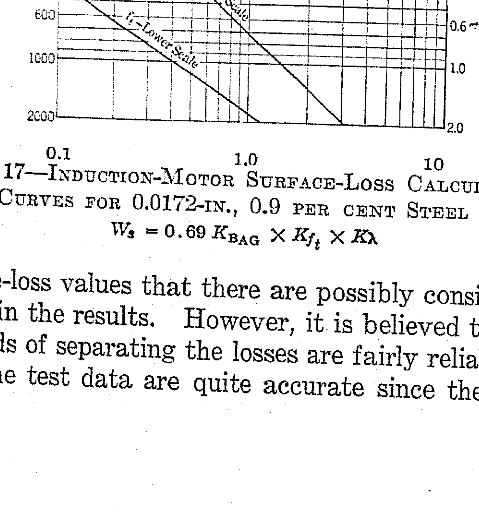

FIG. 17—INDUCTION-MOTOR SURFACE-LOSS CALCULATION CURVES FOR 0.0172-IN., 0.9 PER CENT STEEL

$W_s = 0.69\, K_{BAG} \times K_{f_t} \times K\lambda$

surface-loss values that there are possibly considerable errors in the results. However, it is believed that the methods of separating the losses are fairly reliable and that the test data are quite accurate since they were taken under exceptionally good laboratory conditions with great pains taken to have all instruments accurately calibrated. When there was the slightest doubt about the accuracy of a test run it was repeated. Care was taken in the preparation of the laminations and it is believed that the test results represent approximately the best conditions which may be expected from the various materials. When extrapolations of the test data have been necessary they have been made as carefully as possible, taking into account not only our data but other available data as well. There should be no large errors due to these extrapolations. The greatest chance for uncertainty is in connection with the use of these data for large machines where the slot pitch is large. It is possible that under these conditions our extrapolations may be somewhat inaccurate as no reliable test data were available covering large slot pitches. For the thicker laminated pole material it is possible that there were some errors

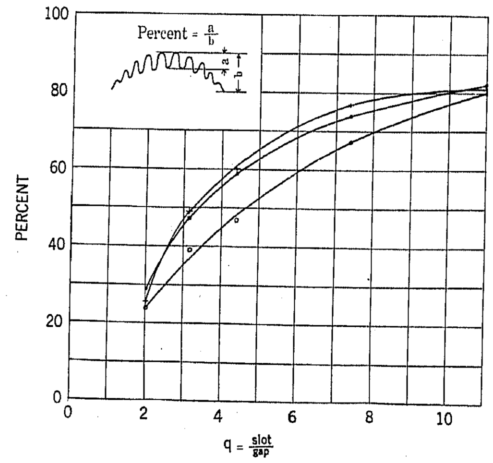

FIG. 18—CURVES SHOWING HIGH-FREQUENCY FLUX PULSATION AT AIR GAP

• Calculated (Carter's formula)
O Ballistic tests
+ Oscillographic tests

at the high inductions and high frequencies due to heating effects. However, under these conditions the test results were obtained very rapidly and it is believed that no serious errors have been introduced due to this cause.

Comparison with Previous Results. No comparisons will be attempted between our results and those of other observers except in the case of Adams data, due to the fact that the conditions of test were so different that comparisons are practically impossible. We shall, however, make some comparisons between the exponents of the various variables as obtained by previous investigators and our own. Referring to the formulas 2, 3, 4, 5 and 6 and to Tables C and E of our data it will be noted that there is a very close agreement in regard to the exponent of B, both from actual tests and from theoretical considerations, the average exponent being approximately 2. There is also a very

great uniformity in the exponent of frequency, the average result being about 1.5. q does not appear in Carter's formulas. A direct comparison cannot be made with Adams' theoretical formula because he uses the product of a and k in place of q. In Adams' empirical formula his q exponents are about an average of our extreme values. Due to the fact that he assumes a constant q exponent, his formulas give results which are too high for small and for large values of q, while a very good check is obtained for average values of q. The q exponent for Wall and Smith's empirical curve is much too high as pointed out by Adams. This is due to the fact that under their peculiar test conditions

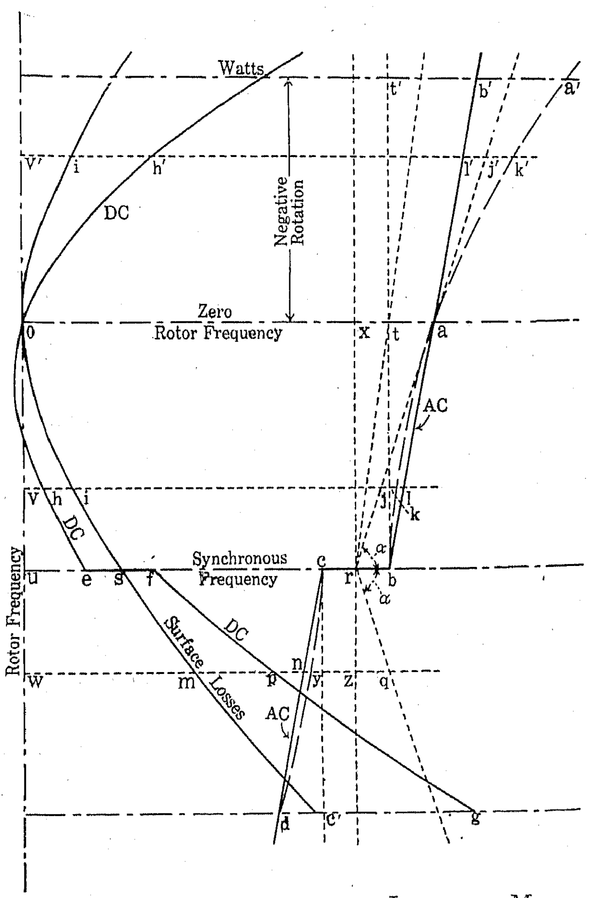

FIG. 19—METHOD OF SEPARATING INDUCTION MOTOR CORE LOSSES

Curve marked $A\ C$ is input to stator
Curve marked $D\ C$ is input to rotor

large flux pulsations existed in the poles and frame of the machine which were included in the estimated pole-face losses. Our exponent of λ is slightly higher than given by both of Adams' formulas. In his theoretical formulas he gives λ to the first power and in his empirical, λ to the 1.05. λ does not appear in the Wall and Smith formulas and although it appeared originally in the Carter formulas it cancels out when f_t is substituted in this formula for V by the relation,

$$f_t = K\,V/\lambda \tag{9}$$

Adams' formula as well as Carter's used V (peripheral velocity) rather than f_t (tooth frequency) and we reduced his V to the latter value in order to obtain his exponent of λ. With respect to the Adams' and Carter theoretical formulas there is an agreement for the eddy-current losses for the ρ exponent and the μ exponent. With regard to the thickness, however, Adams has an exponent of 1 and Carter of 2. It has been shown by our experimental data and may be shown from theoretical considerations that the exponent of t is a function of the thickness of the material. Therefore both formulas are approximately correct in this respect, depending upon the range of thickness covered. The Carter formula for hysteresis loss has too small an exponent for frequency. This should range from 1 to 1.5, depending upon the skin effect.

Latour's[11] formulas for high-frequency losses throw some valuable light on this discussion. They are as follows for hysteresis and eddy losses, respectively.

$$W_h = \frac{\omega^2\, a \sin\tau}{4\, m\, \rho} \quad \frac{\dfrac{\sinh 2\, m\, \alpha\, a}{\alpha} + \dfrac{\sin 2\, m\, \beta\, a}{\beta}}{\cosh 2\, m\, \alpha\, a - \cos 2\, m\, \beta\, a}\, B^2 \text{ app} \tag{10}$$

$$W_e = \frac{\omega^2\, a}{4\, m\, \rho} \quad \frac{\dfrac{\sinh 2\, m\, \alpha\, a}{\alpha} - \dfrac{\sin 2\, m\, \beta\, a}{\beta}}{\cosh 2\, m\, \alpha\, a - \cos 2\, m\, \beta\, a}\, B^2 \text{ app} \tag{11}$$

where

$2\, a$ = thickness of laminations in centimeters.

ρ = resistivity of laminations in abohms.

$$2\, m^2 = \frac{4\, \pi\, \mu\, \omega}{\rho}$$

$$\alpha = \sqrt{1 + \sin\tau}$$

$$\beta = \sqrt{1 - \sin\tau}$$

$$\sin\tau = n \times 4\, \mu$$

It is difficult to draw any conclusions from such formulas as these, but fortunately when the laminations are thick enough, or the frequency high enough to make $\sinh 2\, m\, \alpha\, a$ nearly equal to $\cosh 2\, m\, \alpha\, a$, the eddy losses are then proportional to $\dfrac{\omega^2\, a}{4\, m\, \rho}$ and the hysteresis losses to the same value times $\sin\tau$. We reach this condition for 0.0172 0.9 per cent silicon steel at a frequency of the order of 500 cycles. Now under these circumstances if we can assume constant permeability the eddy losses vary as follows:

$$W_e \text{ varies as } t,\ \rho^{-1/2},\ \mu^{-1/2},\ f^{1/5}$$

It will be seen that this checks Adams' theoretical

formula 2 for all the factors. Similarly the hysteresis loss varies as follows:

$$W_h \text{ varies as } t,\ \rho^{-1/2},\ \mu^{-1/2},\ f^{1/5}$$

It will be seen that this checks Carter's formula **3b** in no respect.

We believe that it is necessary to have a factor corresponding to either the slot pitch or the slot width in these formulas, since it is obvious that the depth of penetration of the flux is a function of this factor. The wider the slot, the deeper the flux penetration, other things being equal. Adams' formulas are the only ones containing this factor when proper corrections are made, consequently we believe his formulas are the only ones which are capable of giving approximately correct results.

No comparisons have been made between Mr. Lamme's and Mr. Hanssen's formulas and the others because of the very large differences in the type of formulas. Mr. Hanssen's formula applied only to induction motors and includes both surface and tooth pulsation losses.

Comparison Between Salient Pole and Induction Motor Losses, and Between A-C. and D-C. Induction Motor Losses

A comparison between the salient-pole and induction-motor losses can be made only for the 0.0172 material since this was the only material tested in both types of machine. Assuming the following conditions, calculations have been made by the surface-loss formulas as obtained by the two methods of test.

TABLE F.

$B = 30$

$f_t = 930$

$q = 3.7$

$\lambda = 0.711$

$K = 0.81$

$W_s = 0.69 \times 0.333 \times 0.36 \times 2.6 \times 1 \times 0.81 = 0.174$ (Fig. 17)

$W_s = 0.20 \times 0.343 \times 0.283 \times 1.59 \times 0.575 = 0.177$ (Fig. 5)

The comparisons between these formulas is made on the assumption that the salient pole machine data are for a flat-top flux wave and the induction-motor data for a sine-shape flux wave. Assuming that the losses vary as the square of the flux density, for the same average induction the salient-pole conditions should give 81 per cent of the loss for the induction-motor conditions. Therefore this factor ($K = 0.81$) has been introduced into the induction-motor formula. The check between these two formulas is remarkably good, especially considering the fact that the two sets of data were obtained on such widely different types of machines, and that the results were obtained and calculated by two different observers. The check will not be quite as good at high inductions because a somewhat different B exponent has been assumed for the induction motor conditions than for the salient-pole conditions.

The comparison between the a-c. and d-c. test results as shown by Figs. 14 and 15 is not quite as good as we would have liked to see. It is believed that the d-c. losses are somewhat low due to the fact that the rotor giving the largest air gap had somewhat higher fundamental-frequency losses than the other rotors due probably to a different condition of the dies or greater stacking pressure, etc. Since all of the surface losses obtained under d-c. conditions are influenced by the extrapolation for the fundamental-frequency losses, it is evident that if this large-air-gap rotor had high losses the estimated surface losses would be too low. We have considered that the a-c. loss results are more nearly correct than the d-c. loss results and for most of our calculations have used them in preference to the d-c. results. Considering the very wide variations which may occur in different machines due to different conditions of shop practise this discrepancy is not serious.

A considerable number of large a-c. and d-c. salient-pole machines have been tested for losses under commercial conditions with two or more air gaps and the difference in losses under the two conditions compared with the difference in pole-face losses as calculated by our formulas. The check results were not especially good due, we believe, chiefly to the notoriously inaccurate results which may be expected from ordinary test-floor conditions. However, the differences were in some cases higher than the calculated and sometimes lower, the mean results checking fairly well. We believe that in general pole-face losses may be calculated much more accurately than they can be determined by ordinary test-floor methods.

Advantages of Method of Presenting Final Results. As previously mentioned the exponential formulas as derived by Adams and others are too complicated for ordinary routine design calculations. The time involved in looking up logarithms is prohibitive except in certain special cases. We believe that our method of presentation, making use of curves on double-log paper, Figs. 5-8, so simplify the calculations that there can be no reasonable objection to using functions of this form. No log tables are necessary and all the calculations can be made in a few moments by means of a slide rule when the necessary factors are given. These four fundamental factors are usually already known by the designer since he needs them for other purposes or they can be calculated readily from data which he already has at hand.

Relation Between Hysteresis and Eddy-Current Losses. Referring to Table D on page 24, giving the relation between hysteresis and eddy-current losses for the various classes of material considered, it is seen that the average hysteresis loss amounts to approximately 30 per cent of the total surface losses. If similar data were obtained on samples of these materials for the same range of frequency by means of an Epstein apparatus, using symmetrical pulsations of flux in the ordinary way it would be found that these hysteresis losses instead of being 30 per cent would be very much less. This increase in hysteresis-loss percentage under surface-

loss conditions is due to the fact that the high-frequency hysteresis loops are displaced from their normal position. It has been shown by previous investigations[2,3,4] that such displacements of the hysteresis loops produce very greatly increased hysteresis losses. Moreover, as will be shown a little later the skin effect when present still further increases these hysteresis losses. The eddy-current losses are unaffected by displacement but are simply a function of the maximum amplitude of pulsation. The skin effect when present decreases these eddy-current losses.

Factors Affecting Hysteresis Losses. To elaborate somewhat on the effect of displacement as previously mentioned, Fig. 1 shows approximately the effect of displacing a hysteresis loop of given magnitude from its normal position to a fairly high value of displacement. The symmetrical dotted loop has the same B amplitude as the displaced minor loop. If the amplitudes of pulsation are not too high the displaced hysteresis loss may be calculated approximately for ordinary open hearth steel by the following formula due to Ball[3].

$$W_h = (1.06 \times 10^{-3} + 0.344 \times 10^{-10} B_m^{1.9}) B^{1.6} \times \frac{f}{d \times 7.92} \qquad (12)$$

where,

W_h = the hysteresis loss in watts per cu. in.
f = frequency
d = density
B_m = the mean displaced induction
B = ½ the amplitude of pulsation.

So far as our available data go the effect of displacement on hysteresis is not very different for various kinds of materials. If we are dealing with another class of material we simply increase or decrease the hysteresis loss as calculated by the above formula according to the known difference in the hysteresis characteristics of the material from normal open hearth steel. The relative specific hysteresis loss for Bessemer and 0.9 per cent silicon steel are on the average about 2 to 1. We know the magnitude of flux pulsation at the surface of induction-motor rotors as shown by Fig. 18. We do not, however, know the relative depth of penetration of flux for various conditions and are at present unable to accurately calculate the surface losses from a knowledge of the specific quality of the material. However, knowing the surface hysteresis losses for a given set of conditions we can quite accurately predict the losses for another class of material having a different specific hysteresis loss.

Factors Affecting Eddy-Current Losses. As previously stated the eddy-current losses are a function only of the amplitude of flux pulsation regardless of the displacement of the pulsating hysteresis loops. Various formulas have been developed for calculating eddy-current losses from a knowledge of the thickness and resistivity of the electrical sheet. These all give eddy-current losses which are too low, the actual losses being in general anywhere from 50 to 200 per cent greater at normal frequencies. However, knowing the eddy-current losses for a given sheet the losses for another thickness or resistivity of sheet may be calculated in the usual way, unless the material has abnormally large grain size in which case the actual eddy-current losses will be considerably higher in general than anticipated.[12] Therefore knowing the surface eddy-current losses for a given class of material the losses to be expected for another thickness or resistivity of the material may be calculated with a fair degree of accuracy provided we take into account skin effect as will be explained in the next paragraph.

Skin Effect with Special Reference to Thickness of Material. By the aid of certain formulas recently developed by Latour it is now possible to calculate accurately the effect of frequency on hysteresis and eddy-current losses for materials of various thicknesses and resistivities. These formulas were developed for use at radio frequencies. However, we have checked them at audio frequencies and have found that the shape of the curves as calculated by them is fairly close to results obtained by test although the absolute value may be somewhat different. The hysteresis formula is based on the assumption that hysteresis loss varies as the square of the induction. This is approximately true at an induction of about 70 kilolines per sq. in. (14 kilogausses) only for commercial materials. Below this induction the hysteresis ratio of variation decreases to approximately a constant value of 1.6 and above it increases quite rapidly to much higher values. In order to use these formulas accurately the hysteresis loss should be determined from loss curves for the material in question at the particular induction at which it is desired to work. Also for surface loss calculations the hysteresis loss should be increased according to the displacement factors as previously noted. With these precautions the formula may be used to calculate the increased hysteresis losses to be expected with increased frequency due to skin effect. It should be noted, however, that for the condition of displaced hysteresis loops the permeability value to be used is not that of the material under normal conditions, but the average permeability as given by the upper and lower tips of the displaced hysteresis loops. For fairly high values of displacement this permeability may be quite low. For a comparatively small amplitude of displaced loop and a displacement, say of 64.5 kilolines per sq. in. (10 kilogausses) the permeability will be of the order of 100.

In calculating eddy-current losses this same precaution in regard to permeabilities must be taken. Moreover, the eddy-current losses as obtained by the formula may have to be increased somewhat to get actual values especially at the lower frequencies. This increase may be determined by measuring the eddy-current loss of this particular material in the Epstein apparatus and

12. Grain Growth in Silicon Steel, W. E. Ruder. *Trans. A. I. M. E.* October, 1913, p. 2820.

then calculating the eddy-current loss by the formula under the same conditions. Referring to Fig. 16 the combined effects upon the hysteresis and eddy-current losses of skin effect may be noted. The curves show a very marked concave tendency downward with increasing frequency. This is due to the fact that the increase in hysteresis loss due to skin effect is very much less than the relative decrease in eddy-current loss. For the thicker materials the skin effect may cause the eddy-current losses to be of the order of one half those to be expected by applying the ordinary square law to the frequency. Referring to Fig. 10 and formula **11** it will now be seen why these curves at the higher frequencies show surface losses increasing approximately as the first power of the thickness of laminations instead of the square. If the formula is used for thicknesses and frequencies which will give $\cosh 2\, m \alpha a$ and $\sinh 2 m \alpha a$ approximately equal, it is seen that the eddy-current losses increase approximately as the first power of the thickness rather than the square as previously mentioned. A comparison of the surface loss constants for the three thicknesses of Bessemer steel, Figs. 6, 7 and 8, indicate that these constants are approximately proportional to the thickness of laminations. It is largely on account of this skin effect that with the thicker materials (see Table D, page 24) the eddy-current losses do not change very greatly in percentage as the thickness increases. It should be understood that Table D, page 24, gives results for comparatively low frequencies only. It is evident that at higher frequencies the hysteresis values will be somewhat higher and the eddy-current losses lower than given by the table. Skin effect will probably account for the fact that the B exponent of surface loss is higher for the thick material than for the thin (see Table C). The thicker the material the greater the skin effect. Now as the induction increases the effective permeability decreases, thus producing a relative decrease in the skin effect which means increased eddy loss. We believe that it is chiefly due to this effect that we have a B exponent of 3.1 for 0.125 Bessemer sheet as against a value of 2.4 for the 0.0281 material.

Depth of Penetration of Pulsating Flux. As previously stated we do not know exactly the way in which the high-frequency pulsations of flux decrease as we penetrate below the surface. However, from the loss characteristics of unsymmetrical hysteresis loops as known from another investigation the results of which it is hoped we may be able to present at a later date, the depth of penetration may be calculated if we assume a certain law of attenuation. For instance, if we assume that the flux pulsations decreases linearly with the distance below the surface we can show that in order to obtain the losses which actually exist the effective penetration must be something over an inch. The work of Hele-Shaw[13] indicates that under some conditions the pulsations may actually extend from one pole to the next through the rotor iron.

Effect of Unequal Slot and Tooth Widths. This investigation has been confined to the case of open slots. When considering the case of partly closed slots two main factors must be considered: (1) the effect of the narrow slot on the depth of penetration of the pulsating flux; (2) the decreased flux density opposite the tooth tips.

1. Where the slot and tooth widths are unequal it would probably be much more accurate to use the slot width σ instead of the slot pitch λ in the double log curves. This would automatically take care of the first factor.

If it is desired to use the λ curve it is only necessary to multiply the value of the slot width σ by 2 and then look up the K value corresponding to $2\,\sigma$. This procedure is probably not permissible when saturation effects are of any magnitude.

2. In order to correct for the change in flux density Carter's fringing coefficients may be used. The correction to be applied to B_{AG} will be as follows.

$$B_{AG}\ (\text{cor.}) = \frac{B_{AG} \times \lambda_1}{g_1} \qquad \textbf{(13)}$$

where λ_1 = tooth pitch
g_1 = gap factor

By this formula a correction is made to the induction corresponding to the actual effective width of tooth as compared with the effective width on the assumption that the tooth and slot widths are equal. After applying formula **13** it is then only necessary to use the new value of B_{AG} in connection with the original B_{AG} curve.

It will be noted that in the salient-pole test machine the tooth and slot widths are not equal although open slots were used. For small differences from unity ratio no serious error is introduced for normal salient pole inductions due to the fact that the teeth have begun to saturate, thus making the pulsation amplitude less than for lower values of induction. We see therefore that if the tooth is wider than the slot the saturation effect is less and the amplitude of pulsation greater, thus largely compensating for the difference from unity of the tooth to slot ratio.

Effect of Saturation. For the inductions ordinarily used in induction motors probably saturation effects are not large but for salient pole machines where the inductions ordinarily run much higher, we undoubtedly have large saturation effects. As the teeth begin to saturate the amplitude of the high-frequency pulsations at the air gap begin to decrease. This results in a smaller surface loss than would otherwise occur. It is well known that as we go to high inductions hysteresis losses increase much faster than the square of the induction. When we consider also that the amplitude of pulsation increases with the induction it is easy to see that the hysteresis pulsation losses may increase as the fourth or fifth power of the induction under certain conditions. However, we find that the surface losses

13. Hydrodynamical and Electromagnetic Investigation regarding the Magnetic Flux Distribution in Tooth Core Armatures, H. S. Hele-Shaw, A. Hay, P. H. Powell. I. E. E. Vol. 34, page 21 (1904).

actually increase only as some power between the square and the cube and that the slope of the double-log curve between loss and air-gap induction is usually nearly straight. This means that the tendency for the rate of change of losses with induction to increase is counterbalanced by the effect of saturation in reducing the amplitude of pulsation. Of course, if it were possible to go to sufficiently high inductions the surface losses would actually begin to decrease.

Insulation between Laminations. It has been shown that by enamelling the laminations of pole material there is a slight decrease in surface losses. Apparently the eddy losses are decreased and the hysteresis losses increased by nearly an equal amount. Evidently if some of the laminations are in electrical contact with each other in spots long path eddy currents are set up which tend to damp out the pulsating flux. The increase in losses due to these long-path eddys just about equals the decreased loss due to hysteresis and the short-path eddys. Of course if the contact between laminations were very good the result would be increased surface losses for we would approach the condition of solid poles. That the unenameled poles do not have higher losses is due to the fact that the sheets have a fairly high resistance scale and are not pressed together very tightly at the air gap.

Effect of Punching. For material which has not been annealed subsequent to punching there will be quite appreciable increased hysteresis losses due to the punching strains. These strains may affect the quality of the material for a distance of $\frac{1}{8}$ in. or more from the punched edges depending on the thickness, nature of the material, and sharpness of the dies. In this area the hysteresis losses may be increased very considerably, appreciably affecting the surface losses since the greater part of the surface losses occur quite near the surface. However, since the resistivity of the material is affected but very slightly by annealing and the eddy losses are much the larger part of the surface losses, annealing will not very greatly decrease the surface losses.

Summary

An attempt has been made to separate the various factors which govern the surface or pole-face losses of laminated material in rotating machines and to determine the way in which these losses vary with each factor. By a separation of the hysteresis and eddy-current components it has been shown that while the eddy losses are responsible for the larger part of the surface losses the hysteresis losses are very appreciable and can not be neglected. As a basis for this analysis test results were obtained on a salient pole machine supplied with various sets of poles arranged for variable air gap and on a special three-phase induction motor supplied with a number of smooth-core rotors having slightly different diameters. The rotors of the two machines contained no windings except exploring coils and were direct-connected to d-c. drive motors, the input to which gave the losses for the various conditions of test. By a suitable separation of the fundamental-frequency losses from the total losses the surface losses were given. This was done either by varying the air gap and extrapolating to infinity air gap (zero surface losses) or by a special method for the induction motor described in detail in the appendix.

The surface losses were assumed to be a function of four main aproximately independent variables, namely air-gap induction B_{AG}, tooth frequency f_t, ratio of slot width to air gap q and the slot pitch λ. These losses are approximately an exponential function of B_{AG}, f_t and λ but the ratio of variation of losses with q decreases as q increases. As a rough approximation the surface losses vary as the square of the air-gap induction, as the 1.5 power of tooth frequency, from the second to the first power of q with increase in q, and as the first power of the slot pitch. These losses can most readily be represented as a function of these variables by plotting on double-log paper, three of the functions being straight lines. Knowing these four variables which the designer can determine very readily the surface losses can be calculated in a few moments by a simple multiplication on a slide rule. This avoids the use of logarithms in spite of the fact that the functions are exponential.

It is shown that by plotting the surface losses in watts/cycle against frequency it is possible to obtain an approximate separation of the hysteresis and eddy components. The hysteresis loss at 1000 cycles is about 30 per cent of the total surface losses for all the materials investigated. Due to the skin effect this separation may be somewhat inaccurate since the hysteresis loss increases faster than the first power of the frequency and the eddy loss less rapidly than the square of the frequency. Due to the uncertainty as to the effective permeability it is impossible to accurately calculate this skin effect. The net result of skin effect is decreased losses since the decrease in eddy losses is considerably greater than the increase in hysteresis losses. Due to the fact that the high-frequency pulsation hysteresis loops are usually greatly displaced from the normal symmetrical position, the hysteresis losses are much higher than would be the case for symmetrical loops for the same frequency and amplitude.

In addition then to the four factors B_{AG}, f_t, q and λ which govern surface losses for any given material and are a function of the design of the apparatus we have the four fundamental factors: Hysteresis coefficient n, resistivity ρ, thickness of individual laminations t and the effective permeability μ which are determined by the nature of the laminated material. The factor n governs not only the hysteresis loss but according to Latour's formulas when skin effect is appreciable also affects the eddy losses somewhat. The eddy losses are inversely proportional to ρ for small frequencies but for high frequencies are inversely proportional to the square root of ρ. Also the eddy losses are proportional to the

square of t for low frequencies and thin material and to the first power of t for high frequencies or thick material. With a given flux density for low frequencies μ is not a factor, but for high frequencies the eddy losses are inversely proportional to the square root of μ and the hysteresis losses directly to the square root of μ. The permeability also influences the attenuation of the pulsating flux but we have no data in regard to this factor and it is probably not very different for the different classes of commercial materials.

It is shown that for chamfered poles approximately correct results may be obtained by assuming that the air gap is that corresponding to the minimum gap. The change in field form largely compensates for the increased gaps at the tips of the poles.

Enamel on the individual laminations of salient poles only slightly decreases the surface losses although it may materially affect the relative hysteresis and eddy losses. In most cases it is an extra manufacturing expense which would be unwarranted.

Due to the hardening of the material near the edges caused by punching the hysteresis surface losses are undoubtedly increased unless the punchings are subsequently annealed. We have no specific data in regard to this effect with reference to surface losses but would expect an increase in hysteresis losses due to punching of from 5 to 10 per cent.

It is believed that the surface-loss curves here presented may be used with considerable confidence at least for open slots where the slot and tooth widths are approximately equal. Our results check those of Adams and his collaborators with a reasonable degree of accuracy although they used different methods of test. Again our results for the salient pole machine and for the induction motor check very well with each other although the methods of test and the conditions for the two types of machines were quite different. In other words we believe that our curves are based on all of the important fundamental factors which govern surface losses. Finally, the check results which we have on commercial machines show a fair average check although some of the individual machines show considerable discrepancy between calculated and test results. This, we believe, however, is due largely to inaccuracies in the commercial testing rather than in the formulas.

From the calculation curves given we believe it is possible to predict from the fundamental characteristics of a new material, namely, n, ρ, t and μ, the surface losses to be expected under any normal conditions of use. Even a simple Epstein test under standard conditions should give sufficient data for an approximate estimate for the surface loss constant.

It is hoped that this paper will bring out further test data on surface losses in order that a more accurate check on existing formulas may be obtained so that this type of loss may be calculated with the same degree of accuracy and confidence with which we now calculate transformer losses.

Appendix

This method depends upon the fact that when the rotor of a polyphase induction motor with the stator excited is driven from an external source there is a sudden decrease in stator input when the rotor goes through synchronism. This decrease is equal to twice the rotor hysteresis loss at zero speed. There is, of course, an equal increase in input to the motor which drives the rotor. For a more complete discussion of these phenomena see Alger and Eksergian.[14] Now at synchronous speeds if we add one-half this sudden change in input to the net input to the driving motor just before synchronism is reached we have the surface losses of induction motor for this particular rotor speed and induction. By changing the frequency of the stator applied voltage and noting the input to the rotor just below and above synchronism, data may be obtained for surface losses for any rotor speed. However, since we desired fundamental-frequency stator and rotor loss data as well as surface-loss results we found it more convenient to keep the stator frequency constant and make the separation of losses by the procedure above to be described. As a matter of interest the discussion covers negative as well as positive rotor rotation and it has been found that surface loss results check very well for both directions of rotation. If the surface pulsation frequency is high with reference to the fundamental frequency there should be no appreciable difference, the surface losses depending then only on the tooth frequency.

Referring to Fig. 19 the following points should be noted before considering it in detail:

1. This diagram refers to any polyphase induction motor with the rotor driven by an external source of power. The curve marked d-c. refers to the input from this external drive.
2. It holds for one given applied stator-frequency and voltage, the rotor-frequency being the only variable.
3. Instrument losses and $I^2 R$ losses in the stator windings are subtracted from the observed a-c. input before plotting.
4. Friction, windage, armature $I^2 R$ and brush $I^2 R$ losses are subtracted before plotting the d-c. input (assuming a d-c. motor drive).
5. The ordinates are a-c. and d-c. input in watts. The abscissas are rotor speeds plotted in terms of stator-frequency, namely, for a stator applied frequency of 60 cycles and a four-pole stator, the rotor speed at synchronism will be 1800 rev. per min., but would be plotted as 60. If desired rotor speeds in rev. per min. could be used for abscissas.

Now considering the sketch in detail, the full line $b' \, a \, b \, c \, n \, d$ represent the a-c. input to the stator ($I^2 R$ losses subtracted as noted above) where $b' \, a \, b$ is parallel to $c \, n \, d$. The d-c. input (minus friction, wind-

age, etc. as noted above) is $h' \, o \, h \, e \, f \, p \, g$. It will be noted that there is a discontinuity in the input curve at synchronous frequency. The decreased a-c. input ($b \, c$) is of course equal to the increased d-c. input ($e \, f$). This sudden change in wattage is equal to twice the product of the rotor torque (due to rotor hysteresis) and the synchronous frequency, namely, it is equal to twice the rotor-hysteresis loss at zero rotor speed. The points r and s bisect $b \, c$ and $f \, e$ respectively. $x \, r \, z$ and $t' \, t \, b$ are drawn parallel to the x axis so that $t \, x$ equals $b \, r$. The lines $t \, r$ and $a \, r$ are straight lines. $r \, q$ is drawn making an angle with the vertical equal to the angle between $a \, r$ and the vertical. Now $t \, x$, $b \, r$, $r \, c$, $f \, s$ and $s \, e$ each equals the rotor hysteresis loss at zero rotor speed and $a \, t$ is the corresponding eddy-current loss. This eddy-current loss will include any copper losses which may be due to circulating currents in the rotor windings if these happen to be closed, also eddy-current losses in the rotor and stator copper. The sum of the stator-hysteresis and eddy-current losses are equal to $x \, o$, or $r \, u$ and are practically constant for all rotor speeds. For any rotor speed below synchronism (positive or negative rotation) the actual rotor-hysteresis loss is equal to the vertical intercept between $t \, r$ and $x \, r$, that is, it is directly proportional to $1\text{-}\omega_r$ where ω_r is the rotor frequency expressed in terms of synchronous frequency. It therefore decreases from $t \, x$ at zero speed to zero at synchronous rotor speed. Above synchronous frequency it again increases at the same rate with frequency at which it decreased. For negative rotation it increases at the same rate as indicated by extending $r \, t$ back to the left.

Before discussing this diagram further it will be well to define a few useful terms.

The *apparent* fundamental-frequency rotor-hysteresis loss is the product of the fundamental-frequency rotor-hysteresis torque and the synchronous frequency. This is a constant and is represented by the vertical distances $t \, x$, $b \, r$, $r \, c$, etc.

The *true* fundamental-frequency rotor-hysteresis loss as noted above is zero at synchronous rotor-frequency and increases uniformly with the frequency on either side of that point. It is represented by the difference between $t \, r$ and $x \, r$.

The *fictitious* fundamental-frequency rotor-hysteresis loss is equal to the difference between the true and the apparent hysteresis loss and is represented by the vertical distance between lines $t \, r$ and $t \, b$ or what is the same thing, the vertical distance between $a \, r$ and $a \, b$. It will be noted that for negative rotation the fictitious hysteresis loss is changed in sign.

The *apparent* fundamental-frequency rotor-eddy-current loss is equal to the eddy-current torque multiplied by the synchronous frequency. It is represented by the vertical distance between the lines $b' \, a \, b$ and $t' \, t \, b$, for rotor frequencies at the left of synchronous speed and is represented by the difference between $c \, c'$ and $c \, n \, d$ at the right of synchronous speed (negative slip). It will be noted that the apparent eddy-current loss increases at a constant rate with rotor frequency at the left of synchronous frequency and decreases at a constant rate for rotor frequencies at the right of synchronous frequency. This is due to the fact that the rotor torque is zero at synchronous frequency, but increases linearly with the slip frequency.

The *true* fundamental-frequency rotor-eddy-current loss increases as the square of the slip or in proportion to $(1-\omega_r)^2$ where ω_r is expressed in terms of synchronous frequency. Knowing the true eddy-current losses as represented by $a \, t$ at zero rotor frequency, the true eddy-current loss for any other frequency may be calculated by assuming this square law. $a' \, k' \, a \, k \, b$ was calculated on this basis. The true eddy-current loss is then the vertical distance between $a' \, k' \, a \, k \, b$ and $t' \, t \, b$. For frequencies above synchronous speed (negative slip) $c \, y \, d$ is drawn as the reverse of $b \, k \, a$ and the true eddy-current loss is the vertical distance between $c \, c'$ and $c \, y \, d$.

The *fictitious* fundamental-frequency rotor-eddy-current loss is the difference between the apparent eddy-current loss and the true eddy-current loss. It is the vertical distance between lines $b' \, a \, b$ and $a' \, k' \, a \, k \, b$. It will be noted that this difference reverses sign at zero rotor frequency. For frequencies above synchronism (negative slip) the fictitious eddy-current loss is the vertical distance between $c \, y \, d$ and $c \, n \, d$.

It is at first rather confusing to note that although the true rotor-hysteresis loss is decreasing with increasing rotor frequency between zero rotor frequency and synchronous rotor frequency, the apparent rotor-hysteresis loss (product of rotor hysteresis torque and synchronous frequency) as represented by the difference between $t \, b$ and $x \, r$, is a constant. A similar confusion exists in considering the true and apparent eddy-current losses. This action may perhaps be made clearer by considering that the stator is revolving at synchronous speed and that the rotor hysteresis torque is represented by friction between the stator and rotor. As long as the rotor speed is below the stator speed the hysteresis torque will be positive and constant. Above synchronous speed the torque will be constant and negative, thus accounting for the sudden change in apparent hysteresis loss. A similar analogy may be used for the eddy-current effects. In this case, however, the coefficient of friction is not constant but is assumed to vary and is directly proportional to the slip frequency being zero at synchronous speed.

In order to separate the surface losses from the other losses let us examine the conditions at a rotor speed represented say by the vertical line $l \, v$ and let us remember that the surface losses must be supplied by the rotor[14]. The rotor itself is driven partly by the input to the drive motor and partly by means of energy transferred across the air gap from the stator. Let us also

14. Induction Motor Core Losses, Alger and Eksergian, JOURNAL A. I. E. E., October 1920, p. 906.

remember that the total a-c. and d-c. inputs as shown by Fig. 19 are equal to the sum of the stator and rotor iron losses. We have, therefore, only to subtract the known stator losses, plus the known fundamental frequency-rotor losses from the total input to give the surface losses. This can most easily be done graphically. In addition to the stator losses and the true fundamental frequency rotor losses the a-c. end is supplying fictitious hysteresis and eddy-current losses which later losses must be subtracted from the a-c. input and added to the d-c. input to give the surface losses. The fictitious hysteresis loss is represented by the difference between lines $t\,b$ and $t\,r$ or by the difference between $a\,b$ and $a\,r$ as mentioned above and is equal to $l\,j$ for the chosen rotor speed. The fictitious eddy-current loss is $l\,k$. Therefore to find the true surface loss $i\,v$, add $l\,j$ plus $l\,k$ (equals $i\,h$) to $h\,v$. This can be done most readily by means of a pair of dividers.

For negative rotation a similar procedure is used. Assume a rotor frequency as represented by $k'\,v'$. Then the fictitious hysteresis loss is represented by the difference between the apparent and true hysteresis loss as before and is equal to $j'\,l'$. The fictitious eddy-current loss is $k'\,l'$. It will be noted that here the true hysteresis and eddy-current losses are greater than the apparent losses. Therefore the fictitious values must be subtracted from the d-c. input since the d-c. drive motor is supplying some of the fundamental frequency a-c. losses. Therefore the true surface loss is $i'\,v' = h'\,v' - (j'\,l' + k'\,l')$.

For rotor frequencies above synchronism (negative slip) the d-c. motor is supplying not only the surface losses but the true rotor losses plus a part of the stator losses. In other words the d-c. end is supplying the true and apparent fundamental-frequency rotor hysteresis losses and the true and apparent eddy losses. Let us consider the rotor frequency $q\,w$. Then $q\,z$ represents the true rotor-hysteresis loss plus the apparent rotor eddy-current loss (reversed) and $z\,y$ represents the apparent rotor-hysteresis loss plus the true rotor eddy-current loss. Therefore in order to obtain the surface loss $m\,w$, $q\,y$ (equals $p\,m$) must be subtracted from $p\,w$. $y\,n$, or course, corresponds to the fictitious eddy-current losses as did $l\,k$ for the lower rotor frequency.

As a check on this method surface loss data were calculated as above for a given rotor, using test data obtained with applied stator frequencies of 20, 40 and 60 cycles. The results are as follows:

TABLE G.

Rotor Frequency	Surface Losses			Stator Frequency
	20	40	60	
20	29	29	28	
30	54.3	52.5	52	
40		77	81	
50		115	114	
60		147	150	

The stator voltages were of course altered in proportion to the applied frequency in order to keep the inductions constant. These checks which are well within the accuracy of the test methods show that reliable results may be obtained with a single stator frequency rather than using stator frequencies corresponding to the rotor frequencies at which data are desired.

Discussion

TOOTH PULSATIONS IN ROTATING MACHINES
(Spooner) and
SURFACE IRON LOSSES WITH REFERENCE TO LAMINATED MATERIALS
(Spooner and Kinnard)

Philadelphia, Pa., February 6, 1924

G. E. Luke: These two papers on iron losses are very worthy papers in that their purpose is to give the designing engineers a knowledge of the stray losses which occur in a machine. These stray losses are very large in the case of some induction motors and in the case of turbo-generator and other high speed electric machines. The old d-c. machines had solid pole faces; the losses on those were large. The only thing that limited them and kept them down within reasonable value was the fact that they used large air gaps. But as the designs were improved, the air-gap was brought down, particularly with the use of the commutating pole, and it was necessary to reduce these pole-face losses; this necessitated a laminated structure.

These papers give a valuation of those losses and how they can be reduced by changing the size of the laminations or by insulating the laminations in the pole. The tooth-pulsation losses are of especial importance because they occur in the tooth zone where the losses are the greatest, due to the copper loss and the high densities found there, and they may be appreciable; sometimes larger than the fundamental loss.

These losses are important first, as regards efficiency; probably of greater importance, though, with regard to heating of the machine. In other words, they limit very seriously the output of the machine as far as temperatures go, and if we can evaluate those losses, it will give the designing engineer a better idea of what his machine will do. In mechanical engineering we have a figure which we call "factor of safety;" it might be called "Factor of ignorance" because it is a figure which takes into account imperfections in design or material.

In electrical engineering we have a similar figure which we use in the magnetic circuit. We obtain losses on the raw material and multiply by that factor, which takes into account the stray losses, and if we can reduce that factor we will put the design on a higher plane and be able to predetermine the performance of any machine to a higher degree of accuracy.

P. L. Alger: I wish to comment on Messrs. Spooner's and Kinnard's paper on surface losses, making two points particularly. One point concerns the relation of the paper to the previous art; and the second concerns the application of the dimensional theory to the results of the paper.

Articles have been published in recent years giving methods for calculating the line-frequency losses and demonstrating their practical accuracy. Mr. Spooner's article now gives us methods for calculating surface losses with reasonable accuracy. Recent papers by Rosenburg and others have given methods for calculating the losses in the adjacent metal parts. These three types of loss, together with the pulsation losses, make up practically the whole core loss of commercial machines. Therefore, if we may hope to have a paper upon pulsation losses, as good as Messrs. Spooner's and Kinnard's paper on surface losses, in the near future, we ought to be able to calculate core losses with satisfactory accuracy. I hope such a paper will be forthcoming before long.

The second point I wish to make is that if you make use of the dimensional theory (which states that the quantity measured is not be affected by any change in the units employed), by applying it to Messrs. Spooner's and Kinnard's results, you may derive some rather interesting conclusions. They find, for example, that the total loss varies approximately as the 2.5 power of the density, when the different exponents for the different conditions are averaged. If this is so and if it is assumed that in calculating these losses theoretically the only factors to be considered in addition to those which they have varied are the resistivity, the thickness of the laminations, and the saturation value of density, then it must follow from the dimensional theory that the loss will vary as the minus 0.5 power, or the inverse square root, of the saturation value of density. Thus we have a measure of the gain it is possible to make by changing the saturation value of the density. Similarly, assuming their exponents to be correct for the variation of the loss with the frequency and with the tooth pitch, it follows that the loss should vary approximately as the minus 1.5 power of the resistivity. The ordinary theory, based upon a constant permeability shows an exponent for the resistivity of minus 0.5. This variation as the minus 1.5 power makes the resistivity very much more important in reducing the losses than we would expect from the conventional theory.

Finally, it may be shown that the losses must vary about as the 1.7 power of the lamination thickness, if the exponents derived for the other variables are really true. Since their tests on variations in lamination thickness show the square law to hold at low thicknesses and the first power law to hold at greater thicknesses, it also follows that their exponents for density and tooth pitch cannot be true over the whole range.

In brief, the whole range of their experiments may be coordinated, made more reasonable, and extended by applying the dimensional theory to their results.

I have prepared a table of the exponents derived in this manner which gives the values expected for the several cases.

TABLE OF EXPONENTS

Experimental					Deduced		
Material	B_{AG}	f	λ	q	B_{SAT}	ρ	t
.0172 M. A.	2.2	1.7	1.1	1.7	− .2	−1.3	1.9
.0281 Bessemer	2.4	1.6	1.3	2.15	− .4	−1.4	1.7
.0625 "	2.6	1.6	1.3	2.2	− .6	−1.4	1.7
.125 "	3.1	1.6	1.3	2.3	−1.1	−1.4	1.7
Solid Steel*	2.5	1.5	..	1.4	− .5	−1.5	..
.0172 M. A. (Induction Motor)	1.9	1.55	..	..	0.1	−1.45	..

*Taken for unpublished experiments. The other figures are from tables *C* and *E* in Messrs. Spooner and Kinnard's paper.

I. F. Kinnard: In closing this paper, I have no further remarks to make, excepting to explain in a little more detail Fig. 11 on page 270. You will note that this is a comparison between the losses obtained, using chamfered poles and ordinary poles with a uniform gap. The basis of the comparison was using minimum air gap; that is, in the case of the chamfered poles we used minimum air gap at the center and average induction. The reason for this I think I can make clear:

Assume a uniform air gap, and a non-uniform gap due to chamfered poles. For the reason that in both cases, we are using average air gap induction over the entire pole face, it is easy to see that in the case of the chamfered poles the losses are increased somewhat due to the greater flux density at the center. These higher losses are satisfactorily accounted for, if, instead of using average length of gap in our formulas, we use the minimum gap in all cases.

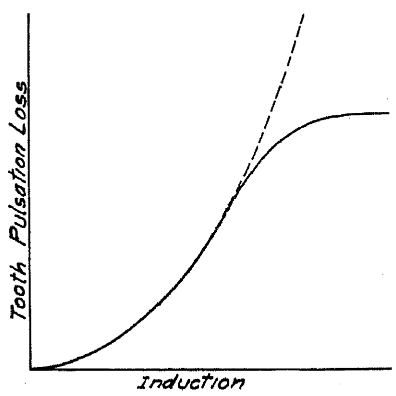

FIG. 1—TOOTH PULSATION LOSSES SHOWING EFFECT OF SATURATION

Although this is an approximation, it is sufficiently accurate for practical purposes.

T. Spooner: I would like to say a word in regard to the practical effect of saturation on tooth pulsation losses. In the paper we did not mention losses specifically but this investigation was made, of course, with the definite purpose of applying the results to the calculation of losses. Suppose we plot from experimental results tooth pulsation losses against air gap induction, we shall obtain a curve similar to Fig. 1. If there were no tooth saturation, the curve would continue as shown by the dotted line but due to the reduction of tooth pulsations caused by saturation, the rate of increase of loss decreases and if the induction were carried high enough, the losses may actually decrease.

Saturation is then an important factor in reducing tooth pulsation losses. This, of course, applies only to induction motor rotors with no windings or with slip ring windings, but does not apply to squirrel cage rotors.

Methods for Testing Current Transformers

BY FRANCIS B. SILSBEE

Associate, A. I. E. E.

Physicist, Bureau of Standards

Review of the Subject.—*In the sale of electric power especially in large amounts at high voltages, current transformers are universally used to operate the meters from the readings of which the bills are made out. While the secondary currents of such transformers reproduce fairly faithfully on a smaller scale the conditions of current strength and phase existing in their primary circuits, meter engineers are realizing more and more the importance of checking up the accuracy of this link in the chain of measurements.*

Many possible methods have been suggested for testing the accuracy of current transformers. Some are delightfully simple and inaccurate while others involve relatively intricate connections and sensitive apparatus, but give correspondingly more exact results. The present paper gives a critical discussion of the various possible methods with data as to the advantages and disadvantages of each, in order to serve as a guide to the meter engineer in selecting the method best suited to the particular working conditions of accuracy, speed, volume of testing, and intelligence of labor, existing in his plant. In all eleven distinct methods are described, and supplementary suggestions are made concerning various types of detecting instruments, etc.

INTRODUCTION

As a result of the continued growth of electric power systems, and the improvement in accuracy of metering methods which has accompanied it, more and more attention is being given both by utility companies and public service commissions to the testing of the instrument transformers which are almost invariably involved in the measurements on which large amounts of electric energy are bought or sold. The choice of which testing method to adopt in any given case depends very greatly upon a variety of conditions, such as the volume of testing work to be done, the accuracy required, the skill of the staff available for making the measurements, and the working conditions such as steadiness of the supply voltage, freedom from stray magnetic fields, etc.

It is the purpose of the present paper to assist the meter engineer in choosing a testing method by assembling in convenient form brief descriptions of the various methods which are now available for testing current transformers, and indicating the advantages and disadvantages of each method.

There will be given first a brief classification of the available methods, together with a discussion of the more important properties of the electrodynamic type of instrument used in many of the testing methods. This will be followed by a brief description of each method with an indication of its advantages and disadvantages, and its probable accuracy, and later by a more detailed discussion of the various forms of detecting instruments which may be used with many of the methods. In Appendix A is given a bibliography which, though by no means complete, lists the more important methods for testing current transformers. The numbers in parenthesis scattered through the text refer to the articles in the bibliography where the particular subject referred to will be found discussed in more detail.

GENERAL CONSIDERATIONS

A typical vector diagram for a current transformer is shown in Fig. 1. If the primary current I_1 is divided by the nominal ratio of the transformer, and reversed in direction, we obtain the vector I', which is here represented as being slightly larger than the secondary current I_2 and as lagging behind it by a small angle β. The ratio of the effective values of I_1 to I_2 is defined as the current ratio of the transformer, while the ratio of the effective values of I' to I_2 is defined as the ratio factor and is the quantity by which the nominal ratio of the transformer must be multiplied in order to obtain the true current ratio. The phase angle of the transformer is defined as the angle by which the secondary current leads the reversed primary current. In order to determine the performance of a current transformer completely as part of a measuring equipment, it is necessary to know the values of ratio and of phase angle for all the conditions of use. Unfortunately these two values depend to a considerable extent upon the frequency, the secondary burden, and the value of

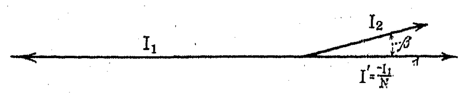

FIG. 1

the currents flowing in the transformer windings, and it is therefore necessary to duplicate all of these conditions when the transformer is tested. Since all testing methods require the insertion of some additional apparatus into the secondary circuit, it becomes a matter of very considerable importance in accurate work to insure that proper allowance for this has been made and that the transformer is tested with a burden closely equivalent to that on which it is used in service. It is convenient to determine the ratio and phase angle at secondary current values of 0.5, 1, 2, 3, 4 and 5 amperes since interpolation between these points is satisfactory for determining intermediate values. It is found that well-made transformers having several primary windings which may be connected in series or parallel have substantially the same ratio factor and phase angle with all connections. In transformers of the hole type in which the primary is inserted by the user of the transformer, this constancy of ratio and

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phase angle is not as perfect as in the preceding case, but the variation in these quantities seldom exceeds 0.3 per cent in ratio and five minutes in phase angle.

In the use of a current transformer for the measurement of current, phase angle is, of course, of no importance and it is necessary to measure only the ratio of the transformer. On the other hand, when the transformer is used with a wattmeter or watthour meter on circuits of power factor other than unity, the effect of the phase angle becomes of importance. The effect of this angle has been worked out by Robinson (28) and others (10, 21, 24, 26), and can be expressed by the equation

$$P = \frac{P' R \cos(\theta + \beta)}{\cos\theta} = P' [R(\cos\beta - \tan\theta \sin\beta)] \quad (\mathbf{1})$$

where P = true power in primary circuit
P' = apparent power as observed in the secondary circuit
$\cos\theta$ = apparent power factor as observed in the secondary circuit
R = ratio of current transformer
β = phase angle of current transformer

In some cases where it is known that the transformer will be used on circuits having a particular power factor, it is desirable to make the test in such a manner as to determine not the ratio and phase angle separately, but the particular combination of these two quantities which is included in the brackets in the right hand member of equation (**1**.) This type of test gives the over-all correction factor which must be applied to power or energy measurements at the particular circuit power factor. Any of the methods described below which use electro-dynamic instruments or watthour meters in the measurement, permit of this type of test.

Methods for testing instrument transformers may be classified (6) into (A) absolute and (B) relative methods as shown in Table I. In the absolute methods the

TABLE I

A Absolute
- *a* Deflection
  - Two Ammeter Method
  - Two Wattmeter Method
- *b* Balanced
  - Mutual Inductance Method
  - Resistance Method
  - Baker Test Ring Method

B Relative
- *a* Deflection
  - Interchanged Ammeter Method
  - Interchanged Wattmeter Method
  - Interchanged Watthour Meter Method
- *b* Balanced
  - Differential Wattmeter Method
  - Bridge Circuit Method
  - Null Bridge Method

ratio and phase angle of a single transformer are determined directly from the observations, while in the latter the constants of the transformer under test are compared with those of a standard transformer of the same nominal ratio which has been previously tested by an absolute method. The absolute methods are suitable for two classes of work (*1*) in standardizing laboratories when the highest accuracy is needed in testing transformers which are to be used as standards and (*2*) in relatively crude measurements under emergency conditions when the standard transformers needed for the relative methods are not available. As is usually the case with any type of measurements, the relative methods are in general simpler and require less sensitive and delicate apparatus for the same accuracy than do the absolute methods, and are therefore to be preferred in most cases.

The methods of either type may be further classified as (a) deflection or (b) balanced methods. In the former the magnitude and phase (or their equivalents) of both the primary and secondary currents are observed separately and the ratio and phase angle computed from these observations. In the latter the effects of the two currents to be compared are opposed and only their vector difference is measured. In the true null methods this difference is completely balanced against a known vector, while in other balance methods of what may be called the "semi-null" type the vector difference is obtained from the deflections of suitable instruments. As in other types of measurement the

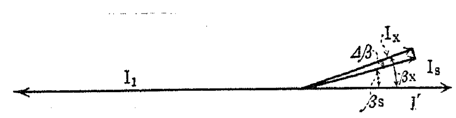

FIG. 2

balance methods have decidedly greater accuracy than the deflection methods and the semi-null methods are usually slightly quicker to operate than the null methods.

Fig. 2 shows a vector diagram applicable to any relative method of testing. The primary current I_1 passes through the primary coils of both transformers and produces currents I_s and I_x in the secondary coils of the standard and test transformers respectively. The differential current ΔI or some effect proportional to it can be measured in terms of I_s. The observations then give the ratio I_s to I_x and the phase difference $\Delta\beta$ between these vectors. The true ratio $\frac{I_1}{I_x}$ is found by multiplying I_s/I_x by R_s (the ratio, I_1/I_s, of the standard transformer). The true phase angle $\beta_x = \Delta\beta + \beta_s$.

In order to avoid repetition later it may be well at this point to review briefly the principal properties of two-circuit electrodynamic instruments (indicating wattmeters and electrodynamometers) since these are often used in transformer testing. Instruments of this type are used for two main purposes (1) as a guide in setting the proper phase relation between the currents in two different circuits, and (2) to measure currents or

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There will be given first a brief classification of the available methods, together with a discussion of the more important properties of the electrodynamic type of instrument used in many of the testing methods. This will be followed by a brief description of each method with an indication of its advantages and disadvantages, and its probable accuracy, and later by a more detailed discussion of the various forms of detecting instruments which may be used with many of the methods. In Appendix A is given a bibliography which, though by no means complete, lists the more important methods for testing current transformers. The numbers in parenthesis scattered through the text refer to the articles in the bibliography where the particular subject referred to will be found discussed in more detail.

GENERAL CONSIDERATIONS

A typical vector diagram for a current transformer is shown in Fig. 1. If the primary current I_1 is divided by the nominal ratio of the transformer, and reversed in direction, we obtain the vector I', which is here represented as being slightly larger than the secondary current I_2 and as lagging behind it by a small angle β. The ratio of the effective values of I_1 to I_2 is defined as the current ratio of the transformer, while the ratio of the effective values of I' to I_2 is defined as the ratio factor and is the quantity by which the nominal ratio of the transformer must be multiplied in order to obtain the true current ratio. The phase angle of the transformer is defined as the angle by which the secondary current leads the reversed primary current. In order to determine the performance of a current transformer completely as part of a measuring equipment, it is necessary to know the values of ratio and of phase angle for all the conditions of use. Unfortunately these two values depend to a considerable extent upon the frequency, the secondary burden, and the value of

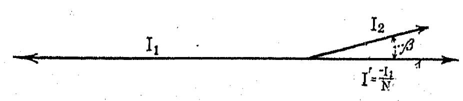

FIG. 1

the currents flowing in the transformer windings, and it is therefore necessary to duplicate all of these conditions when the transformer is tested. Since all testing methods require the insertion of some additional apparatus into the secondary circuit, it becomes a matter of very considerable importance in accurate work to insure that proper allowance for this has been made and that the transformer is tested with a burden closely equivalent to that on which it is used in service. It is convenient to determine the ratio and phase angle at secondary current values of 0.5, 1, 2, 3, 4 and 5 amperes since interpolation between these points is satisfactory for determining intermediate values. It is found that well-made transformers having several primary windings which may be connected in series or parallel have substantially the same ratio factor and phase angle with all connections. In transformers of the hole type in which the primary is inserted by the user of the transformer, this constancy of ratio and

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phase angle is not as perfect as in the preceding case, but the variation in these quantities seldom exceeds 0.3 per cent in ratio and five minutes in phase angle.

In the use of a current transformer for the measurement of current, phase angle is, of course, of no importance and it is necessary to measure only the ratio of the transformer. On the other hand, when the transformer is used with a wattmeter or watthour meter on circuits of power factor other than unity, the effect of the phase angle becomes of importance. The effect of this angle has been worked out by Robinson (28) and others (10, 21, 24, 26), and can be expressed by the equation

$$P = \frac{P' R \cos(\theta + \beta)}{\cos \theta} = P' [R(\cos \beta - \tan \theta \sin \beta)] \quad (1)$$

where P = true power in primary circuit
P' = apparent power as observed in the secondary circuit
$\cos \theta$ = apparent power factor as observed in the secondary circuit
R = ratio of current transformer
β = phase angle of current transformer

In some cases where it is known that the transformer will be used on circuits having a particular power factor, it is desirable to make the test in such a manner as to determine not the ratio and phase angle separately, but the particular combination of these two quantities which is included in the brackets in the right hand member of equation (1.) This type of test gives the over-all correction factor which must be applied to power or energy measurements at the particular circuit power factor. Any of the methods described below which use electro-dynamic instruments or watthour meters in the measurement, permit of this type of test.

Methods for testing instrument transformers may be classified (6) into (*A*) absolute and (*B*) relative methods as shown in Table I. In the absolute methods the

TABLE I

A Absolute
- *a* Deflection
  - Two Ammeter Method
  - Two Wattmeter Method
- *b* Balanced
  - Mutual Inductance Method
  - Resistance Method
  - Baker Test Ring Method

B Relative
- *a* Deflection
  - Interchanged Ammeter Method
  - Interchanged Wattmeter Method
  - Interchanged Watthour Meter Method
- *b* Balanced
  - Differential Wattmeter Method
  - Bridge Circuit Method
  - Null Bridge Method

ratio and phase angle of a single transformer are determined directly from the observations, while in the latter the constants of the transformer under test are compared with those of a standard transformer of the same nominal ratio which has been previously tested by an absolute method. The absolute methods are suitable for two classes of work (*1*) in standardizing laboratories when the highest accuracy is needed in testing transformers which are to be used as standards and (*2*) in relatively crude measurements under emergency conditions when the standard transformers needed for the relative methods are not available. As is usually the case with any type of measurements, the relative methods are in general simpler and require less sensitive and delicate apparatus for the same accuracy than do the absolute methods, and are therefore to be preferred in most cases.

The methods of either type may be further classified as (a) deflection or (b) balanced methods. In the former the magnitude and phase (or their equivalents) of both the primary and secondary currents are observed separately and the ratio and phase angle computed from these observations. In the latter the effects of the two currents to be compared are opposed and only their vector difference is measured. In the true null methods this difference is completely balanced against a known vector, while in other balance methods of what may be called the "semi-null" type the vector difference is obtained from the deflections of suitable instruments. As in other types of measurement the

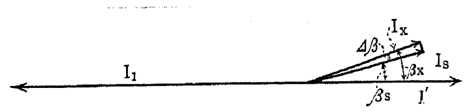

FIG. 2

balance methods have decidedly greater accuracy than the deflection methods and the semi-null methods are usually slightly quicker to operate than the null methods.

Fig. 2 shows a vector diagram applicable to any relative method of testing. The primary current I_1 passes through the primary coils of both transformers and produces currents I_s and I_x in the secondary coils of the standard and test transformers respectively. The differential current ΔI or some effect proportional to it can be measured in terms of I_s. The observations then give the ratio I_s to I_x and the phase difference $\Delta \beta$ between these vectors. The true ratio $\frac{I_1}{I_x}$ is found by multiplying I_s/I_x by R_s (the ratio, I_1/I_s, of the standard transformer). The true phase angle $\beta_x = \Delta \beta + \beta_s$.

In order to avoid repetition later it may be well at this point to review briefly the principal properties of two-circuit electrodynamic instruments (indicating wattmeters and electrodynamometers) since these are often used in transformer testing. Instruments of this type are used for two main purposes (1) as a guide in setting the proper phase relation between the currents in two different circuits, and (2) to measure currents or

differences of currents, such as ΔI of Fig. 2. The fundamental equation for the deflection of an instrument of this type is

$$I_f I_m \cos\theta = k\,D \qquad (2)$$

where I_f is the current in the fixed coil, I_m is the current in the moving coil, θ is the phase displacement between these two currents, D is the deflection and k is a constant of the instrument. As the phase of one of the currents, say I_m, is rotated with respect to that of the other current, the deflection of the wattmeter will rise from zero to a maximum value and then decrease through zero to a negative maximum and then return to its original value. An indication of zero on the instrument corresponds to the value of $\cos\theta = 0$ in equation (2) and hence to $\theta = 90$ deg. It may be noted that the accuracy of locating the zero position is much greater than that of locating the maximum, and therefore the former method should always be used where possible as a means of setting phase. If full-scale deflection of the instrument corresponds to 150 divisions, then an error of one division at the zero mark corresponds to an error in the phase relation between the currents of only 0.4 deg. while the same error at the maximum position corresponds to 6.5 deg.

When used for the purpose of current measurement, one coil is excited by sending through it a current of known magnitude and phase, while the current to be measured is sent through the other coil. The instrument then becomes in effect an ammeter and serves to measure the component of the current in the second coil which is in phase with that in the first. When the fixed coil is excited and the current to be measured is sent through the moving coil, we have the equation

$$I_m \cos\theta = \frac{k\,D}{I_f} \qquad (3)$$

If an ordinary wattmeter, the scale of which is graduated in watts, is used with a separate exciting current I_f in the fixed coil, the value of the constant k is given by $k = w/R_v$ where R_v is the resistance in ohms of the voltage circuit and w is the watts per division. If, on the other hand, the voltage circuit is excited with an auxiliary voltage V, the component of fixed coil current in phase with this voltage is given by

$$I_f \cos\theta = \frac{w\;D}{V} \qquad (4)$$

In order to determine completely any current such as I_f or ΔI of Fig. 2, two readings are necessary in which the wattmeter is excited from two sources of current of different phase. After such readings have been obtained it is possible by graphical construction or by trigonometry to determine both the magnitude of the unknown current and its phase relation to either of the auxiliary current sources. From two pairs of such measurements of I_1 and I_2 or of ΔI and I_2, the vector relation of I_1 to I_2 can be obtained in terms of these readings, and gives the desired constants of the transformer. The mathematical treatment of the relations involved is given in Appendix B. The resulting equations are rather complicated and burdensome in the general case, but fortunately they reduce to very simple forms when one of the auxiliary currents is in phase with the secondary current of the transformer under test and the other auxiliary current is in quadrature therewith. It is therefore advisable wherever possible to insert some form of phase shifting device in the circuits supplying the auxiliary current in order that one of them may be brought into phase with the secondary current of the transformer. The use of the general equations of Appendix *B* should be limited to exigencies when such a phase-shifting arrangement is not available.

In cases which frequently arise in practise where the auxiliary circuits exciting the dynamometer are obtained from a three-phase system, the angle φ in Appendix *B* becomes 60 deg., and the solution reduces to equations

$$R = \frac{A_1}{A_2 \cos\beta} \text{ or } (1 + A/A_2)\ \frac{1}{\cos\beta} \qquad (5)$$

$$\tan\beta = \frac{2\,(B_1 - N\,B_2) - 1.732\,(A_1 - N\,A_2)}{A_1} \qquad (6)$$

$$= \frac{2\,B_\Delta - 1.732\,A_\Delta}{A_2 + A_\Delta} \qquad (6)$$

The symbols used in this and the following equations are defined in Appendix *B*. It will be seen that these are fairly simple if one of the currents can be brought into phase with the transformer secondary current, and the need for a two-phase source of current can be avoided by using these equations instead.

Equation (3A) of Appendix *B* is

$$R\,(\cos\beta - \tan\theta\sin\beta) = A_1/A_2 \qquad (3A)$$

It will also be noticed that the left hand member of this is identical with the factor given by equation (1) above for the correction to the wattmeter or watthour meter operating at a circuit power factor $\cos\theta$. Consequently any of the wattmeter methods may be used with the auxiliary voltage, making an angle θ with the transformer currents to determine the over-all correction factor for that particular power factor.

It appears from the preceding paragraph that all of the testing methods which involve wattmeters or watthour meters in the measurement have the following advantages and disadvantages in common:

Advantages:

(1) They automatically check the polarity marking.

(2) They permit of test giving combined effect of ratio and phase angle at any specified power factor.

(3) The instruments used are of simple and generally available types except when high sensitivity is desired.

Disadvantages:

(1) They require an auxiliary source of voltage.

(2) If phase angles are to be measured this must be a polyphase source preferably two-phase.

(3) It is highly desirable to provide for shifting the relative phase of the auxiliary voltages.

ABSOLUTE METHODS

The term "absolute" may be applied to those methods in which the constants of a single transformer can be determined directly without reference to any standard transformer of the same nominal ratio.

Two-Ammeter Method. (6, 17, 24, 28) By far the simplest possible absolute method is the use of two ammeters, one to measure the primary current and the other to measure the secondary current. The ratio of the transformer is given directly by the ratio of the ammeter readings. Of course the phase angle cannot be determined by this method. The accuracy is limited by the calibration errors of the two instruments and by the accuracy with which they can be read simultaneously. With carefully calibrated instruments it is possible to obtain an accuracy of ½ per cent at full rated current, but because of the non-uniform scales of all a-c. ammeters the accuracy falls off to a very poor value at lower currents. This source of inaccuracy can be reduced only slightly by changing the range of the ammeter used in the primary circuit. This expedient is not feasible in the case of the secondary ammeter since the burden imposed on the transformer by inserting in its secondary circuit the coils of an ammeter of lower range than 5 amperes is excessive, so that the results obtained when such lower range ammeters are used are decidedly different from the performance of the transformer when in actual service. Other objections to this method are that its range is limited by the available range of self-contained a-c. ammeters which is about 500 amperes, the method gives no check on the polarity marking of the transformer, and the results give only the ratio of the transformer and not its phase angle.

Two-Wattmeter Method. (6, 17, 23, 24) In this method the current coils of two wattmeters of appropriate range are connected in series with the primary and secondary circuits of the transformer respectively, and the voltage circuits of the wattmeters are connected in parallel with one another and to one or the other phase of an auxiliary source of voltage. If the auxiliary voltage is in phase with the secondary current, the ratio of the watt readings gives the ratio of the transformer. If the phase of the auxiliary voltage is approximately in quadrature with the primary current, the phase angle of the transformer is given by

$$\tan \beta = \frac{B_1 - N B_2}{A_1} = \frac{W_1 - N W_2}{E I_2} \qquad (7)$$

where W_1 and W_2 are the readings of the primary and secondary wattmeters respectively. I_2 is the secondary current, E the effective value of the auxiliary voltage, and N the nominal transformer ratio (see Appendix B).

This method is also simple and quick, and after a set up has once been checked out, it serves to check the polarity of other transformers which may be tested with it. Furthermore, by exciting the voltage circuits of the wattmeters from an auxiliary voltage which is displaced in phase by an angle θ from the transformer currents, the resultant readings give directly the combined error due to ratio and phase angle when used on a circuit of power factor $\cos \theta$. As in the preceding method the range is limited by the range of available wattmeters which is about 200 amperes. The accuracy may be as high as 0.2 per cent at full current and drops off only in proportion to the current instead of dropping off in proportion to the square of the current as in the preceding method. A disadvantage which it has in common with most other methods using electrodynamic instruments as detectors is that it requires a polyphase auxiliary source of voltage.

We now come to the class of absolute balance methods in which some effects of the primary and of the secondary currents are balanced against one another as a means of determining their ratio and phase displacement.

Mutual Inductance Method. (13) In this method the primary and secondary currents respectively are sent through the primary windings of two large mutual inductors. These are chosen so that the value of each mutual inductance is inversely proportional to the nominal value of the current which it carries, and hence the secondary electromotive forces induced are approximately equal. The secondary coils of the two mutual inductors are connected in series opposition so that the difference in the induced e. m. f. is applied to a detector. A fine adjustment is provided on one of the mutual inductances so that an exact balance can be obtained. In order to compensate for the phase angle of the transformer, a small resistance of a few hundredths of an ohm is connected in series with the secondary circuit of the transformer under test, and the voltage drop across a variable portion of this is inserted into the detector circuit.

This method is suitable for precise laboratory work and can be made exceedingly accurate. It is also easy to construct the mutual inductors with a considerable number of individual coils which may be connected in series or parallel. It is thus possible to obtain a wide range of values of mutual inductance and consequently a wide range of transformers can be tested.

On the other hand, it is very essential that the inductors be astatic since any electromotive forces induced in their secondary windings by a stray field would introduce an error directly in the ratio determination. This feature has been very successfully obtained in the apparatus designed by Fortescue, and now used by the Westinghouse Co. In this equipment a range of from 5 to 5000 amperes is obtained, and the mutual inductors have been made astatic by the use of toroidal coils for

both primary and secondary. Such a uniformly wound toroid produces no external magnetic field, and consequently will have no electromotive force induced in it even when placed in a strong external field providing the conductor producing this field does not pass through the opening of the toroid. By using a relatively large number of turns in the secondary coils, it is possible to obtain a secondary electromotive force as great as 4 volts, and an accuracy of 0.01 per cent can be obtained even while using a rugged and relatively insensitive detector. In the equipment mentioned above the necessary burden inserted in the secondary coils of the transformer by the testing equipment varies from 0.1 to 0.3 of an ohm impedance, depending upon the connections which are used. Complete astaticism requires that the marble core on which the coils are wound shall be machined very accurately to dimensions, and as a result the entire equipment is rather expensive, and so far as the writer is aware has not been duplicated in any other laboratory.

In any such apparatus it is important to make certain that the secondary electromotive force from the mutual inductor is in quadrature with the primary current.

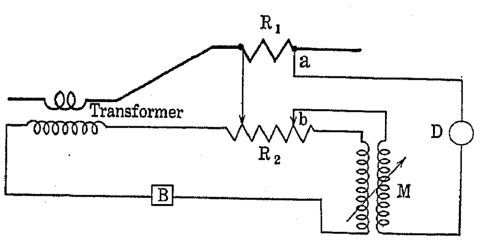

Fig. 3

Often this relation does not exactly hold in cases where the primary circuit consists of conductors of large cross section such as are necessary when heavy currents are to be measured. Fortunately the same toroidal construction which makes the inductor astatic also contributes to reduce this phase defect. (31).

Resistance Method. (1, 6, 9, 19, 20, 21, 24, 25, 28, 29, 30, 33). This method is perhaps more generally used than any other absolute method for the precise testing of current transformers. In its various modifications it is used by the National Standardizing Laboratories of the United States (1), England (33), France (19), and Germany (25), and also by several commercial and university laboratories. The primary and secondary currents are passed through four-terminal resistance standards of such value that the potential drops across the two are equal, and in most equipments of the order of one volt. The potential terminals of the shunts are then connected in series with the detector instrument so that the $I R$ drops are in opposition. The phase angle of the transformer is determined in several different ways at the various laboratories, depending upon the type of detector used. In one group of laboratories the unbalanced e. m. f. resulting from the fact that the primary and secondary currents are not in phase is balanced by inserting in the detector circuit a third source of e. m. f. in quadrature with the primary current. This e. m. f. may be obtained in several ways, the simplest of which is from the secondary of a small mutual inductor (1, 9, 30), the primary of which is connected in either the primary or the secondary circuit of the transformer under test. The compensating electromotive force may be obtained if preferred from a separate phase of the generator supplying the test current (19), or from a shunt circuit containing a condenser connected across the potential terminals of the primary resistor (29). With any of these arrangements the resistances and the quadrature electromotive forces are adjusted until the detector shows a condition of balance and the ratio and phase angle are then computed from the settings by the formulas

$$R = R_2/R_1 \tag{8}$$

$$\tan \beta = \frac{\omega M}{R_2} \text{ or an equivalent formula.} \tag{9}$$

In the second group of methods the effect of the phase angle is determined by the residual deflection of the detecting instrument after the $I R$ drops have been balanced and the detector has been excited from an auxiliary source in quadrature with the transformer currents (20, 25, 28). Slight modifications in the method as described are introduced by the type of detector which is used, but these will be discussed in a later paragraph. The method is capable of an accuracy of 0.01 per cent and can easily be made direct reading except for possible corrections to the values of the primary and secondary shunts. By suitably changing the nominal values of these shunts the method can be made very flexible, and the range can be extended indefinitely. The detector can be made sufficiently sensitive so that the voltage across the shunts is only 0.25 volt at full current, and the burden introduced into the secondary circuit can be kept as low as 0.1 ohm.

The principal objections to this method are the fact that it requires a sensitive and therefore delicate detecting instrument, and that the shunts must be carefully constructed to minimize residual inductance, and to avoid skin effect. The construction of non-inductive shunts for currents up to 1000 amperes is relatively simple but becomes increasingly difficult beyond that point (31). Precautions must also be taken to avoid errors from capacity currents which may circulate through the detector from the primary source of current.

The Baker Test Ring Method. (4) Instead of balancing the electromotive force produced by a pair of mutual inductances or a pair of resistances, as was done in the previously described methods, this method opposes the magnetomotive forces produced by the primary and secondary currents. A special test ring of well laminated transformer steel is provided with primary and secondary windings quite similar to those

of a current transformer. A third winding is placed next to the iron core and has a large number of turns (about 6000). The primary and secondary currents of the transformer under test are led through the corresponding coils around the test ring in such a direction that their magnetomotive forces are in opposition. Any departure of the currents from their nominal ratio produces a slight flux in the ring which in turn induces an electromotive force in the third winding. This third winding is connected to one coil of a wattmeter or other type of electrodynamic instrument, the other coil of which is excited from any suitable auxiliary source. The number of turns through which the secondary current of the transformer circulates is then varied until the indication of the wattmeter is reduced to a minimum. Since the number of turns of the secondary coil can be varied only in integral steps, the procedure in using the apparatus is to record the reading of the wattmeter for two different values of secondary turns near the point which gives the minimum reading. Thus, for example, if the balance point lies between 179 and 180, the readings of the wattmeter for both of these numbers of secondary turns is observed. The wattmeter is then excited from some other aux-

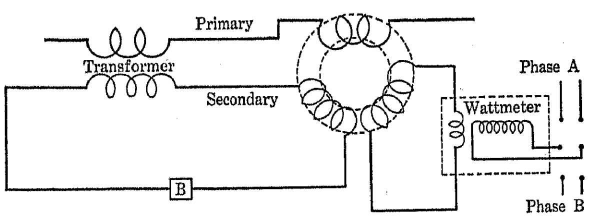

FIG. 4

iliary voltage having a different phase from the first, and a similar pair of readings is taken. From these readings it is possible by a simple graphical construction to lay off on the drawing board the relative location of the primary and secondary current vectors with respect to the auxiliary voltages, and by interpolation between the observed points to determine the ratio and phase angle.

This method has the advantage of requiring only simple and inexpensive apparatus, and of being quite flexible in range since the primary winding on the test ring usually consists of only a few turns which can be easily changed. The computation of the results is, however, somewhat complicated and liable to lead to blunders and the accuracy is not as high as that of the two previously described laboratory methods. Furthermore, as in all methods employing electrodynamic instruments, a separate polyphase auxiliary source is required.

The burden which the apparatus adds to the secondary circuit of the transformer consists of the resistance of the secondary coil of the test ring, which in most cases can be several hundredths of an ohm and in addition there is an effective burden resulting from any electromotive force induced in the coil by the flux arising from lack of perfect balance of the magnetomotive forces.

Relative Methods

If a standard current transformer with known ratio and phase angle and of the same nominal ratio as the transformer which it is desired to test is available, it then becomes feasible to use any one of several additional methods which have certain advantages over those previously described. Of course any of the absolute methods can be used to compare a transformer with a standard transformer of the same nominal ratio, but only those will be mentioned here in which the fact that the nominal ratios of the transformers are the same, permits of a notable simplification or increase in accuracy.

Interchanged Ammeter Method. (6, 24) A simple relative method of testing transformers is analogous to the two-ammeter absolute method first discussed, and consists in connecting two transformers with their primaries in series and with a 5-ampere ammeter and any other desired burden in the secondary circuit of each transformer. Simultaneous readings are then taken of the two secondary ammeters. The ammeters are then interchanged and a second set of readings taken. The ratios of the two transformers are then inversely proportional to the average readings of the ammeters connected to them.

It will be seen that interchange of the ammeters, which is made possible by the fact that transformers have the same nominal ratio, eliminates any error arising from a difference in calibration of the two instruments, and the absolute calibration of either instrument is of only minor importance in the test.

Of course this method gives only the ratio of the transformers and not their phase angle, and the accuracy falls off very badly at low values of the current. For the reason previously mentioned, it is not possible to use low range ammeters in making the test at the lower points on the curve. In actual tests with this method it is very important so to arrange the circuits that the secondary windings of the transformers are not open-circuited while current is flowing in the primary when the two ammeters are interchanged. A switch connected directly across the terminals of each transformer which can be used to short-circuit it while changes are being made in its secondary circuit is a valuable addition to any current transformer testing set-up.

Interchanged Wattmeter Method. (6, 24) This method bears the same relation to the two-wattmeter absolute method as the interchanged ammeter method does to the corresponding absolute method. It is quite simple to check the polarity of the transformers and by using an excitation for the voltage circuits of the wattmeters having a specified phase relation to the transformer currents, the results give directly the

relative performance of the transformers on a circuit of that particular phase displacement. The accuracy, however, is limited to the accuracy of reading the wattmeters and is rather poor with small currents.

Interchanged Watthour Meter or the Agnew Method. (3, 6, 10, 11, 16, 21, 24) This method is very similar to that just described except that a pair of induction watthour meters are used instead of the two electrodynamic wattmeters. The advantage of these meters is that by obtaining their registration over a run of several minutes duration, it is possible to obtain readings of greater percentage accuracy than can be done on the scale of an indicating instrument. This method was originally described by P. G. Agnew (3) and has been the subject of a number of other papers which have suggested slight improvements. At least one of the watthour meters which are used must have graduations marked on the disk, or be of the so-called "rotating standard" type in order that fractions of a revolution may be read off. It is usually desirable to operate the meters at a speed somewhat greater than normal since their accuracy is not impaired and the time of the test is shortened. This condition can be obtained by moving the drag magnets of the meters nearer the axis or by shunting them. By careful work an accuracy of better than 0.1 per cent can be obtained by this method, but the long time required for obtaining the readings of the watthour meters at each point renders the method somewhat slow. The computations involved are rather long but can be considerably simplified by the use of formulas and methods published by Craighead and Weller (10) or by Crothers and Kartak (11).

The three preceding methods are all of the deflection type, inasmuch as the entire primary and secondary effects are observed separately and compared in the computation. The advantages of relative methods of testing appear more definitely, however, in the balance methods, since we here have quantities of substantially equal magnitude and phase to be balanced.

Differential Wattmeter Method. (35) This method is based upon the principle of opposing the magnetic fields due to the secondary currents supplied by the standard and test transformers respectively. The secondary currents are sent through duplicate current windings of a wattmeter in such a direction that their magnetic effects are opposite. The moving coil of the wattmeter is then excited from an auxiliary source of voltage which may be either in phase or in quadrature with the currents in the transformers. A comparison of the indications of this instrument with that of an ordinary 5-ampere wattmeter connected with its current coil in the secondary circuit of one of the transformers and its voltage coil in parallel with that of the differential instrument serves to give the ratio and phase angle of the transformer from the equations (11) and (14) or (15) of Appendix B.

The equality of the two-current coils can be checked by a second observation in which they are interchanged or by connecting them in series opposition and sending current through them from a single transformer. This method is simple and, like other electrodynamic methods, can be used to check polarity and to obtain the effective ratio at any desired circuit power factor. The disadvantage of the method is that it requires a somewhat special instrument which can have relatively few turns in the current coils so that the sensitivity is low. As a result if an ordinary 10-ampere range wattmeter which has two current coils in parallel is used with the coils separated, the sensitivity will be of the order of 1 per cent per division at 5 amperes. It is, however, often possible to overload the current coils of a wattmeter and a suspension-type dynamometer instrument is amply sensitive to give an accuracy as great as 0.01 per cent. As a rough factory check on the performance of transformers to determine any serious mistakes in construction it is probable that this method is very useful.

Modifications of this method which hardly require separate consideration are (1) the use of a polyphase

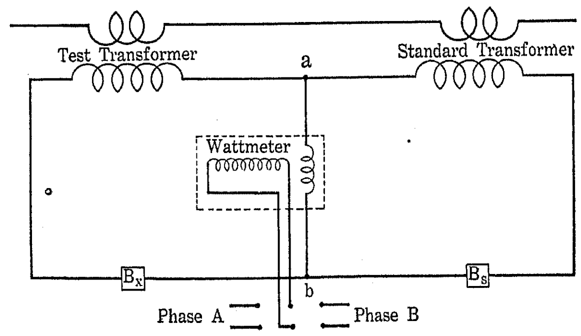

FIG. 5

wattmeter with its elements connected in opposition, and (2) the use of a differentially-wound watthour meter from which the light load compensation has been removed (35).

Bridge Circuit Method. (6, 12, 32). Instead of subtracting the magnetic effects due to the standard and test transformers, it is simpler to subtract the currents directly. This can be done by connecting the standard and test transformers with their primaries in series and with their secondaries also in series in such a direction that both transformers tend to circulate current in the same direction through both secondary burdens, as shown in Fig. 5. A "bridge" circuit is then connected between the points *a* and *b* with the result that any difference in the secondary currents of the two transformers tends to flow across this bridge and can be measured by a suitable detecting instrument placed in the bridge circuit.[1] The detector in the bridge circuit

1. This method was first described by W. A. Folger and was published in the *Proceedings* of the Pennsylvania Electrical Association in 1916, where, however, it is not widely available. It was suggested independently by the author in 1917 and published as part of Scientific Paper No. 309 by the Bureau of Standards.

can be a fairly sensitive low-range pivoted wattmeter excited from a suitable auxiliary source.

When the difference in the secondary currents of the two transformers is ΔI and the impedance of the detecting coil circuit is Z, the effective burden on one transformer is increased and that on the other transformer is decreased by an amount $Z \times \frac{\Delta I}{I}$. When the transformers are nearly equal in ratio this quantity is negligible, but when comparison is made between transformers of very different performance, the shift of the burden may become appreciable. It can be minimized by keeping the impedance of the detecting coil as low as possible. It has been found that the current coil of a one-ampere wattmeter or the voltage coil of a 30-volt wattmeter makes a satisfactory detecting circuit. It is convenient to shift the spring holders of such an instrument so as to put the zero in the middle of the scale. The scale can then be made to read directly the percentage difference in ratio of the transformers. This procedure also places the moving coil more nearly in the

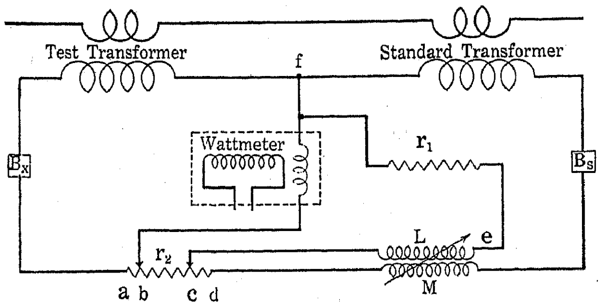

Fig. 6

position where it has zero mutual inductance with the fixed coil, and the slight errors caused by such inductance are reduced. With care this method is capable of an accuracy of 0.1 per cent in ratio and 3 minutes in phase angle.

Null Bridge Method. (32) This method is a modification of the one just described in which the uncertainty in the secondary burden resulting from impedance of the detecting instrument, and also the slight errors which may be produced by mutual inductances between the two coils of the detecting instrument have been eliminated. As shown in Fig. 6 this method makes use of a second bridge circuit, *c e f* which is made to carry the difference in current of the two transformers. The differential current is caused to circulate through this bridge circuit instead of through the detector by suitable adjustment of the resistance r_2 and mutual inductance M. When a balance is obtained the ratio and phase angle of the test transformer are given in terms of those of the standard transformer by the following equations:

$$R_x/R_s = 1 + a + b^2/2 - b\,c\;.\;.\;. = 1 + a \text{ approx.} \qquad \textbf{(10)}$$

$$\tan(\beta - \beta) = b + a\,c = b \text{ approx.} \qquad \textbf{(11)}$$

$$\text{when } a = r_2/r_1 \quad b = \frac{\omega M}{r_1} \quad c = \frac{\omega L}{r_1}$$

Unless the transformers differ widely in performance only the first terms of these formulas are of importance, and the slide wire resistance r_2 can be made to read directly the difference in ratio of the two transformers. The detecting instrument in this case may be either a vibration galvanometer or a separately excited electrodynamic instrument. In the latter case it is desirable, though not absolutely essential, that the phase of the exciting current be adjusted to coincide with that of the currents in the transformer. The magnitude of the exciting current, however, need not be known since the detector is used only as a null instrument.

In any testing method it is theoretically possible to determine the algebraic sign of the phase angle from the direction of the deflection. It is, however, in almost all cases much easier to determine this sign from a simple test based on the fact that adding non-inductive resistance in the secondary of a current transformer tends to increase the ratio and, except in very extreme cases, tends to advance the phase of the secondary current. Hence if the addition of such resistance increases the phase-angle reading the angle was originally positive before the resistance was added. If the addition of resistance decreases the phase-angle reading or reverses its direction, then the angle was originally negative. A similar test with any relative method shows which of the two transformers has the greater ratio.

Detectors

In the deflection methods the measurement is of course made directly by commercial pivoted instruments, and no unusual or sensitive detectors are needed. In the balance methods, however, special types of detector are used either as null instruments or in some cases also as deflection instruments, which serve to measure a small difference between two larger currents.

The absolute methods in general require relatively highly sensitive and therefore somewhat delicate instruments. To bring out more clearly the difference between relative and absolute methods, the case of the resistance method may be compared with the bridge circuit method. In the typical case of the resistance method the available voltage drop across the shunts is fixed at say 0.25 volt in order to keep down the burden inserted in the secondary circuit. At a secondary current of 0.5 ampere the voltage drop is only 0.025 volt, and if the ratio is to be determined to an accuracy of one part in 10,000 the detector must be sensitive enough to indicate an unbalance of 2.5 microvolts. This practically necessitates a suspension type instrument, and most detectors of this sensitivity are read by a mirror and scale arrangement.

In contrast with these, the relative methods require

in general detectors of much lower sensitivity. Thus, for example, in the case of the bridge circuit method, to obtain an accuracy of one part in 10,000 at one-tenth of rated current, the detector must show a visible indication when 50 microamperes pass through it. To compare this with the preceding case we must see what resistance the detector may have without loading the transformers with more than the assumed burden of 0.05 ohm. Even if the transformers under comparison differ in ratio by as much as 5 per cent, a current of only 0.25 ampere will flow through the bridge circuit at full current. The bridge may therefore have as much as 1 ohm resistance without adding more than 0.25 volt drop at the transformer terminals. With this resistance a sensitivity of 50 microamperes corresponds to 50 microvolts, which is 20 times the voltage which the detector for the absolute method would have to detect. Because of this relationship it is feasible to use pivoted wattmeters, such as those intended for measurements on circuits of low power factor. There are also on the market certain suspension types of pointer galvanometers which are fairly rugged, and which are decidedly more sensitive than is usually necessary, consequently by the use of such galvanometers the burden inserted in the secondary circuit can be made less than the 0.05 ohm assumed above.

For the absolute methods requiring high sensitivity, the available types of detectors are (1) the vibration galvanometer, (2) the separately excited electrodynamometer, (3) the separately excited quadrant electrometer, and (4) the d-c. galvanometer used with a synchronous rectifying commutator.

The direct-current galvanometer with rectifying commutator (7, 30), can be made the most sensitive of these instruments, and for the sensitivity required in the absolute testing methods, a fairly rugged galvanometer can be used. There are, however, a number of possible sources of error connected with its use which require a little care in their elimination. The friction of the brushes on the commutator tends to produce local heating, and thus spurious thermoelectric currents in the circuits. This error can, however, easily be eliminated by using as a zero point the reading of the galvanometer when the a-c. side of the commutator is short-circuited. Hunting of the synchronous driving motor, necessity of shifting the brushes to the proper commutating plane, and the irregularities in contact resistances are also sources of annoyance. This method also of course requires a synchronous source of fairly constant voltage from which to operate the driving motor.

A vibration galvanometer (34) can also be made very sensitive and has the great advantage that when it indicates a balance, the observer is certain that no currents of fundamental frequency are circulating in the galvanometer circuit. This is in contrast to the other forms of detector which are not sensitive to currents in quadrature with the phase of their exciting current or with the phase of the rectifying brushes. While the vibration galvanometer balances the fundamental components of the voltages produced by the primary and secondary currents in the transformer, any serious distortion of the wave form by the transformer is indicated by a blurring of the image of the lamp filament as reflected from the galvanometer mirror. No auxiliary source of voltage is required when the vibration galvanometer is used.

On the other hand, such galvanometers are mechanically rather delicate and are sensitive to mechanical disturbances, such as are caused by moving machinery in other parts of the building, so that they must be mounted on some type of vibration-free platform. Furthermore, very high sensitivity is only available over the particular narrow band of frequency to which they are tuned, and they are therefore not at all suitable for use where the frequency of the test circuit is liable to fluctuation. Another disadvantage in the use of a vibration galvanometer is that the observer cannot tell from its indications whether an observed lack of balance is due to the incorrect setting of resistance or of inductance. This objection is, however, more theoretical than actual since a relatively small amount of practise enables the observer to judge, by the rate at which the deflection of the galvanometer varies with the motion of his control handles, which variable requires adjustment; and the time required to obtain a balance with the vibration galvanometer is probably less than with the other types of instrument.

The separately excited electrodynamometer (26, 28) can be made amply sensitive for absolute methods of current transformer testing by using an instrument of the suspension type. With it the methods can be used in which the phase angle of the transformer is obtained by reading the deflection of the instrument when it is excited in quadrature with the primary current. When excited in one phase it is of course sensitive only to lack of balance in that phase, and the observer knows instantly which control to adjust. The direction of the deflection also indicates in which direction the adjustment should be made.

On the other hand, this instrument requires an auxiliary polyphase source of voltage for exciting its fixed coil, and in most methods it requires a phase shifting device for adjusting the phase of its auxiliary voltage. When it is used to measure phase angle by its deflection, it must be calibrated by some auxiliary circuit.

The separately-excited quadrant electrometer (25, 33) is very similar in its functions to the electrodynamometer, and is used to a considerable extent in Europe where such instruments are used in other branches of instrument testing work. So far as the author is aware it has not been used in this country for transformer testing. In order to obtain sufficient sensitivity for current transformer testing, the instrument has to be of

a highly refined and sensitive type, and is suitable for use only under the best laboratory conditions.

Supply of Current

Any testing equipment for current transformers must be able to supply the full-rated primary current to the transformer. Since it is usually undesirable to draw such a large current from the 110-volt supply available in the laboratory, a stepdown transformer is generally used. For this purpose it is sometimes feasible to use an additional current transformer supplied on its 5-ampere winding and delivering heavy current through what was intended to be its primary winding. In tests by the relative method in the field where it is possible to connect the two transformers in series by connecting the standard transformer across a disconnecting switch or some other gap in the line, it is quite feasible to use the actual load on the circuit as a source of primary current. Moderate fluctuations in this current will not seriously interfere with the measurement since the relative ratio and phase angle of the transformers change only slightly for a considerable change in operating current.

The wave form of the supply circuit is not of much importance because most current transformers reproduce very faithfully, in their secondary, even a very distorted primary current wave, consequently the ratio of the fundamental components of the currents is very nearly the same as the ratio of the root-mean-square values, and tests by the various methods described in this paper give substantially the same results. It is usually convenient to read the current in the transformers by means of an ammeter in the secondary circuit. Because of the small deflection of a-c. ammeters on low currents, however, it is often desirable to insert an ammeter in the primary circuit to measure the current when the secondary current is one ampere or less.

The low resistance of the primary circuit which carries the heavy current and the appreciable reactance of the primary windings of the transformers combine to produce a considerable phase shift in the primary current as compared with the source of voltage which supplies the stepdown transformer. For this reason it is very desirable to employ some form of phase-shifting device so that the auxiliary voltages used in exciting the detector instruments can be adjusted to be in phase or in quadrature respectively with the primary current. Such a shift of phase is essential in cases where the phase angle is determined by reading the deflection of the electrodynamometer instrument, or in cases where the test is used to give the combination of ratio and phase angle corresponding to the effective ratio of the transformer on a circuit of specified power factor. In the null method the need for a phase shifter is not so urgent but nevertheless it adds greatly to the convenience and speed of operation.

Brooks Type Transformer

The methods which have been discussed above are directly applicable only to transformers of the usual type with single primary and secondary windings. When used with compensated transformers of the Brooks type (8) certain modifications are necessary. These transformers have both a main and an auxiliary secondary coil and the method of test must be such as to compare the vector sum of the currents in the two secondary coils with the primary current, or with the secondary current of a standard transformer.

The resistance method can easily be modified by inserting into the detector circuit the voltage drop across a third shunt, which is connected in series with the auxiliary coil of the Brooks transformer. Since under normal conditions the current supplied by the auxiliary coil is only a small fraction of the main secondary current the third shunt need not be adjusted to have precisely the same resistance as the main secondary shunt, but may be left at the nominal value of the latter.

The interchanged watthour meter and differential wattmeter methods can be applied to a Brooks transformer if the instruments used are provided with an additional winding to receive the auxiliary coil current. Care must, however, be exercised to avoid introducing an excessive mutual inductance within the instrument between the main and auxiliary secondary coils since this will affect the performance of the transformer. This effect of mutual inductance is probably sufficient to make the Baker test ring method inapplicable for such transformers.

The bridge circuit method can be applied to the Brooks type transformer by connecting the auxiliary coil directly across the terminals of the detector. Since the e. m. f. across the detector is small this does not seriously affect the operation of the transformer. Trouble arises, however, in the null bridge method if there is any contact resistance at the slide wire since the $I R$ drop of the auxiliary current flowing through this contact is likely to affect the detector indications. This source of error can be avoided by using a detector which has an additional coil magnetically equivalent to that used as the "bridge" but insulated from it. Even with this modification the null bridge method does not measure the true effective ratio of the Brooks transformer with rigorous accuracy. The errors, however, are very small since they involve the product of the small percentage difference of the transformers under comparison, multiplied by the small percentage compensation that the auxiliary coil supplies to the main secondary coils.

The fact that transformers of this type have such very small ratio and phase-angle errors over a wide range of current renders them very useful as standards for use with the relative testing methods.

Miscellaneous and Indirect Methods

In addition to the direct methods of measurement described above, certain others have been suggested as being of value in special cases. For instance an a-c. potentiometer (14) may be used to measure separately the voltage drops across the shunts as used in the resistance method, and the resulting triangle of vectors can be solved to give the ratio and phase angle. A power-factor-meter or phase-meter (15) can be used to measure the phase of the primary and secondary currents respectively with respect to a polyphase voltage system, and the difference gives the phase angle of the transformer. Suitable modification of the instrument may be made so that the pointer magnifies the angular displacement by a factor of 5 or more.

In the case of transformers of very large primary current rating it is sometimes necessary to resort to the indirect process of measuring the magnetizing and core loss components of the exciting current of the transformer and computing the ratio and phase angle of the transformer from these data (5). Numerous formulas are available (2, 5, 18, 21, 22, 27) for making this computation, but the results are liable to considerable error because of lack of knowledge of the leakage reactance of the secondary winding. The presence of this reactance makes the terminal secondary voltage of the transformer when in normal operation differ materially from the induced voltage due to the core flux; and thus makes it difficult to determine at what flux density the transformer normally operates. In cases of this kind it is usually preferable to insert a number of primary turns, and thus reduce the primary current rating for the purpose of the test.

Summary

The foregoing survey of the available methods shows that the two-ammeter and two-wattmeter methods are the simplest and least accurate. The relative methods all combine considerable accuracy with fairly simple and rugged apparatus, and are probably to be preferred (1) for testing in the field or station where laboratory facilities are limited and (2) for production tests where large numbers of transformers of the same nominal ratio are to be inspected. The balanced absolute methods are of the highest accuracy but are applicable mainly in the laboratories of government and educational institutions and of the large manufacturers and central stations where they can be applied to the testing of transformers which are later to be used as standards for some relative method.

Acknowledgement

This paper has been prepared under the auspices of the Bureau of Standards, and is published with the approval of the Bureau.

Appendix A

The following list contains references to the more important articles dealing with methods for testing current transformers and related matters. No attempt has been made to include articles on the theory, design or application of current transformers.

1. Agnew, P. G. and Silsbee, F. B. Trans. A. I. E. E., vol. 31, p. 1625-1912. The Testing of Instrument Transformers.

Describes one form of resistance method using vibration galvanometer as detector.

2. Agnew, P. G. Bureau of Standards Scientific Paper, No. 164-1911. A Study of the Current Transformer with Particular Reference to Iron Loss.

Gives equations for performance in terms of magnetizing current.

3. Agnew, P. G. Bureau of Standards Scientific Paper No. 233-1914. A Watt-hour Meter Method of Testing Instrument Transformers.

Describes interchanged watthour meter method.

4. Baker, H. S. *Electrical World*, vol. 57, p. 234, Jan. 26, 1911. Testing of Current Transformers.

Describes Baker test ring method.

5. Barbagelata, A. *Atti dell Assoc. Elettr. Ital*, vol. 14, p. 639-1910. Indirect Tests on Instrument Transformers for Large Currents.

6. Barbagelata, A. *L'Elettrotecnica*, vol. 8, p.165—1921. The Testing of Measuring Transformers.

Describes and classifies nearly all possible methods.

7. Bedell, F. *Journal Franklin Institute*, p. 385, Oct. 1913. The Use of the Synchronous Commutator in a-c. Measurements.

8. Brooks, H. B. and Holtz, F. G. Trans. A. I. E. E., vol. 41, p. 382-1922.

A Two-Stage Current Transformer.

9. Campbell, A. *Proc. Phys. Society* (London), vol. 22, p. 207—1910. On the use of Mutual Inductometers.

Describes resistance method using vibration galvanometer as detector, and mutual inductance in primary to compensate for phase angle.

10. Craighead, J. R. and Weller, C. T. *General Electric Review*, vol. 24, p. 642—1921. Watthour Meter Method of Testing Current Transformers for Ratio and Phase Angle.

(Note: Certain errors in the table as printed in this paper are corrected in the similar table given under "Wattmeters" in the 2nd (1922) edition of Pender's Handbook.

11. Crothers, H. M. and Kartak, F. A. *Electrical World*, vol. 73, p. 516 (I) March 15, 1919; vol. 74, p. 119 (II) July 19, 1919; vol. 75, p. 319 (III) February 7, 1920; vol. 75, p. 1369, (IV) June 12, 1920.

Field Testing of Instrument Transformers.

12. Fogler, W. A. *Proc.* Pa. Elec. Assoc.—9th Conv. p. 234—1916.

Describes bridge circuit method using fixed coil of special separately excited wattmeter as detector.

13. Fortescue, C. Le. G. Trans. A. I. E. E., vol. 34, p. 1599—1915. The Calibration of Current Transformers by Means of Mutual Inductance.

Describes in detail mutual inductance method.

14. Gall, D. C. *Electrician*, vol. 83, p. 603—1919. Testing Transformers by the a-c. Potentiometer.

15. Gifford, R. D. *Electrician*, vol. 75, p. 166—1915. Measures transformer phase angle by phase-meter.

16. Knopp, O. A. *Electrical World*, vol. 67, p. 92, Jan. 8, 1916. Standardization of Current Transformers.

Describes modification of two watthour meter method.

17. Knopp, O. A. *Electrical World*, vol. 75, p. 993—1920. New Current Balance for Calibration Work.

Describes special multi-range current transformer which may be used to extend the range of an ammeter or wattmeter in the two ammeter method.

18. Kuehler, R. *Elekt. Zeit*, vol. 42, p. 1418—1921. Calculation of Instrument Transformer Errors.

19. de la Gorce, P. *Lumiere Elec.* vol. 34, p. 300—1916. *Bull. Soc. Int. Elec.*, vol. 6, p. 299—1916.

Describes resistance method using vibration galvanometer as detector and e. m. f. from auxiliary source to balance phase angle. Method used at Lab. Cent. d'Electricite, Paris.

20. Laws, F. A. *Electrical World*, vol. 55, p. 223—Jan. 27, 1910. Determination of the Constants of Instrument Transformers.

Describes resistance method using separately excited dynamometer as detector.

21. Laws, F. A. Electrical Measurements—McGraw-Hill Co., 1917. Chapter XII contains a good general discussion of instrument transformers and testing methods.

22. McNaughton, A. G. L. *Journal* Institute of Electrical Engineers, vol. 53, p. 269, 1915.

Develops equations for ratio and phase angle of current transformer in terms of magnetizing current.

23. Makower, A. J. and Wust, A. *Electrician*, vol. 79, p. 581, July 13, 1917. Phase Lag in Current Transformers.

Describes two wattmeter method.

24. N. E. L. A. Meterman's Handbook, 1923.

Chapter on instrument transformers contains an excellent discussion of testing methods.

25. Orlich, E. *Elektrotech Zeit*, vol. 30, pp. 435, 466—1909. Anwendung des Quadranten Elektrometers zer Wechselstrommessungen.

Describes resistance method using electrometer as detector. 0.5 volt drop at full current.

26. Handbook for Electrical Engineers, H. Pender John Wiley and Sons.

Article on Transformers, Instrument.

27. Price, H. W. and Duff, C. K. University of Toronto, Bull. 2, Sec. 4, p. 167—1921. Papers on Current Transformers.

A collection of several papers on the performance and testing of transformers with special reference to wave form, hole-type transformers, etc.

28. Robinson, L. T. PROC. A. I. E. E., vol. 28, p. 981—July 1909.

Describes (1) two ammeter method and (2) resistance method using dynamometer or synchronous commutator as detector.

29. Schering, H. and Alberti, E. *Arch. f. Elektrotechnik*, vol. 2, p. 263—1914. Simple Method of Testing Current Transformers.

Describes resistance method using vibration galvanometer as detector and shifting phase of $I_1 R_1$ by a condenser to balance phase angle.

30. Sharp, C. H. and Crawford, W. W. TRANS. A. I. E. E., vol. 29, p. 1517—1910. Recent Developments in Exact a-c. Measurements.

Describes in detail resistance method using d-c. galvanometer with synchronous commutator as detector. Also suggests mutual inductance method and indirect method.

31. Silsbee, F. B. Bureau of Standards Scientific Paper No. 281—1916. A Study of the Inductance of Four Terminal Resistance Standards.

Discusses construction of shunts and mutual inductors for large alternating currents.

32. Silsbee, F. B. Bureau of Standards Scientific Paper No. 309—1917. A Method for Testing Current Transformers.

Describes bridge circuit method and null bridge method.

33. Spilsbury, R. S. J. *Electrician*, vol. 86, p. 296, March 11, 1921. A New Method of Testing Current Transformers.

Describes resistance method using an electrostatic detector after "stepping up" the unbalanced voltage by 1:100.

34. Wenner, F. Bureau of Standards Scientific Paper No. 134—1909. A Theoretical and Experimental Study of the Vibration Galvanometer.

35. Wescott, E. C. *Electrical World*, vol. 76, p. 433, August 28, 1920. Differentially Wound Watthour Meter for Testing Current Transformers.

Appendix B

TRIGONOMETRIC RELATIONS IN CURRENT TRANSFORMER TESTING WITH ELECTRODYNAMIC INSTRUMENTS

Case I. Deflection Methods. As an example of deflection methods we may consider a test by the two-wattmeter method in which I_1 and I_2 are measured by two wattmeters. The voltage circuits of the instruments are excited first on phase A giving readings (reduced to a current basis) of A_1 and A_2 respectively. The voltage circuits are then excited from phase B and the corresponding readings are B_1 and B_2. The vector diagram is shown by Fig. 7.

Let θ = angle by which A leads I_2
φ = angle by which B leads A
R = ratio of transformer
β = phase angle of transformer
N = nominal ratio

Then
$$R = I_1/I_2 = \frac{A_1 \cos\theta}{A_2 \cos(\theta + \beta)} \quad \textbf{(1A)}$$

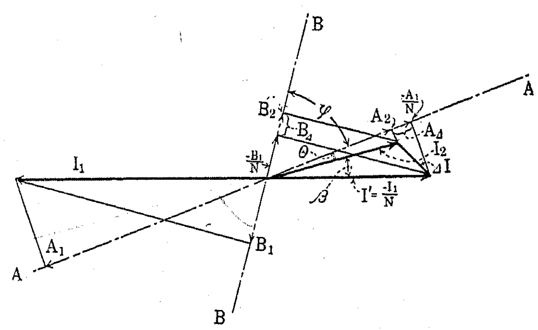

FIG. 7

$$= A_1/A_2 \frac{1}{\cos\beta - \tan\theta \sin\beta} \quad \textbf{(2A)}$$

or
$$R(\cos\beta - \tan\theta\sin\beta) = A_1/A_2 \quad \textbf{(3A)}$$

and
$$A_1 \cos(\varphi + \theta + \beta) = B_1 \cos(\theta + \beta)$$

whence
$$\tan\beta = \frac{\cos\theta - A_1/B_1 \cos(\theta + \varphi)}{\sin\theta - A_1/B_1 \sin(\theta + \varphi)} \quad \textbf{(4A)}$$

In the general case θ can be determined from the current I_2 as measured by an ammeter and the wattmeter reading A_2 by the equation

$$\cos\theta = A_2/I_2 \quad \textbf{(5A)}$$

similarly
$$\cos(\varphi + \theta) = B_2/I_2 \quad \textbf{(6A)}$$

After θ has been computed from (**5A**) it can be used in (**6A**) to compute φ and these two values can be inserted in (**4A**) to compute β. The values of β and θ can then be used in (**2A**) to compute the ratio. This intricate process is fortunately very greatly simplified if the phase of the auxiliary source can be shifted to make $\theta = 0$, and if the other auxiliary phase comes from a symmetrical

2-phase or 3-phase system so that $\varphi = 90$ deg. or 60 deg. respectively.

For $\theta = o$ eqs. (**2A**) and (**4A**) reduce to

$$R = \frac{A_1}{A_2 \cos \beta} = A_1/A_2 \text{ approx.} \tag{7A}$$

$$\tan \beta = \frac{-B_1 + A_1 \cos \varphi}{A_1 \sin \varphi} \tag{8A}$$

which is equivalent to

$$\tan \beta = \frac{-(B_1 - N B_2) + \cos \varphi \, (A_1 - N A_2)}{A_1 \sin \varphi} \tag{8B}$$

since $B_2 = A_2 \cos \varphi$ when $\theta = o$

For a 2-phase system $\varphi = 90$ deg. and (**8B**) reduces to

$$\tan \beta = \frac{-B_1 + N B_2}{A_1} \text{ or } = \frac{-B_1}{A_1} \tag{9}$$

since the adjustment of θ to zero should make the reading $B_2 = o$.

For a 3-phase system $\varphi = 60$ deg. and (**8A**) reduces to

$$\tan \beta = \frac{-2\,(B_1 - N B_2) + 1.732\,(A_1 - N A_2)}{A_1} \tag{10A}$$

Case II. Balance Methods. As an example of balance methods we may consider a test made by the bridge circuit method. Here observations are made on the secondary current obtaining A_2 and B_2 as in Case *I*; and on the differential current ΔI obtaining A_Δ and B_Δ. See Fig. 7. The ratio is then given by

$$R = \frac{N\,(1 + A_\Delta/A_1)}{\cos \beta - \tan \theta \sin \beta} \tag{11A}$$

$$\tan \beta = \frac{B_\Delta + A_\Delta \cos \varphi - A_\Delta \sin \varphi \tan \theta}{(A_2 + A_\Delta) \sin \varphi + \tan \theta\,(B_\Delta + A_\Delta \cos \varphi + A_2 \sin \varphi \tan \theta)} \tag{12A}$$

For $\theta = o$ the equations reduce to

$$R = \frac{N}{\cos \beta}\,(1 + A_\Delta/A_1) \tag{13A}$$

$$\tan \beta = \frac{B_\Delta + A_\Delta \cos \varphi}{(A_2 + A_\Delta) \sin \varphi} \tag{14A}$$

For a 2-phase system $\varphi = 90$ deg. and (**13A**) reduces to

$$\tan \beta = \frac{B_\Delta}{A_2 + A_\Delta} \tag{15A}$$

For a 3-phase system $\varphi = 60$ deg. and (**13A**) reduces to

$$\tan \beta = \frac{2\,B_\Delta + 1.732\,A_\Delta}{A_2 + A_\Delta} \tag{16A}$$

Since in practise it is possible for the angles θ and φ to vary slightly from their nominal values, it is of importance to consider what errors will be introduced into the results by such effects. The magnitude of such errors also depends in absolute methods on the phase angle of the transformer and its departure from nominal ratio, and in the case of relative methods on the difference in phase angle and ratio of the test and standard transformer. These differences will be assumed to have the rather extreme values of 2 deg. and 4 per cent respectively in the following computations:

The error in ratio caused by using the simple equations (**7A**) or (**13A**) in place of (**2A**) amounts to only about 0.06 per cent for an error of 1 deg. in θ, and is roughly proportional to β and to θ for small values of these angles. Since the ratio is determined entirely by readings on phase A, an error in the angle will have no effect on the ratio. The error in phase angle when using equations (**8B**), (**9A**), (**10A**) or (**14A**), (**15A**), (**16A**) in place of equation (**4A**) caused solely by an error of 1 deg. in θ, but with φ having its correct value amounts to 2.3′ and is roughly proportional to θ and to the departure from standard ratio. However, if equation (**8A**) is used an error of 1 deg. in θ will enter directly as an error of 1 deg. in β. If θ is approximately adjusted to zero, an error of 1 deg. in φ will make an error of 2.3′ in β when the test is made on a 2-phase system. On a 3-phase system the error may be greater or less than this depending on the phase relation of ΔI to phase A and in the worst case for the specified limits amounts to 3.5′.

An inspection of equations (**7A**) and (**11A**) shows the great difference in accuracy between the absolute and relative methods when the same instruments are used in both. An error of 1 per cent in reading A_1 or A_2 in eq. (**7A**) will produce an error in the ratio of 1 per cent. In eq. (**11A**) on the contrary the term A_Δ/A_2 is only 0.04 on the assumption previously made. Therefore, an error of 1 per cent in either A_Δ or A_2 will produce an error of only 0.04 per cent in the ratio. An error of 1 per cent in the readings will in most instances produce an error of 1 per cent in the phase angle. Since this angle seldom exceeds 2 deg. or 120′ the error caused by the reading would only be 1.2′ in such a case.

Discussion

I. M. Stein (Philadelphia): On the first and second pages of Dr. Silsbee's paper, a figure of 0.3 per cent is given, as being a maximum for the ratio error in "loop-through" types of current transformers. A number of people are using this type of transformer for standards, and accordingly, the figure given is of particular interest.

We know that there is a difference in the construction of transformers of the "loop-through" type, some being very poor and some very good, a great deal depending upon the way the secondary winding is distributed around the core, and something depending upon whether there are any joints in the laminated core.

If the figure is a maximum for the poorest type, there is not so much to worry about. If it is the figure representing the variations which may be expected in the best type, then it would seem a rather large error for a standard, and I should like to learn from Dr. Silsbee whether the best transformers of that type show errors as large as 0.3 per cent.

Referring now to the vibration galvanometer—the paper indicates in two places that the vibration galvanometer is a delicate instrument.

It is probably true that a number of vibration galvanometers are very delicate, particularly if made to cover a very wide range of frequencies, and perhaps if made considerably more sensitive than is necessary for ordinary measurements on current transformers.

I am familiar with vibration galvanometers that are extremely

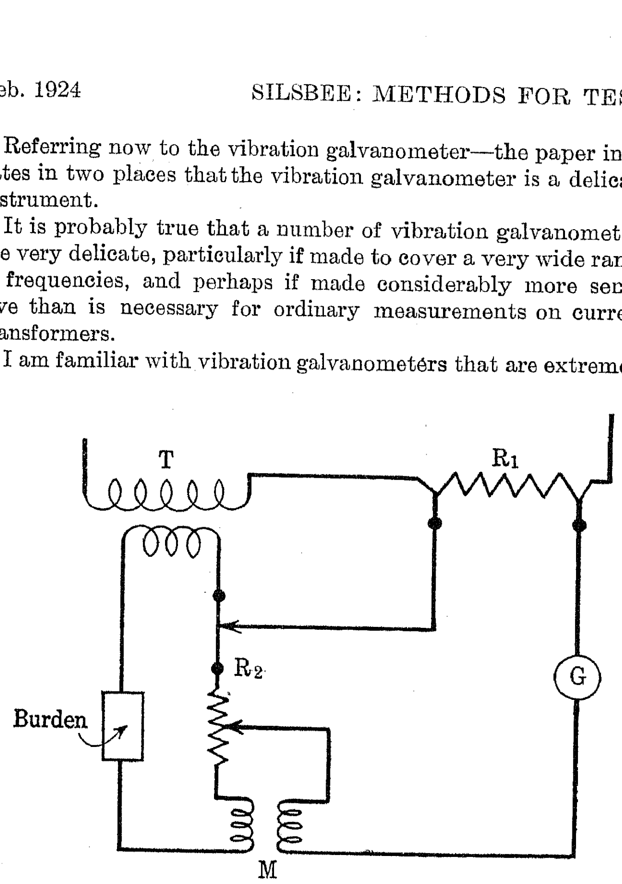

Fig. 1

rugged in construction. They are more rugged than the galvanometer that is used in portable testing sets, the suspension being about twenty-five times as strong as those of the portable galvanometer. I don't think Dr. Silsbee meant to indicate any delicate construction in the vibration galvanometer. I should like to know what interpretation should be placed on his referring to the vibration galvanometer as a delicate instrument.

In connection with the absolute-resistance method, the paper states that precautions must be taken to avoid errors due to

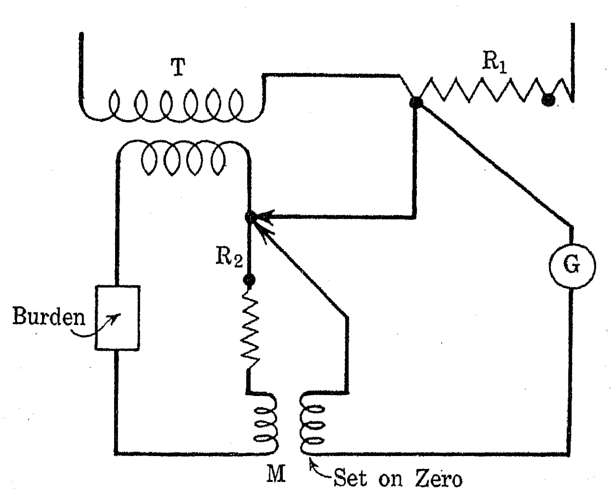

Fig. 2

capacity current which may circulate through the detector from the primary source of current. The precaution applies to tests where you have a sensitive detector, which will pick up very small leakage currents.

The thought occurs to me that if we have leakage in testing where we are using a fairly low voltage for supplying the transformer under test, we must have some leakage when the transformer is installed on a very high-voltage line, say 50,000 volts. Is there any capacity current there which will affect the accuracy of the transformer, particularly at light loads?

I appreciate that the instruments used in service are far less sensitive than those used in testing, but I haven't seen any figures or heard of any results of measurements made to show the probable magnitude of the error.

I should like to add another precaution in dealing with heavy current and sensitive detectors—I refer to stray fields. I think that anyone who has worked with instruments of the kind mentioned, or circuits of the kind mentioned, knows that the effects of stray fields must be eliminated before accurate measurements can be made. In eliminating stray fields, I should like to put in a good word for the vibration galvanometer, when used with the circuit Dr. Silsbee has shown.

In Fig. 1, shown herewith, T is the current transformer being tested, R_1 the primary non-inductive resistance, R_2 is the secondary non-inductive resistance, and M is the mutual inductance. As R_1 and R_2 are made up, the terminals are very close together, and the leads all need to be twisted together to avoid picking up stray fields. The inductance is made astatic, but it may be somewhat affected by stray fields.

If the two leads at R_1 are put under the same binding post (Fig. 2) and the same thing is done at R_2, it involves only a very slight movement, because the binding posts are very close together. If now the mutual inductance is set to zero, then with the full primary current going through the circuit, there should be no vibration of the detector. The circuits must be so arranged to avoid any deflection of the galvanometer under these conditions before you can start to test.

The vibration galvanometer is an excellent detector for this work because as Dr. Silsbee has said, it responds to any current of the fundamental frequency regardless of the phase of that current. You can do a similar thing with a detector requiring shifting the phase of the field current, but that is not so easy, and the very motion of the phase shifter may influence the detector and cause trouble. The vibration galvanometer is an excellent exploring device for getting rid of stray-field errors before starting the measurements.

W. H. Pratt: There is just one word of caution that I think it is well to express; that is, that we should not in practical testing expect from the more convenient methods of relative testing an accuracy approaching the accuracy that can be obtained by the more exact methods. That is I think clearly pointed out in Dr. Silsbee's paper, but nevertheless the figures that sometimes appear as representing what can be obtained by the convenient methods, are a little bit dangerous; that is, they are a little bit better, perhaps considerably better, than are often obtained in the hands of many who may be expected to use these methods.

Frank V. Magalhaes: I would be glad to offer a constructive suggestion to the author of the paper. It would have been very desirable to have Dr. Silsbee add or consider the factor of time as well as the order of accuracy involved in the use of the various equipments and methods. For example, it might be understood that the methods which Dr. Silsbee calls the balanced absolute methods and which employ equipments more or less elaborate would also take considerable time to obtain the measurements. This is not so. With a properly designed primary equipment and a trained operator it is possible to obtain within a few minutes the precise readings of ratio and phase angle which Dr. Silsbee describes in detail.

Dr. Silsbee's paper is a very well prepared comparison of the accuracy of the results that may be obtained by the different equipments or the different methods. There are, however, other factors which would affect the decision of any individual or organization which might be considering the purchase of such a checking equipment for instrument transformers.

The public utilities are probably the largest class of users of instrument transformers in the country. These transformers are used either in connection with the watthour meters or with the power relays, both requiring accurate adjustment.

The various utilities operate under varying rules, depending on the locality. These rules may be the State Public Service orders or municipal regulations in connection with the accuracy and periodicity of check of watthour meters. Certain of these rules require periodic checks of the instrument transformers. These tests may be made on the transformers as received new and prior to installation or may be made in position and connected in service.

Careful consideration would need to be made of various factors, such as the number of transformers involved, the cost of removing them from service for possible check, the period of time between checks and the degree of accuracy desired on the check itself to enable decision to be made as to the type of equipment to be purchased and used.

If the measurements were to be made only on new transformers as received from the manufacturer and they were received in sufficient quantity, these conditions would justify the purchase of one of the more elaborate or expensive equipments, giving balanced absolute results which are of a high order of accuracy and can be very quickly obtained.

If, however, the equipment is required for a public utility of relatively small size with a limited number of instrument transformers all in use and these are by local rule subject to a short period between tests, it would probably be cheaper not to remove the transformer from service but to purchase and use one of the portable sets that do not give the precise results of the laboratory equipment, but provide sufficiently accurate results for the purpose and could be used to test the transformers in position.

One other point that I believe might have been discussed or emphasized by Dr. Silsbee and that is the number of operators necessary to use the various equipments.

It might be considered that the more elaborate equipments giving the precise readings would require several operators. This is not so as equipments of this character can, after they are set up and properly calibrated, be operated very satisfactorily and quickly by a single well trained operator.

J. R. Craighead: There is in Dr. Silsbee's paper. I think, a little tendency to confuse the sensitivity of a method with the accuracy obtainable from the method.

Without going into the details of the methods presented I wish to suggest that the reader keep in mind that the sensitivity which a method will produce is a distinct thing from the accuracy and that the sensitivity mentioned in certain of these cases should not be read as accuracy because the accuracy of the method is dependent upon the certainty of calibrations of the various parts of the apparatus, the accuracy of adjustments, the certainty of temperatures and many other things while the sensitivity is dependent only on the theory of the method, and the relation of the quantities selected for comparison to the sensitivity of the detector.

Perry A. Borden (by letter): The only one of the methods described by Mr. Silsbee for testing current transformers, with which I can claim to have had any great experience, is the Baker Test Ring; but since this combines the precision of an absolute method with the convenience and simplicity of a relative method, I trust I may be pardoned the seeming narrowness of my viewpoint. For fourteen years I have found this system most satisfactory for all classes of current transformers, whether in the laboratory, the workshop or the field. Even with the roughest readings an accuracy of 0.1 per cent is obtained; and with reasonable care ten times that degree of precision may be expected. An excellent dynamometer for use with the ring consists of a Weston Model 310 "low-power-factor" wattmeter, with its moving coil fed directly from the fine wire winding on the ring, and its field excited from normal load current, taken alternately from two phases of a three-phase system. (Higher precision could be obtained with an instrument of the zero-reading type, which would obviate mutual inductance between internal circuits).

In actual practise it is usual to take three or more readings at each value of current, when the system becomes self checking; for, if any source of error has intruded itself into the observations it is almost certain that the several points as plotted will depart from their ideal condition of even spacing along a straight line. For general work it has been found practicable to plot the readings on isometric paper, thus doing away with the need for a drawing board and special scales; and under these conditions the computation becomes very simple, and the possibility of error practically nil.

The true secondary burden on a current transformer interconnected with others on a polyphase metering system is almost impossible to duplicate in the laboratory; and the constants of transformers so installed will differ according to whether they be installed on the leading or the lagging phase of the system. It is highly desirable, therefore, that where the measurement of large blocks of power is concerned, the transformers be checked under actual operating conditions. The Baker method lends itself particularly to work of this class; for, while the outfit is easily portable, the possibility of completely insulating the primary from the secondary winding makes it possible to connect the apparatus directly into the feeder under test, and obtain an absolute determination of current-transformer constants with the circuit actually carrying its load. This, I believe to be regular practise in the plant of the Ontario Power Company at Niagara Falls.

F. B. Silsbee: Mr. Stein asked if this figure of 0.3 per cent possible variation in the pole-type transformer would be applicable to other than bad ones. I think the answer is that it is only in the poorer types of transformers that a variation of this magnitude can be expected to result from different locations of the primary cable. We have made tests at various times at the Bureau on pole-type transformers, trying to place the leads through the holes, in the most widely different positions. In the worst transformers we have come across, have had as much as a 0.5 per cent difference. In the better grades, where the winding is distributed, the error to be expected is decidedly less than the figure in the paper. I put this figure in merely as a caution.

In the matter of capacity currents in the transformer, that is a very interesting point, and I don't think any one knows the answer definitely. Of course, in testing the transformer, especially by the absolutely balance methods, instruments are used which are much more sensitive than the ammeter and wattmeter used in the transformer service. In the higher voltage ranges it is barely possible such a variation would come in.

Stray fields are one of the things always found around heavy current set-ups, and I thought that stray field precautions were so axiomatic they need not be mentioned. The way Mr. Stein outlined is the handiest way to test for stray fields, and is the way we have been doing it at the Bureau.

Mr. Magalhaes raises the question of time. One reason for not laying more stress on it in the paper is the difficulty of estimating the time it takes for a given job because this time depends so greatly on the skill of the operator. Another point is that in most cases a large fraction of the total time of the test is employed in making the connections and in adjusting the burden to properly duplicate the burden at which a test is to be made. All of this time is an overhead common to all methods. After this time is allowed for there is little difference between them as Mr. Magalhaes has pointed out for the balanced method. With the two watthourmeter methods you have to take time to let the meters grind out a fair number of revolutions and this

time is considerable, and is referred to in the paper. Practically all of the methods can be worked with one observer, although it is often convenient to have two men on the job.

Mr. Craighead's comments on the difference between sensitivity and accuracy are well taken. One of the fundamental things to care for in all of this work is to have the burden on the transformer correspond to that at which the standard transformer was tested, and to that at which the transformer under tests is to be used. Any departure from this introduces an error quite apart from the sensitivity of the method and must be guarded against.

Recent Developments in Kilovolt-Ampere Metering

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and

A. R. RUTTER

Review of the Subject.—*The problem of measuring kilovolt-amperes power factor has received considerable attention from the rate maker and the electrical engineer during the last few years. This paper describes two late developments in kv-a. demand meters. One meter is of the indicating type and indicates total kw. hours, kw. demand and kv-a. demand. The other meter is of the recording type and gives a record of kw. demand kv-a. demand, kw. hours, and kv-a. hours.*

THE problem of measuring voltamperes presents difficulties to the electrical engineer which are not as readily solved as might first be suspected. The difficulties lie in the fact that to measure the product of volts and amperes through the complete range of lagging and leading power factors requires either a shifting of the phase relation between the voltage and current in order to measure the quantities by the wattmeter principle, or the measurement of the two components, reactive and power, of the voltamperes which then must be added vectorially to obtain the apparent power.

Assuming that there is a demand for voltampere measuring devices, without considering the scope or possibilities of such a demand, it is the purpose of this paper to discuss the design of two recent developments in kilovolt-ampere-demand meters. The meters described are of the type which combines the registration of two demand meters. While each kv-a. meter employs the same method of obtaining the energy and reactive components, different schemes are used to obtain the vector sum of the components, or in other words, different mechanical devices are used to obtain the "square root of the sum of the squares."

The methods of measuring watthours are widely known because of common application; the measurement of reactive voltampere hours is not so generally used, and hence data concerning the various methods are not available from as great a number of sources. A complete report on the subject will be found in the 1921 Report of the National Electrical Light Association Meter Committee.

Of the various possible methods of measuring the reactive component probably the most outstanding one is the "Compensator" or auto transformer method. This method facilitates the measurement because it uses a standard watthour meter which is calibrated as a watthour meter. Since this method is preferred in connection with the use of the new volt-ampere meters a brief analysis is given here.

To measure the reactive component with a watthour meter it is necessary to connect the meter so that the potential impressed on the voltage coil is in quadrature to the voltage used for the watthour measurement. The reactive component compensator gives the quadrature relation and proper magnitude of voltages for use with

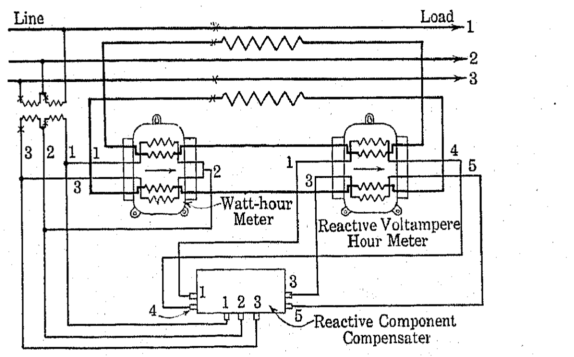

FIG. 1—CONNECTIONS FOR MEASURING VOLTAMPERE HOURS WITH A REACTIVE COMPONENT COMPENSATOR

commercial watthour meters by using two small auto-transformers with suitable taps and connections. In making the connections for reactive component measurement it is necessary to take into consideration the phase rotation since the direction of rotation of the meter changes with phase rotation as well as lagging and leading power factors.

The vector diagram for the connections which are shown in Fig. 1 are given in Fig. 2.

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It will be observed in the diagrams that the small auto-transformers have taps at 57.7 per cent and 115.5 per cent of the primary winding and that the voltages thus obtained are combined so that the resultant voltages are equal in magnitude to the impressed voltages but are in quadrature to the impressed voltages.

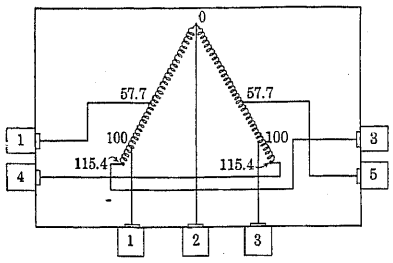

FIG. 2—INTERNAL CONNECTION OF REACTIVE COMPONENT COMPENSATOR

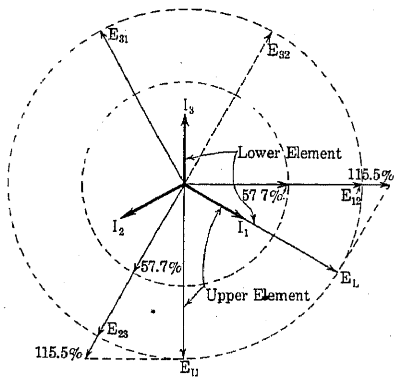

FIG. 3—VECTOR DIAGRAM OF REACTIVE COMPENSATOR VOLTAGES

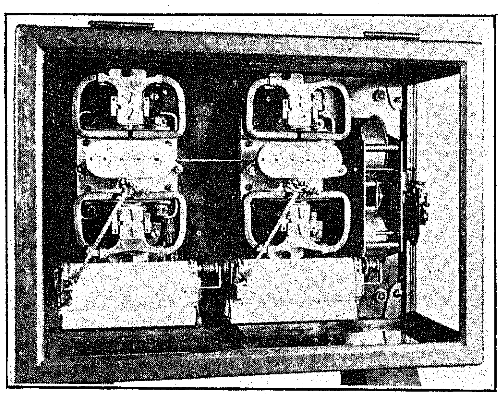

FIG. 4—DUPLEX *R A* DEMAND METER

The voltage used on the upper element of the R.Va.H meter is 57.7 per cent of *E*12 plus 115.5 per cent of *E*23, which is equal to the voltage *E*12 but is 90 deg. behind this voltage. Likewise the voltage on the lower element is equal to the voltage *E*32 but lags it by 90 deg.

It is manifest therefore from the brief description of the reactive voltampere hour meter that it is possible to measure the reactive component practically as easily as the energy component. However the obtaining of the apparent energy or the "square root of the sum of the squares" presents a more difficult problem for which a number of solutions have been offered. Two of the most recent developments are described below.

TYPE *RS* KV-A DEMAND METER (PANTAGRAPH TYPE)

It should be understood that watthour demand meters can be substituted for the two meters in Fig. 1.

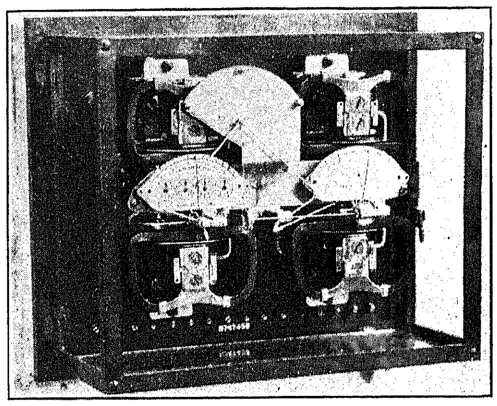

FIG. 5—TYPE *R S* KILOVOLT-AMPERE DEMAND METER

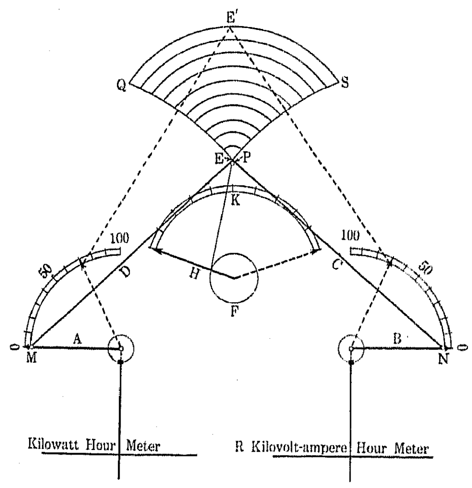

FIG. 6—SCHEMATIC PLAN OF INDICATING MECHANISM OF THE TYPE *R S* KV-A. METER

Fig. 4 shows the arrangement of two demand meters in one case and operated by one clock so as to obtain simultaneous records. When two demand meters are so used it is possible to obtain directly or by calculation the following data:

1. Kw. hours.
2. R kv-a. hours
3. Apparent power factor at which 1 and 2 were obtained.
4. Maximum kw. demand
5. Maximum *R* kv-a. demand.

6. Apparent power factor at which 4 and 5 were obtained.

7. Kw. demand during any time interval on chart.

8. *R* kv-a. demand during any time interval on chart.

9. Apparent power factor for 7 and 8.

While each of these items is valuable in studying the load history of an installation, all are not essential in billing for electric service. In most of the present rate

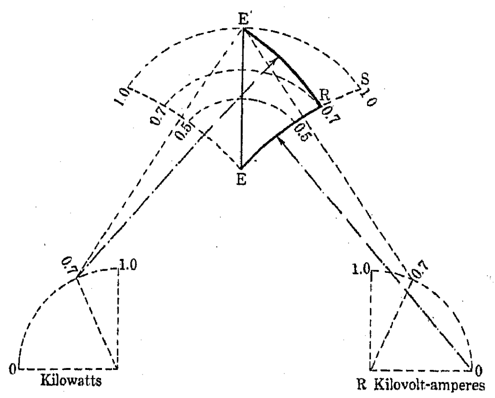

FIG. 7

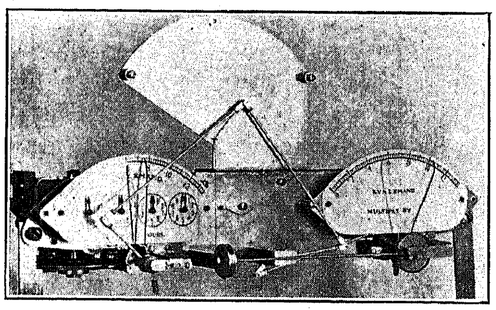

FIG. 8—FRONT VIEW OF REGISTER FROM TYPE *R S* METER

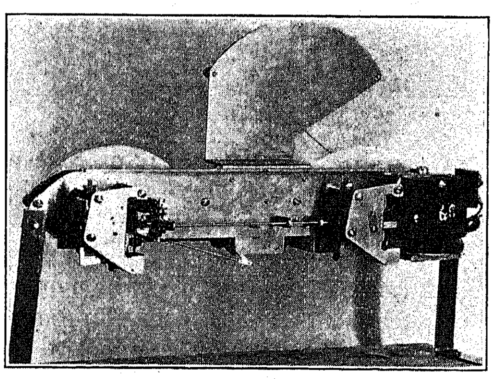

FIG. 9—REAR VIEW OF THE REGISTER FROM TYPE *R S* METER

schedules which take in consideration power factor and kv-a., the important items are kw. hours, maximum kw. demand, maximum kv-a. demand and power factor at time of maximum demand. The type *R S* meter shown in Fig. 5 indicates the kw. hours, the maximum kw. demand and the maximum kv-a. demand. Although the meter has been developed only in the indicating type the principle employed is equally applicable in a graphic type of meter similar to the type *R I* described later in this paper.

The principle employed in the type *R S* meter is a pantagraph arrangement of levers operated by the kw-h. meter and *R* kv-a. *H* meter. A schematic diagram of the operating principle of the meter is given in Fig. 6. *A* and *B* represent pointers for demand registers placed on the kw-h. meter and kv-a. *H* meter. These pointers are placed to the additional use of acting as levers in the pantagraph arrangement of levers *A*, *B*, *C*, and *D*, which are pivoted at *M*, *N* and *E*. During the operation of the meter the point *E* moves over the

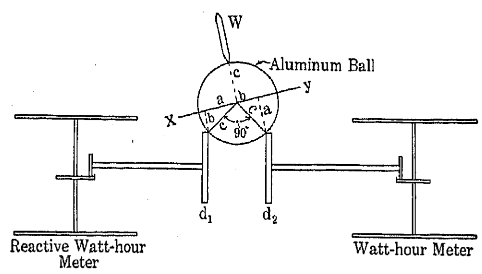

FIG. 10

Radii *a* and *b* to axis of rotation are proportional to respective speeds of wattmeters. Wheel *W* is therefore of a speed proportional to *c*, which is $\sqrt{a^2 + b^2}$ and is therefore proportional to kv-a. *W* is geared to the pointer in meter charts; it also drives dials which register kv-a. hours.

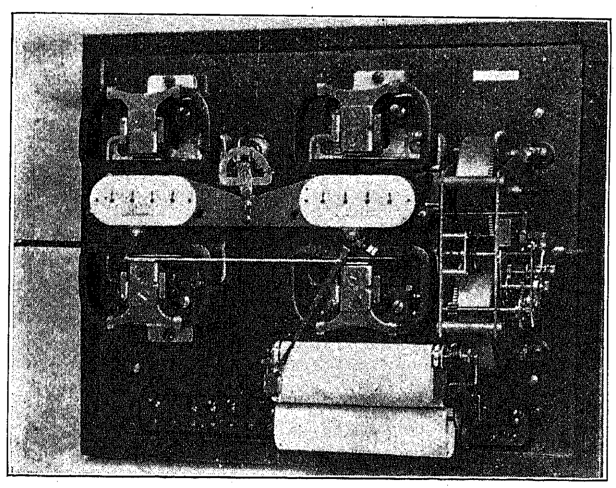

FIG. 11—TYPE *R I* KV-A. DEMAND METER

surface *E S E' Q*, the path of the point depending upon the relative movement of the levers *A* and *B*. A cord is attached to the point *E* and passed through a guide (*P*) to a pulley (*F*) on which the kv-a. pointer (*H*) is mounted. The movement of the point *E* to some point such as *E'* advances the kv-a. pointer (*H*) proportional to the length of cord *E E'*. Now the length of *E E'* is equivalent to the hypothenuse of a right triangle whose sides are equivalent to the indications of the kw-h. and *R* kv-a. *H* meters respectively.

This is more clearly illustrated in Fig. 7 where the point *E* in moving to *E'* may be considered as moving along the path *E* to *R* to *E'*. The base of the right

triangle (E-R) is proportional to the indication of the kw-h. meter while the altitude ($R\,E'$) is proportional to the indication of the R kv-a. H meter. If it is assumed that the reactive component is zero, the point E would move along the line $E\ R$ to the point R which would give a kv-a. indication equal to that of the kw. register, or in other words, a unity power factor indication. Each arc drawn on the surface $Q\ E\ S$ with E as center represents a given value of kv-a. The various radial

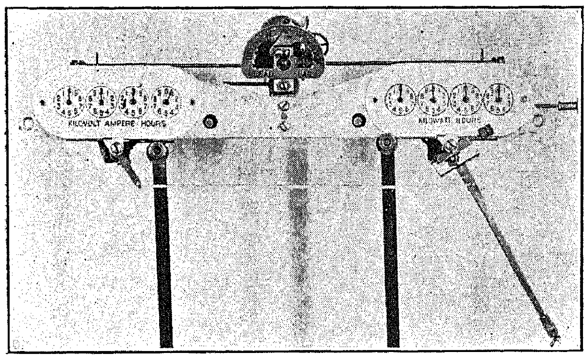

FIG. 12—TYPE *R I* DEMAND METER, FRONT VIEW OF REGISTER

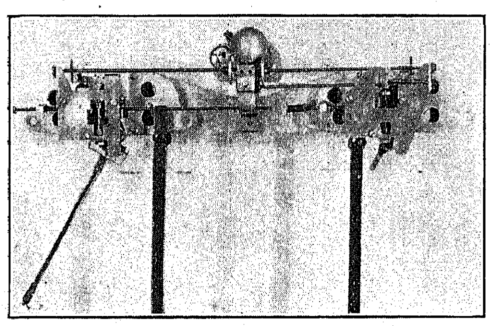

FIG. 13—TYPE *R I* DEMAND METER, REAR VIEW OF REGISTER

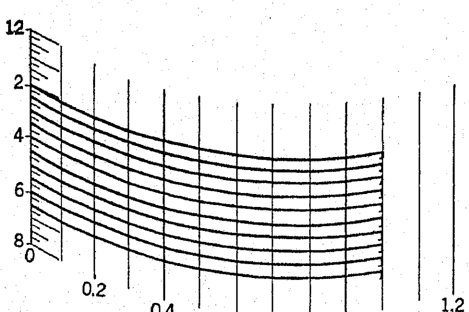

FIG. 14—UNITY POWER FACTOR RECORD

lines such as $E\ S$ - $E\ E$ and $E\ Q$ represent power factor. Hence, the apparent power factor for any kv-a. indication can be determined by observing the position of point E. The approximation of the true kv-a. by the pantagraph method used in the Type $R\ S$ meter depends upon the choice of the length and location of levers and the angles of deflection of the levers. In the design of the meter these variables have been selected so that the maximum difference between the true and the indicated kv-a. demand is well within the accepted limits of accuracy for watthour meters.

The construction of the $R\ S$ meter utilizes commercial polyphase watthour meters for the meter elements and parts of commercial demand registers in the construction of the register. The time interval is determined by the same type of small induction motor as in the commercial form of demand register and the gravity resetting of demand pointers is arranged similar to the same device. Figs. 8 and 9 show front and rear views of the kv-a. demand register.

TYPE *R I* KV-A. DEMAND METER (BALL TYPE)

Another method of combining reactive and power components of volt-amperes make use of a valuable

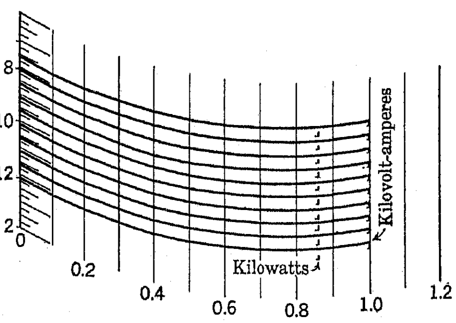

FIG. 15—85 PER CENT POWER FACTOR RECORD

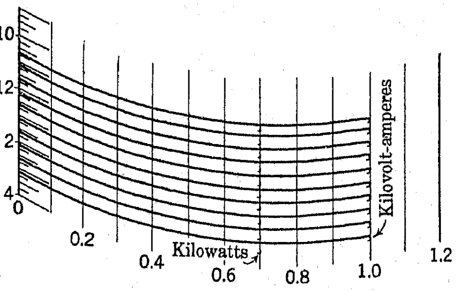

FIG. 16—70 PER CENT POWER FACTOR RECORD

property of a sphere driven by two disks and driving a third disk in which combination the angular motion of the third disk is the vector sum of the motion of the two disks. Referring to Fig. 10, the aluminum ball is supported by disks d-1 and d-2 running in the same direction at varying speeds. The ball assumes an axis of rotation $X\ Y$ such that perpendiculars a and b are proportional to the respective speeds of the disks. The disks are spaced 90 deg. apart so that the two triangles a-b-c are similar and equal. It is evident that in the right triangle a-b-c that c the radius of the sphere is the hypotenuse of the triangle and is therefore the vector sum of a and b. In the type $R\ I$ voltampere meter d-1 and d-2 are driven respectively by reactive and watthour meters and a wheel W is mounted in a movable frame so that it rolls on a great circle of the spher

whose plane is always perpendicular to the axis $X\ Y$ and the angular motion of W is then proportional to the radius C of the sphere.

The frame which carries W is arranged so that it can swing about an axis passing through the center of the ball perpendicular to the plane in which $X\ Y$ moves.

Through suitable gearing the integrated angular motion W is communicated to the pen and leaves a record on the paper chart. The pen is reset at regular intervals as in other recording demand meters. It is

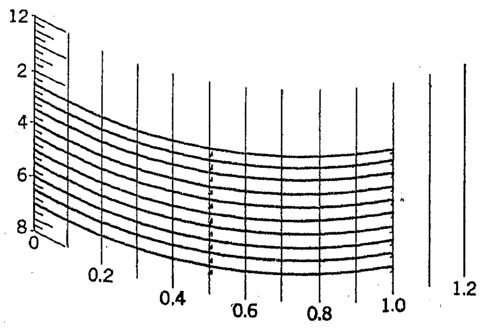

FIG. 17—50 PER CENT POWER FACTOR RECORD

obviously desirable to have both kw. and kv-a. demands readings on one chart, and this was accomplished by arranging a mechanism on the kw. element which causes the pen to hesitate a fraction of a section during the process of resetting and make a mark on the paper which indicates the kw. demand. Since kv-a. is always greater than kw. there is no possibility of this stop advancing ahead of the kv-a. mechanism.

Figs. 14 to 18 illustrate the records obtained under

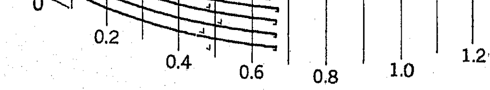

FIG. 18—CONSTANT POWER FACTOR AND VARIABLE LOAD

several conditions of power factor. In Fig. 14 with 100 per cent power factor the kv-a. and kw. indications are coincident and show a demand of 1000 watts for the time interval which was 15 minutes on the particular meter used on these tests. In Fig. 15 the kv-a. demand is still approximately 1000 watts but as the power factor is approximately 85 per cent the kw. record is reduced to about 850 watts. In Figs. 16 and 17 the kw. demand is still less with a corresponding power factor of 70 per cent and 50 per cent. Fig. 18 is a record of gradually decreasing demand with a constant power factor of about 70 per cent. The right hand dials are arranged to register kw. hours and the left hand dials kv-a. hours, hence the apparent power factor for the month can be obtained. Referring again to Fig. 10 a/c is the ratio of kw. to kv-a. and is the cosine of the angle through which moves the frame which carries the wheel W, therefore a pointer mounted on this frame will give an indication of the power factor at any time.

From the above it is manifest that the meter gives a direct reading of indicated power factor, total kw. hours, total kv-a. hours, and kw. and kv-a. demand for the time interval. The power factor for any time interval particularly at the time of maximum demand can be readily obtained by inspection of the chart.

Discussion

W. H. Pratt: Volt-ampere and volt-ampere-hour measurements seem to be occasioned almost wholly by the problem of making suitable rates for electrical energy when used at low power factor; that is of making rates that will stimulate the raising of the power factor in such cases. From this viewpoint devices of this kind should be in a measure self-eliminating, for once the power factor fairly approaches unity there is little or no excuse for the added complexity of apparatus above that of the watthour meter.

The practical problem does not involve the measurement of volt-ampere hours through a wide range of power factor, in fact almost all cases that arise can be met by a watt-hour meter construction excited at a displaced phase. It is often advisable to use a three-element watt-hour meter for the sake of symmetry of connections.

Since the cosine of small angles remains near unity, it is possible to cover a range of 23 deg. with a simple watt-hour meter excited at a suitably displaced phase with maximum errors of one per cent. This represents a range of power factor of from 92 to 70 per cent, or 80 to 50 per cent, ranges amply large for including possible maximum demand connections for a very large portion of practical cases. A simple duplication of parts with a selective register makes the double angle possible.

In the types of apparatus described in the paper it does not seem fair to expect that errors arising from mechanical causes can be less than one per cent and they are likely to be of an irregular character.

One condition which is quite common does not seem to be well met by the ball types of device. That is the regular or irregular pulsation of power factor arising from such loads as machine tools—for instance, wood-working machinery. The roller on the ball would never be where it should be. This criticism would vanish if the roller moved at a speed such as shown in the moving picture, where everything was going fast. But if the roller moves at a leisurely rate, as I understand it does, it seems that in a large number of cases the reading will not be sufficiently accurate.

A. E. Knowlton: Both the kilovolt-ampere metering devices presented in the paper represent commendable progress. It should be noticed, however, that as proposed in the paper, they rest upon a method of potential connections for obtaining the reactive component which is inherently inaccurate on unbalanced loads. An analysis and experimental study of these inherent errors of the "compensator" or "phasing-transformer" scheme of getting quadrature potentials (reported in *Electrical World*, Vol. 82, No. 26) shows the possibility of several per cent

of error in power factor, reactive component, and total kilovolt-amperes under certain conditions of unbalanced inductive load. If the loads, which are so important as to require the integration of kv-a. and indication of kv-a. demand, become materially unbalanced, then much of the value and precision of these new devices is lost through the inherent inaccuracy of the means of obtaining the potentials impressed on the reactive meter elements. The very necessity of observing phase sequence in making the connections to the reactive meter also underlies the fact that any unbalance of volt-amperes is registered in the wrong sense by the reactive meter.

The article does not refer to the type of adjustments provided for the purpose of correcting errors which arise in these two meters while in service. Much of their value to the industry depends on the simplicity and directness of those adjustments which must often be made by a meterman who is none too well qualified to deal with delicate and complex instruments. It would appear that the adjustments at the sphere and disks to compensate for dial friction, slippage, etc., would be materially different in nature from those made by the average meterman handling standardized watt-hour meters.

A. E. Kennelly (by letter): The two types of demand meters described, namely, the pantograph type and the ball type, are both ingenious and interesting. They serve to construct mechanically the magnitude of a hypothenuse from the two side components of a right-angle triangle.

As a matter if terminology, it is suggested that the three sides of the power triangle might logically be called: (1) the active power, or active watts, for the base, (2) the reactive power, or reactive watts, for the perpendicular, and (3) the vector or apparent power (vector watts or the volt-amperes) for the hypothenuse. The reactive power is just as truly power as the active power, except that it does not leave the circuit. The indicating instruments for measuring these three powers would logically be named (1) the active wattmeter (ordinarily, for brevity, the wattmeter), (2) the reactive wattmeter, and (3) the vector wattmeter or volt-ampere meter.

When the corresponding energy components are referred to, they might be called (1) the active energy, or active watt-hours, (2) the reactive energy, or reactive watt-hours (3) the vector energy, or vector watt-hours, or the volt-ampere hours. In this case, the reactive watt-hours would be fictitious, because their true summation value is zero. The corresponding instruments might be named (1) the active watt-hour meter (ordinarily abbreviated to watt-hour meter), (2) the reactive watt-hour meter, and (3) the vector watt-hour meter, or volt-ampere hour meter.

A. R. Rutter: Mr. Chairman and Gentlemen: In connection with the kv-a. paper, I believe the title of the paper points out that the purpose of the paper was to discuss two recent developments, and that the title indicates that there should be future developments in kv-a. metering.

Now, in considering either of the types mentioned in the paper, we should consider the commercial accuracy and the commercial demand of the device as well as the theoretical accuracy, somewhat in line with Mr. Pratt's remarks on testing of instrument transformers.

Now, it is one thing to speak of the theoretical accuracy, and another of the practical commercial accuracy. In regard to the measurement of reactive component it was the intention of the authors to point out that there are a number of schemes or methods of measuring the reactive component. A number of these methods are inaccurate with unbalanced currents, and unbalanced voltages. The particular scheme mentioned, that of the reactive component compensator is described as the outstanding method, and by that we meant the outstanding method at the present time. The method of measuring reactive component with the commercial watthour meter and the use of a reactive component compensator is probably the most widely used method at the present time.

Mr. Smith made some remarks indicating that the development of a true sine-meter is quite possible; in fact, sine-meters are being used I believe by the Philadelphia Electric Company. There are at least two different schemes that might be used to give a theoretical accuracy for a reactive meter.

In connection with the comparison of say, the type $R\,I$ meter with other commercial kv-a. meters, I believe that we should take into consideration the commercial accuracy; that is, that the meter employs standard watthour meter parts and standard apparatus in practically all details.

In regard to the adjustment of the meters, both the type $R\,S$ and the type $R\,I$ are adjusted as standard watthour meters. The adjustment of the pantagraph arrangement in the type $R\,S$ or the adjustment of the ball mechanism in the type $R\,I$ depends upon the workmanship in the construction of the register. In other words, these quantities are fixed.

In connection with Professor Knowlton's comment on reactive metering, I would like to call attention to Professor Knowlton's paper in the December, 1923, issue of the *Electrical World*. In an article entitled, "Tests on Accuracy of Reactive Metering," Professor Knowlton has given a very thorough discussion and summary of reactive metering, and in connection with the commercial application of reactive meters, I believe he has summed up the question of using sine-meters or reactive component meters, using the reactive component compensator, with the remark that of the cost of meter administration and the proficiency of meter men in installing, reading and testing meters are to be given due consideration along with the accuracy of registration, there is a question whether any, except method B, of the methods outlined can be wholly condemned as too inaccurate for use in the majority of loads.

In designing these meters the designer used the reactive scheme feeling that it was the most widely known method and except with extreme cases of unbalanced voltage no serious error would be introduced. Of course, as we have pointed out on a number of occasions if the demand presents itself, a true sine-meter can and will be developed to measure the reactive component.

Mr. J. M. Jones of Pittsburgh has presented written questions on the type $R\,S$ meter. Question No. 1: "Is the ball used only for indicating power factor?"

The ball is used in the type $R\,S$ meter to give the graphic indication, to indicate the power factors, and to integrate the kv-a. hours. In other words, the ball drives the kv-a. mechanism.

The next question has been answered by the first: "Does this ball have any effect on the graphic part of the meter?"

It drives the graphic part of the meter; that is, it drives the pen. Of course, the clock drives the chart.

"Are standard watthour parts used in this meter?"

Standard polyphase watthour meter parts are used. The comment was made that the meter must be such that the same meter man can maintain them that maintains the polyphase watthour meters.

In this connection, I might remark that the meter is a demand meter and the meters should be maintained by the men that maintain the watthour demand meters or other demand meters. In other words, demand meters require a higher maintenance than the ordinary watthour meters.

"With rapidly fluctuating loads and power factor can the ball meter indicate this rapidly changing load?" Mr. Smith will asnwer that question.

"What is the cost, with and without the graphic?"

In this connection I might say that the relative cost of the meter is approximately double that of the single watthour demand meter.

"Can a demand contact be added?"

Demand contacts could be added to the kv-a. register or to the watthour register.

B. H. Smith: Mr. Pratt, Mr. Hart and also Mr. Rutter mentioned Mr. Jones' question on the ball mechanism: "What is the effect of the ball shifting on the registration of the meter?" By actual test we have found in a given time interval, (in a fifteen-minute interval), if the power factor changes from one hundred to fifty per cent, the error at any load is about one-third of one per cent, so it is very small.

If the power factor comes back again to one hundred per cent in the same interval, the error is corrected, so that over all, there is no error due to the shifting of the ball mechanism. This is very readily checked by counting the revolutions of the watthour meter, and the reactive watthour meter, taking the square root of the sum of the squares and calculating what the kv-a. reading should be.

That ease of checking also lends itself to checking by metermen of the operation of the meter, and Mr. Knowlton's question read by Mr. Warner, referring to adjustments by meter men, the ball mechanism itself will not need any adjustments if it is running properly, and we can determine if it is running properly by counting the revolutions of the disk and reading the record on the chart.

Mr. Magalhaes can see the ball in this meter and we would like to have every one look it over.

The ball is one inch in diameter of solid pure aluminum and weighs about twenty grams, a little over half an ounce.

Later tests included in report to the A. I. E. E. Meter Committee give further information on some of the above points.

Automatic Transmission of Power Readings

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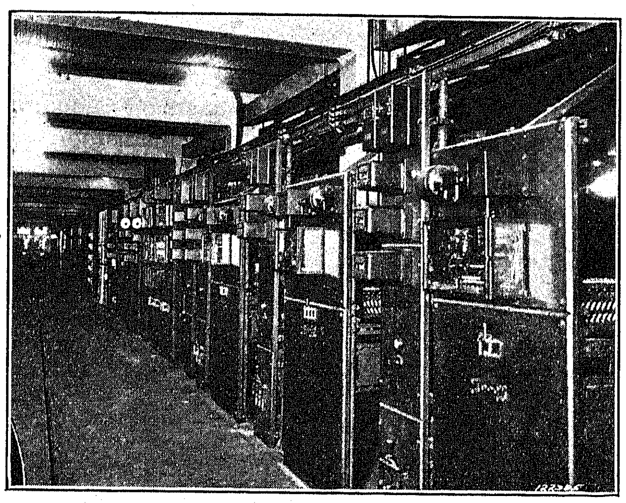
Review of the Subject.—*The need of information at one point as to power conditions in large systems for load dispatching, billing, or other purposes has led to the development of various methods of transmitting power readings over long distances. The development of superpower emphasizes this need and the perfection of means of communicating complete information as to power and load conditions will hasten the development of superpower.*

We wish to discuss various automatic methods which may be employed for remote metering. Our paper will describe the following methods:

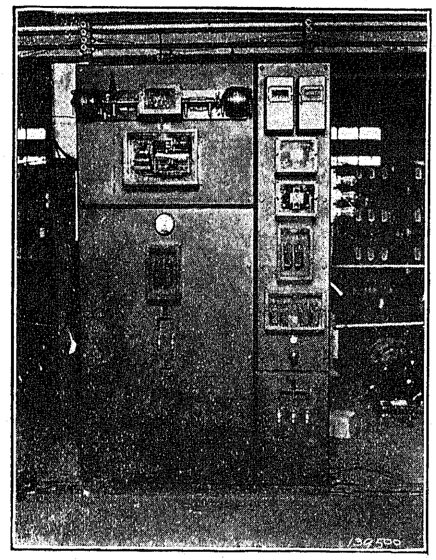
1. Frequency
2. Inverse Current
3. Potentiometer
4. Position
5. Voltage
6. Current
7. Impulse

Frequency Method

One of the earliest developments in the field of long distance metering has now been in operation on the Pacific Coast Electrified Section of the C. M. & St. P. R. R. for several years. Its purpose there is to register the total amount of power received at several substations from a hydroelectric company, to indicate and record this total at the dispatcher's office and then to send back controlling impulses over the line which regulate the trolley voltage and maintain as steady an electrical load as possible and prevent the occurrence of heavy peaks. A pair of No. 8 copper wires was installed over the whole length of the electrified section especially for the metering system, and over this line is transmitted at from 1000 to 2000 volts alternating current whose frequency varies anywhere from 20 to 60 cycles depending upon the load measured. If the load is heavy the wattmeters through suitable



Test Set Up—C. M. & St. P. R. R., Frequency Method



Apparatus of Intermediate Station—Frequency Method

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

speed regulating apparatus increase the speed of small a-c. generators until just the right frequency is sent out equivalent to the measured power. At each station the frequency of the metering circuit is regulated so as to be equivalent to the total power from all stations up to that point, and then at the dispatcher's office the frequency is translated back to watts in a local 60-cycle circuit. This is accomplished as follows: The incoming frequency operates a small synchronous motor whose speed is measured by a speedometer which in turn is balanced against a Kelvin balance wattmeter until the watts are made equivalent to the incoming frequency and thus the total power consumed by the railroad. The local circuit then leads to recording meters, watthour, recording demand, plain indicating wattmeters and a regulator. Thus the dispatcher has immediately available a complete and up-to-the-minute record of the power conditions of the road. This system has given exceptionally good service, but there are several factors involved which preclude universal adoption. First the amount of power required to actuate the metering apparatus and which must be transmitted over the lines is about 40 watts. This amount of power cannot be transmitted over existing telephone or signal lines and involves a special line costing several hundred dollars per mile for copper alone. In addition the sending and receiving apparatus is not suitable for installation on the premises of the average power consumer due to size, energy consumption, and attention required.

Inverse Current Method

Another early installation was on the Montana division of the C. M. & St. P. R. R. Here a current, inversely proportional to the load, is transmitted over a special metering circuit. The wattmeters in each substation measure the a-c. power and regulate an automatic rheostat until resistance is introduced in the metering circuit proportional to the load. All stations are connected in series and a constant voltage is maintained in the dispatcher's office at one end of the circuit. The total resistance in all the stations and the line resistance reduces the current in the circuit to a minimum for full scale readings of the meters in the dispatcher's office, of about 250 milliamperes. As the load decreases this current increases and the fluctuations are recorded in the dispatchers office on graphic milliammeters and various regulating instruments.

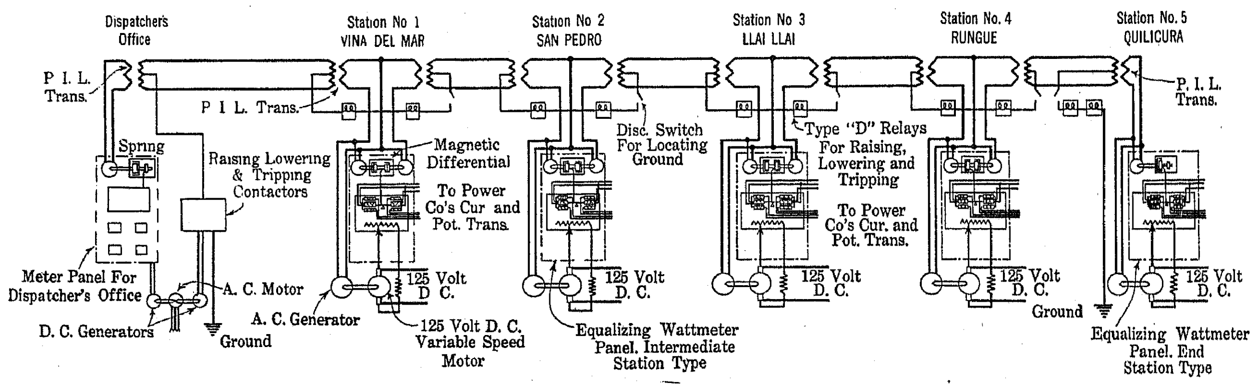


Fig. 1—Schematic Diagram of Indicating and Limiting Equipment

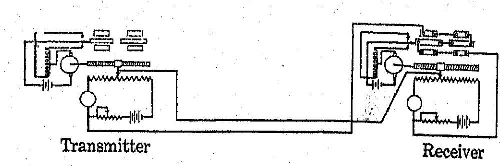


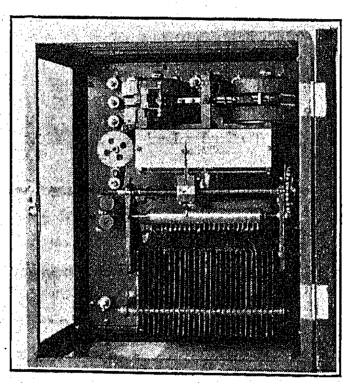
Fig. 2

Potentiometer Method

A standard relay-type graphic wattmeter, either single-circuit or totalizing, is equipped with a slide resistance and a slider is fastened on the pen carriage. A voltage is applied to the slide and the voltage drop from one end of slide to slider is transmitted to the receiver.

The receiver consists of a contact-making galvanometer which operates a slider across a resistance. A voltage is applied to this resistance which opposes the voltage applied to the transmitter slide. Consequently

Totalizing Transmitter—Potentiometer Method



Receiver—Potentiometer Method

the slider on the receiver takes up a position where the receiver voltage is equal to the transmitter voltage and no current flows. If both resistances are in equal steps, the position of the receiver slide is the same as the position of the transmitter slide.

This method is very accurate as long as equal currents are maintained in the local slider circuit. It is a "Nul" method so that it is independent of transmission line resistance, the only effect of a high-resistance line being a lower accuracy in the receiving instrument.

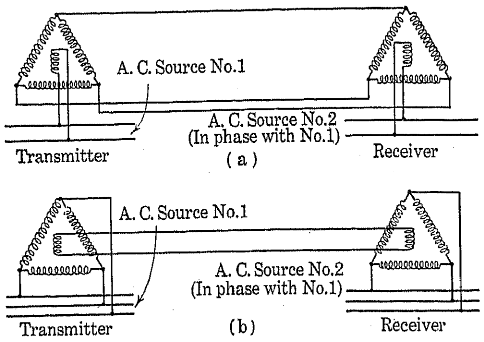


Fig. 3
a—Single-phase excitation.
b—Three-phase excitation.

The main objection to this method is the complicated connections required to give indications and totalization. A second slide and slider is placed on the receiver operating in parallel with the first slider. A constant voltage is applied to this second slide resistance. It requires a voltage regulator to keep this battery voltage constant. This voltage from one end of the slide to the slider is applied to a voltmeter whose scale is marked in kilowatts for indicating purposes and these voltages, as taken from several receivers, are connected in series for totalizing purposes.

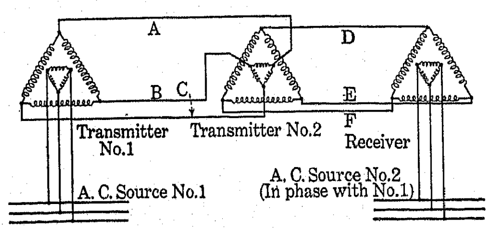


Fig. 4

This is the method employed at the Springdale station of the West Penn Power Co. Totalizing graphics are used for measuring purposes and there are several single-circuit receiving instruments in various parts of the power station. By gearing a long pointer on the worm which operates the receiving slider, a large diameter boiler room indicator is obtained. Alternating-current control is used on this installation.

Fig. 2 shows schematic diagram of this method.

Position Method

Fundamentally this method consists of the use of one of the various types of position transmitters, so connected as to take up positions corresponding to the power measured, and a proper receiver at the receiving end. The Kelvin balance relay type graphic meter may be used to measure the power and the position transmitter properly geared to the mechanism which drives the pen across the paper.

There are various types of position transmitters which would apply here but we believe the induction type has a wider application than most and will, therefore, discuss it in more detail.

If two induction motors are connected as shown in Fig. 3 any motion produced in one will be closely followed by the other. A vane type of synchronoscope may be substituted for the receiving motor and its pointer will follow the rotation of the transmitter rotor. Thus the dial may be marked in kilowatts if the transmitter is connected mechanically to the pen mechanism of the above mentioned graphic.

For totalization, a totalizing graphic could be used for making the measurements and the transmitter connected to it. Another method of totalizing would be to

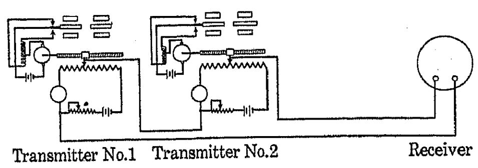


Fig. 5

connect the transmitters on several single-circuit graphics in concatenation so that the positions are additive. This requires the use of a three-phase supply as shown in Fig. 4.

If Transmitter No. 1 rotates, this causes a phase rotation in the rotor windings of Transmitter No. 2 which acts as a transformer so that the receiver is rotated through a corresponding angle. Now if Transmitter No. 2 is rotated this adds an angular displacement in wires D—E—F, so that its motion is added on to that of Transmitter No. 1 and the receiver shows the sum of the two.

Voltage Method

This method consists of mounting a slider on the pen carriage of a relay-type graphic wattmeter which will operate over a slide wire to which constant voltage is applied. The drop across the slide is transmitted to a voltmeter calibrated to read kilowatts. If it is desired to totalize several of these the voltages are connected in series so that the receiving voltmeter reads the sum. See Fig. 5.

Adjusting rheostats and a small ammeter are shown in the slide circuit so that adjustments may be made to take care of changes in battery voltage.

Current Method

This method is similar to the potentiometer method but is much simpler. The power to be transmitted is measured by a Kelvin-balance-wattmeter element with the control spring omitted. A d'Arsonval meter element has its moving element mechanically connected to the moving element on the Kelvin balance. The contacts on the Kelvin balance control a motor-operated rheostat which varies the current in the d'Arsonval meter element until its torque is equal and in opposition to the torque of the Kelvin balance. This current is therefore proportional to the power being measured.

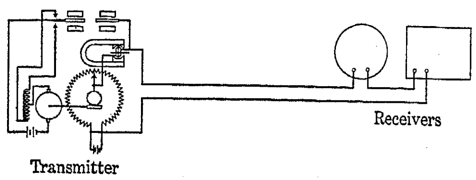


Fig. 6

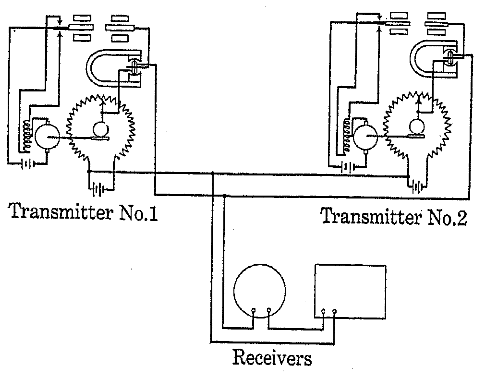


Fig. 7

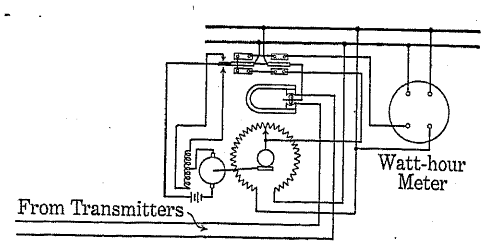


Fig. 8

A standard direct-current ammeter serves as a receiving instrument and may be indicating or graphic. Several instruments may be connected in series both in the generating station and at the remote point without changing the calibration of those already installed or destroying the accuracy of each individual receiving instrument. This is done in some cases to widely distribute the readings.

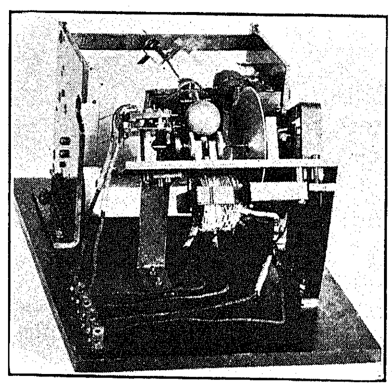
If it is desired to totalize the power from several generating stations, the direct-current control circuits from the various transmitters are connected in parallel so that the receiving instrument measures the sum of all currents.

Fig. 6 shows a simple transmitter and receiver outfit. The contact making Kelvin balance operates the motor which, in turn, operates the slide arm. Since the torque of the Kelvin balance increases in equal steps for equal increases in power, the counter torque produced by the d'Arsonval meter element must increase in uniform steps. Since the scale of a d'Arsonval meter is uniform the curve of transmitted current against power measured will be a straight line passing through zero.

It should also be noted that since the operation of the apparatus depends on current, it is independent of control voltage.

Fig. 7 shows connections for totalizing several transmitters.

In some cases it is desirable to obtain watthour meter readings at the receiving end. This is accomplished by using a transmitter at the receiving end connected in reversed. The direct current is brought into the



Graphic Receiver—Impulse Method (Rear View)

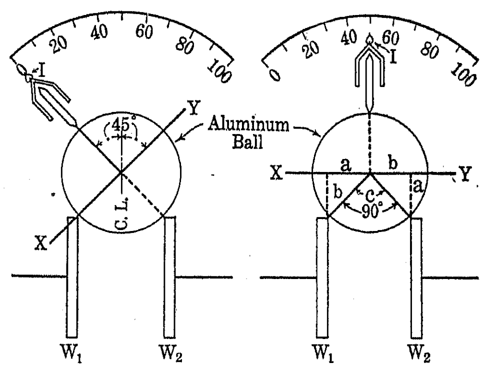
d'Arsonval meter element and the rheostat controls the alternating current from an auxiliary source which passes through the Kelvin balance and watthour meter in series. Fig. 8 shows this connection.

This method allows the transmitting of readings over telephone lines without interfering with conversation up to distances of twenty-five miles.

Direct-current Impulse Method

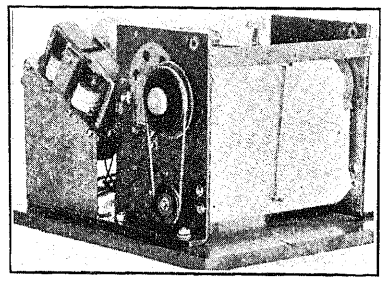
Another solution of long distance metering problems is found in the method of transmitting indications by means of a series of d-c. impulses the frequency of which is proportional to the sending meter speed. A system of this kind has now been operating for some months on a street railway system in connection with supervisory Control of automatic substations from a central load dispatcher's office. Power is measured on polyphase watthour meters in each substation. These meters are equipped with commutators on the main shaft, and with brushes and brush rigging similar to that on a d-c. watthour meter. Alternate opposite

pairs of segments are short-circuited so that for every revolution of the shaft a circuit is alternately made and broken four times. The circuit is completed through a telephone relay whose contacts are connected so as to transmit impulses of alternating polarity of not over 100 volts over standard telephone wires to the dispatcher's office. In the office the impulses are received and actuate a polarized excapement mechanism which allows a tooth of the escape wheel to go by for each complete cycle of impulses or four teeth for each revolution of the sender meter. Dials are geared to the escapement wheels and register the total watthour reading. Indicating readings are accomplished by a ball mechanism which compares the speed of the escapement wheel with a constant speed obtained from a small synchronous motor. Referring to Figs. 9 and 10, an aluminum ball is supported so as to be driven by wheels $W-1$ and $W-2$. If the speed of $W-1$ is zero, the ball will rotate about an axis $X-Y$ inclined 45 deg. from the horizontal. If the two wheels have equal speeds the axis will be horizontal and at any point between these limits the position of the axis is an indication of the ratio of speeds of $W-1$ and $W-2$. In order to determine the characteristics of this relation; it is necessary to consider the triangle $a-b-c$, and $a-1$, $b-1$ and $c-1$; a and b being radii drawn from the point of contact fo the wheels $W-1$ and $W-2$, to the axis of rotation and, therefore, proportional respectively to the speeds of the two wheels. Since the angle between c and $c-1$ has been fixed at 90 deg. the two triangles are similar and equal and the sides, a, b, and c are equal to $a-1$, $b-1$ and $c-1$ and c/a is tangent of the angle which measures the departure of the axis from the zero position. Therefore the receiver meter, the position of whose indicating pointer is determined by this axis has its scale divisions marked proportional to the tangents of angles from 0 to 45 deg. over which range the scale is near enough uniform for practical use.



FIGS. 9 AND 10—REMOTE METERING "RECEIVER" ANALYSIS OF BALL MECHANISM

The time required for the pen or pointer to travel from one point to another with change of load is about 8 seconds, but as with other meters the time has a logarithmic characteristic and tests show that with a peak load lasting five seconds approximately 90 deg. of the actual value will be indicated. A 3-second peak



GRAPHIC RECEIVER—IMPULSE METHOD (FRONT VIEW)

will register about 60 per cent but if the current holds up for about eight seconds the indication will be complete, thus we have an indicating wattmeter which quickly and accurately responds to change in speed of

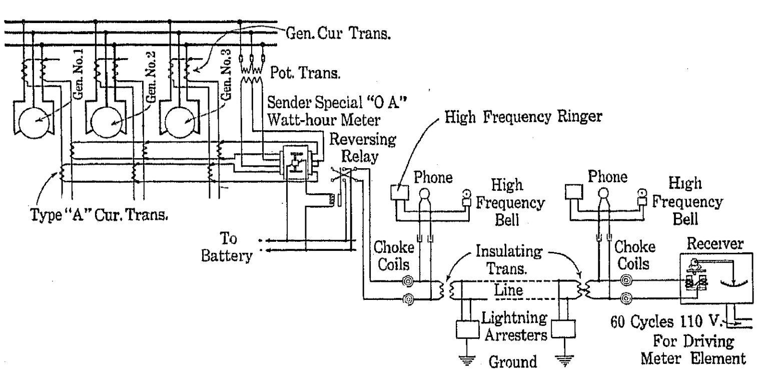


FIG. 11—IMPULSE METHOD. SCHEMATIC DIAGRAM

the sender meter at the distant station. If the distance is so great that not enough energy is received to actuate the receiver mechanism a sensitive telegraph

LARGE BOILER ROOM INDICATOR

relay is interposed which will operate on one or two milliamperes. Where the above system is operated over telephone lines, speech may be transmitted at the

same time. There is a faint noise from the meter impulses although with proper precaution this can be reduced so that it is not seriously objectionable. It is necessary to connect the telephone instruments to the line through condensers and use high-frequency calling apparatus which will also operate through the condensers as shown in the diagram, Fig. 11. This procedure is necessary, otherwise the meter impulses would be short-circuited by the telephone instruments. In order to prevent the telephone current from entering the metering apparatus, choke coils are interposed ahead of the latter which allow the low-frequency impulses to pass but keep out the relatively high voice frequencies.

As in the telegraph industry the need arises for transmitting several messages or indications at once over one pair of wires. This is accomplished by duplexing when one indication is required each way, or by quadruplexing where two indications are required each way. The principle involved with metering as in telegraphy involves polarized relays which respond to reversal of direction of current and marginal relays to increased quantity of current. The home relays do not respond to outgoing messages because the current divides in a differential winding between the line circuit and an artificial local circuit. In this class of service difficulty is encountered on outside lines, but with underground cables good results are obtained.

In general this method compares well in results obtained and in distances transmitted with the telegraph industry.

Discussion

H. P. Sparkes (Pittsburgh): In connection with remote load indication we have been watching several different systems. In the Pittsburgh territory, with regard to this movement, it means this: When several big power plants are tied together, the growth of that system will eventually develop into a miniature superpower system. This necessitates some method of the load dispatcher knowing the exact load on each station.

Now, if you will go into the matter you will find that it is necessary for the system load operator continually to call his power stations and find the load over the territory. In case of trouble, quite often they can't get the stations, due to line interference or some form of trouble out on the power line. Or they haven't time to get them. At a recent meeting this same point was brought up in the Ohio Electric Light Association. They wanted to know the possibilities of such a scheme on a large power system tied in together with several operating companies. In doing so, it will be necessary there to transmit these readings over several hundred miles. A telephone company, a company at least operating the master telephone system there, is at present installing super-telephone equipment to take care of this new development.

As a result, if the superpower program actually comes through, this remote load indicator problem will be solved by that time, and we shall have something by which the load operators can tell what is going on in various parts of the superpower system.

As to the actual operation of these outfits, I have been carefully watching one system in Pittsburgh to determine whether the thing was really successful by using the various schemes with reference to the Potentiometer system in Pittsburgh. I might say we have operating there one scheme which is very successful. It was used last year at the February meeting of the Institute to transmit the loads of the West Penn Power Company's plant from Springdale to the Chamber of Commerce.

A peculiar incident happened during that time. Everything was running, the meter was operating perfectly, and suddenly dropped to zero. I immediately thought something had happened on the local telephone wires as we were using local wires through the local telephone exchange. About ten minutes later the operator on the System whose load was being used for remote indication came in and asked how it was working.

I said, "We are having trouble."

He said, "No, we had it. We had an excellent load on the plant but lost everything. The telephone wires immediately dropped to zero and gave me that indication."

You can see how valuable the immediate notice of loss of power is to an operating company. That system has been operating satisfactorily. It was one of the first systems installed in this part of the country, and has auxiliary indicators operating in the boiler and turbine room, about 37 in. in diameter which may be seen from any part of the plant. One of these indicators is located in the boiler room where considerable dust is sifting through the air. The meter is enclosed in a steel case and is dust-proof.

P. MacGahan (by letter): The importance of the paper by Messrs. Smith and Pierce lies in the fact that the future superpower system will require the transmission of instrument readings to a remote point. Conversely, the development of such instruments will greatly facilitate and perhaps advance the day when the super-power system will be brought into use.

One of the earliest installations of the remote transmission of instrument readings was that at the Springdale Power Plant of the West Penn Power Company near Pittsburgh where the graphic recording wattmeter readings on a switchboard were transmitted to a load dispatcher's office and to a large boiler room indicator approximately 36 in. in diameter, of the illuminated-dial type. This installation works on the potentiometer principle described in the paper.

In later installations it was found desirable to provide for the totalization of readings in several circuits either in the same plant or in different powerhouse locations or in entirely separate plants. The current equipment described in the paper seems to facilitate obtaining this desired result. By adding the desired milliamperes a totalized indication or graphic record can be obtained, at any desired location.

When several circuits are in one location, the previous practise has been to construct a totalizing graphic wattmeter consisting of one meter element in each circuit, all tied together mechanically so as to add the resulting power. This system has the disadvantage that whenever an additional unit is to be installed all the previously equipped units have to be changed and the complete totalizing graphic instrument recalibrated. With the proposed electric totalization method, however, a separate measuring unit would be located at each circuit and a totalizing unit installed in such a way as to allow additional units to be installed at any future time without disturbing the original outfit.

The electric transmission of power readings is intimately associated with the subject of remote control on automatic stations, thus it would seem desirable to have a system for the remote transmission of power readings such as would function without interference with supervisory control over the same connecting wires.

Perry A. Borden (by letter): It is fitting that at this time, when transmission and distribution networks are daily becoming more intricate, there should have been placed before the Institute such a summary of the existing methods of remote indication and totalization of power loads as has been presented by Messrs. Smith and Pierce. By way of further broadening our horizon of such matters I should like to add a brief description of a system

which has recently been placed upon the market by a British manufacturer. This is known as the Fawcett Remote Power Indicator; and, while it might be said to fall within the fifth class enumerated, it possesses features which would almost justify its consideration under a separate class.

The Fawcett indicator makes use of a form of thermal wattmeter, similar to that employed in the Lincoln demand meter. In this meter the difference of temperature of two heating elements is proportional to the watts in the circuit being metered; and, it will be remembered, that in the Lincoln meter expanding elements are made use of to give an indication of this temperature difference. In the Fawcett indicator the temperature difference is made to actuate a pair of thermocouples, thus giving at the point of metering a direct electromotive force representative of the value of the power measured. While, in its present form, this device makes use of a direct deflecting galvanometer in its indicator, it would appear that such a method, if combined with a balancing recorder, would present great possibilities for American practise, particularly in large urban distribution systems.

B. M. Jones (by letter): This paper is on a subject which has in recent years become very important, both to the power companies and to the consumers. At the present time, our company is using a method of obtaining the power factor as outlined in Fig. 1 of Messrs. Smith and Rutter's paper. This gives us very good results, although not the most accurate possible. To put in service in consumers' stations equipment to record power factor (or its equivalent) in the most accurate way possible appears hardly worth while for several reasons; one of which is that the equipment would be special, complicated, elaborate and expensive. Another is that it would not be possible for the general run of meter testers who now work on standard watthour meters and a few other meters, to take over the adjustment of these meters without experiencing some trouble and difficulty. Another point against using the most accurate means of measuring power factor is the fact that the instruments for obtaining these measurements would more than likely have to be tested or recalibrated in the laboratory, which would mean that they would have to be taken off the job, transported to the laboratory, tested, retransported to the job and reinstalled; all of which takes time, costs money, and in addition leaves a loophole for disturbing the adjustment during transportation and reinstallation.

Professor A. E. Knowlton of Yale University, published an article in the December 29, 1923, issue of *Electrical World*, on the subject "Tests on Accuracy of Reactive Metering" which was of great interest. In this paper, Professor Knowlton goes into the question from two or three different angles, and two or three methods of obtaining the power factor are thoroughly covered. His conclusions are very interesting, especially the third conclusion, in which he states that, "If the cost of meters, the cost of maintenance of meters, and the proficiency of metermen in installing, reading and testing the meters are to be given due consideration along with the accuracy of registration, there is a question whether any of the methods except Method B (outlined in his paper) can be wholly condemned as too inaccurate for use with the majority of loads."

To the writer's mind, this is the most important conclusion of Professor Knowlton's paper, and is the one in which power companies are primarily interested just now. Most power companies have thousands of meters of very similar types and the metermen are very proficient in installing, maintaining and testing these meters. To run in a new instrument of complicated design, and call upon them to install, maintain and test it will doubtless bring on errors which may counteract the increase in accuracy. The scheme outlined in Fig. 1 of Messrs. Smith and Rutter's paper and also Fig. 3 of Professor Knowlton's paper is one which uses standard watthour meters with the phasing transformer, and will undoubtedly meet with the approval of the meter-maintenance crew who have to live with the equipment and make it operate with a fair degree of accuracy. This equipment is made by several manufacturers and can be obtained on very short delivery and at quite reasonable prices, and can be tested the same as standard watthour meters.

There are a few points and questions that the writer wishes to bring out regarding the "R. I." kv-a. demand meter which was discussed in detail in Messrs. Smith and Rutter's paper, a sample of which they exhibited.

1. Is the ball used only for indicating power factor?
2. Does this ball have any effect upon the graphic part of the meter?
3. Are standard watthour parts used with these meters?
4. Meters must be such that the same metermen will maintain them who now maintain polyphase watthour meters?
5. In very rapidly fluctuating loads and power factors, can the ball meters indicate the rapid changes accurately? This is very important to most companies on such loads as coal-mining hoist loads and similar loads where the hoist load is a great portion of the total load and is on for a few seconds, 15 or so, and off for 15 to 30 seconds.
6. What is the cost of this meter with and without the graphic parts?
7. Can a demand contact be added to this instrument to operate the usual demand meters?
8. What simple method can be used for testing this meter in the field so that the metermen will know that it is fairly accurate without having to take it to the laboratory for each test? The writer considers this very important, as the matter of taking this meter to the laboratory every time to make a test is objectionable and is to be avoided. If a simple test or check can be made on the meter regularly and it be taken to the laboratory periodically for a thorough test once a year or every six months or so, it is much to be desired.
9. How much does the ball slip and what method can the meter testers use to determine if the ball is slipping? This is especially important where there is a great amount of dust in the consumer's plant where these meters have to be installed.

The question of power-factor clauses in rates is a very important one and has come up in recent years, and is now written into quite a few new contracts that some power companies are making. The ability to obtain accurately and conveniently a record of the power factor for the joint benefit of the consumer and power company is greatly to be desired, and with the present equipment available, this can be done. This is largely due to the perseverance of the manufacturers' meter designers.

B. H. Smith: Referring to discussion by letter from B. M. Jones in answer to his questions 1 to 9, they may be answered as follows:

1. As mentioned in the text of the paper, the ball is used not only for operating the indicating power factor mechanism, but also combines the reactive component and power component meters so as to give a quantity which is proportional to kv-a. hours. This ball mechanism is the heart of the meter and is the mechanism which makes a kv-a. reading possible.
2. The ball drives the pen mechanism which registers kv-a. demand. The kw. indication on the pen is, however, obtained directly from the kw. element, as explained in the text of the paper.
3. The meter elements are standard polyphase watt-hour meter parts.
4. With proper instructions it is expected that this meter can be maintained by the same men who now maintain polyphase watt-hour meters.
5. In rapidly fluctuating loads there is no likelihood of errors in this meter, but with rapidly changing power factors the kv-a. element reads somewhat low, due to the motion required for the ball mechanism to adjust itself. It is found that with power factors changing once a minute from 100 to 0, the error is about

four per cent, and changing from 100 to 70 every minute, the error is about one per cent. If the power factor changes every few seconds, the error would be somewhat greater.

6. The cost of this meter is in the neighborhood of $300. It is not available without the graphic parts.

7. Since the meter gives complete demand readings, it is not necessary to add demand contacts.

8. A simple method of testing this meter in the field is to allow the kw. element to rotate a proper number of revolutions at unity power factor and see if the pen gives a correct reading. A similar test may be made at zero power factor, counting the revolutions of the reactive element.

9. Since, with the meter running properly, the pen furnishes practically no load upon the ball mechanism, there is no tendency for any slipping. but this point can readily be checked by the test given under 8. Complete instructions for the operation and maintenance of this meter are covered by the instruction book which is furnished with the apparatus.

R. F. Pierce: In regard to the Fawcett scheme, although I am not intimately familiar with all the details of it, yet it seems asif, using a thermocouple and small e. m. fs. that are generated thereby, a very delicate receiving instrument would have to be used and the line resistance for considerable distances would cut down the sensitiveness of the response. Of course, a potentiometer method of taking readings could be used, but that is not within the scope of this paper. We are covering automatic methods and to use a very sensitive hand-operated potentiometer method which might be necessary with long distances with this scheme would not give to the operator at all times a knowledge of the load conditions at various points.

The Quadrant Electrometer for the Measurement of Dielectric Loss

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Associates, A. I. E. E.

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Review of the Subject.—*Walker, Skinner, Addenbrooke, Rayner, Orlich, Schultze, Thielers and others* have given a great amount of useful information on electrometers for the measurement of dielectric loss. We have found instruments made somewhat after the design of Skinner and Rayner, so rugged, and so useful as laboratory instruments, even when the instrument is subjected to severe jars and mechanical vibration, and capable of such great accuracy that we have spent considerable time developing methods of use and investigating the sources of error. Our instruments are essentially high-voltage instruments as the needle may have impressed upon it voltages up to about 8500 volts. One of these instruments has been in use almost continuously since 1913, a period of ten years, and for the last seven years it has been continually in use without needing repairs of any sort, in spite of breakdowns of the load being measured, and no adjustments have been made except an occasional turning of the quadrant leveling screws.*

We have developed what we believe to be a new zero method of measuring power factor which has many advantages, and have also used the electrometer as a detector in a high-voltage bridge. We have also gone thoroughly into the errors of the electrometer, both for the deflection and zero methods, and have included these errors in our equations, so that we believe they may be taken care of practically.

Our excuse for publishing this information is that we believe it may be of use to others, and may lead to the more general recognition of the many advantages to be procured in using electrometers.

Part I—Theoretical

1. Symbols

The following symbols will be used:

E = the voltage on the load taken as numerically equal to the transformer voltage.
I = the load current or the charging current of specimen.
I_h = charging current flowing from the needle to the high quadrant.
I_l = charging current flowing from the needle to the low quadrant.
r_1 = resistance across the electrometer quadrants.
r_2 = resistance inserted in the needle circuit in the zero method, *i. e.*, the potential resistance.
C_1 = total capacity to ground of the high quadrant and all connected parts.
C_2 = capacity in the needle circuit.
$\cos \varphi$ = power factor of the load.
$\cos \psi$ = power factor of the circuit in which I_h flows.
$\sin A$ a term included to take care of phase shift in voltage of the point of the transformer winding to which the needle is connected.
d = deflection of the electrometer.
n = ratio of the transformer voltage to the needle voltage.
K = a constant.
f = frequency.
ω = $2\pi f$.

2. Deflection Method

The usual method of using an electrometer to measure power or power factor is shown in Fig. 1A (diagram of connections) and in Fig. 1B** (vector diagram). The usual equation is:

$$\frac{n\,K\,d}{r_1} = E\,I\cos\varphi - \frac{n-2}{2}\,I^2\,r_1 \qquad (1)$$

In this and all the following vector diagrams, the load current represented by $A\,X$ is taken as the vector of reference. The voltage impressed upon the load, $A\,N$, lags behind $A\,X$ by the angle $X\,A\,N$ or φ. $N\,B$ is the voltage across the quadrants, D is the mid-point of the quadrant resistance, and B is the grounded end. $A\,B$ is the transformer voltage. In Fig. 1B, the voltage across the quadrants being a pure resistance drop, is made parallel to $A\,X$. The needle is attached to the transformer (or potential divider) at the point O. $O\,B$ is equal to E/n and $O\,D$ is the effective voltage on the needle, *i. e.*, the voltage to which the deflection is proportional according to Maxwell's theory. The usual procedure is to ground one set of quadrants as shown in Fig. 1A. In what follows this set will be spoken of as the "low quadrant" and the other set the "high quadrant."

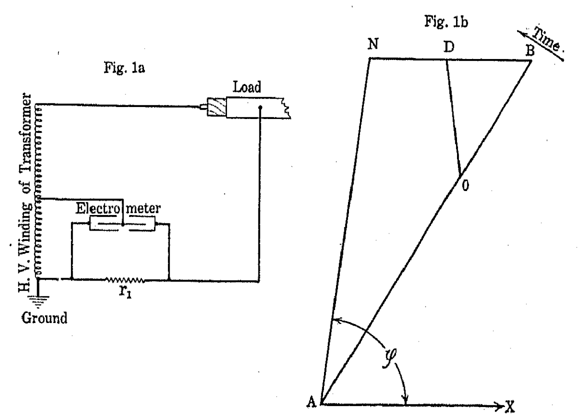

FIG. 1

In actual practise, the impedance across the quadrants is never a pure resistance, inasmuch as the capacity to ground of the high quadrant and all connected parts, the capacity to ground of the low-voltage electrode of the specimen and the distributed capacity of the resistance itself form a shunt across the quadrant resistance. If the load current is large or the sensitivity of the electrometer very high, a large deflection is obtained with a relatively small value of r_1, and it is often quite possible to neglect this capacity shunt; when, however, very small loads are to be measured

*Transactions of A. I. E. E. 1922, p. 601, Simons: "Bibliography of Dielectrics" for reference to articles by these authors and others.

**Note that time is shown revolving counter-clockwise, and thus direction of heating and lagging is reverse of usual.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

especially with very insensitive electrometers, this capacity shunt must be considered. In many of our measurements, the part of the deflection due to the capacity to ground was larger than that due to the power factor of the load. The rigid equation which takes into account the capacity to ground is as follows:

(See Part II, paragraph 18 for its derivation.)

$$\frac{n\,K\,d\,(1 + r_1^2\,C_1^2\,\omega^2)}{E\,I\,r_1} = \cos\varphi + r_1\,C_1\,\omega\,.\,\sin\varphi - \frac{n-2}{2}\cdot\frac{I\,r_1}{E} \quad (2)$$

The diagram of connections and the vector diagram are shown in Figs. 2A and 2B. In Fig. 2A the entire capacity to ground is shown shunting the resistance. The effect of this capacity on the vector diagram 2B, is to make the quadrant voltage, $N\,B$, lag by an angle α, such that $\tan\alpha = r_1\,C_1\,\omega$.

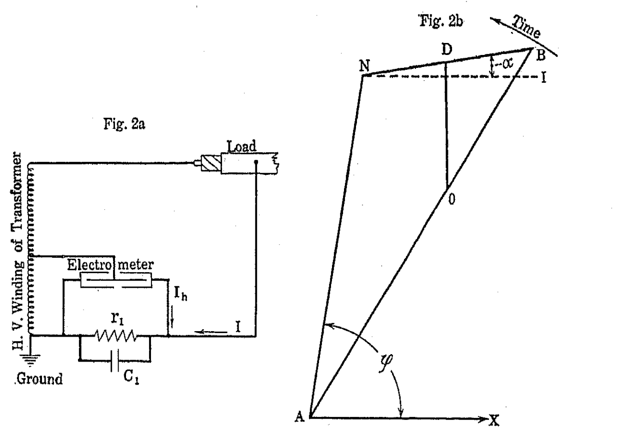

Fig. 2

For most practical cases, $r_1^2\,C_1^2\,\omega^2$ is negligible and $\sin\phi$ may be taken as unity. The equation then reduces to:

$$\frac{n\,K\,d}{E\,I\,r_1} = \cos\varphi + r_1\,C_1\,\omega - \frac{n-2}{2}\cdot\frac{I\,r_1}{E} \quad (3)$$

In the above equations, I_h the charging current flowing from the needle to the high quadrant, has been neglected. This current flows through the resistance and capacity across the quadrants and therefore produces a deflection. The relative importance of this deflection depends on the ratio of I to I_h, and in the case of extremely small specimens where I and I_h are more nearly equal it cannot be neglected.

In addition to the effect of I_h on certain terms of the equation, the energy loss between the needle and high quadrant, whether due to ionization, dielectric loss in the medium in which the electrometer is immersed (usually air but sometimes oil) or direct leakage is also measured.

Another error due to phase shift of the needle voltage is introduced by the use of a potential divider. The high-voltage winding of the transformer may be provided with taps at 1/2, 1/4 and 1/8 voltage or the potential divider of Orlich and Schultze[1] may be used. In either case a plus or minus phase shift of the needle voltage will take place and an error will be introduced which may be a function of the angle of shift, which error we have called $\sin A$. The phase shift due to resistance of the needle suspension might also be included in this term. This error may be calculated exactly and is equal to the resistance of the needle suspension multiplied by the susceptance of the circuit from the needle to the quadrants. In our electrometer this error is negligible and therefore will not be included in the term $\sin A$.

The following equation has been developed (see Part II, paragraph 20, for derivation) to include the errors mentioned above:

$$\frac{n\,K\,d}{E\,I\,r_1} = \cos\varphi + (I_h/I)\cos\psi + (1 + I_h/I)\,r_1\,C_1\,\omega + (1 + I_h/I)\,\frac{[n\,I_h - (n-2)\,I]\,r_1}{2\,E} + \sin A \quad (4)$$

In the above equation it is assumed that $\sin\varphi$ may be taken equal to unity and that $r_1^2\,C_1^2\,\omega^2$ is negligible. In order to apply this formula it is necessary to know the value of I_h for every deflection, as this value is a function of the position of the needle in the quadrants. The method of determining the various quantities appearing in the equation will be found under the zero method.

3. Zero Method for Measuring Power Factor

There are so many well known advantages in the use of a zero rather than a deflection method, that it is hardly worth while to mention them at this point. When applied to the electrometer the zero method obviates the necessity of designing an instrument with a "straight line" constant, eliminates the necessity of calibration, and gives readings almost independent of small voltage fluctuations. Another particular advantage of a zero method is that Maxwell's law for the deflection of an electrometer has been somewhat challenged, especially for large deflections. For a zero method using a reasonably symmetrical instrument Maxwell's law must hold very accurately. The method consists in *bringing the deflection back to zero by the insertion of a resistance in the lead to the electrometer needle*, the power factor being calculated from the value of resistance used. The diagram of connections and the vector diagram are shown in Figs. 3A and 3B respectively.

The effect of the resistance r_2 may be explained as follows. Referring to Fig. 3B the deflection of the electrometer is proportional to the scalar product of the voltage across the quadrants, $N\,B$, and the needle

1. Orlich and Shultze, *Archiv fur Elektrotechnik*, 1912., Vol. 1, page 1.

voltage $T D$. The greater the angle between these two vectors, the smaller will be the deflection and the deflection will be zero if they are perpendicular. If r_2 is inserted in the needle circuit a resistance drop is introduced in quadrature with the needle voltage, causing $T D$ to swing to the right with D as a center, increasing the angle between $N B$ and $T D$ and thereby decreasing the deflection. When r_2 has been increased to a sufficient value to bring the electrometer deflection back to zero, the needle voltage $R D$ becomes perpendicular to the quadrant voltage, $N B$, $T R$ being the voltage drop across r_2.

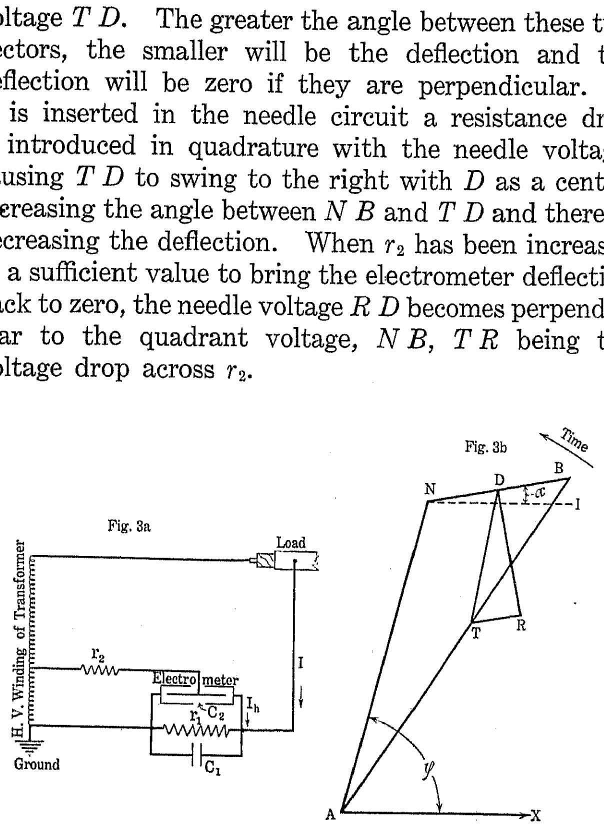

FIG. 3

If n is large and the needle voltage falls to the left of $R D$ (when $r_2 = O$) the deflection will be negative and the insertion of r_2 will merely shift the needle voltage further to the left and increase the negative deflection. Special methods as described in Section 15 must then be used.

The rigid equation applying to the method as outlined above is:

$$\cos \varphi\,(1 + r_1 C_1 \omega \,.\, r_2 C_2 \omega)$$

$$= \sin \varphi\,(r_2 C_2 \omega - r_1 C_1 \omega) + \frac{n-2}{2} \cdot \frac{I r_1}{E} \quad (5)$$

(See Section II, paragraph 19 for derivation).

For most practical cases $r_1 C_1 \omega .\, r_2 C_2 \omega$ is negligible and $\sin \varphi$ can be taken as unity, and we have the following working formula:

$$\cos \varphi + r_1 C_1 \omega - \frac{n-2}{2} \cdot \frac{I r_1}{E} = r_2 C_2 \omega \quad (6)$$

For a given set of conditions, a certain voltage drop across the potential resistance, r_2, is necessary in order to bring the deflection back to zero (See Fig. 3B). The small value of the charging current of the needle would necessitate the use of a very large potential resistance. To avoid this, a condenser, C_0, of 0.004 μf. capacity, was shunted between the needle and ground, which made it possible to bring the deflection back to zero, with a few thousand ohms. In Fig. 3B, one side of C_0 is shown connected to the mid-point of r_1. If this were actually done, the charging current of the auxiliary condenser would flow through one-half of the quadrant resistance introducing an important error. In all our measurements one side of C_0 was connected directly to ground as in Fig. 4. When this is done, it can be shown analytically by a method similar to that used in Part II, Paragraph 19, that the quantity $\dfrac{I r_1 \,.\, r_2^2 C_2^2 \omega^2}{E \sin \varphi}$ should be added to the right hand side of (**5**). The value of this term is so small as to be negligible in our case. If a resistance were inserted between C_0 and ground of such value that the voltage drop across it is equal to half the voltage drop across the quadrants, the above error is approximately compensated for. Experiment showed that this resistance was not necessary.

For very small loads the value of I_h may be comparable to I, so that the former can no longer be neglected. For this case the following formula (see Part II, paragraph 20) has been derived:

$$\cos \varphi + (I_h/I) \cos \psi + (1 + I_h/I)\, r_1 C_1 \omega$$

$$+ (1 + I_h/I)\, \frac{[n I_h - (n-2) I]\, r_1}{2 E}$$

$$+ \sin A = r_2 C_2 \omega \quad (7)$$

This equation is a general one which will be used in all that follows. For $n = 2$ and $n = 1$, equation (**7**) takes the forms:

½ voltage on needle:

$$\cos \varphi + (I_h/I) \cos \psi + (1 + I_h/I)\, r_1 C_1 \omega$$

$$+ (1 + I_h/I)\, \frac{I_h r_1}{E} + \sin A = r_2 C_2 \omega \quad (8)$$

Full voltage on needle:

$$\cos \varphi + (I_h/I) \cos \psi + (1 + I_h/I)\, r_1 C_1 \omega$$

$$+ (1 + I_h/I)\, \frac{(I + I_h)\, r_1}{2 E} = r_2 C_2 \omega \quad (9)$$

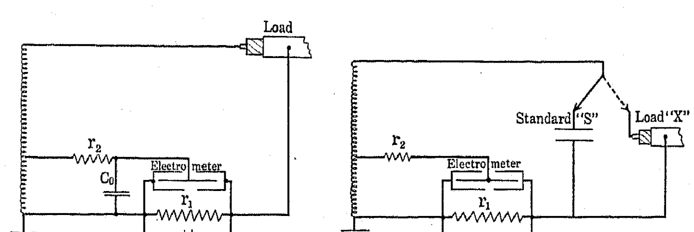

FIG. 4 AND 5

4. OPERATION AND SOLUTIONS OF THE ZERO METHOD WITH CIRCUIT CONSTANTS I_h, $\cos \psi$, $\sin A$ AND $r_1 C_1 \omega$ UNKNOWN

In the majority of practical cases of measuring dielectric losses of cables, the relatively simple conditions of equation (**6**) obtain. The power factor may be quite accurately determined by taking two balances one with a quadrant resistance r_1 and the other with a quadrant resistance $r_1/2$, after the manner of Ray-

ner's deflection method. That is, from these two readings it is possible immediately to eliminate the second and third terms of equation (**6**) and solve for $\cos \varphi$.

This method, however, would not work for very small specimens when the fuller equation (**7**) applies, as may be seen by inspection. Furthermore, it is assumed that C_1 is constant, while actually there may be a slight change between the two readings inasmuch as the distributed capacity to ground in the quadrant resistance is an element of C_1, and the method involves changing the resistance.

A method of overcoming these difficulties has been developed involving the use of a zero loss air condenser of about the same capacity as the specimens. A reading is first taken on the zero loss condenser, using quadrant resistance equal to r_1. Remembering that $\cos \varphi = O$, we may make the following equation:

$$(I_h/I) \cos \psi + (1 + I_h/I)\, r_1\, C_1\, \omega + (1 + I_h/I) \frac{[n\, I_h - (n-2)\, I]\, r_1}{2\, E} + \sin A = r_2\, C_2\, \omega \quad \mathbf{(10)}$$

If we now reduce r_1 to $r_1/2$, and balance with r_2' of potential resistance, there may be a slight change in capacity to ground, which will be called C_r. We then have:

$$(I_h/I) \cos \psi + 1/2\, (1 + I_h/I)\, r_1\, (C_1 + C_r)\, \omega + 1/2\, (1 + I_h/I) \frac{[n\, I_h - (n-2)\, I]\, r_1}{2\, B} + \sin A = r_1'\, C_2\, \omega \quad \mathbf{(11)}$$

Now assume that we increase the capacity to ground by means of an air condenser in such a way as to make $r_2'\, C_2\, \omega$ exactly half of $r_2\, C_2\, \omega$. Calling this added capacity C_x we have

$$(I_h/I) \cos \psi + 1/2\, (1 + I_h/I)\, r_1\, (C_1 + C_r + C_x)\, \omega + 1/2\, (1 + I_h/I) \frac{[n\, I_h - (n-2)\, I]\, r_1}{2\, E} + \sin A$$
$$= 1/2\, .\, (I_h/I) \cos \psi + 1/2\, (1 + I_h/I)\, r_1\, C_1\, \omega + 1/2 (1 + I_h/I) \frac{[n\, I_h - (n-2)\, I]\, r_1}{2\, E} + 1/2 \sin A \quad \mathbf{(12)}$$

and therefore,

$$(1 + I_h/I)\, r_1\, C_x\, \omega = \frac{-\, I_h}{I} - \cos \psi - (1 + I_h/I)\, r_1\, C_r\, \omega - \sin A \quad \mathbf{(13)}$$

Now balance without C_x in circuit on a load whose capacity to ground is C_1', whose power factor is $\cos \varphi$, and whose load current is closely equal to I using a quadrant resistance r_1, and we have

$$\cos \varphi + (I_h/I) \cos \psi + (1 + I_h/I)\, r_1\, C_1'\, \omega + (1 + I_h/I) \frac{[n\, I_h - (n-2)\, I]\, r_1}{2\, E} + \sin A = r_2''\, C_2\, \omega \quad \mathbf{(14)}$$

Now add to the high quadrant, a capacity to ground equal to C_x and change r_1 to $r_1/2$. Then we have

$$\cos \varphi + (I_h/I) \cos \psi + 1/2\, (1 + I_h/I)\, r\, (C_1' + C_r + C_x)\, \omega + 1/2\, (1 + I_h/I) \frac{[n\, I_h - (n-2)\, I]\, r_1}{2\, E} + \sin A = r'''\, C_2\, \omega \quad \mathbf{(15)}$$

Substituting from (**13**) for the term involving C_x, we get

$$\cos \varphi + 1/2\, (I_h/I) \cos \psi + 1/2\, (1 + I_h/I)\, r_1\, C_1'\, \omega + 1/2\, (1 + I_h/I) \frac{[n\, I_h - (n-2)\, I]\, r_1}{2\, B} + 1/2 \sin A = r_2'''\, C_2\, \omega \quad \mathbf{(16)}$$

Doubling (**16**) and subtracting (**14**) from it we obtain the final equation.

$$\cos \varphi = (2\, r_2''' - r_2'')\, C_2\, \omega \quad \mathbf{(17)}$$

Inasmuch as I_h, $\cos \psi$, $\sin A$, $r_1\, C_1\, \omega$, and C_r do not appear in equation (**17**), it may be used to solve for the power factor when the circuit constants are unknown.

5. Operation and Solution of the Zero Method by Comparison with a Standard

The diagram of connections for this method as developed by us is given in Fig. 5. The low-voltage electrodes of the load and of the zero loss condenser used as a standard are permanently connected to the high quadrant maintaining constant capacity to ground except as noted hereafter. (See Section 6). Two balances are taken one with high voltage on the standard and the other with high voltage on the load of unknown power factor. If the charging currents of the standard and the load are approximately the same, equation (**7**) will apply to both readings, $\cos \varphi$ being equal to zero for the first balance. Subtracting equation (**7**) as applied to the first reading from the same equation applying to the second reading will therefore give the unknown power factor. If I is very large compared to I_h the charging currents of the standard and the load need not be equal if $n = 2$.

If the auxiliary condenser C_0 used in the needle circuit is of such capacity that $C_2\, \omega = 1$ or 1×10^N where N is any whole number, and two resistance boxes are used for the potential resistance, the application of this method is simple, convenient, and quick. A balance is first obtained on the zero loss condenser, with one of the potential resistance boxes, the voltage is shifted to the load and another balance obtained using the other resistance box. The unknown power factor can then be read directly from the resistance setting on the second box without calculation.

6. The Error Due to Shunting Capacity

The above method of measurement is based on the assumption that the total capacity to ground of the high quadrant and all connected parts is held constant by permanently connecting the low-voltage electrodes of both the standard condenser S and the unknown load X to the high quadrant. This is not usually the

case, however, as may be seen from what follows. Both S and X are in reality made of three condensers shown diagrammatically in Fig. 6.

Let C_a = capacity between high and low-voltage electrodes.

C_b = capacity between high-voltage electrode and ground (including the guards which are also grounded).

C_c = capacity between low-voltage electrode and ground (including the guards which are also grounded).

With high voltage on S, the total capacity to ground is

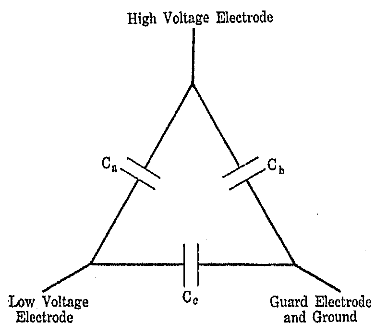

FIG. 6—DIAGRAM OF THE THREE CONDENSERS OF WHICH A CONDENSER IS COMPOSED

made up of the capacity to ground of the high quadrant and all connecting parts, the distributed capacity to ground of the quadrant resistance, the C_c capacities of S and X and in addition, since the high-voltage lead of X is disconnected, a capacity which we shall call C_{ab} made up of the C_a and C_b capacities of X in series and therefore equal to:

$$\frac{C_a \times C_b}{C_a + C_b}$$

In the same way when voltage is on X, there is an additional capacity to ground made up of the C_a and C_b capacities of S in series. Only in case these two additional capacities are equal is the capacity to ground of the system a constant.

In our measurements the above condition does not hold. It was perfectly possible, however, to measure the a, b and c capacities of both S and X, as shown in Sec. 8, to permit of the necessary corrections by substitution of the values so obtained in the equations. If S is a guarded condenser only one set of measurements is necessary unless the temperature varies within indefinite limits. Then as C_a of X must be measured anyway to obtain the load current, only two extra measurements must be made, the C_b and C_c capacities of X. The necessity of making these measurements is eliminated however, if S is of variable capacity. The C_a capacity of the standard can then be made exactly equal to that of the unknown, and if the C_b capacity of the condenser which is not connected to the high voltage is short-circuited by connecting its high-voltage electrode to ground, the additional capacity to ground is always C_a, a constant. The construction of a variable standard air condenser to be operated at high voltage is a somewhat formidable proposition, but it is believed by us by no means impossible of achievement. The relatively small maximum capacity required would greatly lessen the difficulties in construction.

In many cases it may be inconvenient to use any of the methods outlined above. The power factor of a given load can be determined by one reading if I_h, I, $\cos \psi$, $\sin A$ and C_1 are known. The methods used in obtaining these quantities will now be outlined.

7. DETERMINATION OF I_h

Two methods were available for the measurement of this current. In the first, a non-inductive resistance of 1,000,000 ohms was placed between the high quadrant and ground. The voltage drop across this resistance with 7000 volts impressed on the needle was then read on a low-voltage electrostatic voltmeter, from which the current was readily calculated. I_h varies with the position of the needle in the quadrants and the value when the deflection is zero should be used in the equations. It was therefore necessary to put another resistance of 1,000,000 ohms in series with the low quadrant to maintain zero deflection of the instrument. The use of a three-electrode vacuum tube as an amplifier greatly reduced the amount of resistance required in the first method. With two stages, amplification of over 700 times was obtained, thereby reducing the resistances in series with the quadrants to about 1400 ohms. If a condenser of known capacity is shunted across the quadrants and the procedure of Section 13 carried out, the value of I_h can be obtained

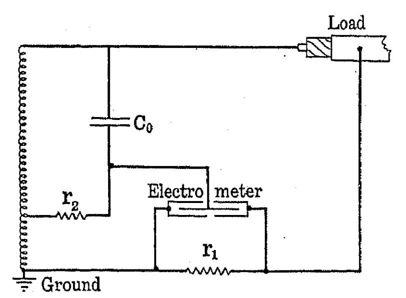

FIG. 7—DIAGRAM OF CONNECTIONS SHOWING LOCATION OF THE AUXILIARY CONDENSER C_0 IF THE INITIAL DEFLECTION IS NEGATIVE

by solving equation (**19**). The value thus obtained was used to check the first method.

8. DETERMINATION OF $r_1 C_1 \omega$

Assuming that the capacity to ground of the quadrant resistance does not change appreciably between r_1 and $r_1/2$, $r_1 C_1 \omega$ can be measured by taking two balances with $n = 2$ on a given load, the first with r_1 and the second with $r_1/2$. Applying equation (**8**) to both readings, subtracting the second equation thus

obtained from the first and dividing by $(1 + I_h/I)$, we obtain an equation which can be directly solved for $r_1 C_1 \omega$.

9. Determination of sin A

If we make the same assumption regarding the capacity to ground of the quadrant resistance as in the last paragraph, sin A may be easily obtained. First determine the value of $r_1 C_1 \omega$ by the method of the last paragraph using a zero-loss condenser as the load. Then take two balances on the same load, one with $n = 1$, and voltage on the needle and load equal to E, and the other with $n = 2$, voltage on the needle of E and voltage on the load of $2E$ this keeping I_h constant. Two equations can now be written and solved for sin A.

10. Determination of cos ψ

If $r_1 C_1 \omega$ and I_h are known, the value of this quantity at any voltage E can be obtained from equation (**9**) by taking a reading on a zero-loss air condenser with $n = 1$. The equation can then be solved for cos ψ.

11. Determination of cos ψ and $r_1 C_1 \omega$

Cos ψ may be determined in one way absolutely independently of our ability to calculate exactly $r_1 C_1 \omega$ as described in paragraph 8, if only the constant, K, of the instrument be known, as of course it would have to be known if used for the deflection method. Incidentally $r_1 C_1 \omega$ becomes accurately known.

Load the instrument with a zero-loss condenser. Make n equal 1 and take a reading at voltage E by the deflection or zero method, preferably the latter. Calling the reading A, then

$$A = (I_h/I) \cos \psi + (1 + I_h/I)\, r_1 C_1 \omega + (1 + I_h/I) \frac{(I + I_h)\, r_1}{2E}$$

Remove the voltage from the zero-loss condenser by disconnecting its high lead but not its low lead. Then keeping everything else the same as before, put the voltage E on the instrument and take deflection, d, and we then have:

$$\frac{K d}{r E I_h} = \cos \psi + 1/2 \frac{I_h r}{E} + r_1 C_1 \omega + r_1 C_{ab} \omega \quad \textbf{(18)}$$

C_{ab} being the shunting capacity of the zero-loss condenser which is now added since the voltage is removed from the high-voltage electrode (see paragraph 6). But C_{ab} is known or can be measured as explained in paragraph 8, and therefore from the above two equations, cos ψ and $r_1 C_1 \omega$ may be calculated, as they are the only unknowns.

12. Determination of $r_1 C_1 \omega$, cos ψ and sin A

We have found it most convenient to determine the capacity to ground as well as cos ψ once for all by means of measurements with two zero-loss condensers of different capacities. Our main purpose in this was to check the general equation for the electrometer, but it also gave a very convenient method of calculating these quantities. One actual set of readings is given in the experimental section under Part III paragraph 23, and will therefore not be described in detail here.

13. Measurement of Capacity by the Electrometer

Capacities may be very conveniently measured by the electrometer used as a zero instrument by merely balancing the deflection of the electrometer with voltage connected to a condenser of zero or constant power factor, and then connecting the condenser of unknown capacity between the high quadrant and ground and obtaining another balance. An examination of equation (**7**) will show that the only change introduced by the insertion of additional capacity to ground is in the term containing C_1, and therefore we may write immediately

$$(1 + I_h/I)\, r_1 C_x \omega = (r_2' - r_2)\, C_2 \omega \quad \textbf{(19)}$$

in which C_x is the unknown capacity, r_2 is the potential resistance which was used to balance before C_x was inserted, and is r_2' is the potential resistance after the insertion of C_x.

This method of measuring capacity has been used in general and in particular for the measurements of C_a, C_b and C_c of our condensers, which is described in detail in Part III, paragraph 22.

14. Effect of Ratio of Potential Division

Most of our measurements have been made with either full voltage or half voltage on the needle. The advantage of using full voltage on the needle is that the phase shift of the needle voltage is eliminated but the disadvantage is that, speaking in terms of power rather than of power factor, one-half the loss in the quadrant resistance is also measured. In general, as pointed out many times before there are distinct advantages in working with half voltage on the needle, because the loss in the quadrant resistance disappears entirely (except in so far as due to I_h) as may be seen from equation (**7**), and this is often of considerable importance, especially in those cases where the simpler equations (**3**) and (**6**) can be applied.

If n equals 2, *i. e.*, half voltage on the needle, the instrument will always deflect (neglecting sin A) unless cos φ, cos ψ, and C_1 are all zero. The direction of this deflection will be defined as the positive direction. If n is very large and the loss small, the instrument may give a negative deflection, as may be seen from equation (**7**). In this case, the insertion of resistance r_2, instead of bringing the negative deflection back to zero will merely increase it in a negative direction.

15. Negative Deflections

If the initial deflection is negative, the zero method must be modified, and several alternatives are available.

(A) The deflection may be brought back to zero by adding capacity between the high quadrant and ground, or preferably, the deflection may be made positive by

this means, and then brought back to zero by the potential resistance r_2, making the above described zero method available. This may become objectionable, however, when it makes the value of $r_1 C_1 \omega$ very large, inasmuch as the part of $r_2 C_2 \omega$ due to load power factor may be very small, and slight errors in $r_2 C_2 \omega$ might make a large error in the power factor.

(B) Another method of accomplishing the result is to change the position of the auxiliary condenser in the zero power factor method from that shown in Fig. 4 to its position in Fig. 7. Speaking in terms of instantaneous values, it will be seen that the direction of current in resistance r_2 would be reversed by this method, and that therefore the needle voltage would be shifted in the opposite direction.

(C) A method based on Miles Walker's suggestion of adding resistance in the high-voltage lead of the specimen is also available. Connect the low-voltage electrodes of S and X together and to the high quadrant, exactly as explained in Section 5. Put voltage on S which must have the same load current I as the unknown X, and add resistance in the high lead until a zero deflection or a slightly positive deflection is obtained, which can be brought back to zero by the usual potential resistance r_2. Now shift the voltage to X, keeping the same resistance in the high lead and again bring the deflection back to zero by increasing r_2. Then, for the simplest case the following equation will apply.

$$\cos \phi = (r_2' - r_2)\, C_2\, \omega$$

Unfortunately the equation above will not apply unless not only the load currents of S and X are the same but also the currents from the high-voltage electrodes to the guards and ground, *i. e.*, C_b must be the same. A correction must also be made due to the fact that the total capacity to ground is not constant, as explained in Section 6.

(D) Another method due to Russell[2] might be used. In this method the deflection is brought back to zero by varying n. It may be seen from equation (4) that the deflection will be zero if:

$$n = \frac{2\,E\,(I \cos \varphi + I_h \cos \psi)}{r_1\,(I^2 - I_h^2)} + \frac{2\,(E\,.\,r_1 C_1\,\omega + E \sin A + I\,r_1)}{r_1\,(I - I_h)} \qquad (20)$$

Assuming that all the circuit constants were known and that this value of n had been determined, it would be possible to solve at once for $\cos \varphi$ in the above equation.

Incidently we have here a check on the derivation of the main formulas presented in this paper. Equations (4) and (7) for the deflection and zero method were derived independently but it will be noted that one side of each is the same. This must necessarily be the case since the two methods coincide for the value of n giving zero deflection as both d and r_2 are then equal to zero.

There are great difficulties in applying this method as varying n with a given value of E means varying the needle voltage, and to accomplish continuous variation of n means a transformer with an infinite number of taps on the high-voltage winding, or difficulties with a high-voltage potential-divider.

(E) When the initial deflection is negative due to a large value of n, probably the best method is to use the connections due to Petersen[3] although this method has not been tried out practically. Quite constant results have however, been obtained with the deflection method and Petersen condensers, with $E = 40{,}000$ volts and the needle voltage of only 1000. Petersen obtained the needle voltage by means of a potential divider composed of two condensers in series, with the needle connected to the mid-point as shown in the diagram of connections, Fig. 8A. The method as suggested by us would consist in placing these two condensers between the *midpoint* of the transformer and ground. With this arrangement the great advantage of using $n = 2$ would be obtained regardless of the ratio of potential division and the initial deflection will always be positive. A resistance inserted between the mid-point of the transformer and the first of the potential condensers shown in Fig. 8A would serve for r_2.

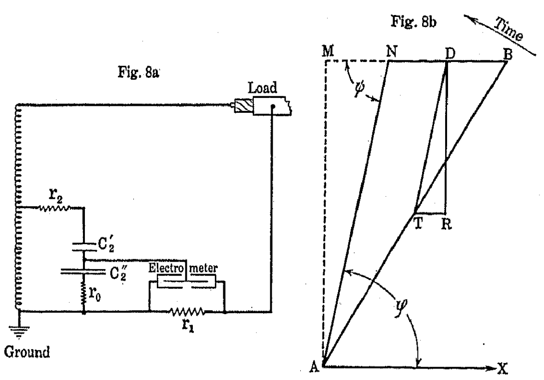

Fig. 8—Petersen Series Condensers Used as Potential Divider

One end of the potential condenser system used in the Petersen method is connected to the mid-point of the quadrant resistance, the center of symmetry. The charging current of these condensers will therefore flow through one half of the quadrant resistance to ground, causing an appreciable error for which correction must be made. This difficulty could be largely avoided if the condenser system was only virtually connected to the mid-point of the quadrant resistance, that is, if a resistance, r_0, was inserted between the low-voltage condenser and ground of such value that the voltage drop across this resistance would be equal to one-half

2. A. Russell, Alternating Currents, 1914. Vol. 1, page 320.
3. Petersen, *Archiv fur Elektrotechnik*, 1912, Vol. 1, page 95.

of the voltage drop across the quadrant resistance. Dielectric losses in the potential condensers and the capacity to ground of the various parts would cause a slight residual error.

The method outlined in this section presents one great advantage in that it can be applied for very high values of n, or what is the same thing, dielectric loss measurements at high voltage could be made with a very sensitive low-voltage electrometer. In such an instrument, I_h would be negligible and due to the high sensitivity, the value of r_1 required for an accurate balance would be so small that the term $r_1 C_1 \omega$ would become negligible. The simple conditions shown in vector diagram 8B would then exist, the voltage drop across the quadrants $N B$, being parallel to the vector of reference $A X$ the load current, angle α vanishing. All the correction terms would disappear and $\cos \varphi$ would equal $r_2 C_2 \omega$ where C_2 equals the capacity of C_2' and C_2'' in series. In practise C_2'' would be variable and adjusted to give the proper needle voltage. It is important to mention that this method is based on the assumption that it is possible accurately to make the virtual connection of the potential condenser system to the mid-point of the quadrant resistance. The simplicity of the solution is due to the fact that angle $T D R = 90 - \varphi$, in Fig. 8B. If the virtual connection is not exact, the vertex of this angle will not fall at D thus introducing an error. The small values of resistance, however, required with a sensitive low-voltage electrometer would probably make this error negligible.

Instead of connecting r_2 between the transformer and the condenser C'_2, it might be connected between the condenser C'_2 and the point where the electrometer needle is connected, in other words r_2 would then be at a relatively low potential. The solution would be the same, and the advantage of having r_2 at a lower potential would be obtained.

(*F*) Another way of handling a negative deflection is to diminish r_1 until the deflection becomes positive, then balance as usual with r_2. As recently pointed out by Addenbrooke if n is greater than 2, a zero method is available by merely varying r_1. In the very simple case when C_1 is negligible, this would give excellent results. It would be very difficult to apply, however, to a comparative method where two balances are made one on a standard zero-loss air condenser and one on the load, as the change in r_1 would change the value of the term $r_1 C_1 \omega$. Furthermore C_1 which includes the distributed capacity of r_1 would vary with r_1.

16. Bridge Method of Electrometer

The electrometer may be used in a zero method according to the connections shown in Fig. 9, in which it will be seen that high voltage is placed upon both S and X, each connected to ground through a resistance, the upper end of each resistance being connected to one set of quadrants of the instrument. A resistance is inserted in the high lead of S. The procedure is the following. First connect both specimens to high voltage, and ground the needle of the instrument. Adjust r_1' and r_1'' until the electrometer reads zero. Secondly, connect the needle to the desired potential point and cut in resistance r_2 in series with S until the electrometer again reads zero. In the simplest case, the usual equations of an a-c. bridge will apply. It will be noted that this is in effect merely the two balances, first capacity and second power factor, in a power factor bridge, the electrometer being used as the detector. This method will not be developed further, although it is obvious that the same difficulties will be presented due to the capacity to ground and the needle charging current, if the specimens are so small that these are effective.

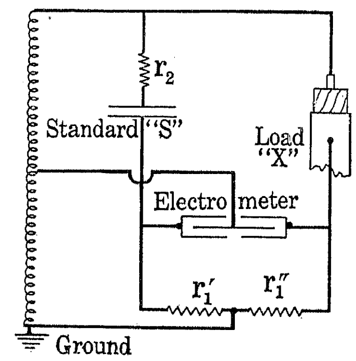

Fig. 9—Diagram of Bridge Method

17. Conclusions

In concluding, we hope that the complexity of some of our general equations and of some of the methods of measurement described will not create an impression that the quadrant electrometer is necessarily a very complicated instrument for the measurement of power factor. In attempting to obtain expressions for some of the smaller errors, we have possibly not emphasized sufficiently the great simplicity of the instrument for most practical cases. In the majority of practical measurements, we have no doubt that the method outlined in the first paragraph in Part I, section 4, will be amply sufficient. In fact, with rather large samples of cable, measuring with half voltage on the needle, the power factor may often be obtained directly from the setting of the potential resistance, r_2, according to equation (6), since the third term will vanish and the second term will probably be negligible.

Any conclusion would be incomplete without a word of appreciation of the late Charles W. Davis, who actually initiated the present work on the quadrant electrometer, practically designed the instrument, and suggested the possibility of developing the zero method just described. His continued interest and enthusiasm were of the utmost assistance in carrying this work to its conclusion.

The theoretical and mathematical sections of the

paper are the work of D. M. Simons, and the experimental section, of W. S. Brown.

Part II—Mathematical

(18) Derivation of Equation of Deflection of Electrometer Neglecting Needle Charging Current

The diagram of Fig. 2 B shows the vector diagram. $A X$, the load current is the vector of reference. $A N$ is the voltage on the specimen, lagging by angle φ, $\cos \varphi$ being the power factor of the load. This voltage will be represented by E. Let the voltage across the quadrants $= E_q = N B$. Let the transformer voltage be $E_0 = A B$. Let the needle voltage be $E_n = O D$.

Maxwell shows that the deflection of the electrometer is proportional to the scalar product of the voltage across the quadrants by the voltage from the needle to the mid-point of the quadrants, or in our case, we may say that the deflection is proportional to $E_q \times E_n$. The method of solution will therefore consist in finding the symbolic expressions for these two vectors and taking their product, which will be done as follows:

$$\dot{E}_0 = \dot{E} + \dot{E}_q \tag{1}$$

$$\dot{E} = E \cos \varphi + j\, E \sin \varphi \tag{2}$$

where I is the vector of reference.

The load current I flows through the resistance r_1 and capacity C_1 which are in parallel across the quadrants of the electrometer. The admittance of this circuit is therefore equal to

$$1/r_1 + \frac{1}{j\,.\,1/C_1\,\omega} = 1/r_1 - j\, C_1\, \omega$$

Therefore $\dot{E}_q = \dfrac{I}{1/r_1 - j\, C_1\, \omega}$

$$= \frac{r_1 I\,(1 + j\, r_1\, C_1\, \omega)}{1 + r_1^2\, C_1^2\, \omega^2} \tag{3}$$

Now $\dot{E}_n = \dot{E}_0/n - \dot{E}_q/2$, and from (1) therefore

$$\dot{E}_n = \dot{E}/n - \frac{n-2}{2}\, \dot{E}_q \tag{4}$$

Substituting (2) and (3) in (4), we obtain:

$$\dot{E}_n = 1/n \left[\left(E \cos \varphi - \frac{n-2}{2} \cdot \frac{r_1 I}{1 + r_1^2\, C_1^2\, \omega^2} \right) + j \left(E \sin \varphi - \frac{n-2}{2} \cdot \frac{r_1 I\,.\,r_1\, C_1\, \omega}{1 + r_1^2\, C_1^2\, \omega^2} \right) \right] \tag{5}$$

If d is the deflection of the electrometer and K is a constant, according to the Maxwell theory $K\, d = \dot{E}_n \times \dot{E}_q$. We may therefore substitute the values of these two vectors from (3) and (5), and after taking the "power product" and simplifying, we obtain:

$$\frac{n\, K\, d\,(1 + r_1^2\, C_1^2\, \omega^2)}{E\, I\, r_1} = \cos \varphi + r_1\, C_1\, \omega\,.\, \sin \varphi - \frac{n-2}{2} \cdot \frac{r_1 I}{E} \tag{6}$$

Equation (6) has also been derived geometrically, directly from the vector diagram.

19. Derivation of Equation for Zero Method with Electrometer, Neglecting Charging Current of Needle

As in section 18, E, E_0 and E_q, represent the load, transformer, and quadrant voltages respectively, represented by the vectors $A\,N$, $A\,B$ and $N\,B$ in Fig. 3B. Let E_p be the voltage on the potential elements, namely the resistance r_2 and capacity C_2 in series, shown by the vector $T\,D$. Let I_p be the current flowing in this circuit. Let E_n be the needle voltage, represented by $R\,D$, $T\,R$ being the voltage drop across r_2.

From the diagram, it will be seen that

$$\dot{E}_0 = \dot{E} + \dot{E}_q \tag{1}$$

$$\dot{E}_p = \dot{E}_0/n - \dot{E}_q/2 = \dot{E}/n - \frac{n-2}{2}\,.\,\dot{E}_q \tag{2}$$

$$\dot{E}_q = \frac{r_1 I\,(1 + j\, r_1\, C_1\, \omega)}{1 + r_1^2\, C_1^2\, \omega}$$

From equation (3) section 18 (3)

$$\dot{E} = E \cos \varphi + j\, E \sin \varphi \tag{4}$$

Substituting (3) and (4) in (2), we get

$$\dot{E}_p = 1/n \left[\left(E \cos \varphi - \frac{n-2}{\cdot 2} \cdot \frac{r_1 I}{1 + r_1^2\, C_1^2\, \omega^2} \right) + j \left(E \sin \varphi - \frac{n-2}{2} \cdot \frac{r_1 I\,.\,r_1\, C_1\, \omega}{1 + r_1^2\, C_1^2\, \omega^2} \right) \right] \tag{5}$$

For simplicity, let $M_1 = 1 + r_1^2\, C_1^2\, \omega^2$ and $M_2 = 1 + r_2^2\, C_2^2\, \omega^2$.

r_2 and C_2 being in series, the impedance of the potential circuit is $r_2 + j \dfrac{1}{C_2\, \omega}$, and therefore

$$\dot{I}_p = \frac{\dot{E}_p\,.\, C_2\, \omega\,(r_2\, C_2\, \omega - j\, 1)}{M_2} \tag{6}$$

and $$\dot{E}_n = j\, \frac{1}{C_2\, \omega} \times \dot{I}_p = \frac{\dot{E}_p\,.\,(1 + j\, r_2\, C_2\, \omega)}{M_2} \tag{7}$$

The symbolic representation of $\dot{E}_n$ may therefore be obtained by substituting the value of E_p from (5) in (7). Due to its length, this will not be done here.

If the electrometer is to read zero, E_n must be perpendicular to E_q. If two vectors are perpendicular, the slope (imaginary coefficient divided by real coefficient) of one must equal the negative reciprocal of that of the other. Setting up this condition and simplifying algebraically, we obtain finally

$$\cos \varphi\,(1 + r_1\, C_1\, \omega\,.\, r_2\, C_2\, \omega) = \sin \varphi\,(r_2\, C_2\, \omega - r_1\, C_1\, \omega) + \frac{n-2}{2} \cdot \frac{r_1 I}{E} \tag{8}$$

In most practical cases, $r_1\, C_1\, \omega\,.\, r_2\, C_2\, \omega$ is negligible, and if $\cos \varphi$ is small, $\sin \varphi$ may be taken as unity, and

we obtain an approximate equation similar to that of the deflection method:

$$\cos \varphi + r_1 C_1 \omega - \frac{n-2}{2} \cdot \frac{r_1 I}{E} = r_2 C_2 \omega \quad (9)$$

Equation (8) may also be obtained easily geometrically.

20. Derivation of General Equation for Zero Method of Electrometer

This derivation is so excessively long algebraically that certain portions will be merely outlined. One equation for instance contained 48 terms, each composed of a product of from 3 to 6 factors of the form $\cos \varphi$, $r\,C\,\omega$, etc. It is believed therefore that an outline will be sufficient.

In the vector diagram of Fig. 10, I the load current is the vector of reference, namely $A\,X$. $A\,B$ or E_0 is the transformer voltage. $A\,N$ or E is the load voltage, $\cos \varphi$ being the power factor, φ being the angle $N\,A\,X$. E_q or $N\,B$ is the voltage across the quadrants of which more will be said later.

The voltage on the potential circuit, r_2 and C_2 in series, is $T\,D$ or E_p, and I_p is the current in this circuit. The effective voltage on the needle to which a deflection would be due is $R\,D$ or E_n.

Let I_h be the current from the needle to the high quadrant, due to the voltage $R\,N$ or E_h. If there is any loss in this circuit, I_h will not be perpendicular to E_h, but will lead it by an angle ψ shown as angle $H\,N\,R$. I_h will therefore be in the direction $H\,N\,L$. If there were only resistance across the quadrants, the voltage drop across the quadrants due to I_h would be in the direction $N\,L$. Inasmuch as the capacity across the quadrants C_1 also exists, the drop due to I_h will lag behind $N\,L$ by some angle α and will be represented by $N\,E$. The drop across the quadrants due to the load current I would be in the direction $N\,I$ (parallel to the load current $A\,X$) if C_1 were not present. Due to C_1 it also lags behind $N\,I$ by the same angle α and may be represented by $N\,J$. The total drop across the quadrants mentioned above, $N\,B$, is therefore the vector sum of $N\,J + N\,E$.

The equation will now be developed. By inspection,

$$\dot{E}_0 = \dot{E} + \dot{E}_q \quad (1)$$

$$\dot{E}_p = \dot{E}_0/n - \dot{E}_q/2 = \dot{E}/n - \frac{n-2}{2} \cdot \dot{E}_q \quad (2)$$

$$\dot{E} = E \cos \varphi + j\,E \sin \varphi \quad (3)$$

$$\dot{E}_n = \dot{I}_p \times j \frac{1}{C_2\,\omega} \quad (4)$$

Since I is the vector of reference, $\dot{I} = I + j\,O$. Let the angle between I and I_h be β, namely the angle $I\,N\,L$, which angle must later be eliminated. Then $\dot{I}_h = I_h \cos \beta + j\,I_h \sin \beta$, and the total current through the quadrant resistance and capacity will be $\dot{I}_0 = I + I_h \cos \beta + j\,I_h \sin \beta$.

Now $$\dot{E}_q = \frac{\dot{I}_0}{1/r_1 + \dfrac{C_1\,\omega}{j}}$$

Let $M_1 = 1 + r_1^2\,C_1^2\,\omega^2$
and $M_2 = 1 + r_2^2\,C_2^2\,\omega^2$ for simplicity (5)

Then substituting $\dot{I}_0$ in (5) and simplifying, we obtain:

$$\dot{E}_q = 1/M_1 [\,(I\,r_1 + I_h\,r_1 \cos \beta - I_h\,r_1 \,.\, r_1\,C_1\,\omega \,.\, \sin \beta) + j\,(I\,r_1 \,.\, r_1\,C_1\,\omega + I_h\,r_1 \,.\, r_1\,C_1\,\omega \cos \beta + I_h\,r_1 \sin \beta)] \quad (6)$$

Now $$\dot{I}_p = \frac{\dot{E}_p}{r_2 + j\dfrac{1}{C_2\,\omega}} \quad (7)$$

Therefore $$\dot{E}_n = \frac{\dot{E}_p \,.\, (1 + j\,r_2\,C_2\,\omega)}{M_2} \text{ from (4)} \quad (8)$$

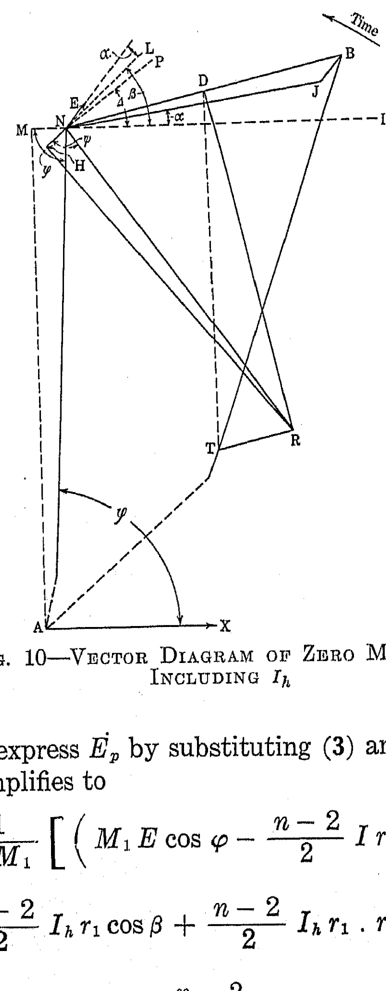

Fig. 10—Vector Diagram of Zero Method Including I_h

We may express $\dot{E}_p$ by substituting (3) and (6) in (2), which simplifies to

$$\dot{E}_p = \frac{1}{n\,M_1}\Bigg[\Bigg(M_1\,E \cos \varphi - \frac{n-2}{2}\,I\,r_1 - \frac{n-2}{2}\,I_h\,r_1 \cos \beta + \frac{n-2}{2}\,I_h\,r_1 \,.\, r_1\,C_1\,\omega \sin \beta\Bigg) + j\Bigg(M_1\,E \sin \varphi - \frac{n-2}{2}\,I\,r_1 \,.\, r_1\,C_1\,\omega - \frac{n-2}{2} \,.\, I_h\,r_1 \,.\, r_1\,C_1\,\omega \cos \beta - \frac{n-2}{2}\,I_h\,r_1 \sin \beta\Bigg)\Bigg] \quad (9)$$

The value of $\dot{E}_p$ from (9) is now substituted in (8) and the expression for $\dot{E}_n$ obtained. This will not be done here due to its length, but it will be called equation (10).

The condition that the deflection of the electrometer shall be zero is that E_n and E_q are perpendicular, or analytically that the slope of one vector is equal to the negative reciprocal of that of the other. We have the expressions for these two vectors in (6) and (10). If the above equation is set up and simplified algebraically (this is the long equation mentioned in the beginning) we obtain the following:

$$[I \cos \varphi + I_h \cos (\varphi - \beta)] (1 + r_1 C_1 \omega \,.\, r_2 C_2 \omega) = [I \sin \varphi + I_h \sin (\varphi - \beta)] (r_2 C_2 \omega - r_1 C_1 \omega) + \frac{n-2}{2} \cdot \frac{r_1 (I^2 + 2 I I_h \cos \beta + I_h^2)}{E} \quad (11)$$

The above equation (11) will of course reduce to equation (8) of Section 19, if $I_h = O$. This equation is rigid, and no simplifications have so far been made. It can not, however, be used in its present form due to the presence of the unknown angle β, which must now be eliminated.

To get rid of β, draw the line $N P$ perpendicular to $N R$. By inspection, angle $L N P = 90° - \psi$. Let angle $P N I = \Delta$. Then $\beta = \Delta + (90 - \psi)$.

$$\text{Therefore} \quad \tan \beta = \frac{\tan \Delta + \cot \psi}{1 - \tan \Delta \cot \psi} \quad (12)$$

This equation gives β in terms of the constant of our circuit, ψ, and the unknown angle Δ, which will now be evaluated. Tan Δ is the slope of the line $N P$, which is by hypothesis perpendicular to $N R$, or E_h. The solution will therefore consist in finding the expression for E_h and tan Δ will be the negative reciprocal of its slope.

From the figure, it will be seen that

$$\dot{E}_0/n = \dot{E}_{r2} + \dot{E}_h + \dot{E}_q \quad (13)$$

where $\dot{E}_{r2}$ is the voltage across r_2, or $T R$. Therefore

$$\dot{E}_h = \dot{E}_0 - \dot{E}_q - \dot{E}_{r2} = \dot{E}/n - \frac{n-1}{n} \dot{E}_q - \dot{E}_{r2} \quad (14)$$

$$\dot{E}_{r2} = r_2 \dot{I}_p = \frac{r_2 C_2 \omega (r_2 C_2 \omega - j\, 1) \,.\, \dot{E}_p}{M_2} \quad \text{See (7)} \quad (15)$$

$\dot{E}_{r2}$ may therefore be obtained by substituting $\dot{E}_p$'s value from (9) in (15). $\dot{E}_h$ may then be obtained by substitution of (3), (6) and (15) in (14).

As stated above, tan Δ is the negative reciprocal of the slope of E_h, and if the above process be carried out, it will be found that:

$$\tan \Delta = - \frac{\begin{gathered}2 M_1 E \cos \varphi - (2n - 2 + n r_2^2 C_2^2 \omega^2)(I r_1 \ldots \\ \ldots + I_h r_1 \cos \beta - I_h r_1 \,.\, r_1 C_1 \omega \sin \beta) \ldots \\ \ldots - 2 M_1 E r_2 C_2 \omega \sin \varphi + \ldots \\ \ldots + (n-2) I r_1 \,.\, r_1 C_1 \omega \,.\, r_2 C_2 \omega \ldots \\ \ldots + (n-2) I_h r_1 \,.\, r_1 C_1 \omega \,.\, r_2 C_2 \omega \cos \beta \ldots \\ \ldots + (n-2) I_h r_1 \,.\, r_2 C_2 \omega \sin \beta\end{gathered}}{\begin{gathered}2 M_1 E \sin \varphi - (2n - 2 + n r_2^2 C_2^2 \omega^2) \ldots \\ \ldots (I r_1 \,.\, r_1 C_1 \omega + I_h r_1 \,.\, r_1 C_1 \omega \cos \beta + I_h r_1 \sin \beta) \ldots \\ \ldots + 2 M_1 E r_2 C_2 \omega \cos \beta \ldots \\ \ldots - (n-2) I r_1 \,.\, r_2 C_2 \omega \ldots \\ \ldots - (n-2) I_h r_1 \,.\, r_2 C_2 \omega \cos \beta \ldots \\ \ldots + (n-2) I_h r_1 \,.\, r_1 C_1 \omega \,.\, r_2 C_2 \omega \sin \beta\end{gathered}} \quad (16)$$

If the value of tan Δ from (16) is substituted in (12), and the resultant value of β is substituted in (11), a rigid equation of the power factor as measured by this zero method will be obtained, as no approximations have been introduced so far. Unfortunately, the complicated expression for tan Δ does not collapse if this is done, and therefore the rigid solution if obtainable would have no practical value due to its complexity. It will therefore be necessary to introduce approximations at this point. The method of solution will therefore be to simplify the expression for tan Δ. This will in most cases introduce no appreciable error, for β itself is not a very important part of equation (11).

Examination of the various terms of (16) shows that it is made up of quantities of very disproportionate magnitudes. E is very large, while I_h, $r_1 C_1 \omega$, $r_2 C_2 \omega$, cos φ, and sin β are very small. Sin φ, cos β and M_1 are close to unity. The first term in the denominator of tan Δ reduces to $2 E$, which is very large, while all the other terms contain at least one of the very small quantities. The entire denominator may therefore be taken as $2 E$. The first term in the numerator is $2 E \cos \varphi$, which contains one of the small quantities. All terms containing two or more of the small quantities may therefore be dropped. We may therefore state that:

$$\tan \Delta = - \cos \varphi + \frac{(n-1)(I + I_h) r_1}{E} + r_2 C_2 \omega \quad (17)$$

Some of the terms dropped may however become significant if n is very large. Inspection of the figure, especially when it is elongated to its practical proportions shows that β must be a very small angle. We may therefore substitute (17) in (12) as follows: Since tan Δ cot ψ is negligible:

$$\tan \beta = \sin \beta = - \cos \varphi + \frac{(n-1)(I + I_h) r_1}{E} + r_2 C_2 \omega + \cos \psi \quad (18)$$

and $\cos \beta = 1$.

The substitution of these values in (11) will give a complicated but very accurate equation for cos φ. For all practical purposes however, the equation may be still simplified by taking sin φ as unity and dropping the term $r_1 C_1 \omega \,.\, r_2 C_2 \omega$ and thereby obtain the following final equation:

$$\cos \varphi + (I_h/I) \cos \psi + (1 + I_h/I) r_1 C_1 \omega + (1 + I_h/I) \frac{[n I_h - (n-2) I] r_1}{2 E} = r_2 C_2 \omega \quad (19)$$

Before applying this formula to the measurement of extremely small samples, consideration should be given to whether or not $\sin \varphi$ may be taken as unity and $r_1 C_1 \omega \cdot r_2 C_2 \omega$ may be dropped. If this is not justifiable, the procedure outlined in the paragraph should be adopted. Large values of n may also introduce errors in $\sin \beta$ in its approximate form.

Equation (**19**) has been also independently derived by an entirely geometric method directly from the vector diagram, on the assumption that the angles are so nearly zero (or 90 deg. as the case may be) that the sines of large angles and cosines of small angles may be taken as unity.

A similar method of procedure for the case of measurement in parallel of two specimens of different power factors, $\cos \varphi_1$ and $\cos \varphi_2$, and different charging currents, I_1 and I_2, gives the following equation which is sometimes of value.

$$\frac{I_1 \cos \varphi_1}{I_1 + I_2} + \frac{I_2 \cos \varphi_2}{I_1 + I_2} + \frac{I_h}{I_1 + I_2} \cos \psi$$

$$+ \left(1 + \frac{I_h}{I_1 + I_2}\right) r_1 C_1 \omega + \left(1 + \frac{I_h}{I_1 + I_2}\right)$$

$$\frac{[n I_h - (n-2)(I_1 + I_2)]\, r_1}{2E} = r_2 C_2 \omega \quad \mathbf{(20)}$$

The derivation of the general equation for the electrometer used as a deflecting instrument, given in (**4**) of Part I of this paper, will not be developed due to similarity to the above derivation. The method used was to find E_q which is exactly the same as (**6**) of the zero method in this section, and finding the effective needle voltage E_n which is the same as E_p given in equation (**9**) of the zero method. $K d$ then equals the scalar product of these two complex quantities and an equation is obtained which contains the unknown angle β. β is then eliminated by a method similar to that used above, though somewhat simpler, and equation (**4**) of Part I of this paper is obtained.

Part III—Experimental

21. DESCRIPTION OF APPARATUS, SOURCE OF POWER

The variations in frequency and voltage on the lines of the local Central Station were too great to permit of the accurate electrical measurements required. These variations especially when combined with the effect of factory vibration on the instruments made rapid observations impossible. The wave form also departed from that of the sine wave.

In view of this condition, a complete outfit was installed for the production of high-voltage alternating-current of constant voltage, constant frequency, and good wave-shape. Two motor generator sets were used. The first of these consisted of a 10-h. p., 2-phase, 440-volt, 60-cycle, squirrel-cage induction motor, direct-connected to a 5.5-kw., 125-volt, direct-current compound-wound generator. In the second set, a -h.p., 1700-rev. per min., 110-volt direct-current shunt-wound motor was direct connected to a 5.0/2. kw., 3-phase 220/110-volt, 60/25-cycles, 1800/750-rev. per min. special sine-wave generator. Central station service was used to supply the alternating-current induction motor of the first set through a starting compensator, fuses, relays, etc. The direct current obtained from the generator of this set was used to supply the direct-current motor of the second set and the alternating current obtained from one phase of the sine-wave generator was used to feed the primary of a 5-kv-a., 110/220-50,000-volt, 60-cycle single-phase testing transformer of special design. This transformer was provided with taps at ½, ¼ and ⅛ voltage.

In order to obtain greater constancy of voltage and frequency of the sine-wave alternating current, a Tirrell voltage regulator was connected to the d-c. generator and a small speed regulator was mounted on the shaft of the d-c. motor. By this means the variations were reduced to a very small amount. Table I gives a comparison of the wave shape of the sine-wave generator at no load and at full load and zero power factor, as compared with a sine-wave.

TABLE I

	No Load Ordinates		Full Load Unity Power Factor Ordinates		Full Load Zero Power Factor Ordinates	
Degrees	Voltage Wave	Equivalent Sine-wave	Voltage Wave	Equivalent Sine-wave	Voltage Wave	Equivalent Sine-wave
0	0.00	0.00	0.00	0.00	0.00	0.00
15	2.50	2.58	2.57	2.58	2.50	2.58
30	5.02	5.00	5.03	5.00	5.00	5.00
45	7.12	7.07	7.08	7.07	7.18	7.07
60	8.70	8.66	8.62	8.66	8.72	8.66
75	9.66	9.66	9.65	9.66	9.75	9.66
90	9.85	10.00	10.05	10.00	9.90	10.00

The maximum deviation of the voltage wave from the equivalent sine-wave is only about 3 per cent.

QUADRANT ELECTROMETER

The design of the instrument as constructed in the laboratory of the Standard Underground Cable Company is of marked simplicity. The base which consists of a large iron casting 20 inches in diameter is supported on three legs which serve for leveling screws. This base in turn supports another casting, or sub-base on ball bearings. This sub-base which carries the entire instrument is centered and free to turn, thus providing adjustment of the instrument with respect tothe scale. On a glass plate 18 inches in diameter, ¼-inch thick supported on three steel rods ¾ inches in diameter, screwed into the sub-base, rests the upper portion of the instrument from which the moving system is suspended. Lateral and vertical motion of this upper portion on the glass plate permits of centering of the needle in the quadrants. The needle can also be made to take up any desired position in the quadrants

by revolving this portion of this instrument on its axis. The needle is constructed in the shape of a figure eight, the major axis being 6⅝ inches and the maximum width 3⅜ inches. It is made of thin sheet aluminum with an edge formed of ¼-inch aluminum tubing. The suspension is a phosphor bronze strip 12 inches long and about 20 by 2 mils in cross-section. A small frame soldered to the lower end of the suspension carries a mirror with one meter radius of curvature, used to reflect a beam of light on a 50-centimeter glass scale. The quadrants are of cast aluminum. They were made by putting a 2-inch hole in the center of two aluminum disks 14¼ inches in diameter, and then cutting each of them into four equal sectors. The quadrants are supported on glass, the nearest points of support being 3 inches apart. The spacing between the upper and lower quadrant is 2 inches and between adjacent quadrants 1/16 inches. Damping is accomplished by means of a light aluminum vane moving in a cup containing a mixture of glycerin and water. The damping cup is supported on a glass plate which is provided with leveling screws resting on the sub-base.

The sensitivity of the instrument may be defined by stating that the constant of the instrument, K, is about 3500.

The Quadrant Resistances, r_1

These resistances aggregating 210,000 ohms were all Curtis wound non-inductive units. Two boxes containing 100,000 ohms each in 10,000 ohm steps and a third box, containing 10,000 ohms in 9 steps of 1000 ohms, 9 of 100 ohms and 10 of 10 ohms were used. This arrangement permitted a very fine adjustment. These boxes were mounted on a table at the right of the electrometer scale and were completely surounded by a grounded metallic shield, through which protruded long, hard rubber tubes fastened to the dials and provided with pointers running over numbered scales to permit of reading the resistance setting from the outside of the shield. A positive "click" on all boxes prevented possibility of error.

The Potential Resistances, r_2

A three-dial decade box with 9 steps of 1000 ohms, 9 steps of 100 ohms, 10 steps of 10 ohms and a 50,000 ohm box containing 10 steps of 5000 ohms were used for the potential resistance. These resistances were also Curtis wound non-inductive and were surrounded with a metallic shield connected to the transformer terminal supplying the needle voltage. They were suspended upside down over the observer's head and were provided with long handles, scales and pointers, similar to those on the quadrant resistances.

The Potential Condenser, C_2

A 12,000-volt Dubilier condenser with a capacity of about 40×10^{-10} farads was used for C_0. The capacity of this condenser, was not, however, taken as C_2 in the equations as C_2 must be the capacity which determines the entire current through r_2. This capacity includes the Dubilier condenser, the capacity to ground of the guarded lead from the resistance box to the needle and the capacity from the needle to the quadrants and the electrometer housing, which when determined by measurement of the current at 7000 volts came out 42.13×10^{-10} farads. Before the sine-wave generator was installed, the value of C_2 varied considerably depending upon the amount of potential resistance r_2 in the circuit. This variation was probably due to the damping out of some of the harmonics by resistance. It was therefore necessary to plot a curve showing the value of C_2 as a function of the value of r_2. With the sine wave generator, however, the value of C_2 as calculated from the total impedance of this circuit, the voltage and the charging current, came out practically a constant independent of the value of r_2.

The ratio of transformation of the transformer as furnished by the manufacturer was exactly 110/50,000 and this ratio was used to determine the secondary voltage. The primary voltage was read on standardized electrodynamometer voltmeters and was controlled by a resistance in the field of the sine wave generator. Frequency was read on a carefully calibrated Frahm vibrating reed instrument with 1/2 cycle reeds.

Three methods of measuring current were used. In the first method the voltage drop across the quadrant resistances was measured by a Kelvin 0-110 volt multicellular electrostatic voltmeter, connected through a mercury switch. A double-pole single-throw switch in the quadrant leads served to disconnect the quadrants, thus avoiding the small error due to the charging current between the high and low quadrants; and a double-pole, double-throw highly-insulated switch served to connect the electrostatic voltmeter either across the resistance or across a standardized electrodynamometer voltmeter supplied with current from the sine wave generator. Rapid and accurate calibration of the electrostatic voltmeter was thus obtained. Two highly sensitive vacuum thermocouples and a Paul microammeter and a specially constructed milliampere current transformer with a very sensitive ammeter were used in the second and third methods. These served principally as a means of checking the value of current as determined by the first method.

Air Condensers

Two air condensers of practically zero loss were available. Air condenser No. 1 was composed of 4 parallel galvanized iron plates about 3 feet square and separated by about 2 inches. The two inner plates were connected together forming the high-voltage electrode and the outer two plates also connected together formed the low-voltage electrode. The low-voltage plates were in two sections, the central ones of about 4 square feet in area forming the low-voltage

plates proper, and the outer ones the guards. Hard rubber strips securely fastened to both sections served as a means of support for the inner sections. The entire condenser with the exception of three small holes, one for the high-voltage lead, one for the low-voltage lead and one for connection to the guard electrode, was surrounded by a grounded metallic screen in order to shield the low-voltage electrode and to make its capacity to ground definite and constant. Voltages up to 20,000 were applied to the condenser without flashover and no ionization could be detected as high as 17,000 or 18,000 volts. These facts in connection with the careful guarding led us to believe that the losses in the condenser at 7000 and 14,000 volts were extremely small.

The other condenser, No. 2, of later and better design, was cylindrical in form. Two concentric copper tubes formed the electrodes. The inner one or high-voltage electrode was three inches in diameter. The outer tube was in three sections, the central one being the low-voltage electrode and the outer two the guard rings. The separation used was 1/8 inches. The entire condenser was surrounded by a heavy copper tube with steel caps at both ends and suitable high and low-voltage terminals. This condenser was always operated under 100 pounds pressure of CO_2 in order to cut down any loss due to ionization. Due to the general design, the presence of CO_2 and the absence of solid dielectric between the high and low-voltage plates, both being supported by the guards we believe the losses in the condenser to be practically zero at 14,000 volts.

22. Experimental Check of the Measurement of Capacity

In Section 13 a description was given of a method of measuring capacity by shunting the unknown condenser between the high quadrant and ground. This method was used to measure C_a, C_b and C_c of air condensers Nos. 1 and 2.

In the case of A C No. 1 (A C will be used for air condenser), the procedure was as follows: 14,000 volts was impressed on A C No. 2 alone, using one half voltage on needle, r_1 = 100,000 ohms, $f = 60$, and the resultant deflection was brought to zero by a value of r_2 = 18,600 ohms, $r_2 C_2 \omega$ was equal therefore to 0.02955. With the high voltage impressed on A C No. 2, the various electrodes of A C No. 1 were then connected between the high quadrant and ground as indicated in Table II, which gives the connections, the value of potential resistance required in each case to bring the deflection back to zero, and further calculations shown in columns 3 and 4. Column 4 refers specifically to the right hand side of equation (19), Section 13.

It will be noted by comparison with Fig. 6, that the readings shown in the above table measure in each case the sum of two capacities, which are: $C_a + C_b$, $C_b + C_c$, and $C_c + C_a$ respectively for the three readings.

TABLE II

Connections of A C No. 1 (1)				
To High Quadrant	To Ground	r_2' (2)	$r_2' C_2 \omega$ (3)	$(r_2' - r_2)x$ $C_2 \omega$ (4)
h-v electrode	l-v electrode and guards	31,600	0.05019	0.02064
h-v and l-v electrodes	guards	33,630	0.05342	0.02387
l-v electrode	h-v electrode and guards	31,100	0.04940	0.01985

Now $I_h = 30 \times 10^{-6}$ amperes, $I = 0.565 \times 10^{-3}$ amperes (A C No. 2 being the load in this case), and therefore $1 + I_h/I = 1.053$. Substituting all these values in equation (19), we find that:

$$C_a + C_b = 5.197 \times 10^{-10} \text{ farads}$$
$$C_b + C_c = 6.008 \times 10^{-10} \text{ farads}$$
$$C_c + C_a = 4.997 \times 10^{-10} \text{ farads}$$

We have therefore that $C_a = 2.11$, $C_b = 3.10$ and $C_c = 2.90$, all in 10^{-10} farads.

The charging currents corresponding to the C_a and C_b capacities of A C No. 1 were also measured by means of a non-inductive resistance shunted by a low-voltage electrometer, from which we obtain by calculation that $C_a = 2.1$ and $C_b = 3.0 \times 10^{-10}$ farads, giving a close check. The value of C_c could not be measured by this means as the air condenser would not stand a high enough voltage between the low-voltage plates and the guards.

A C No. 2, the voltage being impressed on A C No. 1, gave the following values of capacity:

$$C_a = 1.07 \times 10^{-10} \text{ farads}$$
$$C_b = 0.35 \times 10^{-10} \text{ farads}$$
$$C_c = 5.2 \times 10^{-10} \text{ farads}$$

These values are the results of numerous measurements and are checked by charging current readings, minor variations of as much as 2 per cent being averaged out in the values given above, which will be used in the following sections.

23. Check of General Equation (7) by Six Readings

Equation (7) of Section 3 is of course the fundamental and basic equation of this paper. The main distinction between it and the old equations which neglect the needle charging current, is a series of terms containing the ratio of needle charging current to load current. In order to check equation (7), therefore, it is necessary to have two condensers with as small a capacity as is practicable with the sensitivity of our instrument, in order to make I_h/I as large as possible, and the two should have different capacities in order to have a sufficient number of equations.

We therefore performed a series of measurements on our two air condensers in various connections and at different voltages. In all the following cases the low-voltage electrodes of both condensers were permanently

connected to the high quadrant. $r_1 = 100{,}000$ ohms and $f = 60$. Table III shows the connections and voltages of the two readings.

TABLE III

Reading Number (1)	Connections: High Voltage on (2)	Voltage (kv.) on Load (3)	Voltage (kv.) on Needle (4)	C_1 (5)	$1 + I_h/I$ (6)
1	A C No. 1 & A C No. 2	7	7	$8.10 + C_x$	1.0357
2	A C No. 1	7	7	$8.364 + C_x$	1.0539
3	A C No. 2	7	7	$9.355 + C_x$	1.0626
4	A C No. 1 & A C No. 2	14	7	$8.10 + C_x$	1.0179
5	A C No. 1	14	7	$8.364 + C_x$	1.0269
6	A C No. 2	14	7	$9.355 + C_x$	1.0531

It will be noted that $n = 1$ for the first three readings and $n = 2$ for the last three. We know I_h, and the charging currents of the condensers, and can therefore obtain the term $1 + I_h/I$ as shown in column 6, remembering that for readings 1 and 4 the charging current is the sum of the charging currents of the two condensers. We know the capacity to ground, C_1, except for the instrumental capacity to ground. In readings 1 and 4 the capacity to ground in addition to the instrumental capacity is the sum of the C_c capacities of A C No. 1 and A C No. 2. In readings Nos. 2 and 5 it is the same only increased by the C_{ab} of A C No. 2, while in readings 3 and 6 the capacity is the same as in readings 1 and 4 except that it is increased by the C_{ab} capacity of A C No. 1.

Referring first to readings 1, 2 and 3, the general equation in the form of equation (**9**) of Section 3 applies directly to readings 2 and 3, while equation (**20**) of Section 20 applies to reading 1 where the load is composed of two condensers. We may safely assume that the power factor of A C No. 2 is zero. There are therefore three unknowns in the equations namely $\cos\phi$ of A C No. 1, $\cos\psi$, and that part of the total capacity to ground which is due to the instrument, quadrant resistance, and leads, which will be called C_x. We have three equations and three unknowns and can therefore solve. The three equations are very closely satisfied by $\cos\varphi = O$, $\cos\psi = 0.00484$, and $C_x = 2.513 \times 10^{-10}$ farads.

Referring now to readings 4, 5 and 6, equation (**20**) of Section 20 applies to 4, while equation 8 of Section 3 applies to readings 5 and 6. There is an additional unknown, $\sin A$, introduced here, but we may substitute the values of $\cos\varphi$, $\cos\psi$ and C_x in three equations, and obtain three solutions for $\sin A$, which give as an average $\sin A = -0.0008$.

The easiest method of demonstrating the accuracy of this determination is to substitute all these values in equation (**7**) of Section 3 and calculate r_2. A comparison of these calculated values of r_2 with the actual values of potential resistance used will show the accuracy of the equations, according to Table IV:

TABLE IV

Reading	Potential Resistance r_2: Actual Value used	Potential Resistance r_2: Value Calculated from Equation (7)	Error: Ohms	Error: Per cent
No. 1	30,200 ohms	30,252	+ 52	+ 0.17%
" 2	30,080 "	30,191	+ 111	+ 0.36
" 3	33,040 "	33,046	+ 6	+ 0.02
" 4	25,350 "	25,324	− 26	− 0.10
" 5	26,260 "	26,262	+ 2	+ 0.01
" 6	29,290 "	29,472	+ 182	+ 0.62

The maximum error is 0.6 per cent. This is really quite accurate if the sensitivity of our electrometer is considered.

24. Check of Actual Power Factor Measurements

The ability of the instrument to measure power factor by the zero method was investigated by measurements on an artificial load whose power factor could be readily calculated. This load consisted of an air condenser in series with a variable non-inductive low-capacity resistance placed in the high-voltage lead of the condenser in order not to change the capacity to ground for the various readings. This resistance was completely shielded thereby introducing a small shunting capacity around the resistance which both calculation and test showed to be negligible. Five power factor readings were made with air condenser No. 1 and five with No. 2, each reading requiring two balances, one with the resistance short-circuited and one with the desired amount of resistance cut in. As equation

TABLE V
ARTIFICIAL LOAD OF AIR CONDENSER NO. 2 IN SERIES WITH RESISTANCE

Lad o f Air Condenser No. 2	r_2	$r_2\,C_2\,\omega$	Measured Power Factor	Calculated Power Factor	Per cent Error	Actual Watts Measured
Alone	18,530	0.029433	..	..	..	0.0
4,843 ohms	18,690	0.029687	0.000254	0.000259	− 2.0%	0.00155
21,798 "	19,330	0.03070	0.00127	0.00117	+ 8.6	0.00702
58,120 "	20,500	0.03256	0.00313	0.00311	+ 0.7	0.01861
118,650 "	22,570	0.03585	0.00642	0.00636	+ 0.8	0.0379
239,780 "	26,620	0.04228	0.01285	0.01285	0.0	0.0766

Impressed Voltage = 14,000 r_1 = 100,000 ohms
Needle Voltage = 7,000 f = 60 cycles

TABLE VI
ARTIFICIAL LOAD OF AIR CONDENSER NO. 1 IN SERIES WITH RESISTANCE

Load of Air Condenser No. 2	r_2	$r_2\,C_2\,\omega$	Measured Power Factor	Calculased Power Factor	Per cent Error	Actual Watts Measured
Alone	12,910	0 02051	..	..	..	0.0
4,843 ohms	13,530	0.02149	0.00098	0.00095	+ 2.8%	0.00596
21,798 "	15,550	0.02470	0.00419	0.00428	− 2.1	0.0271
58,120 "	20,000	0.03177	0.01126	0.01141	− 1.3	0.0721
118,650 "	27,320	0.04340	0.02289	0.02332	− 1.8	0.1465
239,780 "	42,010	0.06673	0.04622	0.04711	− 1.7	0.2976

Impressed Voltage = 14,000 r_1 = 100,000 ohms
Needle Voltage = 7,000 f = 60 cycles

(8) of paragraph 3 applies to these readings, the power factor for each reading can be obtained by subtracting the $r_2 C_2 \omega$ with short-circuited resistance from the $r_2 C_2 \omega$ with resistance cut in. The actual watt loss measured is equal to the square of the current through the resistance in the high lead times the value of that resistance. The current flowing through the resistance in the high lead is made up of two parts, one due to the C_a capacity of the condenser, the other to the C_b capacity, since the guards were grounded. The calculated power factor of the artificial load is therefore $r\ (C_a + C_b)\ \omega$, where r is the resistance in the high lead.

The foregoing two tables give the results of the power factor measurements, the resistance in the high lead being shown under the load.

Discussion

S. J. Rosch: I would like to ask the authors concerning the limitations of the apparatus and the measurements. For example, can we use it on 22,000 or 33,000 volt cable?

If I remember correctly, Mr. Brown referred to a limitation of 0.3 watt, in other words he claimed to have measured as high as 0.3 watt. If we take the care of a 33,000 volt cable and measure its dielectric loss at say 80° or 90° centigrade, the power factor of a good cable at that voltage and temperature would run about 3 to 4 per cent and the corresponding loss would amount to approximately one watt per foot.

I am also under the impression that the value of this paper would have been enhanced considerably, had the authors given some relative data on measurements obtained with both the quadrant electrometer and the dynamometer wattmeter methods. Curves of this nature would in my estimation, have given a better conception of the value of the method advocated.

D. M. Simons: Mr. Doyle and Mr. Rosch both bring up the question of our not having shown any actual measurements of losses in cables. We did not believe that the present paper was an appropriate place for showing data of this kind, since this is an article on an instrument rather than on cables, and our only interest was in losses which we could measure and which we could also calculate, and thereby check the accuracy of our equations.

The method Mr. Doyle shows is very interesting. The more methods we have the better off we are, in measuring the extremely low losses which are difficult measurements at best. I might add that in our company we have two separate methods: in our Perth Amboy laboratory we have a special bridge for measuring losses, and in Pittsburgh we use the quadrant electrometer. It is very convenient to be able to check measurements made by two such different methods.

In reply to Mr. Doyle's question, we have used the electrometer practically entirely for research work, and not for routine measurements; we have used it almost entirely on single-phase work. Seven or eight years ago, however, we made three-phase measurements with the electrometer, using the deflection method. It of course might be used equally advantageously with the zero method.

Mr. Rosch asked whether the electrometer may be used for measuring losses in reels of 22,000-volt and 33,000-volt cables, especially at the high temperatures. This question was apparently asked particularly because we showed in our tables the results of measuring extremely small losses. The electrometer can, of course, be used very conveniently for the measurement of larger losses and higher power factors, and, in fact, in these cases some of the difficulties mentioned in our paper practically disappear, the effect of the shunting capacity and needle current being negligible, and the measurements can usually be made with extreme ease and rapidity. In other words, the same instrument used to measure a 10th or 100th of a watt at low power factor can easily measure kilowatts, the only requirement being that considerably lower values of quadrant resistance will be required.

Recent Advances in the Design, Manufacturing and Testing of Static Condensers in Power Sizes

BY R. E. MARBURY
Asscoiate, A. I. E. E.
Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Review of the Subject.—*The static condenser is particularly desirable as a power factor corrective device since it requires no attention. To fully capitalize this advantage static condensers must be reliable.*

The reliability of insulation subjected to a constant potential stress, depends on the ratio between the working stress and the disruptive stress.

This ratio has been kept large in all electrical apparatus and years of successful use have followed.

Static condensers are described, which have been designed with insulation factors of safety such as have proved dependable in other apparatus for years.

These ratios have been maintained and the product still kept on a reasonable commercial basis by increased economy in construction and use of material and by accurate check of insulation quality in every unit.

* * * * *

A STATIC condenser is a device for storing electrostatic energy. It is similar in characteristics to an inductance, which stores electromagnetic energy, except that the energy stored and returned to the line is 180 deg. out of phase with the energy stored and returned by an inductance.

The static condenser is an ideal device for compensating for the lagging power factor of inductive loads since the energy for the magnetic field is stored locally as it is released, and is returned at the proper time to rebuild the magnetic field. The line is thus relieved of the need for handling anything but power current.

Wherever insulation is subjected to a potential stress

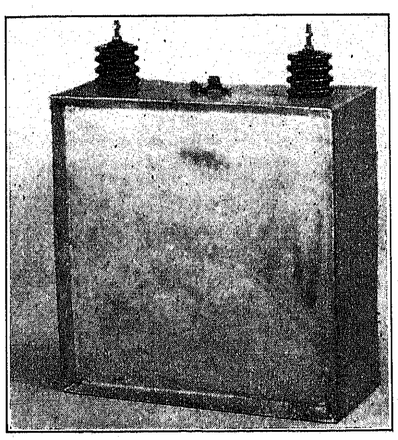

FIG. 1—2 KV-A., 2300 VOLT, 60-CYCLE CONDENSER UNIT TYPE "L. D."

Case dimension 6" x 13½" x 13½". Working stress 145 volts per thousandth of an inch.

as is the case with all electrical apparatus, a static field is established therein. If this potential stress is alternating, electrostatic energy is stored and returned to the source every half cycle. Insulation has been used for years to space electrically current carrying parts. In most cases its function is not only an electrical separator, but also a mechanical separator. Often times the mechanical stresses control the design of insulation, especially where large bodies of metal come in contact with it while expanding and contracting, or when it is subjected to bending or vibration. In addition to the mechanical and electrical stresses placed upon it, it is usually called upon to operate at relatively high temperature. Because in most apparatus, such as transformers, motors and generators, the insulation cost is relatively low as compared with that of other materials, and because the cost of assembly is great, high working stress for the insulation results in little saving in unit cost, while the occasional failure introduced causes a considerable contingent loss. The condition for greatest manufacturing economy for such apparatus is thus the use of low working stress for the insulation.

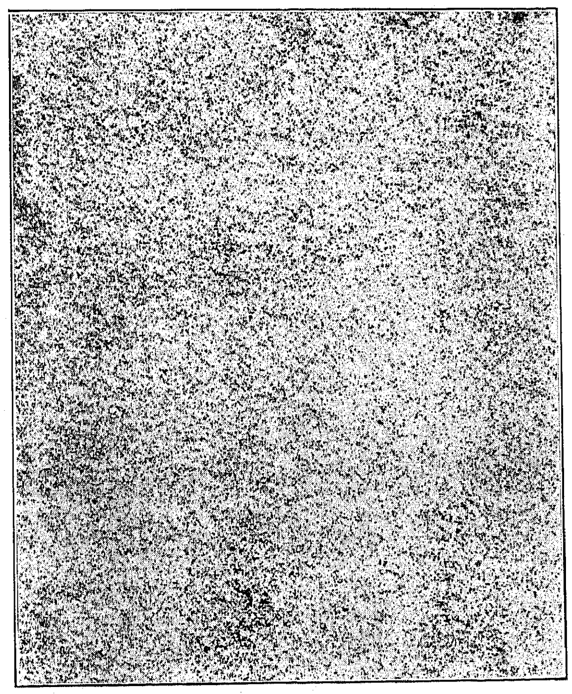

FIG. 2

Sample of thin paper manufactured to be free from holes. The holes in the paper are photographed as black specks by special process. Many of the holes are caused by the pulp being drawn through the screen used in manufacture. The image of the screen may be seen in the arrangements of holes.

When the purpose of the insulation is to store

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

electrostatic energy as in the case with a condenser, the whole situation changes. Here the only active material is the insulation. All accessories such as bushings, case, packing, etc., contribute nothing toward the storing of energy. The insulation, therefore, must for best economy, be worked at or near the maximum safe stress over the entire area.

Early in the development of condensers it was recognized that the conditions under which insulation is used in condensers are more favorable than in other electrical apparatus. The condenser units for power factor correction consist of four stacked sections connected in parallel and assembled in a vacuum tight sheet steel case having one opening for impregnation.

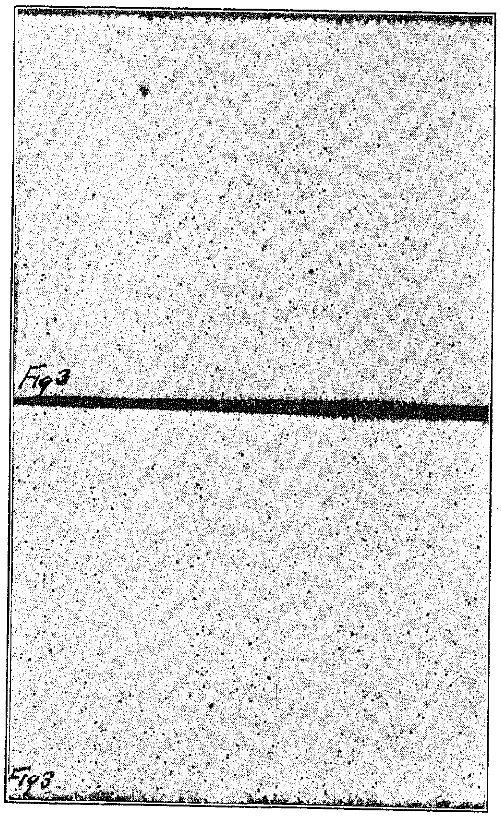

Fig. 3

Samples showing holes in the average thin paper manufactured carefully to be as free of holes as possible.

The sections are made by the inter-leaving paper and metal sheets, alternate sheets of metal extending in opposite directions and soldered to form opposite polarity terminals. Conductors are smooth plates, with no mechanical vibration or bending. Moisture can be more easily removed, thorough impregnation effected and low losses obtained. The operating temperature of the material is very low since the only losses are dielectric losses. The units are entirely filled with oil and sealed, this preventing air or water from entering the insulation during shipment. Early attempts to capitalize on these more favorable conditions led to the construction of condensers designed to work at voltage stresses many many times greater than those used in other apparatus. Laboratory samples have been made to operate thousands of hours at 1100 volts per thousandth of an inch, while commercial units designed with stresses of 400 volts per thousandth of an inch have operated over long periods of time.

When such high dielectric stresses are used, a number of manufacturing difficulties arise. For

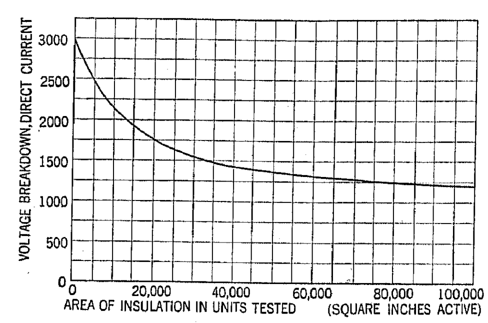

Fig. 4

Curve showing the large variations in breakdown voltage of thin condenser tissue, with area involved in the condensers tested. Tests are made on the unimpregnated material to limit variations to those caused by porosity and foreign conducting particles.

relatively low voltages such as 2300 high stresses mean thin dielectrics between plates. Thin dielectrics with high stresses necessitate utilizing the barrier action of the paper to increase the ultimate strength far beyond that of the oil alone. In order to make use of the barrier action of the paper over large areas of insulation

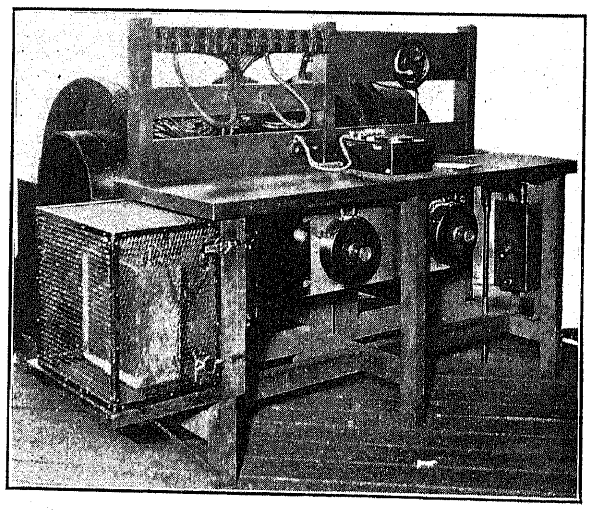

Fig. 5

Equipment for measuring dielectric losses at rated voltage and frequency. The air core inductance is shown at the rear of the rack. The condenser on test is shown in the screened enclosure. The variometer on the right is operated by the handwheel, through a worm reduction.

practically perfect paper is required. The quality is, therefore, largely dependent on porosity in this case.

In addition to the structural requirements the paper must be practically free from metal particles since with very thin dielectric material the size of the conducting

particles is quite comparable to the dielectric thickness and the percentage increase in stress on the insulation where they occur is very great. The difficulties encountered due to metal particles lining up and reducing the breakdown or due to porous paper, increase with the size of the units due to a law of averages and the size of units that can be manufactured on the high stress basis is very limited.

The net result of the early development work using very high stresses showed that it is possible to over capitalize on the more favorable conditions and that we should not lose sight of the practical conditions that have caused us to choose the conservative insulation stresses used in other apparatus. Certain tests have been established which are conceded to insure a sufficient factor of safety for insulation in ordinary apparatus. These tests are based not only on the operating voltage, but also take account of abnormal overvoltage and surge conditions which occur in service. Since the condensers are to operate under conditions very similar to those on which the established tests are based, it is clear that condensers which pass these tests will have the same factor of safety as other apparatus so tested.

It has been found that, due to the high quality of insulation that can be put into condensers less insulations can be used to meet these established test requirements, than in the case of other apparatus. The standard test applied to all static condensers of our present design is 2¼ times normal operating voltage plus 2000 volts for one minute. Advantage has been taken of the more favorable conditions under which condenser insulation is worked, only to the extent that, while the standard test requirements have been met, it has been done by using less insulation and working with higher dielectric stresses than would

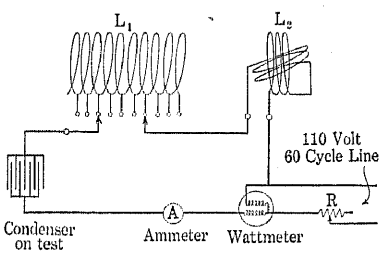

FIG. 6
Diagrams of connections of dielectric loss measuring equipment.

be desirable under the conditions with ordinary power apparatus. Moreover a much greater factor of safety is obtained due to the high quality of material as will be seen from the fact that the average 2300-volt condenser breaks down at 12,000 to 18,000 volts.

The use of large amounts of working material necessary to these low stresses has been made economically feasible by refinements in design which eliminate all unnecessary material and accessory parts as well as the reduction of cost of the paper itself. Since condensers are practically all insulation their cost is directly proportional to the insulation cost and a minimum can only be obtained by using material that gives maximum dielectric strength per dollar invested in insulating material. This does not necessarily mean that the most expensive material is the most economical as the electrical quality of the paper does not increase as rapidly as its cost when attempts are made to produce ideal material. When the cost of the material is plotted against electrical insulation value it is found that the

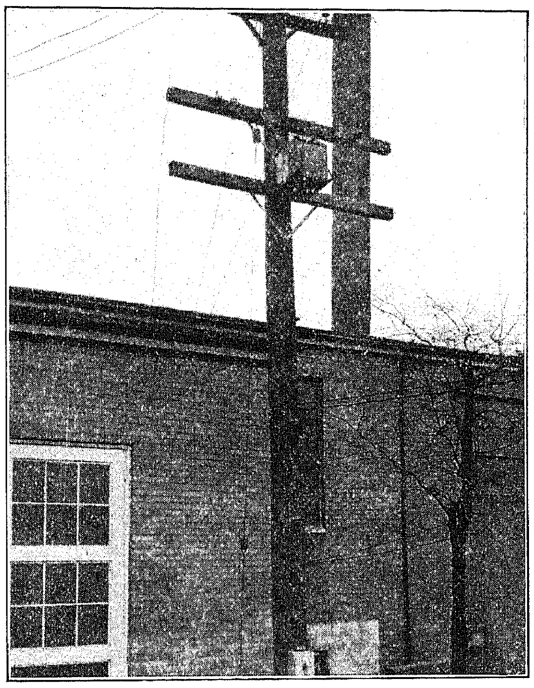

FIG. 7
Three 2-kv-a. units which have operated continuously on a 2200-volt line directly exposed to all weather conditions for over a year.

highly refined materials are not commercially justified and in condensers just as in cable, transformers other apparatus the quality of the material is not so important as the relation between quality and cost. The selection of material is, therefore, an economic study.

An investigation was made as to the exact characteristics required in the paper with the object of reducing the cost due to unnecessary and expensive refinements. This, together with experience over a considerable period has resulted in the use of a very pure wood pulp paper that can be carefully controlled commercially, in so far as foreign materials and quality are concerned.

In carrying out the impregnation of the units, they are assembled along a pipe manifold by means of the pipe thread connection in the center and placed into an oven at 125 deg. cent. Moisture is removed by vacuum and oil is put in while vacuum is maintained. The entire process requires 100 hours. During the treatment over one pound of water is removed from 22 pounds of paper in each 2-kv-a. unit. The process cannot be completed on units having leaks which insure

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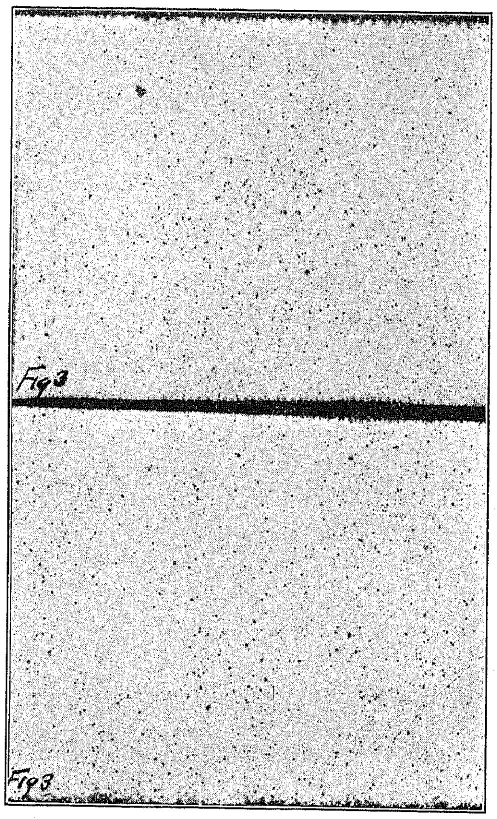

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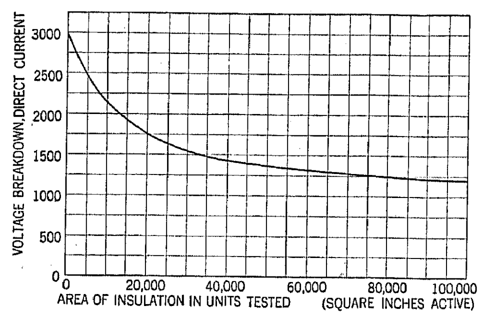
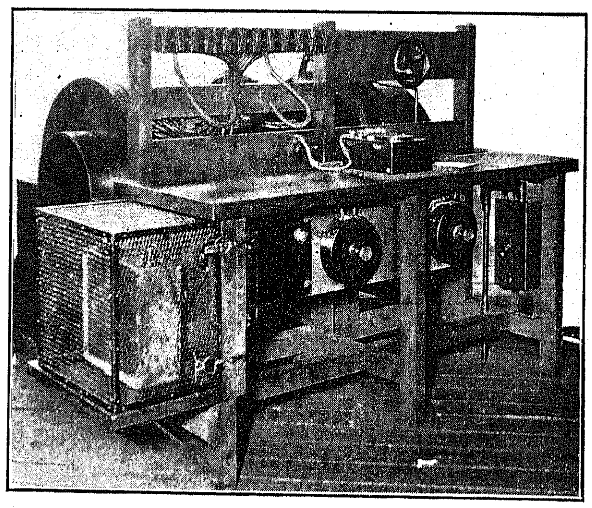


Fig. 4

Curve showing the large variations in breakdown voltage of thin condenser tissue, with area involved in the condensers tested. Tests are made on the unimpregnated material to limit variations to those caused by porosity and foreign conducting particles.



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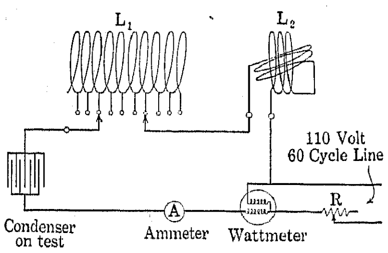

Fig. 6

Diagrams of connections of dielectric loss measuring equipment.

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The use of large amounts of working material necessary to these low stresses has been made economically feasible by refinements in design which eliminate all unnecessary material and accessory parts as well as the reduction of cost of the paper itself. Since condensers are practically all insulation their cost is directly proportional to the insulation cost and a minimum can only be obtained by using material that gives maximum dielectric strength per dollar invested in insulating material. This does not necessarily mean that the most expensive material is the most economical as the electrical quality of the paper does not increase as rapidly as its cost when attempts are made to produce ideal material. When the cost of the material is plotted against electrical insulation value it is found that the

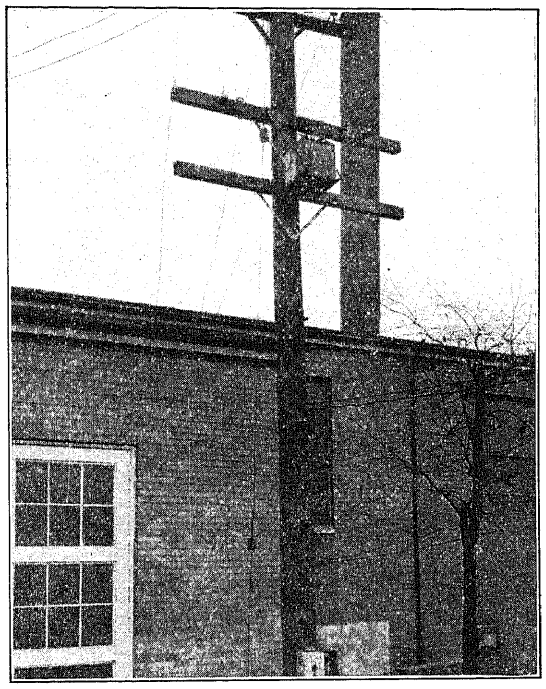

Fig. 7

Three 2-kv-a. units which have operated continuously on a 2200-volt line directly exposed to all weather conditions for over a year.

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An investigation was made as to the exact characteristics required in the paper with the object of reducing the cost due to unnecessary and expensive refinements. This, together with experience over a considerable period has resulted in the use of a very pure wood pulp paper that can be carefully controlled commercially, in so far as foreign materials and quality are concerned.

In carrying out the impregnation of the units, they are assembled along a pipe manifold by means of the pipe thread connection in the center and placed into an oven at 125 deg. cent. Moisture is removed by vacuum and oil is put in while vacuum is maintained. The entire process requires 100 hours. During the treatment over one pound of water is removed from 22 pounds of paper in each 2-kv-a. unit. The process cannot be completed on units having leaks which insure

oil tight units when completed. In order to check the degree of moisture removal before running the oil in a test is made for rate of water omission by means of a liquid air trap.

After the units come from the impregnating ovens they are measured for capacity, the exact values being stamped on the nameplates. They are then tested for dielectric losses first to insure that the impregnating process has been carried out properly and second to maintain the quality from the standpoint of power loss in service. After the losses are measured and it is established that the units are of proper quality each is given an over potential test of 2¼ times rated voltage plus 2000 for one minute. This is r. m. s. voltage and the tests are made on 25 cycles to reduce the loss in the metal parts in the condenser.

The average dielectric loss is ½ of 1 per cent of the kilovolt-ampere. There are many ways of measuring

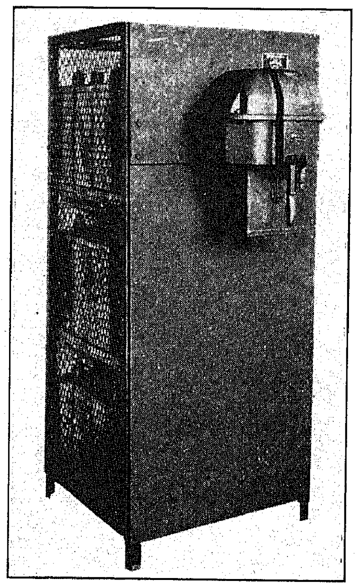

FIG. 8—36-KV-A., THREE-PHASE, 60-CYCLE, 2200-VOLT EQUIPMENT

losses on high voltages at practically zero power factor, but most of them require the use of delicate instruments and extreme care in the handling. For commercial use a method is needed by which every unit can be quickly and accurately measured so that the quality of the product can be closely controlled. The development of such a measuring device has contributed greatly toward the solution of the problem of condenser manufacture.

At the present time such an equipment is in use. The loss is measured under operating conditions with an error of not more than 3 per cent, and less than one minute per condenser is required for the measurement. An inductance of approximately 6 henrys for 2300 volts 60 cycles and having only about twice the loss as the condenser, is connected in series with the condenser to be tested. A 60-cycle voltage is applied and a portion of the inductance which is constructed as a variometer is adjusted so that the whole circuit has roughly unity power factor. The current is adjusted so that the condenser operates at its rated kilovolt-ampere and the loss measured by means of an ordinary wattmeter. By reference to the calibration curve of the inductance, the inductance losses are determined

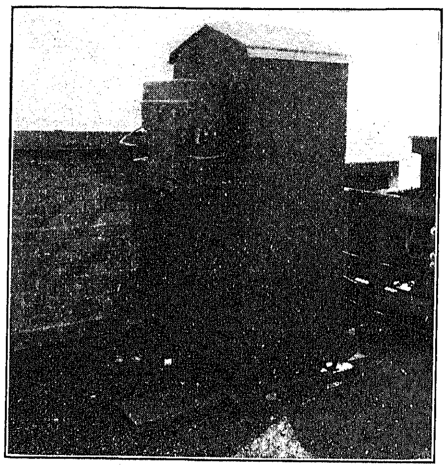

FIG. 9—36-KV-A., THREE-PHASE, 60-CYCLE, 2200-VOLT OUTDOOR INSTALLATION

for the particular current used, and these are subtracted from the total leaving the loss in the condenser.

The inductance was calibrated by means of a pyro-electric wattmeter. This calibration was double-checked by measuring the losses of the inductance and a condenser separately at the low power factors, and then connecting them in series and measuring the

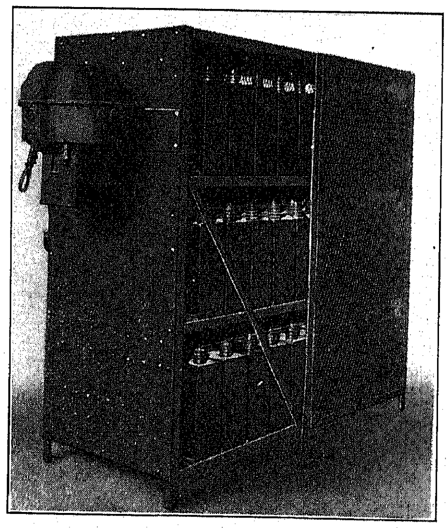

FIG. 10—120-KV-A., THREE-PHASE, 60-CYCLE, 2200-VOLT EQUIPMENT

losses at unity power factor. Extremely accurate tests were made and the sum of the first two checked the third. Such checks were made over the whole working range of the equipment. This establishes the accuracy of the method of measurement.

To meet the various application requirements

assemblies are made of units mounted in structural steel frames and enclosed by expanded metal screens. On the front panel of each is mounted a standard circuit breaker the leads of which connect to vertical bus bars behind the panel. Connections are made between these vertical bus bars and the units with small copper straps to form star, delta or two-phase circuits. The units in the frames are connected to one another by short copper links which may be removed to cut out any desired number of units when seasonal changes of load demand less corrective kilovolt-ampere.

Four standard frame sizes are available for maximum capacities of 36, 60, 96 and 120 kv-a. respectively. These frames are identical except for the depth which varies to accommodate different numbers of units. Larger installations are made up of combination of these standard frames arranged as desired and connected in parallel. This provides great flexibility in application and also permits change of correcting kilovolt-ampere by simply operating the breakers on the individual frames. With large outfits such as 600 and 1000 kv-a. this is very desirable.

Although the obviously large and growing field of application for dependable static condenser equipments has been at times a temptation to begin commercial manufacture prematurely it is now felt that it has been best to delay for complete assurance of quality backed by close control of materials and process with reliable and accurate checks at all stages of manufacture and with test standards for factor of safety of insulation strength which are in general use and which have been proved to insure good service.

Discussion

J. R. Craighead: In any engineering development which leads to the construction of devices of broadly new characteristics, there are usually two stages. The first stage consists in using available materials to produce the device. These materials are rarely exactly adapted to the purpose which they are to serve. The result of this step is usually a costly, heavy, and sometimes unsatisfactory device.

The second stage then must automatically follow. This stage includes the examination of the materials individually, to determine their characteristics, and frequently the development of a better grade of materials to serve the specific purpose more fully. The result of this stage is a device which eventually meets good engineering practise, suitable in cost and in service as related to cost.

In the development of the static condenser for power factor correction, we have completed the first stage and are now reasonably well along in the second stage. Condensers with commercial papers of various thicknesses, various impregnating materials, and various structures (paraffin being at least typical of the general nature of the impregnating materials used) constituted the result of the first stage. These condensers stood up satisfactorily in service under some conditions, but in order to get general satisfactory service from them, it was necessary to reduce the stress per mil in the insulation to the general order of the stress per mil in cables, otherwise they were not permanently satisfactory. The result was that size and cost made the commercial prospect of a condenser that would stand up rather uninviting. Consequently, it became necessary to begin on the second stage.

The second stage consisted of examination of the materials and development of the most suitable. Oil has been very largely substituted for other impregnating materials. We have subjected the type of oil selected to new methods of treatment in order to secure better conditions regarding dryness, purity, and dielectric strength.

Some study has been given to substitutes for paper, of which there is a number available, but the general results indicate the superiority of paper when properly made and applied. We have investigated the manufacture of paper, with a view to perfecting the quality, as to chemical purity, absence of conducting particles, and absence of holes, which tend to cause breakdown.

We have investigated the oil in bulk, as to dielectric strength and considerable improvement has been secured in the quality. We have also investigated this in comparison with the same oil when partially immobilized by the presence of dividing material such as paper.

The result has shown the immense advantage attainable by the use of a number of thin sheets immobilizing the oil very perfectly as compared with a much smaller number of heavier sheets.

The mechanical arrangement of the active parts of the condenser and their relation to electrostatic troubles have also been studied. The results of these studies are embodied in the present condensers which we manufacture.

Regarding Mr. Marbury's paper in detail, it appears to be chiefly a plea for the construction of condensers with a liberal thickness of insulation, as opposed to a construction with insulation of less thickness, but more carefully selected. This is actually a purely commercial question. The factor of safety in the condenser can be made equally great by either means. Consequently, it is not a question of which condenser will break down if the engineering is equally correct in both cases, but it is a question of which condenser will cost the more in the long run, considering first cost, failure or destruction, size of space occupied, and necessary conditions of handling.

Our own experience has indicated the superiority of condensers using a better grade of insulation, when backed up by careful engineering and extremely careful factory practise. We have made considerable investigations along the line of cheaper and more abundant insulation and the results obtainable have not appeared superior to our present method.

Eventual design in condenser practise will be determined by experience in the field rather than by the results of any shop tests that are now available. Meanwhile, the subject is of great interest for discussion before the Institute.

In the last column of page 3, in Mr. Marbury's paper, his language implies the possession of a large amount of interesting data. These data apply to subjects which are not yet by any means fully settled, and it would be of great value if he could contribute some of that to the files of the Institute.

E. K. Shelton: Mr. Marbury's paper calls attention to a development which has become of vast importance in the last few years, due to the intensive consideration given to the problem of low power factor loads. The possibilities in the power factor correction field put the static condenser in a position of importance in the industrial application of electric power.

The development problem involved in the design of static condensers upon analysis is found to be concentrated in the dielectric, its characteristics, treatment, and use. Several references are made in the paper to the advantages of a high working stress, but we are informed that a final selection was made of conservative stresses as used in other apparatus. A working stress of 145 volts per mil is given for the condenser construction as described. As a point of fact, working stresses in other electrical apparatus reach a maximum of 60 volts per

mil so that it is evident a value of 145 volts per mil is already two to three times conservative stresses used in other electrical apparatus. It is further indicated that a wood pulp paper is used. This is a type of paper commonly used in cable and transformer work. Let us see what this question of the selection of the working stress means.

We must keep in mind that the application of static condensers to power factor correction work is largely an economic problem involving reliability, a reasonable initial cost, and the question of size or weight. Reliability obviously must be obtained in any case. The initial cost must be reasonable to permit of a satisfactory return on the investment, else other means of correction will be used or the penalty accepted. Size or weight are factors of importance as this is a device that is added to an existing industrial establishment and must, in the majority of cases, be installed in an already rather completely utilized space. It is quite evident that by mere bulk application of the usual quality of insulating materials as commonly used in other types of electrical apparatus, and with operation at stresses equivalent to those uti ized in such aparatus, condensers can be built that may have reliability but at the expense of size and cost. In considering the advantage gained from the use of a higher stress, it is only necessary to remember that the corrective capacity obtained from a given unit area and thickness of condenser dielectric varies directly with the ratio of the squares of the applied voltages. Thus, an increase of 100 per cent in the working stress on the same dielectric means four times the corrective capacity obtained. If, by the use of a higher quality material, one-half the thickness of dielectric can be used for a given voltage, the same ratio holds, since twice the active area can be obtained in a given volume. This means one-fourth the bulk and weight for a given rating of equipment. Assuming any given economic limitation in size or weight, such an increase in working stress extends the range of application to four times the original limit. Certainly this is argument enough for a real engineering development that such advantages may be realized.

I have been intimately connected with a very extensive development on this same problem. Early work showed that ordinary grades of thin paper were entirely unsuited to this work as they would not give satisfactory results at the higher operating stresses. Such limitation, however, is no argument for retarding real engineering advance on a development of this nature. Paper, while suited for insulation work, was not originally developed with such application in view. Intensive co-operation with the paper manufacturers has already resulted in the production of papers of a quality never before obtained, and this on a basis that makes them entirely available commercially for this work. Conducting particles have been reduced to a very low value. By the selection of the proper materials, a dense paper has been produced that shows a remarkable comparison from the porosity standpoint with the best grades of wood pulp or Kraft papers. Experimental data between 2½ mil wood pulp paper and ½ mil paper of the type developed shows $^1/_3$ to ¼ the porosity for the ½ mil paper. A comparison of five sheets of this ½ mil paper with one sheet of 2½ mil wood pulp paper so that equal thickness is obtained shows approximately 1/15 the porosity for the 5-sheet construction. The advantage gained from the use of such high quality papers is evident. The use of a large number of these thin sheets for a given thickness of dielectric actually decreases the possibility of difficulties due to line-up of conducting particles or holes. A given corrective capacity of condenser with a few sheets of comparatively thick papers means a much larger active area over which this lining up may occur, combined with the greater percentage effect of partial lining up on two or three sheets.

Equally important advances have been made in the treatment of these materials. Methods that are entirely satisfactory when the ordinary stresses in other electrical apparatus are considered, prove utterly inadequate at these higher values, but engineering advances in any line necessarily involve changes and improvement. Advances already made along these lines have shown their value not only to the condenser problem but to the whole art of insulations in general.

Following out this line of development, condensers have been manufactured on a commercial scale that have been in successful operation for periods considerably over a year. 2300-volt units of this type have a breakdown voltage averaging 15,000 so that a factor of safety of approximately 6 is obtained. The normal working stress is 350 volts per mil. In comparison with the 11 pounds of paper per kv-a. required with the 145 volts per mil stress, only one and one-half pounds are used per kv-a. The development work that has already attained such remarkable results has directly benefited the whole electrical art. A definite advance has been made in the direction of more efficient utilization of insulations and in our understanding of dielectric phenomena. The future promises further progress along these lines.

J. E. Shrader: It was my privilege to have done some of the research work along the lines of the development of static condensers when the Westinghouse Company took up this problem, and since I have been called upon, probably something regarding the early work which was done might not be out of place.

It has been realized, not only in the manufacture of condensers, but in all lines of insulation, that the amount of moisture in the dielectric is a factor of very great importance, and also, that to get out the very smallest fraction of the moisture, the dielectric properties are increasingly proportionate.

The early practise was to boil the stacked condenser in oil at atmospheric pressure. This did not appear to me, when I took hold of the problem, to be a very satisfactory method. We first dried the stacked paper condenser in a vacuum at about 125 deg. cent., at which temperature no chemical breakdown of the paper occurred. After prolonged treatment, the oil was admitted into a vacuum space where it was heated before coming into contact with the condenser. This is what we call the spray process. In this process the oil was drawn under vacuum, heated, and allowed to spread through an open space in the vacuum, where it was released in the form of spray. Tests upon this method have shown that not more than a few ten thousandths per cent of moisture was still present. The oil was now introduced into the dried paper and there was no chance of moisture being absorbed in the process.

Any one who has studied the subject knows that by exposure to moist atmosphere, paper or oil will absorb a considerable amount of moisture. For this reason, the condenser was packed inside a vacuum tight vessel and after impregnation it was not again exposed to atmospheric moisture.

C. N. Johnson: The design of the static condenser has been well covered in Mr. Marbury's paper. The problem of the application of the condensers would seem to present a topic for an interesting paper at some future Meeting.

The need of a device for correcting power factor on lines supplying industrial load, is becoming more evident each year. With a reliable corrective device now available, which requires very little energy for its operation, its general use may be expected where a study of conditions shows that the installation cost is justified by the reduction in power losses and improved line condition.

Advances in meter design are keeping pace with the development of corrective devices. At this Meeting a paper is being presented showing some remarkably new developments in metering on a-c. lines. This metering device meets a general demand for a means of indicating the actual kv-a. load, and should bring about a more general use of power factor corrective devices.

Everett S. Lee: In the second column on page 2 of Mr. Marbury's paper we find him making reference to the increased dielectric strength which may be obtained over that of the oil itself by making use of the barrier action, wherein the oil is held

immobile to eliminate that motion which we find in oil in bulk when it is subjected to an electric field.

I think you are probably all familiar with the fact that if you have a bulk of oil and electrodes immersed in the oil, and subject that bulk to an electric field, there will be a motion of the oil; and any imperfections which may be in the oil are brought in the vicinity of the electrodes, with consequent breakdown at a low value.

Now, if we can utilize the principle whereby, if we put barriers in between those electrodes (and in a condenser those barriers are in the form of a number of very thin sheets of material which in turn have been well impregnated with good oil) we get thin films of oil separated by thin sheets of impregnated material, which means that the entire mass is held stationary, as it were, with very small liability of this movement which we find in bulk oil. Of course, we derive greater benefit from that consideration if we use a relatively large number of thin sheets, rather than a relatively small number of thicker materials. Consequently, that is one of the reasons why it seems quite desirable to, as far as possible, make use of that characteristic which we know is true and certain. At the same time, using the larger number of sheets, we decrease the liability of imperfections in the individual sheets lining up in such a way as to cause a path where the dielectric strength may be low.

In speaking of oil, I would call your attention to the fact that care is needed in its use as has been brought out by Dr. Shrader, just as care is needed in the selection of the paper which we employ. During the handling of the oil we must be very careful that it is not brought in contact with the air at such temperatures and in such a way that air may be absorbed by the oil, otherwise, although we so arrange the condenser cases that air may not come in, we still may have some air therein. Also, we must be sure that moisture is not contained in the cases, and that in the treating of the oil at such temperatures as we may use, no deteriorating effects are brought about.

Cooperation between the condenser manufacturer and the manufacturer of the oil has brought about a much better product, just as cooperation between the condenser manufacturer and the manufacturer of paper has brought about a much better product, and this cooperation we feel has been well worth while to the art and to the industry.

I dwell on these points particularly, one reason being the statement made in Mr. Marbury's paper that the loss in condensers, of a manufacture with which he is familiar, is about one-half of one per cent, whereas it is quite easy to obtain losses in the neighborhood of two-tenths of one per cent in condenser units where some of these other factors are not only taken into account, but are actually employed in the design and in the manufacture of the units.

The method of test which Mr. Marbury has outlined seems to have very desirable features, and if the accuracy which he has suggested can be obtained as easily as he has said, it is a method which probably will come into use. I would not, however, have you feel that the use of an electro-dynamometer wattmeter need require a whole laboratory force to take measurements therewith, as we find that it is just as easily possible to use such a wattmeter, following methods which have been outlined for years, I believe first perhaps by the late Dr. Rosa, of the Bureau of Standards. Those methods, together with the use of the dynamometer wattmeter, can be used by factory men in measuring losses in condensers in from one-half to one minute per unit.

I note in connection with the tests, that the test voltages used seem to be those as used in connection with electrical machinery; that is, about two times normal plus a thousand, whereas you may recall that in cables the values are two and one-half times normal for five minutes, rather than for one minute, as in the case of electrical machinery. In other words, there may be variations in the kinds of tests that are applied to different kinds of apparatus, and I believe there is a tendency on the part of many people who are studying these problems at the present time to say: "Perhaps we can get a more suitable test by using the lower values of potential applied for a longer time, rather than the higher values of potential applied for a shorter time;" that is, when the frequencies are at the value for which the apparatus is designed.

It seems that in the art today there is such a need for better insulation that if we can obtain better materials and if we can better the performance of insulating materials through these more refined manufacturing methods, it is much better that we try and do it, rather than to bring the condenser art down to the same level as we find in other types of apparatus. We feel that at the present time there is much more need for more serviceable insulation in other branches of the art than there is in the condenser game, and if, in studying these questions through the manufacture of condensers, we can better the insulation that may be available for other branches of the art, we certainly have gone a step in the right direction, rather than to allow ourselves to use what we may now have and just continue as we are, without having at least put our minds and abilities and resources to work to discover something better.

J. D. Stacy: The paper presented by Mr. Marbury gives a very comprehensive description of the problems of manufacture of condenser units in power sizes, and touches briefly on the question of application of these units to power circuits. It is in connection with the application problem that I wish to point out that since a condenser draws a current proportional to the frequency of the applied voltage, any harmonics present in the voltage wave will produce harmonic currents which, under certain conditions, may predominate, and the corrective effect of a leading current may be destroyed totally or in part by the introduction into the system of these harmonic currents which are just as detrimental in the loading of lines, cables, transformers, and generating equipment as the lagging current for which we attempt to correct.

Referring to Fig. 1, it is apparent that resonant conditions for any odd harmonic above the third may occur with series reactance of 5 per cent or less. These values of reactance lie within the range of line reactances encountered in distribution circuits and the chances are excellent that in applying condensers without reactance, many specific cases will be found where resonant or nearly resonant conditions with harmonics are present.

Experience has shown that line reactance does affect the operation of a condenser equipment. I know of two systems on which condenser equipments installed on very short feeders did not operate satisfactorily, but these same equipments, when moved to the ends of long feeders connected to the same bus as before, gave excellent satisfaction.

Fig. 2 shows a voltage wave form of a 2300-volt distributing bus of a large operating company on which it was desired to install a static condenser, together with condenser current wave forms with several values of series reactance. Calculations of the combined harmonic currents based on readings taken with a hot-wire ammeter, indicate that the combined harmonic currents with no series reactor were 73 per cent of the fundamental current, and with 3.5 per cent series reactor, 214 per cent of the fundamental.

Contrasting with these very poor wave forms, it is evident that when a 20 per cent reactor was used, the current wave form was quite comparable with the voltage wave form.

The effect of harmonic currents on the line current of a circuit which has had sufficient leading current of fundamental frequency added to correct the power factor from 70 per cent to 90 per cent is shown in the diagram Fig. 3. With combined harmonic currents equal to 50 per cent of the fundamental frequency leading current represented by $R\ A$ in quadrature with $O\ R$, the resulting line current is given by $O\ A$ and the equivalent power factor is 87.5. With 100 per cent harmonic

current, the line current is given by $O\ B$ and the equivalent power factor is 81.5 per cent. With 214 per cent harmonic current, which is the value just mentioned as applying for the wave form with 3.5 per cent reactance in Fig. 2, but which is by no means the maximum value obtained in tests on other systems, the line current is represented by $O\ D$ and the equivalent power factor is 59.8 per cent. In other words, by the installation of a static condenser under such a condition, we have increased the line current and lowered the power factor.

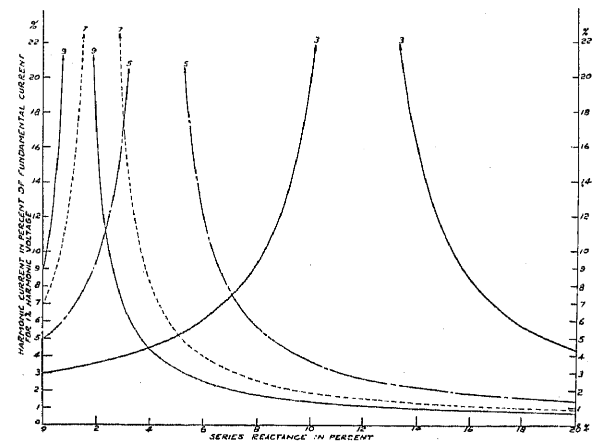

Fig. 1

The only practical method of being certain that a condenser can be furnished to perform its duty under all ordinary system voltage conditions is to furnish with it reactance sufficient to reduce all harmonic currents to a small percentage of the fundamental.

E. F. Northrup: Mr. Marbury's paper is a forceful, and concise statement of facts and information gained from wide experience and based on sound knowledge.

The power static condenser is almost universally thought of in connection with power factor correction on light and power circuits. There is however a development in the industry—

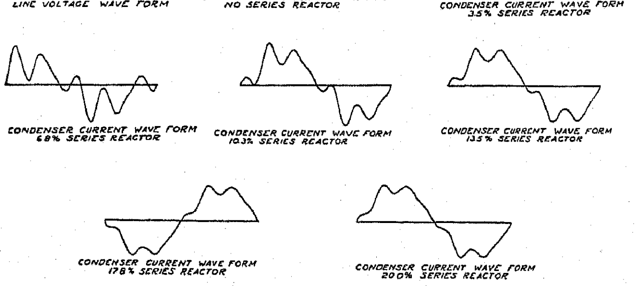

Fig. 2

that of ironless inductive heating-in which the static condenser plays a primary role.

All heating by high frequency currents—10,000 cycles or thereabouts—is only made possible through the employment of the static condenser. Ironless inductive heating using current of such moderate frequency as can be obtained from turbo or motor generator sets gives promise of being widely used, and here also static condensers find important application for power factor correction to secure economy.

It has been completely established that using ironless induction the rate of heating is satisfactory even when using currents of a frequency which can be readily obtained from alternators of substantially standard design.

The sole unfavorable feature of ironless inductive heaters is connected with the low power factor which is more or less inherent in the method. Heaters employing currents of medium frequency when applied to the heating of non-magnetic material have ordinarily a power factor of about 40 per cent. From a cost standpoint we must consider the equipment furnishing the power as made up of two units;

One to supply the in-phase component of the volt-amperes and one to supply the wattless component.

Or one may, of course, use a single unit, the generator, which is made large enough to supply both the in-phase and out-of-phase components. For a 40 per cent power factor uncorrected a single generator to take care of both components would need to be two and one-half times the size of a generator required to supply the same power at unity power factor.

In estimating the cost of an installation for ironless inductive heating it must always be inquired whether in a particular case

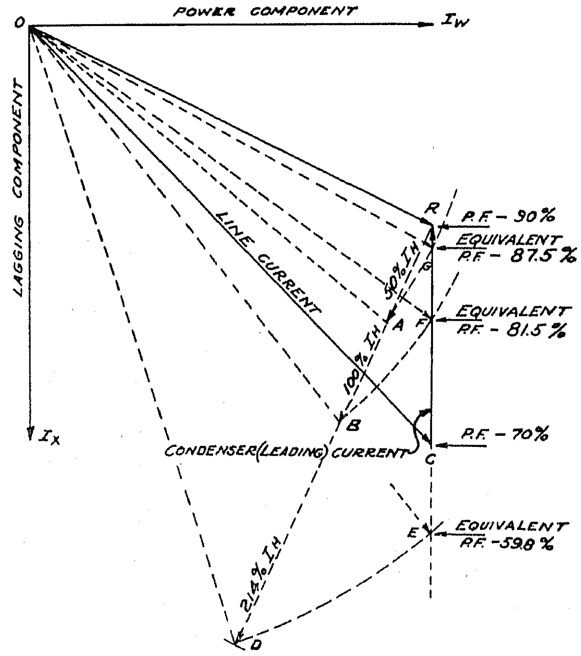

Fig. 3

it is more economical to use a single unit to take care of both components, that is, a unit having the kv-a. rating of the heater; or to supply two units,—a generator to develop the in-phase component of the power, and a static condenser to supply the out-of-phase component.

Any progress made on lowering the cost and in increasing the reliability of static condensers for supplying the out-of-phase component of the kv-a. supplied to ironless inductive heating devices has a directly favorable bearing on the economic feasibility of this highly important branch of the industrial use of electric power.

These remarks apply whatever frequency of current may be used for the ironless inductive heating. While the capacity of the condensers shunting the line to bring the power factor to unity decreases as the frequency chosen increases, the kv-a. of condenser required is independent of frequency used. Hence, whether one uses high, medium, or low frequency for inductive heating, it is of prime importance to so design the inductor that the wattless component shall be as small as possible, but when this has been done, it becomes of great importance to secure the needed kv-a. of condenser at the lowest cost possible.

If such cost becomes sufficiently reduced it will prove economical in all cases to use 2 units;—1 to supply the power and 1 to supply the wattless component of the kv-a. rather than make use of a single generator of capacity sufficient to furnish the entire kv-a. needed.

Mr. Marbury may well include, therefore, as a most important use for the static condenser its adaptation to the art of ironless inductive heating—now in steady progress toward extenive use.

G. D. Robinson: I am sure that there are some who are interested in the use of these condensers with d-c. circuits. If to be used with direct current apparently a condenser should be given a d-c. test.

A breakdown test performed on one of these units has led me to believe that the d-c. voltage which we may use with one of these condensers falls far short of the a-c. voltage.

I would like to ask Mr. Marbury what ratio exists between the a-c. and d-c. ratings of these units. Also, what is the ratio between the voltage to be applied on a-c. test and the voltage which may be applied on d-c. test

H. L. Curtis: The method which is outlined here indicates that Mr. Marbury makes a check by measuring both inductance and capacitance together and then each independently. I wish to call attention to the fact that that may not be a check at all. I have been working on methods of measuring condensers of this value for two or three years, and I have tried absolutley independent methods for making the measurements, and it is not easy to get those which will check. I merely wish to call attention to the fact that in order to get satisfactory checks it is necessary to use methods which are quite independent one of another.

Ralph E. Marbury: Mr. Craighead has given a brief history of the development of a static condenser with which he is familiar. While such a resumé is largely historical, there are certain points on which opinions differ.

The art of condenser manufacture is new. In all radically new apparatus questions arise which cannot be quickly answered. With static condensers the most important problem is the selection of insulating material and safe working stresses. The safe working stress varies with the quality and quantity of material used, operating temperature and character of service. The cost of the condenser depends largely on the cost of insulation. The quantity of insulation required depends on the working stress.

As Mr. Craighead points out in the early development of static condensers, wax impregnated insulation was soon found to be uneconomical, on account of the very low safe working stress.

The outstanding advantage of oil impregnated paper insulation was that the safe working stress was higher than that for solid insulation. The safe working stress for wax impregnated material was not definitely known, and while laboratory tests demonstrated instantly the superiority of oil and paper, time alone could establish the limits.

The first outstanding application of static condensers was for the correction of power factor. While the low power losses give condensers an advantage over other forms of corrective equipment, probably the most important feature is that of small maintenance and no attendance. To fully capitalize this chief advantage the device must be reliable.

For a given material the life of the condenser is indefinitely long below a certain critical working stress. When this stress is exceeded the life of the device becomes shorter, and the performance variable. This critical stress may differ in individual condensers manufactured under identical conditions, due to variations in the material itself or to other factors. This same characteristic would be noticed in most any material worked near the limit or where the limit varied with operating conditions.

Even if all the factors establishing this critical stress were definitely known and perfectly controlled, it would be unwise to work too near this maximum safe stress. Since, however, this is not the case we can only be governed by experience, and in choosing a stress we should introduce a sufficient margin to take care of all conditions that may arise.

We have available much data on insulation stress in similar apparatus extending over many years. These data cover working stresses from 20 to 100 volts per mil. During the development of static condensers thousands of laboratory hours were obtained of insulation as used in condensers, with stresses up to 1100 volts per mil, or over twice the average breakdown of the average insulation used in other apparatus.

If we should combine this laboratory experience with other insulation history we might select as a normal stress for static condensers 400 to 500 volts per mil. This would be expecially true if our choice were influenced by the fact that the size varies inversely as the square of the ratio of stresses. If the working stress is divided by 2 the size will be multiplied by 4.

The problem may be handled commercially in one of two ways. First:—A high stress may be adopted based on laboratory experience alone, and gradually reduced over a long period of manufacture until an entirely reliable device is provided. Second:—A stress definitely well below the safe stress may be adopted, and gradually increased over a long period, until the maximum of perfection is obtained.

In either case the final result would probably be the same, but years of experience in other apparatus has demonstrated the advantages of the second plan.

At first it appeared impossible to proceed along the second plan without excessive costs, and that it would be necessary to start at a comparaitvely high stress such as 300 volts per mil. In order to entirely follow the plan outlined, however, a stress of 145 volts per mil was chosen without regard to cost, and from this point the development became an economic study.

It is the object of this paper to briefly describe the work done which made possible a complete line of static condenser worked at stresses of 145 volts per mil, and at costs no greater than originally expected for 400 volts per mil, designs even though about seven times as much material was required.

A study of materials was made covering the entire range of cost and quality. Some paper cost as much as twelve times that of another. On examining the cost curves it was found that cost depends more on thickness than stock used. This is tied up with the tonnage that can be produced with a given outlay of paper making machinery. The electrical quality is about the same for either linen, wood or cotton fiber. The cost increases as the thickness is reduced, due to the reduction in tonnage, at the mil, to the necessity for using more costly stock, and to the increase in waste. The dielectric losses are hardly effected by the thickness or kind of stock, so long as impregnation and chemical purity remain constant.

For a given dielectric thickness the breakdown is increased by reduction of porosity of the paper or by greater subdivision of the oil by thinner and thinner sheets. If the breakdown is plotted against cost of a given area of material, the thicker more porous and less expensive materials prove far better.

The net result of this economic study of materials, together with improvements in mechanical construction method of impregnating and testing was the two-kv-a. 2300-volt 60-cycle unit, designed with a working stress of 145 volts per mil, and with a cost of no greater than originally expected for a 400-volt per mil design. Many thousands of these units have been manufactured and some have operated nearly three years, and not a single insulation failure has occured. The fact that 145 volts per mil was well within the safe limits has been fully established in service.

With the complete elimination of insulation failure and the ability of the condenser to withstand all line surges and periods of over voltages, individual fuses were no longer necessary. The increased surface of the units for radiation and the elimination of individual fuses made possible a very compact frame assembly, the overall dimensions of which were not a great deal more than those of the early experimental 400 volts per mil type.

In the early stages, dielectric loss measurement was a laboratory process, and it was unsafe to make loss measurements until

The weight of the units per kv-a. was reduced by the elimination of all metal with the exception of the containers and metal conductors.

after over-potential tests had been made, due to the possibility of damaging very delicate low power-factor wattmeters in the event of failure of the unit. Such improperly impregnated units were therefore found and destroyed on the voltgae test. A very reliable and fool-proof means was developed for making loss measurements, where an improperly impregnated condenser could not damage the equipment if failure occurred. This made it possible to select and return for retreatment such units without damaging them, except in rare cases where manufacturing errors were made in putting in the insulation.

This loss measuring equipment reduced factory scrap to a very low figure.

Other methods were developed for controlling the quality of the condensers throughout the entire manufacture.

All units are given a one-minute overpotential test of 2¼ times rating plus 2000 volts for one minute. This test is made generally on 25 cycles.

I do not wish to leave the impression that research ceased when it became possible to work at low stresses economically. Units have operated in the laboratory where their daily performance could be observed and a continuous study of materials is carried on.

One point that has been fairly well established is that a material which breaks down at 400 volts per mil. and is operated at 100 volts per mil. gives more dependable service than one which breaks down at 1600 and is operated at 400, even though the ratio between working stress and operating voltage is the same in each case initially.

Mr. Shelton emphasizes the importance of the porosity of paper. Low porosity is a desirable characteristic, except in cases where the cost of obtaining it overbalances its value. The writer has never seen ordinary paper manufactured here or abroad which was entirely free of holes. This can only be accomplished by subjecting the paper to chemical treatment which lowers its electrical quality. When thousands of square inches of material are used in a single condenser, holes may line up somewhere with even the best paper yet produced. When low stresses are used, and low porosity not depended upon, such a line up has little effect on the condenser, neither does the line up of a few metal particules.

I wish to agree with Mr. Lee that the dielectric losses can be reduced to 0.2 per cent by very careful treatment. This is desirable when very high stresses are used since the stored energy per unit volume of material is great. We work toward an average of ½ per cent on complete equipments. Even with this average many units run even lower than 0.2 of 1 per cent losses.

Since entirely satisfactory performance has been obtained with loss average of 5 watts per kv-a., the only other reason for holding it lower is power cost. The difference in power loss even at three cents per kw-hr. would not justify the special care required, to bring losses down to even 0.3 per cent.

I do not mean to create the impression that the electro-dynamometer is an impractical device. This instrument has been found to be most satisfactory for low power-factor measurements. Any one familiar with this device knows, however, that as the voltages at which tests are made is increased, the resistance of the potential circuit must be increased, and that this resistance, which sometimes must be as large as one megohm, must be free from distributed capacity and inductance, as a slight amount of either will cause large errors. In addition, the arrangement of the resistances with respect to other parts of the circuit is of great importance. In measuring condensers varying in capacity from ½ to 150 microfarads, and at voltages from 20 to 5000, considerable manipulation is needed, requiring a technical knowledge of the principles of the instrument. In addition should an accident occur, such as attempting to test a short-circuited or defective condenser, the meter will be seriously damaged.

The scheme of operation of the inductance test equipment has been fully covered in the paper. However, I might add that should a defective condenser be connected to the test no harm can come to the meters, as the voltage will not build up. If the condenser breaks down on test, the voltage drops to a low value, since the device operates on the principle of series resonance.

In reply to Mr. Curtis, I wish to point out that the calibration of the inductance was a laboratory operation which extended over two months and every precaution was taken to obtain a very accurate calibration. It is true that a measure of condenser losses and inductance losses separately and combined will not always show up a phase angle error in the potential circuit. However, this check was not solely relied on as evidence of correct measurements, but merely made as a final check to note any inconsistency. Once a complete calibration was made on the coil, the special wattmeter is no longer required. A condenser is simply connected into the circuit as shown in Fig. 5 of the paper. The screened enclosure is swung over the unit operating as it latches the safety switch. The current is set at the correct value for the unit on test and the watts loss read on the meter, a standard Westinghouse wattmeter. Assume a typical case of a total of 32 watts. By reference to a calibration curve, the loss in the inductance is determined which is of the order of 20 watts. To this is added the meter losses which in this case would be 3 watts. The loss in the condenser then is $32-(20+3)=9$ watts or 0.45 per cent, since the unit is measured at two kv-a. It might be of interest to note that several thousand pounds of copper were required to build a two kv-a. 2300-volt inductance with a loss of only 20 to 25 watts. This, of course, was an air core construction.

I would like to ask Mr. Stacy why the condenser did not operate without reactors on a short feeder. I am wondering if the difficulty was that of not obtaining the power factor expected, or failure of the insulation in the condenser due to heating of harmonic currents.

In applying reactors with static condensers it should be noted that a given reactor is suitable for only one value of condenser and that either the condenser capacity cannot be changed freely or the reactor must be changed by means of taps every time the kv-a. of the condenser is changed.

The writer has observed the performance of many condensers where no reactors were used and in no case has the use of a reactor been found necessary or even desirable. In all cases the power factor was that to be expected, as well as the currents in the condenser leads. It is quite possible that under certain conditions reactors may be indispensible. It does not seem necessary to make a general use of them. In fact their use should be avoided wherever possible. We are adding condenser capacity to correct for electromagnetic fields. If we add reactors we must add still more condenser kv-a. The type of condensers described above would not be damaged under the worst conditions by harmonic current. They have been used to pass a high 500-cycle superimposed current when operating at rated 60-cycle voltage, in special applications.

In reply to Mr. Robinson's question regarding d-c. application, I wish to say that we have found it desirable to work at higher stresses on d-c. than on a-c. Condensers for a-c. service are rated for commercial lines. The voltage may go up to fairly large values at times. Thus for a-c. service a large factor of safety is incorporated in the design. In addition, the critical stress for d-c. is much higher than for a-c.

For d-c. service the problem is different. Since there are no standardized d-c. voltages for radio broadcasting, the condenser can only be rated for absolutely maximum. The user must take into account poor voltage regulation which varies with every outfit, over a very broad range. A condenser for 3500 farads direct current is therefore maximum rated. If there is a large ripple in the direct current the peak value should be within the d-c. rating. All d-c. conde nsers are tested at twice d-c. operating voltage for one minute. It is intended that the rating should not be exceeded in service by surges, etc.

The d-c. working stress varies between 275 and 460 volts per mil, depending on the rating. As the voltage rating is increased, the working stress is reduced to allow for less uniform voltage distribution through the material.

Effects of Time and Frequency
on Insulation Test of Transformers

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Review of the Subject.—*Permanently grounded transformers must be given the insulation test by inducing the necessary voltage across the windings.*

The A. I. E. E. Standards specify that the time of induced voltage tests be the same as for high potential test. In certain cases where the transformers are of very large capacity the induced voltage must be made at a frequency several times higher than normal. Since the dielectric strength of most insulating materials decreases with an increase in the frequency, an investigation has been made to determine the proper and fair length of time to make induced voltage test when the frequency is higher than normal.

Following are the main points brought out in the investigation.

Above a certain voltage, **time** *of voltage application as well as* **voltage** *causes failure of insulation. The dielectric strength can be expressed as a function of both* **time** *and* **voltage** *by an equation of the form*

$$Kv. = A\left(a + \frac{1 - a}{\sqrt{T}}\right)$$

in which A is the kilovolts necessary to cause failure in one minute, "a" is a constant representing the ratio of strength for infinite time to the one minute strength and T is time in minutes. The value of "a" varies for different materials apparently depending mostly on the dielectric loss.

The breakdown by creepage over solid insulation with the electrodes either on the same side or on opposite sides of the sample (arranged in such a manner that the solid insulation is under considerable stress) is not affected by **time** *nearly so much as is the puncture voltage of solid insulation.*

The behavior of oil without barriers is so erratic that no very definite relation can be obtained between **time** *and dielectric strength. In general* **time** *decreases the strength quite rapidly for the first few seconds after which the effect decreases and probably disappears entirely after two or three minutes. The momentary strength ranges from 25 to 30 per cent higher than the one minute strength.*

The effect of **time** *on the strength of solid insulation and oil in series is about the same as for solid insulation alone until the oil distance exceeds the solid insulation thickness after which it begins to be the same as for oil without barriers.*

The strength-time curves for solid insulation are of approximately the same shape for all frequencies from 60 to 420 cycles, although the strength decreases with an increase in frequency F approximately in accordance with the formula $kv. = 1.75 / F^{0.137}$

Failure by creepage over the surface of solid insulation which is under no stress (i. e., with the electrodes on the same side of the barrier) takes place at approximately the same voltage for all frequencies from 60 to 420 cycles; but if the electrodes are so arranged (on opposite sides) that the insulation is under considerable stress the failure voltage decreases with an increase in frequency in about the same order as the puncture voltage of solid insulation does.

The rupture voltage of oil is the same for both 60 and 420 cycles.

The effect of frequency on the puncture voltage of solid insulation and oil in series is the same as for solid insulation until the oil distance exceeds the thickness of solid insulation, after which the effect decreases and as the oil distance increases the effect approaches that for oil without barriers.

By considering the effects of both time and frequency on dielectric strength it is shown how to determine the proper length of time to make the voltage strain at higher frequencies equal to the strain at 60 cycles for one minute.

INTRODUCTION

SECTION 6362 of the 1922 edition of the A. I. E. E. Standards reads as follows:

> *Testing Transformers by Induced Voltage*—Under certain conditions it is permissible to test transformers by inducing the required voltage in their windings in place of using a separate testing transformer. By "required voltage" is meant a voltage such that the line end of the winding shall receive a test to ground equal to that required by the general rules.

The increased number of permanently grounded transformers within the last few years has resulted in an increased use of the induced potential method for testing the insulation. In case the transformers are single phase, and when required to impose a strain equal to twice line potential on the end of the windings the induced voltage must be $2\sqrt{3}$ or 3.46 times normal. To keep the exciting current within a reasonable value at this high induction, the frequency must, of course, be greater than the rated frequency.

To maintain normal 60-cycle density at 3.46 times normal voltage requires 60×3.46 or 208 cycles. An idea of the large generator capacity required for testing some of the largest transformers of today when using, say, 208 cycles may be had from the following example: Assume a 30,000-kv-a., 60-cycle single-phase transformer to be given 3.46 times normal induced voltage. If the exciting current is 5 per cent of full-load current and to maintain normal iron density the necessary single-phase 208-cycle generator capacity would be $30{,}000 \times 3.46 \times 5 / 100 = 5200$ kv-a. As the generator should be three phase and as the single-phase rating is usually taken as 70 per cent of the three-phase rating the total capacity required would be $5200 \times 100 / 70 = 7450$ kv-a. This rating of course would be required only for intermittent service and an overload could be had, which would permit of reducing the continuous rating to possibly 5000 kv-a. Even this generator capacity of this odd frequency is out of the question for test purposes.

Since the exciting current decreases very rapidly with iron density the most practical thing to do in testing very large units, of course, is to use a generator of higher frequency than 208 cycles. In certain cases this has been done and with the consent of the customer, 420

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cycles has been used with a reduction in the time of voltage application to give a total of 7200 cycles. This investigation shows that this test amply meets the standard insulation requirements.

It is obvious that, except for liquid insulations, to produce the same voltage strain, the *time* should be reduced as the frequency is increased, since dielectric loss increases with frequency and the heating is increased, and consequently the dielectric strength is reduced. But just how much the *time* should be reduced to make the tests at different frequencies of equal severity had not been extensively investigated on built up transformer insulations until recently. Based on the tests which had previously been made and on the results shown by F. W. Peek (pages 179, 182, 183 and 184 in *Dielectric Phenomena in High-Voltage Engineering*) we have considered that the voltage strain at different frequencies was approximately the same if the time was inversely proportional to the frequency. In fact it is logical to expect that within certain time limits the strain should be somewhat more severe for the higher than for lower frequencies for the reason that there is less chance for the heat in the solid insulation to escape. The results of this investigation bear this out as will be shown later.

The object of this paper is to give the results of a large number of tests made on solid insulation, built-up insulations and oil, and based upon these results to show the relation between the time of voltage application at different frequencies to produce approximately the same voltage strains on the classes of insulating materials commonly used in oil-immersed transformers. It is hoped that the data, especially the strength-time curves, will be useful not only for transformers but for other types of electrical apparatus.

As would be expected, it was found that the ratio of time necessary to cause failure at any two frequencies changed as the time of voltage application increased. For example referring to Fig. 23 and assuming for the moment that these curves which show the relation between *time*, frequency and dielectric strength, are correct, it will be seen that a test for 30 seconds at 25 cycles is equivalent to a test for 4 seconds at 120 cycles. In other words the ratio of time here is 7.5:1. However, these curves also show that a test for three minutes at 25 cycles is equivalent to a test at 120 cycles for 10 seconds. Here the ratio of time is 180 / 10 or 18:1 which is more than twice that for the first case. Likewise as the time chosen at the lower frequency for comparison becomes longer the ratio between this time and the time that gives an equivalent voltage strain at the higher frequency becomes larger and larger until finally the ratio becomes infinity.

Factors Considered in Experimental Observations

Test Factors.—Since the ratio of time changes for different frequencies it was of course necessary to determine the variation of dielectric strength with "time of voltage application" as well as the effect of frequency on the dielectric strength for a given time and voltage.

b. Insulation Factors.—When a transformer fails on insulation test it is usually due to one or a combination of the following causes:

1. Puncture of solid insulation
2. Creepage over solid insulation
3. Rupture of oil and
4. Puncture of solid insulation and oil in series.

The above four causes of failure have been investigated in determining (1) dielectric strength vs. time of voltage application and (2) dielectric strength vs. frequency.

Empirical Formula of Strength-Time Curves

An examination by the writer of all available strength-time curves made at different frequencies showed that these curves can best be expressed by an equation of the form first suggested by Peek (see page 179 *Dielectric Phenomena in High-Voltage Engineering*). The equation is:

$$E = \left[a + \frac{(1-a)}{4\sqrt[4]{T}} \right]$$

In which E is the ratio: "dielectric strength at any time divided by the strength for one minute," and a is a constant representing the ratio: "one minute dielectric strength divided by the strength for infinite time," and

T = time in minutes.

The above equation means, of course, that the dielectric strength decreases with time of voltage application in accordance with an exponential equation (of a straight line on double logarithmic paper) until whatever causes[1] failure, ceases to affect the strength. In other words the curve becomes parallel to the axis of time before the strength reaches zero, otherwise electrical apparatus having insulation under a constant moderate stress would fail in time due merely to this stress. We know, however, that this does not happen.

It would, of course, be expected that the value of a in equation (1) would be different for different materials depending probably in part on the density and partly upon dielectric losses. It is natural that the higher the loss the more quickly will the curve flatten out. The following tabulation gives maximum, average, and minimum values of a, and the densities for some of

1. Space and time do not permit of attempting to discuss why *time* and frequency affect the dielectric strength of insulation. This question, however, is being actively studied and it is hoped that at some future time something can be given on this very interesting and important subject of the mechanism of breakdown.

the usual insulating materials used in transformers and tested in this investigation.

Materials in oil	a	Density
0.003 in. untreated cable and kraft papers........................	0.85	0.7 to 0.8
0.012 in. varnished cambric (1 to 10 layers)........................	0.675	1.12
0.005 in. varnished bond paper (6 or 7 layers)......................	0.675	1.12
Combination of 3/32 in. pressboard and 3/32 in. oil ducts in series.....	0.675	
Pressboard at 75 and 100 deg. cent..	0.675	0.95—1.05
Pressboard at 25 deg. cent.........	0.50	0.95—1.05

It will be noted from the above that the values of a do not appear to bear any definite relation to the

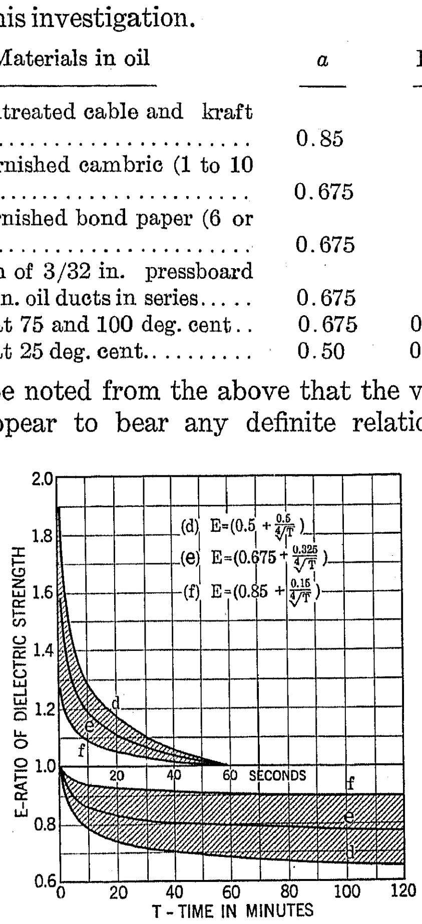

FIG. 1—STRENGTH-TIME CURVES PLOTTED FROM FORMULA: $E = \left[a + \frac{(1-a)}{\sqrt[4]{T}}\right]$ IN WHICH "A" IS A CONSTANT DEPENDING ON THE MATERIAL

density of the material. Probably dielectric loss is one of the main factors since paper generally has a high loss as compared with pressboard especially at 25 deg. cent.

Table No. 1 gives in tabulated form the ratios of dielectric strength calculated for different times T and for maximum average and minimum values of a in equation. (**1**).

TABLE NO. 1
EFFECT OF TIME ON RATIO OF DIELECTRIC STRENGTH CALCULATED BY EQUATION. (1) FOR E =

T-Time in: Sec.	Min.	$\left(0.85 + \frac{0.15}{\sqrt[4]{T}}\right)$	$\left(0.675 + \frac{0.325}{\sqrt[4]{T}}\right)$	$\left(0.5 + \frac{0.5}{\sqrt[4]{T}}\right)$
1	0.0167	1.27	1.587	1.90
3	0.05	1.167	1.36	1.556
5	0.0833	1.13	1.278	1.43
10	0.167	1.084	1.182	1.28
20	0.333	1.048	1.105	1.16
30	0.5	1.03	1.062	1.095
60	1.0	1.00	1.00	1.00
	2.	0.976	0.95	0.92
	5.	0.95	0.89	0.833
	10.	0.934	0.858	0.78
	30.	0.914	0.814	0.714
	60	0.904	0.792	0.68
	120	0.895	0.77	0.65
	infinity	0.85	0.675	.50

Fig. 1 shows the data in Table No. 1 plotted in curve form. The shaded area or envelope of these curves represents the variations in dielectric strength with time of voltage application, that is likely to be obtained from representative Class A insulating materials used in electrical apparatus.

If desired to express the dielectric strength in kilovolts rather than by ratio values the form of the equation becomes

$$\text{Kv.} = A\left[a + \frac{(1-a)}{\sqrt[4]{T}}\right] \quad (2)$$

in which A is the one minute dielectric strength in kilovolts. a, and T have the same significance as in equation (**1**).

To have some convenient yard stick for measurement as we go along, all test data shown later will be compared with the values calculated by equation (**2**) using either the minimum, average or maximum value of a according to the one that fits best.

EXPERIMENTAL 60-CYCLE STRENGTH-TIME CURVES

Unless otherwise stated the voltage was applied to the sample by closing the field switch of the generator with the rheostat set for some predetermined value. An investigation showed that this is the only method

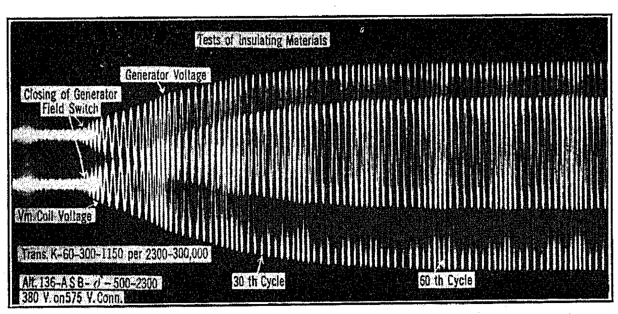

FIG. 2—OSCILLOGRAM SHOWING RATE AT WHICH 60-CYCLE TESTING TRANSFORMER VOLTAGE INCREASED TO A STEADY VALUE AFTER CLOSING GENERATOR FIELD SWITCH

It will be noted that no surge took place

of obtaining full voltage in a short time without producing an initial surge such as we know happens at times when full excitation is thrown on the primary side of a testing transformer. Tests showed also that a disturbance took place when full voltage was thrown across the electrodes by closing the circuit on the secondary side of the transformer while the transformer was excited.

The time given is that from the instant of closing the field switch until failure occurred. An oscillogram of the initial voltage waves, Fig. 2, shows that it required approximately one second for the voltage to build up to a constant value. The true value of time is probably from one-half to one second less than that shown.

All tests in this investigation were made with the

samples in a vertical position. The pressboard in all cases contained approximately 45 per cent of rag material.

The 2-in. and 4-in. diameter electrodes had a 3/32-in. and ½-in. curvature radius respectively on the edges. All kilovolt values given are the r. m. s. values of the peak of the wave as determined by spark gap.

a. Solid Insulation. Table II gives the comparison of calculated values and results of tests made on oil-treated, three-mil kraft papers.

It is, of course obvious, that, since the value of the constant a used for calculation was obtained from these tests, a comparison between the calculated and test results shows only how the curves agree with each other and how closely they can be expressed by an empirical formula.

TABLE II.
VARIATION OF DIELECTRIC STRENGTH WITH TIME OF APPLIED VOLTAGE. 60 CYCLES

2-in. round edged electrodes in oil at 25 deg. cent. Effective sine wave values. Average of 10 tests. Material 0.003 in., oil treated kraft paper.

Test No. 1—7 layers

Time in Sec.	Puncture Kv. Test	Calc.*	
2	24	25.2	
7.5	22.5	23.3	
16	22	22.4	
21.5	21.5	22	
60	21.1†	21.1	
95	21	20.8	
No break in 180	20.5	20.4	(one trial)

Test No. 2—6 layers

Time in Sec.	Puncture Kv. Test	Calc.*
4	19	19
10.1	18	18.2
24.5	17.5	17.5
46.5	17	17
60	16.9†	16.9
160	16.5	16.4

Test No. 3—6 layers

Time in Sec.	Puncture Kv. Test	Calc.*	
1	24	22.2	
4	22	20.2	
11.5	20	19	
49	18	17.7	
60	17.6†	17.6	
99	17.0	17.3	
168	16.5	16.9	
No break in 2 hrs.	16.0	15.8	(one trial)

*Kv. = one min. Kv. $(0.85 + 0.15 / \sqrt[4]{T}$ This has the same constants as given by Peek for paper covered cables referred to previously.

†Estimated by interpolation.

Figs. 3 and 4 show two sets of strength-time curves obtained on single sheets of 3/32-in. pressboard in 25 deg. and 75 deg. cent. oil. The 75 deg. curve checks the average value of a while the 25 deg. curve agrees better with the minimum value of a. The writer has found that pressboard at 25 deg. cent. shows the greatest variation in strength with *time*, of any material tested, but at 75 and 100 deg. cent. (as will be shown later) the variation in strength corresponds to the average value of a. In making any tests at higher than room temperatures, the material was subjected to the tested temperature for at least 24 hours before being tested in order that the condition of the material would not change while undergoing the test.

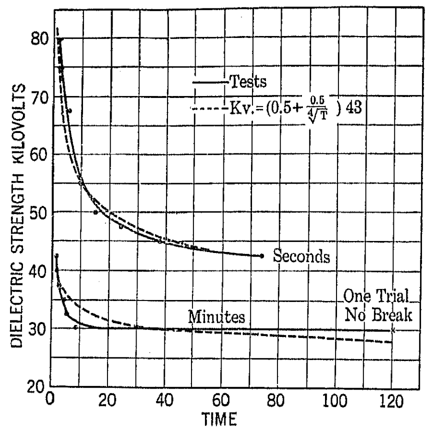

FIG. 3—60-CYCLE STRENGTH-TIME CURVE

One layer of 0.0935-in. oil-treated pressboard in oil at 25 deg. cent. Each point average of 10 tests. Time for voltage to build up (approx. 1 sec.) included 2-in. diam. electrodes

However, for most other materials such as varnished bond paper having a smooth film of oxidized varnish over the surface and for varnished cambric, the test curves check best the average value of the constant a from 25 deg. to 100 deg. cent. Figs. 5 and 6 give results of tests made on varnished bond paper and

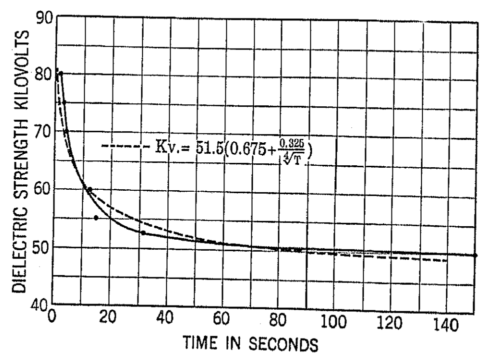

FIG. 4—60-CYCLE STRENGTH CURVE

One layer of 0.0935-in. oil-treated pressboard in oil at 75 deg. cent. Each point average of 10 tests. 2-in. diam. electrodes, time for voltage to build up (approx. 1 sec.) included.

varnished cambric in 25 deg. oil. These test points check the average value of a. Figs. 7 and 8 show the results of tests made on different numbers of layers of black varnished cambric in 30 deg. and 85 deg. cent. oil for longer periods of time than shown in Fig. 5. These curves also check average value of a fairly well from approximately one minute to about two hours.

(Sufficient tests were not made at this time, namely in 1915, for these latter curves to be of much value up to approximately one minute of time.) Apparently temperature of the oil has very little effect on the shape

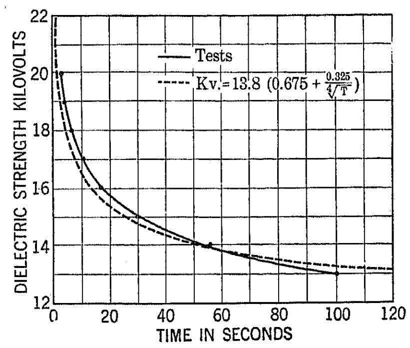

FIG. 5—STRENGTH-TIME CURVE OF 3-MIL BLACK VARNISHED BOND PAPER

Total thickness per layer 5 mils. 2 layers under No. 10 transil oil at room temperature. 2-in. electrodes. Each point average of 10 tests at 60 cycles. Time for voltage to build up (approx. 1 sec.) included.

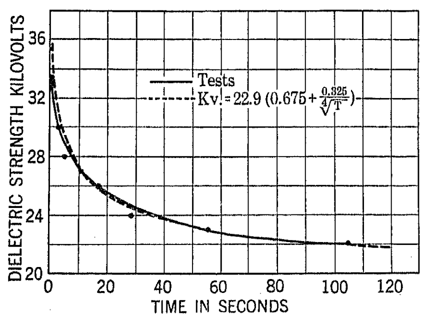

FIG. 6—60-CYCLE STRENGTH-TIME CURVE

0.012-in. black varnished cambric, 2 layers under oil at room temperature, 2-in. diam. electrodes, each point average of 10 tests, time for voltage to build up (approx. 1 sec.) included.

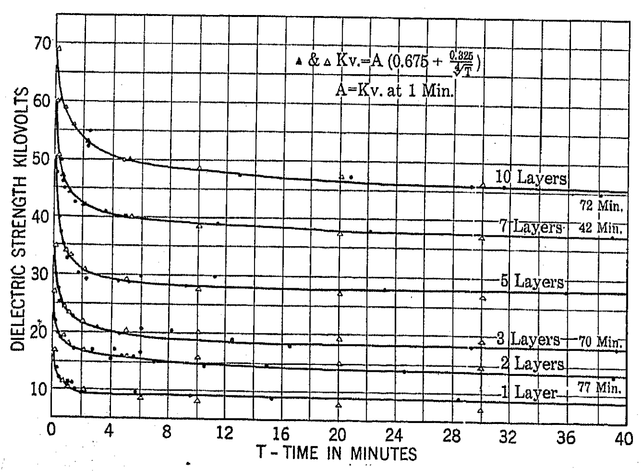

FIG. 7—60-CYCLE STRENGTH-TIME CURVES OF 0.012-IN. BLACK VARNISHED CAMBRIC

of the strength-time curves of black varnished cambric.

b. Creepage over Solid Insulation.—The 60-cycle data on creepage is discussed along with the 420 cycle creepage data under section 5 b.

c. Rupture of Oil.—Oil alone is so erratic that no very definite strength-time curve can be obtained. However, the average of a large number of tests show that the strength decreases quite rapidly after the first few seconds of voltage applications and after this the effect becomes less and probably disappears after two or three minutes time.

Fig. 9 gives the results of time tests made on No. 10

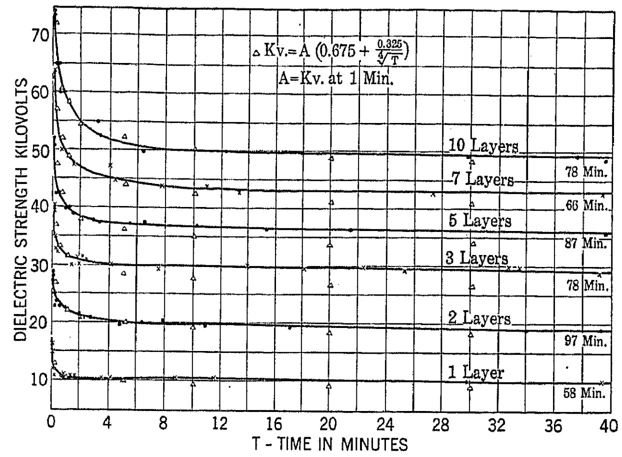

FIG. 8—60-CYCLE STRENGTH-TIME CURVES OF 0.012-IN. BLACK VARNISHED CAMBRIC

transil oil in which was submerged two 4-in. (10-cm.) round edged electrodes spaced 0.375 in. apart. The voltage was applied by closing the field switch of the generator. The scattered points illustrate well the erratic nature of oil as has already been pointed out by Hayden and Eddy.[2] (However when used in series

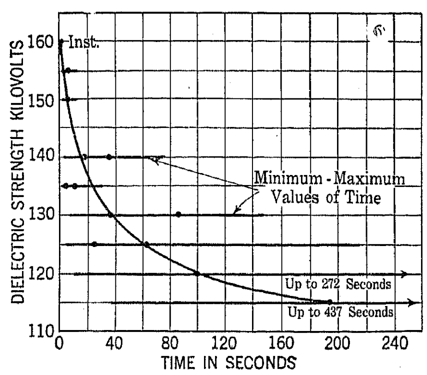

FIG. 9—STRENGTH-TIME TESTS MADE ON NO. 10 TRANSIL OIL AT 25 DEG. CENT.

Two 4-in. (10 cm.) diam. round edge electrodes spaced 0.375 in. (0.95 cm.) apart. Time for voltage to build up (approx. 1 sec.) included, each point average of 10 tests. Dielectric strength of oil 26 kv. tested with 1-in. disks, 1 in. apart.

with barriers of solid insulation, oil is one of our most reliable insulating materials).

The data shown in Table III is interesting in that it shows the effect of increasing the applied voltage. The *one second tests* were made by closing the

2. Three Thousand Tests on the Dielectric Strength of Oil," Hayden and Eddy, TRANS. A.I.E.E., Vol. XLI, 1922, p. 394.

generator field switch and holding it for approximately 1.5 seconds. The other tests were made by bringing up the voltage by means of the rheostats in the field. To make sure that the conditions were the same for each period of time, the tests were alternated with time, that is, each one-second test was followed by a rapidly applied and standard one-minute tests respectively. This was repeated until the series was completed. In each case (except the first column) the voltage was started at approximately 50 per cent of the breakdown value. For the one second tests (1st column) the voltage was started from 25 to 30 kv. less than the failure voltage, and increased in 5 kv. steps, but with rest periods between each step.

TABLE III.

EFFECT OF TIME OF VOLTAGE APPLICATION ON RUPTURE STRENGTH OF NO. 10 TRANSIL OIL

Sine wave, 60-cycle voltage—distance between 4-in. electrodes 0.375 inches. Dielectric strength of oil 25 to 30 kv. tested with one inch disks 0.1 in. apart

Duration Full Voltage Approx. one second	Voltage Increased 10 to 15 kv. per second	Voltage Increased one kv. per 5 seconds
165	164	119
165	144	109
155	170	134
150	132	136
160	168	125
150	166	135
170	174	117
160	170	94
165	165	133
170	172	140
165	150	137
135	140	110
172	168	128
165	162	130
170	158	123
167	170	141
Average 163 kv.	160 kv.	125 kv.
Per cent 130	128	100

At this point the dielectric strength of the oil was unavoidably lowered with sand to put out a fire.

120	108	78
110	122	89
118	88	86
100	110	92
125	98	80
130	118	83
125	95	91
110	100	92
95	138	97
140	145	96
134	118	108
110	138	94
Average 126 kv.	115 kv.	99 kv.
Per cent 127.5	116.5	100

The above shows that the short-time or momentary values were from 27.5 to 30 per cent greater than the approximate one-minute values, shown in last column. This is in fair agreement with the difference shown in Fig. 9 between the instantaneous and one minute values. Just why the strength should be 25 to 30 per cent higher for one second than for one minute duration of voltage is not fully known. One possible explanation is that it requires *time* for whatever impurities are present to fill up and bridge the gap. If the oil was absolutely pure it might be found that time of voltage application would have no effect on the rupture voltage. But commercial oils however are seldom, if ever, free from small particles of lint, etc., floating around.

d. Puncture of Solid Insulation and Oil in Series. The 60-cycle strength-time curves are given and discussed with the 420-cycle curves under section 5 d.

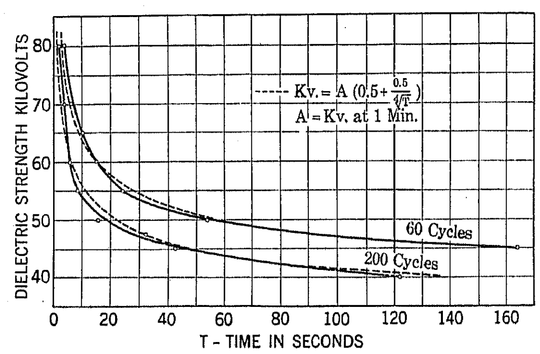

FIG. 10—60 AND 200-CYCLE STRENGTH-TIME CURVES

One layer 0.0935-in. oil-treated pressboard, in oil at 25 deg. cent. Two 4-in. (10 cm.) diam. electrodes. Each point average of 10 tests. Time for voltage to build up (approx. in 1 sec.) included

EFFECT OF FREQUENCY ON:

a. Strength-Time Curves of Solid Insulation. Up to the present we have discussed tests at only one frequency. Fig. 10 shows curves made at 60 and 200 cycles on single layer samples of 3/32 in. oil treated pressboard in oil at 25 deg. cent. The curves show

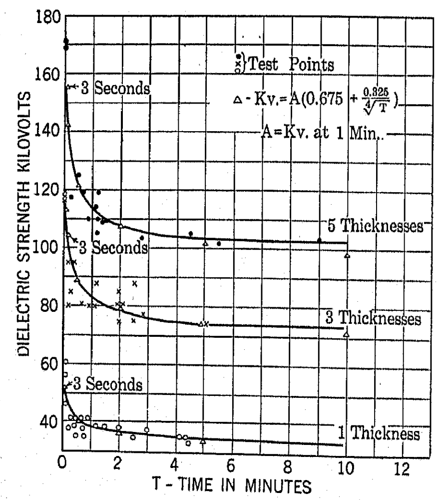

FIG. 11—60-CYCLE STRENGTH-TIME CURVES

0.0935-in. oil-treated pressboard, in oil at 100 deg. cent. two 4-in. (10 cm.) diam. electrodes, each test shown. Time shown from inst. voltage (rapidly applied) became constant until failure

approximately the same percentage difference in kilovolt values. This is an interesting fact even though it may not be known just why it is so. Perhaps some day it can be explained when the mechanism of insulation failure is fully understood.

Figs. 11 and 12 show strength-time curves made on

various numbers of layers 3/32-in. pressboard at 60 and 420 cycles.

To prevent absorption of moisture, the material was immersed under oil immediately after vacuum treatment and removed only as needed for test. The voltage for this series of tests was raised as rapidly as possible by the rheostats in the field circuit of the generator. The time shown on the curves is that which elapsed from the time the voltage reached a steady value and held at this value until break-down. The r. m. s. values of the peak values determined by spark gap were used in plotting the curves. Each test point is shown.

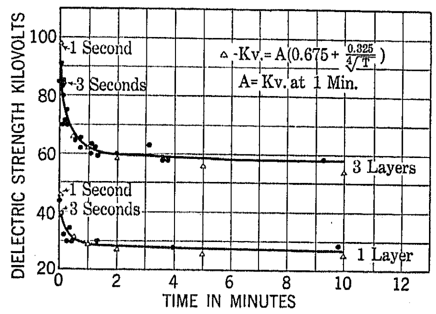

FIG. 12—420-CYCLE STRENGTH-TIME CURVES

0.0935-in. oil-treated pressboard, in oil at 100 deg. cent. Two 4-in. (10 cm.) diam. electrodes, each test shown, time shown from inst. voltage (rapidly applied) became constant until failure

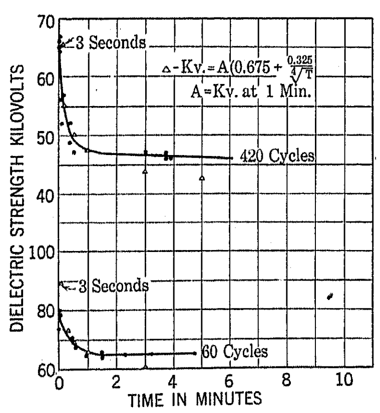

FIG. 13—60 AND 420-CYCLE STRENGTH-TIME CURVES

7 layers 0.012-in. black varnished cambric, in oil at 100 deg. cent.

It will be noted that these curves are of approximately the same shape for both frequencies and check fairly closely the average value of a in equation (2). It is interesting to note that the 420 cycle points are not so scattered as the 60-cycle points. This seemed to be characteristic of all tests made at the higher frequencies on all insulations except oil without any barriers.

Fig. 13 shows the results of tests made on black varnished cambric at 60 and 420 cycles. These curves show that the 420-cycle strength is approximately 75 per cent of the 60-cycle strength.

Fig. 14 gives the results of one and fifteen minute tests made at 60 and 420 cycles on various thicknesses of pressboard. These curves show that the 420-cycle values are approximately 75 per cent of the 60-cycle values for all thicknesses tested.

b. Variation of Dielectric Strength of Solid Insulation for any Given Time. Table IV gives a summary of the 420-cycle dielectric strength expressed as a percentage of the 60-cycle strength.

TABLE IV.

SUMMARY OF 420-CYCLE DIELECTRIC STRENGTH IN PER CENT OF 60-CYCLE STRENGTH

Time						
Momentary	1 min.	5 min.	15 min.	Material	Shown in Fig.	Remarks
76	80	80	..	3/32-in. P.B.	12	1 layer
76	75	78	..	3/32-in. P.B.	12	3 layers
82	72	71	..	0.012-in. B.V.C.	13	7 layers
not observed	72.5	not observed	70	1/32-in. P.B.	14	Average of 2 to 8 layers
..	71.0	..	..	3/32-in. P.B.	14	Average of 1 to 3 layers

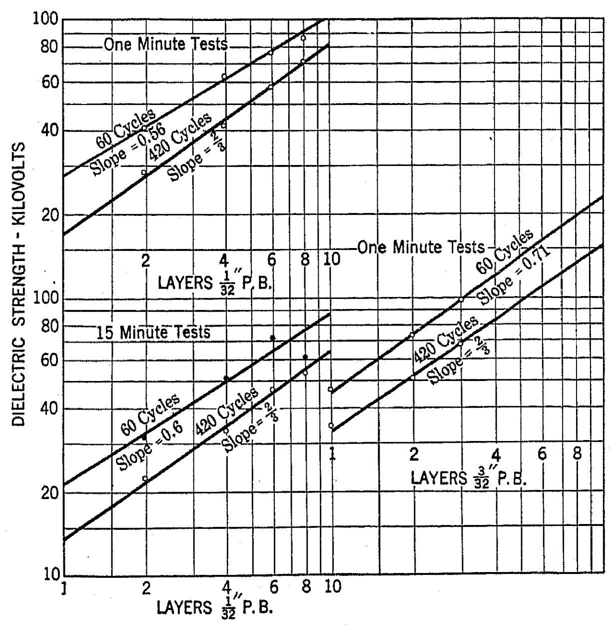

FIG. 14—EFFECT OF FREQUENCY AND THICKNESS ON DIELECTRIC STRENGTH OF PRESSBOARD AT 25 DEG. CENT.

4-in. (10 cm.) diam. electrodes

While the general average of the 420-cycle kilovolt values for solid insulation is approximately 75 per cent of the 60-cycle values it will be noted that the tendency is toward a little higher percentage for momentary values. However for practical purposes it appears that we can say that the percentage is constant regardless of time of voltage application.

The determination of the relation between the dielectric strength of solid insulation at 60 and 420 cycles, does not give the effect of other near and intermediate frequencies. To determine this, a large number of tests were made at 25, 60, 200 and 420 cycles, on solid insulation under as nearly as possible the same conditions. It was found that for all these frequencies at both 25 and 50 deg. cent. the dielectric

strength can be expressed by an exponential equation of the form kv. $= K'/F^n$ where K' is a constant, F is the frequency and n is a numerical value. For 60, 200 and 420 cycles at 100 deg. cent. an equation of the same form holds, but for 25 cycles at 100 deg. cent. the values were approximately 35 per cent higher than the 60-cycle values, whereas to fall in line they should be only 12 to 15 per cent higher. At first it was thought that this departure from the equation might be due to an error into tests, but check tests showed that it was correct. These tests are given in Table V.

TABLE V.
EFFECT OF FREQUENCY ON DIELECTRIC STRENGTH OF 3/32-IN. OIL-TREATED PRESSBOARD—10-CM. ELECTRODES—VOLTAGE INCREASED UNIFORMLY AT RATE OF 1 KV. PER 5 SECONDS—IN OIL

Layers	Freq.	No. Shots	Temp. deg. Cent.	Kv. by Vm. Coil Min.	Kv. by Vm. Coil Max.	Kv. by Vm. Coil Avg.	Avg. by Spark Gap	Ratio of kv. to 60-cyc. kv.*
5	60	5	100	121	132	122.4	129	
3	60	10	100	90	94	92.	94	
1	60	10	100	50	55	52.5	54	
1	25	17	99	60	82	68.3	72	1.33
3	25	16	99.5	117	144	126.4	127	1.35
(ck)3	25	10	99	120	138	126	126.5	1.345
3	25	10	27	104	117	109.4	110	1.11
3	60	10	26	73	101	97	99	
1	420	10	100	40	45	42	41	0.76
3	420	10	100	72	76	75	73.5	0.78
3	420	6	27	76	80	78	76	0.77
1	200	11	98	36	41	39	43	0.80
3	200	10	100	70	74	72	80	0.85
(ck)1	200	4	100	41	43	42	46	0.85
(ck)3	200	4	99	78	80	79	87	0.925
5	200	3	100	96	98	97	108.5	0.84

* For same number of layers.

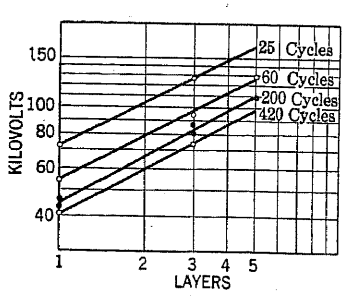

FIG. 15

Effect of frequency of dielectric strength of 3/32-in. oil-treated pressboard in oil at 100 deg. cent. Voltage increased at rate of 1 kv. per 5 seconds, 4-in. diam. electrodes.

The kv. values shown in Table V are plotted vs. layers of pressboard on double logarithmic paper in Fig. 15. When these values (expressed as a ratio of 60-cycle values) are again plotted against the reciprocal of the frequency it will be seen, Fig. 16, that except for the 25-cycle point around 100 deg. cent. they all fall in a straight line.

For practical purposes we can neglect the 25-cycle points at 75 deg. and 100 deg. (since we are concerned here only with frequencies ranging from 60 to about 500 cycles) and use an equation derived from the straight line.

The equation of dielectric strength vs. frequency, or of the line in Fig. 16, is:

$$E = 1.75/F^{0.137} \quad (3)$$

where E is, for any given time, the ratio of strength to 60-cycle strength and F is the frequency in cycles per second.

Although no claim is made for accuracy beyond about 500 cycles it is interesting to note that this

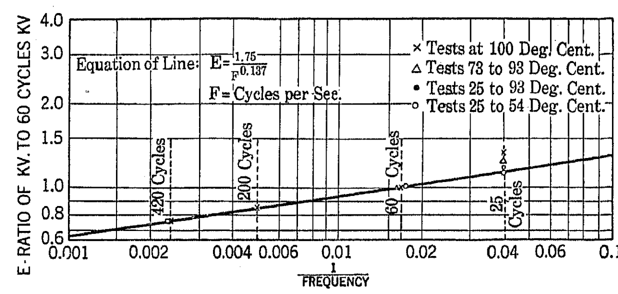

FIG. 16—EFFECT OF FREQUENCY ON RATIO OF DIELECTRIC STRENGTH OF SOLID INSULATION

method of calculation holds fairly closely in comparing Peek's results of tests made at 60 and 90,000 cycles as shown in Table VI.

TABLE VI.
DIELECTRIC STRENGTH AT 60 AND 90,000 CYCLES—FROM F. W. PEEK, PAGE 184, HIGH VOLTAGE ENGINEERING
OILED PRESSBOARD

Time	60 Cyc. test kv.	90,000 cyc. test kv.	90,000 cyc. kv. values calculated by equa. (3) from 60-cyc. test
inst.	35.5	9.5	13
inst.	39.5	6.1	14.4
one min.	31	7.3	11.3
	37	4.1	13.5
	VARNISHED	CAMBRIC	
inst.	53	19.5	19.4
inst.	42	13.5	15.3
inst.	42	10	15.3
one min.	46.5	17.8	17
	31	10	11.3
	31	7.5	11.3

b. Creepage Over Solid Insulation. Tests made with the electrodes on the same side of the sample piece, *i. e.* when the solid insulation was under no stress, gave very erratic results, corresponding more nearly to those obtained on rupture voltage of oil alone. Neither time nor frequency seemed to have any material effect on the failure voltage. But when the electrodes were placed on opposite sides of the test sample (that is when the material was under a stress) frequency had approximately the same effect on the arcover voltage as it had on the puncture voltage, *i. e.* the 420-cycle strength was approximately 75 per cent of the 60-cycle strength. As shown by the data given in Figs. 17 and 18, *time* had very little effect at either frequency.

c. Rupture Voltage of Oil at 60 *and* 420 *Cycles.* While it was not expected that the dielectric strength of transformer oil would vary with such a small difference in frequency as 60 and 420 cycles, tests were made using 4-in. (10 cm.) round edged electrodes spaced 0.375 in. apart in oil that tested 27 kv. with standard 1-in. electrodes spaced 0.1 in. apart. The voltage was increased at the rate of 1 kv. per 5 seconds. The results of these tests are shown in Table VII.

TABLE VII.
60- AND 420-CYCLE RUPTURE VOLTAGE OF NO. 10 TRANSIL OIL
Number of shots at each frequency 25. Temp. of oil 28 deg. cent.

Frequency	Kv. by Vm. Coil min.	Kv. by Vm. Coil Max.	Kv. by Vm. Coil Avg.	Kv. by Spark Gap
60 cycles	86	117	97	98
420 cycles	80	130	106.8	99

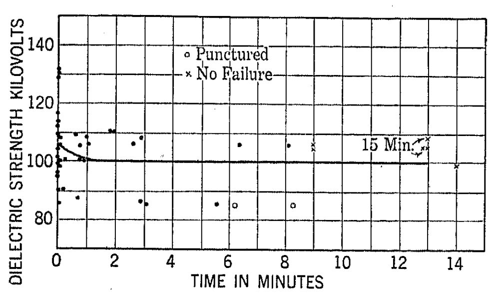

Fig. 17

60-cycle creepage tests on oil-treated, 0.0935-in. pressboard, in oil at 100 deg. cent. 4-in. diam. electrodes on opposite sides of barrier of 5 layers of pressboard. Total creepage distance 1.47-in. approx. stress on insulation 235 volts at 110 kv.

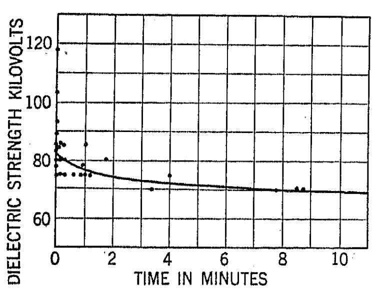

Fig. 18

420-cycle creepage tests on 0.0935-in. oil-treated pressboard, in oil at 100 deg. cent. 4-in. diam. electrodes on opposite sides of barrier of 5 layers of pressboard. Total creepage distance 1.47-in. (approx.). Stress on insulation 171 volts per mil at 80 kv.

Apparently frequency within the limits of 60 and 420 cycles has a negligible effect on the strength of oil.

d. Strength-Time Curves of Solid Insulation and Oil in Series. Referring to Figs. 19 and 20 it will be seen that the 60- and 420-cycle curves are of approximately the same shape as those (Figs. 11 and 12) for solid insulation, that is, they check the average value of a in equation (**2**). The oil duct was equal to the thickness of the pressboard sheets.

But when the oil duct is three times the thickness of the pressboard sheets, the strength-time curve taken at 25 deg. cent. Fig. 21 departs somewhat from the curve taken on solid pressboard at 25 deg. cent. (which we have seen checked the minimum value of a) and agrees more nearly with the average value of a up to about one minute. After one minute the curve apparently flattens out like the one for oil without barriers.

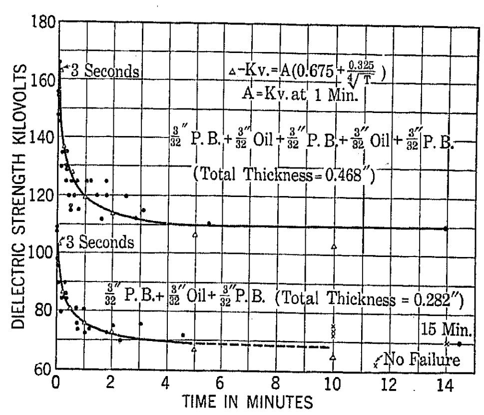

Fig. 19

60-cycle strength-time curves of oil-treated pressboard and oil in series at 100 deg. cent. oil ducts bridged by pressboard spacers. Time shown from inst. voltage (rapidly applied) became constant until failure, 4-in. diam. electrodes.

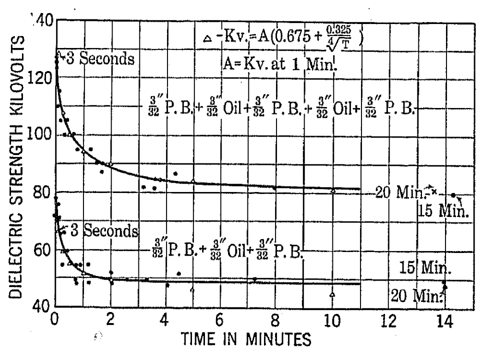

Fig. 20

420-cycle strength-time curves of oil-treated pressboard and oil in series at 100 deg. cent. Oil ducts bridged by pressboard spacers. Time shown from inst. voltage (rapidly applied) became constant until failure, 4-in. diam. electrodes

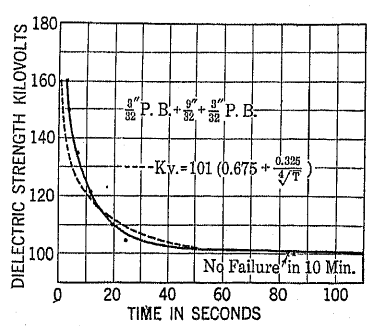

Fig. 21

60-cycle strength-time curve of solid insulation and oil in series at 25 deg. cent. Average of 10 tests, 4-in. diam. electrodes, time for voltage to build up (approx. 1 sec.) included. Material 3/32-in. oil-treated pressboard.

This is, as will be shown later, the worst condition to meet, *i. e.*, it requires the least reduction in time at the higher frequencies to make the strain equal to a 60-cycle test for one minute.

e. Variation of Dielectric Strength of Solid Insulation and Oil in Series for any Given Time. Table VIII gives the results of tests made at 60 and 420 cycles on four different combinations of oil and pressboard in series.

TABLE VIII.

DIELECTRIC STRENGTH OF PRESSBOARD AND OIL IN SERIES 4 IN. (10 CM.) DIAM. ELECTRODES. VOLTAGE INCREASED UNIFORMLY 10 KV. PER MINUTE FROM 55 KV. EXCEPT IN FIRST CASE

(Values taken from Strength-Time Curves Figs. 19 and 20). Each Value Average of 10 Tests.

Case No.	Thickness of Pressboard Sheet*	Oil Duct	60-cycle Kv.	420-cycle Kv.	Ratio: 420 cyc. kv. / 60 cyc. kv.
1	3/32 in.	3/32 in.	76 120	52 95	0.75
2	3/32 in.	3/16 in.	111.7	98.1	0.88
3	3/32 in.	9/32 in.	125.6	113.6	0.903
4	3/32 in.	3/8 in.	129.8	117.3	0.903

*Adjacent to each electrode

Assuming that the dielectric strength of solid insulation and oil in series is a function of the reciprocal of the frequency (the same as for solid insulation) the

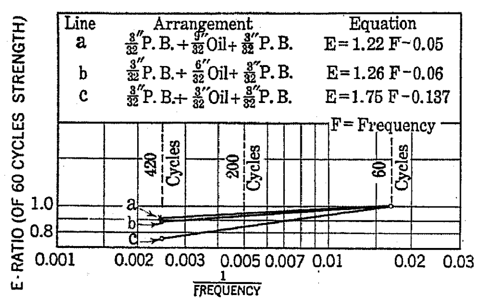

FIG. 22

Effect of oil in series with solid insulation on ratio of dielectric strength at different frequencies, 4-in. (10 cm.) diam. elec. points taken from data in Table 9.

results shown in Table IX and plotted as shown in Fig. 22 enable us to find the effect of frequency between the limits of 60 and 420 cycles on different arrangements of solid insulation and oil in series.

The equations for these conditions are as follows:

Arrangement	Equation
3/32 in. P. B. + 3/32 in. oil + 3/32 in. P. B.	$E = 1.75\ F^{-.137}$
3/32 in. P. B. + 3/16 in. oil + 3/32 in. P. B.	$E = 1.26\ F^{-.06}$
3/32 in. P. B. + 9/32 in. oil + 3/32 in. P. B.	$E = 1.22\ F^{-.05}$
3/32 in. P. B. + 3/8 in. oil + 3/32 in. P. B.	$E = 1.22\ F^{-.05}$

Relation Between Time Frequency and Dielectric Strength of Transformers

The question that naturally presents itself is: What condition shall we take to base the *time* on for various frequencies used in making induced potential tests?

The two extreme conditions are (1) oil alone which requires the same length of time no matter what frequency is used and (2) creepage over solid insulation which is under a fairly high voltage stress, *i. e.* with the electrodes on opposite sides of the barrier and where *time* had a small effect but the strength decreased with an increase in frequency. This condition would really require that the test voltage be reduced for the higher frequency even though the time be reduced. But the test voltage could not be reduced even if the time be kept the same because the test would not disclose any weak points in the oil distances.

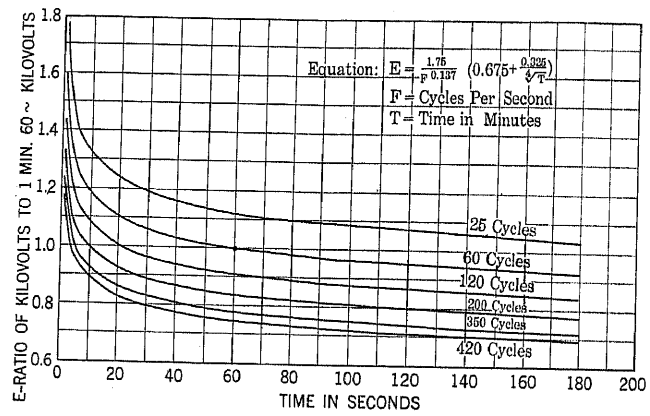

FIG. 23

Effect of frequency and time of voltage application on dielectric strength of solid insulation and solid insulation and oil in series (having equal parts of oil and pressboard) temperature 75 to 100 deg. cent.

General average conditions of either all solid insulation or of solid insulation and oil in series should be a fair basis for estimating the *time* of test.

If equation (**1**) which gives the variation in dielectric strength with time is multiplied by the equation ($E = k / F^n$) giving the variations of strength with frequency the resulting general equation is

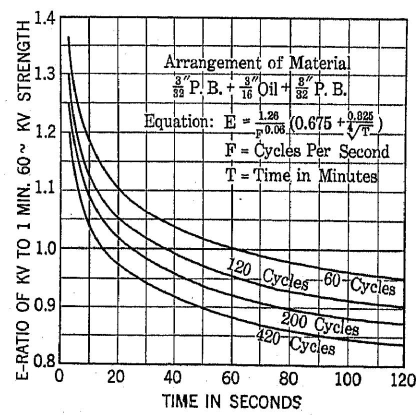

FIG. 24

Effect of frequency and time of voltage application on dielectric strength of pressboard and oil in series at 25 deg. cent.

$$E = k/F^n \left(a + \frac{1 - a}{\sqrt[4]{T}} \right) \quad (4)$$

Fig. 23 shows a series of curves plotted from equation (4) giving the relation between *time, frequency* anp *ratio of dielectric strength* for the average solid insulating materials and for a combination of solid

insulation and oil in series where the oil distance is equal to the thickness of solid insulation.

The curves shown in Fig. 24 are similar to those in Fig. 23 excepting that they are for the conditions where the oil duct is double the thickness of the press board pieces.

Expressed in seconds equation (4) becomes

$$t = \left[\frac{\dfrac{(1-a)}{E\,F^n}}{k} - a \right]^4 60 \qquad (5)$$

in which t is time in seconds, F is frequency in cycles per second, a and k are constants, n a numerical value depending on the material and arrangement of material and E is the ratio of dielectric strength, equal to unity when comparing with 60-cycle strength for one minute.

Table IX gives the relation between time and frequencies to produce the same voltage strain at 60, 200, 350 and 420 cycles.

TABLE IX.

TABULATION OF MATERIAL, TEMPERATURE, CONSTANTS AND TIME TO PRODUCE APPROXIMATELY THE SAME VOLTAGE STRAIN AT DIFFERENT FREQUENCIES—TIME CALCULATED BY EQUATION (5) ASSUMING $E = 1$

Material and Arrangement	Temp. deg. cent.	Value of Constants used in equa. (5)			Time in Seconds (Approx.) cycles			
		a	k	n	60	200	350	420
(1) Solid press board......	25	0.5	1.75	0.137	60	18	11	9.0
(2) Solid press board......	75–100	0.675	1.75	0.137	60	11	6	4
(3) Equal Dist. of P.B. and Oil..................	75–100	0.675	1.75	0.137	60	11	6	4
(4) Two 3/32 in. P.B. barriers separated by one 3/16 in. oil duct........	25	0.675	1.26	0.06	60	26	17	15
(5) Two 3/32 in. P.B. barriers separated by one 9/32 in. oil duct........	25	0.675	1.22	0.05	60	29	21	19

If we choose the constants for the arrangement of material, etc. as shown in case (5) of Table IX which condition requires the least reduction in *time*, the tabulation in Table X gives the duration of equivalent induced voltage test at various frequencies as estimated by equation (5).

TABLE X.

DURATION OF EQUIVALENT INDUCED VOLTAGE TESTS AT VARIOUS FREQUENCIES

Frequency cycles per sec.	Time of Voltage Application in Seconds
60	60
120	38
200	29
208 (3.46 × 60)	28
300	23
350	21
420	19

7. Conclusions

It is evident from the data in tables IX and X that recognition should be taken of the fact that induced potential tests when made at, say, more than double normal frequency, are far more severe that the requirements. In fact it is highly desirable that the *time* be reduced when making tests on large and expensive apparatus at considerably higher than normal frequencies, otherwise the insulation must be increased thus making it unnecessarily expensive.

8. Acknowledgements

It is desired to acknowledge the valuable assistance rendered in the Testing Department by Messrs. N. M. Albert, H. L. Garver, C. F. Green, L. D. Martin and W. F. Weikel in carrying on the long and tedious tests covering a period of three to four months.

Discussion

For discussion of this paper see page 354.

Insulation Tests of Transformers as Influenced by Time and Frequency

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Review of the Subject.—*Many transformers are now being designed for service with one end of the high-voltage winding solidly grounded. These transformers require an overpotential test by induced voltage of either 2.73 or 3.46 times their normal line voltage above ground plus 1000 volts. These tests must be made at more than normal frequency, to avoid too high a flux density in the core, and also to reduce the power required for excitation. It has long been known that the breakdown voltage of solid materials was affected by the length of time of application of voltage. Likewise the frequency of the applied voltage is shown to affect the breakdown voltage for solid insulations, increase in the frequency resulting in a decreased breakdown voltage. The voltage required for creepage failure is shown to be relatively unaffected by frequency. From the results of the tests made it is concluded that induced voltage tests on transformers with graded insulation at higher than normal frequencies should not have the test voltage reduced, but should have the duration of the test shortened to make the severity of the test comparable to the test at 60 cycles on normally insulated transformers.*

WITH the advent of transmission voltages in the magnitude of 220,000 volts, on grounded neutral three-phase systems, there has been an increased tendency to design the transformers with graded insulation, that is, with insulation to ground or between windings in proportion to the actual existing voltages between these parts. With this design, the 60-cycle high-potential test from the high-voltage windings to the other windings and core, based on twice the three-phase line voltage plus 1000 volts, becomes impossible and a substitute in the form of an overpotential test by induced voltage is required. An overpotential test for such transformers frequently used by the Westinghouse Electric & Manufacturing Co. is 2.73 times the normal voltage above ground plus 1000 volts. This test requires the use of a frequency at least 2.73 times the normal frequency of the transformer, to keep the core density down to normal values. Therefore, it is seen that the insulation must be tested at higher than the normal frequency (60 cycles) used in the older high-potential tests. It is the purpose of this paper to discuss the relation of insulation breakdown to the frequency of the impressed voltage as determined experimentally for some special cases.

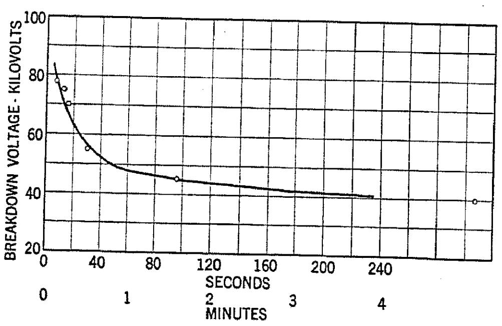

FIG. 1—TIME-VOLTAGE PUNCTURE CURVE AT 60 CYCLES
1/8-in. oil-impregnated fullerboard at 25 deg. cent.

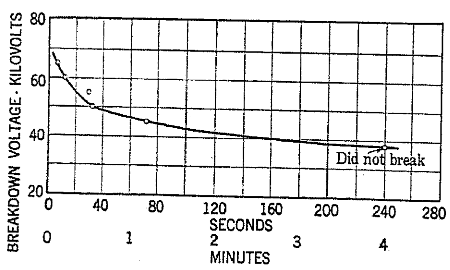

FIG. 2—TIME-VOLTAGE PUNCTURE CURVE AT 140 CYCLES
1/8-in. oil-impregnated fullerboard at 25 deg. cent.

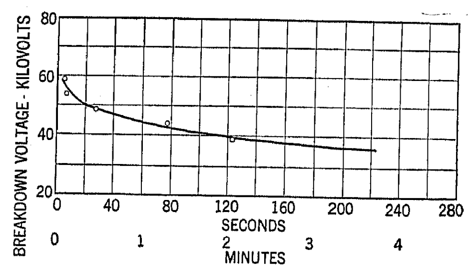

FIG. 3—TIME-VOLTAGE PUNCTURE CURVE AT 220 CYCLES
1/8-in. oil-impregnated fullerboard at 25 deg. cent.

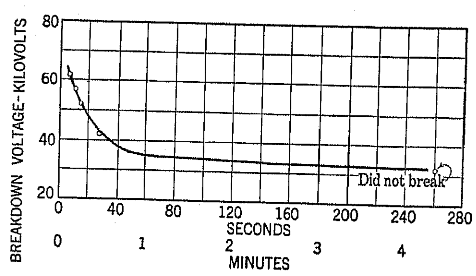

FIG. 4—TIME-VOLTAGE PUNCTURE CURVE AT 350 CYCLES
1/8-in. oil-impregnated fullerboard at 25 deg. cent.

TYPES OF INSULATION FAILURE AND METHODS OF TESTING

There are two possible divisions of failure under oil, one in which oil failure alone is considered, that is, where no solid material is punctured, and one in which there is puncture of solid material. These can still be

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further sub-divided into oil jump or creepage over the surface of solid material and puncture of solid material alone or with oil ducts intervening between the sheets of solid material. The investigation, therefore, naturally divides itself into two parts, and the methods of testing used are divided in this manner.

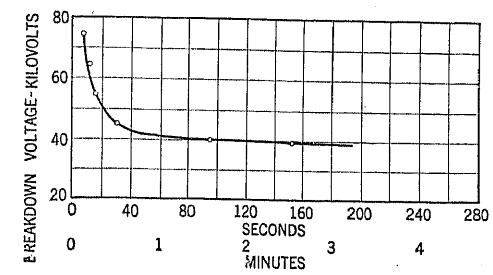


FIG. 5—TIME-VOLTAGE PUNCTURE CURVE AT 60 CYCLES
1/8-in. oil-impregnated fullerboard at 75 deg. cent.

Individual readings in the tests involving oil failure have a tendency to be erratic. In fact, this tendency is so pronounced that it was decided not to attempt to procure average values, but rather to determine the

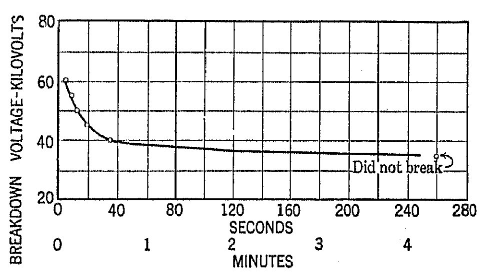


FIG. 6—TIME-VOLTAGE PUNCTURE CURVE AT 140 CYCLES
1/8-in. oil-impregnated fullerboard at 75 deg. cent.

maximum voltage which could be held a given length of time, for a considerable number of tests. To determine this, five minutes was selected as a unit of time, and tests made, starting at a fairly high value and

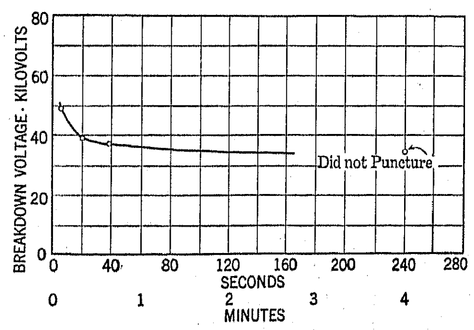


FIG. 7—TIME-VOLTAGE PUNCTURE CURVE AT 220 CYCLES
1/8-in. oil-impregnated fullerboard at 75 deg. cent.

decreasing the test voltage until the voltage was held four or five times for the desired length of time, in succession, with no breaks.

In making puncture tests where solid material was punctured, the effect of the duration of the test on the breakdown voltage was studied to determine the basis for comparison between the different frequencies. The following method was in general used to procure the individual time-breakdown voltage curves.

The field current leads of the generator supplying power were brought to the testing transformer regulating equipment, arranged so that the field current could be adjusted and the field circuit opened or closed. This permitted the most rapid application of voltage without surges by closing the generator field switch. The voltage corresponding to a given

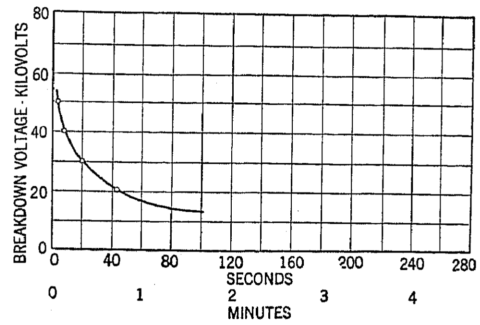


FIG. 8—TIME-VOLTAGE PUNCTURE CURVE AT 350 CYCLES
1/8-in. oil-impregnated fullerboard at 75 deg. cent.

setting of the regulator was then determined by spark gap. This setting was left undisturbed and the voltage then applied to the test piece by closing the generator field switch. Settings up to the continuous voltage strength of the test piece were checked with the spark gap and test piece in parallel, and the readings corrected to suit, as some variation occurred due to the charging current of the test piece. In the individual readings the time was noted at which failure occurred from the time the generator field switch was closed.

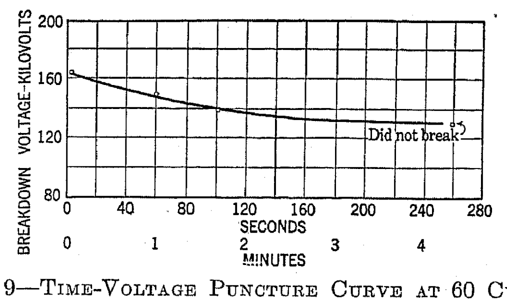


FIG. 9—TIME-VOLTAGE PUNCTURE CURVE AT 60 CYCLES AND 25 DEG. CENT.
3 sheets oil-impregnated 1/8-in. fullerboard with alternate 1/8-in oil ducts. Tested horizontally

In order that an idea of the times required for application of the voltage may be had, oscillograms showing the relation between the voltage, as it built up, and time were taken. These indicate the following times for building up of voltage after the generator field switch was closed: 50,000 volts, 60 cycles, built up in 1 second; 40,000 volts, 140 cycles, built up in 1.22 seconds; and 50,000 volts, 220 cycles, built up in 0.5 seconds. The same machine was used for the

350-cycle tests as for the 220-cycle, and the time should be less for the 350 cycles than for the 220 cycles, since for the same voltage less field current is required. In the worst case, therefore, the data taken give the

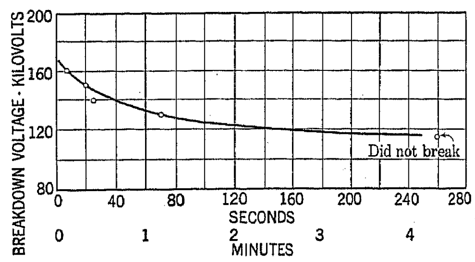


FIG. 10—TIME-VOLTAGE PUNCTURE CURVE AT 140 CYCLES AND 25 DEG. CENT.

3 sheets oil-impregnated, 1/8-in. fullerboard alternate 1/8-in. oil ducts. Tested horizontally.

times to within one or two seconds of time, except for certain tests on fullerboard at 60 cycles and in oil at 75 deg. cent. In these, nearly four seconds elapsed, but it was believed that it did not greatly affect the

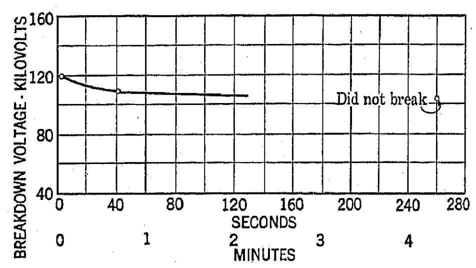


FIG. 11—TIME-VOLTAGE PUNCTURE CURVE AT 220 CYCLES AND 25 CENT. DEG.

3 sheets oil-impregnated, 1/8-in. fullerboard with alternate 1/8-in. oil ducts. Tested horizontally.

reading at one minute intervals of time, and hence these curves were not rechecked.

The wave form in each case was of pure sine form for the 60-cycle and 140-cycle frequencies. It was

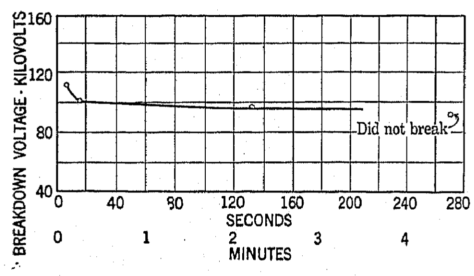


FIG. 12—TIME-VOLTAGE PUNCTURE CURVE AT 350 CYCLES AND 25 DEG. CENT.

3 sheets oil-impregnated 1/8-in. fullerboard with alternate 1/8-in. oil ducts. Tested horizontally.

impossible to make good oscillograms at 220 and 350 cycles, but the wave form is believed to be satisfactory. At all events, due to the method of test calibrating against spark gap, these data are based on crest values of the wave.

CREEPAGE TESTS

Creepage tests were made under oil, between electrodes on opposite sides of a barrier of four ⅛-in. oil-impregnated fullerboard sheets. The electrodes were located by gage ½ in. from the edge. The electrodes were of brass, 4 in. in diameter with ½ in. radius. The barriers were placed in a horizontal position.

The individual readings of the tests at 60 cycles varied so much that it was decided not to try to find average times for a given voltage setting, but rather

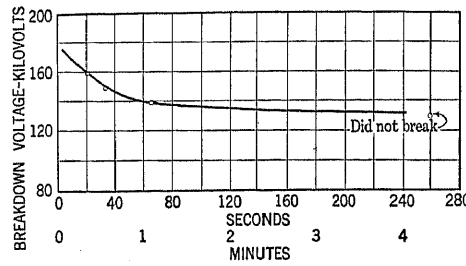


FIG. 13—TIME-VOLTAGE PUNCTURE CURVE AT 60 CYCLES AND 75 DEG. CENT.

3 sheets oil-impregnated 1/8-in. fullerboard with alternate 1/8-in. oil ducts. Tested horizontally.

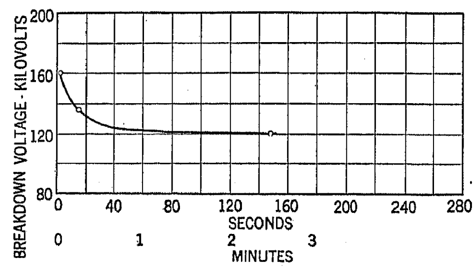


FIG. 14—TIME-VOLTAGE PUNCTURE CURVE AT 140 CYCLES AND 75 DEG. CENT.

3 sheets oil-impregnated 1/8-in. fullerboard with alternate 1/8-in. oil ducts. Tested horizontally.

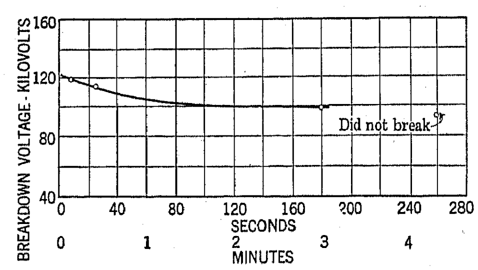


FIG. 15—TIME-VOLTAGE PUNCTURE CURVE AT 220 CYCLES AND 75 DEG. CENT.

3 sheets oil-impregnated 1/8-in. fullerboard with alternate 1/8-in. oil ducts. Tested horizontally.

to find the maximum voltage which could be held fairly continuously (5 minutes) for a number of trials with *no* failures. An idea of the variation of these readings is shown by the following data: Voltage setting, 75 kv.; oil temperature, 25 deg. cent.; time of application, 1 min., 56 secs.; 1 min. 8 secs.; 3 secs.; 4 ½ secs.; 4 min. 41 secs.; 2 min. 1 sec.; 7 min., 36 secs.; 8 secs. The above are the first eight of twenty-eight readings, of which the maximum was

13 min., 14 seconds, and the minimum, 3 seconds. Readings taken at 74 kv. were all of greater length of time than 15 minutes, no failures noted.

A table of the results obtained at other temperatures and frequencies follows:

TABULATION OF CREEPAGE DATA

Voltage held without break in five minutes.

4 in. diameter electrodes, ½ in. from edge of barrier composed of four ⅛ in. fullerboard sheets, placed horizontally.

Frequency	25 deg. Centigrade	75 deg. Centigrade
60 cycles	74 Kilovolts	78 Kilovolts
140 "	80 "	80 "
220 "	81 "	81 "
350 "	72 "	Punctured solid material

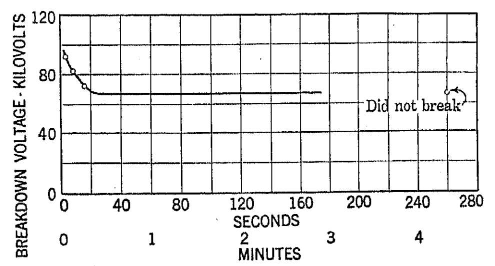


FIG. 16—TIME-VOLTAGE PUNCTURE CURVE AT 350 CYCLES AND 75 DEG. CENT.

3 sheets oil-impregnated 1/8-in. fullerboard with 1/8-in. oil ducts. Tested horizontally.

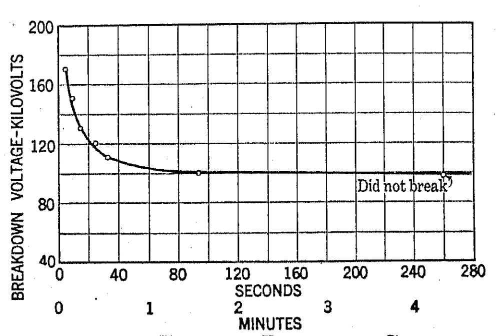


FIG. 17—TIME-VOLTAGE PUNCTURE CURVE AT 60 CYCLES AND 25 DEG. CENT.

2 sheets 1/8-in. oil-impregnated fullerboard separated by 3/8-in. oil duct. Tested horizontally.

VARIATION OF BREAKDOWN VOLTAGE OF BARRIERS UNDER OIL

A number of different conditions were investigated, at two temperatures, 25 deg. cent. and 75 deg. cent. These were, puncture voltage of ⅛-in. thick fullerboard, puncture voltage of three sheets of ⅛-in. fullerboard separated by ⅛-in. thick spacers, giving ⅝-in. puncture distance, and puncture voltage of two sheets of ⅛-in. thick fullerboard, separated by ⅜-in. thick spacers, giving ⅝-in. puncture distance also. The barriers were placed horizontally except for a special test which is reported later. All fullerboard was vacuum-dried and oil-impregnated.

The method of test has been described previously. These data will be reported in the same order as described above, and the typical curve shapes shown for the relation between time and breakdown voltage for any frequency. The electrodes used were four inch diameter brass disks, with one-half inch radius.

Curves of the original data for all the different tests are shown, in curves 1 to 24. It will be seen that the curves are of similar shape, but it is not believed that they are sufficiently so that a representative curve can be drawn.

From these data the relative times can be derived for a given voltage to cause failure at the different frequencies. A tabulation of these results, based on a voltage sufficient to cause failure at the end of one minute at 60 cycles, follows:

Tests on ⅛-in. fullerboard.
Fullerboard placed horizontally in tank.
Curves 1 to 8 inclusive.

Breakdown Voltage	60 cycles	140 cycles	220 cycles	350 cycles
	Times for breakdown at 25 deg. cent.			
48,000	1 min.	50 secs.	34 secs.	20 secs.
	Times for breakdown at 75 deg. cent.			
41,000	1 min.	30 secs.	16 secs.	7 secs.

Tests on Barriers
Three ⅛-in. thick fullerboard plus two ⅛-in.
Oil ducts, alternating fullerboard and oil.
Tested horizontally.*
Curves 9 to16, inclusive.

Breakdown Voltage	60 cycles	140 cycles	220 cycles	350 cycles
	Times for breakdown at 25 deg. cent.			
149,000	1 min.	20 secs.	Inst.	Inst.
	Times for breakdown at 75 deg. cent.			
141,000	1 min.	12 secs.	Inst.	Inst.

Tests on Barriers,
Two ⅛-in. fullerboard plus one ⅜ in.
Oil duct between fullerboard sheets. Tested horizontally, Curves 17 to 24.

Breakdown Voltage	60 cycles	140 cycles	220 cocles	350 cycles
	Times for breakdown, 25 deg. cent.			
102,000	1 min.	42 secs.	18 secs.	14 secs.
	Times for breakdown, 75 deg. cent.			
106,000	1 min.	50 secs.	14 secs.	Inst.

APPLICATION OF DATA TO TESTS ON APPARATUS

From the above data, testing apparatus at higher frequencies increases the severity of the test rapidly. The principal reason for the adoption of the trans-

*1. The tests at 25 deg. cent. at 60 cycles and 350 cycles, were also made with the barriers and oil ducts vertical. It was felt that some increase might result due to freer oil circulation, and it was indeed noted that the shapes of the curves were more nearly the same, and that they were higher than those where the barriers were horizontal. See curve 25 for comparison with curves 9 and 12. The horizontal tests were made as illustrating worst possible conditions.

former with graded insulation is to reduce the amount of insulation required, and if it is found necessary to increase parts of the insulation, merely to meet tests, over that required for the normally insulated transformer tested at the lower frequency of 60 cycles, its

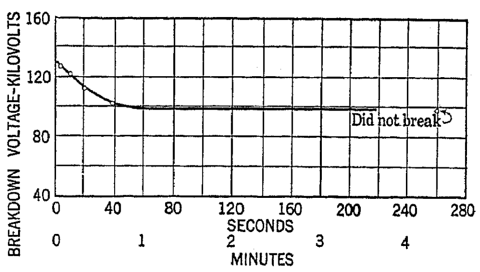


FIG. 18—TIME-VOLTAGE PUNCTURE CURVE AT 140 CYCLES AND 25 DEG. CENT.

2 sheets 1/8-in. oil-impregnated fullerboard separated by 3 8/in. oil duct. Tested horizontally.

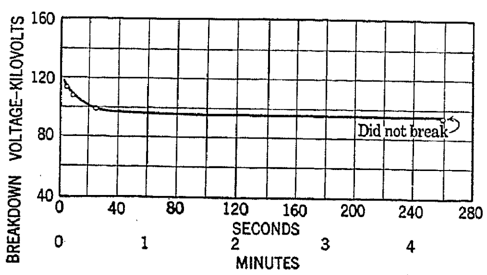
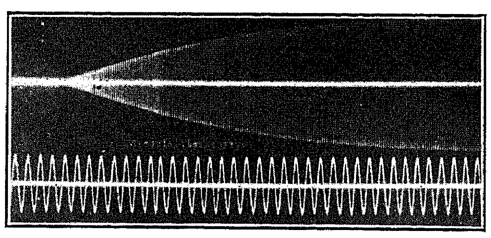
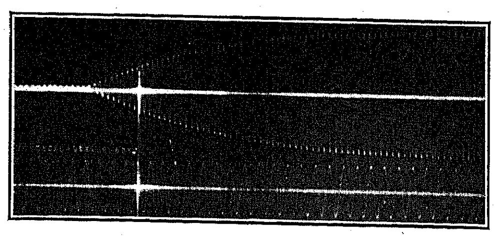


FIG. 19—TIME-VOLTAGE PUNCTURE CURVE AT 220 CYCLES AND 25 DEG. CENT.

2 sheets 1/8-in. oil-impregnated fullerboard separated by 3/8-in. oil duct. Tested horizontally.



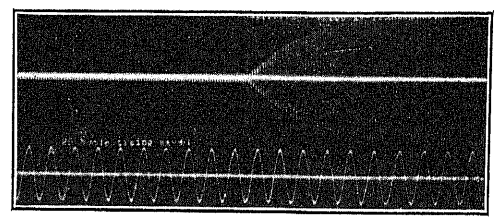
BUILDING UP OF FIELD ON 140-CYCLE GENERATOR TRANSFORMER REGULATOR SET FOR 40-KV.



BUILDING UP OF FIELD ON 60-CYCLE GENERATOR FIELD EXCITED BY 500 VOLTS INSTEAD OF 110 VOLTS

advantage is to some extent at least lost. For this reason the designer is interested to determine what insulation test, if any, would test such apparatus to the same severity as if it could be tested at 60 cycles.

Referring to the results of the creepage data, it would appear that failures due to creepage are not greatly affected by frequency. This is more or less to be expected, for the following reasons: Creepage breakdown, in the absence of foreign conducting material, is probably largely a question of oil failure. For fluids it would also be expected that frequency would not result in quite as large a decrease in the breakdown voltage as for solid material, due to the relative absence of the effect of heat. For air, indeed, the effect of frequency is very slight. Therefore no decrease in the actual voltage



BUILDING UP OF FIELD ON 220-CYCLE GENERATOR SUPPLYING 300-KV. TRANSFORMERS.

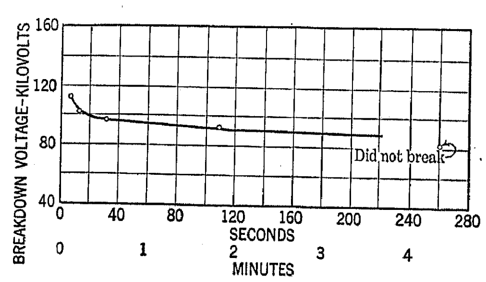


FIG. 20—TIME-VOLTAGE PUNCTURE CURVE AT 350 CYCLES AND 25 DEG. CENT.

2 sheets 1/8-in. oil-impregnated fullerboard separated by 3/8-in. oil duct. Tested horizontally.

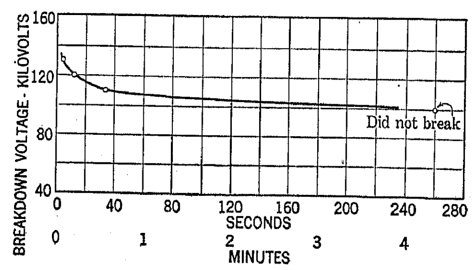


FIG. 21—TIME-VOLTAGE PUNCTURE CURVE AT 60 CYCLES AND 75 DEG. CENT.

2 sheets 1/8-in. oil-impregnated fullerboard separated by 3/8-in. oil duct. Tested horizontally.

applied for test would be desirable. On the other hand, the effect of time is not as great for this type of breakdown. Down to the point at which the applied voltage could be held permanently, the tests were erratic, but the point at which it would hold this voltage appeared to be quite definite. Very slightly above the critical voltage tests of relatively very short time might be obtained. It would be recommended, therefore, that for any test at over 60 cycles contemplated to equal the severity of a 60-cycle, one minute test, that no decrease

in the testing voltage be made, but rather that the duration be decreased to provide for decreased puncture strength of the solid material.

At some places in a transformer insulation failure would involve the puncturing of fullerboard entirely, at others only a portion of fullerboard and oil in series. Observation of the puncture curves would indicate that variations occur with the different percentages of fullerboard which are impossible to reconcile into any kind of average or representative data. Possible reasons for these variations are as follows: For the ⅛-in. fullerboard it is possible that the electrodes, of brass, reduced the heating of the fullerboard due to its large thermal capacity. This possibly would increase the duration of time for a given breakdown voltage and make it longer in the case of the higher frequencies than for the other combinations as tabulated above. For the barriers with the three sheets of fullerboard it is apparent that the higher frequency tests are very severe. For the barriers with two sheets, the conditions are less severe. This might be expected, since if oil is the less affected by frequency, the less the percentage of fullerboard in a given distance, the less the breakdown voltage should be affected by frequency.

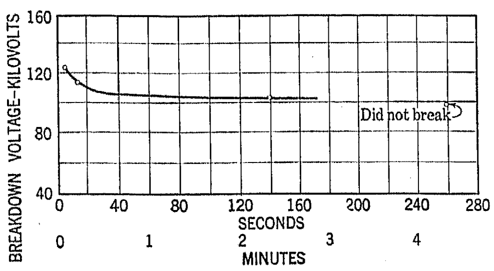


FIG. 22—TIME-VOLTAGE PUNCTURE CURVE AT 140 CYCLES AND 75 DEG. CENT.

2 sheets 1/8-in. oil-impregnated fullerboard separated by 3/8-in. oil duct. Tested horizontally.

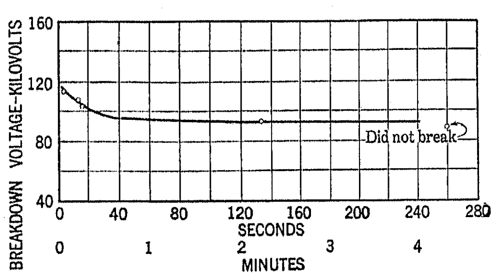


FIG. 23—TIME-VOLTAGE PUNCTURE CURVE AT 220 CYCLES AND 75 DEG. CENT.

2 sheets 1/8-in. oil-impregnated fullerboard separated by 3/8-in. oil duct. Tested horizontally.

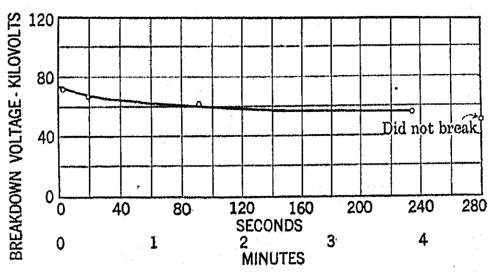


FIG. 24—TIME-VOLTAGE PUNCTURE CURVE AT 350 CYCLES AND 75 DEG. CENT.

2 sheets 1/8-in. oil-impregnated fullerboard, separated by 3/8-in. oil duct. Tested horizontally.

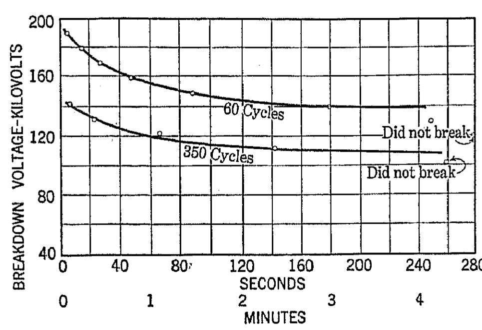


FIG. 25—TIME-VOLTAGE CURVE AT 60 AND 350 CYCLES 25 DEG. CENT.

3 sheets 1/8-in. oil-impregnated fullerboard with alternate 1/8-in. oil ducts. Tested vertically..

In an actual transformer, for the higher voltages, a large proportion of its insulation between windings is of less than fifty per cent fullerboard. It is believed that forty per cent is a fair average of the minimum amount between the windings. From the point of view of making the test on the part least affected by frequency sufficiently severe, it would seem that the condition represented by the two sheets of fullerboard should be chosen. Accordingly it would be logical that the test voltage should be its normal value and that its duration should be decreased in line with the results of the tests, using the lower percentage of fullerboard.

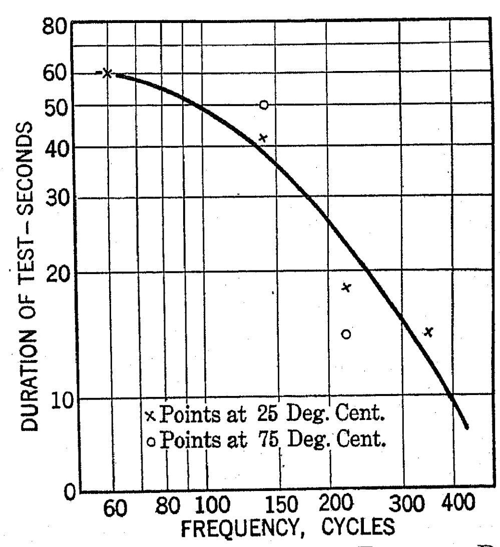


FIG. 26—TIMES FOR PUNCTURE TESTS AT DIFFERENT FREQUENCIES BASED ON VOLTAGE GIVING PUNCTURE IN 60 SECS. AT 60 CYCLES

X Points at 25 deg. cent.
⊙ Points at 75 deg. cent.

Fig. 26 shows points plotted between frequency and time, based on the voltage for puncture in one minute at 60 cycles, for both 25 deg. cent. and 75 deg. cent. These points, being values read from other curves whose individual accuracy may not be over 5 per cent, do not permit drawing a very accurate curve. The curve shown is fairly representative of conditions at both 25 deg. and 75 deg. cent., it is believed. It is perhaps slightly favorable to the apparatus at frequencies up to 150 or 160 cycles, but is decidedly unfavorable at frequencies much over 200 cycles, since apparatus is tested while hot, and the data show the higher frequencies, at 75 deg. cent. to be very severe.

From Fig. 26 the table which follows might then be proposed, for the application of induced test voltages, at higher than 60 cycles. The times are given for 164 and 208 cycles, since these are frequencies which are approximate multiples of test voltages required by the Standardization Rules of the American Institute of Electrical Engineers. The times given for the higher frequencies would require that extra insulation be given to the transformer above that needed if it could be tested at 60 cycles, but this is done to provide for the larger units, which may require testing at these frequencies to reduce the power required for testing.

Duration of Equivalent Induced Voltage Tests at Various Frequencies

Frequency	Time of Application
60 cycles	60 seconds
120 cycles	44 seconds
164 cycles (2.73 x 60)	33 seconds
208 cycles (3.46 x 60)	26 seconds
240 cycles	21 seconds
360 cycles	11 seconds
400 cycles	10 seconds

Discussion

EFFECT OF TIME AND FREQUENCY ON INSULATION TESTS OF TRANSFORMERS[1] (Montsinger)
and
INSULATION TESTS OF TRANSFORMERS AS INFLUENCED BY TIME AND FREQUENCY[2] (Vogel)

Philadelphia, Pa., February 7, 1924

Vladimir Karapetoff: For several years previous to his death, Dr. Steinmetz talked from time to time about third-class conductors and pyroelectric effects, and I often asked myself, why this interest in the third-class conductors. I did not see the point until the appearance of his paper in the "*Electrical World*," giving the so-called *pyroelectric theory of breakdown of insulation*. At the same time, but independently from Dr. Steinmetz, Dr. K. Willy Wagner in Germany was working on the same subject. He came to this country on a visit in 1922 and read a paper (printed in the 1922 A. I. E. E. Transactions) which for a long time will remain a classic on the subject. In other words, where Dr. Steinmetz only had time to go into the theory qualitatively, Dr. Wagner also gave us at least a beginning of a mathematical theory of the breakdown of solid dielectric.

The principle underlying this theory is as follows: Let there be a layer of solid dielectric between two metal electrodes and let this layer be subjected to a d-c. voltage. Let us also assume that somewhere in this dielectric there is a weak filament (or a succession of weak spots which later form a weak filament) along which a breakdown takes place. According to the pyroelectric theory, not only the whole dielectric has a negative temperature-resistance coefficient (which we know only too well to be true), but this weak filament in particular has a very marked negative temperature coefficient, so that the conductivity of this filament or thread increases as the temperature goes up. In this you see immediately an element of instability, which should lead to a breakdown. Being a weaker filament, in the sense of having initially a somewhat higher conductivity than the rest of the material, this filament conducts in proportion more current than the rest of the sheet and consequently becomes warmer than the rest. But, by supposition, when it becomes warmer, its conductivity increases; therefore, it begins to draw more current. Its conductivity again goes up, and so forth. Thus, naturally, a conductor of this kind, applied at a source of sufficient high potential, must ultimately take an infinitely great current. Before it does so, the material is carbonized, or otherwise changes, and we have a breakdown. In other words, according to both Dr. Wagner and Dr. Steinmetz, we have here a pure thermal or pyroelectric effect. There is no mysterious "breakdown," no critical voltage in that sense; it is simply a matter of heating a filament to a point where it becomes a piece of carbon.

Now, if that be so, then by using a source of constant current rather than a constant voltage, we should be able to go beyond the "limit" and yet not break a sample down. Dr. Wagner used wooden blocks for electrodes, with fibers in the direction of the current. Dr. Steinmetz used a Nernst filament, that is, a piece of material of very marked negative temperature coefficient. They both proved that you can go beyond the critical value of the current, but because there isn't enough voltage to cause an infinite current the material can be cooled again and it will be just as good as before, showing that there is no "electric breakdown" but only a thermal effect.

Perhaps it is a one-sided theory; perhaps the theory is only partly true; perhaps there are other factors to be considered. But if this theory be true at least in part, certain mathematical relations follow immediately. You can write a differential equation between the applied voltage and the temperature, and this equation, integrated, gives a function which a material ought to obey. Dr. Wagner uses three different conduction functions to start with, and carries his computations through, correctly stating that we haven't enough experimental evidence as yet to decide which of the three functions is the best. We at Cornell took still another function and also carried the mathematical deductions through to show that there are also other possibilities and that the pyroelectric theory is very elastic in its very foundation.

The papers under discussion interested me greatly as offering new material for a confirmation or a modification of the pyroelectric theory. Dr. Wagner has computed the theoretical effect of the frequency upon the breakdown, and also the effect of the time of application. No matter which of his three functions he takes, the effect of the frequency is linear. If you plot the breakdown voltage against the frequency as abscissa, you obtain a slanted straight line because as the frequency increases, the dielectric loss also increases. It heats the slab and thus prevents more heat from being radiated from the filament sidewise.

Referring first to Figure 16 in Mr. Montsinger's paper, he obtains a straight line when he plots his voltage, not to cycles but to log. of 1 over cycles. This being a straight line, plotting it to the frequency would give a curve of the shape of a hyperbola. So I thought at first that Wagner's theory was inadequate in this case. However, you will notice that the point corresponding to 25 cycles is rather uncertain, and it is the 25-cycle point that makes the curve go up so much. By changing it a

little, we can get a much closer approximation to a straight line. If we take the data given in Fig. 15 (I selected the data for four layers) and replot them to frequency, we obtain an almost straight line, except for 25 cycles.. Taking, furthermore, the first page of Mr. Vogel's paper and combining the ultimate voltages there for different frequencies, I also get practically a straight line. So that on the whole, I see in the new data a confirmation rather than a contradiction of Wagner's theory. I hope that from now on we shall not only multiply experimental data, but also analyse them so as to see whether they confirm or contradict the pyroelectric theory.

Everett S. Lee: Here we have an example of two investigations, made in different places by different men, both to answer the same question, and although they got together, I take it, before they started to make their tests, I am quite sure that they viewed their conclusions in finality before getting together, and if you will read both papers and go through them very carefully, you will note that they are remarkably consistent for this kind of work. One reason is because of the fact that the tests were taken under essentially the same conditions.

In that regard I would call to your attention the fact that at the present time the American Society for Testing Materials is doing quite a bit of work to try to standardize tests on electrical insulating materials. That is of interest to all of you, and if you will place yourself in the position of the man who has to make those tests and go to the standards of our own Institute to try to find out how those tests should be made, or what time should be used in making the tests, etc., you will find nothing there. In other words, it is a branch of the art that has not been taken up and put in the standards of our Institute. So that the American Society for Testing Materials is doing that work, and a considerable amount of the information that Mr. Montsinger and Mr. Vogel compiled has been used in allowing us, in turn, to standardize work requiring the determination of the dielectric strength of insulating materials.

I call your attention particularly to the fact that the curve between dielectric strength and time of voltage application is not a straight line and that tests made for dielectric strength of insulating materials, where the value of time may be low, may bring you on such a portion of the curve that a small difference in time will mean a large difference in dielectric strength; so that if an observer in New York compares some data with an observer some miles distant, those two observers may find that they have different results if they have not taken this into account. So I feel that in these two papers we have been helped considerably in that phase of the work.

Regarding Professor Karapetoff's discussion, we too noted, as he brought out, a tendency for that curve to bend upward at the lower frequencies, and I might state that Dr. Steinmetz had in mind making tests very carefully in the range of 25 cycles down to zero, and that portion of the work which he was carrying on is being continued, and we hope that it will throw additional light upon this particular phase of the question.

I hope that all of you have taken occasion to read the article entitled, "High Voltage Insulation," by Dr. Steinmetz and Mr. J. LeRoy Hayden, which appeared in the January issue of the JOURNAL, because that paper summarizes practically in toto the results of the work which Dr. Steinmetz had carried on in the last few years in his investigation of the mechanism of breakdown of insulating materials.

V. M. Montsinger: I am very glad to have Professor Karapetoff discuss this question from the pyro-electric theory standpoint. I have read very carefully both Dr. Wagner's and Dr. Steinmetz' articles on the pyro-electric theory of failure of insulation. Space, however, did not permit of making comparisons between my results and the formulas given by Dr. Wagner in the way it should be done. Furthermore, additional data on the strength at the lower frequencies, that is, from about 25 cycles down to 0 cycles (direct current) are needed before drawing any definite conclusions as to whether the data substantiates or refutes Dr. Wagner's formula which shows that the strength is a linear function of the frequency. From what tests I have made and according to Mr. Vogel's tests made on single sheets of pressboard (fullerboard) the linear function does not seem to hold very well. For example, taking the data from Mr. Vogel's curves, Figs. 1 to 8 inclusive, covering tests made at 60, 140, 220 and 350 cycles and at 25 deg. and 75 deg. cent. and plotting the 20-sec. kv. values (which points appear to be the most reliable) against the reciprocal of the frequencies, we get practically straight lines on log-log paper, the slopes of the lines being 0.146 at 25 deg. cent. and 0.136 at 75 deg. cent. The average slope of the two lines is approximately 0.141 as against the value of 0.137 which I obtained. From the available data, the indications are that the dielectric strength of solid insulation is not strictly a linear function of the frequency. Further work along this line is being done and it is hoped that enough additional data can be secured to settle this point one way or another.

Short Circuits of Alternating-Current Generators

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Review of the Subject.—*In this paper, the author presents a physical conception and simple non-mathematical analysis of the short-circuit phenomena of alternating-current generators. Formulas are developed for the maximum instantaneous values of the armature and field short-circuit currents delivered by both single and poly-phase generators, which are equipped with and without damper windings.*

A similar analysis covering the rate of decay and the sustained values of the short-circuit currents will be presented in the near future.

THE physical phenomena resulting from the short circuit of alternating-current generators are becoming more or less familiar to the interested electrical designing, and operating engineers. In a general way, it is also well known that the normal flux interlinkages, currents, and induced voltages which are inherently associated with the magnetic and electric circuits of an alternator are thrown into a state of unstable equilibrium when the alternator is suddenly short-circuited at or near its terminals.

Oscillograph records of the armature current delivered by an alternating-current generator under short-circuit conditions show that it reaches a maximum peak value, either during, or at the end of the first half-cycle of the current wave. The peak values of the successive waves gradually decrease in amplitude until a constant or sustained value is reached, usually at the end of two or three seconds. Moreover, if several oscillograms are

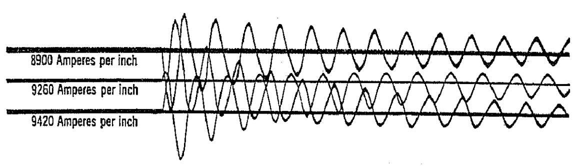

FIG. 1—SHORT-CIRCUIT TEST ON 6250-KV-A. TURBO-ALTERNATOR

60 cycles, 3600 rev. per min., 6600 volts per phase. Star-connected Oscillogram shows current in three phases, 100 per cent voltages.

taken at random, under the same apparent initial conditions, the armature current waves may be either totally, or partially displaced with respect to a zero reference axis during a short time interval immediately following the short circuit. The displacement of the wave is a maximum during the first cycle and then rapidly decreases to zero, so that the wave is, as a rule, approximately symmetrical by the end of the first half second. Similar oscillographic records of the field current show that it also rises in peaks to values several times normal, immediately after short circuit, and then the pulsations decrease in amplitude and are either completely damped out or reach a permanent value, depending on the kind of short circuit, *i. e.*, whether it is a polyphase or a single-phase short circuit.

Fig. 1 shows an oscillogram of the armature currents delivered by a three-phase a-c. generator under short-circuit conditions. The amount of displacement of the current waves, the maximum value of the currents, the rate of decrease of the peak values, and the time required for the waves to become symmetrical are all different for each phase of the generator winding. Fig. 2 is an oscillogram of the field current that flows when a three-phase alternator is suddenly short-circuited. The pulsations which are of line frequency are quickly damped out and the excess direct current gradually decreases to zero value. Figs. 3 and 4

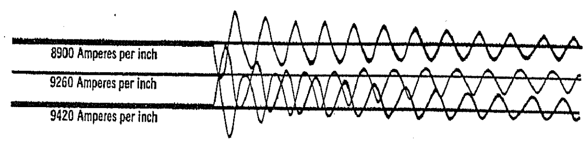

FIG. 1A—SHORT-CIRCUIT TEST ON 6250-KV-A. TURBO-ALTERNATOR

60 cycles, 3600 rev. per min., 6600 volts per phase. Star-connected Oscillogram shows current in three phases, 75 per cent voltage.

show oscillograms of the armature and field currents that flow when a single-phase generator, without a damper winding, is short-circuited. In this case the pulsations in the field current which are of double-frequency decrease in amplitude until a constant value is reached, but do not totally disappear.

From the practical standpoint, the most important phases of the short-circuit phenomena of a-c. generators are:

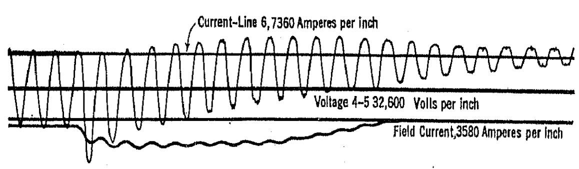

FIG. 2—SHORT-CIRCUIT TEST TURBO-GENERATOR

12,500 kv-a., 13,200 volts, 60 cycles, 3-phase, 1800 rev. per min.

(a) the maximum peak values of the armature and field currents that are reached during the first half-cycle after the short circuit occurs:

(b) the rate of decrease of the peak and r. m. s. values of the current waves for the armature and field circuits; and

(c) the final or sustained values of the armature current.

The maximum peak values of the armature and field

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currents are especially important for they impose the maximum torque on the rotor shaft; the maximum force between the armature conductors of different phase groups; and the maximum force between the end turns of the armature and field windings. The rate of decay of the r. m. s. values of the current determines the amount of energy the auxiliary control apparatus and circuit breakers must absorb before the short circuit is removed, and the successive peak values determines the duty imposed on the circuit breakers in removing the short circuit. The final or sustained values of the armature current determines the heating and consequently, the temperature rise of the armature winding in case the circuit breakers fail to function or the short circuit occurs between the generator and the circuit breakers.

Of the considerable number of articles which have

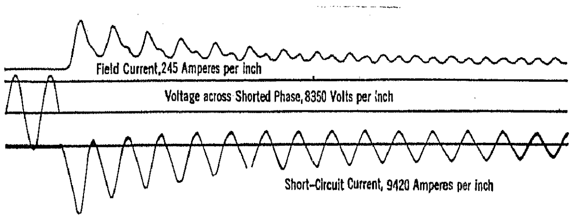


FIG. 3—SHORT-CIRCUIT TEST ON 6250 KV-A. TURBO-GENERATOR

60 cycles, 3600 rev. per min., 6600 volts per phase, single-phase test, voltage across short-circuited phase, current in single-phase test, voltage across short-circuited phase, current in short-circuited phase, and field current. 50 per cent voltage.

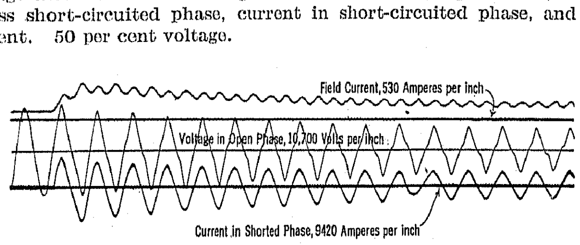


FIG. 3A—SHORT-CIRCUIT TEST ON 6250-KV-A. TURBO ALTERNATOR

60 cycles, 3600 rev. per min., 6600 volts per phase, star-connected, current in short-circuited phases, field current and voltage across open phase. 75 per cent voltage.

been written to explain the short-circuit phenomena of a-c. generators, the most noted and classic is the analysis presented by Dr. P. Boucherot in 1912.[1] However, in this case as well as in most of the others the mathematical considerations have predominated and the interpretation and results can not be effectively grasped by a large number of engineers who are not familiar with the mathematics involved. In view of this fact, it is the purpose of the writer in this discussion to present the physical conception and analysis of the short-circuit phenomena of a-c. generators in a simple manner so that it can be analyzed by the average interested engineer. Nothing but the simplest mathematics will be used in establishing the current and voltage relations. The simplest type of a-c. generator will be considered first, and later modifications will be made to cover actual practical machines.

FUNDAMENTAL FLUX RELATIONS FOR A SINGLE-PHASE GENERATOR

In the simple type of single-phase generator shown in Fig. 5, the stationary and rotating elements are both considered to be perfectly laminated and to have infinite permeability. The stationary element is provided with

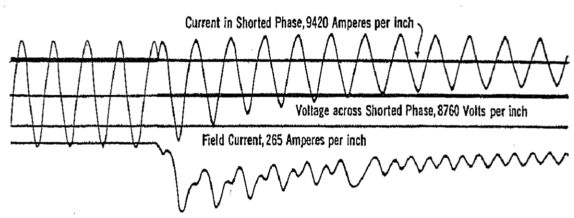


FIG. 4—SHORT-CIRCUIT TEST ON TURBO-ALTERNATOR, 6250 KV-A.

60 cycles, 3600 rev. per min., 6600 volts per phase. Single-phase test. Current and voltage in short-circuited phase and field current. 75 per cent voltage.

an armature winding, $A\,A_1$, which is uniformly distributed over a portion of the armature periphery and has N_a turns, all of which are connected in series. The salient pole rotor has a field winding, $P\,P_1$, of N_1 turns connected in series. When a direct current, I_1, flows in the field circuit a total magnetic flux, I_1, ϕ_1, is produced which interlinks these windings as shown by the dotted lines in Fig. 5. In the above expression, the symbol, ϕ_1, is defined as the total flux produced by the field circuit when one ampere flows in it. This total

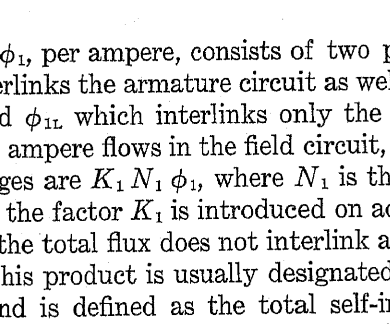


FIG. 5

field flux, ϕ_1, per ampere, consists of two parts; $\phi_{m1\,a}$, which interlinks the armature circuit as well as the field circuit and ϕ_{1L} which interlinks only the field circuit. When one ampere flows in the field circuit, its total flux interlinkages are $K_1\,N_1\,\phi_1$, where N_1 is the number of turns and the factor K_1 is introduced on account of the fact that the total flux does not interlink all of the field turns. This product is usually designated by the symbol, L_1 and is defined as the total self-inductance or coefficient of self-induction of the field winding. In a

1. Note: Since the present paper was written, another noteworthy paper on the short circuit of a-c. generators was presented by Mr. R. E. Doherty in June, 1923. (See JOURNAL A. I. E. E., Oct. 1923).

similar manner the quantity, $\mathcal{L}_1 = k_{1L} N_1 \phi_{1L}$, is defined as the coefficient of leakage self-induction of the field circuit. Similarly, the flux interlinkages of the armature circuit due to one ampere flowing in the field circuit are equal to, $K_{m1a} N_a \phi_{m1a}$, which is defined as the coefficient of mutual induction between the armature and field circuits and is represented by the symbol, m_{1a}.

Coefficient of Leakage Self Induction with Different Rotor Positions

By referring to Fig. 5, it is evident that none of the field flux interlinks the armature winding, when the axis of the field winding is at right angles to the axis of the armature winding, and the coefficient of leakage self-induction of the field winding is equal to its coefficient of total self-induction. When the axis of the two windings coincide, the coefficient of leakage self-induction is a minimum. Since these two rotor positions are displaced 90 degrees, it is obvious that the leakage induction coefficient of the field winding varies between maximum and minimum values at double frequency for the different rotor positions. This variation of the coefficient of leakage self-induction for the different rotor positions is shown by the double frequency curve in Fig. 6.

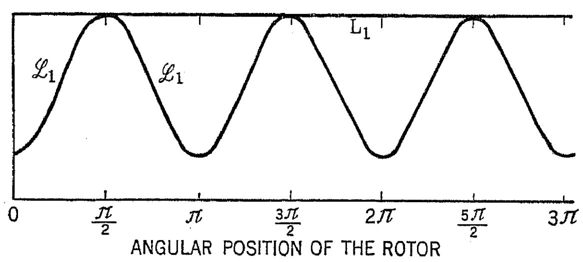


Fig. 6

Coefficient of Mutual Induction with Different Rotor Positions

The shape of the field pole projections is such that the density of the field flux which crosses the air gap and enters the stator core varies approximately as a sine law around the periphery of the armature. Since the field flux is carried mechanically around the stator core when the rotor is driven at a given speed, the flux interlinkages of the armature circuit per ampere in the field circuit vary between maximum and minimum values at the same or synchronous frequency. With the rotor in the angular position shown in Fig. 5, the mutual flux interlinkages of the armature circuit are a maximum. And 90 degrees later, when the field winding is at right angles to the armature winding none of the field flux interlinks the armature winding and its coefficient of mutual induction is zero. When the rotor is in the 180 degree position, the flux interlinkages of the armature winding are the same in magnitude as in Fig. 5, but the flux lines link the circuit in the reverse direction. Since the distribution of the field flux is sinusoidal with respect to the armature periphery, the actual value of the mutual induction coefficient will follow a cosine law as shown by curve 2 in Fig. 7.

Using a similar system of notation, the corresponding induction coefficients of the armature circuit can be written and defined as follows:

$L_a = K_a N_a \phi_a$, coefficient of total self-induction of the armature circuit.

$\mathcal{L}_a = K_{la} N_a \phi_{la}$, coefficient of leakage self-induction of the armature circuit.

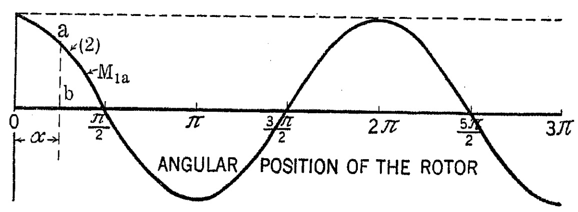


Fig. 7

$M_{a1} = K_{ma1} N_1 \phi_{ma1}$, coefficient of mutual induction of the armature and field circuits.

The coefficient of total self-induction of the armature circuit is not constant for all rotor positions on account of the salient pole construction of the rotor. The leakage induction coefficient of the armature circuit varies between maximum and minimum values according to the same general law as for the similar coefficient of the field circuit. The relation between the total and leakage induction coefficients of the armature circuit is shown by the curves in Fig. 8.

It is well known from the laws of physics that the mutual induction coefficient of two inductively coupled circuits is the same for either circuit with respect to the other; that is, in the case under consideration, $M_{a1} = M_{1a}$, and $K_{m1a} N_a \phi_{m1a} = K_{ma1} N_1 \phi_{ma1}$. Under this condition the curve in Fig. 7 shows the variation of the mutual induction coefficient of either the field circuit with respect to the armature circuit or the armature circuit with respect to the field circuit for any angular position of the rotor.

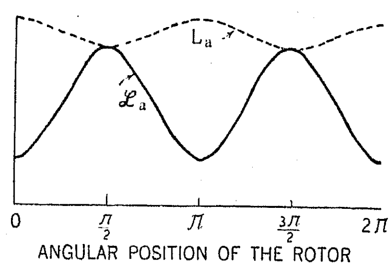


Fig. 8

If the induction and resistance coefficients of the circuits are known, the magnitude of the currents can be readily determined for both stable and transient

conditions when the voltage or flux relations are known. However, under transient short-circuit conditions a better physical conception and simpler analysis of the flux and currents during the first half cycle after the short circuit occurs can be had by assuming the armature and field circuits, both, to have zero resistance. Under this assumption, the instantaneous values of the short-circuit currents for the different circuits can be easily obtained by applying Lenz's fundamental law which can be stated as follows:

"If a closed electric circuit has a given number of flux lines interlinking it, any *change* or *variation* in the number of flux interlinkages will cause a current of sufficient magnitude to flow in the circuit in such direction that the initial flux interlinkages of the circuit are maintained constant."

Maximum Instantaneous Value of Short-Circuit Current

Single-Phase Generator Without Damper Winding. If the armature circuit of the single-phase generator is suddenly short-circuited at the instant the axis of the

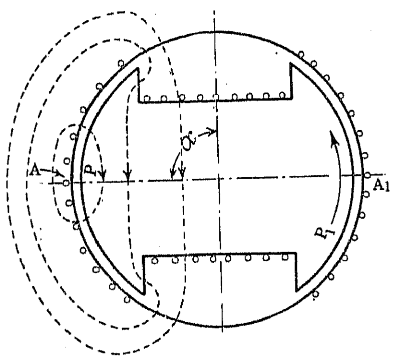


Fig. 9

field winding coincides with the axis of the armature winding, as shown in Fig. 5, the initial flux interlinkages which must be maintained constant, are $I_1 L_1$, and $I_1 M_{1a}$, respectively for the two circuits. At an instant later, when the rotor has turned through a small angle α the initial flux interlinkages of the armature circuit are decreased. Consequently, a current will flow in this winding tending to supply the decreased amount of flux interlinkages. However, when a current flows in the armature circuit it not only produces a flux interlinkage with itself, but also adds additional flux interlinkages to the field circuit. These additional flux interlinkages of the field circuit will in turn tend to be neutralized by an opposing flux set up by a secondary current in the field circuit. At any instant the currents in the two circuits must be of such magnitude that the resultant flux interlinkages of each circuit due to the combined action of all currents are the same as at the instant that the short circuit occurred.

When the rotor is in the angular position shown in Fig. 9, none of the field flux interlinks the armature circuit and the total change of its flux interlinkages are I_1, M_{1a}. At this instant the leakage and total self-induction coefficients of the armature circuit are one and the same as shown by Fig. 8. Consequently, the current, i_a, that flows in the armature circuit must be such that its total flux interlinkages are equal to the change in its initial flux interlinkages.

That is,

$$i_a L_a = i_a \mathfrak{L}_a = I_1 M_{1a}, \text{ or}$$

$$I_a = \frac{I_1 M_{1a}}{L_a} = \frac{I_1 M_{1a}}{\mathfrak{L}_a} \quad (1)$$

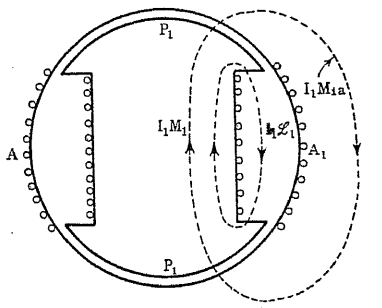


Fig. 10 A

α = 180 deg. $I_1 L_1 = I_1 L_1 + I_1 M_1$

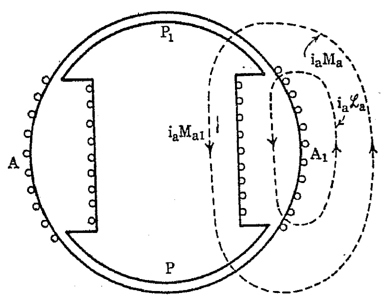


Fig. 10 B

α = 180 deg. $i_a L_a = i_a L_a + i_a M_a$

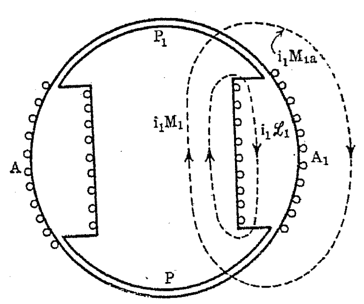


Fig. 10 c

α = 180 deg. $i_1 L_1 = i_1 L_1 + i_1 M_1$

At this instant, there is no current flowing in the field circuit except the normal exciting current, I_1.

When the rotor has turned through 180 degrees as shown in Fig. 10A, the interlinkages of the armature circuit due to the exciting current, I_1, in the field circuit are the same as at the instant of short circuit but interlink it in the reverse direction. Hence, the total change in the normal field flux interlinking the armature circuit is from $I_1 M_{1a}$ to $- I_1 M_{1a}$ or $2 I_1 M_{1a}$. The current,

i_a, that flows in the armature circuit produces a total flux interlinkage, $i_a L_a$, with itself. This total flux interlinkage consists of the leakage flux interlinkages, $i_a \mathcal{L}_a$, and the flux interlinkages $i_a M_a$ due to the mutual flux as shown in Fig. 10B. The corresponding flux interlinkages of the field circuit, due to the normal flux are, $i_a M_{a1}$. Since the interlinkages of the field circuit must be maintained at a constant value $I_1 L_1$, the flux interlinkages of the field circuit due to its secondary current, i_1, must be equal to the mutual flux interlinkages from the armature circuit.

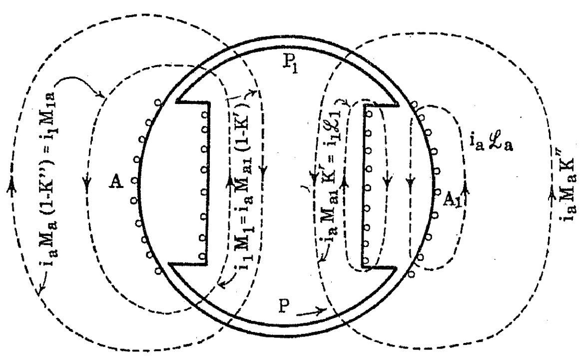


FIG. 11
$\alpha = 180$

That is,

$$i_1 L_1 = - i_a M_{1a}\text{, or}$$
$$i_1 = - i_a M_{1a}/L_1 \qquad (2)$$

For the purpose of analysis, the flux interlinkages $i_a M_{a1}$, of the field circuit due to the armature current can be separated into the two components, $K' i_a M_{a1}$, and $(1 - K') i_a M_{a1}$, where K' is a constant for this particular rotor position. In a like manner, the flux interlinkages of the armature circuit due to its own mutual flux can be divided into two components, $K'' i_a M_a$, and $(1 - K'') i_a M_a$. Fig. 11 shows the components of the flux interlinkages of both circuits due to the currents, i_a and i_1, on the basis that the resultant flux interlinkage of each circuit is the same as at the instant of short circuit. (The flux interlinkages due to the normal field current are not shown for the sake of clearness.)

From Fig. 11 the following flux interlinkage relations for the field circuit are evident.

$$i_1 M_{1a} = - i_a M_{a1} (1 - K') \qquad (3)$$
$$i_1 \mathcal{L}_1 = - i_a M_{a1} K' \qquad (4)$$

Therefore,

$$i_1 M_{1a} + i_1 \mathcal{L}_1 = - i_a M_{a1} (1 - K') - i_a M_{a1} K'\text{, or}$$
$$i_1 L_1 = - i_a M_{a1} \qquad (5)$$

That is, the total flux interlinkages of the field circuit due to the current, i_1, are equal to and neutralized by the total mutual component of the armature flux interlinkages, consequently the only flux interlinkages of the field circuit are $I_1 L_1$ which are the same as the initial value.

From Fig. 11 the following flux interlinkage relations must be satisfied for the armature circuit at this angular position,

$$i_a M_a (1 - K'') = i_1 M_{1a} \qquad (6)$$
$$i_a M_a K'' + i_a L_a = \text{resultant flux interlinkages for the armature circuit} \qquad (7)$$

Since the resultant flux interlinkages of the armature circuit must be equal in magnitude to the total change in its initial flux interlinkages.

$$i_a M_a K'' + i_a \mathcal{L}_a = 2 I_1 M_{1a} \qquad (8)$$

In other words, the magnitude of the armature current must be such that its leakage flux interlinkages plus the leakage flux interlinkages of the field circuit (expressed in terms of the armature circuit constants) due to the proportional secondary current, i_1, equals the total change in the initial flux interlinkages of the armature circuit. It is usually more convenient to express the sum of the leakage flux interlinkages of the two circuits in terms of an equivalent leakage flux interlinkage of the armature circuit alone. Substituting the value of i_1 from equation (2) in equation (6) gives

$$K'' = 1 - \frac{M_{a1}^2}{L_1 \; M_a} \qquad (9)$$

But $\qquad \mathcal{L}_a = L_a - M_a \qquad (10)$

Substituting the values of K'' and $\mathcal{L}_a$ in equation (8) gives

$$i_a (L_a - M_{a1}^2/L_1) = 2 I_1 M_{1a}\text{, or}$$
$$i_a = \frac{2 I_1 M_{1a}}{L_a - M_{1a}^2/L_1} \qquad (11)$$

And from equation (2)

$$i_1 = - i_a M_{1a}/L_1\text{, or,}$$
$$i_1 = - \frac{2 I_1 M_{1a}}{L_a - M_{1a}^2/L_1} \times M_{1a}/L_1 \qquad (12)$$

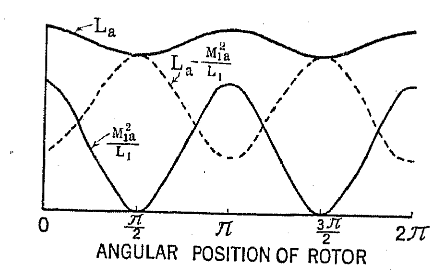


FIG. 12

The quantity, $L_a - M_{1a}^2/L_1$, is defined as the coefficient of the equivalent leakage induction of the armature circuit when the angular position of the rotor is such that the axes of the two windings coincide. The value of this coefficient of equivalent leakage self-induction of the armature circuit can be determined for any other angular position of the rotor from the curves in Figs. 7 and 8 and plotted as shown by curve 3 in Fig. 12. Then the instantaneous values

of the armature and field currents, can be determined from the curve in Fig. 7 and curve 3 in Fig. 12. Figs. 13 and 14 show the approximate wave form of the armature and field currents during the first cycle when the short circuit occurs at the instant the axis of the field winding coincides with the axis of the armature winding.

It is evident from Figs. 13 and 14 that both armature and field currents reach maximum values at the end of the first half-cycle when the short circuit occurs at the instant the armature winding interlinks the maximum field flux. Under these conditions the armature current wave is also totally displaced from the zero reference axis. The maximum value of the armature current under these short circuit conditions, is the maximum possible value that the generator will deliver with the given excitation, for under this condi-

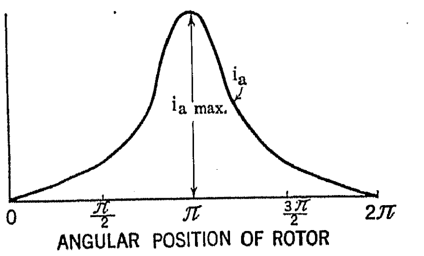


Fig. 13

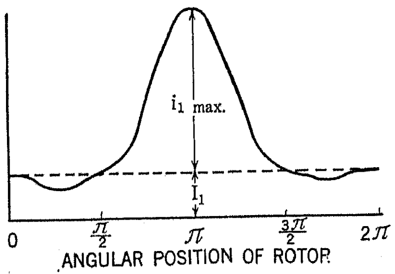


Fig. 14

tion the total flux change (due to rotation) has a maximum value and the coefficient of equivalent leakage induction of the armature circuit is a minimum.

The general equations for the values of the armature and field currents under any short circuit can now be stated as follows:

$$i_a = \frac{\text{Change (due to rotation) in normal field flux interlinkages of the armature circuit}}{\text{Coefficient of equivalent leakage self-induction of the armature circuit}} \quad (13)$$

$$i_1 = -\, i_a \times \frac{\text{Mutual induction coefficient of the armature circuit with respect to field circuit}}{\text{Total self-induction coefficient of the field circuit}} \quad (14)$$

From these general relations the currents can be determined for any short-circuit condition. If the short circuit occurs at the instant the field winding is at right angles to the armature winding none of the

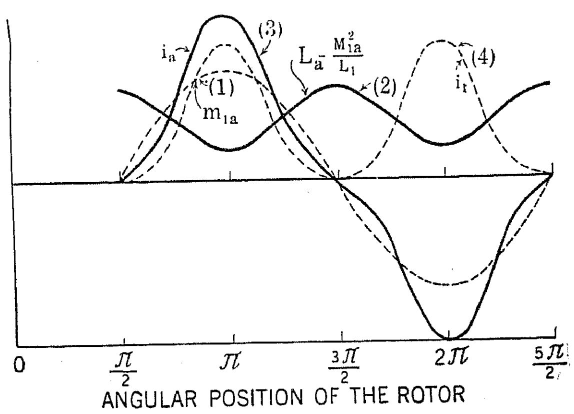


Fig. 15

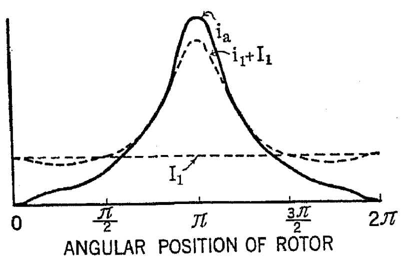


Fig. 16 A

$\alpha_0 = 0$

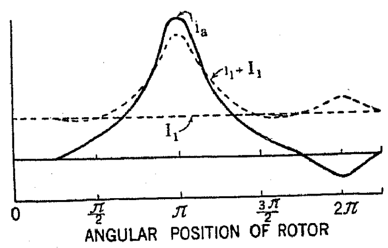


Fig. 16 B

$\alpha_0 = \pi/4$

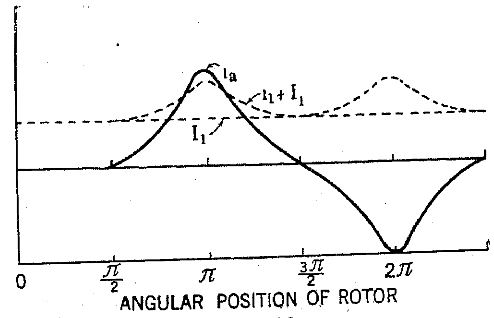


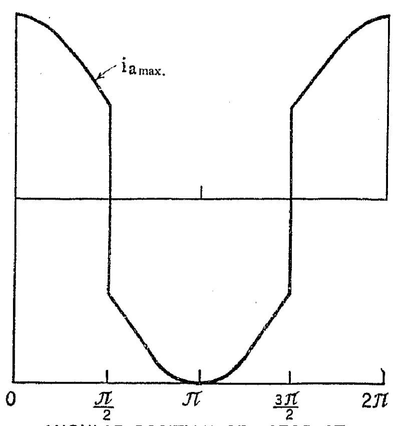
Fig. 16 C

$\alpha_0 = \pi/2$

field flux interlinks the armature circuit. As the rotor changes positions the change in flux interlinkages of the armature circuit will follow approximately a sine law as shown by curve 1 in Fig. 15. The total

equivalent leakage induction coefficient of the armature circuit is shown by curve 2 in Fig. 15.

Curves 3 and 4 represent the general form of the armature and field current waves. In this case, the armature current wave is entirely symmetrical with respect to the zero reference axis. The maximum values of both currents occur at the same angular position of the rotor as in the previous case, but the magnitude of the currents are only one-half of those in the former case.



ANGULAR POSITION OF ROTOR AT INSTANT OF SHORT-CIRCUIT

FIG. 17

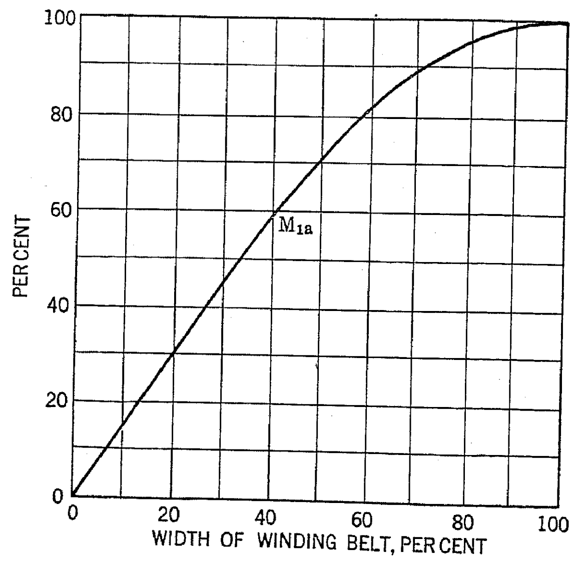


FIG. 18

$$i_a = \frac{I_1 M_{1a}}{L_a - M_{1a}^2/L_1} \tag{15}$$

$$i_1 = -\frac{I_1 M_{1a}}{L_a - M_{1a}^2/L_1} \times M_{1a}/L_1 \tag{16}$$

The curves in Figs. 16A, 16B and 16C show the general form of the calculated currents for both the armature and field circuits when the short circuit occurs at three different rotor positions, namely: $\alpha = 0$, $\alpha = 45$ deg. and $\alpha = 90$ deg.

The curve in Fig. 17 shows the variation of the maximum values of the armature current for the different angular positions of the rotor at the instant of short circuit.

In the preceding discussion the circumferential width of the armature phase band has not been limited to any specific value. It is well known that the flux interlinkages of a single-phase armature winding vary approximately as the sine of one-half the width of the phase belt as shown in Fig. 18, even though the field form does not have a sinusoidal distribution around the armature periphery. The relation between the total equivalent leakage self-induction coefficient, $L_a - M_{1a}^2/L_1$, and the width of the armature belt can be obtained either by calculation or test. The total induction coefficient increases faster than the first power of the phase belt width as shown by curve 1 in Fig. 19. Since the mutual induction coefficient

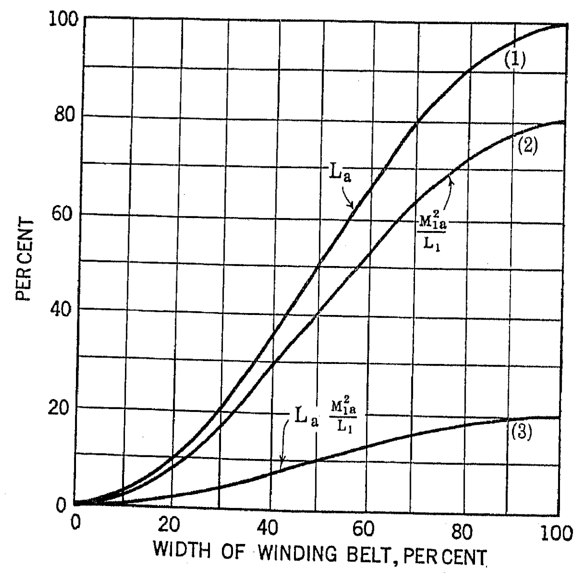


FIG. 19

varies sinusoidally with respect to the width of the phase belt and the magnitude of the field self-induction coefficient is independent of the width of the armature phase belt, the quantity, M_{1a}^2/L_1 will vary approximately as the sine square law as shown by curve 2 in Fig. 19. Curve 3 of this figure shows that the equivalent leakage self-induction coefficient increases faster than the first power of the width of the winding belt. Then by changing the scale of the curve in Fig. 18 to represent total change in flux interlinkages, $K I_1 M_{1a}$, and using the total equivalent leakage induction coefficient of the armature circuit from curve 3 of Fig. 19, the maximum possible instantaneous value of the armature current under particular short-circuit conditions can be determined from Equation 11 and plotted as shown by curve 3 of Fig. 20. This curve indicates that the maximum value of the armature current increases as the width of the phase belt decreases. This relation is based on the fact that the

field flux is constant and the number of conductors in the armature circuit increases directly with the width of the phase belt. Hence, the ordinate $\overline{a_1 b_1}$ gives the maximum value of armature current when one leg of a delta-connected winding or one leg,—terminal to neutral, of a start-connected winding is short-circuited. Ordinate $\overline{a_2 b_2}$ shows the maximum value of the armature current when one leg of a two-phase winding is short-circuited. Similarly, $\overline{a_3 b_3}$ is the maximum armature current delivered when a star-connected winding is short-circuited between two terminals. And $\overline{a_4 b_4}$ shows the current delivered by a single-phase generator with all of the periphery wound.

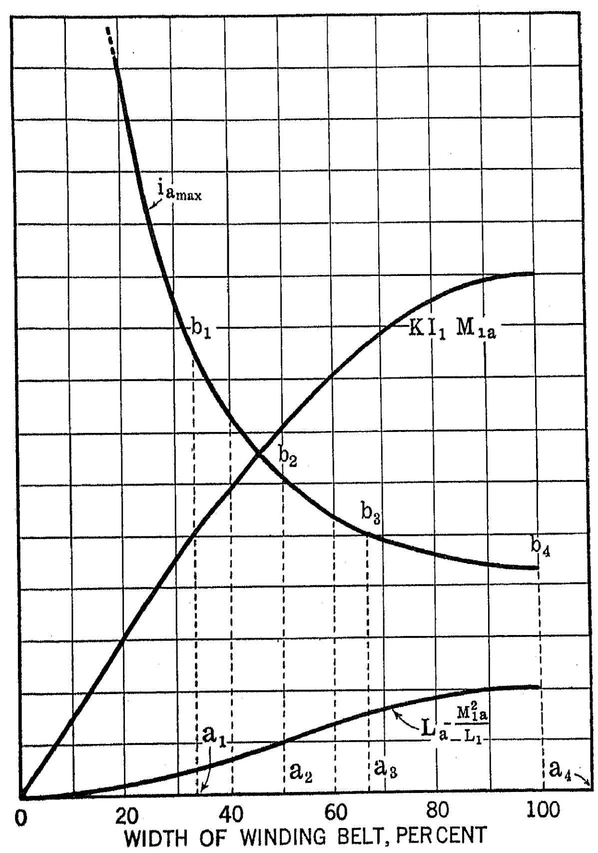


FIG. 20

If the comparison is made between the two-phase and star-connected three-phase machines on the basis of equal and constant voltages, the field flux interlinkages of the two-phase winding will have to be increased approximately 22.5 per cent; *i. e.*, the number of turns will have to be increased 22.5 per cent. The coefficient of equivalent leakage self induction varies as the second power of the number of armature conductors per inch. On the basis of equal terminal voltage, its value for the two-phase winding will be 1.225^2 or 1.50 per cent of its former value. From curve 2 of Fig. 20 the former values for one-phase of the two and three-phase windings are in the ratio of 2 to 3, consequently, the present values on the basis of equal voltages will be proportional to 1.5×2 or 3 for the two-phase winding and also proportional to 3 for the phase group (terminal to terminal) of the three-phase winding. Since the initial flux interlinkages of both cases are equal, the short-circuit current delivered by one-phase of the two-phase winding will be the same in value as that delivered by one phase, terminal to terminal, of the three-phase star-connected winding.

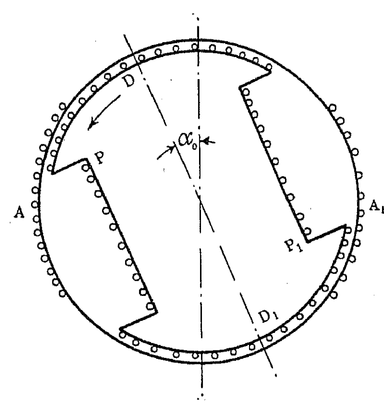


FIG. 21

$\alpha = \alpha_0$

SINGLE-PHASE GENERATOR WITH DAMPER WINDING

If the rotor of the single-phase generator is equipped with a damper winding, $D D_1$, located at right angles to the field winding, $P P_1$, the resultant flux interlinkages of it due to the field current are zero. Under no-load conditions, the flux interlinkages of the armature circuit, due to the normal field flux are the same as for the single-phase generator without a damper winding as was shown in Fig. 7. Hence, in this case,

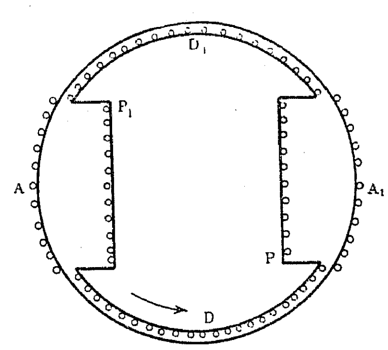


FIG. 22

$\alpha = \pi$

if the armature is suddenly short-circuited at any angular position, of the rotor the initial flux interlinkages, $I_1 L_1$, for the field current $I_1 M \alpha_{1a}$, for the armature circuit, and zero for the damper circuit must be maintained constant for all other angular positions of the rotor. When the rotor is in the angular position, $\alpha = \pi$, shown in Fig. 22, the damper winding, $D D_1$, is at right angles to $A A_1$ and consequently, is

unaffected by the current in $A A_1$ or $P P_1$. Hence, at this position the generator acts as if it had no damper winding and the maximum values of the i_a and i_1 are the same for both cases.

When the rotor is in the angular position, $\alpha = \pi/2$, the axis of the damper winding coincides with that of the armature winding. It is obvious from the previous analysis that the value of the armature current at this instant is equal to the total change in flux interlinkages of $A A_1$ divided by the coefficient of equivalent leakage self-induction of $A A_1$, with respect to the damper winding, $D D_1$. The current in the damper winding will likewise be equal to $i_a M_{ad}/L_d$ and will have its maximum value at this position. Since the field winding is at right angles to both of the other windings, there will be no mutual effect on it and the current, i_1, will be zero. That is, at $\alpha = \pi/2$

$$i_a = \frac{I_1 M_{1a} \cos \alpha_0}{L_a - M_{ad}^2/L_d} \qquad (18)$$

$$i_d = -\frac{I_1 M_{1a} \cos \alpha_0}{L_a - M_{ad}^2/L_d} \times M_{ad}/L_d \qquad (19)$$

$$i_1 = 0 \qquad (20)$$

At all other rotor positions both the field and damper windings act as secondary windings with respect to the armature winding, and the demagnetizing currents that flow in these two windings react on the current in the single-phase armature winding as those of a polyphase armature react on the field winding. The actual value of the coefficient of the equivalent leakage self-induction can be determined from the following simple mathematical relations.

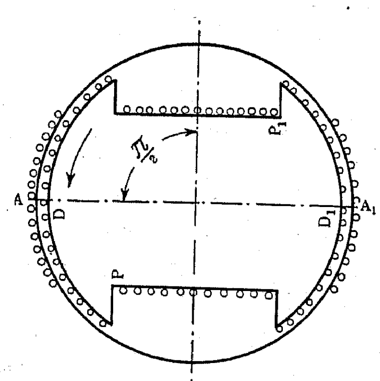


FIG. 23
$\alpha = \pi/2$

$$i_a L_a + i_1 m_{1a} + i_d m_{ad} = I_1 M_{1a} \cos \alpha_0 - I M_{1a} \cos \alpha \qquad (21)$$

$$i_a m_{ad} + i_d L_d = 0 \qquad (22)$$

$$i_a m_{a1} + i_1 L_1 = 0 \qquad (23)$$

Solving these equations gives

$$i_a = \frac{I_1 M_{1a} (\cos \alpha_0 - \cos \alpha)}{L_a - m_{a1}^2/L_1 - m_{ad}^2/L_d} \qquad (24)$$

$$i_1 = -\frac{I_1 M_{1a} (\cos \alpha_0 - \cos \alpha)}{L_a - m_{a1}^2/L_1 - m_{ad}^2/L_d} \times M_{a1}/L_1 \qquad (25)$$

$$i_d = -\frac{I_1 M_{1a} (\cos \alpha_0 - \cos \alpha)}{L_a - m_{a1}^2/L_1 - m_{ad}^2/L_d} \times M_{ad}/L_d \qquad (26)$$

The equivalent leakage self-induction coefficient $L_a - m_{a1}^2/L_1 - m_{ad}^2/L_d$ will be constant for all rotor positions if the two windings on the rotor are identical and have sinusoidal field forms, for the following relations would then be true:

$$L_1 = L_d,\ M_{ad} = M_{a1},\ m_{ad} = M_{a1} \sin \alpha, \text{ and } m_{a1}^2 = M_{a1} \cos \alpha$$

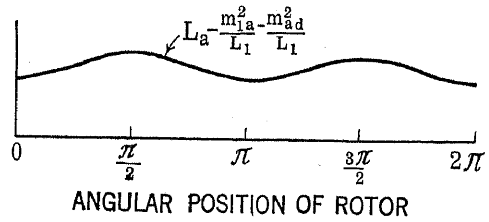


FIG. 24

Then

$$L_a - m_{a1}^2/L_1 - m_{ad}^2/L_d = L_a - \frac{M_{a1}^2 \sin^2 \alpha}{L_1} - \frac{M_{a1}^2 \cos^2 \alpha}{L_1} = L_a - m_{1a}^2/L_1 \qquad (27)$$

However, in actual salient pole machines the two windings cannot be made identical and consequently, the coefficient of equivalent leakage self-induction is slightly pulsating as shown by curve 1 of Fig. 24. Consequently, the damper winding decreases the coefficient of equivalent leakage self-induction of $A A_1$ at the intermediary positions. Hence, the effect of the damper winding at these intermediate positions is to increase the armature current and modify its wave shape, but does not effect the maximum possible value of the armature current.

Two-Phase Generator without Damper Winding

In the case of the two-phase generator with windings $A A_1$ and $B B_1$ placed at right angles to each other the flux interlinkages of $A A_1$ due to the field current, under no-load conditions are the same as for the single-phase generator. The flux-interlinkages of $B B_1$ are the same in magnitude as for $A A_1$ but occur 90 degrees later as shown by curve 2, Fig. 26. The two armature circuits are at right angles and have no mutual effect on each other. If the short-circuit occurs at any position α it is obvious from the previous analysis that the value of the currents, i_a and i_b, must be such that the total equivalent leakage flux produced by each circuit is equal to the corresponding change in flux interlinkages of the respective circuits when the rotor is

in the angular positions, $\alpha = \pi/2$, Fig. 27 and $\alpha = \pi$, Fig. 28 the values of the current in $A\,A_1$ are the same as for the single-phase generator with or without a damper winding, when the short-circuit occurs at the same angular position of the rotor in both cases. Similarly, the currents in $B\,B_1$ at these two positions,

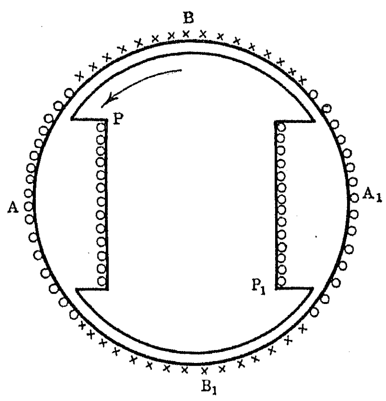


FIG. 25
$\alpha = 0$

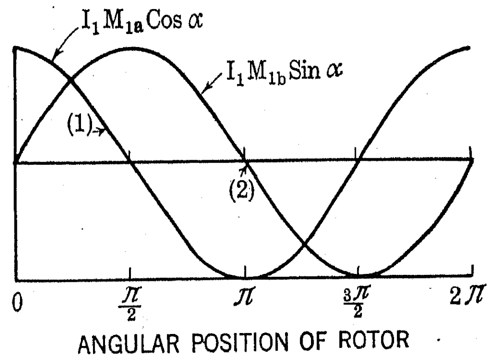


FIG. 26

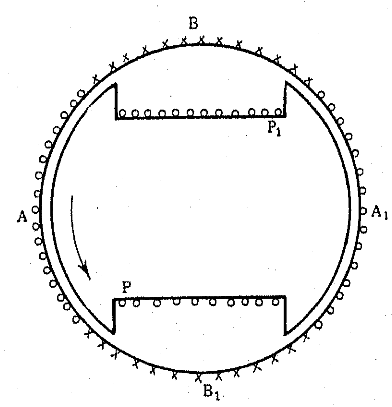


FIG. 27
$\alpha = \pi/2$

are the same as for the case of the single-phase generator, with the short circuit occurring 90 degrees earlier. The currents in $A\,A_1$ and $B\,B_1$ will also reach their maximum possible values at one of these two positions for the total change in flux interlinkages of each circuit is a maximum at one of these positions and the equivalent leakage self-induction coefficients are equal and a minimum.

For all other rotor positions, the equivalent leakage self-induction coefficients of the armature circuits are more complicated to determine on account of the fact that the field circuit $P\,P_1$ acts as an electric coupling between them. That is, if an alternating circuit flows in one of the armature circuits a secondary current, i_1, will flow in the field circuit and this field current in turn reacts on the other armature circuit and produces a secondary current in it. The actual values of the

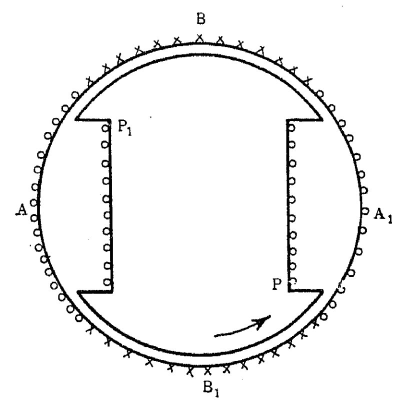


FIG. 28
$\alpha = \pi$

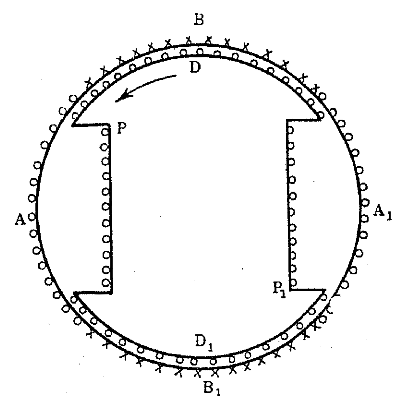


FIG. 29
$\alpha = 0$

coefficients can be readily determined by solving the following simple equations:

$$i_a\,L_a + i_1\,m_{1a} = I_1\,M_{1a}\,(\cos\alpha_0 - \cos\alpha) \qquad (28)$$

$$i_a\,m_{a1} + i_1\,L_1 + i_b\,m_{b1} = 0 \qquad (29)$$

$$i_1\,m_{1b} + i_b\,L_b = I_1\,M_{1b}\,(\sin\alpha_0 - \sin\alpha) \qquad (30)$$

TWO-PHASE GENERATOR WITH DAMPER WINDING

If the rotor of the two-phase generator is provided with a damper winding, $D\,D_1$ located at right angles to the field winding the flux interlinkages of the armature circuits due to the normal field current are the same as for the two-phase generator without a damper

winding. If the short circuit occurs at $\alpha = \alpha_0$ Fig. 30, the initial flux interlinkages of the circuits are:

$I_1 L_1$ for circuit $P P_1$
$I_1 M_{1a} \cos \alpha_0$ for circuit $A A_1$
$I_1 M_{1b} \sin \alpha_0$ for circuit $B B_1$, and
Zero for circuit $D D_1$

When the rotor is in the position $\alpha = \pi/2$ the changes in flux interlinkages are

$I_1 M_{1a} (\cos \alpha_0 - \cos \pi/2)$ for $A A_1$
$I_1 M_{1b} (\sin \alpha_0 - \sin \pi/2)$ for $B B_1$
Zero for $D D_1$ and $P P_1$

The current i_a must be such that its equivalent leakage flux interlinkages with respect to the damper winding are equal to the change in flux interlinkages of $A A_1$. A similar condition must be true for the circuits $B B_1$ and $P P_1$. That is, at $\alpha = \pi/2$.

$$i_a = \frac{I_1 M_{1a} \cos \alpha_0}{L_a - M_{ad}^2/L_d} \qquad (31)$$

$$i_d = - \frac{I_1 M_{1a} \cos \alpha_0}{L_a - M_{ad}^2/L_d} \times M_{ad}/L_d \qquad (32)$$

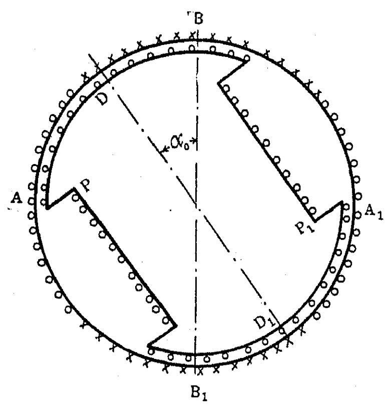


FIG. 30
$\alpha = \alpha_0$

$$i_b = \frac{I_1 M_{1b} (\sin \alpha_0 - 1)}{L_a - M_{1b}^2/L_1} \qquad (33)$$

$$i_1 = - \frac{I_1 M_{1b} (\sin \alpha_0 - 1)}{L_a - M_{1b}^2/L_1} \times M_{1b}/L_1 \qquad (34)$$

Similarly, when the rotor is at the position $\alpha = \pi$ the currents are

$$i_a = \frac{I_1 M_{1a} (\cos \alpha_0 - \cos \pi)}{L_a - M_{1a}^2/L_1} \qquad (35)$$

$$i_1 = - \frac{I_1 M_{1a} (\cos \alpha_0 - \cos \pi)}{L_a - M_{1a}^2/L_1} \times M_{1a}/L_1 \qquad (36)$$

$$i_b = \frac{I_1 M_{1b} (\sin \alpha_0 - 0)}{L_b - M_{1b}^2/L_d} \qquad (37)$$

$$i_d = - \frac{I_1 M_{1b} (\sin \alpha_0 - \sin \pi)}{L_b - M_{1b}^2/L_d} M_{bd}/L_d \qquad (38)$$

Hence at these two positions the currents in the same as for the single-phase generator with winding and the currents in $B B_1$ are the s would flow in $A A_1$ if the short circuit occurr grees earlier. At all other rotor positions, the in the two windings of either element react on cuit of the other element as a polyphase windi on a field winding. Consequently, the equiva

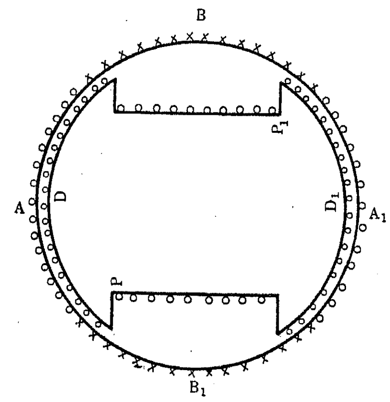


FIG. 31
$\alpha = \pi/2$

induction coefficients of each rotor winding is for all rotor positions and equal in magnitude t

$L_1 - M_{1a}^2/L_a$ for $P P_1$, and
$L_d - M_{1a}^2/L_a$ for $D D_1$

The similar coefficients,

$L_a - (M_{1a}^2/L_1 + M_{ad}^2 \alpha_d)$ for $A A_1$, and
$L_b - (M_{1b}^2/L_1 + M_{bd}^2/L_d)$ for $B B_1$

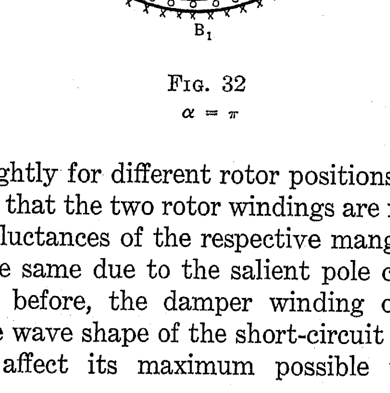


FIG. 32
$\alpha = \pi$

pulsate slightly for different rotor positions on of the fact that the two rotor windings are not i and the reluctances of the respective mangetic are not the same due to the salient pole const Hence, as before, the damper winding only modify the wave shape of the short-circuit curr does not affect its maximum possible valu

maximum possible value of the armature current is the same for a two-phase machine, with or without a damper winding as for a single-phase machine with or without a damper winding and having the same width of phase belt.

Three-Phase Generator Without Damper Winding

The magnitude and variation of the no-load flux interlinkages of any given phase group of a three-phase

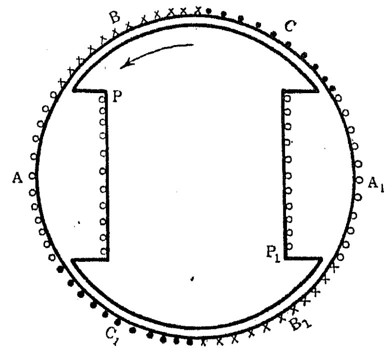


Fig. 33

$\alpha = 0$

winding are the same as in the case of a similar single-phase armature winding for the different rotor positions. The cyclic variations of the flux interlinkages for the different phases are displaced 60 degrees as shown by the curves in Fig. 34. The three-phase generator is not as simple to analyze under load or short-circuit conditions as the two-phase generator on account of the fact that the armature windings have mutual flux relations with respect to one another.

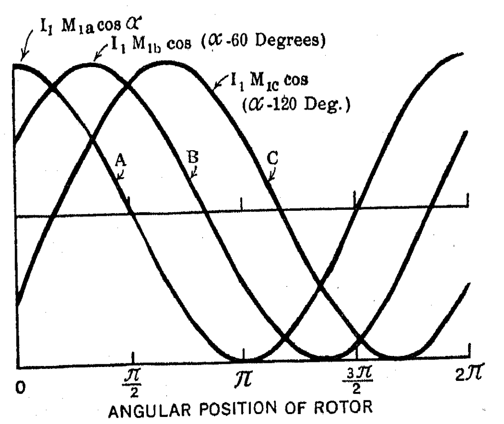


Fig. 34

That is, if one ampere flows in the circuit $A\,A_1$, the flux interlinkages will be L_a, M_{ab}, and $-\,M_{ac}$ for the armature windings, A, B, and C respectively, and M_{a1} for the field winding. The negative sign is applied to the coefficient M_{ac} on account of the fact that the flux interlinks $C\,C_1$ in the reverse direction as compared to the direction of the interlinkages of the other circuits. Using a similar system of notation the induction coefficients of phase B and C are L_b, L_c, and M_{bc}. On account of the salient pole construction of the rotating element the corresponding induction coefficients for the armature circuits are neither equal nor constant for the different angular positions of the rotor.

On account of the complexity of the analysis, the values of the armature currents will be determined for only two short-circuit conditions. One in which the short circuit occurs at the instant the axis of the field winding coincides with the axis of one of the armature windings, and the other in which the axis of the field winding is at quadrature with the axis of one armature winding. If the three-phase short circuit occurs under the first condition as shown in Fig. 33, the initial flux interlinkages of the armature circuits A, B, and C are $I_1\ M_{1a}$, $1/2\ I_1\ M_{1a}$, and $-\ 1/2\ I_1\ M_{1a}$, respectively, on the basis that the windings are delta-connected star-connected with a neutral lead, or each phase group connected independently. After the rotor turns through 180 degrees the total changes in flux interlinkages are $2\,I_1\,M_{1a}$, $I_1\,M_{1a}$, and $I_1\,M_{1a}$ for the re-

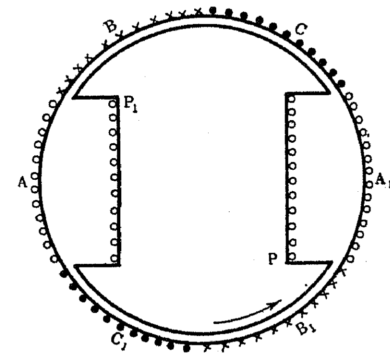


Fig. 35

$\alpha = \pi$

spective A, B, and C armature windings. The currents i_a, i_b, i_c, and i_1 that flow in the windings must be such that the resultant interlinkages of each circuit are maintained at the corresponding initial value. The direction of the current flow in each armature circuit must also be such that the flux opposes the field flux. The current in phase C at this instant will be of opposite sign to that in phases A and B. It is obvious from Fig. 35 that the armature windings are acting in parallel and each furnishing a proportionate part of its required flux interlinkages. Since the armature windings B and C are symmetrically situated with respect to the field winding and phase A of the armature winding, the currents in the A and B phases will be equal in magnitude. The approximate values of the armature currents can be analytically determined by assuming that the corresponding induction coefficients of the armature circuits are equal in magnitude; that is, $L_a = L_b = L_c$ and $M_{ab} = M_{bc} = M_{ac} = 1/2\ M_{1a}$. These relations are only approximately true in the case of salient pole rotors. In this case the currents would

divide between phases A and B in proportion to their changes in flux interlinkages, that is $i_b = 1/2\, i_a$. So far as the current that flows in phase A is concerned the contributory effect of phases B and C can be attributed to phase A as increased induction coefficients. Hence the equivalent coefficients of self-induction of A is $(L_a + M_{ab})$ and the equivalent coefficient of mutual induction with respect to the field winding is $(M_{1a} + M_{ab}) = 3/2\, M_{1a}$. Therefore, it follows by analogy from the analysis of the previous cases that the total equivalent leakage induction coefficient of the phase A is $(L_a + M_{ab} - 3/2\, M_{1a}^2/L_1) = (L_a - M_{1a}^2/L_1) + 1/2\,(M_{1a} - M_{1a}^2/L_1)$ and the value of the armature current is

$$i_a = \frac{2\, I_1\, M_{1a}}{L_a - M_{1a}^2/L_1 + 1/2\,(M_{1a} - M_{1a}^2/L_1)} \qquad (39)$$

It can readily be shown that total equivalent leakage induction coefficients of phases B and C are the same as for phase A and the current values consequently are

$$i_b = \frac{I_1\, M_{1a}}{L_a - M_{1a}^2/L_1 + 1/2\,(M_{1a} - M_{1a}^2/L_1)} \qquad (40)$$

$$i_c = -\frac{I_1\, M_{1a}}{L_a - M_{1a}^2/L_1 + 1/2\,(M_{1a} - M_{1a}^2/L_1)} \qquad (41)$$

It also follows from the previous analysis that the field current is

$$i_1 = \frac{2\, I_1\, M_{1a}}{L_a - M_{1a}^2/L_1 + 1/2\,(M_{1a} - M_{1a}^2/L_1)} \times 3/2\, M_{1a}/L_1 \qquad (42)$$

The equations of the currents for three-phase generator with salient-pole rotors are of the same general form and the values can readily be determined mathematically by solving the simultaneous flux-interlinkage-equations for the different circuits.

Hence in the case of three-phase generators with the three winding connections specified above, the total equivalent leakage induction coefficient, at the rotor position for maximum possible current in one phase, is the same as that for a single-phase machine except that it has an added term. Then for the same total flux change the three-phase armature currents will be correspondingly lower, and the field current will be approximately 50 per cent higher than for a single-phase machine which has the same width of phase belt.

If a star-connected three-phase generator is short-circuited at the terminals when the rotor is in the angular position shown in Fig. 36, the flux interlinkages of the armature winding between terminals, A and B_1 are a maximum and equal to $I_1\, M_{1A}$. The flux interlinkages of the windings a, b and c between the terminals and the neutral are $I_1\, M_{1A}/2$, and zero respectively. Since the interlinkages of the leg c are zero the flux interlinkages of circuits B and C are $I_1\, M_{1A}/2$ and $I_1\, M_{1A}/2$. When the rotor turns through 180 degrees the changes in flux interlinkages are $2\, I_1\, M_{1A}$, $I_1\, M_{1A}$ and $I\, M_{1A}$ for circuits A, B and C. Since, at this position there is no flux change in the leg c of circuits B and C, the current that flows in circuit A and maintains its resultant flux interlinkage equal to its initial value, also satisfies the flux conditions for circuits B and C. It follows directly that the maximum possible value of the current for a three-phase star-connected generator is the same as for a single-phase generator which has a phase belt that covers two-thirds of the armature periphery.

$$i_A = i_a = i_b = \frac{2\, I_1\, M_{1A}}{L_A - M_{1A}^2/L_1} = \frac{2\sqrt{3}\, I_1\, M_{1a}}{3\,(L_a - M_{1a}^2/L_1)} \qquad (43)$$

Similarly, the currents in leg c and in the field winding are

$$i_c = 0 \qquad (44)$$

$$i_1 = \frac{2\, I_1\, M_{1A}}{L_A - M_{1A}^2/L_1}\, M_{1A}/L_1 = \frac{2\sqrt{3}\, I_1\, M_{1a}}{3\,(L_a - M_{1a}^2/L_1)} \times \frac{\sqrt{3}\, M_{1a}}{L_1} \qquad (45)$$

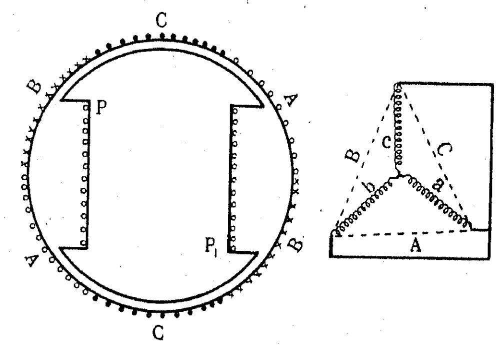
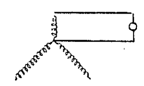
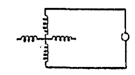
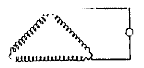
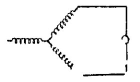
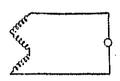
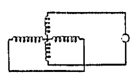
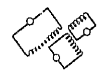
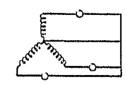
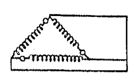
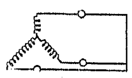


Fig. 36
$\alpha = 0$

Three-Phase Generator with Damper Winding

If the rotor of the three-phase generator is equipped with a damper winding located at right angles to the field winding the maximum possible value of the armature short-circuit current is unaffected by the damper winding for any given armature winding connection. For example, in case the short circuit occurs under the conditions shown in Fig. 33, and the rotor moves to the position shown in Fig. 35 the damper winding is at right angles to phase $a\, a_1$ and hence has no effect on it. Similarly, the damper winding is also unaffected by the other two phases on account of the fact that these windings are symmetrically situated with respect to it and have equal and opposite currents flowing in them. If the winding is star-connected without a neutral lead and the short circuit occurs at the instant shown in Fig. 36 and the rotor is in the position for maximum armature current in phase A, the damper winding is

TABLE I

Case	Phase	Winding Connection	Maximum Peak Value of Armature Current for Constant Conductors per Inch: Amperes	Ratio	Maximum Peak Value of Armature Current for Constant Terminal Voltage: Amperes	Ratio
1	1		$i_{a_{max}} = \frac{2\, I_1\, M_{1a}}{N_a}$	1.732	$i_{a_{max}} = \frac{2\, \sqrt{3}\, I_1\, M_{1a}}{3\, N_a}$	1.0
2	1		$i_{a_{max}} = \frac{2\, \sqrt{2}\, I_1\, M_{1a}}{2\, N_a}$	1.225	$i_{a_{max}} = \frac{2\, \sqrt{2}\, \sqrt{3/2}\, I_1\, M_{1a}}{3/2 \times 2\, N_a}$	1.0
3	1		$i_{a_{max}} = \frac{2 \times 3/2\, I_1\, M_{1a}}{3/2\, N_a}$	1.155	$i_{a_{max}} = \frac{2\, \sqrt{3} \times 3/2\, I_1\, M_{1a}}{3/2 \times 3\, N_a}$	1.0
4	1		$i_{a_{max}} = \frac{2\, \sqrt{3}\, I_1\, M_{1a}}{3\, N_a}$	1.00	$i_{a_{max}} = \frac{2\, \sqrt{3}\, I_1\, M_{1a}}{3\, N_a}$	1.0
5	1		$i_{a_{max}} = \frac{2 \times 2\, I_1\, M_{1a}}{4\, N_a}$	0.866	$i_{a_{max}} = \frac{2 \times 2\, \sqrt{3/2}\, I_1\, M_{1a}}{4 \times 3/4\, N_a}$	1.0
6	2		$i_{a_{max}} = \frac{2\, \sqrt{2}\, I_1\, M_{1a}}{2\, N_a}$	1.225	$i_{a_{max}} = \frac{2\, \sqrt{2}\, \sqrt{3/2}\, I_1\, M_{1a}}{3/2 \times 2\, N_a}$	1.0
7	3		$i_{a_{max}} = \frac{2\, I_1\, M_{1a}}{3/2\, N_a}$	1.155	$i_{a_{max}} = \frac{2\, \sqrt{3}\, I_1\, M_{1a}}{3/2 \times 3\, M_a}$	0.667
8	3		$i_{a_{max}} = \frac{2\, I_1\, M_{1a}}{3/2\, N_a}$	1.155	$i_{a_{max}} = \frac{2\, \sqrt{3}\, I_1\, M_{1a}}{3/2 \times 3\, N_a}$	0.667
9	3		$i_{a_{max}} = \frac{2\, I_1\, M_{1a}}{3/2\, N_a}$	1.155	$i_{a_{max}} = \frac{2\, \sqrt{3}\, I_1\, M_{1a}}{3/2 \times 3\, N_a}$	0.667
10	3		$i_{a_{max}} = \frac{2\, \sqrt{3}\, I_1\, M_{1a}}{3\, N_a}$	1.00	$i_{a_{max}} = \frac{2\, \sqrt{3}\, I_1\, M_{1a}}{3\, N_a}$	1.00

$I_1\, M_{1a}$ = flux interlinkages of one leg of a 3 phase armature winding due to the field current, when the axes of the field winding and the given leg of armature winding coincide.

$N_a = L_a - M_{1a}^2/L_1$ = total equivalent leakage flux interlinkage coefficient of one leg armature winding and field winding when the axes coincide.

unaffected magnetically for the current in leg C in zero and windings A and B are in series and at right angles to the damper winding. In general, the same

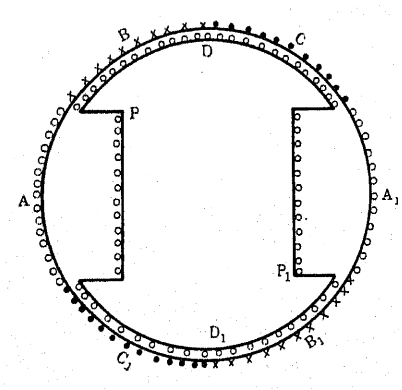


FIG. 37
$\alpha = 0$

conclusions previously given for damper windings of one and two-phase generators under short circuit also apply to the three-phase generator.

The data in Table I give the relative magnitude of the theoretical maximum possible values of short-circuit current delivered by alternating-current generators with different phases and winding combinations on the basis of a given field flux, for (a) constant armature wires per inch of periphery, and (b) constant terminal voltage.

EFFECT OF SATURATION

In general, the iron part of the magnetic circuit does not have infinite permeability but is partially saturated at the condition of normal flux interlinkages. Since under the short-circuit condition which give maximum armature current a major portion of the flux interlinkages must traverse the leakage flux paths, saturation of the iron causes a decrease in the value of the induction coefficients and thus an increased armature current value for a given resultant flux interlinkage. This effect is apt to be quite pronounced at the condition for the maximum possible armature current for approximately double the normal flux interlinkages

must traverse the leakage flux paths and the densities in the iron are necessarily extremely high. It is practically impossible to calculate the effect due to saturation on account of the complexity and indefiniteness of the leakage flux paths and distribution of the leakage fluxes.

Effect of Rotor Construction

In most practical alternating-current generators the rotating element is either solid or built up of reasonably thick laminations which are not insulated from one another, but are short-circuited by assembling rivets and bolts. Hence the iron of the rotor acts as an imperfect or unbalanced polyphase damper winding of high resistance and reactance. Part of this winding acts in parallel with the field winding and consequently, the values of the field current under short circuit will be less than was previously indicated for the thoroughly laminated rotor. This feature modifies the value of the maximum possible armature current only to the extent that it changes the leakage flux interlinkages of the rotor due to the secondary demagnetizing current. At the other rotor positions the values of the armature current are increased and the wave shape modified to a certain extent depending upon the effectiveness of the rotor material as a damper winding.

Effect of Resistance

If the resistances of the electric circuits are not negligible Lenz's Fundamental Law must be modified

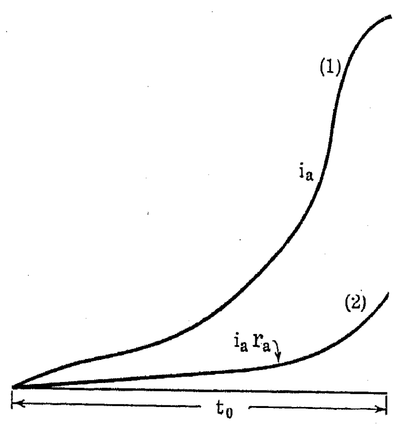


Fig. 38

to take care of the changes in flux interlinkages that are necessary to overcome the $i\,r$ drops of the respective circuits. For instance, in case the armature circuit only has an appreciable resistance there must be a change of its flux interlinkages at any instant proportional to the $i_a\,r_a$ drop. Then referring to the armature current wave under the condition for maximum possible short-circuit current, curves I and II of Fig. 38 show the values of the current and the $i_a\,r_a$ drops for one-half cycle. The total change in flux interlinkages due to the $i_a\;r_a$ drop at the instant of maximum possible current is proportional to the area under curve II for the entire half-cycle. If $i_{a_{max}}\;r_a$ is the maximum resistance drop, the total (decrease) in the flux interlinkages due to the resistance is proportional to $K\,t_0\,i_{a_{max}}\,r_a = \int_0^{t_0} i_a\,r_a\,d\,t$, where K is the ratio of the average to the maximum $i_a\;r_a$ drop. Then applying Lenz's Law it follows from the previous analysis that the total equivalent leakage flux of the armature circuit plus the change in flux

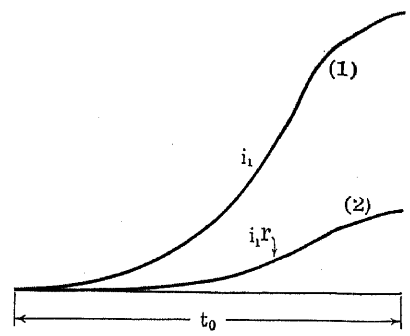


Fig. 39

interlinkages required to offset the $i_a\;r_a$ drop must equal the total change, due to rotation, of its initial flux interlinkages. That is, for the single-phase generator

$$i_a\,(L_a - M_{1a}^2/L_1) + K\,t_0\,i_{a_{max}}\,r_a = 2\,I_1\,M_{1a} \text{ or}$$

$$i_a = \frac{2\,I_1\,M_{1a}}{(L_a - M_{1a}^2/L_1) + K\,t_0\,r_a}$$

$$= \frac{2\,I_1\,M_{1a}}{(L_a - M_{1a}^2/L_1) + K\,t_0\,r_a} \qquad (46)$$

Consequently if the armature circuit has appreciable resistance, a resistance component must be added to the total equivalent leakage induction coefficient, and the maximum values of both the armature and field currents will be decreased.

On the other hand, an appreciable resistance of the field winding also results in a decrease in the short-circuit current values. In the case of a single-phase generator which is short-circuited at the instant of zero voltage or maximum flux interlinkages, the general form of the variation of the circulating current in the field winding, over a period of one-half cycle, is shown in Figs. 16 and 39. Curve II of Fig. 39 shows the $i_1\;r_1$ drop and the integrated area of this curve during the first half cycle, is proportional to the total change in the value of the field flux interlinkages during this time interval. At the end of the first half cycle the actual flux interlinkages of the field circuit are $I_1\,L_1 - \int_0^{t_0} i_1\,r_1\,d\,t$. Consequently the total change (due

to rotation) in the mutual flux interlinkages of the armature circuit is $2\,I_1\,M_{1a} - M_{1a}/L_1 \int_0^{to} i_1\,r_1\,d\,t$ instead of $2\,I_1\,M_{1a}$, as in the case when the field resistance is zero. The maximum values of the armature and field currents are,

$$i_{a_{max}} = \frac{2\,I_1\,M_{1a} - M_{1a}/L_1 \int_0^{to} i_1\,r_1\,d\,t}{L_a - M_{1a}^2/L_1}\text{, and,}$$

$$i_{1_{max}} = -\frac{2\,I_1\,M_{1a} - M_{1a}/L_1 \int_0^{to} i_1\,r_1\,d\,t}{L_a - M_{1a}^2/L_1} \times M_{1a}/L_1$$

If both circuits have appreciable resistance, the decrease in the maximum values of the short-circuit currents will be more pronounced. In this case the maximum current values are,

$$i_{a_{max}} = \frac{2\,I_1\,M_{1a} - M_{1a}/L_1 \int_0^{to} i_1\,r_1\,d\,t}{(L_a - M_{1a}^2/L_1) + K\,r_0\,t_0}$$

$$i_{1_{max}} = \frac{2\,I_1\,M_{1a} - M_{1a}/L_1 \int_0^{to} i_1\,r_1\,d\,t}{(L_a - M_{1a}^2/L_1) + K\,r_0\,t_0} \times M_{1a}/L_1$$

However, in general, the resistances are so small in comparison to the induction coefficients of the electric circuits that the currents will not be materially reduced at the end of the first half cycle. At the outset, the reduction due to the effect of resistance is usually considerably less than the increase due to the saturation effect.

Discussion

P. L. Alger: I have only one comment to make, and that is to remark that I believe the effect of saturation is quite an important one, especially in the application of the same type of theory to the initial currents of induction motors when they are thrown on the line in starting. Saturation makes the actual reactance of the machine much lower than it is when measured under normal conditions, and consequently, the initial current rush is much higher than might be expected, as in 25-cycle transformers, for example, where the ordinary flux is so high as to make the current rush saturate the iron very materially.

I believe that Professor Lyon's A. I. E. E. paper about a year ago, gave a very complete mathematical theory of the subject, taking into account both the resistance or decay effect and the displacement effect, but he did not allow for saturation, and that appears to me to be the most important factor to be added to the theory of this subject.

R. E. Doherty: As Mr. Laffoon has said, the increase in the size of generators and the importance of short-circuit problems in generating and distribution systems makes it important and very desirable that this matter be reviewed at intervals so that the engineers can become at least somewhat familiar with what happens in generators when short circuit occurs.

I think that any one who has gone over this paper will agree that in presenting it from the physical viewpoint the subject is ably handled and an insight given to the phenomena of short circuit in synchronous machines.

I should like to call attention, in a friendly way, to a bit of interesting, if unimportant history. In papers read in 1918, and also in 1920 I made a mistake. I stated the constant linkage theorem which Mr. Laffoon again presents to you, and he makes the same mistake. I said that this theorem follows from Lenz's law. Now, as Professor Karapetoff pointed out in discussing my paper in '19 and '20 the theorem does not follow from Lenz's law. It can be shown, however, that the theorem is perfectly sound. I have given such a proof, based on classical differential equations, in a paper, "A Simplified Method of Analyzing Short-Circuit Problems," at the Swampscott Convention last year.

R. F. Franklin: Expressions for the initial short-circuit current can be derived in a very simple manner by a direct translation of the constant-linkage theorem into mathematics. This method makes it unnecessary to separate the field current into two components as done in the paper.

The linkage theorem, as previously stated by Mr. Doherty is,[2] "*If the resistance of a closed circuit is zero, then the algebraic sum of the magnetic linkages of the circuit must remain constant.*" Take the first case considered, namely, that of the short circuit of a single-phase generator without damper winding. There are only two circuits involved, the field circuit and the armature circuit. Let i_1' and i_a be the instantaneous values of the currents in these two circuits. Then the linkages of the field circuit due to the field current are equal to

$$i_1' L_1$$

and the linkages of the field circuit due to current in the armature circuit are

$$i_a\,M_{1a}$$

Applying the linkage theorem,

$$i_1'\,L_1 + i_a\,M_{1a} = \Omega_1 \qquad \textbf{(a)}$$

where Ω_1 is a constant giving the flux linkages of the field circuit at the instant of short circuit. In a similar manner applying the linkage theorem to the armature circuit,

$$i_a\,L_a + i_1'\,M_{a1} = \Omega_a \qquad \textbf{(b)}$$

where Ω_a is the linkages of the armature circuit at the instant of short circuit. We thus have two equations involving the currents i_1' and i_a which when solved simultaneously give,

$$i_a = \frac{\Omega_a - \Omega_1 \dfrac{M_{1a}}{L_1}}{L_a - \dfrac{M_{1a}^2}{L_1}} \qquad \textbf{(c)}$$

and

$$i_1' = \frac{\Omega_1 \dfrac{L_a}{L_1} - \Omega_a \dfrac{M_{1a}}{L_1}}{L_a - \dfrac{M_{1a}^2}{L_1}} \qquad \textbf{(d)}$$

This method can be used with equal facility for the calculation of the short circuit currents for any winding connection.

It can readily be shown that equations **(c)** and **(d)** correspond to **(13)** and **(14)** except that **(d)** is the expression for the total field current instead of the induced component as in **(14)**. Thus, in equation **(c)** Ω_a is the flux linkages of the armature circuit at the instant of short circuit and $\Omega_1 \dfrac{M_{1a}}{L_1}$ the flux linkages of the armature circuit due to the field circuit for any rotor position after short circuit. Hence the numerator is the change in armature linkages produced by the field circuit due to rotation. The denominator has been defined in the paper as "the coefficient of equivalent leakage self-induction of the armature circuit." Also **(d)** can be written in the form

$$i_1' = I_1 - \frac{M_{1a}}{L_1}\left(\frac{\Omega_a - \Omega_1 \dfrac{M_{1a}}{L_1}}{L_a - \dfrac{M_{1a}^2}{L_1}}\right)$$

or $$i_1' = I_1 - \frac{M_{1a}}{L_1} i_a \tag{e}$$

The two components of (**e**) are the d-c. and a-c. components of the field current respectively, the latter component being the same as (**14**) in the paper.

The inductance coefficients vary with the rotor position as shown in Figs. 6, 7 and 8, so that the current in the two circuits can be calculated for any rotor position by substituting values from the curves of these figures in (**c**) and (**d**) in the manner used by Mr. Laffoon. However, it is easier to express them as trigonometric functions and substitute these functions in (**c**) and (**d**). It is then necessary to calculate only the maximum and minimum values of the inductance coefficients. For instance, the mutual coefficient M_{1a} (Fig. 7) can be expressed as

$$M_{1a} = M_0 \cos \alpha$$

where M_0 is the maximum value of mutual inductance between armature and field circuits. The coefficient L_a (Fig. 8) can likewise be expressed as a function of α. However, the effect of saturation in this coefficient on the resultant short-circuit current is so slight that for all practical purposes L_a can be assumed constant.

I used this method two or three years ago for the calculation of the short-circuit currents for all cases given in Table I. The results I obtained do not check the values in the table. Thus for all three, 3-phase short-circuit conditions, cases 8, 9 and 10, I obtained the same maximum peak value of armature current for constant conductors per inch; and for the single-phase condition, case 1. 1.5 times the current for the three-phase conditions. For the other cases the ratio to three-phase values I obtained are slightly less than those given in the table.

On page 3 the statement is made that, "It is well known from the laws of physics that the mutual-induction coefficients of two inductively coupled circuits is the same for either circuit with respect to the other." This statement, while correct for the condition of no saturation, should be more accurately stated since it is not true when saturation is involved.

N. S. Diamant: I am very glad to note that Mr. Laffoon in his paper makes a free use of the conceptions of the four kinds of "Inductance;" it is gratifying also that he is consistent and employs the usual terminology as given in detail by Boucherot in his excellent paper presented at the International Congress in 1911.

In my paper on "Calculation of Sudden Short Circuit Phenomena of Alternators" presented in San Francisco in 1915, I did not feel that I could make free use of these ideas as they seemed rather novel; though they were by no means new. Briefly we have: (1) the self-inductance L. (2) the leakage inductance L (3) the mutual inductance M and (4) the total leakage inductance reduced to the armature or field; this is designated by Laffoon as N_a or N_f and in my several papers and discussions I have used the symbol λ. I am calling attention to these various coefficients of inductance because there has been and there is considerable confusion on the subject. (See Discussion of Doherty and Shirley's paper in the A. I. E. E. TRANS. for 1918). From a study of transient phenomena and also from the study of the ordinary transformer we learn that the amount of current and the rate of decay of current depends not on L but on the total leakage inductance λ or N as is indicated in the paper.

The simplest physical conception of sudden short circuits, it seems to me is as follows: Consider a two-phase generator having no mutual induction between phases A and B. Just before short circuit there is a certain flux in phase A, say ϕ_a; this dies down to practically zero at the end of the short circuit.

Thus the flux in phase A during short circuit is $\phi_a \, \epsilon^{\frac{r_a}{\lambda_a} t}$ where r_a = armature resistance λ_a = total armature leakage inductance and t = time counted from the instant of short circuit. Similarly, for phase B we have $\phi_b \, \epsilon^{\frac{r_a}{\lambda_a} t}$. Now, $\phi_a = M\, I_f \cos \omega\, t_1$ where M = mutual inductance between field and armature. I_f = field current before short circuit. With reference to the field circuit we have:

ϕ_n = normal flux in the field before short circuit.

ϕ_{psh} = flux in the field at the end of short circuit; *i. e.*, flux in the field during permanent or sustained short circuit.

$\phi_{psh} + (\phi_n - \phi_{psh})\, \epsilon^{-\frac{r_f}{\lambda_f} t}$ = field flux at any instant during short circuit.

Now, returning to Mr. Laffoon's paper we can say that

$$\phi_a \, \epsilon^{-\frac{r_a}{\lambda_a} t} = \text{flux in phase } A \text{ during short circuit.}$$
$$= M\, i_f \cos(\omega\, t + \omega\, t_1) + L_a\, i_a$$

$$\phi_{psh} + (\phi_n - \phi_{psh})\, \epsilon^{-\frac{r_f}{\lambda_f} t} = \text{flux in field during short circuit.}$$
$$= M\, i_a \cos \omega\,(t + t_1) + L_f\, i_f + M i_b \sin \omega\,(t + t_1)$$

For phase B we shall have an expression similar to the one given for phase A. These equations are very simple and they are given in a graphical form in the paper, except that the attenuation factors are entirely omitted. To obtain the armature or field current it is necessary to solve the whole set of equations, and that part may be termed mathematical. Let me repeat that the solution of the equations just given may be mathematical but the analysis and physical interpretation of each equation are very simple. In fact, I think simpler and certainly more complete than that given by the author. It is not necessary to give an extended discussion of the simple physical meaning of the equations as that has already been done. See TRANS. A. I. E. E., 1915; *Electrical World* May 18; June 1; June 29, 1918. Mr. Laffoon uses the conception of a circuit without resistance; this was also used by Doherty and Shirley in 1918. Is it necessary to make this assumption, which is not easy to understand? Suppose the flux changes from ϕ_1 to ϕ_2; the e. m. f. generated by this change will be $\frac{(\phi_1 - \phi_2)}{t}$ and this transient e. m. f. will produce a quantity of electricity $q = i\, t$, where i is a decaying direct current. Thus a direct current is produced in phase A of the armature provided that phase A enclosed an amount of flux ϕ_1 at the instant of short circuit. If the position of the phase A is such that it encloses no flux at the instant of short circuit then there will be no direct current produced in it.

In this way all the phenomena can be explained withouf recourse to a *circuit without resistance*. Another advantage ot this is that it involves an old well-known principle, which is used to determine the apparent resistance of a ballistic galvanometer on closed circuit. I would like to ask if Mr. Laffoon has made use of this principle and how and why did he resort to the conception of the circuit without resistance.

In Table I values for the maximum possible armature currents are given. These do not include the attenuation factor of either the armature or the field, and nothing is said about their effect. In all my work I had found that the effect of these was not negligible; I would like to have Mr. Laffoon's opinion on this since he has accumulated a great deal of experimental data on the subject.

It is stated by the author that damper or amortisseur windings have no effect on the armature current. I would like to know if damper windings have any effect on the attenuation factor, that is, on the rate of decay of the armature current. To what extent do test results check with the approximate and more accurate equations given by Boucherot for alternators with and without damper windings? To what extent do the values for $i_{o\,max}$ given in Table I agree with the expressions given by Boucherot?

C. M. Laffoon: In presenting the analysis of the short-circuit currents of alternating current generators, the mathematical derivation of the general relations was not given on account of the fact that this phase of the problem has already been given considerable consideration by other writers. It was thought that the presentation of the physical conception of the phenomena would facilitate the interpretation of the actual results and mathematical relations by an appreciable number of engineers. That is, the physical or graphical analysis was intended to be considered as a supplement to the mathematical treatment rather than as a substitute for it.

Mr. Doherty's criticism in regard to the writer's statement that

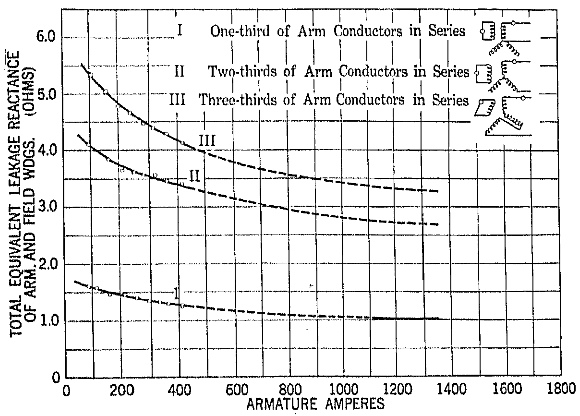


FIG. 1

the constant linkage theorem was based on Lenz's law is well taken. The statement of the constant linkage theorem as given by the writer follows from Lenz's and Kirchhoff's Laws.

It is practically impossible to calculate the effect due to saturation on account of the complexity of the leakage flux paths and the distribution of the leakage fluxes. The effect of saturation is apt to be quite large when the short-circuit occurs at normal voltage, for approximately double the normal amount of flux interlinkages must traverse the leakage flux paths. The following curves show the decrease in value of the total equivalent leakage reactance coefficient of the armature and field windings with armature current for the case of a 6250-kv-a. turbo generator. These test results were obtained by locking the rotor in such position that the axis of the field winding coincided with one phase group of the armature winding, short-circuiting the field winding, and then applying alternating voltage to the armature winding.

The formulas for the maximum peak values of the armature current and the comparison ratios for the different winding combinations, which are shown in Table I of the paper, were determined on the basis that the short-circuit occurs at the instant the axis of the field winding coincides with the axis of the part of the armature winding included between two adjacent leads. All of these cases are treated in considerable detail in the original paper, and it would be necessary to know the assumed short-circuit conditions under which R. F. Franklin obtained his results before attempting to account for the discrepancies. However, at the present time sufficient reliable test data is not available to definitely confirm the relative ratios as given in Table I.

In presenting a quantitative physical conception of the short-circuit phenomena, it seemed advisable to the writer to consider the case of an ideal generator first, and then to show the effect of the different physical constants which are present in the actual generator. The writer is famiiar with the method suggested by N. S. Diamant for determining the value of the resultant flux interlinkages of each circuit at any given time. While this method is sufficiently accurate for most cases, the actual change in flux interlinkages is quite different from that shown by a simple exponential equation. This question is considered by the writer in a second paper on the same subject, which was presented at the Northeastern Regional Meeting, at Worcester, Mass., June, 1924.

A damper winding which is located at quadrature with respect to the field winding has no effect on the magnitude of the peak values of the armature current, but it does modify the wave shape and hence affects the r. m. s. values. If part of the damper winding acts in parallel with the field winding, the maximum peak value of the armature current will be affected to the extent that the total equivalent leakage induction coeffiient of the armature winding is modified. The effect of the damper winding on the rate of decay of the armature current is given in the writer's second paper which was previously referred to.

Dr. P. Boucherot's original paper only considered the case of the single-phase, and two-phase generators; consequently sufficient data was not available for making a comparison with the results given in Table I.

The Economic Development of a Step-by-Step Automatic Telephone Equipment

BY PAUL G. ANDRES

Associate A. I. E. E.

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***Review of the Subject.**—Since Campbell published his paper in 1910, modifications have been made on the step-by-step telephone equipment. This paper covers a review of the economic development of this system with particular emphasis on the technical development in general. The material is grouped under development of circuits and trunking, mechanical design, manufacturing methods, maintenance improvements and possible future developments. Saving in equipment and trunks is considered under the switching selector repeater, director and frequency selecting connector while the reduction in maintenance cost is shown by the reduced number of troubles per line per month when comparing the newer developments with equipment installed fifteen years ago.*

TELEPHONY has together with the electric power industry undergone a tremendous development within the last fifteen years. The rapid advances made in electrical materials, the study and analysis of methods and means of manufacture and the contributions of pure science are finding application in the industry.

Since the inception of the telephone in 1876 the subscriber's needs and operating demands have increased constantly and definitely, tending to complicate the apparatus. Particularly is this true in the case of automatic telephony where the subscriber, because of the nature of the system, expects to obtain an answer from a called party in a minimum time and with the least mental and manual effort. On the other hand the manufacturer has by the most approved and latest methods tried to improve the quality of the product and at the same time tried to make it more economical. Briefly stated the situation resolves itself into two main issues: (a) the increasing needs of the subscriber and operating company and (b) the reduction of cost of the equipment.

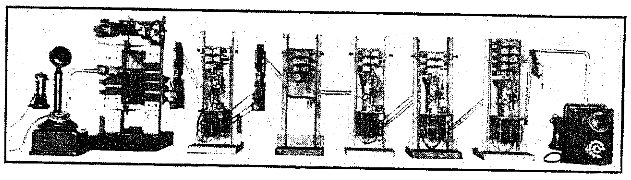

FIG. 1—SIMPLE STEP-BY-STEP SYSTEM

Without regard to the changed economic conditions of the country the subscriber's increasing needs from the start have forced the cost of the equipment up faster than the manufacturer could reduce the cost by redesign and methods. At the present time these needs are still of prime importance and still slightly overbalance the gain made by improvements and developments.

Because of its unit structure the step-by-step telephone system lends itself readily to an analysis of how the needs of the subscriber and operating company are met economically by the use of the latest developments. Each unit performs a definite function practically independently of the other units and consequently may be studied with regard to cost, performance, maintenance, life, and so forth without involving the rest of the equipment.

Inherently the system consists of a series of line switches, selectors, repeaters and connectors operated by a dial at the subscriber's station. Campbell[1] in 1910 outlined the major parts and functions of the elementary step by step system. Fig. 1 shows the parts that constitute this system assigning to each part its proper place in the chain of equipment.

In order to consider the various phases of the development and the result obtained, the subject will be considered with regard to the developments in design of circuits and trunking which represent the ideas to be carried into practise, mechanical design and materials which crystallize these ideas in actual mechanisms, manufacturing methods producing the mechanism in quantities, maintenance improvements or the performance in practise and possible future economic developments.

Developments in Design of Circuits and Trunking

During the past thirty years that the step-by-step system has been in use various developments have been made on the original idea advanced by Strowger. Naturally some of the early work was revolutionary during the time the apparatus was serving its apprenticeship. These changes may well be reviewed as early developments, while the more recent progress is considered under the subject of later developments.

EARLY DEVELOPMENTS

All early exchanges were operated on the three-wire system, which in reality consisted of a two-wire system plus the ground circuit. The change from three-wire to a straight two-wire system, first introduced in 1907,

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

1. A Modern Automatic Telephone Apparatus. W. Lee Campbell, TRANS. A. I. E. E., 1910, Vol. 29, p. 55.

came about largely by the possibilities offered by the two-wire system for purposes of station metering, pay station control, supervision and the like, besides simplifying the telephone wiring with resulting lower maintenance costs. Among other ailments, the three-wire system had the disadvantage of premature release in certain conditions in party line service. It required a good ground connection in the circuit when setting up a call. While in many cases an excellent ground connection is maintained at every subscriber's station at the present time, be it manual or automatic, this is used for purposes of signaling and protection, whereas early automatic stations used such a ground primarily for dialing.

The two-wire system was made possible largely by the adaptation of slow release relays. This type of relay has gradually been developed until it now finds a place on nearly all switches used in the system.

Machine ringing, although first used in manual exchanges, was applied to automatic service because its use resulted in shorter holding time of trunks besides relieving the subscriber from the additional burden of ringing the called subscriber's bell.

Later Developments

Mixed Number Systems. Although mixed number systems had been used in special cases, the Columbus, Ohio installation illustrates the advisability of using such a system resulting in an economy of equipment. As mentioned in the introduction the simple system required a major switch for every digit dialed. In the

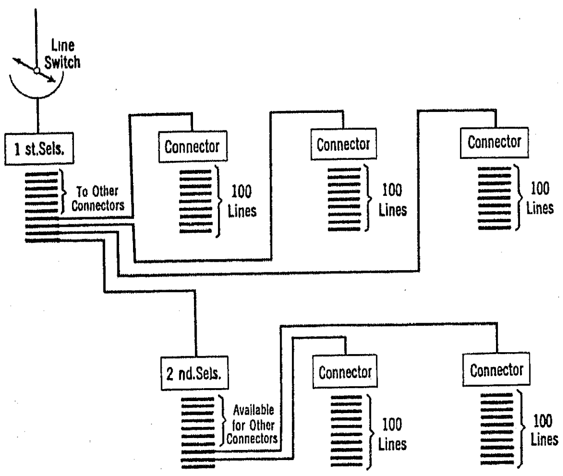

Fig. 2—Mixed Number System

case of the exchange mentioned a four-digit system was used until the growth of the system required additional equipment. By the addition of third selectors, as required, the system was expanded gradually. Fig. 2 indicates how the simple step-by-step system lends itself to future growth by the addition of but small amounts of equipment and no rearrangement of subscriber's numbers. If a proper traffic study has been made and the traffic development can be forecast with a fair measure of certainty, then the installation of the mixed number system is a real economic advantage; if, however, this information cannot be obtained, the possible rearrangement of certain groups in case the system is installed may affect the advantage gained.

Traffic Recorders. The study within recent years of traffic conditions by operating companies, with a view toward the elimination of superfluous equipment, the addition of equipment where needed, the economic distribution of traffic and the economical number and arrangement of trunks, both local and inter-office,

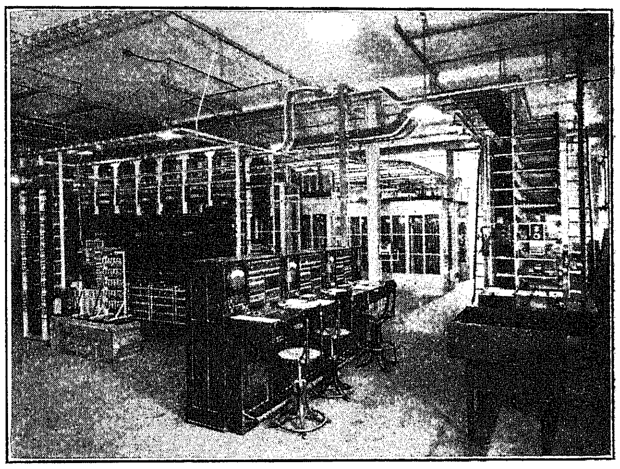

Fig. 3—Traffic Recording Machine in Operation in an Exchange

required that the group equipment be modified to allow the obtaining of this traffic data. Various traffic recording machines have been developed and find increasing application. Fig. 3 shows such a traffic recording machine in operation in an exchange. Numerous theoretical calculations have been made along this line and these machines are being used to obtain the practical substantiation of these formulas.

Line switches. Plunger type line switches on account of their preselecting nature are commonly used for primary line switches. The introduction of secondary line switches invariably effects a saving due to trunk efficiency wherever the traffic is normal. The development of the rotary line switch came about largely through efforts to make a self-contained unit. Because of its unit structure, that is, independence of common mechanism, compactness and reliability, the rotary line switch lends itself admirably for secondary line switch duty.

Switching Selector Repeater. The development of the switching selector repeater was one of the milestones in recent years. This switch as its name implies is a unit combination of a repeater, selector and a discriminating device, intended to repeat impulses to the main office and at the same time to determine whether or not the call is going to or through the main office or whether it is intended for a local number. Eventually the number dialed will indicate to the switch whether or not the call terminates in the local or main office.

Just as soon as the proper combination of digits is dialed, which determines that the call is not intended for the local office, the discriminating feature ceases to function and the switch continues to function simply as an impulse repeater. If, however, the call is intended for a local subscriber, the switch operates as a selector after the last digit of the discriminating part of the number and at the same time releases the trunk to the main office with its attached apparatus and leaves it

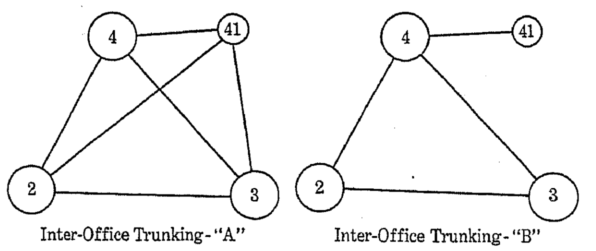

FIG. 4—TRUNKING TO SUB-OFFICE WITHOUT AND WITH SWITCHING SELECTOR REPEATERS

available for other connections. The general method of operation consists in elevating and rotating the shafts of major switches simultaneously in the local and main offices and after the last discriminating digit, releasing the switch or switches in the undesired office.

This device finds its greatest economic application in the case of the sub-office as shown in Fig. 4. In the simple step-by-step system inter-office trunks are required from each office to every other office in the

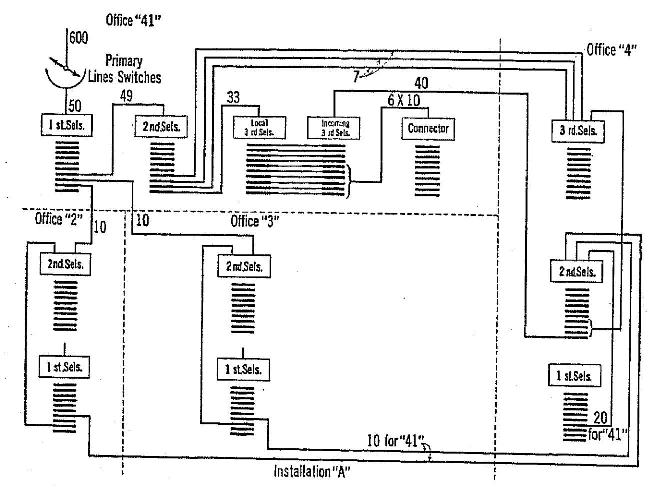

FIG. 5—EQUIPMENT REQUIREMENTS IN SUB-OFFICE WITHOUT SWITCHING SELECTOR REPEATERS

system, but the use of the switching selector repeater requires trunks only to the main office nearest the sub-office.

The economy effected by the use of switching selector repeaters varies with the specific case and depends on the merits of the individual case. The additional inter-office trunks required largely determine and limit the saving. As an illustration of the equipment saving in the sub-office, Fig. 5 and Fig. 6 show the arrangement of the equipment and trunks required in a simple system and a typical installation with switching selector repeaters.

As a basis of calculation let the traffic requirements rest on the following values, where $T\ C$ represent traffic units based on the busy hour traffic.

Office 41 to Office	41	12.84 T C	Office 4 to Office	41	5.92 T C
41	4	5.92	3	41	2.96
41	2	2.96	2	41	2.96
41	3	2.96	41	41	12.84
Total originating		24.68 T C	Total incoming		24.68

The quantities of equipment are as follows:

	Installation A.	Installation B.
1st Selectors	50	0
2nd Selectors	49	0
3rd Selectors Local	33	50
3rd Selectors Incoming	40	40
Connectors	60	60
Outgoing Sec. Line Sw.	0	50

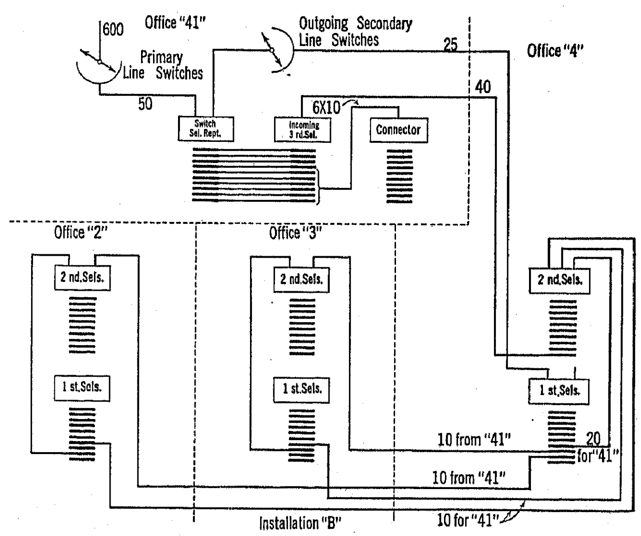

FIG. 6—EQUIPMENT REQUIREMENTS IN SUB-OFFICE WITH SWITCHING SELECTOR REPEATERS

Summed up in brief form installation B replaces 82 selectors with 50 line switches, the relative cost of the two complete being in the ratio of three to one.

Frequency Selecting Connector. Another decided economic advance, particularly in the case of the small community automatic exchange, came about through the development and application of the frequency selecting connector. As shown in Fig. 7 the switch resembles the ordinary connector, except for the addition of two relays and a minor switch, which selects the desired frequency.

Previous to the adoption of this switch party line service depended on a group of party line connectors, the banks of which were multipled together, giving access to one hundred lines for each group. The group was taken from a first selector level and the desired frequency obtained from a level on a second selector. In the case of the frequency selecting connector each

connector selects the proper frequency. Since these connectors can be used indiscriminately for straight, four-party or ten-party line service they can be used on any board resulting in a saving of group equipment. This is very noticeable in the case of mixed service where the exchange is small and party line service predominates.

The Director. A very recent development adapting step-by-step equipment to large metropolitan areas where tandem offices and other complex trunking problems are encountered, is the director. The standard equipment previously described makes it imperative to correlate the office number dialed with a certain and definite level on each of the major switches. Further,

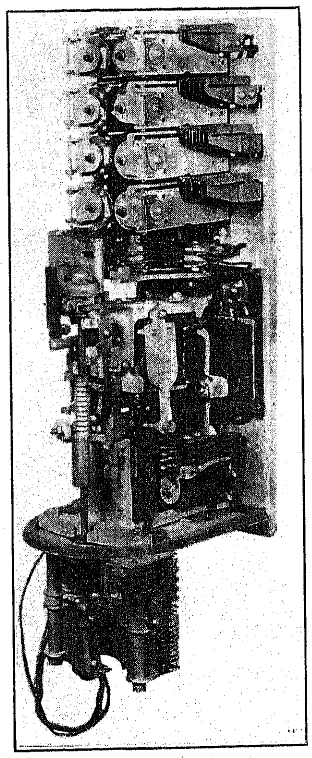

Fig. 7—Frequency Selecting Connector. Minor Switch Just Below Relay Group

in calling an office it may be necessary to pass through tandem offices, each of which would require that the subscriber dial at least one digit. The director is an equipment inserted between the secondary line switch and the first selector to dissociate the numbering from the trunking scheme. It comes into play during the dialing of the number, it rearranges this number and directs the call over the most economical trunk route by any suitable combination of digits and is then disconnected re-establishing the usual through connection.

The actual apparatus as shown in Fig. 8 consists of a director selector, the director, a common impulse machine and a group of relays common to all directors. The director selector in the case of a three-letter office code system is practically identical with a standard selector except for one additional relay, and functions to select one of a number of directors on the first digit called. It releases the director after the completion of the call but remains busy during the duration of the conversation.

The director, Fig. 9, consists of an office code register, four numerical registers, register control, a cross connecting block, sender control switch and a group of relays. By dialing the second and third digits the office

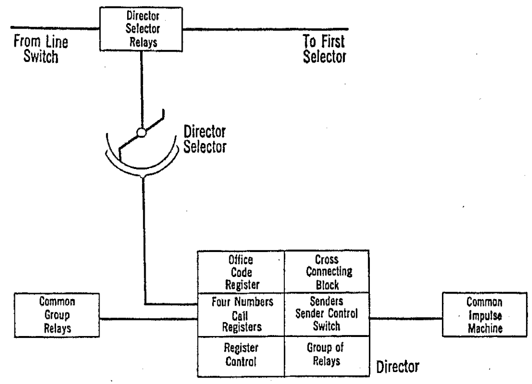

Fig. 8—Director System

code register can connect its four wipers to any one of one hundred sets of four bank contacts making one hundred possible offices available for this director. Since there are ten levels in the director selector this makes a possible one thousand offices available to the subscriber. The numerical register records the successive digits as they are sent in. In the meantime the sender starts to retransmit the number, but due to

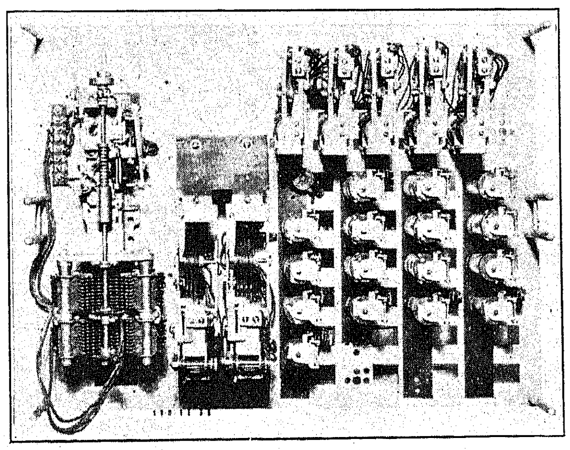

Fig. 9—Director

the arrangement of wiring on the cross connecting block which allows the number dialed in to be converted into any other number of either more or less digits, the call is actually routed over a number combination most favorable to the various offices through which it passes. The common-impulse machine supplies the proper electrical impulse to propagate the call, while the relay equipment consisting of supervisory, and the like, is similar to that customarily in use.

As previously outlined the switching selector repeater, when economically applied to a certain traffic area, effects an economy of switches against a slightly increased number of inter-office trunks. The director, on the other hand, when applied to a large metropolitan area, and in general this is its only economic application, results in a saving of trunks, not only in the number of trunks used, but also in the efficient use of existing trunks.

For a particular case the use of the director reduces the number of second selectors required from 545 to 80 or a saving of 465 selectors. In this particular case the saving in selector equipment approximately counterbalances the additional cost of director equipment. In larger metropolitan areas where the trunking is more complicated and where tandem operation is required, the saving in trunk groups will more than compensate for the cost of director equipment. This is especially true when two or more complete units of 10,000 lines are installed in one building.

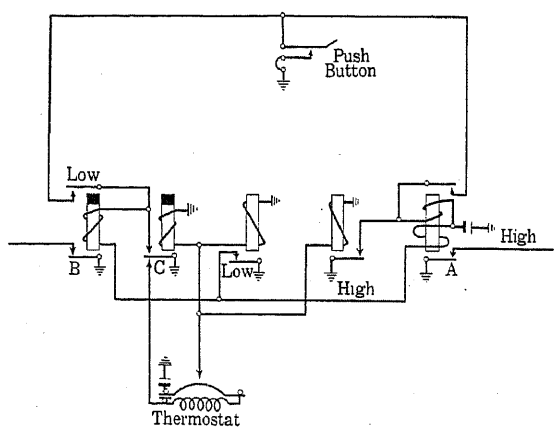

FIG. 10—DIAGRAM OF VOLTAGE LIMITS EQUIPMENT

Voltage Control Apparatus. An interesting power development, resulting incidentally in connection with the development of automatic telephony, is the voltage control apparatus. Since it is desired to keep the battery voltage between 46 to 49 volts the voltage control apparatus must be extremely sensitive and at the same time sturdy enough to stand considerable abuse. The common form of high low-voltage control in the form of a meter element is not reliable, due to the light contact pressure resulting in poor contact particularly after a period of operation when the contacts are slightly pitted or dusty. The most recent device along this line consists of two relays margined to pull up on a voltage in excess of a certain value. Fig. 10 shows the general arrangement of the scheme. A test relay tests the line voltage intermittently, say four or five times per minute. In case the voltage is low no relay pulls up and the circuit condition causes a counter-cell to be cut out; if the voltage is normal one relay only pulls up causing no movement of the counter cell switch; in case the voltage is high the operation of both relays causes a circuit condition to be established which results in the cutting in of a counter cell. The specific application of this scheme is shown on the fifty-line private automatic exchange equipment, Fig. 11A. The two high and low operating relays may be seen in the lower right corner. Fig. 11B gives the front view of the board showing the battery-charging generator

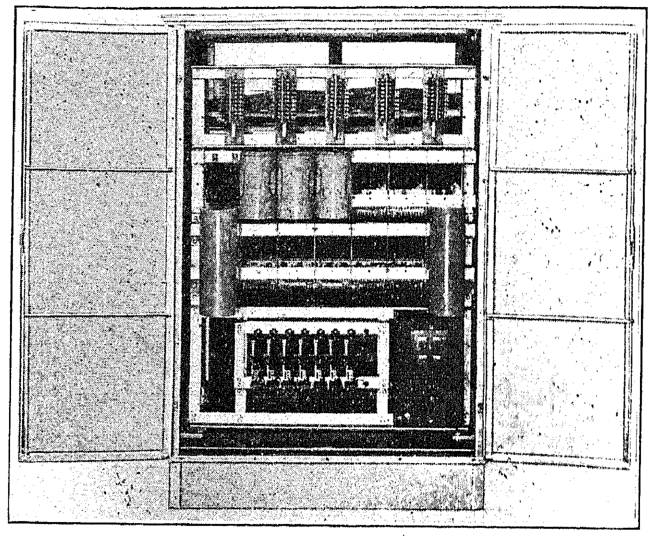

FIG. 11A—FIFTY-LINE PRIVATE AUTOMATIC EXCHANGE. Rear View showing Voltage Control Relays

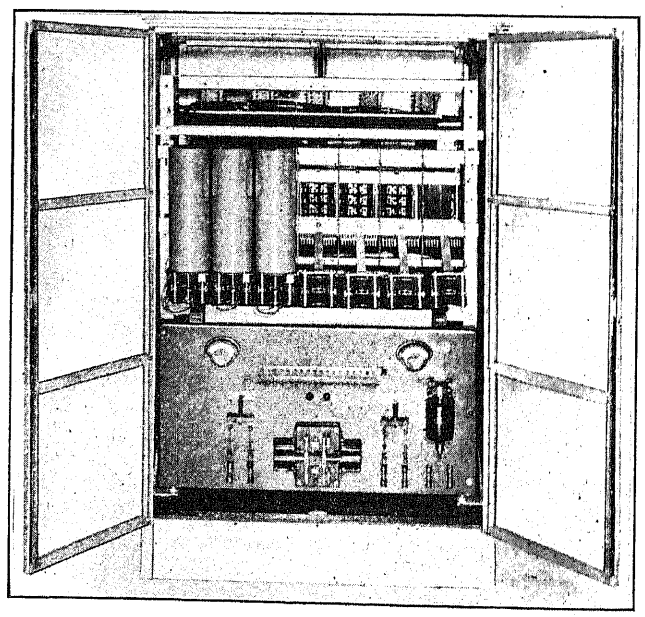

FIG. 11B—FIFTY-LINE PRIVATE AUTOMATIC EXCHANGE. Front View showing Motor Start Switch

start switch and the combination end cell and cut-off switch.

Incidental Developments. Many incidental developments were made during the last five to ten years. The rotary movement on the selector was changed from the common interrupting for a group of selector switches to the self interrupting and later to the relay controlled type.

Although this resulted in a slightly slower speed of hunting an idle trunk this was compensated by a positive stop on the first idle contact and uniformity of operation. Reverting-call switches, code ringing on community automatic exchanges, rotary connectors for private branch exchanges, zone metering, toll switching, measured service and many other parts of the equipment were perfected.

Mechanical Design and Materials

The development of circuits is closely allied with mechanical design and this in turn with materials. A typical case illustrating this point is that of the major switch which was changed in the case of the selector in 1910 and in case of the connector in 1918 from the side switch type to the present type. This change resulted in a switch controlling movement electrically by relay action, instead of mechanically. While the ensuing circuit was made slightly more complex the mechanical structure was materially simplified. The above change was brought about largely by the increasing use of the copper collar or slow acting relay, the time margins of which have been investigated with respect to their performance in step-by-step equipment.

The subsequent change in 1915 of the horizontal for the vertical type relay resulted in a switch which could

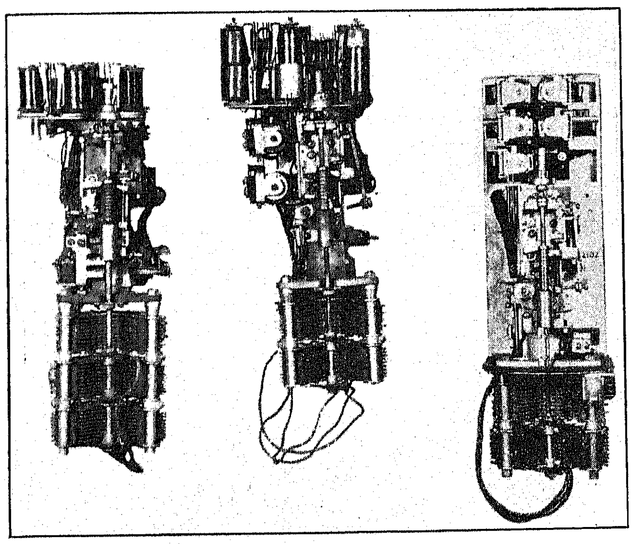

FIG. 12—VERTICAL AND HORIZONTAL RELAY TYPE SELECTORS

be equipped with an individual dust cover. Fig. 12 shows the early styles of vertical relay switches together with the late horizontal relay type switch, and Fig. 13 shows the latter type of switch equipped with the usual dust cover. In both the early and late switches the contacts and springs were mounted in a vertical position allowing any foreign matter as dust and small metal particles to pass through without becoming permanently lodged between the contacts.

Materials used in the construction of the relay and magnets have been improved similar to the changes made in magnetic circuits of the electrical power industry where low retentivity and high permeability are considered advantageous. Extensive tests have been made and are still in progress to find suitable magnetic materials. Various mechanical improvements have been made from time to time on the relay. The substitution of a hinge pin bearing for the pivot point type resulted in a relay with more uniform performance characteristics. This becomes apparent when it is

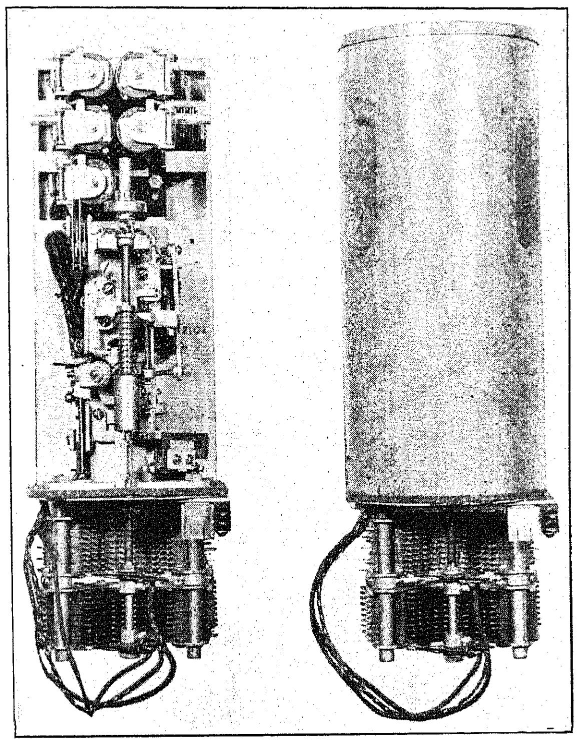

FIG. 13—HORIZONTAL RELAY TYPE SELECTOR WITH DUST COVER

considered that the gap between the armature and the heel piece is on the order of 0.0015 in.

Adaptability of the same type relay for various circuit conditions further led to uniformity of design resulting in a type of relay which may be given any reasonable time characteristics by change in windings. A comparison of the horizontal type of equipment with the vertical type shows this uniformity of design in a striking manner.

The advent and use of high grade modern insulating materials did much toward the production of a higher grade relay and magnet. Formerly coils were made by winding the wire against the fiber heads, using paper insulation next to the core and the leads were brought out through holes in the fiber heads. Notwithstanding the fact that the negative side of the battery is connected to the coil permanently in most cases, electrolysis still occurred between spool heads and the wire. Only by insulating the lead in wires between hard rubber washers, connecting them to termi-

nals on the head and winding the wire on a core insulated with empire cloth could this trouble be eliminated. Phenol fiber, bakelite and micarta are gradually replacing fiber and even though the first cost is higher, the resultant constancy of performance together with long life and freedom from electrolysis result in a more economical equipment.

Enamel-covered wire, particularly in the small sizes of wire, is finding increasing application in the general electrical engineering practise. While enameled wire is commonly used on relays and magnets in order to conserve space and to get the maximum number of ampere turns within a certain space, the protection this wire offers against electrolysis and corrosion is one of its greatest advantages. The adoption in recent years of enamel-covered wire cable has done much toward the reduction of electrolysis and leakage between cable pairs within the office.

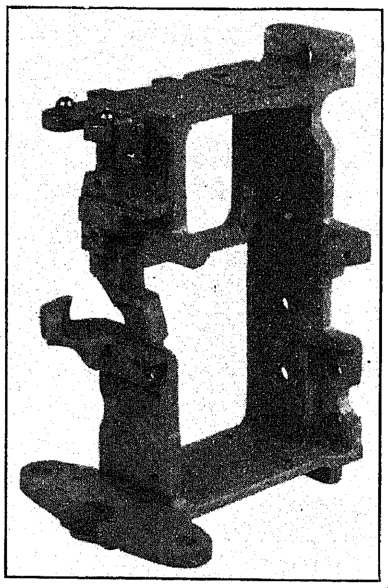

Fig. 14—Cast Iron Switch Frame

The various adjustments require that the switch frame does not change materially during the life of the frame. Numerous die cast metals have been tried, but have been found wanting in permanency. The cast iron frame, Fig. 14, has gradually been developed and has been found extremely constant in retaining its shape. It may be noted at this point that this switch frame is the basis and foundation mechanically of all step-by-step major switches. The use of universal relays (mechanical) mounted on a standard plate together with the magnets mounted on the frame did much toward the standardization of this type of equipment. Major and minor parts are designed to be interchangeable allowing various special circuits to be used on standard equipment in case of some particular individual necessity.

Manufacturing Methods

Mechanical design as a rule must be closely allied with manufacturing methods. Although methods do not change frequently the universal use of the automobile did much toward the change in methods by the evolution of complex machinery for the finishing of small parts. Fig. 15 shows a multiple

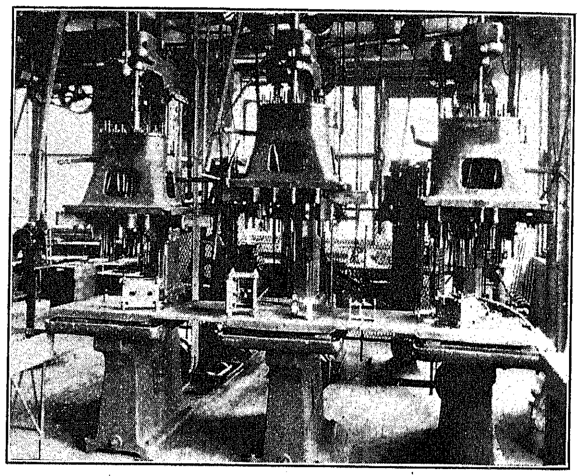

Fig. 15—Multiple Drill Press for Drilling Switch Frame

drill, such as is in common use in the drilling and tapping of holes in the switch frame mentioned above. By means of these multiple drills the frames are drilled quickly and accurately. Many other automatic and semi-automatic machines are used in the manufacture of a step-by-step equipment which are different from machines in customary use, such as contact welders,

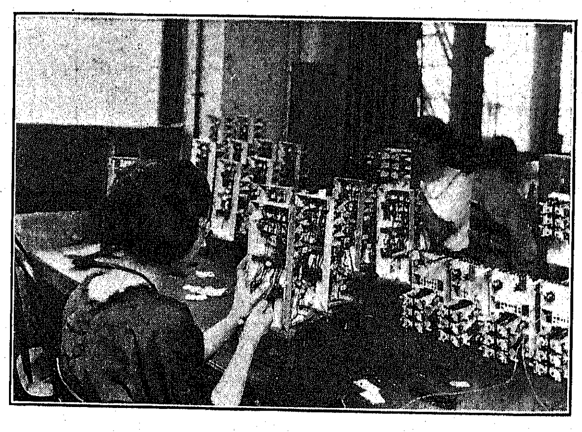

Fig. 16—Connector and Selector Wiring

either single or double contact, special wire braiding machines, and the like.

With regard to manual operations girls have been found to be excellent workers. In the wiring of major switches, Fig. 16, one girl wires the switch completely using two tools. No single tool has been found which will perform as well or obtain as good results as the two

which are in use. The average time of wiring a selector complete is one half hour. This involves the skinning and attaching of 95 connections. In the standard connector there are 215 connections with approximately one hour and fifteen minutes required for wiring. Soldering these connections requires five minutes for a selector and twelve minutes for a connector. After wiring three or four switches from blue prints the prints are found unnecessary and all future wiring is done from memory. Squeezing and alining fixtures, Fig. 17, assure that the spring assemblies are in their correct positions horizontally and vertically and contribute materially toward the manufacture and assembly of these parts which, in order to function uniformly, must be very accurate.

An inspection and engineering inspection department maintain the product at a uniform standard and determine and set all standard adjustments. These

Fig. 17—Spring Alining Fixture

standard adjustments make the performance of the switches commercially uniform. The economic development of the step-by-step system is in a large measure due to the most approved methods of manufacture which allow the full use of circuit development and mechanical design.

Maintenance Improvements

The standard parts with their definite tolerances and standard adjustments resulted in a decided uniformity of service while the proper routining keeps the equipment in first class condition. The change from three wire to two wire, from vertical to horizontal relay equipment and the introduction of dust covers gradually reduced the cost of maintenance per line per month. Fig. 18 gives some interesting comparative data regarding the number of these troubles. The main advantage, however, results in the use of less technical help in maintenance work due to the uniform construction and adjustment of the apparatus as mentioned above. By the use of a jack arrangement of mounting, the switches may be removed for repair in case of necessity and easily replaced by similar reserve switches.

Routining, that is, the periodic testing of all switches and equipment, results in keeping the equipment in first class working order by discovering possible future troubles before they develop into factors which impair telephone service. This service together with the obtaining of service observation data provides an index

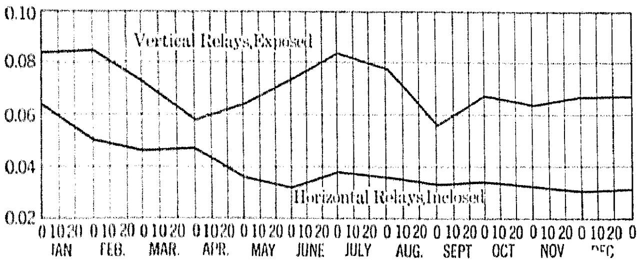

Fig. 18—Comparison of Two Specific Cases of Vertical and Horizontal Relay Equipment from Maintenance Standpoint.

Note Irregularity of Vertical Relay Equipment Curve while Horizontal Relay Equipment Curve Gradually Flattens out.

of the kind of service the equipment renders. Table I gives the tabulated results of service observations in various exchanges together with a final average obtained over a number of years. It is interesting to note that in the final average approximately 22.11 per cent of all lost calls are due to the subscribers whereas only 2.19 per cent are due to total plant trouble. Fig. 19 shows the seasonal variation between the various causes contributing to lost calls.

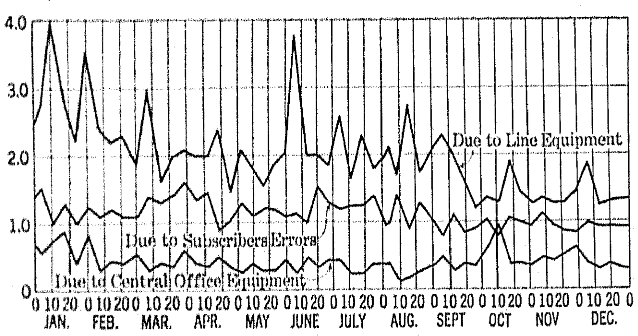

Fig. 19—Seasonal Variation of Per cent of Calls Lost due to Different Causes

In general it may be stated that all development in a measure reflects the maintenance viewpoint. The advances above mentioned on insulating materials, economy of equipment, economy of trunks, etc. invariably result in a lower maintenance cost.

Possible Future Economic Developments

The introduction of better materials, as in the case of the relay, has shifted the proportion of that much of the over-all cost as pertains to first cost and maintenance cost

from the latter to the former, at the same time reducing the sum of the two. By means of intensive circuit study and design coupled with mechanical simplicity the first cost may perhaps be lowered, while keeping the maintenance cost the same or even less.

Special materials are constantly developed. The selection of the proper materials with a knowledge of their capabilities and limitations offers unlimited possibilities in improving the performance and a possible reduction in over-all cost.

A definite value must be placed on lost calls, their significance in a trunking area and their evaluation against a definite amount of equipment and trunks, before equipment and inter-office trunks can be reduced to an economic balance in general. For instance, the calculation of equipment on a lost call of one to one hundred leads to a materially reduced quantity of equipment over that calculated from a lost call of one to one thousand. The service on the other hand is not lowered in the same proportion. The cost of installation and maintenance vary very materially in different localities, and in the same trunking area, therefore, the problem must be viewed in the light of the local situation. The study of local traffic conditions in any particular area by means of traffic recording apparatus to obtain definite and exact data is highly desirable.

The vacuum tube used as an amplifier offers possibilities in that a smaller current may be used and then amplified to the present values. Numerous advantages may accrue from such an arrangement as:

(a) High-resistance subscribers' loops of from 2000 to 3000 ohms.

(b) Replacement of 19 gage and smaller by gages in the neighborhood of 26 to 28 gage cables.

(c) Centralized office equipment in low value real estate areas.

(d) Use of high-resistance transmitters and receivers with possible gains of efficiency and clearer articulation.

Items (b) and (c) particularly indicate possibilities of effecting a saving, but as in the cases mentioned, these changes must depend on the merits of the individual case.

Conclusion

The step-by-step automatic telephone equipment has been developed largely along the lines of filling definite subscribers' and operating companies' needs. The system has undergone but minor changes but the equipment has been improved resulting in a lower maintenance cost and a longer life. Like the locomotive, the automobile and kindred pieces of mechanism, the step-by-step system has been standardized and its operation over a large number of years has been made uniform and reliable. No revolutionary future changes on the system appear imminent but the minor individual parts constituting that system will no doubt be modified to accomplish their specific circuit or mechanical function most economically.

SERVICE OBSERVATION TABLE

	Per cent of Originating Calls		Analysis of Uncompleted Originating Calls			Analysis of Failures		
Exchange	Completed	Uncompleted	Busy	Don't Answer	Failures	Subscribers Errors	Total Plant Trouble	Central Office Equipment Failures
A	79.41	20.59	8.92	8.67	3.00	1.35	1.65	1.32
B	74.06	25.94	11.65	9.92	4.37	2.95	1.42	1.07
C	78.66	21.34	10.96	6.73	3.65	2.19	1.46	1.18
D	74.13	25.87	10.43	8.37	7.07	4.45	2.62	2.10
E	74.60	25.40	9.68	8.20	7.52	5.14	2.38	2.04
F	79.37	20.63	7.74	7.52	5.40	3.63	1.77	1.05
G	69.66	30.34	11.15	9.91	9.28	5.20	4.08	2.75
Average	75.70	24.30	10.08	8.47	5.75	3.56	2.19	1.64

Discussion

G. K. Haspel (by letter): In general, only one connector group is required per 100 lines when these lines serve individual or P. B. X. subscribers, but for party-line service it has been the practise to provide as many groups of connectors per 100 lines as there are parties on the party line, that is, for two-party service, two connector groups per 100 lines are required and for four-party service, four groups.

The comparatively recently developed frequency-selecting connector which selects the ringing frequency (or other party-line station selecting feature), as well as connects to the called line, requires only one connector group per 100 lines, regardless of whether the line gives two-party, four-party or ten-party service.

It is obvious, with the regular connector method, that more connector groups will be required than line switch groups, or, in other words, the number of line switch groups will correspond to the number of hundreds of lines while the number of connector groups will correspond to the number of hundreds of possible stations.

In reducing the number of connector groups so that it corresponds to the number of groups of 100 line switches, the use of frequency-selecting connectors not only reduces the total number of connectors but in many cases also reduces the number of selectors. This saving in equipment is illustrated in the attached sketches showing trunking schemes for an exchange of 800 lines of mixed individual and party-line service.

Fig. 1 herewith shows the switching scheme for 800 lines using frequency-selecting connectors and provides for individual and party-line service up to ten parties per line. Fig. 2 shows a scheme for the same number of lines but for individual and four party service with four connector groups for each 100 lines. In the latter scheme, 32 connector groups are required, making necessary the use of second selectors between the first selectors

and the connectors, which is not necessary in the frequency selecting connector scheme, since the latter has only 8 connector groups.

In Fig. 1, eight levels of the first selectors are used (the "1" and "0" levels are usually reserved for special services) but when more than eight groups of connectors are required, the introduction of intermediate selectors, with access to ten groups each becomes necessary. Thus in Fig. 2, with 32 connector groups, four groups of second selectors are required.

In the two schemes illustrated, the same number of first selectors is required in each case; 137 second selectors are required in Fig. 2, while none are required in Fig. 1; and the number of connectors is greater in Fig. 2 because 32 smaller groups are required as against ten large groups in Fig. 1.

Not only is more equipment required for Fig. 2 but there is also an increase in the battery drain per call, of about 7%, due

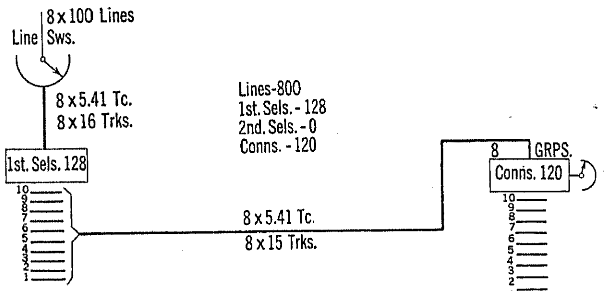

Fig. 1

to the necessity of the second selector in the train of each call. The greater number of switches and associated equipment involved would also increase the maintenance costs.

Further advantages for the frequency-selecting connector scheme, when used for all services (excepting P. B. X. or trunk-hunting service), are:—the use of any line for either individual or party-line service, and by mixing these on each unit a more equal distribution of traffic may be obtained; selective ringing of extension telephones; and the possibility of changing individual to party lines without changing the directory numbers.

The principal objection to the frequency-selecting connector scheme is the difficulty of providing intercepting service for "dead" and changed numbers. This is accomplished by the use of frequency relays bridged across the lines on which calls are to be intercepted and, if the number of calls to be intercepted is large, the cost of the relay equipment is considerable. However, in many exchanges, this intercepting service is not considered necessary, so in such cases the objection does not apply.

With frequency-selecting connectors, each line has a possible ten directory or station numbers and, when less than ten stations are assigned to a particular line, the remaining numbers are useless. This has been called a "waste of numbers," but, since no waste of equipment is involved, the loss is not a material loss, and the objection is not considered valid. Moreover, whether all the

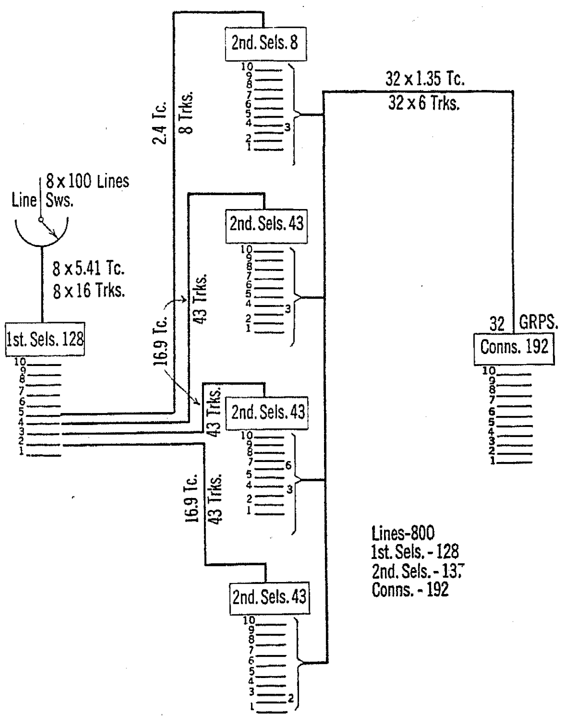

Fig. 2

numbers are used or not, they are available for use, which gives them an actual economical value.

The frequency-selecting connectors are somewhat more expensive per switch than the regular type connectors, but this is often counter-balanced by the decrease in the amount of selectors and connectors required for the entire office or exchange, or, even if the total cost is greater, it is quite possible that the service advantages will justify the extra expense.

High Quality Transmission and Reproduction of Speech and Music

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Review of the Subject.—*Radio broadcasting has drawn attention to the problems involved in obtaining high quality in systems for the electrical transmission and reproduction of sound. This paper gives the general requirements for such systems, discusses briefly the factors to be considered in design and operation and indicates to what extent the desired results can be obtained with the means now available.*

* * * * *

THE primary function of telephone circuits, as normally used in commercial service, is the electrical transmission and reproduction of speech sounds. In considering the operation of such a system, the reproduced sounds are referred to as having two properties, intelligibility and naturalness. While these two properties are not by any means unrelated and are both of importance in all sound reproducing systems, the first is naturally the more important in a commercial communication system. In broadcasting and public address systems, the communication function is supplemented by the function of entertainment and the property of naturalness, therefore, increases in importance in the reproduced speech. Furthermore, the use of music with such systems imposes, in general, more severe requirements upon them because of the wide range of frequencies and intensities required for proper appreciation.

In this paper the fundamental requirements for a system for faithfully transmitting and reproducing sound are outlined, and their applications considered, particularly in connection with broadcasting and the use of loud speakers.

In any system for the electrical transmission and reproduction of sound there are three essential elements: A means for converting sound into electrical energy, usually called the telephone transmitter or microphone; a means for converting electrical energy into sound, usually called the telephone receiver; and means for transmitting the electrical energy from the transmitter to the receiver.

In the operation of such a system, there are three general requirements which it is desirable that the reproduced sounds should meet: First, that they be at about the same loudness as people are accustomed to hearing the original sounds; second, that they be free from appreciable distortion, that is, that the character of the reproduced sounds be so close to that of the original sounds that the ear cannot distinguish between them; and third, that they be free from extraneous sounds. The degree to which these requirements of loudness, freedom from distortion and noise are met is the measure of the quality of the system.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

The discussion in this paper will be directed primarily to the second of the above requirements, that is, the matter of obtaining accurate transmission and reproduction of the original sounds, and the requirements of loudness and noise will be considered only in so far as they have a bearing on the principal discussion. In this connection, it may be noted that with the development of practically distortionless amplifiers, it is possible to compensate for the losses in volume incurred in transmission and in the conversions between sound and electrical energy and thus obtain any degree of loudness desired. The problems of eliminating noise, however, are in many cases difficult, but are too extensive to be within the scope of this paper.

DISTORTION

The sounds which comprise speech and music involve, as is well known, complicated pressure variations. For any small interval, of time, these pressure variations may be resolved into a series of component sinusoidal waves. As the speech or music proceeds, however, the amplitude, the frequency and the phase of these components change. The transmission and reproduction of such sounds may be conveniently considered as a matter of transmitting and reproducing the several component waves.

For a system to be ideal from a quality standpoint, these components must be reproduced unchanged, and no new components introduced. Experience has shown that changes in phase, such as are usually obtained, produce no effects which are noticeable by the ear. Also, as discussed later, all the amplitudes may be diminished or increased uniformly through an appreciable range before the quality is affected.

The requirements then for no noticeable distortion in a sound-reproducing system may be stated as follows:

1. The reproduced sounds shall have the relative intensities of the component frequencies the same as the original sound.

2. The reproduced sounds shall not contain any components of frequencies not present in the original sound.

Failure to meet the requirement set up in (1) is referred to as "frequency distortion." This results

when a system has different transmission efficiencies for the different frequencies.

Failure to meet the requirement set up in (2) is referred to as "non-linear distortion." This results when the relation between the output and input powers is not independent of the magnitude of the input power. Distortion of this type may be obtained from the iron cores of transformers or other coils, from vacuum tubes, from carbon transmitters and from diaphragms or other vibrating mechanical parts. All of these have a load characteristic which generally has a practically linear relation between output and input when operated below certain energy limits, but which shows a non-linear relation between output and input when the input power exceeds these limits. Operation over the non-linear part of such a characteristic results, in addition to changing the intensity relations of the components of the original sounds, in the setting up of components of frequencies which may be different from those of the components in the impressed wave.

Another important factor in the reproduction of sounds which is not generally appreciated is that apparent distortion is obtained if the loudness of the reproduced sounds is materially outside of the range in which the listeners are accustomed to hearing the original sounds. Recent work[1] in hearing has shown that the transmission mechanism of the ear is non-linear in its response even at loudness levels commonly used in speech and music. From this it is seen that the interpretation of complex sounds by the ear is partly accomplished by the "subjective" frequencies introduced by the ear itself. Due to this non-linear characteristic of the ear, when the intensities of reproduced sounds are materially different from those of the original sounds, there is an apparent distortion.

With these requirements in mind, consideration will now be given to the extent to which they can be met with the means and methods now available. In this connection, three electrical transmitting and reproducing systems will be discussed, a high quality telephone circuit, the public address system and the broadcasting system. The first two and the elements used in them have been previously described in some detail. A brief description of them will be given here, however, to show their similarity with the third, which will be discussed more comprehensively, and also to indicate to some extent the evolution of high quality reproducing means. It will be noted that the successful design, maintenance and operation of any high quality system depends upon the development of methods of measuring its operational characteristics, such as the relation between input and output energies over a range of frequencies and intensities.

1. Physical Measurements of Audition and Their Bearing on the Theory of Hearing, H. Fletcher, *Jour.* Franklin Inst., September 1923.

High Quality Telephone Reference System

A number of years ago a telephone circuit was set up in the Bell System Laboratories in which use was made of the various available means to eliminate distortion as much as possible. This was used as a reference circuit in a comprehensive investigation of the effects on the intelligibility of reproduced speech sounds, of variations in the volume of the reproduced sounds, of various types and amounts of distortion and of various amounts and kinds of extraneous noise.[2] This system and the variation of its efficiency with frequency are shown in Figs. 1 and 2.[3] It has also negligible noise and non-linear distortion for the sound powers that it is designed to handle, that is, those corresponding to talking in the normal way over a telephone circuit.

With this system, the fundamental vowel and consonant sounds used in speech are reproduced so well that when a series of such sounds are impressed upon

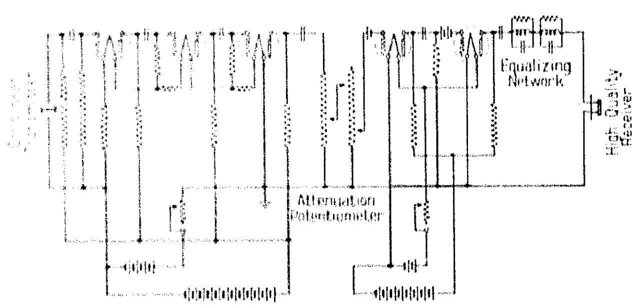

Fig. 1—Circuit of High Quality Telephone Reference System

the system, 99 per cent are correctly understood. This recognition is for the condition when these sounds are combined into meaningless monosyllables, which makes the test much more severe than for these sounds as ordinarily used in conversation where the context aids in the recognition. The degree of recognition obtained with this system is within a few tenths per cent, as good as that obtained by direct hearing. This circuit, therefore, from the standpoint of intelligibility of speech is practically perfect.

This high quality circuit made use of several important developments. First is the condenser transmitter which gives a practically distortionless conversion from sound to electrical energy. This transmitter, which has been previously described,[4] uses a thin metal diaphragm, tightly stretched and placed close to a heavy metal plate. The diaphragm and the heavy plate form an electric condenser and the air film

2. The Nature of Speech and its Interpretation, H. Fletcher, *Jour.* Franklin Inst., June 1922.

3. The "transmission units" used in Fig. 2 and elsewhere in this paper are a logarithmic function of power ratio. The number of transmission units, N, corresponding to the ratio of two amounts of power P_1 and P_2, is given by the relation $N = 10 \log_{10} P_1/P_2$. The power ratio corresponding to N units is therefore $10^{N(.1)}$.

4. Wente, *Phys. Rev.*, June 1917 and May 1922. Crandall, *Phys. Rev.*, June 1918.

between the two serves to damp the vibration of the diaphragm.

Second is the design of vacuum tube amplifiers which are distortionless over a wide range of frequencies and loads. The design of such amplifiers will be discussed in a future paper, so will not be described here, other than to show later some amplifier frequency characteristics which have been obtained.

Third is a telephone receiver having small distortion. For this a permanent magnet type of receiver was used in which the principle of damping the diaphragm by an air film was employed in a manner similar to that described above for the condenser transmitter.

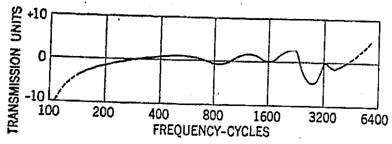

Fig. 2—Frequency Response Characteristic of High Quality Telephone Reference System

Fourth is the use of a network of impedances designed to introduce into the circuit the distortion which compensated for any distortion in the system which it was not practicable to eliminate in the several parts. In this circuit this compensation was needed primarily to take care of residual distortion in the receiver.

Public Address System

The public address system and its applications were described in two papers presented before this Institute in February 1923,[5] and as already noted will be referred to only briefly here. In the public address system which was used at the presidential inauguration in March 1921, the condenser transmitter and high quality amplifiers were used to obtain good quality. In November 1921, this public address system was used with the toll lines to transmit the Armistice Day Service at Arlington, Va., to New York and to San Francisco. At this time use was made of a new design of high quality transmitter, the double carbon button transmitter employing the stretched damped diaphragm of the condenser type. At this time, also, corrective distortion networks were employed to compensate for the distortion of the non-loaded cable circuits which were used to connect to the toll lines. The "volume indicator" for showing the power carried by the amplifiers was also used on this occasion in order to keep them from being overloaded and causing non-linear distortion. This volume indicator, as described in the papers referred to, consists of a vacuum tube amplifier-rectifier operating a quick acting ammeter. In both these loud speaker applications, extensive use was made of single-frequency measuring apparatus for determining the efficiency of the various parts of the system over a wide range of frequencies.

5. Public Address Systems, Green and Maxfield, Journal of A. I. E. E., April 1923.

Use of Public Address Systems with Telephone Lines, Martin and Clark, Journal of A. I. E. E., April 1923.

Broadcasting System

When radio broadcasting started its phenomenal development, this high quality apparatus and the associated testing methods found new applications. It will be noted that the public address and the broadcasting systems are very similar, the main difference being the use of radio in the latter as a convenient means of reaching a large number of receiving stations from one transmitting station.

In Fig. 3 are indicated the essential elements of a radio broadcasting system. In this system, M is the microphone or means for converting from sound to electrical energy, A_1 is the amplifier used to increase the output of the microphone before transmitting it over the wire connection L_1 to the broadcasting station. The amplifier A_2 and the radio transmitter $R\ T$ increase and transform the energy into that which is put upon the antenna. At each of the receiving stations there are required, in general, a radio receiver $R\ R$ for converting the received radio frequency currents into audio frequency currents, an amplifier, and a telephone receiver, either of the type held to the ear or of the loud speaker type.

In regard to the wire line L_1 to the broadcasting station, it should be noted that while much of the material to be broadcast is at present specially produced in a studio closely associated with the broadcasting station, a large and probably increasing proportion of the broadcasting material is produced at points at some distance from the station. In this latter class come (1) material which is not given primarily for broadcasting, such as concerts and speeches for some local audience, (2) material given in a studio located at a point convenient to the artists or speakers, but remote from the broadcasting station and (3) announcements of the progress of athletic games or other sporting events which are made from the place where the games are held.

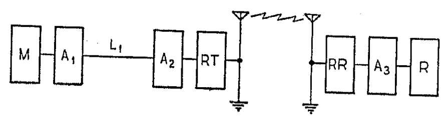

Fig. 3—Schematic Diagram of Radio Broadcasting System

Broadcasting Material

The material used for broadcasting consists in general of speech or music. Speech sounds are extremely complex in their nature and involve frequencies from about one hundred cycles to above six thousand. The first two charts in Fig. 4 show the sound spectra for the sung vowels "ah" and "ā". When these and the other vowels are spoken they are modulated both in pitch and volume from this steady state, the particular manner of starting or stopping them determin-

ing the so-called stop consonants[6]. The unvoiced fricative consonants, "s", "f" and "th", have their sound spectra in the upper frequency regions between 4000 and 10,000. In general most of the energy is carried by the vowel sounds and at frequencies below 1000 cycles, but the fine modulations of the vowels which produce the stop consonants and also the production of the fricative consonants involve frequencies mostly above 1000 cycles. For this reason it is well to bear in mind that the importance of any frequency region for carrying the energy in speech is quite different from that for carrying the intelligibility. On Fig. 5 are shown two curves which contrast this difference.[7] The curve for intelligibility does not directly take into account the naturalness of the sounds. It is found, for example, that while a system, which transmits only the frequency range from 500 to 2000 cycles, reproduces speech which can be easily understood, it leaves much

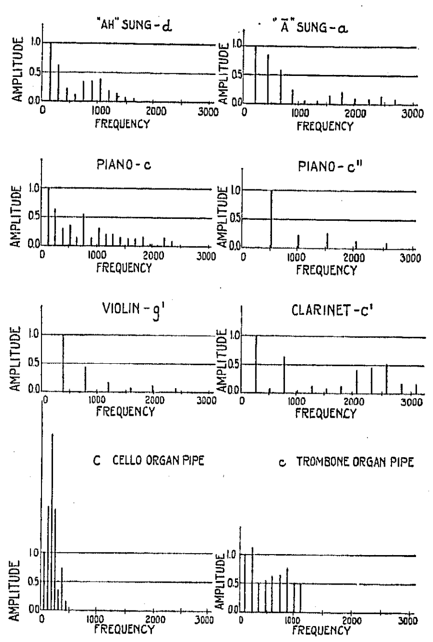

FIG. 4—SOUND SPECTRA OF TYPICAL MUSICAL TONES

to be desired from the standpoint of naturalness. In broadcasting, a broader frequency range is desirable because of the importance of naturalness. Results can be obtained for speech which are good for intelligibility and fairly good for naturalness with a frequency range from about 100 to 3000 cycles, although appreciable improvement is obtained by the extension of the upper end of the range.

The various types of vocal and instrumental music, solo, choral and orchestral, have widely varying characteristics, with fundamental tones as low as 16 cycles and harmonics above 10,000 cycles. The breadth of this range makes its proper handling in a reproducing system extremely difficult, particularly when the large energy of some of the low notes such as used in the pipe organ are taken into account. Fig. 4 gives also charts showing the sound spectra for some typical musical instruments when they are sounded at the pitches indicated. It is very difficult to obtain any quantitative measurements of the importance of the various frequency regions for properly transmitting music, but it has been found that with a frequency range of from about 50 to 5000 cycles good reproduction can be given for most kinds of music. In this connection it may be pointed out that the pitch of musical tones of very low pitch is carried to the ear mainly by the harmonics rather than by the fundamental.[8] For example, with a system not reproducing any frequencies below 100 cycles, the pitch is preserved for notes even as low as 30 cycles. The musical quality is marred, however, when the lower frequencies are not present.

Another important characteristic of speech and music is the intensity range. For speech the range of the average power is of the order of 1000 to 1. In music, such as that given by a symphony orchestra, the corresponding range may be as great as 100,000 to 1. These ranges have an important bearing on the load capacity required for the parts of the broadcasting system as will be brought out later.

PICK-UP OF MATERIAL

In picking up material for broadcasting, that is, in getting the sound energy into electrical energy, the general requirement would seem to be to get to the high quality microphone the sounds in the form in which a skilled listener would wish to hear them if he were free to choose his location with respect to the source of these sounds. In this respect, the skilled listener would be largely governed by hearing the sounds under the accustomed conditions with all undesirable noises, echoes and abnormal reverberations removed. In considering the pick-up of material for broadcasting it should be noted, however, that it corresponds to listening with one ear, that is, the binaural sense of direction which is normally obtained in hearing the sounds directly is lacking. With binaural audition, it is possible to concentrate on one sound source and to disregard somewhat the effect of other sounds coming from different directions or distances. Because of the monaural character of broadcasting it is necessary, therefore, to go even further in reducing noises and reverberation at the transmitter than would be the case for an observer using two ears at the same location.

In picking up sounds, undesirable effects which may be classed as distortion, may be obtained by having either too much or too little reverberation or, where the sounds come from several sources, such as in the case of a quartet or an orchestra, by not having the proper

6. The Nature of Speech and its Interpretation, H. Fletcher, *Jour.* Franklin Inst., June 1922.

7. Curve for energy distribution given in Analysis of the Energy Distribution of Speech, Crandall and MacKenzie, *Phys. Rev.*, XIX, No. 3. Curve for Intelligibility derived from data given in paper mentioned in Note 6.

8. Physical Criteria for Determining the Pitch of a Musical Tone, H. Fletcher, *Physical Review*, September 1923.

relation between the intensities of the sounds which reach the transmitter from the several sources. Since most speeches and musical selections are given indoors, a certain amount of reverberation is generally present. Because of this customary condition, music particularly, without reverberation, such as is obtained in a heavily padded room, sounds "dead." Too much reverberation, on the other hand, causes one tone to drag over into a succeeding one and tends to blur the sounds. In some tests carried out by Prof. W. C. Sabine with rooms in which the reverberation was varied it was found that a group of musicians consistently selected a particular reverberation condition as being most desirable for the piano.[9]

Much of the material that is broadcast is given in a special studio where it is possible to control the conditions. The studio can be placed in a quiet location, it can be treated with absorbing material to give the proper amount of reverberation and the speaker, singers, or musicians can be placed with respect to the microphone so as to obtain the desired balance between the direct sounds and the reverberation and also between the sounds from the several sources where more than one source is used. With the large number of variables involved, it is not as yet possible, however, to give general rules governing all of them.

In regard to the matter of equipping such a room with sound absorbing material, it is seemingly a common mistake to cover as completely as possible the ceiling, walls and floor of a studio with such material. Such a room, in addition to making the music sound "dead," makes it difficult and in some cases impossible for a singer or violinist to keep on the key because they are accustomed to get the pitch of one note from the reverberation of the preceding note. In one studio of about 20 by 30 feet, in which a large amount of experimental work was done to get a suitable reverberation for music, the final arrangement is a hardwood floor with a few rugs, the walls hung with monks cloth and about two-thirds of the ceiling covered with one inch hair felt. The reverberation can be increased when desired by taking up rugs or pulling back some of the wall hangings. In such a room for speaking, however, undesirable reverberation is obtained if the speaker is more than about four feet from the microphone. In connection with the statement regarding the effect of the monaural character of broadcasting on the requirement for the placing of the microphone, it is of interest to note that the reverberation time for this studio, using Sabine's method and coefficients, was somewhat less than that found to be desirable in his tests which were referred to.

There is an increasing demand in broadcasting for the use of material which is not being given specifically for broadcasting, such as a speech by some well-known person or a concert by a symphony orchestra. In such cases it is not usually possible to change the acoustics, so that the problem becomes one of getting the best location for the microphones.

For a speech, the problem is generally not difficult as the microphone can usually be located within about three feet of the speaker so as not to restrict unduly his usual movements. For a symphony orchestra of 75 to 100 pieces the problem presents some difficulties. It is desirable to get the transmitter far enough away from the orchestra so that the paths from it to all the pieces of the orchestra are about equal, in order to get proper balance between the parts, and at the same time, not to be so far away from the orchestra that the incidental noises of the audience are loud compared to the music. Good results have been obtained under these conditions by suspending the transmitter from the ceiling of the concert hall over a point on the floor about thirty to fifty feet from the orchestra and about ten to twenty feet from the ceiling. This brings it over the audience, but far enough away so that noises from it are not bothersome and far enough away from the orchestra to get a good balance between the parts. Also this permits the sound striking the transmitter through reverberation to be sufficiently appreciable as compared to the direct sound. This reverberation gives the impression of the orchestra playing in a concert hall, which, of course, is the natural condition. The scheme of using several transmitters distributed throughout the orchestra, in order to pick up the different parts, is in general undesirable because of the lack of reverberation and the difficulty of getting proper balance between the parts.

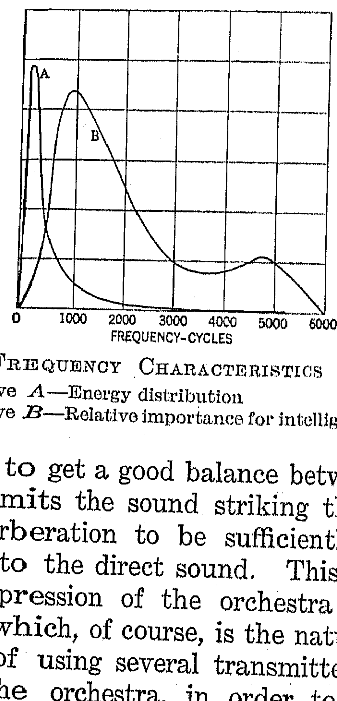

Fig. 5—Frequency Characteristics of Speech
Curve A—Energy distribution
Curve B—Relative importance for intelligibility

Transmitters

Two transmitters or microphones of the air-damped, stretched-diaphragm type have been extensively used for broadcasting, the condenser type and the carbon button type.

The frequency response characteristics of present models of these two types of transmitters are shown in Fig. 6, that designated A being for the carbon and that designated B for the condenser type. Both of these have already been described elsewhere and will not need further consideration here. It should be noted that the condenser type can be designed to have a

9. Collected Papers on Acoustics, Harvard Univ. Press, page 75.

frequency characteristic of almost any degree of flatness desired. Material improvements have been made recently on the carbon type. One of these is the use of a light metal diaphragm by means of which the electrical output for a given sound input has been increased about ten times. A second important improvement in the carbon type has been a change in the acoustic spaces associated with diaphragm to reduce the distortion. The advantage of the carbon type of transmitter is that it requires two stages of amplification less than the condenser type and approaches it closely from the standpoint of freedom from distortion. As a result of the small diaphragm motions used in this transmitter the carbon button is worked far below the saturation point.

Transmission to Broadcasting Station

When material is picked up at a point remote from the broadcasting station, care must be used to avoid distortion in getting it to the station. When, as is usually the case, the point where the material is given and the broadcasting station are in the same city, it is generally possible to get non-loaded telephone cable circuits between the two points. By the use of corrective distortion networks or "attenuation equalizers" with such circuits, uniform transmission efficiency

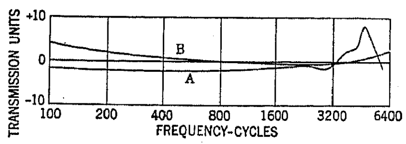

Fig. 6—Frequency Response Characteristics of High Quality Transmitters
Curve *A*—Stretched diaphragm double carbon button transmitter
Curve *B*—Stretched diaphragm condenser transmitter

over the desired frequency range can be obtained, even though the circuits themselves may have considerable distortion.[10] With these equalizers it is possible to equalize such circuits so that the variations of efficiency over the frequency range from the average value are less than one transmission unit.

The high quality transmitters which are used to pick up the material to be broadcast have energy outputs which are so low as to require amplification before they are transmitted to the broadcasting station in order to over-ride extraneous noises which may be encountered. Such amplifiers, in addition to having uniform efficiency for a broad frequency range, must also be capable of giving a large range of amplification and of handling without distortion a wide range of power in order to take account of the variations in the volume of sounds which are impressed upon the transmitter. In picking up speeches, for example, different amplifications may be required for the different loudness of the voices of the speakers. In making a speech, an orator often intentionally changes the loudness of his voice for emphasis. The amplification must be such as to permit low parts to be heard satisfactorily and also the amplifier must be capable of handling the loud parts without overloading. The amplification can be reduced for the loud parts to reduce the power handled, but the power output cannot be kept constant without spoiling entirely the emphasis effects desired by the speaker. In music, the volume of sound varies frequently and over a large range.

Considering this matter from the standpoint of the operator of a radio receiving set, it is desired first, that when the volume of the original sound is at its low point the reproduced sound should be loud enough to override static and other radio frequency interference, incidental noises in his set and room noise at his set. With this condition satisfactorily met, it is desired that the receiving set be capable of handling the maximum volumes of sound without overloading. The sets now available are capable of handling in the order of about a hundredth of this range and to make them handle this larger range would at present be practically prohibitive from a cost standpoint. The same requirement imposed upon the radio transmitter at the broadcasting station would also increase its cost by a large factor. The circuits used between the point where the material is picked up and the broadcasting stations also impose restrictions on this volume range. The lower limit to the power placed upon such circuits is set by the extraneous noise which may exist upon them due to induction from other circuits. The upper limit to the power on the circuit is determined by two factors, one, the capacity of the amplifiers which may be used and the other, the interference which this circuit would cause in other telephone circuits which are in the same cables with it. These circuit requirements, in general, limit the power range which can be satisfactorily handled to a range of about 1000 to 1. From the standpoint of the circuits alone, this range could be increased by special measures which, however, it might not always be practicable to apply.

These conditions, therefore, make it highly desirable to control the volume range given out by the amplifier associated with the transmitter. Some of this control could be exercised at other points in the system, but it is obviously desirable to have it all take place at one point and keep the rest of the system fixed. For this purpose the amplifier associated with the transmitter is equipped with a means for giving a quickly adjustable amplification. To make these adjustments correctly, it is necessary for the operator of the amplifier to know what power is being delivered by it. Use is made here of the "volume indicator," which is bridged across the output of the transmitter amplifier and the amplification of the volume indicator varied by means of a calibrated potentiometer until a standard deflection is obtained. The amplification required to get this deflection is then a measure of the output of the transmitter amplifier. This is supplemented by a monitoring loud-speaking receiver bridged across the circuit

10. Use of Public Address System with Telephone Lines, Martin and Clark. Jour. A. I. E. E., April 1923, page 361.

at the same point. By the aid of these, the operator can check the operation of the transmitter and its associated amplifier and keep the volume of electrical power delivered to the broadcasting station between certain prescribed limits which are far enough apart to give suitable expression to the music or speech. When the sounds striking the transmitter become too loud, the gain of the transmitter amplifier is reduced and when these sounds become too low, the amplification is increased, these changes being made gradually in order to avoid noticeable abrupt shifts in volume. The limits between which the electrical power is kept, are those which have been found experimentally to avoid overloading any part of the broadcasting system and to keep above any extraneous noises in the system.

This adjustment of the gain of the transmitter amplifier to keep the power delivered to the broadcasting set within certain prescribed limits is required also when the pick-up of the broadcasting material is in a studio at the broadcasting station.

Broadcasting Station

In the radio broadcasting transmitter the incoming electrical power is generally amplified before being used to modulate the radio frequency carrier. In this transmitter, the frequency and volume range requirements, discussed for amplifiers, also apply. The amplification obtained in this part of the system should generally be fixed and all necessary adjustments during operation made in the amplifier associated with the microphone. The following discussion of the broadcasting station is from the standpoint of operation, as the apparatus itself is described in another paper.[11]

Operating Requirements. With a fixed setting of the radio transmitter, it is important to determine the maximum power which can be introduced into it without causing noticeable overloading. To do this, there are required a means for indicating power such as a volume indicator, a high quality radio receiving set, a high quality loud speaker and high quality amplifier for operating it and some skilled observers. With the loud speaker, first determine for speech and several kinds of music, the maximum power which can be delivered by the microphone amplifier before overloading is detected. Then with the amplifier connected to the radio transmitter and with the radio receiving set, high quality amplifier and loud speaker, determine what power input into the radio transmitter causes overloading. If this is less than has been previously determined as the overloading point of the microphone amplifier, the overloading is in the radio transmitter. The station should then be operated so that the power delivered to the radio transmitter never exceeds this amount.

As a check on the operation of the station, a monitoring system such as the following should be used constantly. In this system a loud speaker and associated amplifier are connected either directly to the output of the microphone amplifier or to the output of a high quality radio receiving set. In these two connections the reproduced speech or music should sound the same and neither should show any signs of overloading.

This matter of guarding against overloading has been stressed so much because it is a common source of poor quality in broadcasting. Furthermore, it is often a defect in operation rather than in apparatus and as such, constant care is required to avoid it.

Another important factor in good broadcasting is insuring that the system and all its parts maintain their good quality. For this purpose, periodic tests should be made of the complete system with single-frequency currents over the range to be transmitted. For these tests the microphone can be replaced by a source of known amount of current and a measurement made of the electrical output of the high quality monitoring radio receiver. For such tests, use can be made of a "dummy" antenna, if it is not possible or desirable to go out "on the air."

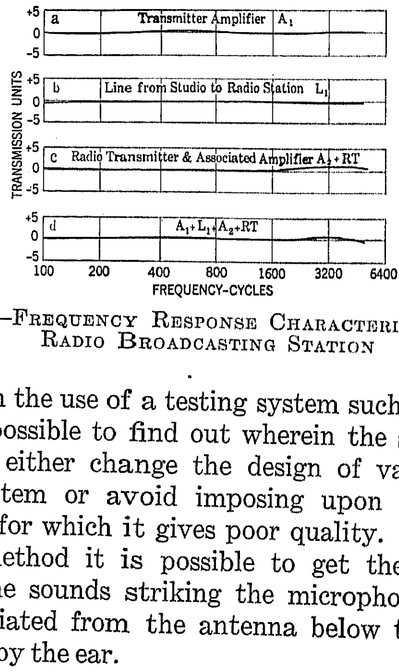

Fig. 7—Frequency Response Characteristics of Radio Broadcasting Station

It is with the use of a testing system such as outlined that it is possible to find out wherein the system falls down and either change the design of various parts of the system or avoid imposing upon the system conditions for which it gives poor quality. By following this method it is possible to get the distortion between the sounds striking the microphone and the energy radiated from the antenna below the amount detectable by the ear.

Frequency Characteristics. The curves of Fig. 7 indicate what can be done in obtaining good quality in broadcasting. These curves are for station WEAF in New York City where the studio and the broadcasting station are in different buildings, the two being connected by a cable circuit about a mile and a half long. Curve a is for the microphone amplifier (A_1 in Fig. 3), curve b for the equalized line between the studio and station (L_1, in Fig. 3) and curve c for the radio transmitter and associated amplifier ($R\,T$ and A_2 in Fig. 3). Curve d is for the system from electrical power of audio frequency leaving the microphone to power from the antenna at radio frequencies. The curve for complete operation of the broadcasting system

11. Transmitting Equipment for Radio Telephone Broadcasting, E. L. Nelson. Presented I. R. E. Jan. 16, 1924.

from sound in the studio to radio frequency power in the air can be obtained by combining curve *d* with the microphone curve from Fig. 5. Curve *d* shows the station as it is now operated. The small variations from the horizontal line can, of course, be eliminated, if worth while, by the use of an attenuation equalizer.

Receiving Station

The apparatus at the receiving station of a radio broadcasting system is required to perform three and preferably four functions. The three are selectivity, conversion of electrical energy from radio to audio frequency and conversion from electrical energy to sound. The fourth is amplification. The first two, selectivity and detection, are the essential functions of a radio receiving set. While it is not within the scope or purpose of this paper to discuss in detail various types of radio receiving sets, some general discussion will be given of the functions of the set in so far as they affect quality. Similar consideration will also be given to the other functions of the receiving station apparatus.

Amplification. The function of amplification is desirable and often necessary in order to bring the energy received by the antenna up to a point where it can produce sounds loud enough to be easily heard. This is particularly the case where loud speaking telephones are used to perform the third function. While it is a relatively simple matter to provide amplification without distortion, it is in performing this function that serious distortion is now introduced at many receiving stations, particularly when the amplification is in the audio frequency range or when it is obtained by regeneration. The provision of amplification without distortion is largely a matter of proper design, based on a knowledge of the characteristics of the tubes used and means for coupling stages together. A common offender in audio frequency amplifiers is the transformer, although with proper design it can be made to function satisfactorily.[12]

Selectivity. In performing the function of selecting the radio wave which it is desired to receive and discarding others, there is a conflict between the degree of selectivity, or sharpness of tuning, and width of the frequency band for the reproduced sounds. If this band width for the reproduced sounds is to be 5000 cycles and both side bands of the radio carrier are to be received, obviously all other waves within a band width of 10,000 cycles will also be received. Further, because it is not possible with the resonant type of selective means to let through without distortion this 10,000 cycle band and at the same time cut off absolutely all other waves near the edges of this band, the set will respond to a wider range of frequencies.

There are, generally speaking, two types of selective means used in radio receiving sets, one a circuit containing one or more adjustable resonant elements and the other a circuit hvaing a fixed selective element with adjustable means for converting the received radio waves into waves of frequencies which will pass through the selective element. With the first type of selectivity, which includes the selectivity obtained with regeneration, the distortion of the reproduced sounds is obviously not fixed but will vary with the sharpness of tuning used. With the second type of set, the selectivity is fixed in the design and involves, therefore, a predetermined compromise between distortion of reproduced sounds and degree of selectivity. Fig. 8 illustrates quantitatively what this compromise entails and also the range of distortion which may be obtained with a set of the variable selectivity type. Curve *A* shows the characteristic for one stage of audio frequency amplification in a particular receiving set, which is seen to cause appreciable distortion only at the low frequencies. As the distortion caused by radio tuning affects only the higher audio frequencies, curve *A* corresponds to a set with no radio selectivity. Curves

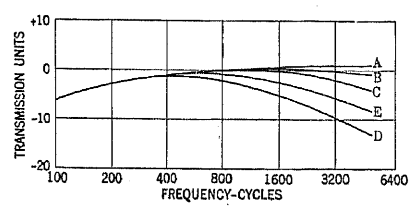

Fig. 8—Variation of Audio Frequency Response Characteristics of Radio Receiving Sets with Different Degrees of Selectivity

Selectivity expressed in terms of attenuation for radio frequency 10,000 cycles from frequency for which set is tuned.

Curve *A*—Attenuation 0 transmission units
Curve *B*— " 10 " "
Curve *C*— " 20 " "
Curve *D*— " 40 " "
Curve *E*—Radio set with fixed selectivity.

B, *C* and *D* indicate the effect of increasing degrees of radio selectivity. These three degrees of selectivity are such that if there is an interfering signal having the same intensity in the ether as the signal being received, but having its carrier frequency 10,000 cycles higher or lower, it will produce an audio signal at the output of the set 10, 20 and 40 transmission units respectively lower than the level of the signal being received. In other words a receiving set having the characteristic *B* which is very desirable from the quality standpoint will be much less selective against interference than one having the higher distortion characteristic *D*. As an example of a practical compromise, curve *E* shows the characteristic of a set of the fixed selectivity type which was designed for general all around use in receiving both local and long distance broadcasting. In this set a frequency 10,000 cycles higher or lower than the frequency to which the set is tuned suffers a loss of 34 transmission units.

The fixed selectivity type of set has some advantage in that its operation is definite and is less likely to give poor quality due to improper operation. The operation of this type of set can be materially improved by employing for the fixed selective element a band pass

12. Telephone Transformers, W. L. Casper. Presented A. I. E. E., Feb. 8, 1924.

filter. Such a filter has the advantage that the characteristic of the transmitted range can be made practically flat for any desired band width and to present a large attenuation for frequencies outside the band. For example, with a well designed filter of this type, the characteristic of the transmitted audio frequency band can be made practically flat up to 5000 cycles and the discrimination against other signals can be made even greater than that given above for curve *D*. This type of selectivity employing a band pass filter was used in the receiving sets of the Catalina Island radio telephone system.[13]

Conversion from Radio to Audio Frequency. In converting the electrical energy obtained from the antenna from radio to audio frequency, there is in general no difficulty from the standpoint of distortion provided the "detector" for making this conversion is worked below saturation.

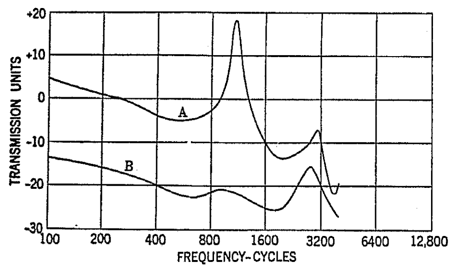

FIG. 9—FREQUENCY RESPONSE CHARACTERISTICS OF TELEPHONE RECEIVERS

Curve *A*—Commercial type
Curve *B*—Specially damped receiver

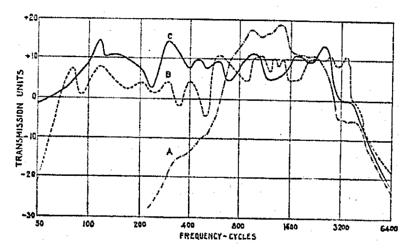

FIG. 10—FREQUENCY RESPONSE CHARACTERISTIC OF LOUD SPEAKING RECEIVER

Curve *A*—Commercial type
Curves *B* and *C*—Experimental models.

Conversion from Electrical Power to Sound. The conversion from electrical power to sound may be accomplished either by the head type telephone receiver or by a loud-speaking telephone, the latter being obviously more desirable for this purpose.

In Fig. 9 is shown the frequency response characteristic of a good type of commercial head receiver when held to ear in the usual manner. It will be noted that this introduces appreciable distortion. A material reduction in this distortion can be obtained with the type of damped receiver used in the high quality telephone system described in the first part of the paper. The characteristic of such a damped receiver is shown also in Fig. 9.

In Fig. 10, Curve *A* gives the frequency response curve for one of the best types of commercial loud speaker. This also introduces considerable distortion, being particularly weak at the lower end of the frequency range. This deficiency while not so serious for speech, is easily noticeable for music. In this figure are given also two curves showing the response characteristic of laboratory models of loud speakers. These are of interest since the means for converting from electrical power to sound are the most serious source of distortion in a system for transmitting and reproducing sounds, and the reproduction given by these models (demonstrated at the time of presenting this paper) is markedly superior to that obtained with the commercially available apparatus, and indicates the future possibilities of broadcasting.

CONCLUSION

From this consideration of systems for the electrical transmission and reproduction of sound, it has been shown that it is practicable to get almost perfect electrical transmission over a broad band of frequencies from the terminals of the pick-up transmitter to the radio transmitter and from there out into the air. With the condenser transmitter and proper associated amplifiers the conversion from sound striking the diaphragm to electrical energy can also be made without appreciable distortion. With a properly designed and operated broadcasting station, therefore, high quality material can be delivered to the receiving stations.

At present the commercial radio receiving sets and the means for converting from electrical energy to sound now generally available can not fully utilize this high quality material. These receiving and reproducing means can, however, be materially improved. The problem now is to make such improvements available in such a form that their cost will not make their use prohibitive. As yet the commercial production of apparatus incorporating such improvements is in the future.

In view of the distortion which exists at the present receiving stations, the question may arise as to the justification for going as far as has been indicated in the other part of the system. The fact is that with the reproducing means now available, material deviations from the frequency characteristics which have been given for the other parts of the system are detectable and the effect of non-linear distortion readily noticed. In a broadcasting system where one element is used for converting from sound to electrical energy and for distributing this energy to a large number of elements for reconverting it into sound, the expense of getting good results in this one element is not prohibitive and, taking into account the whole system, relatively small.

In broadcasting, the novelty of the system was undoubtedly a large factor in its rapid growth and development. Those who make use of the system are, however, becoming more critical of the service which it renders and the quality of reproduction will be of increasing importance in the future.

13. The Avalon-Los Angeles Radio Toll Circuit, Clement, Ryan and Martin, *Jour.* I. R. E., December 1921.

The Function and Design of Horns
for Loud Speakers

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Review of the Subject.—*A discussion of the function of the horn, together with theoretical consideration as to the design of the horn to perform this function in the best way, are given. In the body of the paper, a proof is given to show that the exponential increase in section gives the best results. For this reason, a large part of the appendix is given to the analytical solution of sound propagation in exponential horns. From the results of this work and the well known equations relating first order effects in the telephone receiver, formulas are developed which make possible the design of horns.*

CONTENTS

I. Introduction

With the growth of radio broadcasting and the development of public address systems, the electric loud speaker has come to be an apparatus of general interest. A very important part of a loud speaker is its horn, which when properly designed can greatly increase the acoustic power delivered, and go a long way towards minimizing the distortions arising from the inherent inertia and stiffness of the moving parts of the receiver. The purpose of this paper is to discuss the function of the horn, and how it should be proportioned to best perform this function.

For perfect reproduction from a loud speaker two requirements must be met: (1) a current of a given frequency must produce sound of that frequency and that frequency only; and (2) the power radiated in the form of sound must be proportional to the square of the current independent of frequency. This assumes that the available current is an exact reproduction of the pressure of sound applied to the microphone.

Several things are responsible for the introduction of extraneous frequencies. (1) If the diaphragm is insufficiently damped, the transient in the building up of vibrations will always contain the frequencies corresponding to the free vibrations of the system. This is readily eliminated by damping. (2) Double frequency may be introduced if the alternating variations of flux are not small compared to the permanent flux. This is made negligible by employing strong permanent fields. Modern balanced magnetic constructions of receiver elements were devised principally to provide for a large permanent flux without increasing the mass of the vibrating system excessively. (3) If the diaphragm deflections are large, extraneous frequencies will be introduced because of the non-linear characteristics as to restoring force and magnetic pull. Proper design of the horn will allow ample sound radiation with small diaphragm displacement and thus overcome this difficulty. So the first requirement for perfect reproduction may be closely approached.

The second requirement for perfect reproduction, that of a uniform response characteristic over the acoustic frequency range, can not be met with present day receiver elements. Variations in acoustic power of the order of ten to one between 200 and 4000 cycles are not noticed by the ear, however, and the departure from a uniform response can be kept within this range by proper design of the horn.

Contrary to the prevalent conception, the horn does not merely gather up the sound energy from the receiver and concentrate it in certain directions. Its relation to the diaphragm is much more intimate. It causes an actual increase in the load on the diaphragm, making it advance against a greater air pressure, and withdraw from a greater opposing rarefaction. Anyone can assure himself that the average sound energy in a room is greatly reduced on removing the horn from a good loud speaker. And frequently when the horn is removed the amplitude of vibration of the diaphragm becomes so great that it strikes against the pole pieces. A receiver element without a horn is analogous to a motor without a connected load; or better yet, a receiver element without a horn is like a closed oscillation circuit from which little radiation takes place (radiation resistance zero); while with a horn it is like an open oscillation circuit with an antenna (radiation resistance considerable). The horn is the antenna of the loud speaker.

In order to load, a horn need not resonate. Resonances of course increase the loading at certain frequencies, but cannot be made to provide uniform loading over a wide range of frequencies. That a horn can load without resonating is shown by the increase in sound radiation at the high frequencies where air column resonances are of small intensity. Resonances in the horn due both to material vibration and air column vibration being undesirable because of the distortion they produce, and being entirely unnecessary, should be reduced to a minimum.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

Proceeding now to the detailed consideration of the function and design of horns for loud speakers, we may set down three requisites:

1. A given applied force acting on the diaphragm must cause the air at the throat of the horn to have a nearly uniform velocity over the acoustic frequency range. Proper design of the air chamber above the diaphragm and the initial area of the horn will bring this about.

2. The area of the mouth of the horn must be such that only a small part of the sound is reflected back at that point, because reflections cause air column resonances.

3. The law of increase and the rapidity of increase of section must be such that:

(a) The most complete propagation of sound energy shall take place, and

(b) For a given air velocity at the throat, the power shall be nearly constant as the frequency is varied.

II. Throat Area and Diaphragm Chamber Volume

As we have indicated above, the horn is a radiator of the power which it causes the diaphragm to deliver. We may, therefore, well begin by considering the simplest sound radiating system, namely, a uniform straight tube infinitely long in one direction. Any air disturbance at the end of such a tube is propagated unchanged along the tube. In Fig. 1 suppose the piston is moved to the right. The air immediately ahead is given a velocity. Because of the elasticity of the air it is also compressed in the region next to the piston. Suppose the piston is stopped. There is now a region of pressure and in that region the air has a velocity to the right. The volumetric rate of flow of air into the region just ahead is greater than the volumetric rate of flow out, and so the pressure there rises. As long as the pressure behind is greater than the pressure ahead, the air ahead will be accelerated. When the two pressures become equal there will be no further acceleration. As the air ahead reaches a higher pressure the air behind will be retarded, and will finally reach a condition of zero velocity and normal pressure. Pressure and velocity are thus propagated along the tube. That propagation takes place in only one direction is shown by the fact that the velocity of the air is always in one direction, and that pressure can rise only when air is flowing into a region faster than it is flowing out. If the piston is moved to the left the propagation of disturbance will be in the same direction, but the pressure and velocity of the air will be reversed.

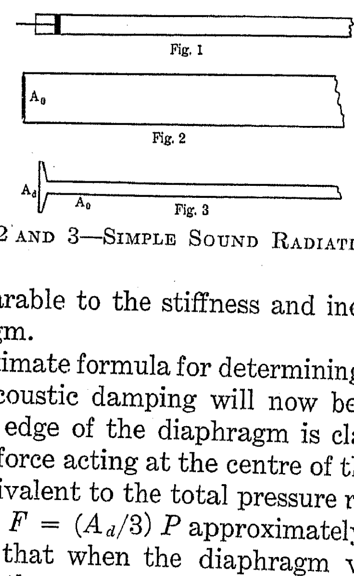
From this discussion it is seen that in a traveling sound wave, pressure and velocity are associated together in time and space phase. Neither can exist without the other. If P is the increase in pressure above atmospheric at any point, and the average air velocity at the same point and at the same time is v, $P = (\rho\, a)\, v$, where ρ is the density of air, a is the velocity of sound in air, c. g. s. units being used.

Suppose in place of the piston there is a diaphragm as in Fig. 2, and for simplicity assume that every part of it vibrates through the same distance. If when it vibrates the air is given a velocity v at a certain instant, the total reaction force over the face of the diaphragm will be that due to the developed pressure, so that

$$F = P\, A_0 = (\rho\, a)\, v\, A_0.$$

If the initial construction of the loud speaker were made in this way it would not be possible to make this force comparable with the stiffness and inertia forces of the diaphragm at the low and high frequencies, respectively. Hence the motion of the diaphragm would be determined almost entirely by its own mass and stiffness, and its motion at its resonant frequency would far exceed that at other frequencies. Hence, also, the power radiated at the resonant frequency would greatly exceed that at other frequencies.

Suppose the arrangement is changed to that of Fig. 3. Here a small velocity of the diaphragm gives the air in the tube a large velocity and much higher pressures are therefore developed. Thus this arrangement, by the principle of the hydraulic press, increases the reaction force on the diaghragm so that it may be made comparable to the stiffness and inertia forces of the diaphragm.



Figs. 1, 2 and 3—Simple Sound Radiating Systems

An approximate formula for determining the quantity known as acoustic damping will now be derived. If in Fig. 3 the edge of the diaphragm is clamped and F is the single force acting at the centre of the diaphragm which is equivalent to the total pressure reacting on it,

$$F = (A_d/3)\, P \text{ approximately.}$$

Assuming that when the diaphragm vibrates with velocity, $\dot{u}$, the volumetric rate of displacement of air is, $A_d/3\, \dot{u}$, the velocity of air in the tube, assuming all the displaced air gets into the tube will be,

$$v = A_d/3 \quad \dot{u}/A_0.$$

From the relation between P and v,

$$F = \frac{\rho\, a}{9}\, (A_d^2/A_0)\, \dot{u}$$

Acoustic damping will be defined as equivalent reaction force per unit diaphragm velocity

$$\alpha_a = F/\dot{u} = \frac{\rho\, a}{9}\, A_d^2/A_0$$

In appendix II α_a is determined more accurately, but for the present this formula will be used to illustrate the point.

That a large value of α_a is desirable may be shown as follows. If β_0 and m are the diaphragm stiffness and mass respectively, and α_0 the internal damping,

$$\dot{u} = \frac{\text{force}}{(\alpha_0 + \alpha_a) + j\,(\omega\, m - \beta_0/\omega)} \quad \text{where,}$$

$\dfrac{\omega}{2\,\pi}$ is the frequency of the sinusoidal force driving the diaphragm. This is analogous to the equation for current in an electrical circuit,

$$\dot{i} = \frac{\text{e. m. f.}}{(R_0 + R_a) + j\left(\omega\, L - \dfrac{1}{\omega\, C}\right)}, \text{ where,}$$

R_0 is the internal resistance of the generator and R_a the load resistance, L and C being series inductance and capacitance. In the electrical case the power dissipated in the load is

$$W = \dot{i}^2\, R_a.$$

Similarly in the acoustic case

$$W = \dot{u}^2\, \alpha_a.$$

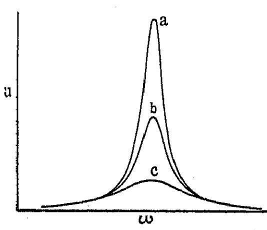


FIG. 4

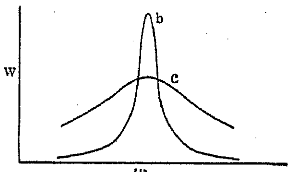


FIG. 5—DIAPHRAGM RESONANCE

If α_a is zero $\dot{u}$ plotted against $\dot{\omega}$ will be as shown in Fig. 4A. The radiated power will be zero. If α_a is increased $\dot{u}$ will be as in Fig. 4B—lower at resonance but about the same at the extreme frequencies. In this case W will be as in curve 5B. Now let α_a be increased still further. The velocity $\dot{u}$ becomes as in 4C, and the radiated power as in 5C. A more uniform radiation of power is secured over the frequency range, and yet the average over the range is about the same.

From the simple formula for acoustic damping it is seen that to increase it we must decrease A_0. Two effects give a lower limit to A_0, however. First, frictional losses in small tubes become great. Second, if the throat is made very small the air instead of being forced out into the tube will be compressed in the chamber above the diaphragm. To prevent this the chamber volume must be decreased, and there is a physical limitation here.

Neglecting frictional losses it is shown in appendix II, that if the radiated power at two extreme frequencies $\dfrac{\omega_1}{2\,\pi}$ and $\dfrac{\omega_2}{2\,\pi}$ is to be $1/n$ times that at resonance $\dfrac{\omega_r}{2\,\pi}$, the area of the throat of the horn and the volume of the chamber above the diaphragm are determined by,

$$A_0 = \frac{\rho\, a}{9}\,\sqrt{n-1}\,\frac{A_d^2\,\omega_1}{\beta_0}$$

$$V_0 = A_0 \frac{\omega_r^4 - \omega_1^2\,\omega_2^2}{\dfrac{\rho\, a}{p_0}\,\omega_1\,\omega_2^2\left(-\dfrac{\omega_r^2}{\sqrt{n-1}} + \sqrt{\dfrac{n}{n-1}\,\omega_r^4 - \omega_1^2\,\omega_2^2}\right)}$$

The following sample calculation will show relative values. Assume

$A_d = 15.5$ cm.²
$\beta_0 = 20 \times 10^6$ dynes/cm.
$\omega_1 = 2\,\pi \times 200$
$\omega_r = 2\,\pi \times 1000$
$\omega_2 = 2\,\pi \times 4000$
$n = 10$

Substituting in the formulas,

$A_0 = 0.2075$ cm.²

If circular in section, the diameter,

$d_0 = 0.514$ cm. $= 0.202$ in.
$V_0 = 0.258$ cm.³

These values of A_0 and V_0 are much smaller than have been used heretofore.

The fact that greater acoustic damping is provided by horns having small throat areas means that the same sound radiation takes place with smaller diaphragm velocities, and hence smaller diaphragm deflections. This will prevent the introduction of extraneous frequencies due to non-linear characteristics of large deflections.

III. FINAL OPENING OR MOUTH OF HORN

The infinite straight tube which we have just been considering loads the diaphragm, and conducts away the resulting acoustic power. It does not, however, communicate this power to surrounding space, and thereby fails to fulfill completely the function of a horn. Evidently the tube must be made to open up into the atmosphere. We are thus led to consider what happens at the open end of a finite tube.

Consider a half wave length of sound, in which there is positive pressure and forward velocity traveling along the tube. While progressing within the tube it is uniformly confined and occupies a constant volume. Hence the pressure and velocity in it remain constant. As it leaves the open end of the tube, however, it expands into an approximately hemispherical shape, Fig. 6A. There is thus an increase in the volume occupied by the wave as it leaves the tube. Evidently a fall in pressure must result. But if the pressure just outside the tube remains lower than that within the tube, the velocity of the air just within the tube will be

increased; this causes the pressure just behind to decrease, and the velocity to increase, and so we have produced a wave traveling back in the tube, which near the open end reduces the pressure and increases the velocity in the oncoming wave. This is the familiar phenomenon of reflection.

The reflected wave not only represents power which does not get out into the air but, depending on the phase in which it reaches the diaphragm, it may produce resonance or dissonance. Obviously the less intense the reflection is, the less marked will be the resonance or dissonance.

It is now easy to see what influence the size of the final opening of the tube has upon the intensity of the reflection. It is evident from Fig. 6A that the larger the section of the tube is, the less will be the relative increase in volume occupied by a half wave length just within and just without the tube, and therefore the less intense will be the reflection. For wave lengths which are less than the diameter of the tube, Fig. 6B, the increase in volume on passing out of the tube is slight, and therefore the reflection is negligible; but for wave lengths which are greater than the tube diameter, Fig. 6A, the increase in volume becomes considerable, and the reflection becomes appreciable.

Thus a horn, to transmit a 500-cycle wave with wave length 68.8 cm. to the atmosphere without reflection would need to have a final opening of diameter about 70 cm.

For many purposes this is an inconveniently large diameter, and it is therefore interesting to determine

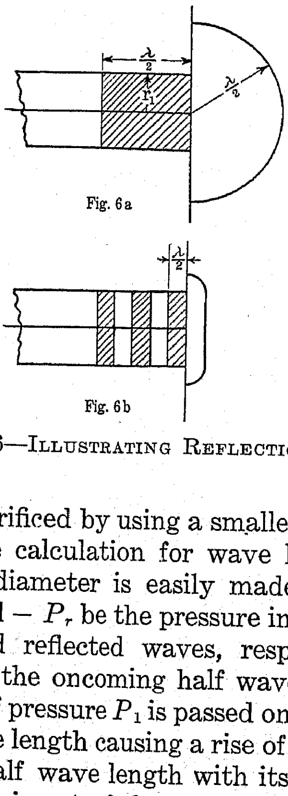


FIG. 6—ILLUSTRATING REFLECTION

how much is sacrificed by using a smaller final opening. An approximate calculation for wave lengths greater than the tube diameter is easily made.

Let P_1, P_2 and $-P_r$ be the pressure in the oncoming, transmitted and reflected waves, respectively. The excess of air in the oncoming half wave length which causes the rise of pressure P_1 is passed on into the transmitted half wave length causing a rise of pressure there. The reflected half wave length with its negative pressure also draws air out of the tube and into the transmitted hemispherical half wave length. This mass of air, which is taken from the half wave length within the tube and transferred to the half wave length outside the tube, causes a reduction in pressure within equal to $P_1 + P_r$, and an increase of pressure outside equal to P_2. Since these two pressure changes are inversely proportional to the volumes involved, we have

$$\frac{P_1 + P_r}{P_2} = \frac{2/3\ \pi\ (\lambda/2)^3}{\pi\ r_1^2\ \lambda/2} = 1/6\ \lambda^2/r_1^2\text{, where}$$

λ is the wave length, and
r_1 is the radius of tube.

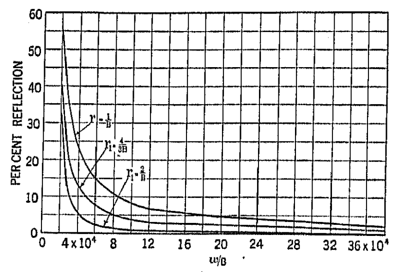


FIG. 6C—VARIATION OF REFLECTION

Furthermore, $P_2 = P_1 - P_r$, so that finally,

$$P_r = \frac{\lambda^2 - 6\,r_1^2}{\lambda^2 + 6\,r_1^2}\,P_1$$

If $\lambda = 68.8$ cm. corresponding to 500 cycles and $r_1 = \lambda/4 = 17.2$ cm.

$$P_r = P_1 \left(\frac{1 - 6/16}{1 + 6/16} \right) = 0.45\,P_1$$

In Appendix IV an approximate determination of reflections is given for the exponential horn in which the area at any distance x from the small end is given by

$$A = A_o\,e^{Bx}$$

where B is a constant which determines the rate of increase. The curves of (Fig. 6C) show how the reflection varies for different values of ω, B, and final radius r_1 of the horn. These curves also indicate that the larger the final radius and the higher the frequency, the less the reflection.

IV. SHAPE OF HORN

We have now shown that to load the diaphragm effectively the initial opening or throat of the horn must be small, and to communicate the resulting acoustic power effectively to the atmosphere, the final opening or mouth of the horn must be large. We must now consider how these two extremities are to be joined; what the length of the horn should be, and according to what law the section should increase.

Consider a uniform straight tube which at a certain point opens up abruptly into a second uniform straight tube of larger section. A sound wave in the first tube propagates in the simple manner described under section II. At the point of sudden change of section more complicated conditions appear, but beyond this point, in the second tube, simple propagation again

takes place. As is well known, the ultimate result of the discontinuity is a reflection, which reduces the energy which passes on to the second tube. Furthermore, the magnitude of this reflection is proportional to the *relative* increase in section at the point of discontinuity, that is to $\frac{A_2 - A_1}{A_1}$, A_1 and A_2 being the sectional areas of the tubes respectively. We may then say that the departure from simple propagation is proportional to the relative increase in section.

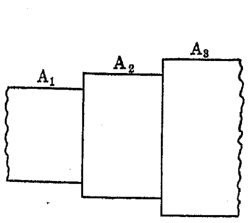


FIG. 7—DISCONTINUING IN STRAIGHT TUBE

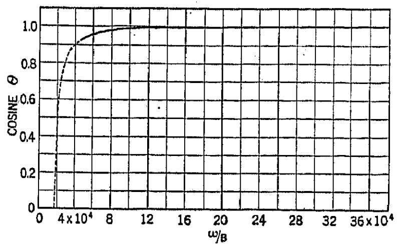


FIG. 8—VARIATION OF LOADING IN EXPONENTIAL HORNS

Returning now to the horn, with continuously increasing section, it seems fairly reasonable to suppose that at each point, the departure from simple propagation will depend on the *relative* increase in section per unit length, that is to $1/A \, \frac{d\,A}{d\,x}$. The initial and final section areas having been determined, the total relative increase in area is prescribed. All this being granted, it seems entirely reasonable to suppose that the best horn will be one in which this relative increase in area is uniformly distributed over its whole length, rather than one in which the relative increase in area is large at some points and small at others. Thus we conclude that for the best horn, $1/A \, \frac{d\,A}{d\,x}$ is constant along its length. To have this property, the sectional area must increase exponentially with the length, that is

$$A = A_0 \, e^{Bx}$$

We may give a more quantitative turn to this argument as follows. Consider two discontinuities in a straight tube as shown in Fig. 7, the areas of the three sections being A_1, A_2 and A_3. A wave having air velocity v_1 in passing from A_1 into A_2 is partially reflected. Let v_{1r} be the velocity in the reflected wave and v_2 be the velocity in the transmitted wave. Similarly in passing from A_2 into A_3 let v_{2r} be the velocity in the reflected wave and v_3 the velocity in the transmitted wave. Let P with the corresponding subscripts represent the pressures associated with each of these velocities.

$$A_2 \, v_2 = A_1 \, v_1 - A_1 \, v_{1r}$$
$$A_3 \, v_3 = A_2 \, v_2 - A_2 \, v_{2r}$$
$$P_2 = P_1 + P_{1r}$$
$$P_3 = P_2 + P_{2r}$$

Because of the constant ratio of pressure to velocity in straight tubes, the last two equations become

$$v_2 = v_1 + v_{1r}$$
$$v_3 = v_2 + v_{2r}$$

From these four equations we may eliminate v_{1r}, v_2 and v_{2r} and obtain

$$v_3 = \frac{4 \, A_1 \, A_2}{(A_1 + A_2)\,(A_2 + A_3)} \, v_1$$

If v_1, A_1 and A_3 are fixed, v_3 will be maximum, if

$$A_2 = \sqrt{A_1 \times A_3}$$

This again leads to the exponential horn.

In Appendix III, with certain approximations, the characteristics of the exponential horn are determined analytically. Pressure and velocity have the same absolute ratio as in the straight tube; but are out of phase by an angle whose cosine is a function of ω/B. Curve Fig. 8 shows the plot of this cosine. It approaches unity (or the straight tube radiation characteristic) if ω is large enough or B small enough. In section I it was shown that a sufficiently uniform power radiation characteristic could be obtained with the straight tube if its section were made small compared to the area of the diaphragm. With the exponential horn having the same throat area, neglecting reflections at the open end, the power will differ from that of the straight tube by the cosine of the angle between pressure and velocity. Since this cosine begins to fall off rapidly below

$$\omega/B = 2.5 \times 10^4,$$

if the radiation is to be uniform down to 200 cycles, corresponding to $\omega = 1257$, B must be less than $\frac{1257}{2.5 \times 10^4}$ or approximately 0.05. This value of B gives a horn which opens up very slowly. Since the initial area must be small and the final area reasonably large, this small value of B will result in a long horn.

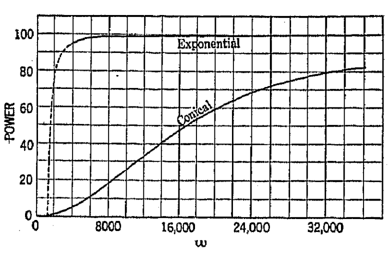


FIG. 9—COMPARISON OF LOADING OF EXPONENTIAL AND CONICAL HORNS

The superior radiating power of the exponential horn is well brought out by comparing it with a conical horn of the same initial and final areas and the same length. Assuming circular sections, let the

initial section radius, $y_0 = 0.254$ cm.
final " " $y_1 = 14.3$ cm.
length $l = 115$ cm.

For the exponential horn, this gives $B = 0.07$. The conical horn will have an angle whose sine is 0.122.

From the equations given in Appendix III, the power propagated at various frequencies for a given velocity in the throat may be calculated. The results are shown in Fig. 9, expressed in per cent of the power for high frequencies which is the same for the two horns. This is a positive indication of the advantage of the exponential horn over the conical.

V. Experimental Work

In the experiments to be described, the several horns shown in the photograph (Fig. 10) were arranged so that a receiver element could be quickly transferred from one to another for comparison. Horn No. 1 is a pyramid 4½ feet long, with initial and final areas 0.0315 and 324 square inches respectively. Horn No. 2 is an exponential horn of the same terminal areas and about the same length. The percentage increase in area per inch is about 20, corresponding to $B = 0.07$ for centimeter units of length. The lower part of this horn may be removed or replaced by the straight extension of diameter 1¼ in. and the same length shown to the right. Horn No. 3 is an exponential horn having the same initial and final areas as horn

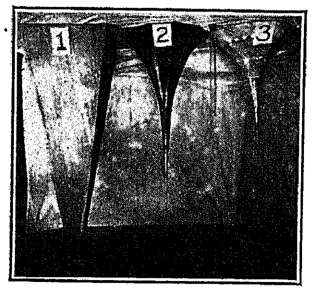


Fig. 10—Horns for Experimental Comparison

No. 2, but the percentage increase per inch is about 35, corresponding to $B = 0.12$.

Experiment I. To compare the loading of conical and exponential horns. Horns 1 and 2 as shown were compared. A receiver having a resonance at 2200 cycles was connected to an audio frequency oscillator. At frequencies below 1500 cycles or above 3000 cycles, it is assumed that the addition of a horn will not materially alter the velocity of the diaphragm. Under such circumstances, the horn having the greatest value of acoustic damping will radiate the most sound.

At each frequency the receiver was placed first on one horn and then on the other. From 250 to 500 cycles the response of the conical horn was weak, while the exponential was very strong. As the frequency was raised the response of the conical increased. Above 3000 cycles the difference was much less marked, being only slightly in favor of the exponential. This test substantiates the curves of Fig. 9.

Experiment II. To compare exponential horns having different rates of increase, horns 2 and 3 as shown were compared. Using the same receiver, and gradually lowering the frequency from 1000 cycles, the two horns radiated nearly equally until about 350 cycles frequency was reached. At this point the response from horn No. 3 fell off abruptly and remained low down to 250 cycles, the lowest frequency of the oscillator used. The quantity ω/B for the small horn at 350 cycles is 18,300. This value of ω/B corresponds closely to the point on curve of Fig. 8 where the loading falls off.

Experiment III. To determine the effect of changing the initial opening, keeping the rate of increase the same, horn No. 2 was used with and without the lower part. A receiver having its diaphragm resonance at 500 cycles was employed. Below and above resonance the response was much greater using the extension. At resonance the rattle and blasting produced without the extension was entirely eliminated with the extension. These results indicate greater loading with the extension.

Experiment IV. To show that increase in loading in Experiment III was due to reduction of area and not to increased length, the upper part of horn No. 2 was used with and without the straight section shown to the right; and then first with the straight extension and then with the exponential extension. Very little difference was noticed with and without the straight section. About the same response was observed below and above resonance and the same blasting at resonance.

Comparing the horn first with the straight extension and then with the exponential extension gave results similar to those of Experiment III, indicating that the addition of the straight part did not increase the loading.

VI. Conclusions

The following are major points in the function and design of horns for loud speakers.

1. The purpose of the horn is to load the diaphragm, and to radiate the power which it causes the diaphragm to deliver. Great loading is required in order to smooth out diaphragm resonances; and to allow ample sound radiation with small diaphragm velocities, thereby reducing extraneous frequencies introduced when diaphragm displacements are beyond the range of linear characteristics.

2. The exponential horn in all probability provides greater loading than any other horn having the same length and terminal dimensions. Experiment I confirms this in the case of the conical horn.

3. The maximum loading of an exponential horn on a given receiver is determined entirely by its initial area, the loading increasing with decrease in area until friction begins to play an important part. This assumes that the chamber volume above the diaphragm is small so that most of the air is forced out into the horn instead of being compressed in the chamber. Experiments III and IV show that decreasing the throat area increases the loading. Experiment II shows that

above a certain frequency, two horns having different rates of increase but the same initial area will provide the same loading.

4. The uniformity of loading of an exponential horn is determined by its rapidity of increase in section—the less the rate of increase, the more uniform the loading. Uniform loading may be expected down to the frequency where $\omega/B = 2.5 \times 10^4$ approximately. Experiment II showed that when ω/B was reduced to 1.8×10^4 the loading fell off considerably.

5. The larger the final opening, the less pronounced will be the horn resonances. The intensity of resonances decreases with increase of frequency. It has been found that, in exponential horns, the resonances are hardly noticeable above the frequency when the horn begins to load if the final section is over 14 in. in diameter.

In loud speakers with horns designed according to the data contained in this paper, the reproduction of music is not just pleasing but close to the original, and the reproduction of voice is not only intelligible but natural.

Appendix I

Mechanical Damping and Motional Impedance

The simple equations for the telephone receiver are:

$$L\frac{d\,i}{d\,t} + R\,i + 10^{-7}\,M\frac{d\,u}{d\,t} = e = E\,e^{j\omega t} \qquad (1)$$

$$m\frac{d^2\,u}{d\,t^2} + \alpha\frac{d\,u}{d\,t} + \beta\,u = M\,i, \qquad (2)$$

when

e = impressed e. m. f., volts
i = current through windings, amperes
L = damped inductance, henries
R = damped resistance, ohms
M = equivalent force per unit current, dynes/amperes
$10^{-7}\,M$ = generated back e. m. f./unit velocity of diaphragm, volts/(cm./sec.)
u = deflection at centre of diaphragm, cm.
m = equivalent mass of vibrating system
α = total damping factor, dynes/(cm./sec.)
β = total stiffness factor, dynes/cm.

M may or may not be complex. Eddy currents and hysteresis which produce a phase difference between flux and current cause M to have an imaginary part.

The steady state solutions of the above equations for an alternating impressed voltage of frequency $\dfrac{\omega}{2\,\pi}$ are,

$$Z = e/i = R + j\,\omega\,L + \frac{10^{-7}\,M^2}{\alpha + j\,(m\,\omega - \beta/\omega)}\text{ ohms.} \qquad (3)$$

It is seen that the impedance is made up of the damped impedance $(R + j\,\omega\,L)$ plus another term which is known as the motional impedance:

$$Z_m = \frac{10^{-7}\,M^2}{\alpha + j\,(m\,\omega - \beta/\omega)} = \frac{10^{-7}\,M^2\,\alpha}{\alpha^2 + (m\,\omega - \beta/\omega)^2} - j\frac{10^{-7}\,M^2\,(m\,\omega - \beta/\omega)}{\alpha^2 + (m\,\omega - \beta/\omega)^2} \qquad (4)$$

This is the equation for the well-known motional impedance circle having diameter

$$\frac{M^2 \times 10^{-7}}{\alpha}$$

and depressed from the horizontal by an angle $-\,2\,\gamma$ where γ is the angle of M, if M is complex. (See Fig. 11.) The above separation of real and imaginary parts is with respect to an axis of slope $-\,2\,\gamma$.

The solution for the deflection is

$$u = \frac{M\,i}{\beta - m\,\omega^2 + j\,\omega\,\alpha} \qquad (5)$$

We shall be more concerned with the velocity of the diaphragm, however, the value of which is given by

$$\dot{u} = j\,\omega\,u = \frac{M\,i}{\alpha + j\,(m\,\omega - \beta/\omega)} = \frac{M\,i}{\sqrt{\alpha^2 + (m\,\omega - \beta/\omega)^2}} \qquad (6)$$

The power put into the diaphragm is given by the average value of $\dot{u}^2\alpha$.

$$W = I^2/2\,\frac{M^2\,\alpha}{\alpha^2 + (m\,\omega - \beta/\omega)^2}\text{ ergs/sec.} \qquad (7)$$

where I is the maximum value of the sine wave of current i. Expressing W in watts or joules per sec.,

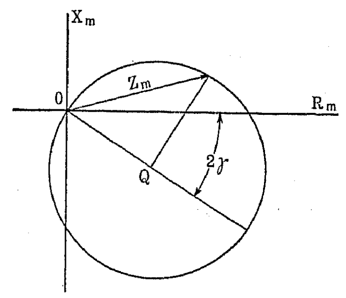


Fig. 11—Motional Impedance Diagram

$$W = I^2/2\,\frac{10^{-7}\,M^2\,\alpha}{\alpha^2 + (m\,\omega - \beta/\omega)^2}. \qquad (8)$$

It is seen from equations (8) and (4) that the factor by which the average square of current must be multiplied to obtain the diaphragm power at a given frequency is the projection $O\,Q$ of the corresponding motional impedance vector on the principal diameter of the impedance circle.

The several constants of the receiver may be determined as described in papers by A. E. Kennelly, R. L. Wegel,[1] and others. Suppose impedance circles are taken on a given receiver element with and without a horn. The damping in the two cases may be represented by $(\alpha_0 + \alpha_a)$ and α_0, respectively. After the power input to the diaphragm with the horn attached has been determined at different frequencies as stated above, the input to the horn is approximately

$$\frac{\alpha_a}{\alpha_0 + \alpha_a} \text{ times the values determined.}$$

The acoustic damping α_a, is large as compared with α_0 in the best loud speakers, and so this ratio approximates unity. For this reason in what follows the acoustic damping will be considered as the total damping.

Appendix II

THROAT AREA AND DIAPHRAGM CHAMBER VOLUME

In the text of this paper the propagation of waves along an infinite straight tube is considered. In Appendix III it is shown that the characteristics of the exponential horn approach those of the straight tube as a radiator of sound over most of the acoustic range of frequency. Therefore, in considering what takes place at the beginning of the horn, the simpler relations between air pressure and velocity which correspond to a straight tube will be assumed.

The following is an approximate determination of the correct chamber volume and initial opening for given loud speaker, when power variation of n:1 between resonance and the upper and lower limits of frequency is allowable. In Fig. 12 let

A_0 = area of throat, cm.^{2}
V_0 = chamber volume, cm.^{3}
p_0 = atmospheric pressure, dynes/cm^2 [$=1.01\times10^6$].
p = pressure developed, dynes/cm.^{2} in chamber and throat
$P_0 = p - p_0$ = change of pressure, dynes/cm.^{2}
$= (\rho\, a)\, v = 41.2\, v$
v = velocity of air in throat
u = deflection of diaphragm
$\dot{u}$ = velocity of diaphragm, cm./sec.
A_d = area of diaphragm.

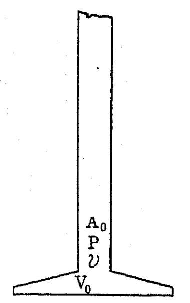


FIG. 12—THROAT AREA AND DIAPHRAGM CHAMBER VOLUME

It will be assumed that when the diaphragm is displaced the change in volume is given by $A_d\, u/3$.

$$p/p_0 = \frac{V_0}{V_0 - A_d\, u/3 + A_0 \int v\, d\, t} \qquad (9)$$

1. The Electromagnetic Theory of the Telephone Receiver, A. E. Kennelly and H. Nukiyama, PROCEEDINGS A. I. E. E., April 1919.

Theory of Magneto-Mechanical Systems, R. L. Wegel, JOURNAL A. I. E. E., October 1921.

$$\frac{p - p_0}{p_0} = \frac{P_0}{p_0} = \frac{A_d\, u/3 - A_0 \int v\, d\, t}{V_0} \qquad (10)$$

$$\frac{d\, P_0}{d\, t} = p_0/V_0 \times A_d/3 \frac{d\, u}{d\, t} - \frac{A_0\, p_0}{V_0}\, v \qquad (11)$$

$$41.2\, V_0 \frac{d\, v}{d\, t} + A_0\, p_0\, v = \frac{p_0\, A_d}{3} \frac{d\, u}{d\, t}$$

$$= \frac{p_0\, A_d}{3}\, \dot{u} = \frac{p_0\, A_d}{3}\, \dot{U}\, e^{j\omega t} \qquad (12)$$

The steady state solution of this equation is:

$$v = \frac{p_0\, A_d/3\, \dot{u}}{A_0\, p_0 + j\, \omega\, .\, 41.2\, V_0} \qquad (13)$$

$$v = \frac{1/3\, A_d\, \dot{u}}{A_0 + j\, \omega \times 4.08 \times 10^{-5}\, V_0}$$

$$= \frac{1/3\, A_d\, .\, j\, \omega\, u}{A_0 + j\, \omega\, .\, 4.08 \times 10^{-5}\, V_0} \qquad (14)$$

Rationalizing denominator,

$$v = \frac{1/3\, A_d\, A_0}{A_0^2 + (4.08 \times 10^{-5}\, \omega\, V_0)^2}\, \dot{u}$$

$$+ \frac{1/3\, A_d\, \omega^2 \times 4.08 \times 10^{-5}\, V_0}{A_0^2 + (4.08 \times 10^{-5}\, \omega\, V_0)^2}\, u \qquad (15)$$

It will be assumed that,

$$F = \frac{P_0\, A_d}{3} \left(= \frac{41.2\, v\, A_d}{3} \right) \qquad (16)$$

$$F = 41.2/9 \frac{A_d^2\, A_0 \times \dot{u}}{A_0^2 + (4.08 \times 10^{-5}\, \omega\, V_0)^2} + 41.2/9$$

$$\times \frac{A_d^2 \times 4.08 \times 10^{-5}\, \omega^2\, V_0}{A_0^2 + (4.08 \times 10^{-5}\, \omega\, V_0)^2}\, u \qquad (17)$$

It is seen that the force acting at the center of the diaphragm must overcome a force in phase with the diaphragm velocity, and one in phase with the displacement. The unit forces are: (1) the acoustic damping,

$$\alpha_a = 41.2/9 \frac{A_d^2\, A_0}{A_0^2 + (4.08 \times 10^{-5}\, \omega\, V_0)^2} \qquad (18)$$

and (2) the increase in stiffness due to the compression of part of the air in the chamber.

$$\beta_a = 41.2/9 \frac{A_d^2 \times 4.08 \times 10^{-5}\, \omega^2\, V_0}{A_0^2 + (4.08 \times 10^{-5} \times \omega\, V_0)^2} \qquad (19)$$

Let α_m = damping when ω is small,

$$\alpha_m = 41.2/9\, A_d^2/A_0 \qquad (20)$$

$$\alpha_a = \alpha_m \frac{A_0^2}{A_0^2 + (4.08 \times 10^{-5}\, \omega\, V_0)^2} \qquad (21)$$

$$\beta_a = \alpha_a \frac{4.08 \times 10^{-5}\, \omega^2\, V_0}{A_0} \tag{22}$$

The general equation for power radiated is:

$$W = \frac{M^2\, I^2}{2} \quad \frac{\omega^2\, \alpha_a \times 10^{-7}}{(\beta - m\, \omega^2)^2 + \alpha_a^2\, \omega^2} \tag{23}$$

β is made up of β_0 the stiffness due to the diaphragm and β_a the increase in stiffness due to the compression of air in the chamber.

Therefore
$$\beta = \beta_0 + \beta_a \tag{24}$$

$$W = \frac{10^{-7}\, M^2\, I^2}{2} \frac{\omega^2\, \alpha_a}{(\beta_0 + \beta_a - m\, \omega^2)^2 + \alpha_a^2\, \omega^2} \tag{25}$$

$$= \frac{10^{-7}\, M^2\, I^2}{2} \frac{\omega^2\, \alpha_a}{(\beta_0 - m\, \omega^2)^2 + 2\, \beta_a\, (\beta_0 - m\, \omega^2) + \beta_a^2 + \alpha_a^2\, \omega^2} \tag{26}$$

$$= \frac{10^{-7}\, M^2\, I^2}{2} \times \frac{\omega^2\, \alpha_a}{\left[(\beta_0 - m\, \omega^2)^2 + 2\, \alpha_a \dfrac{4.08 \times 10^{-5}\, \omega^2\, V_0}{A_0} (\beta_0 - m\, \omega^2) + \alpha_a^2 \left\{ \left(\dfrac{4.08 \times 10^{-5}\, \omega^2\, V_0}{A_0} \right)^2 + \omega^2 \right\} \right]} \tag{27}$$

$$= \frac{10^{-7}\, M^2\, I^2}{2} \times \frac{1}{\left[\dfrac{(\beta_0 - m\, \omega^2)^2}{\omega^2\, \alpha_a} + \dfrac{8.16 \times 10^{-5}\, V_0}{A_0} (\beta_0 - m\, \omega^2) + \alpha_a \dfrac{(4.08 \times 10^{-5}\, \omega\, V_0)^2 + A_0^2}{A_0^2} \right]} \tag{28}$$

$$= \frac{10^{-7}\, M^2\, I^2}{2} \times \frac{1}{\left[\dfrac{(\beta_0 - m\, \omega^2)^2\, [A_0^2 + (4.08 \times 10^{-5}\, \omega\, V_0)^2]}{\omega^2\, \alpha_m\, A_0^2} + \dfrac{8.16 \times 10^{-5}\, V_0}{A_0} (\beta_0 - m\, \omega^2) + \alpha_m \right]} \tag{29}$$

At very low frequencies, the middle term in the denominator is negligible as compared with the first term, and we may write for ω_1 the lower limit.

$$W_1 = \frac{10^{-7}\, M^2\, I^2}{2} \quad \frac{1}{\dfrac{\beta_0^2}{\omega_1^2\, \alpha_m} + \alpha_m} \tag{30}$$

At resonance,

$$W_r = \frac{10^{-7}\, M^2\, I^2}{2} \times 1/\alpha_m \tag{31}$$

If the power at resonance is n times that at ω_1, we have

$$\frac{\beta_0^2}{\omega_1^2\, \alpha_m} = (n - 1)\, \alpha_m \tag{32}$$

$$\alpha_m = \frac{1}{\sqrt{n - 1}} \times \beta_0 / \omega_1 \tag{33}$$

This determines the proper amount of damping at low frequencies, and since we have α_m expressed as a function of the initial throat area we may determine A_0:

$$\alpha_m = 41.2/9\, A_d^2 / A_0 = \frac{1}{\sqrt{n - 1}} \times \beta_0 / \omega_1 \tag{34}$$

$$A_0 = 41.2/9\, \sqrt{n - 1}\, \frac{A_d^2\, \omega_1}{\beta_0} \tag{35}$$

At ω_2 the upper limit of frequency we may write for the power radiated,

$$W_2 = \frac{10^{-7}\, M^2\, I^2}{2} \times \frac{1}{\left[\dfrac{m^2\, \omega_2^2\, [A_0^2 + (4.08 \times 10^{-5}\, \omega_2\, V_0)^2]}{\alpha_m\, A_0^2} - \dfrac{8.16 \times 10^{-5}\, V_0 \times m\, \omega_2^2}{A_0} + \alpha_m \right]} \tag{36}$$

To fulfill the condition that the power at resonance shall be n times that at ω_2:

$$\frac{m^2\, \omega_2^2\, [A_0^2 + (4.08 \times 10^{-5}\, \omega_2\, V_0)^2]}{\alpha_m\, A_0^2} - \frac{8.16\, V_0\, m\, \omega_2^2 \times 10^{-5}}{A_0} = (n - 1)\, \alpha_m\, . = \frac{\beta_0^2}{\omega_1^2\, \alpha_m} \tag{37}$$

$$m^2\, \omega_2^2\, \omega_1^2\, [A_0^2 + (4.08 \times 10^{-5}\, \omega_2\, V_0)^2] - 8.16 \times 10^{-5}\, V_0\, m\, \omega_2^2\, \omega_1^2\, \alpha_m\, A_0 = \beta_0^2\, A_0^2 \tag{38}$$

$$(\beta_0 / m^2 - \omega_1^2\, \omega_2^2)\, A_0^2 + \frac{8.16 \times 10^{-5}\, V_0\, \omega_2^2\, \omega_1^2 \dfrac{1}{\sqrt{n - 1}} \beta_0 / \omega_1}{m} A_0 - (4.08 \times 10^{-5}\, \omega_2^2\, V_0)^2\, \omega_1^2 = 0 \tag{38}$$

Substituting $\beta_0 / m = \omega_r^2$

$$(\omega_r^4 - \omega_1^2\, \omega_2^2) A_0^2 + \left(\frac{8.16 \times 10^{-5}}{\sqrt{n - 1}} V_0\, \omega_2^2\, \omega_1\, \omega_r^2 \right) A_0 - (4.08 \times 10^{-5}\, \omega_2^2\, V_0)^2\, \omega_1^2 = 0 \tag{39}$$

from which

$$V_0 = A_0 \frac{\omega_r^4 - \omega_1^2\, \omega_2^2}{4.08 \times 10^{-5}\, \omega_1\, \omega_2^2 \left(- \dfrac{\omega_r^2}{\sqrt{n - 1}} + \sqrt{\dfrac{n}{n - 1}\, \omega_r^4 - \omega_1^2\, \omega_2^2} \right)} \tag{40}$$

Repeating the two formulas:

$$A_0 = 41.2/9\,\sqrt{n-1}\,\frac{A_d^2\,\omega_1}{\beta_0} \tag{41}$$

$$V_0 = A_0\,\frac{\omega_r^4 - \omega_1^2\,\omega_2^2}{4.08 \times 10^{-5}\,\omega_1\,\omega_2^2\left(-\dfrac{\omega_r^2}{\sqrt{n-1}} + \sqrt{\dfrac{n}{n-1}\,\omega_r^4 - \omega_1^2\,\omega_2^2}\right)} \tag{42}$$

These completely determine the conditions at the throat of the horn.

Appendix III

Propagation in Exponential and Conical Horns

The theory of propagation of waves along a horn is simplified by considering it one dimensional. At any point whose abscissa is x, Fig. 13, let A be the area of a section where at any instant the velocity and pressure of the air are uniform all over the section.

If at any instant the air in the element shown is being accelerated, and ρ is the density,

$$\rho\,A\,(\Delta\,x)\,\frac{\partial\,v}{\partial\,t} = -\,A\,(\Delta\,P) \tag{43}$$

If at a given instant the volumetric rate of flow of air at $x + \Delta\,x$ is greater by $\Delta\,(A\,v)$, the volumetric rate of flow at x, the change of pressure in a time $\Delta\,t$ is given by,

$$\Delta\,P = -\,k\,\frac{\Delta\,(A\,v)\,\Delta\,t}{A\,(\Delta\,x)} \tag{44}$$

Taking limits, the following two differential equations of propagation are obtained,

$$\rho\,\frac{\partial\,v}{\partial\,t} = -\,\frac{\partial\,P}{\partial\,x} \tag{45}$$

$$\frac{\partial\,P}{\partial\,t} = -\,k/A\,\frac{\partial}{\partial\,x}\,(A\,v) \tag{46}$$

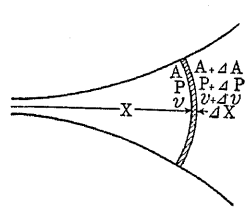


Fig. 13

Let φ be a function of x and t such that

$$v = -\,\frac{\partial\,\varphi}{\partial\,x} \tag{47}$$

$$P = \rho\,\frac{\partial\,\varphi}{\partial\,t} \tag{48}$$

These satisfy equation (45).

Substituting in (46),

$$\rho\,\frac{\partial^2\,\varphi}{\partial\,t^2} = k/A\,\frac{\partial}{\partial\,x}\left(A\,\frac{\partial\,\varphi}{\partial\,x}\right) \tag{49}$$

$$\rho\,\frac{\partial^2\,\varphi}{\partial\,t^2} = k\,\frac{\partial^2\,\varphi}{\partial\,x^2} + k/A\,\frac{d\,A}{d\,x}\,\frac{\partial\,\varphi}{\partial\,x} \tag{50}$$

$$\rho\,\frac{\partial^2\,\varphi}{\partial\,t^2} = k\,\frac{\partial^2\,\varphi}{\partial\,x^2} + k\,\frac{d\log A}{d\,x}\,\frac{\partial\,\varphi}{\partial\,x} \tag{51}$$

In the case of an exponential horn, Fig. 14,

$$A = A_0\,e^{Bx} \tag{52}$$

$$\frac{d\log A}{d\,x} = B \tag{53}$$

$$\rho\,\frac{\partial^2\,\varphi}{\partial\,t^2} = k\left(\frac{\partial^2\,\varphi}{\partial\,x^2} + B\,\frac{\partial\,\varphi}{\partial\,x}\right) \tag{54}$$

$$\text{Let } \varphi = z\,(x)\,w\,(t) \tag{55}$$

$$\rho\,z\,w'' = w\,k\,(z'' + B\,z') \tag{56}$$

$$\frac{\rho\,w''}{w} = \frac{k(z''+Bz')}{z} = \text{constant} = C \tag{57}$$

This is because w contains t alone and z contains x alone.

$$\text{Therefore} \quad \rho\,w'' - C\,w = O \tag{58}$$

The solution of this is:

$$w\,(t) = c_1\,e^{\sqrt{C/\rho}\,t} + c_2\,e^{-\sqrt{C/\rho}\,t} \tag{59}$$

$$k\,z'' + k\,B\,z' - z\,C = O \tag{60}$$

The solution is,

$$z\,(x) = c_3 e^{-B/2\,x + \sqrt{B^2/4 + C/k}\,x} + c_4\,e^{-B/2\,x - \sqrt{B^2/4 + C/k}\,x} \tag{61}$$

Using only the first solution of (59) and for forwardly propagated waves the second solution of (61).

$$\varphi = z\,(x)\,w\,(t) = H\,e^{-B/2\,x} e^{\sqrt{C/\rho}\,t - \sqrt{B^2/4 + C/k}\,x} \tag{62}$$

where H is a constant.

$$P = \rho\,\frac{\partial\,\varphi}{\partial\,t} = H\,\sqrt{C\,\rho}\,e^{-B/2\,x} e^{\sqrt{C/\rho}\,t - \sqrt{B^2/4 + C/k}\,x} \tag{63}$$

$$v = -\,\frac{\partial\,\varphi}{\partial\,x} = H\,(B/2 + \sqrt{B^2/4 + C/k})\,e^{-B/2\,x}\;e^{\sqrt{C/\rho}\,t - \sqrt{B^2/4 \times C/k}\,x} \tag{64}$$

When $x = O$, suppose,

$$P = P_0\,e^{j\omega t} \tag{65}$$

$$\sqrt{C/\rho} = j\,\omega \qquad C = -\,\rho\,\omega^2 \tag{66}$$

$$H\,\sqrt{C\,\rho} = P_0 \qquad H = \frac{P_0}{\sqrt{-\,\rho^2\,\omega^2}} = -\,j\,\frac{P_0}{\rho\,\omega} \tag{67}$$

Substituting,

$$P = P_0\,e^{-B/2\,x}\,e^{j\omega\left(t - \sqrt{\rho/k - \frac{B}{4\omega}}\,x\right)} \tag{68}$$

$$v = P_0\left(\sqrt{\frac{1}{k\,\rho} - \frac{B^2}{4\,\omega^2\,\rho^2}} - j\,\frac{B}{2\,\omega\,\rho}\right)\;e^{-B/2\,x}\,e^{-j\omega\left(t - \sqrt{\rho/k - \frac{B^2}{4\omega^2}}\,x\right)} \tag{69}$$

The value of k will now be found. From equation (44),

$$\frac{\partial\,P}{\partial\,t} = k\,\frac{-\,\Delta\,(A\,v\,\rho)}{A\,\rho\,(\Delta\,x)}$$

$$= k\,\frac{\text{time rate of change of mass}}{\text{mass}} \tag{70}$$

$$= k\,\frac{\partial\log\rho}{\partial\,t}$$

$$d\,P = k\,d\log\rho \tag{71}$$

For adiabatic changes

$$\rho = Q\,(P + p_0)^{1/1.4} \tag{72}$$

$$\log \rho = \log Q + 1/1.4 \log (P + p_0) \tag{73}$$

$$d \log \rho = 1/1.4 \, \frac{d\,P}{P + p_0} = 1/1.4 \, \frac{d\,P}{p_0} \tag{74}$$

$$k = 1.4\, p_0 \tag{75}$$

If a is the normal velocity of sound for plane and spherical waves

$$\rho/k = \frac{\rho}{1.4\, p_0} = 1/a^2 \tag{76}$$

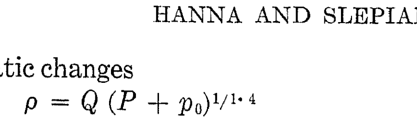


FIG. 14

and

$$\frac{1}{k\,\rho} = \frac{1}{1.4\,p_0\,\rho} = \frac{1}{a^2\,\rho^2} \tag{77}$$

The equations for P and v become

$$P = P_0\, e^{-B/2\ x}\, e^{j\omega\,(t - x/a')} \tag{78}$$

$$v = P_0 \left(\sqrt{\frac{1}{a^2\,\rho^2} - \frac{B^2}{4\,\omega^2\,\rho^2}} - j\,\frac{B}{2\,\omega\,\rho} \right) e^{-B/2\ x}\, e^{j\omega(t - x/a')} \tag{79}$$

where the velocity of sound in the exponential horn is

$$a' = \frac{1}{\sqrt{1/a^2 - \dfrac{B^2}{4\,\omega^2}}} \tag{80}$$

From equations (**78**) and (**79**) it is seen that the absolute relation between pressure and velocity in the exponential horn is the same as in a straight tube.

$$P = (\rho\,a)\,v, \tag{81}$$

but the velocity lags behind the pressure by an angle

$$\theta = \cos^{-1} \frac{\sqrt{\dfrac{1}{a^2\,\rho^2} - \dfrac{B^2}{4\,\omega^2\,\rho^2}}}{\sqrt{\left(\dfrac{1}{a^2\,\rho^2} - \dfrac{B^2}{4\,\omega^2\,\rho^2}\right) + \dfrac{B^2}{4\,\omega^2\,\rho^2}}}$$

$$= \cos^{-1} \sqrt{1 - \frac{a^2\,B^2}{4\,\omega^2}} \tag{82}$$

The value of $\cos\theta$ is therefore a measure of the effectiveness of an exponential horn as compared with a straight tube, as far as radiation along it is concerned. It is a function of ω/B as shown in Fig. 8.

Equation (**80**) indicates that the velocity of propagation of waves in an exponential horn is greater than the velocity of sound in free space, and becomes infinite at a frequency corresponding to $\omega/B = a/2$. This obviously cannot be true on physical grounds, so that we must conclude that the equations derived are only approximations, and very poor approximations for frequencies in the neighborhood of $\omega/B = a/2$. Inasmuch as equation (**80**) follows rigorously from the differential equations (**45**) and (**46**), these must also only approximately represent the actual physical phenomena.

Analysis shows that the considerable departure of equation (**36**) from the physical truth at low frequencies is due to the simplifying assumption that the problem is one dimensional, *i. e.*, that "wave-front" surfaces exist such that on any one of them both the pressure and air velocity are constant and uniform, and that the air velocity is normal to these surfaces. This assumption does not hold strictly for any but conical horns.

We may, however, make use of equation (**80**) which gives, a', the velocity of propagation in the horn calculated on the assumption of "wave-front" flow, as a criterion for determining over what range of frequencies this assumption is approximately valid. For those frequencies for which a' differs only slightly from a, the various formulas derived will be good approximations. In the curves of Figs. 8 and 9 those portions for which a' exceeds a by more than 10 per cent are shown dotted.

For backwardly propagated waves, if the pressure at any point is $P_r\, e^{j\omega t}$ the velocity

$$v_r = P_r \left(- \sqrt{\frac{1}{a^2\,\rho^2} - \frac{B^2}{4\,\omega^2\,\rho^2}} - j\,\frac{B}{2\,\omega\,\rho} \right) e^{j\omega t} \tag{83}$$

This is readily seen if the second part of equation (**61**) is used in the derivation instead of the first.

In the case of the conical horn, Fig. 15, or for spherical waves.

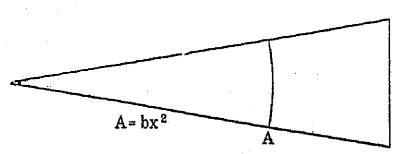


FIG. 15—PROPAGATION IN HORNS.

$$A = b\,r^2 \tag{84}$$

$$\frac{d \log A}{d\,r} = 2/r \tag{85}$$

Equation (**51**) becomes on replacing x by r,

$$\rho\,\frac{\partial^2 \varphi}{\partial t^2} = k\left(\frac{\partial^2 \varphi}{\partial r^2} + 2/r \,.\, \frac{\partial \varphi}{\partial r}\right) \tag{86}$$

$$\text{Let } \varphi = \psi/r \tag{87}$$

$$\frac{\partial \varphi}{\partial r} = -\,\psi/r^2 + 1/r\,\frac{\partial \psi}{\partial r} \tag{88}$$

$$\frac{\partial^2 \varphi}{\partial r^2} = +\,\frac{2\,\psi}{r^3} - 2/r^2\,\frac{\partial \psi}{\partial r} + 1/r\,\frac{\partial^2 \psi}{\partial r^2} \tag{89}$$

$$\rho\,1/r\,\frac{\partial^2 \psi}{\partial t^2} = k\left(1/r\,\frac{\partial^2 \psi}{\partial r^2}\right) \tag{90}$$

$$\frac{\partial^2 \psi}{\partial t^2} = k/\rho \frac{\partial^2 \psi}{\partial r^2} = a^2 \frac{\partial^2 \psi}{\partial r^2} \qquad (91)$$

The general solution is,

$$\psi = f\,(t - r/a) + g\,(t + r/a) \qquad (92)$$

Therefore $\varphi = \dfrac{f\,(t - r/a)}{r} + \dfrac{g\,(t + r/a)}{r}$ (93)

Keeping only $(t - r/a)$ terms which represent forward propagation:

$$\varphi = \frac{f\,(t - r/a)}{r} \qquad (94)$$

$$v = -\frac{\partial \varphi}{\partial r} = 1/r^2 f\,(t - r/a) - 1/r\, f'\,(t - r/a) \qquad (95)$$

$$P = \rho \frac{\partial \varphi}{\partial t} = \rho/r\, f'\,(t - r/a) \qquad (96)$$

Let $f\,(t - r/a) = M\, e^{j\omega(t - r/a)}$ (97)

$$P = M \frac{j\,\omega\,\rho}{r} \cdot e^{j\omega(t - r/a)} \qquad (98)$$

$$v = M\left(1/r^2 + \frac{j\,\omega}{a\,r}\right) e^{j\omega(t - r/a)} \qquad (99)$$

When $r = r_1$, let,

$$P = P_2\, e^{j\omega t} \qquad (100)$$

$$M \frac{j\,\omega\,\rho}{r_1} e^{j\omega(t - r_1/a)} = P_2\, e^{j\omega t} \qquad (101)$$

$$M = -P_2 \frac{j\, r_1}{\omega\,\rho} e^{j\omega r_1/a} \qquad (102)$$

$$P = P_2\, r_1/r\, e^{j\omega\left(t - \frac{r - r_1}{a}\right)} \qquad (103)$$

$$v = P_2^{r_1/r}\left(\frac{1}{a\,\rho} - j \frac{1}{\rho\,\omega\,r}\right) e^{j\omega\left(t - \frac{r - r_1}{a}\right)} \qquad (104)$$

These are the solutions for spherical wave propagation in the forward direction. They may also be used in determining the characteristics of conical horns.

Appendix IV

Calculations of Reflections

The following is proposed as an approximate method of calculating the reflections which take place at the open end of an exponential horn.

At the open end let P_1 = oncoming pressure, P_2 = the transmitted pressure, and P_r the reflected pressure. The corresponding velocities as found in equations (**35**), (**37**) and (**83**) are:

$$v_1 = P_1\left(\sqrt{\frac{1}{a^2\,\rho^2} - \frac{B^2}{4\,\omega^2\,\rho^2}} - j\,\frac{B}{2\,\omega\,\rho}\right) \qquad (105)$$

$$v_2 = P_2\left(\frac{1}{a\,\rho} - j\,\frac{1}{\rho\,\omega\,r_1}\right) \qquad (106)$$

$$v_r = P_r\left(-\sqrt{\frac{1}{a^2\,\rho^2} - \frac{B^2}{4\,\omega^2\,\rho^2}} - j\,\frac{B}{2\,\omega\,\rho}\right) \qquad (107)$$

If r_1 is the radius of the final opening of the horn, it is assumed that the transmitted wave is spherical and of radius r_1.

The following relations hold,

$$P_1 + P_r = P_2 \qquad (108)$$

$$v_1 + v_r = v_2 \qquad (109)$$

$$\frac{v_1 + v_r}{P_1 + P_r} = v_2/P_2 = \frac{1}{a\,\rho} - j\,\frac{1}{\rho\,\omega\,r_1} \qquad (110)$$

$$\frac{\left[\begin{array}{l} P_1\left(\sqrt{\dfrac{1}{a^2\,\rho^2} - \dfrac{B^2}{4\,\omega^2\,\rho^2}} - j\,\dfrac{B}{2\,\omega\,\rho}\right) \\ + P_r\left(-\sqrt{\dfrac{1}{a^2\,\rho^2} - \dfrac{B^2}{4\,\omega^2\,\rho^2}} - j\,\dfrac{B}{2\,\omega\,\rho}\right)\end{array}\right]}{P_1 + P_r} = \frac{1}{a\,\rho} - j\,\frac{1}{\rho\,\omega\,r_1} \qquad (111)$$

$$P_r = P_1 \frac{\left[\begin{array}{l} \sqrt{\dfrac{1}{a^2\,\rho^2} - \dfrac{B^2}{4\,\omega^2\,\rho^2}} - \dfrac{1}{a\,\rho} \\ \quad + j\left(\dfrac{1}{\rho\,\omega\,r_1} - \dfrac{B}{2\,\omega\,\rho}\right)\end{array}\right]}{\left[\begin{array}{l} \sqrt{\dfrac{1}{a^2\,\rho^2} - \dfrac{B^2}{4\,\omega^2\,\rho^2}} + \dfrac{1}{a\,\rho} \\ \quad - j\left(\dfrac{1}{\rho\,\omega\,r_1} - \dfrac{B}{2\,\omega\,\rho}\right)\end{array}\right]} \qquad (112)$$

$$P_r = P_1 \frac{\sqrt{1/a^2 - \dfrac{B^2}{4\,\omega^2}} - 1/a + j\left(\dfrac{1}{\omega\, r_1} - \dfrac{B}{2\,\omega}\right)}{\sqrt{1/a^2 - \dfrac{B^2}{4\,\omega^2}} + 1/a - j\left(\dfrac{1}{\omega\, r_1} - \dfrac{B}{2\,\omega}\right)} \qquad (113)$$

Since we wish to find the values of per cent P_r for different values of r_1 as well as for different values of ω and B, it is convenient to let,

$$r_1 = N/B \qquad (114)$$

$$\frac{P_1}{P_r} = \frac{\sqrt{1/a^2 - \dfrac{B^2}{4\,\omega^2}} - 1/a + j\,B/\omega\left(\dfrac{2 - N}{2N}\right)}{\sqrt{1/a^2 - \dfrac{B^2}{4\,\omega^2}} + 1/a - j\,B/\omega\left(\dfrac{2 - N}{2N}\right)} \qquad (115)$$

Taking a = 34,400 cm./sec. at 20 deg. cent., 760 mm. Hg., P_r/P_1 may be plotted as a function of ω/B for each assigned value of N. Curves of Fig. 6c show this for $N = 1$, $N = 4/3$ and $N = 2$, corresponding to $r_1 = 1/B$, $r_1 = 4/3\,B$ and $r_1 = 2/B$. It is seen that reflections decrease with increase in ω and decrease of B; also that the larger r_1 is, the less the reflection.

Mathematically the above function is minimum if $r_1 = 2/B$, which may be taken as a probable indication that further reduction in reflection will not be great if $r_1 > 2/B$. This value corresponds to a 45 deg. slope between the walls of the horn and the axis.

Discussion

H. Fletcher: The mathematical presentation of the design of horns is one of the best that has appeared in recent years. The equations evolved point out not only some of the criteria for good design, but show the serious limitation of the horn for producing natural reproduction. As pointed out by the authors, the applications of the equations are limited because of the assumptions involved in deducing them.

I desire to emphasize some of the results predicted by these equations and also some of the limitations in the application of them. First, I should like to discuss briefly the criteria the authors set up for good quality transmission. As they mentioned they are essentially the same as given in the paper by Mr. Martin and myself. However, I would interpret somewhat differently the things put down as responsible for the introduction of extraneous frequencies.

The so-called transients set up in the diaphragm are only present when corresponding transients are in the driving force. In the steady state they are absent; so it seems to me better to consider that the diaphragm produces a frequency distorting effect which is the same for all kinds of driving forces. This is particularly true when dealing with speech where there is an almost continuous variation in the form of the waves coming from the diaphragm.

One of the requirements given in the paper for an ideal loud speaker is that the energy radiated at each frequency be proportional to the square of the current. In practically every loud speaker the electrical impedance at the terminals varies over a wide range as the frequency varies, and consequently, maintaining a constant current does not maintain a constant energy input. So it seems to me that it would be better if this requirement were modified to read that the relation between the electrical energy absorbed to the acoustic energy radiated shall be constant for all frequencies.

Also, one of the principal points brought out in the paper is that by loading the diaphragm by making the opening into the horn small, the frequencies introduced by the non-linear characteristics of the diaphragm will be greatly reduced due to the reduced amplitude at which the diaphragm is working. I should like to call attention to the fact that this reduction in amplitude is accompanied by a corresponding increase in the pressure variation in the air chamber which may in some cases produce just as serious overloading as that due to the large diaphragm amplitude. Very frequently in ordinary conversation, the pressure amplitude is greater than 50 dynes. To reach large gatherings this amplitude must be increased at least 100 times, so that the pressure amplitude at the large end of the horn may frequently be greater than 5000 dynes. For the horns discussed in this paper then, the pressure amplitude in the small air chamber would frequently exceed 500,000 dynes or one-half atmospheric pressures. For such large amplitudes, harmonics which are from 50 to 100 per cent of the fundamental, will be created. This difficulty is overcome in the type of loud speaker used in our demonstration by using a large diaphragm and doing away with the enclosed air chamber. The magnetic system is loaded by the large mass of air in front of the diaphragm and consequently no excessively large amplitudes of either pressure or diaphragm motion exist anywhere in the system.

Equations **41** and **42** of the Appendix are important from a design standpoint. It is interesting to note that these equations predict that you cannot build a loud speaker having a horn connected to a diaphragm in the way indicated which will have a uniform response.

For the numerical case discussed on page 3 of the paper, if I interpret the equations correctly, the power radiated at 40 cycles would be only 1/250th of that at resonance. Even for this case, the volume of the air chamber is so small that air friction is beginning to play an important part, and as mentioned, the non-linear distortion of the air chamber is beginning to be very appreciable, due to the excessive values of the air pressure. For the dimensions of the diaphragm and the air chamber given, the distance between the diaphragm and the stationary plate is less than one-fifth of a millimeter. You see, that is a very small gap in order to reach the condition that I just mentioned, that the power radiated be only 1/250th of that radiated at resonance.

This limitation imposed upon the loud speaker, as indicated by these equations, is due largely to the assumption that the diaphragm and the moving magnetic system can be represented by a single degree of freedom and it is coupled to the horn by a single air chamber. If the moving magnet and the air chambers coupling the diaphragm to the horn are composed of several members, this limitation is largely removed. It is always possible to design the driving mechanism and its coupling to any horn of known physical characteristics so that as nearly a uniform radiation at all frequencies as is desired will result. Also, having given a driving mechanism of known characteristics, it is possible theoretically to design the horn and connecting chambers so that approximately uniform radiation will result. However, when one tries to meet these requirements in a practical design, it is found that no materials available have the proper physical characteristics and it is therefore necessary to make compromises.

The discussion regarding exponential horns is very interesting and confirms in a very beautiful way some of the earlier work along this line.

I might call attention to the fact that horns having cross-sections which vary exponentially are not new. The horns used in the public address system and which have been used during this convention are so designed and have been used throughout the country during the past two or three years.

Prof. V. Karapetoff: This problem of horns is a "house-on-fire" problem, in the sense that loud speakers are now being manufactured by the thousand, and while they are being manufactured and sold, we are trying to find out their fundamental theory.

At different times this Institute harbored various orphan branches of natural sciences. While we are harboring acoustics and acoustical papers, I want to be sure that we are carrying on this branch of science in the right way. To me, the right way is that of physics, and it is in this respect that I regret that the authors of this otherwise excellent paper do not even refer to the previous work of physicists on horns.

It was the privilege of this Institute, from the year 1918 to 1922, to have among its fellows one of the greatest physicists of this country—Dr. Arthur Gordon Webster—and one of his great contributions to physics was a theory of horns.

It was my privilege in 1918 to spend a day with Dr. Webster. He showed me two horns, both shaped like the flare of a trombone—one made out of brass; the other made out of plaster of paris. He said: "I'll bet you couldn't tell the sound of one from the other." I was offended, but accepted the challenge—and he won. At that time he said: "I worked out a complete mathematical theory of horns and I checked it with my phone and phonometer"—two wonderful instruments that he had developed—"and it checks very well." Later I saw his papers in the 1919 and 1920 *Proceedings* of the National Academy of Sciences—two classical papers—and I sincerely hope that the authors will at least add a reference to these papers in a footnote, for the benefit of further investigators.

While it is true that a horn has the function of an antenna, are we sure that the horn shape is the best possible shape for an acoustic antenna? Consider an expensive violin or the cello. Either of these is a complicated physical body, with at least four hundred years of development by great masters behind it. The response of these instruments is perfect within a wide range of frequencies. I hope that someone interested in acoustic antennas will try to couple a diaphragm with a violin body and see what results may be obtained.

S. Boyajian: The authors have clearly brought out the

difference in the characteristics of conical and exponential horns, namely that the conical horn intensifies the higher frequencies more than the lower frequencies, while the exponential horn affects them all equally. I would like to ask the opinion of the authors about a possible application of the conical horn to phonographic work. A phonographic record is not true to original; what it lacks is not volume but intelligibility which indicates that the higher harmonics have been recorded much weaker than lower harmonics. Would it not be a good thing, therefore, to use a conical horn so as to intensify the higher harmonics more than the lower harmonics, approaching thereby the original ratio of amplitudes of various harmonics? In the leading types of phonographs with internal horns, the horns consist of a few conical sections. All of the sections do not have the same angle, and the complete horn, therefore, is an intermediate product between the conical and the exponential type, probably nearer the former than the latter. It appears, therefore, that the phonograph manufacturers have (either knowingly or unknowingly) selected a construction which is more desirable for phonograph reproduction than the theoretical exponential horn.

A. Nyman: The paper by Dr. Slepian and Mr. Hanna is a very good and novel treatment of the subject. It gives a mathematical analysis of the subject and proves by experiment that this analysis is correct.

I believe that this same treatment of loud speakers could be extended farther. Two of the essentials of the loud speakers were mentioned: One, that the loudness at different frequencies should be constant; and the other one, I believe, the intelligibility. With regard to intelligibility, there is a quality in loud speakers that has been very little appreciated so far. I call that quality persistence. It consists of the existence of sound emerging from the horn after the applied electrical vibration has ceased in the loud speaker.

Some time ago I was able to conduct some very simple experiments by mechanically listening to a loud speaker after cutting off the applied current, and I found that at certain frequencies, there was an apparent presence of sound.

I believe that a large amount of distortion in the loud speakers is due to this one cause. Quite recently a loud speaker was brought to my attention where this was remedied to a large extent and apparently clarified the sound quite considerably. I believe that similar treatment, as that which has been exercised so far by Dr. Slepian and Mr. Hanna, could be applied to this one quality.

Another quality is directive action of the horn. Everybody is aware that there is such a directive action, and I believe that by proper design of the horn, it could be spread out so that every direction from the horn would receive the same amount of sound.

E. W. Kellogg (by letter): As a means of conversion of electrical into sound energy the loud speaker is extremely inefficient, and one of the chief reasons is the low transfer of energy from the diaphragm to the air. The paper under discussion describes an effort to obtain an approximate impedance fit between the air column and the diaphragm. It would be interesting to know whether it has been possible in practise to obtain very much higher sensitivity in this way than has been obtained with the usual horn proportions.

Another question of practical interest is whether it has been found possible to coil up the horn without impairing its action.

In section II a certain volume is called for in the cavity in front of the diaphragm. The effect of a cavity any larger than necessary at this point would be to reduce the output of some of the highest frequencies. Since it is not evident that such a thing is desirable, this requirement is a little puzzling. Does it not come about from having specified that the output at both ω_2 and ω_1 shall be $\frac{1}{n}$ times that at resonance The resonant frequency 1000 is slightly above the geometrical mean of the upper and lower limits 4000 and 200. Therefore without the cavity, the output at 4000 cycles comes out a little greater than at 200 cycles, and to equalize the two an air cushion is introduced. Would it not be simpler and more satisfactory to impose the condition that at *neither* of the extreme frequencies shall the output be less than $\frac{1}{n}$ times that at resonance? I am asking this question in the interest of simplifying the calculations rather than for its bearing on actual design, for the volume of the cavity as given in the illustrative example is about as small as it would be practical to make it. I would expect that any cavity volume less than that in a quarter wave length of the tube would have very slight effect.

At the end of the paper is a statement that reflections decrease with decrease of B. This is misleading. The curves of Fig. 16 show that for a given value of r_1 and ω, the larger the value of B, the less the reflection. For example, take $r_1 = 20$ centimeters, $B = 0.05$ and $\omega = 5000$, giving $r_1 B = 1$ and $\frac{\omega}{B} = 10 \times 10^4$. The reflection as shown by the top curves is 8 per cent. Now if B is given the value 0.1, $r_1 B$ becomes 2, and $\frac{\omega}{B}$ becomes 5×10^4, for which the reflection as shown on the lowest curve is about 3 per cent.

A word of interpretation of Fig. 16 may be of interest. Each curve represents a certain shape of horn, without saying anything as to its absolute size. For example $r_1 = \frac{2}{B}$, or $r_1 B = 2$, represents a horn ending with a flare angle of 45 deg. to the axis. If we take two such horns, one just twice the size of the other, the larger horn will react to sound waves of a certain length exactly as the smaller one does to sound waves of half the length. The horizontal scale might equally well have been made to read "diameter of bell divided by wave length."

An interesting analogy to an exponential horn is an electrical transmission line in which starting from one end, the inductance decreases exponentially with distance, and the shunt capacity increases so that their product remains constant. The propagation equations for such a line take the identical form shown in Appendix III of the paper. Above a certain frequency waves are propagated along such a line very much as they are over a uniform line, except that the velocity is higher than for a uniform line with the same value of the product $L\,C$. Such a tapered line provides a possible substitute for a transformer for fitting two unequal impedances together. The telephone companies have employed this principle in the form of tapered loading to prevent excessive reflection losses at junctions of loaded and unloaded lines. A horn performs the same function of providing a better impedance fit between the relatively stiff, heavy diaphragm and the very light, or low-impedance air in free space. The equations for the electrical line show an infinite velocity at a certain critical frequency and at all frequencies below this.

The writers express some doubt as to the validity of the equations in Appendix III for frequencies for which the velocity of propagation becomes infinite. I think the difficulty lies, not in the approximations mentioned, but in our understanding of the term "infinite velocity" and in the assumption of zero reflection. In transmission calculations we say that the velocity of propagation is equal to $2\pi f$ divided by the quantity that expresses the change of phase per unit length of line. Now it is quite possible to have the current in phase at all points along a line, for example in a low-loss line less than a quarter-wave-length long, and open-circuited at the far end. A quarter-wave-

length organ pipe also has zero phase difference from point to point. This is true only of steady-state conditions and does not mean that energy is transmitted at infinite velocity. You notice that in the examples I have mentioned, no power is being transmitted to the far end. As soon as power is taken from the end of the line a phase shift appears. This is true also of the horn. The equations given are for a zero-loss line infinitely long, and they show that such a line has a zero-power-factor impedance, or transfers no power, for frequencies below the critical value. Abover the critical frequency, absorption of power at the bell end of the horn is what is required to prevent reflections, and the equations for simple outward propagation represent a possible condition. Below the critical frequency, an infinite line can be simulated only by a zero-power-factor terminating impedance, which is not the condition under which a horn works. Therefore the complete solution, using both values of propagation constant, must be retained for frequencies below the critical value. It will then appear that when power is radiated from the bell, there is a progressive shift of phase along the horn.

L. P. Rundle (by letter): In nearly all of the better grades of radio loud-speaker horns there is at least one right-angle bend; in others there are two and in still others a circle and a half, *i. e.*, six right-angle bends. In this paper no mention is made of bends, what effect they have on the acoustics of the horn or how they should be designed. I should like to know the mathematical treatment and design for bends and curves in the horn. Again no mention is made of the effect of different materials for horns, or the proper thickness in relation to the other distribution of the material, and this is an important item.

No mathematical and physical treatment is given which has to do with the tone color and quality and what, for instance, will make some sound-radiating devices give a sound that is full, rich, and mellow. This is very noticeable in phonographs, as well as loud speakers for radio work, For instance, it is easily noticeable in comparing a Victor and a Brunswick phonograph, in which both have five right angle bends from diaphragm to the end of the horn but which play the same record with a considerable difference in tone quality. I have noticed the same difference in different makes of loud speakers, and to a less extent in the loud speakers of the same make connected to the same receiver set. Now what makes this difference in tone quality?

Another thing that is quite noticeable in some of the loud speakers is that the sound appears to come from the far end of a long hall, which may be due to a sound-image effect and which is a great detriment to any sound-radiating device. How can this be taken care of in the design of horns?

The papers says nothing about the principle of reflected tone or sound, which is one of the principal points in the advertisements of one maker of a very pleasing loud talker. I would like to know if the authors have any data on the design of this type of loud speaker and how it compares with the exponential design of horns.

The equation giving V_o (the volume of the air behind the diaphram) does not tell anything in regard to the shape of that air volume. It may be very thin at the outer edge of the diaphragm. This brings up another point that was not treated in the paper, namely, the limiting size of the throat of the horn and the closeness of the diaphragm backing when friction would cause severe losses. I should like to know if the authors have any data on these points, and if they have made any tests to determine the coefficient of air friction for various materials such as mica, iron, aluminum, fibre, wood, etc. If so, what are these coefficients? It would be interesting to know if the authors have made any experiments on a horn such as coating the inside with various materials such as oils, waxes, graphite, etc., to see what effect it had on the sound-radiating qualities of the horn.

In regard to the shape of the air volume behind the diaphragm of a phonograph, I have found that the shape of that volume is quite important in that it has considerable to do with the sound intensity and somewhat with the quality, this latter evidently being due to the reflections of sound from diaphragm to backing and vice versa, as well as the echoes from rim to rim of the sound box. It seems quite possible that in a sound box of poor design these things will cause stray noises and blasts that blur the quality of tone at certain frequencies.

In the example worked out, N is taken as 10. I should like to know what determines the best value of N to use in the design of a given horn.

I should like to know how the power radiated at resonance, $W\,r = 2\pi \times 1000$, is determined in designing a new horn.

The paper makes no mention of the shape of the diaphragm or the material. Some loud speakers have a corrugated diaphragm, some a concave or arched construction with corresponding curves at the back of the diaphragm and others a mica diaphragm. I should like to know from the authors if they have made any experiments on diaphragms and what effect the size, thickness, shape and material has on the size and shape of V_o and the force required to set the diaphragm in vibration. A mathematical treatment of the physics of vibrating materials of which diaphragms are made would be a proper subject to go with this horn, since the design of the horn is intimately linked with the characteristics of the diaphragm which it loads.

John Minton (by letter): The work which Dr. Slepian and Mr. Hanna have presented is a valuable contribution to existing literature on the subject of horns. Their results apply almost entirely to horns of infinite length, in which case we do not need to consider reflection from the end of the horn. In this case the radiated sound energy can be calculated readily, provided certain simplifying assumptions are made.

Independently of the work of the authors Dr. Goldsmith and I have been engaged in a rather extensive experimental and theoretical study of horns of finite lengths and in a forthcoming paper we shall present a review of certain phases of our work.

In connection with the paper which has just been given, I desire to discuss certain phases of it, particularly those treated in Appendix III. Inasmuch as the publications on this subject are rather meagre, it is desirable, it seems to me, to call attention to certain of these which are of fundamental importance to this subject. This is particularly true because the authors apparently have not made use of such publications and they have, therefore, duplicated much that has been published for some time.

Lord Rayleigh, "Theory of Sound", Volume II, page 158, gave the equations for the pressure, velocity and energy radiation in a straight tube of unlimited length. His three equations for these quantities are respectively:

$$\rho \frac{d\,\phi}{d\,t} = - \frac{\partial a A}{2\,\sigma} \cos k\,(a\,t - x) \qquad (1)$$

$$\sigma \frac{d\,\phi}{d\,x} = \frac{1}{2}\,A \cos k\,(a\,t - x) \qquad (2)$$

$$P = \frac{\rho\,a\,A^2}{2\,\sigma} \qquad (3)$$

where ρ = air density, $k = 2\,\pi$ divided by the wave length (λ), a = velocity of sound, σ = cross sectional area of tube, ϕ = velocity potential and A determines the strength of the source of sound. The source of sound is considered placed at one end of the tube which is closed at the end adjacent to the source. The first two equations show that the pressure and velocity are in phase. The third equation shows that the sound radiation along the tube is inversely proportional to the area of the tube and constant with, or independent of the frequency.

Lord Rayleigh's equations, "Theory of Sound", Volume II, page 113, for the straight conical horns in which the source of sound is located at the vertex are as follows:

$$\rho \quad \frac{d\,\phi}{d\,t} = -\frac{\rho\,k\,a\,A}{\omega\,r}\sin k\,(a\,t - r) \tag{4}$$

$$\omega\,r^2\,\frac{d\,\phi}{d\,r} = A\,[\cos k\,(a\,t - r) - k\,r\sin k\,(a\,t - r)] \tag{5}$$

$$P = \frac{\rho\,k^2\,a\,A^2}{2\,\omega} \tag{6}$$

The additional factors involved are defined as follows: r = the radial distance and ω = the solid angle of the cone. The first two equations show that the velocity leads the pressure by an angle whose cosine is $\frac{k\,r}{\sqrt{1 + k^2\,r^2}}$. That is:

$$\cos\,\theta = \frac{k\,r}{\sqrt{1 + k^2\,r^2}} \tag{7}$$

When the frequency is very high or when r becomes large the expression for $\cos\,\theta$ becomes essentially unity and the propagation is the same as in the straight tube where the current and pressure are in phase. This does not mean that the radiation in the two cases is equal, but merely that the wave propagation is the same as that in plane waves. We may observe, also, that at low frequencies, where $k^2\,r^2$ can be neglected compared with unity, the pressure and current become more separated in phase which reaches quadrature at zero frequency. Later we shall see that the exponential horn does not perform at these low frequencies as well as the straight conical horn.

The radiation along an infinitely long conical horn is given by equation (6). It is proportional to the square of the frequency and inversely proportional to the solid angle, ω. On comparing equations (3) and (6), the ratio of radiation along a tube to that along a straight cone for equal sound sources is thus equal to $\frac{\omega}{k^2\,\sigma}$. That is:

$$\frac{\text{Radiation along infinite tube}}{\text{Radiation along infinite cone}} = \frac{\omega}{k^2\,\sigma} \tag{8}$$

(See Rayleigh, "Theory of Sound," page 159, eq. 7.)

This ratio is equal to unity for a given tube and cone at some particular frequency, when the radiation in the two cases will be equal. For frequencies below this "critical" frequency the radiation along the tube will be the greater; while for frequencies above it, the radiation along the cone will be greater. To illustrate this, suppose $\omega = 0.01$ radian and $\sigma = 1$ sq. cm. Then when $k^2 = 0.01$, or the frequency is about 550 cycles, the straight pipe and cone give equal radiation. The cone is superior above 550 cycles and inferior below this frequency.

We have developed equations for the performance of exponential horns independently of Dr. Slepian and Mr. Hanna, using as the basis for this work the general equations given by Professor A. G. Webster in the *Proceedings* of the National Academy of Sciences, pages 275-282, 1919. Webster derived the same equation for pressure as given by Messrs. Hanna and Slepian in equation (51). Professor Webster also derived a similar equation for the current or velocity. Inasmuch as the authors have made no reference to Webster's publications, I presume they made no use of them. I think that Professor Karapetoff's remarks are appropriate and I am in accord with them.

Webster's equations apply to horns of any profile. He applied them to the following special cases:

(1) Straight pipe (σ = constant)
(2) Straight cone ($\sigma = \sigma_0\,X^2$)
(3) Exponential horn ($\sigma = \sigma_0\,\epsilon^{mx}$)
(4) For the case where $\sigma = \sigma_0\,X^n$ (hyperbolic when $n = -2$)
(5) For the case where $\sigma = \sigma_0\,\epsilon^{mx^2}$

Cases (1) and (2) were treated by him in some detail. The general equations for the pressure and velocity for the exponential horn were given. Case (4) was dealt with and the general equations given. In case (5), however, the method of analysis only was indicated.

If we separate Webster's exponential equations into the direct and reflected waves and put in the time factor, we obtain for the direct wave in the case of a horn of infinite length

$$\rho \quad \frac{d\,\phi}{d\,t} = -\frac{e^{-\frac{mx}{2}}\,\rho\,A\,g\,a}{2\,\sigma_0\,k}\left[\cos\,(k\,a\,t - g\,x) - \frac{m}{2\,g}\sin\,(k\,a\,t - g\,x)\right] \tag{9}$$

$$2\,\sigma_0\,\epsilon^{\frac{mx}{2}}\,\frac{d\,\phi}{d\,x} = A\cos\,(k\,a\,t - g\,x) \tag{10}$$

$$P = \frac{\rho\,A^2\,e^{\frac{-mx}{2}}\,a\sqrt{k^2 - \frac{m^2}{4}}}{2\,\sigma_0\,k} \tag{11}$$

where $g = \frac{1}{2}\sqrt{4\,k^2 - m^2}$ and $\sigma = \sigma_0\,e^{mx}$.

The first two equations for the pressure and current correspond to equations (78) and (79) of Slepian and Hanna. The current equation (10) was written in the form given so that a comparison could readily be made between the cone and exponential horn for the same current or air velocity. The exponential horn is considered closed at the small end.

A comparison of equations (9) and (10) shows a leading phase angle between the pressure and current such that:

$$\cos\,\theta = \sqrt{1 - \frac{m^2}{4\,k^2}} = \sqrt{1 - \frac{m^2\,a^2}{4\,n^2}} \tag{12}$$

This result is identical to that given by Dr. Slepian and Mr. Hanna in their equation (82) n being equivalent to their ω. Our theoretical deductions, therefore, are in agreement for horns of infinite length.

I desire, however, to emphasize some of the deductions that may be drawn from these results. When $\sqrt{1 - \frac{m^2}{4\,k^2}}$ is zero the pressure and current are 90 deg. out of phase and no radiation results. This occurs when $m^2 = 4\,k^2$, or at a frequency, whose wave length is, $\lambda = \frac{4\,\pi}{m}$. If we take $m = 0.07$, then $\lambda = 180$ cm., which corresponds to a frequency of about 185 cycles. m can be made smaller than 0.07 and in this case the frequency at which this occurs would be correspondingly lower. However, too small an m necessitates either too long a horn or too small a final opening in the case of horns of finite lengths.

In the case of the conical horn, however, we saw that it had a finite phase angle all the way down in frequency. At these lower frequencies, then, the conical horn should be superior on the basis of this simple theory.

If we refer to equation (11), the sound radiation at the initial end of the exponential horn is

$$P_{x=0} = \frac{\rho\,A^2\,a\sqrt{k^2 - \frac{m^2}{4}}}{2\,\sigma_0\,k} \tag{13}$$

At frequencies sufficiently high to make $m^2/4$ negligible compared with k^2, equation (13) becomes

$$\underset{\text{(High Freq.)}}{P_{x=0}} = \frac{\rho\,A^2\,a}{2\,\sigma_0} \tag{14}$$

This result is identical with the straight pipe as shown by equation (3), provided the initial opening, σ_0, is the same as the

cross sectional area of the pipe. That is, if $\sigma_0 = \sigma$ (of equation 3) the pipe and the exponential horn are identical in phase relations of current and pressure, as well as in the magnitude of the radiation from the source of sound into them.

To make a comparison under these conditions between the exponential and conical horns, suppose $\sigma_0 = 1$ sq. cm. and $\omega = 0.01$ radian. Under these conditions the exponential horn is inferior to the conical horn at all frequencies above 550 cycles as can be calculated from the relation:

$$\frac{P_c}{P_E} = \frac{k^2 \sigma_0}{\omega} \tag{15}$$

If, therefore, m is 0.07, $\sigma_0 = 1$ and $\omega = 0.01$ and the source of sound used gives the same velocity when used on both horns, the conical would be superior below 185 cycles and above 550 cycles, while the exponential is superior between these two frequencies. Incidently, curves which we have taken of sound pressure and frequency on horns of finite lengths indicate that this result holds approximately even when the source is placed at a finite distance from the vertex of the cone.

It is important, therefore, to observe that in the three cases considered the cross sectional area of the pipe, the solid angle of the cone and the initial area of the exponential horn all enter the power equations in the same manner and have nothing whatever to do with the phase between the pressure and velocity. This is important to observe in any comparative tests that may be made. For this reason it seems to me, if for no other, the demonstration with the various horns made by the authors are not conclusive and are apt to be misleading for by changing slightly the conditions the comparative results are altered greatly.

These results, of course, do not detract from the excellent results which the authors have presented—they merely emphasize certain features and indicate what we owe to Professor Webster for his splendid work in the field of acoustics.

Since these conclusions are of practical value, it is of importance to test them experimentally. Two horns of heavy galvanized iron were made—one being exponential and the other conical. Their lengths were 122 cm., had an initial opening of 1.6 cm. and a final opening of 31 cm. for the conical horn and 38 cm. for the exponential one. As far as radiation is concerned the larger final opening of the exponential horn gives to it a slight advantage. m corresponds to 0.052 and therefore the "cut-off" frequency, so to speak, was about 135 cycles.

Curves of sound pressure at various frequencies taken for these two horns indicate clearly the correctness of our conclusions. There was no question about the result that at those frequencies below 200 cycles the conical horn was much superior in accordance with our theory. These results suggest that the author's conclusions in section IV are probably only partly correct. These experimental results also suggest that if a receiver unit is placed at the small end of a conical horn, the latter perhaps radiates as though the unit were placed at the vertex and not as though it were placed at a finite initial opening. That is, it seems to me, that equations **100** to **104** given by the authors perhaps do not hold experimentally and are perhaps of theoretical value only since the basis of their derivation may be found to be incorrect experimentally.

As stated in the beginning of my discussion, the above results are of theoretical interest only because they deal with horns of infinite length in which no end reflection occurs and hence no resonant phenomena come into play. As already stated, Dr. Goldsmith and I will present in a forthcoming publication[1] experimental and theoretical results on horns of finite lengths, covering certain phases of our work in which the questions of acoustic impedances, resonance, effect of horn on external pressures, energy radiation, end reflection, etc., will be considered in some detail.

1. "Performance and Theory of Loud Speaker Horns," Proceedings Institute Radio Engineers, August 1924.

Merely to illustrate the type of results obtained, I give below two equations. One shows the effect of the conical horn on the external pressure, p_e, and the other shows the effect of the exponential horn. The ear, of course, is a pressure device and for this reason we are more interested in pressures than in other factors. The first equation was derived by Professor G. W. Stewart and published in the *Physical Review*, pages 313-326, 1920. The second equation, derived by us, is similar to Professor Stewart's, except it applies to the exponential horn. p_1 is the pressure at the small end of the horn.

$$p_1 = \frac{p_e}{\dfrac{\sin k r}{k r} + \dfrac{\sigma_2 \sin k (r - \epsilon)}{r \sin k \epsilon} \left(\dfrac{k}{2 \pi} i - \dfrac{1}{c_0} \right)} \tag{15}$$

(Stewart) conical horn.

$$p_1 = \frac{\sqrt{\dfrac{\sigma_2}{\sigma_1}}\, p_8}{\cos g L + \dfrac{m}{2 g} \sin g L + \dfrac{\sigma_2 k Z_0}{g \beta} \sin g L} \tag{16}$$

(Minton-Goldsmith) exponential horn.

Symbols which have not already been defined are as follows: r = radial length of the cone, L = axial length of the exponential horn, c_0 = acoustic conductance of the large end of the horn, $i = \sqrt{-1}$, ϵ is defined by $\tan k t = k r$ and σ_2 is the cross sectional area of the large end of the horn. Z_0 is the acoustic impedance of the large end treated as a fictitious cylinder of definite length and an opening equal to the diameter of the large end. The equation for Z_0 is $p\, a^2 k^2 \left(\dfrac{k i}{2 \pi} - \dfrac{1}{c_0} \right)$ (see our paper referred to, Appendix II, eq. **19**, p. 473) and is similar to the same factor in equation (**15**) above. $\beta = \rho_a k^2$, whose factors have been defined $g = \sqrt{k^2 - \dfrac{m^2}{4}}$.

By means of these two equations we are able to study pressure changes caused by both the finite exponential and conical horns. The effect of resonance, therefore, on external pressures is easily observed. Since experimental data on pressure are available we are able to check the theory. Similar equations have been obtained for the other factors enumerated above, but need not be given here. We believe, therefore, that an extension of Professor Webster's and Professor Stewart's work has led to results which are of prime importance in the study of horns of finite lengths and of various sizes and shapes.

J. Slepian (by letter): The simple experiments which Mr. Hanna has performed are not the only experimental verification of the theoretical results in the paper. Mr. Hanna has measured the increase in loading or acoustic damping produced by a horn, by impedance measurements of the receiver with an a-c. bridge, and has obtained very good checks with the theory. I have also some experimenting curves obtained by Prof. Dayton Miller of Cleveland, which bring out some points very clearly.

Prof. Miller used a specially constructed diaphragm and mirror, which could be attached to any of the various horns which was tested. On sounding a series of organ pipes before the horn, Prof. Miller observed the motion of the diaphragm by means of the mirror, and thus obtained a curve of diaphragm motion against frequency, such as those which I am about to show.

At first sight it would seem that these curves would not give the information desired as to the performance of horns on loud speakers, for what is wanted here is the sound which is produced at various frequencies for a given force on the diaphragm. However, a very interesing and general relation exists which enables us to change over from this data on the properties of a horn as a

pickup of sound, or means for converting acoustic power into diaphragm vibrations, to data on a horn as a radiator, or means for converting diaphragm vibrations into sound.

This being a gathering of electrical men, I shall give the electrical form for this relation; first, this is known as the reciprocal relation for electrical networks. Let A and B, Fig. 1A, be two pairs of terminals joined by a network built up out of simple impedances in any manner. Then if one ampere is made to flow into the terminals A, the same voltage will be produced at B as would be produced at A if one ampere is caused to flow at B. It is very interesting to check this relation for a few simple cases.

The acoustic analog of this relation is then as follows: In a given space containing bounding walls of any description, a source of sound of given intensity at a point A, Fig. 1B, will produce the same pressure on a diaphragm at a second point B, as would be produced on a diaphragm at A if the source of sound were moved to B. A more precise statement of this proposition may be found in Rayleigh's Theory of Sound, Vol. II, p. 131.

It follows then that on sounding organ pipes of different frequencies, but with the same intensitybefore a horn, the curve of pressures developed on an attached diaphragm will be the same as the curve of acoustic loading which the horn would impose on the diaphragm if used as a loud speaker. For an ex-

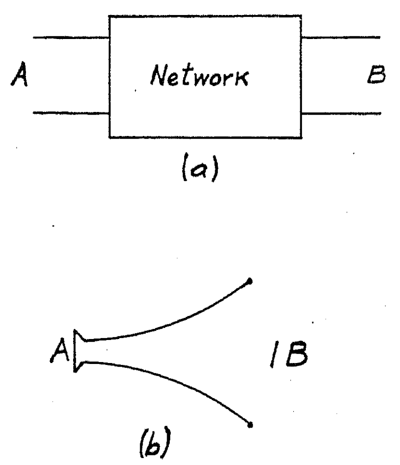


FIG. 1

ponential horn, we should then expect to reproduce the curve, Fig. 8, of Hanna and Slepian's paper.

However, Prof. Miller did not measure the pressure developed at the diaphragm, but the motion resulting from this pressure. We should therefore consider what motion would be produced in the diaphragm by a constant force of varying frequency. The motion of the diaphragm will be determined by its mass, stiffness and damping. The influence of the horn, when it is effective, will be to increase the damping. An important point brought out in Hanna and Slepian's paper is that the loading of a horn for the frequencies for which it is effective, is the same as that which would be produced by an infinite straight tube having the same section as the throat of the horn.

Mr. Hanna has calculated what the motion of the diaphragm would be under a constant force when loaded by an infinite straight tube of the same section as the throat of the horn. His results are shown by the smooth curves in Figs. 2 and 3.

Before discussing these curves, I would like to call attention for comparison to Fig. 10 of Martin and Fletcher's paper. Curve A of that figure is said to be the output curve of one of the best commercial types of loud speakers. The response begins to fall off at about 800 cycles and is one tenth of its maximum at 600 cycles. Now the falling off in response of this type of loud speaker at low frequencies is entirely due to the failure of its horn to sufficiently load the diaphragm, for the electromagnetic force for a given current is just as great or greater at low frequencies as for high. That is, when the acoustic loading of the horn is too small, such large amplitudes of vibration of the diaphragm are necessay to produce appreciable sound that most of the magnetic force is used in overcoming the stiffness of the diaphragm, and little is left for doing work on the air. We should expect then that if a better horn were used, one which would load down to lower frequencies, a much better response curve would be obtained.

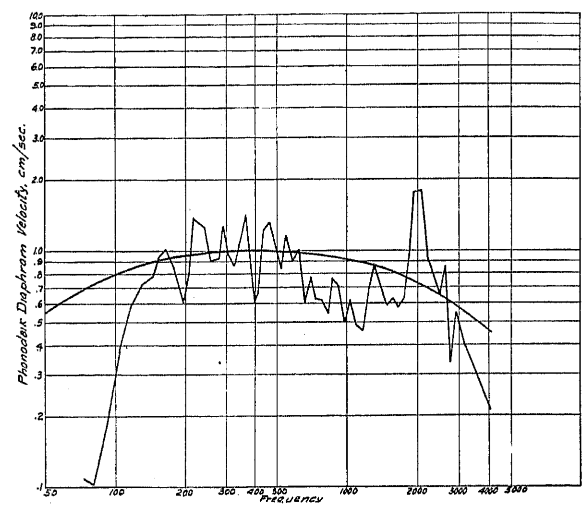


FIG. 2

The curves obtained by Prof. Miller bear this out. Fig. 2 is for a horn whose throat area was 0.81 cm^2. and whose section increased at the rate of 4.5 per cent per cm. of length. We see that it gives the full loading corresponding to an infinite straight tube of the same section down to about 150 cycles, and we must go down to about 100 cycles before its loading effect is reduced to one tenth. This is not only very much better than the curve for

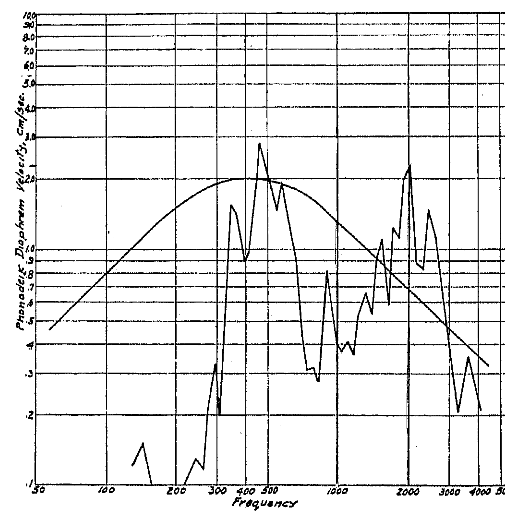


FIG. 3

the commercial type of loud speaker shown by Messrs. Martin and Fletcher, but even compares well with their laboratory models.

Fig. 3 is for an exponential horn having three times the throat area, and two times the rate of increase of section as the horn of Fig. 2. According to the theory, the loading of this horn with larger throat should be only one third that of the other horn, and so the resonances of the diaphragm should be more marked. This is seen to actually be the case. Besides the fundamental

diaphragm resonance, which is the only one which Mr. Hanna considered in calculating his smooth curve, there is also a higher order resonance at about 2000 cycles. Since the rate of increase of section of this horn was about twice that of the preceding horn, the experimental curve should depart from the smooth curve at about twice the frequency for the first horn, and again this is seen to be the case.

The question has been raised as to whether the use of a horn is essential to a loud speaker, or whether sufficiently good results may not be obtained by acting on the air with a sufficiently large diaphragm. The principal objection to a horn is that if it is to be good, it must be quite long. However, the length which a horn must have is determined very largely by the amount of acoustic loading which it must impose upon the diaphragm. A uniform response will be obtained if this acoustic damping force on the diaphragm is about as large or larger than the force required to accelerate the diaphragm because of its mass, or the force required to displace the diaphragm due to its stiffness. Now, with materials capable of being used at present, it is not possible to get acoustic loading large compared to the inertia reaction of the diaphragm if the diaphragm acts directly upon the air.

Thus, suppose a large diaphragm is used, made of material whose density is 2, and whose thickness is 0.01 cm. The mass per cm^2. is then 0.020 grams. Suppose this diaphragm is vibrated at 3000 cycles, generating sound waves. The reaction of the air which gives a damping force on the diaphragm, has for one component the force necessary to accelerate about one quarter wave-length of air, for this amount of air may be said to move with the diaphragm. Now at 3000 cycles, one quarter of a wave-length is only about two and one-half centimeters long, and the mass of a column of this length and one square centimeter in section is only 0.003 grams, which is only 15 per cent of the mass of a square centimeter of the diaphragm. Of the total force on the diaphragm, only 15 per cent is doing useful work on the air, and the rest is consumed in merely moving the mass of the diaphragm to and fro. It is true that this wattless component of force may be counterbalanced by a wattless component of force arising from the stiffness of the diaphragm, but for a finite diaphragm, having no mechanical loss, this compensation can be effective only at certain particular resonant frequencies. Between each successive pair of resonant frequencies, there will be an anti-resonant frequency at which the ratio of force actually acting on the air to total force on the diaphragm will be very much less than the 15 per cent above.

If, for the sake of discussion, we consider an infinite diaphragm, then the resonances disappear, but most of the force on diaphragm is spent in producing waves in the diaphragm itself, which radiate away from the region of application of the force. These waves will be only slightly attenuated by the air. Hence, returning to the finite diaphragm, if there is to be no reflection of these waves at the periphery of the diaphragm and resulting resonances, these waves must be caused to be more strongly attenuated by internal losses in the diaphragm itself, or reflection prevented by losses at the periphery.

In any case, we may conclude, that with a diaphragm of the mass considered, at least seven times as much force must be expended on the diaphragm as actually takes hold of the air, and in practical cases, allowing for losses and anti-resonances, it will be 20 to 100 times as much.

Any electrical engineer will recognize that what is needed is some kind of a coupling or transformer which will compensate for the great disparity in masses of the air and diaphragm. The corresponding electrical problem would be to effectively load a source of alternating voltage having a high internal reactance when only a resistance of relatively small value is available. The obvious solution of this electrical problem is to interpose a transformer which will cause a large current to flow through the resistor with only a small current through the source of voltage. The effective resistance of the load referred to the primary side of the transformer is to the actual resistance in the secondary as the square of the ratio of turns, so that the resistance is thus stepped up to a value where it is comparable with the internal reactance of the source of voltage.

Now the chamber and throat of a loud speaker constitute just the kind of transformer desired. A small velocity of the diaphragm causes the air in the throat to have a large velocity. In the horns described by Messrs. Hanna and Slepian, the ratio of these velocities is 25 or more. The effective mass of the air is thus stepped up several hundred fold to where it is comparable with the mass of even the ordinary iron diaphragm. When loaded by such a horn, 50 per cent or more of the total force on the diaphragm is actually spent on the acoustic load over most of the acoustic range. The mechanical power factor is thus high, and mechanical resonances greatly reduced.

Without a horn, then, for diaphragm materials so far available, the mechanical power factor is very low except at certain resonant frequencies. A very large mechanical input is necessary for a given volume of sound. To reduce the troublesome distorting effect of mechanical resonances, mechanical losses or electrical losses, electrical filter circuits will be necessary. These, however, will not reduce the mechanical input necessary, but rather increase it. A relatively large electrical power plant is then necessary.

Will a large hornless diaphragm save space, then? Possibly, since the large amount of extra electrical apparatus necessary may be made very compact, and may perhaps be put in a somewhat smaller volume than required for a horn. But the saving will be small, and in my opinion, will not be worth while because of the extra complication and expense. Future improvements in the use of light weight diaphragm materials may reduce the amount of electrical apparatus necessary, but a diaphragm of such material would also permit a shorter horn to load it properly. The normal course of development, it seems to me, as we learn how to use lighter diaphragms, would be to use loud speakers with large diaphragms and shorter horns, but the possibility of arriving at a diaphragm so light that it is best used directly on the air without a horn, is quite remote.

Certain Factors Affecting Telegraph Speed

BY H. NYQUIST

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Review of the Subject.—*In considering methods for increasing the speed of telegraph circuits, the engineer is confronted by the problem of transmitting over a circuit the maximum amount of intelligence using a given frequency range, without causing undue interference, either in the circuit being considered or from that circuit to other circuits.*

In this paper the following two fundamentally important factors in connection with this problem are given principal attention:

1. Signal shaping, i. e., giving signals the best wave shape before impressing them on the transmitting medium so as to be able to send signal elements at maximum speed without undue interference into other circuits.

2. Choice of codes so as to transmit the maximum amount of intelligence with a given number of signal elements.

These factors are discussed with the idea of indicating the best methods, as well as to give an idea of what may be expected from these methods.

In connection with the choice of codes, consideration is given to the effect of codes differing in the number of "current values" employed. By this is meant the number of different current values which are employed in a system in forming the different characters. To illustrate, an ordinary land line telegraph circuit makes use of two current values, that is, zero current and the current which flows when the battery is connected to the circuit, or it may use a current value in one direction when battery is connected to the circuit with one polarity and a second current value in the opposite direction of similar magnitude when the battery is connected to the circuit with the opposite polarity. Submarine cables, on the other hand, ordinarily employ three different current values, that is, a definite value in one direction, a similar value in the opposite direction, and zero current. If a quadruplex circuit is analyzed, it will be seen that there are four different current values employed. There is, of course, no theoretical limit to the number of current values which may be thus employed.

The paper also contains a discussion of certain telegraph systems which have been advocated[1], and an endeavor is made to clear up various misconceptions relative to the possibilities of these systems.

Signal Shaping

SEVERAL different wave shapes will be assumed and comparison will be made between them as to:

1. Excellence of signals delivered at the distant end of the circuit, and
2. Interfering properties of the signals.

Consideration will first be given to the case where direct-current impulses are transmitted over a distortionless line, using a limited range of frequencies. Transmission over radio and carrier circuits will next be considered. It will be shown that these cases are closely related to the preceding one because of the fact that the transmitting medium in the case of either radio or carrier circuits closely approximates a distortionless line. Telegraphy over ordinary land lines employing direct currents will next be considered. This will be followed by a consideration of the more complicated case of transmission over long submarine cables.

It will be shown that the waves produced by sending rectangular signal elements through suitable electrical networks which round them off before they are impressed on the transmitting medium are probably best in most cases. Comparison will be made between waves shaped by sending rectangular signal elements through suitable networks and waves made up of half cycles of a sine wave, bringing out the inferiority of the latter.

Direct-Current Telegraph Transmission Over a Distortionless Line

Before proceeding with this discussion, two terms, which will be used in this paper, and which are considered to be of fundamental importance, will be defined—"signal element" and "line speed." It is usually possible, especially when sending is done mechanically, to divide the time into short intervals of approximately *equal* duration, such that each is characterized by a definite, not necessarily constant, voltage impressed at the sending end. The part of the signal which occupies one such unit of time will be called a "signal element." For example, the letter *a* in ordinary land telegraphy will be said to be made up of five signal elements, the first constituting a dot, the second a space and the next three a dash. The "line speed," as used in this paper, equals the number of signal elements per second divided by two. In ordinary land telegraphy the line speed is equal to the dot frequency when a series of dots separated by unit spaces is transmitted.

The discussion will first be limited to the case of direct-current telegraphy over a distortionless line. This case is the simplest and in addition the results will aid in understanding the more complex cases. It may aid in obtaining an understanding of this case to assume that the distortionless line is made up simply of series and shunt resistances.

A distortionless line, such as the one which has been assumed, will transmit all frequencies with equal efficiency from zero upward. In considering applying direct-current telegraph to this line, it will be assumed

1. A. C. Crehore and G. O. Squier. "A Practical Transmitter Using the Sine Wave for Cable Telegraphy; and Measurements with Alternating Currents upon an Atlantic Cable." A. I. E. E. Trans., Vol. XVII, 1900, p. 385.

G. O. Squier. "On An Unbroken Alternating Current for Cable Telegraphy." *Proc. Phys. Soc.*, Vol. XXVII, p. 540.

G. O. Squier. "A Method of Transmitting the Telegraph Alphabet Applicable for Radio, Land Lines, and Submarine Cables." *Franklin Inst., Jl.*, Vol. 195, May 1923, p. 633.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

that the telegraph circuit will have assigned to it only a limited range of frequencies from zero upward, the remaining frequency range being assigned to some other uses, such as ordinary telephone and carrier telephone and telegraph. It will also be assumed that the direct-current telegraph circuit is worked at as high a speed as the frequency range assigned to it will permit.

A number of different wave forms which might be employed to make up the telegraph signal elements will next be examined, consideration being given first to the waves which will be received at the distant end when the different wave forms are impressed at the transmitting end and second to the interference which will be produced in the higher range of frequencies which has been assigned to other uses.

Three forms of voltage waves which will be considered are shown in Fig. 1. A in that figure shows the simplest form of voltage wave, namely, the rectangular form which is produced by applying a battery for a given interval of time and then substituting a short circuit for it. C in the figure is the wave produced by transmitting the rectangular voltage wave A through an electrical network which is the one indicated by the

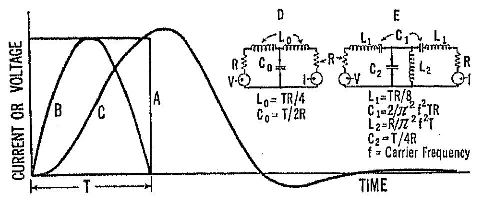

Fig. 1

A - Rectangular Voltage Wave
B - Half Cycle of Sinusoidal Voltage Wave
C - Rectangular Voltage Wave Modified by Being Passed through Network Shown at D or E.

letter D in the figure. (Other forms of networks might also be selected which would produce similar results.) B in the figure is a wave which has the shape of a half cycle of a sine wave. In what follows this wave will be referred to as the "half-cycle sine wave."

In considering the waves which will be received when the above waves are applied at the transmitting end, use will be made of the following general principles, which have been stated by Malcolm[2], for the case of a submarine cable circuit and discussed for the general case in Appendix A.

When a telegraph circuit is worked at a line speed as high as will be permitted by the available frequency range, the shape of the received signal will be practically independent of the shape of the transmitted signal, and further, the magnitude of the received signal will be approximately directly proportional to the area included within the impressed voltage wave.

The area included within the impressed voltage wave being of principal importance so far as the wave received at the distant end is concerned, the areas under the three voltage waves shown in Fig. 1 will next be examined. The areas under waves A and C will be found to be substantially equal while the area under the wave B is only about 0.6 as great. Consequently, it should be expected that waves A and C will be about equally good from the standpoint of the received signals, while wave B will be poorer, producing received signals only about 0.6 as great in magnitude. If the maximum voltage (or power) impressed at the sending end is limited to some given value, the rectangular wave is seen to be the optimum, since this wave has the maximum area. While the area shown under curve C is approximately equal to that under the rectangular wave, the effect produced when a number of signal elements of the same polarity and magnitude are sent in succession is such that the maximum voltage transmitted will exceed slightly the corresponding voltage for the case of the unmodified rectangular wave due to overlapping of adjacent signal elements.

The above comparison of the three waves of Fig. 1 from the standpoint of received signals holds not only for signal elements, but also for complex waves comprising a number of elements. Since for the speeds under consideration the received currents for different shapes of signals applied at the sending end are substantially of the same form, differing, at most, in magnitude, it follows from the principle of superposition that any complex signal, whether built up of elements of one shape or another at the sending end, will produce substantially the same wave form at the receiving end, the differences in the shapes of the elements at the sending end producing differences principally in magnitude of the received waves.

Consideration will next be given to the relative interference which the different wave forms of Fig. 1 will produce in the frequency range assigned to other circuits. Since interference into other circuits results from having the telegraph signal elements contain frequencies which spread into the ranges assigned to other circuits, it is evident that the wave will be the best from the standpoint of interferences which contains the least amount of these outside frequencies. By making use of a method which is discussed in Appendix C, the frequency components of the three waves illustrated in Fig. 1 have been computed and are shown in Fig. 2. The frequency marked $1/2\,T$ in the drawing equals the line speed. T in this connection has the same value as in Fig. 1. The letters in this figure refer to the corresponding waves in Fig. 1, A being the components of an isolated rectangular wave, B the corresponding components for the half-cycle sine wave, and C those for the rectangular wave after it has been transmitted through the network D in Fig. 1. It is seen from Fig. 2 that the rectangular wave form A contains the greatest amount of currents of higher frequencies and is, therefore, the poorest from the standpoint of interference. The half-cycle sine wave contains less of these

2. H. W. Malcolm. "Theory of the Submarine Telegraph and Telephone Cable." The Electrician Printing & Publishing Co., London, March 1917.

higher frequencies although, as will be seen, the high-frequency components are far from negligible. The wave C is the best from the standpoint of interference, since it contains the least amount of these higher frequencies.

From the preceding it is concluded that for the case under consideration, the wave form C in Fig. 1 produced by sending a rectangular shaped signal element through a suitable network is the most suitable. This wave form is almost the optimum from the standpoint of the received signals while from the standpoint of interference into other circuits it leaves little to be desired.

Carrier and Radio

The results for the distortionless line are particularly applicable to the cases of radio and carrier telegraphy because in these cases we have a transmitting medium which is substantially distortionless. We may again make use of Fig. 1 to illustrate three possible voltages, it being understood that these curves represent

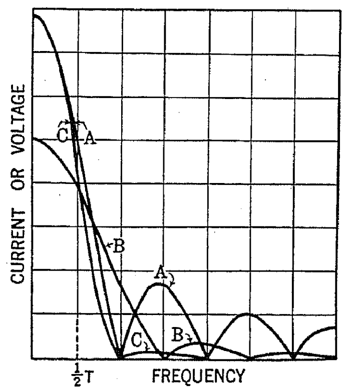

Fig. 2

A - Frequency Components of a Single Dot, Rectangular Wave
B - Frequency Components of a Single Half Cycle of a Sine Wave
C - Frequency Components of a Single Dot, Rectangular Wave Passed through Network Shown in Fig. 1

the envelope or outline of the transmitted currents which are in reality of a frequency considerably higher than the signaling frequency. If now we limit consideration to the case where the carrier frequency is located in the middle of the transmitted frequency band, then this case becomes very similar to the direct-current case and what has been said about the received wave shape being independent of the transmitted one and its magnitude being directly proportional to the area under the transmitted voltage curve still holds. One important difference is that, whereas in the direct-current case the network shown at D, Fig. 1, is used in the alternating-current case having the carrier located in the middle of the free transmitted range, the network shown at E, Fig. 1, is used. A further difference is that in the case of radio where very high frequencies are involved, it may not be practicable to construct the required networks. In that case, however, it is practicable to produce the corresponding direct-current wave and utilize it to modulate the radio wave.

What was said about interference from the circuit in question into other circuits in the direct-current case above also holds for the case of radio and carrier with the difference that, whereas Fig. 2 shows a band of frequencies extending from zero up, the corresponding curve in the case of radio and carrier consists of two such bands. The complete curve for radio and carrier is substantially symmetrical with respect to the ordinate corresponding to the carrier frequency, and the right-hand portion is similar to the curve shown in Fig. 2. It will be obvious that the rectangular wave and the half-cycle sine wave are both objectionable, as voltage waves to be applied to the transmitting medium, because they contain frequency components which may easily extend into the range allotted to neighboring carrier bands. For this reason it is customary in carrier telegraph practise to make use of a transmitting filter to cut off these interfering frequencies. The voltage impressed on this filter is substantially rectangular in outline but after passing the filter it has a shape which is approximately similar to curve C in Fig. 1, and which, therefore, produces less interference than a half-cycle sine wave.

Land Lines

The case of land lines is somewhat different from the case discussed previously because it is not economically desirable to utilize the full frequency range available. In other words, the great expenditure for terminal apparatus that may be proper in the case of submarine cables and long distance radio circuits is not warranted. In land circuits the highest frequencies transmitted are considerably greater than the required line speed. When this is the case, it is usually possible and desirable to make use of the available range to increase the steepness of the received wave. A steep wave front results in prompt operation of the receiving relay and this in turn results in minimum distortion. If a half-cycle sine wave were to be employed instead of the usual rectangular wave or if a network were to be employed which were to round off the wave to the extent indicated in Fig. 1, the received wave would necessarily lose a great part of its steepness and as a consequence the response of the receiving relay would be less positive and the signals would be distorted. It will of course be understood that by means of suitably proportioned networks the wave can be rounded just enough to meet the interference requirement, still retaining sufficient steepness to insure prompt operation of the receiving relay. Therefore, rounding by means of networks is preferable.

If it should be desirable and practicable to utilize the frequency range to its fullest, what has been said above about a distortionless line holds without any substantial modification and it would, in that case also, be more advantageous to use a wave rounded by means of suitable networks than to impress on the line a wave of the half-cycle sine form.

Submarine Cables

In the case of submarine-cable telegraphy, there is a limitation on voltage which has not been emphasized in the simple direct-current case discussed above. The voltage which may be impressed on the cable is limited to a definite value. Moreover, for certain reasons, the cable has an impedance associated with it at the sending end which may make the voltage on the cable differ from the voltage applied to the sending-end apparatus. Inasmuch as the limitation in this case is voltage limitation at the cable, the ideal wave is one which applies a rectangular wave to the cable rather than to the apparatus, because it insures that the area under the curve should be the maximum consistent with the imposed limitations. It would be possible to make the transmitting-end impedance approximately proportional to the cable impedance throughout most of the important range. This would insure that the wave applied to the cable would have approximately the same shape as the wave applied to the apparatus. It would probably be desirable for practical reasons to make this impedance infinite for direct current.

In connection with the submarine cable a special kind of interference is particularly important, namely, that due to imperfect duplex balance. For a given degree of unbalance, the interference due to this source may be reduced by putting networks either in the path of the outgoing current or in the path of the incoming current. These facts, together with the frequency distributions deduced above for each of the several impressed waves as exhibited in Fig. 2, make it apparent that the beneficial reaction on the effect of duplex unbalance, which can be obtained by the use of a half-cycle sine wave instead of a rectangular wave, can be obtained more effectively by the use of a simple network, either in the path of the outgoing or in the path of the incoming currents. Either of these locations is equally effective in reducing interferences from duplex unbalance, but the location of the network in the path of the outgoing current has the advantage that it decreases the interference into other circuits, whereas the location in the path of the incoming current has the effect of reducing the interference from other circuits.

Before leaving the matter of submarine telegraphy, it may be well to point out that it is common in practise to shorten the period during which the battery is applied so as to make it less than the total period allotted to the signal element in question. For instance, if it is desired to transmit an *e* the battery may be applied for, say, 75 per cent of the time allotted to that *e* and during the remaining 25 per cent the circuit is grounded. The resulting voltage is shown in Fig. 3F. From the foregoing, it is concluded that this method is less advantageous than the application of the voltage for the whole period, because while the shape of the received signal is substantially the same in the two cases, the magnitude, being proportional to the area under the voltage curve, will be less. A cursory examination of the literature does not disclose that anything has been published on the experimental side either to confirm or to oppose this result.

Choice of Codes

A formula will first be derived by means of which the speed of transmitting intelligence, using codes employing different numbers of current values, can be compared for a given line speed, *i. e.*, rate of sending of signal elements. Using this formula, it will then be shown that if the line speed can be kept constant and the number of current values increased, the rate of transmission of intelligence can be materially increased.

Comparison will then be made between the theoretical possibilities indicated by the formula and the results obtained by various codes in common use, including the Continental and American Morse codes as applied to land lines, radio and carrier circuits, and the Continental Morse code as applied to submarine cables. It will be shown that the Continental and American Morse codes applied to circuits using two current values are materially slower than the code which it is theoretically possible to obtain because of the fact that these codes are arranged so as to be readily deciphered by the ear. On the other hand, the Continental Morse code, as applied to submarine cables, or other circuits where three current values are employed, will be shown to produce results substantially on par with the ideal. Taking the above factors into account, it will be shown that if a given telegraph circuit using Continental Morse code with two current values were rearranged so as to make possible the use of a code employing three current values, it would be possible to transmit over the rearranged circuit about 2.2 times as much intelligence with a given number of signal elements.

It will then be pointed out why it is not feasible on all telegraph circuits to replace the codes employing two current values with others employing more than two current values, so as to increase the rate of transmitting intelligence. The circuits, for which the possibilities of thus securing increases in speed appear greatest, are pointed out, as well as those for which the possibilities appear least.

Theoretical Possibilities Using Codes with Different Numbers of Current Values

The speed at which intelligence can be transmitted over a telegraph circuit with a given line speed, *i. e.*, a given rate of sending of signal elements, may be determined approximately by the following formula, the derivation of which is given in Appendix *B*.

$$W = K \log m$$

Where W is the speed of transmission of intelligence,
m is the number of current values,
and, K is a constant.

By the speed of transmission of intelligence is meant the number of characters, representing different letters, figures, etc., which can be transmitted in a given

length of time assuming that the circuit transmits a given number of signal elements per unit time.

Substituting numerical values in this formula gives the following table which indicates the possibilities of speeding up the transmission of intelligence by increasing the number of current values.

Number of Current Values Employed	Relative Amount of Intelligence which can be Transmitted with a Given Number of Signal Elements
2	100
3	158
4	200
5	230
8	300
16	400

This table indicates that there is considerable advantage to be secured in going to more than two current values where the circuits are such as to permit it and where the line speed is not lowered as a result. The limitations will be outlined below. It should also be noted that whereas there is considerable advantage in a moderate increase in the number of current values, there is little advantage in going to a large number.

Codes Now in Common Use—Comparison with Ideal

In the case of printer codes, the theoretical results derived correspond closely to practise, as will be obvious from the method of deriving the formula.

In order to compare the theoretical possibilities indicated by the formula with the results which are obtained when non-printer codes are constructed, several codes were assumed, and for each one the number of signal elements required to produce an average letter was deduced. The method of doing this is set forth in Appendix *D*. This work resulted in the following table:

	Signal Elements per Letter	Relative Number of Letters for a Given Number Signal Elements
American Morse (two current values)	8.26	74
Continental Morse (two current values)	8.45	73
Ideal (two current values)	6.14	100
Continental Morse (three current values)	3.77	163
Ideal (three current values)	3.63	169

The column in the above table headed "Relative Number of Letters for a Given Number of Signal Elements" makes possible direct comparison with the results predicted from the formula as given in the table which preceded. It will be noted that the ideal three-current-value code gives an increase in the number of letters for a given number of signal elements as compared with the ideal two-current-value code which is in fair agreement with the theoretical ratio of 1.58:1. It will also be noted that the Continental three-current-value code which is actually in use in the case of submarine cables appears to come quite close to the ideal. In the case of the Continental and American Morse codes, however, where only two current values are used, the results fall short of the ideal, the ratio between the results actually obtained and the ideal being approximately 1.4:1. The reason for this is that a certain proportion of the possible speed is sacrificed in order to make it possible to read the signals by means of a sounder instead of recording them. For instance, the dash has been assumed to be approximately three times as long as the dot. If the signals were mechanically formed at the sending end and recorded at the receiving end, it would be possible to make use of markings 1, 2, 3, etc. signal elements long, as well as corresponding spacings. The ideal codes were so constructed.

It will be seen that the figures deduced for the Continental Morse and the American Morse are substantially identical for two current values. This result probably does not correspond with practise; it is thought that the difference in speed between these two codes is considerably greater, say on the order of 10 or 15 per cent in favor of the American Morse. The discrepancy is due partly to the fact that no account has been taken of figures and punctuation marks in the present computations and partly to the fact that the assumptions as to relative lengths of space is not strictly in accordance with practise.

From the foregoing, it is seen that there is a two-fold gain in changing from the two-current-value American or Continental Morse codes to the three-current-value Continental code. In the first place, there is a theoretical increase in the ratio of 1.6:1 which accompanies the change from the two-current-value to the three-current-value code. In the second place, there is an incidental increase in the ratio of 1.4:1, due to the fact that the present two-current-value codes are longer than would be necessary, if receiving were done by means other than the ear. The total increase in going from the two-current-value Continental or American Morse codes to the three-current-value Continental code is, therefore, in the ratio of 1.6×1.4:1 or 2.2:1, provided the line speed is the same. In this connection it should be noted that in the case of the American Morse, the ratio is probably somewhat less than this for the reasons pointed out above.

Limitations in Applying Codes with More than Two Current Values

Certain inherent limitations which have to do with how much the number of current values can be advantageously increased are as follows:

1. Fluctuations in transmission efficiency of the circuit,
2. Interference,

3. Limitations on the power or voltage which it is permissible to employ.

In addition it may be stated that, in general, whenever more than two current values are employed it is necessary to make the sending and receiving means more complicated and expensive. There may be nothing to gain, therefore, in using codes other than those made up of two current values where the telegraph circuits are cheap.

Considering now the features which limit the number of current values which can be employed, it is believed that the importance of the first factor will be obvious. If the line is subject to fluctuations so that the stronger currents at certain times become less in magnitude than the weaker currents at other times, it will be impossible to discriminate between the different current strengths making up the code, particularly if the fluctuations are rapid.

In connection with interfering currents, it is evident that these may be of such polarity as to add to or subtract from the signaling currents and it is consequently necessary to separate the various current values employed sufficiently so that one current value with the interference added may be distinguished from the next larger current value with the interference subtracted.

The spacing between the current values being determined by the interference and fluctuations in transmission efficiency, it will be seen that the maximum number of current values which can be employed is determined by the maximum power which it is permissible to use.

In the case of land line telegraph circuits operated with direct currents, it is well known that quadruplex circuits are much more seriously affected by fluctuations and interference than are circuits employing only two current values. (A quadruplex telegraph circuit employs four current values for transmission in one direction). In general, it may be said that the possibilities of improving ordinary direct-current operated telegraph circuits in this manner do not appear particularly promising.

In the case of wireless transmission over great distances all three of the above factors are important in limiting the number of current values which can be effectively employed. In the first place, as is well known, large variations take place in the efficiency of the transmitting medium so that the received signals vary considerably in magnitude from time to time. Secondly, the interference, at least at certain seasons, is great enough to make it difficult to distinguish between the current values even when the usual method which employs only two current values is employed. Thirdly, the received power is limited because of the great attenuation suffered by the wireless waves.

In the case of carrier transmission, it may be that there will be a field for the use of more than two current values. The relative cheapness of the line circuits, however, will tend to limit the amount by which it will be economical to increase the cost and complexity of the receiving apparatus. Moreover, it should be borne in mind that no allowance has been made for the effect on the line speed of increasing the number of current values, this being considered outside the scope of the present paper.

Changing an existing network of telegraph circuits so as to employ a code with three instead of two current values would require new types of telegraph repeaters as well as new sending and receiving apparatus, and new operating methods. It is considered to be outside of the scope of this paper to go into a discussion of the details of this matter.

"Sine Wave" Systems

Considerable interest and discussion has been created by suggestions which have been made to use so-called "sine wave" systems of telegraphy. In view of this, a brief discussion of these systems is given below.

A brief analysis of what are the fundamental features of these systems will be given and, based on the results which have been developed in the preceding discussion, comparison will be made of these systems with systems based on other principles. A particular effort will be made to clear up what appears to be fundamentally incorrect assumptions which underlie the arguments which have been advanced in favor of these "sine wave" systems.

Crehore-Squier System. The use of a sine wave envelope to improve the characteristics of telegraph signals was advocated by Crehore and Squier.[3] The words "United States" formed by means of a wave of this type are shown in Fig. 3*d*. The code employed is the same as the ordinary Continental Morse, the only difference being that the signal elements consist of half-cycle sine waves.

In what has preceded, it has been shown that a half-cycle sine wave has a smaller area than a rectangular wave rounded off by passing through an electrical network and, consequently, the sine wave is inferior to the latter from the standpoint of the received signals. From the standpoint of interference into other circuits, it has also been pointed out that the half-cycle sine waves contain more high-frequency components than properly rounded off rectangular waves. Consequently more interference into other circuits will be produced with the wave made up of signal elements consisting of half-cycle sine waves.

Squier System Applied to Submarine Cables. A more recent suggestion by Squier[4] gives the wave shown in Fig. 3*a*. This wave resembles the one advocated by Crehore and Squier in that each signal element consists of a half-cycle sine wave. As has been pointed out, there is no advantage gained by this.

The difference between the two systems lies in the fact that the wave in Fig. 3*a* uses three absolute values

3. Crehore and Squier, loc. cit.
4. Squier, loc. cit. *Proc. Phys. Soc.*

and crosses the axis once every half cycle. The code is the same as the Continental, a space being indicated by a half-cycle sine wave of one unit amplitude, a dot by a half-cycle sine wave of two units amplitude and a dash by a half-cycle sine wave of three units amplitude.

By referring to the figure, it will be seen that the resulting wave resembles a continuous sine wave, except for the fact that successive half cycles differ in magnitude. For this reason, the code may be termed an "unbroken-reversals" code.

In considering the application of this code to submarine cable telegraphy, it is convenient to make use of an analysis which is carried out in Fig. 3. Fig. 3*a* shows the words "United States" written in the code advocated by Squier. Fig. 3*b* shows a constant sine wave whose amplitude is equal to the amplitude of a dot in Fig. 3*a*. Fig. 3*c* shows the result obtained by subtracting the wave of Fig. 3*b* from the wave of Fig. 3*a*. On comparing this last wave with the wave shown in Fig. 3*d*, it will be seen that the two waves are electrically equivalent. They differ only in having the signal elements permuted.

It is thus evident that the wave shown in Fig. 3*a* is

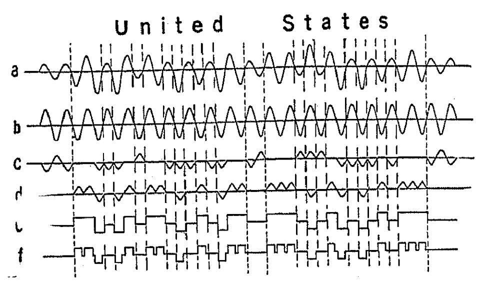

Fig. 3

a - Unbroken reversals code (space = 1 unit, dot = 2 units, dash = 3 units)
b - Constant sine wave, 2 units
c - Wave resulting when substracting *b* from *a*
d - Sine Wave code; note similarity between *c* and *d*
e - Rectangular wave, unmodified
f - Rectangular wave, modified by grounding apex one fourth of the marking time in addition to the spacing time

made up of two components; one being the inert component shown in Fig. 3*b* which transmits no intelligence, and the other the intelligence carrying component illustrated in Fig. 3*c*.

The fact that the component shown in Fig. 3*b* does not carry intelligence from the sending station to the receiving station is made clear when we consider that its value at any moment is predictable and that the component can in fact be produced locally.

The net effect of this component is to reduce the voltage available for intelligence transmission to one-third of the total voltage. For example, if it is permissible to apply 60 volts to a particular cable, 40 volts out of these would be used up in transmitting the inert alternating-current wave and only the remaining 20 volts would be useful for the transmission of intelligence.

Radio and Carrier Telegraphy. Squier has also advocated[5] that the combination of sine wave envelopes, unbroken reversals and a three-current-value code be applied to radio and carrier telegraphy.

The advantages and limitations in applying codes with more than two current values have been fully discussed above, and do not need to be gone into further here. It will be evident that the combining with these of sine wave envelopes and unbroken reversals does no good.

The matter of using sine wave envelopes was discussed above, the discussion pointing out that waves with sine-wave envelopes are inferior to waves produced by sending rectangular shaped signals through suitable networks, both from the standpoint of the received signals, and from the standpoint of interference into other circuits.

The "unbroken reversals" bring in again the use of an inert component. Due to the fundamental difference between cable telegraphy on the one hand, and radio and carrier as usually practised on the other, the inert component in the latter case is somewhat smaller than in the former. In the code advocated by Squier, the current which may be subtracted without greatly affecting the intelligence-carrying capacity of the signals, is about one unit in value, which is the current corresponding to a space. When this current has been subtracted, the space current is reduced from one unit to zero, the dot current from two units to one, and the dash current from three units to two. This subtraction having been carried out, it is seen that the maximum intelligence-carrying component is approximately two-thirds of the maximum current actually employed. (This figure of two-thirds compares with the figure of one-third for the submarine cable.)

In the case of radio, the amount of power which must be radiated from the transmitting station is of particular importance. Since with the system advocated by Squier about two-thirds of the maximum voltage which is radiated is effective in transmitting intelligence, it is evident that about twice as much power must be radiated as would be required if the inert component were not transmitted.

Incorrect Assumptions. Two incorrect assumptions are made in the papers referred to and underlie a considerable portion of the arguments advanced in favor of the systems advocated by Squier.

One of these is that a wave, whose elements are half-cycle sine waves, lends itself to tuning. It is true that in the case of the "unbroken-reversals" code a certain amount of tuning can be secured, but this tuning applies only to the inert unvarying component in the wave, which carries no intelligence. The fact, shown in Fig. 2, that the intelligence-carrying component contains no outstanding narrow range of frequencies to which tuning can be applied should make obvious the error in this assumption.

The other assumption is that a wave, which is ideal for the transmission of power, is also ideal for the transmission of intelligence. As a matter of fact, the transmission of intelligence inherently involves rapid

5. Squier, loc. cit., *Franklin Inst., Jl.*

and unpredictable changes in the current, whereas the transmission of power is best brought about by steady current, either direct or alternating. These two conditions are, of course, incompatible.

Appendix A

Use has been made of the following two principles:

1. In a telegraph circuit in which the line speed is near the maximum, the shape of the received dot is substantially independent of the shape of the impressed dot, and
2. The magnitude of the received current is approximately proportional to the area under the transmitted voltage curve.

The following general discussion of these principles has been furnished by J. R. Carson.

Let the arrival curve, due to suddenly impressed unit battery be denoted by $A(t)$; then the received signal $S(t)$, due to the elementary dot impressed signal $f(t)$ is given by[6]

$$S(t) = \int_0 f(x)\, A'(t-x)\, dx \qquad (1)$$

the upper limit of integration being t for $t < T$ and T for $t \geqq T$. The latter case will alone be considered since the conclusions arrived at in this case are conservative.

Expanding $A'(t-x)$ in (1), we get

$$S(t) = \left[A'(t) - \frac{h_2 T}{2!} A''(t) + \frac{h_3 T^2}{3!} A'''(t) \ldots \right] \int_0^T f(x)\, dx \qquad (2)$$

where
$$h_2 = \frac{\int_0^T x\, f(x)\, dx}{\frac{T}{2!}\int_0^T f(x)\, dx},$$

$$h_3 = \frac{\int_0^T \frac{x^2}{2!} f(x)\, dx}{\frac{T^2}{3!}\int_0^T f(x)\, dx}, \text{ etc.}$$

It follows at once that, provided

$$\int_0^T f(x)\, dx \neq 0$$

and provided the duration T of the signal is sufficiently short, the arrival dot is given approximately by the leading term

$$A'(t) \int_0^T f(x)\, dx$$

and that this approximation becomes increasingly close as the speed of signaling is increased, *i. e.*, as the duration T of the dot is decreased.

6. J. R. Carson. "Theory of the Transient Oscillations of Electrical Networks and Transmission Systems." A. I. E. E. TRANS., Vol. XXXVIII, 1919, p. 345.

The conclusions from the foregoing may be stated in the following propositions:

I. If the speed of signaling is sufficiently high the arrival signal representing the elementary dot is independent in shape of the form of the impressed signal, and is proportional in amplitude to the time integral or "area" of the impressed signal.

It will be evident, however, that if no restrictions are imposed on $A'(t)$ and $f(t)$, the foregoing proposition requires, in general, that the duration T of the dot shall be so small as to make the series expansion rapidly convergent from the start. This, however, requires a speed of signaling very considerably greater than that actually necessary in practise in order that the foregoing proposition shall hold to a good degree of approximation, at least for the types of impressed dot signals specially considered in the present paper. To show this, it is necessary to establish two less general propositions, valid for the types of impressed signals under consideration.

II. If the impressed signal $f(t)$ is everywhere of the same sign, then a value τ exists, such that $0 < / \tau / < T/2$, and such that

$$S(t + T/2) = A'(t+\tau) \int_0^T f(x)\, dx \qquad (3)$$

This proposition follows from the mean value theorem.

III. If $f(t)$ is everywhere of the same sign, and if further it satisfies the conditions of symmetry,

$$f(x) = f(T-x), \; (x \leqq T/2)$$

then a value τ exists, such that $0 < \tau < T/2$ and such that

$$S(t + T/2) = 1/2\,[A'(t+\tau) + A'(t-\tau)] \int_0^T f(x)\, dx \qquad (4)$$

This last equation also follows from the mean value theorem. Furthermore, the conditions stated in proposition III are satisfied by the rectangular wave, the half-cycle sine wave, and the rectangular wave extending through part of the dot provided the reference time $t = 0$ is properly chosen.

Returning to proposition II, let us write

$$S_j(t + T/2) = A'(t + \tau_0 + \tau_j) \int_0^T f_j(x)\, dx,$$

the subscript j indicating the particular type of impressed dot signal, and τ_0 the value of τ for any type of signal, taken as reference. Then

$$S_j(t + T/2) = \left[A'(t+\tau_0) + \frac{\tau_j}{1!} A''(t+\tau_0) + \ldots \right] \int_0^T f_j(x)\, dx \qquad (2a)$$

Now, the condition that proposition I shall hold to a good degree of approximation is that the expansion (2a) shall converge rapidly. Since the maximum possible value of τ_j is $T/2$ and since in practise it is much smaller than $T/2$, the required convergence

obtains for much larger values of T, that is, slower speeds of signaling than that required in the expansion (1). Furthermore, for the three types of signals specifically under consideration τ_1, τ_2 and τ_3 differ from one another by quantities very much smaller than $T/2$ in all actual transmission systems.

If the conditions of proposition III are introduced, the approximation is still closer and proposition I is valid for still lower signaling speeds.

In order to arrive at quantitative ideas of the minimum signaling speeds at which the foregoing proposition is valid, it is necessary, of course, to specify the arrival curve of the transmission system under consideration. An application of the foregoing analysis to representative transmission systems both with and without a "cut-off" frequency has shown that it is valid to a very good degree of approximation for speeds considerably lower than the highest attainable under practical conditions.

Appendix B

Use has been made of the formula

$$W = K \log m$$

where W = the speed of transmission of intelligence

K = a constant

and m = the number of current values employed. The assumptions which underlie this formula and its derivation will now be given.

Let us assume a code whose characters are all of the same duration. This is usually the case in printer codes. If n is the number of signal elements per character, then the total number of characters which can be constructed equals m^n. In order that two such systems should be equivalent, the total number of characters that can be distinguished should be the same. In other words,

$$m^n = \text{const.} \tag{1}$$

This equation may also be written

$$n \log m = \text{const.} \tag{2}$$

The speed with which intelligence can be transmitted over a circuit is directly proportional to the line speed and inversely proportional to the number of signal elements per character provided that the relations above are satisfied. Hence, we may write

$$W = s/n \tag{3}$$

where s is the line speed. Substituting the value of n derived from the equation above, this equation becomes

$$W = \frac{s \log m}{\text{const.}} \tag{4}$$

which may also be written

$$W = K \log m \tag{5}$$

In applying this formula to practical cases it will be found impossible to comply strictly with the condition expressed by equation (1). As an example, consider the comparison between a three-current-value code where each character is made up of three signal elements, and a two-current-value code where each element is made up of five signal elements. It is obvious that the speed with which *characters* can be transmitted in the former case is five-thirds the speed in the latter case for a given line speed. In other words the ratio is 1.67.1 whereas the formula gives the ratio 1.58:1. It should be noted, however, that the former code possesses only 27 characters whereas the latter possesses 32. In other words one *character* of the latter code represents the transmission of more *intelligence* than one *character* of the former. Thus the figure 1.67 for the relative speeds of transmission of *characters* and the figure 1.58 for the relative speeds of transmission of *intelligence* are not incompatible.

It will be noted that the formula has been deduced for codes having characters of uniform duration and that it should not be expected to be anything but an approximation for codes whose characters are of non-uniform duration. To establish the formula for the latter case it would be necessary to make an assumption as to the relative frequencies of the various characters. It seems reasonable to suppose that the formula will give a fair approximation to the facts in this case also, but it should not be expected to be accurate.

Appendix C

The deduction of the curves given in Fig. 2 from the curves given in Fig. 1 requires some explanation. Looked at casually, it would seem as if an isolated dot would not possess any frequency characteristics whatsoever. Nevertheless, if a voltage, such as any of those represented in Fig. 1, is applied to a network capable of being thrown into oscillation, the network will respond to the voltage by oscillating. Suppose, for simplicity, that the network consists of an inductance, a capacity and a very small resistance in series, the response of the network to the application of any of the voltages illustrated is that it oscillates at constant frequency and gradually decreasing amplitude. Further, the response varies when the natural period of the circuit is varied.

There are two ways of looking at this phenomenon. We may say, on the one hand, that the oscillations of the frequency in question are manufactured by the network out of the voltage applied and that the frequency does not exist in the original voltage. On the other hand, we may say that the original voltage contains components at or near the resonant frequency and that the circuit responds to these components because it offers them a small impedance, while it does not respond to other components because it offers them a large impedance. Either of these views is permissible, but it is convenient for the purposes of this paper to use the nomenclature of the second view and to consider the applied voltages to be made up of an indefinitely large number of frequencies. The problem of determining the response of oscillating networks is then solved by deducing the frequency characteristic or the response characteristic of the impressed voltage. This characteristic may be determined by means of the Fourier

integral, whose computation is described in any standard textbook on the subject. The following is intended to outline the considerations, from a physical standpoint, which lead to establishing this integral.

To deduce the frequency characteristic of an isolated dot, it is simplest to start with a long series of dots which are uniformly spaced. If such a series of dots is considered to extend indefinitely, it is possible to analyze the resultant wave into a Fourier series by well known methods. Now, suppose that such a Fourier series has been obtained for a given spacing of the dots. The next step is to increase the spacing between the dots. The result of this is to increase the number of Fourier components in a given frequency range and to decrease the magnitude of each. If this process of increasing the space between the dots is continued indefinitely, we approach the condition of an isolated dot. Moreover, as we approach this condition, the number of components in a given frequency range increases indefinitely and the magnitude of each decreases indefinitely. This limiting result is known as the Fourier integral for the wave in question.

Appendix D

A table has been given in the paper in which the relative efficiency of various codes in transmitting intelligence is listed. The derivation of that table will now be given.

The comparison will include the following codes based on two current values: American Morse, Continental Morse, and the so-called "ideal" two-current-value code. It will also include the following codes based on three current values: Continental Morse and an "ideal" three-current-value code.

The assumption is made that the text is made up of five-letter-words, no allowance being made for punctuation. The following table gives the length of the spaces assumed in terms of signal elements.

	Ordinary Spaces Within Letters	Special Spaces in "Spaced" Letters	Spaces Between Letters	Spaces Between Words
American Morse (two current values).....	1	2	3	4
Continental Morse (two current values).....	1	—	2	3
Continental Morse (three current values)	—	—	1	2

It is assumed that the dashes in the two-current-value codes are of three signal elements, duration except for the letter *l* in American Morse which is assumed to occupy five signal elements. It may be that in practise, the dashes are somewhat shorter than has been assumed but the resulting error is not great. In connection with the relative spacings between letters and words assumed for the Continental and American Morse codes, it is also questionable whether they accord strictly with practise. It may be that these spacings are on the average more nearly equal than the table indicates. However, this assumption affects only the relative speeds obtainable with the American Morse and the Continental Morse and does not materially affect the comparison between codes based on two current values on the one hand and codes based on three current values on the other.

The term "ideal" has been applied to two codes which will next be explained. These codes are constructed on the same principles as the Continental and American Morse codes with an effort to make them as brief as possible without making the reading too difficult. It is thought that the two ideal codes chosen are comparable in the matter of ease of reading. In constructing the two-element code, two steps are involved. In the first place it is assumed that the markings and spacings of any integral number of signal elements' duration can be used so that in addition to the values for markings and spacings assumed above, there may be dashes of two, four, etc., units duration. With these assumptions the 26 shortest characters that can be constructed are next made up. It is found that one character is of 1 unit duration, 1 of 2 units, 2 of 3 units, 3 of 4 units, 5 of 5 units and 9 of 6 units duration. The remaining 5 characters are taken of 7 units duration each. The second step is to ascribe the 26 letters of the alphabet to these characters in such an order that the most frequent letters correspond to the shortest characters. It is most efficient to use the same spacing as was assumed above for the Continental two-current-value code, with the addition that spaces of longer duration than three units may be employed within a letter.

The matter of constructing the ideal three-current-value code is similar. First, the 26 shortest characters are constructed. Two characters can be constructed having a duration of 1 unit, four characters having a duration of 2 units and eight characters having a duration of 3 units. The remaining twelve characters are taken 4 units in duration. Next, the most frequent letters are assigned to these characters in the order of their duration. It is best in this case to use the same assumptions as to spacings between letters and words as was used above in connection with the three-current-value Continental code. The use of spaces within letters is not economical in this case.

A frequency table given by Hitt[7] was used to determine the relative frequency of the various letters. The average duration per letter was computed from this table and corrected for spaces between words and letters. The resultant average duration is as follows:

Code	Signal Elements per Letter
American Morse (two current values)..................	8.26
Continental Morse (two current values)................	8.45
Ideal (two current values)............................	6.14
Continental Morse (three current values)..............	3.77
Ideal (three current values)..........................	3.63

7. Parker Hitt. "Manual for the Solution of Military Ciphers." Army Service Schools Press, Fort Leavenworth, Kansas. Second edition, p. 7.

Discussion

Bela Gati (by letter): The following remarks apply to cable telegraphy. Some experiments with a-c. telegraphy have been made by the writer which are of interest in considering telegraph speed. These tests were made on long lines and oscillograph records of two of the tests which are shown herewith. In one case direct current was employed and in the other alternating current at 500 cycles per second.

In each oscillograph the lower line represents the current at the sending point while the upper line represents the received current. The point *b* shows when the current was started at the sending end. Point *c* shows when it started at the receiving end. Point *d* shows when the current was interrupted at the sending end.

From the oscillograms we see that in the case of the alternating current the received current stops sharply at point *e*, but with the direct current, the conductor discharges afterwards at its

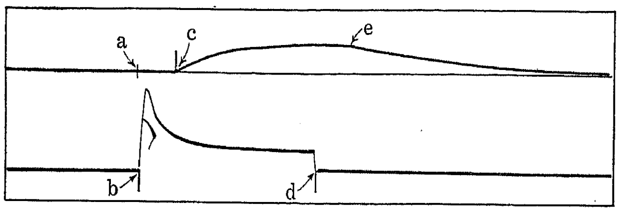

Fig. 1

natural frequency. The signal is lengthened from the point *e* to infinity.

This discharge current is the troublesome feature but there are several ways of lessening the time of discharge. If a cable is not charged to too high a value, especially at the end of the signal, the discharge current will be lessened. The grounding of the circuit at the sending end relieves the sending end of the long discharge and thus makes telegraphy faster.

It is still better to apply an opposing potential. This reduces the strength of the signal but it decreases the charging current. This is the method of Pickard and others.

The best method is to use alternating current. In this case each half cycle counteracts the effect of the previous half cycle. True, the strength of the current is appreciably diminished but the discharge effect is practically eliminated. The first and the

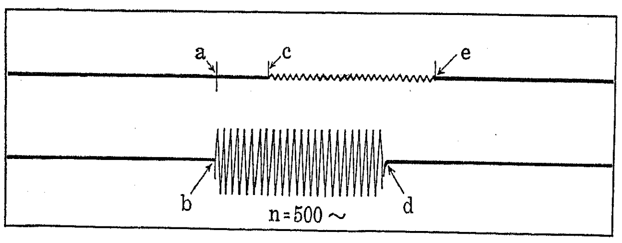

Fig. 2

last half cycles do not have any noticeable effect on the length of the signal.

The received current corresponding to a dot can be obtained from the superposition of two curves, each representing the received current for direct current. One of these curves is displaced along the time coordinate by an amount equal to the duration of the dot and it is plotted with negative values for the ordinate. The result is similar to the ripples obtained with alternating current, if the dot length is made equal to a half cycle. Of course, there are secondary and tertiary phenomena which modify the results and consequently there will be some disagreement between the two curves.

In my opinion the signal should contain ten complete cycles for a dot. This means that telephone frequencies (500 to 2000 cycles per second) are applicable.

The constants used in the experiments from which the oscillograms were taken were as follows: The circuit was 2120 kilometers of bronze wire, 4 millimeters in diameter. For each kilometer: $r = 2.8$ ohm; $l = 1.43 \times 10^{-3}$ henry; $c = 12 \times 10^{-9}$ farad; $g = 10^{-6}$ mho; Zo(= Sending-end impedance as proposed by Dr. Kennelly) = 348.46 (at − 5 deg. 17.85 min.); the attenuation = 41.82×10^{-3} (at 84 deg. 13.09 min.) = $(42.13 + j\,416.07) \times 10^{-4}$; attenuation × length = 8.8; the so-called "cable length" of Dr. K. W. Wagner $= \frac{t}{2}\sqrt{\frac{c}{1}} \times 2120 = 8.57$.

Of course this line is not a cable but it serves to illustrate the phenomena. On a submarine cable the point *e* cannot be exactly distinguished. The conditions are quite complicated and a simple explanation does not apply exactly.

Shunt coils in a cable compensate for the discharge effect and make alternating-current telegraphy possible for a given frequency. There is no reason for the limiting of the frequency to four cycles or ten cycles as is done in the computations of the extremely conservative cable companies. We must apply telephone frequencies giving many cycles for one dot. Only when using such frequencies can the benefits of shunt coils be realizied. The use of shunt coils as applied now at the sending and receiving stations is an entirely different matter from their use with telephone frequency and mounted at proper intervals on the cable. The shunt coils lengthen only the wave length but not the signals. Pupin coils shorten the wave length.

In 1909[2] I first proposed submarine cables with rather high self-induction, 10^{-2} henry. After 15 years, in 1924, the Western Union Cable Company has adopted a cable with such a characteristic.

If but one frequency is considered automatic multiplex printers may be employed (the Murray, for instance) and by this means the cable companies can compete effectively with the new 35-70-cycle channels of the new radio system.

For further information on this subject see the following articles by the present writer: Will the Signal be Lengthened in Alternating Current Telegraphy?—The *Electrician*, London, April 12, 1912; Werdin die Zeichen auch bie Wechselstromtelegraphie verlaengert?—*Elektrotechnik und Maschinenbau*, Wien 1911, Heft, 42.

H. Nyquist: As I understand it, there is very little disagreement between Mr. Gati's results and those obtained in the paper. The principal discrepancy occurs in connection with the matter of improving submarine cable signals by grounding the transmitting end of the cable for short intervals. In the paper the opinion is expressed tentatively that such grounding is not beneficial, whereas Mr. Gati comes to the opposite conclusion. It is to be regretted that his experimental work does not cover this point.

The argument that carrier telegraph at voice frequencies is preferable to direct current for signaling purposes is not, of course, applicable to the subject under consideration, namely, ocean cables as at present constituted. Such cables are incapable of transmitting these frequencies. In the case of the circuits which are capable of transmitting these high frequencies, it is, of course, possible to utilize them for telegraph purposes. In fact, in the case which Mr. Gati considers, namely, where the circuit is assumed to be capable of transmitting 10 cycles of alternating current to form a single dot, it is possible to go even further and to obtain both d-c. telegraph and carrier telegraph simultaneously.

There can further be no question that the introduction of suitable loading extends the frequency range and, therefore, is beneficial, if properly applied. I am under the impression that the reason telegraph companies have not employed loading for submarine cables up to the present is not lack of appreciation of the effectiveness of such loading but rather the lack of suitable methods for accomplishing it.

[2]The C R Law and Rapid Cable Telegraph, *Elektrotechnik und Maschinenbau*, Issue 37, 1909.

Measuring Methods for Maintaining the Transmission Efficiency of Telephone Circuits

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Review of the Subject.—Maintenance of the transmission efficiency of the telephone plant is conducted by a special force using methods and apparatus that have been developed for this purpose. This paper gives a brief description of the transmission characteristics of some of the common types of telephone circuits, outlines a general method for measuring their transmission efficiency and describes several of the most modern types of transmission measuring sets together with a brief mention of the oscillators which supply the power for testing.

* * * * *

THE circuits involved in the transmission of speech in a modern telephone plant, particularly those designed for long distance operation, necessarily involve a considerable amount of complexity. The use of telephone repeaters, the development of long toll cables, the application of carrier systems, and other developments associated with these, while increasing the efficiency and economy of telephone toll circuits, have also increased their complexity and have required the development of more effective means of insuring that the circuits are maintained at all times in good condition and adjustment.

It is the purpose of this paper to present a brief description of the measuring methods which have been developed and put into use, to enable those charged with the maintenance of such circuits to determine rapidly and conveniently whether the circuits are giving the transmission results for which they were designed.

Talking Tests

To a person who is unfamiliar with telephone transmission measurements, the most obvious method of testing a circuit is to talk over it. Such a method is not suitable for routine testing because of the impossibility of obtaining accurate data without taking elaborate and time-consuming precautions. It is impossible to judge at all accurately the efficiency of a circuit by simply listening to some one talk at the other end. Tests have shown that the most skilled observers cannot detect circuit changes which alter the received power by as much as a factor of three when these changes are made between conversations.

The only method of determining the transmission efficiency of a circuit by talking tests is to compare it repeatedly and directly with another circuit of known and adjustable efficiency. Experienced observers are necessary and much time is required to obtain accurate results. In the case of switchboard cord circuits and other central office apparatus, the total transmission loss is often less than the errors of measurement by this method. On long toll circuits, the errors of observation are small compared with the total loss and by careful tests with experienced observers, results sufficiently accurate for some purposes may be attained. The cost of this testing is, however, too great to permit its use on anything excepting important toll circuits.

Testing Line Circuits

The most accurate method of measuring a line circuit, such as a toll circuit, is to connect it as in Fig. 1 between a source of alternating current and a measuring instrument, both of which have impedances approximating the characteristic impedance of the circuit. With such an arrangement the unknown line becomes in effect part of an infinitely long line of its own type and the ratio of the voltages, currents or powers at the ends of the circuit is a measure of its transmission efficiency. In commercial use the toll circuit may be terminated by switching trunks or other circuits which have different impedances from that of the circuit which is being measured. In these cases terminal

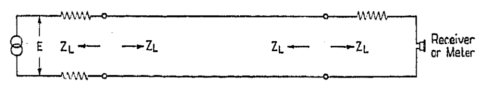

Fig. 1

losses, commonly called reflection losses, will result. As these losses are different for each impedance of the terminating apparatus, it is preferable for maintenance purposes to have them excluded. The measured results can then be readily checked by computations.

The transmission efficiency of the average telephone circuit is not constant with changes in frequency. Because of this a measurement of the transmission efficiency of a circuit with testing current of but a single frequency will not necessarily show the same result as would be obtained if the testing current were supplied by a voice actuated transmitter. In transmission maintenance work we are interested primarily in determining if the circuits are being maintained up to their specified standards. It is possible in many cases to determine this by means of a single frequency measurement or several measurements using testing currents of different frequencies.

Figs. 2 and 3 show the transmission frequency

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characteristics of two telephone circuits which have been chosen to contrast with each other, and to indicate the variations in type of the attenuation characteristic which may be found in a working plant. Fig. 2 shows the characteristics of a 13-gage, heavily loaded cable circuit. This circuit, while not characteristic of the loaded circuits now being installed, is loaded in accordance with the practise of a number of years ago, and is still in operation. Small irregularities in the spacing of loading coils produce irregularities in the transmission frequency characteristic of the circuit. While these irregularities are not serious in the case shown in Fig. 2, they are sufficient to cause a difference at any one frequency of as much as 2 miles of standard cable in two similar circuits which may have exactly the same equivalents for talking. More accurate results can be obtained by making several measurements at frequencies close together and averaging the results. Averages of such measurements on each of two similar circuits are generally in close agreement.

In addition to the irregularities shown on this circuit, it will be noted that the general trend of the curve from 200 to 400 cycles is downward, while for frequencies above 400 cycles it is upward. In this particular circuit the transmission equivalent for talking currents coincides with the single frequency measurement at about 1000 cycles and in this case a single frequency measurement at 1000 cycles would be a good indication of the talking transmission equivalent. A single frequency measurement at 1000 cycles would, however, not be a complete measure of the characteristics of the circuit including its distorting effect.

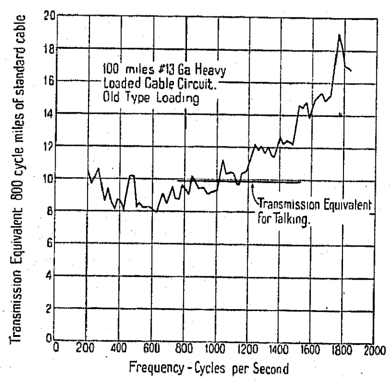

Fig. 2—Transmission Characteristic of 13 Gage Heavy Loaded Cable Circuit

Fig. 3 represents the transmission frequency characteristic of a non-loaded No. 8 B. w. g. open-wire circuit. It will be noted that this curve is quite different from that shown in Fig. 2. This open-wire circuit has no appreciable irregularities in its makeup and its general tendency is to transmit currents of different frequencies much more uniformly than the circuit shown in Fig. 2. In the case of this second circuit a 1000-cycle measurement also agrees with the talking transmission equivalent, but an observer talking over these two circuits would notice a difference in the quality of the transmitted speech. If it is desired to have an accurate picture of the distorting effect of the circuit as well as its volume efficiency, it is necessary to make a number of measurements over the entire voice frequency range.

In a majority of the toll circuits in the telephone plant a single frequency measurement is sufficient for routine purposes, the construction of these circuits being such that any defects which materially increase the transmission loss for talking will be evident at

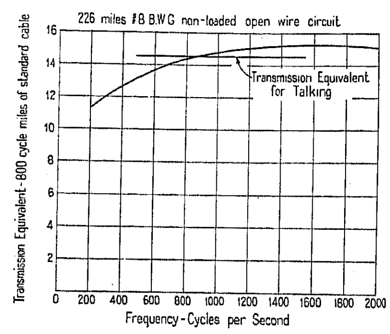

Fig. 3—Transmission Characteristic of No. 8 B. W. G. Non-Loaded Open Wire Circuit

nearly any single frequency in the voice range, although because of irregularities, it is not possible in many cases to determine the loss more closely than 2 or 3 miles of standard cable. However, there are some types of circuits, particularly the most modern types, in which it is possible by the opening of a single wire in a network or filter, or by some one piece of apparatus becoming defective to materially change the transmission-frequency characteristic for part of the frequency range only. In such cases as these, the talking volume as well as the distortion would be altered, although at certain single frequencies the circuit would appear to be normal. For such circuits, it is necessary for routine maintenance purposes, to make measurements at several widely separated frequencies in the voice range.

Testing Central Office Apparatus

The arrangement shown in Fig. 1 and described for line circuit testing can also be used for determining the transmission loss caused by a cord circuit or other piece of central office equipment under service conditions. If the generator and measuring instrument are connected directly together and then are connected by means of the cord circuit, the ratio of the currents in the measuring instrument for the two cases will indicate the effect of the cord circuit on transmission. By making the impedance of the generator and measuring instrument equal to the characteristic impedance of toll circuits of various types, the loss in received power caused by the cord circuit under different conditions can be readily obtained.

While in the case of line circuits it is necessary in some

cases to make several measurements using testing currents of different frequencies in order to determine the condition of the circuit, it is seldom necessary in central office apparatus to use more than one frequency of testing current. The reason for this is clearly shown in Fig. 4, which is the transmission frequency characteristic of a subscriber's operator's cord circuit. Curve A represents a normal circuit and shows the equivalent to vary only a small amount over the entire frequency range, the variation being gradual. If a large number of similar circuits were measured in this manner the same type of curve would be obtained for all of them which were not defective. Because of this, it is reasonable to assume that if two similar cord circuits have the same transmission equivalents for a single frequency of testing current, the two circuits will have approximately the same equivalents for voice frequencies.

Curve B represents the same cord circuit after one of the four windings of the transformer has been short-circuited, thus simulating the condition of a breakdown of the insulation between windings. This curve is also a smooth curve similar to curve A but higher at each point, the equivalent being approximately doubled at the more important frequencies. Because of the fact that a defect in one of the parts of a cord circuit usually increases the equivalent of the circuit at all frequencies, it is evident that a measurement of the equivalent at any one frequency of testing current is sufficient to indicate whether the cord circuit is normal or defective.

Practically all switchboard cord circuits and other central office apparatus such as phantom coils and composite sets have transmission frequency character-

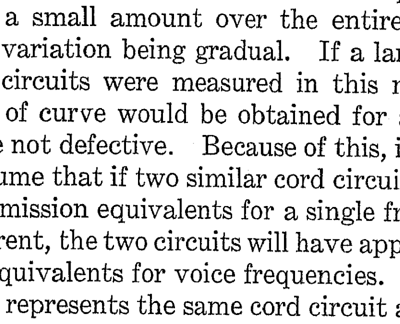

FIG. 4—TRANSMISSION-FREQUENCY CHARACTERISTICS OF SUBSCRIBER'S OPERATOR'S CORD CIRCUIT

istics which vary in the gradual manner of the circuit in Fig. 4 and are affected at nearly all frequencies by a change in the electrical constants of any part.

TESTS WITH ARTIFICIAL CABLE

Although talking tests with artificial cable have not been in general use for a number of years, the method was applied for some time to important circuits and a brief description will be given.

The arrangement generally used is shown in Fig. 5. In this arrangement two telephone sets are connected by means of switches to the ends of the circuit under tests or to an artificial cable, the "length" of which is adjustable. In testing, one observer talks to the other at the opposite end of the circuit, talking alternately through the artificial cable and through the unknown circuit. The artificial cable is adjusted until the same volume of sound is received over each circuit. The number of miles of cable required to obtain a balance is said to be the transmission equivalent of the unknown circuit.

This method requires that both ends of the circuit

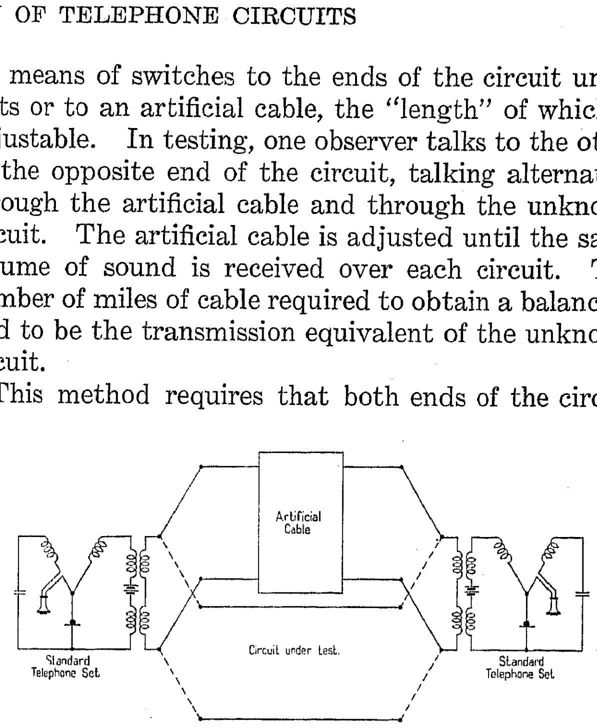

FIG. 5—ARRANGEMENT FOR TALKING TEST USING ARTIFICIAL CABLE

under test be available at the testing point. Long circuits can then only be tested by connecting two of them together at the distant end and measuring them as one circuit. The amount of artificial cable when a balance is obtained represents not only the attenuation in the unknown circuit, but also includes reflection losses which occur at the junction of the unknown line and the substation instruments.

As toll circuits are seldom terminated directly by telephone sets, the reflection losses which occur in practise are generally not those occurring when tests are made by this method. One of the chief advantages of modern testing sets lies in the elimination of these reflection losses by the substitution for the telephone set of terminating impedances which approximate the characteristic impedance of the unknown circuit.

It is difficult to compare, from the standpoint of volume, two circuits which distort the speech in a different manner. Artificial cable has approximately the same distorting effect as some telephone circuits, particularly the oldest types, but it differs considerably from the latest types. In many cases, it is, therefore, difficult to obtain an accurate comparison of circuits by a talking test.

1-A TRANSMISSION MEASURING SET

This was the first successful routine transmission measuring set to be developed to obviate difficulties and delays incidental to tests with artificial cable. Substation sets were eliminated as terminals and replaced with impedances adjustable to the characteristic impedances of representative circuits thus approximating the ideal of Fig. 1. The avoidance of reflection losses by characteristic impedance terminations also

enables the testing to be simplified by the use of single frequencies instead of the voice. This piece of apparatus and the alternating-current generator which supplies the testing current, are readily portable and entirely self-contained, being capable of operation without external batteries. It was designed, primarily, for the purpose of measuring the transmission loss under normal operating conditions of switchboard cord circuits and apparatus, and inter-office trunks. It was not intended for measurement of long toll circuits,

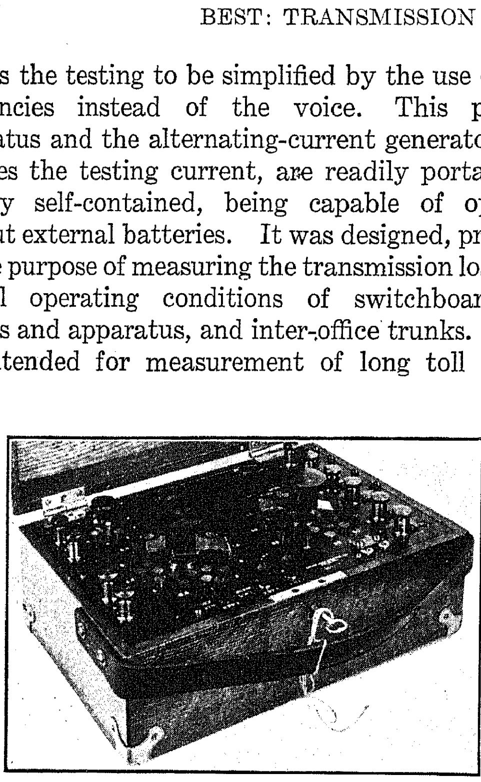

FIG. 6—1-A TRANSMISSION MEASURING SET

although with limitations, it can be used for this purpose. An illustration of this set is shown in Fig. 6.

As shown in Fig. 7, the circuit consists of two branches which are permanently bridged together at one end. An alternating-current source is connected to the junction of these branches and the same voltage is, therefore, impressed upon each. The two branches are similar, except that in the upper one is connected the apparatus or circuit to be measured, while the lower one

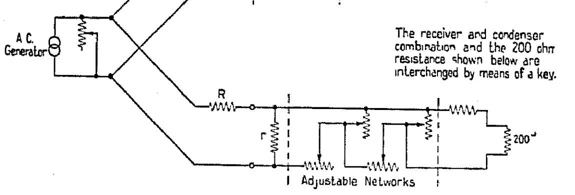

FIG. 7—SIMPLIFIED CIRCUIT OF 1-A TRANSMISSION MEASURING SET

contains an adjustable network for producing different transmission losses. A transmission measurement is made by adjusting the loss of this network until it is equal to that caused by the apparatus to be measured as determined by an equality of sound in a receiver when switched from one branch to the other. As the receiver is switched from one branch a 200-ohm resistance is substituted for it, this arrangement keeping the load on the alternating-current source and, consequently, the voltage at the junction of the two branches constant as the receiver is connected alternately to them.

Each branch is divided into two parts: A sending impedance designated R and a receiving impedance composed of r in multiple with the network to the right of r. The apparatus to be measured is connected between the sending and receiving impedance in the upper branch. The sending and receiving impedances in both branches can, by means of a key, be given any one of three values: 600, 1300 or 2200 ohms non-inductive, corresponding approximately to low, medium and high-impedance circuits, respectively. For transmission measurements, these values are sufficiently close to the characteristic impedance of telephone circuits.

The 1-A transmission measuring set cannot be used for testing circuits between offices, unless they are measured two at a time, the distant ends being connected together, as described in talking tests using artificial cable.

The adjustable network for producing different transmission losses is distortionless and consists of a series-shunt, non-inductive resistance arrangement of constant impedance as viewed from the source of power. It is so designed that the transmission loss can be adjusted in steps which cause the same change in the current as is caused by one mile of standard cable at a frequency of 800 cycles per second. This unit of transmission loss is known as the "800-cycle mile." The distortionless network is advantageous for transmission measuring purposes as it facilitates comparisons of the transmission efficiency of a circuit or piece of apparatus at different frequencies. Tests with artificial cable do not permit this.

The use of a single-frequency alternating current instead of the voice for testing enables comparisons to be made rapidly and quite accurately, it being possible to detect differences in currents as small as 2 per cent. (Approximately 0.2 mile of standard cable.)

The adjustable resistance connected across the junction of the two sending impedances and the alternating-current generator is for the purpose of controlling the sound in the receiver. It has no effect, from an impedance standpoint, on the results.

The two large dials shown in Fig. 6 adjust the loss of the distortionless network in half-mile and 5-mile steps. Directly above these are 3 smaller dials which control the volume of the testing current and the value of the sending and receiving circuit impedances. The binding posts and jacks are for the purpose of connecting the generator, receiver and circuits to be tested.

The actual operation of measuring consists simply in operating a key which connects the receiver alternately to its two positions and adjusting the network dials until the volume of sound is the same for both positions. The transmission equivalent of the apparatus or circuit under test is read directly from the dials.

3-A Transmission Measuring Set

At the time the 1 - *A* transmission measuring set was developed there were no alternating-current measuring instruments available for measuring the power received at the end of a telephone circuit which were sufficiently rugged or practicable to withstand the service required of them. The amounts of power at both the transmitting and receiving ends of a telephone connection are small. The sound energy produced by the voice during a telephone conversation varies over wide limits, an average figure expressed in electrical power units being of the order of 10 microwatts. Only a small part of this power reaches the transmitter, nevertheless, the transmitter being an amplifier, the power generated by it for this case is about 300 microwatts. The received power for an average connection is less than 1 microwatt and conversations may be carried on when the received power is as little as 0.01 microwatt.

The development and commercialization of the three-electrode thermionic vacuum tube made available a means for amplifying these weak currents sufficiently to enable them to be accurately read with standard types of meters. This tube also plays an important part in generators of alternating current for testing purposes. With suitable alternating-current generators and amplifiers available the problem of designing a transmission measuring set consists in arranging the apparatus in such a manner that it may be operated conveniently and rapidly and will measure accurately.

The transmission efficiency of a circuit may, as previously stated, be determined by measuring the currents or voltages at the two ends of the circuit. The ratio of these voltages or currents can then, if desired, be expressed in terms of standard cable. It is, however, much more convenient to have the result obtained directly, without computations. The principles used in the 1 -*A* transmission measuring set have, therefore, been followed in the more recent sets using vacuum tubes.

The 1 -*A* transmission measuring set requires for its operation, the maintenance of equal voltages on the two branches. As no voltmeter is provided, it is necessary to bridge the branches together to obtain equal voltages. This requirement prevents the measurement of circuits between offices unless two circuits are connected together at the distant end. The 1 - *A* set could be used for measuring such circuits if the branches were separated and equal voltages applied to them. In this case the upper branch R of Fig. 7 would be removed and connected to the distant end of the line under test and testing current supplied through it from a generator having a voltage equal to that of the generator connected to the lower branch.

The transmission measuring sets which employ vacuum tubes are arranged so that the voltage across the sending circuit impedances may be maintained at a definite value, a meter being connected in the oscillator circuit. It is, therefore, possible to make measurements on a circuit, in either direction, when a measuring set is connected to each end.

Several types of transmission measuring sets which make use of vacuum tubes have been developed but only the latest types of portable and non-portable sets will be described. The 3 - *A* transmission measuring set, a view of which is shown in Fig. 8, is a portable set designed to replace the 1 - *A* transmission measuring set in common battery central offices where a direct-current supply is available to operate the vacuum tubes. Like the 1 - *A* set, it is intended primarily for measuring central office apparatus and switching trunks. The maximum transmission equivalent which can be measured is 30 800-cycle miles.

Fig. 9 shows the circuit arrangement used in measuring the transmission equivalent of a circuit between two offices, this type of test being commonly called a "straightaway" test, as differentiated from the "loop"

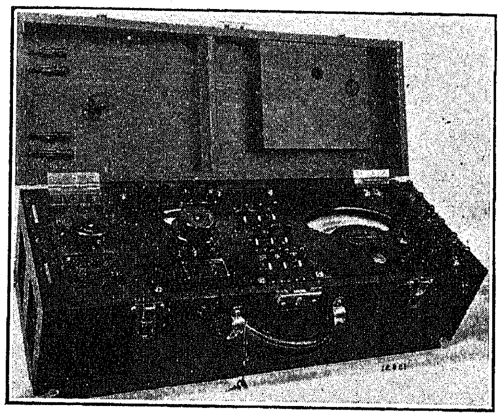

Fig. 8—3-A Transmission Measuring Set

test used with the 1 - *A* set and with talking tests. At the sending end of the circuit only part of the apparatus is in use, as shown. The sole purpose of this apparatus is to impress a voltage across the terminals $C_1\,C_1$. The impedances D_1 together approximate the characteristic impedance of the unknown circuit. The a-c. meter shown in this drawing is a thermo-milliammeter which can, of course, be used in measuring voltage if the resistance of the circuit is known. If the milliammeter were in series with the resistance H_1, shown connected across terminals $C_1\,C_1$, the reading of the meter would be proportional to the voltage. But such an arrangement is undesirable because the impedance of the meter would be important. For practical reasons it is desirable to have the meter where the total current through both H_1, and $D_1\,D_1$, is measured. However, the resistance of H_1 is so small, with respect to $D_1\,D_1$, and the unknown circuit, that practically all of the current is in H_1, and changes in the impedance of the unknown circuit produce a negligible effect on the voltage across

$C_1 C_1$. H_1, then serves the purpose of a generator of low impedance, this arrangement being referred to as a "point source" approximation.

At the receiving end of the circuit the complete transmission measuring set is required. The principal parts of the set are shown in the dotted rectangle to the right of the unknown circuit. They consist of an amplifier-detector unit for amplifying the weak received testing current and converting it into direct current, a meter which is actuated by the rectified current, a sending circuit identical with that used at the sending end of the unknown circuit, and two branches containing networks known as the "calibrating" and "measuring" networks.

The purpose of some of these parts, particularly the calibrating network, will not be evident to those unfamiliar with vacuum tubes. The vacuum tube, while an excellent amplifier and rectifier, cannot be relied on to maintain unvarying characteristics. The characteristics of a tube change slightly with time for several moments after the filament is energized. There is also the effect of long continued use. A still greater change in an amplifier is occasioned by replacing tubes, even in the case of tubes used in telephone circuits where a high degree of uniformity is required. Because of these variations a vacuum tube amplifier acts in a manner similar to that of a meter in which the resistance of the meter and the tension of the springs are subject to variations.

Transmission measuring sets must be designed to operate on power available at a central office. The voltages of the batteries used for telephone work are allowed to fluctuate between fairly broad limits, the difference between the maximum and minimum voltages being about 25 per cent. This change in voltage usually takes place slowly as the batteries are charged, nevertheless the effect of voltage change in the plate circuit of a vacuum tube is shown directly in its amplification.

It should not be inferred that a vacuum tube amplifier is a continuously fluctuating instrument for this is not the case. After the tubes have been operating for about 15 minutes, and at times when the battery voltage is constant, the amplification factor of an amplifier will remain practically constant for some time. The changes experienced are more of an hour to hour or day to day change, but are sufficient to require the provision of an arrangement whereby the amplifier may be regulated.

The amplifier-detector unit is adjusted by connecting the sending circuit directly to its own receiving circuit, the switches being thrown as indicated by the solid lines of Fig. 9, thus bringing the calibrating network into the circuit. The impedances $D\,D$ and H in the sending circuit are set for the same values as $D_1\,D_1$ and H_1 of the sending circuit at the distant end of the unknown line. As previously stated, $D_1\,D_1$ together approximate the characteristic impedance of the unknown line. The sending circuit currents are both adjusted to the same value. The calibrating network is designed to cause the same loss as is caused by 30 800-miles of standard cable, this being the maximum transmission equivalent for which this set is designed. The reduced voltage from the output terminals of the calibrating network is applied to the input of the amplifier-detector unit, is amplified and rectified and produces a deflection on the meter. By means of the "adjusting dial" the deflection is brought to mid-scale. When this condition holds the apparatus is said to be "calibrated", which means that when a given e. m. f. is applied to the terminals $C\,C$ in the sending circuit, and the resulting current attenuated by the calibrating network, a mid-scale deflection is obtained on the meter.

The receiving circuit now being in a condition to measure the unknown circuit, the switches are operated to the dotted position. By this operation the receiving end of the unknown circuit is connected to the transformer K and the calibrating network is replaced by the adjustable measuring networks. While the power in the unknown circuit at the sending end is the same as that supplied to transformer K in calibrating, the unknown circuit attenuates this power so that the power supplied to the adjustable networks is less than that supplied to the calibrating network. Consequently, to obtain a mid-scale deflection on the meter as in calibrating, the loss in the adjustable networks must be reduced, from that of the calibrating network, by an amount equal to that caused by the unknown

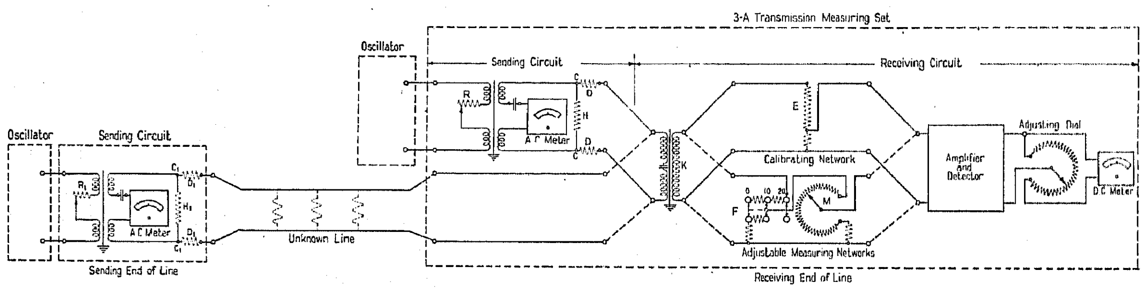

FIG. 9—3-A TRANSMISSION MEASURING SET

Simplified Diagram Showing Arrangement for Straightaway Tests.
Solid Position of switches show corrections for calibrating. Dotted position of switches show corrections for measuring.

circuit. By calibrating the adjustable networks to indicate this reduction, the transmission equivalent of the unknown circuit can be read directly.

As long as the amplification of the tubes remains constant other circuits may be measured without recalibration, the only operation required being the adjusting of the measuring networks until a mid-scale reading is obtained. It is evident that this method of measurement is more rapid and more satisfactory than a method which requires the reading and calculation of currents and equivalents.

In the transmission measuring sets the amplified current is converted into direct current by means of a rectifier or detector, it being customary to use a standard amplifying tube for this purpose. For a given amplifier the deflection obtained on a sensitive d-c. meter using rectified currents is about the same as would hold with a thermal meter if the current were not rectified. The d-c. meter operating on rectified current has an important advantage over a thermal meter in that it operates more rapidly and it is chiefly for this reason that it is used.

There are numerous other details of the 3-*A* transmission measuring set which are of interest and a few of them will be mentioned. Among the most important of these are the transformer *K* and the transformer in the sending circuit. Each of these transformers has an electrostatic shield between the windings, the shield being connected to ground. The purpose of these shielded transformers is to furnish paths of equal admittance to ground from the two wires of the testing circuit, thus preserving the balance of that circuit. The transformer in the sending circuit prevents any unbalanced current being sent into the line, due to unbalances which may exist in the oscillator. The transformer *K*, in the receiving circuit prevents any unbalance currents which may be induced in the unknown circuit from acting on the unbalanced networks and amplifier. It also prevents the receiving circuit from unbalancing the line.

The adjustable network *F* is a constant impedance non-inductive resistance network. The impedance of *F* is 600 ohms at all times as viewed from transformer *K*. This transformer has an impedance ratio of approximately unity, the departure from unity being such that the combination of network and transformer impedance is 600 ohms when measured from the terminals to which the unknown circuit is connected. The impedance of the receiving circuit is normally 600 ohms but can be made 1200 or 1800 ohms by connecting resistances between the transformer and the switch. The impedances in the sending circuit can be given these same values. Two of these impedances differ from those of the 1 - *A* transmission measuring set, being based on conditions now existing in the telephone plant.

The adjustable network *M* is a potentiometer of unusual design. It is made in the form of a circular slide wire, the resistance wire being wound on a mandrel of non-uniform cross-section so that the change in resistance when rotating the drum on which the mandrel is mounted will be logarithmic. The attenuation of a long uniform telephone circuit is represented by the expression $e^{-L\alpha}$ in which L is the total length and α the attenuation per unit length. This is a logarithmic expression and requires a logarithmic variation of the resistance on a slide wire if uniform divisions are to be obtained.

Both networks *E* and *M* are designed on the assumption that the impedance of the amplifier is so high with respect to the networks that it may be considered infinite. This condition is met by the amplifier, the input transformers having high-impedance windings.

The 3 - *A* transmission measuring set operates entirely from the 24-volt central office battery as a source of power for both the filament and plate circuits of the tubes. The energy required by the meter is so small that tubes operating on this low voltage will carry it without danger of overloading them. However the circuit of the amplifier is so arranged that the maximum energy which the tubes will deliver is not sufficient to damage the meter.

Although two meters are shown in Fig. 9 only one is used in the 3 - *A* set. This meter is a microammeter which measures direct current in the detector circuit of the amplifier and, in conjunction with a thermocouple measures, in milliamperes, the current in the sending circuit. It also is used with a shunt to measure filament current. Arrangements are provided so that in case a thermocouple is replaced the combination of meter and couple can be calibrated without the use of an additional meter.

The actual operation of the 3 - *A* set is simple and rapid. The 9 keys shown in Fig. 8 control the sending and receiving circuit impedances, switch from the measuring to calibrating condition, and perform several other minor operations. The two dial handles, mounted concentrically, control the adjusting dial used in calibrating and the network *M* used in measuring. The scale on dial *M* is shown directly above the dial handles.

The dial handle at the extreme left controls a rheostat which regulates the oscillator current in the sending circuit, being designated as R and R_1 in Fig. 9. The vacuum tubes and filament rheostat are in the compartment below this dial handle.

The binding posts and jacks at the extreme right are for the purpose of connecting the circuits to be tested to the measuring set. The jacks are used for cord circuits which terminate in plugs.

4-A Transmission Measuring Set

The 4 - *A* transmission measuring set is a non-portable set designed for routine measurements on toll circuits. It will measure transmission losses as great

as 60 800-cycle miles and will also measure the gain of a telephone repeater. A view of the set is shown in Fig. 10.

The 4 - *A* set resembles the 3 - *A* set electrically in so many details that it is unnecessary to show any schematic circuit. It consists of an identical sending circuit and a receiving circuit which differs only in two important respects, both of these being in the amplifier.

The range of the set being double that of the 3 - *A* set makes it necessary to provide an additional stage of amplification. This extra range means also that the magnitude of the power received from many of the circuits under test is much less than that which is received in the case of the 3 - *A* set. Toll circuits are often exposed to induction from power circuits, sometimes resulting in induced currents of a magnitude which in the case of high transmission equivalents, approaches that of the testing current. Even when the equivalent of the toll circuit is low, such as is usually the case, these induced currents may cause serious errors. The induced currents add to the testing current and cause the toll circuit to appear to have a

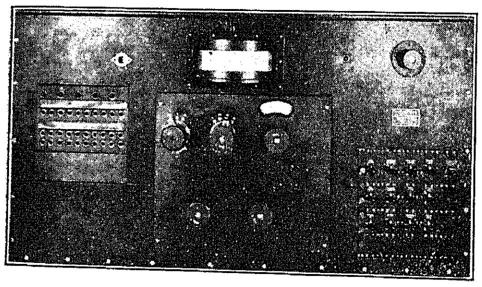

Fig. 10—4-A Transmission Measuring Set

smaller transmission equivalent than it actually has. In order to minimize the effect of induced currents a network of condensers and inductances, known as an electrical "filter" is provided in the amplifier circuit. This filter greatly attenuates all frequencies except those within a narrow band used for testing purposes and enables measurements to be accurately made on nearly all circuits. Special filters may be required when measurements are made over a wide range of frequencies. However, for routine measurements where a single frequency or narrow band of frequencies is employed, little difficulty is experienced when the filter in the 4 - *A* set is used. The filter is located between the first and second tubes in the amplifier. With this arrangement the filter can be changed at will without the resulting changes in impedance appreciably affecting the impedance of the amplifier as a whole, considered from the terminals to which the measuring and calibrating networks are connected. The 3 - *A* set having no filter, is not well adapted to testing circuits which are subject to induction.

In addition to the increased range the 4 - *A* transmission measuring set differs chiefly from the 3 - *A* set in having facilities for enabling the tester to talk over circuits and to the toll operator whose duty it is to establish a connection between the measuring set and the toll circuits.

The vacuum tubes operate from the central office battery as a source of power for the filaments. The plate potential is supplied by the central office battery and an 18-volt flashlight cell battery.

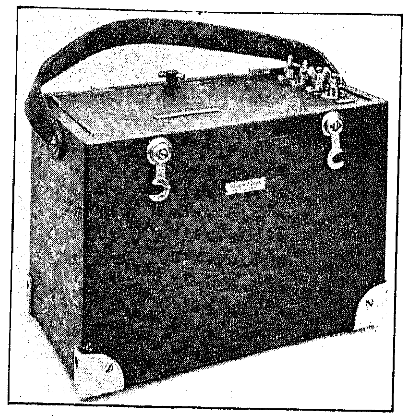

Fig. 11—2-A Oscillator

Oscillators Used with Transmission Measuring Sets

While it is not the purpose of this paper to give a detailed description of the oscillators used with transmission measuring sets a brief statement about each will enable a more complete picture to be obtained. Fig. 11 shows a photograph of the 2 - *A* oscillator which furnishes testing current for the 1 - *A* transmission measuring set. This oscillator contains three dry cells which supply power to operate a vibrating reed type of carbon button generator. A single-frequency current of 800 cycles is obtained.

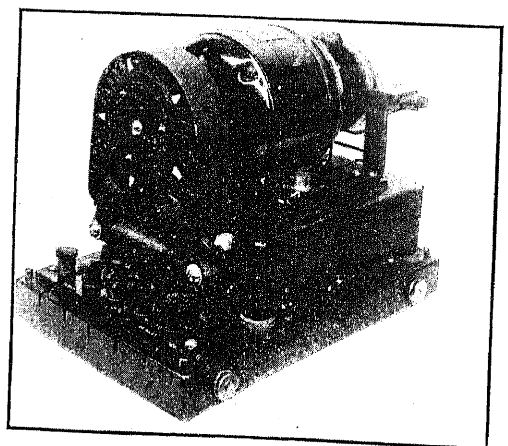

Fig. 12—3-A Oscillator

Fig. 12 shows a photograph of the 3 - *A* oscillator used with the 3-*A* transmission measuring set. This is a motor-driven inductor-alternator which operates

from the 24-volt central office battery as a source of power. It generates a single-frequency current of 1000 cycles. The speed of the motor is regulated by a governor.

Fig. 13 shows a photograph of the 4 - *B* oscillator which is used with the 4 - *A* transmission measuring sets in making measurements at frequencies between 100 and

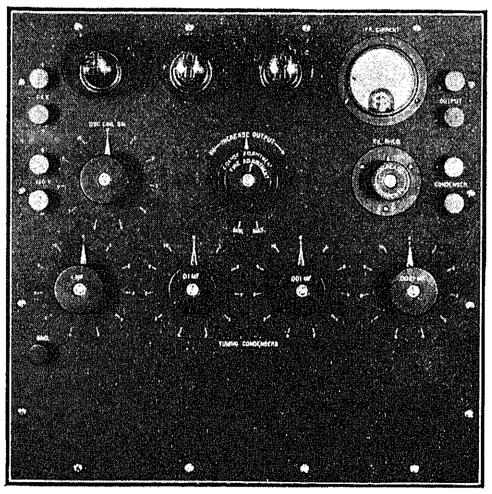

Fig. 13—4-B Oscillator

3000 cycles. This oscillator is of the vacuum tube type and is conveniently arranged so that any frequency may be easily obtained.

Fig. 14 is a photograph of the 5 - *A* oscillator which has been designed for routine measurements on toll

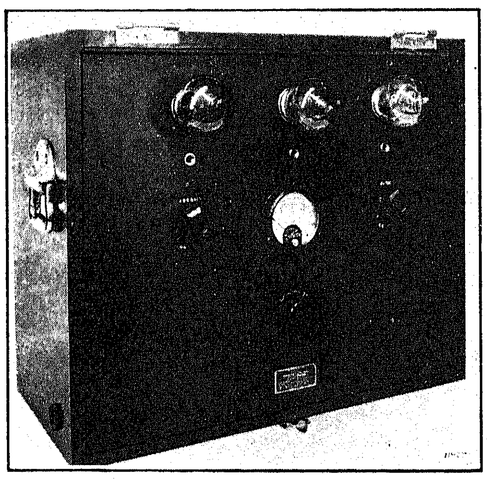

Fig. 14—5-A Oscillator

circuits in cases where it is not necessary to make measurements over a wide range of frequencies. This oscillator is of the "frequency-band" type, the generated current varying in frequency continuously and periodically between 900 and 1100 cycles per second in a manner similar to a siren. The results obtained with this oscillator are the same as the average of a number of single-frequency measurements between these same frequency limits. This oscillator is of particular value in the case of circuits of the type shown in Fig. 2. The frequency is varied by changing the inductance of an inductometer in the oscillating circuit, the shaft on which the moving part is mounted being rotated by a motor.

Conclusion

The development of these transmission measuring sets and associated oscillators has placed in the hands of telephone operating forces convenient and practical tools which enable them to properly maintain the talking efficiency of telephone circuits. Widespread use has demonstrated that the resulting transmission savings and improvements in service are worth many times the cost of doing the work.

Discussion

W. H. Harden: Mr. Best's paper in dealing with the development of telephone-transmission testing apparatus describes three of the common forms of instruments and mentions the oscillators employed to supply the testing current. It may be of interest to outline briefly some of the more important applications which are being made of these testing devices in maintaining the efficiency of telephone circuits in the Bell System.

The three types of instruments considered are the *1-A* and *3-A* transmission-measuring sets which are portable and the *4-A* transmission-measuring set which is designed for permanent installation. The *1-A* type of set has been used extensively during the past eight or nine years and prior to the advent of instruments employing vacuum tubes a majority of the important exchange area circuits in the System were tested with this set. There are at present approximately 150 sets of this type in the Bell System.

When the *3-A* type of set came into use two or three years ago it replaced the *1-A* type for certain classes of work but not all classes. The *3-A* set employing vacuum tubes and visual means of indicating transmission losses allows transmission tests to be made more quickly and accurately than the older type of set. It finds its principal application in testing the larger units of plant, such as common-battery central offices, both manual and machine switching. It is also used to some extent in testing toll circuits. For the smaller units of plant, such as private-branch exchange switchboards, magneto switchboards, etc., where the power necessary for operating the *3-A* set and its oscillator is not available locally, the *1-A* type of set is still generally used. There are at present approximately 100 sets of the *3-A* type in the Bell System.

The testing work using these two types of sets is usually carried on by teams of two men each who travel from office to office. For average conditions involving both large and small central office areas one team can test from 30,000 to 40,000 circuits a year clearing all troubles which are found.

The *4-A* transmission-measuring set, or permanent type, is used extensively in maintaining the transmission efficiency of toll circuits. There are at present some 40 or 50 of these instruments installed at important toll centers about the country. Tests are made straight-away or looped between these centers at frequent intervals and also on the loop basis to the smaller outlying toll centers. The transmission efficiency of toll circuits of all types, such as, 2-wire, 4-wire or carrier, can be quickly checked with the arrangement as outlined.

One thousand-cycle measuring current is now used for all

routine tests on toll circuits and on all tests of exchange area circuits where the *3-A* type of set is employed. The 800-cycle oscillators which are now in the field are still being used for exchange-area testing with the *1-A* type of set and are satisfactory for this work. Whenever it is necesary to check the transmission-frequency characteristics of any circuits the *4-A* type of set is used in conjunction with the *4-A* type of oscillator which can be made to give any desired frequency within the voice range.

This line of transmission-testing apparatus has enabled troubles which cannot be detected by ordinary tests, to be quickly located and cleared. Short-circuited turns in transformers, retardation coils and relays, incorrect wiring of certain equipment and certain defective equipment are examples of the kinds of trouble which require transmission tests for detection. The application of this testing apparatus has gone beyond the experimental stage and routine transmission tests are now an important factor in the maintenance of the telephone plant.

H. H. Nance: At the beginning of the war period there had come into general use vacuum-tube telephone repeaters and toll cables carrying many circuits of different gages and types of loading. The range of telephone transmission was rapidly increasing, and longer and more complicated circuits were being established. Also, at that time the heavy demands for additional circuits, many of which were for the Government's use, necessitated the hurried installation of additional facilities, heavy growth in many instances being to outlying points where comparatively little traffic previously existed. Ofttimes it was a matter of putting together such facilities as could be obtained. All of these factors combined to present a serious problem from the standpoint of transmission maintenance and a heavy burden was placed on the forces maintaining the several thousand circuits then in operation in the plant of the long-lines department of the American Telephone and Telegraph Co. The situation was further complicated, of course, by the fact that skilled personnel required for this work was at a premium.

The development of the *3-A* and *4-A* types of measuring sets brought about a decided improvement in transmission-testing methods. From a practical standpoint the use of a single-frequency measuring current and direct meter readings were of particular benefit and greatly relieved the work of the trained observer. These methods eliminated to a large degree the personal equation, and more accurate and quicker results were obtained and at a very considerable saving in circuit time and testing labor.

The first sets of the *4-A* type were installed at New York and Washington, as these were two long-distance telephone centers of great activity at that time. By means of these sets, the two offices together could measure approximately 200 circuits in a night period of about seven hours, including the directing of repeater adjustments at intermediate points and clearance of trouble found, whereas by the older methods, which had consisted mainly of talking tests using artificial cable, this would have required many nights' work and a testing force several times as large, even if it had been practicable under the conditions to follow the older methods.

Since then, additional measuring sets of the *4-A* type have been installed at a considerable number of points well distributed throughout the long-lines plant and the methods and routine have been systematized to a point where the large number of circuits in this plant is now measured periodically.

A great many circuits, of course, terminate at points where these sets are not installed but measurements on such circuits can be obtained by connecting them through to a testing point by means of other circuits of known equivalents. In some cases the *3-A* or portable type sets, which are likewise well distributed throughout the plant, are used for measuring circuits which cannot conveninetly be measured from the nearest station at which a *4-A* type is installed.

The results obtained in accordance with the present methods and routine have been highly satisfactory, and it has been practicable to maintain the circuits within relatively close limits of variation from the computed equivalents.

The particular work mentioned, that is, the measuring of circuit equivalents from one terminal to the other is, of course, only a part of the transmission maintenance work, since a great many other measurements in connection with that work are made on individual repeaters, networks and sections of circuits at intermediate stations.

R. L. Simpson: The New York Telephone Company is making considerable use of the transmission measuring apparatus which is described in Mr. Best's paper. In each of our seven Plant Divisions we have one or more specially trained forces whose function it is to maintain our circuits up to the proper grade of transmission efficiency. They conduct tests on all of our central-office and private-branch-exchange circuits and equipment.

In general, these forces are equipped with *1-A* and *3-A* transmission measuring sets; and, in addition, we have one of our divisions equipped with a set of the *4-A* type for the maintenance of toll lines which are equipped with through-line repeaters. Periodic transmission measurements are made on a 1000-cycle basis. The *1-A* set is used on small magneto switchboards and P. B. X. switchboards where 24-volt battery supply is not readily available. In our larger central offices where a considerable amount of apparatus is concentrated at one point, and battery supply sufficient to operate the *3-A* set is available, we use this instrument. The *3-A* set, being a visual reading type eliminates to a great extent the personal equation and by eliminating the necessity for personal judgment makes the test quicker.

Before the development of the transmission measuring sets, as described in Mr. Best's paper, a transmission investigation on one of the larger offices in the Metropolitan division of our Company would require an expenditure of both time and money which would probably be out of proportion with the results obtained, but the facility with which circuits can be measured with both the *1-A* and the *3-A* set, makes it possible to accomplish this work both economically and quickly even though the investigation of one of our larger offices would mean the testing of something in the order of 5000 individual circuits, such as cord circuits, operators' telephone circuits, trunks, etc.

These instruments are so designed that they not only indicate the amount of excess loss present when trouble is encountered as compared to the known transmission standard of the particular circuit under investigation, but also make it possible in a large majority of cases to definitely locate the trouble and eliminate it while the circuit is under observation by the transmission tester.

Many of the troubles eliminated by this means are not obvious, nor is their location possible by the usual maintenance methods which are followed by the central office forces. It sometimes happens in the course of the installation of a large number of core circuits involving repeating coils and relays that, due to the amount and complexity of the work required in the installation, some of these coils are installed with one winding reversed. The effect of this would not be apparent on any of the routine tests which are made as these are all on a d-c. basis, but since the repeating coil acting as it does as a transformer with one winding reversed is very inefficient, the effect on the transmission efficiency of the circuit would be detrimental. A transmission measurement of the circuit, together with an investigation, will readily clear up this trouble.

The results of the tests which have been carried out in the New York Telephone Company have been invaluable to us not only due to the fact that we have been able to eliminate some rather obscure troubles, but also due to the fact that we can in many cases anticipate and prevent the reoccurrence of conditions which transmission measurements have shown to react unfavorably on

the performance our circuits are designed to give, through analysis of the reports.

Bela Gati: I always had the opinion that the resulting transmission savings and improvements in service are worth many times the cost of doing the work.

The European government telephone practise has not followed the excellent example of the American Telegraph and Telephone Company as yet; consequently, it is believed to be a great success to speak over the territory of three States in Europe. Government employees are cheap in Europe (about $25 monthly salary for telephone engineers, in Hungary) thus the talking tests are not so expensive.

I agree with Mr. F. H. Best regarding the talking tests. I am glad that the American Telegraph and Telephone Company does not favor it any more, although the company was a great defender of this system and it is the sin of this company (Germans not excepted), that this American method has firm root in our poor Europe.

I wish to complete Mr. Best's objections in one direction. The talking tests do not count in the resonance question. Each 100 kilometers of line has different resonance effects for some speech frequencies, hence the incalculable results obtained by such investigations. In the TRANSACTIONS of the A. I. E. E., Vol., XXX, part II, pages 1679-1680, Multiplex Telephony and Telegraphy by means of Electric Waves Guided by Wires, George O. Squier, Discussion at Chicago, 1911 June 28, I pointed out clearly the resonance effect. If the outgoing current is not in phase with its voltage, then in that case we do not measure the attenuation, but we construct the resonance curve only. I believe this resonance effect disturbs also the results obtained by the various transmission measuring sets. In measuring long lines, the applied filter substitutes in some degree the necessary resonance state at the sending end.

The testing method of central office apparatus is an ingenious one. I quote but one example. Bucarest, the capital City of Roumania was unable to call Budapest, the capital City of Hungary directly, but always needed the intermediation of Brasso. This went on for years. The line signal coils and the clearing-out drop coils were changed by mechanics in Brasso's and this caused the inconvenience. I measured the fault from Budapest, which is a distance of 700 kilometers.

Our subscriber and city lines need measurements also. Paris could not reach Rome because of the interconnecting lines in Lyon. The French administration is not in favor of the secondary lines at all.

In the transmission measuring sets Mr. F. H. Best uses, according to the figures, ohmic resistances only. Kennelly's sending end impedance is needed at the end of the line and this always in the $a - j\,C$ form, $z_0 = \sqrt{\dfrac{r + j\,u\,l}{g + j\,u\,c}}$ compression means a resistance in series with leakance-free condensator. This condensator is missing, therefore the attenuation values differ from the actual ones.

In chapter *3-A*, Transmission Measuring Set, Mr. F. H. Best writes that "no alternating-current measuring instruments were available for measuring the power received at the end of a telephone circuit, which were sufficiently rugged or practicable to withstand the service required of them." I don't know whether there is an instrument which measures the phase displacement at the end of a long telephone line, but I measured the attenuation with my barretter apparatus 15 years ago. I modified Kennelly's barretter, using it in compensation bridge. I measured 100 microamperes telephone-current with Robt. W. Paul's pointer instruments. In loop, I used the same barretter for the outgoing and incoming currents; the d-c. microampere-meter readings were proportional with the square of the current. Keeping the outgoing current always at the same level (50 deg. deflections) from tables previously made the attenuation × length could be read at once.

I measured with these instruments at the bottom of the poles in moonlight, in rainy weather also. We had many auto-accidents, but Paul's microampermeter and the set remained intact.

F. H. Best: As it is not clear as to just what is meant by some of the points raised by Mr. Bela Gati, it may be that his comments can best be answered by referring to the general principle of the method used in the transmission measuring apparatus described in the paper. The measuring apparatus is designed to measure the transmission equivalent of a telephone circuit in such a way that the answer obtained represents the equivalent which this circuit would have were it connected in the center of a long line of its own type. For this reason the apparatus which is connected to the two ends of the circuit under test approximates in impedance the impedance of the circuit under test. For the most precise results this impedance should, of course, be exactly equal to that of the circuit under test. However, the complexity of the testing apparatus necessary to permit an exact impedance match is not warranted. The values of the three impedance terminations included in the testing apparatus have been selected so that in testing any of the circuits in common use in this country some one of these terminations will meet the impedance of the circuit under test sufficiently closely to make the error introduced by the approximation so small as to be negligible. It has not been found necessary to have a reactance component in the impedance of the termination. Loaded circuits are so designed that the reactance component of the impedance at the end of the circuit is small. Non-loaded open-wire circuits have small reactance components. Even in the case of non-loaded cable circuits where the phase angle of the impedance may approach 45 deg. the error introduced by the use of non-reactive terminations is small.

As mentioned on page 4 of the article, the apparatus described is arranged to read the transmission loss or equivalent in terms of the loss in standard cable at a frequency of 800 cycles, the unit being commonly called the "800-Cycle Mile." This cable is the standard cable used for some time in this country, having a resistance of 88 ohms and a capacity of 0.054 $\mu f.$ per mile with $G = 0$ and $L = 0$. This "800-cycle mile" of standard cable, which was a somewhat arbitrary unit, has recently been replaced in the Bell System by a "Transmission Unit," which is described in a paper by Mr. W. H. Martin, published in the June, 1923, JOURNAL of the A. I. E. E. While of approximately the same size as the 800-cycle mile, this new unit is founded on a more logical basis, one unit causing a power attenuation of $10^{0.1}$. New testing apparatus will be calibrated in terms of this unit.

It is not evident what Mr. Bela Gati had in mind in the third paragraph of his comments in which he mentions "talking tests." Such tests have played an important part in development work and have been well justified, but because of the elaborate nature of such tests the American Telephone and Telegraph Company have never used or recommended the making of such tests as a routine method of determining the condition of operating circuits.

In connection with the Barretter mentioned by Mr. Bela Gati for measuring attenuation, it might be of interest to note that the current at the receiving end of a telephone circuit when it is being tested with apparatus described in this paper is in some cases as small as 2 microamperes. The output of the sending apparatus is one milliwatt. For received currents as small as 2 microamperes, it is, of course, necessary to use apparatus employing vacuum tubes. The current levels used in testing are selected to be of the same order of magnitude as the currents impressed on the lines and associated apparatus in telephoning.

Regarding the question as to the proper impedance of receivers, it should be noted that the most efficient receiver is one which matches the impedance of the circuit to which it is connected.

Radio Telephone Signaling

Low Frequency System

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Review of the Subject.—*A signaling system is described which serves to extend to the field of radio telephony the same sort of calling facilities as are now available in wire telephony. By its means the radio attendants or subscribers are enabled to signal each other without requiring that the called party should have to listen with a receiver. The system is intercommunicating with a capacity for a large number of stations on one wave length. It has a range of operation extending as far as a radio channel suitable for the commercial transmission of speech can be reliably maintained, and offers a very satisfactory degree of freedom from interference. Furthermore, it employs simple, standard types of telephone apparatus. Some possible fields of usefulness of the system, its outstanding characteristics, and the apparatus which it employs are described.*

Introduction

In any system of communication, it is desirable that means be provided whereby an attendant or subscriber may be called without requiring that he should have to listen on a particular line. In wire telephone systems, as is well known, means are provided whereby the central office operator may ring a subscriber's bell. In addition, signaling equipment is provided which permits calling the attention of the central office operator at either station on a line to a particular circuit, without requiring her to listen on that circuit. Thus, it is possible for the operator, after she has established the desired connections between certain circuits, to give her attention to others.

The need of such independent signaling facilities for radio systems has not in the past been an important factor, on account of the fact that licensed operators have been required by law to stand watch at all transmitting stations. However, in certain of the various commercial applications of radio telephony which are now being considered, it is undoubtedly true that the licensed operators on continuous watch could be dispensed with, without detriment to the public safety, and, in these instances, signaling facilities would be a distinct advantage.

To anticipate these needs, the development of signaling arrangements suited to practical use with radio systems has been undertaken and much progress has been made. The problem involves the difficulty that the communication channel provided by radio may be less easily maintained in a stable condition than a wire channel. However, it is now felt that it will be practicable to provide signaling facilities adapted to any type of radio telephone service for which a channel suitable for the commercial transmission of speech can be reliably maintained. This will be of considerable advantage, it is thought, in the commercial development of such service.

One promising possibility for the use of this signaling system would seem to be in the field of marine radio telephony. In ship-to-shore radio telephony automatic signaling would serve to lessen the radio operators' duties. Undoubtedly, there would be considerable operating advantage from a commercial standpoint, in placing the ship-to-shore radio telephone service on the same signaling basis as wire lines. This might be expected to become particularly important if the volume of such radio telephone traffic became large.

Among other commercial applications of radio telephony in which signaling facilities are expected to prove advantageous might be mentioned the point-to-point service. A radio telephone system for such a service is being installed by certain interests in Persia as a means of communication between points not reached by wire systems. An automatic signaling system of the type described in this paper is to be employed in this case, and a number of intercommunicating stations are expected to be involved eventually. In the United States there has not, thus far, been much field for the commercial application of point-to-point radio telephone systems. However, where conditions do not favor the installation of wire lines, as in the Persian case, point-to-point radio systems may be expected to find application.

Description of Basic System

The experimental work has indicated that a low-frequency signaling system employing a mechanically tuned alternating-current receiving relay is well adapted to operation in connection with radio systems. Such an arrangement has been shown to be both highly selective and sensitive. These qualities result in giving a very satisfactory degree of freedom from interference, while at the same time the signaling energy level required is so low as to permit operation directly from a radio set employing the smallest available types of vacuum tubes.

With such a system, standard types of apparatus may be employed for the signaling equipment, which is not expensive as compared with the cost of the remainder of the radio equipment. Furthermore, the ordinary

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types of radio transmitting and receiving circuits may be used, without modification.

In testing the system over short distances a simple radio transmitter was used, employing one modulating and one oscillating tube, each of the so-called E type. This transmitter had an output capacity of about 5 watts at wave lengths between 200 and 450 meters.

Any type of radio receiver which is suitable for receiving speech over the particular radio system employed may be used. In the tests, one having two stages of radio-frequency amplification, a detector and one stage of audio-frequency amplification, all of which utilized the small N type vacuum tubes, was employed. The signal-receiving apparatus was operated either from the detector tube or the audio-frequency amplifier, depending upon the signaling range desired.

Fig. 1 shows this scheme in its most simple form which may be made to serve as the basis for a variety of systems, suited to different purposes. As seen in this figure, the outgoing signal is produced by applying an alternating current of a particular frequency to the radio transmitter in the same manner as the speech currents are applied. Modulation of the radio carrier wave at the signaling frequency results. In this particular system, the signaling current frequency employed is 135 cycles.

At the receiving end the radio carrier wave, modulated by the signaling current, is detected in the radio receiver in the usual manner. The output of the detector thus includes a component similar to the signaling current originally sent into the radio transmitter at the outgoing end.

This received signaling current is sent into an alternating-current relay of a type particularly adapted to the purpose, which serves to close a sensitive direct-current relay in a local circuit. This combination of relays acts as a mechanical rectifier.

Fig. 2 represents very roughly the form of the signaling currents at the various stages in the process of operation of the basic system. In this figure, the operation at the transmitting end is shown at A, B and C, while that at the receiving end is shown at D, E, F, and G. Such a graphical representation of these currents cannot, of course, be made accurate in a quantitative sense, but it permits visualizing the character of the signal at different steps in the process of operation.

At A in this figure is shown the direct-current impulse through the sending key and direct-current signal-transmitting relay, which occurs when the key is closed.

The operation of the signal transmitting relay, which occurs when the sending key is closed, supplies 135-cycle current to the radio transmitter as shown at B.

The modulation of the radio carrier wave by the 135-cycle current results in antenna current of the form shown at C. This consists of the radio frequency current varying in amplitude at a rate dependent upon the frequency of the impressed signaling current.

The form of the incoming current in the antenna or loop at the radio receiving station, as shown at D, is the same as that of the transmitted antenna current previously shown at C, but is of course greatly attenuated.

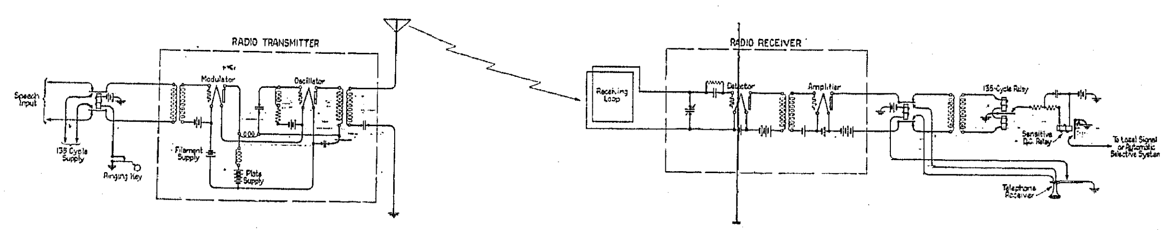

FIG. 1—SIMPLIFIED DIAGRAM OF BASIC SYSTEM FOR 135-CYCLE SIGNALING

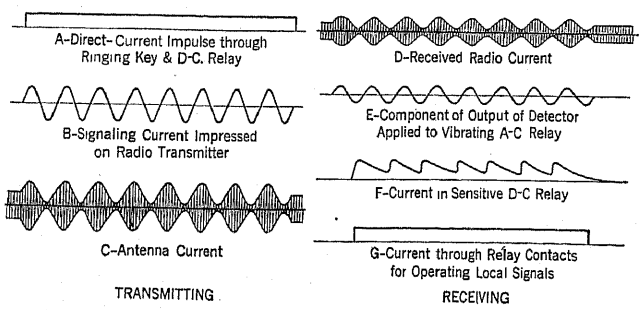

FIG. 2—FORM OF TRANSMITTED AND RECEIVED SIGNALING CURRENTS IN OPERATION OF BASIC SYSTEM

This radio-frequency current, varying in amplitude at a rate dependent upon the modulating frequency of 135-cycles, is impressed upon the detector in the radio receiver. The detector functions in the usual manner, giving as one of the components of its output, 135-cycle current as shown at E. This is similar in form to the signaling current originally sent into the transmitter, as shown at B.

The 135-cycle component of the detector output is sent into the alternating-current relay and causes its mechanically tuned reed to vibrate at a corresponding rate. The vibration of the reed closes the local circuit through its contacts and the sensitive direct-current relay intermittently. This results in a uni-directional pulsating current through the sensitive direct-current relay, as shown at F, and holds the latter closed.

The sensitive direct-current relay, thus operated through the contacts of the vibrating reed, serves to close the circuit through a secondary relay suited to operate directly the local signal. The form of the

current at this point, as shown at G, corresponds closely to the original signal-transmitting impulse through the sending key and direct-current relay, as shown at A. The operation of the secondary direct-current relay in this manner constitutes the conclusion of the final step in the functioning of the basic signaling system. Subsequent steps depend upon the type of signaling facilities desired, as will be described later.

Operation of Alternating-Current Relay

The reliability and range of operation of this system are due in a large measure to the characteristics of the particular type of alternating-current relay used.

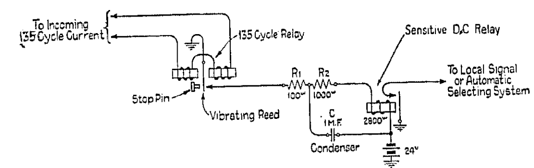

Fig. 3—Secondary Circuit of 135-Cycle Relay

This relay is unusually sensitive, as it will operate on as little power as 30 microwatts corresponding to a current of about 0.25 milliampere. The selectivity of the relay is such that a 4 per cent change in the frequency of the signaling current necessitates doubling the current to give equally effective operation of the relay.

Fig. 3 shows the arrangement employed in associating the vibrating reed relay with the local circuit. It is seen from this diagram that the vibrating reed intermittently closes a circuit associated with a sensitive direct-current relay in such a way that the latter is held closed. The direct-current relay is held operated without vibration, as long as the reed of the a-c. relay is vibrating at the frequency of the signaling current.

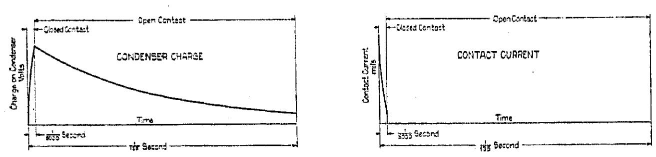

Fig. 4—Characteristic Curves Showing Operation of Secondary Circuit of 135-Cycle Relay

The action of this particular local circuit is such that effective operation of the a-c. relay may be obtained with a very small 135-cycle current. The condenser connected in parallel with the sensitive direct-current relay and the resistance in series with the condenser are so chosen in value that the condenser will take an effective charge from the local battery in a short period of time. Thus, but a small amount of work is required to be done in the form of contact pressure by the vibrating reed to charge the condenser to the degree necessary to operate the sensitive direct-current relay by discharging through the winding of the latter.

The curves shown in Fig. 4 indicate approximately the relation between the condenser voltage and contact current over a period of time corresponding to one cycle, that is 1/135 of a second. From these curves it is seen that the contacts need be closed for only a small fraction of the cycle. Thus, little energy need be expended to effect the operation of the local relay.

Fig. 5 is a typical curve showing the performance of the relay in relation to the frequency of the signaling current under average conditions of adjustment. The mechanical tuning of the reed is largely responsible for

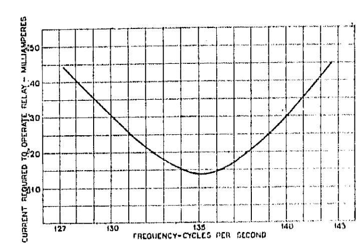

Fig. 5—Typical Curve Showing Frequency Characteristics of 135-Cycle Relay Under Average Conditions of Adjustment

its selectivity. The reed is adjusted so that the natural period corresponds closely to the frequency of the signaling current. It is thus very selective and is relatively free from the ordinary sources of electrical interference such as those caused by telegraph signals, static, voice currents, etc.

Fig. 6 shows the general structure of the relay. The relay is provided with plug connections so that it may be inserted in the circuit or removed from it readily, without requiring soldering of connections. Thus, it is convenient to make necessary adjustments of the

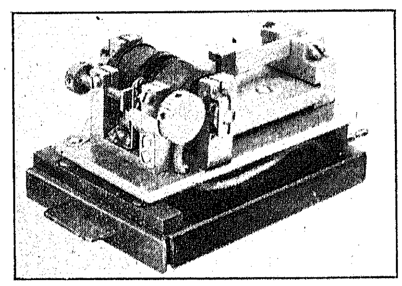

Fig. 6—Structure of 135-Cycle Relay

relay separate from the circuit with which it may be associated in service. The relay is well protected from interference due to mechanical vibration by padding in the mounting. This eliminates rigid mechanical connection between the relay and its external support thereby preventing loss of energy. A stop-pin is also provided which prevents undue vibration of the reed due to transient impulses or excessive currents.

Signaling Current Frequency

The signaling current frequency used in the practical application of this system, as previously mentioned, ha

been 135 cycles. This has had the advantage of being a frequency which is low enough to be relatively free from the various kinds of interference experienced in radio systems and at the same time permit the use of simple and reliable apparatus. The fundamental features of the system, however, are such as to permit its being adapted to use with other low frequencies, if such should be desired.

The inherent advantages in the use of a frequency well below the ordinary voice range for telephone signaling, may be readily made effective in a radio telephone system, since there is ordinarily nothing in such a system to discriminate against the lower frequencies. In fact, as far as the radio characteristics of the system are concerned, what difference there may be is in favor of the lower frequencies, because of the close approach of the modulated carrier wave, in this case, to the carrier frequency itself. While this factor may not be practically important with the short-wave systems now commonly used in radio telephony, it is more likely to be of account with longer waves.

The high degree of freedom from interference which is obtained with this system is in a measure due to the use of a frequency as low as 135 cycles for signaling. This is particularly true with respect to the interference caused by spark and I. C. W. telegraph. The tones from these sources, being within the audible range, are likely to interfere with any system tuned to a normal voice frequency. 135 cycles, however, being well below these frequencies, permits the effective use of both electrical and mechanical tuning in the signal-receiving apparatus to discriminate against interfering currents. This point was well demonstrated in the signaling tests in which it was found that radio telegraph signals similar to those from an I. C. W. or spark transmitter would cause the received speech to become unintelligible when the energy level of the interference was only 20 or 30 per cent of that required to cause the signaling system to fail.

Another advantage in low-frequency signaling occurs when the signaling apparatus is desired to be bridged across the talking circuit. In this case greater efficiency results in the use of 135-cycle signaling, by reason of the fact that it is possible with this low frequency to pass a large proportion of the received signaling energy into the signaling circuit. If a much higher signaling frequency were used, such that it occupied a more important part of the speech range, an undesirable loss might be caused by an efficient signaling circuit.

135-Cycle Supply

The highly selective signal-receiving apparatus which is used in this system permits a greater sensitivity in reception when the outgoing signaling currents are kept closely to the desired frequency. This has been accomplished practically by the development of a 135-cycle interrupter capable of maintaining close regulation and an output of good wave form.

Fig. 7 shows the circuit arrangement of the interrupter which has been developed for commercial use. This employs a tuned vibrating reed actuated by an electromagnet when direct current is applied. The contacts on the reed being in series with the battery circuit, vibration of the reed is effected in the manner of an ordinary buzzer. The actuating circuit of the

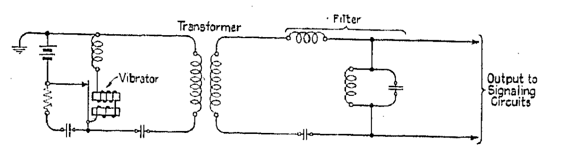

Fig. 7—135-Cycle Interrupter Circuit—Simplified Diagram

vibrator is bridged by the primary side of a transformer in series with a condenser, the secondary side of the transformer being connected to a filter for suppressing harmonics in the output.

Fig. 8 shows the structure of the vibrator for the interrupter. Since the output frequency of the interrupter for a given applied voltage depends upon the

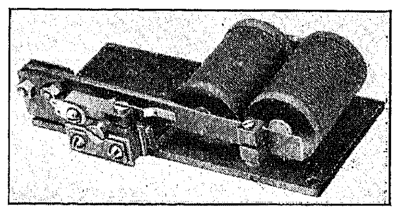

Fig. 8—Structure of Vibrator for Interrupter

natural period of the reed, means are provided for closely adjusting the latter. It will be noted that this consists of a weight, the position of which along the reed is adjustable. This weight is used in making the initial adjustment of the vibrator to give 135 cycles at the normal battery voltage, but variations in adjustment are not required in subsequent operation of the interrupter.

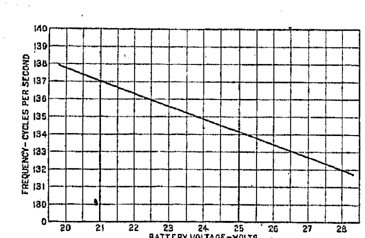

Fig. 9—Typical Curve Showing Relation between Frequency of Output and Applied Voltage for 135-Cycle Interrupter

This matter of frequency regulation is a most important one in the performance of the interrupter. As shown in Fig. 9, the frequency of the output may vary several cycles for a variation in the applied voltage of 20 to 28 volts. This voltage variation represents the maximum which might be expected to occur with an 11-cell storage battery operated on a charge and dis-

charge basis, without any regulating device. If the battery voltage is maintained within closer limits than 20 to 28 volts, as may be accomplished by various means, it is seen from the curve in Fig. 9 that correspondingly closer regulation of the output frequency

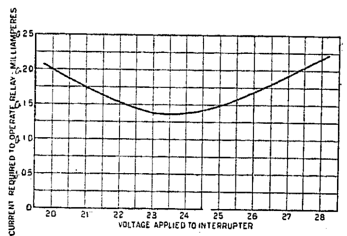

Fig. 10—Typical Curve Showing Effect of Change in Frequency Due to Variation in Voltage Applied to 135-Cycle Interrupter on Current Required to Operate 135-Cycle Relay Under Average Conditions of Adjustment

may be obtained. For example, if duplicate batteries of adequate capacity are available, the applied voltage may be kept within limits such that the output frequency will vary only about one cycle.

average conditions, affect the current requirements of the receiving relay. By keeping the battery voltage within fairly close limits, reliable operation on smaller currents may be depended upon. Where desired, advantage may be taken of the closer frequency regulation obtained in this manner to secure greater signaling range.

The maximum output capacity of this interrupter is about three-fourths of a watt. The output voltage varies from about 25 volts at no-load, to 20 volts when the load is 35 milliamperes and 12 volts for a load of 60 milliamperes.

In certain of the signaling tests over short distances, a simplified form of the above interrupter was used. Under the testing conditions in this case, a smaller 135-cycle output was sufficient for the purpose and it was possible to reduce the size of the filter for the interrupter and to operate it with a 6-volt battery instead of a 24-volt battery.

Automatic Selective Signaling System

This signaling system may be adapted readily to automatic selective operation, whereby any one out of a

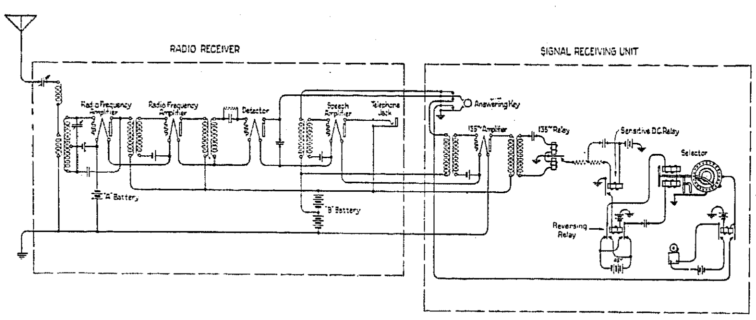

Fig. 12—135-Cycle Selective Signaling System—Circuit Arrangement of Receiving Apparatus

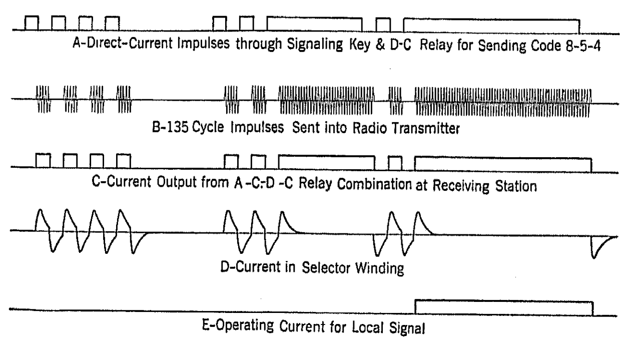

Fig. 13—Form of Transmitted and Received Currents in Operation of Selective Signaling System

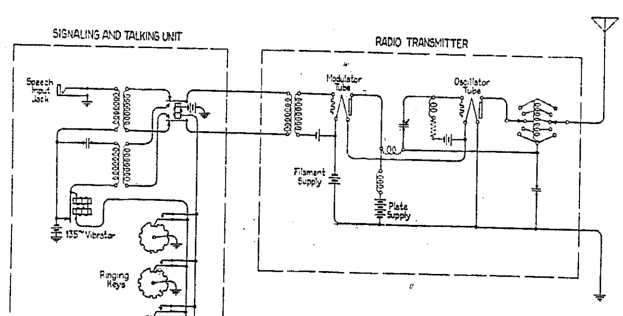

Fig. 11—135-Cycle Selective Signaling System—Circuit Arrangement of Transmitting Apparatus

Fig. 10 combines the voltage-frequency curve for the interrupter and the frequency-current characteristic of the relay to indicate directly how variations in the voltage applied to the interrupter and the consequent changes in output frequency may, under

number of stations on the same wave length may be signaled individually, or all may be signaled simultaneously. This form of the system is expected to be particularly useful in radio telephony, since a number of intercommunicating stations may often be involved as in the marine and point-to-point services. The illustrations which follow will give a picture of a typical arrangement of the automatic selective system.

From Fig. 11 the operation of the circuit at the transmitting end is seen to be as follows: The signaling key for the station which it is desired to call is operated and serves to produce a series of direct-current impulses in the form of a code corresponding to that assigned to the called station. For example, if a certain station is desired whose code signal is 8-5-4, a series of direct-current impulses suitable to indicate this number are sent out. These direct-current impulses operate at first a direct-current signal-transmitting relay which connects to the radio transmitter corresponding impulses of 135-cycle current, in such a manner as to modulate the outgoing carrier wave with current of this frequency

in a coded series of impulses each similar to the single impulse described in connection with the basic system.

As shown in Fig. 12, at the receiving end the radio receiver serves to detect the 135-cycle signals in the manner described in connection with the basic system, and these signals are sent into the alternating-current relay which operates in accordance with the code originally transmitted. This causes the sensitive direct-current relay to operate in accordance with the code and to produce in a local circuit direct-current impulses corresponding to the original impulses sent out at the transmitting end. These impulses then serve to operate a reversing relay which reverses the potential applied to a condenser in series with a stepping mechanism known as the selector, such that when the proper combination of impulses is received, the mechanism closes a circuit arranged to operate the desired local signal.

The functioning of the selective system may be followed in greater detail, by considering the form which the signal takes at various stages. Fig. 13 shows graphically the general form of the signaling currents in the successive steps necessary to transmit a typical code signal, such as 8-5-4.

The form of the original code signal, as applied by means of the sending key to the winding of the direct-current signal-transmitting relay, is shown at *A*. It is seen from this that the number of impulses sent is equal to each of the numbers in the code, either the making or breaking of the current (and consequently either the closing or opening of the signal-transmitting relay) counting as one impulse. This is due to the use of reversals in the final receiving operation, as will be explained later.

Each time the signal-transmitting relay is closed by the direct current applied through the sending key, 135-cycle current is supplied through the contacts of this

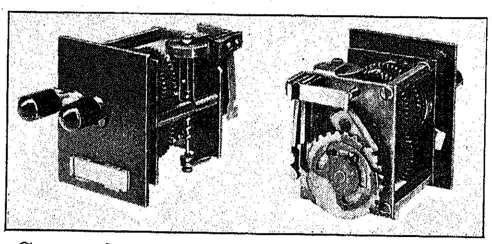

FIG. 14—CODE SENDING KEY FOR PARTICULAR STATION

relay to the radio transmitter, thus the signaling currents impressed on the radio transmitter for the code signal 8-5-4 are as shown at *B*.

These coded impulses of 135-cycle current serve to modulate the transmitted radio wave so that the antenna current is similar to that previously shown for the basic system, excepting that the 135-cycle modulation of the carrier is interrupted in accordance with the code.

The incoming current at the receiving station is similar in form to the transmitted antenna current.

The component of the detector output which is useful in transmitting the signal consists in 135-cycle impulses which are similar in form to those sent into the radio transmitter, as previously shown at *B*.

The operation of the alternating-current relay, as previously explained, serves to operate the sensitive direct-current relay in accordance with the transmitted code. The contacts of this relay close and open the circuit through the reversing relay. The current in the winding of the reversing relay is as shown at *C*.

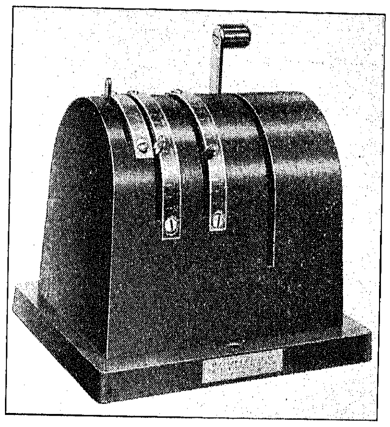

FIG. 15—MASTER CODE SENDING KEY

The reversing relay reverses the potential applied to the condenser in series with the selector winding. The selector is arranged so that it operates on the charge and discharge of this condenser. The operation of this relay reverses the battery connections to the condenser and selector, causing the condenser to discharge and charge to the opposite polarity whenever the relay picks up or releases, and it is this charging current which operates the selector. These reversals take place in accordance with the original code and produce a current in the selector winding as shown at *D*. When the proper code is transmitted, the selector closes a local circuit causing direct current, as shown at *E*, to operate a signal. The final impulse of the series of impulses shown at *D* causes the selector to release and return to normal.

The apparatus employed in this typical arrangement of the system is of a type which has been used in connection with railway dispatching systems and is consequently available in a reliable commercial form. The selecting apparatus consists of the code sending key and the receiving selector mechanism.

Fig. 14 shows the structure of one type of sending key used for this purpose. This key is set to give the desired code for a particular station. Thus, at the sending end it is necessary to have a separate sending key for each station to be signaled.

Fig. 15 shows the structure of a master sending key. This is arranged so that it may be adjusted by the operator to send out any desired code in the system.

Thus, where this type of key is used, but one is required at the sending end for signaling all stations.

Fig. 16 shows the structure of the receiving selector mechanism with the cover removed. This piece of apparatus consists essentially of a polar relay with a ratchet attachment so arranged that successive operations of the relay, at the proper speed, cause the stepping around of a contact wheel. Stop-pins are provided at certain points to prevent the contact wheel from returning to its normal position when the regular sequence of stepping is interrupted at these points. Any interruption of the regular sequence of stepping when the contact wheel is at any other point causes it to release. When the contact wheel has operated over 17 steps a contact is made which operates a signal. Thus, it is seen that to operate the selector so as to give a signal, the direct-current pulses must occur in the proper sequence and the pauses between the groups of pulses must occur at points where stop-pins are located.

The apparatus which has been described is arranged so that as many as 78 stations on one wave length may be signaled separately. The same apparatus can also be arranged so that at each one of the 78 stations, four separate supplementary stations can be individually signaled. For example, if a marine radio telephone system is involved, any one of four stations on each of 78 boats could be signaled separately. With a further slight modification in the apparatus, the same system is capable of being extended to permit the separate signaling of any one out of more than 200 stations. In each of these cases it is also possible to signal all of the stations simultaneously, when desired.

Fig. 17 shows how a system like that which has just been described may be arranged for two-way operation. This embodies a duplication at both ends of the features of the one-way system. With such an arrangement, any one station of a number operating on a given wave length may signal any other station in the same system, without calling in stations not desired. Such a system might be used, for example, between ships equipped with radio telephone systems or between the ships and the shore, where signaling any one of several shore stations might be desired. It might also be used for intercommunication between a number of fixed stations in a point-to-point system.

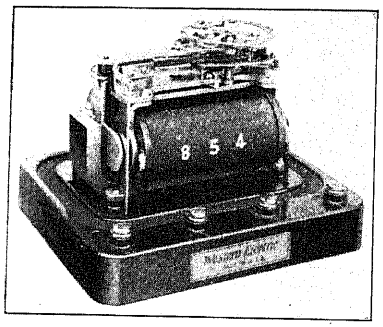

FIG. 16—RECEIVING SELECTOR MECHANISM

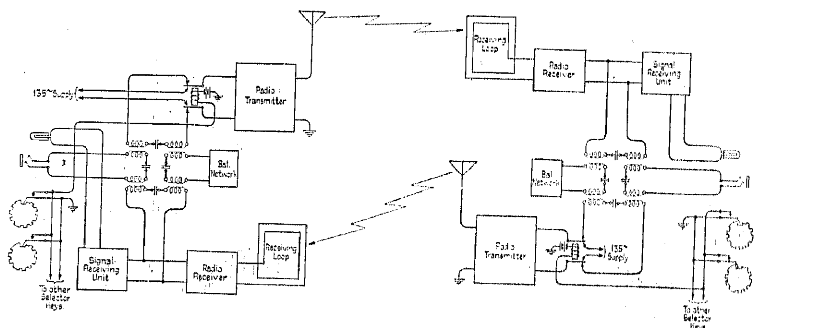

FIG. 17—SIMPLIFIED DIAGRAM OF TWO-WAY RADIO SYSTEM ARRANGED FOR AUTOMATIC SELECTIVE SIGNALING

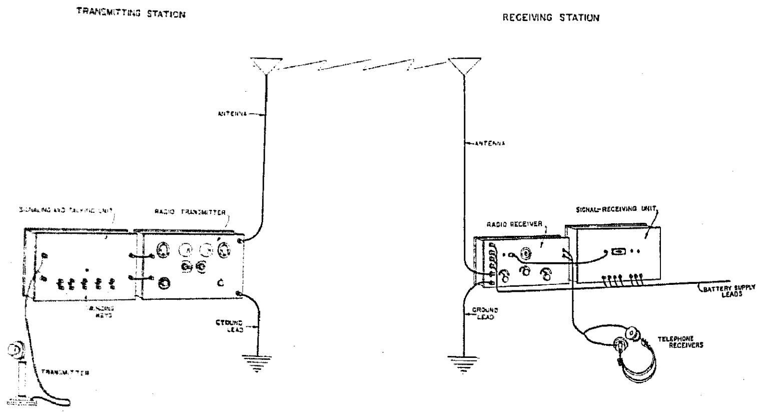

FIG. 18—135-CYCLE SELECTIVE SIGNALING SYSTEM—TYPICAL ARRANGEMENT OF APPARATUS FOR ONE-WAY OPERATION

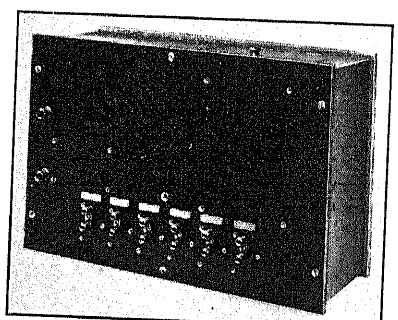

FIG. 19—SIGNALING AND TALKING UNIT

FORM AND ASSEMBLY OF EQUIPMENT

In the possible uses of this signaling system which have been mentioned as likely to have the most immediate commercial application, the signaling apparatus is a part of the radio attendant's equipment rather than the telephone subscriber's apparatus. The apparatus chosen for the purpose is, therefore, in a form which is well suited to central station use, although it has no features which would prevent its being used conveniently at a subscriber's station if the service required it.

In assembling the signaling equipment a uniform panel arrangement has been developed, with the idea that the various units might be used interchangeably in meeting the requirements of different types of installations. To this end, all apparatus has been mounted on panels

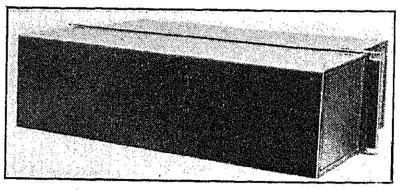

FIG. 20—ASSEMBLY OF 135-CYCLE INTERRUPTER

of a uniform length of 19 inches. The height of the different panels has varied according to the amount of apparatus in each unit, but this vertical dimension has in each case been a whole multiple of the basic dimension of 1¾ inches.

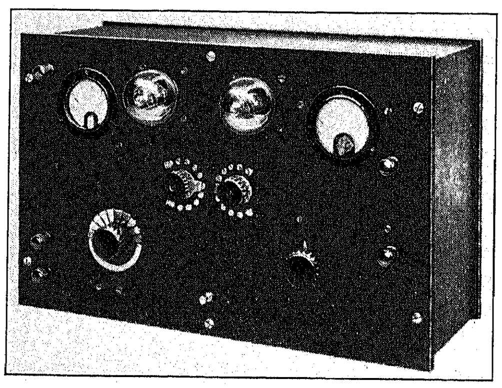

FIG. 21—EXPERIMENTAL RADIO TRANSMITTER UNIT

By this means it is possible to mount the units in any desired manner. If, for example, it is desired to locate the radio and signaling apparatus on a desk or table, each equipment unit constitutes the front panel of a separate box. Fig. 18, shows the various units required

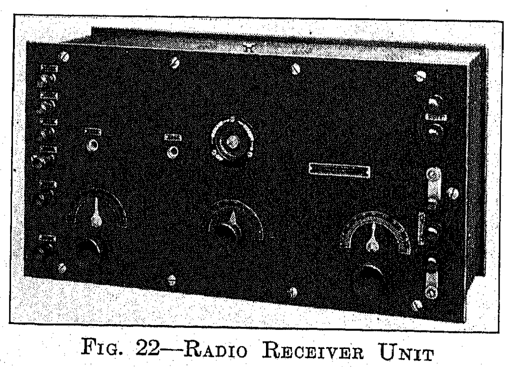

FIG. 22—RADIO RECEIVER UNIT

to make up a complete one-way system mounted in this manner.

Fig. 19 shows in more detail the signaling and talking unit of this group. This unit includes the sending keys, a simplified form of 135-cycle interrupter suited to use with the experimental set, and a six-volt dry cell battery for operating the interrupter and signal-transmitting relay. It also includes an induction coil and talking battery for use with a telephone transmitter.

Fig. 20 shows a separate 135-cycle interrupter unit which also employs the standard panel assembly. Removable covers are provided on both the front and back of the panel to protect the apparatus from

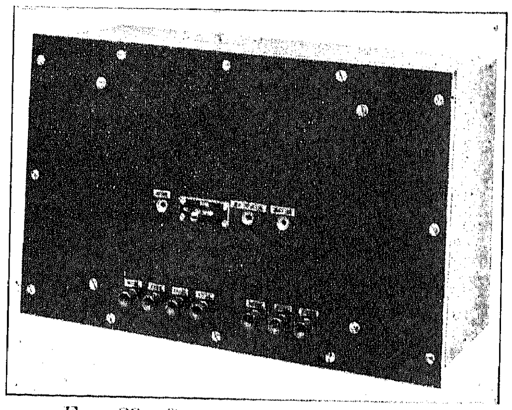

FIG. 23—SIGNAL RECEIVING UNIT

mechanical injury. This interrupter is of a type adapted to commercial use with radio transmitters of various forms and output capacities. It employs the circuit arrangement previously described in the section on "135-cycle Supply."

Fig. 21 shows an experimental radio transmitter unit of the type which was used in making certain of the

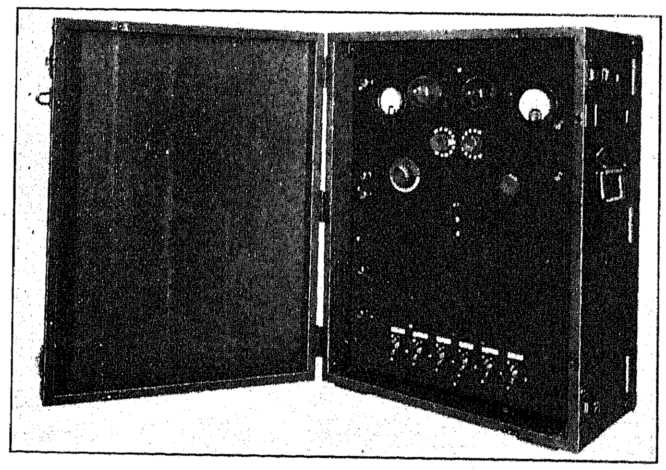

FIG. 24—ASSEMBLY OF EXPERIMENTAL RADIO TRANSMITTER WITH SIGNALING AND TALKING UNIT IN CABINET

signaling tests over short distances. As explained in the section describing the basic system, this transmitter has an output of about 5 watts at 200 to 450 meters.

Fig. 22 shows the assembly of one of the radio receiver units used in the tests. This is of the radio frequency amplifier type employing *N* tubes, such as would be employed in a commercial installation as in the Persian point-to-point system.

Fig. 23 shows the signal-receiving unit for automatic selective signaling. This includes the alternating-current relay, the selector mechanism and the necessary associated relays to respond to the incoming signal when connected to the detector tube of an ordinary radio receiver. This unit also includes an N tube amplifier for use when it is desired to secure the maximum signaling range. In this unit, terminals are provided on the rear of the panel in addition to the binding posts on the front, so that when the panel is removed from the box, rear wiring can be used if desired.

If it is desired to associate several panels together in one cabinet, each panel is detached from the box and mounted on the vertical supports which are provided in the standard cabinets. Fig. 24 shows the experimental radio transmitter together with the signaling and talking unit assembled in one of these standard cabinets which are designed to house equipment of any type mounted on standard panels. Fig. 25 shows the radio receiver and signal-receiving unit in a similar cabinet.

In some cases it may be desirable to mount the radio receiving apparatus, signal-receiving unit, signaling and talking unit and other associated apparatus together on one rack, without employing cabinets. Fig. 26 shows how this might be accomplished, the various panels of standard length being assembled on a standard rack suited to mount many different types of units employing

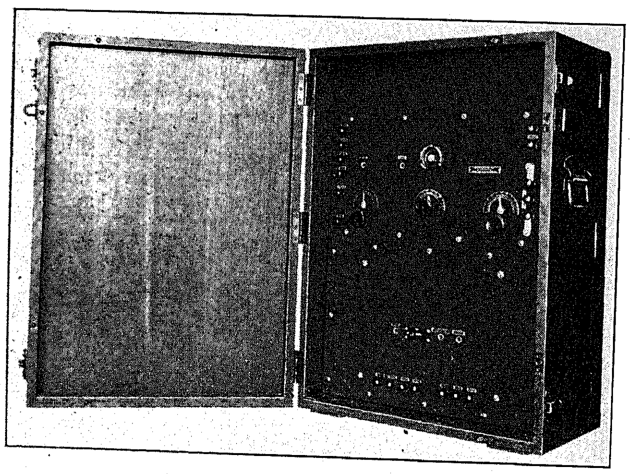

Fig. 25—Assembly of Radio Receiver and Signal Receiving Unit in Cabinet

this type of design. In this case, the radio transmitter is not included with the receiving apparatus, as it is assumed that in a commercial installation it may be of a larger and more powerful type than the experimental one previously shown for purposes of illustration. The 135-cycle interrupter is also shown as a separate unit, since such a separate interrupter may be required if the output of a small interrupter of the type which was included in the signaling and talking unit for convenience in the experimental work, is insufficient.

Review of System Characteristics

In considering the application to radio telephony of the signaling system which has been described, it is of interest to review certain of its outstanding characteristics. These may be briefly summarized as follows:

(1) The selectivity of the signaling system is such that interference of the kinds ordinarily experienced in radio telephony will make speech unintelligible before it will cause the signaling system to fail. Due to this high degree of selectivity, the signaling apparatus will operate if the radio system is adjusted so as to permit commercial transmission of speech, when the field strength is as low as it is desirable to use for the latter. With the types of radio apparatus which have been described, reliable operation of the signaling system has been secured with a field strength as low as 100 microvolts per meter.

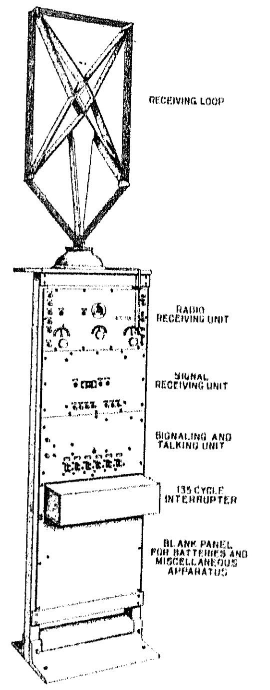

Fig. 26—Typical Assembly Arrangement for Panel Mounted Radio Receiving and Signaling Apparatus on Standard Rack

(2) The sensitivity of the signal-receiving apparatus is such that the energy output obtainable from the smallest available types of vacuum tubes is more than sufficient to operate it satisfactorily. The type N vacuum tube can give an output of several hundred microwatts when operated as an amplifier, while the sensitive 135-cycle relay used in this signaling system will operate with as little as 30 microwatts.

(3) The system readily permits automatic selective signaling, whereby any one station out of a number on the same wave length may be signaled separately, as well as being adapted to use where only one sending and one receiving station are concerned. Seventy-eight stations on one wave length may be signaled separately with the apparatus which has been described while by employing other apparatus of a similar type which is at present available, over 200 stations may be signaled separately. The system is also adapted to permit the simultaneous signaling of all of the stations which may be included in the system on one wave length.

(4) The form and arrangement of the signaling apparatus are practically independent of the type of radio service, the power capacity of the radio system and the wave length used. It is applicable to ordinary radio transmitters and receivers without requiring modification of the radio equipment, and at the same time is simple in form and not high in cost as compared with the remainder of the radio equipment.

Telephone Transformers

BY W. L. CASPER

Research Laboratories of the American Telephone and Telegraph Company, and the Western Electric Company, Incorporated.

Review of the Subject.—In the communication art transformers are used to transfer inductively the energy of speech currents from one electric circuit to another. In addition to this primary function which must be efficiently performed without distorting the speech significance of the transmitted energy, there is a variety of secondary functions such as making possible the super-position of phantom circuits on ordinary telephone circuits, discriminating between speech and telegraph or signaling frequencies, isolating circuits carrying direct current, and preventing inductive interference between adjacent circuits.

A discussion is presented of the frequency range over which telephone transformers must operate efficiently in transferring energy between two circuits and the three most common limiting impedance combinations of these circuits, namely, both circuits resistances, one circuit a resistance and the other a positive reactance and one circuit a resistance and the other a negative reactance. The efficiency with which energy is transmitted is measured by comparison with an ideal transformer which is one that introduces no losses and has the best ratio to connect the two circuits. In studying its action the transformer is replaced by its equivalent T network which affords a ready means of analyzing its losses. The variation of the transformer losses with frequency is discussed for the three above mentioned combinations of circuit impedances and characteristic curves are shown for transformers of different mutual impedances. Characteristics are also given showing the operation of the input transformer into the vacuum tube as the mutual impedance and the transformer ratio are varied. The circuit conditions of the input transformer represent a common special case of the third combination of circuit impedances.

The mechanical construction of various transformers is shown, namely, that of the ordinary battery supply repeating coil, of the telephone induction coil, and of three more recent types of transformers used principally in various vacuum tube circuits such as telephone repeaters, carrier frequency and radio circuits. These transformers are all constructed so as to give the desired accuracy of speech transmission under their particular circuit conditions. The climatic conditions present in the widely distributed telephone plant have been carefully considered and the transformers designed to maintain their initial efficiency over a long period of years.

INTRODUCTION

THE transformers used in the telephone plant are required to transmit speech or signaling currents from one electrical circuit to another in such a way as to obtain maximum transfer of power. They differ in their required action from power transformers in two main respects. They have to transmit milliwatts efficiently instead of kilowatts and must operate efficiently under a variety of conditions of voltage, frequency, etc., instead of under single fixed conditions. These various requirements of operation make it necessary in designing a telephone transformer to proportion it differently from a power transformer.

The requirements of circuits in which telephone transformers are used make it necessary to consider their efficiency in detail over a wide range of frequency and at the same time make sure in each particular case that they have characteristics apart from this efficiency which will enable the circuits to function properly. Such other characteristics, one or more of which may be required of a transformer, are:

1. The efficient transmission of telephone currents while carrying super-posed direct current as in the cord circuit battery supply repeating coil.

2. A high degree of impedance balance between the windings in order: (1) to avoid unbalancing the circuit and rendering it subject to noise or cross-talk troubles; (2) to limit cross-talk in closely associated circuits as in the case of the phantom circuit repeating coil in which the balance required is very precise, the two phantom circuit windings being balanced to about 0.01 per cent; or (3) to prevent sustained oscillations or singing in a two-way telephone repeater as in the case of the repeater output transformer.

3. High efficiency as a low-frequency power transformer, for example, at 16⅔-cycle signaling frequency, as well as high efficiency as a telephone speech frequency transformer.

4. Low efficiency as a power transformer at the frequencies of interruption of direct current used in Morse telegraph as well as high efficiency as an audio frequency transformer. This low power efficiency is necessary in order to reduce troubles, such as, noise interference, false operation of relays and acoustic shock when the same lines are used for both telephone and telegraph.

5. Impedance transformation closely approximating that obtained with an ideal transformer as in the line transformers in circuits in which two-way telephone repeaters are employed. By impedance transformation is meant the modification of the line or network impedances when viewed through the transformer.

6. Impedance transformation for a pair of transformers alike over the transmitted audio frequency range for use as the line and network transformers in two-way repeater operation.

7. Stable impedance transformation even when having been subjected to large magnetizing forces as in the line transformers in two-way telephone repeater circuits in order to maintain the required balance between line and network under all conditions of service.

8. Minimum production of harmonics due to the magnetization characteristics of the iron which would cause interference in the normal transmission frequency range, as in the line transformers on circuits which are used for the transmission of Morse telegraph in addition to speech.

9. To isolate conductively one portion of a circuit from another as in transformers used to connect grounded to metallic lines or to isolate subscriber sets

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from the telephone lines which parallel high-tension power lines and which may therefore be exposed to large inductive disturbances.

10. To maintain the usual conductive isolation and furnish a path to ground for longitudinal currents as may be obtained with a transformer with a shield between primary and secondary windings. By a longitudinal current is meant one which flows along a circuit and returns by some path exterior to that circuit.

As the primary function of the telephone transformer is to transmit telephonic speech efficiently, this paper will be limited to this part of the subject, although the complexity of the telephone plant rarely permits a transformer to be free from the necessity of meeting one or more secondary requirements such as those mentioned above. Two winding transformers only will be discussed.

Frequency Requirements

In the ordinary telephone circuit a range of frequency of about 200 to 2500 cycles is allotted to the transmission of speech, both of these limits varying somewhat with the type of circuit. In the case of long distance lines in which it is desirable to utilize the circuits to the best economic advantage, the frequency range below 3000 cycles is allotted to speech, signaling and telegraph and above that frequency to the transmission of carrier telegraph and telephone. A transformer used for the transmission of speech currents is required to operate efficiently over the entire range of frequency transmitted.

With the present intensive use of the telephone lines, transformers may be required to transmit low-frequency signaling of 16⅔ or 20 cycles, composite ringing of 135 cycles, speech from 200 to 3000 cycles and carrier from 3000 to 30,000 cycles. This carrier frequency is divided into a number of frequency channels which are used for separate telegraph or telephone circuits. A carrier range from 3000 to 10,000 is generally used for telegraph while with carrier telephone a frequency range from 6000 to 30,000 is used.[1] In certain radio telephone circuits transformers are required to operate at frequencies of the order of 50,000 to 100,000 cycles and in radio frequency amplifiers at approximately 1,000,000 cycles. Telephone transformers may be required to operate at any one of these frequencies or frequency bands or they may be required to transmit efficiently two or more of them. Illustrations of this are the ordinary phantom circuit repeating coil which transmits low-frequency (16⅔ cycles) signaling and composite ringing (135 cycles) as well as speech, and the transformers which connect the Key West-Havana cable to the shore lines and which are designed to transmit carrier telegraph up to about 6000 cycles as well as ordinary telephonic speech.

1. Colpitts and Blackwell: "Carrier-Current Telephony and Telegraphy," Trans. A. I. E. E., Vol. 40, page 301.

In radio transmitters used for broadcasting and in Public Address Systems[2] where it is desired to transmit music and to reproduce accurately the exact quality of the speaker's voice, it is necessary that any transformers in the circuit operate efficiently at frequencies at least as low as 50 cycles and as high as 5000 cycles.[3]

In the following the operation of transformers over the voice range of frequencies only will be discussed but it is to be noted that the principles involved apply to transformers for carrier and radio frequencies as well.

Impedance Requirements

Another important factor in determining the design and operation of the transformer are the impedances of the circuits which the transformer connects. The circuit impedances, which are the impedances as measured from the place where the transformer is to be located, may be but a few ohms in magnitude or may be several megohms. They may also vary appreciably in magnitude or phase angle over the frequency range.

The circuit impedances met in the telephone plant are seldom either substantially pure resistances or reactances over the entire frequency range and in considering the action of a transformer between two such circuits at any frequency the actual impedances of the circuits at that frequency must be employed. It is impossible to discuss all possible combinations of circuit impedances here. However, with a knowledge of the action of transformers between three limiting combinations of circuit impedances an indication will be given of their operation under all conditions of importance. These three limiting impedance conditions are:

1. Sending and receiving end impedances both resistances.
2. Sending end impedance a resistance, and receiving end impedance a positive reactance.
3. Sending end impedance a resistance, and receiving end impedance a negative reactance.

It is, of course, to be understood that the sending and receiving end impedances may be interchanged. There are, in addition, three other possible limiting circuit impedance conditions—the sending and receiving end impedances both positive or negative reactances and the sending end impedance a positive reactance and the receiving end impedance a negative reactance. These three impedance conditions are less usual than the first three and it is felt to be unnecessary here to discuss in detail the action of telephone transformers in circuits of this type.

Transformer Efficiency

The telephone transformer is generally used for transmission in both directions and the usual definition

2. Green and Maxfield: "Public Address Systems" Trans. A. I. E. E., Vol. 42, page 247.

3. Fletcher: "The Nature of Speech and Its Interpretation" *Journal* Franklin Institute, Vol. 193, page 729.

Martin and Fletcher: "High Quality Reproduction of Speech and Music," see page 384.

in power work for the primary as the winding which receives the energy from the supply circuit does not hold. The terms primary and secondary are therefore used simply to distinguish between the two windings without regard to the energy flow.

It can be shown that maximum power may be delivered from one circuit to another if the impedances of the circuits are equal in magnitude and opposite in phase, that is, if the resistances are equal and if the reactances annul each other. Maximum power may be delivered from one circuit to another in which the reactances annul each other and in which the resistances are not equal provided an ideal or perfect transformer of proper ratio is used to connect the circuit resistances. In the ordinary case it is not possible to annul the reactances over the entire frequency range to be transmitted and no attempt is therefore made to modify them. Under these conditions transformers are used to connect the two circuit impedances and without annuling reactances the greatest amount of power will be delivered by connecting these impedances by means of an ideal transformer of the best ratio.

By an ideal transformer is meant one which neither dissipates nor stores energy. Such a transformer has infinite primary and secondary self-impedances, infinite mutual impedance, unity coupling factor or zero leakage impedances, and zero d-c. resistances. An ideal transformer for any given circuit condition also has the best ratio to connect the circuit impedances.

It may be stated that when the circuit reactances are not annulled by the addition of reactances of the opposite sign, it is possible to deliver more power to the load impedance at certain frequencies by the use of an actual than by an ideal transformer as the transformer impedances may tend to annul the circuit reactances.

It has long been customary in dealing mathematically with a transformer to use in its place some equivalent network such as a π or a T network.[4] The use of an equivalent T network changes a coupled circuit into a simple circuit in such a way as to make it easier to see the effect of changes of the transformer constants on the transmission of the circuit. The equivalent T network of a two-winding transformer is shown in Fig. 1. in which the junction point of the three arms is considered not accessible and in which P, S and M are respectively, the primary, secondary and mutual impedances.

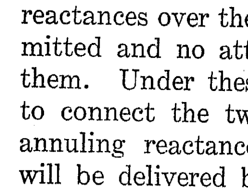

FIG. 1

The series arms a and b of the T network consist respectively of the differences of the primary and mutual impedances and the secondary and mutual impedances. The equivalent T network of the transformer is sometimes shown having series arms $P + M$ and $S + M$ and shunt arm $- M$. These two T's may be derived from the transformer impedances depending on whether the secondary is connected to give a received current in one direction or the other. The T shown here is the most convenient for ordinary considerations. Considering for the present a unity ratio transformer, the arm a will contain the d-c. resistance of the primary and the arm b the d-c. resistance of the secondary. In addition, the leakage impedance will be divided between them. Whether the leakage impedance should properly be considered

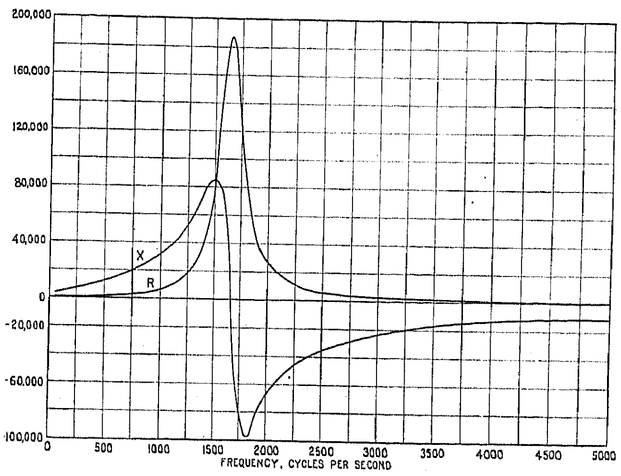

FIG. 2—MUTUAL IMPEDANCE M OF A PHANTOM CIRCUIT REPEATING COIL
X Effective reactance—ohms
R Effective resistance—ohms

principally in the arm a or principally in the arm b or divided equally between them depends on the relative location of the primary and secondary windings. However, in the ordinary case the coupling factor is so near unity and the leakage impedance is so small in comparison with the primary, mutual and secondary impedances that practically no error is introduced if it is assumed to be divided equally between them.

The shunt arm c of the equivalent T network contains the mutual impedance M. This mutual impedance equals $K \sqrt{P_0 S_0}$ where P_0 and S_0 are the primary and secondary impedances less their respective d-c. resistances, and where K is the coupling factor. The coupling factor K has a phase angle in actual transformers but where the coupling factor is nearly unity the angle may usually be disregarded. The mutual impedance is a complex quantity as are also the primary and secondary impedances of all usual transformers. As the mutual impedance is dependent on P_0 and S_0, any factors which enter into P_0 and S_0 will also enter into M. The reactance components of these impedances depend on the number and distribution of turns in the windings and the dimensions and permeability of the core. The resistance component is made up of an effective resistance due to hysteresis, eddy current and dielectric losses.

The distributed or lumped capacities of the windings are best considered in their effective values. These

4. Campbell: "Cissoidal Oscillation" TRANS. A. I. E. E., Vol. 30, Part 2, page 873.

effective capacities may be regarded as shunted across the primary or secondary of the transformer or may be considered as located across M of the arm c of the equivalent T of the transformer. If this effective capacity is considered in shunt with M and combined in it, both the resistance and the reactance of M will have components due to the effect of this capacity. The effective resistance and effective reactance of M will then go through the usual curves for parallel resonance as shown in Fig. 2 for a typical phantom circuit repeating coil.

One other factor frequently enters into the determination of the effective value of M as well as P_0 and S_0. In some circuits, in order to economize on the amount of apparatus necessary, a direct current is allowed to flow through the primary or secondary windings of the transformer. This causes a uni-directional magnetizing force in the core in addition to the a-c. magnetizing force of the speech current. This d-c. magnetization causes a decrease in M from the initial value or value with no d-c. magnetization, depending upon the strength of this magnetization and the reluctance of the magnetic circuit.

The series arms of the equivalent T of the unity ratio transformer are thus impedances consisting of the d-c. resistances and the leakage impedances and are usually relatively small compared with the circuit impedances while the shunt arm is the mutual impedance which is usually large compared with the circuit impedances. The simplicity of the equivalent T network of the unity ratio transformer is very useful and convenient in studying the effect of the transformer in producing losses. With an inequality ratio transformer, however, the arms a and b which equal $P-M$ and $S-M$, respectively, do not appear as small impedances. For instance, if P is larger than S, $P-M$ will be a large positive impedance and $S-M$ will be a large negative impedance and the T network has no decided advantage, from a mathematical standpoint over the ordinary transformer network.

Fig. 3 shows a transformer operating between the sending end impedance Z_1 and receiving end impedance Z_2. Z_2 may be considered less than Z_1. The actual sending and receiving circuits in telephone work are usually quite complex, each consisting of numerous series and shunt elements. According to Thévenins theorem[5] any electromotive force acting through any circuit no matter how complex will produce the same current in any receiving impedance as will some other electromotive force bearing a definite relation to the first electromotive force and acting directly in series with the impedance which would be obtained by a measurement of the circuit looking away from the terminals from which the receiving circuit has been disconnected. From this theorem it may be shown that the complex sending and receiving circuits may be replaced respectively by simple series impedances Z_1 and Z_2 which are the impedances of the complex sending and receiving circuits looking away from the place where it is desired to join them, and a transformer or any other structure studied between these impedances will act identically as if connected between the more complex actual circuits.

5. Comptes Rendus for 1883, Vol. 97, page 159.

If the transformer shown in Fig. 3 were the ideal transformer to connect the circuit impedances Z_1 and Z_2, the best impedance ratio could be found as follows: The current received through the impedance Z_2 is

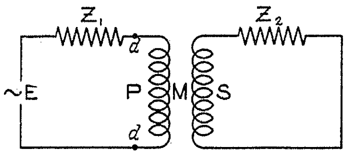

Fig. 3

$$I = \frac{E\,M}{(Z_1 + P)\,(Z_2 + S) - M^2}$$

But since in the ideal transformer P, S and M are infinite pure reactances and the coupling factor is unity and $M = \sqrt{P\,S}$

$$I = \frac{E\,M}{Z_1\,S + Z_2\,P}$$

This expression may be shown to be a maximum when the ratio P/S is equal to the absolute magnitude of Z_1 / Z_2.

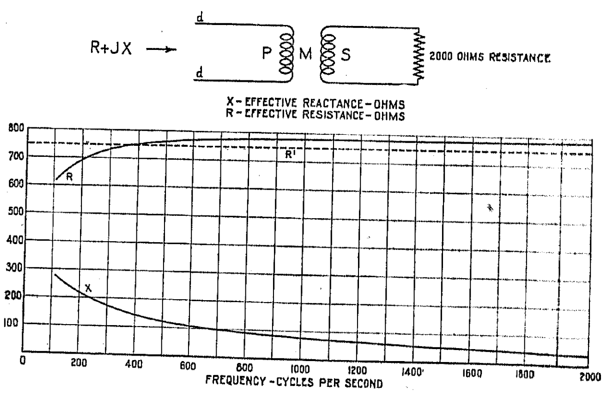

Fig. 4—Impedance Measured at d-d Ratio of Impedances P:S = 1:2.66

A transformer designed for the circuit of Fig. 3 should, therefore, have an impedance ratio $R = P/S = Z_1/Z_2$. With an ideal transformer of such a ratio the impedance at d, d looking toward the receiving end with the sending end open at d, d will equal Z_1. Such an impedance characteristic as measured with an actual inequality ratio transformer having an impedance ratio of 1:2.66 and a receiving end impedance Z_2 of 2000 ohms non-inductive resistance is shown as $R+j\,X$ in Fig. 4. The corresponding characteristic for an ideal transformer of the same ratio is shown as R^1. These characteristics show that an actual transformer in transforming or

modifying the circuit impedance adds both a resistance and a reactance component to the value of the circuit impedance divided by the transformer impedance ratio except at low frequencies where the transformer has a considerable shunting effect on the circuit impedance as explained later on, and where the resistance falls below the value obtained with an ideal transformer and the reactance is increased over its value at higher frequencies.

It may be stated that under certain circuit conditions it is quite important that the transformer give a transformation of the circuit impedance which is nearly ideal in order to limit the impedance irregularity introduced in the line.

For analysis or design work involving inequality ratio transformers the circuit of Fig. 3 may be reduced to a circuit in which the impedances Z_1 and Z_2 have the same absolute magnitude and the equivalent T network of the transformer may be reduced to an equivalent T of an equivalent unity ratio transformer. In this way the disadvantages of the equivalent T network of the inequality ratio transformer are avoided. This transformation is made by multiplying the circuit impedance Z_2 and the secondary impedance S by the impedance ratio R and multiplying the mutual impedance M of the transformer by $\sqrt{R}$ as shown in Fig. 5. As $P_0 = M\sqrt{R}/K$, $P - M\sqrt{R}$ will be a small positive quantity, and as $S_0 = M/K\sqrt{R}$, $S R - M\sqrt{R}$ will be a small positive quantity. $M\sqrt{R}$ will be a large positive quantity. This treatment of the transformer presupposes that P_0 and S_0 have the same phase angle. This is not necessarily precise in the case of all transformers but will hold with sufficient accuracy for the usual type of iron core transformers.

The series arms a and b and the shunt arm c of the equivalent unity ratio T network consist respectively of the d-c. resistances of the primary and secondary plus the leakage impedance, and the mutual impedance, all reduced to terms of the sending end impedance Z_1. In this transformation a circuit such as is shown in Fig. 3 is changed to an equivalent circuit as shown in Fig. 5. This equivalent circuit gives a received current which is less than the received current of the circuit of Fig. 3 by the factor $I/\sqrt{R}$ but the received power is the same in each case. This equivalent T of the equivalent unity ratio transformer consisting of relatively small impedances in series with the circuit and a relatively high impedance as a shunt across the circuit, furnishes an easy means of studying the losses produced by a transformer.

The telephone engineer as well as the power engineer is concerned with the delivery of power. In power work the efficiency of a device is usually expressed in per cent as the ratio of the power delivered to the power supplied or the watts output divided by the watts input. In telephone work it is customary to consider the losses caused by the device rather than its efficiency. These losses are determined by the change in received power caused by the insertion of the device in the circuit. They are expressed in terms of the attenuation of a length (in miles) of standard cable (*M. S. C.*). This unit is such that if two currents I_1 and I_2 flow through the same impedance (load) delivering powers W_1 and W_2 respectively, the number of miles corresponding to the current or power ratios is given by the relation $M\,S\,C = 21.12 \log_{10} I_1/I_2 = 10.56 \log_{10} W_1/W_2$. It can be shown from the above that for small losses a change in current ratio of 1 per cent corresponds approximately to 0.1 mile of standard cable.

In inserting an ideal transformer of best ratio between two circuits of different impedance an increased current is obtained through the receiving end impedance and a transmission gain is effected. If an actual transformer were used in place of the ideal transformer a somewhat lesser gain would usually be obtained due to the losses of the transformer. The transformer loss is determined by the ratio of the received current with the transformer in circuit to the received current with the ideal transformer in circuit.

It is to be noted that the above mentioned ratio of received currents which is used as the basis of the transmission loss of the telephone transformer takes no account of the phase angle of the load impedance and bears no direct relation to the ratio of the power output to the power input. There is, therefore, no very simple relation between the miles loss and the power efficiency. For example, a telephone transformer might have equally low losses when operating into a pure reactance as when operating into a resistance, whereas, the power efficiency approaches zero when the phase angle of the load approaches 90 deg. When operating into a resistance load the current ratio of the telephone transformer approaches the square root of the power efficiency for very efficient transformers.

Transformer Operating Between Resistances

From the circuit of Fig. 5, it can easily be seen that provided the circuit impedances approximate resistances, the smaller $P - M\sqrt{R}$ and $S R - M\sqrt{R}$ are and the larger $M\sqrt{R}$ is, the smaller will be the loss caused by the transformer. The speech current used in most telephone circuits is so minute that the permeability of the transformer cores remains at approximately its initial value regardless of what winding is placed on the transformer. An increase in the mutual impedance will lower the shunt losses, but will also cause an increase in the series losses, as there will be an increase in the d-c. resistance and the leakage impedance in practically the same ratio as the increase in the mutual impedance. If the capacity in the transformer windings is neglected, it will be noted that both the series and shunt arms of the

transformer T contain components of impedances which increase with the frequency and that at zero frequency the shunt loss will be infinitely great and at infinite frequency the series loss will be infinitely great while at intermediate frequencies these losses will be finite. It is, therefore, evident that for a given transformer there will be some frequency at which the transformer operates with minimum losses, and that for operation at a given frequency there is a value of mutual impedance for any given transformer structure at which the losses are a minimum.

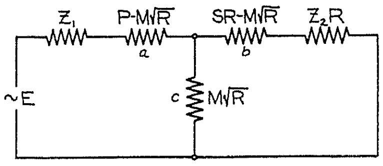

Fig. 5

The transformer of Fig. 5 when operating between non-inductive resistance impedances will produce a loss-frequency characteristic, as shown in Fig. 6, in which the various curves are for a fixed circuit condition and for several different windings or values of mutual impedance. In these characteristics the d-c. resistance of the winding produces a loss which is practically independent of frequency. There is an increase in loss at the lower part of the frequency range due to the increased shunting effect of $M\sqrt{R}$ at the lower frequencies. This loss decreases as the frequency

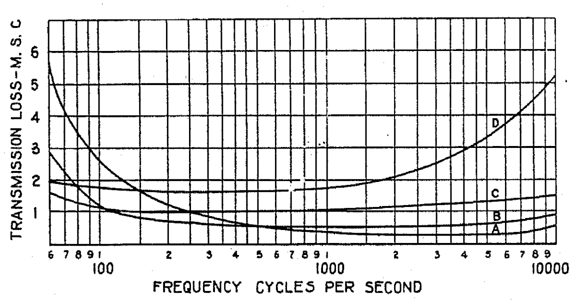

Fig. 6—Transformer Operating Between Resistances

Sending end resistance 6000 ohms
Receiving end resistance 2000 ohms

Transformer	$\frac{M\sqrt{R}}{Z_1}$ at 1000 cycles
A	5
B	10
C	20
D	40

increases. Although in any actual case the departure of the angle of $M\sqrt{R}$ from 90 deg. may cause an appreciable part of the shunting loss it is usually possible to obtain greater reduction in loss by increasing the absolute magnitude of this impedance than by increasing the phase angle. If the transformer has sufficient capacity in and between the windings to affect appreciably this impedance, it may also affect the loss due to it. In some transformers, the effective capacity may be large enough to cause resonance within the frequency range which is transmitted efficiently. This would cause $M\sqrt{R}$ to have a maximum value at the resonance frequency, and the loss due to the shunt arm would go through a minimum value at this frequency. If the capacity is large enough to produce this resonance near the lower end of the transmited frequency range, its effect at the higher frequencies in decreasing the magnitude of $M\sqrt{R}$ might be sufficient to cause an appreciable loss at these frequencies.

The magnitude of the leakage impedances, which, together with the d-c. resistance, make up the series arms of the T, also tends to increase the loss at the upper frequencies, as this impedance increases with the frequency. In cases where both the leakage and the capacity are high, the effect of one may to some extent tend to lessen the loss produced by the other at some frequencies.

It is to be noted that as the leakage impedance generally consists of a larger reactance than effective resistance, this impedance has far less effect in determining the value of received current through Z_2 when the circuit impedances are resistances than does the d-c. resistance.

In the design of telephone transformers to operate with little distortion between resistance impedances, it follows that the transformer impedance ratio is determined as the ratio of the circuit impedances between which the transformer operates, the loss at the lower frequencies is determined principally by the impedance of the shunt arm of the T network, $M\sqrt{R}$, and the loss at the higher frequencies is determined principally by the leakage impedance and the effective capacity. The d-c. resistance adds a loss which is practically constant over the frequency range.

The determination of the windings of such a transformer would be made as follows, assuming for the present that the transformer construction has already been decided upon. The impedance ratio of the transformer is calculated as the ratio of the circuit impedances. Using this ratio, the winding is calculated, choosing such sizes of wire as will make the ratio of d-c. resistance of primary and secondary the same as the impedance ratio, and will completely fill the winding space with an allowance for commercial variations in the winding space, dimensions, wire diameter, winding and insulation. As the transformer dimensions and core permeability (initial or low magnetic density value) are known, the inductance for a given number of turns may be calculated as proportional to the square of the turns or from a trial winding the relation between impedance and number of turns may be obtained. The coupling factor and effective capacity are best obtained by a trial design using the arrangement of winding which is expected to be used in the final design. All the information for determining equivalent T networks is thus available.

The loss curves for different values of mutual impedance may be predetermined from these equivalent networks and the desired winding chosen from these characteristics. Such a series of curves for a transformer operating between resistance impedances is shown in Fig. 6. In the table in this figure is shown the ratio of the shunt arm, $M \sqrt{R}$, of the equivalent T network of the equivalent unity ratio transformer at 1000 cycles, to the sending end impedance Z_1.

Fig. 7 shows the transmission loss characteristic of

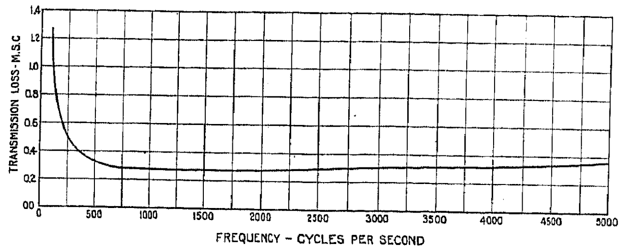

FIG. 7—TRANSMISSION LOSS CHARACTERISTIC OF PHANTOM CIRCUIT REPEATING COIL OPERATING BETWEEN 1830-OHM RESISTANCE LINES

a phantom circuit repeating coil operating between non-inductive resistance lines of 1830 ohms. The mutual impedance-frequency characteristic of this repeating coil was shown in Fig. 2 from which it may be noted that the reactance component of this impedance is negative above a frequency of 1700 cycles per second. Although above this frequency the mutual impedance decreases, its magnitude is so great as compared with that of the line impedance that the loss due to it is practically negligible at a frequency of 5000 cycles per second.

TRANSFORMER OPERATING BETWEEN A RESISTANCE AND A POSITIVE REACTANCE

The case of a transformer operating into a pure positive reactance is, of course, hypothetical and no energy would be delivered unless the reactance had a resistance associated with it. In any actual case, there is always a resistance component to this impedance, but for this discussion it is assumed that the resistance component is practically zero.

In this circuit condition we have one impedance which is independent of the frequency and another which is directly proportional to the frequency. By properly choosing the transformer ratio it is possible to match the circuit impedances at any particular frequency and deliver maximum energy at this frequency. At other frequencies there would be a transmission loss due to this failure to match impedances, and it follows that even with an ideal transformer, it is not possible to obtain uniform efficiency over a range of frequency, and that by selecting the proper ratio, maximum efficiency may be obtained at any desired frequency.

Fig. 8, curve A, shows the transmission loss, in miles due to the introduction of an ideal transformer of fixed ratio between the sending end resistance Z_1 and the receiving end reactance Z_2. This transformer serves to match these impedances at the frequency F_1 and the curve shows the loss above what would be obtained if the impedances were matched at any other frequency. Curve B is a similar characteristic for an ideal transformer, matching impedances at frequency F_2.

Fig. 5 may be considered to represent the equivalent unity ratio circuit (Z_2 being less than Z_1) with the T network of the equivalent unity ratio transformer in the circuit. If, for the present the effect of capacity in the mutual impedance is neglected, the losses produced by the components of the series arms of the transformer operating under these circuit conditions, that is the d-c. resistance and the leakage reactance, are approximately of equal importance. The d-c. resistance causes a loss which is finite at low frequencies and decreases to zero at infinite frequency while the leakage reactance causes a loss which is finite at infinite frequency and reduces to zero at zero frequency. The mutual impedance produces a loss which is infinite at zero frequency and decreases to a relatively small finite value at infinite frequency.

If the mutual impedance contains a capacity component, a reduction of loss may be produced at certain frequencies due to resonance of the capacity reactance with the mutual, leakage and receiving end reactances. This may even give a gain over the characteristic of the ideal transformer for a limited frequency range, but

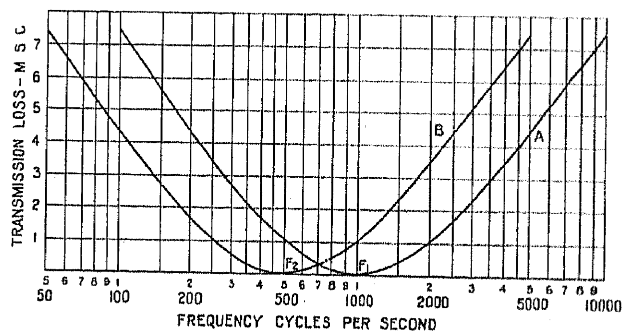

FIG. 8—TRANSFORMER CONNECTING A RESISTANCE TO A POSITIVE REACTANCE

"A" and "B" ideal transformers of impedance ratio R/X at frequency "F_1" and "F_2" respectively.

above the resonant frequency an increased loss is produced which approaches infinity.

Characteristics A, B, C and D of Fig. 9 show measurements of actual transformers having different values of mutual impedance and the proper ratio to match impedances at 1000 cycles. The transmission losses shown are the losses of the actual transformers compared with the corresponding ideal transformer of the same ratio. The ideal transformer, itself, has a loss characteristic causing distortion which varies with the frequency as shown in Fig. 8. The ratio of

$M\sqrt{R}:Z_1$ at 1000 cycles is shown in the table. These transformers are the same as those shown in Fig. 6 as operating between resistances. The losses at the upper frequencies are reduced somewhat by the winding capacity. It may be noticed that of the transformers whose characteristics are shown in Fig. 9, transformer B introduces minimum loss.

In designing a transformer to connect a resistance to a positive reactance, the impedance ratio of the transformer is determined as the ratio of the circuit

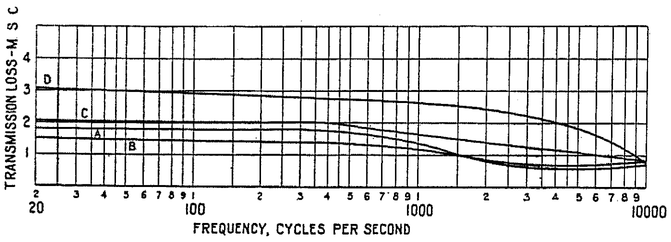

FIG. 9—TRANSFORMERS OPERATING BETWEEN A RESISTANCE AND A POSITIVE REACTANCE

Transmission loss above that of an ideal transformer of the same ratio.
Sending end impedance $Z_1 = 6000$ ohms.
Receiving end impedance $Z_2 = (R + j L)\,\omega$ ohms $= 20 + j\,2000$ ohms at 1000 cycles.
Transformer impedance ratio = 3:1

Transformer	$\frac{M\sqrt{R}}{Z_1}$ at 1000 cycles
A	5
B	10
C	20
D	40

impedances at the frequency at which it is desired to deliver maximum power. The choice of best mutual impedance may be made by assuming windings of different impedance and predetermining their loss characteristics as has been described under transformers working between resistances.

Transformer Operating Between A Resistance and A Negative Reactance

Where a transformer operates between a resistance and a negative reactance, a condition exists where one impedance is independent of and the other is a function of the frequency, although in this case the reactance varies inversely with the frequency. Here again in order to deliver power it is necessary to assume that the receiving end impedance has a small resistance component. The transformer ratio may be made to match these impedances at any frequency, delivering maximum energy, but causing an increasing loss above and below this frequency. The loss characteristic of such a transformer is shown in Fig. 10 which gives the increase in transmission loss of the ideal transformer of fixed ratio over the ideal transformer of the actual ratio of the circuit impedances at all frequencies.

Referring to Fig. 5 and considering Z_2 a negative reactance, there are two frequencies at which resonance takes place. At the first frequency parallel resonance occurs between the impedance $M\sqrt{R}$ and $Z_2 R$ and at the second, series resonance occurs between the leakage reactance components of $P - M\sqrt{R}$ and $S R - M\sqrt{R}$ and the receiving circuit impedance $Z_2 R$. The d-c. resistance causes a loss which, in general, is finite at zero frequency and becomes zero at infinite frequency. The leakage reactance produces a loss which, in general, increases from zero at zero frequency to infinity at infinite frequency going through a minimum, however, and in some cases causing a gain over the ideal transformer at the second resonance frequency. The mutual impedance, in general, produces a loss which decreases from infinity at zero frequency to a finite value at infinite frequency, going through a minimum, however, at the first resonance frequency and possibly even causing a gain over the ideal transformer at this frequency.

Input Transformer

The usual case of a transformer operating under these circuit conditions is the input transformer of the vacuum tube amplifier, the vacuum tube approximating a condenser in its grid-filament impedance.

With the receiving end impedance an ordinary capacity of constant phase angle, it is necessary in order to deliver uniform power over a range of frequency to match impedances at all frequencies or supply a voltage which decreases inversely as the square root of the frequency. In a vacuum tube amplifier, when the output or plate-filament current is proportional to the input or grid-filament voltage, uniform output

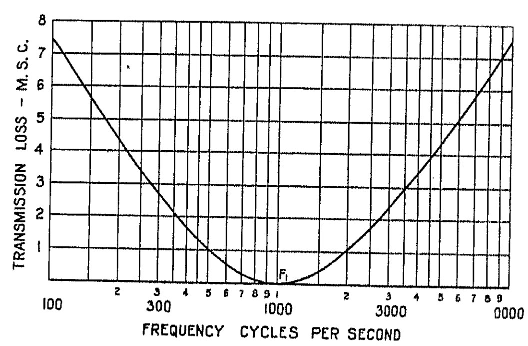

FIG. 10—TRANSFORMER CONNECTING A RESISTANCE TO A NEGATIVE REACTANCE

Ideal transformer of impedance ratio R/X at frequency "F_1" = 1000 cycles.

power will be delivered at varying frequency provided the input voltage is kept constant. When operating into a vacuum tube, therefore, it is not required to match the circuit impedances at all frequencies to limit distortion. An ideal input transformer of fixed ratio will tend to cause the vacuum tube to deliver uniform power over a range of frequency but only throughout that part of the frequency range in which it can maintain constant voltage across the grid-filament impedance.

The receiving end circuit impedance Z_2 is usually larger than Z_1. The equivalent unity ratio circuit and the equivalent T network of the equivalent unity ratio transformer are, therefore, obtained by dividing S and Z_2 by R and M by $\sqrt{R}$ instead of multiplying as in Fig. 5. Such an equivalent unity ratio circuit is shown in Fig. 11.

In considering the losses introduced in the circuit by the input transformer it is not convenient to compare the received current through the actual transformer

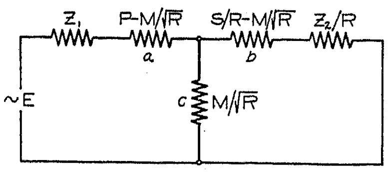

Fig. 11

with that of the ideal transformer of either variable or fixed ratio as neither ideal transformer delivers uniform input voltage to the vacuum tube at all frequencies or causes the amplifier to deliver uniform power. A more satisfactory basis of comparison, particularly for input transformers of different ratios which are intended for the same circuit conditions, is obtained by comparing the ratio of the potential produced across the grid-filament impedance Z_2 (see Fig. 3) to the potential E impressed on the circuit. This ratio gives the amount of effective amplification produced in the amplifier by the input transformer.

The losses of the input transformer have the same general frequency variations as the losses of the transformer operating into an ordinary negative reactance except that at the first resonance frequency the losses produced by the mutual impedance $M/\sqrt{R}$ of the input transformer may be zero but never negative. It is to be noted that the d-c. resistance of the secondary produces zero loss at zero frequency while the loss due to the d-c. resistance of the primary is finite.

Input transformers designed for audio frequencies operate into the high negative reactance of the vacuum tube and therefore the secondary and mutual impedances are necessarily of large magnitude. The capacity of the transformer windings under these conditions becomes of considerable importance and even when limited by careful design usually causes parallel resonance in the arm c of the transformer network in the transmitted frequency range. The same is the case with input transformers designed for carrier frequency operation although the impedances involved are usually not so large. Above this resonant frequency both the arm c and the impedance Z_2/R are negative reactances and their impedances tend to be annulled by the leakage reactance of the arms a and b. The combined effect of the leakage reactances of these arms produces resonance with the transformer and tube capacities which may increase the transformer amplification characteristic even above the value given by the ratio of secondary to primary turns.

Amplification characteristics produced by transformers of different ratio operating from a sending end impedance of 20,000 ohms into a 216-A vacuum tube are shown in Fig. 12. It will be noted that as the impedance ratio of the transformer is lowered, the amplification characteristic flattens out and the frequency distortion is reduced. In the characteristics of some of the lower ratio transformers, a gain in amplification above the ratio of turns of the transformer may be noted at the higher frequencies.

It is to be noted that in the circuit of Fig. 11, a and b are positive reactances while at the high frequencies c and Z_2/R are negative reactances and the circuit approximates a low pass filter. As a filter, it has a cut-off frequency above which it tends to limit transmission and the amplification characteristic falls off quite rapidly.

The d-c. resistance of the primary is of more importance than the d-c. resistance of the secondary. In the

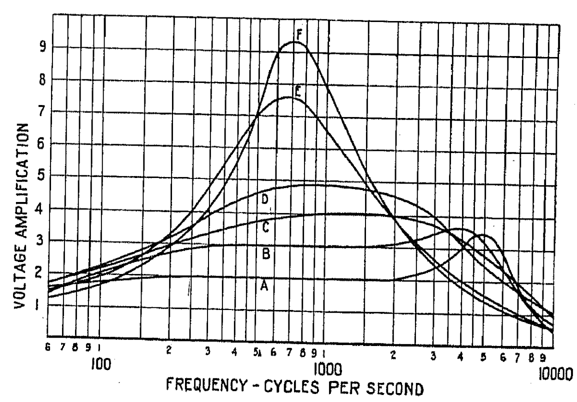

Fig. 12—Voltage Amplification Characteristics of Input Transformers of Various Turns Ratios Operating from 20,000 Ohms Resistance into a 216-A Vacuum Tube

Coils	Turns Ratio
A	1:2
B	1:3
C	1:4
D	1:5
E	1:8
F	1:10

well-proportioned input transformer, the value of the arm c particularly at the resonance frequency becomes very large as compared with the sum of the impedance Z_1 and the impedance of the arm a and at these frequencies the transformer gives practically the amplification represented by the ratio of secondary to primary turns. However, at the lower frequency range, where the impedance of the arm c becomes more nearly equal to the sum of the impedances of Z_1 and the arm a, the d-c. resistance of the primary produces a loss in amplification. The d-c. resistance of the primary is, therefore, a factor in the distortion at the lower frequencies.

For the rapid analysis of an input transformer, it is

customary to consider the d-c. resistance of P as added directly to Z_1 to form Z_0; to consider the total leakage reactance, $+ j X_l$, located entirely in the arm a; to neglect the d-c. resistance of S; to combine the capacity of the vacuum tube and effective capacity of the transformer as determined as located across S to form the reactance $- j X_c$ and to consider the mutual impedance M as the impedance due to the transformer windings exclusive of capacity. Such a circuit as shown in Fig. 13, approximates the actual circuit conditions quite closely in the ordinary case and is useful in design work.

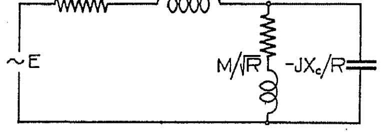

FIG. 13

For an audio-frequency input transformer of a given ratio, the losses or the departure from full amplification depend at low frequencies, principally on the value of the mutual impedance, while at high frequencies the effective capacity of the tube plus that of the transformer has a considerable influence. As the mutual impedance is a function of the number of turns, while the capacity is practically independent of the number of turns, it follows that, in general, for an input transformer operating over a wide band of frequencies, the higher the mutual impedance, the wider will be the transmitted frequency band.

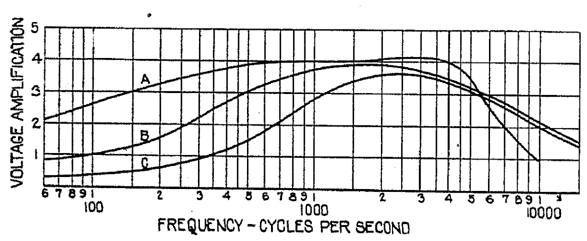

FIG. 14—VOLTAGE AMPLIFICATION OF INPUT TRANSFORMERS OF DIFFERENT MUTUAL IMPEDANCES HAVING AN IMPEDANCE RATIO OF 1:16 AND OPERATING FROM 15,000 OHMS RESISTANCE INTO A 216-A VACUUM TUBE.

Transformer	Ratio $\frac{M \sqrt{R}}{15,000}$ at 1000 cycles
A	1:1
B	1:4
C	1:10

Fig. 14 shows the variation in amplification obtained with input transformers of the same ratio but of different mutual impedances operating between a resistance of 15,000 ohms and a 216-A vacuum tube. It will be noted that as the mutual impedance is increased the width of the transmitted frequency band is increased and the mid-band frequency is lowered. The upper slope of the frequency characteristic is determined principally by the leakage reactance of the transformer and the transformer and vacuum tube capacities. In transformer C the leakage reactance is so proportioned that resonance with this capacity extends the flat portion of the amplification characteristic upward in the frequency range. The lower slope of the characteristic is determined principally by the mutual impedance of the transformer.

This advantage of high mutual impedance explains the use of No. 40 and No. 44 A. w. g. enameled wire for the secondaries in most audio frequency input transformers, as giving the highest impedance secondary windings that can be commercially applied with different methods of winding. With the gage of wire and the secondary impedance thus determined, the possible amplification characteristic will depend on the transformer ratio. With a number of predetermined characteristics of different ratio transformers prepared as shown in Fig. 12, the required windings for the input transformer may be determined to give a desirable compromise between the amplification and the transmission distortion permissible.

From the standpoint of minimum distortion, it

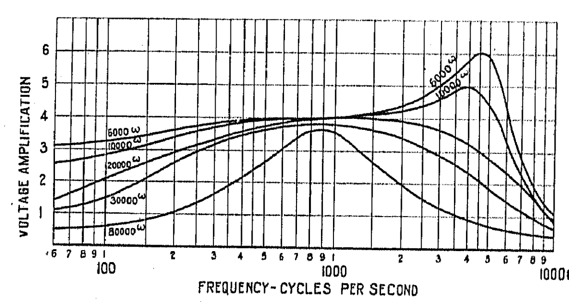

FIG. 15—VOLTAGE AMPLIFICATION CHARACTERISTICS OF AN INPUT TRANSFORMER OF IMPEDANCE RATIO 1:16 OPERATING FROM VARIOUS RESISTANCES INTO A 216-A VACUUM TUBE

should be mentioned that it is desirable to use vacuum tubes of low plate-filament impedance in the amplifier, particularly if this can be done without sacrificing the tube amplification factor. The effect on the input transformer characteristic of operating it from tubes of different plate circuit impedance is shown in Fig. 15.

In the earlier telephone repeaters,[6] a resistance was shunted across the secondary of the input transformer and the grid-filament impedance of the vacuum tube. This resistance was of sufficiently low magnitude to determine the impedance into which the input transformer operated and the transformer was given the proper impedance ratio to match this impedance with the impedance from which the transformer was operated. This resistance served to aid in obtaining a flat amplification characteristic and to fix the impedance measured on the primary of the transformer with the secondary connected in circuit. An impedance characteristic was produced as shown in Fig. 4 instead of the characteristic of the type shown in Fig. 2, which

6. Gherardi and Jewett: "Telephone Repeaters," TRANS. A. I. E. E., Vol. 38, part 2, page 1287.

represents simply a transformer with appreciable effective capacity in the windings. Later on an improvement was made by shunting the primary of the input transformer with a resistance of a value equal to the resistance formerly used across the secondary divided by the transformer impedance ratio. It will be noted that in the first case, the transformer operates into a circuit which is principally a resistance while in the second case, it operates into a capacity.

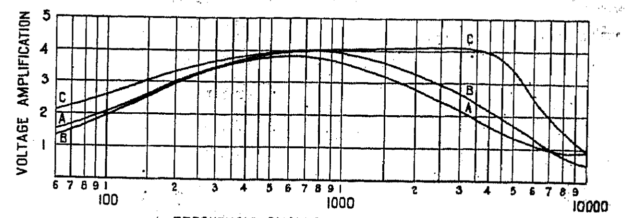

FIG. 16—VOLTAGE AMPLIFICATION CHARACTERISTICS OF INPUT TRANSFORMERS UNDER DIFFERENT CIRCUIT CONDITIONS

Curve	Transformer Impedance Ratio	Resistance Across Low Side	Resistance Across High Side
A	1:64	∞	64 × 15,000
B	1:64	15,000	∞
C	1:16	∞	∞

Measured amplification characteristics of an input transformer operating under both circuit conditions are shown in Fig. 16 which gives, in addition, the characteristic of an input transformer of the same size and construction designed to give the same value of amplification as in the first two cases but without the shunting resistance. The superiority of the latter type of circuit in giving a uniform amplification characteristic is easily seen. This last circuit connection does not have an input impedance characteristic which is practically independent of frequency and it is, therefore, more limited in its uses.

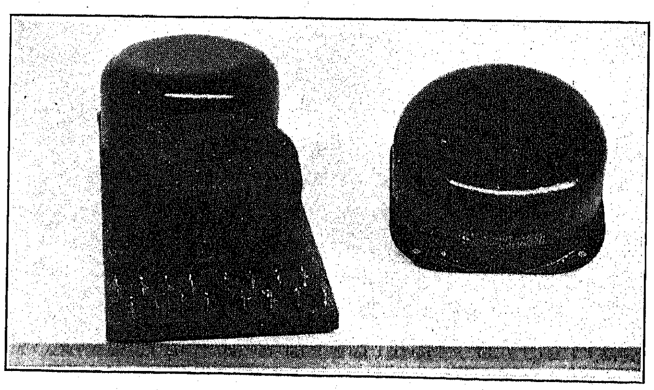

FIG. 17—TOROIDAL REPEATING COIL

In the foregoing, the transformer has been treated from the standpoint of the transmission of telephone currents and those features have been presented which appeared to the writer to be of special interest and usefulness to those interested in its application to this problem. In conclusion, the writer wishes to acknowledge his indebtedness to Mr. Thomas Shaw of the American Telephone and Telegraph Company and to Mr. K. S. Johnson of the Western Electric Company, Inc., with whom he has been associated for a number of years on the general problem of telephone transmission and in the application of the principles described herein.

Appendix

The construction of telephone transformers is covered in an appendix not printed here.

TRANSFORMER CONSTRUCTION

The type of transformer most used in the telephone plant is a toroidal core transformer usually called a repeating coil. This type of transformer has a ring-shaped core of soft magnetic iron wire or silicon steel laminations completely covered with the primary winding over which is applied the secondary winding. Such a transformer, when the dimensions are properly proportioned and the winding is applied so as practically to fill the central hole in the core, is a very efficient structure. The symmetrical distribution of winding limits the stray field, thus preventing cross-talk

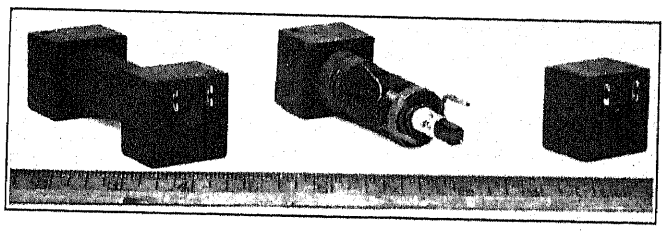

FIG. 18—TELEPHONE INDUCTION COIL

trouble in neighboring circuits. The cost of winding has been reduced to a low value by the development of a winding machine in which a circular shuttle threading through the center of the core is used to hold the wire which is wound on the core by a motor-driven annular part of the machine. The windings are accessible and permit of easy adjustment. This type of transformer is used in telephone installations where the impedances of the circuits in which the transformer is operated are less than 20,000 ohms and where the frequency is relatively low. The usual type of mounting of two repeating coils on a common base is shown in Fig. 17. The case has been removed from the front coil unit to show the construction.

The telephone induction coil used in all subscribers sets is an open magnetic circuit core type transformer. The impedance and frequency requirements of the circuit in which this form of transformer is used are not severe and the value of the transmission is relatively low due to the fact that any one transformer is used a relatively small part of the time and only for conversations from a single station. This, together with the facts that the transformer must be designed to operate with direct current through the windings and

stray magnetic field is of no particular disadvantage permits the use of this relatively inefficient type of transformer shown in Fig. 18. It is to be noted that in this transformer the winding is located about the center portion of the core only as in this location the highest ratio of inductance to d-c. resistance is obtained giving maximum efficiency.

For portable test sets, considerably lighter and smaller types of transformer than are generally used in

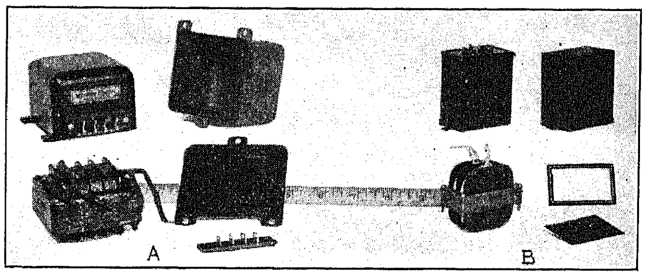

Fig. 19—Shell Type Transformers

the telephone plant are required. Transformer A of Fig. 19 shows the type of transformer used in sets such as the S. C. R. 72 amplifier developed for the Signal Corps of the United States Army. This amplifier set was intended to operate on 1000-cycle telegraph signals transmitted through the ground. The transformers used in this set were required to operate efficiently only in the neighborhood of 1000 cycles and transmission at other frequencies was sacrificed to obtain

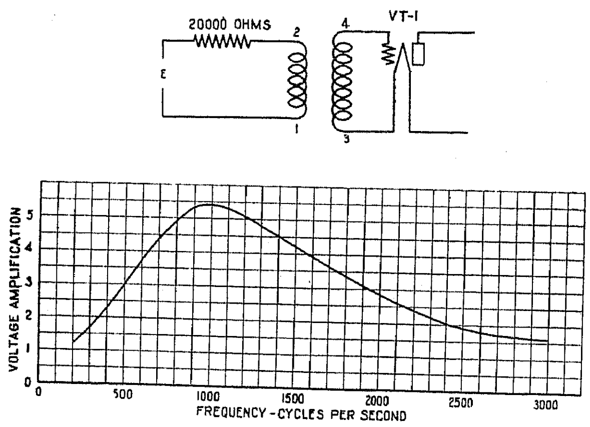

Fig. 20—Amplification Characteristic, 201-A Input Transformer

maximum amplification at this frequency. An amplification characteristic of the input transformer of this set is given in Fig. 20. This type of transformer was widely used in both Signal Corps and Navy radio transmitting and receiving sets throughout the war and weighed about two pounds.

The type of transformer shown as B in Fig. 19 is also of shell type construction and is used in portable telephone field test sets and weighs 12 ounces.

One of the latest designs of shell type transformer and the one which was used for most of the experimental characteristics given herein is shown in Fig. 21. The construction of this input transformer, the No. 224 type, is such that winding and assembly can be readily accomplished and repair easily effected if necessary. The winding space and the core have been proportioned to obtain minimum cost of manufacture. The core consists of I and E shape laminations riveted together to form an I and E part which butt together forming the core. The winding is placed on a spool which fits over the central limb of the E. The two core parts are held together by means of two brackets which are held in place by four machine screws. The terminals are arranged on insulated mounting plates held under the screws of the mounting bracket and serving as mechanical protection to the lead wires.

It may be noted that whereas the form of toroidal repeating coils is core type, that of the transformers employing spool windings is shell type. The core type is used for the toroidal repeating coils as it permits ready adjustment of the windings and the shell type is used in the transformer having spool windings as it permits the use of a winding of small effective capacity.

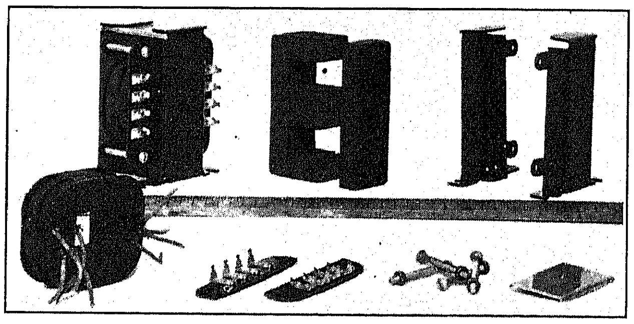

Fig. 21—224-Type Input Transformer

The size of the transformer is governed by three factors, the amount of space available, the cost permissible and the ratio of inductance to d-c. resistance which is required to give the desired freedom from transmission loss.

In the windings of telephone transformers, cotton or cotton and enamel is used for the insulation of the heavier gages of wire while for the smaller gages, enamel or enamel and silk is used. The gage sizes used range from No. 18 A. w. g. or heavier to No. 44. In input transformers, the d-c. resistance of the secondary winding usually causes an inappreciable loss and it is, therefore, desirable to have it take up as little space as possible and the smallest size of wire which can be used with the different commercial methods of winding is employed. The quality of the enamel insulation generally used for this winding is a determining factor in the minimum size of winding which can be used. With the best enamel, it is possible to wind No. 40 A. w. g. wire with little or no interleaving paper and with little special machinery, and have the resulting windings reasonably free from short circuits. How-

ever, with inferior enamel, it is necessary to use a covering of silk in addition to the enamel, or to use interleaving paper between each two layers of the winding. This extra insulation, needless to say, causes a considerable increase in the space taken up by the winding, which is undesirable.

In transformers in which the effective capacity is an important factor, it is frequently necessary to apply the winding in such a way as to reduce the capacity to a minimum. The effective capacity depends on the size of the transformer winding, more care being required to reduce it to a reasonable value in a large than in a small transformer. The capacity of the winding may be conveniently considered as composed of four parts, part one being the sum of the capacities between each two adjacent turns all connected in series from one terminal of the winding to the other, part two the sum of the capacities between adjacent layers

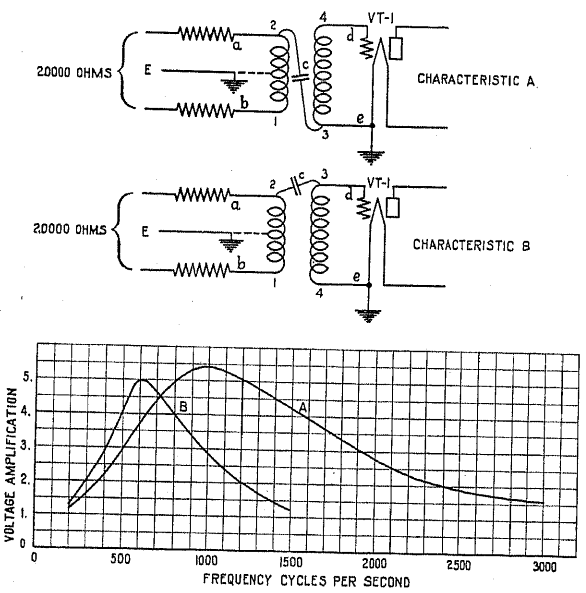

Fig. 22—Transformer Amplification Characteristics with Different Winding Connections

in series, part three the capacities between adjacent winding sections in series and part four due to the capacities between the winding and any other metal or other windings in the neighborhood. These capacities may be considered in effective values as connected across the winding terminals. Of these component capacities, the first may usually be neglected entirely, as it consists of a large number of small capacities in series, this number being only slightly less than the total number of turns. The fourth part is usually the most important as it seldom consists of more than two capacities in series. The second and third parts of the effective capacity are of importance depending on the number of sections and layers and the capacity between adjacent ones. In high ratio transformers, it is usually sufficiently accurate to consider the lower impedance winding as an equi-potential plate and to consider the capacities with regard to the high impedance winding only.

The effect of inter-winding capacities in affecting transmission is given in Fig. 22. This figure shows how sometimes these capacities may be connected between approximately equal potential points in the circuit and thus lessen the effective capacity of the winding. The transformer on which these characteristics were taken is the same as that shown in Figs. 19 and 20. Characteristic A represents the normal connection of windings with the interwinding capacity C located between the parts of the circuit a and e in which it is effectively across one half the impedance of the primary, 1-2. Characteristic B shows the amplification obtained with the secondary winding, 3-4, reversed, terminal 3 being connected to the grid of the vacuum tube instead of the filament. In this case the capacity C is across the entire secondary impedance in addition to one-half the primary impedance, the combined impedance being approximately 200 times the impedance under the circuit connection A. The effect of this inter-winding capacity being connected across points of widely different potential in the circuit in lowering the frequency of the amplification peak is clearly shown.

Transformers are subjected in service to conditions of temperature and humidity which vary greatly. Conditions in exchanges in the tropics and in certain sets for outdoor use are particularly severe while in some other locations there is but little chance of trouble. Under conditions of humidity electrolytic corrosion will take place provided salts which might form an electrolyte on the addition of moisture are present in the completed windings. Corrosion will be accelerated in circuits in which there is a direct-current potential between some point in the winding and another neighboring metallic part. It frequently causes the windings of the transformer to open-circuit particularly when they are wound with the smaller gages of wire. It is rather difficult to obtain materials which are free from slight amounts of salts and to keep the transformer winding free from them during the operations incident to manufacture. Perspiration from the operators hands or the use of soldering salts are common causes of corrosion. To reduce this trouble the windings are imbedded in a moisture resisting compound of oils or waxes which is in itself chemically inert. The moisture-proofing process is usually effected under vacuum after a baking period. The advantages of a carefully worked out moisture-proofing treatment in prolonging the useful life of transformers is so great that it has been universally adopted in all carefully designed transformers.

Discussion

Wm. Fondiller: Mr. Casper has described the ideal transformer, one which would introduce zero loss in a telephone circuit, as represented by an equivalent network having zero impedance series arms and an infinite impedance shunt arm.

In any practical transformer the factor which most largely determines the extent to which this ideal can be approached is the magnetic material used for the core. The ideal core material obviously would be one having infinite permeability and zero losses, as this would make possible 100 per cent coupling, practically zero winding resistance and infinite mutual impedance. Fortunately, however, a magnetic material having characteristics altogether finite in value will permit the construction of quite satisfactory transformers. It may be of interest to indicate what the desirable properties of the magnetic material should be and the extent to which they are attained in materials at present commercially available.

As Mr. Casper has indicated, the speech frequency magnetizing forces operating in these transformers are of low values, generally ranging from 0.001 to 0.05 c. g. s. units. These values are less than 1/100 of those ordinarily employed in power work. On account of the extremely low magnetizing forces, the part of the usual permeability characteristic which is of importance in material to be used in this apparatus is a small portion near the origin. Over this part of the curve the change in permeability with magnetizing force is not very large, consequently, what is commonly called the "initial" permeability may be used to define the relative merits of different materials. The requirements of a desirable magnetic material for the telephone transformer, so far as its action in transmitting speech frequency currents is concerned, are therefore, high initial permeability and low core losses at the corresponding flux densities.

The material which at present meets these requirements most satisfactorily is what is known as high silicon steel transformer sheet. This is a low carbon iron having from 3.5 to 5.0 per cent silicon and is supplied in sheets of 0.014 in. in thickness.

So far as I am aware, no steel mill is at present equipped to make the required tests at very low magnetizing forces. Due to this condition, the product supplied at present by the different manufacturers varies a good deal in respect to some of these magnetic characteristics. The initial permeability ranges from about 400 to 600. Frequently, however, values as low as 250 are observed. The maximum of these permeability values is, of course, desirable, although a steel consistently having an initial permeability of about 500 would be quite satisfactory. Eddy current loss is fairly uniform at about 1.0 erg per gram at a frequency of 1000 cycles per second and a maximum density of 15 gausses. The hysteresis loss under the same test conditions varies from about 1.0 to 5.0 ergs per gram in different samples. When the value of this loss is above 3.5 ergs the results are unsatisfactory. As brought out in the paper by Speed and Elmen, on "Compressed Powdered Iron" in 1921, the hysteresis loss is less in hard than in soft material for the same flux density at very low magnetizing forces. It will be evident from this that some conflict in requirements exists in that it is desired to secure high initial permeability in combination with low hysteresis loss. Until substantially lower hysteresis loss values can be obtained, there is little to be gained from reducing the values at present obtainable for eddy current loss at audio frequencies. The latter can, of course, always be reduced by employing thinner laminations.

The variations just cited make the problem of the design and manufacture of telephone transformers to consistently uniform standards of performance rather difficult. It is hoped that developments, which have been recently instituted in conjunction with some of the suppliers of magnetic materials, will in the comparatively near future make it possible to obtain such materials under suitable magnetic specifications. An important factor contributing to the attainment of this end would be the recognition of the performance requirements of magnetic materials for such uses as Mr. Casper's paper describes, in standardized tests such as those adopted by the American Society for Testing Materials. The present standardized tests for transformer sheets are applicable only to power frequencies and flux densities. What is needed to meet the requirements of communication engineering is a test for permeability and core loss at a flux density of about 15 gausses. This test could be made at a frequency of say, 1000 cycles with a bridge type of testing circuit using an ordinary telephone receiver as an unbalance detector. Simultaneous readings of inductance and effective resistance can be taken in this way from which the permeability and core losses are readily calculated.

I believe that the adoption of a uniform practise for specifying the characteristics of magnetic materials at low magnetizing forces would result not only in a more satisfactory condition as regards the production of telephone transformers, but would also lead to improvements in these characteristics due to a new appreciation by steel manufactures of the effect of their processes on the magnetic behavior of this material.

W. L. Casper: Mr. Fondiller has dealt with the use of silicon steel for the magnetic core material of the speech-frequency transformer giving figures for the initial permeability and the eddy-current and hysteresis losses. The much higher permeability and lower losses of permalloy as brought out in the paper by Arnold and Elmen before the Franklin Institute in 1923 naturally raises the question as to whether permalloy will not supersede silicon steel for this purpose. Obviously, wherever the circuit conditions are such as to require the transformer core to operate only at this initial value and not to require the introduction of gaps in the magnetic circuit these characteristics of permalloy will be of great advantage. For instance, the phantom-circuit repeating coil which is a toroidal-type transformer wound on a silicon-steel core and which weighs about 3½ lb., has been duplicated experimentally in efficiency to talking currents by a transformer of similar construction but having a permalloy core and weighing under two ounces.

As brought out in the paper, telephone circuits, as a rule, have to perform more functions than simply to transmit speech currents and the phantom-circuit repeating coil is required, in addition to this primary function, either to transmit 20-cycle signaling current or be inefficient to Morse telegraph currents. As the amount of energy transmitted under either of these two conditions is vastly larger than the speech-current energy in addition to the frequency being considerably lower, it becomes an involved problem in design as to whether permalloy would prove good, depending largely on the cost of the core material.

Another case where the complex circuit conditions prevent full advantage being taken of the large difference in initial permeability of permalloy over silicon steel is that of the transformers which operate with direct current through their windings as, for instance, the input transformer. The design of such a transformer involves proportioning the core in such a way as to give maximum permeability under the operating conditions which is done by the introduction of an air gap of suitable length. Obviously, the greatest gain by the use of permalloy will be made in input transformers operating from low-space-current tubes. Whereas the economic advantage of permalloy is marked in transformers operating from tubes employing one or two milliamperes space current, its superiority is reduced when this space current is 10 or 20 milliamperes, requiring the cost of permalloy to be not greatly above the cost of its constituents in order to prove satisfactory.

Permalloy is being used at present in a number of those transformers which operate under conditions where maximum advantage may be taken of its high initial permeability and economic studies are in progress to determine under just what other circuit conditions it will prove advantageous.

An Electrical Frequency Analyzer

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Review of the Subject.—*An apparatus has been developed by means of which it is possible to measure and obtain a permanent record of the frequency components of an electric current wave. The device has two frequency ranges: 20 to 1250 cycles and 80 to 5000 cycles; the amount of power required does not in general exceed 500 microwatts; and the time necessary for making a record is about 5 minutes. An attachment is provided which permits of the making of simultaneous harmonic analyses of two complex waves in the same length of time.*

In principle, the process consists in feeding the complex wave to be analyzed into a selective network, the essential feature of which is a sharply tuned circuit whose frequency of tuning is controlled by varying the capacitance in small steps with a pneumatic apparatus similar to that in a player piano. A maximum of response of the circuit occurs at each frequency of tuning which coincides with a component of the complex wave. An automatic photographic recorder of the response to each frequency of tuning is provided by means of which the frequency and magnitude of each component of the complex wave may be obtained. For convenience of operation, an automatic control apparatus is provided, so that it is only necessary to connect the complex source or sources to be analyzed and press a starting button. The completed record of the analysis is delivered after the machine has passed through the entire range of frequencies.

The application has so far been principally to problems in the communication field such as the analysis of performance and distortion at audio frequencies of vacuum tube and mechanical oscillators and amplifiers, analysis of complex telephone waves and speech sounds, and the effect on a complex wave of transmission through electrical and acoustic apparatus. In the power field many applications are obvious, such as for example, quantitative comparison as to frequency content of the voltage and current supplied to and delivered by transformers, voltage and magnetic flux studies in generators and motors, commutation, and the effect of wave-shapes in power transmission line problems and control apparatus.

INTRODUCTION

THE harmonic analyzer described in this paper consists of a variable tuned circuit into which the complex current wave to be analyzed is introduced, and an automatic recording apparatus to register its response as the frequency of tuning is changed.

The first recorded use of a tuned circuit as an analyzer was by Pupin in 1894[1]. He analyzed power waves by measuring the response of circuits tuned to each of the harmonic frequencies. It has been the practise for a number of years to determine the frequency characteristics of currents and voltages on power circuits and noise on telephone lines by means of a variable resonant circuit which includes a telephone receiver for listening.

During the recent war a rapid automatic method was developed for varying the tuning of a circuit in such an analyzer in connection with the analysis of sounds radiated by submarines. The analyzer described in this paper is in principle the same as this apparatus but includes such improvements as were found desirable by experience to increase the speed, dependability and convenience of use. The present apparatus is capable of recording the frequency and magnitude of each component in a complex wave between 20 and 1250 cycles or 80 and 5000 cycles in about five minutes. This analyzer does not measure the phase of the various components but has the advantage that the frequencies need not be simple multiples of the fundamental as is the case with graphical analyzers. With this apparatus it is possible to measure quite accurately component frequencies as close together as about fifteen cycles at the lower end of the range and about 200 cycles at the upper end of the range, and to detect components as close together as three to five cycles at the lower end and fifty cycles at the upper end of the range.

PRINCIPLES OF OPERATION OF THE ANALYZER

Fig. 1 is a schematic diagram of the essential elements of the analyzer circuit. The wave to be analyzed is introduced at the input terminals from which it passes to an input equalizing network and to the variable tuned circuit. The tuned circuit consists of a variable condenser of capacitance C and a coil whose inductance

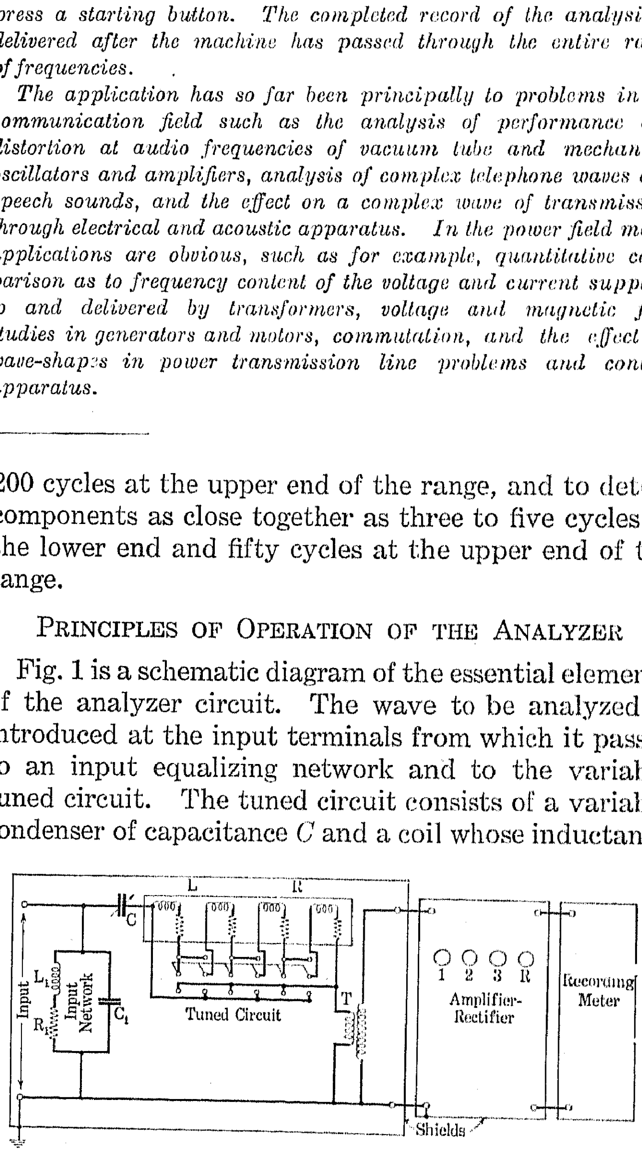

FIG. 1—SCHEMATIC ANALYZER CIRCUIT

is L and resistance R. The value of the capacitance C is varied in small steps by an automatic device to be described in the next section. The inductance L consists of four identical windings on a toroidal core which, by means of a switch, may be thrown in series or in parallel, thereby changing the value of the inductance in the ratio of 16 to 1. With the same range of capacitance values this change in inductance gives the two frequency ranges of tuning, 20 – 1250 cycles and 80 – 5000 cycles. By means of the high-ratio transformer T the response of this circuit is applied to a vacuum tube amplifier-rectifier and registered by means of the recording meter.

1. Resonance Analysis of Alternating and Polyphase Currents, TRANS. A. I. E. E., Volume XI, Page 523.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

This circuit arrangement will analyze a complex wave by virtue of the selective shunting of current by the tuned circuit from the input network. The impedance of the source of the complex wave is in practise maintained high in value at all frequencies compared to that of the input network so that the input wave-shape is independent of the small changes in impedance of the analyzer due to the varying of condenser C. The current fed into the analyzer traverses two paths, the input network and the tuned circuit. The impedances of these paths are respectively,

$$Z_1 = \frac{(R_1 + j\,\omega\,L_1)/j\,\omega\,C_1}{R_1 + j\,\omega\,L_1 + 1/j\,\omega\,C_1}$$

and $$Z = R + j\,\omega\,L + 1/j\,\omega\,C.$$

The transformer T introduces into the tuned circuit a small resistance and inductance, both of which are negligible. The input network impedance Z_1 varies gradually from 0.4 ohms for direct current to about 10 ohms at 5000 cycles. The values of the elements are: R_1 = 0.4 ohms, L_1 = 0.075 milhenries, C_1 = about 15 microfarads. Impedance Z of the tuned circuit depends on the setting of the variable condenser C.

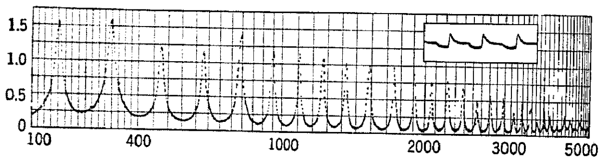

FIG. 2—RECORD OF 160 CYCLE BUZZER OUTPUT

The resistance R of the iron-core coil, varies with frequency; its values for the parallel connection are 0.7 ohms for direct current, 1.5 ohms at 2500 cycles and 4.2 ohms at 5000 cycles. The value of the inductance L for the parallel connection is 23.4 milhenries and is practically constant with change of frequency. For the series connection both R and L are sixteen times as great. The capacitance is varied from about 200 microfarads to about 0.05 microfarads. It will be seen that for each capacitance value there is a frequency, $f_r = 1/(2\,\pi\,\sqrt{L\,C})$, for which the tuned circuit impedance, Z, is R. For other frequencies Z is much greater due to the reactance. An incoming current of frequency f_r is, therefore, largely shunted through the tuned circuit while current of any other frequency passes through the input network. In this way if the capacitance C is varied gradually the tuned circuit will shunt selectively from the input network the successive components of the complex wave.

The special features of design of this analyzer circuit can be better explained by reference to a typical record made by the apparatus. Fig. 2 is the record of analysis of the current from a buzzer which vibrates with a frequency slightly under 160 per second and gives an irregularly shaped wave which is shown in the accompanying oscillogram. In taking this record the windings of the tuning inductance were in parallel so as to give the frequency range 80-5000 cycles. The vertical scale gives approximately the r. m. s. current in milliamperes at each frequency (as read on the horizontal scale) at which a peak occurs. It will be seen that a peak occurs at each multiple of the frequency of the buzzer. The r. m. s. values of input current at the corresponding frequencies as read from the peaks on the record are: 160, 1.6 milliamperes; 320, 1.6 milliamperes; 480, 1.25 milliamperes; 640, 1.2 milliamperes; 800, 1.45 milliamperes; 960, 1.25 milliamperes; 1120, 1.2 milliamperes: 1280, 1.1 milliamperes; 1440, 1.05 milliamperes; 1600, 1.0 milliamperes; 1760, 1.0 milliamperes; etc. The root square sum of all components shows that 4.7 milliamperes was the effective value of the complex current fed into the analyzer.

The fact that the 80 – 5000 cycle records read directly the current at each frequency component is due to the special design of the input network. A small correction is still necessary but can be neglected except where maximum obtainable accuracy is desired. If the input network were a pure resistance the higher frequency components would produce relatively lower peaks because of the falling off of efficiency with frequency of the amplifier-rectifier circuit and the increase in resistance of the tuning coil. The input network was designed empirically so that with constant input current the voltage drop across the input terminals increases with frequency in such a way as to compensate for these high-frequency losses. The tests to determine this were made by taking records of single frequencies of known amounts.

It will be seen that the frequency scale is gradually contracted as the upper end of the record is approached. Owing to the increase in resistance of the coil with frequency, the sharpness of tuning of the analyzing circuit decreases with frequency. Each peak on the record corresponding to a single frequency is a plot of the resonance curve of the variable tuned circuit. The sizes of the capacitance steps are so adjusted that a sufficient number of points, necessary to trace a resonance peak at all frequencies, is recorded. The length of the record and the time required for an analysis are determined by the number of points needed.

When peaks on the record are so close together as to overlap greatly, the reading on the scale is untrustworthy. If, instead of a rectifier and direct-current meter, an alternating-current meter giving deflections proportional to total r. m. s. values, were used, it would be theoretically possible to determine the component frequencies and amplitudes making up any composite peak, provided the number of frequencies could be determined. This procedure, however, would be impracticable. An examination of the theory of the rectifier shows that the problem of separation of the components of a composite peak is in general in-

determinate. The rectifier however resolves adjacent peaks somewhat better than an alternating-current meter.

The analyzer has been most used in the analysis of audio-frequency currents for which the higher frequency range, 80-5000 cycles, is more useful. For the investigation of power problems the lower range would ordinarily be more suitable. In order to simplify the change from one frequency range to the other the tuning inductance only, is changed, leaving the mechanism for varying the capacitance in steps the same for both ranges. Since the inductance change in going from the high to the low-frequency range is in the ratio 1:16 and the change in the frequency range 4:1, the abscissas on the low-frequency records have one-fourth the value of those on the high-frequency records.

Since the smallest frequency divisions at the lower end of the high-frequency records are 20 cycles, these divisions on the low-frequency records are 5 cycles. There are, therefore, four times as many steps of tuning in the same frequency interval on the low as on the high-frequency record. The low-frequency record is therefore not of minimum practicable length. Since the same input network is used with the 20-1250 range as with the 80-5000 range, the low-frequency records are not direct reading in input current, but must be used with a calibration. Our use of the low-frequency range, however, has been so limited as not to justify the preparation of additional equipment for this use of the analyzer.

The apparatus is equipped with a device which permits of making simultaneous analyses of two complex waves. The principal reason for making such double records is to reduce errors in comparing two sources which may vary with time. The device may also be used simply to save time. It operates by connecting alternately to the analyzer the two complex waves in such a way that the record for each wave is traced by points representing alternate tuning condenser settings.

Description

The mechanism of the analyzer is so designed that to take a record it is only necessary, after starting the amplifier and connecting to a 110-volt power source, to attach the leads from the source or sources to be analyzed and press a starting button. The completed record is then automatically delivered in about 5 minutes after which the apparatus returns to the starting condition ready to repeat the operation. This is accomplished by means of pneumatic apparatus operating in conjunction with a photographic recording device.

The pneumatic arrangement is a modification of a piano player mechanism in which a paper roll of standard dimensions is used. By proper perforation of the roll special pneumatic relays are operated in proper sequence to switch the condensers of the tuned circuit, flash frequency lines on the record, stop the mechanism after a record has been completed, rewind the piano roll, and perform other functions necessary to leave the analyzer in the starting condition. Electrical relays for switching the tuning condensers were not found practicable on account of the disturbances induced into the analyzer circuit.

The photographic recording apparatus consists of the camera motor for moving the sensitized record paper at a constant rate, proper arrangement of lenses and lamps for illuminating the mirror galvanometer and tracing the scale and frequency lines, and suitable baths for developing and fixing the record. The record is drawn through the mechanism by means of two motor-driven rubber rollers, which also serve to remove excess solution.

The development of the pneumatic switching apparatus was carried out with a view to making use of as many standard piano player parts as possible. However, it was found necessary to make some modi-

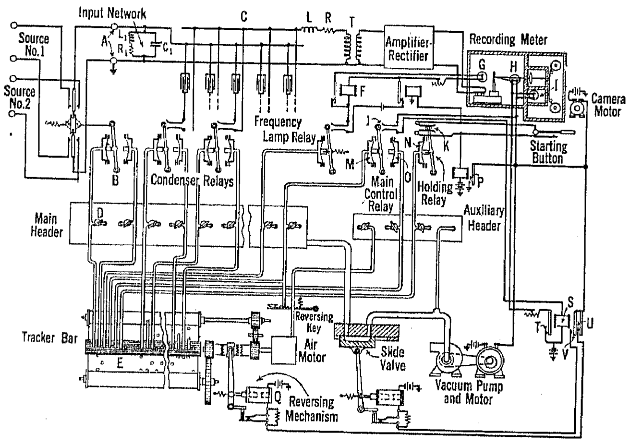

Fig. 3—Arrangement of Pneumatic and Electrical Apparatus

fications in method and apparatus; in particular a new pneumatic motor element (air relay) for switching the condensers at the requisite speed had to be developed.

Fig. 3 is a schematic drawing showing the principal features of the analyzer. In this drawing the vacuum pump is shown driven by an electric motor, and connected by means of pipes to the auxiliary and main headers and relays. This pump maintains in the headers an absolute pressure of about 4 or 5 lb. per square inch. The player piano roll *E* operates the entire mechanism by passing over the tracker bar in the usual manner. The air motor and tracker bar equipment are substantially as supplied by the manufacturers except that the reversing mechanism is arranged to be operated electrically instead of by hand.

The essential features of the air relay which was developed for this analyzer may be better understood by reference to Fig. 4. A cylindrical casting is arranged to mount two flexible diaphragms and two end plates in such a way as to form at each end of the cylinder,

compartments, one side of each of which is a diaphragm. When assembled the two diaphragms face each other and are connected together by a circular spring made of steel strip. In use the two end compartments are partially evacuated thus causing the diaphragms to pull apart, straining the spring. When distended the diaphragms lie against the inner faces of the end plates which are shaped as shown. Obviously if air be allowed to enter either of the compartments the diaphragm belonging thereto will be pulled toward the

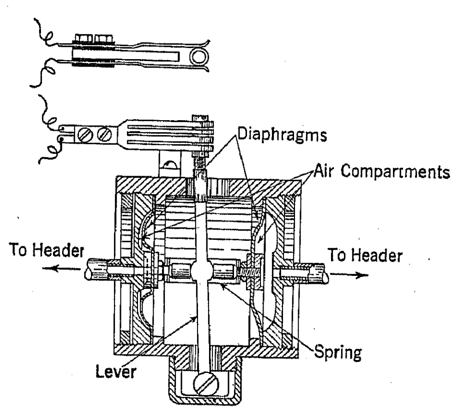

FIG. 4—PNEUMATIC RELAY

other diaphragm by the spring. Passing through the circular spring is a lever pivoted at one end and carrying on the other end an insulated metallic sleeve. This lever is not attached in any way to either diaphragm and will of itself remain in position where last placed. Switch points are mounted in such a way that the sleeve may be forced in or out between them by the action of the diaphragms. This relay has proved very satisfactory in service and is particularly fast in its operation.

Connections between the tracker bar, main header, and the pneumatic relays are made by means of rubber tubing. As shown in Fig. 3 each of these relays requires two rubber tubes leading to the main header and two from the header to the tracker bar. These tubes are connected to the header by means of stop cocks D so connected that the direct passage of air from tracker bar to relay is practically unobstructed but the passage leading from the junction to the header may be made as small as desired by turning the finger valve. As adjusted, the opening to the header is small compared to the size of the tubes so that if air be permitted to enter one of the tube lines (as at the tracker bar), the diaphragm of the relay associated therewith is immediately released. When the tube is closed again, the entrapped air is soon removed through the small opening leading to the header thus restoring the diaphragm to its original position. The relay lever, however, does not follow the diaphragm.

This arrangement possesses the advantage that small openings only are necessary in the player piano roll, and that the opening which connects a condenser into the circuit is not in line on the roll with the opening which disconnects this condenser. Also at the beginning of an analysis by suitable perforations in the roll all air relays can be set simultaneously in the off position (condensers disconnected), thus making sure of the initial conditions. The apparatus is so designed that all the openings causing condenser circuits to close are on one side of the roll and those causing them to open are on the other side.

As before mentioned the tuned circuit is made up of inductance L, having some resistance R, and a bank of condensers designated by C. The function of the "Condenser Relays" is to connect into the tuned circuit any one or any combination of the 25 fixed condensers, thus tuning the circuit in small steps over a wide range of frequencies. The input is fed into this circuit as shown at A, and the degree of resonance, that is the response of the circuit at any particular frequency of tuning, is measured by means of the small transformer T, the amplifier-rectifier and the recording meter.

In addition to operating the tuned circuit a few of the air relays are used to operate the control circuits, mark frequency lines on the chart, etc., uses which required slight modification as indicated schematically in Fig. 3.

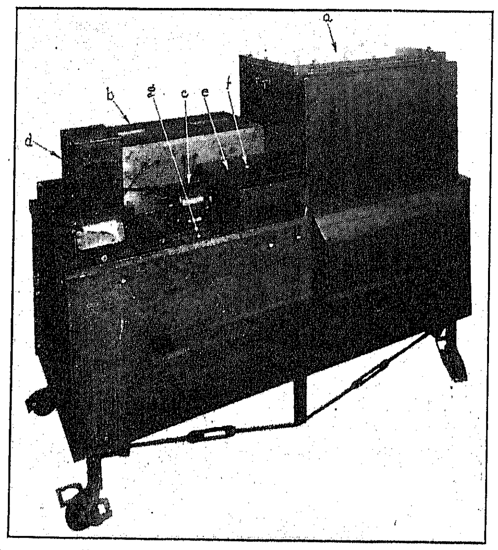

FIG. 5—VIEW OF ANALYZER READY FOR USE

a Input and tuned circuits
b Amplifier-rectifier
c Camera motor
d Recording meter
e Control box
f Starting button
g Reversing key

In two of these control relays only one diaphragm is used, and the switch lever and diaphragm are fastened together by means of a flexible link. It has already been noted that the analyzer is equipped to trace two curves simultaneously on a single record. This is accomplished by means of air relay B which is so arranged as to connect two sources of input alternately to the analyzer. These input connections are alternated rapidly and are effected by appropriate punching of the roll.

The above covers the essential features of the analyzer but there remain a few details having to do with assembly, control, etc., that may be of interest.

Fig. 5 shows the analyzer as completed and ready to operate. The apparatus is assembled on a two-deck, structural-steel table equipped with castors for convenience in handling. Much of the equipment is inclosed for protection against moisture and dust. The recording meter, camera motor, amplifier-rectifier, control relays, and input and tuned circuits are placed

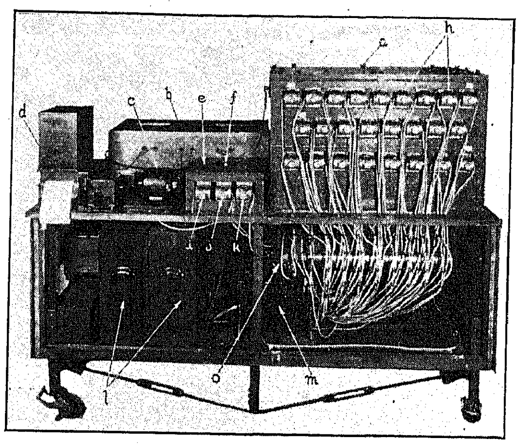

FIG. 6—VIEW OF ANALYZER WITH RELAY SIDE UNCOVERED

a Input and tuned circuits
b Amplifier-rectifier
c Camera motor
d Recording meter
e Control box
f Starting button
h Condenser relays
i Holding relay
j Frequency lamp relay
k Main control relay
l Plate battery
m Air motor
o Main header

on the top. Below are mounted the batteries, the vacuum pump with its motor, and the tracker bar with paper roll mechanism. The arrangement is clearly shown in Figs. 6 and 7 taken with the protecting panels removed. In Fig. 6, *a* is a moisture-proof box containing the input and tuned circuits. The inductance coil is placed in the center of the upper half of this box together with the switch for connecting the windings in series or parallel. The smaller capacitances are of mica and are arranged around the coil and switch assembly in such a way that they may be connected with a minimum length of lead to the air relays which are located on one side of the box. The larger capacitance units are made up of paper condensers and are placed in the lower half of the box. The metal lined box *b* contains the amplifier-rectifier, and at *d* is shown the recording meter. Box *e* contains the control circuits with the necessary relays. The method of mounting the air relays, main vacuum header (attached to underside of table top), air motor, etc., is also clearly shown in this figure. Each air relay is equipped with two rubber tubes leading to adjustable cocks on the header which in turn are connected to the tracker bar. The three-control relays are also shown in Fig. 6. The vacuum pump is shown at *v* in Fig. 7. The piano roll *E* moves over the tracker bar *w* and is reversed by means of solenoid *Q*. In boxes *l* are placed the plate batteries for the amplifier-rectifier.

The control apparatus by means of which the analyzer becomes practically an automatic machine will now be described. Referring again to Fig. 3 it will be seen that there is provided an auxiliary header and an electrically operated slide valve. The functions of these devices will be discussed presently.

The machine is started by pressing the starting button which should be kept closed for a few seconds while normal vacuum is being established in the headers. The air motor then starts and the paper roll *E* begins to travel across the tracker bar. Perforations in the roll are so made that when the roll is in its initial position an opening allows air to enter chamber *N* of the holding relay. As soon as the paper starts, however this opening is closed, chamber *N* is exhausted, and contacts *K* close. This short-circuits the starting button which the operator may now release, and the machine is in full operation. It will be noted that the closing of the contacts of the starting button or contacts *K* puts into operation motors which drive the vacuum pump and the camera apparatus. Simultaneously recording meter lamp *H* and scale-line lamp *I* are lighted. The latter illuminates the record through small holes in an opaque scale strip thus marking horizontal lines due to the motion of the record.

As the roll *E* traverses the tracker bar, appropriate

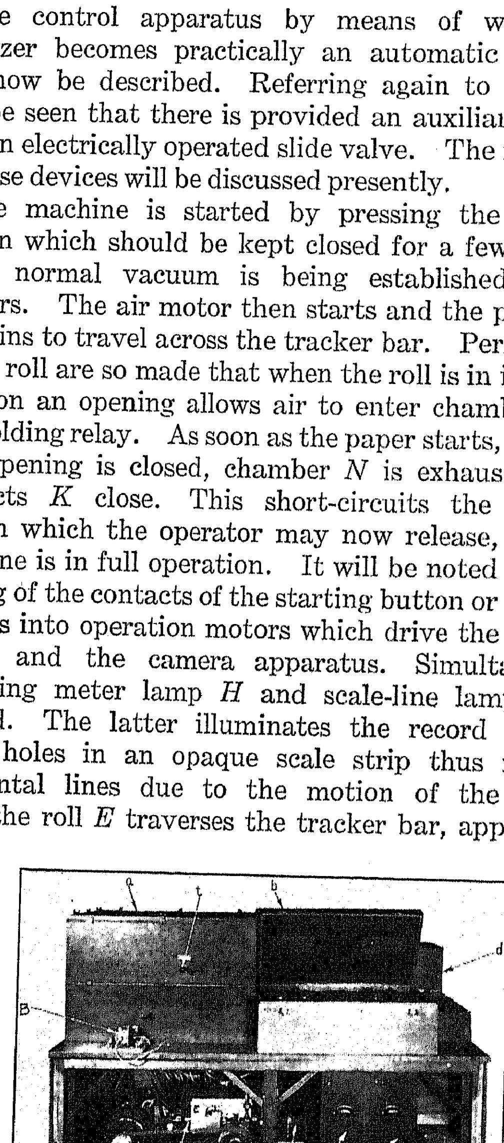

FIG. 7—VIEW OF ANALYZER WITH PUMP SIDE UNCOVERED

a Input and tuned circuits
b Amplifier-rectifier
B Source-alternating relay
d Recording meter
E Paper roll
l Plate battery
n Pump motor
Q Reversing solenoid
t Series-parallel coil switch
v Vacuum pump
w Tracker bar

perforations control the condenser relays so as to switch the proper condensers into and out of the tuned circuit. Proper perforations also control the frequency lamp relay which flashes frequency lines on the record by means of lamp *G*. Relay *F* is inserted in order to make the flash of short duration.

The tracker-bar-paper roll apparatus was received as a unit from the manufacturer and was installed after making modification in the reversing mechanism as mentioned above. This was done in the interest of

automatic control. The paper roll is kept in its proper course over the tracker bar by means of an automatic adjusting device such as used in practically all high grade player pianos.

As the paper progresses over the tracker bar a point is finally reached where the last condenser connections are made and it becomes necessary to rewind the roll and to restore the entire mechanism to its starting condition. This is accomplished by means of a perforation at the end of the record which admits air to chamber *M* of the main control relay, thus closing contacts *J*. Relay *S* then operates since its circuit to ground is completed through contacts *P*. Operation of relay *S* opens contacts *T* thus disconnecting lamps *H* and *I*, and closes contacts *U* and *V*. It will be seen that the closing of contacts *U* operates the reversing mechanism, and rewinding of the roll begins immediately. The closing of contacts *V* operates the slide valve thus releasing the vacuum on the main header, allowing the roll to be rewound with minimum mechanical drag.

It may be noted that means are also provided for rewinding the roll from any point in its forward travel by admitting air manually at the reversing key. This will cause the main control relay to operate so that rewinding will begin. Vacuum is kept on the auxiliary

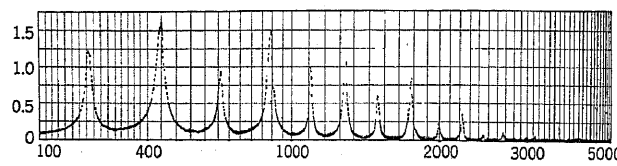

FIG. 8—OUTPUT OF CARBON BUTTON DRIVEN AT AN EXCESSIVE AMPLITUDE

header during the rewind so that control of the analyzer may be maintained to the end of the operating cycle.

When the paper has been completely rewound perforations allow air to enter simultaneously chamber *O* of the main control relay and chamber *N* of the holding relay. This action opens contacts *J* and *K*, thus bringing the entire mechanism to rest in its initial starting condition.

APPLICATIONS

To show the variety of problems in which the analyzer is a useful means of investigation, a few illustrative records have been made and will be discussed. These records were taken in each case to illustrate the use of the analyzer and are not parts of investigations to which they are related. They cannot, therefore, be taken as representative of the performance of the apparatus tested.

One of the uses of the analyzer has been in the study of the performance of microphone buttons. Fig. 8, for example, illustrates the character of the distortion in a button when driven at an excessive amplitude. The button was mounted so that its movable electrode could be driven at a single frequency by a very heavy reed at its natural frequency so that the motion was very nearly sinusoidal. The frequency of the motion was a little less than 450 cycles corresponding to the second peak on the record. The amplitude of motion was 0.001 centimeter or 0.0004 inch which is, of course, much greater than normally obtains in a transmitter. The circuit consisted simply of the button and a battery in series with the analyzer so that the record is an analysis of the current fluctuations in the button. The record shows two series of frequencies generated by the button; a primary series, having for its fundamental the driving frequency, 450 cycles, and a subsidiary series, having for its fundamental half the driving frequency or 225 cycles. The even harmonic components of the secondary series coincide, of course,

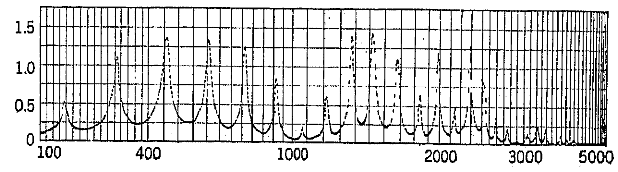

FIG. 9—NOISE IN ROOM AS PICKED UP BY CONDENSER TRANSMITTER

with the frequencies of the primary series. The primary series can be accounted for by the fact that with such large amplitudes the changes in resistance are not a linear function of the amplitude of motion. The subsidiary series is due to the non-symmetrical effect of the inertia of the carbon grains in vibration, the motion being so violent that some of the grains are thrown free from their contacts. For small amplitudes such as those ordinarily encountered in a transmitter, a record would show only 450 cycles, the other frequencies occurring in negligible amount; for intermediate amplitudes the primary series only occurs.

The analyzer has been used in connection with the

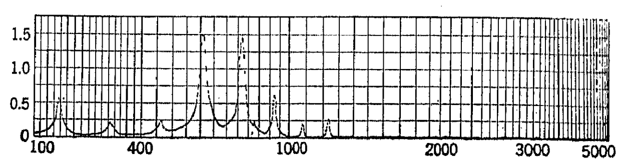

FIG. 10—NOISE IN ROOM AS PICKED UP BY TELEPHONE RECEIVER USED AS A TRANSMITTER

study of sustained sounds and of the performance of acoustical apparatus. Fig. 9 is a record of the noise in a room originating from a buzzer as picked up by a condenser transmitter.[2] The reverberation in the room probably had a large effect on the character of this record. With such a source of frequency the analyzer may be used to study the acoustics of rooms. Fig. 10 is a record of the same noise as in Fig. 9 but as picked up by a common type of telephone receiver

2. "A Condenser Transmitter as a Uniformly Sensitive Instrument for the Absolute Measurement of Sound Intensity." E. C. Wente, *Physical Review*, July 1917.

"The Sensitivity and Precision of the Electrostatic Transmitter for Measuring Sound Intensities." E. C. Wente, *Physical Review*, May 1922.

placed in the same position as the condenser transmitter. A comparison of Figs. 9 and 10 will show the inadaptability of such a receiver for use as a transmitter. The receiver, owing to the resonance of its diaphragm, is seen to be relatively sensitive in the region of 600 to 800 cycles and insensitive at most other frequencies. When this instrument is placed against the ear, as when used as a receiver, the diaphragm resonance is damped so as to give more nearly uniform response.

By means of the calibration of the condenser transmitter and its amplifier, it is possible to make an analysis of the absolute intensity of a sustained sound in the air. This method has been used to study the frequency characteristics of musical instruments. Fig. 11 shows the analyses of three low-frequency organ

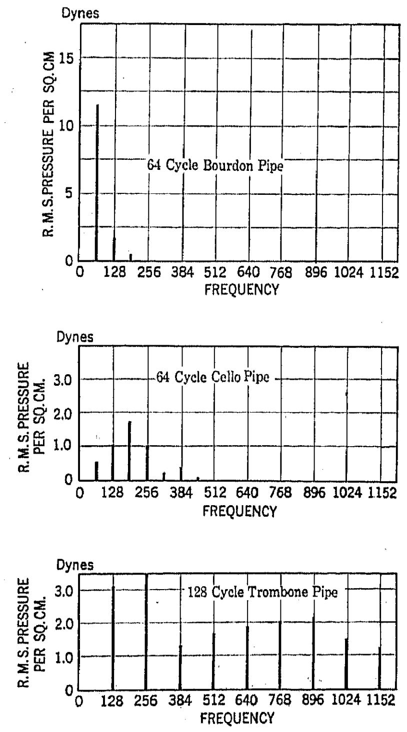

Fig. 11—Analysis of Organ Pipe Tones

pipes. These are plots of r. m. s. pressure change in the sound wave as obtained from the analyzer records. Each vertical line corresponds to a peak on the original record. The upper chart shows the almost pure tone given by a 64-cycle Bourdon pipe. In the case of the cello pipe, also having a fundamental of 64 cycles, the third harmonic is seen to be more prominent than the fundamental or second harmonic. The third chart is for a 128-cycle trombone pipe which was found to be rich in harmonics. The pressure in the single components of the cello and trombone pipes is less than in the case of the Bourdon pipe, and a larger scale of ordinates is therefore used.

To illustrate the use of the attachment which permits the making of two simultaneous analyses, a few double records will be presented. An electric wave filter which has been used in the study of telephone quality was connected to the buzzer source whose output is shown in Fig. 2. Simultaneous analyses of the current delivered to and transmitted through the filter are shown in Fig. 12. This filter is designed to pass all frequencies below 1000 cycles and to suppress all others. The input is represented by a more or less continuous series of peaks along the entire length of

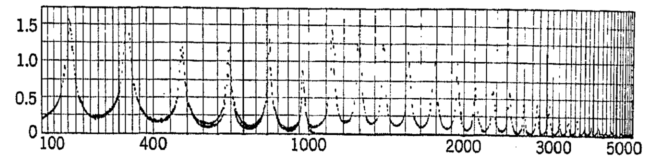

Fig. 12—Record Showing Action of Low Pass Filter

the record. The peaks corresponding to the output coincide rather closely with the input peaks for all frequencies below 1000 cycles and are not detectable for the higher frequencies.

Fig. 13 is a double record showing the analysis of the wave from a buzzer as fed into a common type of loud speaking receiver and the acoustic output as picked up by a condenser transmitter placed in front of

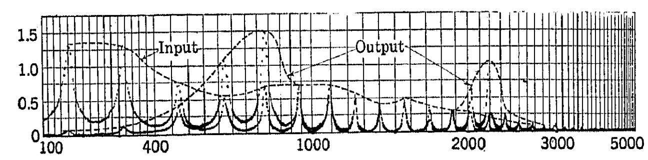

Fig. 13—Records of Electrical Input and Acoustic Output of a Common Type of Loud Speaking Receiver

it at a distance of about 15 inches. The analysis of the input current wave to the loud speaker is shown by the comparatively continuously decreasing series of peaks. The acoustic output is represented by the series having maxima in the neighborhood of 800 cycles and 2200 cycles. This record cannot be taken as an adequate analysis of this loud speaker because of probable reverberation effects in the room.

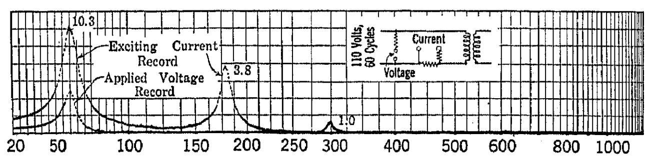

Fig. 14—Record Taken on Transformer at No Load

The analyzer has thus far not been used in the study of power problems. A few illustrative records have been taken, however, on transformers and generators and will be shown as suggestive of the use of this method of attack in such problems.

Fig. 14 is a double record showing applied voltage and exciting current of a small 110-volt, 60-cycle transformer operating at normal voltage and frequency

under the no-load condition. The presence of the well known third and fifth harmonics in the exciting current is clearly shown. Because of the rise in the calibration curve of the analyzer at the low end of the lower frequency range, a scale of ordinates is not shown on this record. Instead, the values of the analyzer current at each frequency are noted on the record. The circuit used in making this record is drawn on the figure. A computation of the components of the exciting current from the record and constants of the circuit shows that at 60 cycles the current was 175 milliamperes, at 180 cycles, 65 milliamperes and at 300 cycles, 17 milliamperes. The total r. m. s. exciting current was, therefore, 187 milliamperes.

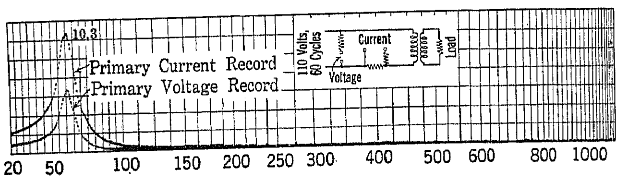

Fig. 15—Record Taken on Transformer Under Load

The operation of this transformer under full load is shown in Fig. 15, where, as before, the primary voltage and current are analyzed. The transformer load consisted of a pure resistance. It will be noted that the third and fifth harmonics have become very small compared with the fundamental. The analyzer currents at each frequency are again noted on the record. In obtaining the analysis of the current it was necessary to further shunt the analyzer. The primary current was 310 milliamperes.

Problems relating to commutation may also be conveniently studied qualitatively and quantitatively by means of the analyzer. The use of an apparatus which will indicate the source and measure the extent of

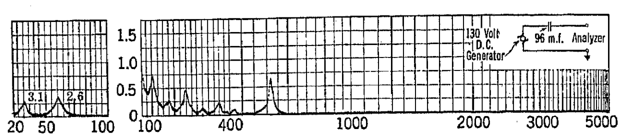

Fig. 16—Record Taken on D. C. Generator at No Load

parasitic frequencies is obvious. Information has been obtained on a small machine direct-driven by a ½-h. p., 60-cycle single-phase motor. Data of importance relating to the generator tested are as follows:

Capacity of Generator	¼ kw.
Number of Poles	2
Speed	1725-1800 r. p. m.
Voltage	125
Field	Shunt-connected
Diameter of Commutator	2.75 in.
Number of Commutator Bars	38
Number of Armature Slots	19
Size of Brush	3/8 in. square
Yoke	Ring type

Records obtained from this machine when operating under no-load and half-load conditions are shown in Figs. 16 and 17, respectively. The corresponding speeds are approximately 1800 and 1750 r. p. m. In order to show what frequencies the machine gives out over the entire range 20 to 5000 cycles each figure is made up of two parts: a portion of a 20-1250 record and a complete record over the range 80-5000 cycles. On each figure is drawn the circuit connecting the d-c. generator to the analyzer. It will be noted that a large condenser is inserted to prevent the passage of heavy direct current through the analyzer.

The consideration of these records leads to the conclusion that there are at least three independent major causes of alternating voltage operating in this d-c. machine. The fundamental frequencies due to these causes are 30, 60 and 570 cycles. It will be noted that the 30-cycle peak occurs only on the no-load record under which condition the average speed is practically 30 revolutions per second. Sixty cycles and a series of its harmonic overtones are seen to be present under both conditions of load. Under load the 60 cycles is augmented whereas its harmonics are reduced. No harmonic overtones of 30 cycles except such as might coincide with the harmonics of 60 cycles

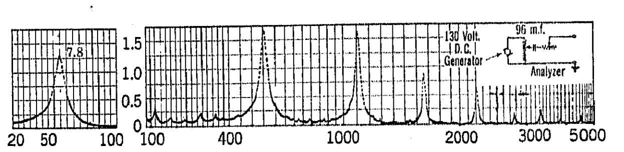

Fig. 17—Record Taken on D. C. Generator Under Load

are found in either case. This indicates the existence of independent causes of the 30 and 60 cycle frequencies, that the 30-cycle cause produces an almost sinusoidal voltage, and that the 60-cycle cause under no load produces an irregular wave which becomes smoother as the machine is loaded.

The no-load record, Fig. 16, shows 570 cycles with no harmonics while the load record, Fig. 17, shows 570 cycles with a complete series of harmonics. This indicates that at no load the cause of 570 cycles feeds a relatively smooth wave to the line while under load this cause feeds an irregular wave to the line. The fact that 1140 cycles is about as strong as the fundamental and that its harmonics are stronger than alternate ones which are overtones of 570 only, suggests the likelihood of a fourth cause having a frequency of 1140 cycles. Small irregularities at frequencies other than those already mentioned occur in the record. These are more prominent under load than at no load and indicate the presence of small, more or less irregular pulses, which increase with load. All of the above frequencies may be accounted for by a consideration of the construction and operating condition of the machine.

The generator was driven by a single-phase, 4-pole,

60-cycle motor which may give rise to torque fluctuations once per revolution, or 30 times per second. Under no load this may produce considerable corresponding fluctuations in speed while under load conditions the generator acts as a damper, eliminating these oscillations.

The 60-cycle peak may be due to any one or some combination of a number of causes, *e. g.*, eccentricity of generator armature, non-uniform winding, non-uniform thickness of mica separators in commutator, high mica between one or more pairs of segments, etc. The records show that for this particular machine in its present condition (new) at normal speed the 60-cycle voltage developed increases considerably with load indicating strongly that the cause is largely influenced by an IR drop somewhere in the machine. The most likely causes therefore appear to be commutator eccentricity, irregular spacing of the segments, or high mica.

The peak at 570 cycles may be accounted for by cyclic variation of flux entering the armature core as the teeth pass the pole faces. At no load the speed is approximately 1800 r. p. m. The number of teeth being 19, it is obvious that there will be 570 fluctuations of air-gap reluctance per second. Under no-load conditions the record shows a comparatively pure wave form for this cause. This is to be expected because of the comparatively uniform distribution of flux under the pole faces at no load. As the machine is loaded, however, the field is distorted and shifted giving rise to an irregular wave form of voltage which is responsible for at least a part of the large harmonic content shown by the load record.

The presence of 1140-cycle peak which is present only under the load condition may be due to the cyclic variation of voltage produced by the commutator bars leaving the brushes. Inasmuch as the speed is roughly about 29 revolutions per second the frequency with which bars leave brushes is about 1100 cycles. This frequency is present under the load condition only, thus indicating that it is due to an IR drop at the brush contacts or to an e. m. f. developed in the short-circuited coil with the brush off the magnetic neutral.

The very small irregularities on the record shown particularly between peaks above 550 cycles on the load record are probably due to slight chattering of the brushes.

It is of interest to note that the so-called frequency of commutation does not appear in either of the records. For this machine this frequency at no load is approximately 346 cycles per second.

From these records it is possible to determine the r. m. s. value of the alternating voltage at any frequency of interest. This is computed from a knowledge of the circuit constants and analyzer impedance. We, thus obtain for the 550 cycle peak (Fig. 17), a value of 0.8 volts and for the 60-cycle peak a value of 1.1 volts.

In general the records taken by means of the analyzer on this commutating machine, confirm quantitatively the well known fact that such machines may give rise to frequencies in the audible range. Consideration of the records indicates that these frequencies may be divided into two classes: First, those pertaining to and controlled by design, and second, those caused and controlled by the physical condition of the machine at any particular time. It is also interesting to note that the driving motor may produce an appreciable effect, particularly under the no-load condition.

Summary

In the above paper there has been given a short statement of the theory and construction of an automatic, recording, electrical frequency analyzer, together with illustrations showing its use and limitations in various fields.

This apparatus has been found very useful in the laboratory in the investigation of many different types of problems chiefly because of the speed with which records can be made and harmonic analyses obtained without computation.

In conclusion the authors wish to express their appreciation to Mr. C. E. Lane and Mr. C. E. Dean, of the Western Electric Company, Inc., for their assistance in the building of this machine and the preparation of this paper.

Discussion

H. Fletcher: I desire to emphasize the tremendous difference in the time required for obtaining analyses by means of this machine and by the method which has usually been employed in the past. After the electrical connections are made, all you have to do is to press a button and then after waiting five minutes you have a photographic record of the analysis, before you are ready for examination. This machine will do in two or three hours what previously required as much as two or three months' work for obtaining the spectrum analyses of such sounds as the output of a horn or organ pipe.

Since the development of the condenser transmitter, which has a practically uniform response with frequency, and of amplifiers, which have similar characteristics, it is now possible to use these new tools for picking up an acoustic wave and transforming it into electrical form without any appreciable distortion. When it is thus transformed it can be sent through the analyzer and in five minutes' time an analysis can be obtained which shows the frequency and magnitude of the components. You can readily see the tremendous advantage such a device will be in research work.

For example, we were trying to find the essential physical thing that caused one to judge the pitch of a musical tone. It is well-known that practically every musical tone has a large number of components, the fundamental and a series of harmonics. We were very much surprised to find that the elimination of the lower frequency range, including the fundamental and first few harmonics, did not change the pitch. For example, musical tones were observed, which had no component frequencies below 500 cycles per second, which still gave a pitch corresponding to a simple pure tone having a vibration frequency of 100 cycles per second. Naturally, when we first observed this phenomena, we were somewhat skeptical of the fact that these frequencies were actually eliminated. This

harmonic analyzer soon gave us convincing proof that they were absent.

Another example will suffice to show the usefulness of this apparatus for acoustic work. In studying the acoustics of rooms, if one took a record of a sound wave under certain conditions, and then had to wait two or three weeks before obtaining an analysis of this wave before making any changes in the room, such as, moving some of the furniture, it is evident that the progress would be very slow. With this device one can get an acoustic picture of what is taking place in the room, move a desk, or something else which you are interested in, and then take another picture, etc., so that in a few days time you can accomplish what used to required months without such a device.

L. P. Ferris: This paper describes a device which may be very useful in making investigations of wave form of power machinery such as may be carried out by large manufacturers. Because of its weight and bulk, the instrument is not suitable for making investigations in the field but for permanent installation in a large test room or laboratory, this would not be a serious drawback. Investigations are now being made of wave shape of power machinery involving the higher harmonics which are difficult to measure with the oscillograph. It is hoped, in these investigations, to obtain information looking toward the reduction of these harmonics because of their contribution to telephone interference. A device built along these lines may be of great assistance in these investigations, by disclosing quickly the magnitudes of the harmonics.

The Multiple-Radial System of Cooling Large Turbo-Generators

BY DONALD BRATT

Formerly with Power Engineering Dept., Westinghouse Electric and Mfg. Co.

Review of the Subject.—*The paper discusses the theoretical basis of a special turbo-generator ventilation system, in which the cooling air divides into several branches, and passes through the stator core radially in and out.*

An extended series of experiments on a full-size model, embodying this system, has lately been carried out by the Westinghouse Co. The tests are described in a paper by C. J. Fechheimer under the Title: "Experimental Study of Ventilation of Turbo-Alternators."

The fundamental questions in regard to the flow of air in any ventilation system are:

1. How high pressure is required to force through a certain volume of air per unit time?

2. How will the air distribute, axially and radially, in the different intake and discharge vents?

3. What will be the "balanced state" of flow, if several branches of air meet and divide in a tube, the intake and discharge taking place normal to the walls of the tube?

These questions are given a thorough analysis, under certain simplifying assumptions, and it is shown that

1. The total pressure required for a certain volume of air per unit time is expressible by means of hyperbolic and trigonometric cotangents of a certain argument, which contains the geometrical dimensions of the air-circuit.

2. The air is distributed according to a simple hyperbolic or trigonometric sine-law.

3. The "balanced state" depends on the solution of a system of simultaneous transcendental equations.

A method of solution is outlined, which is applicable for such cases where the arguments are small. In such cases the transcendental equations reduce to simple algebraic equations.

A numerical example is finally worked out in order to show the application of the derived formulas.

Description of System and Statement of Problem

FIGURE 1 illustrates the radial system of ventilation diagrammatically.

The fans $F\,F$ which are attached to the rotor R set up a static pressure in the end-bells $E\,E$. As a result the air, taken in by the fans at $a\,a$, is forced to flow through the system by way of i or g. g is the air gap, i is a duct back of the core, leading to several intake sections I in the core, in which the air travels radially

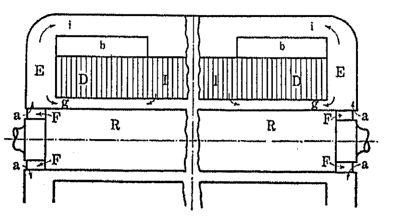

Fig. 1

inwards, dividing in the gap and discharging over the sections D, which are in communication with the external atmosphere at b.

The iron core is assembled as usual in packages, leaving equi-distant vents axially, in which the air alternatively enters and discharges. The arrows indicate the direction of flow.

It is of great importance to know how much air will pass the total system at a given static pressure in the end-bells.

Also, it is necessary to know how the velocity of the air is distributed, as this is of paramount importance relative to the temperature of the iron.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

Any attempt to answer these questions exhaustively would necessarily present very great mathematical difficulties. This is particularly evident, as the air in the gap is partly carried along by the rotation of the rotor. It will first of all be necessary, therefore, to neglect the influence of this rotation, which means the same as to assume that the static pressure in the end-bells is furnished by separate external fans. It has been conclusively proved, by actual tests on a full-scale model, that the influence of the rotation on the total volume of air passing is small, and also that the rotation tends to equalize the velocity in the different vents axially. Thus, the temperature in the iron is probably more evenly distributed under running conditions due to the effect of the rotation, so that our assumption errs on "the safe side."

The fundamental law, governing the flow of an

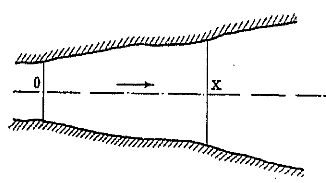

Fig. 2

incompressible fluid is the one first expressed by Bernoulli, stating, that the sum of static head, velocity head and losses, counted from some chosen origin and in the direction of the flow, remains constant. It follows from this, that for any particular circuit $o\,x$ (Fig. 2) we can put the total pressure drop from o to x proportional to the mean air velocity-square at x: the constant of proportionality depending, of course, on x.

Let P = total pressure-drop from o to x
p = static pressure at x
v = velocity at x
γ = density of fluid
g = acceleration of gravity.
then, at any point x we have

$$1. \quad p + \frac{\gamma\, v^2}{2\,g} + \sum_0^x \text{(losses)} = \text{Const.}$$

$$2. \quad P = \sum_0^x \text{losses} = k \times \frac{v^2}{2\,g}$$

The velocity v denotes the volume air per unit time divided by the cross-section at the particular point in question. Hence, no attention is paid to the fact, that the air velocity may vary across the section.

The loss in total head is neglected. Hence, the Bernoulli equation may be written:

$$p + \frac{\gamma\, v^2}{2\,g} = \text{const.}$$

This is necessary in order to obtain reasonably simple results. (Some additional remarks on this particular assumption will be found below).

It is further necessary to assume that the air behaves like an incompressible fluid, that is, that the "principle of continuity" holds.

This is comparatively justified, since the total atmospheric pressure is about 405 in. water, and the total pressure drop in a ventilation-system of this kind rarely would have to exceed 15 in. water, or 3.7 per cent of the total pressure.

The balanced state of flow in a multiple-circuit system will, for its solution, depend on the solution of the air distribution in every particular branch, each branch consisting of one intake and one discharge-length. It is, therefore, necessary to treat one such circuit for itself, assuming the intake- and discharge-lengths, etc., to be known, and then to apply the results to the multiple circuit.

This involves the assumption, that there is, between any two air streams in the tube, a closed wall across the tube, against which wall there is, however, no resultant pressure from either side, since the flow is balanced.

It is evident that such balanced points must exist in a multiple system, but it is difficult to say to what extent the conditions would change, (if they changed at all) if partition-walls were actually introduced.

It is, finally, necessary to assume a constant intake and discharge area per unit length of the gap axially, in order to apply the methods of calculus. This fictitious intake and discharge area per unit length is taken equal to the total cross-section area of one vent-circle axially, at the minimum cross-section, divided by the length of one iron package + one radial vent-width axially.

The preliminary problem has now been reduced to that illustrated by Fig. 3. A tube of, say, circular cross-section, closed at both ends, is to a certain length inserted in a pressure-chamber, while the rest of the tube protrudes into the surrounding atmosphere. The tube is perforated, giving a constant intake and discharge area per unit length of the tube.

It is required to determine the total volume of air passing the tube at a given pressure in the chamber, also to determine the law for the air distribution axially.

ASSUMPTIONS MADE TO SIMPLIFY PROBLEM

It is, perhaps, profitable to summarize the assumptions that have been discussed above.

a. The effect of the rotation is neglected.

b. The variation in air-velocity across the gap, or any other cross-section of the system, is neglected.

c. The loss of head in the gap (or tube) is neglected.

d. The air is incompressible.

e. At any such point in the air gap, where one incoming stream of air divides, or two streams meet, a closed wall is assumed to exist.

f. A uniform intake or discharge-area per unit length of gap axially is substituted for the actual, non-uniform area.

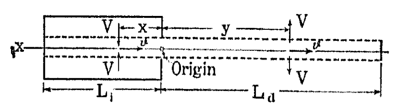

FIG. 3

LIST OF SYMBOLS (FIG. 3).

x = axial coordinate of intake-belt
y = " " " discharge-belt
p = static pressure in tube at x or y
v = axial air-velocity at x or y
V = intake or discharge air-velocity at x or y
a = uniform cross section area of tube
s = uniform intake or discharge area per unit length[1]
P = external pressure (in intake pressure-chamber)
γ = density of air
g = acceleration of gravity

c_i = intake-coefficient, defined by equation $\frac{P - p}{\gamma} = c_i \times \frac{V^2}{2\,g}$

c_d = discharge- " " " " $P/\gamma = c_d \times \frac{V^2}{2\,g}$

L_i = length of intake-belt
L_d = " " discharge-belt

1. The length-unit will henceforth be taken as one vent-pitch axially.

Note: The constants c_i and c_d deserve special attention. The definition

$$\frac{P - p}{\gamma} = c_i \times \frac{V^2}{2\,g}\,; \quad P/\gamma = c_d \times \frac{V^2}{2\,g}\,;$$

means, that the intake air-velocity V can be put proportional to the square-root of the drop in static pressure from the pressure-chamber to the air in the tube, and that the discharge air-velocity likewise can be put proportional to the square-root of the drop in static pressure from the tube to the atmosphere.

The numerical value of c_i and c_d can be determined from the dimensions of the vents in the teeth; the

method of calculation cannot, however, be discussed in this paper.

DERIVATION OF FORMULAS FOR INTAKE AND DISCHARGE

a. Intake. Consider the flow along the axis of the tube! By virtue of the assumptions, we must have the sum of the static head and the velocity head constant at all positions:

$$p/\gamma + \frac{v^2}{2g} = \text{const.} \tag{1}$$

But from definition of c_i we have

$$\frac{P - p}{\gamma} = c_i \times \frac{V^2}{2g} \tag{2}$$

So that

$$P/\gamma = P/\gamma - c_i \times \frac{V^2}{2g}$$

Substitute in equation (1), we get

$$P/\gamma - c_i \times \frac{V^2}{2g} + \frac{v^2}{2g} = \text{const.}$$

and by differentiation

$$v \frac{dv}{dx} - c_i V \frac{dV}{dx} = 0 \tag{3}$$

(observing that P/γ and $2g$ are constants.)
In addition to equation (3) we have the obvious condition that the increase in axial velocity over the infinitesimal length dx shall correspond to the air taken in over dx (principle of continuity.)

$$a\,dv = s\,V\,dx$$

or

$$V = a/s \times \frac{dv}{dx} \tag{4}$$

Hence, by differentiation of (4)

$$\frac{dV}{dx} = a/s \times \frac{d^2 v}{dx^2}$$

Substitute this expression in (3) then

$$v \frac{dv}{dx} - c_i\, a/s \frac{dv}{dx}\, a/s \frac{d^2 v}{dx^2} = 0; \text{ or, simpler}$$

$$\frac{d^2 v}{dx^2} - \frac{s^2}{a^2 c_i} \times v = 0 \tag{5}$$

This equation is linear in v and $\frac{d^2 v}{dx^2}$; it has "constant coefficients," and its solution is well-known to be:

$$v = A\, e^{\frac{Sx}{a\sqrt{c_i}}} + B\, e^{-\frac{Sx}{a\sqrt{c_i}}} \tag{6}$$

A and B are the two integration constants to be determined.

Initial conditions:

for $x = 0$ put $v = v_0$

" $x = L_i$ we have $v = 0$

Thus

$$v_0 = A + B$$

$$0 = A\, e^{\frac{SL_i}{a\sqrt{c_i}}} + B\, e^{-\frac{SL_i}{a\sqrt{c_i}}}$$

$$B = v_0 - A;\ 0 = A\, e^{\frac{SL_i}{a\sqrt{c_i}}} + (v_0 - A)\, e^{-\frac{SL_i}{a\sqrt{c_i}}}$$

$$A = -\left(v_0\, e^{-\frac{SL_i}{a\sqrt{c_i}}}\right) : \left(e^{\frac{SL_i}{a\sqrt{c_i}}} - e^{-\frac{SL_i}{a\sqrt{c_i}}}\right)$$

$$B = \left(v_0\, e^{\frac{SL_i}{a\sqrt{c_i}}}\right) : \left(e^{\frac{SL_i}{a\sqrt{c_i}}} - e^{-\frac{SL_i}{a\sqrt{c_i}}}\right)$$

Substituting in (6) thus

$$v = -v_0 \times \frac{e^{-\frac{S(L_i - x)}{a\sqrt{c_i}}}}{e^{\frac{SL_i}{a\sqrt{c_i}}} - e^{-\frac{SL_i}{a\sqrt{c_i}}}} + v_0 \times \frac{e^{\frac{S(L_i - x)}{a\sqrt{c_i}}}}{e^{\frac{SL_i}{a\sqrt{c_i}}} - e^{-\frac{SL_i}{a\sqrt{c_i}}}}$$

which can also be written

$$v = v_0 \times \frac{\sinh\left(\frac{s(L_i - x)}{a\sqrt{c_i}}\right)}{\sinh\left(\frac{sL_i}{a\sqrt{c_i}}\right)} \tag{7}$$

By aid of (4) then, after differentiation of (7)

$$V = v_0/\sqrt{c_i} \times \frac{\cosh\left(\frac{s(L_i - x)}{a\sqrt{c_i}}\right)}{\sinh\left(\frac{sL_i}{a\sqrt{c_i}}\right)} \tag{8}$$

and for the static pressure, by aid of (2)

$$p = P - \gamma \times \frac{v_0^2}{2g} \times \frac{\cosh^2\left(\frac{s(L_i - x)}{a\sqrt{c_i}}\right)}{\sinh^2\left(\frac{sL_i}{a\sqrt{c_i}}\right)} \tag{9}$$

b. Discharge. For the axis of the tube we have again:

$$p/\gamma + \frac{v^2}{2g} = \text{const. (equation 1)}$$

and by definition of c_d

$$p/\gamma = c_d \times \frac{V^2}{2g} \tag{10}$$

Substituting, then

$$c_d \times \frac{V^2}{2g} + \frac{v^2}{2g} = \text{const.}$$

By differentiation

$$c_d \cdot V \frac{dV}{dy} + v \frac{dv}{dy} = 0 \qquad (11)$$

In this case the axial velocity decreases as y increases, hence (compare 4).

$$a\,dv = -s\,V\,dy$$

or

$$V = -a/s \frac{dv}{dy} \qquad (12)$$

By differentiation of (12)

$$\frac{dV}{dy} = -a/s \frac{d^2 v}{dy^2} \qquad (13)$$

Substitute in (11) then

$$c_d\left(-a/s \frac{dv}{dy}\right)\left(-a/s \frac{d^2 v}{dy^2}\right) + v \frac{dv}{dy} = 0$$

or, simpler (compare (5)

$$\frac{d^2 v}{dy^2} + \frac{s^2}{a^2 c_d} \cdot v = 0 \qquad (14)$$

The solution of (14) is well-known to be:

$$v = A \sin\left(\frac{s\,y}{a\sqrt{c_d}}\right) + B \cos\left(\frac{s\,y}{a\sqrt{c_d}}\right) \qquad (15)$$

Initial conditions:

for $y = 0$ put $v = v_0$

" $y = L_d$ we have $v = 0$

Substitute in (15); then

$$v_0 = B;\ 0 = A \sin\left(\frac{s\,L_d}{a\sqrt{c_d}}\right) + B \cos\left(\frac{S\,L_d}{a\sqrt{c_d}}\right)$$

$$A = -v_0 \frac{\cos\left(\frac{s\,L_d}{a\sqrt{c_d}}\right)}{\sin\left(\frac{s\,L_d}{a\sqrt{c_d}}\right)};\ B = v_0$$

Then (15) becomes

$$v = v_0 \times \frac{\cos\left(\frac{s\,L_d}{a\sqrt{c_d}}\right) \times \sin\left(\frac{s\,y}{a\sqrt{c_d}}\right)}{\sin\left(\frac{s\,L_d}{a\sqrt{c_d}}\right)} + v_0 \cos\left(\frac{s\,y}{a\sqrt{c_d}}\right)$$

or, simpler

$$v = v_0 \times \frac{\sin\left(\frac{s\,(L_d - y)}{a\sqrt{c_d}}\right)}{\sin\left(\frac{s\,L_d}{a\sqrt{c_d}}\right)} \qquad (16)$$

and, by aid of (12)

$$V = v_0/\sqrt{c_d} \times \frac{\cos\left(\frac{s\,(L_d - y)}{a\sqrt{c_d}}\right)}{\sin\left(\frac{s\,L_d}{a\sqrt{c_d}}\right)} \qquad (17)$$

Finally, by aid of (10)

$$p = \gamma \times \frac{v_0^2}{2g} \times \frac{\cos^2\left(\frac{s\,(L_d - y)}{a\sqrt{c_d}}\right)}{\sin^2\left(\frac{s\,L_d}{a\sqrt{c_d}}\right)} \qquad (18)$$

If v_0 is the same both for intake and discharge and p has the same value at $x = 0$ and $y = 0$; which is, physically necessary, we may equate (9) and (18) at $x = y = 0$. thus

$$P/\gamma - \frac{v_0^2}{2g} \coth^2\left(\frac{s\,L_i}{a\sqrt{c_i}}\right) = \frac{v_0^2}{2g} \cot^2\left(\frac{s\,L_d}{a\sqrt{c_d}}\right)$$

or

$$P = \gamma \times \frac{v_0^2}{2g}\left[\coth^2\left(\frac{s\,L_i}{a\sqrt{c_i}}\right) + \cot^2\left(\frac{s\,L_d}{a\sqrt{c_d}}\right)\right] \qquad (19)$$

Equation (19) determines v_0 when P, L_i and L_d are known.

Note. The appearance of trigonometric functions in the formulas for the discharge deserves special attention.

We must have V *positive* for $y = 0$ (meaning discharge) hence it is necessary that $(s\,L_d) : (a\sqrt{c_d}) \leq \pi/2$.

The special case when $\frac{s\,L_d}{a\sqrt{c_d}} = \pi/2$ is particularly interesting. Then

$$v = v_0 \cos\left(\frac{s\,y}{a\sqrt{c_d}}\right);\ V = v_0/\sqrt{c_d} \sin\left(\frac{s\,y}{a\sqrt{c_d}}\right);$$

$$p = \gamma \frac{v_0^2}{2g} \cos^2\left(\frac{s\,y}{a\sqrt{c_d}}\right)$$

hence for $y = 0$ we have $V = 0$; $p = 0$

" $y = L_d$ " " $V = v_0/\sqrt{c_d}$; $p = \gamma \cdot \frac{v_0^2}{2g}$

If, therefore, $\frac{s\,L_d}{a\,\sqrt{c_d}} > \pi/2$ we will get $V = 0$; $p = 0$ for $y = 0$ *and* V *will remain* $= 0$ until such a y' has been reached, that

$$[s\,(L_d - y')\,] : (a\,\sqrt{c_d}) = \pi/2$$

From now on the static pressure starts to increase and the discharge begins.

It is evident, that the idea of a discharge-tube, where no air escapes except over a certain length adjacent to the closed end of the tube, must be in conflict with

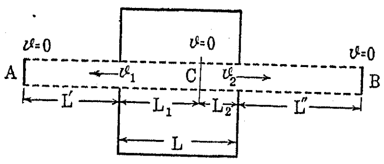

Fig. 4

reality. This is, however, a direct consequence of the assumption that there are no losses of total head in the tube. To overcome such losses a static pressure would always be necessary, resulting in a drop in static head from the tube to the atmosphere, *i. e.*, a discharge would always take place.

A good mechanical analogy is represented by a weight which is pushed over a rough table. The friction makes it necessary to apply a certain force behind the weight, which force would then correspond to the static pressure. If friction was eliminated, the pushing force would drop to zero, and the work represented by the uniform motion of the weight would just equal the work spent to establish the motion, or the work necessary to stop it.

The fundamental equation (1) would strictly have to be written:

$$p/\gamma + \frac{v^2}{2\,g} + \sum_0^x (\text{losses}) = \text{const.}$$

Assuming the losses to be proportional to the square of the axial velocity v and also, for a small length, proportional to that length, we would have

$$\sum_0^x (\text{losses}) = f \times \int_0^x \frac{v^2}{2\,g}\,d\,x$$

where f is a friction coefficient, so that

$$p/\gamma + \frac{v^2}{2\,g} + f \times \int_0^x \frac{v^2}{2\,g}\,.\,d\,x = \text{const.}$$

This equation is evidently much more complicated than (1) and it involves an assumption, which is quite arbitrary, as the law for the losses is not known. It is, therefore, necessary to neglect the losses, that is, to put

$$f \times \int_0^x \frac{v^2}{2\,g}\,d\,x = 0$$

in order to obtain simple results. To correct the error resulting from this approximation, it would be necessary to augment the values of c_i and c_d. These corrections could only be obtained from experiments.

Summary of Formulas

Intake:

$$v = v_0 \times \frac{\sinh\left(\frac{s\,(L_i - x)}{a\,\sqrt{c_i}}\right)}{\sinh\left(\frac{s\,L_i}{a\,\sqrt{c_i}}\right)}$$

$$V = v_0/\sqrt{c_i} \times \frac{\cosh\left(\frac{s\,(L_i - x)}{a\,\sqrt{c_i}}\right)}{\sinh\left(\frac{s\,L_i}{a\,\sqrt{c_i}}\right)}$$

$$p = P - \gamma \times \frac{v_0^2}{2\,g} \; \frac{\cosh^2 \frac{s\,(L_i - x)}{a\,\sqrt{c_i}}}{\sinh^2 \frac{s\,L_i}{a\,\sqrt{c_i}}}$$

Discharge:

$$v = v_0 \times \frac{\sin\left(\frac{s\,(L_d - y)}{a\,\sqrt{c_d}}\right)}{\sin\left(\frac{s\,L_d}{a\,\sqrt{c_d}}\right)}$$

$$V = v_0/\sqrt{c_d} \times \frac{\cos\left(\frac{s\,(L_d - y)}{a\,\sqrt{c_d}}\right)}{\sin\left(\frac{s\,L_d}{a\,\sqrt{c_d}}\right)}$$

$$p = \gamma \times \frac{v_0^2}{2\,g} \times \frac{\cos^2\left(\frac{s\,(L_d - y)}{a\,\sqrt{c_d}}\right)}{\sin^2\left(\frac{s\,L_d}{a\,\sqrt{c_d}}\right)}$$

$$P = \gamma\,.\,\frac{v_0^2}{2\,g}\,.\left[\coth^2\left(\frac{s\,L_i}{a\,\sqrt{c_i}}\right) + \cot^2\left(\frac{s\,L_d}{a\,\sqrt{c_d}}\right)\right]$$

Conditions for Balance in One Intake and One Discharge

If air is taken in over a total length L in a tube similar to the above (Fig. 5) and the discharge conditions at A and B are different, for instance so that the discharging ends of the tube are different in length, the question arises: How will the air subdivide between the two halves or, in other words, where is the axial air velocity = 0 in the tube?

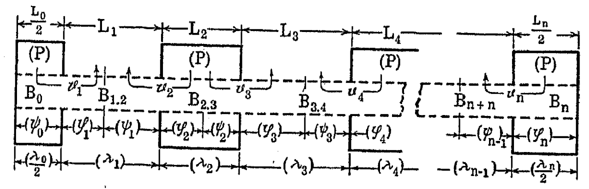

Fig. 5

Note: The quantities ψ φ and λ are here shown as lengths, but are really dimensionless quantities. This is done to show their relation to the actual lengths L_0 L_1, etc.

Notations:

P = total pressure drop
$v_1\,v_2$ = maximum axial air velocities
L = total intake-length axially
$L_1\,L_2$ = intake lengths corresponding to v_1 and v_2
$L'\,L''$ = discharge " " " " " "

Other notations as before.

Consider two separate systems, and a closed wall at $v = 0$ in the intake chamber.

The analytical expression for balance at C is the identity of V at $x = L_1$ and $x = L_2$ (counted from the left and from the right).

By use of equation (**8**) thus:

$$v_1/\sqrt{c_i} \times \frac{1}{\sinh\left(\dfrac{s\,L_1}{a\,\sqrt{c_i}}\right)} = v_2/\sqrt{c_i}$$

$$\times \frac{1}{\sinh\left(\dfrac{s\,L_2}{a\,\sqrt{c_i}}\right)};$$

which immediately gives

$$v_1 : v_2 = \frac{\sinh\left(\dfrac{s\,L_1}{a\,\sqrt{c_i}}\right)}{\sinh\left(\dfrac{s\,L_2}{a\,\sqrt{c_i}}\right)}$$

Further, from equation (**19**) applied to the left and to the right

$$P = \frac{\gamma\,v_1^2}{2\,g}\left[\coth^2\left(\frac{s\,L_1}{a\,\sqrt{c_i}}\right) + \cot^2\left(\frac{s\,L'}{a\,\sqrt{c_d}}\right)\right]$$

$$P = \frac{\gamma\,v_2^2}{2\,g}\left[\coth^2\left(\frac{s\,L_2}{a\,\sqrt{c_i}}\right) + \cot^2\left(\frac{s\,L''}{a\,\sqrt{c_d}}\right)\right]$$

Remembering $L_1 + L_2 = L$; we get for L_1 the equation

$$\frac{\coth^2\left(\dfrac{s\,(L - L_1)}{a\,\sqrt{c_i}}\right) + \cot^2\left(\dfrac{s\,l''}{a\,\sqrt{c_d}}\right)}{\coth^2\left(\dfrac{s\,L_1}{a\,\sqrt{c_i}}\right) + \cot^2\left(\dfrac{s\,L'}{a\,\sqrt{c_d}}\right)}$$

$$= \frac{\sinh^2\left(\dfrac{s\,L_1}{a\,\sqrt{c_i}}\right)}{\sinh^2\left(\dfrac{s\,(L - L_1)}{a\,\sqrt{c_i}}\right)}$$

which must be solved by some cut-and-trial method.

Discussion. For reasons of symmetry we get, when $L' = L'' : L_1 = L_2 = 1/2\,L$. Assume $L' = 0$; then

$$\cot\left(\frac{s\,L'}{a\,\sqrt{c_d}}\right) = \infty$$

and we must have

$$\sinh\left(\frac{s\,L_1}{a\,\sqrt{c_i}}\right) = 0 \text{ or } L_1 = 0$$

Similarly, when $L'' = 0$ we must have $L_2 = 0$ so that, when for instance L' decreases from $L' = L''$ to $L' = 0$, the balance point C will move from $L_1 = 1/2\,L$ towards $L_1 = 0$, and vice versa.

GENERAL MULTIPLE CIRCUIT; STATEMENT OF PROBLEM

The general problem represented by a multiple-radial system of ventilation is illustrated by Fig. 5.

This is merely an extension of the case treated above, when there are several air-branches in the tube, and hence several balance-points.

It is required to find the location of the balance-points. When these are known, the formulas for distribution of velocity are immediately applicable, and the total volume of air passing the system per unit time can be found.

It must be noted, that a solution of this problem can only be found by some cut-and-trial method, like in all cases, where transcendental functions of the unknown occur. Nevertheless, by simplifications, a sufficiently accurate solution can be obtained, as will be shown below, for certain cases.

LIST OF ADDITIONAL NECESSARY SYMBOLS

$\lambda_0/2$ is a fictitious length, corresponding to the drop in pressure, when the air enters from the End-Bell into the gap. (See Numerical Example below). $L_1\,L_2\,\ldots\,L_n$ are given lengths

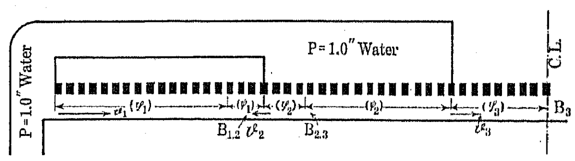

FIG. 6

Note: In regard to the quantities φ and ψ, see note, Fig. 5.

$$\lambda_0/2 = \frac{s\,L_0}{2\,a\,\sqrt{c_i}};\ \lambda_1 = \frac{s\,L_1}{a\,\sqrt{c_d}};\ \ldots\ \lambda_n/2 = \frac{s\,L_n}{2\,a\,\sqrt{c_i}}$$

are thus known, dimension-less quantities.

P = pressure in intake-chambers

$B_{i_1\,(i+1)}$ = balance-points, defined by $v = 0$

$v_1\,v_2\,\ldots\,v_n$ = maximum air velocities axially

$\psi_0\,\varphi_1\,\ldots\,\psi_{n-1}\,\varphi_n$ = unknown dimensionless quantities, determining the location of the balance-points, so that $\psi_0 = \lambda_0/2$; $\varphi_1 + \psi_1 = \lambda$, etc.

Other notations as before.

ESTABLISHMENT OF NECESSARY EQUATIONS TO SOLVE GENERAL PROBLEM

With above notations, using equations (**8**) and (**17**) as analytical expressions for balance, and equation (**19**) for the total pressure-drop to atmosphere, we get:

$$v_1/\sqrt{c_d}\cdot\frac{1}{\sin\varphi_1} \qquad P = \frac{\gamma\,v_1^2}{2\,g}\,(\coth^2\psi_0$$

$$= v_2/\sqrt{c_d}\cdot\frac{1}{\sin\psi_1} \qquad + \cot^2\varphi_1)$$

$$v_2/\sqrt{c_i}\cdot\frac{1}{\sinh\varphi_2} \qquad P = \frac{\gamma\,v_2^2}{2\,g}\,(\coth^2\varphi_2$$

$$= v_3/\sqrt{c_i}\,.\,\frac{1}{\sinh\psi_2} \qquad\qquad + \cot^2\psi_1)$$

$$\cdots\cdots\cdots \qquad\qquad \cdots\cdots\cdots$$

$$v_{n-1}/\sqrt{c_d}\,.\,\frac{1}{\sin\varphi_{n-1}} \qquad P = \frac{\gamma\, v_n^2}{2\,g}\,(\coth^2\varphi_n$$

$$= v_n/\sqrt{c_d}\,.\,\frac{1}{\sin\psi_{n-1}} \qquad\qquad + \cot^2\psi_{n-1})$$

[$= (n-1)$ equations] [$= n$ equations]

Further $\psi_0 = \lambda_0/2$, $\varphi_1 + \psi_1 = \lambda_1$, $\varphi_2 + \psi_2 = \lambda_2 \; . \; . \; .$ $\varphi_n = \lambda_n/2$ [$= (n+1)$ equations]

Unknowns are

$-\ \psi_0$	v_1
$\varphi_1\ \psi_1$	v_2
$\varphi_2\ \psi_2$	v_3
$\cdots$	
$\varphi_{n-1}\ \psi_{n-1}$	
$\varphi_n\ -\ [= 2\,n]$	$v_n\ [= n]$

There are $3\,n$ unknowns, and $(n-1) + n + (n+1) = 3\,n$ equations.

The problem is thus fully determined.

Approximate Treatment and Solution

If $\lambda_0/2\ \lambda_1\ \lambda_2\ .\ .\ .\ \lambda_n/2$ are small enough, we can put $\sin\varphi = \varphi$; $\sinh\varphi = \varphi$; also, therefore, $\cos\varphi = 1$; $\cosh\varphi = 1$ and the equations above will be considerably simplified.

$$v_1/\sqrt{c_d}\,.\,1/\varphi_1 = v_2/\sqrt{c_d}\,.\,1/\psi_1 \qquad P = \frac{\gamma\, v_1^2}{2\,g}\,.\,\frac{\psi_0^2 + \varphi_1^2}{\varphi_1^2\,\psi_0^2}$$

$$v_2/\sqrt{c_i}\,.\,1/\varphi_2 = v_3/\sqrt{c_i}\,.\,1/\psi_2 \qquad P = \frac{\gamma\, v_2^2}{2\,g}\,.\,\frac{\varphi_2^2 + \psi_1^2}{\varphi_2^2\,\psi_1^2}$$

$$\cdots\cdots\cdots \qquad\qquad \cdots\cdots\cdots$$

$$v_{n-1}/\sqrt{c_d}\,.\,1/\varphi_{n-1} = v_n/\sqrt{c_d}\,.\,1/\psi_{n-1} \qquad P = \frac{\gamma\, v_n^2}{2\,g}\,.\,\frac{\varphi_n^2 + \psi_{n-1}^2}{\varphi_n^2\,\psi_{n-1}^2}$$

$$\psi_0 = \lambda_0/2 \quad \varphi_1 + \psi_1 = \lambda_1 \quad \varphi_2 + \psi_2 = \lambda_2\ .\ .\ . \quad \varphi_n = \lambda_n/2$$

Solution:

Remove all the quantities v!

This gives, after some transformation

$$\begin{aligned} \varphi_1/\psi_0 &= \psi_1/\varphi_2 & \text{Put } \psi_0 &= \lambda_0/2 \\ \varphi_2/\psi_1 &= \psi_2/\varphi_3 & \psi_1 &= \lambda_1 - \varphi_1 \\ & & \psi_2 &= \lambda_2 - \varphi_2 \\ \cdots & & \cdots & \\ \varphi_{n-1}/\psi_{n-2} &= \psi_{n-1}/\varphi_n & \psi_{n-1} &= \lambda_{n-1} - \varphi_{n-1} \\ & & \varphi_n &= \lambda_n/2 \end{aligned}$$

then

$$\frac{2\,\varphi_1}{\lambda_0} = \frac{\lambda_1 - \varphi_1}{\varphi_2}; \quad \frac{\varphi_2}{\lambda_1 - \varphi_1} = \frac{\lambda_2 - \varphi_2}{\varphi_3};$$

$$\frac{\varphi_3}{\lambda_2 - \varphi_2} = \frac{\lambda_3 - \varphi_3}{\varphi_4};$$

$$\frac{\varphi_{n-1}}{\lambda_{n-2} - \varphi_{n-2}} = \frac{2\,(\lambda_{n-1} - \varphi_{n-1})}{\lambda_n}$$

This is a system of $(n-1)$ equations necessary and sufficient to determine the $(n-1)$ unknown quantities $\varphi_1\ \varphi_2\ .\ .\ .\ \varphi_{n-1}$

A numerical example will be treated subsequently.

Numerical Example

Fig. 6 illustrates diagrammatically the arrangement of a ventilation-system in a large turbo-generator.

$C - L$ is the center-line axially, which constitutes a line of symmetry, so that the calculation only has to be carried through for one half of the core.

E is the end-bell: G is the gap. R is the rotor surface.

The notations will be the same as used in 10. There are 84 vents axially, arranged so, that

$L_1 = 18$ vents discharge
$L_2 = 16$ " intake
$L_3 = 16$ " discharge
$L^4 = 16$ " intake
$L^5 = 18$ " discharge

Hence the calculation is performed for

$L_1 = 18$ vents discharge
$L_2 = 16$ " intake
$L_3 = 8$ " discharge

$1/2\,L_0$ is a fictitious intake-length corresponding to the drop in pressure when the air enters the gap from the end-bell.

The value of $1/2\,L_0$ will be determined below from the assumption, that the entrance-drop amounts to 20 per cent of the velocity-head in the gap. This value is found to be a good average for an ordinary-shaped entrance.

a. Value of the constants.

From the dimensions of the machine is directly calculated:

$a = 2.1$ ft.² cross-section of gap, including the sunk parts of the slots.

$s = 0.2$ ft.² per vent-circle, taken at the minimum cross-section of the tooth-vent.

$c_i = 1.30$ and $c_d = 1.07$ are values calculated from the shape of the slots and the teeth, number of teeth and shape of vent-fingers. The details of this calculation cannot be given in this paper.

It must be noted, that c_d and c_i include a correction for the loss in total head which at the present time must be considered as uncertain. This example is, therefore, of value only as far as it illustrates the application of the derived formulas.

b. Value of the arguments.

From definition we get $\lambda_1 = \dfrac{0.2 \times 18}{2.1\,\sqrt{1.07}} = 1.65$;

$$\lambda_2 = \frac{0.2 \times 16}{2.1\,\sqrt{1.30}} = 1.34; \quad \lambda_3/2 = \frac{0.2 \times 8}{2.1\,\sqrt{1.07}}$$

$= 0.73$ is obtained from equation (9) putting $x = 0$, hence

$$P - p = \frac{\gamma\, v_0^2}{2\, g} \coth^2 \left(\frac{s\, L_0}{a \sqrt{c_i}} \right) = \frac{\gamma\, v_0^2}{2\, g} \coth^2 (\lambda_0/2)$$

The pressure $(P - p)$ has partly been converted into the velocity-head $\frac{\gamma\, v_0^2}{2\, g}$ and partly been destroyed by the resistance at the gap-entrance, which is put equal to 20 per cent of $\frac{\gamma\, v_0^2}{2\, g}$ hence we can also put

$$P - p = \frac{\gamma\, v_0^2}{2\, g} + 0.2 \frac{\gamma\, v_0^2}{2\, g} = 1.2 \frac{\gamma\, .\, v_0^2}{2\, g}$$

and get for $\lambda_0/2$ the equation

$$1.2 \frac{\gamma\, v_0^2}{2\, g} = \frac{\gamma\, v_0^2}{2\, g} \coth^2 (\lambda_0/2)\ ;$$

or $\coth^2 (\lambda_0/2) = 1.2$

giving $\lambda_0/2 = 1.53$

This would correspond to a fictitious number of intake-vents L_f where

$$\frac{s\, L_f}{a \sqrt{c_i}} = 1.53 \text{ or } \frac{0.2\, L_f}{2.1 \sqrt{1.30}} = 1.53$$

so that $L_f = 18.3$ vents.

c. Equations for the balance-points.

The simplified equations determining the balance-points can now immediately be written up, *viz.*

$$\varphi_1/1.53 = \frac{1.65 - \varphi_1}{\varphi_2} \qquad \varphi_2 = \frac{1.53\,(1.65 - \varphi_1)}{\varphi_1}$$

$$\frac{\varphi_2}{1.65 - \varphi_1} = \frac{1.34 - \varphi_2}{0.735} \qquad \frac{1.53\,(1.65 - \varphi_1)}{\varphi_1\,(1.65 - \varphi_1)}$$

$$= \frac{1.34 - \frac{1.53\,(1.65 - \varphi_1)}{\varphi_1}}{0.735}$$

$$0.735 \times 1.53 = 1.34\, \varphi_1 - 1.53 \times 1.65 + 1.53\, \varphi_1$$

$$\varphi_1 = 3.65/2.87 = 1.27; \text{hence } \varphi_2 = 0.458$$

To find the number of vents axially corresponding to φ_1 and φ_2 we have

$$\varphi_1 = \frac{s\, L_d}{a \sqrt{c_d}} \text{ or } 1.27 = \frac{0.2\, L_d}{2.1 \sqrt{1.07}}\ ;$$

$$L_d = 13.8 \text{ vents}$$

$$\varphi_2 = \frac{s\, L_i}{a \sqrt{c_i}} \text{ or } 0.458 = \frac{0.2\, L_i}{2.1 \sqrt{1.30}}\ ;$$

$$L_i = 5.49 \text{ vents}$$

Note. It is immediately apparent, that the approximate method of solution is *not* applicable in this case, as for instance φ_1 comes out $\varphi_1 = 1.27$.

This is far in excess of the limit, inside which it is admissible to put $\sin \varphi = \varphi$. It will, therefore, be necessary to apply the original formulas and to find a solution by cut-and-trial.

d. Application of original formulas.

The equations determining the balanced condition are

$$\frac{v_1}{\sin \varphi_1} = \frac{v_2}{\sin \psi_1}\ ; \qquad P = \frac{\gamma}{2\, g}\ .\ v_1^2\,(\coth^2 \psi_0 + \cot^2 \varphi_1)$$

$$\frac{v_2}{\sinh \varphi_2} = \frac{v_3}{\sinh \psi_2}\ ; \qquad P = \frac{\gamma}{2\, g}\ .\ v_2^2\,(\coth^2 \varphi_2 + \cot^2 \psi_1)$$

$$P = \frac{\gamma}{2\, g}\ .\ v_3^2\,(\coth^2 \psi_2 + \cot^2 \varphi_3)$$

$$\varphi_1 + \psi_1 = \frac{0.2 \times 18}{2.1 \sqrt{1.07}} = 1.65; \qquad \coth^2 \psi_0 = \coth^2 \lambda_0/2 = 1.20 \text{ (as before)}$$

$$\varphi_2 + \psi_2 = \frac{0.2 \times 16}{2.1 \times \sqrt{1.30}} = 1.33; \qquad \varphi_3 = \frac{0.2 \times 8}{2.1 \times \sqrt{1.07}} = 0.737$$

The most straight forward method of solution is that of a successive approximation. It is, then, best to start at the $C - L$ and to make some first assumption, say that

1. φ_2 corresponds to 5 vents axially
 ψ_2 " " 11 " "

Thus $\varphi_2 = \frac{0.2 \times 5}{2.1 \sqrt{1.3}} = 0.418$

$\psi_2 = 1.33 - 0.418 - 0.922$

and $v_3 = \frac{4030}{\sqrt{\coth^2 (0.922) + \cot^2 0.737}}$

$= 2280$ ft./min. [When P is expressed in inches of water and V in feet per minute, the air taken at a density of 0.074 lb./cu. ft. $4030 \sqrt{P}$ is the value of the constant; it is further assumed here, that $P = 1.''0$ water.]

$$v_2 = v_3 \times \frac{\sinh \varphi_2}{\sinh \psi_2} = 2280 \times \frac{\sinh 0.418}{\sinh 0.922}$$

$$= 925 \text{ ft./min.}$$

$$(4030/v_2)^2 = (4030/925)^2 = \coth^2 0.418 + \cot^2 \psi_1$$

$$\psi_1 = 0.275$$

$$\varphi_1 = 1.65 - 0.275 = 1.375$$

$$v_1 = v_2 \times \frac{\sin \varphi_1}{\sin \psi_1} = 925 \frac{\sin 1.375}{\sin 0.275}$$

$$= 3340 \text{ ft./min.}$$

Finally

$$(4030/v_1)^2 = (4030/3340)^2 = \coth^2 \psi_0 + \cot^2 1.375;$$

giving the first result $\coth^2 \psi_0 = 1.41$.

If assumption 1 had been correct, the final result would have been $\coth^2 \psi_0 = 1.20$.

It is therefore necessary to put

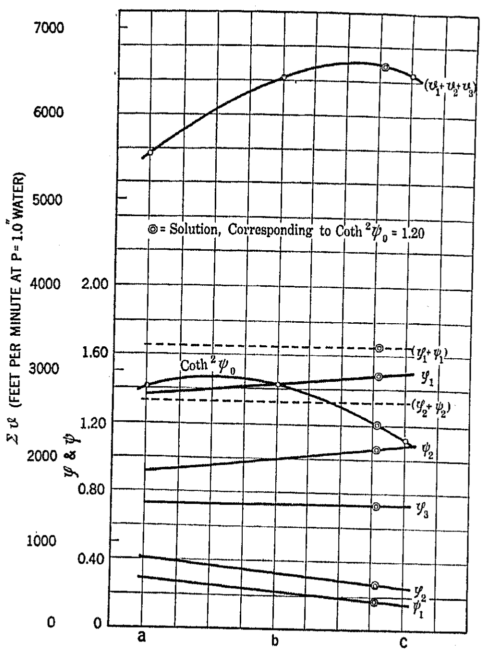

FIG. 7

Table (Fig. 7)

v_1	v_2	v_3	$\Sigma(v)$	φ_3	ψ_2	φ_2	ψ_1	φ_1	$\coth^2\psi_0$
3340	925	2280	5545	0.735	0.921	0.418	0.275	1.375	1.41
3340	751	2330	6421	0.737	0.996	0.334	0.225	1.425	1.43
3620	463	2400	6483	0.737	1.085	0.251	0.129	1.521	1.11

2. $\begin{cases} \varphi_2 \text{ corresponds to } 4 \text{ vents axially} \\ \psi_2 \quad \text{“} \quad \text{“} \quad 12 \quad \text{“} \quad \text{“} \end{cases}$

3. $\begin{cases} \varphi_2 \quad \text{“} \quad \text{“} \quad 3 \quad \text{“} \quad \text{“} \\ \psi_2 \quad \text{“} \quad \text{“} \quad 13 \quad \text{“} \quad \text{“} \end{cases}$

The calculation is exactly the same as before, and the results are given in the table below, also plotted in Fig. 7.

The solution is given on Fig. 7 and corresponds to $\coth^2 \psi_0 = 1.20$.

It is interesting to note, that $(v_1 + v_2 + v_3)$ has a maximum for the balanced condition. This means, of course, nothing else than that the balanced condition is *stable*.

Discussion

The v-values are those corresponding to $P = 1.''0$ water, and must be multiplied by $\sqrt{P}$ for any other value of P.

The φ- and ψ-values, *i. e.*, the balanced condition, is, of course, independent of P.

It has already been mentioned, that the approximate method fails, in a case like this, to give the solution with any reasonable degree of accuracy. This might have been anticipated from the low resistance at the extreme end ($\coth^2 \psi_0 = 1.20$) resulting in a high value of φ_1.

Discussion

For discussion of this paper see page 498.

An Experimental Study of Ventilation of Turbo Alternators

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Review of the Subject.—*When designing a large steam-turbine-driven alternator the volume of air per minute needed to cool the machine is easily calculated if the losses are known, and the allowable air temperature rise is assumed. With data hitherto available it has not been possible to estimate the pressure needed to drive the air through the various paths, nor to predict the distribution of air in the machine. The problem is too complicated to admit of an analytical solution, and even an experimental investigation becomes involved.*

In general, a complete machine is not suitable for an experimental study of air flow, as changes in the series or parallel paths, the introduction of restrictions, etc., can be made only with difficulty, delays and expense. Accordingly, two models, to imitate two radial systems of ventilation, were built. Since ventilation, as such, was to be investigated, no electrical nor magnetic losses were introduced. The stator cores were made of wood, with suitable vent fingers of steel, and with blocks of wood to imitate coils in the slots. The rotor was the same for both models, and was made of steel, ventilated as in a usual machine, but with wood in the slots to imitate coils. Its diameter was such as to give the peripheral speed adopted in the largest turbo alternators; its length was about half of the probable largest rotor of that diameter. An external blower was used chiefly because then the delivered pressure could be independent of the rotor speed. The flow of air in any circuit, such as the flow through the rotor vents, was controlled independent of the flow in any other circuit. Individual radial vents could be closed, etc. Several sizes of air gap were tried. Thus, the influence of the flow as affected by having any path open or closed, or in combination was studied.

The pressure readings were taken by means of an ordinary open-end tube placed in a region of very low velocity, so that the influence of velocity head would be negligible. The total volume was measured by means of an electric thermal meter, with thermocouples for measuring the air temperature rise, instead of the frequently used resistance exploring coils. The distribution of volumes at discharge from the vents was measured by means of a rotating vane anemometer, a special funnel being used, one end of which was inserted between adjacent vent fingers which were extended to the back of the wooden stator core.

Two systems of ventilation were studied. In both, some of the air enters the air gap directly from the end bell; in both, some of the air passes from the end bell through suitable channels at the rear of the core to selected vent ducts, then passes radially inward through those ducts to the air gap, then turns through a right angle, passing through the air gap, and finally turns another right angle and is discharged radially through other vent ducts. This air may be constrained to pass circumferentially or radially through the air gap. The mechanical construction for the two are necessarily considerably different in the arrangement of connections with inlet and outlet vent. In the first model the air moved circumferentially, and in the second it flowed axially through the air gap (Figs. 2 and 3 respectively).

The rotor diameter was 26 inches, the maximum rev. per min. 3600, corresponding to 24,500 ft. per min. In both models 54 slots were used. In Model 1 wedges were present in the vent ducts, whereas they were omitted in Model 2. In Model 1 the upper half was provided with two fingers per tooth, but in the lower, only one finger per tooth was employed. In Model 2 only one finger per tooth was used. For an understanding of the construction, Figs. 4, 5 and 9 should be referred to for Model 1 and Figs. 10, 12, 13, 14 and 15 for Model 2.

To further assist in quantitative analyses, models of tooth and vent sections were built with one, two and no fingers, with and without notches. Air was caused to flow through these small models in both directions and at various velocities, and the pressure drops were measured.

Only a few of the tests are published herewith. The study is not complete, and the paper would be entirely too long if all the tests were included. A few conclusions are:

In Model 1 (for circumferential gap flow):

1. At normal speed the circumferential distribution of volumes is substantially uniform near the end bell, but departs from uniformity as the distance from the end bell increases, the non-uniformity reaching a maximum a little before the extremity.

2. The departure from uniformity is due principally to the influence of rotation, and may cause the velocities in some vents to be quite low. Considerable differences in temperature may be due to such non-uniformity.

3. The non-uniformity causes the total volumes to be appreciably less at normal speed than at standstill, for a given pressure in the end bell.

In Model 2, (for axial gap flow), a few conclusions are:

1. The distribution circumferentially is uniform at a given position axially. The axial distribution is not uniform, being approximately sinusoidal at the discharge. By suitable selection of groups of vents, or other means, the distribution of volumes may be made nearly uniform.

2. The distribution is affected but little by rotation.

3. The proportions may be such that nearly uniform temperatures may be obtained.

4. The volumes are nearly unaffected by rotation.

A mathematical treatment is contained in a companion paper by Mr. Donald Bratt.

FOREWORD

THE purposes of this paper are twofold: To offer suggestions to others who may be struggling to find a means for solving some intricate heat problem, and to present to them in part the results of a certain investigation. It is recognized that the work is far from being complete; that only two of many types of ventilation were studied. As in any research problem, many new avenues open as work proceeds, so has it been in this case. The studies of the data obtained so far are still under way, and the experimental work for the second scheme of ventilation, being conducted in a different manner from that published herewith, is not finished. In this new work, we hope to determine loss coefficients which we cannot obtain with sufficient accuracy from the data on the turbo models.

During the investigation, the author was assisted by Mr. Donald Bratt, who is contributing a companion,

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

mathematical paper to the Institute. The author acknowledges valuable suggestions from him and from a number of co-workers in the Westinghouse Company.

When an attempt is made to analyze the data from temperature tests on electrical machinery one finds the task fraught with many difficulties. Entirely aside from the question of inconsistencies in test data, an accurate determination of the quantitative relationships of the three contributory factors is well-nigh impossible. These factors are:

(a) The generation of heat by losses;
(b) The internal flow of heat by thermal conduction;
(c) The dissipation of heat, chiefly by forced convection, and, to a more limited extent, by free convection and radiation.

In perhaps no machine is the prediction of temperatures more vital than in the large steam-turbine-driven alternator. Leaving out certain quantities which are fairly well fixed by the materials employed, as we have them at present, temperature is the principal limitation which confronts the designer.

Regarding the first factor, assume that available data on the rate of generation of heat by losses are sufficiently well known to enable the designer to predict them.

Consider, then, the second factor, the internal flow of heat. In the large turbo alternator, the stator coils are so long that the heat generated near the middle of the machine by I^2R and eddy currents in the copper can not flow longitudinally to the ends to be picked up there by moving currents of air.[1] Consequently, as regards the copper, that part of the problem is simplified, as only the transverse flow of heat by conduction through the insulation need be considered.

In most large modern steam-turbine-driven alternators, the stator slots are very deep, and the losses in the embedded copper and deep teeth are high. If all that generated heat must flow radially through—say half the depth of the teeth, the temperature drop radially is large, even when account is taken of the comparatively high thermal conductivity of the steel in the plane of the sheet. On the other hand, if the laminations are arranged in thin packages, with that generated heat flowing in a direction normal to the planes of the laminations, the average temperature of the package is only a few degrees higher than at the ends, even though the coefficient of thermal conductivity is comparatively low. Furthermore, the excellent conductivity of the embedded copper secures practically uniform copper temperature throughout that short length. Therefore, with that construction, the thermal drop in the laminations is not an important part of the temperature problem. The only important role played by thermal conductivity is concerned with the transverse flow of heat through the insulation around the coil. The drop in temperature due to that flow can readily be computed, provided the losses and coefficient of thermal conductivity are known.

This, then, brings us to the third factor, *viz.*, the dissipation of heat. In the turbo alternator, practically all the heat is carried away by forced convection, generally known as ventilation. That is the most vital and difficult factor, about which far too little is known. It, in turn, may be broken into five parts: (a) the volume of air needed for cooling: (b) the rate at which the heat is "picked up" from the heated surfaces; (c) the pressure needed to drive the needed volume through the various passages; (d) the distribution of air volumes; and (e) the pressure, delivered by the fan.

(a) The volume of air is very readily calculated from the known losses and permissible air temperature rise; (b) Various data have been published on the rate at which heat is picked up by moving air streams. These data are not always consistent; however, with the high velocities generally used in large modern turbo alternators, the drop in temperature from the iron to the air is usually not large, although it is not negligible. It is beyond the scope of this paper to treat that phase of the subject, and it is to be hoped that more consistent data will be forthcoming. A great deal of work has been done, and more is under way. (c) and (d) of this paper and a companion paper by Mr. Donald Bratt deal with the pressures required to drive a given volume of air through the vent passages, and the distribution of volumes. (e) A paper on "Centrifugal Fans for Electrical Machinery" will appear in an early issue of the A. S. M. E. *Journal.*

Bearing in mind that this paper is concerned only with the air pressures required for given volumes, and the distribution of volumes in a turbo alternator, we shall return to the consideration of the machine in which the stator core is built of thin packages of laminations. With this construction, the laminations at the ends of a package must be kept cool throughout their depth radially by moving air streams. This, in turn, implies radial flow of air.

Perhaps the simplest radial duct system of ventilation of turbo alternators is the one in which all the cooling air passes direct from the end bells into the two ends of the air gap. Then the air flows axially and passes outward through radial vent ducts, distributed in the stator core. With internal fans, either the propeller or centrifugal type may be used to generate the necessary air pressure. The system is illustrated schematically in Fig. 1. This system has been used successfully for many years and is used today for certain sizes and types of design. In some of the large turbos, particularly for 60 cycles, of proportions that are considered normal at the present time, it becomes difficult to cause enough cooling air to flow through the two paths in the air gap. The air-gap size should be based

1. The influence of the longitudinal flow of heat may be computed by formulas in paper "Longitudinal and Transverse Heat Flow in Slot-wound Armature Coils," A. I. E. E., 1921, page 589.

upon the armature ampere-turns, and then it is too small to transmit the volume of air unless the velocities (and therefore the pressures) become prohibitively high, or unless the air temperature rises in excess of that which is considered good practise. The solution then lies in providing additional paths.

If cooling air is admitted at both ends of the air gap, and additional intakes are to be provided, and the air is to pass radially throughout the depth of the vent duct, then the only feasible way for that air to enter is through selected vent ducts at the back of the core. The air then passes radially inward to the air gap, then turns through 90 degrees, then passes through the gap, and finally turns again through a right angle, flowing radially outward through other vent ducts. Although all the cooling air provided for the stator must, as before, pass through the air gap, the number of paths in parallel may be so chosen as to avoid prohibitively high air-gap velocities, and a desirable number and size of vents may also be selected. The static pressure needed may be provided by centrifugal fans at the ends of the rotor, which must generate sufficient pressure to overcome the resistance of the combined series and parallel paths in the air circuit.

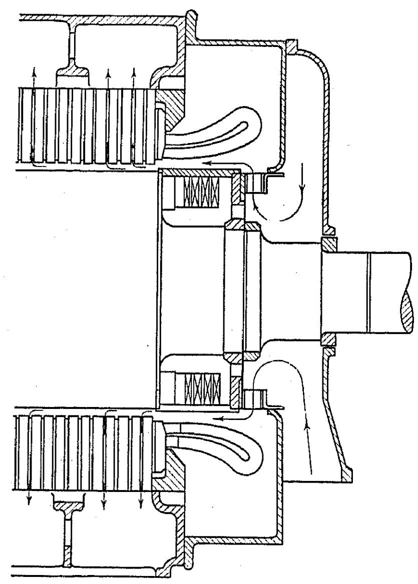

FIG. 1

The air which flows from the back of the core radially inward may be constrained to pass circumferentially or axially through the air gap. The mechanical construction is necessarily considerably different in the arrangement of connecting with the inlet and outlet vents for these two directions of flow in the gap. Diagramatically, these two types of "multiple path radial ventilation" are shown in Figs. 2 and 3. In either, the number of intake or outlet belts[2] is dependent upon (1) the ratio of the gap cross-section to the vent duct section per unit length circumferentially or axially; (2) the total volume of air, (3) the air velocities chosen by the designer; and (4) mechanical considerations.

In either of these multiple radial systems, the flow is very complicated. A solution was impossible with the knowledge of the subject possessed at the outset of the investigation. Yet, in order to design large generators intelligently, it is important to predetermine what the quantitative relations are within a reasonable percentage of error.

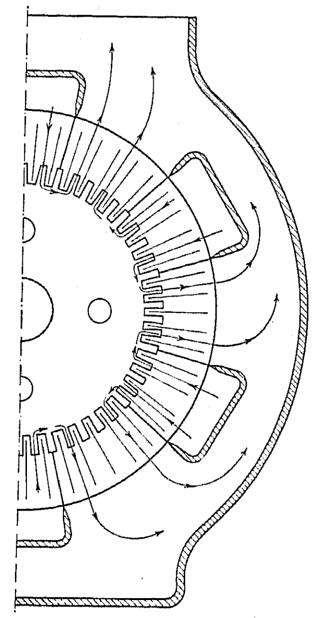

FIG. 2

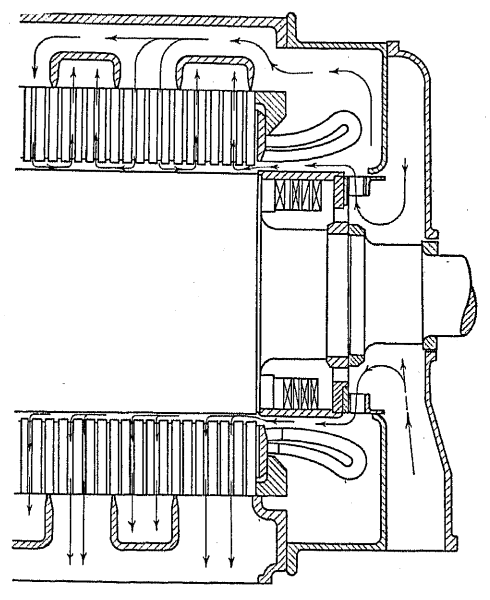

FIG. 3

Accordingly, it was decided in the early part of 1920 to build two models to imitate both systems. Since ventilation, as such, was to be investigated, no electrical nor magnetic losses were to be introduced.

2. A belt refers to a group of vents fed in parallel from the same duct.

The peripheral velocity of the rotating element should be not less than that of a large generator. The construction of the rotor was to be similar, in so far as ventilation was concerned, to that of the largest generators. The length should correspond to about half the length of the probable longest rotor in a machine having the same diameter as the rotor of the model; (half the length, because it was considered desirable to imitate only one-half the machine longitudinally, as the other half would be the duplicate in inverse order). The packages of laminations in the stator were to be imitated by rings of hard wood into which slots of standard sizes were to be cut; between adjacent wooden rings standard vent spacers were to be placed; wood could be used to imitate the coils at the vents, etc. An external blower was to be used, chiefly because the delivered air pressure would be independent of the rotational speed, but also because the static pressure could be measured in a quiet zone where the velocity head was of negligible influence. The whole was to be so made that a maximum of flexibility was to be obtained. Thus, the use of wood in the stator permitted of rapid and inexpensive alterations; steel rings placed over the entrances to the longitudinal vents in the rotor would stop the supply of air, as desired; the entrance to the air gap could be closed by means of a suitable angle ring; the entrances to the stator could be open or closed; several sizes of air gaps were to be used; various restrictions in the air circuit were to be introduced, etc. Thereby, data on the flow in any individual air circuit, such as the stator, the rotor, or the air gap, etc., or a combination, at various air pressures and at various rotational speeds, could be obtained and the results studied.

Ducts, Quantities Measured, Etc.

Before going further into the construction of the two models, and the tests made on them, auxiliary apparatus and instruments will be discussed.

1. *End Bell and Pressure Measurements.* The outer part of an existing cast iron end bell was bolted to one end of the model,[3] it was split, so that half of it could be readily removed for alterations. With the end bell parting horizontal, the upper half could be readily removed. The intake duct was easily attached. The velocities of air in parts of the end bell were so low that an ordinary open-end brass tube could be used for connecting with a manometer for measuring pressures without introducing an appreciable error due to velocity head. The tube was pushed through a sealed wooden plug fastened into one of the upper bolt holes in the end bell. Pressure readings were also taken near the parting farthest from the entrance to the end bell; there the velocities were substantially zero and the pressure readings were practically the same as in the other location.[4]

2. *Diverging Intake to End Bell.* The outlet of the blower was 18 inches square (2.25 sq. ft.), and the cross-sectional area of the end bell at its entrance was a little more than four times as great. In the second model, volumes approximating 15,000 cu. ft. per min. were admitted to the end bell, in certain cases. This corresponds to a velocity of 6660 ft. per min., at discharge from the blower, or a velocity head of 2.73 inches of water. In that case, the static pressure in the end bell was only 3.3 inches of water, and the velocity head there was only $(1/4)^2 \times 2.73 = 0.17$ inch of water. It would not have been feasible to secure that delivery from the particular blower unless a considerable percentage of the velocity head at its discharge were converted into static pressure. It is well known that to recover the maximum velocity head in a comparatively short length, the duct should be shaped like a trumpet with curved sides, and that

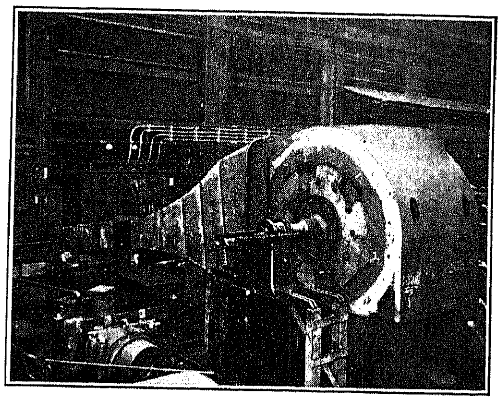

Fig. 4

was the form that was adopted; that intake is shown in Fig. 4, and several other photographs.

3. *Volume Measurement.* The volume of flow was measured by the thermal principle which Prof. Carl Thomas introduced. The air was heated by causing electric current to flow through a suitable resistor in the air stream, the power consumed being measured by familiar indicating electrical instruments. In the Thomas meter, the temperature rise of the air is

3. The lower half of the end bell attached to model 2 appears in Fig. 12.

4. The pressure readings taken on different days should be reduced to some reference standard for density. The standard chosen was 0.074 pounds per cu. ft. which is the density when the barometric pressure is 29.92 inches of mercury, and the temperature is 25 deg. cent. If departures in density, while passing through the apparatus under test, due to changes in pressure and temperature, are neglected, the correction back to the reference standard, based upon the well-known laws of Boyle and Charles, is:

$$\text{Press. corrected} = 0.1003 \left(\frac{273 + t}{P} \right) \text{Press. read.}$$

Here t is the temperature in deg. cent. P is the barometric pressure in inches of mercury.

measured by means of coils in the air stream before and after the air passes through the heater, these exploring coils being connected in a Wheatstone Bridge network. Previous experience showed that those coils were not entirely satisfactory for this kind of work, as the wires were likely to stretch, and with the large volumes, the temperature rises would be too low to obtain accuracy, unless the power consumption in the heater were prohibitive. These objections were overcome by connecting numbers of thermocouples in series; then a comparatively high reading of millivolts on a suitable potentiometer secured accuracy with the low air temperature rise of, say, 1.5 to 3.5 degrees centigrade.[5] If the specific heat of air at constant pressure is 0.2418, and the density is 0.074 pounds per cu. ft. at 29.92 inches of mercury and 25 deg. cent., the volume, as determined with this instrument may readily be shown to be:

$$Q = \frac{0.178\ n\ E\ W}{1000\ e} \quad \frac{273 + t}{P} \text{ cu. ft. per min.}$$

n = number of cold (or warm) thermocouple junctions.

E = microvolts per degree at the average temperature in the volume meter, as determined from the calibration curve.

W = Watts loss in heater.

e = millivolts read on the potentiometer,

t = temperature of the air, degrees cent.

P = barometric pressure, inches of mercury.

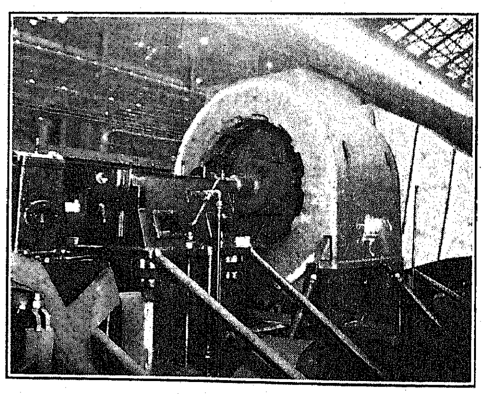

FIG. 5

Obviously, since the density of air changes with its temperature and pressure, its density changes as it passes through an electric machine, as temperature and pressure change. Consequently, the cubic feet per minute are not the same at exit as at entrance, although the weight per minute remains constant. The influences of such effects in this experimental investigation were not considered; it may be of consequence in certain turbos, as say, when the air rise is 25 deg. cent. As given in the equation, P is the barometic pressure, yet the actual pressure at entrance to the model was that plus the pressure read on the manometer, a possible maximum increase of 2.5 per cent above the barometric pressure. That increase also was ignored, and only the barometer reading was used.

While in the type of volume meter used it is not necessary for the velocities to be uniform, it is important

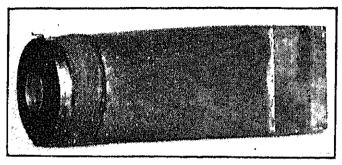

FIG. 6

that the departure from uniformity be not too great. In previous experiments, two sets of adjustable sheet steel planes were placed in a duct through which the the air passed before reaching the volume meter, and these could be adjusted in two directions so as to alter the distribution. They were applied here too, but were subsequently found unnecessary.[6]

4. *Volume Distribution.* The third important quantity to be measured was volume distribution in the model. It was desired to measure velocities or volumes in the intake and outlet vent ducts. There was no recognized way of measuring the velocities in the intake vents, except by means of hot wire anemometers. A number of those devices were made of suitable size; they were calibrated, and placed in some of the ducts on model No. 1. A considerable amount of time and effort was spent in an endeavor to get them to read with reasonable consistency and accuracy, but all attempts were unsuccessful. The hot wire anemometer, as built, is quite sensitive to direction of flow, and a calibration curve taken with the stream is materially different from one against, or normal to the stream. With air in highly turbulent state, as it is bound to be in a vent duct, the instrument readings were valueless. They were consequently abandoned.

The method used for measuring outlet duct volume-distribution was more successful. The vent duct finger spacers were made continuous to the rear, (See Figs. 7 and 8), so that a given volume of air that entered a particular space between adjacent fingers would continue to flow to the rear. The fingers were spaced uniformly at the rear. A small "funnel" which just fit between adjacent fingers was joined to an ordinary rotating vane anemometer, which was held in position

5. The volume meter will be seen in Fig. 4, where it joins the small end of the trumpet shaped diverging duct.

6. For further description see article in *Electric Journal* for May 1923, "Some Elements of Air Flow in Electric Machinery," or in A. S. M. E. Paper on "Performance of Centrifugal Fans for Electrical Machinery."

for half minute readings. (See Fig. 6). The anemometer was calibrated at the laboratory with the funnel, and from the readings, the volume of air per minute passing between adjacent fingers, could be determined by referring to the calibration curve. The sides of the part of the funnel that was inserted between fingers were tapered slightly, so that it could be pushed in tight. Putty was put around that part of the funnel to eliminate leakage, but the readings with and without the putty were substantially the same so that the putty was subsequently omitted. There were two finger spacings for the first, and one for the second, model, so three funnels were made.

In general, a rotating vane anemometer is not reliable. For example, when employed to measure velocities at a number of positions in a duct, the instrument offers obstruction to the flow, and changes the distribution of velocities. If held in the hand at, say, the efflux from a duct, one's arm and hand offer

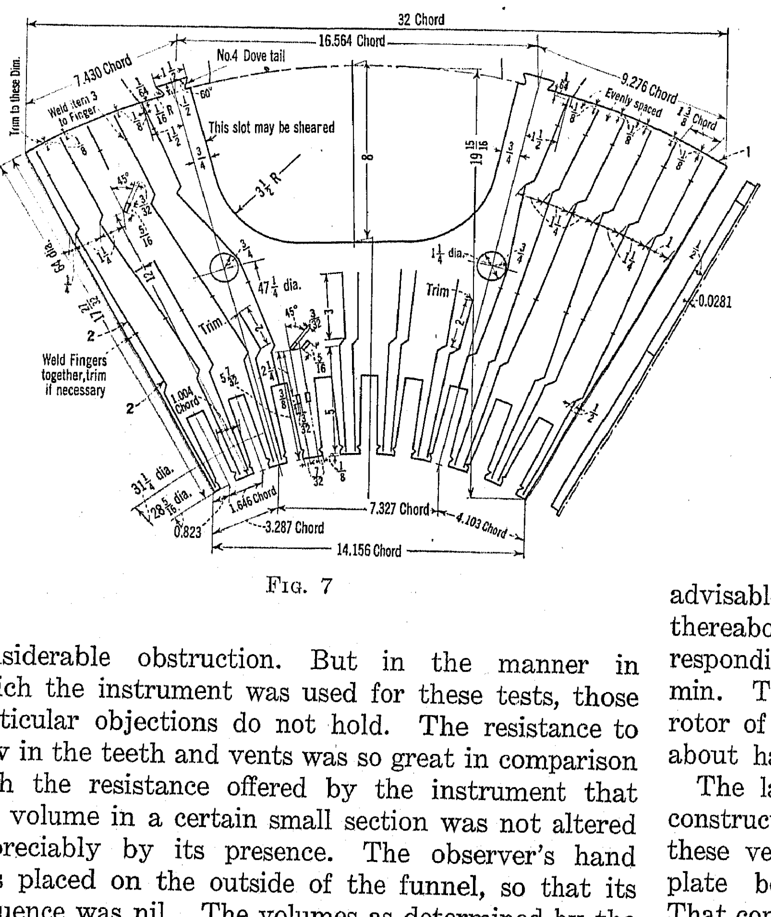

Fig. 7

considerable obstruction. But in the manner in which the instrument was used for these tests, those particular objections do not hold. The resistance to flow in the teeth and vents was so great in comparison with the resistance offered by the instrument that the volume in a certain small section was not altered appreciably by its presence. The observer's hand was placed on the outside of the funnel, so that its influence was nil. The volumes as determined by the volumemeter were compared with the summation of volumes by the anemometer, and in many cases the differences were less than ten per cent, which was considered satisfactory. The anemometer's calibration was checked a number of times during the series of tests (which extended over about ten months) and it was found to change slightly. After quite a time, it was decided that since the anemometer readings were for comparison of volume (or velocity) distribution, and not for the determination of total volumes, further attempts at checking total volumes by this method should be abandoned; the volume meter readings were relied upon for total volumes.

In the various plots of points taken by the anemometer, there are irregularities. While some of these were due to inaccuracies in readings, many were caused by slight dissimilarities between nominally duplicate vent ducts; for example, the finger was pushed more to one side than to the other, and thereby produced an appreciable change in the maximum restricted section. The vent spacers having been made by men who do that kind of work regularly, the irregularities were probably about the same as would be found in a machine, in which the same construction is employed.[7] In a number of cases, the observers were requested to repeat their readings after completing a set and usually the original observations were duplicated within a few per cent. That fact helped to place reliance in them. For the most part, the general shape of a volume distribution curve can be told quite well from a plot of observations, and usually important conclusions can be drawn.

When the tests were started, all of the principal paths for the air to flow were closed, and the volumes for various pressures were measured. These observations were for the purpose of determining the order of magnitude of leakage. When the various passages were closed, the clearances, and therefore the leakages, were about the same as when the model was operated.

Construction of the Turbo Models

1. *The Rotor.* As the peripheral velocity of the rotor was to be at least 24,000 feet per minute, and as it was not considered advisable to exceed 3600 revolutions per minute or thereabouts, the diameter was fixed at 26 inches, corresponding to 24,500 ft. per minute at 3600 rev. per min. The length of the body of the probable longest rotor of that diameter was about 120 inches, so about half that useful length was decided upon.[8]

The large Westinghouse turbo rotors are of plate construction with radial vent ducts at intervals; these vents are only around the slot portions, each plate being milled individually before assembly. That construction was adopted in the models. Fig. 10 will assist in understanding it. Air enters through slots below the main slots, the coils acting as fans. The slots were arranged in four groups to imitate four poles. As in the machines, the plates are held together

7. In the recent large turbos, a different construction has been adopted, one advantage of which is that greater uniformity is assured.

8. The rotor was made a little longer than the stator, in order that an angle iron, put in to stop the flow of air, could be placed over the extension.

by bolts between stub-end forgings. The coils are imitated by treated hard wood blocks driven into the slots, held in position by wedges of the same size as in a machine of corresponding proportions. The influence of the coils at the ends, and of the retaining rings, upon the ventilation of the stator and of the rotor body, were negligible, so those parts were not imitated in the models.

2. *Construction of Model No. 1.* The first model, Figs. 4 and 5, imitated the scheme of ventilation in which the air moved circumferentially in the air gap. The number of stator slots selected was 54, and the number of stator axial intakes or outlets was six.

as applied in model II. In this model, short fiber wedges were screwed to the small wooden blocks. The large wooden rings had previously been slotted to the depth that the wedge is sunk below the inner periphery of the stator punchings in a machine. The composite in the tooth belt simulated that portion of a generator quite accurately.

An estimate of the velocities in the vent ducts back of the teeth indicated that the pressure drops there would be very small in comparison with the drops in the teeth, around the wedges, turning through a right angle, etc. Consequently, the pressure needed for a given volume would be substantially the same if the

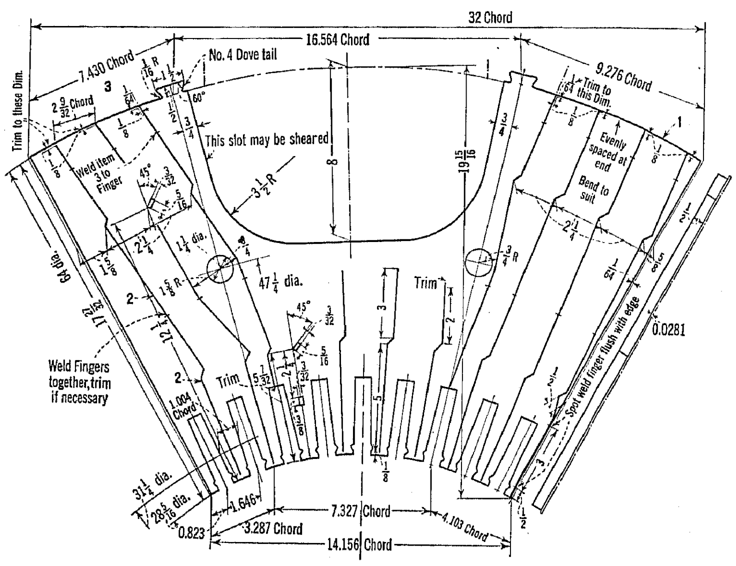

FIG. 8

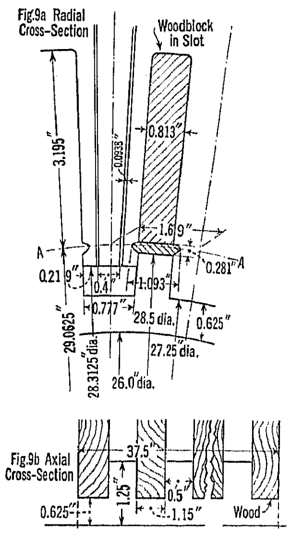

FIG. 9

Dimensions and data
54 slots, 22½" vents axially
Each intake extends over 4 toothpitches
$A - A$ Diameter of restricted area
Total cross sections at $A - A$

Intake	single spacing	0.460 sq. ft
	double spacing	0.469 sq. ft.
	total	0.929 sq. ft.
Outlet	single spacing	0.576 sq. ft.
	double spacing	0.469 sq. ft.
	r total	1.045 sq. ft.

Axial coss-section (Fig. 9B)
(single) = 0.210 sq. ft.
(total) 2 × 6 × 0210 = 2.52 sq. ft.

Then, there were (54 / 6) = 9 teeth in each combination of intake and outlet belts; as the outlet was to carry the intake air plus the air admitted direct from the end bell to the air gap, a larger number of teeth was placed in the outlets than in the inlets, five in each belt of the former, and four in the latter. Further, in order to assist in the study, the vents in the upper half were provided with two fingers per tooth, and in the lower one was used. (See Figs. 7 and 8). To simplify manufacture, a circle of existing punchings with vent fingers attached were nailed on one side of each wooden ring. Small wooden blocks, having the same width as the slot, the same depth as the slot below the wedge, and the same thickness as the vent duct width, were nailed to each wooden ring for the purpose of imitating the coils in the vent duct. They will be seen in Fig. 11

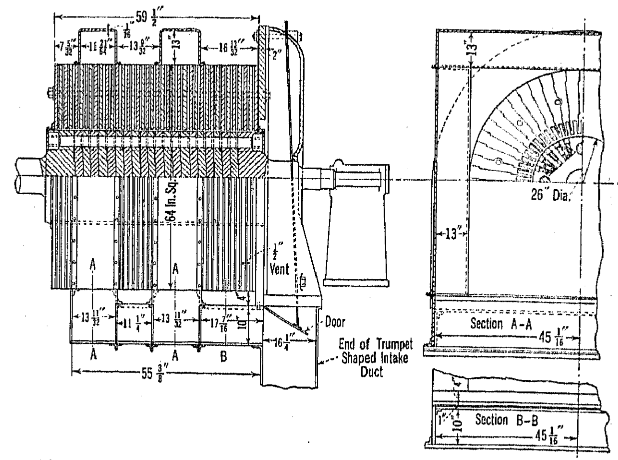

FIG. 10—MODEL FOR INVESTIGATING VENTILATION OF A TURBO-ALTERNATOR

length of the path in the vents back of the teeth were materially shortened. Recognizing this, the construction was considerably simplified by cutting out parts of the wooden rings and punchings at six equidistant places circumferentially, thereby providing ample area for the axial passage of the air from the end bell. (See Fig. 4.)[9] (If the same system of ventilation were used in a machine, these axial passages would be incorporated in the frame.)

As will be seen from Figs. 4 and 5, an existing cast iron frame was used for the first model. This frame had "chimneys" at the top and bottom, and they were so large that a man could climb into them and take anemometer observations at the outer peripheries of the wooden core. The stator of this model was shorter axially than the rotor, as will be seen from Fig. 5. This was because the blower which was used was not capable of delivering more than 10,000 cu. ft. per min. at about 10 inches of water at its maximum safe speed;

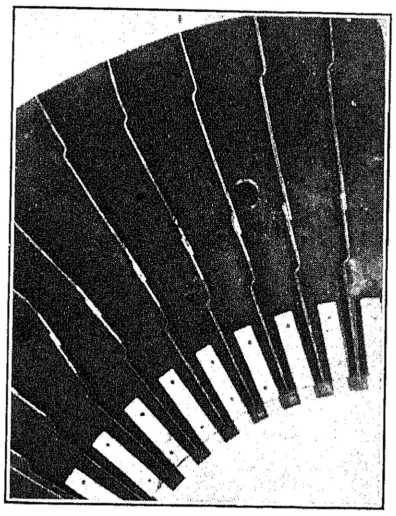

FIG. 11

and the length of the stator was made such that its resistance to the flow of air would approximately correspond to those values. These figures were based upon a preliminary rough estimate of pressure drops. Owing to the direction of flow in the gap, in the first model, it was not so important to make the length about half of the probable maximum, as in the second model. On the other hand, it was important to secure fairly high end bell pressures, in order to determine the combination of pressures and peripheral velocities. These considerations, and a preliminary inspection of an approximate curve of the blower characteristics assisted in determining the model's proportions. In order to prevent escapement of air from those radial vents in the rotor which were beyond the stator, suitable wooden stops were placed in the longitudinal ducts (those through which the air flowed to the radial vents) and those stops were removed when the rotor was used in model II.

For those who may wish to calculate velocities, and to compare results, Figs. 9A and 9B have been included. They show the sections of maximum restriction; the sketch in Fig. 9A shows two fingers, but the areas are given for both one and for two fingers.

3. *Construction of Model No. 2.* In the second model, the system of ventilation shown schematically in Fig. 3

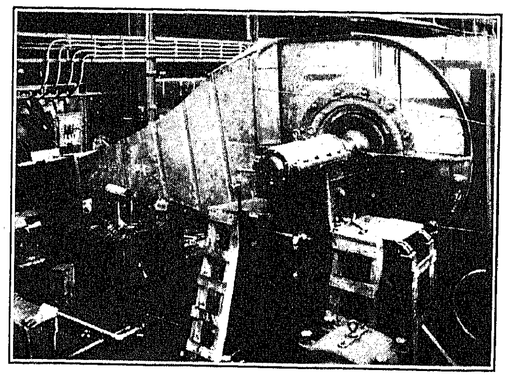

FIG. 12

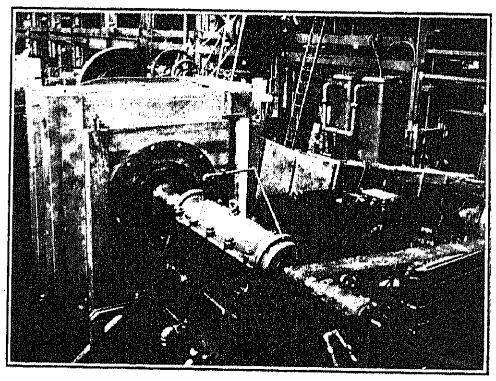

FIG. 13

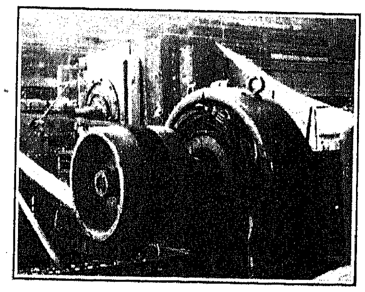

FIG. 14

(in which the air moved axially through the air gap) was simulated. The same rotor was used as for model I. The model is shown to scale in Fig. 10, and Figs. 12, 13, 14 and 15. (In the drawing, part is shown in plan, and part in horizontal section, but the bearings are shown in elevation.) The number of intake belts selected was two (equivalent to four in a machine). There were a total of 38 vents, distributed as follows:

9. In Fig. 4, some of the entrances to the intakes are closed and some are open.

10 in the first discharge belt, adjacent to the end bell; 8 in each stator intake belt; 8 in the discharge belt between the two intakes, and 4 in the final discharge belt. In the preliminary layout, it was assumed that the first six discharge vents would probably take the air which passed into the air gap direct from the end bell; the other 16 discharge vents would be fed from the 16 intake vents (after having passed through the gap). There were provisions for closing any of the vents, as might be desired, to secure other combinations.

In this model, the frame casting was abandoned. The hard wood spacers which imitated the stator

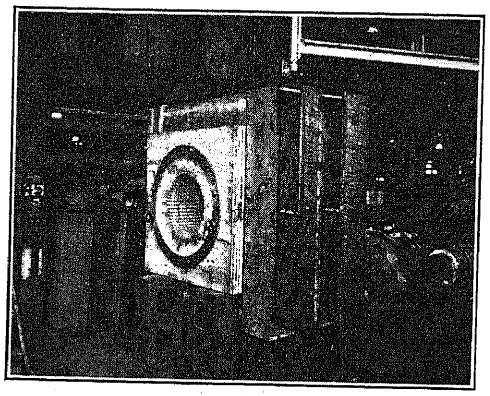

Fig. 15

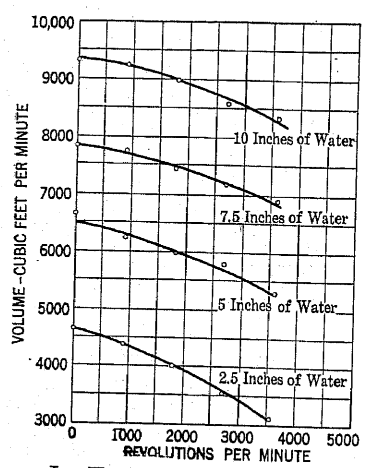

Fig. 16—Model I. Volumes for Various Rev. Per Min. and Static Pressures with Stator + Air Gap Open

punchings were square on the outside, and circular with slots, teeth, etc., inside. The dimensions of the teeth and slots were the same as in model I, shown in Fig. 9A. The vent spacers were made, like in model I, by fastening fingers to punchings, with slots spaced as for 54 per circle, the punchings being circular on the outside (see Fig. 11). Only single fingers were used; these were run to the backs of the punchings and were uniformly spaced there, to assist in volume distribution measurement.[10] Stator slot wedges were omitted in this model. The same method for imitating coils in the vent was adopted as in model I, but the small wooden blocks which imitated the coils in the vents were made of such depth as to correspond to the coil plus the wedge; the wedge notches were absent. The stator was supported on suitable steel structure, so arranged as not to interfere with the flow of the discharged air, except at short distances from the corners of the squares, where the velocities were very low; and the air which would have been discharged there was diverted without affecting the flow in the restricted sections in the teeth appreciably.

10. The anemometer readings were taken only between 3 to 5 adjacent fingers at the top and at one side, as others were not accessible.

The entrance to the air gap could readily be shut off by fastening the angle ring shown on the drawing in Fig. 18. The leakage around the ring was minimized by use of card board rings which at first fit tightly around the outside of the rotor, and subsequently the revolving rotor wore a small clearance space. Leakage around the ring at the other end was similarly reduced.

It was highly desirable to measure the volumes

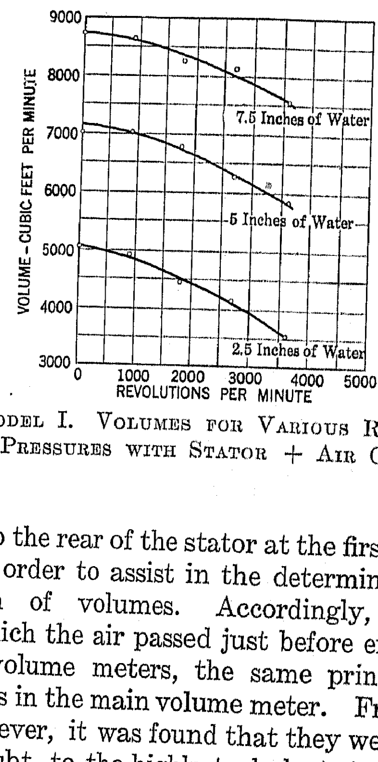

Fig. 17—Model I. Volumes for Various Rev. per Min. and Static Pressures with Stator + Air Gap + Rotor Open

passing into the rear of the stator at the first and second intakes, in order to assist in the determination of the distribution of volumes. Accordingly, the ducts through which the air passed just before entering were made as volume meters, the same principles being employed as in the main volume meter. From readings taken, however, it was found that they were valueless, due, no doubt, to the highly turbulent state, (meaning that the directions of flow were indefinite); it is well known that if the velocities through a thermal volume meter depart too far from uniformity, the readings will be considerably in error. In Fig. 15, some of the wires which were intended to be used for these volume meters will be seen.

Pressure readings were taken at various localities in addition to those in the end bell. They were taken by means of a manometer connected to open end brass tubes pushed through holes in regions of low or negligible velocities. Thus, near the regions in the sheet steel intake, enclosures around the horizontal center lines (fartherest from the entrance), the velocities

were substantially zero. The earliest readings showed an unaccountably high pressure drop from the first position in the end bell to the other positions. A piece of sheet metal was placed vertically in the end bell just beyond the end of the trumpet-shaped duct, so as to shut off the air from the end bell. The drop in pressure for the air going around the corner then assumed a value such as might have been anticipated. The subsequent tests were made with a wooden hinged door in that position, (shown in Figs. 10 and 12); it was held with horizontal steel rods which passed

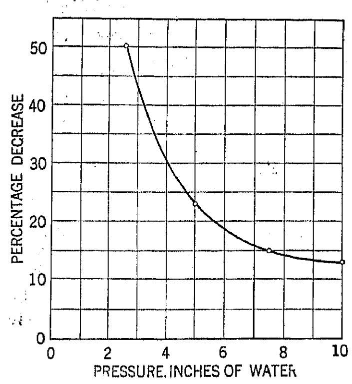

Fig. 18—Model I. Stator + Air Gap Open. Decrease in Volume from Standstill to 3600 Rev. per Min. Due to Rotation

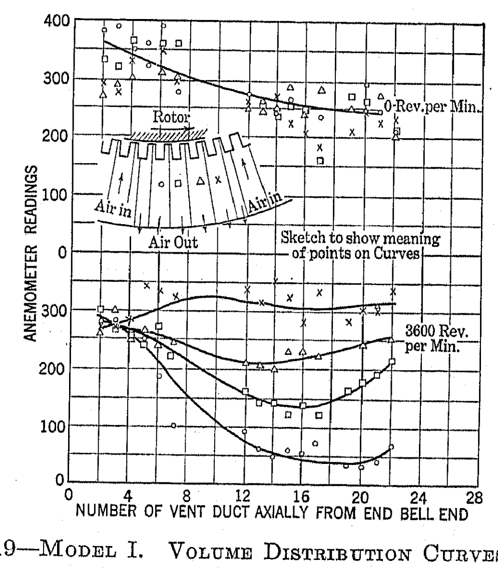

Fig. 19—Model I. Volume Distribution Curves Taken at Pressure of 7.5 in. of Water. Stator and Air Gap in Parallel

through the end bell and could be clamped in any position desired while air was flowing. It is evident that if the door were so placed as to shut off air from the end bell, the unbalanced pressure would tend to make it turn farther toward the end bell; if turned so as to close the main duct which carried air to the stator, the unbalanced pressure would keep it closed. There was one position between these two where there was practically no unbalanced pressure, and the clamps for the steel rods were secured so as to hold it in that position. The pressure drops were then of the order that one might expect.

Some later tests with the rotor removed showed what was the cause of the high pressure drop without the door. An eddy was set up, probably a spiral which whirled around the shaft, much like the familiar

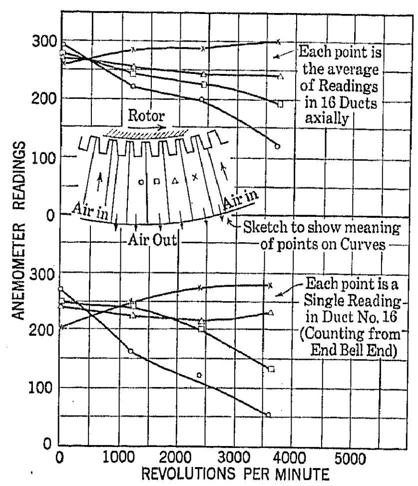

Fig. 20—Turbo Model I. Volume Distribution Curves Taken at a Pressure of 7.5 In. of Water. Stator and Air Gap in Parallel

whirling of water in a wash bowl. It is believed that the eddy was started by the unequal distribution of air discharged from the trumpet-shaped diverging entrance duct; probably caused by the departure of the duct's center line from the horizontal. The direction of

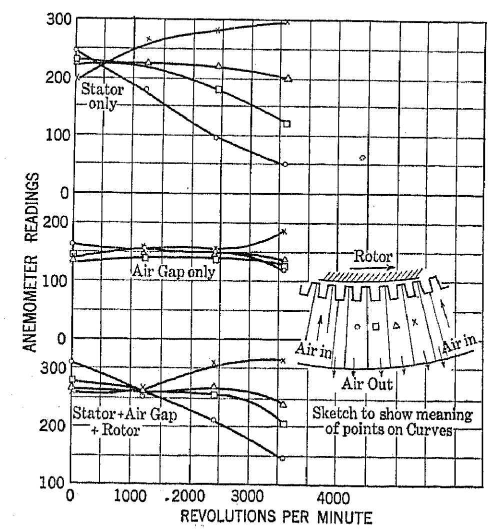

Fig. 21—Turbo Model I. Volume Distribution Curves Taken at Pressure of 7.5 In. of Water. Each Point is the Average of Readings Taken in 16 Ducts Axially

rotation of the eddy (clockwise looking axially, facing the outside of the end-bell) corresponded to the case of more than half the air entering the end-bell above the horizontal center line. In machines as built, such conditions do not obtain, except possibly in the

rare case when an external blower is used, and with a duct similarly connected to the one in the model. The experience is recorded here because the results are the kind that would not be anticipated, and may be of interest and value to others.

TEST RESULTS

Very many tests were made, far more than are included in this paper. Only a few that are representative, and that are believed to be of interest to those who have followed the development of turbo-alternator design are incorporated.

Model I. Many less tests were made on model I than on model II, but there were enough to secure that information which was needed most. A few of the results of those tests are plotted in Figs. 16 to 21 inclusive. It was believed that the deductions which were drawn were sufficient to warrant tests to be made with only one size of air gap. It was also felt that for that diameter, it would not in general be good practise to use more than six intake or outlet belts, as in a machine, the frame would be considerably complicated thereby. The average of the intake or outlet belt represents but one-twelfth of a circle. A change either in air-gap size, or in number of belts would undoubtedly have been accompanied by a change in performance; certainly the distribution of volumes would be more uniform with more intakes and outlet belts. As to the air-gap size, the circumferential distribution would probably have been improved by a larger gap, but with the number of intake and outlet belts chosen the gap velocity was then so low that at standstill only a small percentage of the pressure was consumed there. Furthermore, for electric considerations, the gap used in the model (0.625 inch on a side) is about the size that would be used with a 26-inch diameter rotor in a machine running at 3600 rev. per min.

On the various curve sheets we have designated as "stator" the condition when the air enters from the rear of the stator and travels radially toward the air gap; the condition has been styled "air gap" when the air enters the gap direct from the end bell; the path for that air which passes through the rotor vents we have called "rotor". From Figs. 16 and 17, a picture of the reduction in volume due to rotation can readily be obtained for various end bell pressures. The path is for the "stator" plus "air gap" in Fig. 16, and for the "stator" plus "air gap" plus "rotor" in Fig. 17. It will be seen that the volumes are nearly proportional to the square roots of the pressures when the rotor is stationary, but not so when the rotor is running. In every case, the influence of rotation is to reduce the volume; it is as though a counter pressure were produced by rotation, the percentage reduction in volume being greater, the lower the applied pressure. If there is a counter pressure due to rotation, its percentage is greater the lower the external pressure, and therefore the larger is the percentage decrease in volume. The "counter pressure" is due to a whirl which follows the external periphery o thereby decreases the effective cross se There is another effect of rotation in m subsequently explained.

In Fig. 18 are plotted the percenta volumes from standstill to 3600 rev. per applied pressures, as determined for stator plus the air gap from Fig. 16. seen that there is 50 per cent decrease of water and only 13 per cent with course, the percentage must approac pressure approaches infinity. The per with air supplied from the rotor in ad is nearly the same as without it. A to this system of ventilation is that siderable decrease in volume with in unless rather high pressures are used.

With the stator plus air gap plus more air is supplied than without the Thus, at 3600 rev. per min. and 7.5 i 7550 cu. ft. with air through the rotor a 6850 cu. ft. without the rotor, or ten pe The increase looks reasonable for the ad paths.

Refer now to the volume distri Only data for single finger spacing are p others were taken. In Fig. 19 th readings taken for as many vents a accessible, and for four consecutive p ferentially are plotted for standstill and min. for the one case of air feeding int air gap (at the end bell end) in paralle are various irregularities, the follow may be drawn:

(a) At standstill with low relative there are no differences circumferenti distance axially.

(b) At standstill the volume disch axially decreases with the distance fro entrance.

(c) At 3600 rev. per min. the distribution is substantially uniform ne

(d) At 3600 rev. per min. the distribution departs from uniformity from the end bell entrance increases, apparently reaching a maximum a li extremity.[11]

(e) Considering that the air in the circumferentially in the same directio surface moves, then, due to rotation are decreased materially in the first ven increased slightly in the last vent du mediate values in the intervening ducts.

(f) The distortion is undoubtedly lar for the reduction in volumes with ar

Before discussing these deductions, I

11. We call departure from uniform ci tribution "distortion."

will be examined. In both of these, volume distribution curves are plotted (for single fingers only), and the plots show the influence of change in angular velocity upon the volumes in individual vents, distributed circumferentially. From a number of curves like those for 3600 rev. per min. in Fig. 19, it was ascertained that the distortion is a maximum at about duct No. 16 axially. Consequently, the distortion for duct 16 is plotted in Fig. 20, showing approximate maximum distortion. A glance at Fig. 19 will show that the distortion is less if the average values rather than the individual points are plotted, and these averages also appear in Fig. 20.[12] In Figs. 19 and 20 the curves plotted are for air feeding into the back of the stator and into the air gap at the end bell. The average of 16 readings axially for other combinations are shown in Fig. 21. Thus, for the upper plot, air is fed into the stator only; for the middle plot, air is fed into the air gap only; and in the lowest plot, air is fed into the stator, the air gap and the rotor, all in parallel. From an inspection of Figs. 20 and 21, the following additional deductions may be drawn:

(g) The distortion due to rotation is largely dependent upon the manner in which the air is fed into the gap. If the air is fed in axially, (which we call air gap), the distortion, though present, is of little consequence. If all the air is fed in from the stator, the distortion is a maximum.

(h) The distortion which would obtain with all the air fed from the stator is decreased by feeding in parallel from the air gap. It is still further decreased by feeding in air from the rotor vents in addition.

Comments on the various deductions, in so far as they may not be self-evident, will now be given.

(b) The air entering from the stator and from the air gap, (rotor stationary) meet at right angles, and consequently, much of the velocity heads are lost. This would mean that the velocity head of the air direct from the end bell is dissipated more and more the farther from the end bell,—with a consequent reduction in volume. (The loss due to friction drop may be neglected). This meant too, that, other things being equal, the combined total volume was materially less than if the volume with the air gap alone had been added to that with the stator alone. It is well to bear deduction (b) in mind, as conditions with model II were quite different.

(c) and (d) The air which enters the gap direct from the end bell undoubtedly combines with the air from the stator which nominally travels circumferentially, so that the resultant with the high peripheral rotor speeds, is a spiral, like a corkscrew. Near the end bell the influence of the air passing direct from the bell predominates, and the distortion is of little consequence. Farther on, the velocity head of the axially moving air decreases, but the velocity head of the circumferentially moving air increases as its volume increases, thereby increasing the distortion. A possible explanation for the distortion being greatest before the end (axially) is reached, is that a portion of whatever axial component of velocity that remains is converted into pressure, and as such is useful in producing a flow in the last vent ducts; the influence of axial flow in the gap is to decrease the distortion.

(e) and (f) That the discharge velocities in some vents are decreased, and in others increased, due to rotation, is undoubtedly due to a whirl of air which travels circumferentially with the rotor.

One way of regarding the phenomenon is that the influence of this whirl is to decrease the effective gap section through which the main volume of air flows, which in turn means that the velocity there is increased. Some of the applied static pressure must be consumed in creating this velocity. Therefore, at duct m in Fig. 22 the gap velocity is high and consequently, the static pressure is low, in accordance with Bernoulli's equation. Most of the velocity in a given radial vent duct is dependent upon the difference in static pressures between the entrance and exit to the duct, and if that

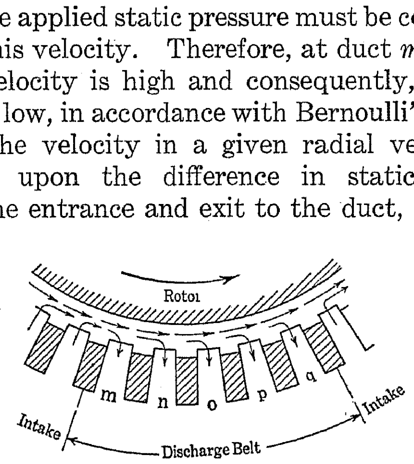

FIG. 22

pressure difference is low, the duct velocity must also be low. The velocity in the gap is lowered from m to n, due to the reduction in volume by that taken out at m. If the sum of velocity and static heads remains constant, then the static pressure at n is greater than at m, and therefore the discharge velocity at n is greater. Of course, there is a loss of head from m to n, but if only some velocity head is converted to static, the velocity in duct n is greater than in duct m. Similarly, for the other ducts; at the last duct in a belt, sufficient velocity head in the gaps may be converted into static to cause the velocity in that duct to be greater than with the rotor stationary. However, the net result is that the loss of head in the gap is sufficient to lower the average outlet velocity, with the rotor running, below that which obtained at standstill. Consequently, the total volume is lowered as a result of the distortion arising from rotation.

(g) The above explanation shows why the air that travels circumferentially through the gap is not uniformly distributed circumferentially, when running. The conditions that obtain for the air that enters the gap axially will now be given. Evidently, if all the vents were outlets, distributed uniformly, the discharge

12. It will be noted in Fig. 19, that readings were taken in 16 out of the total of 22 vents axially. That 16 should not be confused with duct number 16.

velocities would be alike at a given distance axially. The influence of rotation is to skew the air; the air which enters axially moves in a spiral, like a corkscrew and the pitch of the screw decreases as the distance from the end-bell increases. The motion may become nearly circumferential as the far end is approached. Consequently, near the far end, conditions are similar to those which obtain when the air enters through the stator intakes and moves circumferentially through the air gap; that is, distortion at the far end is to be expected. However, the gap velocities, determined from the volumes are low, since most of the air that entered axially escaped before it reached the last vents; as previously explained, if the circumferential gap velocities are low, the pressure differences over the discharge belt are small, and the outlet velocity differences are not large; but distortion is undoubtedly present. An idea of the order of magnitude of this distortion for the average of the readings axially can be obtained from an inspection of the middle set of curves in Fig. 21.

(h) Considering the rotor alone, the volume of air from the rotor vents is small, and being impelled by a high-pressure fan (the rotor coils), the outlet velocities must be nearly uniform. Consequently, the influence of that air is to reduce the distortion arising from the air that enters the stator intakes. Also, the air which travels axially from the end bells tends to reduce the distortion.

It was previously pointed out that in model I all teeth have sharp wedge notches. Laboratory tests on sections of teeth indicated that, for a given pressure, with the notches cut off the wedges, an average of about twice as much air would flow, as with the notches. In most large machines, the wedge notches are eliminated at the vent ducts. If, in model I, this had been done, the pressure drop in the vents for a given volume through them would have been considerably less, and had the gap conditions remained the same, the drop in the gap would have been a greater percentage; consequently, the distortion would have been even greater.

For the intake vent ducts, in which we were unable to secure reliable data, there must be distortion, somewhat as in the outlets. The gap velocities are maximum near the division between an intake and an outlet belt. That represents a low-pressure region, and the largest pressure difference exists between the intake (connecting with the end bell) and that low-pressure region; consequently, those vent ducts have the greatest velocities of any intake vents. That is, the high-velocity region in the intake vents is in the inverse order to the outlet vents. This will be clearer after studying results on model II.

In general, it may be said that the air flow with the type of ventilation in model I is too complicated to admit of a quantitative mathematical solution. Even at standstill, our present knowledge is too incomplete to put into equations the combined flows; and with rotation, the problem is very much more difficult. A solution of the flow with the stator alone at standstill is not very difficult, if comparatively simple assumptions are made; however, that solution is of little value unless the axial flow is combined with it, and especially, unless the influence of rotation is considered. Certain empirical relations were secured co-ordinating pressures, volumes, and rotation, use having been made of data not published in this paper, in addition to the data given. Those equations are not being published at present.

The conclusion reached is that the type of ventilation investigated with model I is undesirable, first because of the decrease in volumes with rotation, and second because of non-uniformity of discharge velocities, due

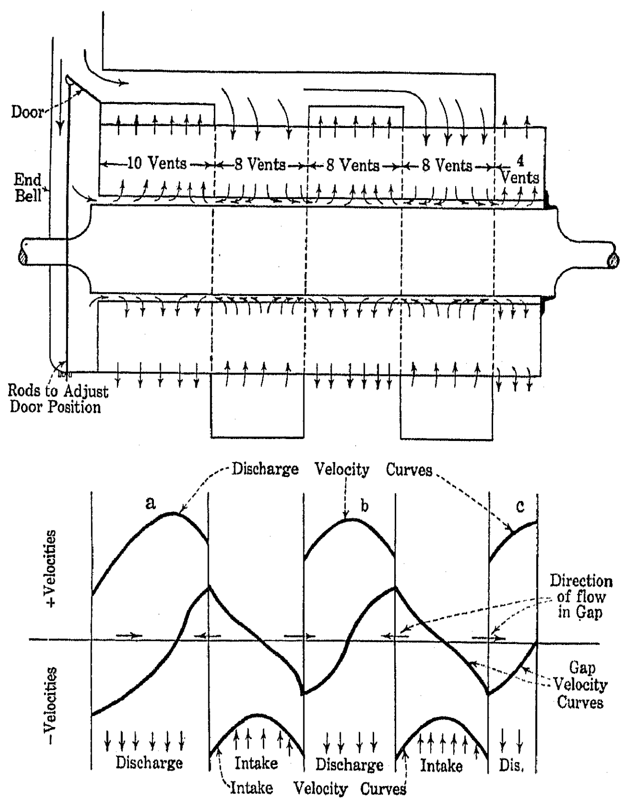

FIG. 23—TURBO MODEL NO. 2 SCHEMATIC DIAGRAM TO SHOW DIRECTIONS OF AIR FLOW AND DISTRIBUTION OF VOLUMES

to rotation. The non-uniformities, (distortion) occur for relatively long distances axially near the middle of the machine. Most of the heat generated in the tooth zones near the middle of a long machine must be carried away by moving air streams in that region; it cannot be conducted axially along the copper to the ends, as the length is too great; only a small fraction of it can be conducted circumferentially from tooth to tooth, because the length of the path is great, the cross-sectional area is small, and the thermal conductivity of the steel is rather low.[13] Consequently, those teeth

13. The thermal conductivity of low loss silicon steel is less than half the conductivity of ordinary steel; and the conductivity of ordinary steel is only about one-ninth of that of copper.

for which the air supply is small will heat considerably, and there will be high local temperatures.

A modification of model I type of ventilation consists in shunting some of the air which is admitted through the stator intakes through other paths of definite areas back of the teeth. If the same velocities are used in the teeth as with model I ventilation, and the same total volumes, then there is less air for the teeth, and the package thickness may be greater. With the same velocities in the teeth, nearly the same pressure of air must be used, and as the circumferential gap velocity is less, there probably will be a little less distortion. On the other hand, the thicker package of laminations means a greater drop in temperature axially; as that drop is proportional to the square of the thickness of the package, it may become an item of considerable magnitude. From an experimental viewpoint, the study of this type of ventilation would first mean the development of an instrument for measuring air volumes (or velocities) in the various paths. No existing instrument is adapted to this. That would have required more time than was at our disposal. Many machines have been built with this type of ventilation, and it is claimed that they are satisfactory.

Model II—*Test Results.* Very many more conditions were tried out in this model than in model I, and the test results appearing in the forms of curves in this paper are those which are believed to be most helpful and to be representative of those conducted. Only single finger vent spacers were used, and there were no wedge notches. The total number of sections of air gap for which tests were made were four. The first gap was the same as in model I, obtained with 26-inch diameter of rotor, with 0.625 inch gap, with stator slots sunk 0.625 inch below the inner periphery, giving a net gap cross-section for axial flow of 0.553 sq. ft. A second gap section of 0.364 sq. ft. was obtained by filling in the upper parts of the slots, making the inner bore of the stator smooth. The rotor was later turned down to 25.25 inches, making the gap section 0.573 sq. ft. Finally, the fillers in the slots were removed, increasing the gap section to 0.763 sq. ft. On every curve sheet for model II the gap section is recorded.

The same designations for the paths for air flow are used for model II as for model I. Thus, it is "air gap" when air is fed directly from the end bell into the air gap; it is "stator" when air is fed into the stator intake vents at the back of the core; it is "rotor" for that air which is passed through the rotor vent into the air gap. Thus, "air gap" plus "stator" means that air entered through those two paths.

A statement is given in the following of the tests conducted on model II, a few of which are published herewith; perhaps others contemplating similar work may obtain a helpful suggestion.

The 0.553 sq. ft. gap tests were made with air gap only, stator only, rotor only, air gap plus stator, air gap plus rotor, stator plus rotor, and air gap plus stator plus rotor, with all vents open, and set up as indicated in Fig. 10 and the upper part of Fig. 23. With the same gap, vents Nos. 5 and 6, and later Nos. 4 to 7, inclusive, were closed at the outside, these tests being made to determine the influence upon distribution. Later the first two vents were closed at the outside. With the 0.553 and 0.763 gaps, tests were made with various positions of the door, (Figs. 10 and 22), to determine the influence of a change in ratio of stator to end bell pressure upon total volumes and its distribution. With 0.553 and 0.573 gaps, the effectiveness of a ring in the air gap to assist to define the division of air was determined. With the 0.553 gap, and stator only, the first six vents were closed, so that the paths of the air were defined, and the pressure drops could be fairly readily determined; other combinations of numbers of ducts with the same general set-up were tried. With the same gap, all the intakes were removed, so that all 38 vents discharged air, the only intake being into the gap at the end bell end.

With 0.364 gap, the wood fillers at the tops of the slots were allowed to project into the end bell to imitate coil ends, and their influence, before and after being cut off flush, were determined. Tests were made with the stator plus air gap, and air gap only. Then a converging cone was placed at the air-gap entrance to assist the air to enter. Various combinations with certain outlet vents closed were tried.

With the 0.573 gap, in addition to tests previously cited various, rather fruitless, attempts were made to explore the velocities in the gap with a specially devised pilot tube. After that method was abandoned, the distribution was determined with the first five vents closed at the inner periphery of the stator. The influence of several kinds of guides at the entrance to the gap was determined.

Some of the tests made with a 0.763 gap were enumerated above. In addition, special restrictions were placed at the end bell entrance to the gap; and the first two wooden "packages" (imitating packages of laminations) were cut back.

Tests were also made with the rotor removed: (a) With air entering direct to the stator bore from the end bell, with the far end closed by card board; the usual entrance to the stator being closed; (b) the entrance to the stator open, the bore at the far end open, and at the end bell end closed; (c), as (b), but the bore closed at both ends.

In many of these tests total volumes, total pressures, and distribution of volumes were measured for various angular velocities. Some very valuable and some very interesting results were obtained; on the other hand, as in nearly all research work, some results have so far not been of any value. The results are so numerous that a paper several times the length of this might be written to cover the results.

A realization of the importance of being able to

estimate pressure drops suggested to us that small models be built of the full size of a tooth vent section from the inner to the outer bore, and this model was to be made so that sharp, round, and blank wedge notches could be added. Pressure drops were to be measured for various volumes, with flow in both directions, with one, two and no fingers. That in itself is quite a large and valuable research, the results of which cannot be included in this paper. The sum of the pressure drops with one finger for the two directions, are about twice as high for a given volume with sharp wedges as with none; in every case the drop is higher (for a given volume) with air flowing toward the inner bore than away from it. The test results for the blank wedges were subsequently checked for both directions of flow by applying formulas to be found in standard reference books on hydraulics. (Air is treated as an incompressible fluid; and for velocities well above the critical, viscosity has negligible influence upon the pressure drops. See author's articles in the *Electric Journal* on "Some Elements of Air Flow," beginning with August, 1922.)

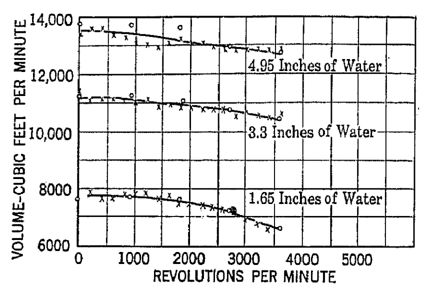

Fig. 24

Turbo Model No. 2. Volumes for various angular velocities and static pressure with stator + air gap open. Rotor external diameter = 26 in. Single air gap = 0.625 in. Stator slots sunk 0.625 in. Below stator inner periphery. Gap area = 0.553 sq. ft.

⊙ = Points taken Nov. 8, 1922.

x = " " Mar. 14, 1923

Total Volume Curves

Refer now to the various figures. In Figs. 24, 25, and 26, volumes of flow are plotted against revolutions per minute for several static pressures. All volumes on these figures were those measured minus leakage. The model as shown in Fig. 10 was used, and was arranged as follows:

(a) The stator was in parallel with a 0.553-sq. ft. section air gap, results plotted in Fig. 24; (b) in Fig. 25, conditions were as for Fig. 24, plus the rotor; and (c) in Fig. 26, conditions were as for Fig. 24 with the rotor turned down to 25.25 in. diameter.

These curves for volumes are somewhat similar to those for model I. The volumes at zero speed are practically proportional to the square roots of the static pressures, and the volumes with the rotor intakes closed decrease slightly with the increase in rotor speed. It should be noted, however, that the decrease in volumes is small, at least as compared with those in model I. In Fig. 24 the percentage decrease in volumes increases as the pressures decrease, as in model I. These decreases for both models (from Figs. 18 and 24), are recorded in Table I.

TABLE I.

Pressure Inches of Water	Percent Decrease in Volume from Standstill to 3600 rev. per min.	
	Model I.	Model II.
4.95	23.5	7
3.3	38.5	8

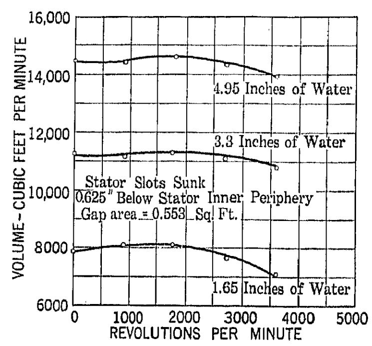

Fig. 25

Turbo Model No. 2. Volumes for various angular velocities and static pressures with stator + air gap + rotor open.

Rotor external Diameter = 26 in.

Single air gap = 0.625 in.

Had we been able to use pressures as are used in machines, of the order of 10 inches of water, the percentage decrease in volume due to rotation with model II would undoubtedly have been of the order of only a few per cent—practically negligible. With the rotor shut off, as before, but with a gap section 38 per cent larger (Fig. 26) the decrease in volume due to rotation is even less—4 per cent decrease with only 2.5

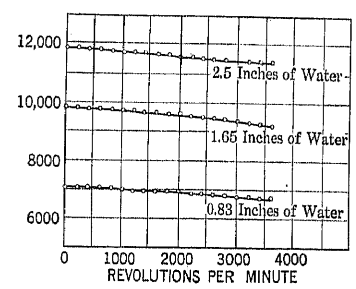

Fig. 26

Turbo Model No. 2. Volumes for various angular velocities and static pressures with stator + air gap open. Rotor external diameter = 25.25 in. Single air gap = 1 in.

Stator slots sunk 0.625 in. below stator inner periphery. Gap area = 0.763 sq. ft.

inches of water; it certainly must be negligible with pressures that are used in most turbos.

If the rotor is provided with radial vent ducts, it would seem that at moderate peripheral velocities, the pressures created by the coils that act as fans, actually increase the volumes (see Fig. 25) around 1800 rev. per min. At higher speeds there is a slight decrease in volume, so that the volume at 3600 rev. per min. is a little less than at standstill. At the low

pressure of 3.3 inches, there in only about 3.8 per cent less volume than at standstill; with the rotor shut off, the decrease was 8 per cent with the same size gap (Fig. 24 or Table I).

As was pointed out, the influence of rotation is to surround the rotor surface with a whirl which has the effect of decreasing the effective gap section, thereby increasing the velocities for the main air streams, and consequently raising the pressure drops for a given volume. As for a given diameter and angular velocity of rotor, the whirl has a certain depth, the percentage reduction in effective gap section due to it decreases as the radial depth of the gap is increased. Therefore, with the larger gap the axial gap velocities were increased less due to rotation, and the percentage pressure drop decreased. In model II, the circumferential distortion due to rotation is absent, and therefore there is no increase in pressure drop due thereto. That is why the percentage reduction in volumes due to rotation in model II, is so much less than in model I.

Relatively small volumes of air flowed through the rotor vents, and it will be seen from a comparison of Figs. 25 and 24 that the volumes at a given pressure were but little higher with the rotor open than with it closed. Separate tests, with the rotor acting alone, showed that the volumes increased materially as the speed increased, due to the fact that the rotor acted as a fan in series with the external blower. At 3600 rev. per min., the volume was nearly as high when the blower was shut down as when it was delivering a pressure of five inches of water. That would be expected, as a peripheral velocity of 24,500 feet per min. corresponds to a pressure of about 37 inches of water, part of which is realizable. At low angular speeds, the air fed into the air gap either from the end bell or from the main stator intakes is augmented by the flow through the rotor; at high speeds, however, the air thrown out of the rotor vents nearly tangentially, at high velocities, is met by air which moves nominally in an axial direction.

Much of the velocity heads in these two streams which meet is destroyed, and consequently the volume for a given pressure is not augmented at the rate that it was for lower pressures. At very high speeds there may be a decrease in volumes. This is an explanation offered for the shapes of the curves in Fig. 25. Of course, if a belt of stator vents is long enough, (axially) the influence of rotation is to cause the air to move spirally; that is, a combination of tangential and axial motions. Attempts have been made to analyze the phenomenon quantitatively, but it is too complicated for us to handle with our present knowledge. In so far as the turbo alternator is concerned, with model II type of ventilation, it is safe to predict the volume at standstill, and take that to be the volume at full speed, or to deduct a few per cent for safety.

Volume Distribution Curves

In Figs. 24, 25, and 26 for total volumes, the points plotted are those for volumes as measured minus leakage. In the distribution curves, the values of volumes as measured for the particular conditions of test, without deduction for leakage, are recorded on some of the sheets.

In Fig. 27 are shown the results of tests made on a turbo which was a duplicate of machines that were built for about 15 years. The machine receives all its air direct from the end bells, at either end, as illustrated in Fig. 1. The points were taken by pushing an open end brass tube into vent ducts at the rear, and temporarily sealing the space around the tube with putty, the other end of the tube being connected to a manometer. The readings thus obtained were taken to be the pressure that was useful in driving the air through the vent when the tube and seal were absent. The square roots of the ordinates on the pressure curve should be

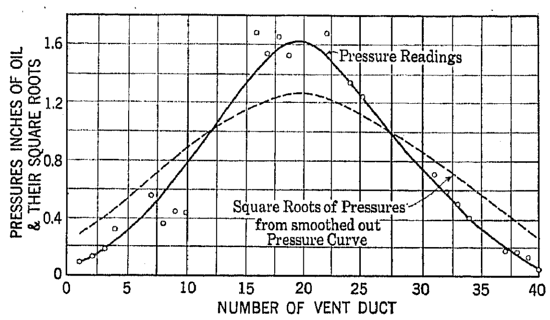

Fig. 27

Volume distribution curves in a turbo alternator with air entering only at the ends of the air gap. Each vent duct 3/8th in. wide. Ratio of summation of min. sections in teeth in 20 ducts to gap section = 2.1. Total length of stator = 65 in. Stator internal diameter = 26 in. Rotor internal diameter = 23½. Rev. per min. = 3600. Propeller type of fans at the external diameter of the rotor beyond the stator were the source of pressure. Readings taken with the use of a brass tube pushed into vent at the external diameter and the vent space around the tube was temporarily sealed. The dotted curve is an approximation of the shape of the volume distribution curve.

proportional to the velocities that obtain in normal operation. The shape of the dotted curve is that which is obtained in a machine with air fed into the air gap at both ends, and in which the ratio of cross-sections of vents to gap is the same and that has similar proportions of teeth and slots. It has long been known that with this type of ventilation the radial flow is a maximum near the center, and a minimum near the ends. It was, in fact, that that led us to anticipate similar distribution, in part at least, for model II, as the air moves axially in the air gap there, too.

Mr. Bratt, in a companion paper, has derived equations for the shapes of these curves, based upon Bernoulli's theorum, and the equation of continuity. Mr. Bratt has shown that, if losses of head in the gap are neglected, the volume distribution curve approaches that of a simple cosine. An inspection of the dotted

curve in Fig. 27 will show that its shape approaches that of a cosine. Perhaps a physical interpretation may be of assistance—to give a general idea as to why the velocities increase toward the middle of the machine.

Bernoulli's equation states that, neglecting losses, the sum of the velocity head and static pressure measured from some reference point in the direction of flow is a constant. (The velocity head is a constant times the square of the velocity). Considering the velocity head in the end bell as zero, and the pressure there a maximum, much of that pressure is converted into velocity head when the air enters the air gap, since the velocity there is high, due to the large volume passing; the static pressure is low. As the velocity in the vents is proportional to the static pressure in the gap, the outlet velocity must be low. But by whatever percentage the volume in the gap is reduced by the air that is discharged through the first vents, the velocity in the gap is decreased; the velocity head also decreases, and in accordance with Bernoulli's theorem, the static pressure rises. The latter means a higher discharge velocity at the next vent; that means a further reduction in gap velocity and velocity head; an increase in static pressure, which in turn causes the air to be discharged at a higher rate, and so on.

Evidently, the volume entering the gap must be the summation of the volumes discharged from all the vents up to the center line. Consequently, if the machine is short, the total volume is relatively small, and the maximum gap velocity must be comparatively low. Therefore, the static pressures at the regions of maximum gap velocities are higher than for the long machine, and the discharge velocities would be correspondingly higher. Thus, suppose the machine whose outlet velocities are indicated in Fig. 27 had one-half the number of vents. If the Bernoulli equation without losses is applicable, the total head in the gap at the middle of the machine is the same as in the end bells; as the gap velocity is zero at the middle, all of the head must be static, which is converted into radial velocity in the vents. If the losses of head in the gap are neglected, there must be the same vent velocity at the middle with the full length as for the half length machine for the same pressure in the end bells. For the vents next to the middle the static pressure in the gap is lower than at the middle vents by the amount represented by the velocity head corresponding to the volume that escaped through the middle vent; since that volume was the same for both lengths, the static pressure (and therefore the outlet vent velocity) is the same for either the long or the short machine, and so on. That is, the outlet velocity curve is substantially the same for the two lengths of machines, with, of course, the extremities of the curve for one-quarter length removed from the full length machine. Actually the two curves would not be exactly alike, because there are losses of head in the gap, and those losses are not the same for the two lengths. The above explanation may be of assistance in the study of some of the subsequent data in curve form.

Consider now the effect of lengthening the machine. If the length be increased to a certain critical value (defined in Bratt's paper as $\frac{S\,L}{a\,\sqrt{C\,d}} = \pi/2$) the velocities in the first vents become zero. If the length be further increased, for the ideal machine without loss in the gap, there will be no discharge in any vents until that critical point is reached. Of course, there are losses of head in any machine, and therefore, there *is* a decrease in total head. This can only take place by a small discharge all along, as then the velocity head decreases in the gap, the static pressure changing but little, until the critical point, where the approximate sine wave discharge begins.

That this is more than theory will be seen from Fig. 28. In this case all the external enclosures were removed from model II, all 38 vents being used for discharge. The only entrance was to the air gap at the end bell end. The manner in which the curves were obtained is given on the figure.[14] The ratio of vent to gap section is far greater than would correspond to the critical length.

At standstill the discharge velocities are nearly uniform until vent number 31 is reached; then the discharge rates increase to the end, along a curve that approximates a cosine. The calculated starting point of the sine curve is at about duct number 32.

14. To make clear the explanation given on Fig. 28, the calculations for a few points are given: At zero speed, the anemometer readings were as follows:

Duct. No.	Anemom. Reading	Sum	Gap Vel.
38	156	156	967
37	155		
		311	1560
36	140		
		451	2800
35	120		
		571	3540
		1518	9400
2	9		
		1527	9460
1	8	1519	9410

The maximum gap vel. = Volume/Area = 5200/0.553 = 9410 ft. per min. The proportionality constant- 9410/1519 = 6.2. The gap vel. at any event position = Sum as given above times 6.2.

To determine the vent duct velocities from the gap velocities, use the equation of continuity: If $-\Delta v$ is the decrement in gap vel. due to taking off the volume through a particular vent duct, the volume decrement $= (-\Delta v)\ a = VA$ where a = gap area, V = outlet vel., and A = vent duct area at minimum section. Then $V = -(a/A)\ \Delta v = -(0.553/0.141)\ \Delta v = -3.92\ \Delta v$.

(V is plotted as positive). If the distance is very short, and if S is the vent area per inch length axially, $S\,d\,x$ is area for short length. The volume $= V\,s\,d\,x = -a\,d\,v$; or $V = -a/s\frac{d\,v}{d\,x}$, a constant times the derivative of the gap velocity.

In the foregoing, no statement was made in regard to losses of head at entrance to the gap. When a fluid passes from a large to a smaller section, with sharp edges, it undergoes a contraction in section for a short length—called "vena contracta." The smaller section means that there is an increase in velocity, and therefore of velocity head. Some of this velocity head is lost when the air subsequently slows down; the high velocity at the vena contracta is accompanied by low static pressure there. In the turbo model, when the edges at the gap entrance were sharp, the contraction phenomenon was undoubtedly present, and therefore the velocity near the first vent was high, and the static pressure low. In fact, the static was so low, in certain cases, as to be below atmospheric pressure; the air then flowed in, instead of out through the radial vent duct. In a few tests, most of which are not being published, the flow *was inward* (negative) in the

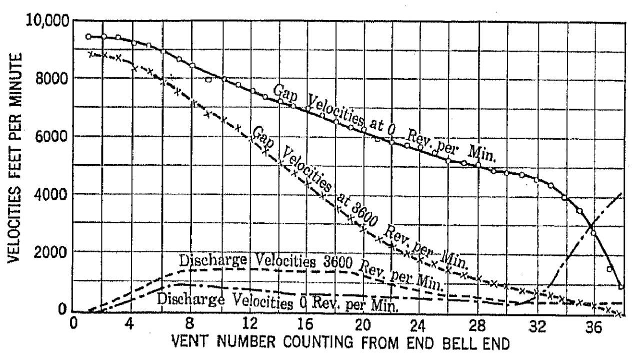

Fig. 28

Turbo Model No. 2. Velocity distribution curves. Air entering air gap only. All 38 radial vents used as discharge ducts. Rotor diameter 26 in.; radial depth of gap 5/8 in. Stator slots sunk 5 8 in. below inner periphery. Gap area = 0.553 ft. Tests made at pressure of 7.5 in. of water in end bell. Volume with rotor stationary = 5200 cu. ft. per min. Volume with rotor at 3600 rev. per min. = 4900 cu. ft. per min. Original test points not plotted, but summation of outlet readings beginning with duct No. 38 are plotted as gap velocities after determining the proportionality constant from total volumes. The derivatives of the smoothed gap velocity curves give the discharge velocity curves.

first vents. In Fig. 28, the anemometer rotated slowly in the reverse direction, indicating negative flow in the first vent, due undoubtedly to negative gap pressure which resulted from the vena contracta incident to the sharp entrance. As the air slowed down with the enlarged area following the vena contracta, the pressure rose slightly in the gap,[14] and therefore the outlet velocities became positive, and increased in value up to about duct 7. It is believed that the reason for the slight fall in discharge velocities after duct 7 is that the gap loss per unit length became less following the highly turbulent state arising from the enlargement after the vena contracta. However, the discharge velocity drops only slightly, and then becomes nearly constant. If V_1 and V_2 are the gap velocities at two points, 1 and 2, P_1 and P_2 the corresponding static pressures, and γ the density of the air, then by Bernoulli's equation:

$$\frac{V_1^2}{2\,g} + P_1/\gamma = \frac{V_2^2}{2\,g} + P_2/\gamma$$

$+$ Losses of head between positions 1 and 2.

The condition for the discharge velocities to remain constant is that $P_1 = P_2$. Then,

$$\frac{V_1^2 - V_2^2}{2\,g} = \text{Losses}$$

That is, the condition for the discharge per vent to be uniform is that the decrease of velocity head per unit length equal the loss per unit length. Actually, the problem is not so simple, since the loss is not proportional to the decrease in velocity head, but more nearly to the square of the decrease in velocities. In other words, the discharge per vent is not uniform.[15]

With rotation, conditions are different than at standstill. With conditions as in Fig. 28, the air is undoubtedly moving practically circumferentially in the air gap for a considerable part of the axial length, so that the axial component of velocity there may be neglected. Undoubtedly, starting at the end bell, the air first moves axially, then spirally, the angle that the spiral makes with an axial line becoming greater the farther from the end bell, approaching, and perhaps practically reaching 90 deg. if the length is great enough. While a crude picture of the flow is obtainable, a quantitative analysis is, at present, impossible.

The test curves will perhaps be better understood from an inspection of Fig. 23. Consider first the discharge velocity belts. There are three: (a) the belt adjacent to the end bell; (b) the discharge belt between the two intake belts; and (c) the discharge belt beyond the second intake belt. If, as before, the losses of head in the gap and influences of rotation are neglected, the discharge velocity curves are cosines. In the simplified case, illustrated in Fig. 23, the maximum velocity of the discharge in belt *b* is at the midpoint axially, and that point corresponds to zero gap velocity; equal volumes flow from the two adjacent intake belts toward the middle of the discharge belt *b*. The discharge velocity curve is proportional to the derivative of the gap velocity (as shown in the foot-note No. 14). Consequently, if the discharge velocity curves are parts of cosines, the gap velocity curves are parts of sine curves, displaced from each other by 90 degrees; the maximum of one corresponds to the minimum of the other. For belt *b* it is assumed that there is a wall at the middle. The point at which the gap velocity becomes zero, (and the discharge a maximum), we have called the "balance point." In belt *c* the conditions

14. The fact that the outlet velocities reverse and rise proves that some of the difference between the velocity heads at the vena contracta, and after filling the gap is not lost; for if all were lost, the static pressure and the outlet velocities would remain constant. This is in keeping with theoretical and experimental results published in works on hydraulics, where the loss of head for a fluid entering a duct with sharp entrance edges is given as 0.4 to 0.5 of the velocity head corresponding to the velocity when the duct is full.

15. The above is put in to give the reader a more complete and a better understanding of the statement made earlier.

are as for one-half of b; in c there is a definite mechanical stop at the far end.

In the first belt a, conditions are somewhat different. Assume that the pressures in the end bell and back of the first intake belt are equal. In general, the loss of head through the intake vents and up to the discharge belt is greater than at the gap entrance, at the end bell end, so that more total head is available for the discharge on the end bell side than at the intake side. It is almost evident, therefore, that the "balance point" in the first belt a, is usually not at the middle of the belt, but is nearer to the first intake belt, as indicated in Fig. 23. The discharge velocities are lower at the end bell side than at the intake side, as will readily be understood from previous explanations.

The intake velocities with the rotor stationary, are at minimum value at the middle and maximum at the ends of the belts. This follows from the following: Gap velocities are zero at the middle, and maximum at the ends of the belts; the static pressures in the gap are, in consequence, at maximum at the middle, and minimum at the ends of the belt; the vent velocities are dependent upon the differences between the pressures back of the core and in the gap, and those differences are minimum at the middle and maximum at the ends of the intake belt. Neglecting losses of head in the gap, those velocity curves are hyperbolic, instead of trigonometric, and are cosines for the vents, and sines for the gap, referred to the balance point.

As in most complex phenomena, the actual curves can not be so simple. There is a loss of head in the gap, and the solution of equations with allowance for that loss complicates them considerably. However, those equations have been solved, but they are not yet ready for publication. The balance point in the first discharge belt is shifted; that, in turn, shifts the balance point of the first intake away from the middle; the location of other balance points are likewise affected until, at the middle of the machine, the balance point is at the middle of the central belt. The influence of rotation also plays its part in distribution that will be subsequently discussed more at length. However, the calculations based upon the comparatively simple assumptions, giving trigonometric and hyperbolic sine and cosine distributions is extremely useful and helpful just as the use of the simple sine wave assumption in alternating-current calculations is of enormous help and usually gives results which are sufficiently accurate.

In regard to the balance point, Mr. Bratt has given methods of calculating its location. Other unpublished methods are available, all of which are more or less approximate; the final check is that the pressure drops in each of the parallel circuits are equal; and that the vent velocities at a balance point, taken from either side, are equal.

Referring now to Fig. 29, it will be seen that the shapes of the outlet velocity curves at standstill are not very different from those shown in Fig. 23. As belt the two curves are much alike, the ordinates being a little higher for the curve with two vents closed. It is substantially the same. This is indicated in Fig. 29, when the first two vents were closed, the distribution for that arrangement, as well as for all vents open, being shown. The other curve, for the first five vents closed, is for a slightly larger gap than the other two; therefore, the discharge volumes per vent should be a little larger. It is close enough to the other two curves to verify the previous reasoning. It is difficult to explain why the outlet volumes with the larger gap are apparently lower than with the smaller gap in the two other discharge belts. There was a lapse of several months between the times at

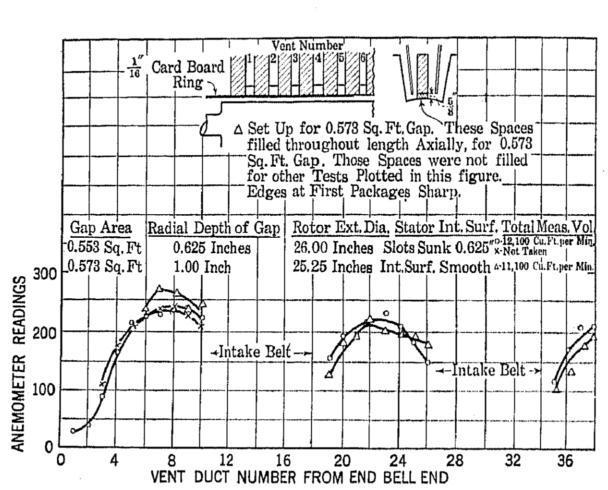

Fig. 29

Turbo Model No. 2. Volume distribution curves. Stator + air gap open. Taken with a pressure of 3.3 in. of water. Rotor stationary. 0 = all discharge vents open gap area = 0.553 sq. ft. x = first two discharge vents closed on outside gap area = 0.553 sq. ft. △ first five discharge vents closed on inside. Gap area = 0.573 sq. ft.

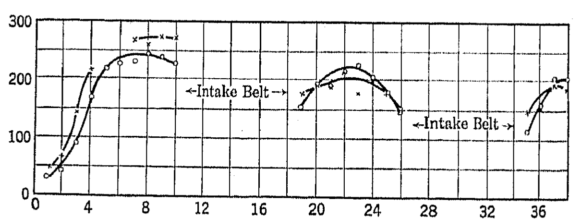

Fig. 30

Turbo Model No. 2. Volume distribution curves. Stator + air gap open. Rotor stationary. Edge at entrance package sharp taken with a pressure of 3.3 in. of water. Radial depth of air gap = 0.625 in. Slots sunk 0.625 in. in addition. Rotor external diameter = 26 in. Gap area = 0.553 sq. ft.

⊙ = all vents open

x = vents 5 and 6 closed at outside 11,700 cu. ft. per min. 12,100 cu. ft. per min.

which they were taken, and the anemometer probably changed slightly in the interim. If that is correct, the volumes for the larger gap, in the first belt, should be higher.

The curves shown in Fig. 30 are also for standstill conditions. The only difference between the conditions under which the two sets of curves were taken were that in one all vents were open, and in the other two intermediate discharge vents were closed. The measured

volumes were substantially the same. In the first was previously explained, the shortening of the first discharge belt a, (effected by closing some of the vents), simply cuts off the first part of the curve, the remainder believed that whatever losses there were in the gap were less from vent 4 to vent 7, when vents 5 and 6 were closed, than when all were open; the reason therefor being that the slowing down was more gradual, like in a duct whose sides diverged more gradually. That the velocities in the first ducts adjacent to the end bell should have been so nearly alike for the two conditions, follows at once from the fact that the volumes were nearly equal.

In Fig. 31 three sets of curves are plotted: Two at

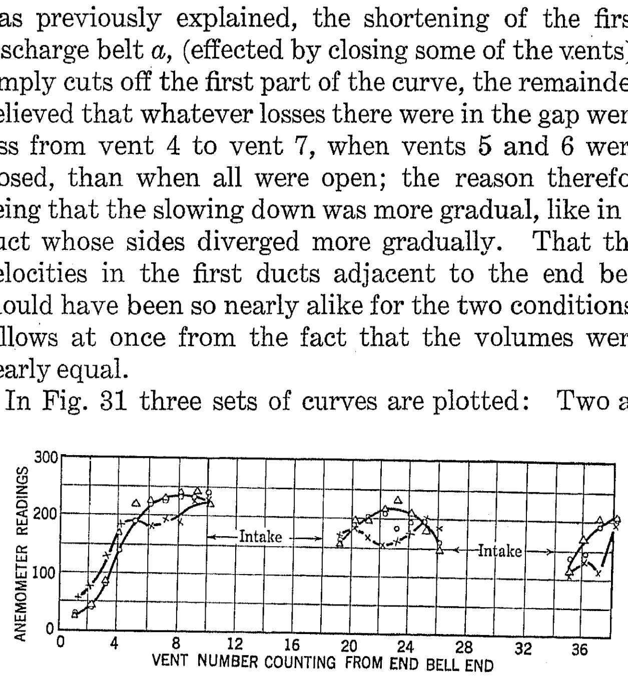

Fig. 31

Turbo Model No. 2. Volume distribution curves. All discharge vents open. Stator + air gap open. Edge at entrance package sharp. Taken with a pressure of 3.3 in. of water. Radial depth of air gap = 0.625. Slots sunk 0.625 in addition. Rotor external diameter = 26 in. Gap area = 0.553 sq. ft.

⊙ = Taken Oct. 24, 1922 rotor stationary 12,100 cu. ft. per min.
Taken Mar. 13, 1923 " " 12,000 " " " "
Taken Oct. 24, 1922 " at 3600 rev. per min. 11,400 " " " "

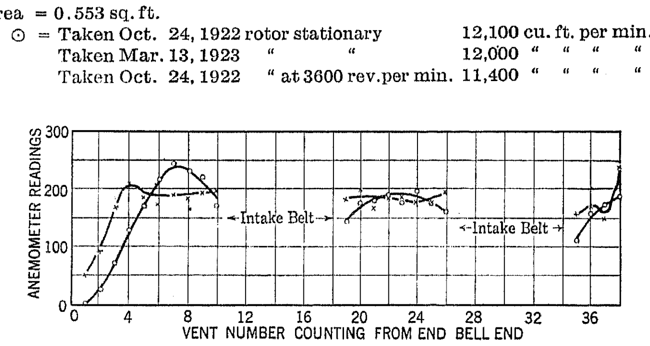

Fig. 32

Turbo Model No. 2. Volume distribution curves. Stator + air gap + rotor open. Edges at entrance packages sharp. Rotor diameter 26 in. Radial depth of gap 0.625 in. Stator slots sunk 0.625 in. below inner periphery. Gap area = 0.553 sq. ft. Test made at pressure of 3.3 in. of water.

⊙ = Turbo rotor speed = 0 12,150 cu. ft. per min.
x = Turbo rotor speed = 3600 rev. per min. 11,650 " " "

standstill, for the same conditions, but taken five months apart, and illustrate that, even though the anemometer readings were somewhat inaccurate, for the most part, the results were dependable. The volumes, too, are almost the same. The general shapes of the curves are sinusoidal. The third curve is for the same conditions except that the rotor was driven at 3600 rev. per min. The distribution is then changed, the discharge volumes are slightly more uniformly distributed than at standstill. There is an appreciable departure from a cosine wave.

In the previous curves, air entered the air gap from the stator intakes and from the end bell. In Fig. 32 air entered from the rotor also. At standstill the curves are much the same shape as previously, and at 3600 rev. per min. there is again more uniformity.

There are several ways whereby the volumes in the first vents adjacent to the end bell can be raised. One way is to shorten the belt axially, the results of which are shown in Fig. 29. Two other ways are illustrated in this paper. For the curves shown in Fig. 33, a ring was shrunk on the rotor, located between the fourth and fifth vents. This ring acted as an effective barrier,

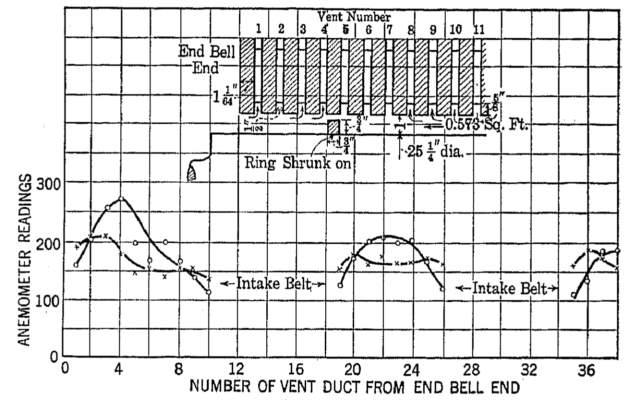

Fig. 33

Turbo Model No. 2. Volume distribution curves. Stator + air gap open, with shrunk-on rotor. Edges at entrance packages sharp. Spaces above blocks imitating coils in vents filled for entire length axially. Making inner bore of stator smooth. Gap area = 0.573 sq. ft. Tests made at a pressure of 3.3 in. of water.

⊙ = Rotor rev. per min. = 0. Total measured vol. = 13,500 cu. ft. per min.

x = Rotor rev. per min. = 3600. Total measured vol. = 11,600 cu. ft. per min.

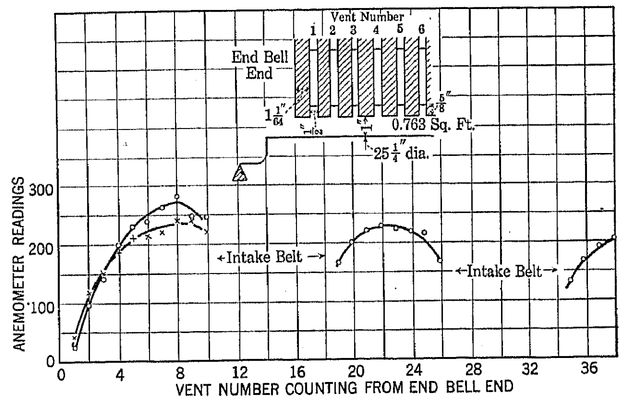

Fig. 34

Turbo Model No. 2. Volume distribution curves with sharp edges at first packages. 0.763 sq. ft. area in gap. Taken at a pressure of 3.3 in. of water in end bell.

⊙ = Turbo rev. per min. = 0
x = Turbo rev. per min. = 3600
Total volume = 14,400 cu. ft. per min. at standstill
" " = 13,500 cu. ft. per min. at 3600 rev. per min.
Stator + air gap.

whereby the balance point was defined. At standstill, as well as at 3600 rev. per min., the volumes in the first vents in the first belts are then a little higher than in the first vents in the middle discharge belt. In the first belt, at 3600 rev. per min. the volume distribution is not so very far from uniform. Tests were also made at standstill with a ring that could be moved to various

axial positions, and it was found that the balance point could be controlled as desired.

The other method which is shown in this paper for raising the velocities in the first vents, consists in increasing the air gap at one or more of the packages of laminations adjacent to the end bell. The sketch in Fig. 35 shows the manner of application to the turbo model: The increase in gap reduces the gap velocity at the first and second vents, thereby raising the static pressure, and consequently, the discharge velocity. The influence of high velocities due to a vena contracta are also reduced. These tests were made with a larger gap than the previous ones, and in order to show the gain, the data for the same gap, but without the first packages cut back are plotted in Fig. 34. While the volumes in the first vents in the first belt (Fig. 35) are slightly less than in the first vents in the middle belt, they are representative of velocities that are sufficiently high to

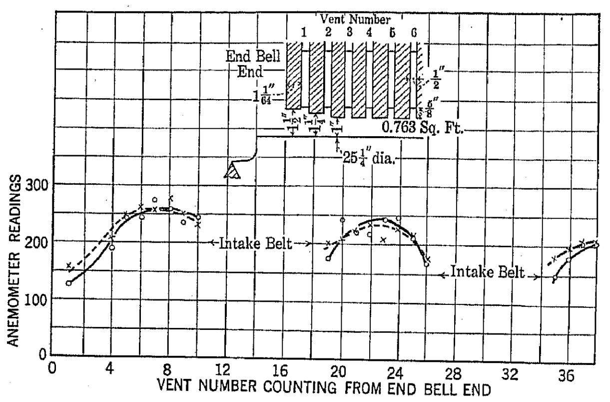

Fig. 35

Turbo Model No. 2. Volume distribution curves with first packages cut back. 0.763 sq. ft. area in gap. Taken at a pressure of 3.3 in. of water in end bell.

⊙ = Turbo rotor rev. per min. = 0
x = " " " " = 3600
Total volume = 14,100 cu. ft. per min. at standstill
" " = 14,000 cu. ft. per min. at 3600 rev. per min.
Stator + air gap.

avoid undue heating. That point will be discussed subsequently.

One interesting point, to be noted from an inspection of Figs. 34 and 35, is that the distribution of volumes at 3600 rev. per min. approaches cosine wave shape, as at standstill. There seems to be no doubt but that the formulas in Mr. Bratt's paper are of considerable assistance, even at normal speed, when the air-gap size is slightly enlarged above that originally used. The influence of the external whirl, which travels circumferentially with the rotor, becomes less the larger the gap, because more radial depth is left for the nominal axial flow in which the influence of rotation is small.

It is of interest to compare the test results on the model with those of actual machine. In Fig. 36, are curves taken in a turbo provided with internal centrifugal fans. A special device used for taking these curves is illustrated in Fig. 37. It consisted of two brass tubes, one within the other between which there was absolutely no leakage. The outer tube was connected to a pump, such as is used for inflating automobile tires, and a suitable gage showed the pressure. The inner tube connected with a U-tube manometer. A small toy balloon was fastened by cords and it could be inflated by means of the pump. The device was pushed into a vent duct from the rear, and into the space between a vent finger and a coil side, when the balloon was not inflated. The pressure was then applied by means of the pump, and the balloon swelled enough to stop any flow through that duct, the

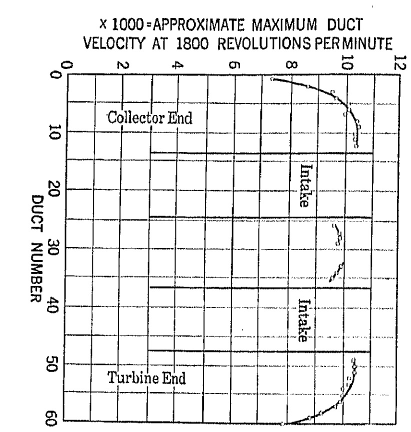

Fig. 36

Air velocities at minimum tooth sections in outlet vents in 25,000 kv-a. 1800 rev. per min., 60 cycles; turbo alternator.

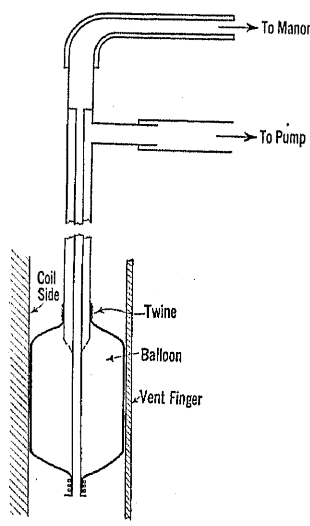

Fig. 37

gage indicating the pressure. (The value of pressure needed had been previously determined when the rotor was not in place). Then the manometer was read. As there was no flow of air, the manometer read the static pressure in the gap at that location, and the square root of that static pressure was taken to be proportional to the velocity in the vent duct when the device was absent. Observations were taken in three positions circumferentially for a given axial position, and averaged. There was usually but little difference between the three readings. The total volume was

determined from the losses and air temperature rise. From a comparison of the mean of the square roots with the mean velocity determined from the volume per vent divided by the vent area, the device was calibrated in terms of velocity.

It is, indeed, interesting to note the agreement in shapes of curves in Fig. 36 with those of the model in which the first discharge was shortened—for example, with Fig. 29. In the case of this alternator it seems that the air gap was sufficiently large to permit of minor influence due to rotation, the curves being similar in shape to those obtainable at standstill.

Other tests were made on this turbo alternator. With the field excited, and after conditions were steady, thermo-couples attached to fiber rods were pushed into vent ducts, for measuring temperatures at various depths radially and positions axially. One device was for air temperatures and the other for iron temperatures.[16] Both devices showed lower temperatures at the first vents next to the end bell, than farther on, even though the volumes through the first vents were lower. There were no stator coils in this machine while the tests were made, but instead wood occupied the same spaces which the coils ordinarily do. These wood fillers did not extend into the end bells as coils usually do; therefore, in the complete machine, the excellent thermal conductivity of the copper, taken in conjunction with the high rate at which heat is "picked up" from the surfaces of the coil ends, reduces the temperature of the machine near the ends still more.

It is believed that the lowered velocities in ducts near the ends deserves further comment. Machines like the one for which approximate volume distribution curves are shown in Fig. 27, have been built, and have been in service for about fifteen years. Those machines are 65 inches long, they have low velocities in the end vents, but they have given no trouble from heating due thereto. Undoubtedly, the longitudinal flow in the copper to the end windings has eliminated danger from that source. The ideal distribution is that which gives uniform temperatures, and is secured by velocities in the first ducts lower than in others. By proper selection of the lengths of the intake and outlet belts, with perhaps one of the several devices added by means of which the first vent volumes are increased, this condition may be approximately attained.

Some of the deductions for model II type of ventilation may be summarized. Most of these have been elaborated upon in the foregoing and require no further comment.

A. The total volumes are affected but little by the angular velocity; the decrease in any case is very much smaller than for model I type of ventilation. For air pressures of the order that are now used in large machines, without air fed through the rotor vents, the decrease is negligible.

16. These devices are described in paper by Symons and Walker. "Heat Paths in Electrical Machinery." I.E.E., 1912.

B. When air is fed through the rotor vents in addition to the air gap and stator, there may actually be a slight increase in total volume above the value at standstill.

C. With large air gaps, the decrease in volume due to rotation is practically negligible.

D. The volume of air passing through the rotor vents is generally affected more by the speed of the rotor than by external pressure. In the model, the external pressure might have been zero, with the rotor running at 3600 rev. per minute, without affecting the volume much. This is important in connection with the cooling of the rotor.

E. As ordinarily built, the volume of air which passes through the rotor has little influence upon the total volume.

F. The axial distribution of volumes in the discharge belts approach cosine curves, with minimum values at the ends of the belts. The corresponding velocities in the air gap approach sine curves, whose zero values correspond to the maximum on the cosine curves. In the intakes the curves are approximately hyperbolic; for radial flow the maximum values at the ends of the belts, and for the gap flow, the zero values correspond to the minimum values on the radial flow curves.

G. The radial velocities are proportional to the derivations of the gap velocities.

H. The discharge vent velocities decrease as the gap velocities increase; the intake vent velocities increase with the gap velocities.

I. It is possible for the flow through one or more discharge vents to be inward (negative) near the beginning of a belt of vents.

J. Other things remaining the same, the shorter the belt of discharge vents, the higher are the minimum velocities, until a critical length is reached; after which, over quite a range of belt lengths, the minimum velocities are nearly independent of the length. With low rotational speeds, or with relatively large gaps, there is then a minimum value in the vents near the entrance to the gap, which then increases to a maximum, then a dying down to a nearly uniform discharge rate until the critical point is reached, where the discharge rate increases along the approximate sine curve.

K. The position of the maximum point on the discharge velocity curve can be shifted by various mechanical means, such as a ring in the air gap, or change of entrance conditions to the belt of vents.

L. The distribution of discharge volumes (axially) is modified by the influence of rotation, the tendency being to make the volumes more uniform. The smaller the gap, the greater is that influence.

M. It is desirable to have somewhat lower volumes in the vents adjacent to the end bells than in other vents, in order that more uniform temperatures be secured longitudinally.

N. The total volume in each belt of intake vents

equals the summation of volumes in the adjacent belt of discharge vents up to the "balance points" in those discharge belts.

O. The air entering the air gap direct from the end bell is arithmetically additive to that volume entering the gap from the first intake belt, and flowing toward the same discharge belt. This is quite different than for model I.

Discussion

MULTIPLE SYSTEM OF COOLING LARGE TURBO-GENERATORS (Bratt) and AN EXPERIMENTAL STUDY OF VENTILATION OF TURBO-ALTERNATORS (Fechheimer)

Philadelphia, Pa., February 8, 1924

S. L. Henderson: The papers of Mr. Fechheimer and Mr. Bratt are of primary interest to the designer; particularly to those designers of large machines which require more than the usual air gap entrance for the admission of cooling air. To others, the papers will indicate the amount of research work which is being carried on to enable the manufacturer to build larger and better machines. To the designer the methods for calculating the air circuits may appear complicated and not easy of application. However, when the importance of the proper ventilation system is considered, the designer may well spend considerable time in laying out this part of his design. When it is realized that the construction cannot be changed once the machine is built and that the design of the machine must stand or fall on the ventilation system as laid out, too much time and care cannot be spent in its layout. In the application of the methods put forth in these papers, as in all designs, a first approximation must be made of the relative widths of the different intake and outlet belts and the pressure drops required in the different paths. When once this is done, the designer can readily see how the proportions need to be changed so that the pressures in the various paths will balance up and also so that the proper velocity distributions may be obtained.

The methods have been used in the design of a number of different machines, and Fig. 36 in Mr. Fechheimer's paper shows how the tests on one of these machines came out. The curves of velocity distribution are somewhat flatter than the calculated values; *i. e.*, the velocities in the radial ducts are more uniform than as calculated. In other words, the calculations are on the safe side.

Edgar Knowlton: A number of years ago the company with which I am associated had occasion to build a machine which was ventilated very much the same as described in Mr. Fechheimer's Type 1. We found this was unsatisfactory, and after several years' operation the machine was changed to a type of ventilation where the air entered both ends of the air gap and was discharged radially through the air ducts. This change was of considerable benefit.

When we first began the use of the system of ventilation whereby the air entered both ends of the air gap, it was very satisfactory for the quantities of air and velocities at which the air entered the gap. Later it was found necessary to chamfer the air gaps, and it was rather remarkable that in some cases the quantity of air was nearly proportional to the increase in the area of the entrance to the air gap.

I would like to ask if Mr. Fechheimer found difficulty in closing the air ducts, and if that were the reason why he used the "toy balloon" method? I consider this a very ingenious device.

I would also like to ask if any negative pressures were found in the air ducts alongside of the armature coils?

We have found some cases where zero pressure existed on salient-pole machines with low peripheral speeds, and in one case recently I had occasion to examine a machine which was somewhat unusual, in that it had − ¾-in. pressure on one side and + 1.5 in. on the other side of the armature coil.

G. E. Luke: This problem of cooling large turbo-generators is getting to be a greater one each year. The aim of the designer is not only to produce a more reliable machine but also to design one with a maximum output per pound of material. The great advances made along this line account in part for the fact that the cost of power to the consumer has not increased and in many cases has decreased in spite of the fact that other costs have materially increased.

In the cooling of an electric machine, we are practically limited to the use of air as a medium for the carrying away of the losses. The reasons for the almost universal use of air are due to its unlimited supply, its mobility or the ease with which it can be forced to flow over the ventilating surfaces, and its high insulating qualities. Other mediums such as water and oil have been used, but only in a few special cases.

The cooling of a machine with air involves three important problems: first, the conduction of heat to the ventilating surfaces; second, the transfer of heat from these surfaces to the air; and third, the proper distribution of air over the cooling surfaces. There are many systems of ventilation used and these two papers cover a detailed investigation of two of these systems experimentally and analytically. They particularly deal with the means for determining the air distribution and for evaluating the relative importance of the various factors determining this air flow. You cannot emphasize too greatly the fact that a properly designed machine necessitates a correct distribution of the air over the ventilating surfaces.

The presence of what are called "hot spots" may be due to an improper air distribution. What is attempted by the designer is to ventilate best those parts of the machine which have the greatest loss. In other words, the air velocities over the surfaces should bear an almost direct proportion to the unit watts loss dissipated from these surfaces.

Thus in Fig. 1 with the radial system of ventilation, the highest air velocities are found in the zone of maximum loss, that is, the tooth zone. The amount of heat loss which can be liberated from a given surface for a given surface temperature rise is almost proportional to the average air velocity such as shown by the solid line curve, Fig. 2.

But the limiting temperatures are the maximum copper temperatures found in the embedded copper. The copper loss must be conducted through the insulation (which is also a good heat insulator) to the iron. Here it is added to the iron loss and both losses are conducted through the iron to the ventilating surfaces. Both steps in this conduction of heat must be accompanied by a temperature difference which may be quite large, especially the gradient necessary to cause the copper losses to flow through the insulation. This temperature gradient from the "hot-spot" to the ventilating surface is always present and is proportional to the loss dissipated and cannot be decreased by an improvement in ventilation. If we plot watts loss in the machine for a given temperature rise of the copper, the curve, Fig. 2, changes from a straight line to a curve as shown in dotted line, due to this internal temperature gradient.

The experimental tests made by the authors are especially difficult. They involve the measurements of very small air pressures and velocities in very limited and inaccessible positions. An analytical treatment of the air flow, such as Mr. Bratt has given us, is valuable in that it permits of the possibility of calculating the effects of the various factors. Many difficult points are found in the air flow problem. Thus, in Fig. 1 the air must change in velocity very greatly as it is forced to flow

through the tooth zone where it is obstructed by the windings, the slot wedges and the ventilating fingers. The authors have found that the pressure required to force a given quantity of air through such a passage depends upon the direction of air flow whether radially outward or inward.

R. B. Williamson: The problem of the ventilation of turbo-generators is a complicated one involving as it does, the handling of large volumes of air in a very restricted space. In order to cool a machine of given size, a certain minimum amount of air must be passed and this air must be distributed so that all parts of the stator will be uniformly cooled, or at least the distribution must be such that the hottest parts will be well within the safe limits permissible for the class of insulation used. Furthermore, this volume of air must be circulated with moderate end-bell pressures such as can be readily developed by centrifugal fans mounted at each end of the rotor.

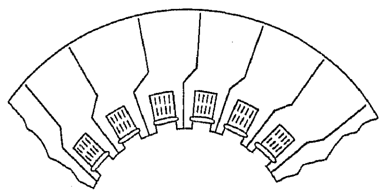

Fig. 1

In general, two methods have been used for distributing the air; these may be termed the axial method and the radial method. In the axial method the air passes axially (parallel to the shaft) from each end bell through the air gap and through longitudinal flues or ducts in the punchings back of the teeth and discharges in the center part of the stator. While this method is satisfactory for relatively short machines, it gives rise to a hot region in the central part of long stators for the reason that the air as it passes longitudinally through the stator becomes heated as it progresses towards the centre, and the air for cooling the central part is hot by the time it reaches these parts. In the radial system, as its name implies, the ducts are radial or annular, the core being sub-divided by a relatively large number of ducts as indicated in Figs. 1 and 2 of the paper. The laminations are thus sub-divided into a number of sections

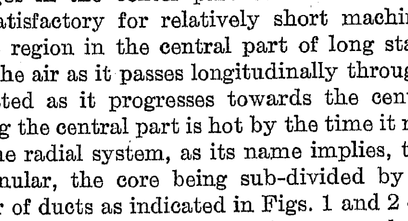

Fig. 2

of small axial width, usually not over 1½ to 2 inches as measured parallel to the shaft. The present paper describes tests of air flow in the different types of radial ventilation referred to as Method I and Method II, both of which provide a number of relatively short paths in parallel for the flow of air after it leaves the end bells. The sub-division of the flow into a number of parallel paths cuts down the distance that the air has to travel from the time it enters the machine until it leaves it, thus permitting the use of moderate air pressure. Again, if the paths are suitably arranged, and the vent spaces in the ducts properly designed, very even cooling can be obtained. As far as my personal experience is concerned, it has had to do almost entirely with Method I which has been used successfully for the past 14 years with machines ranging in size from 1500 kv-a. to 27,000 kv-a. and with stator cores up to 10 ft. in length. The original method provided only two paths at each end of the machine, but about 1910 we began constructing machines with 6 paths at each end somewhat similar to that shown in Fig. 2, except that the vent segments were arranged to by-pass part of the air back of the teeth by making the vent ribs in suitable curved form. In the more recent large machines there are 12 paths at each end of the machine, which with a 72-slot punching gives three teeth opposite each inlet and three opposite the corresponding outlet.

To my mind, the outstanding advantage of the radial system of ventilation lies not so much in the particular arrangement of the parts, but in the fact that with either Method I or II it is possible to introduce cool air directly into the central part of the machine and thus cool this part almost as effectively as the end portions. In fact, tests show that with Method I suitably applied, the air discharged from the centre ducts is no hotter than that discharged from the ends. Further, we have not yet reached any limit to the length of the machine that can be ventilated by this method so far as stator ventilation is concerned. Mechanical features may limit the length and it might also develop that external fans would be needed in some cases, but so far as getting the air through the stator is concerned, I believe, satisfactory ventilation can be secured with the largest stators and by the use of either of the methods described.

After all, the real test of any given method of ventilation is the temperature rise in the centre of the machine as indicated by temperature detectors in the slots and with a sufficient number of such detectors to give indications all around the circumference. Recent tests on large units with ventilation as per Method I, using 12 paths, show that the temperatures are uniform and that there is no difficulty in building large machines with this system of ventilation that will show a temperature rise as indicated by detectors well within 60 deg. cent. The real limitation of large units thus becomes one of rotor rather than stator heating, but the low stator temperature rise possible with the radial system is a distinct advantage, as it reduces the chances of trouble due to expansion and contraction.

An incidental advantage of Method I is that it can be worked out so as to give a very simple and compact construction of the stator yoke. However, this has no bearing on the method itself which our experience indicates gives very uniform cooling with low end-bell pressures.

C. M. Laffoon: I desire to say a few words on the general subject of ventilation for large capacity turbo-generators. At the present time one or two of the larger electrical manufacturing companies are building 12,500-kv-a., 3600-rev. per min. and 43,750-kv-a., 1800-rev. per min. turbo generators for 60-cycle operation.

There is no doubt but that the trend in generator sizes is decidedly upward, on account of the fact that central stations are rapidly increasing in size, and there is a consequent demand for greater generator output per unit of floor space.

In addition to this, the operating companies and purchasers of large capacity units are demanding machines with low temperature guarantees, in the neighborhood of 80 deg. or less, in some cases 60 deg., depending upon the type and quality of the insulation used on the electrical conductors.

It is obvious from these facts that the question of the proper choice of a ventilating system and the predetermination of the temperature rises for any chosen turbo system are of vital importance to both designing and operating engineers. As has been suggested several times this morning, the system of ventilation which should be used is the one in which a permissible amount of cooling air can carry away the generator losses with a minimum and fairly uniform temperature rise throughout the entire machine, provided, that this system of ventilation is not so complicated as to handicap the cost of the machine, and that some efficient, reliable and stable means can be obtained for delivering the air to the different parts of the generator.

Regardless of the type of ventilation that is used, it becomes necessary in large capacity turbo-generators, to resort to the multiple path system. That is, the air must be delivered to

machine at several different axial positions, in order to make a machine economical from the standpoint of cost and to assure a reasonably uniform temperature distribution. Some form of the radial systems that have been discussed here this morning is particularly applicable to machines with low temperature guarantees, because, as has been said before, large volumes of cool air or other cooling medium can be introduced into the machine in the tooth zone where the combined loss per unit volume is inherently high.

As you know, the American practise has always been, in large generators, both water-wheel and high-speed and 25-cycle turbo-generators, to use the radial system of ventilation. The radial system of ventilation is becoming quite generally used by all of the manufacturing companies in this country in order to obtain the lower temperature guarantees. With the radial ventilating system, the air can be presented to the paths at varying pressures, or can be taken into the end bell at constant pressure, if the paths are so designed and proportioned that the air will properly distribute through the machine and give fairly uniform temperatures. However, in either case the problem of actually designing the pressure-generating system for the air or the proportioning of the air paths and the number of passages that are provided, has been very difficult on account of the complexity of the problem and the lack of information available on the subject. The two papers by Messrs. Bratt and Fechheimer mark a definite advance in the published information on the subject of ventilation, and the experimental data obtained from models, as well as from turbo-generators as actually built, should greatly aid and facilitate the solution of this design problem.

It is true that machines using the radial system of ventilation have been built in the past by all of the companies in this country without the aid of this particular information. However, there is no question but that by the use of these data which have been presented here and the mathematical analysis, more comprehensive and more intelligent designs should be made with less trial. The formulas and data as given in these papers are not simplified so that they can be readily used by designing engineers. However, they can be so simplified, so arranged and modified, that they should prove a source of valuable use to engineers who are responsible for the design of large turbo-generators.

F. D. Newbury: Mr. Williamson referred to his successful use of the circumferential system of ventilation. As I was very careful to point out in presenting the paper, the system of Model I is not exactly the same as the system actually used in practise, and furthermore, I wish to emphasize that our tests have no necessary relation whatever to temperature results. We simply attacked one part of the ventilation problem, and if our results have any value, they are valuable in showing relative performances of these two systems, and what may be expected in limiting cases.

Mr. Williamson also referred to the axial system of ventilation. That is a system that was, and has been, and is being very extensively used. It has been used very successfully, but it has the limitation that Mr. Williamson pointed out: that with very long machines, the length of the path through the machine is such that the rise in air temperature may become a serious disadvantage. So, in comparing the results from the first model and the second model, we only wish to draw the conclusion that Model II, representing the multiple radial system, seems to have advantages over the circumferential system of Model I, principally in a more uniform distribution of air, circumferentially, and a substantially uniform distribution of air axially, and, in fact, a very much better distribution of air axially than in the simple radial system, which is in such extensive use.

Mr. Knowlton referred to an experience of his own in connection with a very considerable increase in total volume of air when the entrance at the air gap was increased by cutting back several of the first packages. If you will refer to Figs. 34 and 35, I believe, of Mr. Fechheimer's paper, you will notice that although the air entrance was increased from one inch in Fig. 34, to one and one-half inches in Fig. 35, the total volumes are substantially equal. That illustrates, I think, the real value of these papers—that by confining the work to a particular phase of the problem, confining it to the measurement of pressures and volumes, under various conditions, and not complicating it with temperature results or loss results, we can definitely say that such and such a thing will happen when such changes are made in the ventilating circuit.

Mr. Knowlton also referred to the device illustrated on next to the last page of the paper, which he referred to as the "toy balloon." That, I think, is a very ingenious experimental device—I can say that because I had nothing to do with it—and it was used, as Mr. Knowlton suggested, in order to completely close up the ventilation duct and thereby enable the pressure to be measured without any velocity head being present.

Mr. Henderson referred to the actual use of the results from this work in practise. I think the first impression from these papers is that the numerical calculation work is complicated. Even if that is so, the results obtained from the application of Mr. Bratt's formulas are exceedingly valuable and the work involved is well worth while. Since the results have been available within the last 12 months, the work has been applied to some million kv-a. in turbo-generators, but in that large volume only seven different sizes have been involved; so you can well afford to spend considerable time and work in properly proportioning the ventilation system of a design, when it does involve so much in the way of risk and money and is of such importance in the results to be obtained from the design.

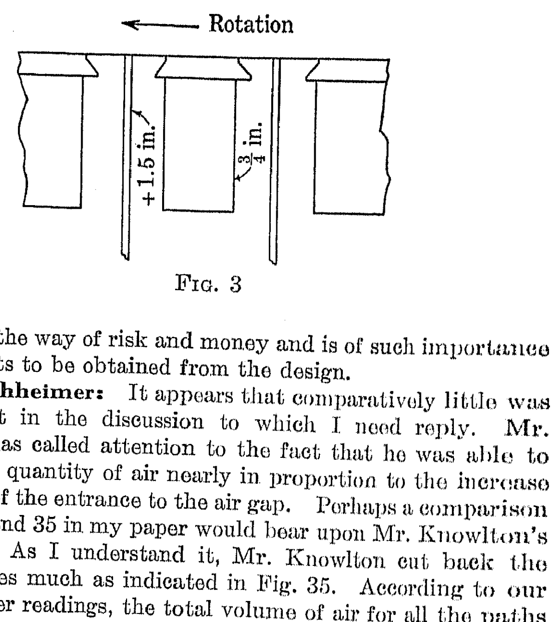

Fig. 3

C. J. Fechheimer: It appears that comparatively little was brought out in the discussion to which I need reply. Mr. Knowlton has called attention to the fact that he was able to increase the quantity of air nearly in proportion to the increase in the area of the entrance to the air gap. Perhaps a comparison of Figs. 34 and 35 in my paper would bear upon Mr. Knowlton's experience. As I understand it, Mr. Knowlton cut back the first packages much as indicated in Fig. 35. According to our volume-meter readings, the total volume of air for all the paths was not changed a great deal, but the distribution of air, particularly for the vent ducts near the end bell, was modified very materially. Undoubtedly, had we been feeding this first section only, the volume-meter would have indicated a considerable increase when the first packages were cut back. The loss at entrance to the air gap expressed as a percentage of velocity head has lowered appreciably by the gradual reduction in section of the air gap. This would be evident to any one who is familiar with a bell-mouth entrance to a duct, which this condition approaches. The fact that the volume-meter did not show more of an increase was in part due to the fact that only a fraction of the total volume of air entered the air gap direct from the end bell, and to some errors in observation. A rather cursory examination of the average values of anemometer readings for Figs. 34 and 35 would not show very great differences.

Mr. Knowlton also mentions that he measured minus $\frac{3}{4}$ in. pressure on one side of the coil and plus 1.5 in. on the other side of the armature coil in a salient pole alternator. I have

obtained from Mr. Knowlton a sketch showing positions in which the readings were taken, and this sketch I am reproducing herewith. There are undoubtedly so many things that enter into measurements of this kind that may mislead one, that it is impossible for me to offer a complete explanation. I can only suggest reasons for the difference. The air leaves the rotor at a slight angle with the periphery, and on the right-hand side of the coil it impinges on the slot wedge so as to destroy nearly all of the velocity head. What air then passes into the vent duct moves past the sharp edge of the wedge beyond which there is a "vena contracta," accompanying which there is usually a considerable reduction in pressure. If the pressure tube had been placed near this "vena contracta," it is easy to understand that the reading would be negative. On the other side of the coil, the volume of air flowing is probably more because all of the velocity head of the air as it leaves the rotor is not destroyed, and it is possible that the pressure tube was placed in such a position that the air impinged upon its opening. Consequently, the reading on the manometer would have been the static pressure plus the velocity head or a fraction thereof.

Of course, on this side, as on the other, there may have been a contraction in section beyond the edge of the wedge, but if the pressure tube were placed, say near the narrow section where the velocity head was greatest, and just before the "vena contracta," it is easy to understand why the reading was + 1.5. I might offer a number of other explanations, but this much is certain—that readings of this character are liable to be very misleading, and considerably in error. That was why we used the "toy balloon" method. The mouth of the brass tube which connected with the manometer was about 3 in. from the air gap, and we felt quite confident that disturbances due to velocity would have negligible influence. Even so, we found discrepancies when the tube was placed on the side of the coil, corresponding to the side where Mr. Knowlton obtained − ¾ in. Our final readings were taken on the other side of the coil, and apparently then we were in a region which was quiet. I believe the picture of what takes place is clarified somewhat by thinking of the moving air streams at very high velocities—as though they were water—and imagining them to splash up into the duct as a result of disturbances in the high velocity stream.

The whole subject of ventilation or air flow is an extremely complex one. It is believed that the work which has been done up to date is only a small part of what may be done, and we hope that others interested in the problem will contribute papers from time to time before technical societies, which will assist the engineering public in solving some of our difficult problems.

Donald Bratt: There is one question that Mr. Knowlton brought up. He spoke of an instance in which he had noticed, in a radial type of flow, that some of the velocities in the extreme end came out negative; that is, the air was actually traveling the other way to what was intended. That is, in fact, the result that we very often find in the flow of air and flow of water in pipes. Sometimes, where a cross-section area changes considerably from a large to a small value, there will be a contraction in section of the jet of water or air, and a very low static pressure. The same thing may actually happen in a radial ventilation system. We have observed in Model II, in many instances, that the flow actually was reversed.

I have not, of course, been able to take care of that sort of condition in my mathematical analysis. It isn't of much importance anyway.

Brush Mounting as a Factor of Satisfactory Operation

BY PHILIP CHAPIN JONES

Member, A. I. E. E.

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Review of the Subject.—*That the brush is an ever present source of trouble on rotating electrical machinery is evidenced by the back numbers of our technical periodicals which are replete with innumerable articles on the subject. These articles cover practically all problems of material and application excepting that of the geometrical design. It is this phase of the subject that the following paper attempts to give.*

Undoubtedly these factors of geometrical design are known as they are readily discovered, their relations are relatively simple, but so far as I have been able to find they have not been published.

The facts brought out by the following paper are: First, that the upper angle of the brush is a function of the lower angle and the coefficient of friction between the brush and the commutator, second, that the lower angle is a function of the pressure desired against the holder, and third, that the trailing brush has little to justify its use. These three conclusions are based on a design which eliminates a resultant moment acting on the brush which would tend to make it bind in the holder.

* * * * *

IF the electrical engineer responsible for the maintenance of rotating electrical machinery is of an inquiring turn of mind he will sooner or later confront the question of brush design. The brush is one of those things which, in itself of minor importance, can so often be the seat of serious trouble. The selection of the proper brush is dependent on two groups of factors, brush quality and brush design. Relating to quality, there is quite a mass of available information, largely in the publications of the various brush manufacturers. Density, hardness, conductivity, and coefficient of friction are tabulated for all the different grades and with the help of suggestions from the manufacturers proper selection is not difficult. In regard to the design of the brush and its position relative to the holder and commutator there is very little information extant that I have been able to find. It is in an effort to relieve this situation that the following brief outline of the essentials of brush design is written.

The function of the brush is to conduct current between a stationary lead and the moving commutator. The resistance of the contact between brush and commutator is proportional to the pressure perpendicular to the plane of contact. The amount of heating at the contact and the amount of current per unit area of contact that the brush can safely carry are proportional to this resistance and therefore the foremost problem of brush design is to keep the correct pressure on the contact surface at all times.

There are, in general, four forces acting on the brush: First, the longitudinal pressure of the brush spring; second, the reaction pressure of the commutator; third, the force of friction on the commutator which tends to move the brush parallel to its plane of contact; and fourth, the resulting side pressure of the brush holder. (Although the weight of the brush itself acts in different directions depending on the relative position of the brush on the commutator, it is neglected because in general it is small in comparison to the other forces acting.) These four forces are shown diagrammatically in Fig. 1. The forces shown are the resultant forces acting on the center of the various brush faces. P is the contact force with which we are primarily concerned, and T is the inwardly acting force of the brush spring—the only force that can be varied at will. The force of the holder against the brush is shown as H. It might, of course, be on either side of the brush. F is the force of friction and is equal to the coefficient of friction (f) of the brush on the commutator, multiplied by the contact force P. $(F = f\,P)$. Since the coefficient (f) is given for various grades of brushes, the total force F can be eliminated as an unknown quantity, leaving only the other three forces, T, H and P, to be dealt with.

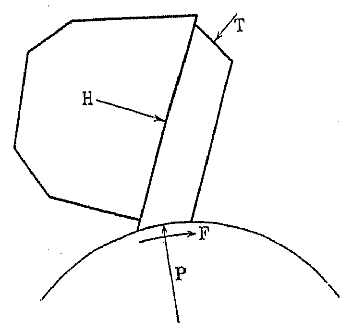

FIG. 1

Fig. 2 again shows the brush of Fig. 1 with the addition of all the critical angles marked as well as the forces. There are only three primary, independent angles — ψ, α and β. Thus, the problem of brush design is to select the values for these three angles which will most effectively keep the contact force P at the desired value under all conditions.

The main obstacle with which we have to contend is friction between the brush and the brush holder which prevents free movement of the brush and thus causes variations in the contact force P. The brush is constantly wearing along the surface of contact and in order that the contact force P may be constant, the brush must be able to move freely outward as well as inward, so that it may follow any irregularities in the

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commutator. The forces preventing a free movement in the inward and outward direction perpendicular to the line H are friction against the brush holder, which is proportional to the side thrust H, and a possible binding of the brush in the holder when the resultant force H does not act at the center of the holder as shown, but above or below this point. This tends to cause the holder to cut into the brush at the bottom or top and thus lock it in one position.

To progress one step further, it becomes evident that

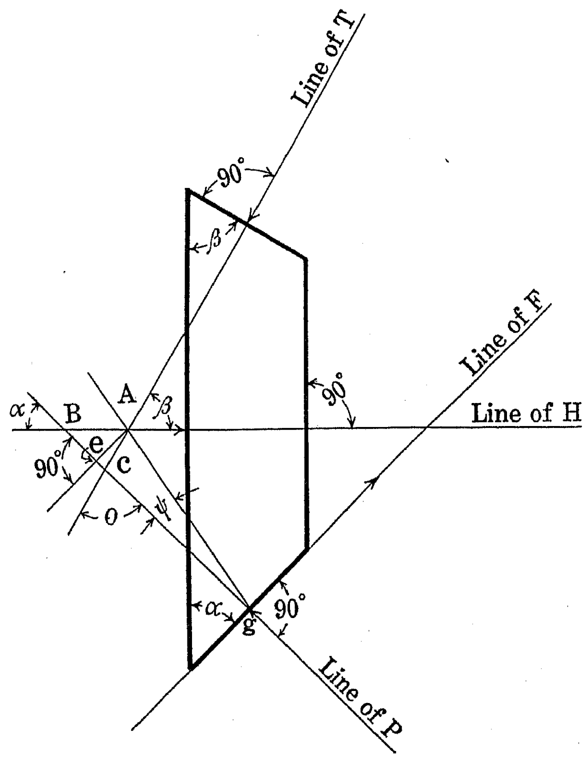

FIG. 2

correct brush design will consist in so selecting the three fundamental angles that the side thrust H not only acts at the center of the holder, to prevent binding, but is as small in absolute value as can safely be obtained with the object of minimizing the friction to radial motion.

An analysis of these forces and angles—given in the appendix—brings out the fact that there is only one independent variable, the lower brush angle α. For any value of α, the upper brush angle β, in order to prevent binding of the brush in the holder, must have a definite corresponding value represented by the expression $\beta = \alpha + \tan^{-1} f$.

Observing this requirement and keeping the contact pressure, P, constant at the desired value, the side thrust H varies as a complex function of α. The pressure of the brush spring, T, need not be changed in order to maintain the contact pressure (P) constant as α varies. (See Fig. 5)

To minimize friction of the brush against the holder it is desirable to hold the side thrust as small as possible. With a trailing brush this side thrust can never be less than ½ the contact pressure so that unless a condition exists that makes this relatively large side thrust necessary there is little excuse for the use of the trailing brush. Our natural tendency is to use the trailing position. We know, if we pushed a stick along the sidewalk ahead of us it would catch, chatter, and altogether behave unmanageably, while if we trailed it, all would be well. In the case of the brush and commutator we are dealing with as nearly perfectly smooth surfaces as can be obtained so that the cases are not at all parallel. As is quite often the case our first impression is in error. Mathematical analysis reveals the true situation and unmistakably endorses the leading brush.

The ideal condition would be to make the side thrust zero. This would require a box-type holder with the brush run in the leading position. The lower brush angle would then have to be about 75 deg. and the upper brush angle 90 deg. For reversing motors—running as much in one direction as in the other—ideal conditions can never be attained and some compromise must be accepted.

Appendix

To arrive at the correct relations between the various forces and angles connected with the brush it is best to treat it as a problem in statics and to take moments of the forces acting around the intersection of T and H. Calling the arm of the force P equal to L_p and that of F, L_f gives the following: $P \times L_p = F \times L_f$ and as $F = f P$ then, $P \times L_p = f P \times L_f$ or $f = L_p/L_f$. It is evident that $L_p/L_f = \tan \psi$ whence it follows that,

$$f = \tan \psi \tag{1}$$

The relation just indicated $f = \tan \psi$ must hold not only when the brush is new but also as it wears down to

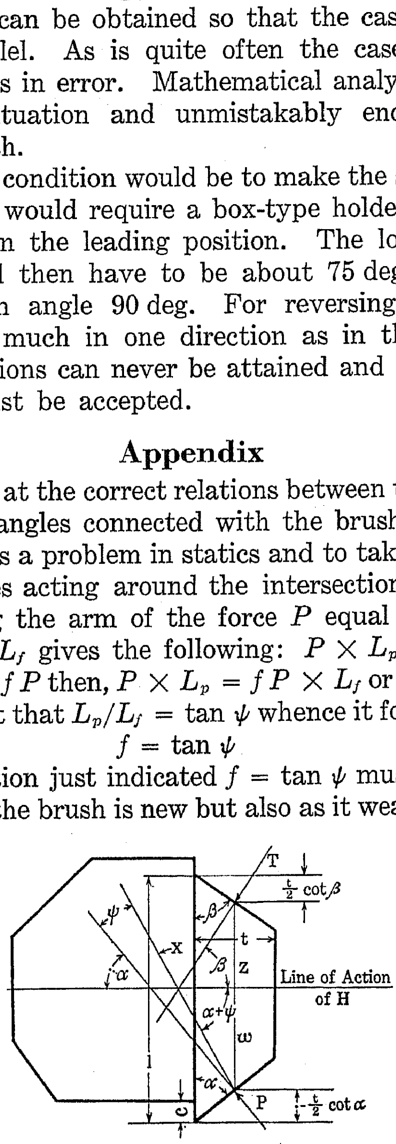

FIG. 3

its shortest length. To bring this about the intersection of the lines of action of T and H must always be on the line X which is one side of the angle ψ, and H must always be half way between the top of the brush and the bottom of the holder. With these requirements in mind it develops that, referring to Fig. 3

$$w \cot (\alpha + \psi) = z \cot \beta$$

where $w = [(1 - c)/2 + c] - (t \cot \alpha)/2 = 1/2\,(1 + c - t \cot \alpha)$

and $z = (1 - c)/2 - (t \cot \beta)/2 = 1/2\,(1 - c - t \cot \beta)$

Substituting these values of w and z in the above equation gives:

$$\cot(\alpha + \psi) = \frac{l - c - t\cot\beta}{l + c - t\cot\alpha}\cot\beta \qquad (2)$$

Now c can be made any value desired as part of the brush holder design. It should of course be quite small. If c is made of such a value that

$$\cot\beta = \cot(\alpha + \psi) \qquad (3)$$

then $1 - c - t\cot\beta = 1 + c - t\cot\alpha$

and $c = (t\cot\alpha - t\cot\beta)/2 = t/2\,(\cot\alpha - \cot\beta)$

$= t/2\,[\cot\alpha - \cot(\alpha + \psi)]$

An idea of the magnitude of c thus obtained may be had by taking $\alpha = 60$ deg. and $\psi = 15$ deg. and $t = 1/2''$ which gives $c = 1/4\,(0.577 - 0.268) = 0.077$ inch.

The great advantage of taking c of this value is that it gives the very simple relation between α, β and ψ of

$$\beta = \alpha + \psi \qquad (4)$$

This enormously simplifies the problem. The value of ψ is known as it is $\tan^{-1} f$. Thus the only angle to be determined is α whence β will follow from equation (4).

The effect of changing α is to change the relative values of H, T and P. The best method of determining the most satisfactory value of α then is to plot the relations between these quantities against α. As P is the pressure that directly affects the problem, it will be best to plot the two ratios of H/P and T/P against values of α.

To obtain these values advantage is taken of the fact that as the brush is stationary the resultant of all forces acting on it is zero. Resolving all forces into tangential and radial components and equating each group to zero gives, for radial forces (parallel to P)

$$P - H\cos\alpha - T\cos\odot = 0$$

where $\quad \odot = 180 - (\alpha + \beta) \qquad (5)$

and for tangential forces (parallel to F)

$$F = H\sin\alpha - T\sin\odot = 0 \qquad (6)$$

From (5), $\quad H = (P - T\cos\odot)/\cos\alpha$ and from (6)

$$H = (T\sin\odot - fP)/\sin\alpha$$

and equating these two values of H gives

$$(P - T\cos\odot)\tan\alpha = T\sin\odot - fP$$

whence may be deduced

$$T/P = (\tan\alpha + f)/(\sin\odot + \cos\odot\tan\alpha) \qquad (7)$$

Returning to equations (5) and (6) and solving for T and equating as was done for H gives:

$$H/P = (\tan\odot - f)/(\sin\alpha + \cos\alpha\tan\odot)$$

These values, it will be evident from a glance at Fig. 2, were obtained by considering a brush running in the leading position, *i. e.* with the direction of the commutator motion such as to tend to push the brush away from the holder. Before plotting these values, therefore, it will be well to examine the brush in the trailing position (commutator motion tending to force the brush against the holder) as shown in Fig. 4.

It will at once be evident that the same relation between f and ψ holds here as it does for leading brushes,

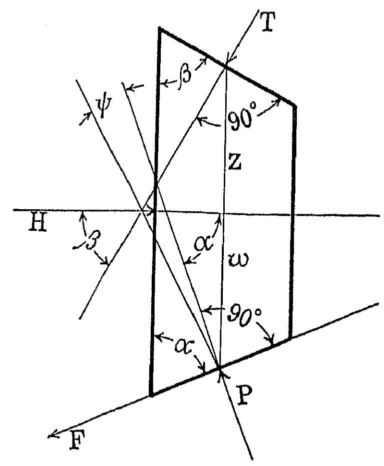

Fig. 4

i. e., $\tan\psi = f$. Here, however, ψ is on the opposite side of P. Thus ψ is always on the side of P toward the direction of F. In order, however, to use the same value for c as before we must have the relation cot

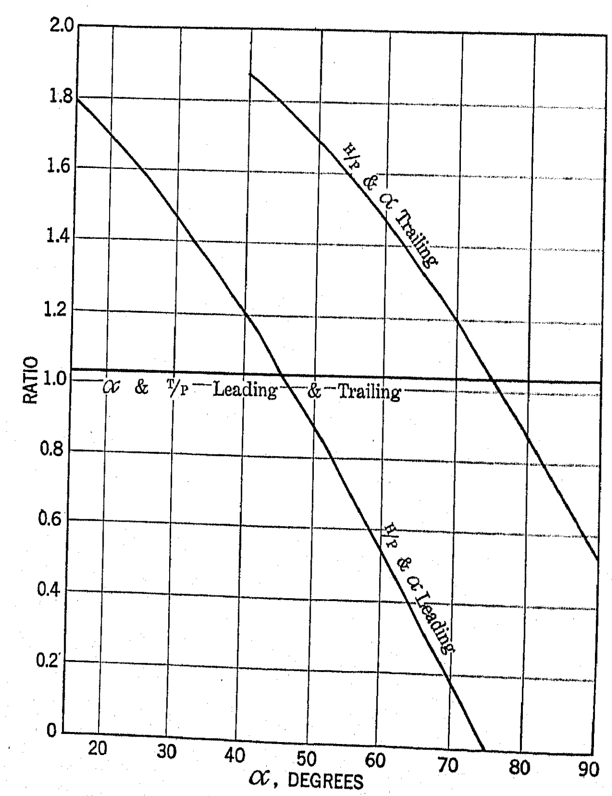

Fig. 5—Brush Data
$f = 0.268$

$(\alpha - \psi)$ equals $\cot\beta$ or $\alpha - \psi = \beta$. This is exactly the same equation as (4) with ψ taken as a negative angle. This relation will be found to hold throughout. All equations for leading brushes will hold for trailing

brushes if $-\psi$ is substituted for $+\psi$. As the tangent of a negative angle is negative f will also have to be considered as negative as $f = \tan \psi$.

This gives the two fundamental equations for trailing brushes as:

$$T/P = (\tan \alpha - f)/(\sin \odot + \cos \odot \tan \alpha) \qquad (9)$$

$$H/P = (\tan \odot + f)/(\sin \alpha + \cos \alpha \tan \odot) \qquad (10)$$

In making a plot some value of f will have to be assumed. The entire range of the coefficient of friction for different grades of brushes runs from perhaps 0.14 to possibly as high as 0.40. An average value would be in the neighborhood of 0.268 which gives 15 deg. as the value of ψ. For this value of f the curves of H/P and T/P are shown in Fig. 5. Inasmuch as no great accuracy of adjustment can be maintained in brush settings the curves shown are sufficiently accurate for all ordinary values of f.

Discussion

R. F. Franklin: I wish to discuss two points concerning Mr. Jones' paper. One is the criterion of zero box pressure; the other, the mathematical analysis of the brush forces.

The condition of zero box pressure which Mr. Jones strives to obtain, will not give satisfactory operation in practise since the slight variations of commutator friction for different points on the commutator will cause the brush to move back and forth in the box and chatter. Practical operation requires a brush angle which will insure the brush always bearing against one side of the box. However, the "box reaction" involved in this condition should not be great enough to prevent free axial movement of the brush in the box. The proper brush angle, therefore, is not the one that will give zero box pressure, but the one that will give a box pressure sufficient to hold the brush in contact with the box for all values of commutator friction and yet not so large as to prevent free axial movement of the brush.

Mr. Jones is correct in his assertion that the brush angle for reversible motors must be a compromise between that best suited for leading and trailing operation. For equal box reaction, the "lower brush angle" for the trailing brush is greater than that for the leading brush, because the tangential friction force in the former case adds to, and in the latter subtracts from, the tangential component of the axial force, $T \sin \beta$ (See Fig. 1). A compromise brush angle for reversible operation would, therefore, give increased box reaction for trailing operation and decreased box reaction for leading operation. If the friction force F becomes greater than the tangential component of $T \sin \beta$, the brush is dragged to the opposite side of the box during leading operation and will acquire what is called a "double fit," *i. e.*, one part of the brush face will be in contact with the commutator for one direction of rotation and another part of the face for opposite rotation. Definite pressure against one side of the box at the top during leading operation can be assured by beveling the top of the brush slightly so that a component of the spring pressure acts against the box.

The other point I wish to discuss is that of the author's analysis of the brush forces. The box reaction cannot be considered as acting at one point near the center of the brush since the contact surface between the brush and the box is a plane. Brushes which have been in service always show wear or polish which indicate that the box reaction is concentrated in a narrow area at the extreme bottom of the box. The method of taking moments, therefore, is incorrect. Instead, the forces acting should be resolved into components acting along and perpendicular to the axis of the brush. Thus the box reaction may be represented by two forces, as shown in the accompanying Fig. 1; force H_1 near the top of the box, due to the spring pressure, and H_2 near the bottom of the box, due to the result of commutator reaction and friction.

The analysis of the forces then becomes very simple. Thus, the expression for the forces acting along the axis of the brush for the case of a leading brush, as shown in Fig. 1, is

$$T \sin \beta = P \sin \alpha + F \cos \alpha \qquad (1)$$

The reaction at the top of the box is

$$H_1 = T \cos \beta \qquad (2)$$

and the reaction at the bottom of the box is,

$$H_2 = P \cos \alpha - F \sin \alpha \qquad (3)$$

Substituting the well-known relation $F = f P$, where f is the coefficient of the friction between brush and commutator, in (1) and (3) the following relations are obtained;

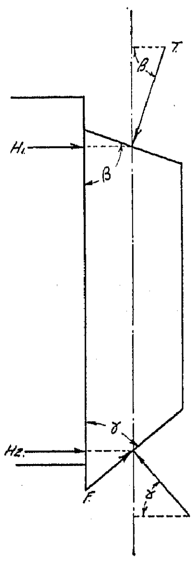

FIG. 1

$$T \sin \beta = P (\sin \alpha + f \cos \alpha) \qquad (4)$$

and

$$H_2 = P (\cos \alpha - f \sin \alpha) \qquad (5)$$

The equations for the trailing brush are obtained by reversing the sign of the friction force. Thus for the trailing brush,

$$T \sin \beta = P (\sin \alpha - f \cos \alpha) \qquad (6)$$

$$H_1 = T \cos \beta \qquad (7)$$

$$H_2 = P (\cos \alpha + f \sin \alpha) \qquad (8)$$

If a square top brush is assumed ($\beta = 90$ deg.) the force equations (2), (4), (5), (6), (7) and (8) can be written in the form,

$$\frac{T}{P} = \sin \alpha \pm f \cos \alpha$$

$$H_1 = 0$$

$$\frac{H_2}{P} = \cos \alpha \mp f \sin \alpha$$

where the upper signs are for the leading brush and the lower signs the trailing brush. These equations are plotted in Fig. 2 for the average value of $f = 0.268$ assumed by Mr. Jones. The curves in this figure should be compared with those of Fig. 5 in the paper.

P. D. Manback: Mr. Jones has presented a very interesting phase of the subject of brush application and one which probably has not been given enough consideration in the past. However, we have made this question the subject of considerable investigation and experiment for some time past. I think the mathematical deductions are very enlightening but may very easily lead one astray when making practical applications of these principles.

It may be true that the weight of the brush need not be taken

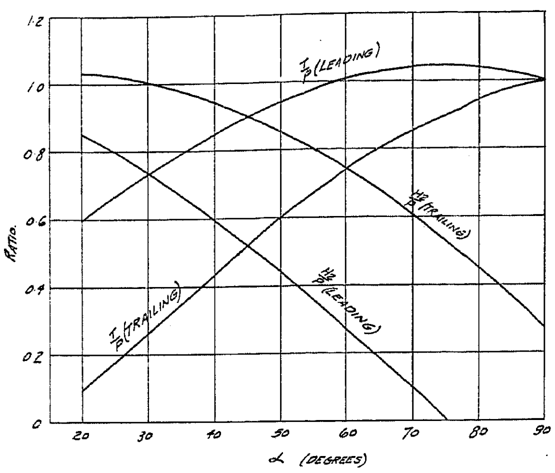

Fig. 2

into account when calculating brush pressure in the case of carbon brushes, but in dealing with metal graphite brushes the weight of the brush is quite considerable and must be taken into account when calculating brush pressure. Also it may be ideal from a theoretical standpoint to operate a brush in a leading position with a commutator bevel of 75 deg. and top angle of 90 deg., but our experience has shown this usually to be bad practise. In the case just cited, the brush is floating in the holder and the side thrust is zero. This means that the force of friction just overcomes the tangential component of the

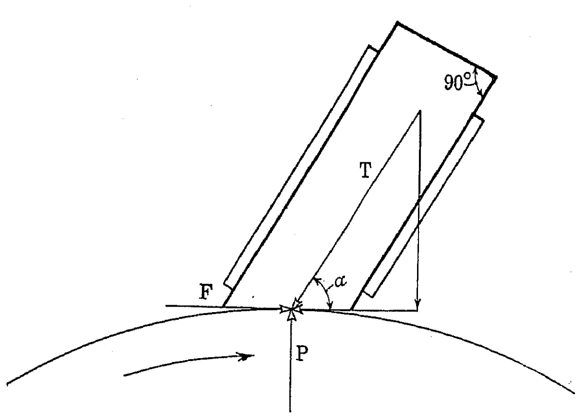

Fig. 3

spring tension. If the frictional force were absolutely constant, the practical case would approach the theoretical very closely, but this is not always true. Some parts of the commutator are not as highly polished as others, so that at one instant the friction may overcome the tangential component of the spring tension, while in the very next instant the friction may be less than the tangential component, thus giving rise to a pronounced brush chatter, harmful to commutator and brushes alike. It has been the object and aim of the brush manufacturer to keep away from this so-called critical angle so as to eliminate the undesirable element of chatter.

It has been found that a very definite relationship exists between the coefficient of friction and this critical angle. This relationship is derived as shown in Fig. 3.

If F, the force of friction, is just equal to the tangential component of the spring tension, then $F = T \cos \alpha$ where T = spring tension. $F = f P = f T \sin \alpha$ then $f T \sin \alpha = T \cos \alpha$ or

$$f = \frac{\cos \alpha}{\sin \alpha} = \cot \alpha$$

Since T, the spring tension, cancels out of the equation, it shows that the relationship holds true regardless of the magnitude of the spring tension, which agrees with practise. If we know the coefficients of friction of various brush grades, this simple relationship enables us to calculate the critical angles which should be avoided for satisfactory operation when the top angle of the brush is 90 deg. It must be taken into account, however, that the coefficient of friction will vary within certain limits, due to irregularities of commutator surfaces.

W. C. Kalb: I might mention that there is one statement in the early part of Mr. Jones' paper which I don't believe conveys quite the meaning he intended, where he says that the resistance of the contact between brush and commutator is proportional to the pressure perpendicular to the plane of contact. He is speaking, as I understand it, of the electrical resistance. Of course, that relationship is inverse.

In his presentation of this subject Mr. Jones has brought out some very interesting factors. It might be said, however, that too much emphasis has been laid on the importance of keeping the resultant force H at an extremely low value and its location central between the top of the brush and the lower edge of the holder. This suggests the approach of this problem from a slightly different point of view and its simplification by the elimination of the angle ψ.

It will be noted that the line of T in Fig. 2 of Mr. Jones' paper meets the line of F at an angle of $\alpha + \beta - 90$. If tangential and radial forces are resolved on the basis of this angle and the angle α, we have

$$P = T \sin (\alpha + \beta - 90) + H \cos \alpha \quad (1)$$

and

$$F = T \cos (\alpha + \beta - 90) - H \sin \alpha \quad (2)$$

or

$$P = H \cos \alpha - T \cos (\alpha + \beta)$$

$$F = T \sin (\alpha + \beta) - H \sin \alpha = P f$$

When

$$H = 0$$

$$f = - \tan (\alpha + \beta)$$

but this would not be a stable brush position, hence $- \tan (\alpha + \beta)$ must be greater f.

This condition must hold at the moment of starting as well as during the running period, otherwise the brushes may be rocked from their position by the high static friction encountered at that instant. The coefficient of static friction may be as high as unity on a brush with normal running friction. So $- \tan (\alpha + \beta)$ should be greater than 1 and $\alpha + \beta$ should therefore be less than 135 deg.

When the machine is running, it is not sufficient that balance of forces maintain the resultant force H slightly above zero. Freedom from chattering is only attained when the component of the force P normal to the side of the holder exceeds the like component of the force F.

That is, we should have

$$P \cos \alpha > P f \sin \alpha$$

or

$$\cot \alpha > f$$

While 0.4 is rather high as a coefficient of friction for a good grade of brush at proper tension, Mr. Jones has shown that it is a figure which may be encountered and therefore the design of the brush holder should be such as to accommodate a brush with this coefficient of friction. $\cot^{-1} 0.404 = 68$ deg., indicating that a larger angle at α is not desirable.

That the limits here suggested are reasonable is supported by the following illustrations from service conditions:

Standards that operate well,

$\alpha = 60$ deg., $\beta = 60$ deg., $\alpha + \beta = 120$ deg.

$\alpha = 60$ deg., $\beta = 50$ deg., $\alpha + \beta = 110$ deg.

$\alpha = 52\frac{1}{2}$ deg., $\beta = 75$ deg., $\alpha + \beta = 127\frac{1}{2}$ deg.

A standard that does not operate well,

$\alpha = 70$ deg., $\beta = 90$ deg., $\alpha + \beta = 160$ deg.

The foregoing discussion is confined entirely to a consideration of leading operation. We are in full accord with Mr. Jones in recommending leading in preference to trailing operation.

Philip Chapin Jones: Mr. Franklin points out that zero box pressure would not be saitsfactory in practise. In my paper I stated merely that "the *ideal* condition would be to make the side thrust zero." I had no intention of recommending it as good practise for the very reasons which Mr. Franklin mentions. As a matter of fact, I find in practise that a value of 75 deg. for β and 60 deg. for α works very satisfactorily.

The second point Mr. Franklin raises I do not believe is quite correct. In undertaking a mathematical investigation of physical phenomena it is necessary to assume certain ideal conditions and then in practise to make such allowances as will provide for a deviation from the ideal conditions. Thus, in this brush analysis I have assumed tacitly that we are dealing with plane surfaces both on the brush and in the holder. Under this assumption any force applied to the brush will react on the holder, not at a point or on a line but over the entire surface of the brush. For the purpose of taking moments, however, it is perfectly correct to consider a resultant force applied at some one point which is the center of gravity of all the differential pressures acting over the entire surface. In case there is no resultant turning moment on the brush, this point of action of the resultant pressure will be in the center of the surface. Inasmuch as I have so taken the various brush angles that there will be no resultant turning moment, the resultant force in the case I am discussing in the paper will be at the center of the surface.

Mr. Franklin's statement that "brushes which have been in service always show wear—in a narrow area at the extreme bottom of the brush" is too broad to be strictly true. This evidence of wearing at the bottom of the brush, however, is merely an indication that there is a turning moment acting on the brush.

Mr. Franklin's division of the box reaction into two forces H_1 and H_2 is arbitrary and entirely unnecessary as long as he is only dealing with components parallel and perpendicular to the axis of the brush. He would have had exactly the same resultant equations had he used only one H no matter at what point he applied it. It is impossible to compare curves drawn from his equations with my Fig. 5 as his equations correspond to only one point on my curve for leading brushes and to none on my curve for trailing brushes. Fig. 5 is plotted, as noted in the text for the condition that $\beta = \alpha + \psi$ where ψ (equal to 15 deg. in this case) is positive for leading brushes and negative for trailing brushes. Mr. Franklin starts by assuming that β is 90 deg. which at once fixes α as 75 deg. for leading brushes under my assumptions.

In my first investigation of this subject I did exactly as Mr. Franklin has but I soon found that I was not covering the subject and that my equations were indeterminate. In my final analysis, therefore, as given in this paper I use moments first to insure a radial movement only of the brush and then use force componants to get the relations between the various forces.

Mr. Manback's remarks call for no further comment from me. In replying to Mr. Franklin's first point, I pointed out that I had no intention of recommending this condition of zero box pressure as the best practical value.

My statement of contact resistance to which Mr. Kalb refers is, I believe, strictly correct but the proportionality, of course, is inverse and not direct.

As regards the second section of Mr. Kalb's comments, I have little to say. Apparently my paper has given the impression that I was recommending a zero box pressure, while, as I have pointed out previously, I was merely indicating it as an ideal limit which could never be reached in practise. The values I do recommend, of $\alpha = 60$ deg. and $\beta = 75$ deg. would, I believe, meet with Mr. Kalb's approval and they also satisfy the other requirements as pointed out in my paper.

Theory of Three-Circuit Transformers

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Review of the Subject.—*The characteristics of three-circuit transformers, the literature of which is very meager, are discussed here in considerable detail. The features of the scheme of treatment are as follows:*

1. The scope and general aspects of the problems of three-circuit transformers are reviewed.

2. Some peculiar phenomena of considerable theoretical interest are cited.

3. An electrical network equivalent to the magnetically interlinked circuits of a three-circuit transformer is developed, useful in visualizing the problem and in predicting by inspection a number of its characteristics.

4. Two physical interpretations of the equivalent network are given to assist the understanding of its principle and its applications.

5. The case of auto transformers interconnecting three circuits is interpreted so that the formulas developed for three-circuit transformers become universally applicable regardless of the presence or absence of metallic interconnection among the three circuits inside the case.

6. Formulas are developed for the calculation of regulation with various loads in the different windings.

7. Formulas are developed for the division of load between two primary circuits, or two secondary circuits in parallel.

8. Formulas are developed for the equivalent effective impedance for short circuits.

9. The behaviour of a three-circuit transformer operating in parallel with a two-circuit transformer is analyzed so as to determine the flow or distribution of load kv-a. in the network.

10. The problem of unsymmetrical loads, particularly that of single-phase line-to-neutral short circuits on a polyphase system is discussed in an appendix, with a simplified method of solution, deriving formulas for some representative cases. When the transformer is interconnecting two polyphase generating systems, the division of single-phase line-to-neutral loads and short circuits between the two systems is considered and solved by the same method and formulas.

11. The theory of three-circuit transformers is extended to four circuits in another appendix illustrated by an example, and is then generalized to n-circuit transformers.

12. For convenient reference, the more important formulas and symbols are collected in another appendix.

The Problem

THREE-circuit transformers present some interesting characteristics, involve a number of new problems and cover a much wider field of application than one might suspect. As examples the following short list of the more important cases may be cited:

Transformers having two primary windings supplying one load from two separate generators or generating systems of different voltages. The systems may be single-phase or polyphase. Problems arise as to, (a) the division of load and short-circuit current between the two primaries, (b) the equivalent effective impedance of the transformer for a short circuit across the secondary lines, and (c) regulation.

Transformers having one primary winding and two separate secondary windings supplying two separate loads of different voltage ratings. One of the secondaries may be a condenser circuit for power factor correction or voltage regulation or both. The system may be single-phase or polyphase. The main problems are those of regulation and the combined load in the primary.

Transformers having one primary winding, one secondary (load) winding and one tertiary winding, the latter not connected to a load but provided for the purpose of magnetically interlacing the different phases of a polyphase bank, as for instance in the case of four-wire Y-Y distribution-transformers and grounded Y-Y banks. The problems to be solved are: Regulation for unbalanced loads (line-to-neutral), and equivalent effective impedance for line grounds, that is, for line-to-neutral short circuits.

Transformers with one primary winding, one secondary winding and one voltmeter winding. The main problem is that of regulation, and the object to be attained in design is to make the voltmeter winding indicate the true voltage of the secondary by turn ratio, or, when this condition is not a fact, to know the necessary correction.

Transformers having two primary windings interconnecting two generating systems and one tertiary winding not connected to any load but provided for the purpose of magnetically interlacing different phases of a polyphase bank. The problems are: Equivalent reactance of the bank for line-to-neutral short circuits, and the division of short-circuit current between the two primaries or generating systems.

Transformers of which the primary or the secondary (or both) consist of two or more coils in multiple. The important problem is the division of load current, on account of serious local overheating which may be caused by unbalanced current division. Ordinarily, parallel coils are balanced for normal operation, but tap connections on any winding may introduce an unbalance, or some other feature, such as many-stranded conductors may involve an unbalance that is not objectionable but requires calculation.

A three-winding transformer in multiple with a two-winding transformer. The important problem is the division of load among the various windings and generators.

Presented at the Midwinter Convention of the A. I. E. E., Philadelphia, Pa., February 4-8, 1924.

Some Peculiar Phenomena

In a two-primary one-secondary transformer, the division of load between the two primaries is not as simple a matter as the usual rule of "inversely as their impedances with respect to the secondary," as one might expect. With certain coil arrangements one primary may carry practically the entire load regardless of the reactance of the other primary with respect to the secondary. And not only that, but simple coil-arrangements are possible in which one of the primaries not only does not carry its share of the secondary load, but itself draws power from the transformer and feeds it to its generator proportional to the secondary load. That generator and prime-mover characteristics affect the load division is of course true and the division of load may be absolutely controlled thereby, as is to be discussed in due order, but the peculiar phenomenon here referred to is one which the characteristics of the transformer windings in certain combinations tend to impose, and holds particularly true if the two generators were on one shaft, or, if the two circuits were connected in multiple to the same busbars.

In certain other cases load division may be indeterminate as far as the reactances of the windings go and be entirely determined by the resistances of the windings. This happens when two primaries are thoroughly interlaced.

In case of a short circuit on the secondary of a transformer which has two independent primary circuits, the total short-circuit current is not necessarily the sum of the short-circuit currents which the two primaries would separately and independently produce.

In a one-primary two-secondary transformer, if one of the secondaries is loaded and the other is idle, the voltage regulation across the idle secondary may or may not be zero and may be positive or negative, *i. e.*, the voltage may be lowered or boosted, depending, as before, on the arrangement of the windings.

A somewhat common illustration of the difference in the regulation of two secondary windings is found in high-voltage testing transformers in which the inaccuracy of obtaining the secondary voltage by ratio from the primary is recognized and a special voltmeter coil is provided. The relative location of this voltmeter coil with respect to the primary and secondary determines the accuracy of the voltage readings taken from it.

A Function of Impedances

Although the characteristics of a three-circuit transformer are functions of the arrangement of the windings, yet, to be able to predetermine those characteristics, it is not necessary to have a drawing of the physical arrangement of the windings, or, in its absence, to dismantle the transformer to expose it to view. The arrangement and relative position of the windings are important only in their effect on the reactances of the windings for the subject under consideration, and thus, if those reactances (or, rather, impedances) are known, the performance characteristics of the transformer can be completely determined.

In dealing with alternating-current problems, it is unnecessary to go beyond the conceptions of resistance, reactance and impedance (in ohms or in percentages) into those of inductance in henries, capacitance in farads, and differential coefficients of currents and voltages. No recourse is taken to them in this paper, and the equations and formulas are developed in terms of the engineering units of resistance, reactance and impedance.

Furthermore, the reactances considered in this paper refer exclusively to those reactances which the transformer offers to the load currents (not those which apply to the magnetizing currents). They are sometimes called load-reactances and sometimes leakage-reactances. In the present paper all reactances and impedances will be understood to be those applying to the load currents, unless otherwise explicitly excepted.

Most of the characteristics of a three-circuit transformer, including the peculiar phenomena mentioned above, may be seen by the aid of an equivalent network which may, therefore, be profitably described at this point.

Equivalent Network of Three-Circuit Transformer

A single phase, three-circuit transformer is shown diagramatically in Fig. 1, as a two-line diagram in Fig. 1 A, as a single-line diagram in Fig. 1 B, with connected apparatus *A*, *B*, *C*, which may be generators, motors, lighting load, or any other kind of electrical apparatus. All that the transformer does between the circuits of *A*, *B* and *C* is to link them with a transformation in voltage and current. This transformation is accomplished at the expense of a magnetizing current taken by the transformer, core and copper losses in the transformer, and an impedance or impedances introduced between the various circuits. In considering such characteristics as regulation, division of load, short-circuit currents, etc., magetizing current may be ignored, and, since the ratio of transformation has no effect on these characteristics, it may be assumed as one-to-one for convenience. If the transformer and load constants are given in ohms, amperes, volts, etc., turn ratios must, of course, be considered in transferring or converting them from one winding to another.[1] However, if all such quantities be expressed as percentages of rated values of corresponding circuits (based on an assumed standard kv-a. load) turn ratios drop out of consideration completely. Since transformer constants are as a rule given in

1. Currents and voltages are reduced from the basis of one circuit to that of another by the inverse of the turn ratios. Resistances and reactances in ohms are reduced directly by the square of the turn ratios.

percentages, and most answers to problems are required in percentage form, and since also it is very desirable not to encumber equations and calculations by factors involving turn ratios, therefore, in all equations throughout this paper turn ratios are entirely ignored and the various constants are understood to be either per cent or converted values. As a rule, there is less chance of error in calculations if percentages are used rather than the converted values of ohms, volts and amperes.

With this understanding, the magnetically interlinked circuits of a three-circuit transformer may be completely represented by the electrically interlinked

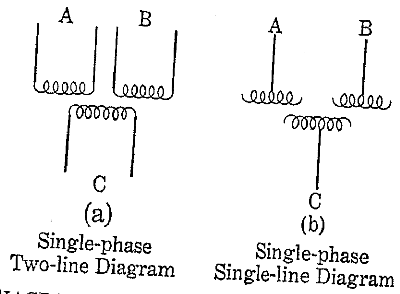
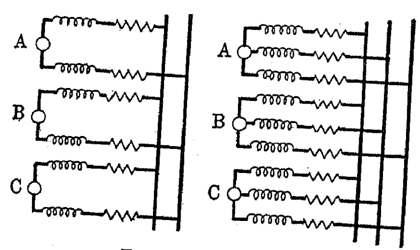
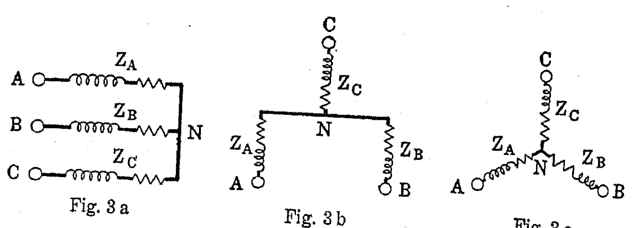


FIG. 1—DIAGRAMMATIC SKETCH OF THREE-CIRCUIT TRANSFORMER



FIGS. 2A AND 2B



FIGS. 3A, 3B AND 3C

circuits of Fig. 2; Fig. 2A representing a single-phase unit (or bank) of three-circuit transformers, and Fig. 2B a three-phase unit (or bank) of three-circuit transformers. Both Figs. 2A and 2B are considerably simplified by the use of a single line diagram as in Fig. 3A. Fig. 3A, therefore, applies to all single-phase and symmetrical polyphase transformers interconnecting three circuits per phase. Fig. 3B is essentially the same as Fig. 3A, but may sometimes be preferred so as to segregate the primary and secondary circuits from each other. Fig. 3C is a perfectly symmetrical diagram and may be preferred by some. It should be carefully noted that Fig. 3C is a connection diagram like the others (not a vector diagram) and that, therefore, its appearance like a three-phase Y must not lead one to think that the circuits A, B and C are 120 deg. away from each other. These three circuits or systems (*i. e.*, A, B and C) are in phase with each other except for what little phase-shift may be produced by impedance drops due to load currents.

It will be noted that the equivalent network amounts to the connection of the three circuits or systems A, B and C to the same busbars through impedances Z_a, Z_b and Z_c, equivalent to the impedance effect of the interconnecting transformer (or auto transformer). The physical significance of this equivalence will be discussed below.

IMPEDANCES OF THE EQUIVALENT NETWORK

The impedances Z_a, Z_b, Z_c of the equivalent network (Fig. 3) are not as a rule equal to each other, and, although they originate in the commonly recognized leakage impedances between pairs of windings of the transformer, they are not numerically equal to them but are determined by them as follows:

It is well-known that power or kilovolt-amperes flowing between a pair of circuits, say A and B, interconnected by a transformer must overcome the leakage impedance Z_{ab} introduced by the transformer. Looking at the equivalent network of such a transformer (Fig. 3C), it will be seen that the impedance to the flow of kilovolt-amperes between A and B is $(Z_a + Z_b)$. Hence, if the equivalent network is to represent the performance of the transformer correctly, it must satisfy the condition that,

$$Z_a + Z_b = Z_{ab} \tag{1}$$

Similarly, it must satisfy the conditions that,

$$Z_a + Z_c = Z_{ac} \tag{2}$$

$$Z_b + Z_c = Z_{bc} \tag{3}$$

Solving these equations for Z_a, Z_b and Z_c, which are the impedances of the equivalent network, in terms of Z_{ab}, Z_{ac} and Z_{bc} (which are standard data of a transformer) we obtain

$$Z_a = \frac{Z_{ab} + Z_{ac} - Z_{bc}}{2} \tag{4}$$

$$Z_b = \frac{Z_{ab} + Z_{ac} - Z_{ac}}{2} \tag{5}$$

$$Z_c = \frac{Z_{ac} + Z_{bc} - Z_{ab}}{2} \tag{6}$$

These equations are naturally vectorial. The resistance and reactance components of the equivalent impedances are, therefore,

$$X_a = \frac{X_{ab} + X_{ac} - X_{bc}}{2} \tag{7}$$

$$X_b = \frac{X_{ac} + X_{bc} - X_{ac}}{2} \tag{8}$$

$$X_c = \frac{X_{ac} + X_{bc} - X_{ab}}{2} \quad (9)$$

$$R_a = \frac{R_{ab} + R_{ac} - R_{bc}}{2} \quad (10)$$

$$R_b = \frac{R_{ab} + R_{bc} - R_{ac}}{2} \quad (11)$$

$$R_c = \frac{R_{ac} + R_{bc} - R_{ab}}{2} \quad (12)$$

In a transformer with three independent windings, $?_a$ is identically the same as the effective $A\ C$ resist-.nce of winding A; R_b the same as that of winding B; nd R_c the same as that of winding C. Equations 10), (11) and (12), however, have not been put in nerely for their symmetrical looks in comparison with he reactances, but in apparatuses, having considerable tray a-c. impedance-losses the effective a-c. resistance f each winding may be difficult to predetermine while he effective resistance of pairs of windings may be asier to predetermine and still easier to test. In ther words, exact values of the equivalent resistances re obtained more accurately from the impedance-vatts measurements per pair of windings with the aid f formulas (10), (11) and (12). Furthermore, in auto ransformers the windings of the various circuits not eing independent of each other, the simplification which applies to straight transformers would not apply ɔ them, while equations (10), (11) and (12) apply niversally. Since in any transformer or auto trans-ɔrmer the copper or impedance loss (watts) for any air of circuits (one acting as primary the other as econdary) is a requisite datum, the convenience and mplicity of these equations may be appreciated.

It may not be amiss to emphasize two considerations t this point:

(a) All impedances used in these equations (whether ı ohms or in percentages) must be those effective at ıe external circuit terminals. In straight transformers ıere is not much chance of error in this matter (especi-ly if the impedances are expressed as percentages) ıt in the case of auto transformers there is a possibility f confusing the impedances between series and common indings of the unit as a transformer and the values ; auto transformer, in which case the auto transformer alues must be used because they are the values fective at the circuit terminals. It is for this reason ıat throughout this paper the word "circuit" is very equently used instead of "winding."

(b) If the various impedances are expressed as ercentages, they must all be based on a common andard kv-a. output which standard output may be ssumed arbitrarily regardless of the capacity of the rcuits. The actual kv-a. loads in the various circuits ay then be given as certain fractions or multiples this assumed standard kv-a. output.

Physical Significance of the Equivalent Impedances

The physical significance of the impedances of the equivalent network may be viewed in two different ways:

Equivalent Impedances considered as the Effective Impedances of the Individual Windings (or Circuits). If the total impedance between the windings of two circuits, say A and B, is Z_{ab}, we may conceive that a portion Z_a of this total impedance Z_{ab} belongs to the winding of circuit A, and the rest of it, which we may call Z_b, belongs to the winding of circuit B, and then of course, as in equation (1),

$$Z_a + Z_b = Z_{ab}$$

We have already seen that in a simple case, such as a transformer with three independent windings, the resistance components of the equivalent impedances are identical with the resistances of the corresponding individual windings, and it is therefore most natural to conceive of the reactance components of the equivalent impedances as the reactances of the corresponding individual windings.

This is an extremely simple point of view and therefore very convenient and useful in practical problems.

In a two-circuit transformer the resolution of its total reactance into primary and secondary reactances is indeterminate[2] and unnecessary: unnecessary for the simple reason that the usual operating characteristics of a two-winding transformer depend on the total impedance and not on its division between primary and secondary. In a three-circuit transformer such a resolution becomes necessary and the problem is rendered determinate by the condition that the resolutions of the three total leakage impedances Z_{ab}, Z_{ac} and Z_{bc} between corresponding circuits be consistent as formulated by the simultaneous equations (1), (2) and (3), and solved by equations (4), (5) and (6).

The equivalent network, then, instead of dealing directly with the total leakage-impedances between pairs of windings or circuits, deals with the leakage-impedances of the individual windings or circuits.

This point of view is quite simple, and very helpful in attacking problems of regulation, division of load, short-circuit currents, and allied problems, and is valid within the scope of all such problems. It is recommended, therefore, to all who are interested in practical applications and do not wish to be encumbered by refinements of more rigorous theory. The scope and limitations of this point of view are critically discussed in Appendix A, after which the second interpretation of the equivalent impedances is given considering them as "mutual impedances" effective for load currents. This second point of view is ab-

2. Unless the complete design of the transformers were available.

solutely rigorous and universal but somewhat more difficult to grasp than the first.

Explanation of the Peculiar Phenomena

A number of the peculiar phenomena mentioned in the foregoing may be explained by an inspection of the equivalent network.

It is possible for one of the reactances in the equivalent network to be negative (See Fig. 4). Equations (7-9) do not contradict this possibility, and experience

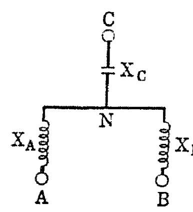


FIG. 4

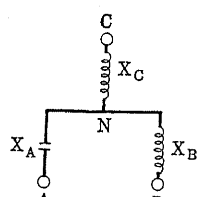


FIG. 5

with concrete transformer designs confirms it. It can be reasoned out, however, that not more than one branch can have a negative reactance, because the net impedance between every pair of windings of the transformer must be positive or inductive and can not be negative or condensive. It also follows from this consideration that the negative reactance must be smaller than either one of the positive reactances in the other branches. As a matter of fact, if a negative reactance occurs, it is very small compared with the other reactances. How a negative or condensive reactance effect can occur in the leakage reactance of the inductive windings of a transformer involves a rather elaborate analysis of leakage flux linkages described in Appendix C. It is this equivalent negative reactance that is responsible for most of the peculiar phenomena mentioned above.

(a) *Explanation of Regulation Peculiarities.* Considering Fig. 4, assume that C is a generator, A is an inductive load, and B an idle secondary. Evidently, the voltage of A will drop under its assumed lagging-power-factor load, because the net impedance from C to A is inductive. However, the voltage of B must rise under the lagging-power-factor load of A because the drop in the condensive reactance X_c by a lagging current will boost the voltage at the point N, and as there is no current and no drop in X_b, the voltage of B will be same as at N and therefore boosted above that of C. To state it in a more general way: With a lagging load in either A or B or both, the voltage of the neutral point N will be higher than the impressed voltage at C. Thus, it may be seen how a given load at a given point may lower the voltage of some part of the network and raise that of another.

(b) *Explanation of Short-Circuit Current Peculiarity.* Let A and B in Fig. 3B be two primary circuits, and C the secondary. If the impedance Z_c were zero, the short-circuit current in C furnished jointly by A and B would be the vector sum of the short-circuit currents which A and B would independently produce. However, if Z_c is the dominant impedance of the network, and Z_a and Z_b comparatively negligible, the short-circuit current in C will be practically the same whether A alone or B alone or both are excited.

(c) *Explanation of Peculiarities in Load Division.* Let A and B in Fig. 5 be two primary circuits, A with a condensive reactance, B with an inductive reactance. If the secondary C is loaded, evidently A and B will divide the load inversely as the impedances of their respective branches. But thè impedances of these two branches have opposite signs, and therefore the currents and kilovolt-amperes in those two circuits will have opposite signs. Thus, if the secondary load is of unity power factor, one of the primary circuits, *viz.*, A, will act as generator, the other, B, as motor, A furnishing power to both C and B. The vector diagram of this is shown in Fig. 6. If the load in C is a zero power factor lagging load, A will furnish a lagging load, B a leading load, the latter being neutralized by the excess lagging load in A.

The foregoing peculiarity in load division is rather of theoretical interest only and does not imply a possible source of difficulty in practise, because division of load between two primary circuits can not in practise be left to the peculiarity of the apparatus but is controlled externally: the division of the kw. component of the load is controlled by the setting of the governors of the prime-movers, the division of the reactive component of the load being controlled by the setting of the voltage regulators of the generators or primary circuits, as will be further discussed below. The peculiarity mentioned above could take place only under two different conditions, *viz.*, (a), if the two primary windings A and B were actually connected in multiple, or (b), if the

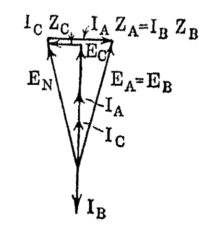


FIG. 6

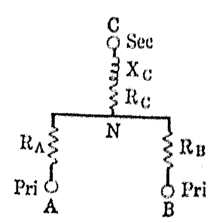


FIG. 7

generators of A and B were mounted on one shaft and their angular position fixed so as to make their voltages in phase in accordance with the vector diagram in Fig. 6 where E_a and E_b, the terminal voltages of A and B, are shown as identical. This condition, of course, does not arise ordinarily in practise except in complicated tie-in networks and can even in those cases be controlled completely by means of regulators.

The second peculiarity in load-division, the case in which load-division is independent of the reactances

of A and B with respect to C, but is determined by their resistances only, is when the reactances of the branches of A and B is zero, the reactance between the two primaries being zero and that from either primary to the secondary being X_c. (Fig. 7.)

With the aid of the foregoing general physical concept of the network of a three-circuit transformer, we are in a position to proceed to the derivation of formulas for the calculation of regulation, load division and short-circuit currents.

Calculation of Regulation

Regulation may be formulated in a number of ways, some absolute, some approximate. We may first review the regulation formulas for a two-circuit transformer and then indicate how they apply to three-circuit problems.

Two-Circuit Transformers. (a) Absolute Formulas.

$$\text{Per Cent Regulation} = \left\{ \frac{E \text{ total}}{E \text{ load}} - 1 \right\} 100 \qquad \textbf{(13a)}$$

$$= \left\{ \frac{Z \text{ total}}{Z \text{ load}} - 1 \right\} 100 \qquad \textbf{(14a)}$$

$$= \left\{ \frac{\text{kv-a. input}}{\text{kv-a. output}} - 1 \right\} 100 \qquad \textbf{(15a)}$$

$$= \left[\sqrt{ \left\{ 1 + m \frac{(\% I R)}{100} + n \frac{(\% I X)}{100} \right\}^2 + \left\{ m \frac{(\% I X)}{100} - n \frac{(\% I R)}{100} \right\}^2 } - 1 \right] \times 100 \qquad \textbf{(13b)}$$

$$= \left\{ \sqrt{ \frac{(R_{load} + R_{trans})^2 + (X_{load} + X_{trans})^2}{R_{load}^2 + X_{load}^2} } - 1 \right\} 100 \qquad \textbf{(14b)}$$

$$= \left\{ \sqrt{ \left(m + \frac{\% I R}{100}\right)^2 + \left(n + \frac{\% I X}{100}\right)^2 } - 1 \right\} 100 \qquad \textbf{(14c)}$$

$$= \left\{ \sqrt{ \frac{(\text{kw.}_{output} + \text{kw.}_{imp.})^2 + (\text{kv-a}_{react.\,output} + \text{kv-a}_{react.\,imp})^2}{\text{kw.}^2_{output} + \text{kv-a.}^2_{react.\,output}} } - 1 \right\} 100 \qquad \textbf{(15b)}$$

Notes on Absolute Formulas. In equation **(13a)** the total voltage is the no-load secondary voltage which also, of course, corresponds to the no-load primary voltage reduced to the same basis of turns. The total voltage might also be called the input voltage, and the load voltage might be called the output voltage.

Formulas **(14a)** and **(15a)**, if not self-evident, can be derived from **(13a)**; **(14a)** is derived by *dividing* the numerator and the denominator of the fraction in **(13a)** by the load-current: **(15a)** is derived by *multiplying* the numerator and denominator of the fraction in **(13a)** by the load-current. In **(15a)** the kv-a. input is exclusive of the magnetizing current and the core loss. These are ignored because they do not change from no-load to full-load and do not produce a change in voltage with varying load. Even ignoring the excitation kilovolt-amperes of the transformer, the input kilovolt-amperes is different from the output kilovolt-amperes on account of the kilovolt-amperes consumed in the impedance of the transformer.

Formulas **(13a)**, **(14a)** and **(15a)** are basic, and although they are not in terms of commonly given data for direct substitution, they can be expressed in terms of such data as shown in formulas **(13b)** deduced from **(13a)**; **(14b)** and **(14c)** deduced from **(14a)**; and **(15b)** deduced from **(15a)**. In **(14b)** the resistances and reactances are in ohms. In **(15b)** the subscript "imp." designates the kw. or reactive kv-a. consumed in the impedance of the transformer.

Of all these formulas, the simplest, most direct and as a rule, the most convenient one is **(14c)** m is the power-factor of the load and n its reactive factor. All percentages and other variable data must, of course, correspond to the actual load, otherwise a conversion or correction factor must be included.

The absolute formulas give the regulation as the difference of two numbers which are very nearly equal, and, therefore, to get this difference, *i. e.*, the final answer, correct to two or three significant figures, the value of the square root term must be computed correct to four or five places. Hence, the absolute formulas as such do not yield much accuracy with a ten-inch slide rule, but they are useful for reference, and they also form the basis of all approximate formulas.

Regulation may be positive or negative. Positive regulation corresponds to drop in secondary voltage at full load, or, rise in secondary voltage at no-load as the A. I. E. E. definition puts it. Negative regulation is then rise in voltage at full-load, or drop in voltage at no-load. The A. I. E. E. wording is preferable, one of the many reasons pertinent to the following discussion being that the rise or drop of voltage from full-load to no-load is also the rise or drop of voltage from (loaded) secondary to primary, from output voltage to input voltage. Corresponding to negative regulation, primary or input voltage (per cent or converted value) will be less than secondary or output voltage; the primary or input kilovolt-amperes (exclusive of excitation kilovolt-amperes) will be less than the load or output kilovolt-amperes and the total impedance (*i. e.*, impedance of load plus vectorially the impedance of the transformer) will be less than the load impedance alone.

Approximate A. I. E. E. Formula. An approxima-

tion based on formula (**13b**) and standardized by the A. I. E. E. is as follows:[3]

$$\text{Per cent Reg.} = m\,(\text{per cent } I\,R) + n\,(\text{per cent } I\,X) + \frac{[m\,(\text{per cent } I\,X) - n\,(\text{per cent } I\,R)]^2}{200} \quad \textbf{(16)}$$

Regulation of Three-Circuit Transformers

Since the load of a three-circuit transformer is not likely to be the same in any two of its circuits, the two-circuit regulation formula cannot be applied directly but must be applied in two steps as is to be described below.

Just as in a three-circuit transformer with two secondary circuits the regulation in the two secondaries need not be alike, so, in a transformer with two primary circuits, if the share of each primary from the total load is to be externally controlled, then, the two primary circuits will have to have different voltages at full load than on no load. Therefore, in a three-circuit transformer there are three regulations to be considered corresponding to the three pairs of windings or circuits.

Referring to Fig. 3B, the regulation between A and C may be calculated in two steps: From A to N (the neutral), and from N to C. Then, designating the first Reg_{an} and the latter by Reg_{nc}, the regulation, Reg_{ac}, between A and C is,

$$\text{Per Cent Reg}_{ac} = \text{Per Cent Reg}_{an} + \text{Per Cent Reg}_{nc} \quad \textbf{(16a)}$$

$$= \text{Per Cent Reg}_{an} - \text{Per Cent Reg}_{cn} \quad \textbf{(16b)}$$

The correctness of this procedure will be recognized considering the fact that regulation is not a vector concept but an algebraic one. It can only be positive or negative; one representing drop in voltage, the other rise in voltage. Regulation between two points (or circuits) can, therefore, be calculated as the algebraic sum of two steps; one from the first point to an intermediate point, and the second, from that intermediate point to the final point, these points and the direction being indicated by the subscripts. Accordingly, if per cent Reg._{an} is positive, per cent Reg._{na} must be considered negative, since, if the voltage rises from A to N, it must drop from N to A. That is the basis of the difference between (**16b**) and (**16a**) which differ in the order of the subscripts of the second term, the two forms being given to emphasize the algebraic character of the formula. However, it is believed that no error is likely to be made in practise due to confusion of signs if the physical facts are clearly kept in sight. As a convenient rule, we can say that if two circuits are dissimilar in duty, that is, one is primary, the other secondary, the regulation between them is the algebraic sum of the regulation from the secondary to the neutral and the regulation from the neutral to the primary (eq. **16a**); but, if both are primary or both secondary, then, the regulation between them is the algebraic difference of their regulations with respect to the neutral point (eq. **16b**).[4]

Example. A three-phase bank of three 5000-kv-a. transformers interconnects a 66,000-volt generating system, a 114,000-volt transmission system and a 13,860-volt synchronous condenser load. The 66,000 volts are stepped up to 114,000 volts through a single winding, that is, through an auto transformer, but that does not in any way influence the calculation of regulation once the impedances effective at the external circuit terminals are known.

Data. The following data are by test, and all the percentages are based on an output of 5000 kv-a. per phase.

Per Cent $I\,R_{PS} = 0.55$; Per Cent $I\,R_{PT} = 0.60$;
Per Cent $I\,R_{ST} = 0.50$

Per Cent $I\,X_{PS} = 8.65$; Per Cent $I\,X_{PT} = 10.6$;
Per Cent $I\,X_{ST} = 8.3$

From these data the impedances of the individual circuits are calculated to be,

Per cent $I\,R_P$

$$= \frac{\text{per cent } I\,R_{PS} + \text{per cent } I\,R_{PT} - \text{per cent } I\,R_{ST}}{2} = 0.325$$

Per cent $I\,R_S$

$$= \frac{\text{per cent } I\,R_{PS} + \text{per cent } I\,R_{TS} - \text{per cent } I\,R_{PT}}{2} = 0.225$$

Per cent $I\,R_T$

$$= \frac{\text{per cent } I\,R_{PT} + \text{per cent } I\,R_{ST} - \text{per cent } I\,R_{PS}}{2} = 0.275$$

Per cent $I\,X_P$

$$= \frac{\text{per cent } I\,X_{PS} + \text{per cent } I\,X_{PT} - \text{per cent } I\,X_{ST}}{2} = 5.475$$

Per cent $I\,X_S$

$$= \frac{\text{per cent } I\,X_{PS} + \text{per cent } I\,X_{TS} - \text{per cent } I\,X_{PT}}{2} = 3.175$$

Per cent $I\,X_T$

$$= \frac{\text{per cent } I\,X_{PT} + \text{per cent } I\,X_{ST} - \text{per cent } I\,X_{PS}}{2} = 5.125$$

The equivalent network of this transformer together with the numerical values of its constants are shown in Fig. 8, which should be kept in view in the following calculations.

3. A. I. E. E. standardization Rule No. 6391-b. We have written out above as per cent $I\,R$ the meaning of "q_r," and as per cent $I\,X$ the meaning of "q_x" used in the A. I. E. E. formula.

4. Regulation of a three-circuit transformer may be calculated correctly by at least three different methods. The scheme of referring all regulations to the meutral point was suggested by Mr. K. K. Palueff. Another method is given in Appendix F.

To calculate the various regulations for the following conditions of load:

Circuit S delivering 5000 kv-a. at 80 per cent lagging power factor

Circuit T delivering 2900 kv-a. at 0 per cent leading power factor.

The regulation from the secondary to the point N by the A. I. E. E. formula (equation (**16**) is

$$\text{Per cent Reg}_{SN} = 0.80 \times 0.225 + 0.60 \times 3.175 + \frac{(0.80 \times 3.175 - 0.60 \times .225)^2}{200} = +\,2.11$$

In calculating the regulation from the terminals of the tertiary circuit T to the point N, we must note that the actual load (2900 kv-a.) is less than the standard load (5000 kv-a.) on which the percentage values of the impedances of its branch are based. The latter should, therefore, be reduced by the ratio (2900/5000 = 0.58) in the calculation of regulation which will be

$$\text{Per Cent Reg}_{TN} = 0 \times 0.58 \times 0.275 - 1 \times 0.58 \times 5.125 + \frac{(0 + 1 \times 0.58 \times 0.275)^2}{200} = -\,2.97$$

In calculating the regulation from the point N to

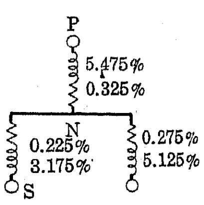


FIG. 8

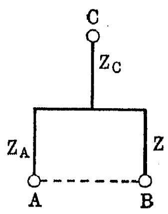


FIG. 9

the terminals of primary P, we must first calculate the actual load in that branch.

Evidently, the output of the branch P is the vector sum of the following four items, *viz.*, (a) kv-a. in the external load of circuit C, (b), kv-a. in the external load of circuit T, (c) kv-a. consumed in the impedance of branch S, (d), kv-a. consumed in the impedance of branch T. Making this calculation in tabular form, we have,

Output of Circuit P per Phase

	Kw.	Kv-a. Reactive
Output of S........	0.80×5000	+0.60×5000
Output of T........	0	− 2900
Impedance kv-a. of S	0.00225×5000	+0.03175×5000
Impedance kv-a. of T	0.00275×.58×2900	+0.05125×0.58×2900
Total...........	4016 kw.	+345 kv-a. lagging

4030.79 kv-a. at 99.63 per cent power factor
8.56 per cent reactive factor

The percentage values of the impedances of branch P are based on an output of 5000 kv-a. per phase, but since the actual output is 4030.79 kv-a. the actual percentages will be (4030.79/5000 = 0.807) times the values given in Fig. 8.

The input of circuit P (exclusive of excitation kv-a.) will be the vector sum of its output and the kv-a. consumed in the impedance of branch P. Thus,

Input of Circuit P

	Kw.	Kv-a. Reactive
Output....	+4016	+345
Impedance.	+0.00325×0.807×4030.8	0.05475×0.807×4030.8
Total......	4026.55	+523 lagging

4060.37 kv-a. at
99.17 per cent power factor
12.88 reactive factor

Since we know the input and output of branch P, we could calculate the regulation from N to P by formula (**15a**) as an illustration:

$$\text{Per cent Reg}_{NP} = \left\{ \frac{4060.37}{4030.79} - 1 \right\} \times 100 = \left\{ \frac{4060.37 - 4030.79}{4030.79} \right\} \times 100 = 0.734$$

The regulations between pairs of circuits then follow as

$$\text{Per cent Reg}_{SP} = \text{Per cent Reg}_{SN} + \text{Per cent Reg}_{NP} = 2.11 + 0.734 = +\,2.84$$

$$\text{Per cent Reg}_{TP} = \text{Per cent Reg}_{TN} + \text{Per cent Reg}_{NP} = -\,2.97 + 0.734 = -\,2.24$$

$$\text{Per cent Reg}_{ST} = \text{Per cent Reg}_{SP} - \text{Per cent Reg}_{TP} = +\,2.84 - (-\,2.24) = +\,5.08$$

or

$$= \text{Per cent Reg}_{SN} - \text{Per cent Reg}_{TN} = +\,2.11 - (-2.97) = +\,5.08$$

DIVISION OF KV-A. LOAD

In a double-secondary transformer of which both secondary windings have the same voltage rating and are actually connected in parallel, also in a double-primary transformer in which the division of the kv-a. load between the two primary circuits is not externally controlled, the division of load will obviously be inversely as the impedances of the respective circuits. Thus, if A and B are those two circuits, (Fig. 3B).

$$\frac{(\text{kv-a.})_A}{(\text{kv-a.})_B} = Z_B / Z_A \qquad (\mathbf{18})$$

Since the load in the third circuit C must be the vector sum of those in A and B, therefore,

$$\frac{(\text{kv-a.})_A}{(\text{kv-a.})_C} = \frac{(\text{kv-a.})_A}{(\text{kv-a.})_A + (\text{kv-a.})_B} = \frac{Z_B}{Z_A + Z_B} = Z_B / Z_{AB} \qquad (\mathbf{19})$$

$$\frac{(\text{kv-a.})_B}{(\text{kv-a.})_C} = \frac{(\text{kv-a.})_B}{(\text{kv-a.})_A + (\text{kv-a.})_B}$$

$$= \frac{Z_A}{Z_A + Z_B} = Z_B/Z_{AB} \quad (20)$$

The two simplified equations, *viz.*:

$$\frac{(\text{kv-a.})_A}{(\text{kv-a.})\ \text{total}} = Z_B/Z_{AB} \quad (21)$$

$$\frac{(\text{kv-a.})_B}{(\text{kv-a.})\ \text{total}} = Z_A/Z_{AB} \quad (22)$$

hold numerically as well as vectorially and are, therefore, very convenient.[5]

It may appear as though, since in a double-primary transformer load division must be externally controlled, and in a double-secondary transformer the two circuits are usually connected to independent external loads, that these load division formulas have no practical application. We may, therefore, indicate here some practical applications for them.

(a) In a double primary transformer in case of a short circuit on the secondary side, the division of short-circuit kv-a. between the two primary circuits follows the foregoing formulas at least initially, because control apparatus can not come into play instantly, and, even after they have come into play, they can not function half as effectively under short circuit as at normal loads.

(b) In some complicated interconnections of systems and apparatus, external control, though possible, is not resorted to on account of increased complication, in which case division of load takes place along the lines indicated above. An illustration of this is afforded in the case of a three-circuit transformer operating in parallel with a two-circuit transformer to be discussed at a later point.

(c) Transformers with only two external circuits will sometimes have two or more circuits in parallel internally. In such cases the parallel circuits are as a rule designed with as perfect symmetry as possible but exceptional cases of dissymmetry may arise in which these formulas will be needed to determine the load taken by each circuit. Eddy current or circulating current problems in some circuits subject to large losses due to such causes incapable of a simple exact theoretical calculation may be approximated by the foregoing formulas.

5. The term (kv-a.) total here means the total output of the two circuits or the input of the third circuit. The input of each primary circuit is its share of this total kv-a. plus (vectorially) the kv-a. consumed in its branch. This qualification is mentioned here to make the principle clear but in many practical cases such refinements of calculation may be unnecessary and the kv-a. consumed inside the transformer may be ignored except in high impedance transformers.

Short Circuit Impedance

In a one-primary two-secondary transformer, the impedance of the unit for a short circuit across either secondary (one at a time) is, of course, the same as that of a two-circuit transformer, since one of the secondaries is assumed not to take part in the short circuit. However, in a two-primary transformer, both circuits being excited, also in a two-secondary transformer having a simultaneous short-circuit across both circuits, the effective impedance of the unit in limiting the short circuit is calculated as follows: Designating the two circuits similar in function as A and B (See Fig. 9), and considering them as though they were connected in parallel as shown by the dotted line in the figure, the effective impedance from combined (A B) to C is evidently

$$Z_{short} = \frac{1}{1/Z_A + 1/Z_B} + Z_C \quad (23a)$$

$$= \frac{Z_{AC}\, Z_{BC} - Z_C^2}{Z_{AB}} \quad (23b)$$

Since in short-circuit problems the resistance of the circuits may in practise be ignored, we may write for the short-circuit reactance of the unit,

$$X_{short} = \frac{X_{AC}\, X_{BC} - X_C^2}{X_{AB}} \quad (24)$$

The share of each one of the two similar circuits of the short-circuit current (or rather kv-a.) follows in accordance with the division of load formulas (equations **21** and **22**).

Example. In the example given for regulation, assume that the secondary and tertiary circuits were simultaneously short-circuited. What would be the short-circuit loads in the primary, secondary and tertiary? Calling the secondary A, the tertiary B, and the primary C, we have the constants,

$X_{AC} = 8.65$ per cent; $X_{BC} = 10.6$ per cent; $X_{AB} = 8.3$ per cent

$X_A = 3.175$ per cent; $X_B = 5.125$ per cent; $X_C = 5.475$ per cent

Substituting these in equation (**24**),

$$X_{short} = \frac{8.65 \times 10.6 - 5.475^2}{8.3} = 7.43 \text{ per cent}$$

(approx.)

The short-circuit current and kv-a. will, therefore, be (100/7.43) that is, 13.5 times normal.

The share of the secondary A and tertiary B of this short circuit will be, by formulas (**21**) and (**22**),

A's share $= Z_P/Z_{AB} \times 13.5 \times$ normal
$= 8.35 \times$ normal (approx.)

B's share $= Z_A/Z_{AB} \times 13.5 \times$ normal
$= 5.15 \times$ normal (approx.)

Since the normal, that is, the kv-a. base, is 5000

kv-a., the short-circuit kv-a. in the different circuits will be,

C (primary), 13.5 × 5000 = 67,500 kv-a. per phase
A (secondary), 8.35 × 5000 = 41,700 " " "
B (tertiary), 5.15 × 5000 = 25,800 " " "

The short-circuit currents are calculated similar to the short-circuit kv-a., with this difference, however, that the normal or basic current is not the same for all three circuits on account of their different voltage ratings. The short-circuit currents may also be derived from the short-circuit kv-a.

Three-Circuit Transformer in Parallel with a Two-Circuit Transformer

Fig. 10 shows a three-circuit transformer with primary C, and secondary circuits A and B, in parallel with a two-circuit transformer with primary M and secondary N, the parallel connection being between primaries C and M, and between secondaries B and N.

Considering the load on the bus $B\,N$, it is evident that this will be furnished over two paths in parallel (shown by solid arrows), one path with the impedance $(Z_c + Z_b)$, the other with the impedance Z_{mn}; the load dividing between these two paths inversely as their impedances.

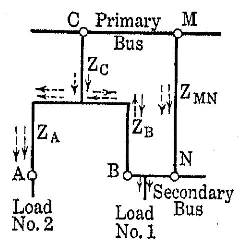


Fig. 10

Considering the load in A, it is evident that this also will be furnished over two paths in parallel (shown by dotted arrows), one path with the impedance Z_c, the other with the impedance $(Z_{mn} + Z_b)$; the load dividing between these two paths inversely as their impedances.

In branches C, M and N, the two component loads add vectorially as shown by the relative directions of the solid and dotted arrows. In branch B, link Z_b, they subtract vectorially. As a consequence of this, we have the seemingly absurd condition that, assuming the load 1 and 2 to have the same kind of power factor (*i. e.*, both lagging or both leading), the removal of one of the loads increases the load in B. The reason for this is, of course, the fact that B acts like a secondary for load 1, but as a primary for load 2.

An interesting feature to observe in this connection is that for a load in A the circuits of C and B acting like two primaries in parallel, the division of load between them is completely determined by the impedances of the network as indicated above, independent of generator and prime mover control apparatus. It desired, it would be possible of course to control this by suitable regulators between B and N, or, between C and bus $C\,M$, or between M and bus $C\,M$.

The foregoing considerations show that the planning design and operation of three-circuit transformers in parallel with two-circuit transformers is a much more involved problem than the parallel operation of two-circuit transformers.

Example. Let the three-circuit transformer be the same as that used in the previous illustrations. (See also Fig. 8.). Let the reactance Z_{mn} of the two-circuit transformer be 6 per cent. Ignoring the resistances to a first approximation, we have for the constants of the network shown in Fig. 10,

$$X_a = 3.175 \text{ per cent}$$
$$X_b = 5.125 \text{ per cent}$$
$$X_c = 5.475 \text{ per cent}$$
$$X_{mn} = 6 \text{ per cent}$$

Let the load on bus $B - N$ be designated $(\text{kv-a.})_1$ and that on A, $(\text{kv-a.})_2$.

$$B\text{'s share of } (\text{kv-a.})_1 = + \frac{Z_{mn}}{Z_{bc} + Z_{mn}} (\text{kv-a.})_1$$
$$= +0.36\,(\text{kv-a.})_1$$

$$\text{" " " } (\text{kv-a.})_2 = - \frac{Z_c}{Z_c + Z_b + Z_{mn}}$$
$$= -0.33\,(\text{kv-a.})_2$$

$$\text{" total load} = 0.36\,(\text{kv-a.})_1 - 0.33\,(\text{kv-a.})_2 \text{ (vectorially)}$$

$$C\text{'s share of } (\text{kv-a.})_1 = B\text{'s share of } (\text{kv-a.})_1 = 0.36\,(\text{kv-a.})_1$$

$$\text{" " " } (\text{kv-a.})_2 = \frac{Z_b + Z_{mn}}{Z_c + Z_b + Z_{mn}} (\text{kv-a.})_2$$
$$= 0.67\,(\text{kv-a.})_2$$

$$\text{" total net load} = 0.36\,(\text{kv-a.})_1 + 0.67\,(\text{kv-a.})_2 \text{ (vectorially)}$$

$$M\,N\text{'s share of } (\text{kv-a.})_1 = (\text{kv-a.})_1 - B\text{'s share} = 0.64\,(\text{kv-a.})_1$$

$$\text{" " " } (\text{kv-a.})_2 = \text{same as } B\text{'s share but positive} = 0.33\,(\text{kv-a.})_2$$

$$\text{" total net load} = 0.64\,(\text{kv-a.})_1 + 0.33\,(\text{kv-a.})_2 \text{ (vectorially)}$$

$$A\text{'s total net load} = (\text{kv-a.})_2$$

When very precise results are desired, especially if the reactances are low and the resistances are appreciable, the actual impedances should be used in such calculations (instead of merely reactances), and their manipulation will be vectorial.

When the three-circuit transformer operating in parallel with a two-circuit transformer has two primary circuits instead of two secondary circuits, the general

scheme of analysis is very similar to the foregoing and will be evident in the light of it. Loads 1 and 2 will correspond to generators 1 and 2, respectively, and in most practical cases the load taken up by the two generators will be governed by external control, in which case the problem is much simpler than the case discussed in the foregoing in which $(\text{kv-a.})_1$ and $(\text{kv-a.})_2$ were not assumed to have any constant relationship to each other.

Influence of Generator and Prime-Mover Characteristics

A generator may influence the operation of a three-circuit transformer to which it is connected, by virtue of its impedance and by the action of its voltage regulator.

If the generator is equipped with an automatic voltage regulator, the generator impedance will not influence either regulation or load-division, but it will influence the short-circuit currents and should therefore be included in the impedance of the circuit to which it is connected. That is, in the calculation of the effective short-circuit impedance, also in the division of short-circuit current,

Z_a is to include also the impedance of the generator connected to A;

Z_b is to include also the impedance of the generator connected to B, etc.

Of course, if the generators are not equipped with any voltage regulators, the modified values of Z_a, Z_b and Z_c as defined in the preceding paragraph apply in the calculation of regulation, and also in that of division of load if it is not otherwise externally controlled.

If the induced voltages of two alternators connected in parallel have the same phase-angle but different magnitudes (a condition which may be brought about by the manipulation of the voltage control of the alternators) their difference, the unbalanced voltage, produces a circulating quadrature current through the impedance of the alternators which is practically all reactive. This circulating current is lagging in one and leading in the other unit. If the two induced voltages are equal in magnitude but out of phase from each other (a condition which may be brought about by the manipulation of the governors of the prime-movers) their difference, the unbalance voltage, is in quadrature with their mean value, and produces a circulating current parallel to the mean voltage. This current acts like a generator or load current in one and as motor or power current in the other. Now, if the two alternators are furnishing kv-a. to an external load, it will be evident that the circulating kv-a. load will add (vectorially) to the share of one unit in the external load, and will subtract (vectorially) from the share of the other unit in the external load. The foregoing statements hold whether the alternators are paralleled directly or through transformers. Three conclusions follow from these considerations:

(a) The total net reactive load in one of the two alternators or primary circuits can be increased or decreased at the expense of the other by producing a circulating reactive kv-a. by the manipulation of their induced voltages.

(b) The total net kw. load in one of the two alternators or primary circuits can be increased or decreased at the expense of the other by producing a circulating power by the manipulation of the governors of the prime-movers.

(c) The share of each alternator or primary circuit in the external load is completely determined by the inverse of the impedances of their respective circuits and this is not really altered by either alternator-voltage or prime-mover governor control. These controls merely superpose a circulating kv-a. so as to bring about a desired resultant load in each unit. This superposed circulating kv-a. becomes apparent when the external load is removed; therefore, the resolution of the total load in either circuit into two components as circulating kv-a. and as its share of the external load is not a mathematical fiction but a statement of actual fact.

Appendix A

Critical Analysis of the Interpretation of the Equivalent Impedances as the Impedances of the Corresponding Individual Windings

Although the resolution of the total leakage impedances Z_{ab}, Z_{ac} and Z_{bc} of a three-circuit transformer into equivalent components Z_a, Z_b and Z_c is absolutely correct and valid yet the interpretation of these latter as the leakage impedances of the respective windings is only a very convenient and useful fiction, not an absolutely, universally and uniquely valid interpretation. It is desirable to point out the limitations of this interpretation before proceeding to the absolute interpretation.

(a) If the interpretation of the equivalent impedances as the leakage impedances of the respective windings were absolute, it would appear as though here we had a simple scheme of settling an old controversy relative to the question as to how much of the reactance of a transformer belongs to the primary and how much to the secondary, a subject on which no two transformer engineers may be expected to agree.

(b) The very scheme of resolution shows that the resolution of, say, the total impedance Z_{ab} into Z_a and Z_b is made dependent on and with respect to the winding (or circuit) C, and that this resolution changes when the impedance of C *with respect to* A and B is altered. The physical meaning of such a resolution *with respect to* a third winding is better grasped from the second point of view (to be described below).

(c) It has been seen that one of the equivalent reactances (which we are identifying with the leakage

reactances of the corresponding individual windings) may be negative, that is, condensive. Obviously, a resolution that leads to a condensive leakage reactance in an inductive winding can not be absolute and inherent but must be relative. This fact also is better understood from the second point of view.

(d) The equivalent network of a transformer with more than three circuits, discussed in another appendix, clearly shows that such a resolution is relative, and valid only for particular classes of problems.

Appendix B

Equivalent Impedances Considered as "Mutual" Impedances

The basic problem in the theory and calculation of the characteristics of three-circuit transformers is to know what effect a load in one circuit has on the other circuit.

To see how the equivalent network and its impedances give us the effect of two circuits upon each other, consider Fig. 3c. Let us assume that C is a primary circuit and A and B are two secondary circuits. If A draws a current from C, there will be an impedance drop between C and A through the impedances ($Z_a + Z_b$). There will also be an impedance drop between C and B due to the current taken by A through the impedance Z_c. If the current was taken by B instead of by A, the drop through Z_c would now be effective for A.

Evidently, Z_c is that portion of the leakage or load reactance of $C A$ which also belongs to B, and that portion of the leakage or load reactance of $C B$ which also belongs to A. To put it a little differently, Z_c is the reactance common to loads in A and B. An inductance common to two circuits is known as mutual inductance, and we may therefore call the reactance common to the loads in A and B load-mutual-reactance between A and B, writing it as M_{ab}. If one prefers to think of mutual-inductance not as inductance "common" to two circuits, but as the coefficient which gives the voltage in one circuit induced by a unit current in the other, that point of view also applies to the present case, because Z_c is the coefficient which gives the voltage induced in A by a unit load-current in B, and vice versa. It will be observed, similarly, (see Fig. 3c) that Z_a is the mutual-reactance between B and C effective for their load currents, and that Z_b is the mutual-reactance between A and C effective for their load currents. The qualification of this mutual-reactance as load-mutual-reactance, or, as mutual-reactance effective for load-currents should be clearly recognized as follows:

In classical literature inductance, reactance, mutual-inductance, mutual-reactance are applied to quantities which relate to currents considered as magnetizing currents. In usual technical literature, especially in transformer literature, reactance has come to be used for the quantity which applies to the load currents of the transformer. When it is desired to be rigorous and explicit, distinction is made by qualifying the reactance as magnetizing-reactance for one and load or leakage-reactance for the other. Following the same analogy, there is no reason why we could not speak of magnetizing-mutual-reactance and load-mutual-reactance. The magnetizing-mutual-reactance between two windings of a transformer is usually of the order of from 1000 per cent to 2000 per cent, while the load-mutual-reactance is of the order of from a fraction of 1 per cent to 10 per cent or more. The reason why such a distinction has not been recognized before is simply because occasions calling for it have been rare. Load-mutual-reactance in a transformer requires at least three circuits, and three-circuit transformers have come into importance only within the last few years.

If the interpretation of the equivalent reactances as "common" or "mutual" reactances is grasped once, its absolute character as compared with the first interpretation will be evident. To make the discussion and proof of this interpretation more rigorous, we shall derive the mutual-reactances effective for load currents independently of the equivalent network and thereby show the identity of the two.

Equivalent Impedances Derived Analytically as Mutual Impedances

Required to find the mutual impedance between A and B effective for their load currents.

The following classical equation for leakage-reactance between two circuits A and B will be recognized

$$X_{ab} = X_a + X_b - 2\,M_{ab} \tag{25}$$

In this equation X_a and X_b are commonly interpreted as self- or magnetizing-reactances and M_{ab} is interpreted as magnetizing-mutual-reactance. Equation (**25**) holds true and can be verified by test regardless of the presence or absence of other windings, and regardless of the condition in which those other windings may be. We may therefore assume a short-circuited winding C on the same core with A and B. Now, one who is ignorant of the existence of this short-circuited winding, C, would call the measured value of X_a the self-inductive reactance of winding A, but one who is aware of the fact, would call this measured value that of X_{ac}, and the measured value of X_b that of X_{bc}. The current with which the measurements are made will be considered by the latter a load-current (not a magnetizing current) and the mutual-reactance M_{ab} will apply only to the load currents. Recognizing the existence of a short-circuited winding C explicitly, he will write:

$$X_{ab} = X_{ac} + X_{bc} - 2\,M_{ab}$$

and solving for M_{ab},

$$M_{ab} = \frac{X_{ac} + X_{bc} - X_{ab}}{2}$$

Comparing this with equation (**6**), the identity of

the equivalent reactance X_c (see Fig. 3c) with the load-mutual reactance of the circuits of A and B becomes evident.

Appendix C

Physical Explanation of Negative (Condensive) Equivalent Impedance

It has already been pointed out that one of the equivalent impedances may be negative. This would be unintelligible if considered as the straight reactance of a winding, and although not immediately self-evident when considered as a mutual-reactance, it yet becomes amenable to explanation.

A negative equivalent reactance considered as a mutual reactance raises the question: Can a circuit carrying a lagging load induce a voltage in another

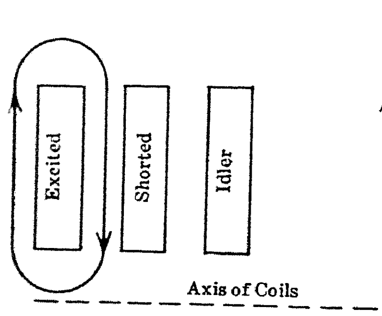


Fig. 11

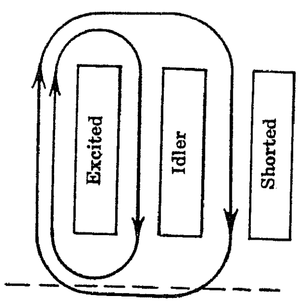


Fig. 12

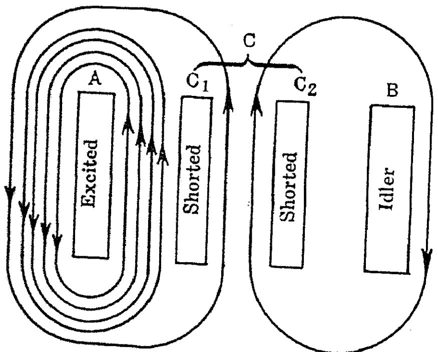


Fig. 13—Total Flux Linkages of A = + 5
Total Flux Linkages of B = − 1

circuit opposite to the voltage induced by the magnetizing current? Or, putting the question another way, can one secondary carrying a lagging load and lowering its own voltage, boost the voltage of the other secondary? Reversing the direction of the windings cannot influence this matter in the least, for, if we assume that the polarity of the idle winding in which the induced voltages are being observed is reversed, then, both the voltage induced by the magnetizing current and that induced by the load current will be reversed and thus the relative phase of the two voltages will remain unchanged. To understand the physical nature and basis of this phenomenon properly, it is necessary to examine the distribution of load- or leakage-flux in some representative cases.

Distribution of Load- or Leakage-Flux

Case I. Load-Mutual-Reactance Zero. Fig. 11 shows the distribution of load flux in a simple case: C is short-circuited, A is excited. The flux linkage being a measure of reactance, it is evident that no load or leakage-flux links B when A is carrying a current. And hence the load-mutual-reactance M_{ab} is zero. In this and the following diagrams the influence of the thickness of the conductor on reactance is ignored to simplify the discussion of the principles.

Case II. Load-Mutual-Reactance Positive. Assume as another case that the arrangement of the windings is as in Fig. 12. The flux linkages and corresponding reactances will be as follows: Assuming the distance between A and B, and that between B and C, equal,

Total net flux linkage of $A = 2 = X_{ac}$
" " " " " $B = 1 = M_{ab}$ (Positive)

In this instance, the load-mutual-reactance between A and B is one half the leakage reactance between A and C, and has the same sign.

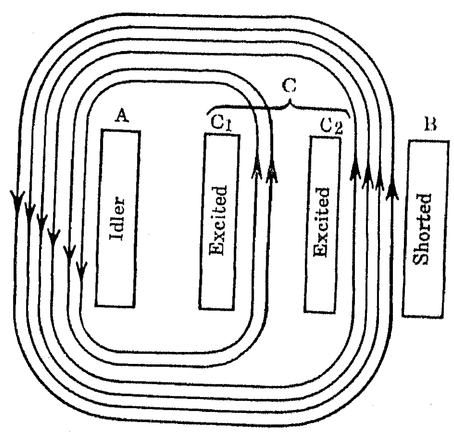


Fig. 14—Total Flux Linkages of C = 5
Total Flux Linkages of A = 6

Case III. Load-Mutual-Reactance Negative. Assume now that C consists of two coils, C_1 and C_2 in series, spaced similar to A and B, (Fig. 13). With C short-circuited and A excited, the distribution of flux and corresponding flux-linkages will be as shown in the diagram.

Assuming the spacings between coils equal:

Total net flux linkages of $A = +5 = X_{ac}$
" " " " " $B = -1 = M_{ab}$ (Negative)

Thus, it is evident that the windings may be so arranged as to produce a load-mutual-reactance which is positive, zero, or negative.

A practical aspect of the last case considered is that if the circuit of A carry zero power factor full load and its voltage *drops* 5 per cent, the voltage of B will be *boosted* 1 per cent.

Case IV. Load-Mutual-Reactance Greater than Total Leakage Reactance. A feature of load-mutual-reactance that is as interesting and important as the possibility of the load-mutual-reactance being negative, is the

possibility of it being larger than the reactance of the coil itself. That is, M_{ab} may be greater than the load or leakage reactance of A or B with respect to the primary. That mutual reactance from one coil to another (of equal turns) can be greater than its own self reactance does sound queer but its truth may easily be established.

Considering Fig. 14, where B is short-circuited and C excited, the flux linkages and corresponding reactances are as follows:

Total net flux linkages of $C = 5 = X_{cb}$

" " " " " $A = 6 = M_{ca}$

That it is possible for the mutual reactance of two coils to be larger than the self reactance of one, is true for the no-load condition as well as for the load condition. Thus, referring back to equation (**25**), if we solve for M_{12}, and ignore X_{12}, as negligibly small, we get

$$M_{12} = \frac{X_1 + X_2}{2}$$

That is, the mutual reactance is practically the mean[6] of the two self reactances and therefore larger than the lesser of the two.

The possible per cent divergence between self and mutual reactances for load currents is far greater than that for magnetizing currents.

Appendix D

Direct Testing of the Equivalent Impedances

The consideration of the equivalent impedances as mutual impedances leads readily to a convenient way of testing them directly without having to use equations (**4-6**) while the first point of view would not help so well, if at all, in this matter. For instance, considering Z_c (see Fig. 3c) as load-mutual-impedance between A and B, it follows that it ought to be possible to test it directly by putting a load current in A and measuring the voltage in B, or putting a load current in B and measuring the voltage in A.

In order that the current may be a *load* current and not a magnetizing current, it is necessary to short-circuit the third circuit C, because the distinction between a magnetizing-current and a load-current in a transformer is that a current in a winding not neutralized by an equivalent opposite current in another winding is a magnetizing current, while a current which is neutralized by an equal and opposite ampere-turns in another winding is a load current. In the present case the short-circuited winding carries the equal and opposite ampere-turns.

In obtaining the equivalent impedances directly by test, in order that their resistance and reactance components may be determined, in addition to the voltmeter reading in the idle winding and ammeter reading in the excited winding, a wattmeter reading should also be taken connecting its current coil in series with either the excited or short-circuited winding and its potential coil across the idle winding. Knowing thus the volts, watts and amperes of the load-mutual-impedance, its resistance and reactance components are calculated in the usual manner.

In as much as ordinarily the three leakage-impedances have to be tested as a matter of course as standard data, the equivalent impedances are preferably calculated from them rather than doubling the work of testing. The discussion of the method of testing them directly has been given here primarily for the sake of the light which it throws on their physical nature and interpretation.

6. This is sometimes given as geometric mean instead of arithmetic mean.

Appendix E

Single-Phase Loads and Short-Circuits on Polyphase Banks

Single-phase loads and short circuits may be divided into two classes, *viz.*, (a) those between lines, and (b) those between a line and the neutral. A review of the former is helpful in formulating the latter, and, furthermore, it may be desirable to review the phenomena of two-winding transformers before proceeding to the more complicated case of three-winding transformers.

In a polyphase network consisting of two or more different banks of apparatus, it is highly desirable to express the constants, that is, the per cent impedance of each bank, based on one assumed standard symmetrical polyphase load, and then to express the effective impedance to various single-phase or unsymmetrical loads in terms of these impedances.

A consideration which the writer finds very convenient and safe in analyzing the impedance of a polyphase network to single-phase loads and single-phase short circuits is to add up the impedance (kv-a.) of every branch of the network for the particular distribution of the assumed load-current and express this sum as a percentage of the assumed load or reference kv-a., which then is also the percentage impedance drop; instead of considering either the per cent impedance volts or the ohms in each branch, because in these latter cases there is always the troublesome question whether certain branches should be considered in series or in multiple, as ohms or voltage drops combine differently in series than in multiple, whereas the kilovolt-amperes consumed reactively can and should always be added just as the kilowatts consumed by the resistances are added up. This procedure is identical with the principle of basing the per cent resistance of a transformer on the per cent impedance watts. In using the reactive kv-a. method for the calculation of reactance drops, it is necessary to note that reactive kv-a. loss (like resistance kw. loss) varies as the square of the load.

The following theorems underly the analysis of

single-phase loads and short circuits on symmetrical polyphase systems.

Theorem I. A *single-phase* load on the lines (not through the neutral) of a symmetrical three-phase system, equal to one third of the rated (or assumed standard) polyphase kv-a. of the system, encounters a per cent impedance drop equal to two-thirds of the per cent impedance of the system for the rated (or assumed standard) kv-a. of the system.

This law holds true whether the load is limited to two line wires (Fig. 15), or to all three line wires, two lines acting as return to the remaining line (Fig. 16). This law also holds true regardless of the kind of apparatus; regardless of the type of connection of apparatus, as long as it is a symmetrical three-phase

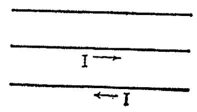


Fig. 15

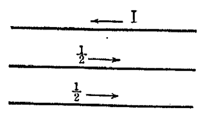


Fig. 16

connection; and, regardless of how composite the system may be. Due to structural peculiarities, generators may slightly vary from the above law, but this variation need not be considered except in very refined work. Transformers may be considered practically free from any such variation.

It will be noted that in the case of Fig. 16, the load voltage is the altitude of the triangle of the line voltages, that is, 86.6 per cent of the line voltages.

Theorem II. The per cent impedance of a three-phase apparatus, as for instance a transformer bank, to a single-phase load from one line to its neutral must be worked out specially for each particular connection. However, on the primary side, this single-phase load is transferred to the lines and therefore the impedance

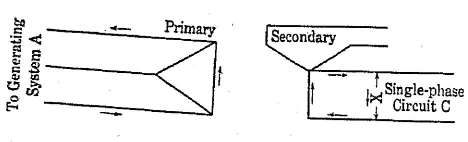


Fig. 17

of the system connected to the primary lines follows the law of Theorem I.

If the neutrals of a number of apparatuses are connected together, as for instance by grounding a system at a number of points, these apparatuses must be considered in parallel for a line-to-neutral load or short circuit.

Theorem III. A line-to-neutral single-phase load distributes itself in such a way as to encounter the minimum impedance drop. When it is possible for the load to distribute itself without magnetizing the transformer core, it does so distribute itself.

Theorem IV. Two interconnected polyphase systems divide any kind of a load, including single-phase loads and short circuits in accordance with the general formulas developed in the foregoing for double-primary three-circuit transformers, as will be illustrated in due course.

The more important connections for line-to-neutral loads and short circuits are the *Y*-delta or delta-*Y* connection, and the interconnected *Y* or Zig-Zag connection, in two-circuit transformers; and, *Y*-*Y*-delta tertiary connection in three-circuit transformers.

Two-Circuit Transformers interconnecting Two Generating Systems, when Loaded Line-to-Neutral, become a Three-Circuit-Transformer Problem. All voltages, currents and impedances in the following discussions are in percentages unless otherwise specified. The per cent marks (%) are omitted for simplicity.

(I) Delta-Primary, Y-Secondary Connection

Assume a single-phase line-to-neutral load (Fig. 17) equal to the kv-a. rating of one leg, or, one-third of the polyphase kv-a. rating of the bank. It is evident that there will be current in only one leg, and will be equal to the rated current of that leg. Designating the per cent impedance of the transformer for rated symmetrical polyphase load, Z_T; and, that of the system A, to which the primary lines are connected,

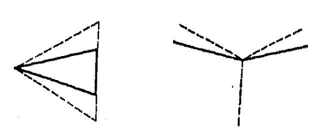


Fig. 18

Z_A; the per cent impedance drop for this single-phase power flowing from system A to the load circuit C equal to the kv-a. rating of one leg, will be,

$$Z_{AC} = Z_T + 2/3\, Z_A \tag{25}$$

Hence, for a line-to-neutral single-phase short-circuit,

$$\text{Short Circuit kv-a.} = E/Z_{AC} \times \text{Rated kv-a.} \tag{26}$$

Current in short circuit

$$= E/Z_{AC} \times \text{Rated line current on } Y \text{ side} \tag{27}$$

The per cent regulation on primary side across the loaded lines follows at once as

$$\text{Regulation} = (2/3)\ (Z_A/Z_{AC})\ E \tag{28}$$

Example. A 15,000 kv-a. bank of step-down delta-*Y* transformers having 8 per cent impedance at rated polyphase load is connected to a 25,000 kv-a. generating system having 20 per cent impedance at its rated load. The latter includes the impedances of the generators, step-up transformers, lines and any other apparatus in series with the system.

Since the kv-a. rating of transformer bank and system are different, it is desirable to reduce the impedances to a common kv-a. basis.

Let the kv-a. basis be 15,000 kv-a. Then, the transformer impedance will be the same as before.

Z_T = 8 per cent

The system impedance will be less in the ratio of 15,000/25,000, making

Z_A = 15,000/25,000 × 20 per cent = 12 per cent

The effective impedance for a 5000-kv-a. single-phase load on circuit C will be:

Z_{AC} = 8 per cent + 2/3 × 12 per cent = 16 per cent

Assuming 100 per cent voltage on the system,

Short-Circuit kv-a. = 100/16 × 5000 = 31,200 kv-a.

Current in the Short Circuit = 100/16 × Rated line current (Y side) for 5000 kv-a. per leg.

Regulation across loaded lines on delta side = 2/3 × 12/16 × 100 = 50 per cent.

The short-circuit voltage diagram for this case is shown in Fig. 18 in heavy lines. Dotted lines show the normal voltages on no-load.

(II) Y-Primary, Delta-Secondary Connection

Referring to Fig. 19 it will be seen that with a single-phase load in circuit C equal to the kv-a. rating of one leg, the load in each leg is only one-third its rated load. The reactive kv-a. consumed in each leg is one-ninth of that at full rated load and the reactive

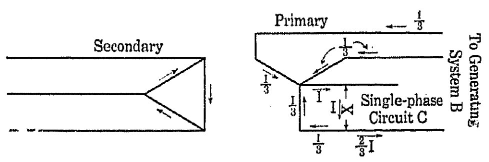


Fig. 19

kv-a. consumed in the three legs is three times this, that is, one-third of the normal reactive kv-a. in one leg at its rated load. The per cent reactance of one leg at rated load is Z_T, and, therefore, the per cent impedance of this bank for a single-phase line-to-neutral load equal to the kv-a. rating of one leg is $Z_T/3$. The impedance of system B to which the primary Y is connected has an impedance 2/3 Z_B for this single-phase load.

Therefore,

$$Z_{BC} = 1/3\, Z_T + 2/3\, Z_B \tag{29}$$

Short-Circuit kv-a. = E/Z_{BC} × Rated kv-a. of one leg (30)

Current in short circuit = E/Z_{BC} × Rated line current (31)

The per cent regulation on the delta side across the short-circuited phase is

$$\text{Regulation} = 2/3\, Z_B/Z_{BC}\, E \tag{32}$$

Example: Assume the same transformers as before.

Let system B be rated 50,000 kv-a. 25 per cent impedance at rated load. On a 15,000 kv-a. base, its impedance will be (15,000/50,000) 25 per cent, or 7.5 per cent.

Then,

$$Z_{BC} = 8/3 + 2/3 \times 7.5 = 7.667 \tag{33}$$

Total short-circuit kv-a. = 100/7.667 × 5000 = 65,200 (34)

The kv-a. in each leg is one-third of this.

The regulation on the delta side across the short-circuited phase is,

Regulation = 2/3 × 7.5/7.667 × 100 = 65.2 per cent (35)

The short-circuit voltage diagram is shown in Fig. 20.

It follows from the regulation formula that if the impedance of the generating system were negligible compared with that of the short-circuited transformer,

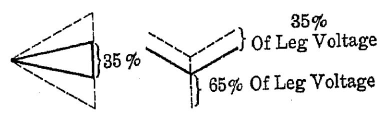


Fig. 20

the line-to-neutral short-circuit on the Y side would not in the least affect the voltages across the delta secondary. This is an interesting feature and has an application in the following case.

(III) Delta-Y Transformer Interconnecting Two Generating Systems, Fig. 21

It was seen in the discussion of Case II that if the generating system on the Y side (System B) had negligible impedance compared with the transformer, the voltages on the delta side would be unchanged from no-load to a line-to-neutral short circuit. It follows, therefore, that in such a case, whether the delta side is connected to a generating system or not would make no

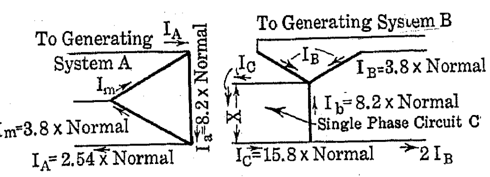


Fig. 21

difference to line-to-neutral loads and short circuits, and all such loads and short circuits will be furnished entirely by the system on the Y side. However, it is not very often that the system impedance can be ignored as compared with the transformer impedance, and it is therefore desirable to take it into consideration, and the problem may be tackled very conveniently as follows:

With the load circuit C, and interconnected generating systems A and B (Fig. 21), it is to be noted that we have a veritable case of a three-circuit transformer, consisting of two primary circuits and one secondary circuit, and, therefore, we may confidently apply the three-circuit transformer formulas to the present case.

As has already been seen, the impedance of the load circuit in conjunction with either generating system is

$$Z_{AC} = Z_T + 2/3\, Z_A \tag{36}$$

$$Z_{BC} = 1/3\, Z_T + 2/3\, Z_B \tag{37}$$

Also $$Z_{AB} = 2/3\, Z_A + 2/3\, Z_T + 2/3\, Z_B \tag{38}$$

The last equation is in accordance with Theorem I, and refers to single-phase kv-a. flowing from one system into the other equal to one-third of the assumed standard three-phase kv-a.

Example: Assume the various constants used in the examples of cases (1) and (2).

$$Z_{AC} = 16 \text{ per cent} \tag{39a}$$

$$Z_{BC} = 7.667 \text{ per cent} \tag{39b}$$

$$Z_{AB} = 2/3\,(12 + 8 + 7.5) = 18.3 \text{ per cent} \tag{39c}$$

The various impedances of the equivalent circuit (Fig. 3c) are by formulas 4, 5, 6,

$$Z_A = \frac{Z_{AB} + Z_{AC} - Z_{BC}}{2} = 13.32 \text{ per cent} \tag{40a}$$

$$Z_B = \frac{Z_{AB} + Z_{BC} - Z_{AC}}{2} = 5 \text{ per cent} \tag{40b}$$

$$Z_C = \frac{Z_{AC} + Z_{BC} - Z_{AB}}{2} = 2.68 \text{ per cent} \tag{40c}$$

The impedance of the whole combination, effective at the short circuit, is, by formula 24,

$$Z_{short} = \frac{Z_{AC}\, Z_{BC} - Z_C^2}{Z_{AB}} = 6.32 \text{ per cent} \tag{41}$$

Assuming 100 per cent line voltage, the short-circuit kv-a. is 100/6.32, that is, 15.8 times 5000 kv-a., which equals 79,000 kv-a. Of this the share of each generating system is,

$$(\text{kv-a.})_A = Z_B/Z_{AB} \times 79{,}000 = 21{,}500 \text{ kv-a.} \tag{42a}$$

$$(\text{kv-a.})_B = Z_A/Z_{AB} \times 79{,}000 = 57{,}500 \text{ kv-a.} \tag{42b}$$

(IV) Interconnected *Y* or Zig-Zag Connection

Referring to Fig. 22, let Z_t be the per cent impedance between the two coils on one leg of the apparatus acting as a single-phase transformer. When the two coils are connected in series as a zig-zag auto transformer, the impedance in per cent of the load voltage (line-to-neutral voltage) becomes $Z_t/1.73$. With normal rated line current in the single-phase line-to-neutral circuit *C*, making the load kv-a. equal to the rated kv-a. per leg, the current in each leg is only one-third normal, the impedance kv-a. in each leg one-ninth normal in the three legs one-third normal. Hence, the effective impedance of the apparatus for circuit *C* is one-third normal, that is $Z_t/(1.73 \times 3) = Z_t/5.2$. If we designate the impedance of the generating system for the rated symmetrical load of the apparatus by Z_A, its effective impedance for a single-phase load equal to the rated kv-a. per leg will be two-thirds of this, that is, $2\, Z_A/3$. The net effective impedance for the circuit *C* is thus,

$$Z_{AC} = 2/3\, Z_A + 1/5.2\, Z_t \tag{43}$$

$$\text{Short circuit } I = \frac{E}{2/3\, Z_A + 1/5.2\, Z_t} \times \text{Rated } I \tag{44}$$

The current in the coils is one-third of this as indicated in Fig. 22.

(V) *Y*-*Y*-Delta Connection

(1) *Y-Primary, Y-Secondary, Delta Secondary.*

(a) Line-to-neutral single-phase load (or short circuit) on the primary *Y*. (Fig. 23).

Since the secondary *Y* circuit takes no part in the

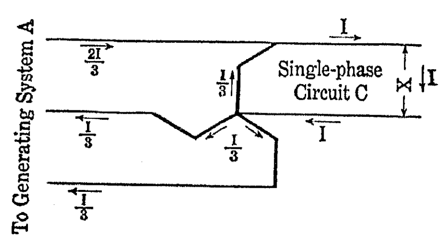


Fig. 22

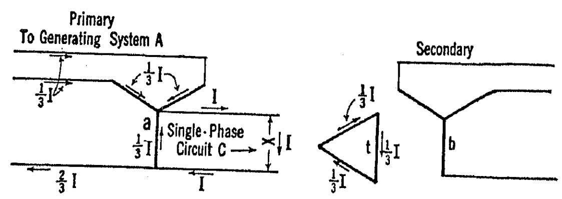


Fig. 23

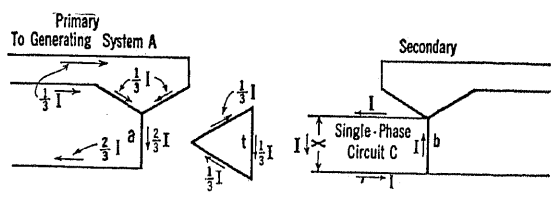


Fig. 24

phenomena under consideration, it will be seen, especially on comparing Fig. 23 with Fig. 10, that this is the same as the case of two-winding *Y*-primary delta-secondary transformer.

Representing the impedance between *Y*-primary and delta tertiary by Z_{at} based on the rated symmetrical polyphase kv-a. of the *Y*, the effective impedance between system *A* and circuit *C* for a single-phase load equal to one-third of the rated polyphase load will be, similar to the case referred to,

$$Z_{AC} = 1/3\, Z_{at} + 2/3\, Z_A \tag{45}$$

where Z_A is the impedance of the generating circuit *A*.

(b) Line-to-neutral single-phase load (or short circuit) on the secondary *Y*. (Fig. 24).

Designating the symmetrical three-phase impedance

between secondary and tertiary by Z_{bt}, the impedance between system A and circuit C, for a load equal to one-third of the polyphase rating, will be

$$Z_{AC} = 2/3\, Z_{ab} + 1/3\, Z_{bt} + 2/3\, Z_A \tag{46}$$

Short circuit kv-a. $= E/Z_{AC} \times$ Rated kv-a. per leg (47)

Current in short circuit $= E/Z_{AC} \times$ Rated line current (48)

The current in the different windings and lines may be readily calculated from the current in the short circuit with the aid of the diagram (Fig. 24). Note

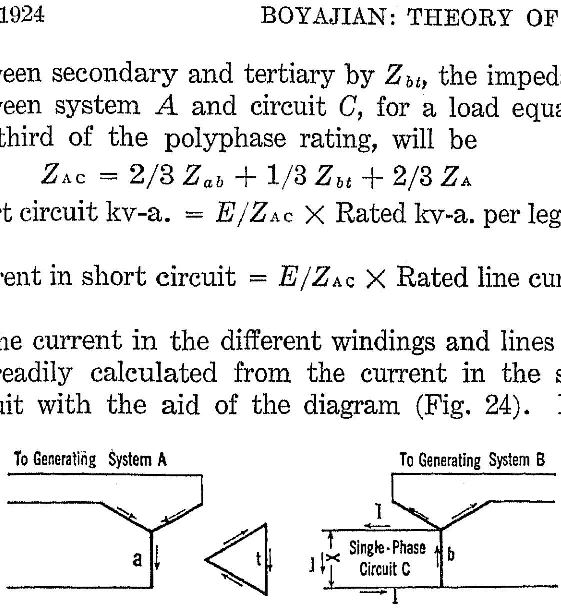


FIG. 25

that the diagram assumes one-to-one ratio on each leg.

(2) *Both (Y)s Connected to Generating Systems, Delta Secondary.* (*Fig.* 25).

Single-Phase Load, Line-to-Neutral, on Either Y

Let the side to which the single-phase load is connected be designated system B; the other side, system A (Fig. 25).

The impedance of either primary system with respect to the circuit C, by itself, would be

$$Z_{AC} = 2/3\, Z_A + 2/3\, Z_{ab} + 1/3\, Z_{bt} \tag{49}$$

$$Z_{BC} = 2/3\, Z_B + 1/3\, Z_{bt} \tag{50}$$

Also $$Z_{AB} = 2/3\,(Z_A + Z_B + Z_{ab}) \tag{51}$$

The equivalent impedances of each circuit will be,

$$Z_A' = \frac{Z_{AB} + Z_{AC} - Z_{BC}}{2} \tag{52a}$$

$$Z_B' = \frac{Z_{AB} + Z_{BC} - Z_{AC}}{2} \tag{52b}$$

$$Z_C' = \frac{Z_{AC} + Z_{BC} - Z_{AB}}{2} \tag{52c}$$

the values of the three impedances on the right to be taken from the preceding equations:

The net effective impedance of the combination of the two generating systems and transformer for the load (or short circuit) in circuit C is, by formula (**24**),

$$Z_{short} = \frac{Z_{AB}\, Z_{BC} - Z_C'^2}{Z_{AB}}$$

The kv-a. drawn by a line-to-neutral short circuit is:

Short circuit kv-a. $= E/Z_{short} \times$ Rated kv-a. per leg.

Of this kv-a. the part furnished by each generating system is (by formulas **21** and **22**).

(Short circuit kv-a.)$_A$ $= Z_B'/Z_{AB}$ $\times$ Total short circuit kv-a.

(Short circuit kv-a.)$_B$ $= Z_A'/Z_{AB}$ $\times$ Total short circuit kv-a.

Knowing the (single-phase) short-circuit kv-a. in each generating system, the line currents follow. It should be remembered that for a single-phase load (or short circuit) as in Fig. 16, the corresponding voltage is 86.6 per cent of the line voltage.

The current in the short circuit is

$I_s = E/Z_{short} \times$ Rated line current

Of this current the part furnished by each generating system is, by formulas (**21**) and (**22**),

$(I_s)_A = Z_B'/Z_{AB} \times I_s$

$(I_s)_B = Z_A'/Z_{AB} \times I_s$

Knowing the currents $(I_s)_A$ and $(I_s)_B$, the corresponding currents in the various parts of the network may be calculated, although the method of calculation by kv-a. is preferable as no turn ratios, etc., need then be considered.

Example. A Y-Y connected auto transformer interconnects a 66,000-volt system (system A) with a 114,000-volt system (system B), the neutral of the auto transformer being solidly grounded: see Fig. 26. An auxiliary delta winding is provided to lower the impedance of the transformer for line-to-neutral loads and also to supply a load.

The various constants of the circuits based on an output of 5000 kv-a. per phase are:

$Z_{at} = 10.6$ per cent

$Z_{bt} = 8.3$ per cent

$Z_{ab} = 8.65$ per cent

Let the generating system impedances for 5000 kv-a. per phase be

$Z_A = 3$ per cent

$Z_B = 2$ per cent

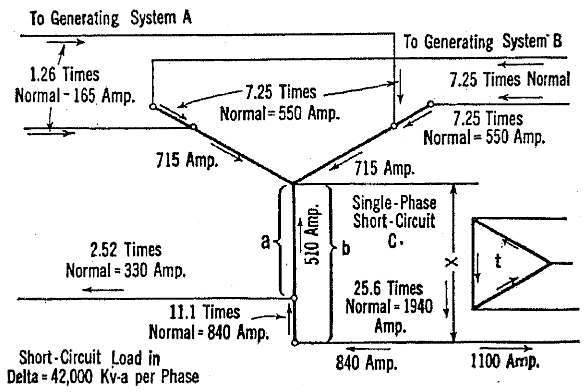


FIG. 26

If one of the high-voltage lines (system B) becomes grounded, (short circuit at C), what will be the short-circuit current and kv-a. and how will they divide between the two systems?

The total impedance between generating system A and circuit C (Fig. 26) ignoring system B, by equation (**46**) which is based on (Fig. 24).

$$Z_{AC} = 2/3 \times 8.65 + 1/3 \times 8.3 + 2/3 \times 3 = 10.5 \text{ per cent}$$

The impedance between system B and circuit C

ignoring system A, is, by equation (**45**) (changing letter a to b, to $A\,B$) which is based on (Fig. 23).

$Z_{BC} = 1/3 \times 8.3 + 2/3 \times 2 = 4.1$ per cent

By equation (**51**)

$Z_{AB} = 2/3\,(3 + 2 + 8.65) = 9.1$ per cent

From these follow the equivalent impedances of the respective circuits: (see Fig. 3c)

$$Z_A' = \frac{9.1 + 10.5 - 4.1}{2} = 7.75 \text{ per cent}$$

$$Z_B' = \frac{9.1 + 4.1 - 10.5}{2} = 1.35 \text{ per cent}$$

$$Z_C' = \frac{10.5 + 4.1 - 9.1}{2} = 2.75 \text{ per cent}$$

The impedance of the total combination effective for the circuit C is, by equation (**24**),

$$Z_{short} = \frac{10.5 \times 4.1 - 2.75^2}{9.1} = 3.9 \text{ per cent}$$

With an effective impedance of 3.9 per cent at a 5000 kv-a. load, evidently the short-circuit kv-a. at 100 per cent system voltage will be 100/3.9 times, that is, 25.6 times 5000 kv-a., or 128,000 kv-a.; and the current *in the short-circuit* will be 25.6 times the normal line current corresponding to 5000 kv-a. per phase, or $25.6 \times 75.8 = 1940$ amperes.

The part of this current and kv-a. furnished by each system is, by formulas (**21**) and (**22**),

Part of System $A = Z_B'/Z_{AB} = 1.35/9.1 = 0.148$
$= 14.8$ per cent $= 0.148 \times 128{,}000$ kv-a.
$= 19{,}000$ kv-a.

Part by System $B = 100 - 14.8 = 85.2$ per cent
$= 0.852 \times 128{,}000$ kv-a. $= 109{,}000$ kv-a.

The currents in the two systems (see Fig. 26) may be obtained either from their share of the short-circuit current or from their share of the short-circuit kv-a. as follows:

System A, delivering 19,000 kv-a. single-phase at 86.6 per cent of 66,000-volt line voltage will have a current of $19{,}000/0.866 \times 66 = 330$ amperes on one line, and half of this in each of the other two lines acting as joint return to the first line (see Fig. 26), System B, delivering 109,000 kv-a. single-phase at 86.6 per cent of 114,000 volts line voltage, will have a current of $109{,}000/0.866 \times 114 = 1100$ amperes in one line, and half of this in each one of the other two lines jointly acting as return to the first line.

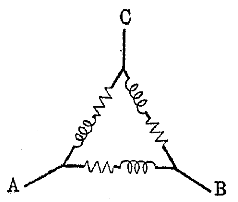
Appendix F

Theory of Four- and N-Circuit Transformers Equivalent Network

The star network described in the preceding pages is not the only equivalent network for a three-circuit transformer. The network could also be a mesh or delta (see Fig. 27) as well as a star or Y. Of course, the values of the links in the delta network are not the same as those in the Y network. For the same reason, the two physical interpretations which hold for the Y or star network do not hold for the delta or mesh network. It was on account of its simpler physical interpretation and also simpler mathematical manipulation that only the Y network was considered in the theory of the three-circuit transformer. However, the mesh network is here mentioned as an introduction to the N-circuit transformer, because it appears that the alternatives available in the case of the three-circuit transformer are not available in the case of N-circuit transformers: that is, the simplification afforded by the use of a star instead of a mesh network is not open to higher number of circuits, but an essentially mesh network must be used.

In constructing equivalent network, the following conditions must be satisfied:

1. The diagram must have as many terminal points as there are circuits.
2. The diagram must be symmetrical for every one of these terminals.
3. Since in an N-circuit transformer there are $N\,(N-1)/2$ leakage impedances to be reckoned with,



Equivalent Mesh Network of Three Circuit Transformer

Fig. 27

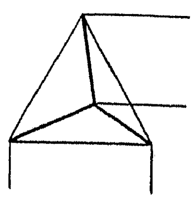


Fig. 28

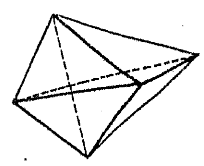


Fig. 29

therefore, the equivalent network must have $N\,(N-1)/2$ variables or impedance links. Diagrams for a number of such circuits, up to eight circuits, are tabulated below. All the diagrams, except the first, are represented by three-dimensional figures, like basket work, to bring out clearly their symmetry. Of course, the essential facts are the number of terminals and the number of links. That the figure is three-dimensional has no further significance whatever, beyond aiding in seeing the symmetry, and may just as well be considered a flat diagram if desired. Every pair of terminals or "corners" are supposed to be connected by a line representing one impedance link. Some of the lines which would be invisible in a solid are omitted so as not to obscure the general appearance of the figure.

Fig.	27	28	29	30	31	32
Number of circuits	3	4	5	6	7	8
" " terminals or corners	3	4	5	6	7	8
" " links	3	6	10	15	21	28
Links terminating at each corner	2	3	4	5	6	7

The value of each impedance link has to be solved as follows:

Considering Fig. 27,

$$\frac{1}{1/Z' + \dfrac{1}{Z'' + Z'''}} = Z_{AB}$$

$$\frac{1}{1/Z''' + \dfrac{1}{Z' + Z''}} = Z_{AC}$$

$$\frac{1}{1/Z'' + \dfrac{1}{Z' + Z'''}} = Z_{BC}$$

Since the number of the impedance links increases faster than the square of the number of the external circuits, these equations increase in complexity very rapidly with increasing number of external circuits. In practical problems some of the labor can be saved by the use of a short-circuit calculating board, but the constants to be set-up on the board, *i. e.*, the values of the impedance links, have to be calculated.

The problem of the N-circuit transformer can be simplified if, instead of solving for the most general case in which no assumption is made as to which circuit or circuits are primary and which secondary, the nature of each circuit and its load is known, as will be illustrated below.

Four-Circuit Transformers. In considering the effect of the load in one secondary on other windings, a primary winding is always implied. In a three-circuit transformer this is unmistakable, because, in speaking of the mutual impedance between one secondary winding and another, there is only one winding left to act as primary. But in a four-circuit transformer, since there are two other circuits besides the two designated in the subscript of M_{ab}, the third circuit must be

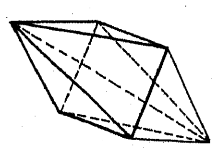


FIG. 30

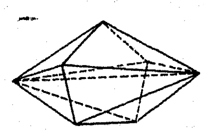


FIG. 31

explicitly stated, such as M_{ab-c} or M_{ab-d}. The general formula for M then is:

$$M_{ab-c} = \frac{X_{ac} + X_{bc} - X_{ab}}{2}$$

$$M_{ab-d} = \frac{X_{ad} + X_{bd} - X_{ab}}{2}$$

etc. Similarly for the resistance components.

Assume that A, B and C are secondary, D primary. The total vector impedance drop in any secondary due to simultaneous loads in all the secondaries will be,

$$e_a = I_a Z_{ad} + I_b M_{ab-d} + I_c M_{ac-d}$$
$$e_b = I_a M_{ab-d} + I_b Z_{bd} + I_c M_{bc-d}$$
$$e_c = I_a M_{ac-d} + I_b M_{bc-d} + I_c Z_{cd}$$

The extension to N-circuit transformers is evident: thus, calling the primary P, and the secondaries a, b, c, etc.

$$e_a = I_a Z_{ap} + I_b M_{ab-p} + I_c M_{ac-p} + I_d M_{ad-p} + \ldots \text{etc.}$$

A point of refinement to be considered in these

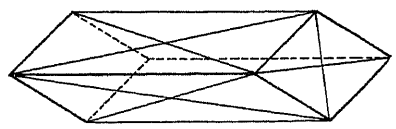


FIG. 32

equations is this, that in obtaining the impedance drop in A, the power factors of the loads in every circuit should be referred to the terminal voltage of A, not to the terminal voltages of the respective circuits. If referred to the terminal voltage of the respective circuits, the result will be only approximate.

Appendix F

COLLECTED FORMULAS AND SYMBOLS

1. *Equivalent Impedance.* Given the total impedance of each pair of windings or circuits as primary secondary, the equivalent impedance of each individual winding or circuit is,

$$Z_a = \frac{Z_{ab} + Z_{ac} - Z_{bc}}{2}$$

$$Z_b = \frac{Z_{ab} + Z_{bc} - Z_{ac}}{2}$$

$$Z_c = \frac{Z_{ac} + Z_{bc} - Z_{ab}}{2}$$

These equations are vectorial. To obtain the resistance components, substitute R instead of Z; for the reactance components, X instead of Z.

If the calculation is to be made in terms of ohmic values, all the values to be substituted above should be "converted" or reduced to a common voltage basis. If the data are available in percentage form, they may be directly substituted.

When a circuit has an independent winding, and stray losses are negligible or easily estimated, the effective resistance of the circuit is the same as the effective resistance of its winding.

In auto transformers and other complicated connections, care should be taken that the impedance data used should be those effective at the terminals of the circuits under consideration.

2. *Regulation.* The equivalent impedance of each circuit is conceived of as the impedance between itself

and the magnetic neutral of the unit, as shown in the diagrams of equivalent networks. Hence,

Calculate (*a*), the regulation from each secondary terminal to the neutral, and (*b*), from the neutral to each primary terminal, by the following formula

$$\% \text{ Reg.} = k\, m\, \%\, I\, R + k\, n\, \%\, I\, X + \frac{(k\, m\, \%\, I\, X - k\, n\, \%\, I\, R)^2}{200}$$

in which k is the ratio of actual load or current in the branch to that load or current on which the $\%\, I\, R$ and $\%\, I\, X$ of the branch are based, m is the power factor of the load in the branch, and n its reactive factor. Regulation may be positive or negative and its algebraic sign must not be ignored. For secondary and tertiary circuits, k, m and n are ordinarily given, but for the primary they must be calculated, and the following equations may be used. Designating primary by "1," secondary by "2" and tertiary by "3."

$$(k\, m)_1 = (k\, m)_2 + (k\, m_3) \quad \text{Approx.}$$
$$(k\, n)_1 = (k\, n)_2 + (k\, n)_3 \quad \text{Approx.}$$

or, more precisely.

$$(k\, m)_1 = k_2 \left(m_2 + k_2 \frac{\%\, I\, R_2}{100} \right) + k_3 \left(m_3 + \frac{k_3\, \%\, I\, R_3}{100} \right)$$

$$(k\, n)_1 = k_2 \left(n_2 + \frac{k_2\, \%\, I\, X_2}{100} \right) + k_3 \left(n_3 + \frac{k_3\, \%\, I\, X_3}{100} \right)$$

$$\% \text{ Reg}_{12} = \% \text{ Reg}_{1n} + \% \text{ Reg}_{n2}$$
$$\% \text{ Reg}_{13} = \% \text{ Reg}_{1n} + \% \text{ Reg}_{n3}$$
$$\% \text{ Reg}_{23} = \% \text{ Reg}_{2n} - \% \text{ Reg}_{3n}$$

ALTERNATIVE METHOD

The following alternative method of calculating regulation may be preferred by some. This gives the total regulation of either secondary, $\% \text{ Reg}_{12}$, or tertiary, $\% \text{ Reg}_{13}$, in one step:

$$\% \text{ Reg}_{12} = k_2\, m_2\, \%\, I\, R_{12} + k_2\, n_2\, \%\, I\, X_{12} + k_3\, s_3\, \%\, I\, R_1 + k_3\, t_3\, \%\, I\, X_1 + \frac{(k_2\, m_2\, \%\, I\, X_{12} - k_2\, n_2\, \%\, I\, R_{12} + k_3\, s_3\, \%\, I\, X_1 - k_3\, t_3\, \%\, I\, R_1)^2}{200}$$

$$\% \text{ Reg}_{13} = k_3\, m_3\, \%\, I\, R_{13} + k_3\, n_3\, \%\, I\, X_{13} + k_2\, s_2\, \%\, I\, R_1 + k_2\, t_2\, \%\, I\, X_1 + \frac{(k_3\, m_3\, \%\, I\, X_{13} - k_3\, n_3\, \%\, I\, R_{13} + k_2\, s_2\, \%\, I\, X_1 - k_2\, t_2\, \%\, I\, R_1)^2}{200}$$

in which formulae, k, m and n have the same meanings as before,

s_2 = Power factor of secondary load with respect to tertiary terminal voltage.
$= (m_2\, m_{23} - n_2\, n_{23})$. For m_{23} and n_{23} see below.

s_3 = Power factor of tertiary load with respect to secondary terminal voltage.
$= (m_3\, m_{23} + n_3\, n_{23})$.

t_2 = Reactive factor of secondary load with respect to tertiary terminal voltage.
$= (n_2\, m_{23} + m_2\, n_{23})$.

t_3 = Reactive factor of tertiary load with respect to secondary terminal voltage.
$= (n_3\, m_{23} - m_3\, n_{23})$.

n_{23} = Reactive factor (*i. e.*, sine) of the angle between secondary and tertiary terminal voltages.
$= 1/100\, (k_2\, m_2\, \%\, I\, X_2 + k_3\, n_3\, \%\, I\, R_3 - k_2\, n_2\, \%\, I\, R_2 - k_3\, m_3\, \%\, I\, X_3)$.

m_{23} = Power factor (*i. e.*, cosine) of the angle between secondary and tertiary terminal voltages.
$= \sqrt{1 - n_{23}^2}$

3. *Division of Load.* If A and B are two circuits similar in function, as both primary or both secondary, and if the division of total kv-a. load between them is not externally controlled, it takes place in accordance with the following formulas:

$$(\text{kv-a.})_A / (\text{kv-a.})_{total} = Z_B / Z_{AB}$$
$$(\text{kv-a.})_B / (\text{kv-a.})_{total} = Z_A / Z_{AB}$$

4. *Short-Circuit Impedance.* With two primary circuits A and B simultaneously excited, and with a short-circuit across secondary C, the effective impedance which determines the total short-circuit current and kv-a. is

$$Z_{short} = \frac{Z_{ac}\, Z_{bc} - Z_c^2}{Z_{ab}}$$

Although the equation is strictly vectorial, sufficiently close results may be obtained ordinarily by substituting reactance instead of resistance in this formula.

5. *Impedance of Generator Circuit.* Since generating systems are supposed to be equipped with proper voltage regulators to maintain the primary voltages constant, the impedances of generator circuits are ignored in the calculation of regulation, but they should not be ignored in the calculation of short circuits unless they are negligibly small. To include the impedance of the generating circuit, add it to the equivalent impedance of the primary to which it is connected.

6. For various cases of unsymmetrical loads and short circuits on symmetrical systems, refer to the body of the paper.

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Discussion

P. L. Alger: Mr. Boyajian's theory of three-circuit transformers applies to the induction motor as well as to the transformer. His three-circuit transformer is the same in theory as the double-squirrel-cage induction motor, and the same phenomenon of a negative reactance which occurs in his diagrams also occurs in the equivalent circuit of the motor.

It is very interesting to review these analogies which occur between related branches of engineering and utilize them in extending one art to keep up with the others. I believe that this paper of Mr. Boyajian's may be of material assistance in extending our knowledge of the complicated phenomena of the multiple squirrel-cage induction motor.

H. L. Cole (by letter): The subject of multiple winding transformers is becoming of greater importance as the networks of power systems increase in number and size. Up to the last few years it has been of importance mainly to the design engineer, by whom the various loading and short-circuit conditions have to be studied in order that such special transformers will meet the electrical, mechanical and thermal conditions imposed upon them in service. The large number of applications of multiple winding transformers in recent years has shown the necessity of studying the relation of the transformer impedances to those of the rest of the system to which it is connected, as brought out by Mr. Boyajian. The paper covers a large field of study, and will prove a useful reference on the subject.

In Appendix E (V) there is one case of short-circuit conditions in which I do not entirely agree with the author's analysis. Equation (**46**), which gives the impedance between system A and circuit C as

$$Z_{AC} = 2/3\, Z_{ab} + 1/3\, Z_{bt} + 2/3\, Z_A$$

should be $Z_{AC} = 2/3\, Z_{ab} + 2/9\, Z_{at} + 1/9\, Z_{bt} + 2/3\, Z_A$

In this way, the effect of the impedance Z_{At}, which is a factor in limiting the current in circuit C, is taken into account.

A. Boyajian: I have checked and reaffirmed the correctness of the equation (**46**) which Mr. Cole has criticised. I am afraid that the alternative equation which he offers is in error.

My equation is given in a form suitable for all interleaved transformers (whether shell or core type), irrespective or whether the tertiary is between the primary and secondary or outside. The equation which Mr. Cole offers is in a form which I believe presupposes a definite location for the tertiary. If we correct the first term of his equation, changing it from $2/3\ Z_{at}$ into $4/9\ Z_{ab}$, it will fit the case of an external tertiary, and is then reducible into my equation. Thus, referring to the attached sketch, and making use of the principle that reactive kv-a. consumed, varies as the square of the current (as explained and illustrated in my paper), we get the following results:

(1), Since the density in the gap between a and b, on the vertical phase, is $^2/_3$ normal, the reactive kv-a. consumed thereby will be $4/9\ Z_{ab}$; (2) the density between b and t, on the vertical phase, is $\frac{1}{3}$ normal, and therefore the reactive kv-a. consumed thereby will be $1/9\ Z_{bt}$; (3), on the other two phases between a and t the density is $\frac{1}{3}$ normal, the reactive kv-a. consumed in each is 1/9 normal, and therefore for the two legs we have $2/9\ Z_{at}$. Adding these three items, the effective reactance of the transformer itself, exclusive of that of generating system A, will be,

$$Z_{eff} = \frac{4}{9} Z_{at} + \frac{2}{9} Z_{at} + \frac{1}{9} Z_{bt}$$

This equation differs from that of Mr. Cole in the coefficient of the first term, having 4/9 instead of 2/3, and is reducible into equation (**46**). Thus, since

$$Z_{at} = Z_{ab} + Z_{bt}$$

to a very close approximation; making this substitution,

$$Z_{eff} = \frac{4}{9} Z_{ab} + \frac{2}{9} (Z_{ab} + Z_{bt}) + \frac{1}{9} Z_{bt}$$

$$= \frac{2}{3} Z_{ab} + \frac{1}{3} Z_{bt}$$

which is equation (**46**), less $2/3\ Z_A$, the impedance of the generating circuit.

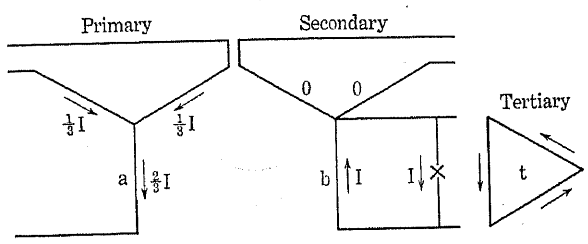


FIG. 1

If the tertiary delta be assumed located between primary and secondary, as shown in Fig. 24 of the paper, we get: (1), on vertical phase, between a and t, $4/9\ Z_{at}$; (2), between b and t, Z_{bt}; and, (3), in the other two phases, $2/9\ Z_{at}$. Adding these three items,

$$Z_{eff} = \frac{4}{9} Z_{at} + Z_{bt} + \frac{2}{9} Z_{at}$$

$$= \frac{2}{3} Z_{at} + Z_{bt}$$

But, $Z_{at} = Z_{ab} - Z_{bt}$. Hence, making this substitution,

$$Z_{eff} = \frac{2}{3} Z_{ab} + \frac{1}{3} Z_{bt}$$

which again is equation (**46**), less $2/3\ Z_A$, the impedance of the generating circuit.

Equation (**46**) being thus applicable to a very close approximation to the case of the inside as well as of the outside tertiary, it is applicable in general, and is desirable on account of its simplicity. It is also evident that Z_{at} enters into the equation indirectly, in terms of Z_{ab} and Z_{bt}, by virtue of its relationship to them.

Hydroelectric Practises and Equipment of the South

O. G. THURLOW AND J. A. SIRNIT
Non-member Member, A. I. E. E.
Both of Alabama Power Co., Birmingham, Ala.

UNTIL a short time ago little thought was given to the hydroelectric possibilities in the South and still less was its economic value appreciated. Coal was cheap and easily obtainable. Consequently, power could be generated cheaper with coal than by water. It is, therefore, an everlasting tribute to those pioneers with vision and courage, who in the face of such adverse conditions ventured into developing water powers and laid the foundations for the great hydroelectric developments that we have today in the South.

Hydraulic Power Resources of the South

The rivers of the Southeastern section of the country and their tributaries, originating on the southern part of the Appalachian Highlands, not having the perpetual snows, depend solely upon the seasonal and periodical precipitation for maintaining their flow. The average annual precipitation, which is greater than in any other part of the United States, ranges between 40 in. and 80 in.

Although this region is for the most part heavily timbered, which tends to prevent sudden run-offs and equalize the river flows, the rivers are subject to high variations in flow. The daily hydrograph of the Coosa River at Lock 12 for the year 1922, illustrates clearly the fluctuations of the river, and is typical of most of the southern rivers. Some rivers have even a greater flow variation, as for example, the Tennessee River, which has a flow ranging from 7200 second ft. minimum to 430,000 second ft. maximum, recorded at Florence, Alabama.

The ratio of rainfall to the run-off has also almost as wide a variation as the river flow itself. It varies with the location and also with the amount of rainfall. While at the headwaters the ratio is 3:2 or better, in the lower foothills it varies from 4:1 to 2:1, the highest ratio prevailing in dry seasons or years, the lowest in wet periods.

The following table gives rainfall and run-off data on the Upper Coosa basin for different representative years:

Year	Rainfall Inches	Run-off Inches	Approximate Ratio
1904—dry	40	9.5	4:1
1906—wet	63	30	2:1
1912— "	63	30	2:1
1916—average	47	18	2½:1

For the most part, these rivers yield only run-off river power with very little primary, but a large amount of secondary power when supplemented with steam power or power from storage reservoirs, can be utilized wholly or in part to good advantage. Practically every river on its headwaters or on its tributaries has one or more possibilities for developing storage reservoirs, a number of which has already been developed. The principal storage developments already in operation are those of the Southern Power Company on the headwaters of Catawba River with a storage capacity of 13,500,000,000 cu. ft. and Georgia Railway and Power Company on the Tallulah River, one at Burton with 5,280,000,000 cu. ft. and one at Mathis containing 1,369,000,000 cu. ft. of storage.

The Alabama Power Company has now in course of development a storage project on the Tallapoosa River at Cherokee Bluffs that will, when completed, have a storage capacity of 60,000,000,000 cu. ft. and will be able to develop approximately 136,000,000 kw-hr.

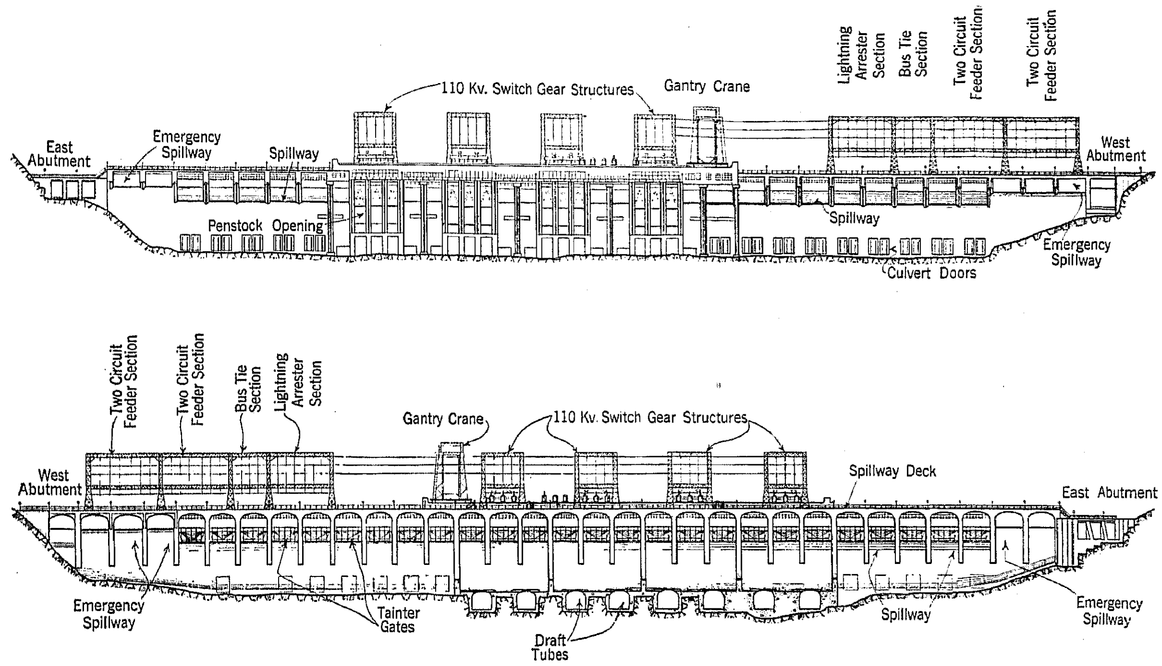
Design of Power Plants

The design of hydroelectric generating plants in the south does not differ materially from the design of similar plants elsewhere, and since practically all the developments are of recent date, with few exceptions, the plants are of modern design and construction. The generating units are of vertical type and in sizes ranging up to 30,000 horse power, depending on water-flow and head.

The majority of the southern hydroelectric plants are low-head developments with exception of the Tallulah Plant of the Georgia Railway and Power Company on the Tallulah River which has a head of 600 ft. and the Ocoee Plant of the Tennessee Power Company on the Ocoee River, with a head of 254 ft. These can be classed as medium high-head developments.

A departure from the conventional design of low-head power houses is represented in the Mitchell Dam Plant of the Alabama Power Company on the Coosa River. As it is well known, the run-of-river plant suffers an appreciable reduction of operating head during flood periods, and to overcome this disadvantage engineers have been forced to install a greater number of generating units where it was possible to do so, to compensate for the loss in capacity of each unit. Of course, such a procedure necessarily makes the installation more expensive with the resulting increase of production costs. In some instances provisions had to be made for admitting water into the draft tube through jets of relatively high velocity which, by accelerating the velocity of the combined turbine discharge and jet water through the draft tube, produces a negative head.

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-11, 1924.



Elevation of Dam and Powerhouse Looking Up-Stream

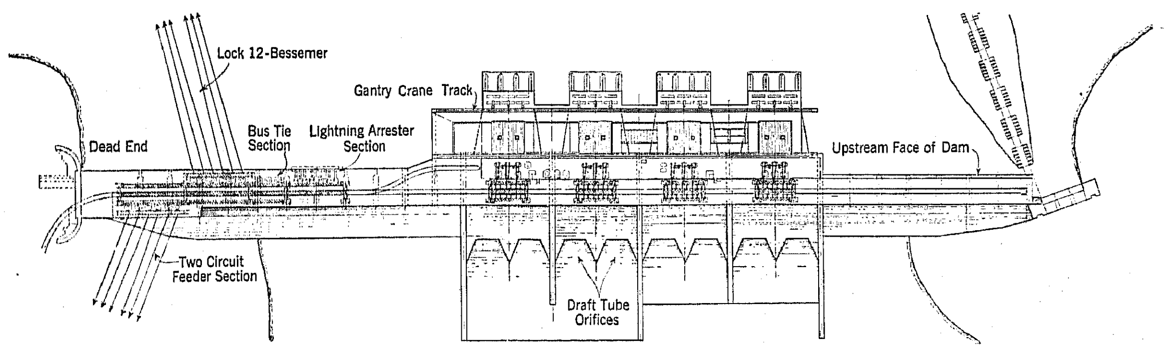


Fig. 1—26 Spillway Gates 30 ft. Wide by 15 ft. Deep—930 ft. Spillway, Including Piers

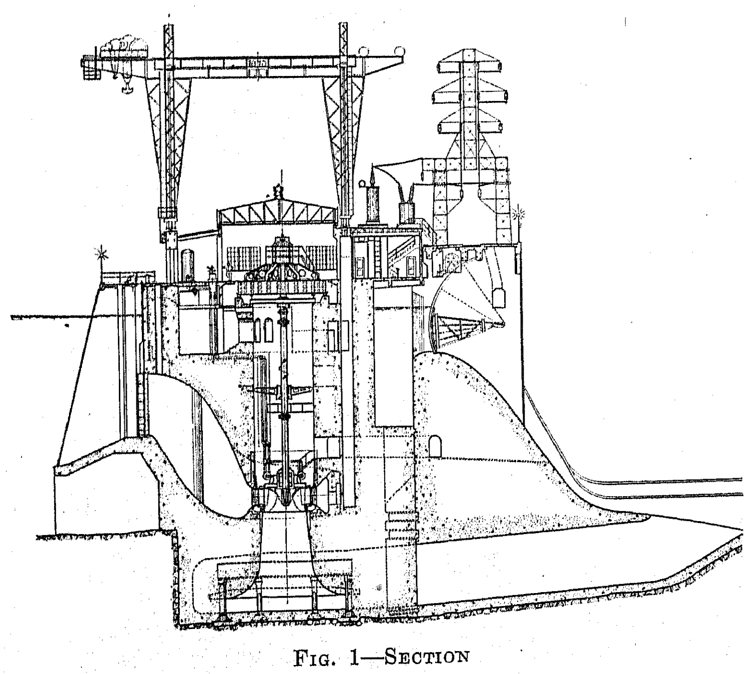


Fig. 1—Section

In designing Mitchell Dam Plant, the idea was conceived that the most effective way to maintain the normal head on the waterwheel, and thereby the full capacity of the unit during flood periods, would be to remove the excess tail-water head from the discharge opening. This resulted in the development of what is now known as the Thurlow back-water suppressor. The introduction of this feature called for an entire change in the general design of the power plant, causing a departure from the established conventional practise.

The principal and most outstanding feature in this plant is the location of the power units on separate foundations in the river on the upstream side of the dam. (See Fig. 1.)

The usual power house building is entirely eliminated. The generator room is covered by a low movable roof, which is designed in two sections, joined on the transverse line and mounted on wheels; each section moving in opposite direction. In normal operation the generator room is completely enclosed, but for handling large parts of machinery, this roof can be opened and a

gantry crane utilized for performing the necessary work.

The entire operating floor is on one floor level. The generators, governors, switchboard, low-tension switching, bus galleries and offices are easily reached without climbing stairs or ladders. This materially adds to the convenience of operators and makes supervision and inspection of the plant and its machinery more effective.

An elevator is also provided with each unit for use of the attendants in inspection of the turbine bearings and main shaft.

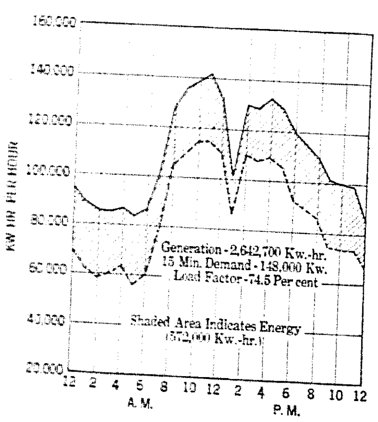


Fig. 3

This power house is designed for operation on the unit system, that is, each generator and its bank of step-up transformers are connected as a unit. There is, however, also a transfer bus provided which enables parallel operation of all or any number of units and from which the station service power and lighting is obtained.

Southern Super-Power System

Comparatively little has been mentioned in print about the inter-connected system of the Southeastern States, yet it comprises one of the largest systems in the United States.

While the proposed Eastern super-power zone is still under discussion, a virtual super-power system has been in existence in the South for several years covering the States of Alabama, Georgia, Tennessee, North and South Carolina. The Southern zone consists of the inter-connected system of seven independent companies as follows:

Alabama Power Company
Columbus Power Company
Georgia Railway and Power Company
Central Georgia Power Company
Tennessee Power Company
Southern Power Company
Carolina Power and Light Company

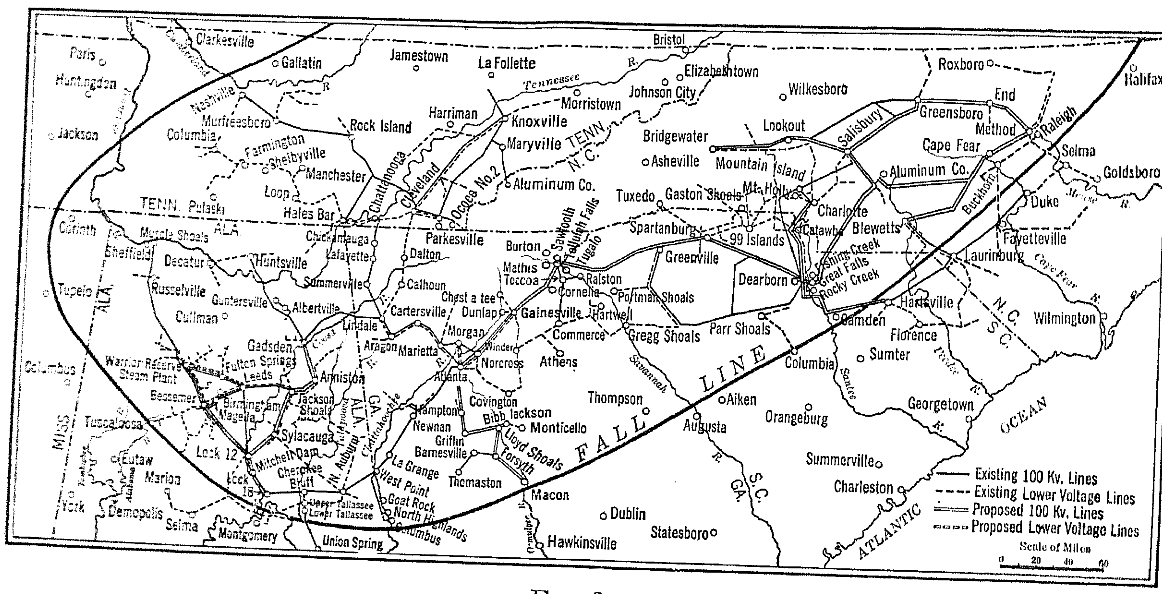


Fig. 2

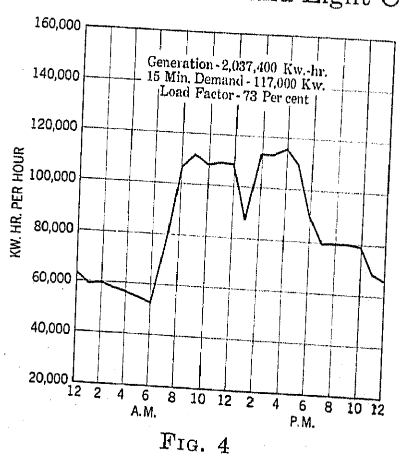


Fig. 4

A map of the Southeastern States showing the territory covered by these systems is shown in Fig. 2.

It will be noted that the inter-connected system forms a complete network over a territory approximately 600 mi. long, and 300 mi. wide at its greatest width.

All large plants, hydro and steam, are connected to this system with an aggregate capacity of about 1,100,900 kw.

The table below shows the various plants with the capacities of the different power companies, that are either in operation or will be in the near future.

POWER-GENERATING STATIONS OF THE POWER COMPANIES IN THE SOUTHEASTERN SECTION

Hydroelectric stations	Kw.	Steam electric stations	Kw.
Alabama Power Company			
Lock 12	81,000	Warrior	70,000
Mitchell Dam	60,000	Gadsden	10,000
		Birmingham	8,000
Total	141,000	Montgomery	5,000
		(leased from U. S. Govt.) Sheffield	60,000
		Total	153,000
Tennessee Power Co.			
Hales Bar	43,300	Parksville	13,000
Parksville	18,650	Nashville	29,600
Great Falls	7,800	Knoxville	6,250
Ocoee No. 2	18,750	Hales Bar (June 1924)	20,000
Total	88,500	Total	68,850
Georgia Railway & Power Co.			
Tallulah Falls	72,000	Atlanta, Butler St.	7,800
Morgan Falls	21,000	Atlanta, Davis St.	11,750
Tulago	50,000		
Dunlap	2,200	Total	19,550
Chestater	900		
Total	146,100		
Columbus Electric & Power Co.			
Goat Rock	17,500	Columbus, Ga.	9,000
North Highland	6,900		
		Total	9,000
Total	24,400		
Central of Georgia Power Co.			
Lloyd Shoals	18,000	Macon	3,000
Total	18,000	Total	3,000
Southern Power Company			
Bridgewater	20,000	Eno	25,000
Lookout	18,000	Greensboro	6,800
Catawba	4,800	Mount Holly	36,800
Fishing Creek	30,000	Greenville	6,800
Mountain Island	60,000		
Great Falls	24,000	Total	75,400
Rocky Creek	24,000		
Wateree	56,000		
Dearborn	45,000		
99 Islands	18,000		
Total	299,800		
Carolina Power & Light Co.			
Buckhorn	2,900	Florence	1,500
Blewett	30,000	Raleigh	3,720
		Goldsboro	1,200
Total	32,900	Cape Fear	15,000
		Total	21,420
Total hydroelectric kw.	750,700	Total steam electric kw.	350,220

The main trunk-line is over 800 mi. long with several thousand miles of branch lines connected to it.

The inter-connection of these systems was accomplished by the building of tie lines and the installation of sufficient transformer and switching capacity at the tie-in points, to take care of the exchanged energy. Few changes had to be made in the individual systems themselves, as, fortunately, all the companies involved had already standardized on a frequency of 60 cycles and most of them operated their principal transmission lines at 110,000 volts.

Although energy has been transmitted a distance of about 600 miles on this inter-connected system, the use of high-voltage trunk-lines of the order of 220,000 volts has not been found necessary. It is obvious, of course, that no great bulk of power could be directly transmitted a distance of 800 mi. at 110,000 volts, but by a process of relaying the energy from generating stations to adjacent load centers, the same benefits have been accomplished. For instance, assume that the Carolina Power & Light Company at the extreme northeastern end of the inter-connected system desires to secure a block of power which is to be supplied by the Alabama Power Company. The latter company delivers the required amount of power to the Georgia Railway and Power Company at the Georgia-Alabama State Line, which will be absorbed on the Georgia system in that vicinity. The Georgia Railway and Power Company will then transmit energy from its Tallulah Falls Plant, at the other extreme end of its system, to the Southern Power Company. This Company will in turn absorb this energy locally and transmit a like amount from generating stations in the vicinity of the tie point of the Carolina Power and Light Company. The systems, relaying the energy, of course, deduct an amount sufficient to cover transmission losses, costs of transmission, etc.

During the Fall of 1921 several of the Southeastern power companies, recognizing the possibilities accruing from the inter-connection of the systems and anticipating a power shortage during the dry weather seasons, arranged to secure the use of the then idle steam plan at Muscle Shoals, Alabama. Accordingly, the Alabama Power Company leased this plant from the United States Government, and that Company in turn contracted with several of the other companies to hold in reserve a certain capacity which could be used by these companies at any time upon reasonable demand.

The Muscle Shoals steam plant is located in the extreme northwestern section of Alabama, and con-

nects with the Alabama Power Company's system through a 90-mi., 110,000-volt, single-circuit, transmission line at the Warrior River steam plant. Although the Muscle Shoals steam plant has a generating capacity of 60,000 kw., the amount available for distribution by the Alabama Power Company is limited to 30,000 kw. by the transmission line capacity between Muscle Shoals and Gorgas Steam Plant.

The value and importance of this arrangement and of the inter-connection were clearly demonstrated in 1922, when during an abnormally dry season in the Southeastern section, relief was given to companies in the Carolinas and Georgia. Some of the companies would have had to curtail power to their customers, had they not been able to secure this standby power. A typical load curve, when power was being delivered from Alabama to the Georgia companies and the Carolinas for 24 hours of the day is shown in Fig. 3, where the unshaded area indicates normal system load on the Alabama Power Company's system, and the shaded area the energy delivered to other systems. For comparison,

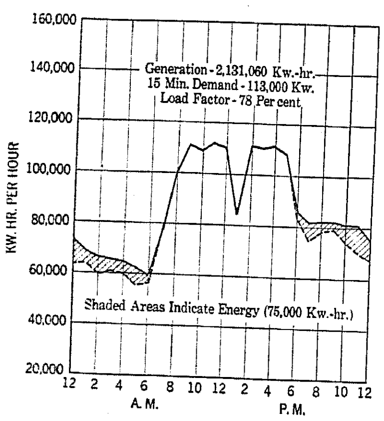


FIG. 5

a typical system load curve of the Alabama Power Company's system for a day when no interchange of energy took place is shown in Fig. 4.

Since the principal reserve capacity of the inter-connected system is at Muscle Shoals, the relaying of large blocks of power to date has been mainly from Alabama to Georgia and to the Carolinas. There has, however, been considerable power interchanged between some of the neighboring companies, but we can only speak of those in which the Alabama Power Company's system took part, namely, between it and the Georgia Railway and Power Company. The benefits derived by inter-connection of large power systems have been further demonstrated by the result of the interchange between these two companies. The hydro plants of the Alabama Power Company are run-of-river plants with comparatively small storage reservoirs, and the flow of the Coosa River on which these plants are situated, is subject to wide variations. The bulk of the Company's load during the low water seasons of the year is carried by steam generating stations. The principal hydro plants of the Georgia Railway and Power Company on the other hand, are backed by storage reservoirs having a capacity to take care of normal seasonal variations in river flow.

During certain seasons of the year it is possible to co-ordinate the operation of the Georgia and the Alabama Plants to secure maximum efficiency on each system. For instance, during the season when there is a surplus of water in the run-of-river plants, and when water is being stored at the reservoir plants, power is supplied from the Alabama system to the Georgia system during the off-peak hours—the normal load on the Alabama system during off-peak hours being considerably less than the capacity of its hydro plants. This interchange, therefore, permits the operation of the flow-of-river plants at a very high load factor and conserves water at storage plants.

During the peak load hours it, of course, becomes necessary for each system to carry its own load and the storage plant draws down a certain amount of its stored water. A typical load curve on the Alabama Power Company's system when supplying off-peak power to Georgia is shown in Fig. 5. By referring to Fig. 4 it will be noted that considerable improvement is made in the load factor on the Alabama Power Company's system. Another method of operation which has proven of advantage is that of supplying energy from the storage plants to the Alabama systems during peak hours, and during the off-peak hours power is returned from the Alabama system to the Georgia system. This reduces steam plant operation and permits the operation of run-of-river plants at high load factor.

Load dispatching on the inter-connected system is quite a simple matter, the principal requirements being close co-operation between dispatchers of the various systems, and sensitive governors in the main generating plants. The Georgia Railway and Power Company, being located centrally with respect to other systems, usually receives and relays the load orders to and from the various companies. An effort is made to predict load requirements several days in advance in order to prevent the unnecessary operation of steam plants.

It is, of course, essential that all systems maintain a frequency of about 60 cycles before successful interchange can be obtained. The frequency is in reality controlled by the system supplying the power, and the load swings of the entire inter-connected system are shared equally between the systems. The installation of accurate frequency-indicating instruments have materially assisted in the operation of the system. Several of the companies have installed at their principal stations frequency devices known as Warren master clocks.

Another requirement of interchanging power between systems is that of maintaining proper voltage conditions in order to bring about the proper distribution of the reactive current among the several sys-

tems. The bulk of the load carried on these systems is at a power factor ranging from 80 to 90 per cent and, therefore, the total reactive component of the load is rather large. For instance, at 80 per cent power factor the reactive component is 60 per cent. At times it is necessary to operate generators under light load or even no-load conditions for voltage boosting purposes.

The application of proper relays on all the systems is essential, in that a line or piece of apparatus in trouble should be immediately disconnected so that the remaining system may be operated without further disturbance.

This inter-connection has also proved of advantage in increasing the reliability of service. Line or apparatus failures upon isolated systems naturally cause interruptions to customers. Upon inter-connected systems it is nearly always possible for the combined generating capacity to take care of any reasonable number of failures. This possibility has been well demonstrated by the use of the tie between Georgia and Alabama in emergency cases. In 1923 a severe sleet storm in northwest Georgia broke down some of the lines west of Atlanta, and the load in western Georgia was immediately picked up and carried from the Alabama Power Company's system until repairs could be made. Likewise, interruptions to customers in Alabama have been prevented by use of the tie-line.

In addition to interchange between power systems, benefits have also been derived from the interchange of industrial plant and central station energy. Such interchange, to a limited extent, has been accomplished between the Alabama Power Company and Tennessee Coal, Iron and Railroad Company. This company generates a considerable portion of its power requirements from the exhaust steam of its rolling mill engines and from steam obtained from boilers fired by blast furnace gas which would perhaps otherwise be wasted.

Besides steel mills, there are other industrial establishments which can undertake inter-connected operation with central stations to advantage. Among such industries would be those requiring heat in the form of low-pressure steam, in the production of which electrical energy can be generated as a by-product. When more by-product energy is being generated than required, the surplus could be disposed of to the central station company, and vice versa, when a deficiency occurred the additional energy required would be purchased from the central station.

As the load on power systems continues to increase, the benefits of inter-connection will become more and more pronounced. To summarize, some of these benefits are as follows:

1. Reliability of service.
2. Reduction of total reserve capacity.
3. Economies due to diversity of stream flow in different water sheds.
4. Economies due to diversity of load factors.

Discussion

For discussion of this paper see page 569.

Hydroelectric Practises and Equipment

on the Pacific Coast

BY SVEND BARFOED

Member, A. I. E. E.

Chief Engineer, Frank G. Baum, San Francisco, Cal.

Review of the Subject.—The paper takes up the physical features having deciding influence on design and construction of hydroelectric plants. It describes the mountain system of the region, the precipitation, the runoff, the streams suitable for power development, and the major characteristics of the development; then takes up a little more in detail the structural features, such as reservoirs and diversion dams, the conduit system, surge tanks and forebays, pressure pipes, power houses and equipment. The choice of impulse and reaction turbines is discussed and some performance curves given; this followed by conditions imposed upon modern plants by the transmission line; switch gear is briefly discussed, followed by a resume on transmission lines and their control. Finally, some results which have been obtained are related, and the paper ends up with the hope that hydroelectric developments and distribution will continue as in the past, rather than have it undertaken by competing municipalities, which by ambitious advertising of cheap power (tax free) would endeavor to attract industries and people to their crowded areas.

It is felt that the hydroelectric power industry on the Pacific Coast has distributed the benefits of cheap electricity to the small and large community alike, tending to more stable development of the entire region.

THE Pacific Coast is taken to mean the three states that border directly on the Pacific Ocean—Washington, Oregon and California. They have a shore line of over 1500 miles and within their borders the climatical, meteorlogical and topographical features vary to an extraordinary degree and almost any climate from the subtropical to the artic may be found. Due recognizance is taken of these facts in the planning and execution of hydroelectric plants and a great variety of types of structures have been built to meet them.

Mountain System

From north to south two parallel mountain chains, separated by great interior valleys, extend from the Canadian border nearly to the Mexican line. Along the ocean is the Coast Range, densely covered with forests in the northern part which gradually taper off towards the south and finally cease in the extreme southern portion where desert conditions prevail. About 100 miles inland is the more elevated and more massive mountain chain called in Washington and Oregon the Cascades and in California, the Sierra-Nevadas. The forest cover is here also much heavier in the north, with the coniferous timber line gradually receding in the south where timber is found only at the highest elevations.

The two mountain chains merge into each other at two points, one in the Siskiyous in southern Oregon and northern California, and the second in the Tehachapi at the lower end of the San Joaquin Valley in California.

These mountain chains, together with the ocean, are the great actors in the distribution of precipitation over the land. The total range in precipitation is from less than one inch in the south to over 130 inches in the north.

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-11, 1924.

Precipitation

Precipitation generally occurs when an area of low atmospheric pressure appears off the southern Alaskan coast and travels in a diagonal path southeasterly across the continent. This low pressure area draws, from the ocean to the south and southwest, air currents which have become heavily laden with moisture from contact with the ocean's surface. During the wet season the land is cooler than the ocean, southerly from it, and the wind, passing over the land, is cooled and gives up its moisture as rain or snow. Near the coast, with the exception of the higher mountains, the precipitation occurs chiefly in the form of rain, also as rain in the large valleys between the mountain ranges, and generally as snow in the Cascades and Sierras. In the south there is a decidedly wet and dry season, with the dry season much the longer, while in the north the difference between the wet and dry seasons is not so pronounced and the wet season is much the longer. There are modifications from local sources. Dry is summer; wet is winter.

The more sparsely the occurrence of precipitation, the greater is the seasonal variation in total amount from year to year.

Runoff

The runoff of the precipitated moisture from the mountain areas is influenced by the structure of the earth's crust, by plant growth, by form of precipitation (rain or snow), by temperature, wind and duration of storms. Some of these features are singly dominating in various sections. The most pronounced is in the lava fields of eastern Oregon and northeastern California. Much of this lava cover is porous to such an extent that there is very little surface runoff, the water at once sinking into the ground and appearing later in the stream channels well regulated by nature. In the granite Sierras there is a minimum of percolation, the heavy snow covers retard the runoff in varying de-

gree, and this natural storage of water is highly important in maintaining stream flows well into the summer months. In the Cascades heavy humus covers and snow have similar effect.

Streams

The western slope of the mountains receives a greater amount of moisture than the eastern. As a general rule, power streams flow west and the greatest number of large developed power sites are found on the western slope of the interior mountain range. There are exceptions to this rule, of course.

South of the Tehachapi the streams have not sufficient flow to make any large power developments, but it is here that some of the early high head plants were built and much of the art was learned.

The largest rivers of the Pacific Coast are the Columbia and the Colorado. They have drainage areas far outside the region and have cut their way through the mountain ranges and plateaus to reach the ocean. These two streams have not been utilized on their main courses and are the great store houses of future power in the west.

Fig. 1—Natural Storage

Storage Reservoirs

Over a large part of the territory, the variation in stream flow has made storage reservoirs necessary for economical power production. The degree to which this has been carried has varied from a maximum in the southern part to a minimum in the regions of the lava-covered area. Also in the north, due to the better distribution and greater amount of the precipitation, the building of storage reservoirs is, at the present time, of relatively less importance than in the south. Storage will grow in importance with time, when streams will be utilized for a greater proportion of the average flow. South of the Tehachapi storage reservoirs have been built large enough to regulate stream flows for 10-year periods or more. The water is here, however, stored primarily for irrigation and whatever power can be had is considered a by-product. Further north in the granite Sierras the regulation of streams is often not complete and storage reservoirs within reasonable cost are relatively small. Here many reservoirs have been built for both power and irrigation, and as far as economically feasible, they have been dimensioned to regulate for the variation of stream flow of one season. The snow melts off uniformly by the end of June, quite independently of the amount. In dry years there is often a deficiency which must be made up by operation of steam plants.

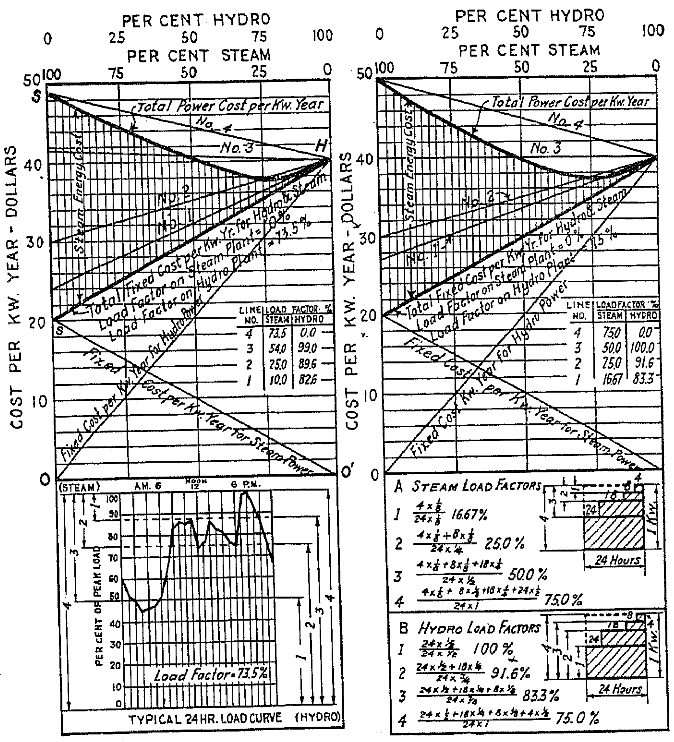


Fig. 2—Economic Division of Water Power and Steam

(A) Steam Load Factors:
(1) 10.0%
(2) 25.0%
(3) 54.0%
(4) 73.5%

(B) Hydroload Factors:
(1) 99.0%
(2) 89.6%
(3) 82.6%
(4) 73.5%

75 0% TOTAL LOAD FACTOR

Assume All Load Over 87.5% Taken by Steam
Steamload Factor (1) = 10.0%
Hydroload Factor (3) = 82.6%

$$\begin{pmatrix}\text{Steam Energy}\\ \text{Cost Per}\\ \text{Kw. Year}\end{pmatrix} = \begin{pmatrix}\text{Percent of}\\ \text{Peakload}\\ \text{on Steam}\end{pmatrix} \times \begin{pmatrix}\text{Steam}\\ \text{Load Factor}\\ \text{Percent}\end{pmatrix} \times \$40$$

$$12.5\% \times 10\% \times \$40 = \$0.50 \text{ Per Kw. Year}$$

Cost of the Steam Energy Generated.

This added to line $s\ H$ (representing the total Fixed Cost of Hydro & Steam) at the proper abscissae, gives a point on the Curved Line $S\ H$ which represents the sum of Fixed & Energy Charges for any division of the Load.

NOTE:

(A) Steam load factors are derived by working from top of Diagram down, making steam supplant hydro in increasing amount.

(B) Hydroload factors are derived by working from bottom of diagram up, making hydro supplant steam in increasing amount.

BASIS:

Hydro cost per kw. year = \$40.
Steam fixed cost per kw. year = \$20.
Steam energy cost per kw. year and 100% load factor of steam plant = \$40.
Demand costs = Fixed costs for hydro and steam, including interest, maintenance and depreciation, and also all operators, coal, water, etc., to bring the plant up to speed and voltage. ready for business.
Energy costs are only those costs which increase with kw-hrs. output.

Large reservoirs have been built in the Sierras for irrigation purposes, city water supplies and for storage of auriferous gravel after gold has been extracted by

hydraulicing. Power is now generally developed through the head created by the dams and can as a rule be profitably disposed of to a nearby network of a utility.

In much of the Sierra territory, water is extensively used for irrigation. The demand for power and irrigation does not coincide throughout the entire irrigation season and a certain amount of conflict exists. In some cases this can be remedied by after-storage and floodstorage at elevations below power developments. The state of prosperity of the irrigationists and the stand taken by yellow newspapers and local demagogues have much to do with each particular case, but eventually, necessity will demand conservation of all water resources and a fair use to the greatest number.

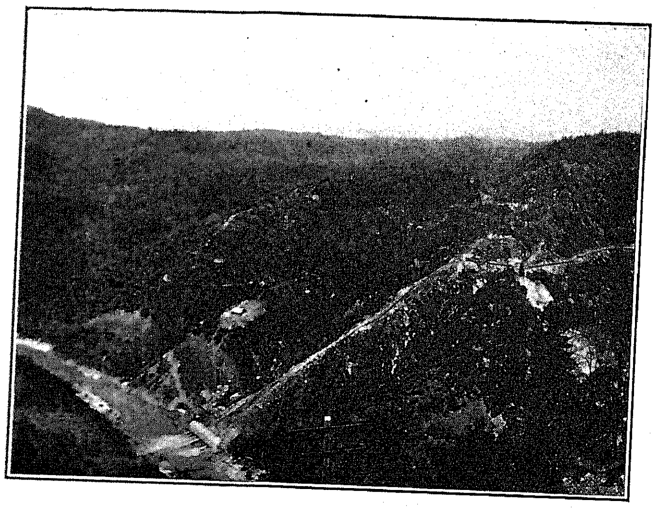


Fig. 3—Colgate Power House
Showing pressure pipe, forebay and flume
Head: 700 ft. Installed capacity: 16,000 kw.
Pacific Gas & Electric Co.

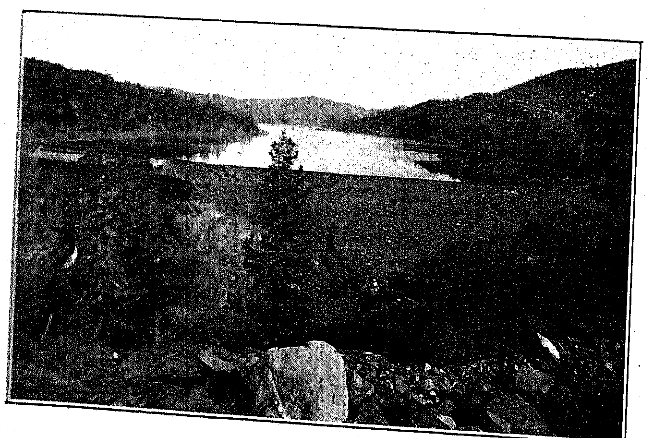


Fig. 4—Fordyce Dam and Reservoir (Rock Fill Type)
Spillway at left
Water is taken by flume and ditch to Lake Spaulding
Pacific Gas & Electric Co.

Character of Developments

A great variety of types of hydroelectric plants have been built in this large territory and each new location has its own peculiar problems to solve, governed principally by the following conditions and requirements: Topography, storage of flood water, size of the transmission network into which it feeds, size of the plant itself, loadfactor of the power market, time characteristic of the water supply, etc. For example, plants which have a natural uniform water supply are natural base-load plants, and those which depend on storage are built to handle relatively low-load factors

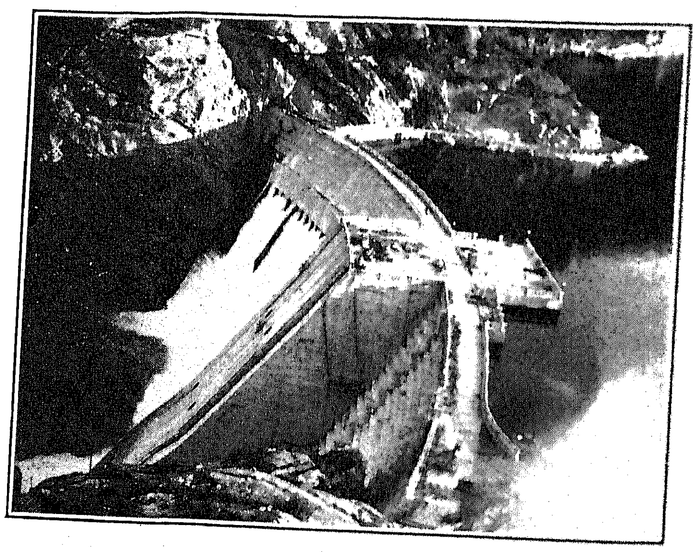


Fig. 5—O'Shaunessy Dam
Gravity concrete type, stores water at Hetch Hetchy for water supply of City of San Francisco.
Dam is built to a height of 226½ ft. above stream level, the ultimate height above stream level will be 312 ft. The foundation extends 118 ft. below stream level. Present structure contains 375,000 cu. yd. of concrete and will store 67 billion gallons of water, and ultimately will store 113 billion gallons.

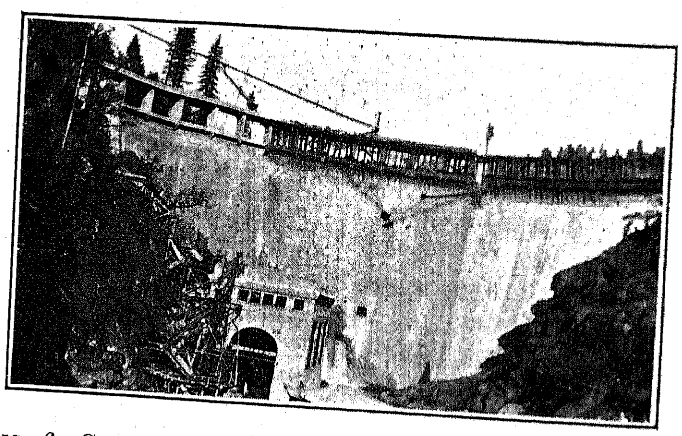


Fig. 6—Constant Angle Arch Dam with Power House at Bullards Bar
Dam stores gravel after gold has been extracted by hydraulicking. Power sold to Public Utility.
Height of Dam from Foundation: 183 ft. To Bridge Floor: 202½ ft.
Thickness at top just below Spillway Lip: 6 ft.
Thickness at bottom: 40 ft.
Overflow: 65,000 sec. ft. Depth of Overflow: 15 ft.
Installed Capacity: 8,000 kv-a.

in varying degree. The lower the load factor for which a plant may be built, the greater its construction cost per average unit of power, and a condition is approached where it pays to operate a steam plant to carry peak loads in conjunction with a hydro plant. The division of load between hydro and steam is best worked out by laws of economy and an example is given in diagrammatic form in Fig. 2.

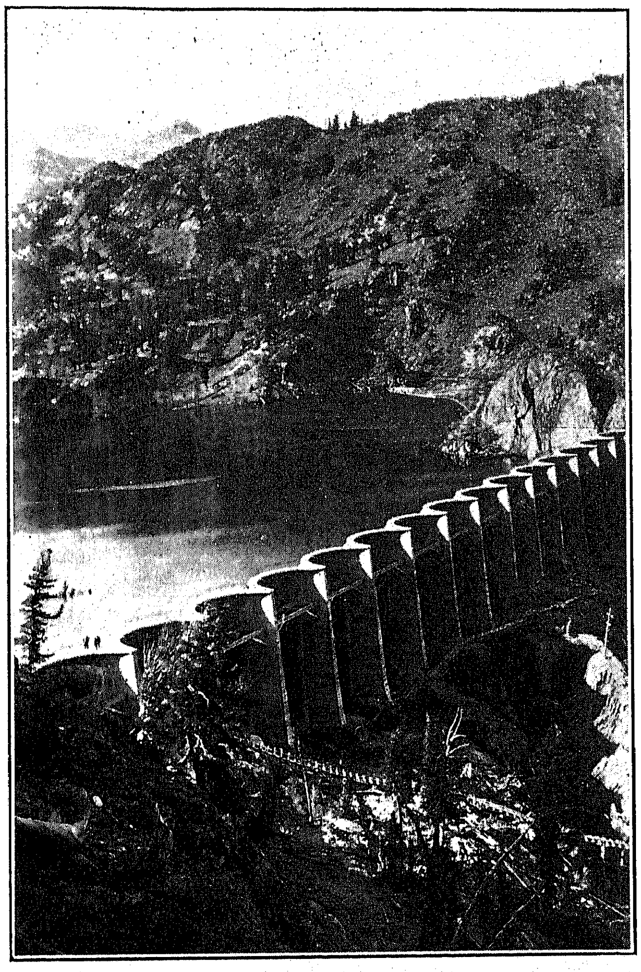


FIG. 7—MULTIPLE ARCH DAM AT GEM LAKE

Constructed at an elevation of 9050 ft. on the eastern slope of the Sierrsa in the vicinity of Mono Lake.
Height: 84 ft.
Length: 720 ft.
Span of Arches: 40 ft.
Stores water for Southern Sierras Power Co.

FIG. 8—ROLLER SECTION AND SLUICE GATES IN DIVERSION DAM FOR UPPER FALLS STATION, SPOKANE, WASHINGTON WATER POWER CO.

Steam plants are further considered to have the important function to give service insurance for large load centers and to build up the load on a system during intervals of construction of hydro plants. The use of steam plants is not uniform over the entire area. Some of the utilities of eastern Washington get along without them. This is possible when the location of the water power is favorable, transmission short, and water supply certain every year, or the minimum flow is above the requirements.

FIG. 9—ELDORADO CONDUIT

Concrete-Lined Ditch with Wood Flume in Distance
Western States Gas & Electric Co.

Interconnection between companies is becoming quite general, although the amount of power which is interchanged is not very great. The equipment required for a true interchange of power is not in all cases suitable without modification, and the interconnected lines are small in capacity.

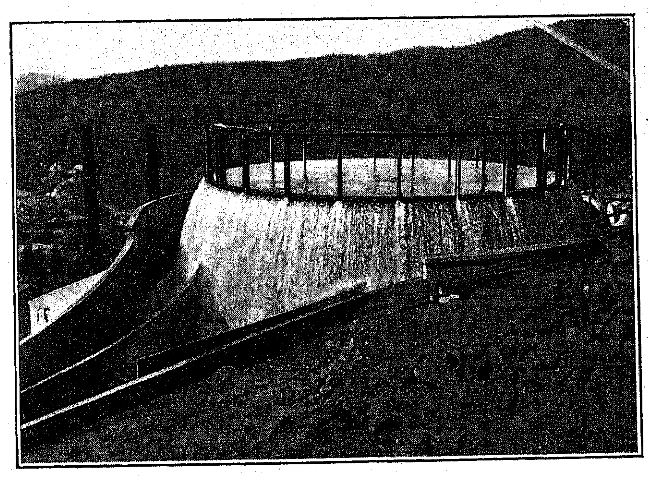


FIG. 10—SURGE TANK

60 ft. Diameter. 60 ft. Depth. Connects to 14 ft. Diameter Tunnel and Two 10 ft. 9 in. Diameter. Pressure pipe Pit River No. 1. Pacific Gas & Electric Co.

Structural Features

Dams. Storage and diversion dams of all types have been built, the choice of type being generally determined by foundation conditions, available construction material, spillway requirements, overflow, and the psychological effect on human beings. The design of dams must now be approved by state authorities. In the early days, water systems were developed

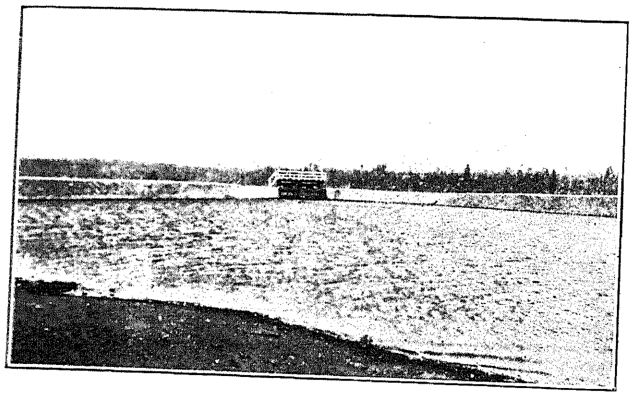


Fig. 11—Forebay
At End of Open Canal.
Connects to 10 ft. Diameter Pressure Pipe.

for washing gold-bearing gravel, and dams built for this purpose are now functioning for the more modern use of power and irrigation. Some of these old dams were forerunners of what is known as the rockfill type of dam. The gravity type of masonry dam has been used here under similar conditions as elsewhere and

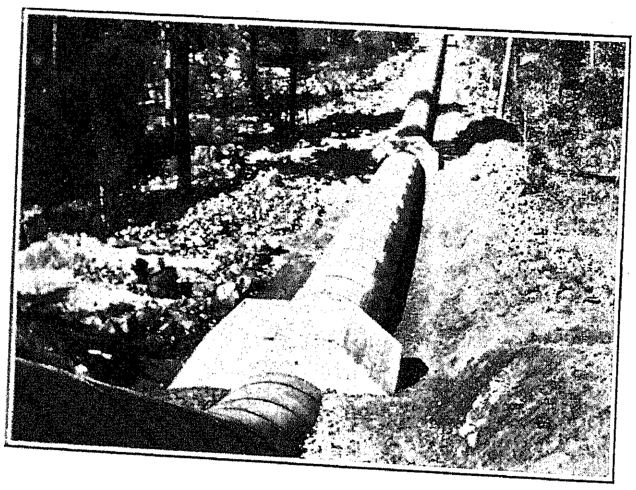


Fig. 12
Riveted Steel Pressure Pipe 9 ft. Diameter.

recently built in large size for power, irrigation and water supply. The earthfill dam is often found and is characterized by the kind of material and method of construction. In regions favored with a good quality of uniform earth, the method is to spread the material in shallow layers and to compact them by wetting and rolling. With a material requiring segregation, the hydraulic-fill method is much in favor. Core walls are used, should more pervious material be encountered.

Since the cost of storage dams is a large item in the total cost of construction, the desirability for a reduction in cost has brought out some new types, namely,

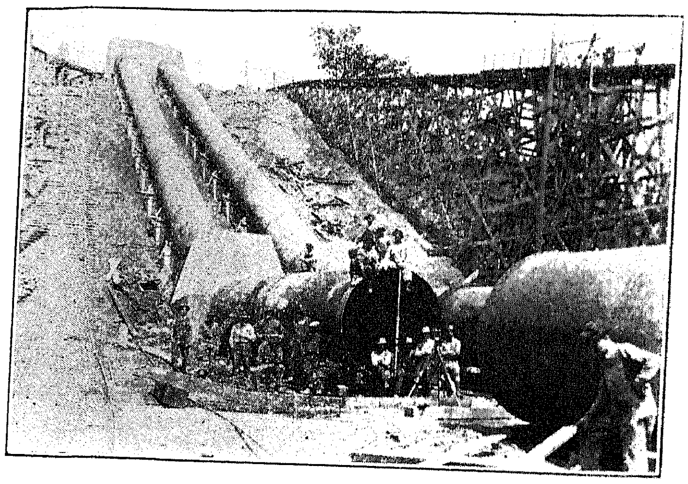


Fig. 13—Pipe Line Construction (Welded)
Pipes are here 8 ft. in Diameter. Expansion joints just below anchors.
Pit River No. 1 Development of Pacific Gas & Electric Company.

the multiple-arch dam and the constant-angle arch dam.

The multiple-arch dam consists of a number of inclined arches, resting on buttresses spaced uniformly across the site. The constant-angle arch dam is a single arch and is especially suited to a V shaped, rather

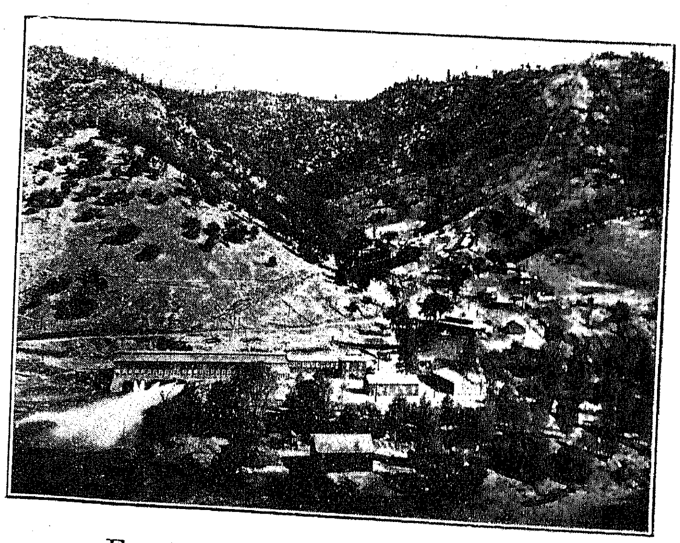


Fig. 14—Electra Power House
Head 1466 ft. / 1266 ft. { Supplied from two sources. 200 ft. apart in elevation.
Installed capacity: 20,000 kw. Pacific Gas & Electric Company.

narrow canyon. It is built up of a series of superimposed arch slices, which, as far as possible, are subtended by an angle of 133 degrees. This gives a minimum of masonry for a given unit of stress, considering the arch slices are uniformly loaded in a radial direc-

tion. This type of dam is in use with overflows as high as twenty feet. Both types require an unyielding rock foundation. Considerable investigation has been made in a theoretical way of the stress distribution in these new types of dams, and some experimental work on a large scale is now underway to supplement the work already done. This is highly to be commended since the material (concrete) is, from

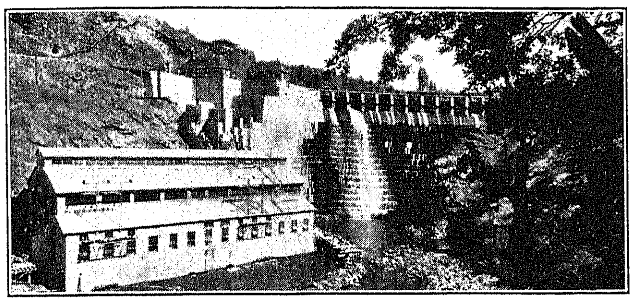


Fig. 15—Copco Power House and Dam
California-Oregon Power Co.

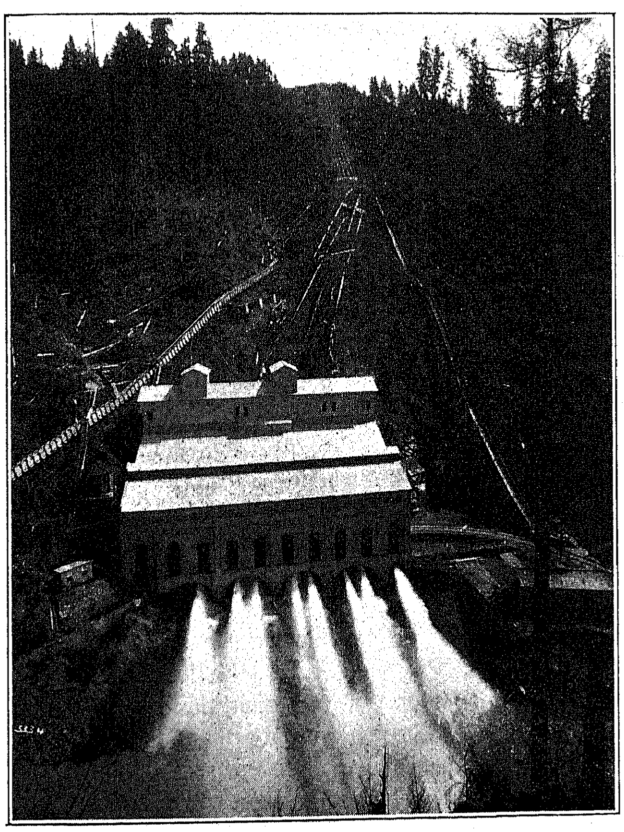


Fig. 16—Electron Power House
Puget Sound Power & Light Company.

the nature of its manufacture, not in the same class as, for instance, steel or cast iron. More than 20 dams of these two types have been built in the west and none has failed. The reduction in cost over a gravity type of dam is due to correct placing of the material to transmit the load to the foundations. In the gravity type the bulk of the material acts merely as weight.

Conduit System

Since the majority of the plants built are of the medium and high-head type, conduits or aqueducts are used to concentrate the fall of the stream at a favorable point. In some cases, such conduits are of considerable length, one, for example, being 24 miles long for a static head of 1900 feet, but in more favorable locations the same head may be created by a conduit of but a few miles in length.

Under the subject of "dams" it was mentioned that water systems were built in the early days for mining purposes. Water released at the dam was taken for many miles in ditch and flume, built along the mountain side to the place of use. Many of these old ditches have been repaired and enlarged since the days of hydraulic mining and now serve for hydroelectric development.

The conduit system has undergone a certain amount

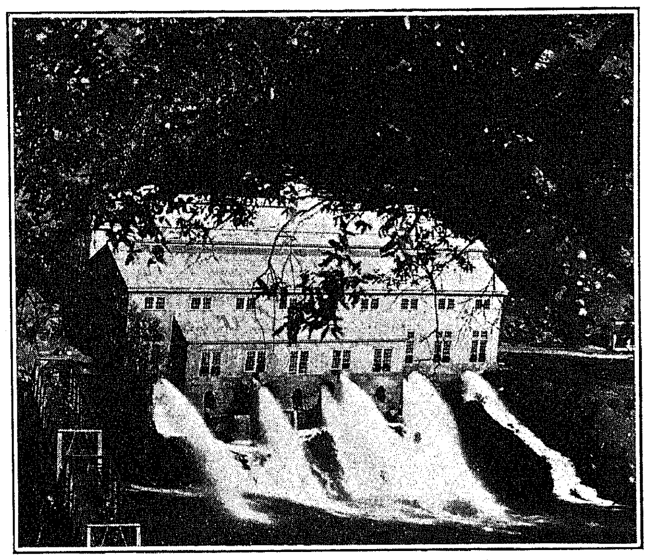


Fig. 17—Stanislaus Power House
Head: 1498 ft. Installed capacity: 40,000 kw. Sierra & San Francisco Power Co.

of evolution, and types of structures have tended toward more permanent construction. Some of the governing factors in the selection of conduit types have been length, topography, climatic conditions, construction cost and service conditions. The size of a conduit for economic power production is so determined that the annual charges, plus the value of the lost power due to friction, are a minimum. Closed and open conduits are used, each having its field. Where the climate is severe, with much snow and danger from slides, modern plants use tunnel, where less severe, pipe lines of steel, concrete or wood are used. The closed system may or may not be operated under pressure. Some plants have open systems, irrespective of above considerations, but generally the open system is used where the conduit is long and the climate mild.

When a conduit is of the closed type and not too long,

it generally terminates in a surge tank; when of the open type, in a forebay. In a closed conduit the cross section should be large enough to carry the maximum load, which at any time may come on the plant. In the open conduit it need only be large enough to carry the average flow of water demanded during 24 hrs., the peak load being carried by the forebay.

Surge Tanks and Forebays

The closed conduit with surge tank has come into

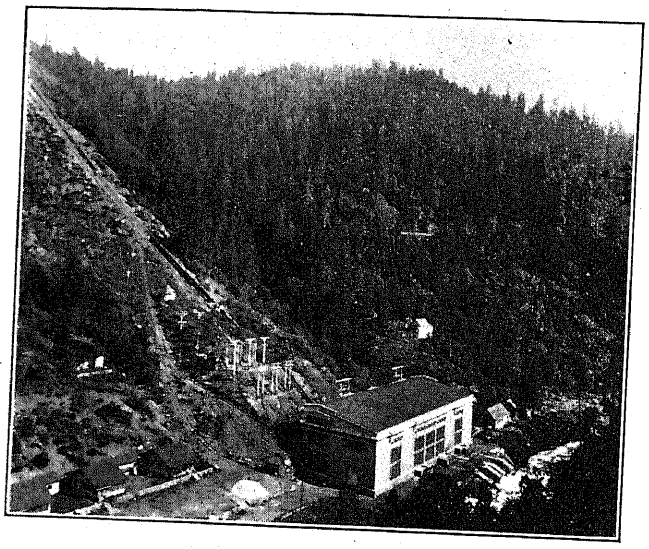


Fig. 18—Drum Power House
Head: 1375 Ft. 3 Units of 12,500 kv-a. Total 37,500 kv-a. Pacific Gas & Electric Co.

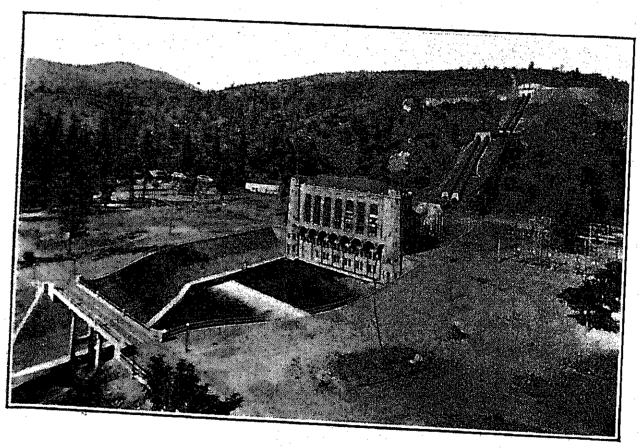


Fig. 19—Pit River No. 1 Power House
Head: 454 ft. Installed Capacity: 70,000 kv-a. Pacific Gas & Electric Company.

more general use during the last few years and the theory of the function of surge tanks in operation is now well understood. The tanks are given various shapes, depending on the selected duty cycle, which is given by the time characteristic of load changes and the general operating conditions of a system, including accidents and the failings of operators. The difficulty in selecting the proper surge tank is not so much with the theory as it is with the duty cycle. It has been built as a plain cylinder, a truncated cone right side up or inverted, or of the shape of an hour glass, Fig. 10.

Some tanks are built with their rims at an elevation high enough to stop completely the flow in the conduit upon complete rejection of load, while others have the rim developed as a spillway with capacity ranging from small amounts nearly to full capacity of the conduit. The conditions found at the intake of the conduit is often the deciding factor, namely, where stored water must be conserved in a reservoir and a cessation of flow desired or where there is no reservoir but the intake is directly on the stream and conservation of water is no object.

Forebays in connection with open canal systems of considerable length are, if possible, made larger than required for 24-hour regulation, in order to give service insurance, should the conduit system become inoperative due to snow or landslides, etc. Unless topographic conditions are favorable, the requirements for forebay construction often necessitate large expenditures. They may be formed by a gully closed by a

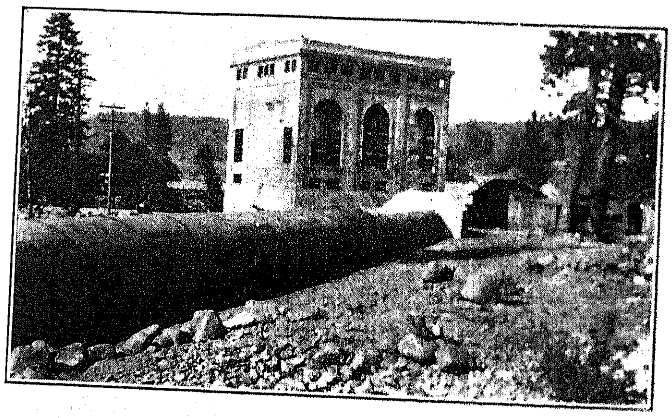


Fig. 20—Hat Creek No. 1 Power House
Pacific Gas & Electric Co. 210 ft. Head. Capacity: 10,000 Kv-a.

dam of any type, or by an excavation made in a suitable and obliging mountaintop, or merely an enlargement of the canal section along the hillside. All forebays, except where the conduit is very short, are equipped with a spillway, generally in the form of a weir, on one or both sides of the conduit just before the forebay is entered. Where water carries sand or grit in suspension, sandtraps are provided. They are designed to retard the water to a velocity which will permit suspended matter to sink to the bottom and to skim off the top layer of clear water.

Pressure Pipes

In most developments the pressure pipe is a considerable item in the total installation and for high heads and long lines it is one of the most important ones, and is given very careful consideration in design, installation and operation. In the great majority of cases and nearly always for heads greater than 150 feet, the pressure pipes are made of steel. Wood stave pipe

has been used to some extent and concrete pipes are few. A combination is sometimes used of wood-stave construction in the upper part and steel below. Skill and care are required in the installation of wood-stave pipe and, if properly done and the staves kept completely saturated, the pipe will give good service for an indefinite time. It is preferred to keep the staves away from the soil by means of cradles of concrete or durable wood (cedar and redwood). Steel pipes are either riveted or hammerwelded. Riveted pipe has been used since the beginning and given excellent service, Fig. 12. The welded pipe was imported to a small extent before the war, but is now made in this country, and some of the recently built plants have used this type for both high and medium heads, Fig. 13.

In pipe of large diameter and light or medium metal,

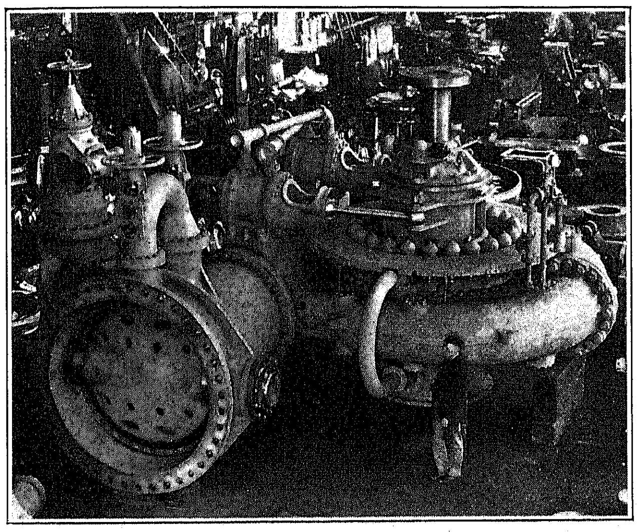


Fig. 21—High Head Reaction Turbine

Capacity: 35,000 Horse power
Effective Head: 857 ft.
Speed: 514 rev. per min.

To be installed in the Oak Grove Power House of the Portland (Ore.) Railway, Light & Power Co. This is at present the world's highest head reaction turbine.

the loss in head, due to friction, is about equal for the two kinds of pipe, but when riveted pipe is small in diameter and the metal heavy, the space occupied by straps and rivetheads is relatively large and the loss greater than for welded pipe which has flanged joints and presents a smooth bore throughout.

Welded pipe is installed with greater ease and, due to the fact that expansion joints can readily be furnished with the pipe and of the same material, it need not be buried. Being above ground, its condition from the outside is always known and repair and painting work greatly facilitated.

Riveted pipe is often installed in an excavated trench and expansion joints are not required. Some installations, with pipe partly buried, use expansion joints of built-up material which have given good service.

The location in the field of a pipe line is very important, but often conditions are such that there is but one choice and the design and construction must over-

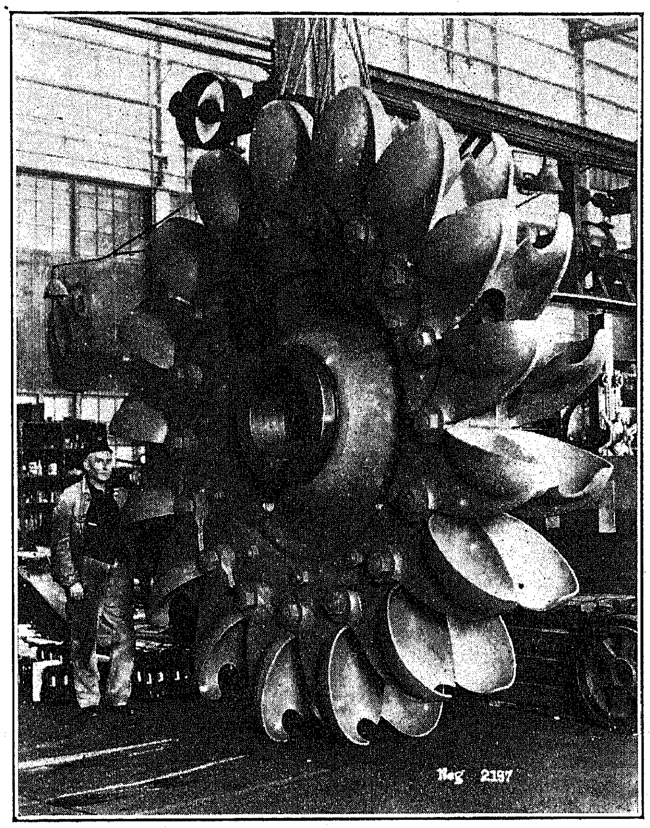


Fig. 22—Impulse Wheel

Capacity of wheel: 12,500 horse power. Two wheels will be mounted on same shaft with generator between them.
Effective head: 1,250 ft. Speed: 257 rev. per min.
Will be installed in Mocassin Power House of Hetch Hetchy development for San Francisco water supply.

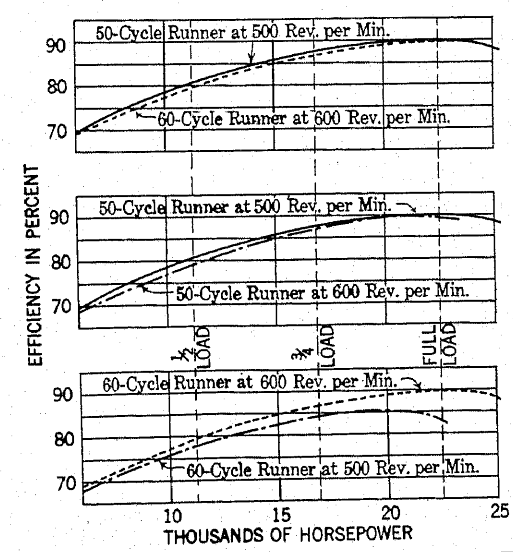


Fig. 23—22,500-Horse Power Pelton Reaction Turbines for Kern River No. 3 Plant. Southern California Edison Company

Effective Head 780 to 820 ft.

come many difficulties. Anchors and their location are of much greater importance with the pipe above

ground than with pipe buried. Pipe lines without expansion joints are suited best to a broken profile and horizontal angles; pipes having expansion joints are suited best to a straight course between intake of pipe and turbine.

In the design and manufacture of riveted pipe the best boiler shop practise is followed and, due to the experience gained with the great number of pipe lines

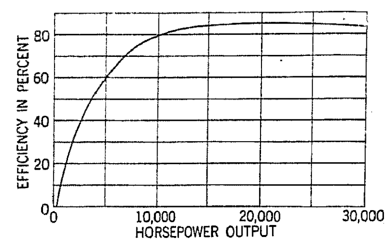


FIG. 24—EFFICIENCY CURVE FOR 25,000-HORSE POWER IMPULSE TURBINE

Effective head 1250 ft. Speed 257 rev. per min.
The Pelton Water Wheel Co., San Francisco, Cal.

installed, the art of building riveted pipe is highly developed on the coast. For the lighter sections, plates are lapriveted in both longitudinal and circumferential joints, and the best practise is to make the downstream end of a course an inside one; for heavier work buttriveting is used. The efficiency of a joint is determined by rules similar to those which govern for boilers. By efficiency is meant the ratio

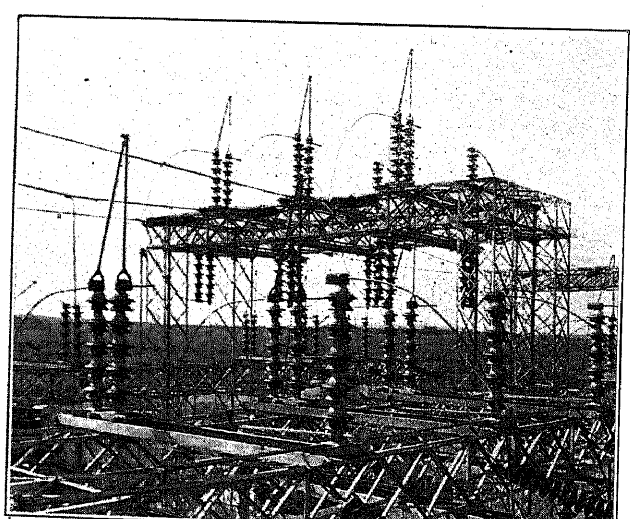


FIG. 25—220,000-VOLT DISCONNECTING SWITCHES
Pacific Gas & Electric Co.

which the strength of a unit length of a riveted joint has to the same unit length of the solid plate.

Careful tests and inspection are made of the rolled material for both riveted and welded pipe, and it is considered imperative to test with suitable overpressure every section of welded pipe at the manufacturer's shop before shipment. In spite of this care, defective material and workmanship creep in and some pipes have burst and caused damage, but the cause of bursting may not always have been due to defects, but to faulty operation or failure of relief valves.

Long pipe lines contain moving water columns of thousands of tons weight, which must never get out of control. If control is lost, disaster is swift and certain. Aside from the control obtained through the governor, stop-gates or valves are placed in long lines both at top and bottom. For short lines and single-unit plants the valve at the bottom is sometimes omitted. In the older plants the gates at the top were simple devices often made of wood; in later plants, valves have been introduced directly in the pipe line, just after leaving the forebay or surgetank. Both slide valves and butterfly valves are used, and the latter seem to be gaining in favor and so far have given very good results. The butterfly valve is sufficiently tight when carefully made, and the body of its closing disc can be made of efficient form with respect to the flowing water.

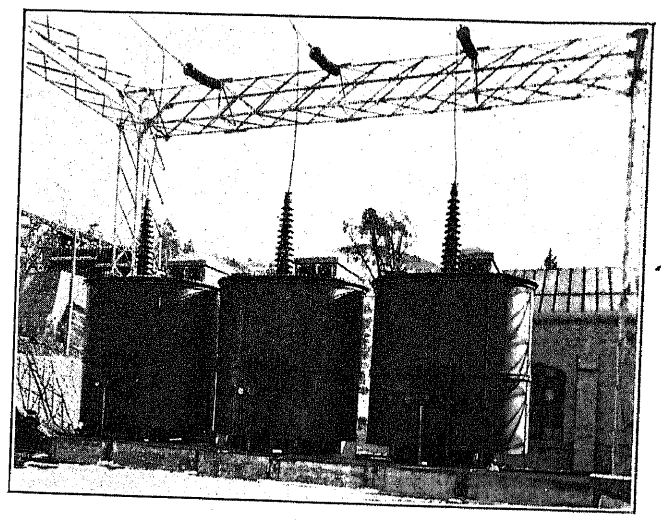


FIG. 26—50,000-KV-A. BANK OF TRANSFORMERS
11,000 volts to 220,000 volts. Grounded neutral, single terminal. At Pit River No. 1 Power House. Pacific Gas & Electric Co.

At the power house end slide valves, butterfly valves and plunger valves have been used with uniformly good results, with the exception of the control for the plunger valve, which has not been positive and powerful enough to the degree required by the high pressures and long lines used here.

POWER HOUSE AND EQUIPMENT

Building. The architectural treatment of power house exteriors and interiors is given considerable attention. It is not always that engineer and architect agree. A power house is neither a bungalow, a spanish hacienda, or a cross between an armory and a church, but should receive its characteristic lines from what it contains and from the purpose it fulfills. When such a view is coupled with the inspiration to be had from nature at the site, a proper basis is laid for a good solution.

Turbines and Waterwheels. Until a comparatively few years ago, plants in the southern half of the territory used the impulse turbine almost exclusively, while in the northern territory, due to more moderate heads, the reaction turbine was generally used. A turning point came with the installation of the Centerville turbine in 1906, which was the pioneer high head machine (559 ft.) with a maximum capacity of 9000 horse power. Since then reaction turbines have continued to extend in use for the higher heads which were formerly covered by the impulse turbine. The reason for this change is the better efficiency which may be obtained and the full use of the available head by means of the draft tube. Before the advent of the successful thrustbearings almost all plants had horizontal shaft arrangements, but now the large modern installations of reaction turbines are vertical. The impulse turbine, however, continues to be arranged with a horizontal shaft.

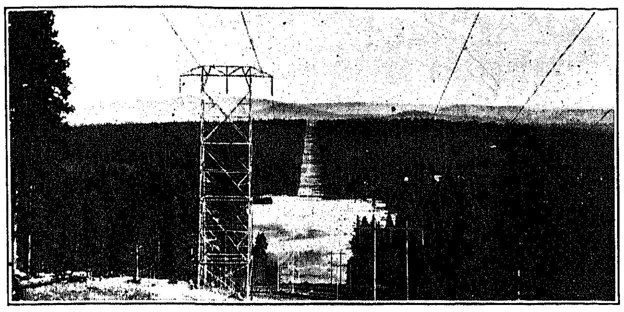


Fig. 27—Pit River Transmission Line
220,000 volts.

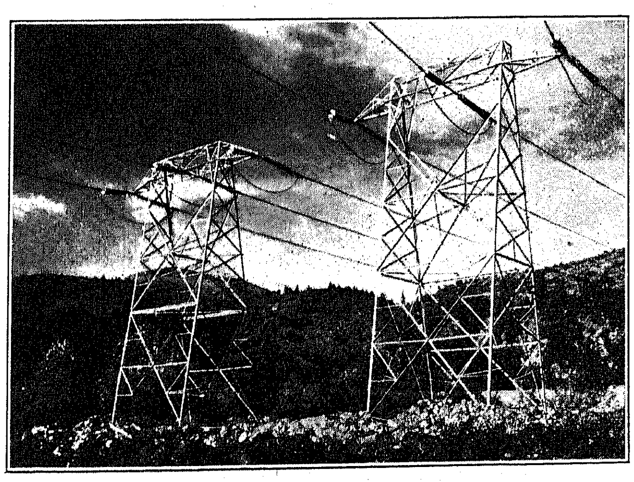


Fig. 28—Anchor Towers, 220-kv. Lines
Moderately severe climatic conditions. 19 ft. center to center of conductors. Pit River Lines. Pacific Gas & Electric Company.

With the higher heads, the reaction wheel is exposed to a great pressure difference between inlet and outlet of the runner, and manufacturers give much attention to details of the leakage path. A recent development is the rubber seal ring. By virtue of water lubrication, which is excellent for rubber, the clearance between the running and stationary parts can be made so small that the surfaces may be said to touch. This development is undergoing time trial on wheels in service and is being watched with interest.

There has been some trouble from vibrations in large reaction turbines, indicating that the water passages do not transform the energy correctly. There may be many causes for this, but it would seem that a turbine must be carefully proportioned to give its output at best efficiency with the amount of water available; not for example with one and one-half times this amount, for if the additional water is to be had only occasionally, the turbine at all other times is working with a poor efficiency. Draft tubes may be the originators of much of the vibration and although there has been great improvement in draft-tube design, some draft tubes do not give the correct impression when viewed in action on large units. Whatever the swirl of the water in the draft tube may be, the tendency to form vortices should be suppressed completely.

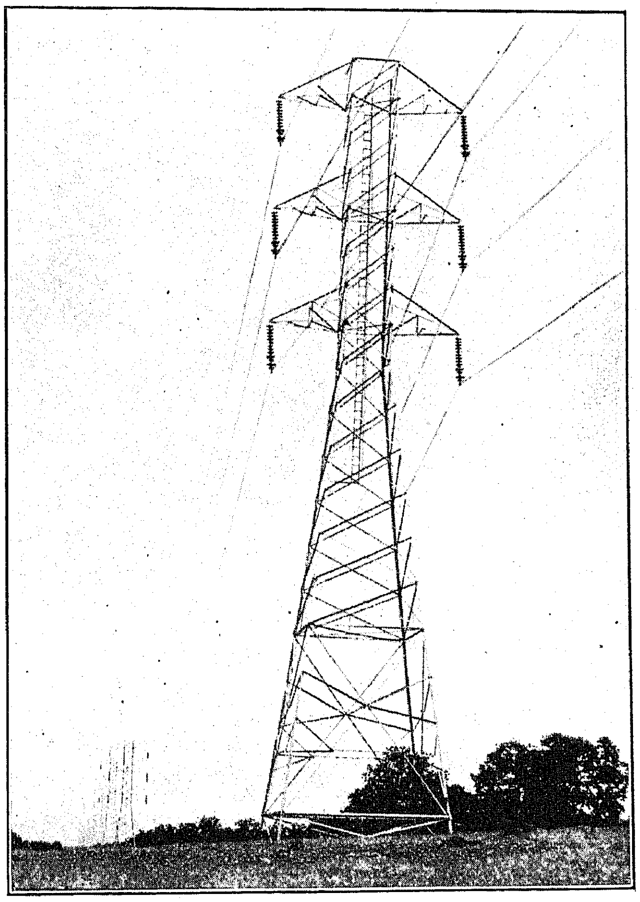


Fig. 29—220,000-Volt Pit River Line
Length of line: 202 mil. Double-circuit transmission tower. Pacific Gas & Electric Company.

For the higher heads and long pipe lines relief valves are used, and depending upon the conditions, are proportioned to discharge from 100 per cent to lesser

amounts of the water used by the turbine. The opening of the valve is mostly governor controlled and opens synchronously with the closing of the turbine guide vanes. The closing is gradual at a predetermined rate given by the desired pressure rise, which is arrived at through computation and by experimental adjustment. In some plants with moderate head and short lines operation has been satisfactory without relief valves. Depending upon the head, the writer uses as a guide the following permissible pressure rises:

For heads up to
150 feet ....40 to 50 per cent
500 " ....20 to 25 per cent
over 800 " ....15 to 20 per cent

The energy of the spouting jet from the relief valves

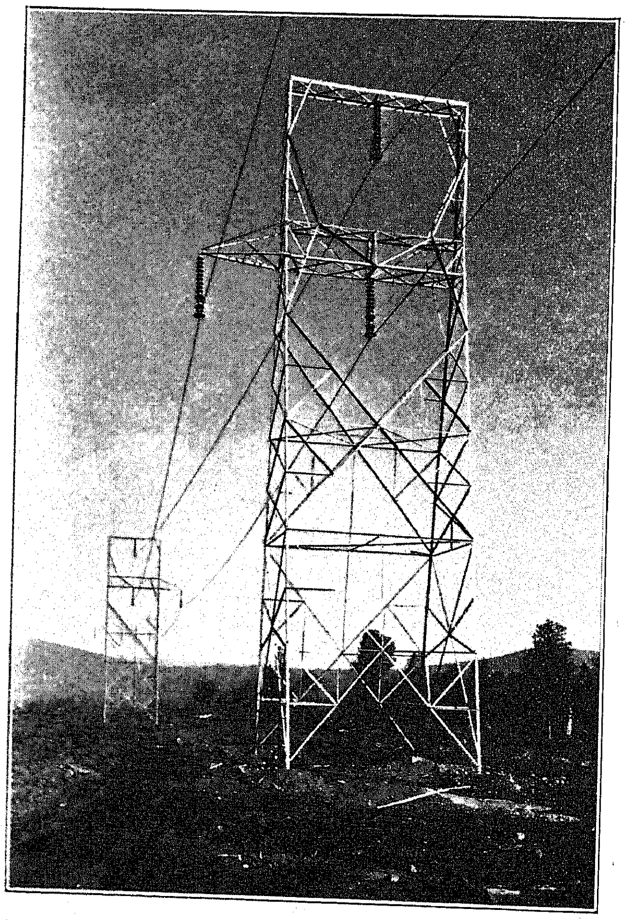


FIG. 30—TRANPOSITION
220,000-Volt Pit River Line. Pacific Gas & Electric Company.

is either dissipated by free discharge or absorbed by vortex baffle chambers. In restricted locations good results are obtained by vertical discharge of the jet into baffles, which direct the jet through an angle of 270 deg. towards the center of the jet from all sides.

The impulse wheel will probably hold its place for heads above 1000 ft. for a long time to come, units of 30,000 horse power are in use at that head, but conditions for best efficiency improve with increasing head. Some efficiency curves are given for both types of wheels to show what is being accomplished at these high heads, Figs. 23-24. The impulse wheel is used for units of small capacity at much lower heads; they have few moving parts and are rugged in these sizes and for that reason preferred for exciter drive.

The governing of impulse wheels is accomplished in different ways. The jet may be deflected by movement of the nozzle body itself, or by a jet deflector which cuts into the stream, or by movement of the needle. When the movement of the needle is under the control

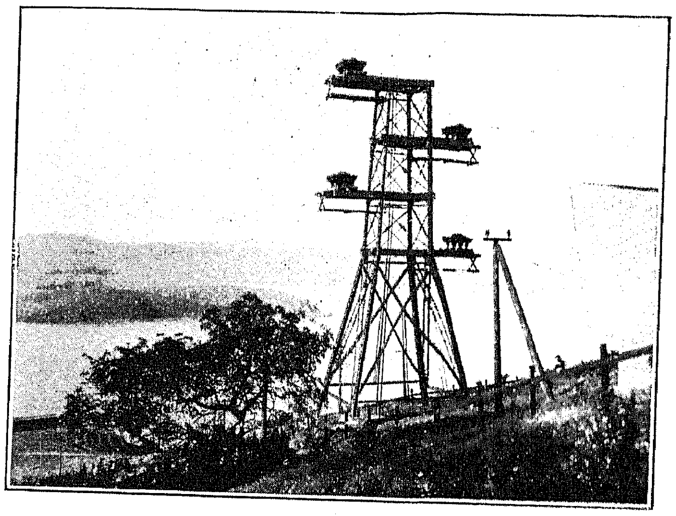


FIG. 31—CARQUINEZ CROSSING
Pacific Gas and Electric Company.

of the governor, an auxiliary nozzle is added, the needle of which is opened when the main needle closes to follow a reduction in load. The closing of the auxiliary needle takes place at a predetermined and adjustable rate through the intervention of a dashpot and heavy springs.

Generators. 60 cycles is the standard frequency, with the exception of a large utility in the south. This utility uses 50 cycles, but has some equipment suitable for both 50 and 60-cycle operation.

The mechanical arrangement of generators is given by the prime mover and is in all respects similar to installations elsewhere. In large plants, by means of suitable housings and ducts, the cooling air is given a definite path to and from the outside, independent of generator room. This method makes the control of air currents in case of accident and the installation of fire extinguishers much easier. Clean air can usually be had without filtering.

As transmission voltages increase and lines become longer, a certain number of the generators of a hydroelectric transmission system are required to handle the charging current of an unloaded line. This charging current is a magnetizing current which produces a field superimposed on the field of the rotating direct current magnets and in phase therewith. The excitation from the exciter machines is, therefore, lowered

for this condition as much as possible, yet it must be positive and definite. The sum must not be great enough to raise the voltage further, which would increase the charging current and build up a still higher voltage. The resulting kv-a. load on the generator must not exceed its capacity.

It has been proposed for many years, and it is now becoming common practise, to protect generators against internal electrical breakdown by means of balanced relays. Governors are usually provided with a load-limiting device, and in addition, when a fault develops in the winding, they are actuated to automatically close the turbine guide vanes to reduce the flow of energy to the fault.

The arrangement of the exciter varies almost with each plant and is considered in connection with its selected prime mover or drive. In some of the large modern plants with the vertical shaft arrangement of the main units, the exciter is mounted directly on top of these and may have a capacity to excite all units or only the machine to which it is attached, depending upon the number of main units in the plant. A reserve exciter set is usually provided in either case, driven independently by a separate small hydraulic turbine and an induction motor. The control of excitation currents for long high-capacity lines is by broad range regulators and both hand and automatically-controlled field rheostats, and has reference to the expected changes in load, the arrangement of synchronous condensers in the network, and the particular characteristics of the line.

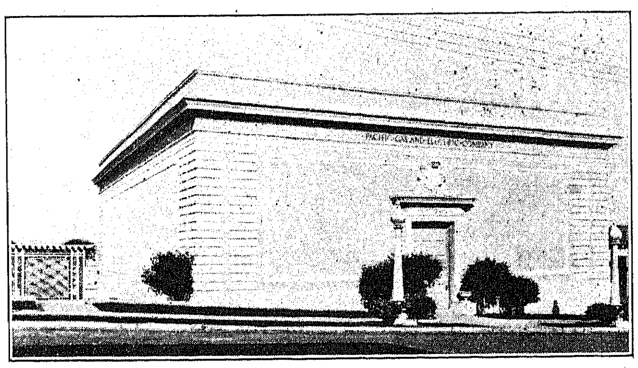


Fig. 32—Typical City Substation
Station K in San Francisco of Pacific Gas & Electric Company.

Switch Gear. All low-tension switch gear, as well as its arrangement, is of the same design as found elsewhere in the country. In some recent installations, all oil switches between the generators and transformers are omitted, resulting in remarkable simplicity. The isolated-phase system has not yet been employed.

The high-tension gear is more and more being installed out of doors for potentials of 110 kv. and higher, and with properly selected insulation, results have been good. The oil switches and breakers for high voltages assume truly large proportions, and the quantity of oil they contain may be as much as for a 20,000-kv-a. transformer. Most of these oil switches have two breaks per pole, some have four or more, and the movement of the contacts may be either vertical or horizontal. Switches are in use with or without the so-called explosion pots. High-tension oil switches may be due for some improvement; they are fitted with ponderous mechanisms and have slow acceleration of moving contacts. It is not expected that the

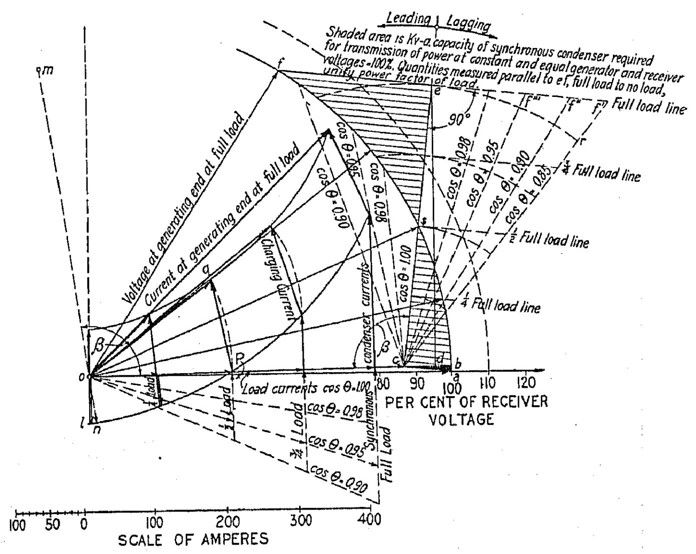


Fig. 33—Regulation Diagram for Transmission Lines

size of oil-switch containers can be much reduced, because of the a priori requirement for electrical clearances.

The chief reason for placing the high-tension gear in the open is, of course, the reduction which may be obtained in first cost, due to the elimination of large housing structures. Some companies use an arrangement whereby a high-tension oil switch may be by-passed by means of disconnecting switches, so that the line may be kept in service while the switch is isolated for inspection, cleaning and repairing. High tension disconnecting switches are generally operated from the ground by grounded mechanisms; that is to say, the ground end of an insulator assembly is usually the point of application for operating, not the live end. Some companies, however, prefer the live end. High-tension oil switches and disconnecting switches have been manufactured on the coast for many years, which is the result of a need made necessary by the early high-tension transmission lines in California. High-tension switch yard arrangements are kept as simple as possible, in order that operators may have clear evidence of operations.

Transformers. With the higher voltages and where it is not too difficult to provide the necessary space, transformers are often installed out-of-doors with the high-tension switch gear. The question of safe insula-

tions in the bushings outside the tank is one which time alone can answer, and some day transformers may be moved inside again.

The grounded 3-phase transmission system is almost universally used, the desire being to secure a stable neutral and fixed potentials from terminals to neutral. The transformers are almost exclusively single-phase, and generally at generating end transformers are two-winding with delta connection on the low-tension side. Usually both terminals of the high-tension windings are brought out, but in some recent high-voltage work one terminal was grounded to the case, so only one terminal is conspicuous through the high-tension bushing. At the receiving end both two-winding and three-winding transformers are used, depending upon connections to other high voltage lines. If connected to other high-voltage lines, the three-winding arrangement is required for star-star connected auto transformers, the third winding serving for circulation of the single-phase triple-harmonic currents and for operation of synchronous condensers. In extra high-voltage work the space required for insulation and electrical clearance from winding to winding, becomes larger and the electro-magnetic interaction of the windings is influenced thereby, and particularly for transformers with a number of taps, care is required in the design to secure the desired regulation and potential distribution. The two-winding transformer is better off in this respect than the three-winding transformer, and an endeavor is being made to reduce to a minimum the number of taps on important transformers.

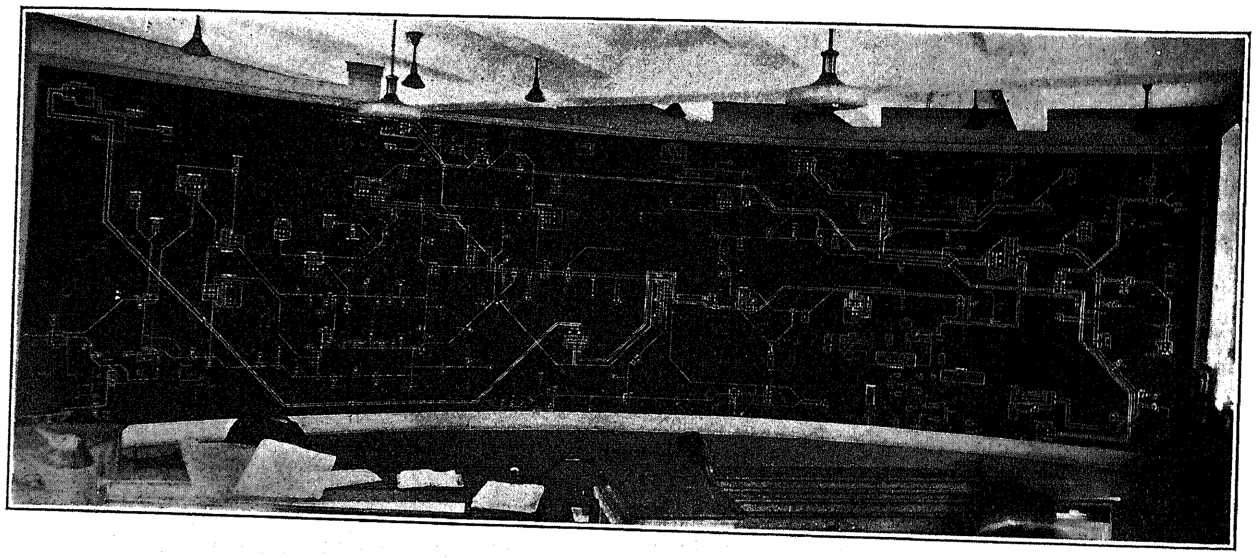


Fig. 34—Load Dispatcher's Operating Board
Transmission System of Pacific Gas & Electric Company.

Transmission Lines

In the beginning, all transmission lines were constructed with wooden poles and pin type insulators up to 60,000 volts, but with the adoption of 100,000 volts, the suspension-type insulators with steel tower construction came into use. In the north, where there is an excellent timber supply, the wooden pole construction is still used for the highest voltages. If development were planned far enough in advance of construction so as to take advantage of careful impregnating methods for preserving timber, there would be small reason, indeed, for not using wood pole construction for any type of line. The popular call for permanent construction, as argued from a civil engineering standpoint, is responsible for the development of the steel tower. In the southern half of the territory the steel tower is used for lines of all kinds, but chiefly from 60,000 volts up. The type of tower used, except for minor details, is the same as used in other parts of the country. Where climatic conditions are favorable it is possible to use a vertical arrangement of the conductors, but where less favorable, they are either offset or arranged in a horizontal plane. The most severe conditions to be met with are found in certain districts in the lower regions of the Columbia River, in the vicinity of Portland.

Insulation of high-tension lines has received a renewed and rather extensive investigation, due to the advent of the 220-kv. lines. This work has not ended, and the companies are engaged in collecting facts concerning the operation of this last develpoment in transmission.

High-voltage lines are controlled by aid of rotating synchronous machinery, located at terminal substations. For this purpose there is installed and under order on the Pacific Coast at least 300,000 kv-a. in synchronous condensers. At the present stage of the transmission art, some of the lines could not be controlled in any other way. A simple diagram is given in Fig. 33, by which a clear vision is obtained of the

factor involved in the control of high-voltage lines. A description of this diagram was given in the A. I. E. E. TRANSACTIONS, 1922, page 790, the notation is the same. High voltages are necessary, as it is not possible to transmit large blocks of power economically over long distances in any other way.

SOME RESULTS OBTAINED

It has been previously remarked that the most important hydroelectric plants were located on the western slope of the interior mountain range. The large centers of population which furnished the principal market for power were mostly located on the coast or in the great interior valleys. So, from the very beginning, it was necessary to transmit the power at high voltages over lines of great length, traversing the territory generally from east to west and from northeast to southwest. As these transmission lines passed through the country, it was possible for various industries and the lesser centers of population to avail themselves of electric service, the result of which is a diversified use of electricity fully as great in the small cities as in the large.

Other than serving the large centers of population, the earlier transmission lines furnished power to the important mining industries of California, and the average motor installation for this purpose has always been very large. A little later came the extensive use of the electric motor in pumping water for irrigation, first from wells and more recently directly from the stream or river by means of large pumping plants. Due to the universal use of electricity, the rates for light and power have tended to become uniform over large territories, so that the location of industrial plants is not primarily chosen because of cheap power in any one center. The advantage of such rates is that they tend to prevent population from thronging to congested areas, where the difficulties arising from social conditions are becoming a problem.

It is hoped for the future that the development and distribution of hydroelectric power will continue as in the past, rather than have it undertaken by competing municipalities which, by ambitious advertising of cheap power (tax free), would endeavor to attract industries and people to their already crowded areas.

It is felt that the hydroelectric power industry on the Pacific Coast has distributed the benefits of cheap electricity to the small and large community alike, tending towards a more stable development of the entire region.

The writer wishes to express his thanks to the Power Companies and others who furnished the photographs and the data given with each.

Discussion

For discussion of this paper see page 569.

Recent Developments in Hydroelectric Equipment

BY WILLIAM MONROE WHITE
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OUR times in this century have often been referred to as the Age of Power. The interest now shown in hydroelectric developments almost justifies our renaming the present age "The Hydroelectric Age."

Remarkable industrial growth has occurred in several sections of our country which have had available cheap hydroelectric power.

The Alabama Power Co., conceived by broad-minded men and nurtured by far-seeing capitalists, is seeking to serve the people of the state of Alabama and adjacent sections, and to provide the very sinews by which the energetic and forehanded business men, within the range of its spreading transmission lines, may develop and establish businesses and industries which will mean prosperity and better living conditions for all. One unit in this great plan of development has recently been placed in regular operation at Mitchell Dam on the Coosa River. For one who is familiar with the hydroelectric developments of North America and who has an appreciation for the genius of those who conceive and build such properties, it is inspiring to stand upon the wooded hills, overlooking the site of Mitchell Dam, and to view the manner in which men have fashioned a mountain of concrete between abutting hills, in combination with foundations, waterways, backwater suppressors, superstructure and buildings for housing the hydroelectric machinery, and yet all

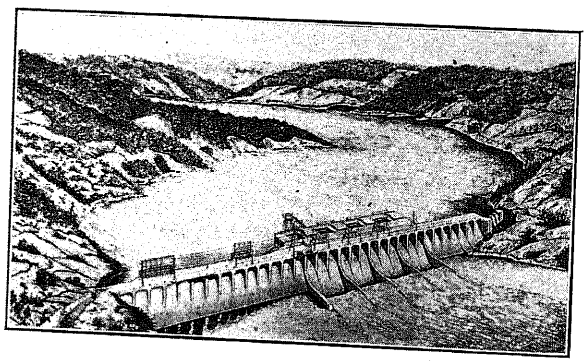

FIG. 1

so devised within the narrow limits of the river's channel at this point, so that no serious obstacle is interposed to the passing of floods which frequently deluge the basin drained by the Coosa River.

The main purpose of this paper is to present to the engineers some facts relating to recent developments in hydroelectric machinery, such as that installed at the power house at Mitchell Dam for economically producing power from falling water. There are installed and now in operation at Mitchell Dam three waterwheels designed and built by the Allis-Chalmers Mfg. Co. of Milwaukee, Wisconsin, each of a capacity of 24,000 horse power when operating under a head of 70 ft., and running at a speed of 100 rev. per min.

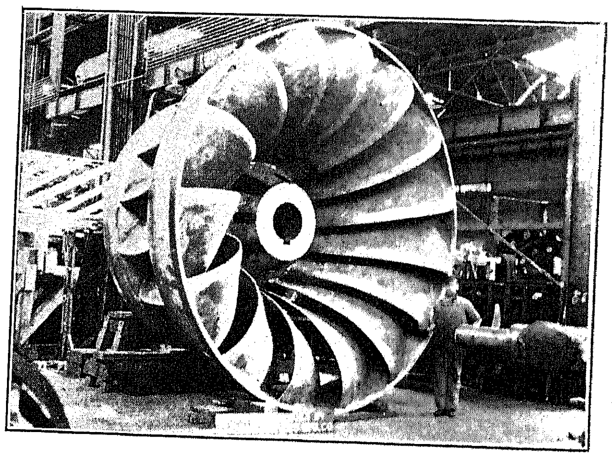

FIG. 2—IRON RUNNER—WEIGHT 105,000 LB. ALABAMA POWER CO. MITCHELL DAM PROJECT

Each of these units is installed in a structure located on the upstream side of the dam, fashioned with waterways of ample capacity, leading to and from the turbine and with walls that support the superstructure. The waterwheels must be located near the level of the tailwater. The generator and electrical equipment are desirably located above the elevation of high water upstream from the dam. This necessitates a considerable distance between the waterwheel and the generator. Notwithstanding this unusual requirement, wheels of normal rational design have been incorporated in the plant, each having a single runner of cast iron, 130 in. in diameter weighing 100,000 lb. This runner is shown in Fig. 2. Fig. 3 shows the shop assembly of the cast iron speed ring and movable cast steel guide vanes for one of the units. Each unit is equipped with outside operating mechanism controlled by cylinders located at the bottom of the pit, and operated by governor actuators located on the generator floor.

The main shaft, 25 in. in diameter of forged steel hollow bored, is divided into three lengths and has a total length of 80 ft. The governor flyballs are direct-connected to this shaft and control the governor actuator by means of a connecting rod. Each governor

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-11, 1924.

actuator is supplied with oil by a rotary gear pump with a capacity of 150 gal. per min. at 150 lb. pressure.

The turbine pit can be reached either by an elevator or by a stairway. The elevator is located between the turbines in a vertical shaft which opens into a horizontal tunnel connecting the three turbine pits.

The Thurlow backwater suppressor[1] installed for the first time, utilizes flood water to increase the head on the turbines. The units have been thoroughly tried

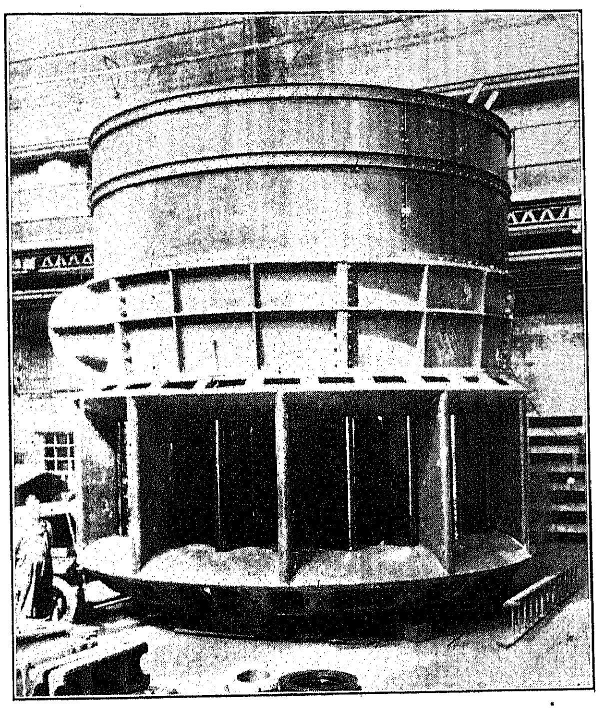

Fig. 3—Concrete Spiral Cased Turbines

out and the backwater suppressor greatly exceeded expectations. Another feature is the outdoor gantry crane and the sliding roof of the power house covering the generators. Two of the units are equipped with the White hydraucone and one with a flattened elbow draft tube. The hydraucone is a concentric form of draft tube which changes the direction of flow of water discharged from the runner in a very short length of tube and at the same time regains energy by changing velocity head into pressure head, with very small losses, due to the fact that it is formed to correspond to the natural shape of a jet of water striking a flat plate or a cone. The engineers of the Alabama Power Company were instrumental in having a large number of comparative draft tube tests made, the report of which was read at the January A. S. C. E. Meeting. It is interesting to note that these tests made by disinterested engineers showed the superiority of the hydraucone type of tube over the elbow type. The writer has advocated this design since 1916.

Recently the first of three 70,000-horse power hydroelectric units was placed in operation at the plant of the Niagara Falls Power Company on the American side at Niagara Falls, New York. It is estimated that the operation of this one unit will release for other duties approximately 1500 men daily, who heretofore had been engaged in mining, hoisting, loading, hauling, switching and firing coal under boilers in order to develop this same amount of energy. Fig. 4 shows a sectional view of the 70,000-horse power combined hydroelectric unit which the Allis-Chalmers Mfg. Company now has under construction for the Niagara Falls Power Company for Station No. 3-C. Two of these turbines are of the Wm. Cramp & Sons manufacture, two of the generators are of the General Electric manufacture and the third complete unit, turbine, governor and generator is of the Allis-Chalmers manufacture. Probably a brief description of these, the largest power-producing units in the world, will further serve to give a general idea of the present day trend in the field of hydroelectric development.

The water for these units is conducted from the forebay at the top of the cliff through butterfly valves connected to concrete lined tunnels driven through the solid rock. At the lower end, a plate steel section of penstock is imbedded in the tunnel and 21-ft. 6-in.-

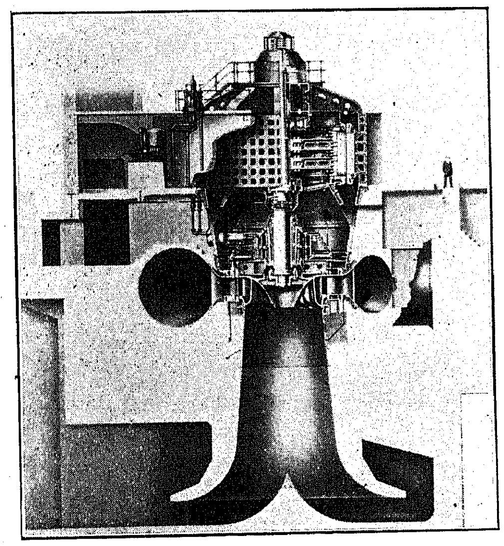

Fig. 4—Largest Power Generating Unit for Niagara Falls Power Company

diameter Johnson type valves are used for shutting off the water from the turbines. The casings on two of the turbines are of cast steel made in sections and bolted together. The third unit is of the circular-section plate-steel type riveted to a cast steel speed ring made in sections. The turbines operate under 213 ft. effective head at 107 rev. per min.

All three units will be equipped with the concentric type of draft tube regainer of the hydraucone or Moody spreading type. The principal features of the Allis-Chalmers hydraulic turbine are a cast steel runner

1. J. A. Sirnit's Paper on "Hydroelectric Power Plant Design" in A. S. M. E. *Transactions*, 1922, Vol. 44.

made in one piece approximately 180 in. in diameter, weighing about 100,000 lb.; a main shaft 34 in. in diameter of forged steel, hollow bored, provided with forged flanges on both the upper and lower end, the runner being bolted to the lower flange with steel-fitted bolts and a main turbine guide bearing of the adjustable, lignum-vitae type, lubricated with water drawn from a settling reservoir at the top of the cliff. The guide bearing is built with four adjustable shoes so that they may be taken up in any direction to compensate for wear. The guide vanes are of cast steel with upper and lower stems cast integral. The upper stems extend through bronze bushings in the cover plate where they are connected to the shifting ring by means of levers and breaking links. These breaking links are so designed that they will give way either in tension or compression when some obstruction prevents the opening or closing of the guide vanes.

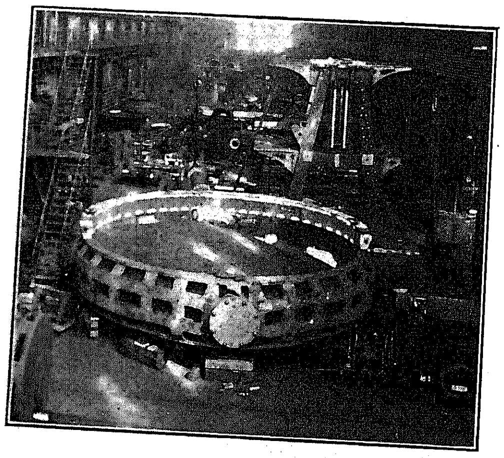

FIG. 5—BUTTERFLY VALVE. ASSEMBLY OF HOUSING WICKET AND OPERATING SHAFT

The generators are rated 65,000 kv-a., 13,200-volts, 25-cycle. The rotors are of cast steel built up in sections with pole pieces bolted to the rim. An auxiliary generator of 650-kw. capacity is mounted within the generator just above the main rotor. This is to provide current for the auxiliary apparatus such as oil pumps, motor-generator sets, cranes, lights, etc. The entire rotating weight of over 700,000 lb. is carried on the Kingsbury thrust bearing. The exterior design of all three generators is practically identical in order to preserve uniformity, although the design of the coils and winding is made independently by the engineers of the General Electric Company and the Allis-Chalmers Mfg. Company.

At the upper end of the penstock, instead of the customary type of square head gate, three 23 ft. 6 in.-diameter butterfly valves are used to shut off the water. They have cast iron housings made in sections and cast steel and plate steel wickets turning on forged steel pivots. The axes of the butterfly valves are set vertically, the operating mechanism being located on the deck above the water level. These valves, which establish a new record for size, are to operate under a head of about 50 ft. Figs. 5-6 show different views of the valves being assembled in the shops of the Allis-Chalmers Mfg. Company.

The largest single contract ever placed for hydraulic machinery is that recently placed by the Quebec

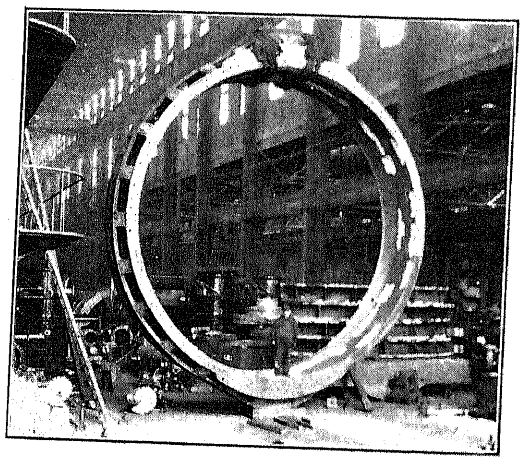

FIG. 6—BUTTERFLY VALVE HOUSING

Development Company covering eight complete 45,000-horse power turbines to operate under 110 ft. head, at 112½ rev. per min., to be installed near Lake St. John in the Province of Quebec, on the Saguenay River, about 125 miles due North of the City of Quebec. The major parts of four additional units are also being installed so that the final power house will consist of 12 units of a total capacity of 500,000 horse power. These units will be of the plate-steel spiral-cased type. Fig. 7 shows a view of the first one of the 12 gigantic casings being erected in the Toronto shops of the Canadian Allis-Chalmers Limited. The inlet diameter is 20 ft. The plate thickness is ⅞ in. at the inlet

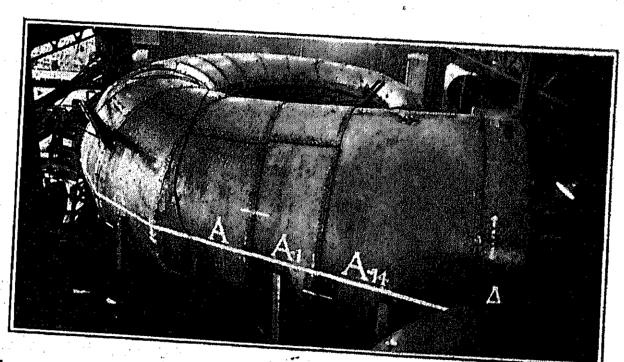

FIG. 7—PLATE STEEL CIRCULAR SECTION SPIRAL CASING. ONE OF TWELVE 40,000 H. P. SINGLE-RUNNER VERTICAL SHAFT TURBINES FOR QUEBEC DEVELOPMENT CO., LTD.

tapering down to ½ in. at the smaller end of the casing.

The past year has seen advances not only in the larger sized hydroelectric units, but even the 200 and 300-horse power open-flume turbines have been improved because of the knowledge gained in constructing larger units. The high-speed Nagler type of runner

is being adopted almost universally, improvements in efficiency having been obtained which make it compare very favorably in that regard with the slower-speed Francis type of runner which it replaces. This, when taken in combination with the improved efficiency of the higher-speed generator at lower cost gives it a distinct advantage. Because of its higher speed, the energy remaining in the water discharged from these runners represents a large percentage of the total. For this reason the design of the draft-tube regainer must be considered carefully. Many tests have been conducted to determine the type of draft tube that will give the best performance with these runners. Frequently the available space, especially in reconstruction projects, is very limited, but wherever conditions allow, we find that the most efficient results are obtained with the concentric or hydraucone type of regainer. Fig. 8 shows a number of high-speed runners being manufactured in the shops of the Allis-Chalmers Company at one time during the past year.

The plate-steel built-up guide vanes used with these

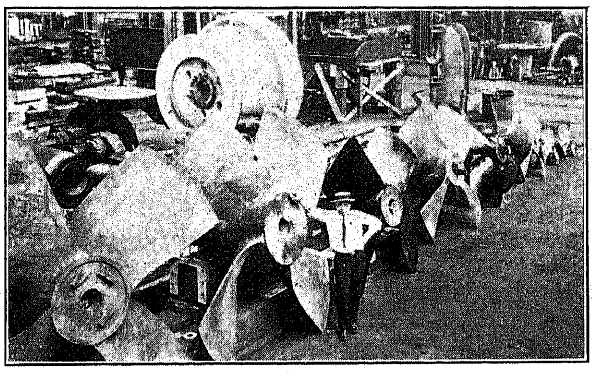

Fig. 8—Nagler High-Speed Runner of Various Sizes, Types and Characteristics for Low Head Developments

high-speed runners have been found to be a distinct improvement. In several cases sticks or pulp wood have become lodged between the vanes distorting the plates considerably. If the old type of cast iron vanes had been used they would have been broken, but with the plate steel construction it was only necessary to remove the damaged vanes and straighten them.

The design of the concrete spiral-casing type of machine has received its share of attention and much time and thought have been expended in improving and simplifying their construction. The writer is in favor of placing the nose of the casing even with, or down stream from, the centerline at the side of the unit. This shortens the path of the water, enables the units to be placed closer together and provides additional thickness in the separating walls between the units where it is most needed. Great care must be exercised in constructing the concrete spiral casing and a large amount of reinforcing steel must be used to properly strengthen it, especially above 60 ft. head. For this reason the writer has long recommended the use of the plate-steel spiral casing for heads as low as 50 ft. For normal settings very little more steel is required to build the spiral casing than is used for the reinforcing of the concrete spirals. It can be readily put together in the field and mass concrete poured around it, doing away entirely with the complicated wooden forms which must be built by expert carpenter or pattern makers. Since the plate steel casings are entirely self-supporting, they may be left practically uncovered. They may be set one almost touching the other, thus decreasing the power house length and doing away with the heavy concrete dividing wall which is necessary with the concrete spiral setting to obtain sufficient strength in case one unit is drained while an adjacent one is in operation. Our Company has constructed plate steel spiral casings for heads as low as 30 ft., and probably established a record for high-head plate-steel spirals during the last year in those built for the New England Power Company, Davis Bridge Plant, developing 19,500 horse power each under 345 ft. head.

A striking example of the use of the plate-steel spiral casing for medium heads is the 20,000-horse power unit for the West Kootenay Power and Light Company to operate under 70 ft. head. In this case, the saving by shortening the distance between the centerline of units, thus decreasing the amount of excavation in the solid rock required for the power house practically paid for the additional cost of the plate steel casing, not taking into account the saving in reinforcing steel and labor in building the forms and constructing the concrete casing.

In the high-head field, undoubtedly, a new record has been established in using a Francis type of turbine for 850 ft. head, these being the Oak Grove turbines of the Portland Railway, Light & Power Company now being constructed by the Pelton Company, where they will use the Moody type of draft tube and rubber sealing rings on the runner and guide vanes.

Impulse turbines have apparently established no new records during the past season, although an additional 30,000-horse power turbine, recently designed and constructed by the Allis-Chalmers Mfg. Company is now being installed in the Caribou Plant of the Great Western Power Company. This is the world's record both for horse power and size of machine, for this type of unit. Experiments conducted in the hydraulic laboratory of our Company upon several different designs of buckets and impulse wheel settings, have proven conclusively that the efficiency of a well-designed impulse unit compares very favorably with that of the best type of Francis wheel and even surpasses it at part gate.

Governors

No radical changes have been made in the fundamental features of hydraulic turbine governors during the last ten years, although many improvements have

been made with the view of simplifying and making this equipment more reliable. The governor equipment for the 70,000-horse power Niagara Falls turbine is similar to that used with the 37,500-horse power units installed in 1919. The flyballs are mounted directly on the main shaft and their motion is transmitted to the governor stand located on the upper generator floor. A large number of plants have been placed in operation, using a motor to drive the flyballs, thus doing away with jack shafts and belts. Fairly satisfactory results have been obtained, although there is always danger of losing the driving current in case of trouble with the generator, in which case the turbine gates will open wide instead of closing. To eliminate this danger a small independent generator mounted on the waterwheel shaft driving only the flyball motor would be ideal, but the cost makes it prohibitive for general use.

In the large power systems existing at the present time, the instantaneous load fluctuations reach such a small per cent of the total output of the system, that there is little actual work for a governor. Their existence is only justified as a safety device, for in case of electrical trouble they will shut the machine down quickly. An interesting development has been the installation of several hydroelectric units not equipped with governors. The generators and oiling systems were designed to withstand prolonged runaway, the units delivering current into a fairly large power system, and regulation being provided by the governor-equipped units in other stations on the system. Sufficient leeway is maintained in the governor-equipped station to pick up any rapid increase in load, slow fluctuations being controlled by manual adjustment of the units not equipped with governors. The three 5500-horse power, 65-ft. head, vertical units of the Brown Company at Berlin, New Hampshire, are the best examples of a complete installation without governors.

Automatic Equipment

Automatic-control equipment for hydroelectric installation has drawn more and more attention during the past few years. We give below the principal functions which the various types of automatic equipment furnished with our governors perform.

Emergency Stop Device

This is a safety feature which is being used on nearly all of the large installations and enables the operator at the switchboard to shut the turbine gates and stop the machine quickly without requiring the assistance of the second or third operator. This is accomplished by closing a switch on the switchboard, energizing a solenoid mounted on the governor stand, which immediately causes the pilot valve to be lifted into the closed position and held there, thus bringing the turbine gates to the full closed position in the minimum time allowable. This emergency stop device may also be connected to temperature indicators in the turbine or generator bearings, or generator windings, to overload or differential relays, or any other recording or indicating device used to indicate abnormal conditions.

Remote-Control Equipment

This equipment is designed to be used with large units where it is especially desirable to have the machine under the direct control of the switchboard operator, even during starting up and stopping, instead of having the second operator start the machine and bring it up to speed by hand as is the common custom. By means of the electrically operated synchronizing device and remote-control load-limiting device the switchboard operator may open the gates slowly and bring the unit up to speed from dead stop, indicating instruments on the switchboard showing him at all times the exact position of the turbine gates and the load-limit setting. With this equipment it is not necessary for the second attendant to touch the governor and the unit may be separated from the line, shut down completely and the brakes applied by the switchboard

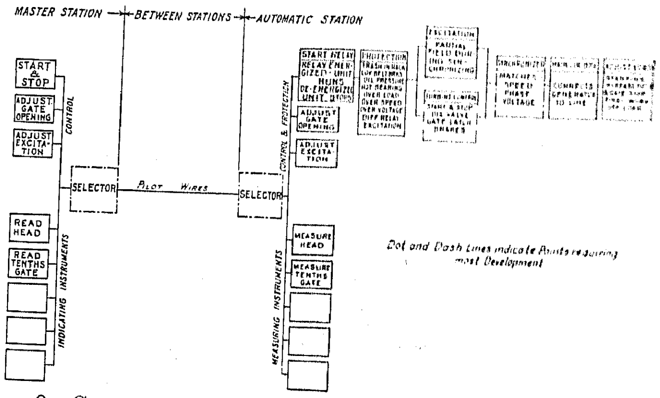

Fig. 9—General Scheme of Complete Automatic Remote Control System

operator. This same equipment may be used for controlling a distant unit from another station, although it will require more connecting wires between the two stations than with the full automatic control equipment.

Full Automatic Control

The most general type of automatic remote control is illustrated by the chart shown in Fig. 9. The features which are present at the master station are shown at the left of the sheet. They consist of the control switches by which operations are performed at the automatic station, indicating instruments which show conditions existing at the automatic station, and a selector mechanism by which these various functions can occur without an excessive number of pilot wires between stations. Pilot wires connect the two stations.

The right half of the diagram shows the equipment at the automatic station. Two types of operations are controlled by the selector. The upper part of the diagram shows those required to control and protect

the unit. The lower part shows the measuring devices which work in conjunction with the indicating instruments at the master station.

In any system power is required at the automatic station before it can be started, so that either the incoming lines must be energized or a storage battery provided.

Operation

Consider the method by which the system operates. To start, the selector at the master station is set by hand or by float to the start position and the starting switch closed. In response, the starting relay at the automatic station picks up. If the protective devices indicate that the machine is in running condition, the turbine starts and the alternator is then synchronized and connected to the line. The time required for starting is one or two minutes.

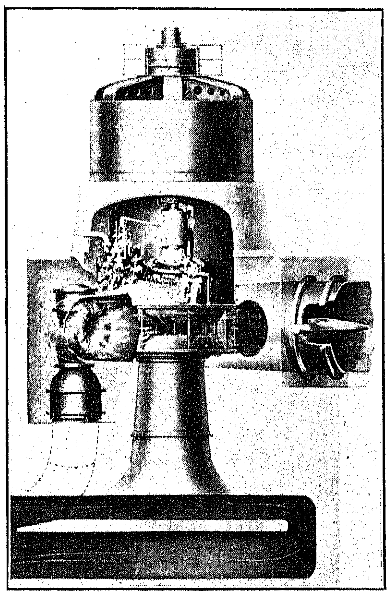

Fig. 10—Sectional View of Hydroelectric Unit, Showing Butterfly Valve, Pressure Regulator, Hydraucone and Integral Type Governor

If desired, equipment may be furnished for adjusting the turbine gates by means of the load-limiting device arranged for remote control. The usual indicating instruments are for head and gate opening but others may be added if desirable.

Power Houses

For the larger type of vertical hydroelectric unit, the one-floor station shown in Fig. 10 seems ideal. The butterfly valve with sealing rings shuts off the water at the inlet of the casing. A governor-actuated pressure regulator prevents excessive pressure rises, due to the use of a long penstock. The plate-steel spiral casing is riveted to a cast-steel speed ring. The governor is of the integral type with the flyball mounted directly on the main shaft and the governor stand located on one of the regulating cylinders, thus doing away with long runs of piping. The generator is supported on a concrete barrel and there is only one floor in the power house, so that one attendant may watch both the electrical and governor equipment. The unit is equipped with the White hydraucone, which type of draft tube has become very popular in the last five years.

From 1911 to 1916, a large number of experiments were made on various types of draft tubes, particularly the hydraucone with runners covering a wide range of specific speeds. Between the years 1913 and 1918, the Nagler was being developed and improved high-speed runner. The insistent demand at that time for higher speeds under low heads was responsible for the rapid advances made in its design. Following this, we made simplifications and improvements in control equipment, including direct-connected flyballs, integrals governors, remote control for butterfly and gate valves, and sealing rings to prevent leakage through butterfly valves.

Recently, attention has been directed our to power house arrangement. The increasing size of units, and the high cost of trained operators have greatly influenced the modern power house design. Every power house arrangement is studied with a view to greater accessibility, increased convenience and the least number of operators. Thus it is necessary to make many changes but with each change are embodied a greater simplicity and sturdiness of design.

Hydroelectric power is the most important source of energy this country has at the present time, so let us give to its proper development the time and thought which it deserves.

Discussion

For discussion of this paper see page 459.

Acceptance Tests for Hydroelectric Plants

BY FRANK H. ROGERS

Wm. Cramp & Sons, Ship & Engine Bldg. Co., Philadelphia, Pa.

TESTS made at site on hydroelectric units are important from the viewpoint of acceptance or rejection of the machinery and are of great value to the power company as the data thus obtained, permit the most economical operation of the units. It is recognized that in most plants water economy is vital, so that high efficiencies of both generator and water turbine must be secured over the range in flow and head occurring at the site. For this reason, the efficiency guarantees made in the various proposals received are rightly given consideration in awarding contracts for the machinery. In any given plant, a fair yearly income value can readily be calculated for each per cent gain in efficiency and such value capitalized at say 10 per cent shows the additional money which can properly be spent for the machines developing the higher efficiency. As contracts are frequently awarded on this basis, tests should be made to determine whether these guarantees have been met.

From the data secured by tests made at site, tables and curves are prepared, which permit the operators to properly divide the load between the various units, resulting in the maximum possible kilowatt hours output for a given flow of water.

In a number of plants, especially those located on international streams, a definite quantity or flow of water is permitted the power company. The water consumption is sometimes checked by taking measurements of flow at stated intervals, with the corresponding output of the generators, and from these readings the total flow is estimated from the total output over a given period. Such a method is both inaccurate and expensive and is unnecessary if efficiency tests are made, for then the generators are actually calibrated water meters and the flow can be determined accurately from the kilowatt output. This phase of the matter will be taken up later in more detail.

METHODS OF TEST

In this article the writer will deal primarily with the methods of testing hydraulic turbines. For many years there was no standard and as most of the contracts contained little or no information as to testing, the engineer who conducted the tests used his own ideas and his own instruments, calibrated at times, but more often used with assumed coefficients. The head on the turbine was measured at various points and the velocity heads in the conduits were often neglected. Hence it is little wonder that arguments arose between the power companies and the manufacturers as to the results obtained. Furthermore, there being no standard, the efficiencies secured at one plant bore no relation to those obtained at another plant.

To remedy these difficulties, a Testing Code for Hydraulic Turbines was prepared under the auspices of the Machinery Builders Society in October, 1917. This code was actually compiled by the engineers of the leading turbine manufacturers in the United States. After completion, a draft of this code was sent to about fifty consulting engineers and engineers of the power companies interested in this line of work. Many valuable suggestions were thus obtained, so that the final code adopted represents the consensus of opinion of a large majority of the manufacturers and engineers in the hydraulic field. Since 1917, this code has been in general use and has been attached to practically all contracts for hydraulic turbines.

A similar code was presented in May, 1922 by the Committee on Power Test Codes of the American Society of Mechanical Engineers under the title of Test Code for Hydraulic Power Plants and Their Equipment. This code is similar and agrees on all cardinal points with the Machinery Builders' Code. It is more general in its scope, covering the testing of the entire plant such as canal, forebay, and penstock, as well as the turbine, and also contains rather full instructions as to the form of preparing tables showing data and results. The methods of measuring power output, water consumption, head and speed are covered by a supplementary code on instruments and apparatus.

As both of these codes have been published, there is no necessity of a detailed description of the methods of testing contained therein, but it might be of interest to briefly outline the various quantities to be measured, in order to determine the turbine efficiency and the instruments and apparatus for making such measurements.

TURBINE EFFICIENCY

The turbine efficiency is defined as the ratio of the brake horse power, or the power delivered by the turbine shaft, to the water horse power, or the energy delivered by the water to the turbine:

$$E = \frac{B\,H\,P}{W\,H\,P} \tag{1}$$

The water horse power is given by the formula:

$$W\,H\,P = \frac{Q \times W \times H}{550} \tag{2}$$

in which

Q = Flow in cubic feet per second
W = Weight of water in pounds per cubic foot
H = Effective head on turbine.

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-11, 1924.

Measurement of Brake Horse Power

In the case of hydroelectric units which will be considered in this article, the brake horse power is determined from the generator output divided by the generator efficiency. Therefore, the generator efficiency must be determined by tests made in the shops of the builder or after installation, either by the direct measurement of input and output, or by the separate loss measurement, the electrical measurements being made in accordance with the Standards of the American Institute of Electrical Engineers 1921, Revision.

The generator efficiency is given by the following formula:

$$\text{Eff.} = \frac{\text{Kw. output at generator terminals}}{\left\{\begin{matrix}\text{Kw.}\\ \text{output}\end{matrix}\right\} + \left\{\begin{matrix}I^2 R\\ \text{armature}\end{matrix}\right\} + \left\{\begin{matrix}\text{Open circuit}\\ \text{core loss}\end{matrix}\right\} + \left\{\begin{matrix}\text{Stray}\\ \text{load losses}\end{matrix}\right\} + \left\{\begin{matrix}\text{Windage}\\ \text{and friction}\end{matrix}\right\}} \tag{3}$$

The above formula is based on the assumption that the generator is separately excited both during the efficiency tests made on the generator and the efficiency tests made on the turbine.

In determining the windage and friction, core loss and stray load losses, the generator may be driven by an independent motor, or where a direct-connected exciter is used, by its own exciter as a motor. If this method cannot be used, it is then possible to obtain these losses by a retardation test. In making such a test the turbine shaft and runner should be disconnected so as to eliminate the windage and friction losses of the turbine shaft and runner. In many plants it is impracticable to disconnect the turbine shaft, due to the fact that frequently no lower generator bearing is furnished. In such cases the windage and friction of the turbine shaft and runner may be calculated from the following formula:

$$K\,W = K\,B\,D^4\,N^3 \tag{4}$$

in which $K\,W$ = Windage and friction loss of shaft and runner

B = Height of turbine distributor in feet

D = Entrance diameter of runner in feet measured at centerline of distributor

N = Revolutions per second

K = Empirical coefficient.

It has been found by actual tests made that a close value for K is 0.000115.

One of the methods used in determining this coefficient was rather interesting and might be worth mentioning. In the Holtwood plant of the Pennsylvania Water and Power Company, the turbines are of the vertical shaft, two-runner type, and it was possible to disconnect the lower runner, so that in the first test, the total windage and friction loss included the generator loss plus the loss for two turbine runners. After disconnecting the lower runner, the test was repeated and in this case, the total windage and friction loss represented the generator loss plus the loss due to one runner. By subtracting these two losses the difference gave the windage and friction loss of one turbine runner and thus gave a ready means of determining the correct windage and friction loss of the generator alone.

Measurement of Head

The effective head on the turbine is the difference

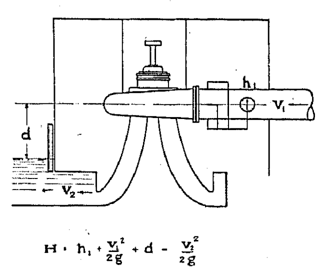

Fig. 1—Effective Head on Hydraulic Turbines

between the total energy contained in the water immediately before its entrance to the turbine and the total energy in the water immediately after its discharge from the draft tube. The total energy at any point is the pressure head plus the velocity head and hence:

$$H = \left\{h_1 + \frac{V_1^2}{2\,g}\right\} - \left\{h_2 + \frac{V_2^2}{2\,g}\right\} \tag{5}$$

If the datum plane is taken at the centerline of the casing and d is the distance from this plane to tailwater,

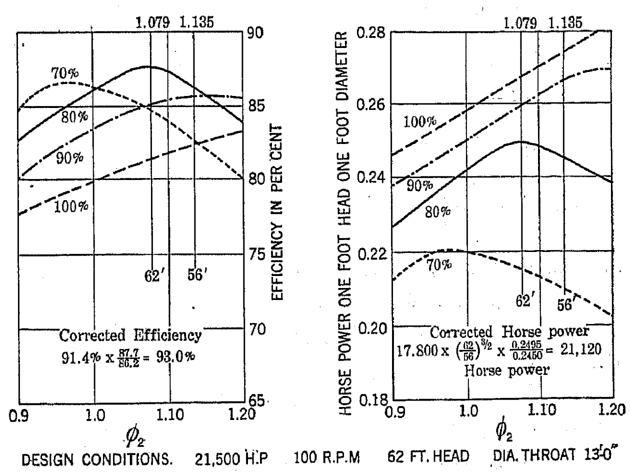

Fig. 2—Performance Curves of Turbine from Laboratory Tests

$$H = h_1 + \frac{V_1^2}{2\,g} + d - \frac{V_2^2}{2\,g} \tag{6}$$

Fig. 1 shows the location of the gages and the velocities to be considered in determining the effective head. At least four piezometers should be provided near the casing intake and connected independently to a calibrated pressure gage, a mercury manometer, or open

glass tubes on a gage board, depending on the conditions. A board, rod or float gage should be located in the tailrace near the draft tube discharge at points reasonably free from local disturbances. It is often found advisable to locate rods or float gages in stilling boxes.

The turbine should be tested at the head and speed specified in the contract. If it is found that the actual head existing during the test differs from the contract head, the speed should be adjusted to correspond. It is a well known principle that if the speed of a turbine is changed in proportion to the square root of the ratio of the heads, the horse power will change in proportion to the three-halves power of the ratio of the heads and the turbine efficiency will remain constant. Therefore, if during the test the load on the generator is made up of a water rheostat, or if it is permissible to operate the generator at different frequencies, the proper correction can be made for any variations in head and the results are absolutely reliable. If, on the other hand, the generator is delivering load to a system where the cycles and hence the speed are fixed, it is impracticable to correctly adjust the speed to suit the head. In such cases the values of power and efficiency as shown by the test must be corrected on the basis of test curves of the same or homologous turbine made at a testing flume. In tests conducted in laboratories, runs are made at each gate opening for a wide range in speeds, and curves are prepared showing speed-power and speed-efficiency for each gate opening. Fig. 2 shows sample curves of such tests. The value ϕ_2 plotted as abscissae is called the speed characteristic and its value is given by the formula:

$$\phi_2 = \frac{V_P}{\sqrt{2\,g\,H}} \tag{7}$$

or ϕ_2 = The ratio of the peripheral speed of the runner to the spouting velocity due to the total effective head.

As an example, consider a turbine designed for a normal full-load output of 21,500 horse power under a head of 62 ft. at a speed of 100 rev. per min. The rated diameter of the runner = 13′ – 0″ and therefore from the above formula $\phi_2 = 1.079$.

Assume for one of the test points actually obtained the following values:

Gate opening	= 80 per cent
Horse power	= 17,800
Effective head	= 56 ft.
Rev. per min.	= 100
Turbine efficiency	= 91.4 per cent

For the above conditions of head and speed, the value during the test of ϕ_2 is given by

$$\phi_2 = 1.079 \times \sqrt{62/56} = 1.135$$

By referring to the two curves shown in Fig. 2 at the two values of ϕ_2, we obtain the ratios to apply to the horse power and efficiency to correct for the speed and then step up the horse power by the three halves power of the ratio of the heads to obtain the results under contract conditions.

Thus $H\,P = 17{,}800 \times (0.2495/0.2450) \times (62/56)^{3/2} = 21{,}120$

$$E = 91.4 \times (87.7/86.2 = 93.0\%$$

Measurement of Quantity of Water

The determination of the quantity of water flowing to the turbine is usually considered the most difficult of all measurements and is undoubtedly the most interesting. Many different methods of quantity determination are used, depending on local conditions, among which might be mentioned: Venturi meter, weir, Pitot tube, current meter, salt solution, screen or diaphragm, color velocity, salt velocity and the Gibson method.

Most of these methods are well known but it might be worthwhile pointing out some interesting features of the Pitot tube and current meter and giving a short description of the salt velocity and Gibson methods, as the two latter, while of very recent origin, have been thoroughly tried out and found to give a high degree of accuracy and reliability.

Pitot Tube

The Pitot tube method should be used in conduits or penstocks where reasonably straight and smooth flow may be expected to occur. Therefore, the tube should be installed in a straight line of penstock, the traverse section being at least ten pipe diameters from any upstream bend and five pipe diameters from any downstream bend or disturbance.

If the axis of the Pitot tube opening is parallel to the direction of stream line flow, the water will rise in the tube to a height exactly equal to the pressure head, plus the velocity head existing at the point of the tube. That the Pitot tube correctly reads velocity head can be readily proven theoretically and this fact has been demonstrated many times by tests made both by still water ratings and in flowing water from nozzles. If the pressure head is read from piezometers in the sides of the penstock, the difference between the tube reading and the piezometer reading, is the velocity head from which the velocity is obtained by the well-known formula:

$$V = \sqrt{2\,g\,h} \tag{8}$$

Fig. 3 shows a type of tube frequently used and on the right side the connections between the Pitot tubes, piezometers, and gage board. Where a considerable straight run of penstock is available resulting in good flow conditions, only two Pitot tubes need be used and four piezometer openings, all connected to independent tubes on the same gage board. Traverses are made across the penstock by both tubes and at each point all six gages are read and the difference between the tube and average piezometer readings, gives the veloc-

ity head from which the velocity for each area of the penstock can be plotted.

The penstock is divided into a number of equal-area annular rings as shown in Fig. 4, the center circle having an area equal to one-half of the area of each annular ring. The tube stations are located at the center of area of each ring and at the center of the pipe, so that each reading taken is of equal weight. The velocities in feet per second are plotted against areas in square feet, so that the area of the diagram, as obtained by a planimeter, gives the quantity of flow in cubic feet per

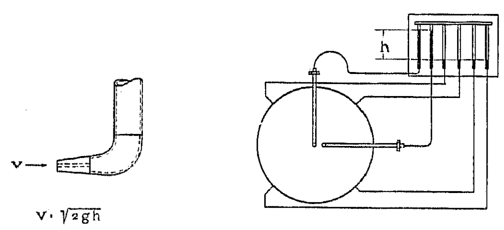

FIG. 3—PITOT TUBE AND GAUGE BOARD CONNECTIONS

second. Where two tubes are used, the average of the two quantities thus obtained is taken as the true quantity. In the case of short penstocks where the flow may be disturbed, additional tubes are used to determine the velocities on radii between the two main tubes. In spite of the fact that the Pitot tube both theoretically and actually reads the correct velocity head, it has been found by many experimenters that the quantity as indicated by the tubes is always greater than the actual flow by from 1 per cent to 4 per cent. depending on local conditions. For example, tests made at Niagara

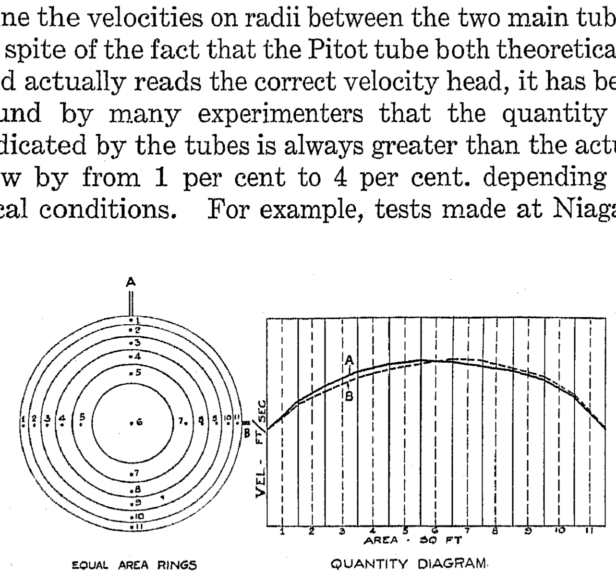

FIG. 4—PITOT TUBE STATIONS AND QUANTITY DIAGRAM

Falls in 1908, in a 5-ft. diameter penstock with Pitot tubes and checked by weir measurements, showed that the quantity obtained from the tubes was about 2 per cent greater than that given by the weir, requiring a quantity coefficient of 0.9755 for the Pitot tube method. A careful series of tests was also made at the hydraulic laboratory of the University of Pennsylvania, measuring the velocities in a 7-in. diameter pipe by Pitot tubes and at the same time measuring the actual flow in large weighing tanks. In these tests, the quantity coefficient of the tube varied from 0.968 to 0.997, depending on the velocity, the coefficient approaching unity as the velocity increased.

Engineers were, therefore, confronted by two facts separately proven, but contradictory.

(1) The Pitot tube gives the correct velocity head without calibration when the axis of the tube is parallel to the stream line flow.

(2) The quantity of water flowing in a conduit as obtained from Pitot tubes gives results always greater than the true flow.

To solve this enigma, a series of tests was made by the writer in 1910 at the Testing Flume of the Rensselaer Polytechnic Institute at Troy, New York. Various types of tubes were tested as shown in Fig. 5. The tubes were attached to a car propelled on tracks above the flume and readings of the tubes were compared with

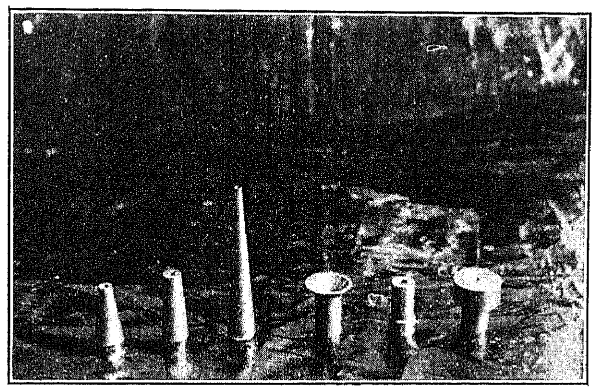

FIG. 5—TYPES OF PITOT TUBE NOZZLES

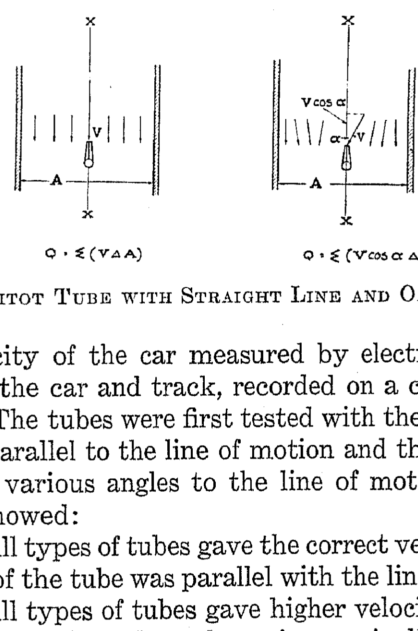

FIG. 6—PITOT TUBE WITH STRAIGHT LINE AND OBLIQUE FLOW

the velocity of the car measured by electric contacts between the car and track, recorded on a chronograph drum. The tubes were first tested with the axes of the nozzles parallel to the line of motion and then with the tubes at various angles to the line of motion. These results showed:

(1) All types of tubes gave the correct velocity when the axis of the tube was parallel with the line of motion.

(2) All types of tubes gave higher velocity than the resolved velocity when the axis was inclined to the direction of motion.

Fig. 6, shows on the left hand side, a tube inserted in a pipe, the axis being parallel to the centerline of the pipe for all positions of the traverse and with straight line flow assumed, so that the quantity is determined from

$$Q = \Sigma\,(V\,\Delta\,A) \qquad (9)$$

and hence the velocity required is the velocity normal to the area A. The pipe shown on the right side indicates a condition of oblique flow and in this case as the quantity is given by:

$$Q = \Sigma\,(V \cos \alpha\, \Delta\, A) \tag{10}$$

the velocity required from the tube reading is the resolved velocity at each point or $V \cos \alpha$.

Now the Pitot tube does not read velocity but velocity head, and hence when the water strikes the tube obliquely at a velocity of V, the tube should read a head of: $\dfrac{V^2}{2\,g} \cos \alpha$, as shown in Fig. 7. The velocity

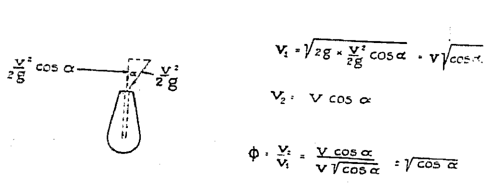

FIG. 7—VELOCITY GIVEN BY PITOT TUBE IN OBLIQUE FLOW

in feet per second calculated from this reading of the tube is given by

$$V_1 = \sqrt{2\,g \left(\frac{V^2}{2\,g} \cos \alpha\right)} = V\,\sqrt{\cos \alpha} \tag{11}$$

and it has been shown above that the velocity required to give the correct flow is

$$V_2 = V \cos \alpha \tag{12}$$

From the above it is evident that for conditions of oblique flow the Pitot tube readings will give velocities too great and hence in determining the quantity, the coefficient of the tube is given by:

$$\phi = V_2/V_1 = \frac{V \cos\ \alpha}{V\,\sqrt{\cos\ \alpha}} = \sqrt{\cos\ \alpha} \tag{13}$$

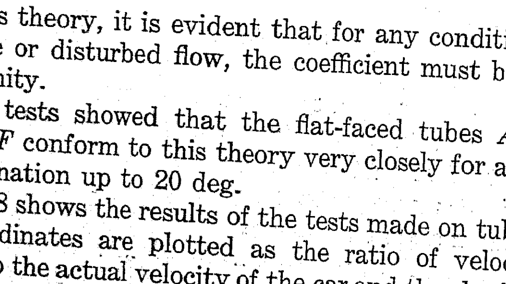

FIG. 8—TEST OF PITOT TUBE AT VARIOUS ANGLES OF INCLINATION TO THE FLOW

On this theory, it is evident that for any condition of oblique or disturbed flow, the coefficient must be less than unity.

The tests showed that the flat-faced tubes A, B, E and F conform to this theory very closely for angles of inclination up to 20 deg.

Fig. 8 shows the results of the tests made on tube B. The ordinates are plotted as the ratio of velocities reads to the actual velocity of the car and the abscissae, as angles of inclination. V_B shows velocities given by the tube B, V_T the theoretical velocity based on resolved pressure, and V_R the resolved velocity to give the correct flow. The tube coefficient ϕ is the ratio of V_R to V_B and varies from unity at zero degrees to 0.96 at 20 deg.

Absolute straight line stream flow is very difficult to produce in laboratory work and is never obtained in practise when water is flowing in pipes or open channels at ordinary velocities. The flowing mass of water is full of local whirls, so that at any given point the velocity varies both in direction and magnitude. Therefore under the usual conditions met in engineering, a coefficient less than unity must be used to correct the quantity obtained by Pitot tube measurements. This coefficient, however, has a small range in actual value, its limits for ordinary conditions being unity and 0.96. The Machinery Builders Code fixes the value at 0.976 as the average obtained from a large number of comparative tests, and the A. S. M. E. Code gives the coefficient as 0.98.

This latter code also mentions the Pitometer as a commercial adaptation of the Pitot principle and states that the coefficient for this instrument is furnished by

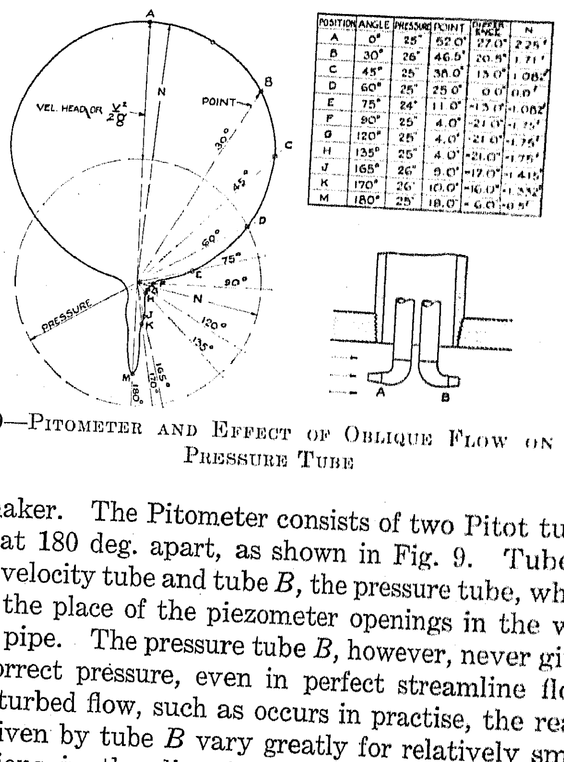

FIG. 9—PITOMETER AND EFFECT OF OBLIQUE FLOW ON THE PRESSURE TUBE

the maker. The Pitometer consists of two Pitot tubes fixed at 180 deg. apart, as shown in Fig. 9. Tube A is the velocity tube and tube B, the pressure tube, which takes the place of the piezometer openings in the wall of the pipe. The pressure tube B, however, never gives the correct pressure, even in perfect streamline flow. In disturbed flow, such as occurs in practise, the readings given by tube B vary greatly for relatively small variations in the direction of flow. This is clearly shown in the polar diagram at the left of Fig. 9. This was plotted by revolving a single Pitot tube through various angles from zero deg. to 180 deg. and comparing its readings with the true pressure as given by piezometers in the wall of the pipe. At 180 deg. or normal position of tube B with perfect flow and a velocity of 12 ft. per sec., tube B showed a pressure reading of 19 in. or 6 in. less than the true pressure, resulting for this assumption in an instrument coefficient of:

$$\phi = \frac{\sqrt{2\,g \times 2.25}}{\sqrt{2\,g \times (2.25 + 0.5)}} = 0.905$$

If the direction of flow varies but 10 deg. from normal, tube B shows a pressure reading 16 in. less than the true pressure, giving a coefficient of:

$$\phi = \frac{\sqrt{2\,g \times 2.25\ .\ (\cos 10°)}}{\sqrt{2\,g\ (2.25 \cos 10° + 1.332)}} = 0.875$$

It is thus evident that the coefficient obtained by still-water ratings will be very unreliable under flow conditions as met in practise.

Current Meters

Current meters have been used very widely for gaging the flow of streams and rivers and also in determining the flow in tests of hydraulic turbines where a canal or flume is available. Current meters should only be used where reasonably straight and smooth flow may be expected to occur, and therefore a long straight run of flume of rectangular cross section should be available. It is always preferable to locate the current meters on the intake side of the plant, rather than in the tailrace, as the flow conditions in the tailrace are usually very much more disturbed.

A great many experiments have been made to determine the accuracy of quantity measurements by current meters, using the still-water ratings of the meters and comparing the readings with the flow determined by some other method of measurement, and it has been found that the still-water ratings are not reliable as oblique or disturbed flow affects the readings of the meters in a somewhat similar manner to the effect of such flow on Pitot tubes. Unlike Pitot tubes, however, it is found that some types of flow meters read too high and other types too low. Thus the cup-type meters invariably read high and the screw type usually read low.

A series of very interesting tests was made by Benjamin F. Groat in connection with the tests on a 6000-horse power turbine at Massena, New York.[1] In these tests Mr. Groat used a Haskell meter of the screw type and a Price meter (large size) of the cup type. The meters were carefully rated from a boat to determine their coefficients. When actually used in the turbine test, however, it was found that a large constant difference in discharge determinations was obtained by means of the two types of meters. In the case of these two particular meters, it was found that the velocities, as determined from the still-water rating of the cup meter, were considerably in excess of the velocities as found from the similar rating of the screw meter and that if one-seventh of the difference between the two velocities was added to the velocity given by the screw meter, the result would be very close to the true velocity at the point in question.

A series of tests made in the Testing Flume of the Rensselaer Polytechnic Institute in 1912, also proved that current meters of the two types mentioned above will indicate velocities quite different from the actual velocities in conditions of oblique flow.[2]

The two meters used in these tests were a Price meter (small size) of the cup type, and a Buff and Berger screw meter of the Fteley-Stearns type. The two current meters were attached to the car in a similar manner to the Pitot tubes as previously described in this paper, and turned at various angles to the direction of motion of the car. The results of these tests are shown in Fig. 10, and are also plotted in a similar manner to the Pitot tube tests. It will be noted that as the angle of inclination is increased from zero degrees, the cup-type meter shown by curves R_L and R_R always registers higher velocities than the true velocity shown by curve $R_C = \cos\ \theta$, whereas the screw-type meter shown by curve R_M always shows lower readings than the true velocity. It is rather interesting to note from these curves that the cup-type meter gave different readings for velocities inclined to the right from those inclined to the left. For ordinary inclinations such as 15 or 20 deg. the true velocity lies about half-way between the velocities

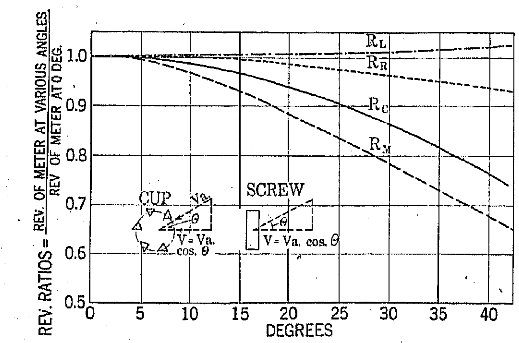

Fig. 10—Velocities by Current Meters in Oblique Flow

indicated by the two types of meters, but for greater degrees of inclination, the screw-type meter shows a less error on the negative side than the cup-type meter shows on the positive side.

It is therefore evident that where current meter measurements are to be made to determine the flow, it is necessary to use two types of meters in the test and to correct the readings from the data previously obtained from still-water ratings of these meters at various angles of inclination.

Salt Velocity Method

The Salt Velocity, sometimes termed the brine velocity or Allen method for measuring the flow of water, has been developed during the last three years by Professor Charles M. Allen of Worcester, Mass. Extensive investigations have been made at the Alden Hydraulic Laboratories of the Worcester Polytechnic Institute, checking the results obtained by a Venturi

1. See B. F. Groat's paper on "Pitot tube Formulas,—Facts and Fallacies" published in *Proceedings* of the Engineers Society of Western Pennsylvania, May, 1914.

2. These tests were described in a paper by L. F. Moody on "Measurement of the Velocity of Flowing Water," published in *Proceedings* of the Engineers Society of Western Pennsylvania, May, 1914.

meter and weir and a number of commercial tests have been conducted on units in power plants, notably the 20,000-horse power turbines of the Laurentide Power Co. at Grand Mere, Quebec, and the 41,000-horse power turbine of the Shawinigan Water and Power Company at Shawinigan Falls, Quebec.

A very complete and interesting description of this method, the results of the laboratory investigations and the commercial tests made, was given in a paper presented by Charles M. Allen and Edwin A. Taylor at the Annual Meeting of The American Society of Mechanical Engineers in New York, December 1923.[3] It may be of interest, however, to give a brief summary of the method and its manner of application in commercial plants.

The theory is simple and is expressed very clearly by the authors of the above-mentioned paper as follows:

"The Salt Velocity Method of Water Measurement is based on the fact that salt in solution increases the electrical conductivity of water. Salt solution is introduced near the upper end of the conduit and the passage of the solution across one or more pairs of electrodes at other points in the conduit is recorded graphically by electrical recording instruments. The passage of the salt solution between two points is accurately timed and the volume of the penstock between the same points is accurately determined. The discharge in cubic feet per second equals the volume in cubic feet divided by the time in seconds."

Therefore in making such a test, it is only necessary to provide for the introduction of the salt solution, two sets of electrodes, each consisting of two metallic bars insulated from each other, and a source of electrical energy, generally 110 volts d-c. or a-c. A recording ammeter or wattmeter is connected in the circuit of

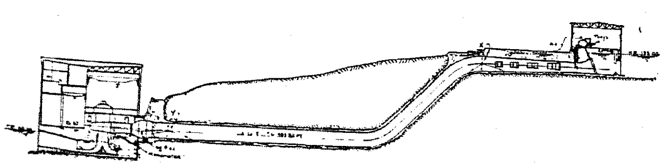

Fig. 11—Test of 41,000 Horse Power Shawinigan Unit by Allen Method

the electrodes and before the salt solution is introduced, the initial steady reading of the meter will be from one half to one ampere. When the salt solution reaches the first set of electrodes, the current passing between the two bars increases, due to the greater conductivity of the water, reaches a maximum value, and then decreases to normal again as the charge of salt passes by, thus producing on the chart, a humped curve. A similar curve is drawn by the meter when the salt solution passes the second set of electrodes. The time is also recorded on the same chart by a standard clock or seconds pendulum connected electrically to a pen or by a hot spark. The time between the centers of gravity of the two curves produced is the average time it has taken the water to travel the distance between the two sets of electrodes. The volume in cubic feet between the electrodes being accurately determined by physical measurements, the quantity flowing in cubic feet per second equals the volume divided by the time.

The general arrangement of the 41,000-horse power

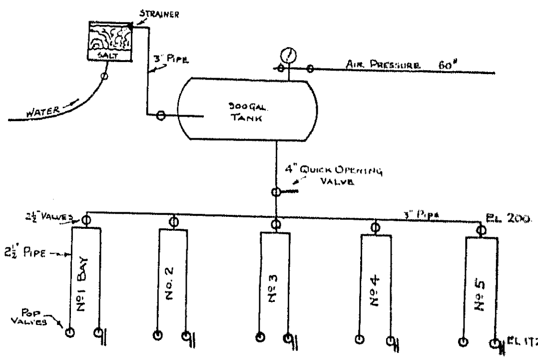

Fig. 12—Arrangement of Tanks for Allen Method

Shawinigan unit is shown in Fig. 11. The penstock was 20 ft. in diameter, about 500 ft. long and fed by five gathering tubes. The salt was introduced at the upper end and its passage recorded at electrodes X and Y. Fig. 12 is a diagram showing the mixing tank, pressure tank, quick opening valve, pop valves and piping. Two pop valves were located in each of the five bays and an introduction electrode was connected to one of the valves in each bay. The main electrodes at X and Y each consisted of two steel bars 4 in. wide, ½ in. thick, spaced 4 in. apart and extending across the horizontal centerline of the penstock as shown in Fig. 13. The electrodes used in the gathering tubes are shown in Fig. 14, and it was possible during the test to compare the discharge measured in these five tubes with the same discharge measured in the 20 ft. penstock between electrodes X and Y. The greatest variation was 0.7 per cent and the average of all discharges checked exactly.

A set of sample curves drawn by the recording Wattmeter is shown in Fig. 15, the upper curve being for practically full gate opening and the lower curve for 61 per cent gate opening of the turbine. The time in seconds is shown on these curves by the dots at the bottom of each curve. The numbers marked on these curves are the test numbers and show which curves are to be taken together for the calculation of the time of salt passage. Thus on the upper curve 250 Int. shows the time of introduction of the salt solution at the upper end of the penstock, 250 X shows the time the salt solution passed electrodes X, and 250 Y, the time the salt solution passed the electrodes Y. The centers of gravity of the curves at 250 X and 250 Y

3. An abstract of this paper "The Salt Velocity Method of Water Measurement" was printed in *Mechanical Engineering*, January, 1924.

are then determined and the time between these two centers of gravity reads from seconds indicated at the bottom of the curve.

The tests made on the 20,000-horse power Laurentide unit were of considerable interest, as this unit is provided with concrete volute casing and with short concrete penstock of varying cross section. This type of setting has always presented great difficulties for accurate water measurements, but the tests by the salt-velocity method were entirely satisfactory, repeat tests checking each other very closely and tests, made with varying apparatus and various methods of computation, giving the same final result. The general layout of this setting is shown in the diagram in Fig. 16. Eight pop valves were located in each of the three bays at the entrance to the penstock. Thus, 24 pop valves were arranged so as to give a uniform distribution of salt over the entire cross section. Six pairs of introduction electrodes were placed about 22 in. downstream from the plane of the pop valves.

The final electrodes shown at *A*, *B*, and *C*, were placed vertically at the lower end of the penstock about 3 ft. upstream from the entrance to the scroll case. These electrodes were of steel plate 4 in. wide by ½ in. thick, and were continuous, extending from the floor to the roof. A rather interesting photograph of the pop valves in operation is shown in Fig. 17. This photograph was taken with the penstock empty and under air pressure of 60 lb. per sq. in. The final electrodes near the entrance to the volute casing are also shown in Fig. 18. A set of sample curves taken during this test is shown in Fig. 19 for gate openings of the turbine of 85 per cent, 75 per cent, and 60 per cent. The center of gravity of the curves, made when the salt passed the introduction electrodes and the final electrodes, is shown by the vertical lines near the bottom of each curve.

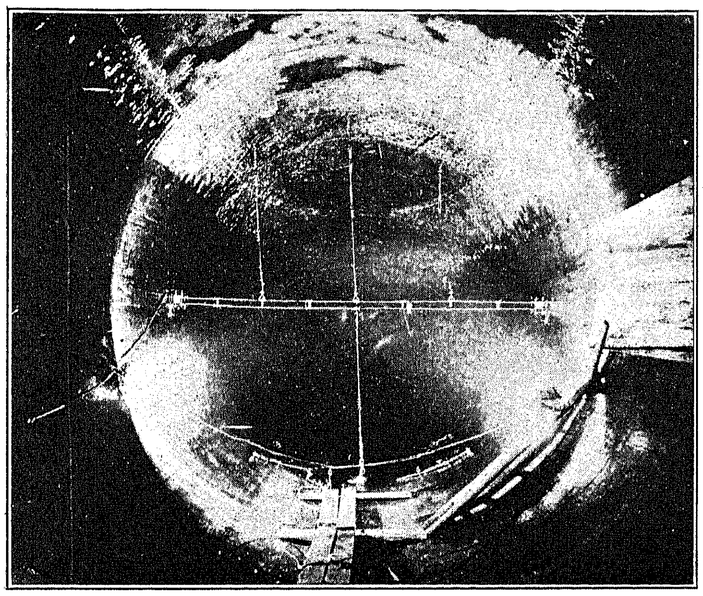

Fig. 13—Test of 41,000 Horse Power Shawinigan Unit—Main Electrodes

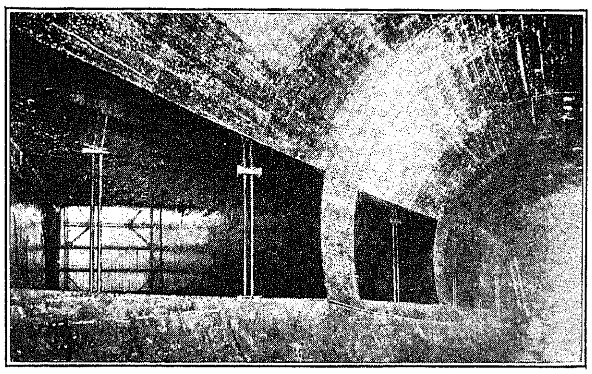

Fig. 14—Test of 41,000 Horse Power Shawinigan Unit—Electrodes in Gathering Tubes

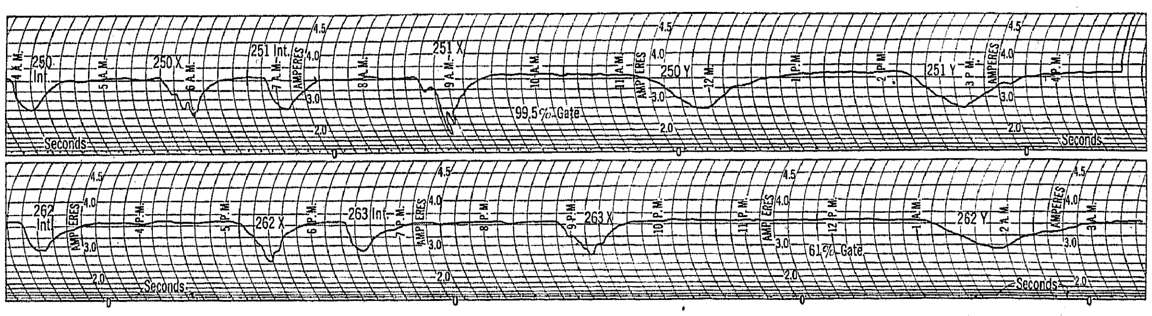

Fig. 15—Test of 41,000 Horse Power Shawinigan Unit—Sample Curves

The Gibson Method

While the Gibson method of water flow measurement has been fully described in a recent paper presented by Norman Gibson at the Annual Meeting of The American Society of Mechanical Engineers in New York City, December 1923,[4] a brief summary of

4. An abstract of this paper "The Gibson Method and Apparatus for measuring the Flow of Water in closed conduits," was printed in *Mechanical Engineering*, December, 1923.

the method and the underlying theory will be of interest.

The method makes use of two well-known principles: the first being Newton's second law of motion, that the force is equal to the product of the mass and the rate of change of velocity of that mass:

$$F = M \frac{d\,v}{d\,t} \tag{14}$$

This equation may also be written:

$$F\,d\,t = M\,d\,v \tag{15}$$

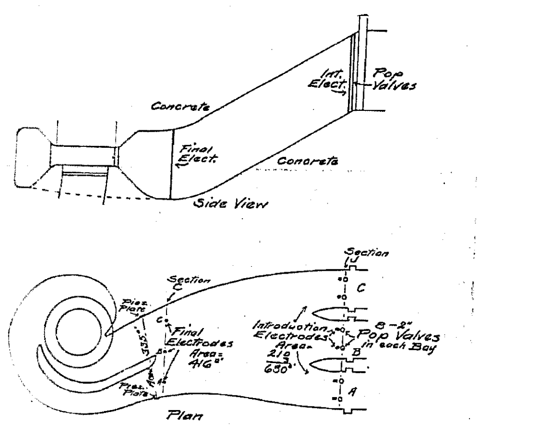

FIG. 16—TEST OF 20,000 HORSE POWER LAURENTIDE UNIT BY ALLEN METHOD

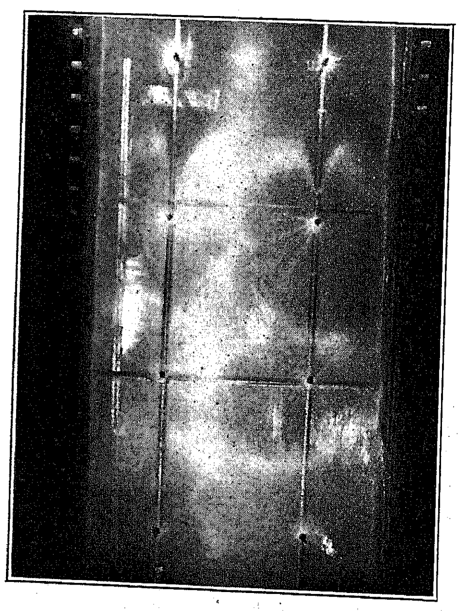

FIG. 17—TEST OF 20,000 HORSE POWER LAURENTIDE UNIT—POP VALVES IN OPERATION

which means that the impulse or the product of force by time is equal to the change in momentum or the product of the mass by the velocity change.

The second principle is the corollary of the first, *viz.*, the relation between the change of pressure and the change of velocity of a column of water expressed in terms of the velocity of the pressure waves which are propagated during the change from one end of the column to the other.

To apply this principle to the determination of the quantity of water flowing in a pipe of "uniform" cross section, consider that a valve or the gates of a turbine at the lower end are closed gradually in a given time, so that the velocity of the water in the pipe is gradually diminished and is finally stopped when the valve or turbine gates are fully closed. During closure, the pressure in the pipe rises and this pressure rise is the manifestation of the force exerted to stop the flow of the mass of water in the pipe. From the fundamental impulse equation (**15**), if the mass of the water in the pipe is known and the time taken to stop its flow and the "average" force exerted in doing so are measured, the velocity of the water, prior to the interruption of its flow, may be readily determined.

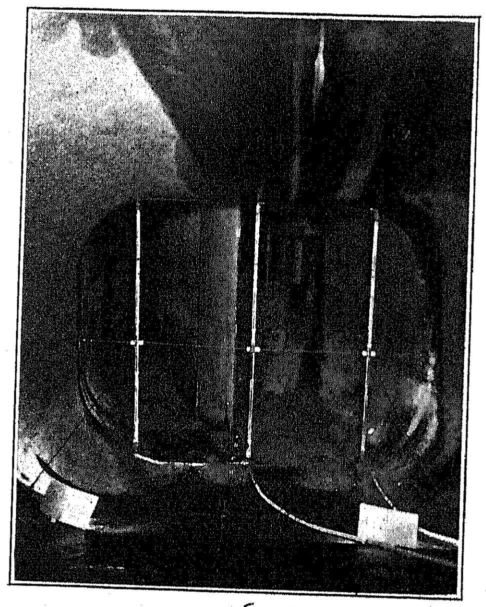

FIG. 18—TEST OF 20,000 HORSE POWER LAURENTIDE UNIT—FINAL ELECTRODES

Let: a = area of pipe in sq. ft.
L = length of pipe in ft.
Q = quantity flowing in cu. ft. per sec.
V = velocity in ft. per sec.
T = time to stop flow in sec.
P = average pressure rise, in ft. during time T
w = weight of water per cu. ft.
g. = accelleration due to gravity.

Then from equation (**15**)

$$P\,w\,a\,T = \frac{L a w}{g} \; X \; V \tag{16}$$

for which

$$V = g \frac{P\,T}{L} \tag{17}$$

The average pressure rise P and the time T are recorded by the specially designed Gibson apparatus, so that it is only necessary to measure the length L of the penstock, to determine the velocity V.

In practise, the test procedure consists of placing the desired amount of load on the unit, and for a period of time observing the head-water elevation, the tail-water elevation, casing pressure, speed, gate opening, and the various electrical measurements necessary to determine the generator output. At a given signal, the turbine gates are closed and the load taken up by the units operating in synchronism with it. The rate of closure need not be uniform, although this operation should take place as smoothly as possible. During the closure the flow in the penstock is being retarded and produces a change in pressure. A continuous record of this pressure change and the time of closure is made by the Gibson apparatus.

Fig. 20 is an illustration of this apparatus and Fig. 21 is a sectional view, showing the various parts in detail. Essentially, the process consists of photographing the oscillations of the mercury column D by means of the light from the lamp Z passing through the water-filled portion of the tube D and projected by the lens J upon a film wrapped around the drum N. A shutter A is used to protect the film when changing the drums. The drum is rotated at a constant speed by the small motor Q. Thus, a diagram (see Fig. 22) is obtained.

On this diagram A marks the beginning of the record, at O the pressure begins to rise as the turbine gates are being closed, at K the gates are fully closed and from K to C the column of water is being restored to equilibrium in a series of damped harmonic oscillations or waves. At Z there is an interval of time sufficiently long to allow these oscillations to subside, and then at F there is a short record which registers the pressure in the penstock under the then existing hydrostatic conditions. It will be observed that the narrow vertical spaces D, where the record has been obliterated, occur on the diagram at regular intervals. These form a record of time usually one second apart. This record is made by the pendulum K (Fig. 21) which swings in front of the aperture opposite the mercury column, thus cutting off the light from the diagram.

In working up the results of the test, it is necessary to delineate the diagram and determine the net area representing the change in momentum. This is accomplished in the following manner:

Referring to Fig. 23, the running condition at the particular gate opening of this test is shown at A on the diagram, and hence a line drawn parallel to the reference line E E will show this pressure. Another line drawn through F will give the static conditions existing after the closure, and therefore, the distance Y between these lines represents the friction and velocity head at this particular gate opening. The

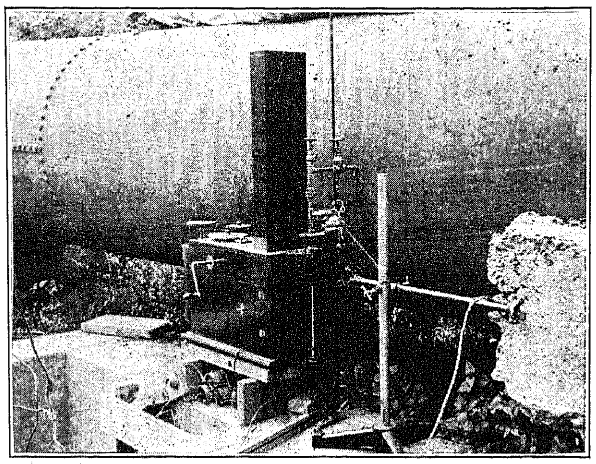

Fig. 20—Gibson Apparatus for Measuring Flow

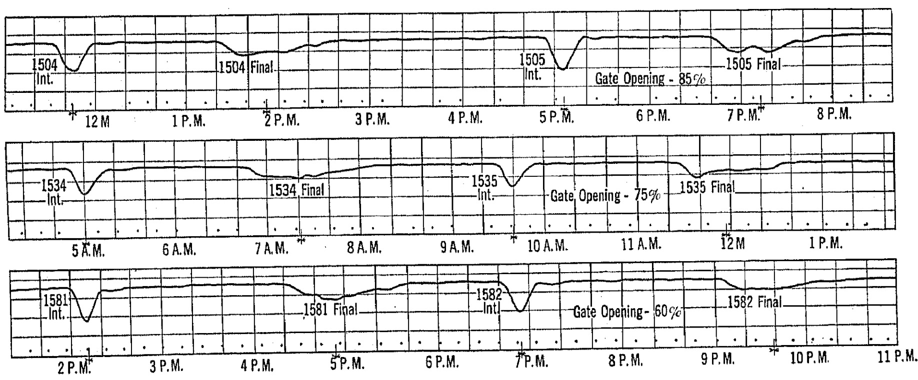

Fig. 19—Test of 20,000 Horse Power Laurentide Unit—Sample Curves

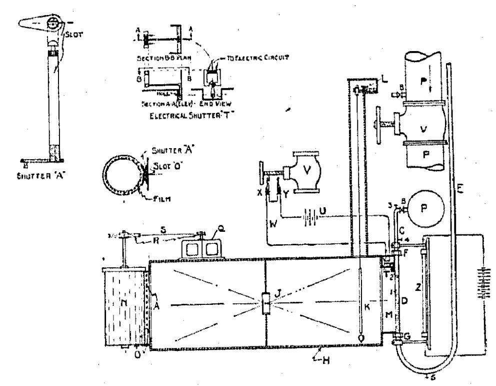

Fig. 21—Sectional View of Gibson Apparatus

point K at which the gates are completely closed is found from a study of the after waves $C\,C$ which, being damped harmonics, follow a general law. By reference to the ratio between successive waves, the distance from W to M may be determined, thus locating the end of the diagram.

In order to separate that part of the gross area of the diagram produced by the recovery of the velocity and

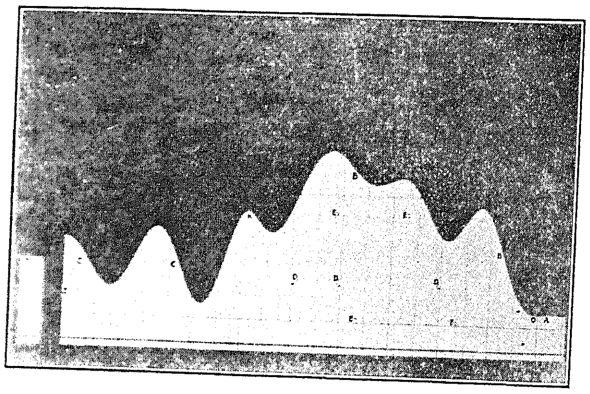

FIG. 22—TYPICAL PRESSURE—TIME DIAGRAM, BEFORE DELINEATION

friction heads, it will be remembered that both of these quantities are proportional to the square of the velocity. Therefore, by drawing an assumed recovery line $O\,M$ the diagram may be divided up into small areas a_1, a_2, a_3, etc., and the successive ordinates h_1, h_2, h_3, etc., obtained, from which the true recovery line $O\,N\,M$ may then be drawn. In numerous cases a record of the surges in the forebay must be taken,

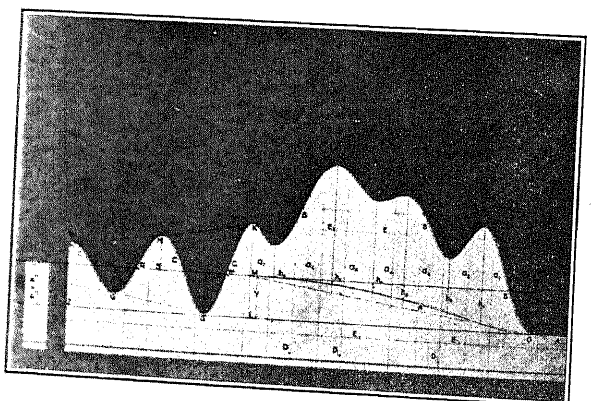

FIG. 23—TYPICAL PRESSURE—TIME DIAGRAM, AFTER DELINEATION

which are then plotted on the diagram and form a correction to the recovery line. The net area of the diagram, $O\,N\,M\,K\,B\,O$, is then measured, and corrected for shrinkage of the print and width of slot.

To substitute this area in equation **(17)** for $P\,T$, it must be corrected by the pressure and time scales used on the diagram so as to give the average pressure in feet of water multiplied by the time in seconds.

Let A = net area of diagram in sq. in.

r = vertical height in in. corresponding to one-ft. pressure rise

S = horizontal length in inches corresponding one sec. of time.

Then $P\,T = \dfrac{A}{r\,S}$, so that equation **(17)** becomes

$$V = \frac{g\,A}{r\,S\,L}$$

and if $K = g/r$

$$V = \frac{K\,A}{S\,L} \qquad \textbf{(18)}$$

For a penstock of uniform area throughout its entire length, this would be the form of the final equation, from which the quantity flowing in cubic feet per second would be determined by multiplying the value V by the area of the pipe in square feet.

In practise, however, the sections of the penstock are usually not of uniform area, but the penstock is made up of a series of different sections of lengths l_1, l_2, l_3, etc., having areas a_1, a_2, a_3, etc., respectively. In such a case the relations given in equation **(16)** hold for each section and the total impulse $P\,T$ for the combined pipe will equal

$$P\,T = 1/g\,(l_1\,v_1 + l_2\,v_2 + l_3\,v_3, \text{ etc.})$$

In this case it will be more convenient to solve directly for Q, which may be done by substituting Q/a_1, Q/a_2, Q/a_3, etc., for v_1, v_2, v_3, etc., thus:

$$P\,T = 1/g\,(l_1\,Q/a_1 + l_2\,Q/a_2 + l_3\,Q/a_3 \text{ etc.})$$

or $P\,T = Q/g \times \Sigma\, l/a$

If $\Sigma\, l/a = F$ and we solve for Q

$$Q = \frac{g\,P\,T}{F}$$

As previously explained $P\,T = \dfrac{A}{r\,S}$ and $K = g/r$.

Therefore, the final equation for penstocks of non-uniform area becomes:

$$Q = \frac{K\,A}{S\,F} \qquad \textbf{(19)}$$

It is thus seen that the quantity is determined from the net area of diagram A, the constants of the apparatus K and S and the physical dimension of the penstock. To the above quantity must be added the amount of leakage flowing through the guide vanes when they are in their closed position, for this amount of flow is not cut off by the closure of the gates. The total quantity is then obtained.

Power-discharge and efficiency-discharge curves of an actual test are shown in Figs. 24 and 25, and serve to illustrate the consistent results obtained with this method of measurement. It is of interest to note that satisfactory proof of the accuracy of this method was

obtained by thorough tests made in comparison with volumetric measurements at the hydraulic laboratory of Cornell University. The mean variation of all tests showed the Gibson method to be only 0.2 per cent high with respect to the volumetric measurement.

This precision is further demonstrated by the curve shown in Fig. 26 made on unit No. 4 at the Queenston Station of the Hydro-Electric Power Commission of Ontario. In this test two independent measurements of discharge were made by different observers, using separate instruments connected some distance apart

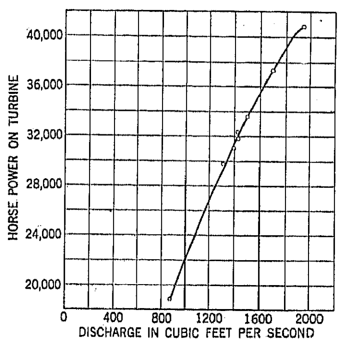

Fig. 24—Power-Discharge Curve of Unit No. 17 of The Niagara Falls Power Company. Tested by Gibson Method

on the penstock. The various constants which include the ratio of the pressure scales, the length of penstock and average areas of the penstock were all different, as shown by the table of constants. An inspection of these curves will illustrate in a convincing manner the close agreement between these independent measurements. Another curve (Fig. 27) shows the performance of a 10,000-horse power hydroelectric unit upon which two tests were made about two months apart, with an independent set of apparatus for each test. In the

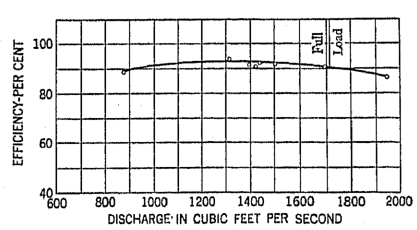

Fig. 25—Efficiency-Discharge Curve of Unit No. 17

first case, the Gibson apparatus No. 5 was connected below the elbow which would for the other methods of measurement be considered as a very undesirable location, owing to the disturbed flow around the bend in the penstock. In the second test, Gibson apparatus No. 1 was connected above the elbow. The lengths of the penstock were therefore different, as were the constants, but nevertheless, the power-discharge curve established by these two tests showed results to be practically identical.[5]

The ability, as demonstrated above, to make repeated tests on the same unit with identical results, using independent equipment attached at different points, not only serves as an indirect check on the work but gives weight to the idea that a waterwheel itself may be considered as a measuring device, and when once calibrated is an indicating flow meter. It is only necessary to establish the power-discharge or the gate-opening-discharge relation by a readily made and economical test such as this method presents. It may

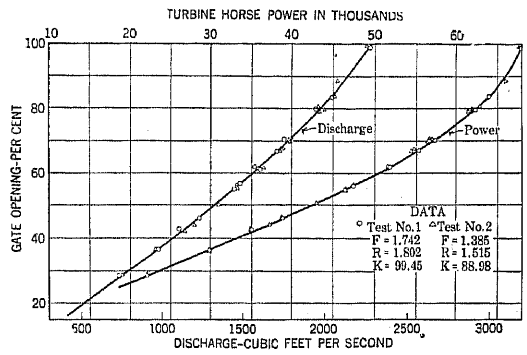

Fig. 26—Power-Discharge Curve of Unit No. 4, Queenston Station of Hydro-Electric Power Commission of Ontario—Check Tests by Gibson Method

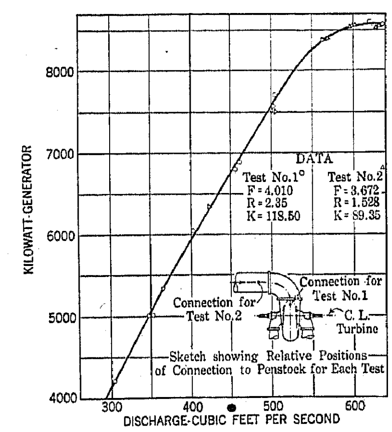

Fig. 27—Power-Discharge Curve of 10,000 Horse Power Unit of The Niagara Falls Power Company—Check Tests Gibson Method

then be desirable to re-calibrate after a period of about two or three years, or if any major repairs are made on the wheel.

Value of Test Data to Power Company

The data secured in accurate tests of hydroelectric units are of great value to the power company as they permit a comparison of the total kilowatt hours produced

5. The data on these tests were presented by E. B. Strowger of the Niagara Falls Power Company, in his discussion of Mr. Gibson's paper at the Annual Meeting of the American Society of Mechanical Engineers, December, 1923.

to the total water consumption for any given period, and this ratio is a direct measure of the economy of operation. In practically all plants, careful records are kept of kilowatt hours output, but in many stations, no careful check of water consumption is attempted. In some cases this neglect is undoubtedly due to excess water supply over plant requirements, so that the output of the station is a question merely of load demand or capacity of units, but such examples of under-developed sites are now rare, especially in the eastern part of this country. Today, the question of monthly or even daily flow of the stream throughout an average year is carefully studied and storage lakes or reservoirs built wherever possible to provide for the units a more constant flow throughout the year, than nature usually furnishes. Storage, therefore, permits over-development as compared to minimum stream flow or even average flow, and results in the use of a large part of the yearly flow of the stream in spite of seasonable variations. Water consumption of the units is therefore a vital matter in most plants and accurate records are kept.

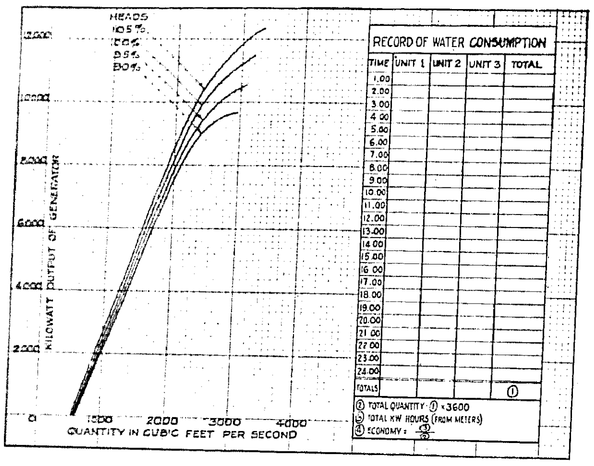

Fig. 28—Calibration Curves of Generator as Water Meter

Efficiency tests should be made on the units as soon as practicable after installation, and curves prepared showing discharge in cubic feet per second plotted against generator output. Typical curves are shown in Fig. 28 for a 10,700-kw. unit operating under a normal head of 60 ft. These curves are actually the calibration curves for the generator used as a water meter. If the head varies appreciably, several power-discharge curves are plotted, one for each head.

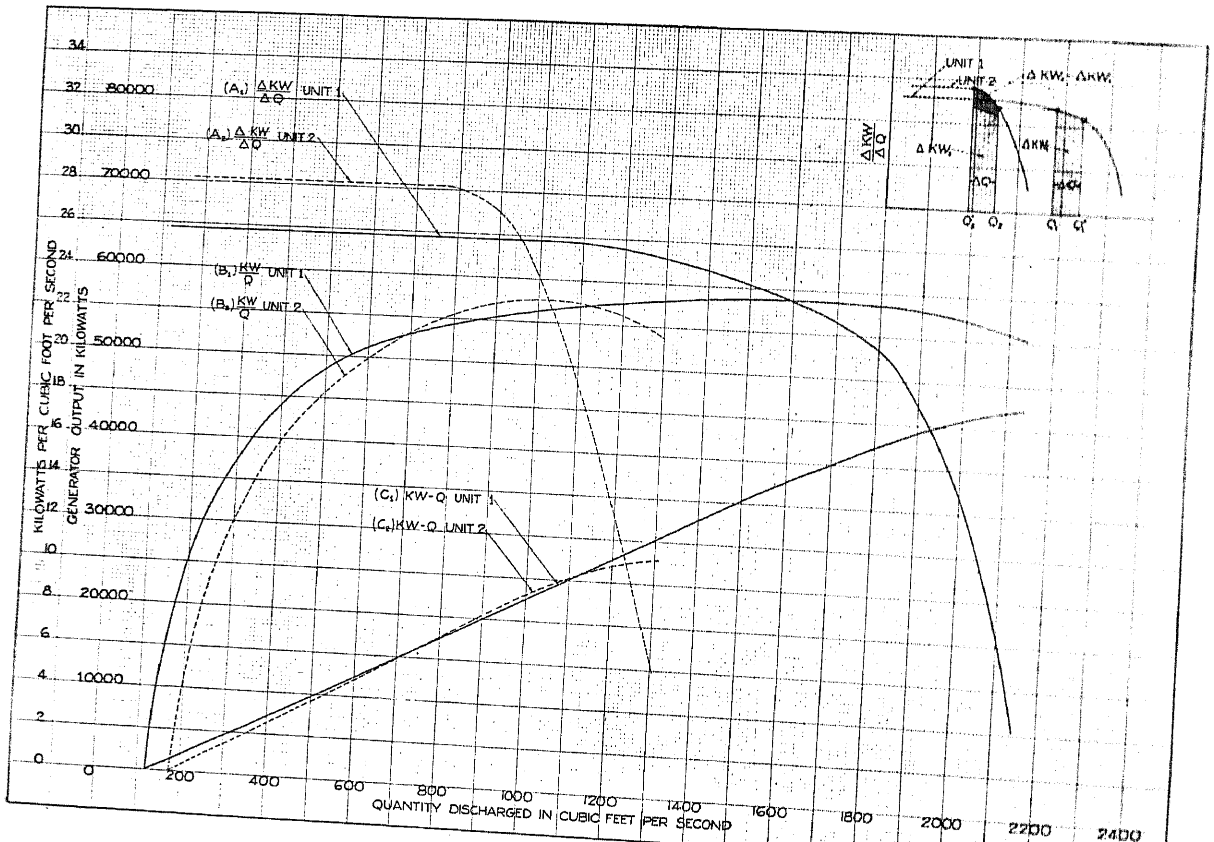

Fig. 29—Determination of Economical Division of Load between Two Units

Records of discharge are frequently kept, such as indicated on the right side of Fig. 28. The kilowatt meters are read on each machine at stated intervals, every hour or half hour as the case may demand, and the corresponding discharge, obtained from the curves, entered on the chart. By averaging the values of quantity and multiplying by 86,400, the total daily water consumption in cubic feet is obtained for each

unit and for the whole station. The total kilowatt hours divided by the total quantity shows the average economy for the 24-hour period.

Maximum economy results, of course, when each unit is operated at its point of maximum efficiency, giving a maximum power output per cubic foot of water, but this ideal condition only exists at one-load demand in a one-unit plant, two loads in a two-unit plant, etc. The actual problem to be solved is, therefore, the correct division of the total load demand between the various units in the plant so as to secure maximum economy.

In plants containing the same size units of exactly the same design, it can readily be shown that the total load should be divided equally between all units in operation. Where the units are of different sizes and of different characteristics, the problem is a little more complicated. Fig. 29 will serve as an example of a plant containing two units, one of 44,000-kw. capacity with low-speed type runner and the other unit of 26,000-kw. capacity with a much higher speed type runner. It is not at all probable that two such runners would be used in the same plant, but this example is chosen to give curves of different appearance so that the readings may be readily differentiated.

From the power-discharge curves C_1 and C_2, new curves are plotted of kilowatt per cubic foot per second B_1 and B_2. Then, the first derivative of each power-discharge curve is drawn, this being expressed as

$\frac{\Delta K w}{\Delta Q}$ and defined as the rate of gain in power to

increase in discharge. These curves A_1 and A_2 are actually the tangents of the angles of slope of the power-discharge curves. If a series of horizontal lines is drawn on this diagram, the intersection of each line with the two derivative curves will give the correct division of discharge and load between the two units for best economy.

For example, a horizontal line at 24 on the vertical scale intersects A_1 at 1520 cu. ft. per sec., corresponding to a load of 35,800 kw. and intersects A_2 at 1003 cu. ft. per sec., corresponding to a load of 23,200 kw. This gives the correct division of a total load of 59,000 kw. and a total flow of 2523 cu. ft. per sec. between the two units for best economy.

The correct division of load between two or more units in a station may therefore be stated by the following theorem: *The load should be so divided between two or more units so as to obtain equal values of the first derivatives of the power discharge-curve of each unit.*

This truth may be more clearly understood by reference to the insert curves shown in the upper right hand

corner of Fig. 29. The derivative curves $\frac{\Delta K W}{\Delta Q}$ for

units 1 and 2 are reproduced plotted against the flow Q. The horizontal dash and dot line shows the correct division of load. Assume a change in division of Q so that the discharge of unit 1 increases from Q_1 to Q_1' or ΔQ, and discharge of unit No. 2 decreases the same amount from Q_2 to Q_2' or ΔQ. It is evident that the shaded area under curve 1 between Q_1 and Q_1' is

equal to $\frac{\Delta K W_1}{\Delta Q} \times \Delta Q = \Delta K W_1$, or the gain in

kilowatts for unit No. 1. The area under curve 2 is the loss in kilowatts for unit 2 and it is self-evident that this loss is a greater numerical value than the gain, so that the net loss in power is shown by the difference between the two areas or $\Delta K W_2 - \Delta K W_1$. From the characteristic shape of the derivative curves, this is true for all values of power or discharge chosen, and therefore proves the theorem stated above.

From a series of horizontal lines drawn on this diagram, a table shown below is thus prepared for the operators convenience to show a correct division of load:

TABLE I

Total Kw.	Unit No. 1	Unit No. 2
24,000	2,000	22,000
29,200	7,200	22,000
34,300	12,300	22,000
39,400	17,400	22,000
43,400	21,400	22,000
49,900	27,700	22,200
52,700	30,200	22,500
55,400	32,700	22,700
61,200	37,700	23,500
66,600	42,100	24,500
72,300	45,700	26,600

It is rather interesting to note that for the curves assumed in this case, unit No. 2 must be operated at very nearly constant load, most of the load changes being taken on unit No. 1. This table is based on the assumption that both units must be in operation. For the lighter loads, better economy is obtained by running a single unit. Thus from a study of curves B_1 and B_2, it is evident that the best results will obtain for the following method of operation:

Loads zero to 14,500 kw. operate unit 1.
Loads 14,500 to 25,700 kw. operate unit 2.
Loads 25,700 kw. to 45,000 kw. operate unit 1.

Discussion

PAPERS ON HYDROELECTRIC POWER DEVELOPMENT

(Thurlow and Sirnit, Barfoed, White, Rogers)
Birmingham, Ala., April 7, 1924

W. S. Lee: Mr. Barfoed's paper, like any one touching on hydro-electrical practise and the whole series of operations on the West Coast, could be divided and make good subjects for forty papers.

I am going to take some exceptions to his arch dam in comparison with the ones in Southern use. It is true this type has developed in California on account of the distance which materials have to be hauled as dams of this design have less cement and material to be transported. In the South, I think, the gravity section is the proper one to use.

It is not the practise in the South to use dams of this design, and the reason for it is heavy overflow of water.

I would also take exception to the use of the core-wall in earth dams. A core-wall in an earth dam is of absolutely no use. It is not strong enough to hold anything and with a wet blanket

of earth on one side and a dry blanket of earth on the other it will move, crack, and leak, thereby, destroying its only value.

P. M. Downing: Steam plants in the hydro-electric system have for several recent years been in use on the Coast. The length of the transmission lines in the West is generally quite great, and there can be no question that the steam plant is quite essential as an economical factor in handling peak loads.

I don't want the impression to get out that the people on the Coast have the feeling that multiple arch dams are suitable among all kinds of conditions. There are certain places where one or two-arch dams are much more economical than gravity dams. The kinds of dams used are dependent entirely on the conditions. Very few of the dams of the multiple-arch type are ever used where there is heavy run-off; there the gravity type of dam is certainly more desirable. We don't have the heavy run-offs during the whole year; our maximum flows come along in May, generally, when the snow is melting. Occasionally in winter months, if there is a light snow and a warm rain on top of that snow, there will be quite heavy run-offs; but that is the exception rather than the rule. We do not have the severe conditions to meet there that you have to meet in the streams in the South. So that I say the dams that are selected depend upon the conditions.

Geo. A. Orrok: I am very glad to hear Mr. Downing bring out the fact that where you have a long distance high-tension transmission line it is good practise to run the line at the maximum load all the time.

I have listened to Mr. White and the other gentlemen talk about the pitting. Mr. White pays particular attention to the wearing away of the runner—he calls it pitting; I call it cavitation, but it amounts to the same thing. Cavitation has been one of the most difficult troubles to overcome. I have been told quite frequently that the European manufacturers have learned how to overcome cavitation and that they had no more cavitation. I had occasion last summer to visit Italy and Switzerland, and I was greatly concerned to find in a corner of almost every plant three or four old runners with very marked cavitation. In one plant I found five runners that had been replaced. The old runners had been welded.

It is my own opinion that until a new runner design has been made and tried out in actual service, no one can guarantee that it will not cavitate. In a conversation with Mr. White this morning, he brought out the fact that this might be a chemical action, and that the theory of cavitation which was brought out in connection with the marine propeller is the proper theory. In hydraulic runners it usually is some slight error as malformation or misapplication of the curve of the vane. A vacuum is formed in the wake of the vane where the occluded oxygen of the water may collect. I recollect three runners made from the same set of cores where this cavitation did not occur in the same place. Of the three runners made from one core and put in three separate settings in the same hydro station, one cavitated on the back of the vanes exactly in the center, the second one on the inside of the outside rim, and the third one cavitated near the hub. There have been two other runners made since then from the same box which I understand show signs of cavitation but not in the same place.

When the curve of the vane is changed to meet the condition, the cavitation will cease, and after that you can make any number of runners from that same core and there will be no more trouble.

In making some heat transfer tests, I had occasion to work up a method for testing the amount of the occluded oxygen in water. In this case the water was salt. This method has been adopted as a standard method and I believe it makes a lot of difference whether the water that is flowing through the wheel contains more or less occluded air. In a test on a centrifugal pump several years ago, we got an efficiency of eighty-nine per cent, until we noted a lot of air bubbles in the water and found we had a perfect plant for mixing air with water. The water weighed nearer fifty pounds per cubic foot than sixty-two and a half. That would make some difference in the cavitation as well as efficiency.

P. M. Downing: On many of the plants operating in the West, where we use impulse wheels, we have an action very similar to the cavitation that takes place in the reaction turbines, generally within the buckets on both sides of the splitter. The action there is very similar to the cavitation that takes place in the reaction runner. I would like to ask if there is any close relationship between the causes of pitting in these two different types of wheels?

W. S. Lee: What Mr. White and Mr. Orrok have been discussing takes place on the opposite side of the runner. We have in operation today wheels that have been in operation eighteen years, on which the pitting occurs on the under side of the runner. If it does not cut through, we bore a small hole of ¼ in. in size; this often stops the pitting.

Geo. A. Orrok: Do I understand Mr. Downing to say that the cavitation spots are rough just as though the iron had been dissolved out leaving the graphite of the cast iron showing?

F. M. Nash: I have had to contend with the question of this eating-in or erosion in both impulse and reaction turbines and have found it about as bad in one as in the other. The impulse turbines using a 3 in. nozzle under 1800-ft. head had to have both needle tip and nozzle tip renewed two or three times a year. With the reaction turbines, we had considerable pitting on the back side and near the outer edge of each runner vane, just as Mr. White says, but the worst wear occurred on the clearance rings of a 500-hp. horizontal double-runner reaction turbine under 340-ft. head. This ring was of cast iron and the erosion destroyed the clearance-ring metal for a depth of 1 in. and more in a radial direction. This caused a very ragged and deeply pitted condition that had to be repaired. Repairs were made by dismantling the turbine, cutting back the clearance ring with torch and chisel for a radial depth of 1 in. or more and then welding in by use of torch a ring made up of 1¼-in. x 2-in. iron to give it something near the original setting. This work was all done in the field and no attempt was made to true it in a lathe. The repairs proved to be fairly satisfactory.

All the pitting was considered to be due to chemical action of the particles of air on the iron under a slight vacuum, and not due to the wear of either water or sand and silt. Furthermore, all the pitting of this kind that I have had occasion to observe has always occurred in water wheels or turbines under a relatively high head. Under heads of from 20 to 40 ft. even though the water is heavy with silt the year around, I have never known this pitting to occur.

W. F. Dawson: One would be rash, after noting the serious troubles still experienced with cavitation to claim an immediate remedy, but I am strongly of the opinion that marked improvement could be secured if these parts could be chilled-cast, or at any rate, chilled about those surfaces where electrolysis, due to cavitation, is expected. Many of these runners are cast in manganese "bronze" or more properly manganese brass, which contains approximately 45 per cent of zinc. Zinc has a strong tendency to segregate, particularly when poured in ordinary sand moulds, but if the moulds are lined with chills of thicknesses, say equal to one or two times the thickness of the casting, the segregation is prevented and a clean casting of uniform grain structure is thereby obtained. Chilled castings not only prevent the segregation of the zinc and other materials in the alloy, but also makes a stronger and tougher casting. The elastic limit is frequently raised 30 per cent and I have seen cases where the elongation was increased from 2 to 3 per cent to 10-15 per cent.

While I have not had actual experience with chilled iron castings, I believe that this material can also be chilled to advantage, but in that case the material would have to be carefully annealed after casting.

It is fairly well known, but should be continuously emphasized,

that corrosion occurs due to electro-chemical action between elements separated from each other in the electro-chemical series whenever an electrolyte is present. If the chilled casting or any substitute process maintains a perfect chemical combination of the elements, the tendency to corrode is greatly reduced as compared with those castings in which the elements are segregated.

Geo. A. Orrok: I have never had the opportunity of seeing an impulse turbine bucket that was cavitated. If you take cast iron and put it under a magnifying glass, you can pick out the cast-iron crystals; the carbon standing up by itself, and you can take your knife and pick the graphite out of the cast iron. In cast steel you get that same thing, but certainly the harder crystals of the cast steel stand better while the softer ones are dissolved out by oxygen.

I would like to say one thing with regard to this chilled iron. I have seen chilled iron buckets which cavitated in the same way. I don't know what has been done with manganese bronze, but steel plate buckets cavitate in the same way when the curves are not right.

is available, the thrust bearing in the first unit has been running to date without connection to it. This is possible because of the high efficiency of the water cooling coil. The inevitable agitation of the oil by the thrust-bearing runner, below the top baffle, is utilized to secure this result.

The virtual elimination of loss of oil results from the use of the air-seal ring in conjunction with the baffle plates around the coil. This ring, which is shown in the illustration, keeps air out of the oil bath and films, thus increasing the bearing's factor of safety. The free surface of the oil is protected from all disturbances. Consequently evaporation losses and oxidation of the oil are reduced to a minimum. Their formation being prevented, oil spray and oily mist are kept out of the electrical windings.

With this unit operating normally, an inspection of the bath through the windows provided shows that the oil is clear and solid, holding no air in suspension; also that in spite of the high rim speed at which the bearing operates, the free oil surface is to all appearances as smooth and free from bubbles as during a prolonged shut-down.

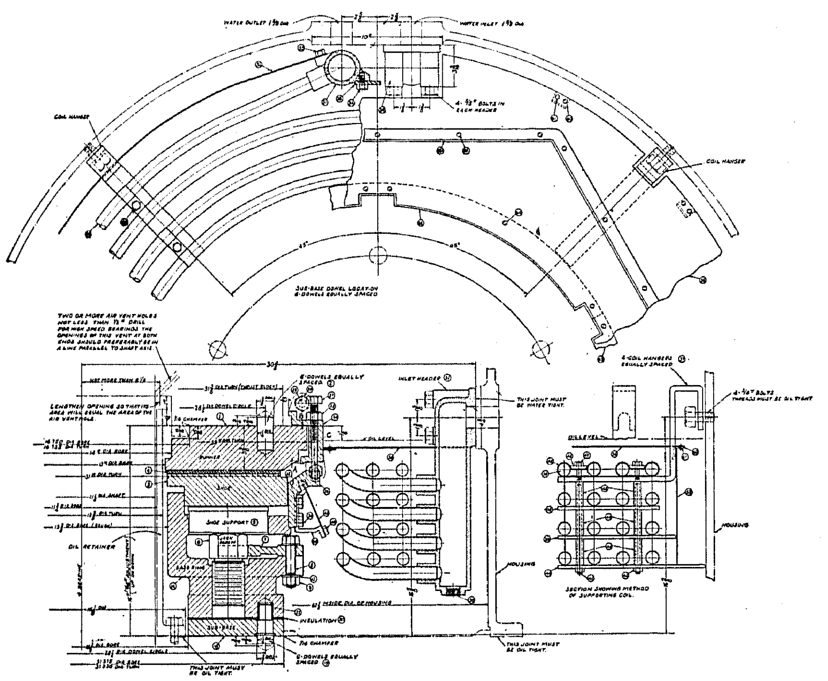

FIG. 30.—THRUST BEARING FOR 70,000-H. P. GENERATORS

Each of the three 70,000-h. p. Units of the Niagara Falls Power Co. uses a 69-in. Kingsbury bearing, illustrated above, which carries 1,250,000 lb. load at 107 rev. per min.

Arthur B. Lakey (by letter): The author has mentioned that the entire rotating weight of his generator, exceeding 700,000 lb., was carried by the Kingsbury thrust bearing. This figure, however, is far short of the total thrust load. When hydraulic load and revolving weights of generator and turbine are all included, the load on each thrust bearing reaches a total of 1,250,000 pounds, the highest bearing load on record.

The first of the three Kingsbury thrust bearings for these 65,000-kv-a. units went into service with the General Electric—I. P. Morris unit on December 18, 1923.

The illustration shows the distinctive features of the bearings which has a diameter of 69 in. across the babbitted faces of the pivoted segments. The unit bearing pressure is 483 lb. per square inch of the net surface of the segments. At the normal speed of operation, 107 rev. per min. the thrust bearing friction loss per unit is about 45 h. p. or 1/16 of 1 per cent of the unit's rated output.

Certain design features of these Niagara thrust bearings are deserving of comment. Although an oil-circulating system

The type of bearing equipment described, with cooling coils and air seals included, is virtually standard on recent important medium-speed and high-speed vertical work. In most of these installations no oil circulation is used for the thrust bearings.

When assembling this 65,000 kv-a. Niagara unit, the jack screws supporting the Kingsbury shoes were utilized to lift the rotor from the blocks and to adjust its height to secure the proper vertical clearance in the turbine. The screws proved much more convenient for this purpose than the station cranes.

The illustration shows the insulation which, also in line with Mr. Kingsbury's recent practise, was included in the Niagara thrust bearings as a protection against possible damage from shaft current.

Later in his paper, Mr. White mentions several other developments using turbines with Francis and high-speed runners. All these units use Kingsbury bearings, mostly fitted with cooling coils and largely employing air-seal rings.

The entirely successful performance of the unit already

operating at Niagara under the previously unheard of load of 1,250,000 lb. has strengthened the conviction that thrust bearings are no longer a limiting factor in hydroelectric design. In fact, a generator has already been projected in which the estimated thrust load is to be 1,600,000 lb. When constructed this giant bearing will not represent any greater relative step in advance than the Niagara bearing which has already proved itself in service.

F. H. Rogers: The various points raised by Mr. Mitchell and others are of considerable interest, as they bring out a method of operation in a hydroelectric station which the writer believes has been in more or less general use, but which unquestionably is in error. That is, in most stations where three or four units are installed, it seems to be the general impression that the most economical method of operating these units is to run as many of the units as possible at their points of best efficiency and to take up the balance of the load by running one or two units at a considerably reduced load.

Without studying this matter carefully, it might appear that this method of operation would be the best as most of the units in the station would be running at their maximum efficiency. However, the fallacy of this is very fully proven and is shown clearly diagrammatically in Fig. 29. The point at issue is that although the majority of units which are run at their best efficiency are certainly developing power in the most economical method, the remaining unit or two, as the case may be, which are running at part load, are running so inefficiently that this loss more than balances the gain in the units operated at their best efficiency, and that the most economical results, taking the plant as a whole, follow when all units are run at the same value of the differential curves showing the rate of gain in power to increase in discharge.

It might be pointed out that the particular examples shown in the paper in Fig. 29 are for units operating under the same head, although these units are of different characteristics and of different size. The general theorem, however, applies to units operating under different heads and in different stations, provided that we are interested in the maximum total output of all the plants in kilowatts for a given total quantity of water discharged through all of the plants.

It might further be pointed out with interest that the curves shown in Fig. 29 are plotted for a constant effective head on the turbine for all flows. In reality, however, in any given station, the effective head on the turbine decreases as the load is increased due to the greater friction loss through the intakes and penstock. If this point were taken care of in plotting the curves in Fig. 29, the power output Curves C_1 and C_2 would bend down more rapidly as the quantity increased and this, of course, would have the effect of bending down the differential Curves A_1 and A_2 more rapidly as the quantity increased. These differential curves show very clearly the inadvisability from an efficiency viewpoint, of running any hydraulic turbine appreciably beyond its best efficiency point, for it can be seen from the differential curves that the gain in power which is equal to the area under the differential curves, rapidly decreases as the load is increased beyond the best efficiency point. In most plants, therefore, when water economy is of importance, it is now considered advisable after making an efficiency test on the unit, to block the turbine gates definitely at a point slightly beyond the maximum efficiency, so as to prevent the operators running the machines beyond this point where the economy is falling off at so rapid a rate.

S. Barfoed: Construction of hydroelectric undertakings is very much influenced by natural conditions. Therefore, structures selected for one location will seldom fit in another. One cannot favor certain types. But it is an engineer's duty to be fully informed about economical methods and structures, so that he can give the public the benefit of saved capital in the use of them. It is obvious that no structure should be chosen for a site where it does not fit or where suitable materials can only be had at great cost.

On southern streams, of which Mr. Lee speaks, no doubt he is right to choose the gravity type. A multiple-arch dam is not an overflow dam, but a single-arch dam properly designed is. Apparently the canyons of southern streams are not of the shape which will at all permit consideration of the single-arch dam. In the west many streams flow in canyons of ideal shape for such dams. A saving in capital expenditure results and the structure has at the same time a much higher factor of safety than a gravity dam.

Earth dams are by no means standardized. With previous foundations of great depth, the corewall is a successful item in their construction.

Tests on 22-Kv. and 4-Kv. Lightning Arresters

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Review of the Subject.—*The development, within recent years of high-voltage rectifying tubes has made possible the construction of an apparatus which may be used to produce very high-voltage surges of steep wave front, believed to be comparable in potential and capacity to surges caused by lightning discharges taking place near transmission lines. With such an apparatus it has become possible to conduct tests on lightning arresters which show clearly the kind of performance these arresters are giving in service.*

The Duquesne Light Company has had on its system a great number of lightning arresters of the horn gap and series resistance type. The number of failures on installations these arresters were intended to protect and the number of failures on the arresters themselves have led the engineers of this company to go rather deeply into the lightning-arrester question and into the problem of learning, before an arrester is installed, just what degree of protection it is likely to afford.

To get the information desired it was found necessary to make a series of tests on the various arresters now in the market. In making these tests arbitrary requirements were set up as being necessary in an arrester for satisfactory operation on this system. These requirements were as follows:

1—High speed of discharge

2—Ability to break down dynamic current

3—Ability to operate repeatedly

The protective value of arresters meeting these requirements was felt to be proportional to the discharge capacities of the arresters.

This paper sets forth the results of the tests made on 22-kv. and 4-kv. lightning arresters.

THIS paper is a report of a series of tests on 22-kv. and 4-kv. lightning arresters to determine the relative value of the different types for protection of a power system.

The tests were designed to bring out the following characteristics:

(a) Relative time required to start discharge through gap and arrester.

(b) Relative discharge capacities.

(c) Relative ability to break dynamic current following discharge.

(d) Ability to handle repeated discharges.

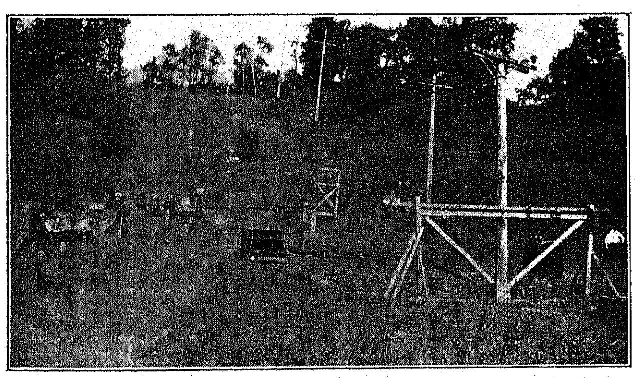

FIG. 1—VIEW OF TEST FIELD SHOWING 22-KV. ARRESTER SET-UP

The first and second groups of tests were made to determine the relative time to discharge and the discharge capacity of the arresters. Fig. 2 shows a diagram of the connections used in this test. A transformer was used to charge a set of condensers of 0.2 μ f. capacity, kenetrons being used to rectify the current. Each lightning arrester in turn was then connected in the test circuit, as shown in the sketch. A resistance shunted by a sphere gap was placed in series with the arrester to measure the discharge currents. One sphere gap was so placed as to measure the voltage of the surge and another placed across the arrester

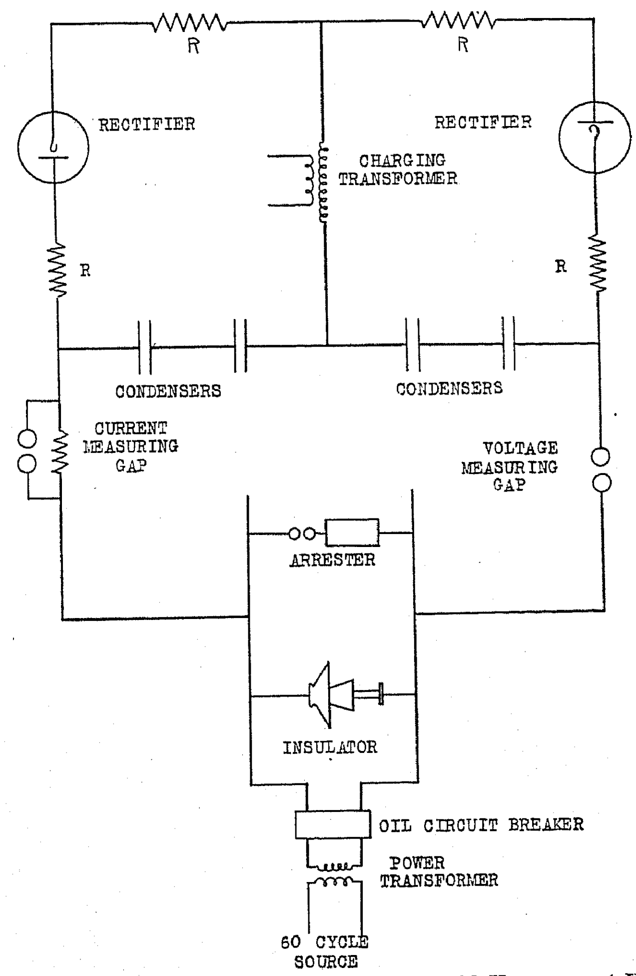

FIG. 2—DIAGRAM OF CONNECTIONS FOR 22-KV. AND 4-KV. LIGHTNING ARRESTER TESTS

itself. In the first group of tests this latter gap replaced the insulator shown in the diagram and was used

to measure voltage across the arrester. The breaker in the circuit from the power transformer was left open. Within this group a separate set of tests was made to determine the effect of resistance in series with the arrester. With the same set up as before, the resistance on several arresters was varied and the voltage across the arrester measured, the surge voltage being made the same, as nearly as possible, on each test. It was found that the voltage across the arrester increased with increase of series resistance.

The results of these tests are shown in Table I.

TABLE I
DISCHARGE CAPACITY OF 22-KV. ARRESTERS

Arr. No.	Surge Kv.	Arrester Kv.	Resistor Kv.	Resistor Ohms	Surge Amps.	Arrester Resist.	Remarks
	100	64.5	89.5	270	332.0	81.8	
1	100	71.0	69.0	90	767.0	43.9	
	150	76.0	150.0	270	555.0	69.6	
	150	92.0	107.0	90	1190.0	46.0	
	100	100.5	12.0	270	44.5	1189.0	
2	100	99.5	6.5	90	72.2	718.0	
	150	110.5	56.5	270	210.0	300.0	
	150	117.0	40.0	90	445.0	156.0	
	100	106.5	46.5	270	172.0		
3	100	117.5	34.5	90	383.0		
	150	156.0	73.5	270	272.0		
	150	177.0	41.5	90	461.0		
	100	99.0	24.0	270	89.0	1111.0	
4	100	106.5	18.5	90	205.0	520.0	
	150	152.0	147.5	270	545.0		Bushing flashed
	150	168.0	140.0	90	1550.0		Bushing flashed
	100	105.5	101.5	270	376.0	284.0	
5	100	109.0	92.0	90	800.0	136.0	
	150	131.0	157.0	270	580.0	226.0	
	150	140.0	135.5	90	1500.0	93.0	
	100	91.0	20.5	270	76.0	1200.0	
6	100	99.0	9.0	90	100.0	990.0	
	150	127.0	39.0	270	144.5	879.0	
	150	147.5	19.5	90	216.0	682.0	
	100	111.0	10.0	90	111.0		2 in. gap
7	100	106.5	23.0	270	85.0		
	150	169.0	121.0	90	134.4		
	150	164.0	37.0	270	137.0		
	100	116.0	13.5	90	150.0	771.0	2 in. gap
8	100	109.0	28.0	270	103.7	1050.0	
	150	164.5	22.5	90	250.0	656.0	
	150	150.5	41.0	270	152.0	990.0	

In the second series of tests the pin type insulator shown in Fig. 2 was used in place of the gap across the arrester, in order to demonstrate directly the degree of protection secured. The insulator was a 22-kv. pin type, having a dry flash-over value of about 80,000 volts. Before starting these tests, the surge generator was used to impress high-voltage impulses on the insulator. The insulator flashed-over each time, showing that the voltage was high enough for the test proposed. Each lightning arrester in turn was then connected in parallel with the insulator and the impulses set up as before. A flash-over of the insulator or a flash on the arrester, other than across the gap, was considered a failure of the arrester.

Another set of tests was conducted within this group, also, to determine the effect upon arrester performance of series resistance in the circuit, used to take the place of the surge impedance of the line. In some of the tests on each arrester the surge impedance of the circuit was 224 ohms while in the other tests it was zero. The latter condition was found to be considerably more severe than the former and was thought to be worse than would be met at any time in actual service.

The results of this test were as shown in Table II.

TABLE II
PROTECTIVE VALUE OF 22 KV. LIGHTNING ARRESTERS

Arrester Number	Series Resistance	Insulator Flashovers Per Cent	Remarks
1	224	0	
	0	0	
2	224	0	
	0	0	
3	224	0	
	0	50	
4	224	0	Bushing of arrester flashed over
	0	0	Bushing of arrester flashed over
5	224	0	
	0	0	
6	224	0	Resistance rods partially flashed over.
	0	100	
7	224	25	Resistance rods on arrester flashed over.
	0	100	Change in series resistance made no difference. Change in gap reduced number of flashovers.
8	224	50	Careful arrangement of resistors was necessary to prevent flashing at resistor.
	0	0	Flashed over insulator on arrester or over arrester resistances.

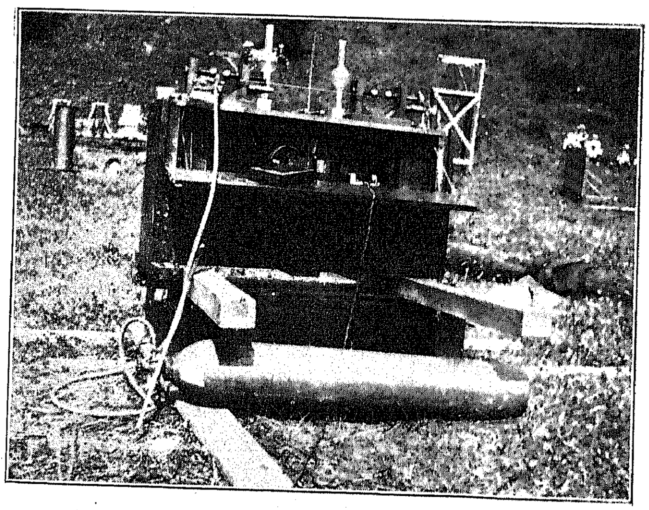

FIG. 3—IMPULSE GENERATOR USED IN FIELD TO MAKE DYNAMIC TESTS

The third group of tests was made to determine the capacity of the lightning arrester to interrupt dynamic current after a static surge had caused an arc across the gap of the arrester when connected to a power line. To make these tests the surge generator was connected to the power line, giving the same effect as closing the breaker in the power circuit as shown in Fig. 2. The connections on the power circuit were as shown in Fig.

4, the 15-ohm resistor in the neutral being in series on the initial test on each arrester. This precaution was taken to limit the short-circuit current in case of failure of the arrester. Each arrester that withstood

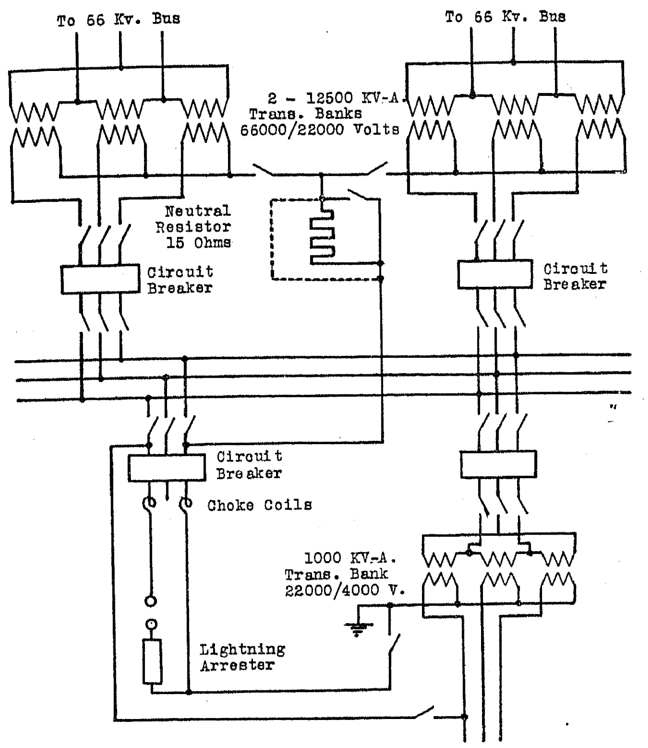

FIG. 4—DIAGRAM OF CONNECTIONS AT SUBSTATION WHERE DYNAMIC TESTS WERE MADE

this test satisfactorily was then tested with the resistor short-circuited to eliminate any effect the resistor might have.

The results of these tests were as shown in Table III.

TABLE III
DYNAMIC TESTS—22-KV. ARRESTERS

Arrester Number	Number of Tests	External Resistance	Surge Voltage	Remarks
1	3	15 ohms	40 kv.	Cleared
	3	0	47.5 kv.	Cleared
2	3	15 ohms	47.5 kv.	Cleared
	3	0	47.5 kv.	Cleared
3	3	15 ohms	47.5 kv.	Cleared
	3	0	47.5 kv.	Cleared
4	3	15 ohms	47.5 kv.	Cleared
		0	47.5 kv.	Cleared
5	1	15 ohms	47.5 kv.	Cleared. Fuse blown.
	2	15 ohms	47.5 kv.	Failed to clear. Circuit was opened by breaker.
6	3	15 ohms	47.5 kv.	Cleared
	5	0	47.5 kv.	Cleared
7	1	15 ohms	47.5 kv.	Cleared circuit when resistance rods were blown from tubes.
	1	15 ohms	47.5 kv.	Failed to clear. Circuit opened by breaker.
8	2	15 ohms	..	Failed to clear circuit.

DESCRIPTION OF ARRESTERS

The following is a very brief description of the arresters tested.

1—Electrolytic or aluminum cell, 91 trays, laboratory sphere gap.

2—Carbon disc pile, 4 columns of discs, sphere gap.

3—Oxide film; 96 cells; sphere gap.

4—Series water resistance; sphere gap and horn gap.

5—Multiple horn gap with shunt resistance. Fuse ahead of arrester.

6—Multiple gap; series resistance. Circuit-breaking device to increase gap.

7—Horn gap with series resistance, carborundum covered rod to extend arc.

8—Horn gap with series resistance.

RESULTS

Arrester No. 1—showed highest speed and highest discharge capacity, protected insulator and stopped flow of dynamic current.

Arrester No. 2—showed second highest speed and third highest discharge capacity of valve-type arresters. Protected insulator and stopped flow of dynamic current.

Arrester No. 3—showed second highest discharge capacity and third highest speed of valve-type arresters. Stopped dynamic current and protected insulator each time when surge impedance was in circuit but only 50 per cent of time when there was no surge impedance in circuit.

Arrester No. 4—showed slow speed of discharge and intermediate discharge capacity on surges at 100-kv. potential. Interrupted dynamic current. Bushing of arrester flashed over on 150-kv. surges, emphasizing lack of speed in discharging.

Arrester No. 5—showed high discharge capacity and protected insulator perfectly but would not interrupt dynamic current without blowing fuse, thus making it inoperative for further surges. The discharge current went across gaps to ground in the static tests. In the dynamic tests this resulted in short-circuit.

Arrester No. 6—showed intermediate discharge capacity and interrupted dynamic current. Some of the series resistors were wholly or partly flashed over on each test with surge impedance. The insulator was, however, perfectly protected when there was surge impedance in the circuit, but flashed over each time when the surge impedance was removed.

Arrester No. 7—showed low discharge capacity, failed to protect insulator and would not interrupt dynamic current.

Arrester No. 8—showed low discharge capacity, protected insulator part of time but would not interrupt dynamic current.

CONCLUSIONS—22-KV. ARRESTERS

Among the requirements to make an arrester satis-

factory for service, the three following are of major importance.

First, it must operate before the surge voltage impressed on the system can cause a flash over or breakdown of the insulation the arrester is intended to protect.

Second, it must interrupt the dynamic current which will have a tendency to flow through the arc across the arrester gap.

Third, it must be in condition for repeated operations.

If a lightning arrester meets these three requirements, then its value as a protective device will be directly proportional to its discharge capacity. Therefore, using the maximum discharge capacity needed at 100 per cent, the percentage of protection afforded by any arrester may be determined. Thus, since the surge current is equal to the surge voltage, divided by the surge impedance, there is only one unknown in the equation

$$I = \frac{E}{\text{surge impedance}}$$

This unknown is the voltage.

Knowing the surge impedance, we may assume several values of voltage generally agreed to be reasonable. For example, let us assume the voltage to be 150-kv. A series of tests was made on the arresters at this voltage with a surge impedance of 270 ohms.

This value of impedance is low but was so chosen to place rather severe duty on the arresters. The point at which it would be desirable to have the arresters discharge may be taken as 35 kv. maximum. Then the surge will have to be reduced by the arrester from 150 kv. to 35 kv. or 115 kv. Under these conditions the surge current will be

$$\frac{E}{Z} = \frac{115{,}000}{270} = 426 \text{ amperes.}$$

However, as voltage reflection takes place at an open end, current reflection similarly takes place at a short-circuit and, since the discharge of an arrester results in a short-circuit on the line, the current to be discharged by the lightning arrester is twice that shown in the calculation, or 852 amperes. Therefore, a lightning arrester with a discharge capacity of 852 amperes at 150 kv. would be a 100 per cent arrester.

Referring to Table I, we may see that the discharge capacity of the electrolytic or aluminum cell lightning arrester at 150 kv. is 555 amperes. As these tests were single-phase in nature, this arrester may be considered to afford 65.2 per cent protection on single-phase surges. Using this method of calculation and the values of surge current shown in Table I, we find that as to discharge capacity, the lightning arresters tested rank as follows for single-phase surges of 150 kv. and surge impedances of 90 ohms and 270 ohms.

Discharge Capacity—22-Kv. Arresters
150-kv. Surge—High Frequency

Z = 90 ohms	Z = 270 ohms
1—46.5 per cent	1—65.2 per cent
2—17.4 per cent	2—24.7 per cent
3—18.1 per cent	3—31.9 per cent
6— 8.5 per cent	6—17.0 per cent

However, as lightning surges may be considered as occurring simultaneously on all three-phase wires, the true value of an arrester is determined by its operation under such conditions. Arresters No. 1 and No. 3 use a single ground leg for all three-phase legs, while Arrester No. 2 uses a ground leg for each phase leg and the resistance-type arresters have each leg connected directly to ground. Therefore, the discharge capacities of Arresters No. 1 and No. 3 on three-phase surges as compared to the other arresters is 50 per cent of the discharge capacity indicated by the tests. So far as can be seen by the writer, there are no engineering difficulties in the way of connecting Arrester No. 3 in the same way as Arrester No. 2. If this connection were applied to Arrester No. 3 the table above, written for three-phase surges, would be as follows:

Discharge Capacity—22-Kv. Arresters
150-kv. Surge—High Frequency

Z = 90 ohms	Z = 270 ohms
1—23.3 per cent	1—32.6 per cent
2—17.4 per cent	2—24.7 per cent
3—18.0 per cent	3—32.0 per cent
6— 8.5 per cent	6—17.0 per cent

Three-phase tests could not be made with the equipment available, so that these figures could not be verified.

As a result of these tests, conclusions on the different types of arresters as protective devices have been drawn as follows:

Of the lightning arresters tested, Arrester No. 1 is the most satisfactory for removing surges from lines and equipment. It has a very high speed of discharge and a heavy discharge capacity.

Arresters No. 2 and No. 3 are of about equal value as protective devices, No. 2 having a greater speed of discharge and No. 3 having the heavier discharge capacity, provided the same connections are used on both arresters. Neither of these arresters affords the same quality of protection as No. 1.

Arrester No. 6 was the only other one to meet the requirements originally set up. This lightning arrester has a comparatively low discharge capacity with low values of surge impedance and its inability to protect the insulator in some cases was probably due to this characteristic.

Of the other arresters tested, each failed in some respect to meet the requirements set forth. Arrester No. 4 has a low speed of discharge which allows flash-

overs to occur at a weak point in the insulation. This arrester has a fair discharge capacity and would perhaps be satisfactory on surges of sloping wave front. The cause of the slowness of the arrester discharge is thought to be the series resistance. Arrester No. 5 shows very desirable discharge characteristics but cannot be satisfactory so long as the fuse operates to clear the dynamic current, leaving the arrester inoperative on further surges. Arresters No. 7 and No. 8 are felt to be unsatisfactory because of their failure to protect the insulation and to clear dynamic current.

Tests on 4-Kv. Lightning Arresters

The tests on the 4-kv. lightning arresters were similar to those made on the 22-kv. arresters, except that the second group of tests—showing protection to an insulator—was omitted. The results of the discharge capacity tests were as shown in Table IV and the results of the dynamic interrupting capacity tests were as shown in Table V.

TABLE IV
DISCHARGE CAPACITY OF 4-KV. ARRESTERS

Arrester	Surge Kv.	Arrester Kv.	Resistor Kv.	Resistor Ohms	Surge Amperes	Arrester Ohms
1	40	18.0	15.1	10.3	1460	9.2
	60	18.0	27.1	10.3	2620	5.1
2	40	13.3	12.1	11.2	1080	12.3
	60	15.6	18.0	11.0	1637	9.5
3	40	20.1	9.6	11.2	856	
	60	27.7	16.8	10.9	1540	
4	40	30.7	7.0	11.2	625	49.0
	60	34.3	10.8	10.8	1000	34.3
5	40	32.2	4.6	11.2	411	78.3
	60	45.6	9.3	10.9	853	53.5
6	40	27.5	19.0	11.3	1680	
	60	30.5	29.2	11.1	2630	
7	40	25.6	9.6	11.2	856	29.8
	60	34.9	20.3	10.9	1862	18.7
8	40	21.6	8.6	11.2	766	28.2
	60	28.3	14.5	10.9	1330	21.3

TABLE V
DYNAMIC TESTS—4-KV. ARRESTERS

Arrester Number	Number of Tests	External Series Resist.	Surge Voltage	Results
1	3	15 ohms		Cleared
	3	0		Cleared
2	3	15 ohms		Cleared
	3	0		Cleared
3	3	15 ohms		Cleared
	6	0		Cleared
4	3	15 ohms		Cleared
	3	0		Cleared
5	Dynamic tests not completed			
6	1	15 ohms		Failed to clear circuit. Arc held between horns until breaker operated to clear circuit.
7	1	15 ohms		Failed to clear circuit. Arc held between horns until breaker operated to clear circuit.

Description of Arresters. A brief description of the arresters tested follows.

1—Carbon disk pile, single column of disks, enclosed gap.

2—Multiple gap, two shunting resistance rods.

3—Oxide film, twenty cells to ground, sphere gap.

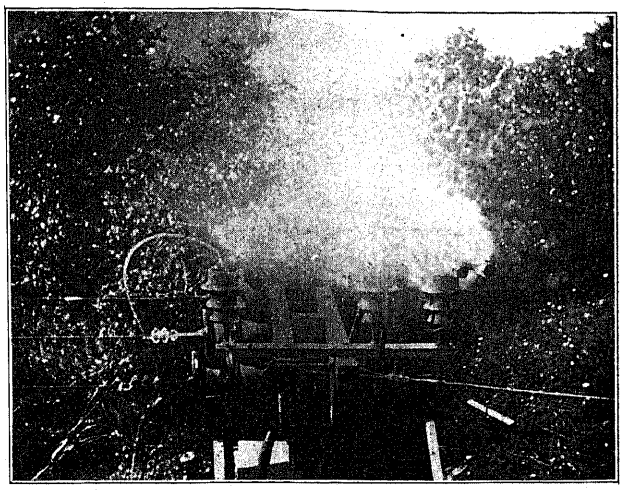

Fig. 5—Failure under Dynamic Test of Horn-Gap Arrester with Series Resistance

4—Multiple gap, series resistance. Whole arrester encased in porcelain.

5—Series water resistance, sphere gap.

6—Porcelain bushing over iron pipe. Horn gap between line and ground, no series resistance.

7—Multiple gap, series resistance, circuit breaking device to increase gap.

8—Multiple gap and series resistance rod encased in porcelain.

Results

The results obtained from the tests on the 4-kv. lightning arresters were as follows:

Arrester No. 1—showed rapid speed and high discharge capacity. This arrester interrupted dynamic current very satisfactorily.

Arrester No. 2—showed high speed of discharge intermediate discharge rate and interrupted dynamic current saitsfactorily.

Arrester No. 3—showed intermediate speed of discharge, a fair capacity of discharge and interrupted dynamic current satisfactorily.

Arrester No. 4—showed low speed and discharge capacity but interrupted dynamic current satisfactorily.

Arrester No. 5—showed low speed and discharge rate. The dynamic tests on this arrester were not completed.

Arrester No. 6—showed high discharge rate but was incapable of interrupting dynamic current.

Arrester No. 7—showed intermediate discharge capacity but was incapable of interrupting dynamic current.

Arrester No. 8—showed intermediate speed and capacity of discharge. The dynamic tests were not completed on this arrester.

Conclusions—4-Kv. Arresters

Ordinarily on 2200-volt and 4000-volt service lightning arresters are spaced rather closely and the lines are placed beneath the higher voltage lines on the poles. Both of these conditions tend to make a very high discharge capacity in the lightning arrester less essential on low-voltage circuits than on high-voltage circuits. The magnitude of the line voltage itself makes it possible to use series resistance in the arrester low enough to give fair discharge capacity and yet high enough to interrupt the dynamic current tending to follow the static discharge across the gap of the arrester when flash-over occurs. In other words, the lightning protection problem becomes much less difficult of solution on low-voltage circuits than is the problem on higher-voltage circuits.

In making the tests on all of the arresters, it was felt that the most accurate results were secured when the resistance in the arrester was of approximately the same value as that of the arrester resistance. However, it was also found that the arresters which operated satisfactorily with low-circuit resistance retained about the same ranking as to discharge capacity under such conditions as when tested with the higher values of circuit resistance. In view of these facts, only one set of discharge-capacity tests was made on the 4-kv. lightning arresters. These tests were made at surge potentials of 40 kv. and 60 kv. No tests were made on the electrolytic or aluminum cell arrester because its principle of operation was found to be satisfactory on the 22-kv. lightning arrester tests and it was felt to be the best surge protective device available in the 4-kv. field.

The dynamic test on the 4-kv. arresters showed very successful operation on most of the types tested. Numerous operations of the arresters were performed very successfully and the manner in which the surge was cleared indicated that satisfactory results would probably be obtained in service with any of the types that withstood the test.

Discussion

E. E. F. Creighton: It seems to me that Mr. Young's tests are in the direction to satisfy the users of lightning arresters. The users have felt for some years the need of tests, aside from those of the manufacturers, developers, and designers of lightning arresters. It is important to recognize the limitations of the tests as carried on outside of a thoroughly equipped laboratory. On one hand, there is the possibility of separating the sheep from the goats. There are on the market types of lightning arresters which are really not dischargers at all and have no particular value. The operator should be able with his tests to separate arresters of good discharge rate from those that have practically no discharge rate.

On the other hand, I hold out no hope at the present time that the operators will so refine their methods of tests as to distinguish between types of lightning arresters all of which have a reasonable discharge rate. It is not yet possible, by any method we know, to distinguish fine differences between lightning arresters. There are no casual tests of a few days' duration that I would accept to take the place of a year of tests involving hundreds of thousands of discharges under as many varied conditions as we can devise. May I repeat that these statements have nothing to do with the work that Mr. Young has done, because the value of his tests is to show whether an arrester has a sufficient discharge rate to take off lightning or not.

K. B. McEachron: For many years the large manufacturing companies have been making tests on lightning arresters of various types in order to ascertain as nearly as possible how the different arresters compare in characteristics. To do this, use has been made of the discharge of condensers through the arresters under test. Different values of capacity and voltage have been used by the different investigators to represent as well as may be, the duty an arrester would be expected to perform in service. Although no agreement has been reached among the several investigators as to the proper values of the constants to be used, all appear to be agreed that the use of a discharge of this general type is proper for the determination of the instantaneous voltage across an arrester. Such a test determines only the peak value of the discharge and does not give any information as to how long such peak voltages continue. Very often lower voltages longer applied will cause more damage than higher peak voltages of much shorter duration. Peak voltages as measured by sphere gaps tell only a part of the story, and thus comparisons based on such tests only partially determine the relative protective values. Of course, the value of the instantaneous voltage with any particular arrester tested will depend not only on the arrester but on the constants of the circuits used. This is true because in general the wave fronts developed by different impulse generators will differ, and as a result the same arrester will discharge at different voltages depending on the speed of operation of the arrester.

With a certain wave front, tests made on different arresters as reported in this paper by Mr. Young are comparable, although it should be remembered that changes in wave front will affect different arresters differently.

To be of greatest value the report of a test of this character should give more details than given, concerning the conduct of the test, and the exact nature of the apparatus used when tested. Such data will aid in making comparisons with other tests made elsewhere.

Since much of the value of the arrester is based on the instantaneous voltage across the arrester when small currents are flowing, and on its ability to discharge large currents without excessive rise in voltage, the measurements given by Mr. Young in Table I are important.

With every arrester tested, except the electrolytic, the voltage across the arrester was found to be nearly equal and in some cases more than the applied voltage. In addition, voltages are also observed across the series resistance which in some instances are also equal to the applied voltage. This is a condition which does not tend to inspire confidence in the test results. By referring to Fig. 2 which is the connection diagram for the impulse test, an explanation for some of the variations in test results may be found. Presumably the gap called the voltage measuring gap was set for either 100 or 150 kv. as the case might be. The actual condenser voltage was higher than the setting of the gap by an unknown amount depending on the arrester in circuit. This is true because the arrester with its gap is in series with the main measuring gap, and the voltage across each gap will depend upon the relative capacity of the two gaps in series. In order that the voltage across the condenser be correct, a high-resistance water tube should have been placed across the arrester and its gap.

There are certain details of test which effect the results considerably, for instance, when reading the gap across the series resistance, the gap in parallel with the arrester should be opened to prevent sparking and vice versa It is assumed that such precautions were taken. The nature of the series resistance is important in that its resistance must be constant at all frequencies. The water tube is the most satisfactory resistance.

It has been my experience in taking a large number of impulse volt-ampere curves with as small a physical circuit as possible, that the sum of the instantaneous voltages across the arrester and the series water tubes was approximately equal to the applied impulse. Cases have arisen due to reflections, oscillations, or other causes, where this approximate equality was not attained.

It is difficult to draw conclusions based on the data given in Table I, because of the apparent lack of agreement in the results. This condition is well illustrated in the case of arrester No. 4. This arrester is a type which should have constant resistance instead of being reduced from 1100 to 520 ohms when the series resistance was reduced from 27 to 90 ohms.

To secure the best comparison it is desirable to make tests on as large a group of arresters as possible to determine the variations between arresters of the same type. Great care should also be taken in making the tests to obtain the most accurate results possible.

The results of tests given in Table II are interesting and are, perhaps, more comparable than those given in Table I. Although the condenser voltage is not given, it is probable, as the author states, that the test made without any series resistance represents a condition more severe than would be met in practise. Such a discharge may well equal 10,000 amperes or more, the actual value, of course, depending on the arrester resistance and the circuit constants.

Table III shows the results of dynamic tests. In a test of this character it is very desirable to take oscillograms so that the exact operation of the arrester may be studied. From the standpoint of the effect of the arrester operation on the system, the valve-type arresters have shown themselves to be superior in that little or no dynamic current flows following the condenser discharge.

The simple calculation made to determine the current to be discharged by the 22-kv. arrester assumes that the arrester resistance during discharge is zero which is, of course, not strictly accurate. This calculation does give a value which indicates the order of magnitude of the maximum current and arrester under the assumed condition should discharge. A more accurate calculation is probably not worth while for the purpose of this paper especially when the actual surge voltage may vary greatly from the assumed value.

It is interesting to note that in spite of differences in detail the conclusions regarding the value of the different types agree in general with tests which we have made. The actual values obtained are in many cases very different, although the agreement is much better for the general tests given in Tables II and III.

The results of the tests on the 4-kv. arresters and the conclusions drawn are interesting. The discussion here given concerning the testing methods applies with equal force to these tests. It should be noted however, that better agreement is found among the results in Table IV than in Table I. Mr. Young concluded that satisfactory protection would be obtained from any of the types which met the test satisfactorily. In general where the density of the arresters is high these conclusions will probably be true. More detailed tests would no doubt have shown more definite differences between various types.

H. G. Brinton (by letter): In Mr. Young's paper, certain calculations of arrester efficiency were made. It was assumed that a surge voltage was applied to two resistances in series. One of these resistances represented the arrester impedance and one represented the surge impedance of the line. The voltage distribution was calculated from Ohm's law. This method is incorrect as the current and voltage relations are not the same as in a circuit involving a true surge impedance which is a factor depending upon the distributed inductance and capacitance of the line. The correct general formulas derived by methods ordinarily used for traveling waves are given in an article by Faccioli and Brinton on "High Frequency Absorbers" in the "*G. E. Review*" for May 1921. The particular formula for each special case can be obtained from the general formula.

In case the arrester is tapped off a line of uniform surge impedance Z and the arrester impedance is practically a pure resistance R, the ratio of arrester voltage to the voltage of the traveling wave or surge voltage is equal to

$$\frac{2\,R}{2\,R + Z}$$

The arrester voltage is 1/5th the surge voltage when the arrester resistance R is 1/8th the surge impedance. Using the above mentioned incorrect method for this same case, the resistance R is found to be 1/4th the surge impedance instead of 1/8th.

In case the arrester is at the end of the line with a transformer, we may assume that the surge impedance of the transformer is about 10 times that of the overhead line and the formula given above is changed to

$$\frac{20\,R}{11\,R + 10\,Z}$$

In this case the arrester voltage is 1/5th the surge voltage when R is one ninth (1/9) the surge impedance of the line.

In the case of the aluminum arrester we have to consider the effect of capacitance as well as resistance in the arrester, although the arrester films puncture and short circuit the capacitance if the voltage rises sufficiently. Taking the simpler case of an arrester tapped off a line of uniform surge impedance the general formula for the ratio of arrester voltage to surge voltage (assuming film has not yet punctured) reduces to

$$1 - \frac{Z}{2\,R + Z}\,\epsilon^{-\frac{t}{c(R + Z/2)}}$$

Assuming that $Z = 300$ and $R = Z/8 = 37.5$ and $C = .033$ m. f. = capacitance of 75 cells in series and further assuming that $t = 1$ micro-second with a wave 1000 feet in length, the magnitude of the above voltage ratio is 0.32. Thus, the arrester voltage is about ⅓ instead of $^1/_5$ the surge voltage, as it would be if the arrester were a pure resistance. This assumes that the arrester voltage is not high enough to puncture the film. From the formula for condenser voltage it can be calculated that the voltage on the films is about ½ the arrester voltage under the above conditions. After the films puncture the action is that of a pure resistance.

W. F. Young: To the writer's mind, the four characteristics of lightning arresters which are of most interest to operators are those named at the beginning of the paper. A statement in absolute terms of the manner in which any arrester meets these conditions is of academic interest to any one buying or selling arresters, but to the operator the thing of practical interest is a comparison of the arresters that are available with respect to these various characteristics. It was to get this comparison that the tests described were made. It is felt that the result desired has been secured.

Because of the wide divergence in design of equipment to bring about the same result and the obvious conclusion which is to be drawn therefrom that somebody is wrong, some hesitancy was felt by the Duquesne Light Company management concerning the publication of the results obtained. However, it was finally agreed that the results might be made known, along with an easy way of interpreting them, provided conclusions drawn were based upon arbitrary requirements. A considerable

amount of protest was expected from the manufacturers of lightning arresters. Some of this protest has been forthcoming. It is possible that certain of the manufacturers have been silent for the reason that the conclusions have been agreeable to them either as the type of arrester made by them stands high on the list or because they do not pretend that their equipment gives maximum protection, but rather that it gives a degree of protection for a comparatively low price.

The real purpose of the paper has been to accomplish what Mr. Creighton indicates has been accomplished, namely, to separate the sheep from the goats. No pretense is made that the method of calculation is exact. Neither is any pretense made that the characteristics of any arrester tested are exactly the same as the characteristics of other arresters of a similar type. It is maintained, however, that the analysis proposed, when applied to any set of arresters subjected to complete comparative tests, will result in a classification of the arresters which will be correct.

It is recognized that such a classification of arresters is not sufficient information upon which to condemn any of two or three types of arresters of similar characteristics. It is felt, however, that the results obtained in these tests now shifts the problem from a study of the arrester itself, to a study of the requirements of an arrester, *i. e.*, to a study of the nature and magnitude of the surges which the arrester may be required to discharge.

Mr. McEachron's discussion has brought out several points of error which it is absolutely essential to guard against if correct results are to be obtained in a test of this sort. It is necessary for instance that a surge of uniform shape be applied to the various arresters to secure comparative results. Several different types of surge, however, may be applied to all of the arresters to eliminate, as far as possible, error due to lack of knowledge of the form of wave imposed upon lines by lightning discharges. It is interesting to note that while such changes in wave form will give considerably different results, the same general classification of arresters usually prevails when any surge, which may be considered an imitation of that imposed by lightning, is used. It is also interesting, but not surprising, to note that the general results obtained agree with those obtained in other tests although there was no connection between the persons making the tests.

In this paper effort was made to avoid detail as much as possible as it was felt that the thing of real interest was the general classification of arresters obtained in tests made by an operating company. It may be stated here, however, that the test circuit differed from that shown in Fig. 2 in that the arrester under test was shunted by a very high resistance leak.

Lightning Arrester Application from the Economic Standpoint

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FOR a good many years, it has been recognized that practise in application of lightning arresters varies and that different engineers determine their choice and use of arresters on widely different base lines. While this may be partly explained by differences in climatic conditions, the variation is without question largely due to the fact that a clear relation has not been established between cost of and benefit from arresters.

Such a relation is difficult to determine because the wide variation in conditions, even in a single locality, makes necessary the consideration of a large number of installations as a unit. Mr. D. W. Roper's extended experience and close analysis of results over many years with thousands of installations shows that even several hundred arrester-years experience cannot be depended upon to give closely accurate results.*

However, the fact that the acquiescence to this lack of a basis for arrester application is an untenable position for arrester manufacturers has been brought definitely to our attention, as a result of the bringing out in 1922 of the two parallel lines of autovalve arresters. Previous to this time, valve type arresters had been made only in the large capacities, intended for protection of important installations, and too expensive or otherwise unsuited for use at the smaller and less important installations. Arresters, designed with the particular object of being low enough in cost to be suitable for use where the valve types were prohibitive, were of so greatly reduced protective value as to be naturally and logically considered as of an entirely different order, just as a fuse is a different order of overload protector from an oil circuit breaker. A comparison between an electrolytic arrester and a former distribution arrester, both for 6600-volt service, will illustrate this.

The electrolytic arrester has a relief voltage at steep wave front of approximately 18 kv., instantaneous value (11.5 kv. for the "AL" type arrester), a resistance of approximately 19 ohms, at ordinary summer temperatures, and a counter voltage of 9.3 kv. The distribution arrester had a relief voltage of approximately 28 kv. and a resistance of approximately 400 ohms. Since this arrester was not of the valve type, it, of course, had no counter voltage. The comparison is well shown by curves 1 and 2 of Fig. 1. For higher voltages the difference is still greater and for this reason we have never felt justified in making the distribution-type arrester for higher voltages.

With the autovalve arresters, the case is quite different. The station type is designed to duplicate the electrolytic arrester in protection. The relief voltage is approximately 21 kv., instantaneous value, the resistance approximately 15 ohms, though this varies with voltage, decreasing as the voltage increases, and the counter voltage is approximately 13 kv. The distribution type is exactly the same as the station type, except that the electrode area is one-fourth as great and therefore the resistance, at the same overvoltage value, is four times as great. The comparison is shown by curves 1 and 3 of Fig. 1, no curve being given for the station-type autovalve arrester, since this closely parallels the electrolytic.

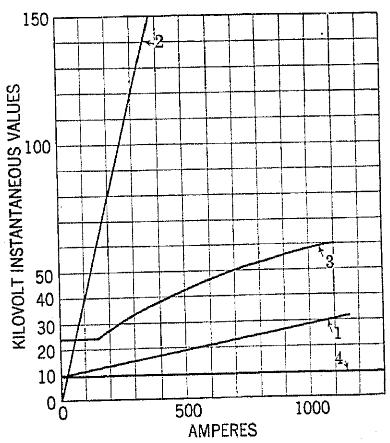

FIG. 1—VOLT-AMPERE RELATIONS SHOWING COMPARATIVE PERFORMANCE CHARACTERISTICS OF SEVERAL ARRESTER TYPES

Curve 1—Standard 6.6-kv. Electrolytic Arrester
Curve 2—Former standard 6.6-kv. Distribution Arrester of the Gap and Resistance Type
Curve 3—Standard 7.5-kv. Distribution Autovalve Arrester
Curve 4—Line Voltage

It is impossible at the present time to express the comparison shown by these curves in terms of directly comparative protective values, but since the object of an arrester is to hold surge voltages down to a safe value, and since this can only be done by permitting surge current to flow through the arrester without excessive rise in voltage, it is obvious that the arrester with the lowest voltage value in the volt-ampere curve is the best one.

It is known that injury by surge voltages begins at some overvoltage value such as 2.5 to 3 times line voltage, and that it increases with increase in voltage, probably faster than in direct proportion to the excess over the minimum injury voltage. It is probable

*TRANSACTIONS A. I. E. E., 1916, Vol. XXXV, p. 655.
*TRANSACTIONS A. I. E. E., 1920, Vol. XXXIX, p. 1895.

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-10, 1924.

that arresters may need to carry surge currents as high as 1000 amperes completely to prevent injury, but that such severe conditions occur only a few times per season, while the less severe surges, requiring the arrester to carry 100 amperes or so, are far more frequent. True relative values cannot be established without more complete data than we now have as to the effect of the surge voltages on insulation and as to the frequency with which surges of different magnitudes occur. For use in applying arresters, however, it may be assumed without fear of very great error that, presuming on the fulfillment of the intention of complete elimination of injury from surge voltages by use of the electrolytic arrester or of the station type autovalve arrester, the distribution autovalve arrester will do one-half as well, that is, in the long-run, eliminate one-half the failures which would occur if no arresters were used. No attempt is made to place a value on the relatively small protection given by the old distribution type, since it is no longer made.

As soon as the changed conditions brought about by the autovalve development were recognized, the need for better general application data was made apparent by a diversity of inquiries, indicating such a wide range of conceptions as to make a logical recommendation seem worthwhile even if it must be based on estimates.

The following attempt to reduce the problem wholly to an economic basis resulted.

General Considerations

The general approach to the problem was an attempt to evaluate the extent and cost of failures to the average system, if entirely unprotected, with the assumption that the yearly expense so incurred may justifiably be spent for lightning arresters. This assumes that the proper application and use of autovalve lightning arresters will reduce the trouble from surge voltages to a negligible quantity. As stated above, it is thought that this is a reasonable assumption, in case the type "SV" station arresters are used. When the type "LV" arresters are applied, as a compromise of protection for the sake of reduced cost, the assumption is that the damage due to surge voltages will be reduced to 50 per cent of the unprotected value instead of practically to zero as with the type "SV" arresters.

Based on records of experience, it is assumed that on an unprotected system in territories of average lightning conditions, 7½ per cent of the transformers installed will be injured by surge voltages each year. Put in another way, each transformer will be injured by surge voltages on the average of once in 13 years.

In calculating the cost of such injuries, two major points were given consideration, namely, the actual cost of restoring service and repairing the injured transformers, and the estimated valuation of the loss of service.

The cost of restoring service and repairing the injured transformer consists in locating the trouble, replacing the injured transformer with a spare unit from stock held for this purpose, transporting the spare transformer to the point of installation and the injured transformer back to the repair shop, and examining, testing and repairing the transformer. Estimates of these expenses are divided into two parts, a more or less fixed charge to cover all except the actual repairs, and the cost of repairs, which is assumed to be equal to 7½ per cent of the initial cost of the transformers.

The evaluation of the cost of loss of service is based on the supposition that cost to the power user must be given the same weight as cost to the power suppliers. It has been many times demonstrated that this idea of determining business policies from the standpoint of the customer is sound. As an average, the cost of power is approximately 5 per cent of the total cost of manufacturing process.

For short time interruptions, the power user's expenses continue without decrease and he thus loses 20 times as much as the power supplier.

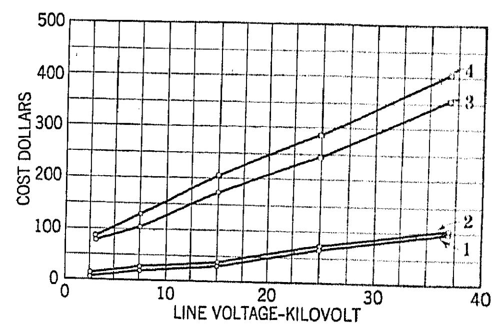

Fig. 2—Arrester Costs
Curve 1—Type *L V* Arrester—First Cost
" 2— " " " —Installed
" 3— " *S V* " —First Cost (Per Phase)
" 4— " " " —Installed (" ")

Based on these two points, the cost of loss of service is taken as 20 times the value of the revenue loss, due to the interruption.

To determine the value of lost revenue, it is assumed that the average duration of a service interruption is 5 working hours, that is, 5 hours always taken from the normal revenue producing period. Average loads are taken to be for lighting service, full load four hours per day and for power service full load eight hours per day and six days per week. The average cost of power to the user is assumed to be 6 cents per kw-hr. for lighting and 2 cents per kw-hr. for power.

As a matter of interest, to check the results of the analysis, the following data are also of interest. Of the total operating company expense, approximately 35 per cent may be taken as interest on the investment and 65 per cent as "operating cost," which includes all other items. Of the total investment 38 per cent may be taken to be in generating station, 18 per cent in transmission lines and substation and 44 per cent in the distribution system. Approximately one half of

the investment in the distribution system may be considered to be in distribution transformers. This means that 7.7 per cent of the gross revenue may legitimately be credited to the initial investment in the distribution transformers.

The cost of arresters used is the installed cost, consisting of initial cost plus an assumed average cost of installation.

Data and results are given below:

Normal Lightning Arrester Costs

Type "LV" Arresters		Type "SV" Arresters	
Rating kv.	Cost Dollars	Cost Dollars	Cost per Phase Dollars
2.5	\$5.67	\$230	76.5
7.5	17.81	310	103.
15.0	30.00	520	173.
25.0	65.00	735	245.
37.0	97.50	1065	355.

Values for cost per phase are given in order that the values for the arresters may match up with the values for single-phase transformers.

Cost of Installation

Type "LV,"a ll voltages, \$7.50 per arrester
Type "SV"

Rating kv.	Cost	Cost per Phase
2.5	\$25.00	\$8.50
7.5	75.00	25.00
15.0	100.00	33.00
25.0	125.00	41.00
37.0	150.00	50.00

Cost of Arresters Installed

Rating kv.	Type "LV" Cost	Type "SV" Cost per Phase
2.5	\$13.17	\$85.00
7.5	25.31	128.00
15.0	37.50	206.00
25.0	72.50	286.00
37.0	105.00	405.00

These costs are shown on curve Fig. 2.

Transformer Costs

Trans. Rating kv-a.	2,300-220 Volts	6,900-2,300 Volts	12,300-2,300 Volts	22,000-2,300 Volts	33,000-2,300 Volts
1½	33	63			
5	64	97	154	266	
10	102	144	187	204	347
25	197	257	297	392	428
50	324	391	427	532	562
100	486	559	615	741	780
200	708	794	891	1002	1100
250		935	974	1120	1240
500		1360	1420	1622	1740

These represent the normal cost for standard single-phase, 60-cycle distribution transformers, except that these 250-kv-a. and 500-kv-a. prices are standard single-phase, 60-cycle power transformers.

Fixed Cost for Restoring Service

2300 Volts—1½ to 25 kv-a.—1 phase—\$10.00 to \$15.00
2300 Volts—25 to 200 kv-a.—1 phase— 43.00
High Voltages—Up to 500 kv-a.—1 phase— 45.00

Gross Yearly Revenue

Transformer Rating kv-a.	Lighting Load Kw-hr. per Yr.	Lighting Load Dollars per Yr.	Power Load Kw-hr. per Yr.	Power Load Dollars per Yr.	Average Dollars per Yr.
1½	2,200	132	2,700	54	90
5	7,300	438	9,000	180	300
10	1,460	876	1,800	360	620
25	36,500	2190	45,000	900	1,550
50	73,000	3650	90,000	1,800	2,700
100	..	..	180,000	3,600	3,600
200	..	..	360,000	7,200	7,200
250	..	..	450,000	9,000	9,000
500	..	..	900,000	18,000	18,000

Cost of Failure

Assumption:

Yearly failures with unprotected system	7½ per cent
" " " complete protection	0 per cent
" " " Type "LV" "	3¾ per cent
Cost of repairs of damaged transformers of original cost	7½ per cent
Service loss per failure.........	5 hours
Normal average operating time (yearly)..................	2000 hours
Cost of loss of service equal....	20 times lost revenue
Average cost of loss of service per interruption equals $\frac{5 \times 20}{2000}$ = ...	1/20 yearly revenue
Pro rata average yearly share per transformer............	7½ per cent of cost of one failure.

Cost of arresters for complete protection may equal pro rata yearly cost of failure capitalized at 15 per cent. Type "LV" arresters may cost one half the pro rata yearly cost of failure capitalized at 15 per cent.

Cost of Failure

Transformer Rating kv-a.	Fixed Charge	Repair Cost	Revenue Loss × 20	Total	$\frac{\text{Total} \times 0.075}{0.15}$
		2300 Volts			
1½	10	2.48	4.50	16.98	8.49
5	11	4.80	15.00	30.80	15.40
10	13	7.65	31.00	51.65	25.83
25	15	14.80	77.50	107.30	53.65
50	43	24.30	135.00	202.30	101.15
100	43	36.40	180.00	259.40	129.70
200	45	53.30	260.00	458.30	229.15
		6900 Volts			
1½	45	4.72	4.50	54.22	27.11
5	45	7.35	15.00	67.35	33.68
10	45	10.80	31.00	86.80	43.40
25	45	19.30	77.50	141.80	70.90
50	45	29.30	135.00	209.30	104.65
100	45	41.90	180.00	266.90	133.45
200	45	59.60	360.00	464.60	232.30
250	45	70.00	450.00	565.00	282.50
500	45	102.00	900.00	1047.00	523.50

Transformer Rating kv-a.	Fixed Charge	Repair Cost	Revenue Loss × 20	Total	Total × 0.075 / 0.15
		13,200 Volts			
5	45	11.55	15.00	71.55	35.78
10	45	14.00	31.00	90.00	45.00
25	45	22.30	77.50	144.80	72.40
50	45	32.00	135.00	212.00	106.00
100	45	46.20	180.00	271.20	135.60
200	45	66.80	360.00	471.80	235.90
250	45	73.00	450.00	568.00	284.00
500	45	106.50	900.00	1051.50	525.75
		22,000 Volts			
5	45	19.90	15.00	79.90	39.95
10	45	22.80	31.00	98.80	49.40
25	45	29.40	77.50	151.90	75.95
50	45	39.80	135.00	219.80	109.90
100	45	55.80	180.00	280.50	140.25
200	45	76.50	260.00	481.50	240.75
250	45	84.00	450.00	579.00	289.50
500	45	121.50	900.00	1066.50	533.25
		33,000 Volts			
10	45	26.00	31.00	102.00	51.00
25	45	32.10	77.50	154.60	77.50
50	45	42.20	135.00	222.20	111.10
100	45	58.50	180.00	283.50	141.75
200	45	82.50	360.00	487.50	243.75
250	45	93.00	450.00	588.00	294.00
500	45	132.00	900.00	1077.00	538.50

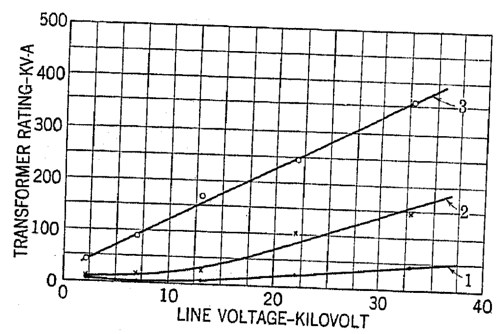

FIG. 3—TRANSFORMER SIZES FOR ECONOMIC BALANCE OF ARRESTER COST AND BENEFIT
Curve 1—Type *L V* Arrester—Grouped Installation
" 2— " " " —Isolated "
" 3— " *S V* "

MINIMUM TRANSFORMER SIZE FOR WHICH PROTECTION IS WARRANTED

The last column in the tables immediately preceding is the capitalized value of the pro rata average yearly cost of failure. This is equal to the maximum permissible cost for an arrester to give complete protection on the basis of an exact economic balance between cost of arrester and savings by use of an arrester. From these values and the figures for the cost of Type "SV" arresters per phase, a determination is made for the minimum size of transformer with which it is justifiable to use type "SV" arresters.

In the same way, the minimum-size transformer, which it is economically justifiable to protect with type "LV" arresters, is determined from the cost of the "LV" arresters and from values equal to ½ the value in the last columns of the tables, showing capitalized value of pro rata average yearly cost of failures.

In the case of network systems in which type "LV" arresters are installed, close enough together so that each transformer is virtually protected by four "LV" arresters, as later discussed under "EXCEPTIONS," complete protection is afforded and the minimum-size transformer which it is economically justifiable to protect with type "LV" arresters is determined from the full values in the last column of the tables, showing capitalized value of pro rata average yearly cost of failure.

Transformer sizes so determined are given in Table I and curve Fig. 3.

Voltage	Transformer Size for Economic Balance: Type "SV" Arrester	Type "LV" Arrester Isolated Installation	Type "LV" Arrester Grouped Installation
2,300	41.5	10	4.0
6,900	90.0	16	1.5
13,200	170.0	27	6.0
22,000	247.0	105	23.0
33,000	363.0	148	45.0

Similar calculations for 25-cycle transformers show that the sizes for economic balance are from 5 per cent to 10 per cent smaller than in the corresponding cases with 60-cycle transformers.

DISCUSSION

To get an idea of the meaning of these transformer sizes and the arrester cost, the following further calculations are made.

On the assumptions made, the average yearly loss per transformer is $\frac{5 \times 20}{2000} \times 0.075 = 0.00375$ times the gross yearly revenue.

Crediting 7.5 per cent of the gross yearly revenue to the distribution transformer, this means that the average yearly loss represents 5 per cent of the income which may justly be credited to that transformer. This in turn means that 5 per cent of the revenue, which may justly be credited to an individual distribution transformer, should be spent for lightning protection.

From the foregoing data, a determination is made as to the percentage which the first cost of the lightning arrester is of the first cost of the transformer of a size for economic balance of cost of arrester and benefits from arresters. The percentage figures are given below.

Voltage	Type "SV" Arrester Percentage	Type "LV" Arrester Isolated Installation Percentage	Type "LV" Arrester Grouped Installation Percentage
2,300	27.2	5.6	10.3
6,900	19.7	9.4	28.3
13,200	21.4	9.8	18.7
22,000	21.9	8.6	17.1
33,000	24.2	10.4	18.3

This means that it is justifiable to spend for lightning arresters for complete protection something like 25 per cent of the initial cost of the transformers to be protected and that in the case of type "LV" arresters, it is permissible to spend from 10 per cent to 25 per cent of the initial cost of the transformers to be protected, depending on installation conditions.

Exceptions

It will, of course, be realized that the conclusions reached in this analysis are only applicable to average cases, and that particular conditions in individual installations will make wide departure from these conclusions permissible. Several of the major exceptions are discussed below.

Although an individual transformer is best protected when the lightning arrester is connected to the system directly at the transformer, considerable benefit is derived from additional arresters, connected to the same circuit, if they are not too far distant. Thus, where the density of installation is sufficiently high, a network can be completely protected by the use of type "LV" arresters.

There are many cases of industrial load in which the actual damage, resulting from loss of service, is not adequately represented by 20 times the cost of the loss of revenue. Examples of this are the kind of processes in which a failure of power supply results in loss of valuable material in process, as for example, in a steel mill, a foundry, and a bakery. Under such conditions, it is not justifiable to omit the best available type of lightning arrester under any circumstances. Type "SV" arrester should always be used regardless of other considerations.

There is a tendency in many systems to safeguard loads where continuity of service is vital, by the use of inter-connections which provide for supply to these loads from more than one source. Wherever a transformer failure does not involve a shut-down and is less than that necessary for re-placement of the transformer, the justifiable expenditure for lightning arresters is reduced, the amount of reduction depending on local conditions, as for example, load requirements and the time for restoring service.

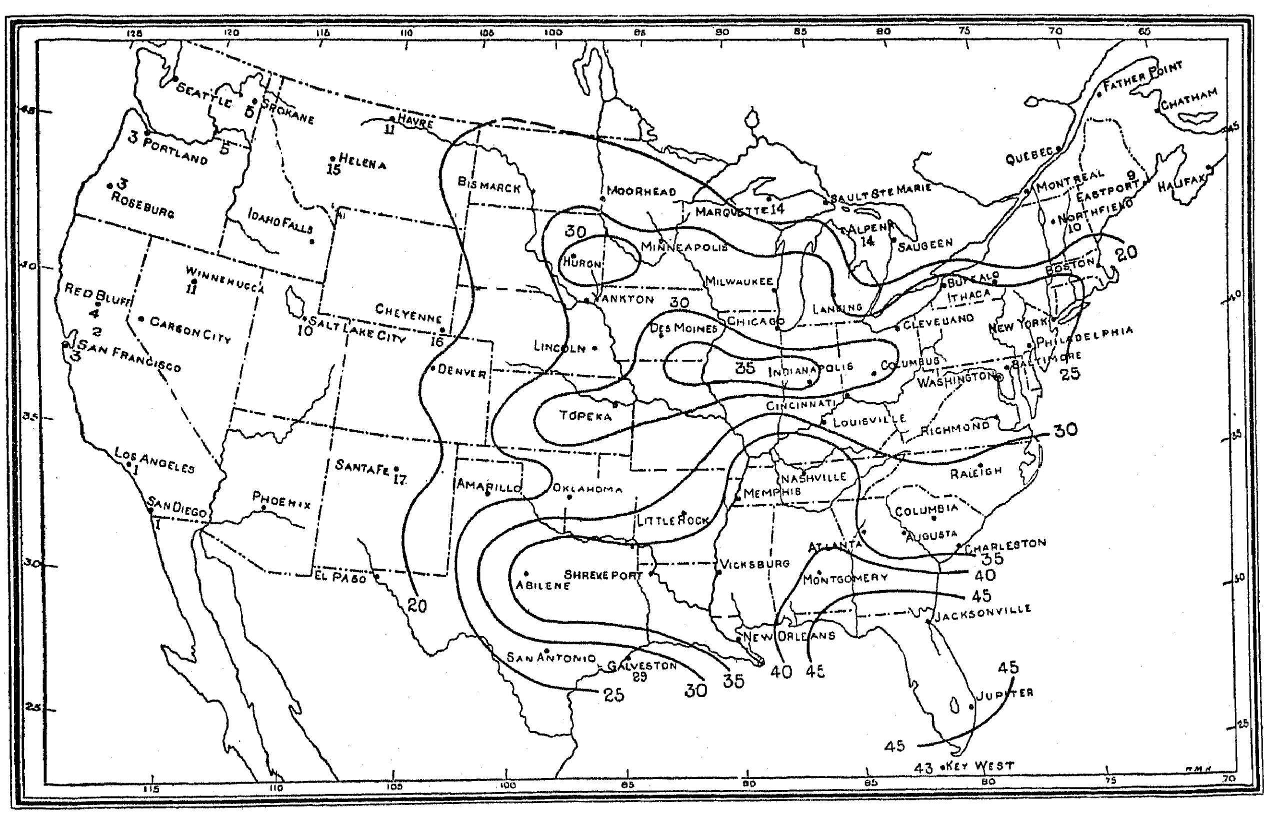

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FIG. 4

The intensity of lightning conditions varies very widely over the country. Ordinarily, for any particular location, a general idea of the intensity of these conditions is in the mind of the operating engineer. An interesting supplement to this direct information is given by the map, Fig. 4, which shows the distribution of thunder storms throughout the United States over a long period of years. Some comparisons between different sections of the country can be made with this map.

Appreciation is expressed of valuable assistance rendered during this study by Mr. E. C. Stone.

Discussion

K. B. McEachron: In the fourth paragraph in the first page, the statement is made that the relief voltage of the aluminum arrester for 6.6-kv. service is 18 kv., and for a certain distribution arrester is 28 kv. In the next paragraph is stated that the station type auto-valve has a relief voltage of 21 kv. instantaneous value. It is also stated that the comparison is well shown by the curves in Fig. 1. If the relief voltage is taken as that at which current first begins to flow then, according to the figure referred to, the distribution type (Curve 2) has a relief value of zero volts and the type *S V* and *L V*, 10 and 24 kv. respectively. From an examination of the curves, is it not true that these curves do not represent test results, but rather calculated curves based on theoretical considerations It is stated that the station type "closely parallels the electrolytic and therefore no curve is shown." As shown, however, this cannot be the case, since as stated in Mr. Atherton's paper before the Institute in 1923, the arrester is designed for a so-called counter voltage of 25 per cent above the crest value of the rated voltage of the arrester. In this case, this gives about 13,000 volts, which is the value given in the paper. It should be remembered that this critical voltage is without the series line gap which adds materially to the voltage at which discharge begins.

At 60 cycles this increase may amount to 25 per cent or more, while with a steep wave front a still larger multiplying factor is usually required. In this connection, I would like to have Mr. Atherton explain how the value of 21 kv. is obtained.

Curves, such as shown in Fig. 1, are misleading unless it is stated that the values are based on assumed theoretical conditions, and that they cannot represent operating results because the series gap—always used—has been omitted.

It is desirable to correct an erroneous impression as to the characteristics of the multigap arrester, which may be inferred by a study of Fig. 1. Curve 2 represents the volt-ampere characteristics of a 400-ohm fixed resistance. In practise the resistance used in many thousands of arresters is not constant as assumed by Mr. Atherton, but is reduced to a fraction of its 110-volt resistance when subjected to high-voltage steep-wave-front impulses. This means that in actual operation the instantaneous voltage at the higher currents will be very much less than indicated and is not correctly represented by a straight line. Well designed arresters of this type compare very favorably with the valve type of arresters when compared on the basis of instantaneous voltages during discharge. There are other features of the gap-resistance type which make it inferior to the valve types.

In making this study of the economics of lightning-arrester application Mr. Atherton finds it necessary to make certain assumptions. Among other things he states that the type *S V* arrester may be considered as giving perfect protection. For the purpose of this paper any other arrester might have been used or perhaps better yet a hypothetical arrester which might be called arrester No. 1 and the arrester giving half protection designated as arrester No. 2. It has not been established that any type affords the assumed protection, nor do I believe that the art has yet been developed to such a point that no failures in service can occur.

The experience of Mr. Roper in Chicago, referred to in the paper, shows that increasing density of the arresters results in decreased losses of apparatus. In general until the number of arresters exceed the number of transformers the best results will be secured by placing an arrester at each transformer. If additional arresters are required it is probable that the greatest benefit will be obtained by placing the additional protection along the line rather than placing several arresters in parallel at one transformer installation.

In making tests using impulse voltages to determine arrester characteristics, it is well to bear in mind, that laboratory tests will, in general, give varying results, and to be directly compared, all the tests should be made on the same impulse generator. Tests made by different investigators, should be compared with caution.

It is also true that although a line is drawn as the volt-ampere characteristic of an arrester, as a matter of fact, with present-day testing methods, the curve is a broad band whose width is determined by two factors. The first is the inherent variation between arresters and the second is the range of division of sparking on the measuring spheres. These variations do, in some cases, produce a deviation of as much as 10 per cent above and below the average curve.

A. L. Atherton: Mr. McEachron's comments and questions in regard to Fig. 1, "Volt-ampere relations showing comparative performance characteristics of several arrester types," merely point out that such a characteristic does not indicate the relief voltage, or voltage at which the discharge starts, and to this extent such curves fail to give a complete indication of protection value. Since the lightning-arrester performance is discontinuous, the relief voltage cannot be shown on the curve of operating characteristics, but must be covered by a separate statement, as I have done.

The value of 21 kv. cited, is merely the crest value of double-line voltage which is fairly representative of this type arrester. This value is secured by test and, since the time lag is very small, it is not very greatly different for power frequencies and for steep wave fronts.

Volt-ampere characteristic curves of lightning arresters as determined by present-day testing practice will not be precise, but will, as Mr. McEachron has stated, cover a band of considerable width, due not only to variations in the testing procedure and measurements, but also to variations in the product. A central line, however, clearer to use, and, if the limitations are recognized, just as dependable.

Operating Experiences with the Relaying of the Duquesne Ring

BY H. P. SLEEPER

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Review of the Subject.—*A paper was presented by the writer at the A. I. E. E. Spring Convention, held at Pittsburgh, April 24 to 26, 1923, and was published in the July 1923 issue of the Institute Journal, on the subject: "Selective Relay System of the 66,000 Volt Ring of the Duquesne Light Company." That paper described in detail the systems of relaying which are used to protect the Duquesne Ring for both short circuit and ground faults. That paper also described the service tests which were made on the ring to test the relay protection.*

Nearly a year has passed since the ring was put into operation with the protection described, and ample opportunity has been given to study the protective schemes under actual service conditions and to prove their practicability. This period of service has included one of the worst, if not the very worst, lightning seasons that has ever been experienced in the history of the Duquesne Light Company. Electrical storms in the Pittsburgh District, are unusually severe, both as to intensity and duration, due probably to the geographical location of the district, situated as it is at the junction of two rivers. Storms in this vicinity almost invariably seek the Ohio River Valley and follow it up to the point of junction of the Allegheny and Monongahela Rivers, after which they will continue up one of these streams. Storms of several hours duration are not at all uncommon during the height of the lightning season. These have resulted in many insulator flashovers during the past season, and the ground relays have given an excellent account of themselves. Several line short circuits have occurred, and the proper relays functioned correctly in every case.

This paper gives a log of all relay operations on the ring for the first eleven months' service, and it will be noted that every case of trouble on lines has been cleared by the proper relays. A study of all operations leads to certain conclusions, as to the proper means of improving the present protection, and these are described in detail. All troubles and difficulties have been frankly described and nothing withheld. It is hoped that other operating companies, which are using, or contemplate using similar types of protection may derive some value from the discussion and data presented.

THE Duquesne Ring consists essentially of a double-circuit 66,000-volt loop surrounding the City of Pittsburgh. (See Fig. 1). Its circumferential length is approximately eighty miles, and the two circuits are paralleled and sectionalized at six step-down sub-stations, and two generating stations. Power is fed into the ring at opposite ends by the Colfax and Brunot Island power plants, the capacity of each plant being 120,000 kw.

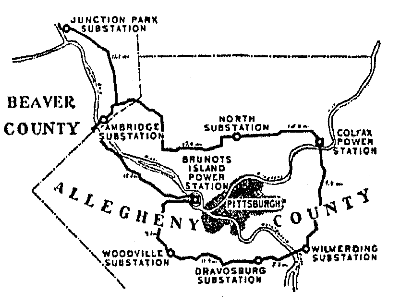

FIG. 1—MAP OF THE DUQUESNE RING

The short-circuit protection of the lines is secured by the use of cross-connected reverse-power Westinghouse Type "CR" relays. The ground protection is effected by means of balanced-current Westinghouse Type "CD" relays connected in the neutral of the balancde lines. These lines are balanced in pairs in every case, except at Junction Park, where no balance is made, and where straight reverse power protection with interlocked overload relays for directional ground protection is used.

The busses and transformer banks at each station are protected by differentially-connected overload relays. The low-tension busses are protected by means of overload relays only on the low-tension transformer breakers.

The line protection is shown diagrammatically in Fig. 2; a relay diagram of the system is shown in Fig. 3;

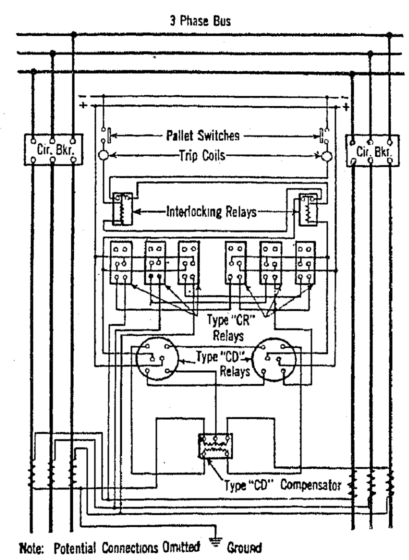

FIG. 2—DIAGRAM OF CONNECTIONS OF LINE RELAYS

and a schematic diagram of the relay protection of a typical station is shown in Fig. 4.

The relay settings used during the past year were as follows: Line short-circuit relays, 7-ampere current tap; 0.4 second definite minimum time; current transformer ratio 400/5. Since each 4/0 circuit is rated at 30,000 kw., this current allows each circuit to carry 200 per cent load before tripping, and hence

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requires an unbalance of 560 amperes between balanced phases to operate the relays. As the minimum short-circuit current on the system is approximately 3000 amperes, this leaves an ample margin of safety.

The line ground relays are set to operate on an unbalance of 40 amperes. As the minimum ground current is approximately 600 amperes, positive operation of these relays is assured. The ground current on the 66,000-volt system is limited by the 63-ohm ground resistor at each of the two generating stations.

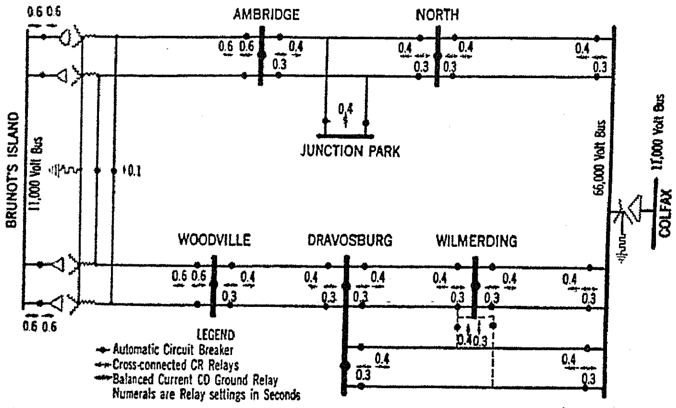

Fig. 3—Schematic Diagram of 66-Kv. Ring System, Showing Relay Protection of Lines

The high-tension bus differential relays are set to operate on 0.7 ampere current tap, 0.75 second. With 400/5 current transformer ratio, this causes these relays to operate on a minimum of 56 amperes, which allows the same relays to give short circuit and ground protection on the busses. The value of the time setting for the differential relays was arrived at after several incorrect operations of bus differential relays had occurred. It was observed at certain stations that the bus relays would operate each time that a line fault occurred adjacent to that particular bus. The line relays would first operate correctly to open the line breaker. Immediately, as soon as the line breaker opened, the bus differential relays, which were then set at 0.1 second, would close contacts and isolate the bus. The cause was readily surmised and proved by test to be due to the action of the pallet switch of the breaker. This is a heel and toe switch, the function of which is to operate mechanically when the breaker is opened, to short circuit the current transformers in the breaker and to open the parallel circuit of these current transformers with the transformers in the breakers remaining in circuit on the bus, thus leaving a true balance on the bus differential relays. Thus, when the breaker opens under load or short circuit, it is necessary that the arc on the breaker contacts be broken simultaneously with the operation of the pallet switch contacts. Any time difference which may exist will create an unbalance in the bus differential relay circuits, and if the time of duration of this unbalanced condition is equal to or greater than that of the time setting of those relays, it is obvious that they will close contacts. As the time of duration of the arc in the breaker is problematical, and will vary somewhat with faults of different characteristics; and moreover as the heel and toe pallet switch has a brief period, during which the blade is closed on

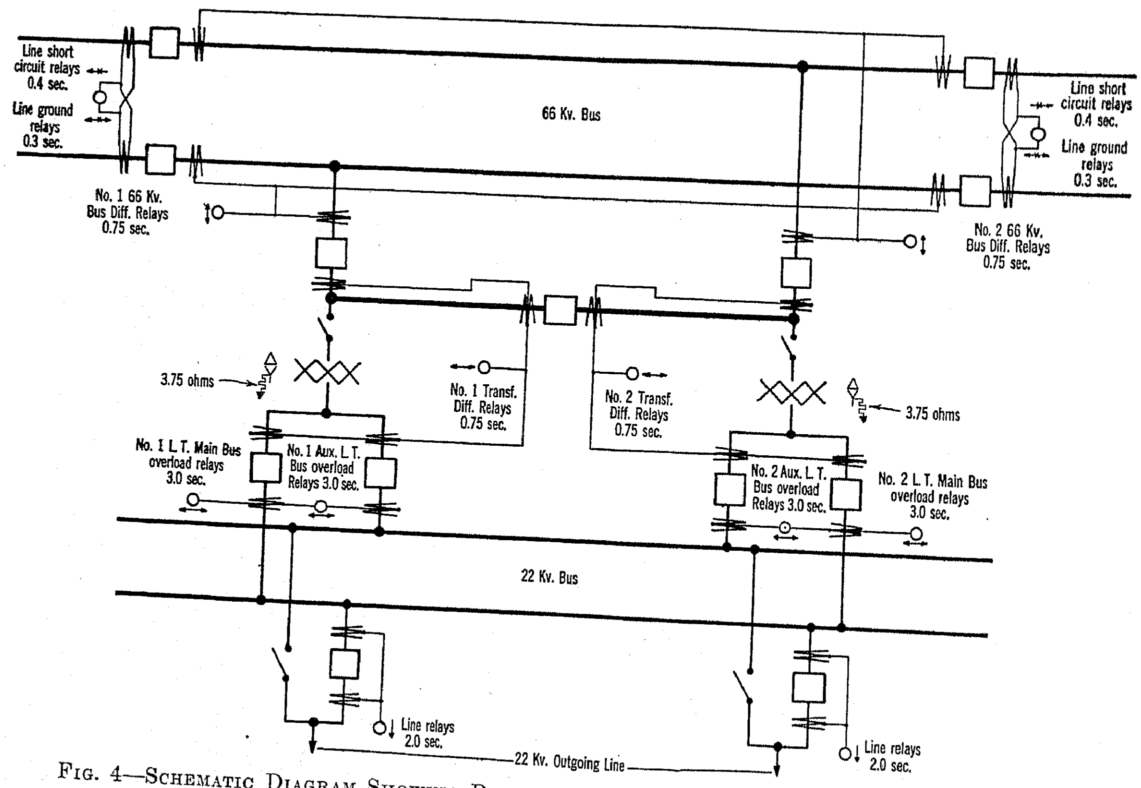

Fig. 4—Schematic Diagram Showing Relaying of Typical Substation-Present Layout

TABLE I

LOG OF 66-KV. RELAY OPERATIONS

Case No.	Date	Cause	Breaker Openings	Relays Operated	Correct Relay Operations	Incorrect or Unaccountable Relay Operations	Total Relay Operations	Remarks
1	3-23-23	22-kv. bus short on bank tapped to North - Colfax No. 2	North-Colfax No. 2 at Colfax North-Colfax No. 2 at North 2 remaining No. 2 differential breakers at North	CR CR Bus diff. No. 2	2	1	3	Time settings too low on bus differential relays caused them to operate incorrectly. See discussion in text. No load lost
2	3-24-23	Unknown at time. Later, one reversed - current transformer found, and grounds on secondary wiring. Operations occurred simultaneously with 22-kv. line trouble	3 breakers on No. 1 bus at Ambridge 3 breakers on No. 2 bus at Ambridge	Bus diff. No. 1 Bus diff. No. 2 Bus diff. No. 1		3	3	Bus differential relays at this station gave trouble for some time. It was difficult to locate, but was finally found as indicated under "cause" in this case Lost some local load at Ambridge
3	3-27-23	Strain insulator on Brunot Island-Woodville line pulled apart, allowing this phase to fall on next lower phase, burning both phases down	Brunot Island - Woodville No. 1 at B. Island Brunot Island - Woodville No. 1 at Woodville	CR CR	2		2	No load lost
4	3-30-23	Suspension insulator on middle phase of Brunot Island - Ambridge No. 1 line pulled apart, allowing this phase to fall on lower phase, burning down one phase	Brunot Island - Ambridge No. 1 at B. Island Brunot Island - Ambridge No. 1 at Ambridge	CR CR	2		2	No load lost
5	4-29-23	Unknown at time. See remarks	Brunot Island - Woodville No. 2 at B. Island Brunot Island - Woodville No. 2 at Woodville 2 remaining breakers of No. 2 bus differential at Woodville 2 remaining breakers of No. 2 transformer differential at Woodville	CD CD Bus diff. No. 2 Trans. diff. No. 2	2	2	4	A subsequent failure developed evidence of a punctured strain insulator on the Brunot Island-Woodville No. 2 line at Woodville. The incorrect operation of the differential relays was due to the low time settings, as in Case 1 No load lost
6	5-12-23	Unknown at time. See remarks	Brunot Island - Woodville No. 1 at B. Island Brunot Island - Woodville No. 1 at Woodville	CR CR	2		2	A very severe wind storm was in progress at the time. A flash was reported to have been seen on one of the towers, and it is probable that some debris was blown into the line, causing a short circuit No load lost
7	6- 6-23	Unknown at time. See remarks.	3 breakers on No. 1 bus at Ambridge 3 breakers on No. 2 bus at Ambridge	Bus diff. No. 1 Bus diff. No. 2		2	2	These operations occurred at the time a ground appeared on the 22-kv. system at this station It is believed to have been caused by the defects reported under "Cause" in case 2. No load lost.
8	6- 6-23	Unknown at time. See remarks.	3 breakers on No. 2 bus at Ambridge	Bus diff. No. 2		1	1	Same as Case 7. No load lost.
9	6-14-23	Man working on tower on North - Colfax No. 2 line caused insulator to flash over.	North-Colfax No. 2 at North. North - Colfax No. 2 at Colfax.	CD CD	2		2	No load lost.

TABLE I—*continued*

Case No.	Date	Cause	Breaker Openings	Relays Operated	Correct Relay Operations	Incorrect or Unaccountable Relay Operations	Total Relay Operations	Remarks
10	6-19-23	No cause found.	Colfax - Dravosburg No. 2 at Colfax.	CD?		1	1	It is probable that this was an operation of the ground relays at Colfax, as there was a lightning storm at the time. No other breakers opened, and the breaker stayed in when closed. See discussion in text. No load lost.
11	6-22-23	Electrical storm	Ambridge-North No. 1 at Ambridge. Ambridge-North No. 1 at North Jct. Park-Ambridge No. 1 at Jct. Park	CD CD CO	3		3	A broken string of insulators was found on this line. No load lost.
12	6-22-23	Electrical storm	North - Ambridge No. 1 at Ambridge	CD	1		1	Other breakers on this line were open at the time. No load lost.
13	6-22-23	Electrical storm	North - Ambridge No. 2 at North	CD	1		1	It is believed that this occurred simultaneously with Case 12, and is therefore correct, as the two lines were running in series, since North-Ambridge No. 1 was open at North at the time. No load lost.
14	6-22-23	Electrical storm	North-Colfax No. 1 at North North-Colfax No. 1 at Colfax North-Colfax No. 2 at North North-Colfax No. 2 at Colfax	CR CR CD CD	4		4	This was caused by a direct stroke of lightning on a tower. See discussion in text. No load lost.
15	6-23-23	Electrical storm	North-Colfax No. 1 at North	CD		1	1	This is a parallel case with Case 10. No load lost.
16	6-24-23	Electrical storm	3 breakers on No. 2 bus at Ambridge	Bus diff. No. 2		1	1	This is a parallel case with Case 2. No load lost.
17	6-24-23	Electrical storm	Ambridge-North No. 1 at Ambridge Ambridge-North No. 1 at North Jct. Park-Ambridge No. 1 at Jct. Park	CD CD CO	3		3	Several damaged strings of insulators were found on the line after this storm. No load lost.
18	6-24-23	Electrical storm	3 breakers on No. 2 bus at Ambridge	Bus diff. No. 2		1	1	This is a parallel case with No. 2 and occurred simultaneously with a ground on a 22-kv. line fed from this station. No load lost.
19	6-24-23	Electrical storm	3 breakers on No. 2 bus at Ambridge	Bus diff. No. 2		1	1	This is a parallel case with No. 2. No load lost.
20	7- 5-23	Electrical storm	Brunot Island - Ambridge No. 1 at B. Island Brunot Island - Ambridge No. 1 at Ambridge	CD CD	2		2	No load lost.

TABLE I—*continued*

Case No.	Date	Cause	Breaker Openings	Relays Operated	Correct Relay Operations	Incorrect or Unaccountable Relay Operations	Total Relay Operations	Remarks
21	7- 6-23	Electrical storm	North-Ambridge No. 1 at North North-Ambridge No. 1 at Ambridge North-Ambridge No. 2 at North North-Ambridge No. 2 at Ambridge Jct. Park-Ambridge No. 2 at Jct. Park Brunot Island - Ambridge No. 2 at B. Island Brunot Island - Ambridge No. 2 at Ambridge Brunot Island - Ambridge No. 1 at Ambridge	CD CD CD CD CO CD CD	7	1	8	All of these breaker openings were simultaneous or nearly so. The only incorrect operation was the opening of Brunot-Island No. 1 at Ambridge, and its cause is unknown, but would seem to point to improper operation of the interlocking relay. Local load lost at Ambridge and Jct. Park.
22	7- 6-23	Unknown	North-Colfax No. 1 at Colfax	CR?	1		1	A heavy voltage surge was recorded on this operation, which would indicate a short circuit. A severe electrical storm accompanied by a high wind was in progress at the time, and it is possible that debris was blown in the line, causing a short circuit which was relieved by the single breaker opening recorded, which would be possible if the fault were very close to Colfax. The disturbance was entirely relieved by the breaker opening recorded. No load lost.
23	7- 6-23	Electrical storm	3 breakers on No. 2 bus at Ambridge	Bus diff. No. 2		1	1	This is a parallel case with No. 2, and occurred simultaneously with a ground on the 22-kv. system at that point. No load lost.
24	7-10-23	Electrical storm	3 breakers on No. 1 bus at Ambridge 3 breakers on No. 2 bus at Ambridge	Bus diff. No. 1 Bus diff. No. 2		2	2	This is a parallel case with No. 2. Only a small amount of local load lost, as 22-kv. bus was unaffected.
25	8-21-23	Electrical storm. Defective lightning arrester on Dravosburg-Woodville No. 1 line.	Dravosburg - Woodville No. 1 at Dravosburg Dravosburg - Woodville No. 1 at Woodville	CD CD	2		2	No load lost.
26	8-21-23	Electrical storm	Colfax-Dravosburg No. 2 at Colfax Colfax-Dravosburg No. 2 at Wilmerding Colfax-Dravosburg No. 2 at Dravosburg Colfax-Wilmerding No. 1 at Colfax Colfax-Wilmerding No. 1 at Wilmerding Wilmdering - Dravosburg No. 1 at Wilmerding Wilmerding - Dravosburg No. 1 at Dravosburg	CD CD CD CD CD CD CD	7		7	No load lost.
27	8-27-23	Electrical storm	Brunot Island - Woodville No. 1 at B. Island Brunot Island - Woodville No. 1 at Woodville	CD CD	2		2	No load lost.

TABLE I—*continued*

Case No.	Date	Cause	Breaker Openings	Relays Operated	Correct Relay Operations	Incorrect or Unaccountable Relay Operations	Total Relay Operations	Remarks
28	9- 4-23	Electrical storm	North Ambridge No. 1 at Ambridge Jct. Park-Ambridge No. 1 at Jct. Park North-Ambridge No. 2 at North North-Colfax No. 2 at North	CD CD CD CD	2	2	4	It is difficult to analyze this case, as it is not certain that the reported breaker openings were correct. No load lost.
29	9-10-23	Lightning - arrester lead broke loose on Brunot Island-Woodville No. 1 and swung into adjacent phase at Woodville; simultaneously, a strain insulator flashed over at Woodville on Brunot Island-Woodville No. 2.	Brunots Island-Woodville No. 1 at B. Island Brunots Island-Woodville No. 1 at Woodville Brunots Island-Woodville No. 2 at B. Island Brunots Island-Woodville No. 2 at Woodville	CR CR CR CR	4		4	See text for further discussion. All synchronous load was lost.
30	10- 7-23	Lightning arrester failed on North-Colfax No. 1	North-Colfax No. 1 at North North-Colfax No. 1 at Colfax	CD CD	2		2	No load lost.
31	10-28-23	22-kv. bus short circuit on bank tapped to North-Colfax No. 2	North-Colfax No. 2 at North North-Colfax No. 2 at Colfax	CR CR	2		2	No load lost.
32	11- 1-23	22-kv. line breaker failed and involved 22-kv. transformer bus at North	3 breakers on No. 2 transformer bank at North	Trans. diff. No. 2	1		1	The No. 1 22-kv. transformer breaker also opened correctly on overload to clear No. 1 bank, but the only load lost was the local 4-kv. service.
33	11-10-23	22-kv. short circuit on No. 1 transformer bus	3 breakers on No. 1 transformer bus at Wilmerding	Trans. diff. No. 1	1		1	No load lost.
34	11-20-23	Breaker bushing failed on No. 2 line at Jct. Park, and involved the tie disconnects.	Jct. Park-Ambridge No. 2 at Jct. Park North-Ambridge No. 2 at Ambridge North-Ambridge No. 2 at North North-Ambridge No. 1 at Ambridge North-Ambridge No. 1 at North	CO CD CD CD CD	5		5	No load lost, as low-tension bus remained hot from loop feeders.
35	12- 1-23	Colfax Dravosburg No. 2 was closed at Dravosburg before the ground switch at Colfax was opened.	Colfax-Dravosburg No. 2 at Dravosburg	CR	1		1	No load lost.
36	12-16-23	Bushings failed on both sides of No. 2 line breaker at Junction Park.	Jct. Park-Ambridge No. 2 at Jct. Park North-Ambridge No. 2 at North North-Ambridge No. 2 at Ambridge Jct. Park-Ambridge No. 1 at Jct. Park.	CO CD CD Bus diff.	4		4	No load lost. Ambridge was carried on the No. 1 line from North, and Jct. Park on the 22-kv. loop lines.
37	12-16-23	Insulator flashover at Jct. Park. Same cause as case 36.	Brunot Island - Ambridge No. 2 at B. Island	CD	1		1	This was an unnecessary operation, and was due to the fact that Brunot Island-Ambridge No. 1 was out of service for repairs. This opened the ring between Brunot Island and Ambridge but no load was lost, as Ambridge was carried from North.

TABLE I—*continued*

Case No.	Date	Cause	Breaker Openings	Relays Operated	Correct Relay Operations	Incorrect or Unaccountable Relay Operations	Total Relay Operations	Remarks
38	12-16-23	Insulator flashover at Jct. Park.	North-Ambridge No. 1 at North	CD	1		1	Lost local load at Ambridge and Jct. Park.
39	12-16-23	Insulator flashover at Jct. Park may have occurred.	North-Ambridge No. 1 at North	CD		1	1	This is probably a parallel case with No. 10, as the trouble was cleared by this single breaker opening. It is a doubtful operation. No load lost.
40	1- 7-24	Bus short circuit on No. 2 transformer bus at Dravosburg, caused by failure of line breaker.	No. 1 Trans. breaker at Colfax No. 2 Trans. breaker at Colfax	CO CO	2		2*	The transformer differential relays did not operate, and the trouble was removed by the operator at Dravosburg by hand. All load lost.
41	1- 8-24	Bushing flashed over on line side of No. 1 breaker at Junction Park	Jct. Park-Ambridge No. 1 at Jct. Park North-Ambridge No. 1 at North North-Ambridge No. 1 at Ambridge	CO CD CD	3		3	No load lost.
42	1- 8-24	Trying to close on above fault at Ambridge	North-Ambridge No. 1 at Ambridge	CD	1		1	No load lost.
43	1- 8-24	3 reactance coils flashed over on Brunot Island-Ambridge No. 2 at B. Island, simultaneously with a failure of a cable feeding off the same bus.	Brunot Island - Ambridge No. 2 at B. Island Brunot Island - Ambridge No. 2 at Ambridge	CR CR	2		2	No load lost.
44	1-16-24	Bushings failed on both sides of No. 2 breaker at Junction Park.	North-Ambridge No. 1 at North North-Ambridge No. 1 at Ambridge North-Ambridge No. 2 at North North-Ambridge No. 2 at Ambridge	CD CD CD CD	4		4	Lost local load at Junction Park.
45	1-26-24	22-kv. short circuit at Wilmerding, fed radially from Colfax on Colfax - Wilmerding No. 2 line.	Colfax-Wilmerding No. 2 at Colfax Colfax-Wilmerding No. 1 at Wilmerding	CR CR	2		2	During 22-kv. circuit breaker test, one transformer bank was isolated at Wilmerding, and fed radially from Colfax. This removed the balance between Colfax-Wilmerding No. 1 and No. 2, and each opened correctly at its feeding end when the test breaker failed to clear at Wilmerding. No load lost.
46	1-29-24	Bushing failure on bus side of No. 2 breaker at Junction Park	North-Ambridge No. 2 at North North-Ambridge No. 2 at Ambridge	CD CD	2		2	The 66-kv. tie breaker at Junction Park was open at this time, hence the operations recorded cleared the ground from the 66-kv. system. No load lost.
47	2- 4-24	Kite string fell onto Dravosburg - Wilmerding No. 2 line.	Dravosburg - Wilmerding No. 2 at Dravosburg Dravosburg - Wilmerding No. 2 at Wilmerding	CD CD	2		2	No load lost.
Grand Totals 47	3-15-23 to 3-15-24				87	21	108	

*One failure.

both sets of contacts to avoid open circuiting the current transformers; it is obvious that some period of current unbalance will exist each time a breaker opens. It is, therefore, necessary to set the time of the bus relays in excess of any such possible discrepancy. The maximum time difference found was between 0.2 and 0.3 seconds. The setting of 0.75 second therefore has a considerable factor of safety.

The transformer differential relays are set at 0.7 ampere current tap, and 0.75 second definite minimum time. The same condition of unbalanced currents will exist on these relays when one of the breakers in this differential opens, as in the case of the bus differential relays, discussed above.

The low-tension bus overload relays are set for a current tap which will allow the bank to carry 200 per cent load without tripping; and a time setting of three seconds. This time setting is used to be selective

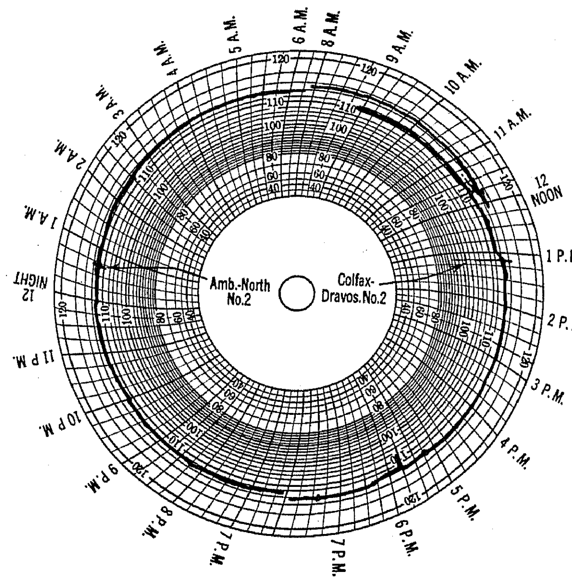

FIG. 5—VOLTAGE CHARTS SHOWING DISTURBANCES RESULTING FROM NORMAL OPERATIONS OF 66-KV. RELAYS GROUND RECORD AT LEFT—SHORT CIRCUIT RECORD AT RIGHT

with the two-second settings on the outgoing feeders from the low-tension bus.

Table I gives a log of total relay operations on the ring for the eleven months period March 15, 1923 to February 15, 1924. During that time a total of 39 line faults occurred, all of which were successfully sectionalized by the relays. Of these 39 faults, it is interesting to note that 12 were short circuit and 27 were ground faults. This division is not strictly accurate, as operation indicators were not installed on all relays during the entire period. However, it is believed to be substantially correct, and was determined by checking such indicators as were installed, and by a study of the phenomena observed at the locations of the faults, and also by the observed and recorded disturbances on the system. The latter is a very dependable factor, as was demonstrated when the initial service tests were made. A ground fault on the 66-kv. ring, which exists as a ground only, does not give a noticeable voltage disturbance on the low-tension busses, and a maximum voltage variation of the system voltage of four volts has been recorded on a 110-volt meter, which is fed from the 11-kv. bus in the center of the high-tension transmission network. The disturbance is usually less than four volts, varying from two to three volts on a normal operation of the ground relays. Fig. 5 shows a voltage chart, recording a ground fault on the ring which was sectionalized by the ground relays without any noticeable voltage fluctuation, giving a drop of about three volts with arise of one volt. The same chart shows a record of a 66-kv. line short circuit which was sectionalized by the short-circuit relays. This shows a voltage drop of twenty volts, followed by a rise of three volts. It is probable that the actual drop was less than this value, as the meter has considerable overtravel. Hence the voltage variation which occurs at the opening of any section of the ring has been found to be a very dependable criterion as to the nature of the fault.

In every case of line fault the trouble was successfully sectionalized by the ring relays, and the ring was left intact. The ring was opened at one point several times, but the other side remained closed and the power plants were not separated by the action of the automatic relays. Some of the cases, however, involved operations which were unnecessary, and some which were not entirely accountable. One failure was experienced.

The following general divisions may be made of the total number of cases of trouble which occurred on the ring:

Total faults on the ring	43
Faults giving 100% correct relay operation	36
Faults involving incorrect or unnecessary relay operations	6
Other incorrect operations	11
Faults involving relay failures	1

An analysis of the above leads to some interesting conclusions. First, of a total of 43 faults which have occurred on the ring, all but one have been successfully cleared by the relays. This gives an overall efficiency for relay operation of 97.7 per cent. It is also interesting to note five of these faults were double circuit faults. That is, the trouble which occurred involved both of the balanced circuits, either simultaneously or nearly so. It is obvious that this is the worst type of fault to encounter with a system of balanced protection, which depends inherently upon the inequality in fault energy in the balanced circuits. Yet, in each of these cases, both defective units were removed successfully from the system. Three correct operations have occurred on transformer differential protection. On the basis of relay operations, a total of 108 have occurred, of which 87 were correct and 21 incorrect or unaccountable.

The matter of unnecessary breaker openings is one of the utmost importance, particularly on a double-circuit system like the Duquesne Ring, where two

important factors enter. First, if any breaker opens unnecessarily, it leaves its mate (the line against which it is balanced) in a precarious condition unnecessarily. Second, if any section of the lines is operating singly (only one line in service, its mate being defective or out of service) and it opens unnecessarily when a fault occurs on the opposite side of the ring, it may separate the two power plants, situated as they are at the opposite ends of the ring. It is therefore obvious that it is of utmost importance to prevent such incorrect operations, which may endanger the unity of the system. Exactly how many of the operations listed as unnecessary were correct or incorrect is impossible to determine definitely. Several operations occurred, such as Cases 10, 15 and 39, where only one breaker opened on a section of line, and then went back into service when closed and carried load. This of itself gives no definite indication, since in every case where insulator flashovers occurred and the line was sectionalized by the ground relays, the lines went back into service immediately and carried load in a rain storm. Usually an inspection of the line would find a small part of one insulator petticoat slightly chipped. Therefore, the fact that the lines which opened at one end only, went back into service immediately is not an indication that no fault existed. In fact, all indications point to the fact that a ground fault did exist. This gives rise to the point that a flashover was apparently extinguished by the opening of one end of the circuit, potential being continuously supplied from the other end.

The hazard of unnecessary breaker openings on a system of this kind is very clearly shown in Cases 13 and 37. In each of these cases one line of a balanced pair had been automatically opened at both ends, due to a line fault, or else the one line was out of service for repairs. Under these conditions, a fault occurred on another section of line, and this caused the good section of line, operating singly, to open at the feeding end, as all relays are set to operate in a given time, which is, of course, the same for either single or double-line operation. These operations resulted in splitting the ring open on one side, which, however, did no harm, since the ring was closed on the other side. If, however, the ring had been operating with a single-line section on both sides, and a fault had occurred, it would have opened both single-line sections and separated the power plants. Obviously, the lesson to be learned from these cases is that single-line operation must be automatically rendered selective with the remaining balanced sections. This may be called Modification No. 1 and is referred to later.

Two other cases of trouble have been experienced which may prove of particular interest to users of balanced protection, and which have led to Modification No. 2. These cases both have to do with the use of interlocking relays. These relays, which are shown in Fig. 2, are connected in the conventional manner, so that the automatic tripping of breaker No. 1 on a pair of balanced lines operates the direct-current interlocking relay on line No. 2. The contacts of this relay open instantaneously, and lock out the tripping circuit of its breaker until a definite time interval has elapsed, when it is restored. This time interval is sufficient to allow the faulty line to clear at the opposite end and thus remove the fault from the system. If some such precaution is not taken, it is obvious that the good line of the pair will open at No. 1 end simultaneously with the opening of the bad line on No. 2 end. It is evident that the success of this scheme depends entirely upon the proper functioning of the interlocking relay. If this relay fails to trip, the good line of the pair will be incorrectly opened. If this relay trips but fails to reset, the second line is left without protection at that end. Both of these conditions have been found to exist, although fortunately under test and not service conditions. At best, a direct-current time relay requires considerable maintenance, and the timing feature is not accurate. For the latter reason it is desirable to set them for a liberal time interval, so as to assure the positive clearing of the bad line at both ends to hold the good line in service. For this reason, and because the particular type of interlocking relays supplied were especially accurate and dependable at that point, these relays, as used in 1923, were set for a re-setting time delay of eight seconds. Under these conditions, two cases occurred involving the operation of the relays.

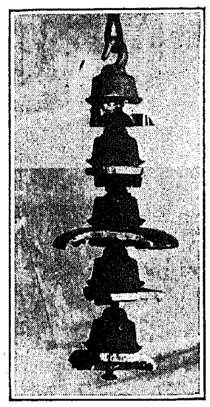

Fig. 6—Line Suspension Insulator Subjected to Short Circuit by a Direct Stroke of Lightning

Case 13 occurred during a severe electrical storm in July. A stroke of lightning hit at or very near one of the 66-kv. double-circuit steel towers about one-half mile from the North Substation on the North-Colfax lines. The severity of the discharge is shown by the fact that on the following day it was found necessary to replace 20 suspension insulator strings on this line in the vicinity of the stroke. (See Fig. 6.) At the tower struck, all three insulators flashed over on No. 1 circuit, and at least one insulator string on No. 2 circuit. This action presumably was simultaneous. The operators at both North and Colfax

report that both lines opened simultaneously. This is, of course, theoretically possible, assuming a three-phase short circuit on one line and a single-phase ground circuit on the other line, in which case the short-circuit relays on one line and the ground relays on the other line would be unbalanced and operate simultaneously. To do so, it would be necessary for both sets of relays to close before the interlocking relay had operated to render one line temporarily non-automatic. Moreover, it will be noted that there is a difference of 0.1 second in the time settings of the short-circuit and the ground relays. Hence, it is highly probable, that the short-circuited line opened first and the ground circuit remained on No. 2 line during the 8.0 second period, that this line was rendered non-automatic, due to the proper functioning of the interlocking relays, and that this line was tripped as soon as the trip circuits were restored. No distress, however, was apparent on the system, except a severe voltage dip when the initial short circuit occurred.

The next cast was not so fortunate. This is recorded as Case 29. The initial fault was occasioned by a lightning-arrester lead coming loose at the Woodville Substation platform and swinging into an adjacent phase of the Brunot Island-Woodville No. 1 line. The short-circuit relays functioned properly, and the line was cleared at both ends. However, almost simultaneously with the short circuit on No. 1 line, a fault developed on a strain insulator at Woodville on Brunot Island-Woodville No. 2 line. This line was then non-automatic by the action of the interlocking relays, resulting from the opening of No. 1 line. The fault on No. 2 line developed almst immediately into a short circuit and for eight or nine seconds the system was in distress. The voltage dropped to a low value and the two power plants hunted, resulting in considerable surging. At the end of the eight second period, the interlocking relays restored the trip circuits of the line and it was tripped automatically at both ends. All synchronous load was dropped, but no other damage resulted, nor did any simultaneous failures develop. As a consequence of this breakdown, resulting as it did in sustained distress, it was decided to reduce the time delay on the interlocking relays from eight seconds to four seconds, Since that time, two similar cases of trouble have developed. Cases 34 and 44, and no particular distress has occurred on the system, while only a small part of synchronous load was lost, and that only in the immediate vicinity of the trouble. As previously stated, these two occurrences resulted in the adoption of Modification No. 2.

A third and fourth modification have been made as a result of Case 40. This resulted in the only failure to date of any of the 66-kv. relaying, and it is interesting to note that the failure occurred on differential apparatus protection rather than on the balanced line protection, where perhaps the probability might be considered greater, particularly considering the larger number of operations which occur on the line relays. This failure was the result of a rather extraordinary sequence of events which could hardly be foreseen. A 22-kv. line breaker was closed on a fault to test the line, after the breaker had opened once automatically. The breaker apparently cleared the fault at the contacts, but threw fire and oil from the vent in the case. This caused a short circuit on the bus side of the breaker bushings, and the overload relays on the low-tension bank breaker operated to trip the breaker. The 22-kv. bus was split at the time. This breaker failed in opening, blowing off two tanks and short circuiting the low-tension side of the transformer bank. This placed the fault within the bank differential protection, which failed to operate. The fault was cleared when the operator stripped the board, opening all high-tension switches. An investigation later showed the transformer differential protection to be in perfect condition. A very thorough investigation brought forth the following explanation, which, however, could not be definitely substantiated, since the evidence was destroyed. It seems probable that when the transformer breaker was tripped, the breaker did not fully open, with the result that the heel and toe pallet switch remained in the neutral position. This left the differential relays short circuited and hence inoperative, thus accounting for their failure to close contacts.

As a result of the experiences to date with this relay system, certain conclusions have been reached as follows:

1. It must be possible to operate the ring with one or more single-line sections in circuit, without danger of dropping these sections when faults occur on other sections. (See Modification No. 1).
2. Some system of interlocking must be used which will be capable of more accurate time settings than the present, and especially one whose failure to function properly cannot leave any lines without protection. (See Modification No. 2).
3. Every system of primary relaying must be provided with a system of back-up protection which will act if the first system fails, a "second line of defence," as it were.

These features are now being applied to the ring, and are described in the following, under Modification No. 3.

4. Means must be provided to give protection for faults occurring inside the tanks of circuit breakers. (See Modification No. 4).
5. It must be possible to automatically remove any single unit from service at a station without killing this station. (See Modification No. 5).

Modification No. 1 consists of the addition of a set of overload relays, which are automatically thrown into service when one line of a pair is removed from circuit. These relays then give overload protection on the remaining line, and their time is set high enough to be selective with the remaining fast-time balanced relays. This will prevent the unnecessary opening of all good single-line sections when faults occur on the other paired sections. At the same time, the single-

line sections are left with ample protection for themselves. The modified protection is shown schematically in Fig. 7. The balanced line protection is the same as before, shown in Fig. 2, except that the direct-current time delay closing interlocking relay has been eliminated. This relay has been replaced by an instantaneous direct-current multicontact relay, the function of which is to shunt out of the tripping circuit the contacts of either the fast-time balanced relays, or those of the long-time overload relays according as the station is working with one or two lines in service. The action of the direct-current relay is controlled by the position of the breaker. If the breaker is closed, the pallet switch to this relay is open, and the back contacts of the relay shunt the tripping contacts of the long-time

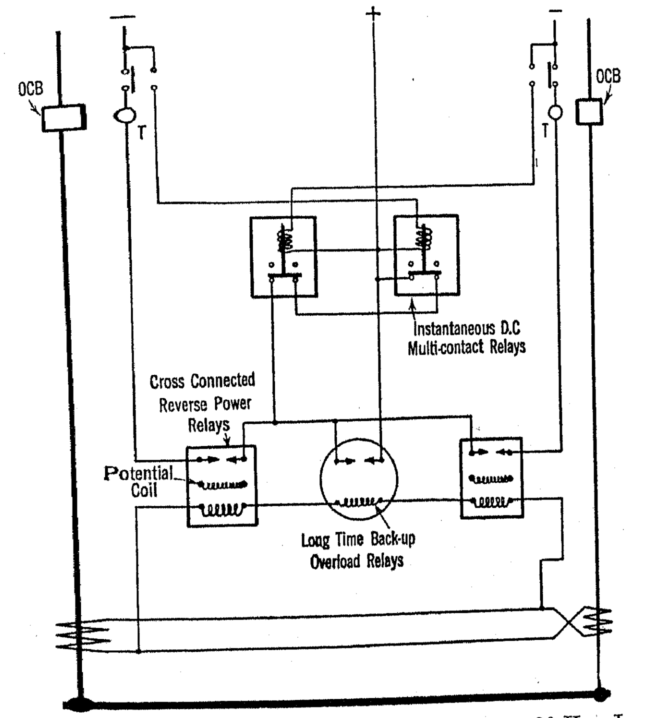

FIG. 7—SCHEMATIC DIAGRAM OF IMPROVED 66-KV. LINE PROTECTION

overload relays, provided the direct-current relay on the other line is in the closed position. This leaves in circuit the contacts of the balanced relays only. But if the one line is open and the other line closed, the pallet switch in the former is closed, thus energizing the direct-current relay, which takes the shunt off the trip contacts of the long-time overload relays, and shunts out the trip contacts of the balanced relays. This places the desired long-time setting on the relays of the single line, and gives it selective operation for subsequent faults on balanced sections.

Modification No. 2 follows as a consequence of Modification No. 1, and secures the employment of accurate and dependable interlocking relays. This is accomplished by the same relays as were added in the preceding modification to secure selective single-line operation. The time delay is secured through the action of the long-time back-up alternating-current overload induction relay, which is automatically thrown into service when one of the balanced lines is tripped out, either automatically or otherwise. This relay starts to trip out the good line, but the bad line clears at the other end before the long-time overload relay can close contacts. This scheme has the additional features:

1. That the good line is never rendered non-automatic;
2. That the failure of the direct-current relay, either to close or open, does not cripple any protection, but merely destroys the selectivity;
3. That all units in the interlocking scheme are very positive in their operation;
4. That very accurate time-setting of the interlocking feature is accomplished.

Modification No. 3 has been adopted to secure a second line of defense to two different systems of relaying: the line relays and the station protection. This is shown in Fig. 8, and consists of reverse-power relays on each 66-kv. transformer breaker, set to trip for power flowing from the ring to the station bus. The trip circuits of these two sets of relays, however, are crossed, that is, the relays on No. 5 transformer breaker trip breaker No. 6, and the relays on No. 6 transformer breaker trip breaker No. 5. This is done to secure back-up protection for the line circuit breakers, and is accomplished as follows. Referring to Fig. 8, let us assume that a fault has developed on the upper right hand line. The breaker at the other end has opened correctly but breaker No. 1 at this station has for some reason failed to open automatically. This places a balanced fault on the ring, which cannot be detected by any of the remaining balanced types of protection. Such an occurrence would shut down the system, unless special means were provided to relieve it. This is accomplished by the No. 1 66-kv. bus-splitting reverse-power relays which respond to the power which will pass over the bus tie feeding into the fault through line breaker No. 1. These relays will trip breaker No. 6, which leaves all the station load on the good line, and by splitting the bus, has taken the balance off the lines. This passes the trouble back to the next adjacent station, where the line relays will operate to clear the fault from the system. The second function of these bus-splitting relays is to act as back-up protection for any faults which may occur at the station. Thus, if any of the bus or transformer differentials fail to function properly when in trouble, these relays will act as overload protection to take the trouble off the ring.

Modification No. 4 has been adopted to take care of faults inside breaker tanks. It is done, as shown in Fig. 8, by overlapping the protection around each circuit breaker. Thus, on a line breaker, the relays are connected to the current transformers on the bus side of the breaker, and the bus differential relays are connected to the current transformers on the line side of the breaker. Thus, a ground circuit inside a

tank will operate both the line and bus relays, and thus clear both sources of feed to the fault.

Modification No. 5. The purpose of this modification is to prevent a 22-kv. bus short circuit from killing the station. It consists of the installation of a bus tie breaker in the main 22-kv. bus, and the protection of each section of this bus by means of bus-differential relays. (See Fig. 8). An additional set of over-load relays is inserted between these busses to act as bus-splitting protection. The purpose of this is to complete a second line of defense for the 22-kv. outgoing line breakers. Thus, if line breaker No. 14 fails to open for a fault on its line, this will put the trouble on No. 2 section of the 22-kv. bus, which will then be cleared by the operation of the bus-splitting overload relays opening bus tie breaker No. 12 and the operation of No. 2 main bus overload relays opening breaker No. 11. This will clear the fault from the system and leave the station hot on the No. 1 section of the low-tension bus. If the fault had been on the bus itself, such as by the failure of a line breaker No. 14, this would have operated the No. 2 22-kv. bus differential relays and opened breakers 11, 12, 14 and the breakers of any other outgoing lines from this section of bus.

All of the improvements described above are now in the process of being installed at the ring stations. It is not assumed that this will mean that perfect operation may be expected hereafter, but it is believed that these improvements are a big step in the right direction and, that the improved performance derived will amply justify the added investment.

As a whole, the performance of the ring relaying during the past eleven months is considered extremely successful. Of a total of 43 major faults which developed, 42 were successfully sectionalized and only on two occasions was a major outage permitted to result. The single failure which occurred was the result of defective apparatus which could not be controlled. This contingency will be eliminated in the future by the safety valves which are being added. The action of the balanced line relays has been very pleasing. All cases of line trouble have been relieved with a very small amount of damage and disturbance. This is, of course, due to the sensitivity of the relays and their rapidity of action. Every case of insulator flashover during the past season has been disconnected so quickly that a negligible amount of injury has resulted at the fault and the lines were returned to service in every case under adverse conditions. This even includes the case of the direct lightning stroke of Case 14, where, although considerable damage was done to the porcelain, it was nevertheless possible to put the lines back into service immediately as the copper was practically undamaged.

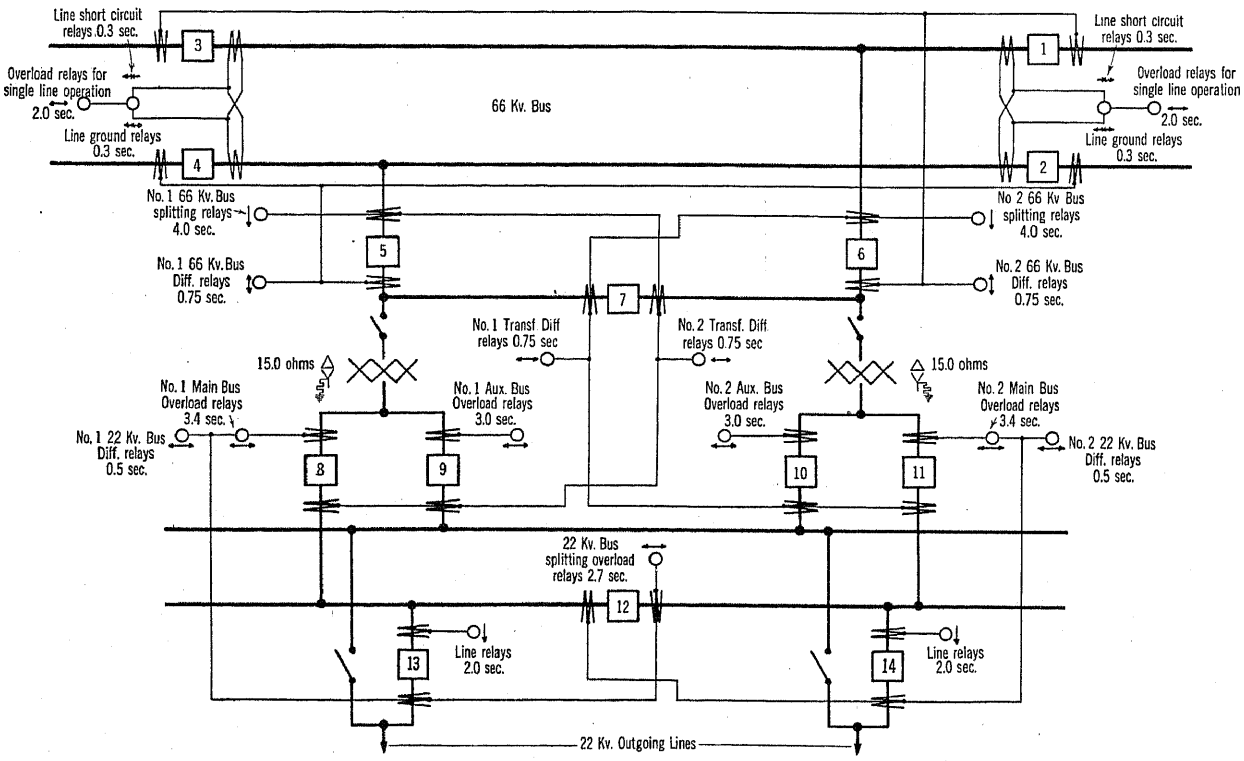

FIG. 8—SCHEMATIC DIAGRAM SHOWING RELAYING OF TYPICAL SUBSTATION—ULTIMATE LAYOUT

As a result of the past eleven months' experience, many valuable lessons have been learned. These have been carefully weighed and acted upon. Systems of this kind, with this type of protection, are still young and present many problems which are novel and interesting. It is, nevertheless, confidently believed that the performance to date will compare extremely favorably with any equivalent system during the same time. It is further believed that with the additional safety features added, as described in this paper, the coming year will considerably better the record to date.

New Type of High-Tension Network

An Interconnecting System for the Supply of Electric Power over Large Areas

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Review of the Subject:—*The purpose of the paper is to present for consideration a new method of interconnecting sources of power and load centers in a large district with a well developed and well distributed load. The central idea is the superposition of a high tension network of single-circuit lines over the whole district for the purpose of supplying a medium in which current may flow in any general direction as changing conditions may dictate. This is similar to the underground network of the Edison Companies in the large cities.*

With such a layout available, power anywhere in the district may be fed into the network and it may be taken out at any other point without serious loss of energy.

The network is connected directly to load centers as well as to existing generating centers and thus greatly assists the present distribution lines distributing power and also stabilizes the potential at each load center reached.

The charging current of the network may largely neutralize the lagging component of the load with favorable design.

A concrete illustrative network is worked out in considerable detail, covering the present load with an equal amount of new future load in the interconnected systems of Alabama, the Carolinas, Georgia and Tennessee, with a branch to the Appalachian Power Company in Virginia. The result shows a very effective, efficient and low cost system for this territory.

The plan is applicable to other districts as well.

Details are given as to the layout taken and the operating characteristics of the system.

INTRODUCTION

THE purpose of the present paper is to propose a new type of inter-connection for high-tension networks for the economic supply of electric power over large areas having an already well developed load. No judgment can be formed on such a proposal without considering all pertinent factors, such as feasibility, cost, economy operating quality, technical design, etc.

There is a natural evolution in the development of the supply of power to any growing industrial district. In the beginning, supply centers start at favorable points, either sites for cheap power or favorable for advantageous load, and grow and spread until the district is fairly well covered with power systems, at least where the active industries exist. As these systems come in contact at their extreme points, they make connections and exchange power to their mutual advantage. In fact, one system may to a material extent depend on power from another system for supplying increments of its load as well as for emergency power.

There comes a point, however, as the load of the district grows, where the favorable sites for local power plants, especially water power, are exhausted in much of the district and resort must be had to local steam plants or more distant water power.

It will usually be the case that within any one such large district some few points will be much more favorable for the development of power in large blocks than other points, calling for the transmission and distribution of this power over the district. When this point is reached, a radical change in the nature of the interconnection and the general supply system is required. To propose a suitable system for such a situation is the object of this paper.

Up to the present time no true general high-tension supply network has been installed. The report of the U. S. Geological Survey, 1921, entitled "A Super-power System for the Region between Boston and Washington," by W. S. Murray and others, Professional Paper 123, proposes an interconnection for the Northern Atlantic States, but the present system differs materially therefrom and is believed to have some marked advantages for many districts.

The point of view taken in the layout of the type of network proposed herein, is to utilize the transmission lines made necessary by the transmission of power from the most favorable sources of water or steam power in bulk, to serve at the same time to distribute this power over the district as widely as possible and to gain such other advantages as can be secured at the same time at small cost, for example, the improvement of voltage regulation and power factor.

The total advantage that can be gained over the trunk-line conception of interconnection, *i. e.*, using heavy lines between large power centers, is surprisingly great and can be realized only by studying concrete examples. In the paper following, these matters are gone into in some detail and a concrete example worked out to permit intelligent judgment to be made.

Part I

SPECIAL FEATURES OF THE NEW NETWORK

The underlying features of the new type of high-tension network, which is predicated on the combination of the supply of power from large central stations, located at the most favorable generating points, with the ready delivery of power at all points in the district where it may be needed, are the following:

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-11, 1924, also presented at the Northeastern District Regional meeting, Worcester, Mass., June 4-5, 1924.

Laying out the high-tension lines to reach as many substations at load centers as possible and using single circuit lines for this purpose, thus securing a true network.

Increasing the capacity of the distribution lines to deliver power in their own territory by feeding power back along these existing distribution lines from the various points of connection to the network, thus shortening the average local transmission and increasing the available feeds.

Establishing each point at which the high-tension network is connected to the existing distribution lines as a point of constant voltage, as well as a new center of distribution of high-tension power.

Utilizing favorably situated local power stations to stabilize the network voltage and to hold the established voltages fixed, regardless of the amount of or distribution of the flow of power and taking advantage of the slope of potential along a line with the prevailing direction of the flow of power to improve the line-power factor.

Co-ordinating the network voltage and charging current for any particular network to improve the power factor in the distribution lines and providing for control of line power factor at individual substations.

The result of the application of these principles is a network covering a district in all directions by many individual circuits and touching all the important sources of power and load centers;—a network of such a nature that any power available at any point may be freely drawn at any other, so that the most advantageous sources of power may be developed, regardless of their location and load centers be established, regardless of their relation to the location of the generating plants;—such a system that the capacity of the existing distribution circuits is very greatly increased and their power factor improved without additional expense and the voltage regulation of local distribution lines is stabilized at many new points.

This high tension network has the advantages of the low voltage underground net works of the Edison Companies in large cities as far as efficient transmission and flexible distribution of power is concerned.

Aside from the North Atlantic district, of which so much has been written, there are several districts in this country where some such high-tension network as here described will be of great advantage. The Southeastern States with their well-known combination of systems and the States south of the Great Lakes from Buffalo to Chicago, and as far South as the Ohio River and others may be cited.

Typical Network

In order to make the value of this type of network more evident and to give a concrete example of how an actual installation would work out, a layout suitable for the Southeastern States, including Southern Virginia, North and South Carolina, Georgia, Alabama and Tennessee, has been worked out in some detail, but based on more or less arbitrary assumptions of load.

This network transmits new power, generated largely by hydro-electric plants, located in Alabama and Tennessee, to the amount of nearly 1,000,000 horse power, including 350,000 horse power from Muscle Shoals, and distributes it over the whole district, wherever it can be best used, this all being in addition to the present 800,000 or 900,000 horse power now generated. A large part of this load is located in the States of North and South Carolina, while the bulk of the power is generated far west of this.

To cover periods when water is low and steam must be used, a large steam plant is taken at the center of the system of the Appalachian Power Company in Virginia and another near Knoxville, where steam plant conditions are favorable. Furthermore, it is assumed that some power will be taken for general use from seasonal-storage reservoirs in a dry time.

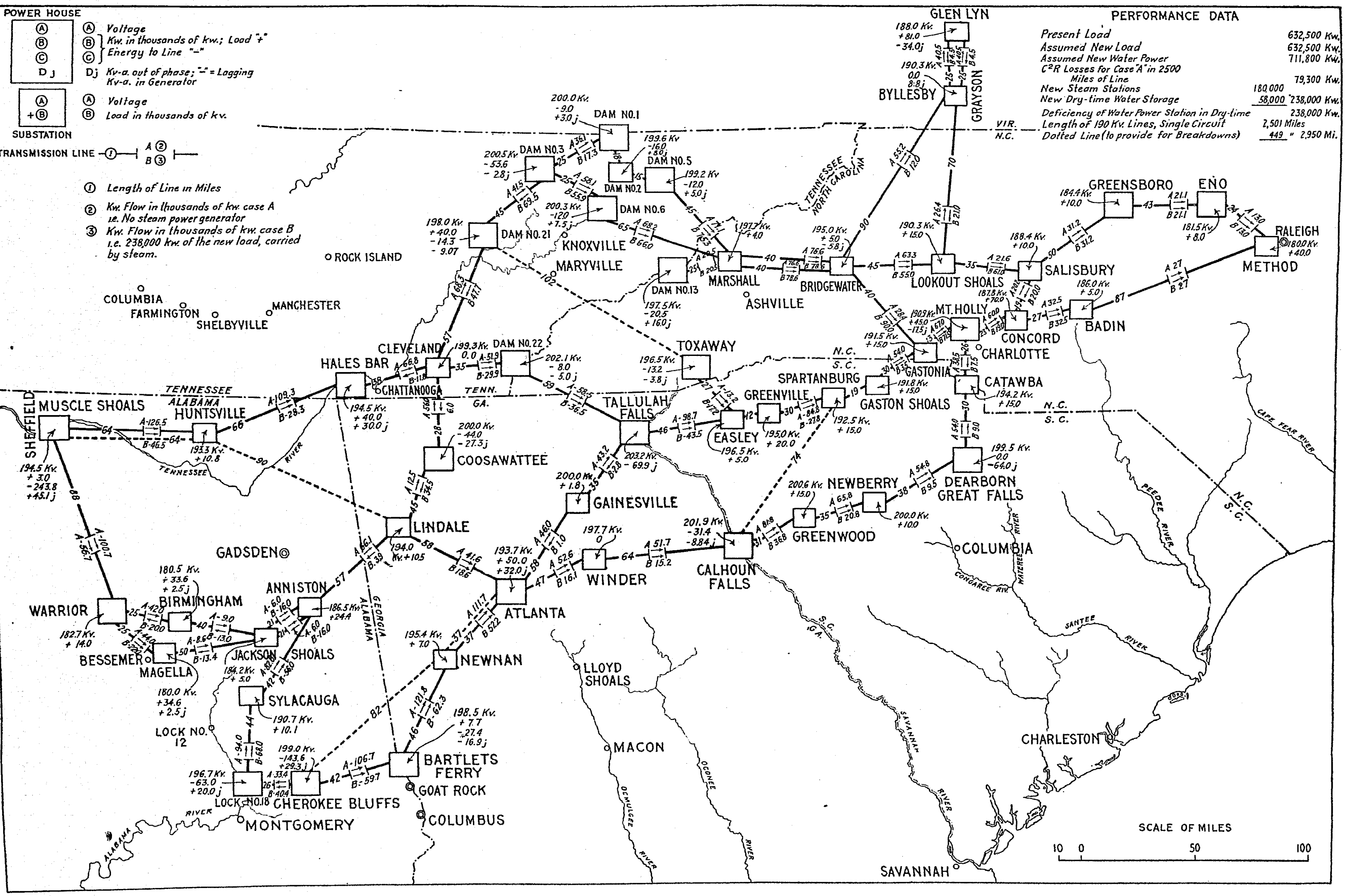
It is expected that this amount of new power will be required by the growth of the district in from seven to ten years.

This network which operates on an average voltage of 190 kv. has a total line loss at peak load of about 11 per cent and also has satisfactory voltage regulation. The average $C^2 R$ line loss may be less than half of this, on account of the large portion of time when the load is light. Furthermore, the power factor in the distribution lines is raised to about 95 per cent on the average. As it happens, no synchronous condenser need be added in this layout. The capital cost, including transmission lines, high-tension oil breakers and step-down transformers, will be from $30,000,000. to $40,000,000 or or from $45. to $60. per kw, delivered to the local distribution lines.

The actual locations of the high tension lines, the load carried by each, and the new power taken from the net work at each substation and generating point are shown in the accompanying map. The details of the layouts, will be found described at length in Part III.

Before discussing more in detail the advantages and economies gained for this district by this network, a brief discussion of some of the considerations underlying an organization to handle such an installation will be appropriate.

Obviously, to secure the greatest benefit from the layout under consideration, its operation and control must be under one authority. This suggests a *Network Organization* of some sort. It is further necessary that the savings and advantages be divided equitably among the several constituent companies and that the relationships be such that the most economical use of the water power and of the installation may be made without its being to the advantage of one system over another; also that new developments shall be made at the most favorable points and in the most economi-



POWER HOUSE
Ⓐ Voltage
Ⓑ Kw. in thousands of kw.; Load "+"
Ⓒ Energy to Line "–"
Dj Kv-a. out of phase; "–" = Lagging Kv-a. in Generator
SUBSTATION
Ⓐ Voltage
Ⓑ Load in thousands of kv.
TRANSMISSION LINE
① Length of Line in Miles
② Kw. Flow in thousands of kw. case A i.e. No steam power generator
③ Kw. Flow in thousands of kw. case B i.e. 238,000 kw. of the new load, carried by steam.
PERFORMANCE DATA
Present Load 632,500 Kw.
Assumed New Load 632,500 Kw.
Assumed New Water Power 711,800 Kw.
C²R Losses for Case "A" in 2500 Miles of Line 79,300 Kw.
New Steam Stations 180,000
New Dry-time Water Storage 58,000 238,000 Kw.
Deficiency of Water Power Station in Dry-time 238,000 Kw.
Length of 190 Kv. Lines, Single Circuit 2,501 Miles
Dotted Line (to provide for Breakdowns) 449 " 2,950 Mi.
GLEN LYN
BYLLESBY
GRAYSON
DAM NO.1
DAM NO.2
DAM NO.3
DAM NO.5
DAM NO.6
DAM NO.13
DAM NO.21
DAM NO.22
KNOXVILLE
MARYVILLE
MARSHALL
ASHVILLE
BRIDGEWATER
LOOKOUT SHOALS
SALISBURY
GREENSBORO
ENO
RALEIGH
METHOD
BADIN
CONCORD
MT. HOLLY
CHARLOTTE
GASTONIA
CATAWBA
GASTON SHOALS
SPARTANBURG
GREENVILLE
EASLEY
TOXAWAY
TALLULAH FALLS
DEARBORN
GREAT FALLS
NEWBERRY
GREENWOOD
CALHOUN FALLS
COLUMBIA
WINDER
GAINESVILLE
ATLANTA
NEWNAN
LLOYD SHOALS
MACON
BARTLETS FERRY
GOAT ROCK
COLUMBUS
CHEROKEE BLUFFS
LOCK NO.18
MONTGOMERY
LOCK NO. 12
SYLACAUGA
JACKSON SHOALS
ANNISTON
BIRMINGHAM
BESSEMER
MAGELLA
WARRIOR
GADSDEN
LINDALE
COOSAWATTEE
CLEVELAND
CHATTANOOGA
HALES BAR
HUNTSVILLE
MUSCLE SHOALS
SHEFFIELD
ROCK ISLAND
COLUMBIA
FARMINGTON
SHELBYVILLE
MANCHESTER
CHARLESTON
SAVANNAH
TENNESSEE
ALABAMA
GEORGIA
TENN.
GA.
N.C.
S.C.
VIR.
NORTH CAROLINA
TENNESSEE RIVER
CAPE FEAR RIVER
PEEDEE RIVER
SANTEE RIVER
WATEREE RIVER
CONGAREE RIV.
SAVANNAH RIVER
OCONEE RIVER
OCMULGEE RIVER
ALABAMA RIVER
SCALE OF MILES
10 0 50 100

cal manner, without unfairness to any particular company.

While it would not be appropriate in this paper to prescribe any definite corporate organization for this situation, for this would necessarily be an evolution after much consideration of all the special factors existing in this as in any district, it may be of some value to discuss briefly one possible plan.

Controlling Organization

It may be taken that the *Network Organization* sells, buys, generates power, constructs and owns or leases the new power stations. The Network Organization is controlled by the constituent companies, the equity of each being proportional to the maximum load taken from the network, plus its agreed capacity to supply power to the network on demand. These quantities are kept independent of such power as is generated and consumed by any one local company without regard to the network as at present. As the system grows, a new apportionment of equity will be made yearly among the constituent companies, adjusting for any increase in load or supply capacity. This keeps the interests of the several companies proportional to the part they play in the network operations, as well as their shares in the profits.

The Network Organization should be able to finance improvements, credits for which might be guaranteed by that company in whose territory the improvements lie, so that the best possible rate of interest might be obtained.

In negotiating contracts with constituent companies for the delivery of power, the Network Organization would have to offer a price lower than that at which the local company could generate its own power, which would give a more or less practicable method of arriving at an equitable selling price for power and one in line with market conditions, which is desirable. Such a contract could be of any suitable form for the service.

Contracts for the sale of power might well be made also with outside consumers.

Contracts by the Network Organization to buy power or to buy water-power sites should be based on maximum power for which the Network Organization may call and not for the amount of power that may actually be delivered in any particular. This is so that the network load dispatcher may be able to draw power from the most economical source without regard to the monetary interests of any of the particular companies. The competition among the various constituent companies should enable the Network Organization to buy the cheapest power or develop the most favorable sites. It should be the policy to make the operations of the Network Organization profitable, so that its credit would be of the best. The constituent companies would get the net proceeds in any case.

Part II

Advantages and Economies

In this section the advantages and economies of the 190 kv. network, shown on the map and described in detail in Part III, are discussed:

1. All the advantages of diversity, as applying to any interconnection, are obtained, such as daily-load-curve diversity, reduction of spare capacity, divergence between the differing periods of activity of different industries, either seasonal or due to swings of "boom-times" and "hard-times," railroad electrification, etc., also diversity due to difference in the dry periods of water power at different parts of the district. While it is beyond the scope of this paper to study this particular matter in a numerical sense, it may be pointed out that with a diversity gain of 10 per cent on the total peak output, here taken as 1,250,000 kw. on the 100 kv. lines, the additional power that might be sold would be 125,000 kw., with no additions to the general installation.

When the opportunity to reduce spare capacity is considered, it may well be that 10 to 15 per cent in the capital cost of generating units, power house, etc., may also be saved, at least in all new work. This may well amount to two to three million dollars in the final installations. This is in addition to the value of the above 125,000 kw. which may be worth from $2,500,000 to $3,500,000 a year; an amount sufficient alone to cover fixed charges on the cost of the network, transformers and high-tension breakers.

2. The fact that power generated at any point may be used at any other, without material loss and within any reasonable limit, with the advantage of unitary control for the system, would permit power to be sold considerably nearer the total water power available than with the component systems all operating separately. With this factor may be taken the material reduction in the continuous discrepancy between the growth of the load and the new generating capacity installed, which is a material factor, getting greater as larger units must be installed. If these two factors aggregate 10 per cent it means another 125,000 kw.;—if 15 per cent, 187,500 kw., which at $25.00 a kw. year and 10 per cent fixed charges, would cover to a capitalization of $30,000,000 to $45,000,000.

The advantage of the factors so far considered might be appraised also by considering that a plant of 75 to 80 per cent of the size otherwise necessary would be sufficient for carrying any definite load.

3. If the same quantity of power be transmitted by direct trunk lines from the same water powers and steam plants to selected single points at the centers of the chief loads, about three quarters of the same mileage will be required for the actual transmission, but when the necessity for spare lines for individual trunk units is taken into account, it is doubtful if the trunk lines

would not require more total length of circuit. This is because in the network properly laid out, very few links are long enough or important enough to require a spare. The delivery of this power to 35 step-down stations by the network system will thus cost no more for the high-tension lines than trunk-line distribution to a half dozen points. It should perhaps be stated that there may be all gradations between the two types of interconnections and that probably there is little likelihood that a trunk-line type pure and simple would be installed in this region.

To distribute the amount of power assumed to the 35 substations from the delivery points of the trunk lines would require perhaps 25 to 30—No. 0000 cable—100 kv. circuits, 50 miles long, aggregating with spares, 1500 to 2000 miles and costing $10,000,000 to $15,000,000. The necessity for this expenditure is largely avoided by the network and the delivery of power is made in a much more satisfactory and reliable manner. The statement here made must, of course, be based on averages and is largely of the nature of an estimate.

4. In the matter of voltage regulation, a very great advantage is secured by the network. In the first place, the voltage is stabilized at 35 load points, largely at points of distribution lines remote from their supply, and consequently with a naturally variable voltage. This means that the voltage at intermediate points on the distribution lines will be very materially steadied. Furthermore, with the power factor raised to 95 per cent in the distribution lines, the drop will be cut nearly in half. This improvement in the regulation is a major factor, but no effort is here made to give it a money value.

5. Perhaps the most important advantage of the network, shared only to a less degree by the trunk line transmission, is the opportunity to develop the most favorable power station projects to the exclusion of less favorable ones, which may happen to be nearer the load. If it be assumed that the large favorable sites may be developed for $30.00 a horse power less than local ones, this would mean for one half of the 1,000,000 new horse-power $15,000,000. This advantage may amount to a very much larger sum of money. As no such amount of local water power as 1,000,000 horse power is available, the absence of interconnection would mean local steam plants, adding 0.5 cents more or less to the cost of the power or the building of both steam power plants and part-time water-power plants, a still more expensive plan.

6. If the separate development of local companies means the addition of enough steam power with its higher cost to force an increase in power rates, with all the dissatisfaction that follows, and this can be avoided by the development of the distant water power, this factor alone would be of very great moment—as a matter of fact, this seems to be not an improbable contingency.

7. It is possible to concentrate a load of 100,000 or 200,000 kw. at almost any point in this system with only a small and local addition to the network. In case of war, this power could be secured by curtailing other uses, if necessary, at almost any desired points. This would be of extreme value in case of a war, since large amounts of power could be obtained at specific points and at very short notice, this power being drawn from any available supply at any point in the network. This would be useful in such work as the making of nitrates, poison gases or other special chemicals that might be required. During the late war the lack of any such quickly available large blocks of power greatly impeded the quick development of our war activities. In time of peace, the same thing could be accomplished, for example, by using secondary power, by power from the large Tennessee or Alabama storages, by operating steam plants or by adding additional generating capacity in small amounts wherever advantageous opportunities offer, such as completing plants in which provision has been made for future extension. 200,000 kw. of concentrated secondary power at $10.00 a year would be worth $2,000,000 a year, which would pay interest on the full cost of the network and step-down transformers and breakers.

It may be that a much larger amount of secondary power might be used.

8. An examination of the map will show that the flow of power can be in almost any direction in the district over favorable routes. The power from eastern Tennessee may go to the neighborhood of Charlotte, Virginia, Birmingham, or Atlanta. Muscle Shoals may send power to Charlotte, Virginia, Atlanta or Birmingham and similarly with the steam plants. This facility of the network is of a fundamental importance as it is not necessary to predetermine before installing a new station where the block of new power is to be used, as is the case with the trunk line system. If the center of load shifts greatly, no material disadvantage results.

9. The unitary control and the ability to distribute large blocks of power in any direction over the network are of the greatest advantage. Where it is necessary to pass water over a dam when the local companies' load does not call for it, either at night or in the day time, the power of all such water can be absorbed in the larger system and the various reservoirs and pondages may be used to store water at other points. It may well be that new reservoirs could be added at favorable points for the specific purpose of saving energy in this manner. As far as the usefulness of the reservoir is concerned, it may be located at any point in the system without material disadvantage.

If it be determined to equalize the flow of the Tennessee River up to a certain point for purpose of navigation, this need cause no loss, since the power may be absorbed elsewhere as just stated.

10. With the trunk-line type of transmission, the steam plant must be at the receiving ends of the trunk lines, or if placed at favorable points for the generation

of steam, additional and often long transmission lines must be installed. This is a most serious factor.

11. The supply of energy to new districts, which may be a little remote from existing lines, is easily provided, for the voltage here chosen is low enough so that step-down transformers and switches will be of reasonable expense, even for capacities as small as a very few thousand kw. The cost of tap high-tension lines, if layed out with this in view, can be made very reasonable. The network is so complete that for the district covered and a fringe of 50 miles wide on each side, the high-tension supply can be economically brought to any point.

General Conclusion

A great deal of ground is covered in this paper and the importance and far reaching character of many of the considerations pointed out have not been adequately emphasized, on account of the limitations of space. It seems clear to the author that the interests of the power companies, the power users, and the community as a whole, require that the applicability of such a network as here proposed be carefully studied in all those districts where the industrial load has reached the proper development.

If important new lines are installed in such a manner as not to fit the plan of this type of network, a great future advantage may be lost. It would not be for the interest of the community as a whole, for development to start along lines which might sacrifice great future advantage, merely because it might be of a temporary advantage to some particular company.

The principal idea to be here emphasized is the importance of an early and fundamental study of the best ultimate method of interconnection, before inadvertantly a district is committed to some disadvantageous plan. If the study here made be a proper criterion, the advantage to be gained by the use of the best suited system, as distinguished from the mere enlargement of existing systems along conventional lines, is very great, affecting the cost of power, the reliability of operation and the conservation of our coal, in addition to the freedom with which power-using communities can be located at most favorable points without regard to the location of power stations.

Part III

Technical Basis of Network Performance

The discussion above has been based in part upon a definite performance of an assumed high-tension network, covering the Southeastern States. A statement will now be given of the basis on which this performance has been determined.

High-Tension Lines

The high-voltage circuits are single circuit steel tower lines. The line conductor for each phase is composed of two or three cables strung side by side and electrically in parallel. Either two No. 0000 or three No. 00 aluminum cables, steel reinforced, can be made, if properly spaced, to give substantially the actual electrical line constants assumed. The limits of the present paper will not permit a discussion of the mechanical characteristics of such a line, interesting and important as they are. This is the resistance equivalent of about a 400,000 cm. aluminum steel cable or 85 per cent of the resistance of No. 0000 copper. The outside diameter of the equivalent cross section of two No. 0000 aluminum steel in one cable would be about 0.83 inch and the maximum voltage permissible for this single cable would be about 190 kv. However, the dividing of the conductor will raise the corona limit so that it will be well over 200,000 volts. This statement of the raising of the corona limit by the dividing of the conductor has both a theoretical and experimental basis. It will not be further discussed here. The line spacing is taken as 15 ft. average. While this is less than is sometimes proposed, this value is considered ample. One of the factors justifying the lesser spacing is the relatively tight stringing appropriate to the steel cove lines.

The electrical line constants used are as follows:

R = 0.23 ohms per mile equivalent conductor, of two No. 0000 aluminum cables.

Reactance = 1.59 ohms per mile x $\frac{2}{3}$ = 1.06, this being for one mile of two double cables, single phase.

(Allowing for a reduction of $\frac{1}{3}$ from the divided conductor).

Charging current = 0.267 x 3/2 amps. per mile = 0.4 per 100,000 volts for two double cables, single phase.

(Allowing for an increase of 50 per cent for the divided conductor).

Neutral dead grounded at each station.

Assumed Loading

As a starting point, loads were more or less arbitrarily assigned to about 35 substations in the territory covered and aggregating approximately the sum of the present peak loads of the companies now serving the district to represent the existing loads;—this total comes to 632,000 kw. The list of stations and the load assigned to each is shown in Column 1 of the table. Capacities were then arbitrarily assigned to the principal *existing* generating stations, more or less representing their maximum outputs, in the aggregate appropriate for carrying the assumed loads. Capacity taken for the steam units is shown for the several stations in Column 2 and capacity for hydraulic stations in Column 3.

For the purpose of this study, it was then assumed that at the time supposed to be appropriate to the full use of the network, an additional or new load equal to the present would have developed at each substation, these new loads being shown in Column 4 in the table.

TABLE I—DATA FOR STATIONS IN THE NETWORK

					Case A		Case B			
	Normal maximum load—present	Rated steam power—present	Rated hydro power—present	Maximum new load assumed	New hydro assumed	Out of phase kv-a. at each power hse. + means leading in generator	Hydraulic storage taken equivalent to steam	New steam assumed	Deficiency of water assumed at dry time	High-Tension voltage; - maintained constant—in kv.
	1	2	3	4	5	6	7	8	9	10
STATIONS	Kw. and kv-a. values divided by 1000									
Carolina										
Badin	5			5						186.0
Bridgewater	5		25	5		− 5.8	10.0			195.0
Calhoun Falls					31.4	− 8.4			8.5	201.9
Catawba	15		230	15						194.2
Concord	7			7						187.8
Dearborn—Great Falls						−64.0				199.5
Easley	5			5						196.5
Eno	8	31		8						181.5
Gastonia	15			15						191.5
Gaston Shoals	15		22	15						191.8
Greensboro	10	8		10						184.4
Greenville	20	8		20						195.0
Greenwood	15			15						200.6
Lookout Shoals	15		21	15						190.3
Marshall	4			4						197.7
Method	40	3		40						180.0
Mt. Holley	45	45	70	45		−17.5				190.9
Newberry	10			10						200.0
Salisbury	10			10						188.4
Spartanburg	15			15						192.5
Toxaway					13.2	− 3.8			4.0	196.5
Virginia										
Glenlyn	81	60		81				90		188.0
Grayson-Byllesby			40							190.3
Alabama										
Anniston	24.4			24.4						186.5
Birmingham	33.6			33.6		+ 2.5				180.5
Cherokee Bluffs					143.6	+29.3			40.0	199.0
Huntsville	10.8			10.8						193.3
Jackson Shoals	5.0			5.0						184.2
Magella	34.6			34.6		+ 2.5				180.0
Mitchell Dam—Lock No. 18					63.0	+20.0			33	196.7
Muscle Shoals	3.0			3.0	243.8	+45.1			125	194.5
Sylacauga	10.1			10.0						190.7
Warrior	14.1	100		14.0						182.7
Georgia										
Atlanta	50.0	10	30	50.0		+32.0			12.5	193.7
Barletts Ferry	7.7	8.0		7.7	27.4	−16.9			15.0	198.5
Coosawattee					44.0	−27.3				200.0
Gainesville	1.8			1.8						200.0
Lindale	10.5			10.5						194.0
Newnans	7.0		150	7.0		−69.9	15.0			195.4
Tallulah Falls										203.2
Winder										197.7
Tennessee										
Cleveland					9.0	+ 3.0	23			199.3
Dam No. 1					16.0	+ 8.0				200.0
Dam No. 2					53.6	− 2.8		90		199.6
Dam No. 3					12.0	+ 5.0				200.5
Dam No. 5					12.0	+ 7.5				199.2
Dam No. 6					20.5	+16.0				200.3
Dam No. 13					14.3	− 9.1	5.0			197.5
Dam No. 21	40	9		40	8.0	− 5.0				198.0
Dam No. 22							5.0			202.1
Hales Bar	40	45	100	40						194.5
	632.5	339	854	632.5	711.8		58	180	238	

$C^2 R$ Line loss 711.800 − 632.500 = 79.300 Kw.
As per cent of power generated 12.5 per cent
As per cent of power delivered 11.1 per cent

To supply the new load and transmission-line losses, it was assumed that new hydro stations would have been developed in the most favorable locations, these being shown in Column 5. The principal new plants are at or near Cherokee Bluffs on the Tallapoosa River and the big storage powers of the Upper Tennessee, together with Muscle Shoals and also Coosawattee. The general condition with the present power, carried by the present units in the present manner, together with the new 632,000 kw. carried by the new hydro-electric plants, but all connected with the same system, is called Case A.

As an alternative, to show a typical low water con-

dition, with 238,000 kw. of the new load carried by steam plants, the conditions of Case B are set up, the locations and loads of the steam plants being shown in Column 8. These conditions are the same as Case A, except that the water power in certain of the new stations is assumed to be short in the aggregate amount of 238,000 kw., corresponding to the amount of steam power generated. The actual locations and amounts of water shortage assumed are shown in Column 9 in the table.

It is assumed that the simultaneous water shortage in the *present* plants is cared for at the same time by the *present* steam plants. It is believed that Case B here assumed is more exacting on the network than the situation that would occur in actual service.

It is assumed that a certain amount of hydraulic power, taken above the normal high water requirements, will be drawn from two or three of the best seasonal storages, in lieu of an equal amount of steam power and this storage-water power is shown in Column 7 of the table.

New 100,000 kw. steam power plants are located at Glenlyn near the center of the Appalachian Power Company territory and near Knoxville in Tennessee.

The points at which the equivalent of steam power is taken from the storage are the following:

Bridge water	10,000 kw.	N. C.
Tallulah Falls	15,000	Ga.
Dam No. 1	23,000	Tenn.
Dam No. 21	5,000	Tenn.
Hales Bar	5,000	Tenn.

It is assumed that a 100,000 kw. steam plant is located near Dam No. 3 on the Clinch River.

Voltage Control

It has been further assumed that the high-tension voltage at all regulating points; that is, all points where synchronous apparatus of important capacity and under control of the system operator is located, will be points of fixed voltage, regardless of load conditions and for dry or wet seasons, but that the particular voltage at any particular point may be chosen at any suitable value within certain limits, here between 180,000 and 200,000 volts, as may best fit the power-factor conditions in the line. While these limits were set up for the layout of this case, it was found as the calculations progressed that the voltage slightly exceeded the upper limit at certain points, but it was not considered worth while to recalculate the system to bring down these high voltages within the limit, as might readily have been done by slightly changing the power-factor assumptions at certain regulating stations.

The control of voltage at load points between the regulating stations may be handled in two ways. First, it may be assumed that the variations in voltage at these intermediate-load points, which will be probably on the average a fraction of one per cent and only rarely and at a very few points over 1 per cent, are negligible and no provision made to correct for the variation. Second, an automatic variable ratio device may be used at each station, such as an induction regulator. With the small variations of the voltage to be expected, only a very small capacity regulator will be required, well within the limits of present regulators. As in favor of the use of automatic regulation at all points, it should be noted that while the high-tension voltage will vary very little, the drop in the step-down transformer which carries a lagging load may be several per cent.

The tendency of variations in load, or in the distribution of load, to change the voltage at various points is corrected by changing the power factor at which power passes from one regulating station to another. For example, if the voltage at station A is 190 kv. and at B 187 kv. with a load of 50,000 kw. passing at 99 per cent power factor, the same drop of 3000 volts will exist when only 25,000 kw. is passing, if the power factor be right, in this case say 80 per cent. To control this power factor, there must be an appropriate out-of-phase current flow from synchronous apparatus in A to that in B as is well known. This function will be automatically attended to by voltage regulators.

In the table the out-of-phase kv-a. required of each regulating station for Case A is shown in Column 6. It will be found that no machine is called on to operate when well loaded at any power factor less than 0.85 lagging. Leading-power factors are all much higher.

Power Factor of System

For the purpose of making a numerical calculation certain further assumptions were made.

1st. That the *new* load taken by the *new* network is at 80 per cent power factor, high tension.

2nd. That a lagging load will be taken at the load points, in addition to the above, sufficient to raise the power factor of the *present* load fed by the present distributing lines to 95 per cent.

As a matter of fact the charging current to the 190 kv. line will in the aggregate nearly neutralize the total lagging kv-a. of the load and transformers, but as the distribution of the lagging kv-a. and the leading kv-a. is different, the neutralization is far from complete.

Exception to the above assumptions was made in the case of Method and Glenlyn Stations where the lagging kv-a., taken by the high tension, was slightly less than the amount above specified.

Calculations of Performance

On the basis of the above assumptions, the actual voltage, current, kw. and kv-a. values were calculated for the whole system for Case A. The formula used for this purpose was the so-called "Split Capacity" formula, described by the author in a paper presented before the Institute on June 29th, 1909, entitled "Calculations of the High-Tension Line." Any suitable formula, taking into account the capacity of the

line, may be used. Of course, it was necessary to obtain an approximately correct distribution of currents between the various parallels of the network. For the purpose of clearness and convenience the following statements are made.

In using the split capacity formula, any out-of-phase kv-a. at the starting point of the calculation (normally the receiving end of the line) of such a direction as to cause a slope of potential going-up from the starting point is to be marked + and vice versa. Also any energy delivered to the starting point of the calculation is to be marked + and vice versa. By observing this rule and remembering that the quantities $C^2 R$ and $C^2 p L$ are always positive and added to the appropriate quantities at the starting point to find the totals at the finish point, a line may be calculated from either end with identical results.

When a synchronous machine is connected to a transmission line, containing a material amount of capacity, we have the following:

a. Current, flowing from the machine (whether a motor or a generator), to the line and tending to cause a slope of potential upward from the point of connection of the machine, is such as to strengthen the field of the machine and may be called "field boosting" current similarly with kv-a., and

b. Current or kv-a. flowing from the machine into the line tending to cause a slope down in the line away from the machine, tends to weaken the field and may be called "field bucking" current or kv-a. This current is called leading if the machine is a generator, but must be called lagging if the machine is a motor or synchronous condenser, because the positive direction of current flow in these motors and condensers is taken as *into* the machine, not *from* the machine, as with generators.

If another synchronous machine is connected to the other end of the line, then:—

c. Cutting resistance out of the field-exciting circuit raises the potential of the machine, but reduces the amount of out-of-phase "field boosting" kv-a. flowing to the line, throwing it into the other machine, thus tending to bring the power factor of the first machine nearer unity and vice versa. If, however, the field has already been raised so high that "bucking kv-a." is flowing to the line, this field-bucking current will be increased by further raising of the field, and vice versa.

It would not be feasible to secure scientific accuracy in such a calculation as this with a reasonable amount of labor, but the voltages here given are believed to be correct within less than 1 per cent, except for two or three points where numerical mistakes have been found too late for correction. There errors are, however, not sufficiently important to materially effect any of the station quantities. The power actually flowing in the several lines is indicated in the transmission lines on the face of the map and are marked "*A*".

This calculation permits a determination of the total $C^2 R$ losses in the new network for this maximum load condition. The losses exclusive of corona or transformers are 79,300 kw. or about 12 ½ per cent of the delivered power and 11.1 per cent of the generated power for the maximum condition. The average year around losses would be probably about 35,000 kw. In this calculation no allowance is made for the saving in line losses in cases like the line from Atlanta to Tallulah Falls, where the current will tend to flow one way over a given route in the network and is now the other way in the present parallel distribution line. In such a case, of course, the oppositely flowing currents cancel, leaving only the difference flowing partly in one line and partly in the other. This will affect a large local saving in line loss on both lines.

Case B—The Station voltages and power factors of the system as a whole were not calculated in detail for Case B on account of the amount of labor involved. To give an approximate idea of the effect of the change in the location of the sources of the power on the energy flow in various lines, a determination was made of the change in amount of *power flow* for each line—neglecting $C^2 R$ line losses, however. The resultant energy flow values are shown on the map in the appropriate transmission lines, marked with the prefix "*B*". Since line $C^2 R$ losses are neglected in the *B* values, these amounts will not be as accurate as the *A* figures.

An examination of the values on the map will show that in many lines as compared with case *A* the current is reduced in Case *B*—in some reversed and in a few increased. The net result will be to slightly reduce the $C^2 R$ losses.

The effect of these changes in current flow on the *voltages* at the various stations should be further discussed.

Consider, for example, the lines between the new steam plant of the Appalachian Power Company, assumed to be installed at Glenlyn for new reserve power and Mt. Holley on the Southern Power Company's system, both regulating stations. In Case *A*, 81,000 kw. is delivery to Glenlyn—in Case *B*—9,000 kw. is sent south from Glenlyn. Considering the branch line from Glenlyn to Grayson, Lookout Shoals, Salisbury, Concord and Mt. Holley, the effect of starting up the steam plant is to superimpose upon the Case *A* flow of 40,500 kw. north, a 45,000-kw. flow south, giving a net flow of 4500 kw. south. This 45,000-kw. counter flow varies a little in the several sections of the branch line on account of the side circuits, as will be seen from the map, but this 45,000 will be about right to represent the average flow. This will tend to reduce the average current and the slope of voltage appropriate to Case A, which is from Mt. Holley down toward Glenlyn.

Suppose now that this 45,000 kw. energy current be sent at 80 per cent power factor, so that the out-of-phase component tends to increase the slope of the potential from Mt. Holley toward Glenlyn. This will then tend to balance the change in the potential slope

caused by the 45,000 kw. load counter current and leave the voltage slope unchanged.

However, since in this case some of the line sections carry cross currents toward the Tennessee powers, the effect of these various corrective currents will be different in different line sections and while the terminal voltages may be unchanged, the intermediate voltages will be affected, as is shown by the following table, in which the second column shows the voltages appropriate to the special assumptions under Case B just described.

	Case A	Case B Special
Glenlyn	188.0 kw.	188.0
Grayson	190.3	190.6
Lookout Shoals	190.3	194.3
Salisbury	188.4	186.7
Concord	187.8	187.7
Mt. Holley	190.9	190.9

Here it is seen that the maximum departure from Case A voltage is at Lookout Shoals. If, however, an out-of-phase kv-a. of 12,500 lagging is taken locally at Look-out Shoals this will drop its voltage to approximately the Case A voltage without disturbing the Mt. Holley or Glenlyn voltages. This will, however, lower the voltage at Salisbury until it is much below the case A voltage. But by taking an out-of-phase current of the proper amount locally at Greensboro, the voltage at Salisbury can be raised without affecting the voltage at Lookout Shoals or Mt. Holley. In this case, the out-of-phase current from Greensboro must go in proper proportions to Lookout Shoals and Mt. Holley and then be absorbed by the synchronous apparatus at these stations. This will correct all the voltages and be close enough for all practicable purposes. A similar analysis may be made for the other branch line via Bridgewater or for other lines, such for example, as the line from Dam No. 3 to Cleveland and Anniston where a heavy reversal of flow occurs over a long distance. This line can be more easily controlled than the line just discussed, because there is less cross current flow and the generating stations at the ends are of larger capacity and will handle larger out-of-phase currents.

While the numerical verification has not been made for Case *B* as a whole, on this particular point, there is no doubt that by properly controlling the power factors at the control stations, all the various station voltages can be kept constant within 1 per cent or less, without calling for any generator to carry full load at less than 85 per cent power factor lagging. In this case it should be remembered that when the steam plants are operating, there will be a largely increase electrical capacity available, and that an underloaded generator can carry a much lower-power-factor current than a fully loaded one.

In general, the use of synchronous condensers would greatly broaden the range of control of voltage and they may be necessary in some networks. In the present case, however, it is apparently possible to operate with constant voltages and reasonable power factors without condensers.

Again, in a network there is always a tendency for the currents to so distribute themselves as to have the least possible loss of voltage.

These operations here described are automatic if the voltages are kept constant at the regulating points by adjusting the field strengths of the synchronous apparatus and require no intelligent intervention.

The change in conditions here assumed between Case A and Case B is the extreme and occurs only once a year and comes on gradually, giving opportunity for adjustment by changing taps, if necessary, as the change develops.

The effect of light load will be less difficult to handle than Case B for their out-of-phase currents are more effective, ampere for ampere, in correcting voltage slopes and there is ample machine capacity to handle them.

A high-power-factor load, having the characteristics of decreasing to a lower-power-factor load, as with induction motors, when off the peak, tends to hold the potential slope along the line relatively constant.

Relation of Network and Distributing Lines

Since the plan here proposed throws the generators, distributing lines and the low-tension network in parallel with the high-tension network between stations common to both networks, the method of control of the division of current between the high tension network and the distribution lines, *e. g.*, the present 100-kv. lines, should receive some consideration.

Assume that a present power house is located at *P*, Fig. 1, feeding loads at *M* and *N*. If the line voltage be taken as 110 kv. and it be assumed that a load of 30,000 kw., 80 per cent power factor, is carried at *N* and an additional load of 20,000 kw., 80 per cent power factor, be carried at *M*, a voltage of 117 kv. will be required to maintain 100 kv. at the load. If, now, a connection be made at *N* to a constant potential network, the 30,000 kw. load is taken from the transmission line by the network, relieving the distribution line, provided suitable voltage adjustments are made as will be later explained. Furthermore, since the point *N* is now a source of energy, the load at *M* will be carried jointly by the network and the generator at *P*. Suppose, now, that we increase the load at *M* until the voltage at *M* is the same as that originally existing at *N*. This will require a load of approximately 160,000 kw. at *M* which, together with the original 30,000 at *N*, makes 190,000 kw., not including a possible increase at *N* above 30,000, showing an enormous increase in the effectiveness of the 110 kv. line for distribution purposes, by connection to the network at the point *N* instead of at *P*, as is ordinarily proposed. This extreme case is taken to bring out the principle

involved, that is, the great advantage in delivering new power at the load ends of the distributing lines.

As already stated, in the layout here proposed, the present distributing lines are frequently in parallel with the high-tension network for certain sections. It is proposed to control the power factor of the current passing over the distribution lines. In the case where the distribution line is connected to the network only at the other end than the generator end, the true power passing over the line can also be controlled by adjusting the governor of the generator prime mover. If there are several distributing lines thus connected to the same generator and to the network, it will be possible to control only the total power sent to the network, but the division of this power over the several distribution lines will be determined by the electrical constants of the lines and the network and the power-factor adjustments at the various points. The distribution of the true power and the power factors at various points cannot be separately controlled in the system proposed without special devices, when the generator is connected to the high-tension network as well as the load stations.

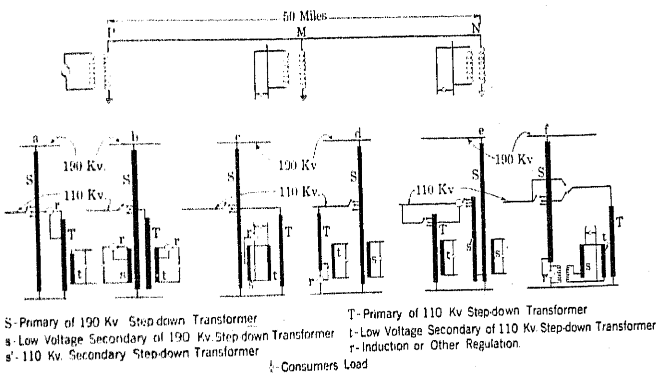


Fig. 1

Returning to the control of the power factor in the distribution lines, it will be noted that if a loaded distribution line be connected at its terminal to the step-down network transformer on a voltage tap giving exactly the same voltage as the line terminal, no current will flow to or from the network. Suppose now that the line has a change of load or change of power factor so that its terminal voltage is different. If the line now be connected to a second voltage tap in the step-down transformer, corresponding to the 2nd terminal voltage, no current will flow. If, however, the line carrying the second load be connected to the first tap, the potential of the line terminal will be changed and current will flow to or from the network until the line load is the same as in the first case; for the line, with the generator voltage fixed, can have this definite terminal voltage only with this particular line condition.

The above explanation ignores the effect of differences in power factor between the line terminal and the tap. It is the interplay of the reactance and resistance of the 110 kv. line with the tap voltage of the step-down transformer that really determines the division of true power between the 110 kv. and the 190 kv. lines. Similarly, whenever the line terminal is connected to a different voltage tap in the step-down transformer, current will flow into or out of the network until the current in the distribution line is of such a power factor, when carrying the appropriate amount of true power, that the terminal voltage of the line naturally is that of the tap, and so in general. Since the line must carry an appropriate amount of load the power factor of this load at the terminal point will be determined by the particular voltage point at which it is connected to the network. Once this power factor is determined at the terminal, the power factor will also be determined at the other points in the distribution line. When load is connected to intermediate points in the distribution line, the power factor that is controlled by the tap voltage is the mean or equivalent power factor. If, as will theoretically always be the case, the voltage of the step down transformer, fed by the network, is changed by the flow of current into the line where it is connected, the power factor of the current in the distribution line will be that corresponding to the terminal voltage at which the system finally settles down.

To put it another way, since the object of controlling the power factor of the 110 kv. is to improve the voltage regulation of this line, it is merely necessary to connect the line terminal to the proper tap and its voltage is fixed and the power factor will take care of itself.

Clearly, changes of low-tension voltage, due to changes of load and load power factor at the line terminal point, can be compensated for by changing the taps on the high-voltage winding of the low-voltage step-down transformer; so can changes be made in the voltage of the network—or correction of voltage may be made on the low-tension side of the step-down transformers or individually on important feeders.

If we assume that the network voltage is constant and that the distribution line terminal is connected to a particular tap in the step-down transformer, the voltage of the line terminal is fixed and hence the load at that point will be determined by the power factor of the line current and vice versa and the power factor of the current flowing into the network will vary with changes of load.

With the 110-kv. line terminal connected to some definite tap in the step-down transformer, if the network voltage does not vary greatly, nor the range of power factor become too wide with the actual load changes found in service, a very simple and practicable operating arrangement results, in which the *average* power factor can be controlled by choosing once for all a suitable tap on the step-down transformer. Means should be provided to then adjust the low tension bus-bar potential to the value desired when the tap has been chosen to secure the proper power factor.

The transformer ratio, represented by the tap under discussion, has to do with the *relative* potential of the

generator and that of the step-down transformer tap. Thus the adjustment of voltage may be made, where other conditions permit, either at the generator end of the line, in the step-up transformers or on the generator bus, or at any other point in the line.

If, on the other hand, the network voltage varies too widely or the range of power factor with a fixed voltage tap is too great, a more sensitive and very flexible system may be secured by providing means for automatically changing the tap during operation in response to a suitable regulator, set to maintain constant potential or constant power factor, as may be desired, for the action of the regulator is here equivalent to a change in tap.

With the tap changing device, which might well be an induction regulator, connected between the distribution line terminal and the step-down transformer tap, automatic control from a voltmeter would maintain constant voltage on the line terminal or control from a power-factor meter would maintain constant power factor. In either case, the quantity not controlled by the regulator could be controlled *on the average* by properly selecting the power factor or the voltage which the regulator should maintain constant.

Both power factor and voltage could be simultaneously controlled by having two independently controlled regulators.

In all these cases where there is a change in taps to control both power factor and voltage, means should be provided for correcting the effect of a change in the taps on the low tension service busbar.

A number of connections may be used for this control, for example, diagrams *a*, *b*, *c*, *d*, *e*, and *f* in Fig. 1, which are largely self-explanatory. In all cases the automatic regulator can be controlled to give constant voltage, preferably on the low-tension bus to which local feeders are connected. In case *e* the regulation can be controlled by the power factor at any suitable place to give constant power factor on the 110 kv. line. Where automatic control is provided for one factor, manual adjustment should be provided for the other.

In all cases except *e*, Fig. 1, an auto transformer is used for stepping down from the 190 kv. to the 110 kv. and an auto connection might just as well be used here. Regulators are just as effective in the low tension circuits where they are more readily installed, except that for automatic-power-factor adjustment there must be a change in ratio between the 190 kv. and the 110 kv. lines.

In cases *d* and *f*, the regulator is on the ground connection of the high tension windings, thus relieving insulation strains. In case *f*, a small balancing transformer is used with its high-tension winding connected around the regulator and its low-tension winding in series with the low-tension winding of the 190-kv. transformer to equalize its voltage with that of the low-tension winding of the 110 kv. step-down transformer. A special winding on the regulator or a tap on the regulator winding would accomplish the same thing.

While all windings have been shown star, delta windings may be used in many cases. In cases *d*, *e* and *f* the 110 kv. transformer can operate without the 190 kv. transformer. Case *f* is the most flexible.

As a matter of convenience the variations in power factor, required for various loads in a typical 50 mile 110 kv. transmission with No. 0000 conductor, are shown in Fig. 2, in which lines of constant power factor are shown connecting terminal voltage with load. It is assumed that the line voltage at the generator is maintained constant at 110 kv.

From these curves it can be seen how much the 110 kv. line voltage at the terminal must vary with the power factor fixed as the load changes; or by following the horizontal line corresponding to any voltage, it may be seen how much the power factor will change for any range of load. This is rather an extreme case and no doubt an actual installation would show much less variation. The case where load is carried at intermediate points is more complex but will involve no serious practical difficulties.

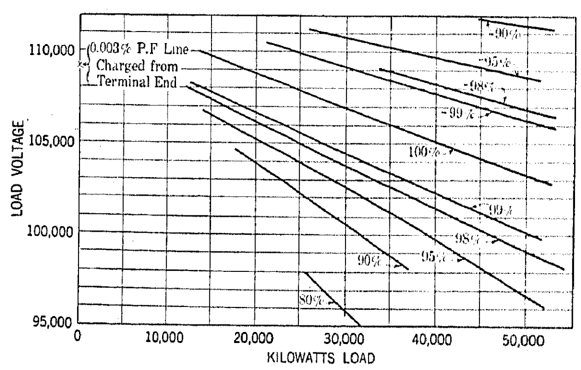


Fig. 2

Many other matters of interest connected with this use of regulators and method of voltage and power-factor control might be touched upon, but the most essential factors have been discussed and space will not permit more.

Part IV

Operating Features

The operating requirements of this system as they involve new or difficult features may be considered as follows:

Control

It is obvious that from a technical point of view and as far as day to day operating is concerned, the system must be under the absolute control of one dispatcher, at least as far as generators connected, field charge and governor adjustments, load carried, operation of all high tension connections, etc., are concerned. The maximum use of facilities further demand that control of water in rivers and reservoirs, the taking on of load

and dropping of load where necessary should be under single control.

Power Distribution

The distribution of power among the several generating stations is a fundamentally important one, for not only must the frequency be rigidly maintained but the dispatcher must be able to say how much load should be taken by particular machines so that water may be properly conserved.

Theoretically, the best results will be obtained from the point of view of the maintenance of frequency, by having a single station do all the regulating and setting all the others on a fixed gate and it may be that, as a matter of fact, one station may be able to regulate the frequency in this system. This will result, if the changes in load on the system as a whole occur so slowly that the load on the fixed gate machines can be adjusted rapidly enough to keep the proper margin for regulation at the regulating station. There are perhaps three or four stations or local groups of stations that could supply a regulating margin of 5 per cent which might be sufficient. This would mean, however, a large amount of idle apparatus at one point which would be undesirable, especially where there is no pondage to save water.

Furthermore, since governors of individual units must in any case be arranged so as to perform in consonance with others in a similar manner on load variations, they may about as well be in different stations. From these and other considerations, it is probable that the best plan is to have a single regulating station in each of the major systems, all with their governors carefully designed to be adjustable as to sensitiveness to changes (that is quickness of action); as to range of load change with change of speed; and as to normal speed. They may then be so adjusted that under operating conditions they will operate as nearly alike as possible. The setting of governors can be adjusted from time to time during operation, so that very close frequency regulation can be obtained. These regulating points should preferably be at the large storage reservoirs, so as to conserve water and to have water always available for regulating purposes. It is not likely that there will be any serious trouble on the distribution of load.

In this, account must be taken of the penstocks and water columns feeding the regulating generators but fortunately in this district there will probably be available in each system generators supplied directly from reservoirs and dams with very short water columns. It will do no harm to have a certain amount of variation in the sensitiveness of quickness of action of the governors, since the momentary overload capacity of the quickest acting will hold the frequency until the others catch up.

Instead of setting the machines which are not expected to govern on a fixed gate, their governors may be adjusted to have a wide range of speed with small changes in load. Then by changing the "normal" speed setting of the governor at the switchboard from time to time as conditions change, the load may be maintained at the prescribed value substantially as though with a fixed cut-off.

By this arrangement the following advantages are obtained—

a—Frequency will be maintained by a few specially sensitive governors, operating in stations selected as the most favorable.

b—Should anything happen to break apart the network, each sectional system would have its own system-regulating station and furthermore other local machines will come to the rescue of the regulating station on account of their wide speed range governors, above described, though with a slight drop in frequency, if the disturbance is very serious. In the case of a plant "importing" a large amount of power, it would be necessary to drop some load, if disconnected from the network at all points.

c—Regulation would be obtained without disturbing the economical use of stored water, for the *average* rate at which water is passed through the regulating station can be controlled at will by the load settings of the fixed-gate machines.

d—No very heavy blocks of power would have to be passed backward and forward over the network, as with change in load with regulation concentrated at one point system.

Favorable regulating points would seem to be
Bridgewater (with generating capacity enlarged)
Tallulah Falls,
Cherokee Bluffs (with the large storage developed)
Dam No. 3 including the large storage.

Voltage Regulation

A plan for making every station on the network a voltage stabilizing point is covered in Part III above and need not be further discussed here.

Power Factor Control

In Part III above is described a plant for controlling the *average* power factor of any network station, although the power factor must be allowed to vary within certain limits to secure voltage control.

Reliability

Any general scheme of transmission and distribution must offer a high order of reliability of service. This proposed scheme is exceptionally favorable from this point of view. In the first place, a breakdown in any one transmission line link will affect only stations on this particular link, for no single link is essential to the network as a whole. The segregation of a single link is simplified because with the wide separation of power houses and the absence of large concentrated loads, no short circuit can occur in the network of a magnitude

to exceed the safe capacity of oil breakers. The single-circuit lines contribute to this result.

In the second place, since it is usually considered that with two circuits feeding a station, either circuit being able to carry the whole load, a reasonable degree of reliability is secured. Even if these circuits are on one tower, the network plan is especially well guarded, since any station is fed from two or more directions so that no cause affecting one supply line would also affect the other. In addition, most substations are connected to other stations by the low-tension distribution lines giving still another source of emergency power. With the trunk-line type of circuit a break in the line will affect a large amount of power and a number of stations.

Protective Relay System

At first thought the securing of a satisfactory relay layout for protection against disturbances may seem difficult, but this is not likely to be the case.

When once a satisfactory means is found to properly disconnect a bad circuit at one station, the same means may be used for most stations regardless of their number. Special stations, however, would require special treatment. Considering one of the long single lines of the proposed system, containing several intermediate stations, a line short circuit or ground will ordinarily cause a great fall in the potential at the point of the trouble and the potential of the line will slope up in both directions to nearly normal at points at some distance. Suppose now that inverse-time-limit-overload relays of the normal type be used, and that a device be added for rendering the devices inoperative for a certain period more or less proportional to the momentary line voltage. Then, upon the occurence of a ground the inverse-time-limit relays will all start to run, but the one nearest the ground will be rendered operative first on account of its lower line voltage. If it be assumed that the time of action of the retarding device be 0.2 sec. for 30 per cent voltage at the relay and 2.0 sec. at 90 per cent voltage, a difference in perhaps 20 per cent in the momentary voltage between two consecutive stations would permit time enough to cause selective opening of circuits.

At points where the voltage is over 90 per cent, normal selective action comes only from the difference in the inverse-time-limit relays, but this is not important for a short circuit or ground in which the voltage drops only 10 per cent at the nearest station is very rare. It should be remembered that at each branch line and at each station there will be a dividing of the short circuit current so that there will be a marked difference in line current at points only one or two stations from the short circuit permitting selective action.

In addition to the above, a directional relay should be placed in each line at each station so that when power is flowing in *excess* away *from* the station in one line, it will open the tripping circuits of the breakers in the other lines. This will prevent any good line being cut off on the wrong side of any station.

When a high tension short circuit occurs at a station inside the breakers, the last mentioned power relay will not operate and all the breakers will open quickly on account of the low line of voltage.

When a low tension short circuit occurs the reactance of the transformer will prevent its pulling very heavily on the high tension so that any desired local form of low tension protection may be used, except that inverse time overload relays should be used, set to act quickly to clear a heavy short circuit such as would occur only at or near the buses.

The above scheme of relay protection is suggested as one feasible method. It is, of course, to be understood that the best scheme for this network should be the subject of long and careful study and many alternative schemes can no doubt be devised.

Metering

The metering of power should not be very difficult or expensive, although it involves a number of problems of some interest.

Power should preferably be measured at the step-down side of the network transformers both for load power and for generator power;—also the maximum demand kw. and reactive kv-a., on graphic meters, set with accurate timing. By summing up these kw. and reactive kv-a. for all the stations in a particular system, taking account of positive and negative values, the total net *power* supplied the system from the network will be obtained and its equivalent power factor.

This plan means that in cases where the network carries a certain amount of power from a local generator to a local load, formerly carried by the local distribution circuits and in the same direction as the main flow of through power in the network, the network is charged for the increase in the $C^2 R$ line losses, while with the through power flowing in the opposite direction to the local flow, the network gains the decrease in $C^2 R$ line losses.

If an accounting of this variation in line loss is desirable, in place of taking the numerical sum of all the kw. in or out at the several stations, the values of the several kw. *loads* may be separately combined and the total value of the kw. generated obtained and a slightly higher rate per kw-hr. set on the load kw. than on the generated kw.

In addition, there should be taken the readings of kw. and reactive kv-a. in the high tension line at the incoming side of those stations located nearest the boundary line of the district on all through lines passing from one local system to the next. The best readings that can be obtained from bushing-type transformers, two to each phase of each breaker, (taking advantage of calibration curves obtained in place, if necessary) should be sufficient. These readings will not be usually as good for determining the charges for power to any system as the

readings recommended above, since where there is much through power, the power supplied to the local company, as measured on incoming and outgoing lines, will be a relatively small difference of two large quantities and hence relatively inaccurately measured.

Part V

Costs

Only the briefest estimates of costs can be here made, but some idea of the magnitude of the expenditure involved is important. The elements of most interest in this study are the transmission network, the step-down transformers, the oil breakers for substations and other high-tension gear. These costs may be approximated as follows:

2500 miles single circuit, 190 kv. transmission line @ $8000. Exclusive right of way...........	$20,000,000.
2/3 x 1,250,000 kv-a., 190 kv. auto transformers @ $3.00 in place..........................	2,500,000.
200 oil breakers 190 kv. with dead grounded neutral and miscellaneous switchgear, outdoor stations, etc. in place..............................	4,500,000.
500 miles additional 190-kv. line @ $8,000.........	4,000,000.
If straight transformers instead of auto transformers be used add........................	1,500,000.

All for the delivery at 110 kv. or low-tension feeder voltages of 632,000 new kw.

Discussion

Robert Treat: Those who have had occasion to try to calculate the division of current in parallel high-voltage and low-voltage lines, will have great admiration for Mr. Thomas' courage in undertaking the solution of a problem of the magnitude of this one. Those who have not been brought in contact with such computations, probably have very little conception of the difficulty of the task.

Some time ago, in our office, we were faced with a similar problem, except that it was very simple compared with the one which Mr. Thomas has undertaken to solve. A brief description of the problem and some of the important things we learned in working it out, will be of interest.

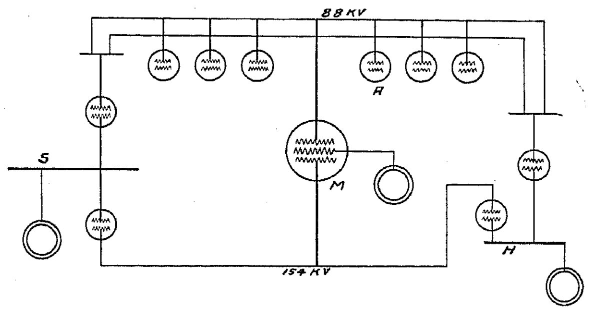
Referring to cut herewith a large source of power S, supplies a double-circuit 88,000-volt transmission line from which are distributed a number of variable loads. At the far end of the line, there is connected a hydro-station H, whose available capacity varies from full load to nothing. In parallel with the 88-kv. lines, is a 154-kv. transmission line running from H to S. There is one other connection between the 154 kv. and the 88-kv. systems at a point M near the middle of the transmission, which is made through a three-winding transformer bank.

It was required to determine under certain limiting conditions of load and with different amounts of power available from H, the variation in voltage on each of the substation buses. In order to calculate this voltage variation, it was first necessary to know the division of the power and wattless current in the system. The manner of procedure was as follows. An assumption was made that the load at a certain substation A came partly from M and partly from H. Calculations were then made step-by-step on the 88-kv. system from A to H; and then through the three-winding transformers, and the 154-kv. line to H, to see whether the voltages on the low-tension bus at H arrived at by the two calculations were the same.

It was soon apparent that mere equality of the magnitude of these voltages was not sufficient, that they must also be in phase, naturally, because they were the same bus voltage. So it was necessary to keep track of the phase angles of the voltages during each step of the computation, *i. e.*, we assumed a certain load on the lines adjacent to A toward H, and calculated the drop to the next substation. At this substation it was necessary not only to determine the amount and power factor of the current transmitted toward A, but also to know the phase angle of its bus voltage with reference to the voltage on the bus at A. This process had to be carried out through each step of the way to the low-voltage bus at H; then similar calculations were carried out through the other portion of the loop, A-M-H. Obviously, the criterion of the correct assumption as to the distribution of load and wattless in the two systems, was that the voltage on the low-tension bus at H when calculated over the two sides of the loop, came out the same both in magnitude and in phase.

It was also necessary to carry out the same process in the loop formed by Stations M and S, and sometimes, it was found that assumptions which appeared to give satisfactory results in one of the loops, were not right when we considered the other. This was because the two loops had one part in common, the three-winding transformers at M. Sometimes it was necessary to make the computations for one set of conditions several times, before arriving at an estimate of load division sufficiently close for the purpose.

One of the interesting things which we found out from these calculations was that the 88-kv. line from H was carrying about



Transmission Network

154,000 and 88,000 volt systems constituting two loops having one part in common.

twice as much load as the 154-kv. line. This was undoubtedly because of the higher reactance in the 154-kv. transformers at H and M as compared with the lower voltage single transformation in the bank at H. The losses in the 154-kv. line were low, it is true, but there was some question as to whether, with the very small load which it was carrying, its investment was justified; whether it also might not properly be an 88-kv. line, and, therefore, less expensive, and for the particular conditions which we were investigating, carry more power. In the example investigated, the justification for the higher voltage line is found in future conditions which operate to increase its loading.

It was also found that with no regulators or tap-changing equipment in the system, it was impossible to change the distribution of power current and wattless current over the two circuits of different voltage; they were fixed, once and for all, by the circuit constants. In order to get more wattless current to flow in either system, it was necessary to be able in some manner, to secure the effect of changing a transformer tap, thus changing the apparent voltages of the two systems which are tied together through the transformer, thereby shifting the wattless. A shift in the wattless also changes the angle of the voltage drop in the two circuits; thereby the actual load in the two circuits is also changed somewhat, but the two components, power and wattless, are both changed with only that means of adjustment, *i. e.*, by transformer taps there is no independent control of power and wattless.

In order to get more power current to flow in the 150-kv. line, it would be necessary in some way to introduce a phase shift in the voltage similar to the phase shift which is inherent in a three-phase regulator. That would shift the distribution of power current between the two circuits, which in turn, would change the voltage drops, thus affecting the distribution of wattless current to some extent. Here, again, it is seen that it is impossible to change the power current without, at the same time, changing the wattless current.

Complete control over the division of power current between the two circuits, and the independent control of the division of wattless current, demands some means of independently varying both the magnitude and the vector angles of the voltages applied to the two circuits; *i. e.*, it is necessary to secure independently, the effect of a phase shift, as by a three-phase regulator, and the effect of changing transformer taps.

This whole question, of course, has very many factors, and as Mr. Thomas suggests in his paper, it requires much further study. I agree with him that it should be fully analyzed from all possible angles, in order that as much as possible may be learned about the problem before attempts are made to carry out the program which he suggests.

W. S. Lee: Mr. Thomas deserves the appreciation of transmission engineers and the power companies at large for this study. It is impossible for Mr. Thomas or any other man to take a system or a group of transmission systems and lay down on a sheet of paper, transmission lines that will fit all the conditions. That can't be done. Unfortunately, load will come at some unexpected place; power plants may be developed at one point or another, and they, of course, must be coordinated in his system of transmission lines. His paper does give us a great deal of food for thought, in that we should continually study the possibilities of connecting all these companies together.

H. Cole: I think there are a number of important problems which have to be worked out in connection with the tying together of large transmission systems. It has seemed to me that some sort of tap-changing transformer or other device which would allow different voltages to be carried on either side of the point of interconnection would be very useful.

H. L. Wills: What is the use of considering these interconnected networks as individual propositions? The only reason we use a synchronous condenser or some method of that kind for holding down the voltage rise is because we haven't enough of that system. If we had that system so big that it was practically one plate, it wouldn't matter where we put our power into it; we would have the same thing as one plate of a condenser—and that is what you are working towards, according to my notion.

W. E. Mitchell: From an operating standpoint this problem is not quite as simple—while it works and we have been working it very satisfactory for the past two years—as it might appear to be. The problem of regulation, to push the power whichever way you want it to go, is not so difficult. We had a good deal of trouble in the beginning getting the governors in the different systems to a speed of control that was more or less similar for the load changes in each system, so that with a swing in load, no particular system would try to pick it all up. The result was, from that standpoint, that we were able to carry through from our system over to the east of us, and they were able to pass on farther to the east, loads of 30,000 to 35,000 kw. without any serious trouble. Strangely enough, our troubles all came at night-time when the loads became lighter and when we were trying to keep steam plants here loaded up and the hydraulic plants just floating in for regulation purposes. Under those conditions, we found that the swings were very much greater and that unless the governors had been carefully set and readjusted at night-time the swings of load instead of going 20,000 eastward, we would find were coming 10,000 westward when we didn't want it at all.

But that isn't as bad as the other problem, the problem of voltage regulation and the loading of the wattless current onto the one who is trying to give the power. You come in to help a gentleman out of his troubles, and he promptly doubles the burden on you and doesn't take anything away from you at the same time. Now that, we have found, is serious, and that is a question entirely of voltage.

That can't be controlled, so far as I can see, at the generating plants alone. It has to be controlled at the load centers, and, as far as we can see on our system today, that necessitates condensers.

We are studying our system; Mr. Lee is studying his system; Mr. Edgar is studying his system, looking forward ten years, and when we look forward ten years, we double our present loads. So there is nothing extravagant in Mr. Thomas' suggestion here. If anything, it is too conservative in the quantities considered.

H. S. Fitch: It is not at all simple to adjust plant frequencies so that the load may be distributed as desired. The first point is to adjust the settings of all governors repeatedly until there is obtained an easy fluctuation of load on the least efficient plant, and base load which is constant probably throughout the whole twenty-four hours in the day on the most efficient plant.

The next thing to solve is the wattless current. On our system, by means of penalizing the consumer, forcing him to pay a higher rate for poor power factor, we do obtain some help by having him install synchronous apparatus, but our problem has been met more generally by spotting synchronous condensers at the low-voltage points on the system. We have had to come to the point at the less efficient power stations of disconnecting the turbines from the generators, to take care of this wattless.

The third point is the relay problem. Any interconnected system on which there is a great amount of network must have a relay system which will function so that if one plant is shut down, the load is transferred from that section; in other words, if line trouble occurs between plants and they separate one from another, are the plants unequally loaded so that you are forced to transfer loads by dropping them? Or, again, if you are forced to shut down during the hours of the night or on Sunday, will a condition be created whereby the load must be interrupted again to transfer it to the plants which are running? These problems on relays are large ones and require a great deal of thought on the subject of interconnected networks.

F. M. Nash: I think I can give an illustration of what sometimes happens when you have a long line and try to get two hydro-electric generating stations on the opposite ends of the line to parallel successfully. The bigger station of 10,000 kw. in this particular instance, after a case of trouble, was allowed to charge the line. Then the smaller station at the extreme opposite end of the system was synchronized before any load was taken on anywhere. This was done two or three different times, and in each instance when the lower station had synchronized there began, very shortly afterwards a slow oscillation on the meters that gradually increased to such a point that the two stations would go out of step. It was an urgent condition and the only way we got the plants operating again in parallel was to get the large plant to charge the line, then pick up a large synchronous motor load near the center of the line, and then synchronize the smaller plant.

As the present-day tendency is in the direction of still longer lines and higher voltages, this particular incident is to my mind splendid evidence of the necessity of using synchronous motors or synchronous machines of some kind to hold the high-tension net works together and maintain satisfactory operating conditions in the generating stations.

Howard A. Stanley: There has been one point made which, I think, perhaps needs a little attention. On Page 12 of Mr. Thomas' paper, he says: "The segregation of a single link is simplified because with the wide separation of power houses and the absence of large concentrated loads, no short circuit can occur in the network of a magnitude to exceed the safe

capacity of oil breakers." That is about all there is, I think, in the paper on the question of short circuits.

I wonder if in this network we are not trying to accomplish two mutually conflicting objects. No matter how much reactance you may have, it is entirely possible to have a greater concentration of kv-a. in a short circuit a hundred miles from the generating station than at the station itself.

We have, in Fall River, a network which compared with what we are talking of here is something in the nature of a toy, 23,000 volts—serving approximately 30,000 kv-a. at the present time. It actually works out, in our network, that short circuits remote from the stations are more severe than those at the station.

W. A. Moore: I am speaking from the viewpoint of a manufacturer taking a large block of power.

You said that you would regulate the voltage at a substation by changing the fields possibly in the nearest generating station. That is a problem we are struggling with today on a smaller scale. We are trying to hold a certain voltage condition. I am wondering just how often you would have to change the fields, that is, would you change them at seven o'clock in the morning, at noon time and at night? Or do you find that for actual operation you would have to keep a man on the board, varying the fields every five or ten minutes just to compensate as people 'phone in and tell you that the load at a certain place has changed?

P. H. Thomas: Use an automatic voltage regulator at the generating station. The nearest power house would be the place to do it.

I would like to say here that the advantage that is gained by this particular scheme is that the total transmission, the total amount of power handled by the high tension is overwhelmingly large with regard to any particular station. You have a 220-kv. line 100 miles long between two stations. No local load you can take off the middle of that will affect that voltage so you will notice it. By having a big thing dominate the whole, the variation produced by the local users doesn't amount to anything. If the voltage of this system is regulated every 100 or 150 miles, that is all that is necessary. It wouldn't vary more than half a per cent. from any load you take off in between. The voltage control should be automatic. If the voltage is maintained constant by voltage regulators in the general stations, the regulation generator takes care of itself.

Mr. Moore: That leads to another question. Suppose this sub-station is big enough to have some regulating equipment. Then you would, at times of light load, lower the power factor purposely so as to get the voltage drop.

At the present time here in New England some of the power contracts call for a certain average power factor, and you get a bonus or get penalized, depending on that average. At the time of low load and you lower the power factor purposely, then where the wattless energy is read on a reactor meter, a large reading of wattless power is recorded, and this in turn penalizes you.

C. R. Oliver: It depends upon whether you are trying to maintain the voltage or improve your power factor. If you have a contract like that, let the power company regulate the voltage.

L. W. W. Morrow: The big question in regard to the scheme proposed by Mr. Thomas is "Will it increase service reliability and decrease the cost of installation?"

The long-line bulk-power transmission line has been installed in several instances and its problems have been solved very successfully by using synchronous condensers. But instead of this method Mr. Thomas makes a proposed equivalent to taking power from Niagara Falls and sending it through an interconnected network in multiple to the loads in the New York district. It is a question as to whether this proposal is a better economic and operating proposition than the heretofore used bulk power line having no intervening loads. Under abnormal conditions for example a short circuit on one of the light load taps of the network system might interrupt service to heavy load points and introduce more chances for service trouble than occur on the long line system having no intervening taps between the source and the load.

Then as regards the cost of installation it would seem that the use of single circuit multiple link lines would result in a cost comparable to that incurred in building bulk power double circuit lines because greater mileage is involved in the network system. And, although power factor and regulation are worked out satisfactorily on the network proposed by Mr. Thomas without using synchronous condensers it is a question as to whether this investment could be avoided in the general case.

Another element involves the question of voltage regulation under abnormal and normal conditions of load in a large territory. Mr. Thomas has several fixed but different values of voltage at several locations in the network which retain practically constant values for changing loads under the conditions he assumes. But experience with large networks indicates a continual shifting of loads on the system, the necessity to change the sources of power to conform to water, fuel, and load conditions and the necessity to handle troubled parts of the system quickly and easily. Under these conditions it would seem difficult if not impossible to retain constant voltage values at the different load points on the system. Apparatus are not yet developed to automatically give a wide range of voltage at a substation nor have operating facilities been developed to a degree which will permit a load dispatcher to control excitation, governing and transformer ratios at multiple points on a system in a very short time.

Then again experience indicates that the satisfactory operation of an interconnected system requires that each system in the network have the same standards of construction, service and insulation. This condition is seldom found and there would seem to be difficulties in instituting the type of system proposed in territories now occupied unless this condition was fulfilled.

These points are brought up in connection with this notable paper only as points apt to be raised by engineers more accustomed to the old methods and I am sure Mr. Thomas can answer them.

J. C. Damon: Mr. Thomas presents a pioneer system as regards voltage and extent, but some of the advantages of this network over a trunk line system have been practised.

The West Penn Power Company has a 22,000-volt distribution network which if mapped to an enlarged scale would be similar to that presented and the experience of this system largely bears out what Mr. Thomas expects in economy of lines to reach customers and the ability to feed in both directions to the customer. Relaying such a network has been very difficult but recent developments are solving the problem.

Another question discussed is being demonstrated right on the systems shown on Mr. Thomas' map. The ring of transmission lines shown on the southern portion of the map has been closed within the last few weeks by a new connection between the Alabama Power Company, and the Columbus Electric & Power Company, and the existing connections between the Columbus Electric & Power Company, Georgia Railway & Power Company and the Alabama Power Company. Before the closing connection was made, an investigation showed that the power stations around that ring could control the voltage, as well as the flow of power and wattless current between companies, within commercial limits without any new synchronous condensers. One assumption differed from those made by Mr. Thomas. That assumption is that, knowing beforehand when the light-load periods come, the station voltages can be lowered somewhat to avoid the necessity of drawing heavy lagging current to hold down the voltage over these lightload periods.

F. L. Hunt: The problem that Mr. Thomas has worked out, is, of course, one that we must all solve. We are facing it now, and we have attempted, to some extent, to solve it along

the line of economics. It seems to me the solution must be economically sound.

I noticed in one place Mr. Thomas suggested that the system of operation must be worked out only with regard to the whole, regardless of how it might effect one company. Such a plan would be difficult to work out in practise.

Mr. Thomas, in the case which he has assumed, has allowed 100 per cent growth on each sub-station, and, states that the network should be laid out with that assumption to take care of the future conditions. It seems to me that is a little optimistic. They don't always grow so evenly as that, and if you map out a big network on that basis and then the growth is entirely different, the result is not so good as pictured.

C. E. Skinner: Mr. Thomas' scheme presents one rather serious objection from the standpoint of the manufacturers of transformers. This is on account of the very large number of voltages varying by small steps from 190 kv. to over 200 kv. Either the transformers must be provided with a considerable number of taps or transformers in the different stations will not be interchangeable. Interchangeability of transformers is, of course, very desirable for such a system.

C. R. Oliver: Looking at the paper purely from an operating man's standpoint, it struck me as unique and radical in some parts, because of the results on our system; a small interconnected one in this paper. Before I take up the points brought out in the paper, I would like to mention one or two things that Mr. Thomas discussed outside of his paper this morning. One was that by running the generators on the system—it would be possible to eliminate any synchronous condenser apparatus.

Now, our experience here in New England has been probably the same as that of most of the other companies similarly connected. We have a peak load of 140,000 kv-a., and it has been necessary for us to install 47,000-kv-a. of synchronous condensers. That was after we had used all the correcting capacity of the generators, and we had a hard time convincing our financial people that we needed those condensers. And since we have had them running we have had an awful time to get one out of service for ordinary repairs, because the operating people are using them all the time.

I am wondering on this larger system, using the old stations and the old generators designed for probably only 80 per cent factor, how Mr. Thomas was arranging to keep the power factor constant regardless of the flow of current and regardless of the direction and the amount of current. It is something we haven't been able to do on our system in New England.

P. H. Thomas: The power factor wasn't kept constant.

C. R. Oliver: One other point Mr. Thomas mentioned was that the advantage of the loop over the double-line transmission was that the loop would give so much more capacity because of being fed in by both routes. It seems to me as though the loops must be designed of unlimited capacity. A case I had in mind that particularly illustrates the point I want to make is that on your map at Muscle Shoals, where there is 243,000 kv-a. capacity, with a line running east towards Huntsville and a line running south towards Birmingham, Alabama. Now, in case of trouble on the 65-mile line between Muscle Shoals and Huntsville, that single-circuit line would be called on to transmit 226,000 kv-a. 204 miles to get it back up around Chattanooga.

Now, the General Electric and Westinghouse Company, to date, haven't even been able to figure a double line able to carry that power. I am wondering how that would be taken care of unless that water is wasted and the load is supplied from some other station.

I am particularly interested in how Mr. Thomas is going to maintain these various voltages that he has laid out, all the way from 185,000 up to 195,000, with a generating voltage of 220,000. We have light-load conditions; Saturday afternoon conditions; Sunday conditions and night conditions, and I am very curious to find out how that voltage would be maintained unless we had an extensive amount of tap changes on the transformers. I don't see how the system can be worked unless we use synchronous condensers and hold the voltage flat.

The other point that our company is particularly interested in, is his suggestion of multiple conductors. We have been doing quite a lot of studying on the super power transmission line, and our engineers have not been able, to date, to solve this problem, which is the problem of suitably mounting and handling multiple conductors on transmission lines.

In your design of towers, you claim that a 15-foot spacing is ample between conductors. Now, the insulator manufacturers tell us that the length of insulators on 220,000 volts will be at least six feet. With a 15-foot spacing between wires we will have 7½ feet between the wire and steel. The conductor, under extreme wind conditions, will swing out to a 45-degree angle. The clearance then will be less than two feet between the 220,000-volt conductor and the steel. We felt that was too close.

With regard to the method of regulating the voltage all over this loop, particularly the transfer of power, in case of power shortage from the extreme southern portion to the extreme northern portion—I am anxious to find out how you are going to do it. Apparently, it is not going to be so simple, with all of the various power companies connected, to transfer power back and forth unless you have unlimited capacity of line and unlimited capacity of transformers.

The other point I am particularly interested in is regarding the question of costs. You have outlined 2500 miles of single-circuit line at a cost of \$8000 per mile, exclusive of right of way. A line which we finished a month ago, which is a super power line, cost us more than \$8000 a mile for material alone, regardless of any labor or right-of-way or anything else.

Your 200 oil circuit breakers, at \$4,500,000 would come to \$22,000 apiece. The latest quotation we had from the manufacturer, was \$30,000. And your figure not only included the oil circuit breakers, but miscellaneous switchgear, outdoor stations, etc. On a recent estimate we made, this came to \$75,000 per breaker.

Farley Osgood: Mr. Thomas has shown us, theoretically, what we would like to do. He has heard practical criticisms from the operating men which may alter the results as he shows them in his paper. I do not think this casts any discredit upon the paper, nor do I think we owe any the less to Mr. Thomas for having developed it in the way in which he has, because if he can point out to us the ideal, with our practical field operation we can lead ourselves into the avenues of finding out how the ideal scheme will have to be modified, so Mr. Thomas' work for us, as operators, is truly accomplished.

I think that I can appreciate his paper as much as anybody, because in New Jersey we are working on this same problem ourselves. Fortunately, it does not have to be decided tomorrow, but with this basic information which Mr. Thomas has prepared for us, we are able to save a great deal of time and to point out to ourselves the difficulties that we are likely to get into.

The practical criticisms which you have heard, I would like to concur in, because our experience matches up with the statements which have been made.

There has been some reference made to the voltage regulation and control, and we in our system get a good deal of help, particularly in power factor improvement, by using, whenever it is possible to do so, our idle generators as synchronous condensers. We have developed a fairly quick coupling arrangement, by which the generators can be disconnected from their turbines, and it is our practise to throw our generators on to the line as synchronous condensers whenever the steam end of the unit is to be taken out for repairs for any extended period, say even for longer than a day or two. We have been doing this for a number of years and, if I am not mistaken, we were the first group to establish this as a universal standard practise of operation.

In our northern network of several hundred thousand kilowatts, we have many prime-mover elements, the generator ends of which are used as synchronous condensers whenever they can be released for such service, and the improvement in the load carrying capacity of the system will vary from 25 per cent to 50 per cent of the kv-a. rating of any generator thrown on as a synchronous condenser.

There is no reason, where a generator is driven by a water wheel, particularly if it is a horizontal unit and easily uncoupled, why it could not be used in this manner. In a system as outlined by Mr. Thomas this same scheme could be applied, and all the free generators that would naturally be shut down on account of high cost of driving them, could come on the line as synchronous condensers, and the result to the network must be a very considerable benefit.

You men here in New England are studying your problems as they relate to power obtained from the St. Lawrence, and we in New Jersey are most interested in your study, as it is not beyond the possibility that we might participate in that same development should it be carried through the Metropolitan section, including New Jersey, and on through to Philadelphia.

In the matter of fear of coordination of operation, we must not forget that the people who are going to make these interconnections, while they are not all controlled by one holding outfit, at least, are neighbors, and neighbors are supposed to be friendly, and in the electrical fraternity I am glad to say that is very largely true. And if a network can be devised which will give a group lower costs, the fellows who are operating that group are going to go along with the combination and play the game in the best arrangement of load dispatching that may be worked out mutually.

No engineer ever went into a conference at which some fellow in the group did not say something that he did not believe or that he did not like, but generally the conference comes through with a unaminous opinion, and I have not the slightest doubt that that will be the practical outcome of the conferences which will bring about this network operation on an economical basis as soon as it can be set forth properly, physically, to warrant the money that it will cost.

The thing that I fear most is that, as engineers born full of enthusiasm and who seem to develop more each year, we might be too hasty in recommending to our financial people the accomplishment of this problem outlined. So—not in the way of any disparagement, but rather in the way of a note of warning—let us be sure in making our recommendations to our financial people, first, that we are sound and positive in the engineering accomplishment which we are going to get, and second, that we do not make the recommendation before it will pay.

Long transmission lines are all right, but they should not be built before they will pay their way. We have had many discussions on the cost of transmission lines versus cost of transportation of coal, and in most of the studies in our section of the country one hand about washes the other—in fact, it is mostly in favor of the transportation of the coal by freight.

If we have a large investment for a long period, not earning its way, we are going to get into discredit with the people who back us in these schemes. Our duty is to convince the financiers that they must furnish us money far enough ahead of the load to make us sure that we will be ready to take care of it when it comes. Let them figure the cost of carrying the financial burden meanwhile. We, on our part, will be prepared to show them accurately when the load is expected, and that when connected it will pay.

Otherwise, we shall get into ill repute and we shall have difficulty in getting money for the most excellent engineering enterprises which are in front of us, which are the most costly of any which have come before our electrical body thus far, and which are the most important to the industrial and operating men, and which should be gone into carefully, co-operatively, with our financial backers.

P. H. Thomas: Mr. Mitchell has asked why I have been able to secure stabilization of voltage without the use of synchronous condensers.

There were three reasons. The first and most important one is this: I have taken the liberty of establishing the voltage at each point at the value which will be most favorable for this purpose. I have not taken a level-voltage system. There has been a great deal of talk lately about the constant-voltage system, meaning the level-voltage; that is to say, a system in which the voltage is the same at all points. When you have established such a condition,you have committed yourself to a large and unnecessary expense, at least, in any case where the flow of power is predominantly in one direction. In the transfer of 100,000 kw. for 100 mi., there is a natural drop of voltage along that line for that power. If you establish the voltages at the two ends as normal which gives this natural slope of voltage, you don't need any synchronous condensers; but if you undertake to maintain the voltage at both ends at the same value, you must have synchronous condensers at one end, the receiving end. You notice in my set up I have freely adjusted the voltages to the flow of the load. That is the biggest factor in eliminating the necessity for synchronous condensers, and it is a very powerful one.

In the second place, there is no distinction between leading current which you get from a condenser and what you get from a generator. By taking lagging current on the generator stations, we reduce the natural voltage drop produced by power coming from the line. I have used the present generators up to 85 per cent lagging factor, where that was necessary to make a favorable power factor for having the power flow across the system. The conditions of this set-up come very close to requiring some synchronous condensers. A worse power factor load would call for some synchronous condensers, but their capacity would be very much less than you would expect.

As the third feature, I have worked in the new idea of "divided conductors". It isn't entirely new; I proposed the scheme either in 1911 or 1912, but it is new so far as most of you are concerned, I think. Here is a case where that idea is of extreme importance. Instead of using a conductor ⅞ in. in diameter, I use three conductors in parallel of the same total cross-sections, spaced 12 or 15 in. apart to increase the electro static capacity of the line and reduce its reactance. At the limit, I think, you can get about double the electrostatic capacity with the three conductors that you get in the one, and about half the inductance. The power in this set-up is taken as flowing over a line where the inductance is two-thirds of normal and at the same time the discharging current is 150 per cent of the normal. The result is obviously to reduce greatly the line drop and to help the power factor.

Those three points, taken in this case, eliminate the synchronous condenser. That is the answer to that particular question.

W. S. Lee: What are you going to do with a voltage regulator on one system which tries to halt the whole load?

Mr. Thomas: The voltage regulator only controls the voltage; it won't affect the distribution of load very much. Give a definite case, Mr. Lee.

Mr. Lee: A power station situated 50 or 60 mi. from a load center, with a capacity of 80,000 kw. in the station, with a voltage regulator on it. When it boosts that load and another load is perhaps 80 miles on the other side, and it is lagging, what is going to happen?

Mr. Thomas: The generator must be big enough to stand the maximum that can come on, that is all. I don't see any complication there. It seems to me the flow of lagging current automatically takes care of itself.

Mr. Lee: Regulation in putting the voltage up means a bigger flow of current, that opens the governor wide. Then that plant goes out of regulation.

Mr. Thomas: A raise in the voltage doesn't change the load on an induction motor materially. The load is the mill.

Mr. Lee: It does on all we handle.

Mr. Thomas: If the voltage regulator keeps the voltage constant, there shouldn't be any change in the load.

Mr. Lee: Certainly; but if the voltage regulation can have this effect I think the regulation does more harm than good.

Mr. Thomas: That is the problem of the governor, I should say.

Mr. Lee. We would take the governors off, to run your system better.

Mr. Thomas: I will venture to say this type of network will work in much better shape than any other form of a regulator system that you can get. I think the problem you speak of is primarily a governor problem.

Mr. Lee: I think it is a governor problem, and I think you want a rather slow-acting governor.

Mr. Thomas: I am not saying it ought to be slow or fast, but that should be decided by whatever governor you use. That doesn't hinge on the character of the network. The governor should not be too sensitive. But the point I would make is that all the governors should be alike, equally sensitive.

With regard to the calculation of the line—there are 62 different transmission lines. The way I got at it was this. I took the map and drew the lines, noted their lengths, and assumed that the drop in any particular line was proportionate to the power that would flow over it. That is not strictly correct, because the power factor is not exactly the same all over the system. However, it varies only a few per cent., and for the purpose of this discussion, it was legitimate to assume, I think, that the drop in any line was proportionate to the length and the power flowing over it.

These lines are all of the same conductor. The size of this conductor was determined by corona and it happened to come out about right for the energy losses.

After establishing the load taken at each station and the power to be supplied from each generator, I made a trial distribution of the current through the network, to see whether the drop in each branch was the same as in other parallel branches. Of course, it wasn't, at the first trial.

Then I shifted the values and finally succeeded in getting, by trial, a series of values within about one per cent. checking up, so that the drop between any two points was the same by different routes, taking the drop as the product of the length times the power.

Then I started at the load end, with the known power at any one particular station, and made a guess at what the proper power factor would be, that is, at what field strength the local generator would be operated. That enabled me to calculate the voltage at the other end of that local line. If that were a generating station I would add in the power or subtract the load, make a guess at the right amount of wattless power and work back over the arrangement.

I had to make one or two tries at that, but finally a satisfactory set of values was found. It was, I might say, a case of shrewd guessing. However, each particular line was calculated exactly when the original assumption was made.

In regard to the distribution of power between the steam and hydraulic stations, I would like to say this. It no doubt strikes you all as a tremendously complicated system to operate, but as I have thought it over and as it has grown in my mind—and it is of two or three years' growth—I don't look upon it as a complicated system. It may be a difficult thing to get started, but picture, in your minds, the ordinary Edison direct-current network in a large city like New York or Boston. In many ways those are the simplest things to operate. They are networks. The power distributes as it likes. If they find the cables are getting too heavily loaded at one point they put in another cable, either parallel or feeding to some other substation, and they keep the voltages about right.

Now, as regards the operation of the distribution of power. Supposing we are starting at a time when water is high and hydraulic plants can carry all the load. The water wheels will have automatic governors. Now, if the water goes down, pretty soon there comes a point where there isn't quite enough water to produce a sufficient backing for regulation. You see that the water is perhaps drawing down in the different ponds, and the load dispatcher tells the steam plant to start up and put a machine on the line. A machine is put on the line and probably that machine begins to govern then, and as the water still drops, or the load becomes higher—the governor on the steam machine will open up and take more steam—and as the water drops off more and more they will take off a water wheel, put on a steam machine, and as any interconnected system now distributes the power by the automatic control of the prime-mover governors, so would this system.

As far as any practical interchange of power for synchronizing purposes or distribution of load, is concerned, this network is a perfect connection; that is, the drops are so small they wouldn't prevent interchange of power or prevent the developing of a large synchronizing power.

This ideal is theoretical, but it is the basis on which successful operation rests in all of our systems today; and I don't see any reason why it shouldn't work out in the same way here.

Now, I would like to make this distinction. At the present time there are interconnections in the South. Most of the companies touch one another. But that is, in no sense, comparable with this network. Each one of these systems is a self-sustaining system, or almost self-sustaining. One company may have an outlying sub-station on one side, which touches a line on another system. They make a connection between. If there is a little surplus power in one or the other, they can pass the power backwards and forwards. But it is limited by the capacity of the individual circuits with which the systems connect and by the fact that those systems are operated for the local power and not for the sake of supplying power to one another. It is like trying to pull a heavy load with a small string. If, however, they are connected firmly together, then the automatic interchange will be complete and it won't be so delicate and won't be so likely to fall apart at any time.

In regard to that voltage regulation, it looks very complicated, but it isn't. Supposing the system is running and we have our power houses; each power house has its power factor; each load has its power factor. We will say the peak of the load is on. The load drops off, and perhaps around Charlotte, Concord, Gastonia, Mount Holly, the voltage will begin to creep up because the load is creeping off and the generators, we will say, are maintaining the same voltage.

As that happens, the voltage regulator, when it feels the voltage rising, will change the field, until it makes a little difference in the power factor, and then your voltage goes back to where it was. Nobody worries about it.

Now, if at midnight you find that these machines are drawing a big lagging load to keep the power factor down at Mount Holly when at Tallulah Falls it is very high, there is no objection to the man at Tallulah Falls lowering his field. It will change his voltage locally, but that won't bother him. Then the voltage regulators at Mount Holly will reverse and change the power factor back to whatever is necessary to keep that voltage.

The control of power factors at all of these stations is absolutely in the hands of the load dispatcher, if he is willing to change the generator field settings in the different places. Any time you lower the voltage at the station, you change the power factor.

I assumed fixed voltages here to convince you that if you wanted to have constant voltage you could do it—that the system permits it—that the theory provides that you can control your voltage. As a matter of fact, if I were operating the system, I wouldn't have exactly constant voltages. I would adjust them. The main load centers I wouldn't allow to shift

very much, but in the distant power houses, I would have the voltage shift up and down somewhat, and by such a method as that, you can eliminate such objections raised as to running many light generators at night, etc.

Mr. Osgood made an excellent point with regard to the use of generators as synchronous condensers by cutting off the prime mover. That is really economy. You are getting something for nothing when you do that. The only thing I will say is that in this case you will get an added advantage from that sort of thing where the generators lie in the sub-stations because you correct the power factor at the sub-station and the lower power factor doesn't have to be taken over the high-tension line. In a remote generating station, if that is where your synchronous condenser is located, that merely relieves your other generators of the bad power factor load.

I didn't intend to say very much about the costs, in this paper, and Mr. Oliver isn't the first one to criticize the $8000 per mile as the cost of these circuits. I feel very confident that if Mr. Oliver were given the task of building 2500 mi. of single-circuit line in Alabama and Tennessee, and he had five or ten years in which to do it, and if those lines merely had to do what these lines have to do, he could do it for that price. Remember that conditions will be very, very different. I have taken some actual figures for the cost of towers, etc., and I feel confident that can be done.

Now, of course, there are all kinds of conditions in which work is done. Where conditions are favorable, in foreign countries with better labor conditions, and that sort of thing, that $8000 cost could be bettered. But conditions around here are very different. A man doesn't have a free hand—he can't do as well as he might in other places.

The question was raised at the beginning of the discussion as to how it happened that we could have such large amounts of power transmitted such long distances and when it came to a short circuit not have much, if any, power to handle. Mr. Stanley, I believe, pointed out that it wasn't the case down at Fall River. This is the reason. We are working on a larger scale. We have longer distances here. We have high voltages. Now, you who have studied high-voltages lines, know that a 220-Kv. line will transmit a very large amount of power a long distance if the voltage is maintained. As a result of the reactance of the line, you can get a high degree of efficiency because the charging current in that high-voltage line neutralizes the effect of the reactance, and as long as the charging current will neutralize the effect of reactance in that long line, you will get efficient transmission.

But should you get a short circuit so that the voltage is killed at that point, then the reactance is no longer neutralized by the capacity effect, and you are working with a reactance, pure and simple, at least on one end, but the net result is that you are working through a large reactance and the amount of power taken is very small.

Take, for example, a case which I have in mind of a 500-mile transmission at 220-kv.—a very large circuit. You short circuit one end of that with full voltage on the other end and the amount of power from the generator is about 7000 kw., on a circuit which has between 140,000 and 150,000 kw. as a maximum load.

Most of the jumps between stations in this set-up are 20, 30, 40 and 50 mi. If you get a short circuit, a ground, there, you will find, by actual calculation, that the amount of power that will flow is small and is below the minimum limits of a 190,000-kv. circuit breaker.

That is a hard idea to get through at first, but you will realize it as you go into it. But that comes from the fact that we have a very high voltage and far distances. It is not true with our present installations. Conditions are not the same as they are in normal networks. These long-distance networks have a radically different character, and the great difficulty is to get enough synchronizing power between stations, and not too much.

The question of reliability is another very important one. It was in my mind, as it is in yours. But I feel pretty well satisfied, from what thinking I have done, that this network is far more reliable than the so-called trunk-line type of transmission. The principal reason is this. Supposing you have a receiving end here and a sending end here, with a transmission line between. If something happens to the center of that line, or any part of it, and you get a ground, you lose the voltage—there is practically no flow of power between one end and the other. You have lost your chance for holding synchronism.

On the other hand, where you have a scheme like this, where there are two or three different routes, you may get just as heavy a short circuit as you want on one route and there may be no interchange of synchronizing currents across that link; but the other link holds them in synchronism. And after a while, whenever you are ready and a good circuit comes back, they are still in synchronism.

That isn't the only thing to consider, but that is one of the essential points in keeping continuity of service in a network, and that, it seems to me, is pretty nearly enough to throw the balance in favor of the double link connection—seperate link sections between stations.

I maintain, also, that a single circuit, feeding a station from both directions, is safer than two circuits coming from a single direction, because no one accident can sever both connections. With the two circuits you feel pretty secure. But as you analyze it, you are safer where you have a single circuit which can be fed from both ends.

Of course, the question of relaying is a very important one, and I don't think there is time to go into it. But it is taken up in the paper to considerable extent. I have thought of it further than is in there, and while some new devices will be required, I believe that a relay system, at least as successful as the present one, can be devised. For example, where you have a series of stations on a long single circuit and you get a ground, it is a difficult thing to cut off the last end where the fault is and not cut off some other section. But we have an idea to work with there, which is a relatively new one, by which we can have time limits, on relays, and the relay will open quickest where the voltage on the circuit is the lowest.

On these long circuits, the voltage builds up very rapidly with distance. It is full voltage on one end and 50 or 75 mi. away there may be a ground on the other end. If the relay that has the lowest voltage on opens first, that will clear the bad section. That isn't entirely worked out.

Aluminum was used in this conductor because of its larger bulk, so we could use a smaller size conductor equivalent resistance.

I did make the statement that I don't think it is safe to infer from present troubles in interconnections, because, with very few exceptions, there are no existing interconnections of the character of this.

Suppose you do build a trunk line to a center and your load leaves it—your trunk line is worthless. This system has the same conductor all over it, and it will permit, as I say, a change of 250,000 kw. in the supply of steam or water power, which is the balance that shifts with the seasons, from one side to the other, without any distrubance of voltage that the operating man will notice. The network is the thing which can stand an unexpected shifting in load much better than any other type that is proposed.

Mr. Skinner objects to too many taps on the transformer.

I don't think this as an actual installation would call for abnormal transformers. It is true, there would be a variation of about 10 per cent in the voltage that would require two or three taps. We would size it up as to what was going to be a good voltage for that station, and put it up two or three per cent,

or down two or three per cent. The thing would go by steps. You would have to have some taps. But you have two chances for taps—a chance on the 220-kv. transformers, the lower transformers, and also the chance to lower these drops that are taken. You could make a little more to Easley and a little less to Greenville, or a little less to Easley and a little more to Greenville. There are a great many devices you can use to avoid that particular trouble.

Mr. Oliver asked a question in regard to Muscle Shoals and cutting out the power there. That brings up again the point that I raised a while ago. Until you have studied this thing, you don't realize the different numerical relations that exist. Supposing the line did fall from Muscle Shoals to Huntsville. That means 238,000 kw. would have to go over the 88 mi. of line to Warrior over a single circuit. Warrior is a steam plant. Warrior has something like 100,000 kw. capacity. From that point the circuit splits. There are two circuits. The normal load is about 42,000 kw. in each one of those. It runs down to about 6000 kw. when you get to Anniston.

Now, that 88 mi. would take the 200,000 kw. without any trouble. The load, after that, divides in two—it adds 100,000 on those other circuits. But there is a steam station in Birmingham. At Jackson Shoals there is a steam station, and that power will merge in there and sluff away, and when you read your ammeter you will find there is a little higher load, but you won't find it in other parts of the system.

But for fear that would be considered too great a risk, I have dotted in a third circuit. You will notice the dotted line from Muscle Shoals to Huntsville, which runs down to Lindale and Atlanta. I felt, myself, that the thing was too precarious, so there is a whole spare circuit which is not used in the calculation on drop, which I have installed for the sake of giving more than two circuits to take care of Muscle Shoals. That is too much power to tie up to two circuits.

The same way at Cherokee Bluffs. While the dotted line from Newnan and Atlanta is not used in the calculation, it is put in for good measure, because you shouldn't tie up too much power to so few circuits.

Carrier Telephone on Power Lines

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WHILE the use of power lines as the transmission medium for telephone currents is a comparatively recent development, the fundamental problems involved and the methods employed do not differ essentially from the somewhat older arts of multiplex or carrier telephony over telephone lines, and radio telephony. The differences lie chiefly in the range of carrier frequencies which is best adapted to each scheme, and the means which are employed to connect the telephone equipment to the transmission medium.

In any telephone system, the fundamental requirements are as follows:

a. The operation from the user's standpoint must be reliable, simple and safe, the received signal must be of ample volume, free of disturbing noise, and a sufficiently faithful reproduction of the original sound to be clearly understandable.

b. The maintenance of the equipment must be simple, so that an excessive amount of neither time nor skill is required to keep the system in operation.

c. The cost of the equipment, both as regards investment and operation charges, must be such that it is at least as economical as other methods of accomplishing the same result.

When these fundamental requirements are interpreted in terms of the particular problems encountered in the design of carrier equipment for power lines, it is found that there must be provided, from the transmission-engineering standpoint, sufficient amplifying capacity to care for both normal and abnormal losses in transmission. Distortion of the wave form of the speech waves must be kept small, and sufficient selectivity must be provided to completely separate the desired frequency bands from the undesired ones. The carrier equipment must be well protected from accidental exposure to the power-line voltage. Duplex operation (without push-button control) similar to ordinary telephone practise, should be provided, as well as selective signaling. The equipment must be so designed as to be cheap to manufacture and easy to install, operate and maintain.

A knowledge of the transmission characteristics of power lines and associated power apparatus at carrier frequencies is necessary for the understanding of the transmission problems peculiar to power-line carrier-telephone systems.

While the theory and practise developed in connection with commercial telephone systems both at voice frequencies and at the higher frequencies employed by multiplex-carrier telephone systems on telephone lines[1] have made it possible to predict the order of magnitude of the characteristic[2] or surge impedance and the attenuation of the line alone, it has seemed desirable to check the computed values with actual measurements, made in accordance with methods employed in tele-

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1. Article of E. H. Colpitts and O. B. Blackwell, A. I. E. E. TRANS., Vol. 40, 1921, p. 205.

2. The characteristic or surge impedance (Z_0) of a transmission line may be defined as the impedance of an identical line of infinite length. It may be determined on a line of finite length by application of the formula

$$Z_0 = \sqrt{Z_{open} \times Z_{short}}$$

where Z_0 = characteristic impedance
Z_{open} = impedance of open-circuited line
Z_{short} = impedance of short-circuited line

phone transmission engineering. These measurements also include the effect of grounded guard wires and the power equipment associated with transmission lines, which is more difficult to predetermine.

In Fig. 1 is shown a curve of the ratio of received current I_R to transmitted current I_T, plotted against frequency, for a power-transmission line of 100 miles[3] in length.

This curve is plotted from experimental data obtained on an actual 110,000-volt line, the line being free of all power equipment. The use of this ratio of output to

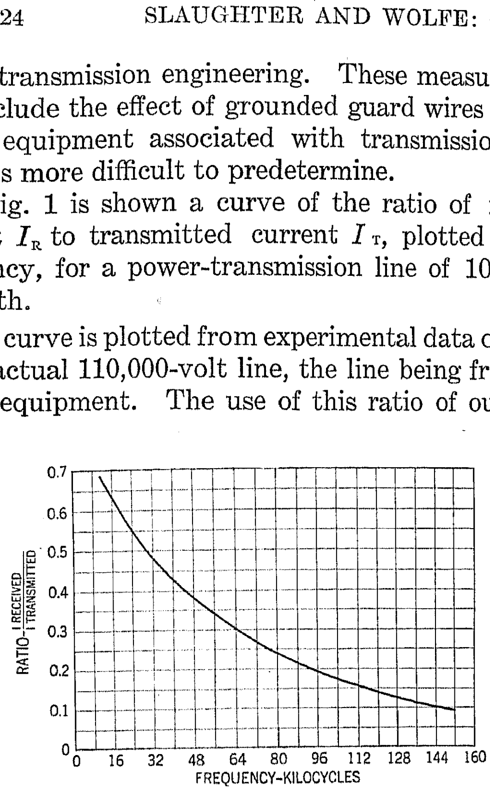

FIG. 1—ATTENUATION OF A TYPICAL POWER TRANSMISSION LINE OF 100 MILES IN LENGTH AT CARRIER FREQUENCY

input current, as a measure of the power loss in the transmission line, is, of course, predicated upon the assumption that the line is uniform and is terminated at each end in its characteristic impedance.

Although the exact attenuation of a line is dependent upon the geometry, leakage resistance, conductance and other factors, the values given in Fig. 1 may be applied to most 110,000-volt transmission systems in this country to obtain approximate values of attenuation.

The experimental data are at present meager but it seems probable that the effect of grounded guard wires may be ignored in calculating the approximate attenuation of high-voltage transmission lines.

In Fig. 2 is shown the impedance versus frequency characteristics of a typical 6600 to 110,000-volt transformer. Two curves are shown, one representing impedance with the primary open and the other with the primary short-circuited. These curves are erratic between 10,000 and 50,000 cycles showing the presence of resonant effects. However, above the latter frequency, the curves coincide and the shape of the curve indicates that the impedance is determined by the distributed capacity of the winding. The coincidence of these curves leads to the conclusion that above 50,000 cycles the impedance of the transformer is independent of the apparatus connected to the low-potential side.

The information contained in Figs. 1 and 2 gives us a basis for determining the characteristics of a transmission line and its associated power apparatus. Knowing the length of a particular line, and the extent and location of power equipment connected to it, it is possible to arrive at an approximate value of the attenuation of the system. This point, however, is not as important as the other conclusions which may be drawn from a study of the line and transformer characteristics.

Since the attenuation of the line alone is smooth and not excessive for frequencies below 150,000 cycles, it is apparent that from the standpoint of this attenuation, the frequency employed is relatively unimportant. However, a study of the characteristics of power transformers indicates that for frequencies below 10,000 cycles these transformers act as transformers and introduce large carrier-frequency losses, due to the transfer of carrier-frequency power to the low-potential network. For frequencies between 10,000 and 50,000 cycles resonant peaks are likely to occur, resulting in excessive attenuation for particular frequencies, while above 50,000 cycles, neither of these objections applies.

In selecting the frequency band best suited for carrier telephony on power circuits, the characteristics of both the line alone and the transformers indicate that the use of frequencies in the band between 50 and 150 kilocycles will result in satisfactory operation. From the standpoint of interference, this frequency band is also desirable. The frequencies are too high to interfere with commercial telephone systems since the present

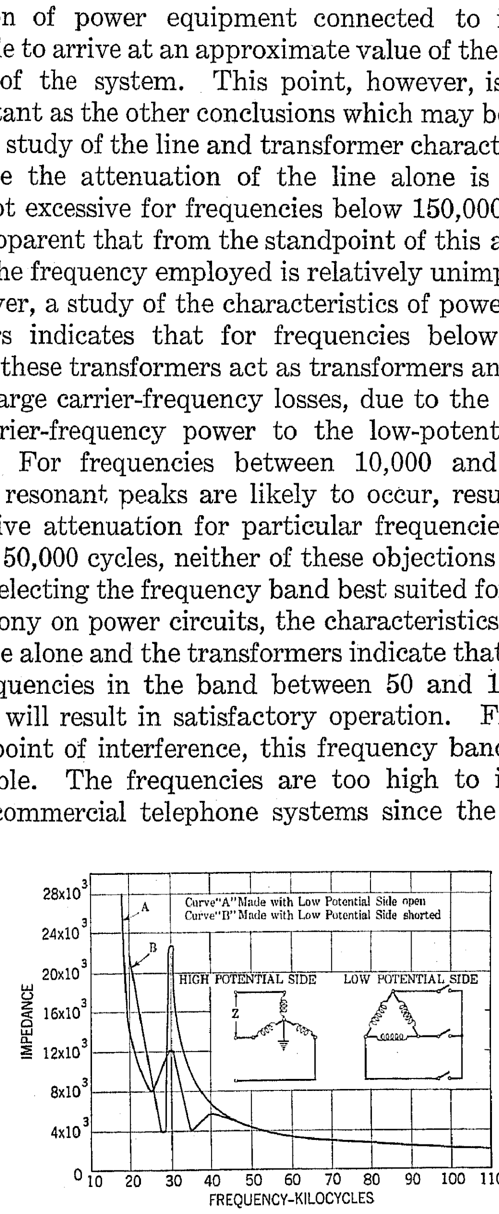

FIG. 2—IMPEDANCE CHARACTERISTICS OF A TYPICAL 6600: 110,000-VOLT TRANSFORMER BANK AT CARRIER FREQUENCIES

limiting frequency for multiplex systems is about 35,000 cycles; and they are too low to interfere with broadcasting or marine radio communication. They are, however, in the band of frequencies employed for certain classes of radio-telegraph service.

From the standpoint of coupling the carrier equipment to the power line, this frequency band proves again to be desirable, since coupling by means of capacity becomes easier as the frequency is increased.

In the development of this carrier equipment, it was early decided to employ a full metallic high-frequency

3. This curve may be reproduced for lines of a length L different from 100 by the following formula.

$$I_{rec}/I_{trans} = \frac{1}{\log^{-1}(L/100 \log X)} \quad \text{for any frequency } f_1, \text{ where } 10{,}000 f_1 < 150{,}000$$

cycles where X = the reciprocal of I_{rec}/I_{trans} as obtained from the curve

circuit rather than a ground-return circuit. The trend of the communication art has always been away from ground-return circuits because the attenuation of ground-return circuits is greater than that of comparable full metallic circuits. As the ground connection itself is unstable, such a circuit is subject to interference from other ground-return circuits and will itself produce interference in adjacent communication circuits. In addition to these objections, a ground-return circuit is generally noisy, particularly when it is associated with power equipment.

The provision of proper means for connecting a carrier telephone circuit to the high-voltage transmission lines is one of the most difficult problems involved. The coupling device must offer a safe and efficient means for transferring high-frequency energy to and from the power lines.

The use of inductive coupling did not seem feasible since it required a special transformer having a 60-cycle difference of potential between windings of approximately 65,000 volts on a 110,000-volt line. The use of existing power transformers to secure this coupling is not feasible because they do not act as transformers at the carrier frequencies.

The capacity coupling seemed to offer the most feasible solution of the problem. The first type of condenser considered was the capacity secured between a single wire parallel to the power-line conductor and the conductor itself. This type of condenser has two fundamental weaknesses. In the first place, it has what is known as distributed capacity and in the second place, its capacity to ground is generally larger than the capacity to the power conductor. The use of this form of condenser lends itself best to a tuned circuit, similar to that employed for radio-transmitting circuits. The major part of the energy which in a radio-transmitting circuit would be radiated into the ether is transferred from the antennas to the power-line conductor by virtue of the capacity existing between them. There are two serious objections to this type of circuit, the less serious objection being that since it is a tuned circuit, the frequency band which can be transmitted is very narrow which makes this arrangement not so well adapted to duplex operation as the filter arrangement described below. The second objection is the fact that a portion of the energy delivered to the antennas is radiated and may be picked up by a suitable radio receiving set. Since secrecy is not a primary object of this type of communication, this would not be objectionable but for the fact that it may require the licensing of carrier equipment by the Government and its operation by licensed radio operators. From the standpoint of reception, this type of coupling is also undesirable. It has many of the characteristics of a radio receiving set and will readily pick up radio signals lying in its frequency range. The use of concentrated capacity, such as in an ordinary fixed condenser, avoids these difficulties, but it introduces the problem of insulating the condenser plates. The development of a satisfactory condenser has been undertaken by a well-known manufacturer of power-line equipment, and this condensor will, no doubt, be available for all commercial installations.

The coupling capacity may be employed either as a part of a series-resonant circuit or as part of a filter circuit. The former lends itself to the transmission or reception of a narrow-frequency band but is not efficient for the simultaneous transmission and reception of signals of two different frequencies. Filter circuits[4] can be designed to pass a relatively wide band of frequencies with a small attenuation. The filter also simplifies the problem of static 60-cycle charges since it provides a path to ground for low frequencies without grounding the high-frequency circuit. Fig. 3 shows the characteristic of a band-pass filter, suitable for use as a means of coupling the carrier currents to the power line.

The power required for telephone transmission over power lines is largely determined by the magnitude of the disturbing currents or "noise" which are of frequencies in the range of the carrier equipment. If no

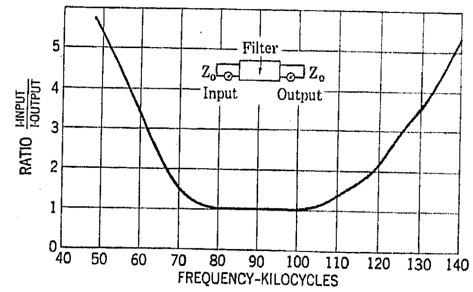

Fig. 3—Attenuation Characteristic of a Band Pass Filter Similar to that Shown in Fig. 6

"noise" were present the carrier transmitter would not be called upon to increase the power delivered by the telephone transmitter, but would have to merely change the frequency band. The receiving equipment would then furnish all the amplification needed to counterbalance losses in transmission over the system. However, in actual practice noise exists in the form of more or less infinitesimal electrical disturbances at all frequencies, the level of this noise generally decreasing as the frequency increases.

Based on a knowledge of noise conditions on transmission lines and with other practical considerations in mind, the receiving gain has been limited to a voltage amplification of about 100, which corresponds to a power amplification of about 10,000. Since it is desirable to deliver about 0.01 watt to the telephone receiving circuit, a carrier-frequency power as low as 0.000001 watt at the input to the receiving circuit will produce satisfactory results under favorable conditions. To allow for less favorable conditions, it is assumed that a power of 0.00001 watt is required. Based upon this

4. U. S. Patents 1,227,113; 1,227,114 of 1917.
G. A. Campbell, Wave Filters for high frequencies.

assumption and the curve given in Fig. 1, it is evident that an output of one watt from the transmitting circuit is sufficient for operation over 265 miles of transmission line, when the carrier frequency is 150 kilocycles and there is no loss in the coupling device or the associated power equipment. Actually, the loss due to other transmission lines connected to the line on which the carrier is operated, the loss in power equipment, and loss in the coupling device, may reduce this normal range to 100 miles or less. It seems probable, therefore, that a relatively small output from the transmitting circuit will be sufficient for normal operation on the majority of transmission lines.

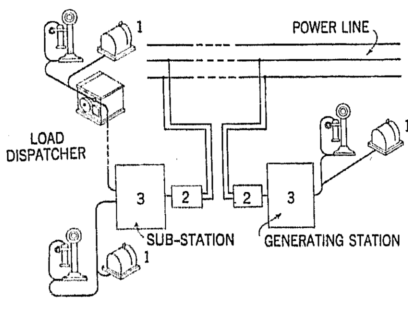

Fig. 4—Power Line Carrier Telephone System
1. Ringing Keys
2. Coupling Devices
3. Carrier Equipment—See Fig. 5

However, when emergencies arise, telephone service is of the utmost importance, and, therefore, a larger transmitting output should be available, sufficient to insure satisfactory operation under unusual line conditions and actual line failures.

In order that duplex operation may be secured, it is necessary to operate the transmitting and receiving circuit simultaneously, which means that the receiving circuit, designed to operate on an input of 0.00001 watt, must operate in parallel with the transmitting circuit which may deliver as much as 50 watts to the power line. This large power ratio makes the separation of the transmitted and received currents a difficult problem, which has been solved only because of recent progress in the art of designing electrical filters. By employing frequencies for transmitting and receiving, which differ by about 20,000 cycles, it is possible to prevent the transmitting circuit from interfering with the receiving circuit. For this purpose, a high-pass filter and a low-pass filter, similar to those described in recent literature, are employed.[5] The high-pass filter passes frequencies above 100,000 cycles and attenuates frequencies below that frequency, while the low-pass filter passes frequencies below 80,000 cycles and attenuates frequencies above 80,000 cycles.

Fig. 4 is a block schematic diagram of a carrier telephone system, developed for use on high-voltage power lines. The design of this equipment, both electrical and mechanical, has profited by the research and development work in connection with the arts of communication at speech frequencies, carrier frequencies, and radio frequencies.[6] As a result, the circuits employed and the equipment used in these circuits have the background of wide experience.

The transmitting circuit comprises an oscillator-modulator circuit which will provide sufficient output for operation on the majority of power lines. The circuit also provides a power amplifier which may be connected in the circuit by a simple operation, for use in emergencies. The operation of the circuit, while employing the 50-watt power amplifier, is in no way different from the operation without the amplifier. The transmitting circuit is entirely automatic in its operation, and is energized only when the circuit is in use.

The receiving circuit consists of a two-stage high-frequency amplifier followed by a negative grid detector. There is no tuning required in this receiving circuit. The circuit is not regenerative, this having been avoided because of the lack of stability in circuits of this type.

Signaling on this carrier circuit is entirely automatic and where more than two stations are involved, it is selective. The operation of the ringing key sends out a predetermined train of pulses at the carrier frequency. These pulses, when received and rectified, operate a selector of a type commonly used for telephone dispatching on railroad lines.

The voice-frequency telephone circuits are standard circuits suitable for either two-wire or four-wire operation. The circuit may be extended to a telephone

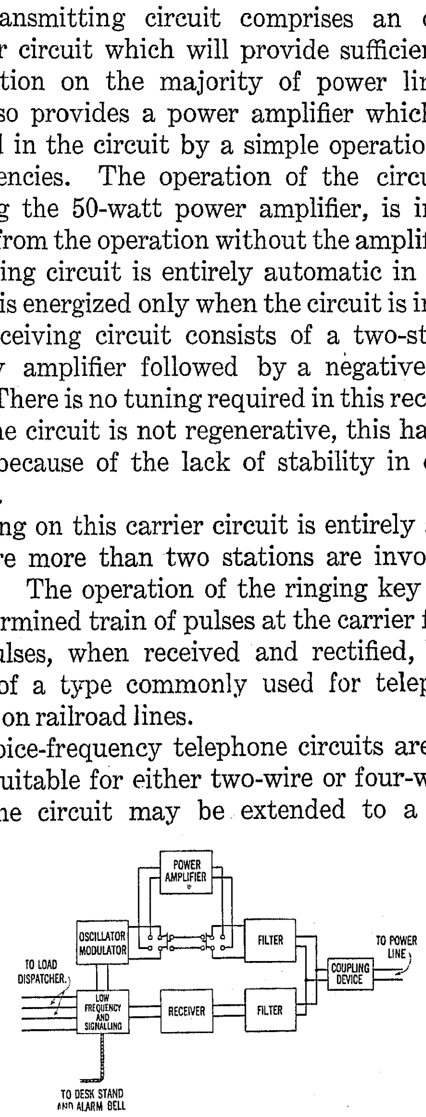

Fig. 5—Block Schematic of a Power Line Carrier Telephone Terminal

located at a point several miles from the carrier equipment with full control of the carrier, as is desirable in the case of the load dispatcher, or without control of the carrier; that is to say, where the load dispatcher is located at a point some distance from the carrier terminal, he may be provided with the same control and talking facilities as for the terminal proper. After considering the relative merits of the control arrange-

5. G. A. Campbell, B. S. T. J., November, 1922.
O. J. Zobel, B. S. T. J., Vol. 11, No. 1, pp. 1-46.
O. J. Zobel, J. H. Carson, B. S. T. J., Vol. 11, No. 3, pp. 1-52.

6. H. W. Nichols—Lloyd Espenschied, I. R. R. *Proc.* Vol. II, pp. 193-239, June, 1923.
Lloyd Espenschied, I. R. E. *Proc.* Vol. 10, pp. 344-368, October, 1922.
L. M. Clement, F. M. Ryan, D. K. Martin, I. R. E. *Proc.* Vol. 9, pp. 469-505, December, 1921.

ments, employing the talking circuit for control purposes also, as compared with a separate pair of wires for the control circuit, it has been decided that the separate-control circuit results in a much simpler and more reliable arrangement. Accordingly four wires, two for talking and two for control are required. (See Fig. 5). Where it is not desirable to provide control of the carrier equipment, the talking circuit may be extended through a standard *P B X* board in a manner similar to that

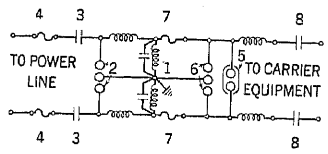

Fig. 6—Band Pass Filter Suitable for Coupling a Carrier Telephone System to a Power Transmission Line

This Equipment may all be Outdoors:
1. Low Resistance Coil with Mid-Point Grounded
2. Heavy Duty Static Spark Gaps
3. Condensers capable of Withstanding Power Line Voltage
4. Fuses capable of Breaking a Power Arc at the Line Voltage

5. Vacuum Spark-Gaps
6. Static Spark-Gaps
7. Fuses

(5–7: This equipment similar to that employed for Protecting Wire Telephone Systems Paralleling Power Lines.)

8. Mica Condensers capable of Withstanding a Potential of 10,000 Volts

employed for ordinary wire-telephone circuits. Two or more carrier systems of this type may be operated in tandem where circumstances make it possible and desirable to do this. No special equipment is required in extending the carrier circuit in any of the above ways.

The power supply for the carrier equipment may be obtained from any source which is convenient. The tube filaments require 24 volts d-c. and a current drain of only a few amperes, which can ordinarily be supplied best from storage batteries. The plate circuits of the receiver and the low-power transmitter require 150 volts d-c., with a current drain of a fraction of one ampere. This can be obtained from storage batteries, or a motor-generator driven from the 24-volt supply required for the filaments. The high-power transmitter requires 800 volts d-c., for plate potential and this can be obtained best from a 150-watt motor-generator set driven from the 24-volt source. This provides a power supply independent of the transmission line, an important point where transmission line failures or interruptions of power supply from other causes are likely to occur. Where the latter consideration is not important, both the 150 volts d-c., and 800 volts d-c. may be secured from a-c. rectifiers.

The vacuum tubes employed in this equipment are standard tubes, familiar in telephone and radio practise. The average life of the tubes employed in the receiving circuit is about 10,000 hours which is equivalent to continuous operation for a little more than a year.

Some of the tubes in the transmitting circuit have a shorter life but since they are not continuously operated, replacements will not be necessary on the average more than once a year.

The protection of the personnel operating the carrier equipment, and the carrier equipment itself, from the high voltages employed on the power line is of vital importance. In Fig. 6 is shown diagramatically the method employed in coupling the carrier equipment to the power line. The condenser (3) may be a fixed condenser or the distributed capacity secured by suspending a wire parallel to the power-line conductors. In either case, this condenser is designed and tested so that it has the accepted factor of safety required for work of this character. In event of a failure of this condenser, a momentary short-circuit current flows through the fuse (4) to ground at (1). The reactance shown, connecting this condenser to ground, is negligible at power frequencies since its inductance is a fraction of 1 millihenry. In spite of the low inductance of this coil, it is possible that an appreciable voltage may be developed across it and, therefore, heavy-duty static spark-gaps, having about ¼-in. clearance, are connected across this coil. On the carrier-apparatus side of this coil the fuses, vacuum-gap and air-gap commonly used for protection of wire telephone circuits paralleling high-voltage power circuits, are used and as a final measure of protection, a high-voltage mica condenser (*S*) separates the coupling device from the carrier equipment. This circuit has been laid out to secure the

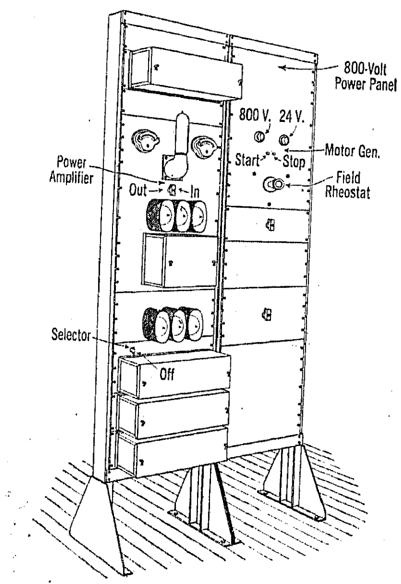

Fig. 7—Carrier Telephone Terminal Equipment

maximum possible protection both from static-power potentials and from conditions which may arise because of the failure of the coupling condensers. The fuse shown at (4) is designed to break the power arc at the line potential.

The carrier equipment, which has just been described, is suitable for power networks involving one or more transmission lines, extending for 100 miles or more from the load dispatcher's position. The exact length or extent of a transmission system over which this equipment

will operate satisfactorily is, of course, difficult to predict and each installation must be considered separately.

The number of carrier terminals which may be operated on one power-transmission system is practically unlimited. It is easily conceivable that 10 or more carrier stations might be operated on a single line. This system would then compare with a commercial party-line telephone system, except that every station may call every other station without passing the call through a common point. In some cases, it may be desirable to pass the call through a common point. The change from one standard ringing key to another standard key accomplishes this result.

For power cables, the use of carrier telephony at frequencies of 100,000 cycles is not practical except in very short lengths of cable. However, lower frequencies may be employed for transmission on cables, although the coupling to the power conductor will be considerably complicated for lines of very high voltages.

To provide for those power systems, composed of several power lines operated by different companies but connected in parallel, the carrier frequencies may be located at various points in the band from 50 kilocycles to 150 kilocycles so as to avoid interferences between carrier systems.

Discussion

H. L. Wills: When Mr. Slaughter announced that he was going to talk on one watt, I said to myself, "Good heavens, that's the same old thing all over again; that is what they always do—they keep going lower down in the scale of power, and we keep going the other way, and then they expect us to live together decently when we are both so far apart!"

So it brings to my mind this thought: What are the advantages to be gained by the use of approximately one watt in our telephonic intercommunication over the high lines? I would like to ask Mr. Slaughter if he thinks that is going to be ample to do our work?

Then there is the matter of coupling. I would like to ask if there is going to be any real gain in the use of condensers for coupling?

Then you know we have talked a lot about the telephone people getting off the ground with the power engineers, and the power engineers seem now to be getting back on the ground again. Now here comes along an organization, and they are getting off the ground again. What is the advantage of the full metallic circuit?

It seems to me that in starting out on a program of intercommunicating power lines or interconnecting power lines, our intercommunication system, if it is going to follow along our lines, should be ample; should be such that it can be used universally, and we should not spend time or effort in setting up a great number of systems and then later on in our stage of development find that some or most of it is all wrong.

Leonard F. Fuller: This paper describes work in which the interphase, or all-metallic, carrier-current circuit was employed, as shown in Fig. 1, in which 1 and 2 are coupling wires, or high-voltage coupling condensers, through which energy from transmitter T is fed to line conductors a and c and then to receiver R through coupling means 3 and 4.

The circuit comprises T, 1, 3, R, 4 and 2.

If either conductors a or c open, test has shown that although the circuit is interrupted metallically, it remains closed electrically because of the capacitance between the unbroken conductor b and each portion of the interrupted conductor. The ground of the broken conductor at each end of the break seems to make little difference.

When both conductors a and c are broken and fall to earth, the condition is serious.

In Fig. 2, is shown the ground-return carrier-current circuit, in which 1 and 2 are coupling wires, or coupling condensers, arranged to feed into line conductors a, b and c operating in

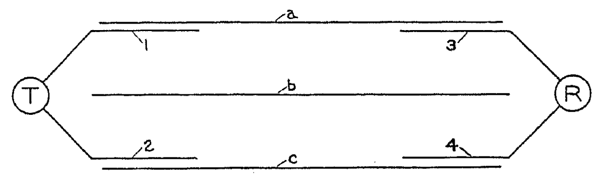

FIG. 1

parallel. In this case, the earth completes the carrier-frequency circuit, and the interruption and grounding of any two of the three conductors has very little effect upon the energy arriving at the receiver from the transmitter T.

In the case of twin-circuit lines, five of the six line conductors may be interrupted and grounded without seriously interfering with the carrier communication.

This, in itself, is a feature distinctly in favor of the ground-return method, but on the other hand, the power required for communication between two points on a transmission system is very much less with the interphase method. Thus, if the power used with the interphase connection is increased to that necessary for successful operation with the ground return method, the transmitter of the interphase system will be able to operate through even more serious line interruptions than is possible with the ground-return circuit.

The losses occurring in the case of the ground-return method, which require more power output from the transmitter, probably occur in the earth portion of the circuit.

It is possible that one reason why the interruption of all conductors is a more serious matter with the ground-return method than with the interphase system is because the group of line conductors which operate in parallel carrying the carrier-frequency current are usually considerably farther above the earth than the distance between phases, thus making the reactance of the circuit through which the carrier current is circulating much higher in the case of the ground-return method. Thus, a higher quadrature voltage is required to circulate a given current, and when all lines are down, the e. m. f. which must be induced at the receiving end of the break to produce a suitable

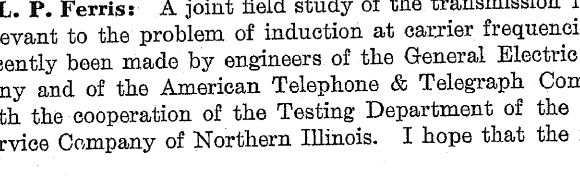

FIG. 2

received signal is much higher in the case of the high-reactance ground-return circuit.

For these reasons, the interphase method of operation is rapidly coming into favor for this type of communication.

L. P. Ferris: A joint field study of the transmission factors relevant to the problem of induction at carrier frequencies has recently been made by engineers of the General Electric Company and of the American Telephone & Telegraph Company, with the cooperation of the Testing Department of the Public Service Company of Northern Illinois. I hope that the results

of this work may be made generally available in the near future. Some of the results relating particularly to the questions of line and transformer characteristics may be of interest now in connection with the discussion of the present paper.

Fig. 3 shows the ratio between the current received at the end of a 17-mile section of a 33,000-volt transmission line to the current sent into the line, for a "metallic circuit" of one pair of wires and also for a circuit made up of the three wires of a line with ground return. These curves are not smooth because the terminating resistance departs from the exact characteristic impedance of the circuit, as defined in the paper. At the time these tests were made the characteristic impedance of the circuit had not been accurately determined. The irregularities in the curve are probably due in a large measure to this departure from the correct characteristic impedance, under which conditions reflections from the far-end of the circuit are to be expected. Because of the much larger attenuation in the grounded circuit, these reflection irregularities are less pronounced in the lower curve. Particularly at the high frequencies this curve becomes relatively smooth.

The curves in Fig. 1 bring out clearly the large difference in the attenuation in the "metallic" circuit and the circuit of the three-wires-to-ground. At 85,000 cycles the current received in the grounded circuit is 1/9 and the power, therefore, about 1/80 of that received in the "metallic" circuit for the same current and power transmitted. As noted in the paper under discussion, this difference is an important advantage in favor of the metallic circuit for this type of system.

The values of the current ratio above unity for the upper curve in Fig. 1 are due to the terminal irregularity and, of course, do not represent an increase in power although the received current may be larger than that transmitted.

In a foot note at the bottom of page 1 the authors give a formula by which the current ratio for a line of any length may be calculated from the ratios for a different length taken from the curve. This formula may be written in several other forms. For example, if R_2 is the ratio $\frac{I_r}{I_t}$ for length L_2 and R_1 is the corresponding ratio for L_1, then the equation may be written $R_2 = R_1^{\left(\frac{L_1}{L_2}\right)}$. This may also be written as $R_2 = \log^{-1}\left(\frac{L_2}{L_1}\log R_1\right)$. Either of these forms is useful in making simple calculations, and the first of the two may be particularly advantageous where use is made of the log-log slide rule.

Because of the losses in power transformers bridged across the circuit, the question of the impedance presented by apparatus of this character at the carrier frequencies is of considerable importance. Messrs. Slaughter and Wolfe show the variation of the impedance between two terminals of a grounded Y-delta transformer bank, with the secondary side sort-circuited and with it open. It is of interest to examine the variation of impedance of a single transformer with frequency, and also of the Y-connected bank when the impedance is measured from the three wires of the line to ground. Fig. 2 gives the impedance-frequency characteristic of an 8333-kv-a. transformer rated at 21,000 to 12,000 volts. This is a single-phase unit operated in a 25,000-kv-a. bank on a 33-kv. system. The curve shows the impedance-frequency variation for five resistances across the secondary terminals. These are 0, 17.3 ohms, (which is the resistance required to give full load current of unity power factor) 1000 ohms, 6800 ohms and infinite resistance The effects of resonance of the transformer windings at certain frequencies are shown very clearly The curves for low secondary impedance show low values of primary impedance in the lower frequency range with humps appearing at about 43,000 and 70,000 cycles. The high secondary impedance curves show large values in the lower frequency range, trailing off to low values at the higher frequencies Where a metallic circuit is used and transformers connected in star, the bridged impedance presented to that circuit will be twice the impedance of one transformer, as may be seen from Fig 2 of the paper where the impedance is measured between two terminals with two transformer windings included between them, In the range above 30,000 cycles, the lowest bridged impedance obtained for any value of secondary impedance shown in (Fig 2) is approximately 1500 ohms, or the equivalent of 3000 ohms for two transformers in series. For the lower secondary impedances which are much more likely to obtain under practical conditions the primary impedances are generally considerably higher.

Fig. 3 gives the impedance of a bank of three transformers of the same type as used for the measurements on the single unit just discussed. For this test the bank was connected Y-delta. The delta was closed and the impedance was measured between the three line terminals in parallel and the neutral. It will be observed that this transformer bank introduces a bridged impedance between the three wires and ground varying between 500 and 6500 ohms in the range of above 30,000 cycles. In this range the impedance at any frequency is less than twice the impedance of a single transformer at that frequency as shown by (Fig. 2). From this and other cases giving similar results, it is believed that the use of a two-wire metallic circuit is to be preferred to that of the circuit between the three wires and ground from the standpoint of bridged losses due to Y-grounded transformer banks.

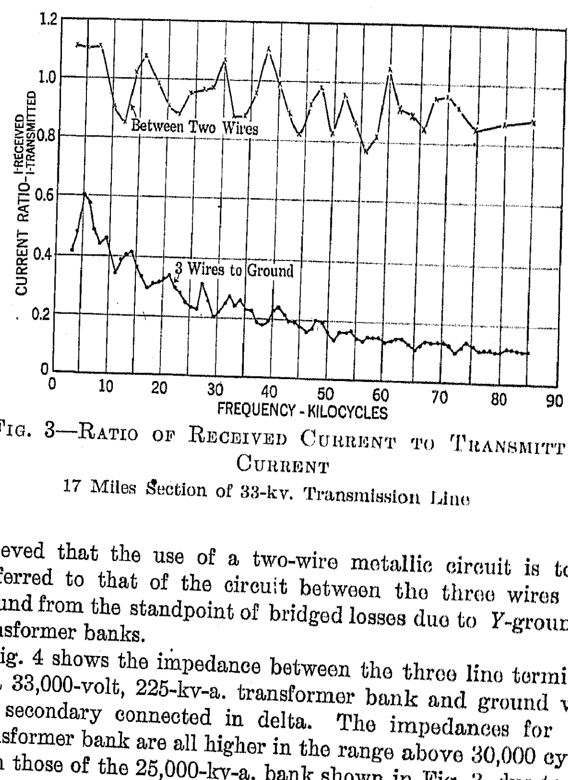

FIG. 3—RATIO OF RECEIVED CURRENT TO TRANSMITTED CURRENT

17 Miles Section of 33-kv. Transmission Line

Fig. 4 shows the impedance between the three line terminals of a 33,000-volt, 225-kv-a. transformer bank and ground with the secondary connected in delta. The impedances for this transformer bank are all higher in the range above 30,000 cycles than those of the 25,000-kv-a. bank shown in Fig. 3, due to the lower power rating of these transformers, and similarly higher impedances were found for the single units comprising this bank. The three curves on this slide show the primary impedances under the following conditions:

1. Primary neutral isolated, leads from secondary delta open.
2. Primary neutral grounded, leads from secondary delta open.
3. Primary neutral grounded, leads from secondary delta short circuited.

In the major part of the carrier range these curves show that grounding the neutral to the case reduces the impedance to about half its value with neutral isolated.

Fig. 5 is shown in order to exhibit the manner in which the impedance of the three wires of a line in multiple to ground varies with frequency. The dotted curve shows the impedance

to ground with the far-end of the circuit clear of ground, whereas, the solid curve shows the impedance with the far-end grounded. The peaks of these curves are, of course due to reflections from the open or short circuited distant end of the line and the frequency intervals between peaks are determined by the velocity of phase propagation and the length of the section measured. Because of the increase in attenuation as the frequency is increased the magnitude of these peaks diminishes towards the

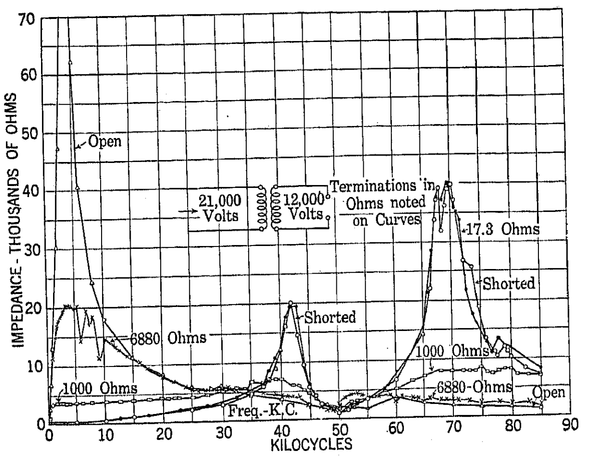

FIG. 2—TRANSFORMER IMPEDANCE
8333-kv-a. Transformer, 21,000-36,300 Y/12,000 Volts.

higher frequencies. It may also be noted that at the upper end of the frequency range these curves are approaching a common value, which is the characteristic impedance of the circuit or the impedance that the circuit would present if it were infinitely long. As noted at the bottom of page 1 in the paper the characteristic impedance of the line (or the impedance with which it must be terminated to avoid reflection) is given by the formula

$$Z_c = \sqrt{Z_o \times Z_s}$$

where Z_c is the characteristic impedance, Z_o is the impedance

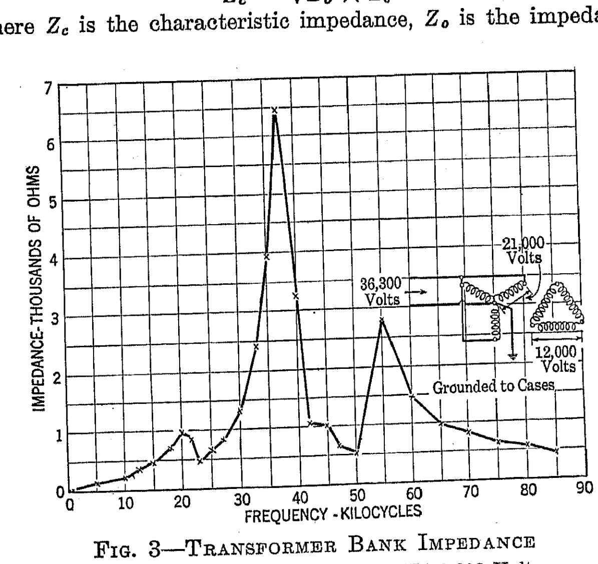

FIG. 3—TRANSFORMER BANK IMPEDANCE
3-8333-kv-a. Transformers, 36,300 Y/12,000 Volts.

with the far-end of the circuit open and Z_s is the impedance with the far-end of the circuit short-circuited. The characteristic impedance for this line has been calculated from the open and short-circuited impedances and plotted on the curve for reference. It varies somewhat with frequency and lies in the range between 400 and 470 ohms. This slight variation is explainable by variations in the gage and arrangement of the conductors along the line.

It should be pointed out that not only the impedances but also the attenuation factors for high tension lines vary over a considerable range for different types of circuits and also vary considerably for circuits of the same type, depending upon circuit conditions. Variations are produced by the presence of other wires (including ground wires) on the same lead which absorb energy. It is very difficult to say just how these variations are produced or to give any reliable method of calculating their effects. For approximate calculations, however, it is permissible to assume that the attenuation factors for one line may be applied to another line of similar construction when the appropriate circuit lengths are taken into account.

G. Y. Allen: The matter of attenuation was rather interest-

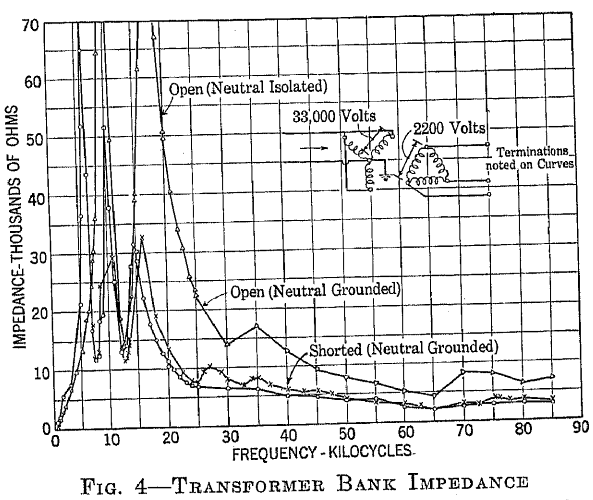

FIG. 4—TRANSFORMER BANK IMPEDANCE
3-75-kv-a. Transformers, 33,000-11,000 2420-2200-1980 Volts.

ing to me. Fig. 1 of Col. Slaughter's paper shows the attenuation for a typical power line with no connected apparatus and with an impedance connected at the receiving and transmitting ends of the line corresponding to its surge impedance. Under those conditions, of course, there will be no reflections along the system and the attenuation can be easily calculated.

Fortunately or unfortunately our pioneer experience was obtained on the lines of the Duquesne Power Company of

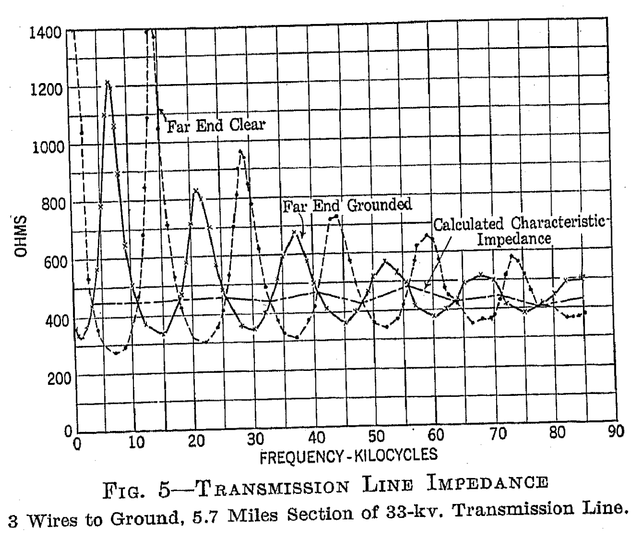

FIG. 5—TRANSMISSION LINE IMPEDANCE
3 Wires to Ground, 5.7 Miles Section of 33-kv. Transmission Line.

Pittsburgh. In contradistinction to a straight line, with no connected apparatus, the Duquesne System consists of distribution rings of various voltages to which are connected a vast amount of apparatus. In attempting to apply carrier-current telephony to this system, it was very difficult to predict the results from day to day. The apparent reason is that with so

much connected equipment and with such a complex network the constants of the line changed rapidly, even from hour to hour, and the performance of such a network cannot easily be calculated. We found, for instance, that it was necessary to change the wavelength that was used in order to insure communication even during as short periods as one hour.

I merely bring out this point to emphasize the fact that the calculation of the performance of a network is not always as simple as Fig. 1 in Col. Slaughter's paper would indicate. As a matter of fact, our experience indicates that the majority of networks are much more complex than the one Col. Slaughter has considered. It is on account of this indeterminate factor that the Westinghouse Company has constantly advocated the use of surplus power to reduce the probability of interruption of communication to a minimum. It is a well appreciated fact that communication is most essential in time of trouble. If carrier-current telephony is to serve the power industry as it must be served, it must operate during abnormal line conditions.

There is one other thing in connection with coupling that I desire to touch on. Col. Slaughter states that the efficiency of condenser coupling is much greater than antenna coupling. This is probably true. Our results have indicated that instead of getting approximately 80 per cent of the energy in the oscillating circuit on the system, which is probably the efficiency of condenser coupling, antenna coupling may give considerably less. We appreciate this relation and our only reason for continuing to use antennas is on account of the reduction of hazard, and also due to the fact that on high-voltage lines we have not yet seen a condenser that we feel will stand up. We have some thoughts worked out on paper for the improvement of antenna coupling and our present policy is to continue to use antenna coupling for potentials of about 30,000 volts.

E. R. Craft: It seems to me that what the power companies are going to need is a means of intercommunication and intra-communication which will render the most reliable service at the least possible cost. Whether or not it will be antenna-coupled or condenser-coupled, it is going to be worked out eventually and adopted.

It is not a question of using one watt or 500 watts of energy, but of obtaining the system fundamentally most efficient for the purpose. Then there can be used whatever amount of power is required.

In considering the type of system that is going to be employed by any individual operating company, thought must be given to how this is going to work into the other systems with which it must connect sooner or later. Take the Southeastern district, for instance. The system is not the system of the Alabama Power Company nor that of the Georgia Railway and Power Company. It is the power network of the entire district.

W. V. Wolfe: Considering Mr. Wills' discussion, his first question was: "What are the advantages of low power over those of higher power?" Mr. Craft has already answered that in a certain measure and there are only one or two points which I want to make in connection with that. The first is this: A telephone system must be duplex in its operation, in order to be a telephone system. The problem of making a telephone system duplex increases materially as the power which you use in transmitting increases. As we have pointed out in our paper, you have to operate your transmitting circuit in parallel with your receiving circuit. The ratio of power required to operate the receiving circuit to that required for the transmitting circuit may be of the order of 10,000 to 1, and obviously as you increase that transmitted power, the difficulties of keeping those two things separated increase very materially.

Mr. Wills' second question was with regard to the coupling wire versus the condenser. In the first place, the coupling wire does not give as safe a method of coupling to the power line as the condenser does, first, because the coupling wire extends 1000 or 1500 ft. in parallel with the power conductor itself. It is true that it is put up in the same manner, of the same size conductor, and in every way as nearly as possible the same construction as the power line itself, and the possibilities of its failure are no greater than those of the failure of the power line, but power lines do fail, and this 110,000 volts or higher may come in contact with the antenna wire over this range of 1000 or 1500 ft. By using the condenser, we put that possibility of failure in one spot: we don't extend it over 1500 ft.; and in addition to that, the condenser, because of the different methods with which it is used in coupling to the power line, gives a scheme which is very much safer than the coupling wire.

In connection with that I want to say here that in our demonstration or proposed experimental work at Magella, we had the failure of a condenser, probably due to the condenser being damaged in shipment, but at any rate, regardless of the reason for the failure of the condenser, this condenser failed while the man was talking on the circuit. This man who was talking on the circuit when the condenser failed is in the room this evening; he was listening on the circuit at the time of failure. The only thing that happened was that he got a little click in the receiver, and he was called out by the operator of the station who had heard the fuse go out. I don't think we can assume the same degree of safety with the antenna wire strung parallel to the power line wire for a distance of 1000 or 1500 ft.

Mr. Wills also asked the advantages of the full metallic circuit over the ground-return circuit. Dr. Fuller has pointed out many of those advantages; our friend from the Westinghouse Company has also pointed out some of those advantages. There is one thing which I believe neither of those gentlemen has pointed out, and that is this: That the antenna scheme of coupling on a full metallic or a ground-return basis leads to a certain loss due to radiation of power from this coupling device. With the ground-return circuit, the radiation, as measured in experimental work which we have done, is of the order of four times that encountered in the full metallic circuit. That in itself is not important, except for the fact that the business of radio communication, of broadcasting, is developing at a tremendous rate in this country, and the government is very anxious to keep radio where radio belongs. It is only a question of time till a carrier system or any other system which radiates into the ether will have to meet certain government requirements. They may not be the same requirements that are asked now for a radio station, but they will certainly have to be regulated by the government.

One question which Dr. Fuller brought up was the question of one wire failing on a full metallic circuit. The only thing I can say there, and the only thing I believe we can say for any carrier circuit or any communication or power circuit is this: That as long as the circuit is intact, you will be able to talk through it. What constitutes a carrier circuit may be something entirely different from what constitutes a power circuit. A failure of one wire may, under certain conditions, constitute a failure of a carrier circuit; under other conditions it may not constitute a failure of a carrier circuit. I may say, under certain conditions, we have not only been able to talk through it with one-wire conditions, but we have talked with three wires opened and grounded. It may result in a failure of the carrier circuit, but does not always result in such a failure.

Another thing in connection with the ground return and the full metallic circuit is this, as pointed out by Dr. Fuller and Mr. Allen: The fact that in the ground-return circuit, if you take the transformer off the line, you are going to change the character of the ground-return circuit from a transmission standpoint, and that change may be of an order of magnitude which may result in a failure of the carrier system to operate. We don't find that condition to be true in the full metallic circuit. We have at all times operated our experimental metallic circuits without information as to whether there was a transformer on here or on there; it made no difference.

There is just one other point I want to mention, and that is the cost of condensers as compared with the cost of antenna schemes. I think the point there has been overlooked. The important thing in a communication system, which may mean the successful operation or the failure of the operation of a power system, is not the cost; the cost of these condensers being small as compared with the cost of the power apparatus used in a big power network. If they are going to mean the difference between a successful carrier system and an unsuccessful carrier system, then I do not believe that any power company can afford to let the economics of that thing hold them up.

High-Voltage Oil Circuit Breaker Tests, Alabama Power Company System

BY J. B. Mac NEILL
Associate, A. I. E. E.
Electrical Engineer, Westinghouse Electric & Mfg. Co.

IN conjunction with the Alabama Power Company, a series of 50 high-voltage oil circuit breaker tests was made during May and June of 1922. The Bessemer substation of the power company was chosen as the location for these tests due to the large power concentrations that were possible on short circuit, the possibility of maintaining system voltage under short circuit to the greatest extent and because it was felt that less interruption to service would occur if tests were made here than elsewhere.

As far as is known the Alabama Power Company was the first to conduct a comprehensive set of tests with necessary oscillograph apparatus and facilities for observation on 44,000 volts and 115,000 volts. Of the fifty tests made on Westinghouse breakers, 36 were made on 44,000 volts and fourteen were made on 115,000 volts. The maximum generating capacity connected to the system on 44,000-volt tests was 145,000 kv-a. The maximum generating capacity connected to the system on 115,000-volt tests was 188,000 kv-a. in addition to a tie with the Georgia Power System. The maximum power actually opened was 3660 amperes at 44,000 volts, and 2850 amperes at 110,000 volts, which give approximately 250,000 kv-a. three-phase at 44,000 volts, and 542,000 kv-a. three-phase at 110,000 volts. A glance at the power system set-up and the amount of generating apparatus connected will show that these tests were made under power conditions comparable to those on the larger of the systems throughout the country.

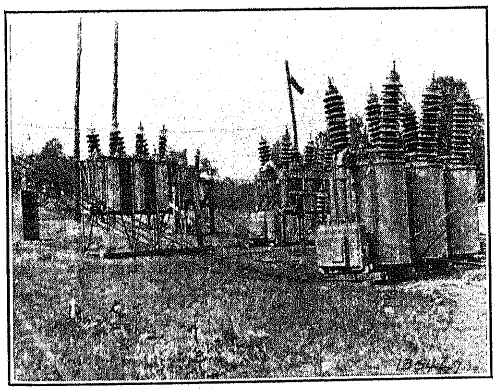

FIG. 1—TEST YARD SET-UP

Three types of circuit breaker were tested on 44,000 volts, these being the old 45,000-volt type *G B* breaker, the modern 50,000-volt *G*-11, and the modern 73,000-volt *G*-11 breaker. Two breakers were tested on 110,000 volts, namely the 115,000-volt modern *G*-11 breaker and the old 110,000-volt type *G A* breaker. It was originally intended to test the 115,000-volt type *G*-2 breaker, but this breaker was in active service in the substation and as the maximum power available did not tax the capacity of the lighter breakers, it was not felt necessary to test the *G*-2 breaker. All breakers tested were of the outdoor type, electrically operated with plain break contacts, and equipped with ordinary accelerating devices.

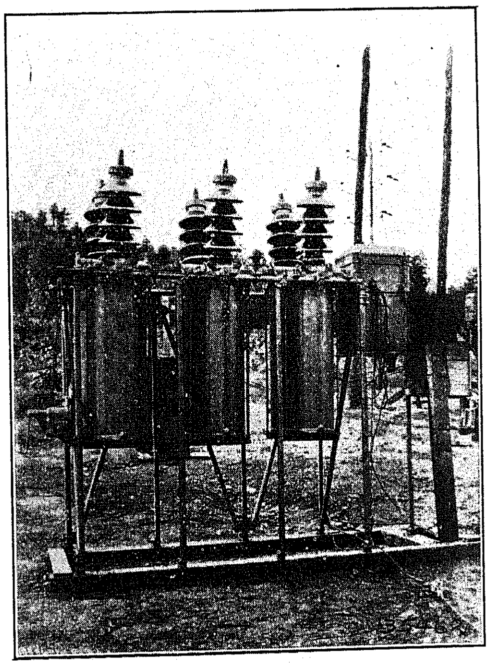

FIG. 2—50,000-VOLT G-11 BREAKER

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-11, 1924.

TABLE I
SUMMARY OF TESTS ON
50,000-Volt G-11 Breaker
on 44,000 Volts

Test No.	Opened Circuit	A		B		C	
		Current opened 1-st. Arc. Cycle r. m. s. Amps.	Cycles of Arcing	Current opened 1-st. Arc. Cycle r. m. s. Amps.	Cycles of Arcing	Current opened 1-st. Arc. Cycle r. m. s. Amps.	Cycles of Arcing
1	O. K.	..	..	..	..	1350	5.5
2	O. K.	2050	6.5	1870	4.5	1670	6.5
3	O. K.	1840	4.5	1910	..	1860	3.5
4	O. K.	1570	5.5	1935	..	1775	5.0
5	O. K.	2120	6.5	2650	..	2520	5.5
6	O. K.	2040	4.5	2650	..	2580	10.5
7	O. K.	1140	3.0	..	..	..	..
8	O. K.	2470	..	..	..	2585	6.5
9	Yes (Split Tank)	3140	8.0	3600	..	..	..
45	O. K.	2370	4.0	2700	..	..	..
46	No Test	..	..	..	..	..	..
47	O. K.	..	..	2800	..	2580	..

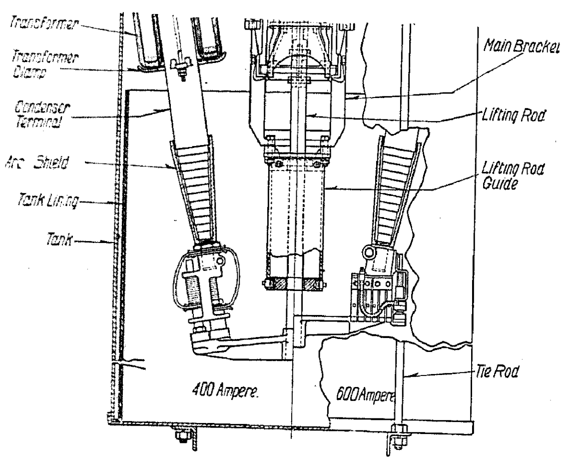

FIG. 3—50,000-VOLT G-11 CONTACTS
73,000-VOLT G-11 CONTACTS

The method of making the test has been described in another paper but we will mention that the current transformers used with the oscillographs were given a special calibration test at the factory after the tests were over, and all current values have been corrected accordingly.

TESTS ON 50,000-VOLT TYPE G-11 OIL BREAKER

Table I gives a summary of the test data on this breaker. Fig. 2 shows the breaker as arranged for test. Fig. 3 shows the standard type of contact used. This breaker is equipped with baffled vents and on heavy short circuit threw some oil through these vents. While there is nothing of particular interest in connection with the tests on the standard breaker, test No. 9 on a special arrangement of the breaker may be of some interest. For this test the breaker had been equipped with one muffler per pole, the idea being to prevent an oil spray coming through the vent. This test represented practically the maximum available on the 44,000-volt system, the oscillograph reading showing 3140 amperes on the *A* phase, and 3600 amperes on the *B* phase. The back pressure in the tank due to the muffler caused the *A* phase tank to split along the bottom seam as shown in Fig. 4. The conclusion to be drawn therefore from the application of mufflers to relatively lightly constructed high-voltage breakers is that a point in rupturing capacity is reached where the internal stresses caused thereby are beyond the strength of the breaker. The breaker when equipped with mufflers is not capable of relieving it-

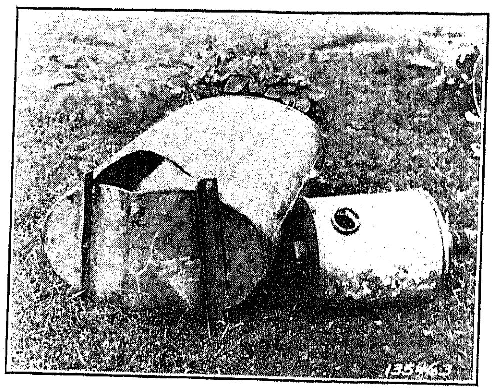

FIG. 4—MUFFLER AND TANK—50,000-VOLT G-11 BREAKER

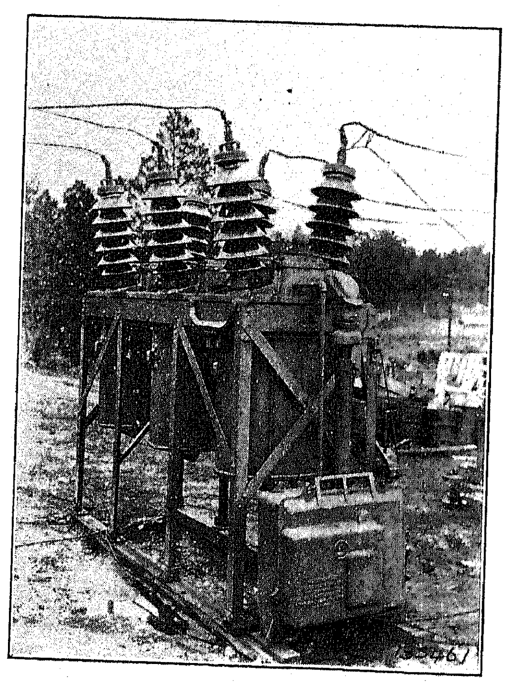

FIG. 5—73-KV. G-11 BREAKER

self as readily as with the open vent and as a result the breaker is damaged at a current below what it would have opened with the open vent. This statement has no reference to the application of mufflers to breakers constructed to withstand high internal pressures.

Deterioration of the contacts was limited to rather slight pitting of the arcing tips and there was no damage to the tank lining. Several specimens of oil were taken from tanks during the test and they indicated that

TABLE II
SUMMARY OF TESTS ON
73,000-Volt G-11 Breaker
on 44,000 Volts

Test No.	Opened Circuit	A Current opened 1-st. Arc. Cycle r. m. s. Amps.	A Cycles of Arcing	B Current opened 1-st. Arc. Cycle r. m. s. Amps.	B Cycles of Arcing	C Current opened 1-st. Arc. Cycle r. m. s. Amps.	C Cycles of Arcing
10	O. K.	See test No. 9 for Approximate Currents					
11	Yes						
12	O. K.	3300	12.5	3660	..	3330	12.0
43	Yes	3000	13.0	3220	..	2840	12.0
44	O. K.	2680	..	3140	..	2740	8.0

after the oil had settled a while it still retained insulating value suitable for further service, being in general well up towards normal dielectric strength.

Tests on 73,000-Volt Type *G*-11 Breakers

This breaker is rated at 2400 amperes at 73,000 volts

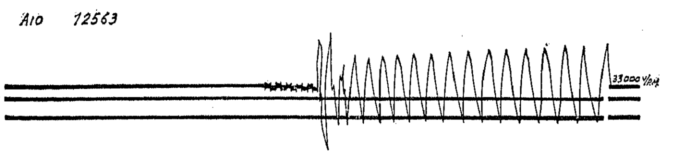

Fig. 6—Showing Voltage Reestablishment on 44,000-Volt System with No Ground Near Point of Short Circuit

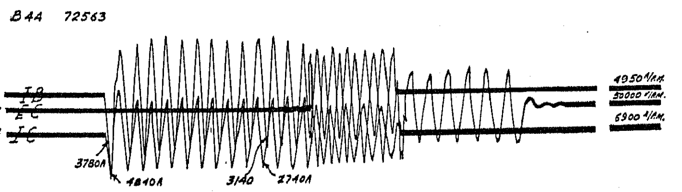

Fig. 7—Showing Voltage Reestablishment on 44,000-Volt System with Ground at Short Circuit

and the maximum current opened on it was 3660 amperes at 44,000 volts. The breaker is shown in Fig. 5 and contact construction is shown in Fig. 3. A summary of the tests on this breaker is given in Table II. Three of the tests were single interruptions of a three-phase ungrounded short circuit and the last two formed a duty cycle consisting of two interruptions at a two minute interval on a three-phase grounded short circuit. On test No. 43 considerable oil was thrown from all tanks, probably amounting to as much as ¼ in. from each tank. This was due apparently to a partial gas explosion in the top chamber which forced oil between the frame and the tank. On the succeeding test two minutes later, the oil throw was much less, probably a half gallon total from all three tanks. On the other tests the breaker threw slight amounts of oil.

In this connection, the 44,000-volt tests were made on this breaker with the short circuit ungrounded. Fig. 6 is of interest as showing a typical voltage reestablishment under these conditions, of approximately 50 per cent overvoltage. With the short circuit ungrounded, the only ground on the 44,000-volt power system was through a single transformer bank at the Lock No. 12 generating station. In contrast to this result, note the oscillogram shown in Fig. 7 taken during test No. 44 with the system grounded at the short circuit in addition to being grounded back at the

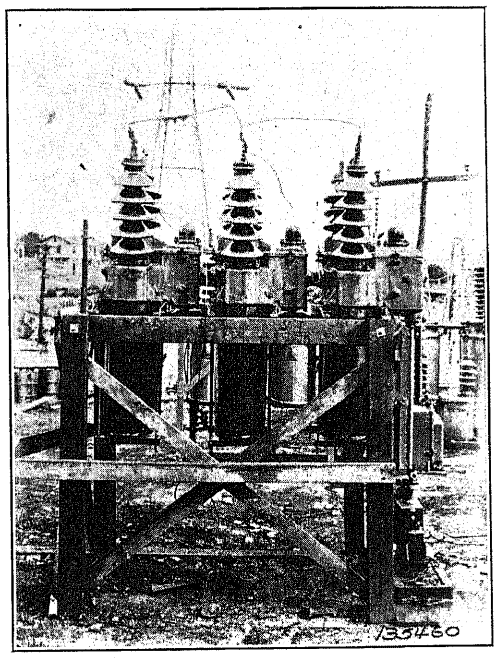

Fig. 8—45,000-Volt G. B. Breaker

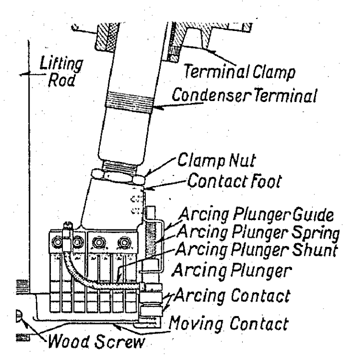

Fig. 9—G B Breaker Contacts

generating station and with approximately the same power at short circuit as in the oscillogram preceding. The complete absence of overvoltage surge with the system thoroughly grounded is of particular interest.

Tests on 45,000-Volt *GB* Breaker

The type *GB* breaker shown in Fig. 8 is an old breaker that had been on the Alabama Power Company's line since approximately 1913 and it was desirable to find out its rupturing capacity as it stood, and

TABLE III
SUMMARY OF TEST ON
GB—45,000-Volt Breaker
on 44,000 Volts

Test No.	Opened Circuit	A: Current opened 1-st. Arc. Cycle r. m. s. Amps.	A: Cycles of Arcing	B: Current opened 1-st. Arc. Cycle r. m. s. Amps.	B: Cycles of Arcing	C: Current opened 1-st. Arc. Cycles r. m. s. Amps.	C: Cycles of Arcing
13	O. K.	..	..	..	..	784	4.0
14	O. K.	740	4.5	..	..	..	..
15	..	..	..	..	..	..	..
16	O. K.	734	5.0	..	..	..	..
17	..	..	..	..	..	..	..
18	O. K.	..	..	..	..	805	4.0
19	Yes	2260	6.5	1870	..	1710	7.0
20	O. K.	1495	4.5	..	..	..	..
21	O. K.	1820	7.0	2060	..	1850	..
22	O. K.	1760	7.0	1800	..	1680	..
23	O. K.	1010	3.5	..	..	..	..
35	O. K.	2230	8.0	2500	..	2465	7.5
36	O. K.	..	..	2700	..	2660	10.0
37	O. K.	..	..	2820	..	2660	7.0
38	O. K.	2595	13	..	..	..	..
39	Bkr. did not close	2770	..	2875	..	..	..
40	O. K.	2825	..	2980	8	..	..
41	Yes	2670	..	2935	..	..	..
42	Did not open	2740	..	2950	..	..	..

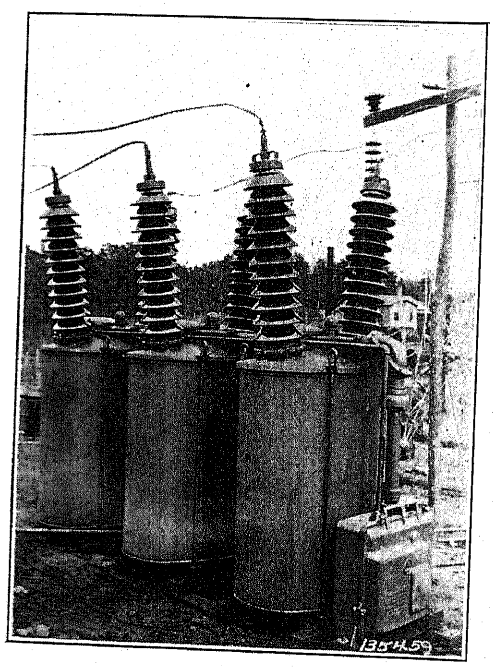

FIG. 10—115,000-VOLT G-11 BREAKER

with some improvements. The contact construction of this breaker is shown in Fig. 9 and the test results are given in Table III.

The breaker was tested on 44,000 volts, 60 cycles, three-phase, with grounded neutral, using a maximum total connected generating capacity of 139,000 kv-a. The breaker without modifications opened up 2000 amperes with the short circuit ungrounded with considerable emission of smoke and throwing of oil at this value.

The breaker was then altered by the substitution of some improved contact details and tested by seven single interruptions of three-phase short circuits, mostly grounded, one interruption of a single-phase short circuit to ground and one repetition on a two-phase short circuit to ground on a duty cycle consisting of two interruptions at a two minute interval, starting from the closed position. A maximum of 2900 amperes was interrupted with more or less throwing of oil on the various tests.

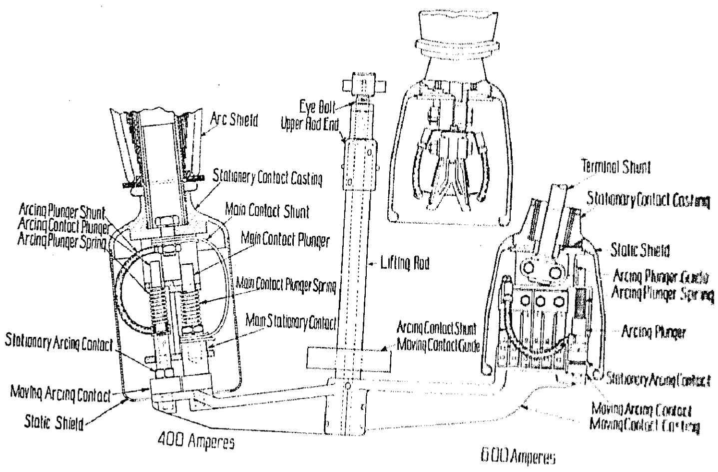

FIG. 11—115,000-VOLT G-11 CONTACTS

TABLE IV
SUMMARY OF TESTS ON
115,000-Volt G-11 Breaker

Test No.	Opened Circuit	A: Current opened 1-st. Arc. Cycle - r. m. s. Amps.	A: Cycles of Arcing	B: Current opened 1-st. Arc. Cycle r. m. s. Amps.	B: Cycles of Arcing	C: Current opened 1-st. Arc. Cycle r. m. s. Amps.	C: Cycles of Arcing
24	O. K.	894	4.0	900	5.0	..	..
25	O. K.	985	2.5	1030	..	1060	..
26	O. K.	1470	3.0	..	..	..	..
27	O. K.	1495	5.0	1540	..	1080	..
28	O. K.	1810	5.0	1850	..	1790	..
29	O. K.	2030	7.0	..	..	2225	..
30	O. K.	2500	7.0	..	..	..	..
48	O. K.	..	..	..	..	2640	13.0
49	O. K.	..	..	..	..	2850	8.0

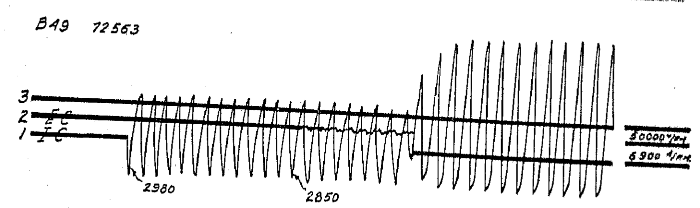

FIG. 12—115,000-VOLT G-11 BREAKER. OPENING 2850 AMPERES AT 115,000 VOLTS

Two attempts were made to close this breaker against short circuit, but they were not successful as the arrangements of the control wiring in connection with the oscillograph operation could not be properly worked out in the limited time. In the last one of these attempts, after the breaker had opened several

short circuits, and the oil was at a relatively low value, imperfect contact in the breaker caused heavy arcing and emission of flame. The breaker was mechanically undamaged except for burning on the arcing tips.

Tests on 115,000-Volt Type *G*-11 Breakers

This breaker is shown in Fig. 10 and its contact

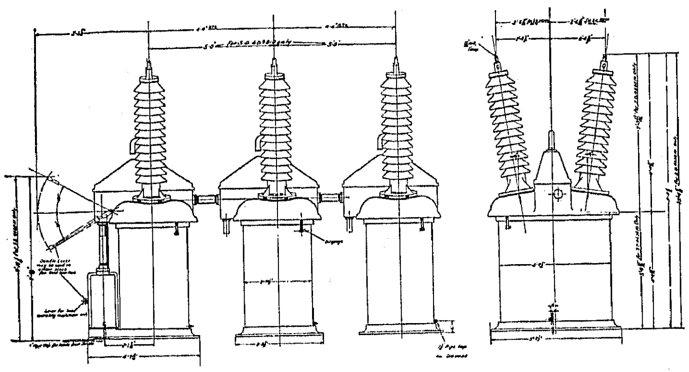

Fig. 13—G. A. 110,000-Volt Breaker

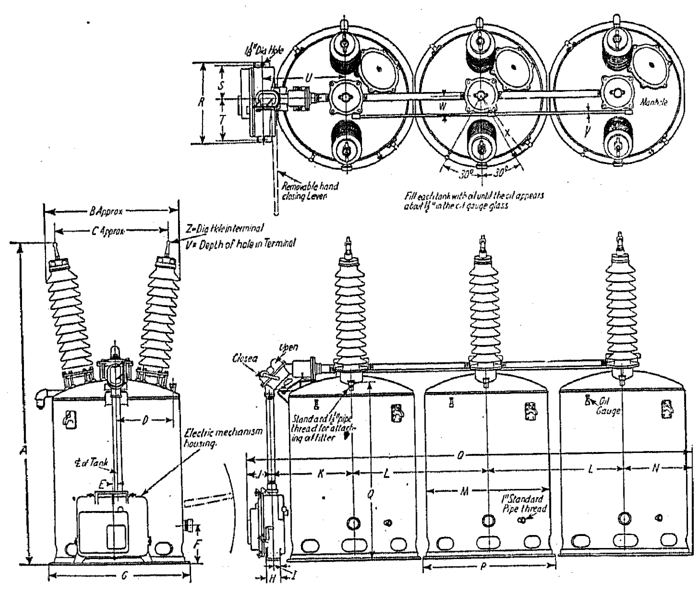

Fig. 15—115,000-Volt G-2 Breaker

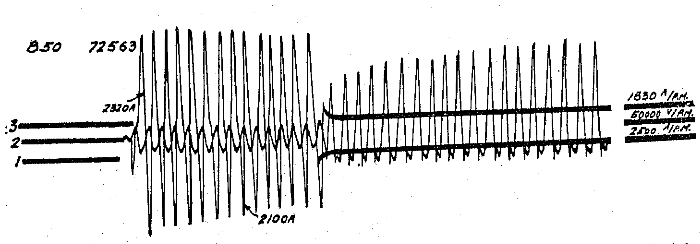

Fig. 14—110,000-Volt G A Breaker. Openihg 2100 Amperes at 115,000 Volts

construction shown in Fig. 11 while the tabulation of test data is given in Table IV. This breaker was tested on a 115,000-volt, 60-cycle, three-phase, grounded neutral system, using a maximum total connected generating capacity of 188,000 kv-a. in addition to a tie in with the Georgia Power Company which transmitted a relatively small amount. Two single interruptions were performed on a three-phase grounded short circuit; two on a short circuit from one phase to ground, one duty cycle consisting of a single interruption of a three-phase short circuit to ground followed at a two-minute interval by an interruption of a single-phase short circuit to ground; and one duty cycle consisting of three single interruptions at two-minute intervals of successive three-, two- and single-phase short circuits to ground in the order stated.

The maximum current interrupted was 2850 amperes in a single-phase short circuit to ground. At the end of the test the oil in the tanks was clear and the contacts in good condition. No oil was lost in any of these tests with the possible exception of a few drops at a terminal flange or so. These tests serve to emphasize the adequate construction of this breaker in every way and also indicate that a thoroughly grounded system such as this 110,000-volt system gives easier switching duty than one that is ungrounded or im-

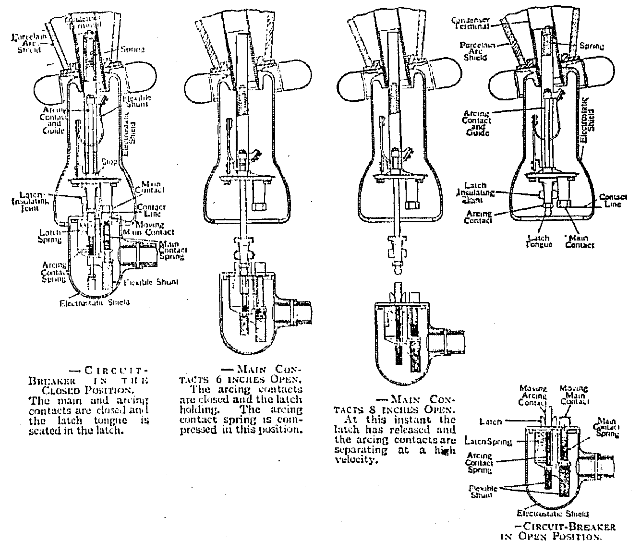

Fig. 16—High-Speed Contacts for 115,000-Volt G-2 Contacts

perfectly grounded. Fig. 12 shows an oscillogram taken in test No. 49 with the breaker opening the maximum single-phase short circuit to ground that was possible on the system, or 2850 amperes in the arc.

Tests on 110,000-Volt *G A* Oil Breaker

The 110,000-volt type *G A* breaker shown in Fig. 13 was purchased in 1913 and has been continuously in operation at the Magella substation of the Alabama Power Company. This breaker had originally an interrupting capacity of approximately 1900 amperes at 110,000-volts and was tested on the 115,000-volt, 60-cycle, three-phase, grounded neutral system, using a maximum total connected generating capacity of 169,000 kv-a. The breaker performed three single interruptions and one duty cycle consisting of two

interruptions at a two-minute interval, on a three-phase grounded short circuit. The maximum current interrupted was 2100 amperes. The only sign of distress was a slight leakage of oil from around the terminal clamps in one or two tests. It was not possible to determine the number of cycles of arcing in this breaker as the oscillograph equipment was located 14 miles away at Bessemer and the drop through the line from the point where the potential transformers were connected was so large as to make impossible the detection of the initial cycles of arcing. However, referring to Fig. 14, which shows the breaker opening a dead short circuit with 2100 amperes in the arc, and making allowance for the operating time of the mechanism, we find that the arc probably endured for 10 cycles.

115,000-Volt Type *G*-2 Breaker

As a matter of interest Fig. 15 shows the 115,000-volt type *G*-2 breakers located in the switch yard at the Bessemer station. Fig. 16 shows the contruction of the high-speed contacts used with this high-power breaker. Due to limitations of power available on short circuit, it was not thought necessary to test this breaker and it is felt that in the near future arrangements can be made for such a test. The high speed of contact separation on the *G*-2 breaker increases its rupturing capacity greatly by reducing the number of arcing. Its rupturing capacity is further increased by the mechanical strength of structure which consists of a boiler iron tank with riveted and welded top and botton domes.

Discussion

For discussion of this paper see page 655.

Circuit Breaker Tests at Bessemer, Ala.

On 300-Ampere, 110,000-Volt Breakers

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THE tests herein described were made possible through the cooperative efforts of the Alabama Power Company's officials, engineers and operators. The writer desires to express his appreciation to them, and to call attention again to the value that

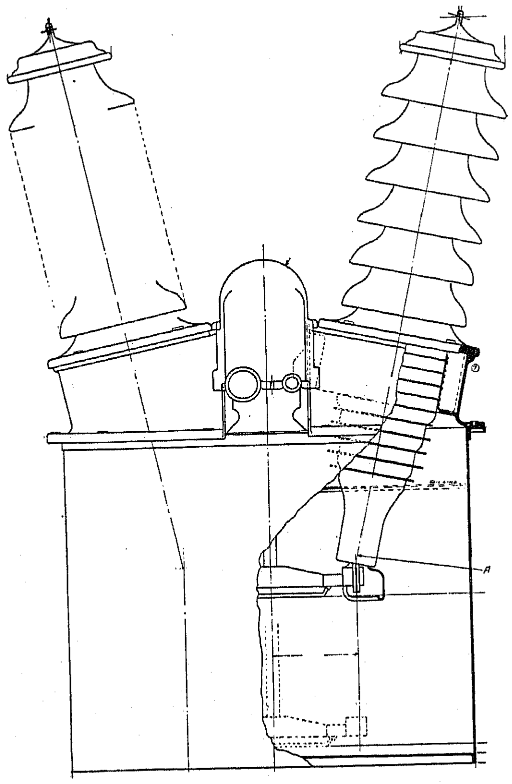

FIG. 1

properly conducted field tests have in the manufacture of apparatus.

With the increase in capacity of the generating equipment of the Alabama Power Company, the power available under short-circuit conditions became so great at various substations that it exceeded the rated interrupting capacity of the oil circuit breakers on the 110,000-volt circuits. The Alabama Company had installed on its system a large number of type *F K O-21A* and *F K O-22 A*, 300-ampere, 110,000-volt oil circuit breakers and the General Electric Company was asked if the rating of these breakers could in any way be increased and the breakers made more satisfactory to handle the increased load.

In this paper the results of the tests on the rebuilt 110,000-volt breaker, together with the tests on plain break and explosion chamber 44,000-volt breakers, are described. The 110,000-volt breakers as originally supplied to the Alabama Company were of the plain break contact-finger type shown in Fig. 1.

The arrangement of the system, the general method of test, and a discussion of the effect on the system are described in another paper presented at this session.

110,000-VOLT TESTS

These breakers were rebuilt to equip them with explosion chambers. This necessitated increasing the height of the oil tank. New insulating linings were also added. The plain break contacts originally used are shown on Fig. 2 and the explosion chamber contacts with which they were replaced and on which the tests were made are shown in Fig. 3.

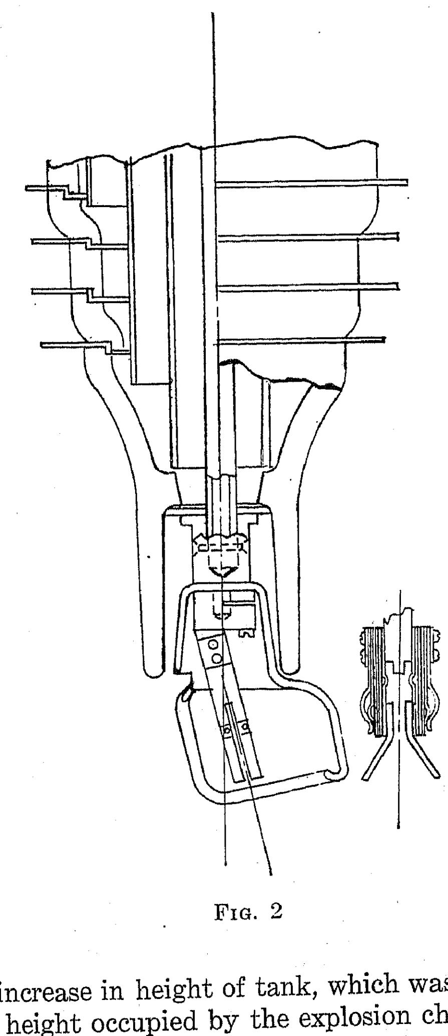

FIG. 2

The increase in height of tank, which was to provide for the height occupied by the explosion chamber, was done by the Alabama Power Company by means of an acetylene weld. The remainder of the materials necessary to make the changes were supplied by the General

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-11, 1924.

Electric Company, all the work of installation being done by the Alabama Power Company. After the specified changes had been made on a single pole unit, tests were made upon it in order to observe its action under various short-circuit loads, and to determine that it had a safe interrupting capacity of 450,000 kv-a. under their circuit conditions. The system arrangement and line reactances upon which the tests were made are shown in Fig. 4. These tests were made during January, 1922.

During the tests, the test breaker was tapped from one line of a Y-connected grounded transmission system, namely on phase 3 of line *B* from Magella to Bessemer.

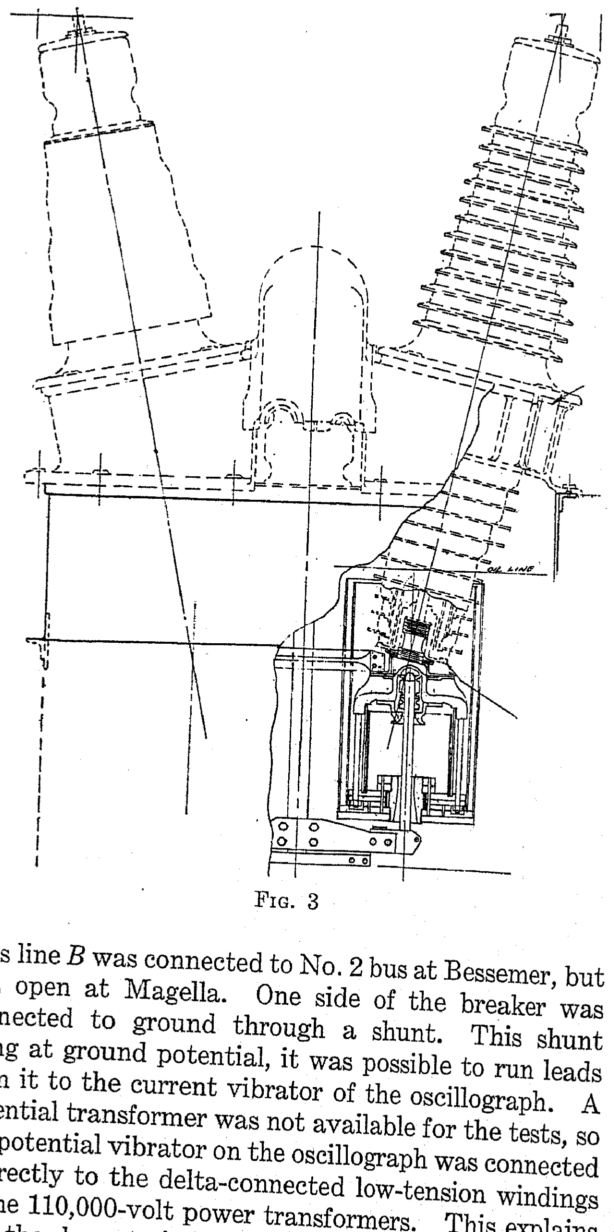

Fig. 3

This line *B* was connected to No. 2 bus at Bessemer, but was open at Magella. One side of the breaker was connected to ground through a shunt. This shunt being at ground potential, it was possible to run leads from it to the current vibrator of the oscillograph. A potential transformer was not available for the tests, so the potential vibrator on the oscillograph was connected indirectly to the delta-connected low-tension windings of the 110,000-volt power transformers. This explains why the characteristic arcing record is not shown on the oscillogram.

The short circuit was thrown on by service-breaker No. 806 and was opened by the breaker under test. In no case was the breaker under test used to close as well as to open the circuit and in this respect conformed to the Baltimore tests which preceded it. It should also be noted that all tests were made single-phase from line to ground and that the neutral of the power transformers were therefore grounded.

To avoid any possibility of injury to the oscillograph operator, the oscillographic apparatus was located upon an insulating stand.

The attempt was made in some of the tests to obtain pressure records by means of an indentation recorder—steel ball on a copper plate—but it was so slow in operation that the attempt was given up. The test records show the currents, voltages and speeds.

In considering these tests, it should be realized that an old style breaker structure was under investigation, one which had been built without any especial preventive means to guard against oil throw, and which in comparison with our present day standards was wide open for the escape of oil.

During the test, circuit connections were changed and generators added or taken off as necessary to give the desired interrupted current. The following table is a summary of the results of the tests.

R. m. s. Amps. in Arc	Equivalent kv-a. 3-Phase	Arc Length Inches	Contacts Speed Ft./Sec.	Oil Throw	Signs of Distress	Approx. Generators Connected
220	38,100	4.6	4.87	None	None	70,000 kv-a.
445	84,800	3.95	4.65	"	"	80,000 kv-a.
1215	231,000	8.35	5.56	1 pt.	2	70,000 kv-a.
1225	233,000	8.74	..	None	"	114,000 kv-a.
1560	297,000	7.87	5.48	1 pt.	"	70,000 kv-a.
1710	326,000	7.65	6.64	1 pt.	"	86,000 kv-a.
1720	327,500	8.35	5.75	1 pt.	"	144,000 kv-a.
1935	368,300	7.46	5.75	2 qts.	"	114,000 kv-a.
2010	382,500	8.86	6.25	1 qt.	"	114,000 kv-a.
2180	415,000	7.35	6.65	1 qt.	"	114,000 kv-a.
2220	422,500	7.66	6.30	1 pt.	"	144,000 kv-a.
2220	422,500	8.23	6.40	1 pt.	"	144,000 kv-a.
2225	424,000	10.00	6.55	1 pt.	"	144,000 kv-a.

The arc length was determined from the speed recorder and the design data of the breaker. The lack of a potential transformer across the breaker contacts detracts somewhat from the value of the films, but, nevertheless, there are some that are of interest in showing the nature of the results. Fig. 5 shows an interruption with a current value of 2740 amperes as the r. m. s. of the first peak of short circuit and 2220 r. m. s. amperes in the first half cycle of arc. *A* is the voltage across line 1 and 2, *B* is the current in line 3, and *C* is a speed record.

It will be noted that the decrement current was small and this is characteristic of all of the tests.

Fig. 6 shows the current and speed records on an interruption having a maximum r. m. s. at first half wave of short circuit of 2370 amperes, and 1710 amperes in the first half wave of arc.

Fig. 7 shows the current and speed on an interruption of 1610 amperes r. m. s. first half cycle at short circuit and 1235 r. m. s. first half cycle of arc. The

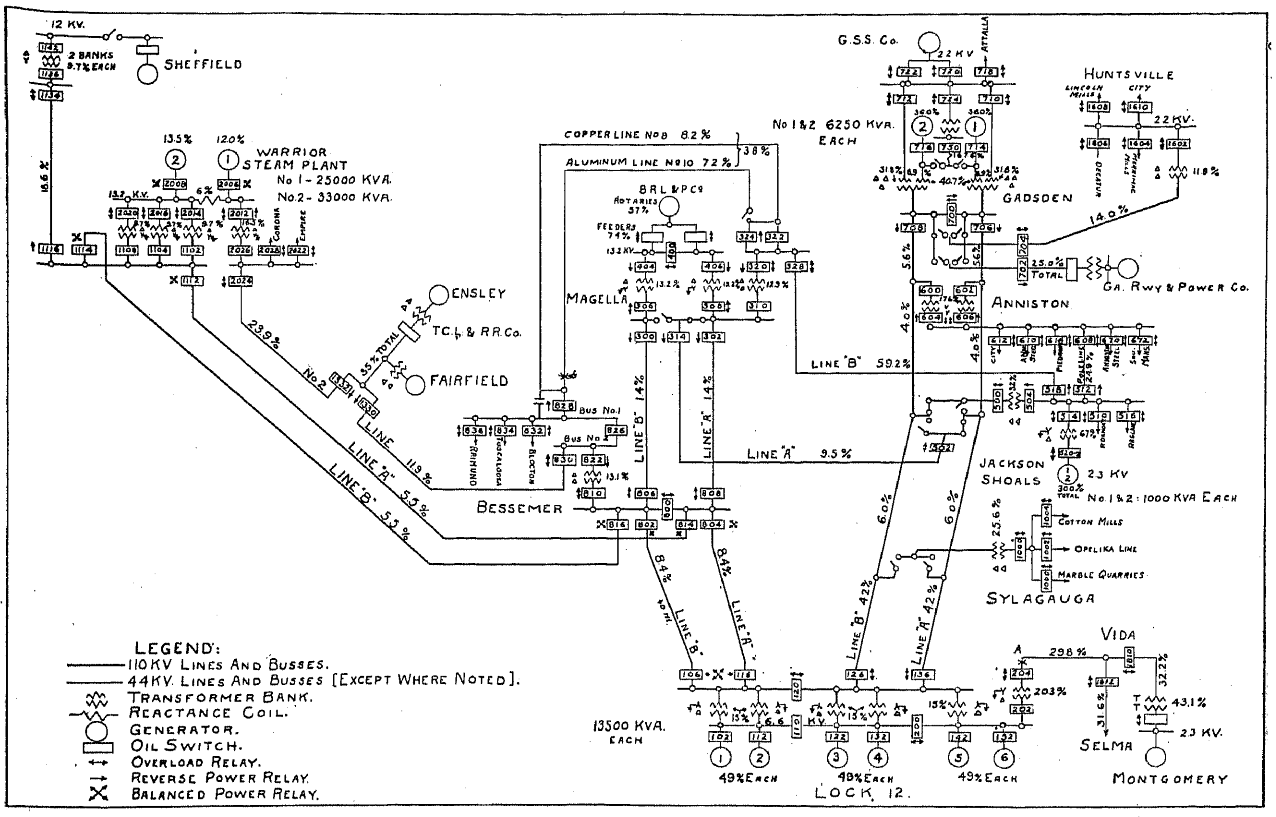

FIG. 4

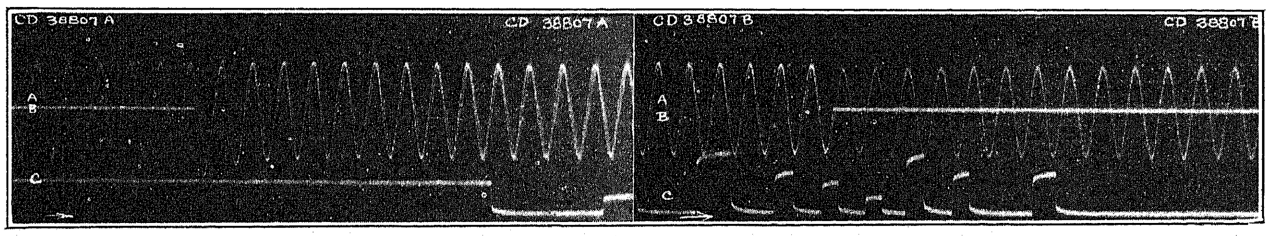

FIG. 5

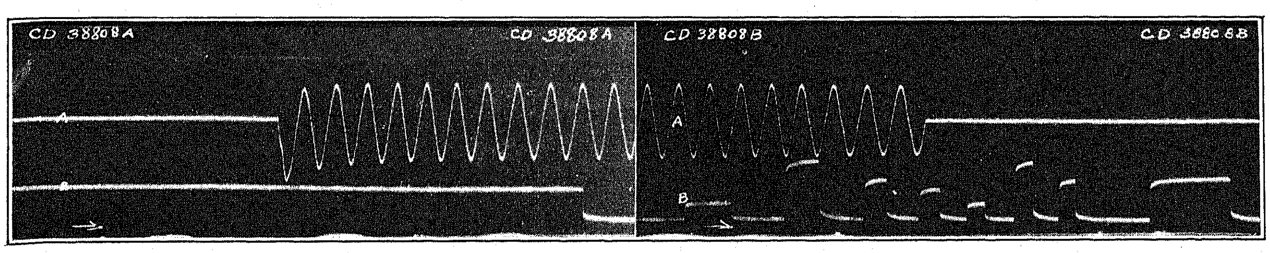

FIG. 6

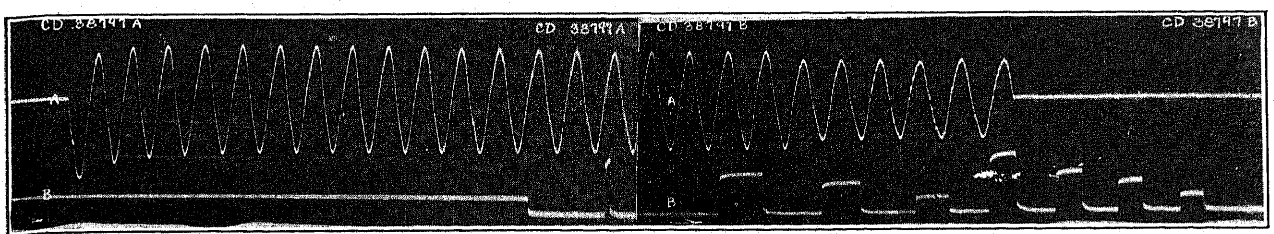

FIG. 7

falling off of current during arcing was due to breakers 106 and 308 opening before the short circuit was cleared by the test breaker.

44,000-Volt Tests

The tests at 44,000 volts were made both at Lock 12 and at Bessemer during January, 1922. The breakers

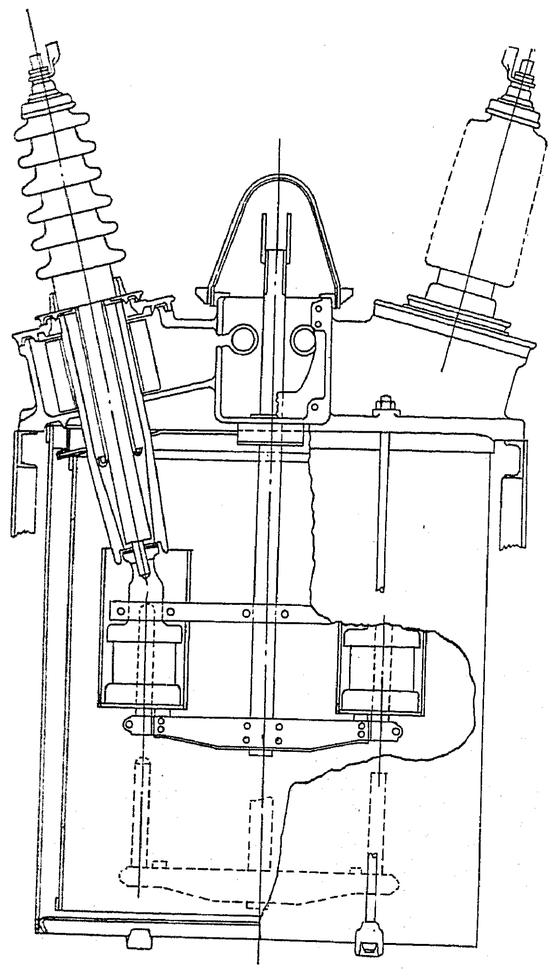

Fig. 8

tested were a 50,000-volt *F H K O*-36 *B* breaker, which had been arranged so that the explosion chamber contacts with which it is normally equipped could be replaced by plain break contacts. Both the plain break and explosion chamber contacts in this breaker were tested at Bessemer but only the explosion chamber was tested at Lock 12. The 50,000-volt explosion chamber breaker is shown in Fig. 8. The plain break contacts are shown in Fig. 9.

The 44,000-volt tests at Bessemer were made single phase by tapping the breaker onto phase No. 3 on the aluminum line from Bessemer to Magella and all tests were from No. 3 line to ground. All short circuits were closed by breaker No. 826 as shown on Alabama Power Company's diagram, Fig. 4.

On the lock 12 tests, the breaker tested was tapped onto the Montgomery circuits and the short circuits were thrown on by a transition model *K*-36-*A*, 50,000-volt breaker. All tests were made single phase from line to ground and in all cases both at Bessemer and Lock 12 the circuits from which the test line was tapped were carrying the ordinary commercial load. This latter fact should be kept in mind in considering the results of the test.

Oscillogram Fig. 10 shows an explosion chamber recorded when interrupting 1404 arc amperes. *A* is the voltage across the arcs and *B* the current interrupted. It should be noted that the arc records have all of the characteristics of the plain break arc and none of those of the explosion chamber arc. Fig. 11 shows another oscillographic record of an explosion chamber interrupting 1055 arc amperes and the remarks made above apply here also. Both of the above records were made at Bessemer. Fig. 12 shows an oscillogram of plain break contacts, when interrupting 1005 arc amperes and Fig. 13 shows the same breaker when interrupting 980

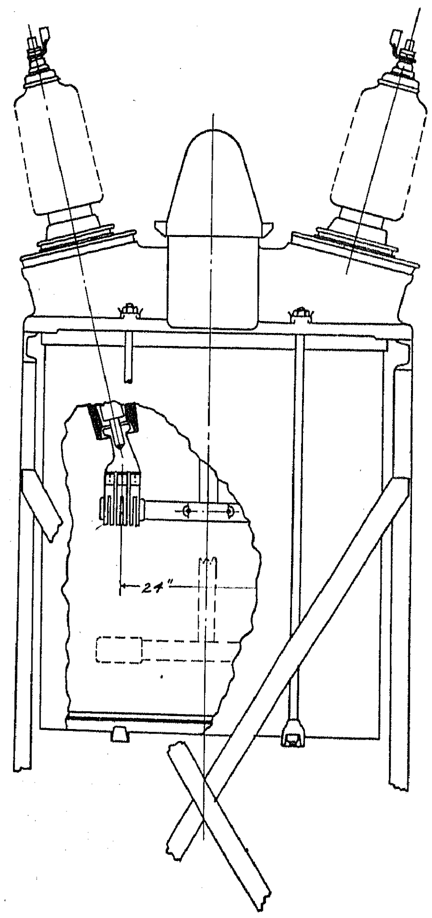

Fig. 9

arc amperes. Both the above oscillograms show the regular characteristics of the plain break arc and should be compared with the explosion chamber arcs of Figs. 10 and 11.

The results of the Lock 12 tests, as would be expected from the test conditions, did not differ from those made at Bessemer, and no especial remarks need be made

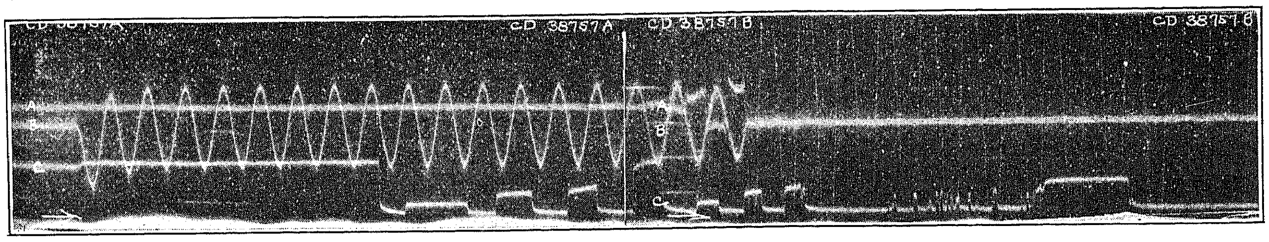

FIG. 10

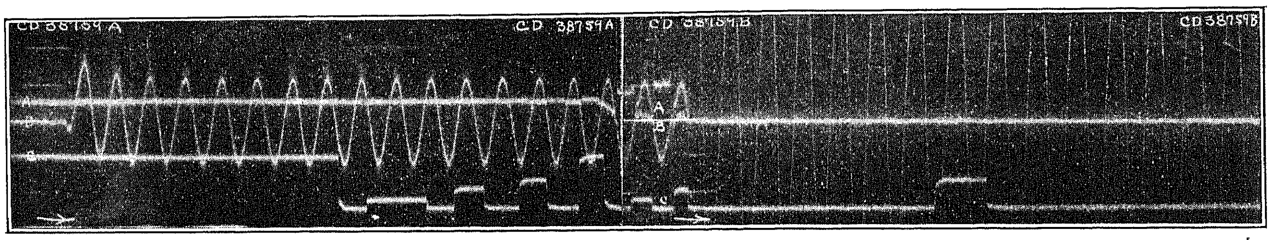

FIG. 11

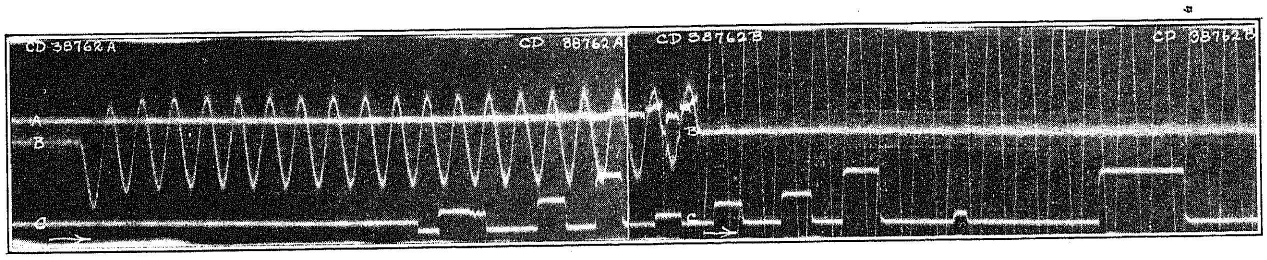

FIG. 12

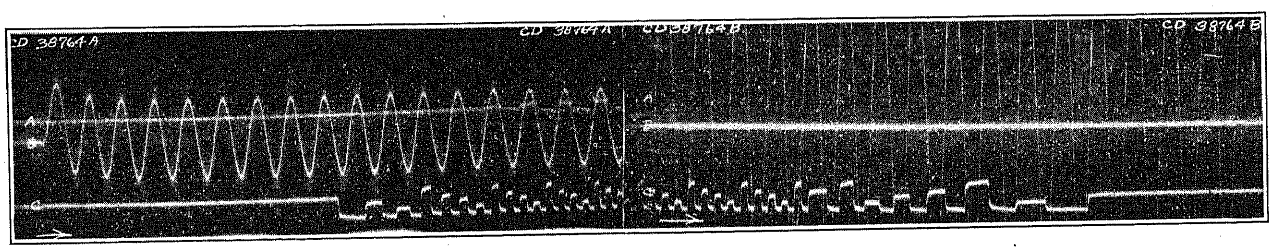

FIG. 13

THE 44,000-VOLT TESTS

Location	Amperes First Half Wave of Short Circuit	Amperes First Half Wave of Arc	Equiv. kv-a. Interrupted at Arc 3-Phase	Arc Duration Half Cycle (60-Cycle)	Speed Ft. Sec.	Oil Throw	Type of Contact
Bessemer	1082	805	61,300	4	6.2	None	Explosion Chamber
"	920	865	65,800	7	7.1	"	" "
"	1093	895	68,000	7	7.1	"	" "
"	1300	1040	79,000	8	6.85	"	" "
"	1138	1080	82,100	8	6.85	"	" "
"	1265	1055	80,200	9	7.4	"	" "
"	1220	1090	82,900	5	6.6	"	" "
"	1340	1090	82,900	8	..	..	" "
"	1340	1005	76,400	7	5.7	"	Plain Break
"	1130	1060	80,560	9	5.8	"	" "
"	1180	980	74,400	8	7.2	"	" "
"	1135	980	74,400	9	7.2	"	" "
"	980	980	74,400	2½	6.0	"	" "
"	1142	980	74,400	4½	6.66	"	" "
"	1185	980	74,400	7½	7.0	"	" "
"	474	405	30,750	..	..	..	" "
"	816	695	52,700	4½	6.65	"	" "
Lock 12		1100	83,600	8½	7.0	"	" "
" "		1065	81,000	7	6.9	"	" "
" "		1065	81,000	7½	7.3	"	" "

concerning them, except that they were stopped by the failure of a power transformer, failure taking place about 0.30 of a second after the test breaker had cleared the circuit. Inasmuch as the currents interrupted during these tests were comparatively small, the results, as would be expected under such conditions, do not show the full advantage of the explosion chamber. At such low currents the explosion chamber breaker functions very much as a plain break breaker.

In order to be efficient, the explosion chamber breaker must operate at high pressure in the chamber, hence a breaker designed to have an interrupting capacity of 3850 amperes at 44,000 volts, (the real interrupting capacity of the explosion chamber breaker tested) does not begin to show its remarkable current-interrupting capacity until approximately the safe limit of the plain break contact in the same tank is reached. Up to about that time it functions largely as a plain break breaker and is given the break distance required by such a breaker. From that point on, the gas pressure acting on the rod increases, the breaker speed increases, the arc becomes shorter and at maximum rating the arc duration is a minimum. The designer has full control of these characteristics and the breaker as a whole can be largely designed to meet any particular condition. The ability to control the speed of operation of the contacts by means of the gas pressure in the explosion chamber is of great operating value, as this pressure takes the place of accelerating springs, but offers no resistance to closing, as do the springs.

Conclusions

The tests herein reported indicate in the case of the 110,000-volt *K*-22 breakers that the interrupting ability of the breaker could be materially increased by the addition of explosion chambers. Although no data on the operation of the breakers without explosion chambers were obtained during the tests, the performance of such breakers was observed under actual service conditions prior to the test, so that a fairly accurate knowledge of their interrupting ability was obtained.

As has already been stated, the current available on the 44,000-volt tests was not sufficient to stress either the plain break or the explosion chamber breakers. It is not believed that the tests at Bessemer produced as severe a duty on the breaker as might be obtained under other conditions with the same current interrupted. This statement is based on the general knowledge of the factors affecting interrupting duty.

These factors, as determined by the system itself and the method of tests, operate in such a way that in my opinion the duty on the breakers was not maximum for the currents interrupted. Upon another system or in another location, the arc lengths might have been much longer at the same currents interrupted and that would mean more gas, greater pressures and decreased factor of safety.

All should realize that 1,000 amperes, for instance, interrupted at a definite voltage does not necessarily mean a definite stress to the circuit breaker. It does not mean a definite arc length nor a definite quantity of gas generated, as everything depends upon the existing conditions at the time of interruptions.

The manufacturer must necessarily design circuit breakers to safely rupture their rated current under the worst obtainable conditions. There are certain locations, however, where the conditions are such that the duty is light and it is reasonable for the power company to take advantage of such conditions in the breaker installations, provided it is understood that a change in conditions may make the breaker unsafe.

If any discounting of the absolutely safe interrupting rating of the breaker by the manufacturer is made by the power company it should be on its own responsibility, because it is the power company who has the knowledge of and control over the conditions.

It is my opinion that all primary breakers should be constructed so that they are safe with a definite minimum factor of safety under the worst obtainable conditions.

Discussion

For discussion of this paper see page 655.

Oil Circuit Breaker Investigation

as Carried on with a 26,700-Kv-a. Generator

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Although oil circuit breakers have been used for a number of years and many successful designs produced, the work of the designer has been handicapped by the lack of definite design constants, based on experimental results. On some of the earlier designs, field tests were relied upon for the confirmation of the interrupting rating. Such tests were of great value, but, due to the inherent erratic behavior of oil circuit breakers as a current interrupting device, the data obtained was not usually of a general or fundamental nature. To remedy this situation and to permit continuous and consistent research on the interrupting characteristics of alternating-current circuit-controlling devices, a special testing equipment was installed.[1]

Testing Equipment

This station contains a three-phase specially built

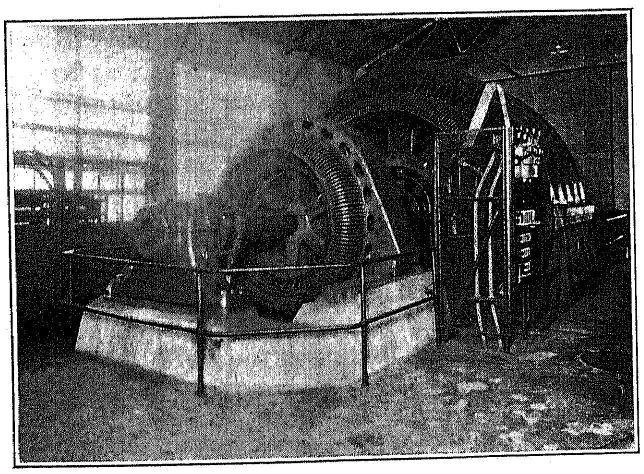

Fig. 1—Testing Generator at 1-10-26,700-300-25 13,200/7620/6600/3810 Bldg. 60E

26,700-kv-a., 25-cycle alternator of low reactance. The windings are arranged for connection to give 13,200, 7620, 6600 or 3810 volts. A three-phase 1500-horse power direct-connected induction motor is used as the driving power. The generator and driving motor are shown on Fig. 1.

The short-circuit current supplied on short circuit is controlled by means of reactors having ten taps and a maximum value of 6.1 ohms per phase.

In addition to the generator, high-voltage transformers of low reactance are provided. These transformers permit three-phase testing at any voltage up to 44 kv. and single-phase testing up to 132 kv. These transformers with the bus construction are shown on Fig. 2.

The measuring equipment, in addition to the usual

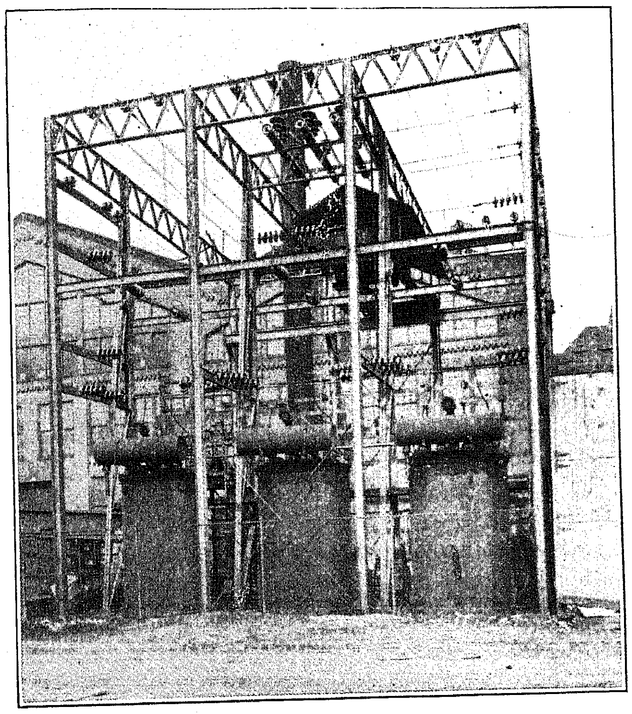

Fig. 2—High Voltage Testing Equipment of High Capacity Testing Station Bldg. 60 E

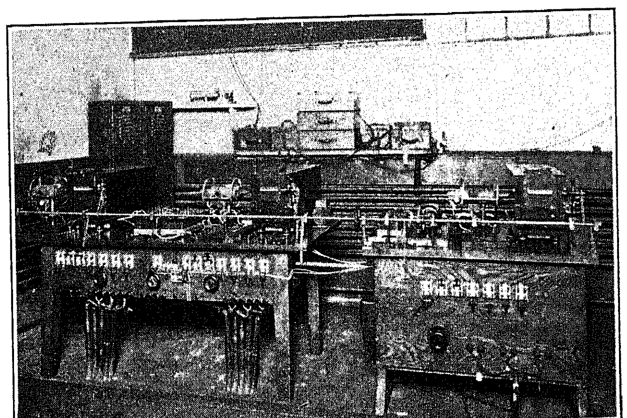

Fig. 3—Three Oscillographs on One Driving Shaft as Used in Control Room of High Capacity Testing Station, Bldg. 60 E

meters, consists of three oscillographs, pressure recorders, speed recorders, and such other apparatus necessary for the collection, analysis and measurement

1. C. E. Merris, *General Electric Review*, June, 1923.

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-10, 1924.

of gas. The oscillographs are shown on Fig. 3. The general diagram of connections of the station, together with detailed connections of the oscillographs, is shown in Figs. 4 and 5.

This equipment will produce short circuits approximating 300,000 kv-a., three-phase, at 13,000 volts.

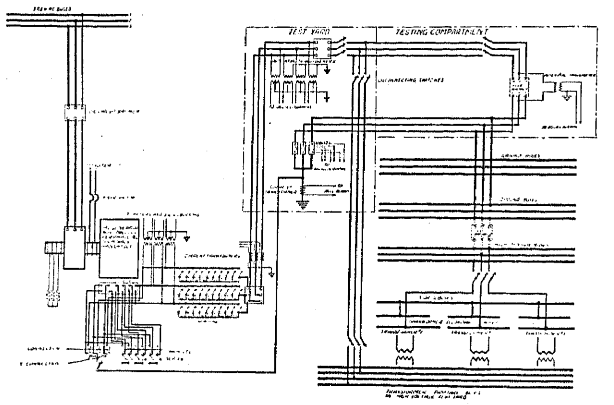

Fig. 4

The equivalent three-phase kv-a. at 13,200 volts can be increased to approximately 600,000 kv-a. by single-phase test to ground at 7630 volts. Approximations of short circuits very much higher than 600,000 kv-a. can be obtained by testing one break on single-phase tests.

In order to provide protection against injury to the

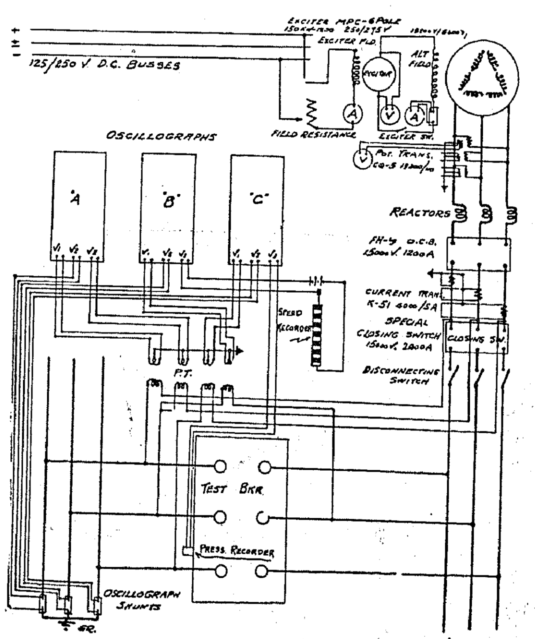

Fig. 5

observers from burning oil or flying particles during the tests, a bomb-proof of brick, steel and concrete was constructed. This bomb-proof has one side open so that observations can be made from a safe distance.

It is the purpose of this paper to discuss some of the characteristics of oil circuit breakers as determined from the tests with this testing equipment, and to indicate as far as possible the effect of some of the factors considered.

The Factors Investigated

Break Distance. The satisfactory operation of any oil circuit breaker depends upon the break distance. This break distance must be ample to interrupt the arc under the severest condition of operation or there will be a permanent gas generation which will quickly result in the destruction of the breaker. The required break distance for any given voltage and interruption is determined by the circuit connections, *i. e.*, grounded or ungrounded, power factor, connected shunt load, amperes interrupted, pressure in the oil tank, tank cross-section, etc., and it is evident that to have an absolutely safe breaker, the worst conditions must be assumed. This condition is fortunately a dead short circuit at the generator terminals on an ungrounded system without shunt load, and it is under these conditions that our rating tests are made. If the breaker is to be used on a line where easier conditions exist, then the breaker may have a larger factor of safety than under its rating condition. Cases have been observed where the break distance of breakers was inadequate for operation in generating stations but proved to be satisfactory in substations because, due to the arrangement and connections of the system, the arc length obtained for a given ampere interruption was less.

Speed of Break. The interrupting capacity of a breaker depends upon the speed of break, but one cannot say that the higher the speed, the greater the interrupting capacity in every case. The interrupting capacity of a breaker depends not only upon the quantity of gas generated, but upon the speed of generation, and it may well be that a given breaker, if operated at higher speed will have a less interrupting capacity. The higher speed may well result in a longer arc, in more gas and more pressure than if operated at the lower speed, and this condition has been observed in test. Before we can determine the effect of a speed change, we must know many factors relating to that particular breaker.

In what has been written about speed of break, it has been assumed that the moving contact was traveling at practically uniform speed. As a matter of fact, however, every breaker will have its own speed characteristic and this characteristic at no load may be decidedly different from the full interrupting capacity speed. In fact, at some load, the speed may not only slow down but actually stop and reverse in direction so as to reclose the breaker. There are several reasons for this behavior and none of the plain break breakers can be considered as entirely unaffected by it. Whether the defect is a serious one in any particular case can only be determined by actual test of the breaker under severe conditions.

In the case of fairly low voltage breakers, operating to interrupt large current, the actual speed of the moving contact may have little relationship to the interrupting capacity of the breaker, as such breakers interrupt the arc by the magnetic blowout effect instead of the physical separation of contacts. It may be found, however, that the heaviest stress is not produced by the largest current interrupted and that more gas and a greater pressure is produced when interrupting a lesser current than that of the maximum rating. It is needless to say that the breaker must be safe when interrupting these smaller currents and that this fact must be considered in the rating of the breaker.

Oil Head. The head of oil over the contacts influences the interrupting capacity of the breaker, as it largely determines the pressure above and below the oil surface and therefore tank rupture. It also determines the arc stabilizing, shock to the entire breaker structure, oil throw and gas ignition. Too much oil in the tank is as bad as too little. The correct quantity to use can only be determined by repeated tests at all loads up to the interrupting capacity rating of the breaker. In testing oil circuit breakers on skids, it is frequently noticed that the breaker jumps clear from the floor at the instant of interruption. This, of course, is due to the kinetic energy in the oil as a result of being blown by the arc gas, which is expended when the oil mass strikes the top of the breaker. This shock may be so severe as to break the top casting of the breaker.

Air Space above the Oil. The proper air space above the surface of the oil will vary with each individual breaker and the correct quantity has to be settled by the designer as a result of his observations of the action of various breakers under test. It should be noted that the tank pressure is not the only factor to be considered as affected by the air space, as there are also oil throw, arc stabilizing, secondary explosions, and gas ignition. Oil head and air space must be considered together.

Various Types of Venting. The venting of an oil circuit breaker is an important problem, as it affects the oil throw, tank pressure, and gas ignition. It has come to be recognized that the modern high class breaker must limit the ejection of oil or incandescent gases into the room at the breaker, and in order to accomplish this end a thorough investigation of the problem was made. As a result of these tests, the oil-throw problem is well in hand but it should be realized that small and inexpensive breakers will not be free from oil throwing at extreme loads. The non-oil-throwing breaker is a comparatively modern product, and the vast majority of breakers now in use were designed with little regard to the question of oil throw. Hence their construction does not readily lend itself to the rebuilding into a non-oil-throwing type.

Determination of Allowable Tank Pressure. In oil circuit breaker design, the maximum instantaneous pressure to which the structure may be safely subjected is of great importance, as it determines the safe interrupting capacity of the breaker. The foregoing statement refers particularly to those breakers having tanks of other than circular shape. Such information can only be had as a result of repeated tests when utilizing suitable recording instruments in connection with a source of power such as our large testing generator. Any calculations of static stresses which the structure will withstand are difficult to make and the results obtained, due to the lack of proper constants, are wide from the actual permissible pressures.

Method of Tank Construction and Best Material. Here also the use of the testing generator was invaluable as it settled questions which had been debated previously, but without any definite result, and the results of these tests are sure to show up in future records of performances.

Methods of Tank Support. Definite results have been obtained from the investigation of the method of supporting the tank and in all new breaker design advantage is taken of the findings from the tests.

Tank Lining Investigation. The question of the lining of the oil tank and, in fact, whether it should be lined or not, is one of great importance. The tests made with the large testing generator have definitely answered this question. They have shown that linings are in most cases necessary and that there is a decided difference in the efficiency of various linings. Just why one type is best in one case and another type better under other conditions has been determined.

Contact Investigation. One of the most important questions in oil circuit breaker construction is that of the contacts, both main carrying contacts and arcing contacts, and more attention is given this one feature than any other single feature entering into the breaker construction. The investigation of this feature included not only the burning and carrying capacity of the contacts, but also the heat generating and dissipating characterisitcs of the connected studs and bus bars, the effect on the brush by the shock of closing of the breaker, the degree of over-travel the brushes will stand, the permanency of the brush structure, the specification for the brush metal to give best results, the best contact pressure and area of contact, and the design of contacts so that they will not be affected by the magnetic stresses under short-circuit condition.

The burning of the contact members of an oil circuit breaker is a very important factor in the entire oil circuit breaker investigation and probably receives more attention, while the breaker is undergoing tests, than any other single feature. Repeated tests are made at the standard number of duty cycles. This means that the breaker is tested in closing on a short circuit, as well as opening the short circuit and under duty conditions, the equivalent of the rated interrupting capacity of the breaker and worst possible circuit conditions. Contacts are also tested to destruction in order that we may

have definite information as regards the maximum continuous duty the contacts will stand.

The brush heating is the chief feature of the circuit breaker which may be adversely affected by the action of the operating company. This may come from the improper adjustment of pressure at installation, by the use of insufficient bus bars section, by the heat insulation of that section, by the insulating tapings, by the running of cables carrying large currents causing eddy current losses, by installation of tanks close together, which decrease heat radiation, and by the installation in unventilated cells in locations where the ambient temperature is high.

Ability of Breaker to Withstand the Shock of Oil Throw. Without the generator of large capacity, it

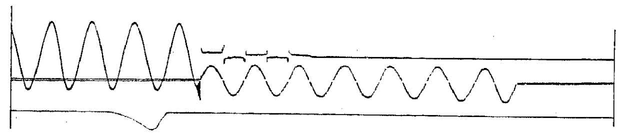

Fig. 6

would have been impossible to test for this condition which in the past has been responsible for serious breakages.

Reclosing Characteristics of the Breaker. The conditions under which the breaker contacts slow up or actually reclose have received special consideration and tests and facts have been discovered which were not suspected at the time of starting the investigation.

Oil Throw and Gas and Oil Ignition. These features have been investigated in the case of the old style breakers; the newer breakers do not have these defects. It is, of course, impossible to stop the throw of oil and gas in the old type breakers without practically rebuilding them; however, the tests have shown how it would be possible to construct the breakers and permit the oil throw while preventing ignition.

Secondary Explosions. Attention was given to the cause and magnitude of secondary explosions. The tests were made with a bomb and also upon full-size oil circuit breakers and the large testing generator. Oscillograms, Figs. 6 and 7, show such a secondary explosion. They show that the breaker interrupted the circuit with ease, that in 0.014 sec. after interruption, pressure developed in the air space (Fig. 6), that the maximum pressure was reached in 0.004 sec., that the pressure below the oil, due to the necessary acceleration of the oil mass, was delayed 0.007 sec. behind the air space pressure. The breaker was not injured. These secondary explosions are nearly always caused by a static spark igniting the explosive gas mixture and may come while the breaker is open or closed. Their cause is well understood and if the station operator takes proper care of the breakers, there should be no such explosion in the newer breakers.

Acceleration and Retardation of the Moving Contact Member. The acceleration of the moving contact member under short-circuit conditions is extremely important and equally important is the retardation, especially in the case of the explosion chamber breakers. With these latter breakers any desired speed of opening may be readily obtained. The breakers are specially designed to give the desired opening speed and means have been developed to satisfactorily decelerate moving parts.

Investigation of Magnetic Stresses Produced in the Breaker. Magnetic stresses may cause the lifting of the brush or the throwing back of some types of arcing contacts, at "Make." Such stresses may also cause movement of bushings and studs. The current limits of various designs have been determined and designs for higher duty developed.

Arc Stabilizing Tests. If the contact blocks under oil are not separated a sufficient distance or there is insufficient distance between these blocks and the metal part of the operating rod or cover or tank, an arc is liable to be stabilized across these insufficient distances and cause the destruction of the breaker. The safe distances depend upon the voltage, circuit conditions and amount of current interrupted and each type of breaker is tested many times under the most severe condition of operation, in order to prove that the distances to prevent stabilizing are ample.

Oil Viscosity and other Oil Characteristics

The characteristics of the oil used in oil circuit breakers are important factors in the satisfactory operation of the breaker, and the oil supplied with the General Electric oil circuit breakers is rigidly held to specification. Miniature tests are only made in the research laboratories, but tests are also carried out with the large testing generator. That oil is best which

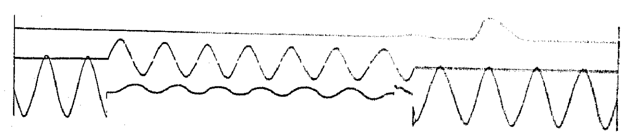

Fig. 7

produces the least quantity of fixed gas for a given interruption, which has the least carbon production, the most suitable oil viscosity for the particular breaker in which it is to be used, the highest dielectric strength for the given interruptions, the smallest quantity of oil vaporized, the greatest percentage carbon precipitation, the least absorption of moisture, and the highest flash point. The oil affects the circuit-breaker operation in ways little realized by those not intimately connected with the breaker investigation.

Other arc-extinguishing liquids, as well as the oil, are under constant observation by means of the full-size tests. Miniature tests show interesting results, but definite conclusions can only be obtained by

comparing such tests with tests made on full-sized apparatus.

Duty Cycle Tests

All breakers are tested at the new proposed duty cycle, *i. e.*, two open-close-open at the rated interrupting capacity, or if that cannot be obtained, at capacities which may be used to interpolate and thus obtain the full duty.

Tests have also been made at duty cycles other than standard, in order to determine the relative severity of these supplementary cycles.

Gas Production and the Resulting Stresses Upon the Breaker. An extended investigation into the gas generated by the arc has been carried on for months past with the large testing generator and will be continued for months to come. In these tests, the gas volume generated, the speed of generation, the pressures above and below the oil level, the current and voltage at the break, are all recorded on the oscillograph. The effect on the breaker structure are also recorded. These data

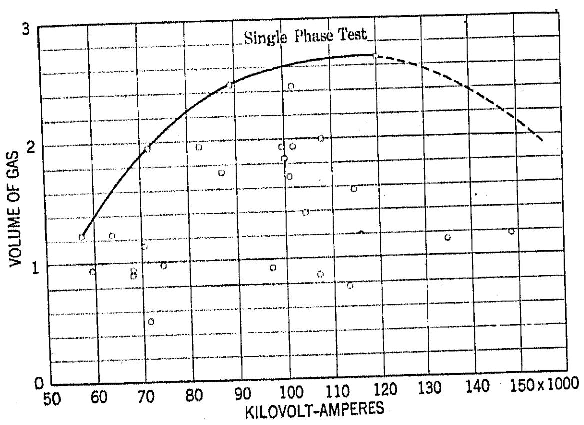

FIG. 8

are used in obtaining an empirical formula with which to calculate the interrupting capacity of any existing breaker and the design information for any new breaker of any proposed interrupting capacity. In this connection, it can be stated that, theoretically, the quantity of gas generated depends upon a large number of variables. For instance, the r. m. s. current and voltage varies throughout the arcing period, the current decreasing, the voltage at arc increasing with the time. The resistance of the gas stream and the dielectric strength of the gas vary with the pressure and temperature of the gas, and since the gas generation is a heat phenomena, this pressure affects the $I^2 R$ losses and the dielectric strength affects the duration of arcing. Then there is the effect of the magnetic blow-out which affects the arc duration. The power factor and shunt load both affect the recovery voltage, which in turn affects the re-establishment of the arc at each zero value of the current wave, the available stored energy—electromagnetic and electrostatic—in the circuit, which may be discharged through the arc at the zero current value and aid re-establishment of the arc and other causes—all of which combine to make the gas production extremely fluctuating. The extent of this fluctuation is shown in Fig. 8, on which has been plotted the volume of gas generated and current in the arc at definite voltage and circuit conditions. It is evident that our empirical formula for interrupting capacity determination must be based upon the curve of the extreme points, as plotted from the tests, and that such a curve can only be obtained by means of a large number of tests made with a generator able to produce the conditions at the desired rating.

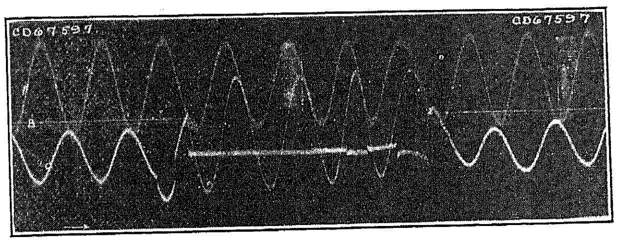

FIG. 9

Investigation into the Action of the Oil Circuit Breaker under Various Conditions. That the action of the oil circuit breaker is very erratic is soon realized when one attempts a systematic investigation of its action. Individual short circuits vary widely in effect, when all conditions of the short circuit are made as nearly identical as possible. That is, you may take a given generator, running at a definite speed, and excited to the same voltage, with the same impedance in circuit, and short circuited by the same breaker, under the same conditions of grounding, and the gas generated may vary

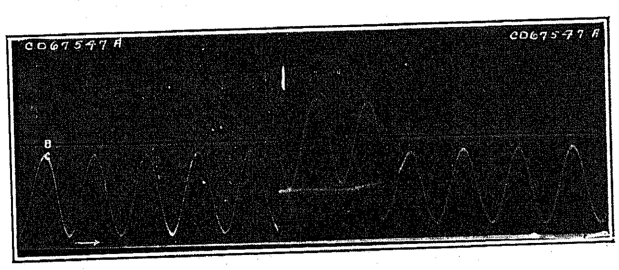

FIG. 10

several hundred per cent. The gas generated is the final measure of the efficiency of a given breaker, but the speed of generation must, of course, be taken into consideration. If fluctuations to the extent indicated above are observed under constant and controlled conditions, what must be the variation during the ordinary short circuit on commercial systems where the field excitation, power factor, shunt load, conditions of grounding, and other factors vary widely?

In general on commercial-systems, the factors mentioned above are usually combined so that the interrupting conditions are less severe than they are under the

controlled conditions of test. Therefore, it is safe to assume that breakers which will pass tests with the testing generator equipment will give satisfactory service under normal operating conditions. By normal operating conditions is meant conditions limited by the normal generator and circuit characteristics and would not, for instance, include a heavy lightning stroke or cross with a higher voltage line.

The service condition where an oil circuit breaker in a generating station controls a single feeder is, of course,

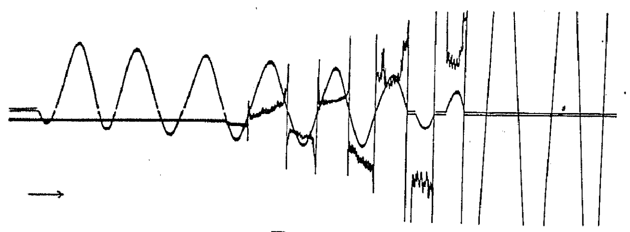

Fig. 11

in general equivalent to the condition obtained with the testing generator and should produce equivalent results.

There is, however, considerable difference in the operation of breakers on ungrounded systems, as compared with the operation on systems with the neutral grounded and a short circuit to ground.

Oscillogram Fig. 9 shows a short circuit upon an ungrounded system and oscillogram Fig. 10 shows a short circuit with the same apparatus upon the same system with the neutral grounded and a ground at the breaker.

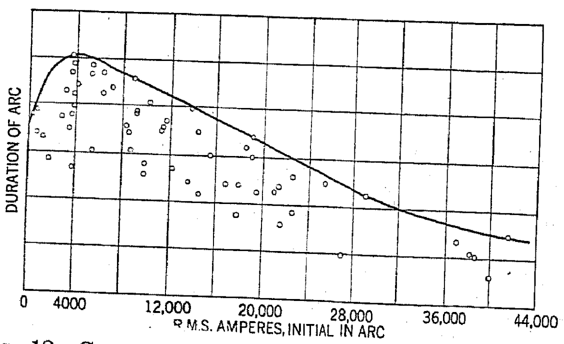

Fig. 12—Curve Showing Arc Duration as a Function of Current Interrupted by a Type F K Oil Circuit Breaker (Plain Break) Test Conditions Equivalent to Those Prevailing at Baltimore Test but Made on Basis of 13,200 Volts, 3 ϕ

The feature to be noticed is the recovery voltage, which is characteristic of the two conditions and is considerably greater in the case of the ungrounded system. The greater recovery voltage means a longer arc and more gas generated in the breaker before final interruption of the circuit. By recovery voltage is meant the instantaneous voltage rise at the instant of circuit interruption, that is, it is the voltage which tends to re-establish the arc at the zero value of the current wave. In this connection, it might be said that while we state broadly that the interruption of the circuit always takes place at the zero value of current, this statement should not be taken too literally. It takes a certain voltage to maintain an arc; the longer the arc, the higher the required voltage, so that a time must come at every operation when the circuit tends towards interruption and before the absolute zero value is reached. This fact will largely account for the "kick" which will be made manifest by the use of spark gaps, but is not shown by the oscillograph, it being a steep wave-front phenomenon. The action of this phenomenon, however, is to aid in re-establishing the arc.

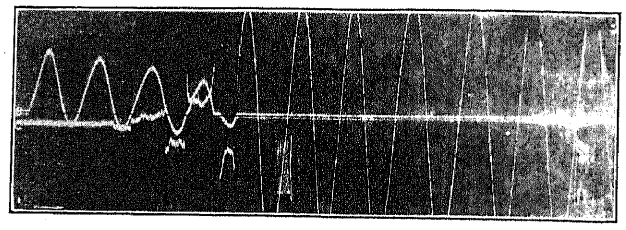

Fig. 13

That the circuit may be re-established some considerable time after the zero current value, is shown in oscillogram Fig. 11, in which case about 60 electrical deg. have elapsed before re-establishment. This case is not unusual, but on the contrary, is quite frequently noticed. The arc is, of course, an energy phenomenon and the current and voltage in the arc are substantially in phase, but at the instant of final circuit interruption there is a change in the relationship of the two and the lag of current is then determined by the circuit as a whole. This is important, because it in a way explains why the low power factor conditions are the harder

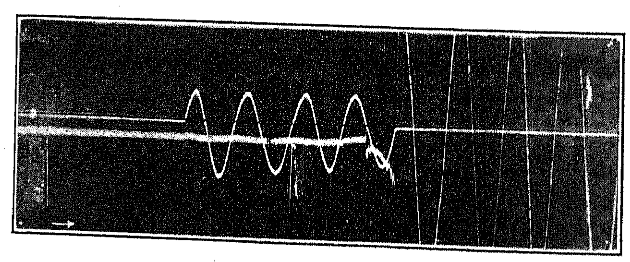

Fig. 14

to interrupt. At zero current value and 90 deg. lag, the maximum voltage is available to re-establish the arc, while at unity power factor there is zero voltage at zero current. This difference caused by power factor variation is really one of time, only because if we assume a case of unity power factor and a 25-cycle circuit, the same voltage is applied at the expiration of 0.01 second, as would have been applied instantaneously at zero power factor. During this 0.01 second, the gas has had a chance to cool, thereby increasing its dielectric strength, and the gap has been increased (assuming an opening speed of 5 ft. per

second) by 0.6 in. Both of these factors act to increase the interrupting capacity of the breaker in the case of unity power factor.

Investigation of the Magnitude of Current Interrupted upon Circuit Breaker Action. In the case of fairly low voltage, the magnetic blow-out effect has to be considered in connection with the interrupting capacity of the breaker. Curve No. 12 shows such a study.

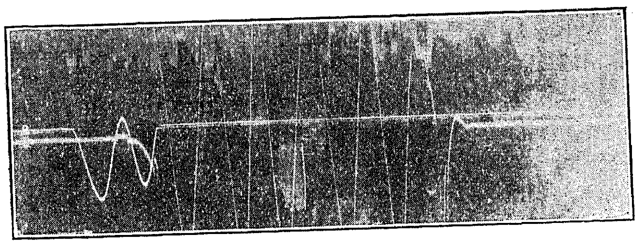

Fig. 15

This curve shows that the maximum arc length corresponds to a fairly definite current value and that any increase in current above this value acts to decrease the arc duration.

The logical deduction is that at some current value, less than the maximum interrupting rating, the breaker may fail. This deduction is correct and many breakers

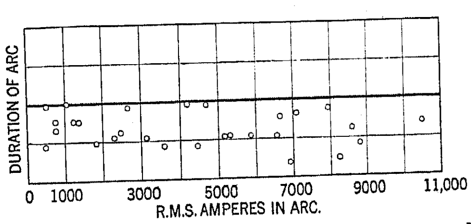

Fig. 16—Curve Showing Arc Duration as a Function of Current Interrupted by a Type F K H Oil Circuit Breaker (Explosion Chamber) Test Conditions Equivalent to those Prevailing at the Baltimore Tests but Made on the Basis of 15,000 Volts, 3 ϕ

are undoubtedly stressed more at part rating than at their maximum rating.

Comparison of the Plain and Explosion Chamber Breakers. Oscillogram Fig. 13 shows a plain break interruption, and Curve Fig. 12 shows the plot of arc lengths and currents on such a breaker. The erratic behavior of the plain break breaker is striking but is characteristic of this class of breaker. Oscillogram Fig. 14 shows an explosion chamber interruption in the same tank at substantially the same current and with the same mechanism, while oscillogram Fig. 15 shows the same explosion chamber breaker interrupting substantially double the current at 1:73 times the voltage of the plain break breaker in Fig. 13.

Curve Fig. 16 shows the plot from tests of half cycles and currents of an explosion chamber breaker.

The foregoing oscillograms and plots are characteristic of the two types of breakers and comments are not needed as to the story they tell in reference to breaker efficiency, an efficiency which increases with the voltage increase in the case of the explosion chamber breaker.

Testing of Breakers to Operate under Special Conditions. For this class of work the testing generator equipment is invaluable, as we are able to obtain results and definitely settle questions which it would be impossible to do without such an equipment. Miniature tests are of little value unless they can be compared directly with tests made on regular apparatus under operating conditions.

In order to be proven safe, every breaker must be tested under full maximum operating conditions or must be compared with a similar breaker which has operated under these conditions.

Conclusions

What has preceded shows the large number of variables which enter into the determination of the oil circuit breaker interrupting capacity. It shows that each variable depends upon the others and that the breaker as a whole must be judged from results obtained when actually performing under loads which vary from the smallest up to its maximum rating and under repeated operations at each value. A single shot at any particular load is far from conclusive and the only safe rating is that obtained from plotting many tests made at all capacities.

For this work, the large testing generator has proven invaluable. It has made clear phenomena previously not understood. It has brought out facts not dreamed of until they were shown up by the tests. It has pointed the way as to what features to avoid and what improvements to make, and its influence is showing in the design and performance of our breakers and will continue to show its value in the research which is already planned for years ahead.

Discussion

For discussion of this paper see page 655.

High Voltage Circuit Breakers

The Operator's Viewpoint—Giving Practises, Experiences and Opinions

BY J. S. JENKS

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THE practises of the station operator in regard to circuit breakers are largely governed by his previous experience, his system and conditions on his system. For instance, a very large system may cover a very large area and have a very great capacity connected, but be of such a character that there is very little interconnecting, with very simple switching, and it may have no very great power capacity concentrated at any one point. On the other hand, the system may be very great in power capacity and cover such a small area that the potential of the system is rather low, and on account of the dependability of low potential and the lack of the necessity of numerous ties, the concentration of power is easily limited. In the first case, we find potential so high, and the breaker has to be so large to safely handle the potential, that lightning is not much of a factor, and the current is so small that it is really insignificant in such a large breaker. In the second case, the currents are greater and the breakers of necessity have to be large and substantial to withstand the great magnetic stresses, but such systems generally are confined to a small, closely built area, largely underground, where lightning is a small factor, and they are not interconnected to a great extent on account of the greater reliability of the low potential of the system. The breakers are not subject to such excess capacities, hence the low-potential breaker problem is not such a difficult one.

In addition to the above-mentioned system, we have the system which consists of large capacities covering an area of such size and such power density that high-voltage, high-capacity lines are necessary for transmission, and a medium voltage with high current values is necessary for primary distribution. Such systems are usually highly interconnected within themselves and also with neighboring systems, hence the switching is not only complicated but is subject to great concentration of capacity, and is subject to and very susceptible to lightning and disturbances which are particularly severe on the breaker, where the current's concentration is great and the potential high enough to make arc extinguishing difficult.

The experience of the operator with breakers depends on the system; how it is built and how operated. The more exacting the service, the more experience the operator gets with breakers. For instance, if the service is not exacting, breakers can be adjusted so they will not open until the intensity of the trouble reduces, and then a number of breakers can be allowed to function and greatly reduce the individual strain and probably cut off the power from a breaker in which the arc would re-establish with disastrous results if it remained energized. If service would allow breakers to rest a while after opening so the gas might escape and the oil settle the contacts, and the oil in their vicinity cool, the breaker would perform with less trouble. On the other hand, if the system supplies much service upon which life and property depend, the breaker cannot be nursed but must be sacrificed, if necessary, for the sake of service.

My experience has been with a system and breakers in the latter class, hence I will confine myself largely to the circuit-breaking problem of the West Penn System, and endeavor to give you practises and experience with that system from its conception to its present development. The West Penn System consists of five power stations having a capacity of 226,000 kv-a. connected by and serving the public over 215 miles of 66-kv. circuits (122 miles of which are constructed and fitted for 132 kv., and shortly will be operated as such); 1350 miles of 25 kv.-circuits (25 kv. also includes 22 kv.) and 661,000 kv-a. in high-voltage transformers, covering an area of 12,000 square miles. In addition, this system is interconnected with several other large systems, embracing the American Gas & Electric Company, Ohio Power Company, Northern Ohio Traction Company, Cleveland Illuminating Company and the Duquesne Light Company—who are connected with the Pennsylvania and Ohio Power Company, and the Harmony Electric Companies, making a total connected power station capacity of 1,092,000 kv-a., with 280 miles of 132-kv. circuits; 850 miles of 66-kv. circuits, 2370 miles of 25-kv. circuits and 3,447,000 kv-a. in high-voltage transformers. Connections are now planned and rights-of-way partly secured, which will add to this group the systems of the Penn Public Service and the Potomac Edison, both of which are already connected with the Penn Central and the Keystone Power Corporation, connecting power station capacities of 1,280,000 kv-a. with 375 miles of circuit above 100 kv., 937 miles between 50 and 100 kv., 4200 miles between 20 and 50 kv., and 3,865,000 kv-a. in high-voltage transformers.

With such systems connected as they are, you can

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understand that the switching problem of the West Penn System, with which these larger capacities are connected, is no mean undertaking.

In 1902 the West Penn started with 6000 kw., 38.5 miles of 22-kv. circuit and 10,000 kw. in transformers, using single-pole stick breakers, which I will describe at some length for the benefit of those who are not familiar with this apparatus. These breakers had a marble base 45 in. long, mounted on two pin-type insulators and having two terminals on the front 36 in. centers. The top terminal was made in hook form so that breaker would hang thereon. The breaker consisted of a handle about 72 in. long, having a contact and carbon-protected eccentric fuse clamp at the upper end which hooked on to the top terminal, said clamp being opened by cord which hung beside the handle. At the center of the handle there was another contact which fitted on the lower terminal on the base, and also was connected by a hinge to a movable arm about 30 in. long, having a conductor from the hinge to the end, where it terminated in a carbon-protected screw fuse clamp. Springs held the arm in both the open and closed position. To set the breaker a fuse 10 in. long, of the desired capacity, was fastened in the clamp on the arm, and then in the eccentric clamp, the spring putting a slight tension on the fuse. The breaker was then hung on the base and functioned automatically by the fuse melting and the arm swinging down, creating a break of 72 in., and was opened by hand by pulling the cord which released the fuse, allowing the arm to swing down the same as when the fuse melted.

We placed in series with these breakers plug switches so wired that the lower breaker contact would be disconnected by the plug to eliminate the hazard of handling the breaker with the arm energized while hanging down, after breaker had been opened, automatically or by hand, to allow the placing of the breakers in proper contact before any energy passed, and to allow the operator to get in a position of safety in event the breaker opened as soon as service was re-established.

This type of breaker was fairly satisfactory for a time, but as the system grew in line and transformer capacity trouble developed from the breakers only opening part of the phases, which resulted in insulation breakdowns from surges and the floating neutral which was not grounded, the transformers all being two-phase low and Scott three-phase high, the current also became too great for the contacts and the fuses had to be so large that when they blew they created such a quantity of metallic vapor that the arc would re-establish several times and under some line and transformer conditions would not break even with the 72 in. opening, and it was necessary to open other breakers or the plug switchers by hand to break the arc.

As a result of these limitations in 1906, when our transformer capacity had grown to 15,000 kv-a. and the transmission system consisted of 107 miles of 22-kv. circuit it was decided to discontinue the practise of installing stick breakers and place oil breakers in all of the new developments and some of the more important old ones. This breaker differed from the ordinary ones in that the top, bushings and tanks were made of wood fiber, and it had two additional tanks in which were located oil-insulated series transformers having three ratios. Inverse time-limit relays were mounted on the breaker beside the tripping coil. These breakers were remote manual control. The control consisted of a ¼-in. tiller rope, which could be run for long distances and around all kinds of corners by the use of small pulleys, and operated by a toggle handle which locked by passing over center, tripped free at the breaker, hence the control did not have to actuate in the case of an automatic operation. This breaker was a 300-ampere, 50-kv. test, and used for 22-kv. service, but at once proved it did not have sufficient capacity and gave trouble from lightning, due probably to the fact that the West Penn System had the second worst lightning condition in the United States, as shown by records compiled at the time. The line insulation, having a wet test value of about 75 kv., was about 50 per cent greater than the breaker, and the lines were not protected by lightning arresters, there only being an arrester on the bus.

I wish to call your attention to the method of installing these breakers, which consists of mounting direct on the substation wall, enclosing with ⅝-in. transite board and connecting by means of detachable-handle plug switches so arranged that the breaker could be taken out of service without interruption by plugging in a breaker jumper and removing the breaker leads, which were also connected by similar plugs.

In order to relieve the lightning stresses and to disconnect a breaker or the entire substation in event of trouble, we designed and built a combination pole-top switch and arrester, which was placed on each line that entered a substation. This was unique, in that it had a horn which was in circuit, forming the horn for the switch, the arrester and a one-turn choke coil which greatly assisted the arrester to function, due to the power flowing up the arrester horn.

We quickly discontinued the use of this switch for 22-kv. service and placed them in 6600-volt service, where they are still in use in their original condition, except that the fiber tanks were replaced by steel as the fiber eventually leaked badly, and while the leakage was stopped by encasing the tanks with tin, they proved too weak mechanically.

We then installed a 400-ampere breaker of the same make which also had top, tanks, posts and bushings of wood fiber, and tested to 125 kv. This breaker differed from the smaller breaker in that it had four breaks per pole with a series trip coil located between and connected with the center stationary contacts, which were supported by posts. This breaker was mounted and connected the same as the smaller breaker. The enclosure was similar except there were

partitions between the poles. The manual control was the same but, in addition, some breakers were electrically operated.

At the same time the lightning arrester arrangement was changed and an arrester placed on each line to protect the breaker.

During the period from 1912 to 1920 these breakers were controlled by a very simple and effective system of impedance relays designed and built by ourselves, which locked all line breakers, making them non-automatic until the potential dropped in the zone where the breaker was located, to correspond with the setting of a low-voltage relay which unlocked the breakers and allowed them to operate from overload.

This breaker was universally used by us for all 22 and 25-kv. service from 1906 to 1920, except the breakers at the main power stations, where there were breakers of a different make having two breaks per pole greater carrying capacity, a lower electrical test and about the same rupturing capacity.

During this period, the system grew from 6000 kw. to 98,433 kw. installed, and from 107 miles of 22-kv. circuits to 757 miles of 25 kv. and 26 miles of 66-kv. circuits; transformers connected from 15,000 to 161,343 kv-a. and the 25-kv. line insulators increased to a wet test of 90-kv.

All this, of course, increased the duty on the breakers, with the result that all breakers on the 25-kv. system had to be reinforced. The four-break breakers were the first to need attention, due to the weakness of the fiber tanks, which had been encased in tin to prevent breakage but with no idea of strengthening them, hence the tanks were replaced with steel tanks. At the same time it seemed advisable to strengthen future breakers, hence iron tops and moulded insulation posts and bushings were used. The tanks were made of 1/16 in. steel and baffles were provided to retard the throwing of oil, as the tanks were provided with springs to allow them to lower slightly to relieve pressures which might burst the tanks, as there was no convenient way of placing a vent in the top.

The moulded insulation-fitted switches stood a slightly higher potential test than the fiber, but did not stand up nearly so well in service. The posts gave us so much trouble we made a general replacement with wood or canvas bakelite. The bushings would stand the original test and later fail in service, due principally to the conductor being misplaced, broken or burned off in the bushing.

As the system grew and was fed from numerous points, the relays mentioned before proved inadequate. In order to provide current to operate modern relays, it was necessary to provide series transformers, which we built and placed on the breaker bushing. As the modern relays would increase the duty on the breakers, we speeded up the break by removing the series trip which released a weight that fell on the tripping lever, and placed a 12-volt, d. c. trip coil which acted directly on the tripping lever, thereby reducing the time of breaker operation 50 per cent.

At the same time, the tanks were reinforced by braces on the sides; the tank liners which were suffering from the arc at the arcing contacts were replaced by double-arc resisting liners, and the arcing contacts were placed on the inside of the main contacts to get them further from the tanks. There was also placed in the transite enclosure of each pole a container of carbon-tetro-chlorid to smother out any burning oil which might be thrown out of the tank.

These breakers, which originally would only break about 1200 amperes are still in use giving remarkable service in location, where the short-circuit current is limited to about 2500 amperes.

Connellsville Station breakers were rebuilt and their capacity about doubled by speeding up the break, reinforcing the tanks and replacing the light iron tops with a heavy iron top having a large chamber above the top, which acted as an oil separator and muffler. When these breakers again reached their limit, they were replaced by similar breakers of modern design, having heavy steel tops and tanks. The insulation value was allowed to remain about the same and this proved too low, as it was only about one-half of the line value. The consulting engineers who made this breaker change believed the breaker would have sufficient insulation, as the neutral of the system was just then being grounded at Connellsville through a 28.8-ohm resistor, but this did not prove the case and the insulation of the breakers has had to be reinforced.

We have had considerable experience with many other breakers made by different manufacturers, which we inherited or that were placed on our system by our customers, or consulting engineers of our own. All of these breakers have given trouble when their capacity was approached, and it has been necessary to limit the capacity back of them by moving them to another location, or back them up by high-capacity breakers.

In 1919, when a new station was started which would have an ultimate capacity of 300,000 kv-a., we took up with the breaker manufacturer the design of a higher-capacity breaker of the 25-kv. class, and as the result, contracted for a large number of breakers for both indoor and outdoor service, which would have a potential test of 140 kv., a current capacity of 600 amperes and a rupturing capacity of 16,000 amperes at 25 kv., under A. I. E. E. rules.

These high-capacity breakers were installed in Springdale Station in reinforced concrete compartments 10 ft. 6 in. by 11 ft. 8 in. to confine any fire or damage, but open at the top to provide large vent area to prevent the structure from being wrecked in case of an explosion. The outdoor switches are located on concrete bases under steel structures on 16-ft. centers.

All switching is single with an auxiliary bus for each transformer bank, controlled by a breaker arranged so that any circuit or group of circuits may be, without

interruption of service, transferred by disconnecting switches to the auxiliary bus and breaker in case of trouble, or to facilitate inspection and repairs, for oil breakers need servicing. In fact, they remind me of motor trucks. I was investigating trucks once, when we were about to purchase a large fleet. One of the dealers insisted on my going to see his establishment, which he claimed was the largest and most complete service station in the county, where they had the most complete stock of parts and large forces of competent engineers and mechanics on duty twenty-four hours per day, 365 days per year to give service, "and you will need it," he said. Well, I didn't purchase his truck but another, and found that it needed service, too. So it is with breakers. They all need service, and we have found that servicing breakers is one of the biggest dividend payers we have. In fact, this has so impressed some of our breaker-service men that it was intimated to me, if we had enough breakers our servicing would save so much we could pay our dividends without collecting from our consumers.

I see there is some interest in this savings department. It is under the supervision of a very high-grade relay and breaker expert. His organization consists of relay and breaker engineers and high-grade relay and breaker mechanics. They are organized into crews and are provided with motor trucks equipped for and supplied with all kinds of instruments, apparatus and tools for testing and doing every kind of relay and breaker work. This organization checks, adjusts and tests all new relay or breaker installations before being put in service; inspects and tests all relays and breakers at regular intervals; repairs and rebuilds relays and breakers whenever necessary; checks every important relay or breaker operation, examining the relay and breakers to determine their condition, particularly contacts, insulation and oil; investigates—with the idea of determining the cause—every faulty relay or breaker operation on our property and the important ones on other properties. By this method we are able to keep our breakers and relays in infinitely better condition than we could by having the regular construction or repair forces who are not especially trained to do the work. That is what we call "servicing breakers."

All disconnect switches indoors have a dry test of 101 kv., and outdoors 150 kv., and are all provided with a type of lock that breaks the seal of the blade in the jaws, and bars it out at least ⅛ in., in order to make it operate easily and prevent damage to disconnects as the result of sticking.

In January, 1923 it became apparent that we would need breakers of still greater capacity on our 25-kv. system. As our plans for connecting additional properties acquired and interconnecting with other large utilities with high-voltage transmission made it possible to develop short-circuit current too great for the 16,000-ampere breakers, the matter was again studied and contract made for a large number of breakers, guaranteed to have a potential test of 140 kv., current capacity of 600 amperes and a rupturing capacity of 25,000 amperes at 25 kv.

The three 25-kv. breakers which will most generally be used on our system are the 2500, 16,000 and 25,000-ampere breakers. In the case of the middle size breaker the conduit runs are long and expensive as the relays are mounted in substation buildings, and in some cases buildings have to be erected to protect the relays, while with the larger breaker the relays can be placed in the operating mechanism box, which is made large to accommodate same and thus save the cost of the conduit run and relay housing. Therefore, the cost of the two larger breakers installed is about equal, even though the breakers are quite different.

In the case of the higher potential, we have followed the manufacturer's standard more closely, using 73-kv. breakers for 60 to 69-kv. service, and 135-kv. breakers for 120 to 132-kv. service, because practise has shown that these classes are not so affected by lightning disturbances, and the cost of extra breaker insulation in these classes is so great that we allowed the breaker to have a lower insulation value than the lines and breaker auxiliaries, which are of the unit type and may have a defective unit which would lower the over-all insulation if a reasonable margin of safety were not allowed. Further, we are depending upon the arresters to protect the breakers.

The same general wiring plan is followed with these breakers as with the 25-kv. breakers, but the disconnect switches are of a very different type. Those disconnecting the breaker are two-post, horizontal-break, steel-protected, self-alining jaws, gang-operated, mechanically interlocked with the breaker so they cannot be operated except when the breaker is open, and of such rigid construction that several hundred pounds can be hung on the end of each blade and the switches will retain proper alinement and make proper contact. In fact, I have hung on the end of the blade when these switches were frequently operated, so I might see how they functioned under severe service, and they worked perfectly. Horizontal break was used to economize in head room and to keep the number of posts to a minimum, as the blades are connected directly to the breaker leads and are dead whenever the switch is open, there was no objection to the reduced clearance between phases.

In the case of the disconnecting switch to transfer from the main to the auxiliary bus where it may be completely energized in the open position, vertical-break, gang-operated switches were used to maintain the phase spacing. These switches, likewise, have self-alining jaws with sleet protection, and are so counter-weighted that they would take the open position any time the control failed. They also have horns and are guaranteed to break full-load at normal potential.

We do not expect to use them for such service, but they are located on top of the structure so they could be so used if necessary.

When completed, the high-voltage switch gear of our largest substation, located at Charleroi, will consist of twenty-seven 37-kv., and thirteen 135-kv. high-capacity breakers and their auxiliaries, representing half a million dollars.

All operators have had more or less trouble which originated in or outside of the breaker itself. These difficulties will again depend largely on the system of which the breaker is a part, where located and how operated. Let us first think of those troubles which originate in the breaker itself, and then those which originate from causes outside of the breaker, and try to suggest ways of correcting improper happenings.

The principal cause of trouble originating in the breaker itself is a reduction in the insulating value of the various insulating mediums, and that reduction is generally due to the presence of moisture. I have often been asked "how does the moisture get in?" and counter with "what keeps the moisture out, that stays out?" and as I could never find what keeps it out, will give you a few ways in which it gets in.

We pour it in with the oil, yes, even if the oil does test up very high, moisture is generally present in such small particles and so separated that it is not detected by the normal methods of testing but can be detected by some forms of heat tests. These moisture particles are collected on the surfaces of breaker members by the electromagnetic field, which generally holds the moisture in the small globule form where it deposits until it is absorbed by insulating material or the breaker is de-energized, when the small globules combine and form large globules which descend by gravity and capillary attraction until they come in contact with insulating material which will absorb them, or descend through the oil to the bottom of the tank, if the oil does not have such an affinity for moisture that it will again absorb it, in which event the cycle repeats. This may be prevented by using an oil which has very little affinity for moisture and is free from moisture when placed in the breaker. But what about the moisture which accumulates in breakers from condensation or is taken in by the breaker breathing? We all know that breakers breathe, but I am afraid none of us realizes to what extent moisture is taken in by breathing. Breathing was brought very forcibly to my attention a short time ago. When investigating breaker failures, it was found that the breakers were taking in a great deal of moisture, in fact, to such an extent that water would gather in the bottom of the tanks and freeze, and could be lifted out and the oil still test fair. Just how much moisture, had been taken in could not be determined, but you may get some idea from the fact that the bushings themselves breathed and accumulated water very rapidly. One bushing, the conductor of which was a one-inch inside diameter, brass tube 4 ft. 8 ⅜ in. long, accumulated about a teaspoonful of water in a week's time. A hydrostatic test showed there was no leak at the bottom end where the contact was screwed on, but the top end where the terminal was screwed on leaked slightly at 5 lb. per square inch, hence the moisture got in by breathing through a leak that took 5 lb. to develop. If this tube, having a cubical content of 42 cu. in., sealed as tight as it was, and wrapped up as the conductor of a 73-kv. breaker bushing is, breathes a teaspoonful of water a week, will someone tell me how much moisture a breaker can absorb in a week's time if it has a cubical content of 83,160 cu. in., with no wrappings whatever to protect it from atmospheric changes, a vent that offers no resistance to breathing, and a top on which snow and ice accumulate, assisting in the condenser action and even covering the vent so the breathing is through snow or ice? Well, if you don't know, I will tell you—enough to ruin the best breaker ever built, and so far, we have never really made an effort to keep moisture out of breakers. I know of one instance. Shield and drains were placed in breakers to shed and conduct the water from the more vital parts, rather than take steps to remove and keep out the moisture.

Can moisture be kept out of a breaker? Let us return to the tubes in the terminals. While the majority of these tubes accumulated water, some accumulated oil only, while others both oil and water, which showed they were breathing at either or both ends. Some were about full, which showed the content stood high above the oil in the tank, while others were absolutely dry, which proved they had not breathed at all and no moisture had been condensed on the inside.

Breakers can and should be made moisture proof and provided with quick opening vents, which would relieve gases but prevent moisture from entering the breaker, and then the many failures which originate from moisture in breakers will be a thing of the past, providing proper oil is used.

Other causes of trouble originating in the breaker itself are the many mechanical weaknesses which develop, some from expansion and contraction, others from material deterioration, but generally due to the design, particularly the very sudden starts and stops which create excessive strain and shock, resulting in the failure of many parts and very faulty latch operation. We have had cases where we had to put on a latch to latch the latch, and others where the latches would latch, but on account of their having to be so sensitive, would often vibrate off. While all these mechanical failures are serious, the worst are those which result in the indicating switch giving a false indication. The majority of the mechanical failures can be prevented by a substantial, simple mechanism, using the principle of the crank traveling through about 180 deg., so arranged to give the breaker a retarded movement at both ends of the closing travel and accelerate the middle of the travel, so the total closing time remains normal, the time curves being spiralled as a modern

railroad spirals its curves. On opening, the minimum of the mechanism should trip free very speedily but come to rest through a spiralled motion, at the same time interlocking with the closing mechanism ready for closing. The latch should be positive in action and not depend upon spring or gravity devices which could kick or vibrate open. The false indications can be prevented by arranging an indicating switch operated from the movable contact in each breaker pole. For about a year we have been buying high-voltage breakers only that were so fitted.

When thinking of those troubles which originate from causes outside of the breaker, let us include breaker failures for which the breaker is solely responsible but which follow outside disturbances. The most common causes of such troubles are lightning, surges and over-capacity. The first two generally result in electrical breakdowns but there have been cases where breakers have been wrecked by lightning as if blown up from an explosion, when no electrical breakdown was apparent. Electrical failures generally happen to the bushing, although we do have electrical breakdowns from the contacts through the oil and lining to the tank, from those which take a path to ground via the insulator supporting the movable contact and also from the top of the bushing to the breaker mechanism, wrecking it and rendering the breaker inoperative. Bushing failures are usually the most disastrous of the electrical failures and if we only stop to consider a second the duty expected of a breaker bushing, I am sure we would realize that breaker bushings should be more liberally designed and tested than any other bushing for like service, as the breaker bushing is frequently the end of a circuit where the electrical stresses build up very high, as there is no beyond over which the stresses may distribute, or discharge, as is the case with bushings such as transformer, inlet and arrester bushings. The remedy for these troubles is more liberal insulation, which, it is true, may mean a larger breaker. "But how much more liberal?" I have been asked. That depends upon your system, its size, how the lines are insulated, how effective the arrester equipment and what kick it is possible to get when switching. More than a quarter of a century of experience has shown that the most satisfactory bushing for all kinds of service is one that is free of stresses caused by the method of support, takes its load under compression and has a loose insulated conductor fastened at one end only. Such a bushing is hardly feasible for extremely high potential and some types of outdoor equipment.

Over-capacity failures are the most disastrous. I have seen breakers that were completely demolished by explosions; others that were completely destroyed by the fire following an over-capacity explosion, and frequently great damage done to both indoor and outdoor substations. An over-capacity failure is any failure which results from a breaker being susceptible to greater capacity than it can successfully interrupt, that is, the capacity may be in excess of that for which the breakers were designed, or may be considerably less, and the breaker capacity reduced even below such values by defective oil. I remember a case of oil carbonizing greatly as the result of a single-breaker operation. Under normal switching, the oil did not seem to deteriorate at all but if the breakers opened on a short circuit, the oil would carbonize to such an extent that its test of 30,000 volts would be reduced to less than 7000 volts, and if the breakers were put back in service without correcting the oil and were subject to a second short circuit it would result in a failure even though the short circuits were only a small fraction of the breaker capacity. This resulted in the necessity of changing oil after every short circuit before the breaker was put back in service.

Other oils vaporize, volatilize and decompose very readily and create gases which have a very low dielectric strength and are so inflammable that they ignite as soon as they come in contact and form certain mixtures with the air, or are ignited by arcs which are formed or maintained as the result of the low dielectric strength of such gases. I have seen breakers which had, apparently, functioned properly and opened the circuit but which were wrecked as the result of gas being ignited by an electrical breakdown outside of the switch, caused by the low dielectric value of the gas.

All oils, however, are not bad. Different oils will make a difference in the functioning of a breaker. The best oil, however, is far from being a perfect circuit-breaker fluid. The expression brings mirth, but let's laugh on and try to imagine some of the characteristics of a perfect circuit-breaker fluid.

A perfect circuit-breaker fluid should be non-vaporizing or non-volatilizing, non-inflammable and non-absorbent of moisture, yet have a specific gravity considerably less than water; be non-corrosive, non-poisonous, non-adhesive and not subject to capillary attraction, but be capable of high heat conductivity so that the heat might be transferred directly to the container without motion of the fluid. It should extinguish the arc at the first electrical zero and prevent it re-forming; should have a very high and permanent dielectric strength and low viscosity; should not change through a temperature range from 100 deg. minus to 900 deg. cent. Its dielectric strength should have a negative coefficient, when in contact with arcs, which would dampen out arcs as a resistor to reduce shock, then return to its original state.

With such a breaker fluid the circuit-breaker problem would be very much simplified, but since we do not have such a fluid and have to be content with oils of various grades, the best of which rapidly deteriorates in any of the present day breakers, let us consider what kind of a breaker should be provided to give good service with poor oil, as I feel that it will be infinitely easier to design and build a better breaker than it would be to always procure the better oil.

In attempting to produce a better anything it seems logical that we should first familiarize ourselves with what has been done or thought out by others in the line in which we are interested, and as we are all more or less faimiliar with the standard lines of breakers in common use today in this country, I will pass that and consider some of the breaker designers' ideas not in general use, to see if we might not determine some things to do and some things not to do, and as the operating mechanism is such a simple problem I will pass that also and confine myself to those things which pertain to the inside of the breaker.

The thought that seems to have been uppermost in the minds of designers to interrupt a circuit, has been to extinguish or displace the arc by interposing between the breaker contacts fluid, solid or gaseous barriers. The most common of these devices depends upon the unconfined arc at the breaker contact creating the force to cause an oil barrier to extinguish the arc. This idea has not proven very effective, due to the fact that there is more power in the arc itself, repelling the barrier, than can be produced to establish a barrier and extinguish the arc. Hence, mechanical and chemical means have been employed to try to accomplish the same end. These consist of movable contacts being of such form as to direct a stream of oil into the arc, and depend on the inertia of the oil, which, of course, will not produce sufficient force to overcome the arc. Another plan contemplated a piston or diaphragm attached to the movable contacts, closely fitted to the wall of the tank, which had a by-pass from the bottom to a point near and directed at the contacts. This device was not effective because the underside of the piston had a larger area than the top and the pressure in the neighborhood of the arc, which was connected to the bottom of the tank by a by-pass was greater than the pressure above the piston. The explosion from the arc, the inertia of the oil and the dashpot effect of the piston all tend to retard the breaker action and, if speeded up by great external mechanical force, would create a void or partial vacuum into which the arc would be drawn. Such a breaker would not be very successful.

Other plans provided a pump which would start and force a stream of oil in the path of the break, but it was impossible to start a pump and a stream of oil quick enough to be of any value. Then tests were run with the pump kept in continuous operation, but as stated before, the power in the arc was too great for the pump and the stream of oil which had to pass through a body of oil was easily diverted by the arc, and the breaker did not prove any better than a breaker having still oil.

A closed oil-pressure system provided for the interruption of the circuit in a confined stream of oil. This meant that the pump had to be so large and the power consumed so great that it was a failure, or the oil-confining chamber was so small that it was damaged by the arc before extinguishing could take place. The power of the arc was also greater than could be maintained by a reasonable pump.

Another design which would seem worthless is one having cylinders of insulation in which are mounted the stationary contacts. The movable contacts are mounted on the end of plungers which fit said cylinders and are held against the stationary contacts by springs. High-pressure oil is admitted to the cylinder and forces the plunger out, opening the contacts, and at the same time is supposed to completely fill the cylinder, thereby preventing an arc from forming.

We also have numerous explosion-chamber types wherein the oil and arc are confined until the movable contact, gas and oil are expelled, as the result of a pressure being built up in the chamber, due to metal and oil volatilizing, thus extinguishing the arc, and while this is the most practical of such devices I feel that the greatest value of this device is in the directing of the results of the power interruption. In order to speed up the extinguishing of the arc and increase the capacity, it has been proposed to circulate and direct oil into the arc by the means of a pump. Here again we have the objection to and insufficiency of the pump.

A breaker design which appears to me to have merit is one having series breaks arranged so that the early opening of some contacts produces arcs in enclosures, forming gas which expels oil from the enclosures right in the break of later opening contacts extinguishing the arcs and interrupting the power to the early opening contact and the circuit.

A large number of breaks in series have not been received with much favor in this country but foreign designers have used large numbers of breaks in series, simultaneously opened, with very great success, and in cases of great single-phase capacities shunt a part of the breaks with resistance, to increase the breaker capacity by retarding the potential rise following the current zero, thus preventing the arc from re-forming. We have an American designer who has suggested a large number of breaks in series opened successively, the idea being that with the successive opening of contacts an opening will occur without arcing when the current is at zero, hence the oil insulation will not have deteriorated, no arc will establish and the circuit will be interrupted.

Other designers provide for injecting solid barriers into the arc to break it. In fact, one came to my attention that was designed to cut the arc off, as you would cut a piece of paper with a pair of shears, by closing refractory barriers between the contacts as soon as they parted.

We also have the gaseous barriers which consist generally of containers filled with various gases or fluids, which would be gasified by the heat of the arc or an explosive, and extinguish the arc, the gas to be liberated by several means, such as puncturing with a sharp instrument actuated by the breaker mechanism, the

melting of a fuse plug by the arc, the bursting of the container by expansion or explosion of its contents from the heat of the arc or the explosion of explosives placed therein and set off by a fuse lighted by the arc. It has also been suggested that the arc be blown out by an explosive. Just imagine placing a sufficient charge of explosive in a breaker to insure the blowing out of a million kv-a. arc! Well, don't get scared, none of these devices is practical in its present state and I do not think any of us will be using any of them, but we may soon be using a breaker in which the arc is extinguished by steam generated by the arc.

The steam reminds me of a breaker designed some time ago, which had an insulated tank with a stationary contact at the bottom and a movable contact which was raised completely out of the tank. The bottom half of the tank was filled with water, the top half with oil, hence when the contacts were separated, the water functioned as a resistor and the current was finally broken in the oil, but as the oil might become wet from its contact with the water, the movable contact was raised completely out of the tank to prevent any leakage through wet oil.

The inflammable gases which accumulate in the top of a breaker have been a real menace, and designers have different ways of correcting this hazard. One suggests injecting into the top of the breaker inert gas or vapor to prevent the formation of explosive mixtures, also to wash the breaker of all gaseous products. Others would separate the gas in small cells where it would be cooled, and conducted away in volumes so small it would not be dangerous. Others have elaborate cooling systems. A German turned his military training to good account and placed in the top of the breaker a wire entanglement to cool the gases and to prevent any sparks or flames from getting beyond the entanglement and igniting the gas.

I have only found where one designer made any effort to prevent breathing and eliminating moisture due to condensation, and I fear he has "foozled" for he has suggested an unlined, reinforced concrete pit large enough to hold all three poles of a breaker—including the operating mechanism, series and shunt transformers, and deep enough that the temperature of the oil will remain practically constant all year, and provided with a cast iron cover above the ground, having a manhole which serves as a vent.

This designer, apparently, did not know that oil quickly penetrates and disintegrates concrete, and that it would be impossible to permanently waterproof a pit, as he suggests, so moisture would not penetrate it; that the enormous cast iron top with the extreme difference of temperature above and below it would cause it to sweat very profusely, and a manhole vent would do some breathing.

But what is the real trouble with the breaker? The trouble is not with the breaker; poor, innocent thing, it is just what we made it and I have never seen a breaker that was as good as it could be reasonably made. The real trouble is with us.

(1) The operator has not determined his breaker needs.

(2) The manufacturer has not fitted himself, by research work, to meet the operator's needs.

(3) The operator and manufacturer have not co-operated to develop, operate and service breakers as they should.

These are the three cardinal points in the breaker problem. If we get into the matter conscientiously, the job is done.

Discussion

PAPERS ON OIL CIRCUIT BREAKERS

(MAC NEILL, HILLIARD, JENKS)

BIRMINGHAM, ALA., APRIL 9, 1924

J. D. Hilliard: On Mr. Jenk's paper, I heartily concur in his final conclusions that the operator has not determined his breaker needs. It seems to me rather absurd that the whole line of stations should be held to the expense of building a breaker at a definite rating, to take care of the severe conditions of some particular circuit. It seems to me the thing to do is to get after the particular circuit and ease up on the condition.

He mentions, in his various types of breakers, the series break, but it is a debatable question whether the series break has any advantage over the ordinary breaker.

"The manufacturer has not fitted himself, by research work, to meet the operator's needs." That, with the General Electric Company, has been done away with; we have fitted ourselves.

"The operator and manufacturer have not co-operated to develop, operate and service breakers as they should." That is undoubtedly true, and there should undoubtedly be many more tests made in the field than have been made in the past.

A series of tests was made upon plain-break and explosion-chamber breakers, and operating in the same plant, with the same mechanism, by the same generator, under the same circuit connections and load. Broadly speaking, we found that in that particular test the maximum amount of gas with the explosion chamber at 1200 amperes, at 63,000 volts, was about one-third of the minimum amount with the plain break; that the maximum amount with the explosion chamber was about one-tenth of the maximum amount with the plain break. With increased duty the explosion-chamber breaker has a speed increase due to the increased pressure in the explosion chamber. The plain breaker, shows very little change in speed. Whatever change there will be, however, in the plain-break breaker is a decrease in speed, rather than an increase. That comes from the increased pressure in the tank acting upon the rod passing through the tank, and also upon other factors which all tend to decrease the speed of operation. The rod passing through is acting as a piston, and the force on that piston is acting opposite to the force of acceleration given by the spring and this tends to slow up the breaker.

A series of tests was made recently by the A. E. G. Company of Germany to determine arc duration. In a two-break explosion-chamber breaker at 80,000 volts the longest arc duration was 9 half cycles on a 50-cycle circuit; at 70,000 volts the arc duration was about 7 half cycles. In the plain-break breaker, with six breaks in series the maximum arc duration at about 67,000 volts

was 25 half cycles. These tests bear out the tests we have made in Schenectady very closely.

J. B. Mac Neill: In the years 1913 and 1914, the Westinghouse Company had available, due to cancellation by a large customer, a 15,000-kv-a. low-reactance alternator on which a considerable number of tests were made in our factory. Our experience since has indicated that the results of those tests were to some extent not borne out by subsequent field tests.

We account for that by the inherent rapid decrement characteristic of a single machine when subjected to short circuit. This decrement takes place in the short-circuit current and also in the terminal voltage, so that the arc kv-a. delivered to the breaker decreases approximately as the square of either current or voltage.

Mr. Hilliard's remarks regarding the tests at the General Electric factory are very interesting. I was surprised, in looking over his paper and in looking over my paper, to see the apparently relatively large concentration of power that was available at Bessemer for the Westinghouse 44,000-volt tests. Mr. Hilliard draws the conclusion that the power available was not adequate to stress either the plain-break or the explosion-chamber breakers. We reached 3600 amperes at 44,000 volts,

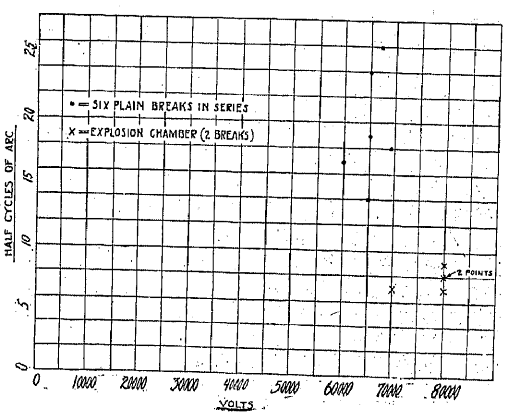

FIG. 1—ARC DURATION IN PLAIN-BREAK AND EXPLOSION-CHAMBER OIL CIRCUIT BREAKERS

Arc duration with different voltages at 50 cycles per minute. The length of time is given in half cycles. The plain-break breaker has six plain breaks in series; speed, 5 m. per sec.; amperes, 234-290. The explosion-chamber breaker has two breaks; speed, 3.23 m. per sec.; amperes, 330-410.

and that would seem to me to be adequate power for those purposes.

On page 4 of Mr. Hilliard's paper, regarding the ability of the breaker to withstand the shock of oil throw, in 1915 the Westinghouse Company, in conjunction with the Detroit Edison Company, made a series of tests at the Connors Creek Station, where there was available approximately 12,000 amperes at 24,500 volts on short circuit which still stands as one of the highest power tests ever made. At that time the conditions in the top chamber of the breaker, the possibility of breaking castings due to the upward impact of the oil, and more especially the actual explosion in the top chamber, of gases hot enough to ignite with oxygen, were rather fully gone into before the tests were over.

In 1920, we made at our factory, under the direction of our Research Department, a series of tests involving bombs. Those tests were under the direction of Mr. Escholz and Dr. Rodman, and were of considerable interest and some help in determining breaker structures. The particular difficulty with those tests was controlling the rate of propagation of the disturbance, but by certain investigations of powder proportions the resulting and mixtures which burned with different rates of rapidity and it was possible to simulate in some way the action that obtains in high-power breakers.

I just wanted to add to Mr. Hilliard's comment on the bottom of page 4 regarding secondary explosions, that they are nearly always caused by static sparks. Cases are on record where these explosions were caused by hot gases such as hydrogen igniting with the oxygen in the top chamber and not static sparks, though undoubtedly there are cases where static sparks are the cause of the trouble.

On the subject of oil viscosity, the Westinghouse Company has conducted a series of researches in the last two years on the subject of oil viscosity and has found that the question of flash-point has been much overstressed as affecting circuit-breaker operation. We have found that the lighter oils of late years have given very successful operation in breaker structures. Dr. Rodman has been in charge of those experiments. Briefly, years ago the high flash-point oils did not contain the same proportion of highly volatile matter that the lighter oils did, but due to the activity of the people who make gasoline nowadays, those extremely volatile constituents are removed, with the result that the lighter oils are now considered adequate by us and even superior to the heavier oils for circuit-breaker use. You will be interested to know that within the last year we have therefore standardized for indoor circuit breakers on the use of transformer oil.

In my paper, I deal somewhat with the extremely high reestablished voltages experienced at Bessemer on the 44,000-volt system. These are probably due to the fact that this 44,000-volt system is not as completely grounded as the 110,000-volt system and this in the minds of our engineers accounts for the tremendous kick we got upon opening the arc. It is doubtful whether anything of the same magnitude of these same voltage kicks can be obtained unless there is connected up a large high-voltage system with many transformers and interconnected lines which have a tendency to cause surges and re-establish the arc.

W. E. Mitchell: I am glad that we have had two types of papers presented here, because the two sides of the story must be worked out together to get the right answer. We have had the operator's viewpoint and we have had the manufacturer's viewpoint, and only as these two are co-ordinated will we get a satisfactory answer.

I think that the design is absolutely the function of the manufacturer. About all the operator can do—and it is what he should do, as Mr. Hilliard and Mr. MacNeill both brought out—is to the best of his knowledge and ability give the facts to the manufacturer as to what he requires. But, gentlemen, when that is all said and done, what we want are circuit breakers that will open the circuits, exactly as we want transformers that will stand the bumps.

If I had any criticism to make of the manufacturers, it would be that they have in the past years spent too much time trying to convince the operators that what they had manufactured was exactly right, and there was nothing whatever wrong with it—instead of correcting obvious faults that have been brought out in operating practise. It may have been the operator's fault; he may have put the switch in a place where the duty was a little too severe, but after all, what we must have are oil circuit breakers so substantial in design that they will stand some abuse.

Today the oil circuit breaker is immeasurably better than it was three or four years ago, but it still has altogether too much trigger work and too many jim-cracks on it. When you have to come down to adjusting breakers to two thousandths or four thousandths of an inch, to get them to operate properly, and if when you fail to do that you break the operating rods or links, or something else, it is too delicate a piece of mechanism to give to the ordinary electrician. We are not running a laboratory; we are giving service to our customers and what we must have are very rugged switches. As I looked through one of the factories a month ago and saw some of the changes that have been made, getting away from the old elliptical tank which was formerly used, to a round, welded tank with cast steel tops of such

construction that it would not blow up every time there was an explosion, I felt much better; and I realized that really great advances have been made in the past four years. We must get switches that won't be wrecked when opening a short circuit.

Probably you can come back from a design standpoint and say, "Tell us exactly what duty you want," and I will admit we ought to tell you, but only within broad limits, gentlemen. We want to get things simplified into big classes. Then we want the switches to come onto our system so that we can work them, so that ordinary men and not factory men can put them up, can repair them and can give service to the people whom we are supposed to give service to with them.

J. M. Oliver: On the Alabama Power Company's system considerable trouble has been caused by secondary explosions in oil circuit breakers, especially in those of rather old design. There were several instances in which the breaker ruptured the circuit without very much evidence of distress and probably a minute after the opening, due to some unknown cause, an explosion would occur which practically wrecked the switch. We believe that oil circuit breakers should be designed to withstand the maximum pressure which can obtain in the switch tank, as it is very difficult to prevent secondary explosions.

I would like to hear a little more discussion on the question as to whether the heaviest duty on an oil circuit breaker is at the generating station or at the substation. Mr. Hilliard pointed out that low power factor causes a heavier duty on the breaker, and others maintain that on the substation breaker the recovery voltages being higher, the duty is greater.

On our system the maximum short-circuit current at this time is about 800,000 kv-a. Looking ahead for several years we find that short circuits as heavy as 1,500,000 kv-a. may be expected, and we begin to question whether the system should continue in operation interconnected as one network, or whether it will be necessary to separate the system into two or more parts. We find it desirable to operate as one network in order to secure the best efficiency in plant and line operation. If it is impossible to secure oil circuit breakers which will successfully handle short circuits as high as 1,500,000 kv-a., it will of course be necessary to look for some means of reducing the short-circuit duties.

P. M. Downing: The function of the operating company is to give service—not service when conditions on the system are absolutely right, but service under all conditions. If the manufacturers are unable, with their present equipment, to provide facilities to the operating companies with which they can give service, the operating people, I think, feel that it is up to the manufacturers to provide those facilities, if it is humanly possible to do so, to give that service.

We realize that the operating companies and the manufacturers are up against two problems. First, to provide a switch that will function properly from an electrical standpoint, and the other problem is to provide that switch at a price that the operating companies can afford to pay. Only very recently, within the last two years, the company with which I am associated had occasion to purchase a good many 220,000-volt oil switches. I am not at liberty to quote prices, but the prices of those switches were not low; they were a very important item of expense.

So the manufacturers must get this idea firmly fixed in their minds: That systems cannot be designed around the oil switches. You can't arrange your circuits so as to meet the convenience of the designers or the manufacturers. The public demands service. You must take on any load that comes along, and supply that service. If the present design of switches is inadequate to handle the loads that are carried, then they must be changed, because after all, when you consider that there is a legal and a moral responsibility resting upon the operating companies to give service, it is up to the manufacturers and to the operating companies to provide facilities for giving that service.

E. E. George: I just wanted to express my appreciation of Mr. Jenks' remarks about the need of restoring service promptly. I think it is quite a step in the right direction. The schedules we usually have,—of one minute or two minutes,—for restoring service, ought to be obsolete; they are a relic of the old days when you couldn't get a call through central in less than a minute, when the operators closed all the switches by hand, before we had any dependable relays that we could time accurately. Instead of talking about restoring our service in thirty or sixty or ninety seconds, we ought to speed up to suit the fastest customers, instead of trying to accommodate the slowest ones.

From the standpoint of public relations, probably the residential customer is the most important. He can't protect himself; he has to depend upon the electric company to do it for him. This involves no problem the electric company can't take care of by relay, and at a relatively small expense.

Instead of talking about twenty to eighty seconds before coming back on the line after an interruption, we ought to be able to get back in twenty or thirty cycles.

Mr. P. H. Thomas: Mr. Hilliard, I understood you to say that it was probable that some of the tests made here in the Alabama Power Company on the 44,000-volt breakers were under conditions not as severe as might occur in practise. Could you give some idea what the nature of those conditions are?

Another question: What is the action that actually suppresses the arc—is it the pressure developed, or what is it?

A question to Mr. Jenks: With the study he has made of new ideas, has he something definite to recommend to us better than what we now have in switches?

G. H. Middlemiss: I think there is one paragraph in Mr. Hilliard's paper, of considerable significance. Tests have been made at duty cycles other than standard, in order to determine the relative severity of these supplementary cycles.

There has been a lot of discussion of late as to the standard duty cycle which should be adopted, and I am frankly of the opinion that it is going to be very difficult to secure a duty cycle which is universally acceptable to operating engineers all over the country, due to varying conditions of service and other factors at each particular location.

In our own operation we close some breakers at two-minute intervals, some at one-minute intervals, and others at less. If these breakers are operating at somewhere near their maximum rating of interrupting capacity, we would like to have the manufacturers tell us what multipliers to use in connection with these breakers, so that we could determine their rating at the duty cycles other than standard.

Another point in connection with contacts is this: We cannot dismantle the breaker every time it functions in accordance with a duty cycle and find out what is wrong with the contacts, or if the contacts are all right. We have other troubles to worry about. We would like to have investigations made, and have the manufacturers give us an idea of how many standard duty cycles the breaker will stand at capacities somewhat near its rating, before we will have to pull the breaker down and redress the contacts, and make other adjustments.

J. D. Hilliard: Answering Mr. Thomas' questions as to what really causes arc interruption in an oil circuit breaker, there is no doubt the arc is interrupted by the dielectric strength of the gas in the arc stream and this strength is a function of the temperature and pressure of that stream. In the explosion-chamber breaker we deliberately produce a high pressure and the gas stream is very effectively cooled by the atomized oil injected into the stream as it escapes from the chamber. We also have a high speed produced by the chamber pressure and all of these act to produce a remarkably efficient breaker.

The interrupting capacity of any oil circuit breaker depends upon the quantity of gas generated by the arc within the breaker and the speed of the gas formation.

The rate of gas formation at a given electrode speed varies as the *square* of the time of arcing, not *directly* as the time of arcing, as is generally believed. This can be readily seen if we draw a

triangle in which the base represents the duration of the arc in half cycles, the altitude the length of arc in inches, the speed of break being assumed as constant. If we assume that the arc length is the same irrespective of any slight speed variation, then it is evident that the area of the triangle, which represents the gas volume, varies as the square of the half cycles of duration of arc.

If the recovery voltage of the circuit is such as to increase the arc length, then it is probable that the arc will hang after the end of the stroke has been reached and gas will be generated at such a rate as to destroy the breaker. The only safe way is to have the break distance such that the arc is certain to be ruptured by the time the end of the stroke is reached. In oil circuit breakers, where large quantities of gas are formed, the oil is violently projected against the top of the breaker and will lift the breaker from the floor unless it is bolted down.

It is practically impossible to have the gas escape from the tank as fast as generated especially as the projected body of oil plugs the outlet and it is, therefore, necessary to make the tank and top sufficiently strong to stand the gas pressure. The volume of gas is considerably greater than it is generally thought to be, rates of generation exceeding 100 cubic feet per second having been observed in tests. This rapid gas formation greatly stresses the tanks and tops and it is necessary that the construction be made very substantial or disaster will result.

There are various ways of decreasing the quantity of gas generated and the one the General Electric Company has found to be most effective is the explosion chamber. In this breaker we deliberately use high pressures within the chambers to increase the dielectric strength of the gas and to supply a cooling blast of atomized oil which also increases the dielectric strength. The gas is moreover projected vertically downward into the cool body of oil in the outer tank which again acts to cool it and by the time it has risen to the surface of the oil it is at such a temperature that ignition cannot take place from the heat, and moreover a gas mixture containing over 60 per cent. gas cannot be exploded, and at the interrupting rating of the breaker the mixture is always richer than this, so there can be no secondary explosion until diffusion has taken place. We build our newer high-class breakers of such a strength that they will withstand a secondary explosion without damage should one occur.

Answering Mr. Thomas' question as to the conditions at the Alabama Company test, these were all made single-phase at Bessemer, and both single-phase and three-phase at Lock 12, but on the three-phase tests the neutrals were grounded and the short-circuiting breakers were also tied into the ground so that the maximum voltage obtainable across any break was 57 per cent. of line volts.

During all the tests made at Baltimore, I never observed a recovery voltage equivalent to the initial voltage, whereas if the ground had been taken off either the neutral or the short-circuiting breaker, many observations would have been made showing a recovery voltage in excess of line volts. In Schenectady we test with but a single ground so as to obtain the highest possible recovery voltage. It might, of course, be possible to have surges or resonant conditions which would boost the recovery voltage, but no such evidences were observed in the Baltimore or Alabama Power Company tests.

In reference to the quantity of gas generated in an oil circuit breaker, Fig. 2 herewith shows the relative volumes very clearly. The figure shows the result of tests with an explosion-chamber and a plain-break type of breaker when tested in the same tank under the same load conditions and with the same operating mechanism.

The tests were made in such a way that the measurements of gas volume were correct within two per cent., the gas being measured very carefully after the rate of leakage, which was very small, had been found by test. The method consists in catching the gas and measuring it under standard conditions, *i. e.*, at atmospheric pressure and temperature. The measurements were made upon the fixed gas and any vaporized oil which might be later condensed was not measured. However, the pressure measurements, made in the outer tank of a circuit breaker check the volume measurements and show the remarkable decrease in pressure in the outer tank due to the use of the explosion chamber.

The oil question is one which needs considerable more thought than is given it by the ordinary station man. It is very important that the oil specified by the maker of the breaker be used. We have made a large number of tests upon oil and other liquids and find a wide difference in viscosity, freezing point, quantity of gas formed, carbon precipitation, etc.

The minimum gas formation may be represented by 8, the maximum by 90 when interrupting the same load, and while the pressures are not represented by the same figures, they do correspond in a way. The reason they do not conform closer is undoubtedly due to the presence of the vapor of the liquid which later condenses and consequently is not measured. There is, of course, no such great variation as indicated above in the oils we ordinarily use in the circuit breakers.

J. V. Jenks: Mr. Hilliard questioned my statement that the

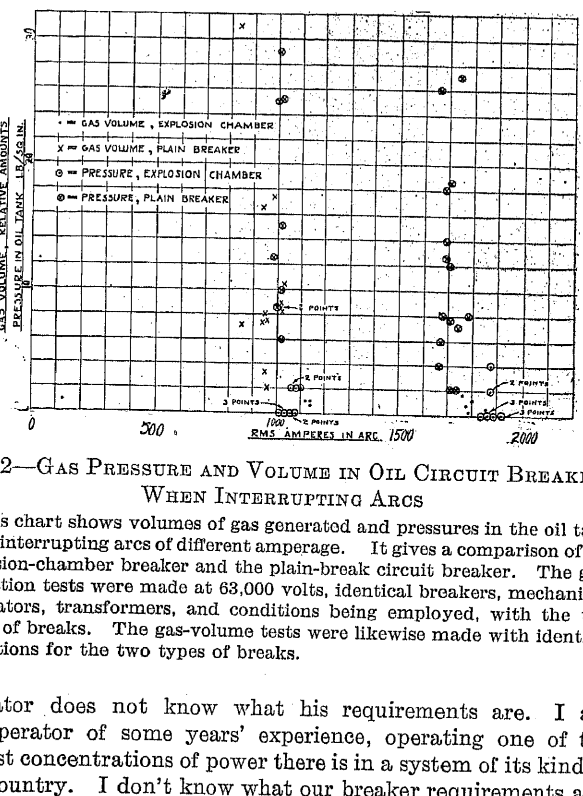

Fig. 2—Gas Pressure and Volume in Oil Circuit Breakers When Interrupting Arcs

This chart shows volumes of gas generated and pressures in the oil tank when interrupting arcs of different amperage. It gives a comparison of the explosion-chamber breaker and the plain-break circuit breaker. The gas-formation tests were made at 63,000 volts, identical breakers, mechanism, generators, transformers, and conditions being employed, with the two types of breaks. The gas-volume tests were likewise made with identical conditions for the two types of breaks.

operator does not know what his requirements are. I am an operator of some years' experience, operating one of the largest concentrations of power there is in a system of its kind in the country. I don't know what our breaker requirements are, and I have never met any other operator who did, and on that basis I made the statement.

Mr. Hilliard said that my statement, that no manufacturer had properly equipped himself, is incorrect. I bought from the General Electric Company 51 circuit breakers, guaranteed to break a three-phase arc of 1,000,000 kv-a. I asked the General Electric Company to produce a test to substantiate that guarantee. They said they were not in a position to do it. Mr. Hilliard, himself, admits that his test on his generator which can produce that amount of power might be somewhat different from line conditions; that the resonance or surges of the line might affect it.

We operators don't buy a circuit breaker to be tested and to serve up against a generator—we buy one to be served and serve the public miles away where there are many possibilities of resonance and surges and transformers and other apparatus which which might kick into that breaker.

Mr. Hilliard said there was no advantage in the additional multiple break. The facts are these: That the operator has no facilities for determining many of these theoretic things; we are not equipped to make these tests, and should not be asked to. About the only thing we really have, that means anything, is the "eating of the pudding."

We have on our system General Electric breakers of two breaks, which have a break much more than twice the length of break on the four-break breakers. They never have given, and I don't think it is possible for them to be made to give, the service the four-break breakers do.

Mr. Thomas asked if I had any idea as to what a breaker ought to be. I am going to tell you, in short, the specification which will largely govern the next half million dollars' worth of breakers the West Penn Company buys.

The operating device shall consist of a small motor, which will draw from the supply battery, which is always provided to insure continuity of operation, the minimum current.

The operating mechanism and all breaker units will be rigidly tied together with a steel structure, so that any failure of foundation will not interfere with the operation.

All the multiplying levers will be removed from the breaker proper and put in the operating housing, which will be an all-open-housing, so that all parts possible may be reached without entering the tank. This housing will be large enough to accommodate all necessary relays.

The operating mechanism will be trip-free, leaving as little of the inert mass to be moved by the breaker as possible.

The hand-closing device will be so arranged that a minimum of power will close the switch; also provided with a trip-free mechanism. The trips shall be positive and resetting upon the opening of the breaker. I can point out to you cases where circuit breakers are operating with three trips, three trigger arrangements—one really to hold the contacts, another one to hold the first and another one to hold the second. All of those things are supposed to function and operate properly almost instantaneously. The breaker comes in with a bang, and the triggers are supposed to catch and latch, but they don't do it. We have to adjust to thousandths of an inch, and yet we can't get the results desired. If we get a slight excess pressure on a trigger, the breaker won't trip properly.

So there is no excuse for the tripping mechanism not being made in such a way that it resets itself when it has plenty of time to do it, and then that the mechanism comes up against that locking device and is carried into the closed position.

The tripping device should be so arranged, interconnected with the operating mechanism, that the breaker would be instantly reclosing. Of course, the type of contact we are using now has to stop, but the mechanism does not need to stop. It can be started and continue on, and close the breaker again, re-establishing service as it ought to be re-established at once, not in a minute or two minutes, or some other period. That is necessary for several reasons: One of them is that in order to segregate a system, we find from time to time it is necessary, in order to conserve investment, to install air-break apparatus. Air-break apparatus should disconnect faulty pieces in small units where you cannot afford to put a costly circuit breaker, an attendant, and everything else that goes with it. These air breaks will open simultaneously with the oil breaker, but the oil breaker being a little speedier and so relayed, will interrupt the circuit. While the main circuit is interrupted, the air break will open the branch circuit to the part in trouble, and the oil breaker will immediately re-establish service again on the remainder of the system which is not at fault.

The method of transmitting power from the operating mechanism to the various breaker units should be by a torsion shaft, which can be closely fitted and packed to prevent gas passing from one unit to another and to prevent the entrance of moisture.

The series transformer should be sectional-wound, making it impossible to get a number of different ratios. It should be confined in a part of the supporting casting which normally supports a bushing; be sealed absolutely from the gas in arc chambers; removable integral with the bushing and connected on its low-voltage side by weather-proof conduits entirely outside of the breaker. The bushings, themselves, should be very liberally designed and of a very rigid nature, so that great magnetic stresses will not damage them.

The tank should be top-attached, to eliminate the possibilities of voids at the top which occur when tanks are supported from the bottom. Switch tanks do bulge and bolts do stretch, and when a tank bulges or a bolt stretches, if supported from the bottom, it leaves an opening around the top of the tank. The top of the tank should be sealed by a gasket on the order of a pump-sucker, which would close tighter with pressure, instead of opening on a slight movement of the tank at the top. The tank should have a bump bottom with an internally packed drain at the lowest possible point. It should be equipped with permanent lifting facilities in the case of those tanks which are lowered for inspection.

All openings to the tank or top should depend upon gaskets only for preventing capillary attraction of moisture. The idea of manufacturers building a breaker with a top of such form that the water remains on it and is prevented from going into the breaker by only a gasket, is something that the operator ought to insist on having corrected.

Linings should be of fire-resisting material. Particularly an inner lining should be designed to withstand arcs rather than have any particular insulating value. The lining next to the tank should have a high dielectric strength. Liners should be supported in a manner which would keep them from the moist oil in the bottom of the breaker, where they absorb the moisture, and which reduces their dielectric strength.

At least six, magnetically directed, barriered contacts should be provided, to assure series breaks which will speed up the break, which will keep the gases segregated, so that they can be cooled and conducted to the air chamber in the top of the breaker without danger of ignition from over-temperature.

The top of the breaker should be provided with a suitable baffle arrangement, which will absolutely prevent oil from being discharged from the vent. The vent should be self-hermetically sealed, and the breaker should be provided with a dependable breather. The manufacturers have for years done all within their power to keep moisture out of a transformer, which in reality can stand more moisture than a breaker. Because of the temperature in the transformer and the viscosity of the oil generally being kept at fairly constant value, the moisture does get to the bottom of the tank without doing much damage. The oil is not agitated in the transformer as it is in the breaker, where the moisture is stirred up. Yet, they build breakers with unprotected vents, so arranged that a very light snow will build snow all round them and then the breakers breathe through snow—and we do know that breakers breathe. It seems to me that instead of trying to keep the moisture out of the breaker and make the breaker the most dependable thing, we have lost sight of some very essential things which we have found in other electrical apparatus.

Such a breaker should, without throwing oil, still be fit for service at its rating after opening at least five times consecutively, under any system condition, irrespective of power factor, grounds or number of phases. If we get such a breaker—and I believe it can be developed at not much greater cost than what we are paying for the present-day breaker—I think that our breaker problem will be largely solved.

A New Self-Excited Synchronous Induction Motor

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Review of the Subject.—*Two early forms of the synchronous induction motor are described and their performance analyzed with special reference to synchronizing torque. A new self-excited form of this type of motor devised by the author is then described, its mode of operation and characteristics are elucidated and the principles governing its design explained with the help of vector diagrams. An analysis of the synchronizing torque shows that whereas prior machines all exhibit an alternating synchronizing torque of either slip, or double the slip frequency, the synchronizing torque in the new motor is substantially unidirectional and pulsating. The machine is particularly well suited for smaller units and operates with unity or leading power factor throughout its normal load range without taking an excessive current at any load or requiring any adjustment other than that automatically provided by its inherent regulating capacity.*

* * * * *

It is believed that the first synchronous induction motor was conceived and built by E. Danielson in 1901, while in the employ of the Allmaena Swenska Elektriska Aktiebolaget. (See E. T. Z., year 1901, page 1065). Danielson used a standard polyphase slip-ring motor with a three-phase winding on the revolving secondary in conjunction with an external source of

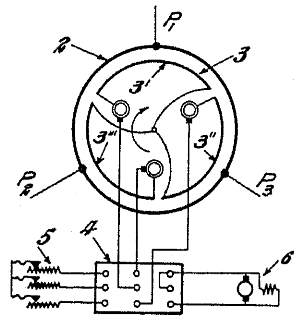

Fig. 1—Separately Excited Synchronous Induction Motor Danielson, 1901

unidirectional current. A two-pole machine of this type is diagrammatically illustrated in Fig. 1 where 2 is the primary three-phase stator winding, 3 the secondary three-phase rotor winding connected to slip-rings, 4 a three-pole two-way switch, 5 the starting resistances and 6 the source of unidirectional current. The machine was started in the usual way by inserting resistances 5 between the slip-rings and reducing these resistances to zero in one or more steps. When up to speed the switch 4 was thrown to the right, shortcircuiting two of the rotor phases in series and connecting all three to the d-c. exciter 6 as more clearly shown in Fig. 2.

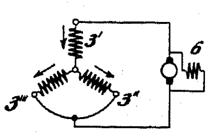

Fig. 2—Synchronizing and Exciting Circuit of Danielson Motor

Danielson found that with a unidirectional excitation which allowed of a 50 per cent overload as synchronous motor and yielded a leading power factor of 0.86 to 0.9 at full load, the synchronizing torque was about equal to the half-load torque. It was also observed that when loaded beyond its load limit as synchronous motor, the machine did not stop like an ordinary synchronous motor, but continued to operate as an asynchronous motor. Under these overload conditions the speed fluctuations were very marked and the motor would not go back to synchronism unless the load was reduced to about half normal, notwithstanding the fact that the machine was considerably over-excited.

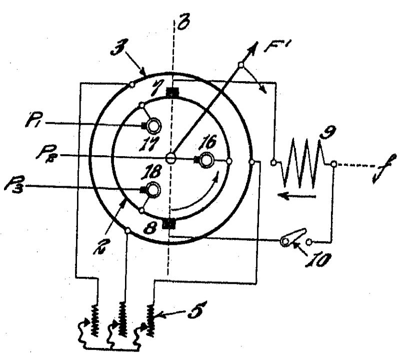

Fig. 3—Self-Excited Synchronous Induction Motor. Fynn, 1906

Whether the excitation could be sufficiently increased to synchronize the motor with full-load torque was not ascertained. It is not thought that materially different results have ever been noted with this type of machine.

The next step in this field was probably taken by the writer, who devised the machine diagrammatically shown in Fig. 3. (See B. P. No. 11, 298 of 1906). In the two-pole embodiment which is illustrated, the high-tension three-phase winding 2 is on the rotor and provided with slip-rings for connecting to the supply P_1, P_2, P_3. The three-phase low-tension winding 3 is on the stator and connected to the adjustable starting resistance 5. The unidirectional excitation is taken from the rotor and conveyed to the exciting winding 9

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located on the stator. To this end the high-tension winding is connected to a commutator as well as to slip-rings, and brushes 7, 8 carried by the secondary, cooperate with the commutator and are connected to the exciting winding. The axis f of this exciting winding is displaced 90 electrical degrees from the brush axis b. At synchronism the machine acts like a synchronous converter as well as a motor and delivers unidirectional current to the winding 9. In this way an external or independent exciter is dispensed with and the operating characteristic is considerably modified.

Let us now analyze the performance of these early machines a little closer. Fig. 4 shows another diagrammatic view of the Danielson motor. The three-phase rotor winding is here shown in the form of three belts, the direction of the direct current in the active part of the belts being indicated by dots and circles. The dots indicate current directed down through the plane of the paper and the circles denote the opposite direction of the current. The axis of the unidirectional rotor ampere-turns, or of the magnetization F produced by the direct current in the rotor, is f. It does not change its position with relation to the rotor windings unless the rotor connections or the polarity of the direct-current source is changed and neither happens in normal operation.

When the high-tension winding 2 is connected to the supply, a primary revolving flux F' is set up, as is well understood, and when the secondaries are closed, as happens in Fig. 1 when switch 4 is thrown to the left, secondary currents are induced in them and while they bring the motor very close to synchronism, they cannot possibly pull the machine into step or synchronize it, for the simple reason that these induction-motor torque-producing currents diminish very rapidly as synchronism is approached and become zero at synchronism. These currents are not indicated in Fig. 4.

When the machine is near synchronism, Danielson introduces his unidirectional excitation, the connections then being as shown in Fig. 2. The phases 3'', 3''' are connected in series and short-circuited while the phases 3', 3'' and 3', 3''' are closed over the exciter 6, which has a certain amount of reactance. This destroys the induced-current balance in the rotor and therefore the constancy of the induction-motor torque, and increases the slip. So far as the exciter 6 is concerned, phases 3'' and 3''' are connected in parallel and the two of them are in series with phase 3', which produces the distribution of unidirectional ampere turns shown in Fig. 2 by arrows and in Fig. 4 by dots and circles. Notwithstanding the unbalancing of the induced currents in the rotor as to phase and magnitude, all of these still become zero at synchronism. The approximate shape of the resulting unbalanced induction-motor torque T_i is shown in Fig. 5. Its magnitude diminishes as synchronism is approached and it becomes zero at synchronism. Because of friction and windage losses, the useful induction-motor torque becomes zero before synchronism is reached.

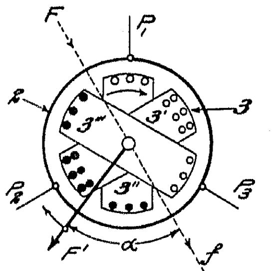


FIG. 4—SHOWING HOW SYNCHRONIZING TORQUE IS PRODUCED IN DANIELSON MOTOR

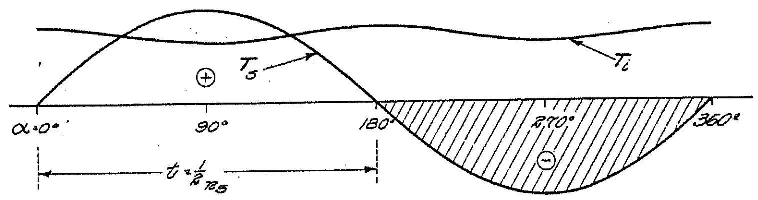


FIG. 5—INDUCTION MOTOR TORQUE T_i AND ALTERNATING SYNCHRONIZING TORQUE T_s NEAR SYNCHRONISM. DANIELSON, 1901.

To bring the rotor into synchronism, it is necessary to dispose of a torque which will accelerate the rotor and the load from the asynchronous speed, at which it is being driven by the induction-motor torque, right up to the synchronous speed. In so doing, this torque must overcome the inertia of the rotor and of the load. The greater the inertia and the "slip" and the shorter the time available for the acceleration, the greater this synchronizing torque must be.

Disregarding the hysteresis torque, which is very small in an induction motor, the only possibility of developing a synchronizing or "pull in" torque in the Danielson motor lies in the added unidirectional rotor magnetization already described.

The unidirectional ampere-turns on the rotor can and do produce a torque with F'. The difference in speed between their axis f, which revolves clockwise with the rotor, and F', which revolves clockwise at synchronous speed, is equal to the "slip" of the rotor, and the interaction can be most conveniently studied by assuming that the rotor stands still and that F' revolves clockwise at slip frequency. It is at once clear that F' can produce a torque with the unidirectional rotor ampere-turns whenever its axis does not coincide with the axis f of the latter. Referring to Fig. 4, let α be the angle denoting the displacement between f and the axis of F'. Beginning to count this displacement at the instant when F' coincides with and is of the same direction as f, we find that for $\alpha = 0$ the synchroniz-

ing torque is zero. As F' continues on its way the revolving flux begins to enter that side of the rotor which carries upwardly-directed unidirectional current, thus producing a clockwise and here positive torque, which reaches a maximum when F' is displaced 90 electrical degrees from f and goes back to zero when F' coincides with, but opposes f. A further movement of F' causes this flux to enter that side of the rotor which carries downwardly-directed unidirectional current,

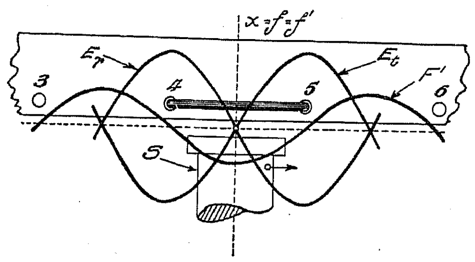


FIG. 6—PERIODICAL SPACE RELATION BETWEEN STATOR COILS AND ROTOR POLES AT NO-LOAD IN IDEAL SYNCHRONOUS MOTOR

producing a counterclockwise and here negative torque, which reaches a maximum when F' is again at right angles to f. In Fig. 5 this torque is represented by the curve T_s of slip frequency. The time t, during which this torque is positive, equals $\frac{1}{2\ n_s}$ where n_s is the slip frequency. The amplitude of T_s for a given rotor winding depends on the magnitude of F', which is practically constant near synchronism, and on the

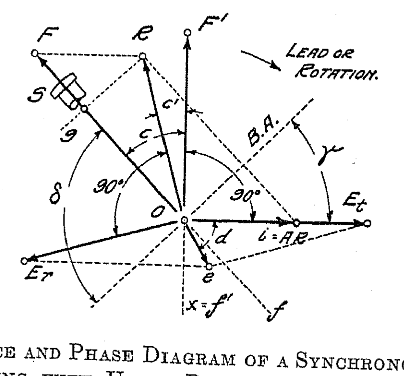


FIG. 7—SPACE AND PHASE DIAGRAM OF A SYNCHRONOUS MOTOR OPERATING WITH UNITY POWER FACTOR (i = 30)

Also applies to new motor when the brush-axis of the latter is at right angles to the total unidirectional magnetization produced by the secondary.

magnitude of the unidirectional current which is also constant so long as the resistance of the exciting circuit or the unidirectional voltage of the exciter are not intentionally varied.

The usual simplifying assumption is made throughout this paper that all fluxes have sine distribution and all e. m. fs. and currents vary according to the sine law.

While T_s is negative it can, of course, not accelerate the rotor into synchronism. In fact, it retards its speed and makes it all the more difficult for the positive wave to bring about synchronism. This negative torque has to be met by an increased induction-motor torque and said torque cannot increase unless the rotor speed is lowered. The only hope lies in the positive wave of T_s.

So long as the machine operates as an induction motor the revolving flux F' produced by the primary lags about 90 electrical deg. behind the terminal voltage E_t as shown in Fig. 6. The primary back e. m. f. E_r is then generated by F' in the windings located in the stator slots 3, 4, 5, 6. A certain amount of lagging magnetizing current is necessary to produce F' and the power factor of such a machine is low on that account, particularly when the motor operates at no-load. If now, at no-load, the rotor is driven synchronously and so excited by unidirectional current as to produce a flux F of same magnitude and direction as F', then the machine will operate as before, except that the rotor must now preserve its synchronous speed and the power factor will now be unity because it will no more be necessary for the primary to carry a magnetizing

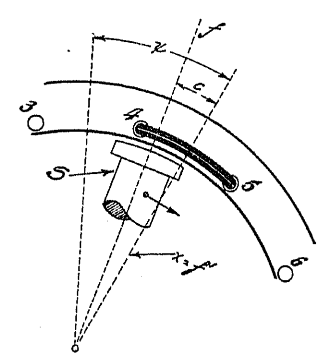


FIG. 8—PERIODICAL SPACE RELATION BETWEEN STATOR COILS AND ROTOR POLES IN PRACTICAL SYNCHRONOUS MOTOR

current. The flux F, thus produced by the unidirectional rotor magnetization, will replace the original revolving flux F' produced by the stator and will perform all of the latter's functions.

The machine has been converted from an asynchronous to a synchronous one and it is seen that the axis f of the pole S coincides with the axis f' of F' and with the axis x of the primary stator coil 4, 5. This, however, is an ideal condition, true only in case the reactance of the motor is nil, which is never the case. In a practical machine unity power factor cannot be secured even at no-load unless E_r is so out of phase with E_t as to produce a resultant e leading E_t by an angle d equal to the lag of the primary current i behind e, which lag depends upon the impedance of the primary. This is well shown in the phase and space diagram of Fig. 7 in which the impedance of the primary has been chosen unduly large in order to secure a clear diagram. It is here convenient to measure all space displacements from the axis x of the coil 4, 5 of Fig. 6 and to deal with fluxes rather than e. m. fs. Thus F' which lags 90 deg. behind E_t is a measure of the latter. The resultant

flux R leads the back e. m. f. E_r by 90 deg. and is a measure for same. The angle c' measures the space displacement of R from x and the phase displacement of E_t and E_r, the actual phase difference between the last two always being equal to $(180-c')$ deg. The angle c measures the space displacement of the unidirectional flux F, therefore of the axis f and of the pole S, from the axis x. It is seen that E_r must lead E_t by less than 180 deg. and the pole S must move back, against rotation, by some angle c in order to secure unity power factor

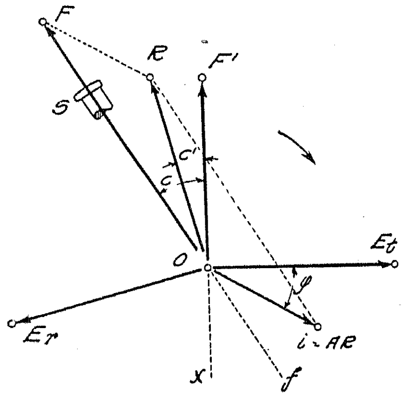


FIG. 9—SPACE AND PHASE DIAGRAM OF A SYNCHRONOUS MOTOR OPERATING WITH LEADING POWER FACTOR (i = 30)

for even very small currents. This is also shown in Fig. 8, where ψ measures the pole pitch. Under these conditions the axis f of the pole S will be c degrees short of the coil axis x, or of the axis f' of the revolving induction motor flux F', when E_t passes through zero on its way from a negative to a positive maximum. The angle c is greater than the corresponding angle c' for R or E_r because of the influence of the armature reaction.

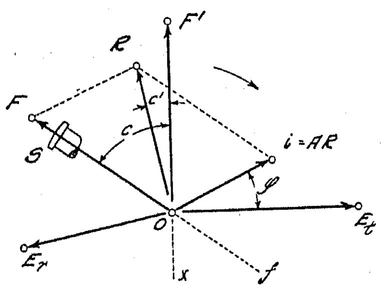


FIG. 10—SPACE AND PHASE DIAGRAM OF A SYNCHRONOUS MOTOR OPERATING WITH LAGGING POWER FACTOR (i = 30)

To secure a leading primary current the unidirectional excitation must be increased, which will bring about a reduction of the angle c, see Fig. 9, but an increase in maximum torque. Fig. 10 shows the relations for a lagging current of same magnitude as in Figs. 7 and 8 and Fig. 11 illustrates what happens when unity power factor is secured for a larger current than that in Fig. 7. For a given excitation c' increases as the torque is increased. In a machine with constant self-induction coefficient and without armature reaction, the torque reaches a maximum when c' = 90 deg., *i. e.* when E_r leads E_t by 90 deg.; thereafter it falls off and the operation becomes unstable. For these conditions $c = c'$.

In practise c is always larger than c', the lower the power factor the greater is the difference in favor of c, as well shown by Figs. 9, 7 and 10. Maximum torque is actually reached for values of c' smaller than 90 and for values of c of about 90 deg. When the load on the machine is increased while the excitation is kept constant, the rotor is retarded more and more, c and the torque increasing and the power factor decreasing until the maximum torque is passed, when the machine falls out of step. Because the value of c is limited the overload capacity of an over-excited motor, Fig. 9, is greater than that of an under-excited machine, Fig. 10.

This consideration shows, among other things, just what the synchronizing torque must accomplish. Fig. 5 shows that this torque is exerted over an arc equaling one pole pitch ψ. The time in which this arc is covered depends on the slip of the rotor and increases as the slip diminishes. This time is $\frac{1}{2\ n_s}$ where n_s is the slip fre-

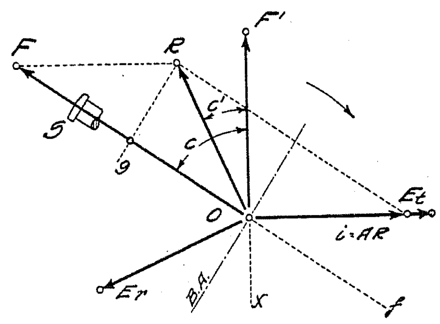


FIG. 11—SPACE AND PHASE DIARGAM OF A SYNCHRONOUS MOTOR OPERATING WITH UNITY POWER FACTOR (i = 43)

quency per second. Within $\frac{1}{2\ n_s}$ seconds or less, T_s must accelerate the rotor and the load to synchronous speed and bring about a certain angular space relation between S and F'. When synchronism is reached the space angle between the axis f' of the revolving flux F' and the axis f of S or of the unidirectional rotor magnetization must be equal to or less than c_{max}, the latter lagging behind the former. The angle c_{max} is that which, for the prevailing excitation, corresponds to the maximum available synchronous torque. If acceleration is completed when S lags c_{max} degrees behind F', then the synchronous motor torque will take control of the action of the rotor. If the maximum synchronous torque is just sufficient to deal with the prevailing load, the rotor will continue to operate with a periodical space lag of c_{max} degrees with respect to x. If the maximum synchronous torque is in excess of that required by the pervailing load, the rotor will be drawn into its proper periodical space relation to the axis x,

i. e. the angle c will be suitably reduced and the rotor will continue to run synchronously.

The difficulty with this type of machine appears to lie in the fact that if it is built with an induction motor air-gap and without defined polar projections, conditions necessary to a good asynchronous motor starting and operating performance, it becomes extremely sensitive to armature reaction, calls for a comparatively small number of no-load exciting ampere-turns, corresponding to the small asynchronous motor magnetizing current, and requires a very large change in exciting ampere-turns from no-load to full-load, in order to keep the power factor at all constant. Even with exciting ampere-turns corresponding to a leading power factor at full load, the synchronizing torque is only about 50 per cent of the full-load torque and if the load is thrown off without changing the unidirectional excitation, the machine takes an unduly large leading current. Synchronous conditions can be much improved by sacrificing the induction motor characteristics, but even then this type is not suitable for smaller sizes because of the necessity of a separate exciter.

Turning now to Fig. 3, when this machine is connected to the supply, a revolving flux F' is produced and revolves synchronously in space so long as the rotor, here the primary, is at rest. At this time the brush voltage is of full frequency. When the primary moves it does so in a direction opposed to that in which F' rotates and as the primary speeds up so does F' slow down in space, *i. e.*, relatively to the stator. When the rotor is as near to synchronism as the secondary 3 can bring it, F' revolves with slip frequency with respect to the stator but still synchronously in so far as the rotor conductors are concerned. The brush voltage is now a single-phase current of slip frequency. The maximum value of this voltage is the same throughout, as long as F' does not change, but its frequency is always equal to that with which F' moves with respect to the stator because the brushes are stationary. When F' coincides with the brush axis b, the brush voltage is zero, when it is displaced 90 electrical deg. from said axis, it is a maximum, when F' is directed from left to right, brush 7 is positive and vice versa. The curve e_b of Fig. 20 shows how the brush voltage changes; it is plotted against α, the angle by which F' is displaced from the brush axis, the zero value of α being assumed to exist when F' coincides with the brush axis and is directed from brush 8 to brush 7. The positive voltage value is that corresponding to its direction in normal operation.

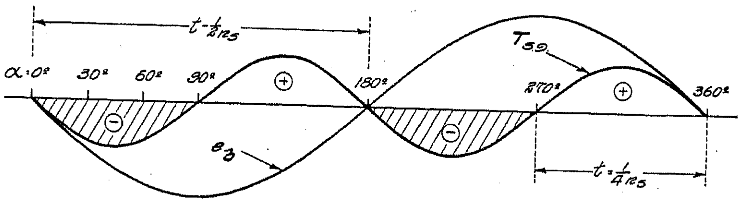
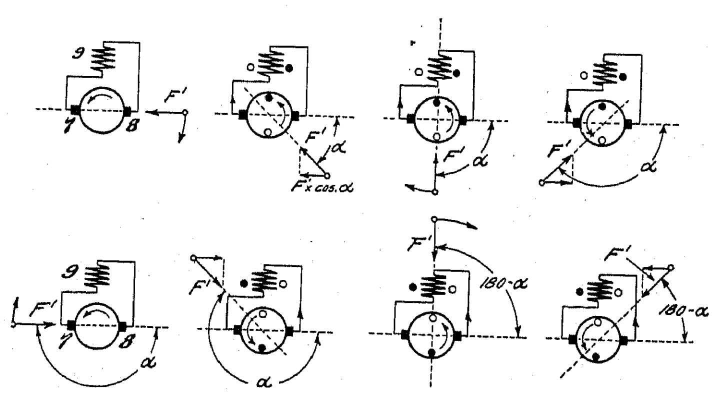


FIG. 20—SYNCHRONIZING VOLTAGE AND ALTERNATING SYNCHRONIZING TORQUE IN FYNN 1906 MOTOR



FIGS. 12 TO 19. SHOWING HOW THE ALTERNATING SYNCHRONIZING TORQUE IS PRODUCED IN FYNN 1906 MOTOR

This torque forms one component of total synchronizing torque of the new motor.

If switch 10 of Fig. 3 be now closed, the alternating brush voltage will send a current of slip frequency through the exciting winding 9 and an additional torque, over and above the prevailing induction motor torque, will be developed between the primary revolving flux F' and the ampere-turns in 9. It is true that when 10 is closed an additional secondary circuit is provided, which is inductively responsive to F' and which, therefore, will have generated in it secondary induction-motor torque-producing currents closing over the brushes 7, 8, but this torque, such as it is, may be disregarded, since it merely adds to the induction-motor torque produced by 3 without qualitatively changing the performance.

As a first approximation, it is permissible to assume that there is no phase difference between the brush voltage and the current it produces in 9, because the slip frequency is very small even when the full-load induction-motor torque is being developed and further diminishes as synchronism is more closely approached.

Figs. 12 to 19, inclusive, give a clear view of how the synchronizing torque is produced in this case. The brush voltage is always proportional to *sine* α and the component of F' at right angles to the axis of the exciting winding 9, which is the only part of F' available for producing torque with the ampere-turns in 9, is proportional to *cosine* α. The synchronizing torque T_s is proportional to *sine* α $\times$ *cosine* α, as may be seen, for instance, by reference to Fig. 13. Bearing these conditions in mind, it is only necessary to follow the movement of F' relatively to the brush axis and in each case note the direction of the brush current.

In Fig. 12 the brush voltage e_b is zero and the torque is also zero because the current in 9 is zero. In Fig. 13 the brush voltage and current are negative, producing the current, distribution in, 9 shown in the figure by dots and circles. The horizontal component of F' is the

only one which can produce torque with the ampere-turns in 9. The brush voltage, and therefore the ampere-turns in 9, are proportional to *sine* α, since $\alpha = 45°$ *sine* $\alpha = 0.707$. The horizontal component of F' is proportional to *cosine* α or to 0.707 and the torque T_s is, therefore, proportional to one half. In this case this is the maximum value of the torque. As appears from the direction of the ampere-turns in 9 and from that of the horizontal component of F', this maximum torque is clockwise and here negative.

In Fig. 14 the torque T_s is zero, although e_b is a maximum because the axis of F' coincides with that of 9. When $\alpha = 135°$, Fig. 15, T_s has reached a positive maximum. The changes of e_b and of T_s with varying α are plotted in Fig. 20.

It is seen that for otherwise equal conditions the self-excited motor of Fig. 3 develops a maximum synchronizing torque of half the value and double the frequency of the maximum synchronizing torque produced by the separately-excited motor of Fig. 1. Being of double the frequency a positive, or useful, half cycle of the synchronizing torque of Fig. 3 lasts but half

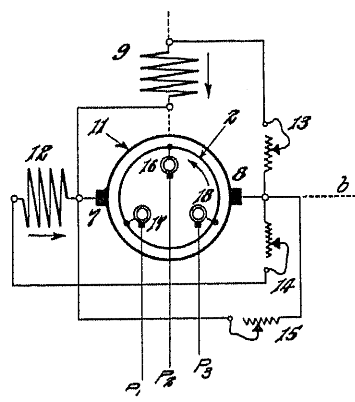


FIG. 21—SELF-EXCITED SYNCHRONOUS INDUCTION MOTOR. NEW FYNN

the time, $\frac{1}{4\ n_s}$ seconds, of the corresponding torque wave of Fig. 1. The synchronizing requirements are, however, exactly the same for both machines. They have been discussed in connection with Figs. 6-11 inclusive.

Another difference between the two machines is that in Fig. 1 the armature reaction only affects the magnitude and space position of the resultant magnetization by adding to or subtracting from the unidirectional magnetization F, whereas in Fig. 3 this reaction also affects the brush voltage through its effect on the resultant magnetization.

The self-exciting feature of Fig. 3 is, however, vital in the case of machines with an output smaller than that corresponding to, say, 150 horse power at 1000 rev. per min., and this type was, therefore, thought worthy of further effort. The writer presently devised the machine shown in Fig. 21, which overcomes many of the drawbacks of the earlier forms and exhibits some quite interesting and novel characteristics.

Fig. 21 diagrammatically illustrates the new motor in its simplest and two-pole form. The rotor is the primary member, it carries a polyphase winding 2 adapted for connection to the supply P_1, P_2, P_3 through the slip-rings 16, 17, 18 and a commuted winding 11 with which cooperate brushes 7, 8 carried by the secondary member which is here the stator. In order to avoid all uncertainty as to the brush position it is assumed throughout that the brushes bear directly on the commuted winding. The secondary carries an "exciting" winding 9, connected to the brushes 7, 8 by way of the adjustable resistance 13 and displaced 90 electrical deg. from the brush axis, and a "neutralizing" winding 12 coaxial with the brush axis and connected to the brushes by way of the adjustable resistance 14. In addition, an adjustable resistance 15 is connected to the brushes 7, 8 to shunt the commuted winding 11.

This machine can be operated with and without the resistance 15. Assuming that 15 is not used, the slip-rings are connected to the supply with the circuits of the windings 9 and 12 open or each closed over a predetermined amount of resistance. The polyphase rotor currents produce a revolving flux in the usual manner and the latter generates secondary torque-producing currents in the stator windings 9 and 12 displaced by 90 electrical deg. which are, therefore, in the most favorable position for producing a uniform torque with the primary flux. The secondary currents generated in 9 as well as in 12 close over the brushes and the rotor winding 11 and can be regulated by manipulating the adjustable resistances 13 and 14, whereby the torque developed and the current taken by the motor can be adjusted like in a slip-ring induction motor. In this manner the motor reaches a speed very close to the synchronous. At such time the primary revolving flux moves with slip frequency with respect to the windings 9, 12 and the brushes 7, 8. The low periodicity single-phase brush voltage sends a considerable current through the windings 9 and 12 and the secondary conduced ampere-turns thus set up in said windings, reacting with the primary revolving flux, develop a powerful synchronizing torque in a manner to be more fully explained. The synchronizing torque, developed by the windings 9, 12 which first did duty as polyphase secondaries of a polyphase asynchronous motor, brings the motor into step, producing a further change in the reactions within the machine. As soon as synchronous speed is reached the primary revolving flux becomes stationary in space; it can, therefore, be replaced by a unidirectional magnetiation, stationary in space. Similarly, the primary armature reaction is stationary in space so long as the load and the excitation are constant, this reaction can therefore also be opposed by a unidirectional magnetization. When the load or the excitation is changed, the axis of the armature reaction alters its position in space and then continues stationary. Because the primary revolving flux is now stationary in space, the brush voltage is unidirectional and the magnetizations pro-

duced by the stator windings 9 and 12 are, therefore, also unidirectional at synchronism. Broadly speaking, 12 opposes the primary armature reaction or at least a component of same, and 9 produces the excitation with or without the help of 12 as will hereafter appear.

Small machines can be started and operated without the use of adjustable resistances in the circuits of the windings 9 and 12. For larger machines adjustable resistances should be used and reduced in one or more steps as the speed increases. It is quite possible to use but one adjustable resistance, placing it in a conductor carrying the current of 12 as well as that of 9. This single resistance can, for instance, be located between brush 7 and one terminal of 12 and one of 9. For still larger motors, or when the starting conditions are particularly severe or the winding 11 operates close to the limit of good commutation, it is advisable to use the adjustable resistance 15 to shunt the commuted winding at starting. At starting, the voltages generated by the primary revolving flux in 9 and 12 are considerably higher than the voltage generated by it in 11 and appearing at the brushes. Furthermore, the periodicity of the stator-generated voltages is, at starting, equal to the frequency of the supply, with the result that the iron-surrounded commuted winding 11 offers considerable reactance to the secondary currents. For these reasons, it is quite possible to shunt the commuted winding with a value of non-inductive resistance 15 which will allow large starting-torque-producing secondary currents to pass, while restricting to a small value that current which the brush voltage is able to force through it. When the resistance 15 is used, its value should be increased with increasing speed and made infinite, preferably before synchronism is reached.

Instead of using two independent windings 2 and 11

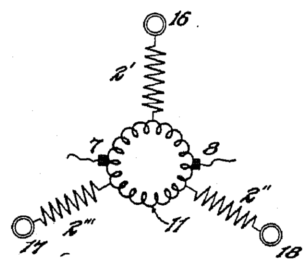


FIG. 22—PRIMARY WINDING OF SELF-EXCITED SYNCHRONOUS INDUCTION MOTOR. FYNN 1906

on the primary, it is more economical and more efficient to combine them, for instance, as shown in Fig. 22, where the commuted winding 11 is located in the center or zero point of the three-phase star connected winding 2′, 2″, 2‴. The winding 11 here operates under the conditions prevailing in a synchronous converter with a single winding on the primary. It is known that under load the losses in such a winding are less than if the direct current was suppressed without changing the value of the alternating current or vice versa. By making use of this condition, the losses in the primary can be somewhat reduced and the arrangement shown in Fig. 22 also permits of the best possible utilization of the winding space on the primary, with due regard to commutation requirements.

It may be stated that nothing, insofar as the principle of operation is concerned, is changed when the primary is located on the stator instead of the rotor. When the primary is stationary, the revolving flux always revolves synchronously with respect to it, and

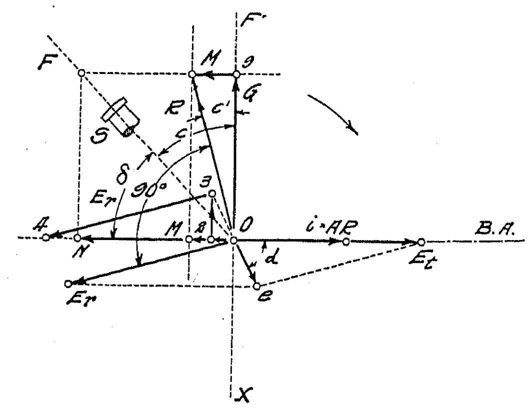


FIG. 23—SPACE AND PHASE DIAGRAM OF NEW MOTOR FOR FULL-LOAD UNITY POWER FACTOR AND RESULTANT MOTOR MAGNETIZATION LAGGING ABOUT 90 DEGREES BEHIND THE BRUSH AXIS

therefore in space, and the secondary revolves in the same direction as this revolving flux. In asynchronous operation the speed of the secondary is a little short of that of the revolving flux and at synchronism the speeds of the two are the same. When the primary is located on the stator, the brushes cooperating with the commuted winding on the primary must revolve with the secondary.

The various windings may be variously dimensioned according to the desired characteristic, but to fix ideas, let it be assumed that 9 and 12 are designed to produce, with given values of the resistance 13 and 14, a number of conduced ampere-turns which will insure unity power at the terminals of the motor at full load. The rotor winding 2 must be able to carry the load current and be adapted to the voltage and the periodicity of the supply. The winding 11 must be dimensioned with two ends in view which, however, are not antagonistic. On the one hand it is necessary to secure perfect commutation at starting as well as at synchronism; on the other hand, the brush voltage should be as low as possible so that the number of turns in the windings 9 and 12 on the secondary can be kept down to a value which will not make the value of the voltage generated in 9 and 12 at starting so high as to be dangerous to life or to the insulation. In addition to this, the resistance of one or both of the windings on the secondary must be made so low that when the resistances 13 and 14 are short-circuited the number of available conduced ampere-turns

on the secondary is sufficient to produce the desired synchronizing torque. For reasons which will presently appear, it is best in this connection to rely on 12 rather than on 9 and to dimension 12 accordingly.

The phase and space diagram of Fig. 23 will help to make clear the conditions which govern the design of the windings 9 and 12. Let E_t represent the terminal e. m. f. and i the full-load current. For unity power factor at the motor terminals, the phase of i will coincide with that of E_t. The armature reaction ampere-turns or the armature reaction flux $A\,R$ is of the same phase as i and may be represented by the same vector. Because of the impedance of the primary the current i will cause an ohmic drop 0-2 and a reactive drop 2-3, the first displaced 180 degreės from i, the second lagging 90 deg. behind i. The impedance drop 0-3 must be equaled and opposed by the resultant e. m. f., e acting on the armature circuit in order to produce the current i lagging d degrees behind e. The angle d is determined by the time constant of the armature which here is $\frac{2\text{-}3}{0\text{-}2}$. Now e must be the resultant between the terminal e. m. f. E_t and the back e. m. f. E_r which settles the phase and magnitude of 3-4 or E_r. The resultant flux R (or the corresponding ampere-turns) must lead E_r by 90 deg. and its magnitude is determined by that of E_r. The dimensioning of the windings 9 and 12 depends on the relation desired at a given load, say at full load, between the axis of the armature reaction and that of the brushes 7, 8. A number of solutions are

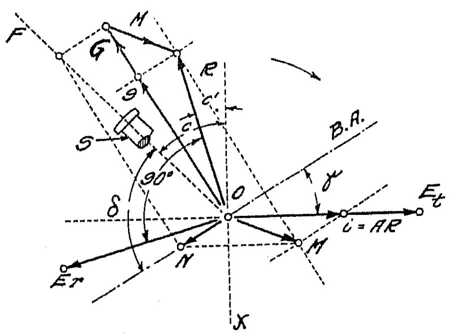


Fig. 24—Space and Phase Diagram of New Motor for Full-Load Unity Power Factor and Resultant Motor Magnetization Lagging Less than 90 Degrees Behind the Brush Axis

possible, each giving a different operating characteristic, particularly as to maximum torque and as to power-factor change with load. Fig. 23 shows the case for which the armature reaction flux coincides with the brush axis at unity power factor and full load. To secure this condition the flux N produced by the conduced ampere-turns of the winding 12 must exceed the armature reaction flux $A\,R$ by the vector M and the flux G produced by the conduced ampere-turns of the winding 9 must be such that the vectorial sum of G and M equals R. If the brush axis coincides with i then N must be in phase opposition to i since 12 is coaxial with the brush axis; and G must be at right angles to and lead N since 9 is placed 90 electrical deg. ahead of 12. The end of the vector M is found as the intersection of a parallel to N drawn through the end of the vector i with a parallel to G drawn through the end of the vector R. The flux available for the generation of the brush voltage is 0-9 which is here equal to G. The resultant flux R is larger than 0-9. This diagram settles all design details for this particular case.

But unity power factor at full load, *i. e.*, for the same

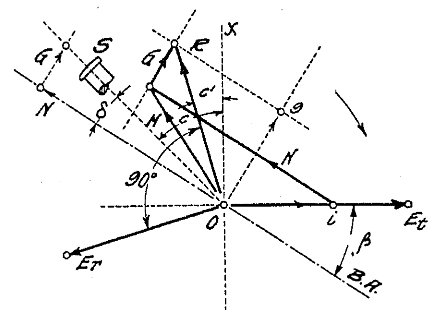


Fig. 25—Space and Phase Diagram of New Motor for Full-Load Unity Power Factor and Resultant Motor Magnetization Lagging More than 90 Degrees behind the Brush Axis

current as that sssumed in Fig. 23, can be secured with the axis of the armature reaction leading the brush axis. When the armature reaction axis is said to lead the brush axis, it is displaced from the latter *in* the direction of rotation of the primary as shown by γ in Fig. 24. N is coaxial with the brush axis and the resultant M, of N and $A\,R$, combined with G which leads N by 90 electrical deg. must produce R. The magnitude and phase of the latter must, of course, be the same as in Fig. 23, since the load and the power factor are the same. The end of the vector M is found as the intersection of a parallel to N through the end of i with a parallel to G through the end of R. This determines the magnitude of N and G, in other words, the ampere-turns in the windings 12 and 9. The flux available for the generation of the brush voltage is 0-9; it is less than R and less than G.

It is seen that N becomes zero and G a maximum as soon as the brush axis is displaced 90 electrical deg. from the line joining the ends of the vectors i and R. This limiting case is that of the machines shown in Figs. 1 and 3 and is positively characterized by the phase and space diagram of Fig. 7, except that the brush axis line has no significance in connection with Fig. 1.

Similarly, unity power factor may be had at full load with the axis of the armature reaction lagging β degrees behind the brush axis. This may also be expressed by saying that the axis of the armature reaction is displaced by β degrees from the brush axis in a direction opposed to that of the rotation of the primary member. This case is illustrated in Fig. 25 and the end of

the vector M is found as in Fig. 24. The flux available for the generation of the brush voltage is 0-9, which, in this case, is larger than G but again smaller than R.

When, in continuing its clockwise rotation, the brush axis comes to occupy a position parallel to the line joining the ends of the vectors i and R, the ampere-turns in winding 12 or the flux N becomes a maximum and the ampere-turns in winding 9 or the flux G zero, R is now simply the resultant of N and i or $A\,R$ and $N = F$. This, however, does not mean that there is no flux displaced by 90 electrical deg. from the brush axis. As before, this flux is the component 0-9 of R and is here much smaller than R. Even in this limiting case a brush voltage is available and the machine is operable, but its operation is likely to be unstable because but a slight variation in the position of the brush axis can cause sharp variations in the power factor, even when the load is constant. This limiting case is shown in Fig. 26.

It is now seen that beginning with the limit case of Fig. 7, where the brush axis is displaced 90 electrical

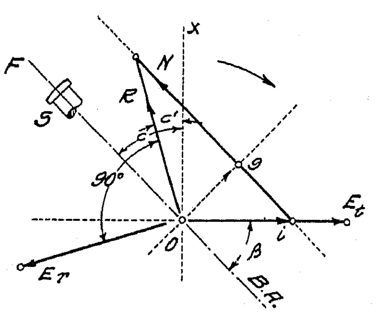


FIG. 26—SPACE AND PHASE DIAGRAM OF NEW MOTOR FOR FULL-LOAD UNITY POWER FACTOR AND BRUSH AXIS COINCIDING WITH TOTAL UNIDIRECTIONAL MAGNETIZATION PRODUCED BY THE SECONDARY

deg. from the axis of the resultant unidirectional magnetization produced by the secondary member and shows the maximum lag with respect to the axis of the armature reaction, the windings 12 and 9 can be dimensioned to cause this axis, under given load and power factor conditions, to occupy any position within 90 electrical deg. measured in the direction of rotation. When unity power factor is desired at full load, the better proportions to be given to the windings 12 and 9, from the point of view of maximum synchronous operating torque and power-factor change with load, are those which cause the brush axis either to coincide with the axis of the armature reaction or to lead the latter by less than the maximum permissible angle. For the constants chosen, this maximum leading angle is shown in Fig. 26 and happens to be in the neighborhood of 45 electrical deg., it always occurs when the brush axis coincides with the axis of the resultant unidirectional magnetization produced by the secondary member. To secure the one or the other condition, it is necessary to suitably select the ratio of ampere-turns in 9 and 12, yet a bare statement of this ratio does not positively characterize the operating characteristics of a machine, unless the ratio of resultant flux and armature reaction flux, or of their equivalents, is also given. Thus, for the machine under consideration, the brush axis coincides with the armature reaction axis at full load and unity power factor, when the ratio of "neutralizing" ampere-turns in 12 to "exciting" ampere-turns in 9 is about unity. If the full-load ampere-turns of this machine were 50 per cent higher, the above ratio for unity power factor at full load and for coincidence between armature reaction and brush axis would be about one and one half.

A very welcome feature of this motor is the fact that when the windings 12 and 9 are proportioned in the manner set forth, the power factor will have acceptable values at all loads without it being necessary to make any adjustments whatsoever with changing load. This is an absolutely necessary condition in all smaller sizes and is only little less important in connection with large motors. By large is meant anything exceeding an output corresponding to about 150 horse power at 1000 rev. per min. The power factor may be kept at near unity for all loads in excess of about one fifth of full load, in which case, it will be lagging at lighter loads, or, and this is usually the better choice, it can be made leading at no load and about unity at full load. In no case will the machine take an excessive leading current and hand adjustments during normal operation are, therefore, not necessary.

The next point of interest is the synchronizing ability of the new machine and the relation of its winding 12 and 9 and of the ratio of the ampere-turns they produce to the magnitude and conformation of the synchronizing torque.

Reference to Figs. 3 and 21 shows that the "exciting" winding 9 occupies in both machines exactly the same position with reference to the brush axis b. The variation of the brush voltage e_b must be identical in the two motors and it follows that so far as winding 9 of Fig. 21 is concerned, the synchronizing torque it produces and the voltage to which it is due are correctly represented, at least as to form, by the curves $T_{s.9}$ and e_b of Fig. 20. The magnitude of $T_{s.9}$, other conditions being equal, is proportional to the number of ampere-turns carried by 9, and that of e_b can be and is assumed to be the same in both motors.

Just what the "neutralizing" winding 12 contributes to the synchronizing torque may be determined by following the line of thought adopted in deducing the shape and magnitude of $T_{s.9}$. The simplifying assumption will again be made that the frequency of the brush voltage e_b is so small very near synchronism that there is no phase difference between said voltage and the current it determines through the winding 12. Figs. 28 to 35, inclusive, really tell the whole story.

Starting with that position of the slowly revolving primary induction motor flux F' in which its axis coin-

cides with the brush axis and this flux is directed from brush 8 to brush 7, and assuming that the positive voltage (and current) direction is that which corresponds to its direction in normal operation as shown in Fig. 27, we can plot in Fig. 37 the volt and torque curves as they are deduced from Figs. 28 to 35. For the conditions of Fig. 28, $\alpha = 0$ and both e_b and $T_{s \cdot 12}$ are zero. After the primary flux F' has moved through $\alpha = 45$ deg. in a clockwise direction, the voltage generated at the brushes is negative, as indicated by the dot and circle within the armature, and proportional to *sine* α. The torque between F' and the ampere-turns in 12, due to e_b, is at this time proportional to e_b and to that component $F' \times$ *sine* α which is displaced 90 electrical deg. from the axis of 12. For $\alpha = 45$ deg. $T_{s \cdot 12} = 0.707^2 = 0.5$. The direction of the torque exerted on the rotor, deduced from the direction of $F' \times$ *sine* α and the current distribution in 12 shown by dots and circles, is seen to be counterclockwise or positive. Throughout this paper the full circles or dots indicate downwardly-directed current.

In Fig. 30 the primary flux has progressed through located and connected as 12 in Fig. 21 produces a maximum torque equal to just twice that produced by a winding located and connected as 9 in Fig. 21.

Fig. 36 is added for the purpose of an easy check as to the direction of the torque in Figs. 28 to 35 and also in Figs. 12 to 19. Fig. 36 reproduces the conditions of Fig. 30. The flux F' emanating from the rotor is directed upwardly, the winding 12 is distributed on the stator 20, as shown by the dots and circles, which also indicate the direction of the current in the conductors of 12. It is seen that the interaction of F' and the ampere-turns in 12 exerts a clockwise torque on the stator, and consequently a counterclockwise, and here positive, torque on the rotor.

In Fig. 31, where $\alpha = 135$ deg. $T_{s \cdot 12} = 0.5$ and is still positive. When $\alpha = 180$ deg., e_b and $T_{s \cdot 12}$ are both zero.

The next position of F' relatively to the brush axis is again of the greatest interest. In Fig. 33, $\alpha = 225$ deg., e_b has changed its sign and is now positive *but* $T_{s \cdot 12}$ is *again positive* as is seen by reference to the direction of $F' \times$ *sine* α and to that of the currents in 12.

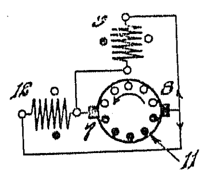
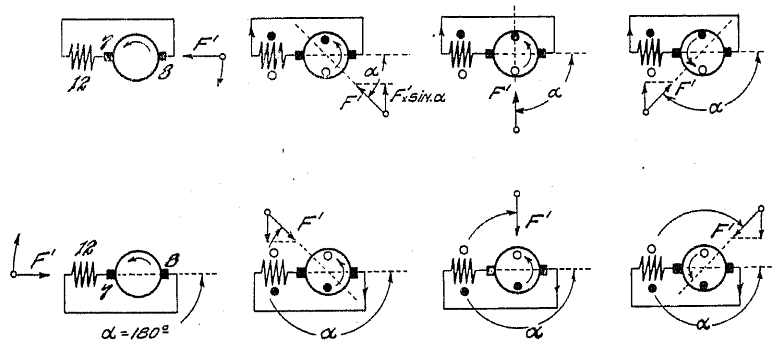


Fig. 27—Direction of Brush Current in Rotor and Stator of New Motor at Synchronism



Figs. 28 to 35—Showing how the Unidirectional Pulsating Torque Component is Produced in the New Motor

amother 45 deg. and $\alpha = 90$ deg., $e_b =$ *sine* $\alpha = 1$ or a maximum and the full flux F' is effective in producing a synchronizing torque with all of the ampere-turns in 12. This torque is still positive and now proportional to the maximum value of e_b and to all of F'. This is the highest instantaneous value of synchronizing torque it is possible to produce in such a motor, because the voltage e_b responsible for the ampere-turns in 12 is a maximum at a time when the space relation between said ampere-turns and the co-operating flux is at its best.

The very important fact is hereby made clear that, when in a self-excited synchronous-induction motor a secondary winding is located coaxially with and connected to the brushes cooperating with the commuted winding on the primary, such a winding produces in conjunction with the primary revolving flux a synchronizing torque proportional to *sine*^{2} α. The maximum value of this torque is proportional to the product of the maximum value of the ampere-turns of said winding and the full magnitude of the revolving flux. This means that, under otherwise equal conditions, a winding

The second wave of $T_{s \cdot 12}$ is identical with the first as to shape and direction, as shown by Figs. 32 to 35 and 28.

Another remarkable feature in connection with this winding 12 is that, located as it is with respect to the brushes to which it is connected, it produces, in conjunction with the primary induction motor flux, a varying *but always positive synchronizing torque* independently of the direction of the alternating brush voltage.

A third feature of importance is that each wave of this *pulsating* torque $T_{s \cdot 12}$ lasts $\dfrac{1}{2\,n_s}$ seconds and not $\dfrac{1}{4\,n_s}$ as is the case for each wave of the *alternating* torque $T_{s \cdot 9}$ to which winding 9 contributes.

The motor of Fig. 21 evidently produces two synchronizing torques, one due to the winding 9, the other due to the winding 12. The former alternates with

double the slip frequency. It is of interest to consider the resultant synchronizing torque and to determine how 9 and 12 should be dimensioned in order to produce the most advantageous resultant synchronizing torque as to value and shape.

The torque values deduced in connection with Figs. 20 and 37 have been deduced on the assumption of an equal number of ampere-turns in the windings 9 and 12 of Fig. 21. This condition happens to correspond to that of Fig. 23, where the armature reaction axis coincides with the brush axis and a combination of Figs. 20 and 37 will, therefore, give the resultant synchronizing torque available under the conditions fixed by the vector diagram of Fig. 23 where the ampereturns in windings 9 and 12 are equal. Fig. 38 shows the resultant torque. It is positive during $\frac{3}{8\,n_s}$ seconds and negative during $\frac{1}{8\,n_s}$ seconds, the maximum negative torque is but 18 per cent of the maximum positive torque.

To fairly compare this performance with that of a corresponding Danielson motor, the latter must have the same design constants and a unidirectional exciting winding with a number of ampere-turns equal to the square root of the sum of the squares of the ampere-turns in 9 and 12, under which conditions both machines will show unity power factor at the same load. The maximum positive equals the maximum negative synchronizing torque in the Danielson motor and is 18 per cent greater (1414 to 1200) than the maximum positive synchronizing torque of the new motor of Fig. 21 when the comparison is based on equal ampere-turns in the windings 9 and 12. See Figs. 23 and 38. The positive torque in the new motor lasts $\frac{2 \times 3}{8\,n_s}$ as against $\frac{4}{8\,n_s}$ seconds per slip cycle in the 1901 machine. The negative torque of Fig. 21 is only 15.6 per cent (220 to 1414) of the negative Danielson and lasts $\frac{2 \times 1}{8\,n_s}$ as against $\frac{4}{8\,n_s}$ seconds per slip cycle.

If the comparison is made on the basis of the ampere-turn relation in the windings 9 and 12 of Fig. 21 defined by the vector diagram of Fig. 26, it is found that the positive maxima are the same in both machines but that the newer machine develops *no negative torque at all* and each of its unidirectional impulses is available for the same time interval as each of the alternating impulses in the motor of Fig. 1.

As to the motor of Fig. 3, it develops an alternating synchronizing torque like that produced by the Danielson motor except that it has double the frequency and half the maximum amplitude of the latter and is, therefore, inferior in this respect.

The result of these considerations is that excellent synchronizing conditions will be secured whenever the relation of ampere-turns in 12 and in 9 is so chosen that, at full load and unity power factor, the brush axis either coincides with the axis of the armature reaction or leads the latter by any angle right up to the limiting case of Fig. 26, where the ampere-turns of 9 are zero and the synchronizing torque a maximum. There is, therefore, no conflict between operating and synchronizing requirements and the machine can be readily designed to satisfy most exacting specifications as to both of these features. Within the limits named, the proportioning of the ampere-turns in the windings 9 and 12, in other words, the choice of the angle between the brush axis and that of the resultant unidirectional magnetization due to said windings, is just a matter of design.

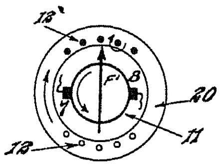


FIG. 36—EXPLANATORY DIAGRAM TO BE READ IN CONJUNCTION WITH FIGS. 12 TO 19 AND 28 TO 35

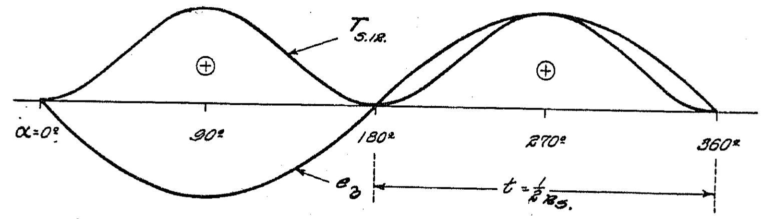


FIG. 37—SYNCHRONIZING VOLTAGE AND UNIDIRECTIONAL PULSATING SYNCHRONIZING TORQUE COMPONENT. NEW FYNN MOTOR

The advantage of a pulsating over an alternating synchronizing torque is not to be measured simply by the maximum values of the two, because the retarding effect of the negative wave of an alternating torque is another and very important factor which reduces the effect produced by the corresponding positive wave. As a consequence, the new machine with its substantially unidirectional torque will quite readily synchronize with a torque equal to its maximum load torque as a synchronous motor.

The synchronizing torque is also produced when the machine falls out of step upon an increase of the load beyond the maximum, which the motor can deal with synchronously and here the effect of an alternating as against a pulsating torque can be very clearly observed. The machine with a pulsating synchronizing torque runs much more uniformly and exhibits, for otherwise

equal conditions, a higher asynchronous overload capacity.

One more word about the brush voltage. Having brought the motor to synchronism, it becomes unidirectional and, as the load varies, it is subject to variations in magnitude brought about by the effect of the varying armature reaction on the other unidirectional magnetizations present in the machine and the consequent change in magnitude and displacement of the resultant magnetization with respect to the brush axis. The degree to which e_b varies and the way in which it varies in synchronous operation depend on the relation of the ampere-turns in the windings 9 and 12.

In Fig. 7 the axis f of the total magnetization F produced by unidirectional ampere-turns on the secondary, is displaced by 90 electrical deg. with respect to the brush axis. This diagram is one of the limiting cases previously referred to and represents the conditions of operation of Fig. 21 when the ampereturns in the winding 12 are zero and those in 9 are responsible for all of the magnetization F. In this case the brush axis leads the resultant magnetization on R by less than 90 electrical deg. and the brush voltage increases as the load decreases. See Figs. 7 and 11. This fact is most readily recognized by assuming that the machine is perfect and requires no current to run it light. For zero current E_r must equal and oppose E_t, then R will coincide with x and F with R. Not only will R increase a little, but all of R, instead of the component 0–9 only, will be effective so far as brush voltage generation is concerned, because R is then perpendicular to the brush axis. This increased exciting voltage increases F and tends to increase R. To keep the latter at its proper value, which is unavoidable, the motor takes a demagnetizing or leading current which increases as the load decreases. If the excitation is set for unity power factor at full load and the resistance of the exciting circuit is not changed as the load varies, the motor will take an objectionably large leading current at no-load if designed with due regard to a good starting performance and high-weight efficiency.

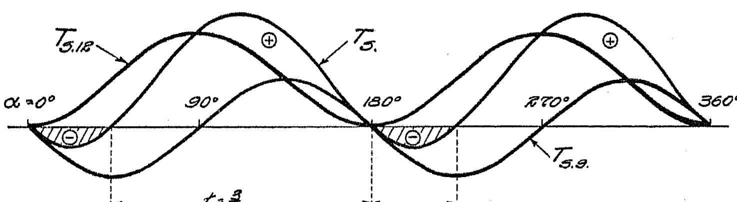


Fig. 38—Resultant Synchronizing Torque of New Fynn Motor When the Ampereturns in the Exciting and Neutralizing Windings are about Equal

The other limiting case is that of Fig. 26. Here the axis f of the total magnetization F, produced by unidirectional ampere-turns on the secondary, coincides with the brush axis. This diagram represents the conditions of operation of Fig. 21 when the ampere-turns in the winding 9 are zero and those in 12 are responsible for all of the magnetization F. In this case the brush axis leads the resultant magnetization R by more than 90 electrical deg. and the brush voltage increases as the load increases. Making the same simplifying assumption as in the case of Fig. 7, it is at once seen that for zero current and coincidence between x, R and F the brush voltage should be zero since R then coincides with the brush axis. To keep R at its proper value the motor will, however, take a magnetizing or lagging current and bring about a corresponding adjustment of the vectors R, F, N and i. The brush voltage varies with R and with the sine of $(c - c')$.

Generally speaking, the brush voltage e_b is proportional to $R \times \text{sine}\,[\,(c - c') + \delta]$ where δ is the angular displacement of the axis of F with respect to the brush axis. The angle $(c - c')$ varies with the load and with the power factor.

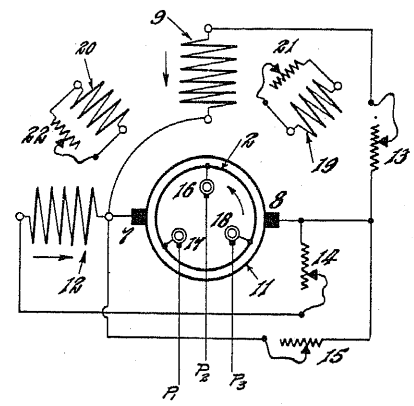


Fig. 39—Self-Excited Synchronous Induction Motor New Fynn

Between the above limits lies a wide range of ratios of ampere-turns for 9 and 12, which enable the designer to cause the power factor to vary with the load in almost any desired manner and even to closely approach a nearly constant power factor for all loads. It is thought that a moderately leading power factor at noload and a slightly leading power factor at full load will usually be the most acceptable power factor performance, due regard being given to overload capacity and high synchronizing torque. For the machine constants forming the basis of Fig. 23 a somewhat higher ratio of "neutralizing" to "exciting" ampere-turns than that shown in Fig. 23 would yield such a power factor curve, together with excellent synchronizing and high overload capacity.

This variation of the brush voltage, which is the unidirectional exciting voltage at synchronism and the

alternating synchronizing voltage at sub-synchronous speed of the machine shown in Fig. 21, particularly its rise with rising load, makes it possible to operate this motor efficiently and safely with high power factor at all loads and without it being necessary to resort to external automatic or hand-operated regulating devices. The fact that the brush or exciting voltage can be made to rise with increasing load by suitably proportioning the windings 9 and 12 makes it possible to run the machine with unity or even leading power factor at full load, without the risk of being confronted with an unduly large leading current at noload. Unity or leading power factor at full load means high overload capacity and good weight efficiency. This automatic regulating capacity is inherent in the machine of Fig. 21, its mechanical expression is the momentary acceleration or deceleration of the primary member, consequent on a change in load which brings about different periodical space relations between the brush axis, or the unidirectionally excited secondary member which carries the brushes, and some selected point on the primary member. When the stator is the primary, then it is the secondary which is subject to this momentary acceleration or deceleration. This phenomenon has been fully analyzed in connection with Figs. 6, 7, 8, 9, 10 and 11, also in connection with Figs. 23, 24, 25 and 26, it is there measured and expressed by the angle c. Briefly stated, it is the changing periodical space relation c, between the axis x of any coil on the primary and the axis f of the resultant unidirectional magnetization on the secondary member, brought about by changes in load or excitation, which determines and governs the inherent automatic regulating property of the new machine. The meaning of "changing periodical space relation" is this. For certain load and excitation conditions the axis of the pole S may, in its clockwise rotation, occupy with respect to the axis x of the coil 4, 5 the position shown in Fig. 8 at the time when the terminal voltage E_t passes through zero on its way from a negative to a positive maximum. This space relation is indicated by the angle c and occurs at time intervals of one period of the supply frequency. If the load is changed the angle c is changed, the periodically recurring relative position of the axes f and x is changed and the periodical space relation of the two is said to have changed. This phenomenon is common to all synchronous motors but in this one it is made use of to automatically keep the power-factor variations within bounds or, in other words, to compound the motor.

Having acquired an insight into the mode of operation and the principles governing the design of the machine under reference, it will not be necessary to do more than indicate the changes which can be made to adapt the motor to varying conditions of service. Thus, where the starting requirements are particularly severe, or the machine is large, it will be of advantage to relieve the commutator as much as possible at starting and to this end the arrangement shown in Fig. 39 can be used. Its distinguishing feature is the use of additional secondary windings 19 and 20 adapted to be closed over adjustable resistances 21 and 22. Any polyphase arrangement of secondary windings inductively responsive to the polyphase primary windings will answer the purpose and it is more economical as to stator space to displace the axes of these auxiliaries from the axes of the windings 9 and 12. The initial

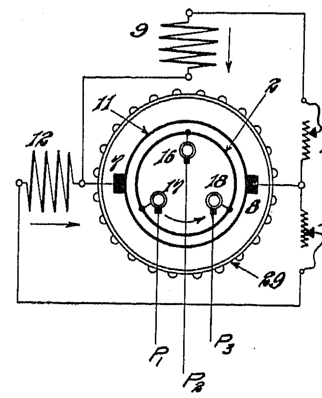


FIG. 40—SELF-EXCITED SYNCHRONOUS INDUCTION MOTOR NEW FYNN

starting effort can be secured by means of these auxiliary windings with the circuits of 9 and 12 closed over resistances of such value as to keep the current through the commutator down to a harmless figure. It is also possible to leave the circuits of 9 and 12 open during the initial starting period, closing them or reducing the resistances in circuit with them only after the motor has reached a sufficient speed.

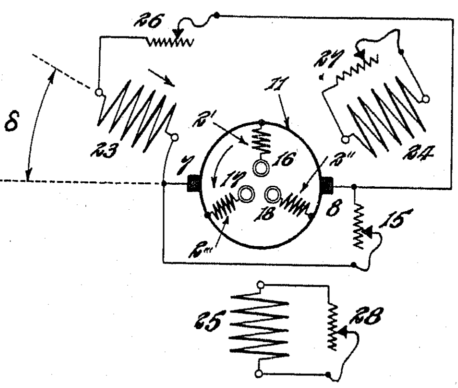


FIG. 41—SELF-EXCITED SYNCHRONOUS INDUCTION MOTOR NEW FYNN

Another way of applying auxiliary, and preferably polyaxial, secondary windings is shown in Fig. 40 where a squirrel cage 29 is located on the stator and given that resistance which will give the desired starting torque with or without the help of 9 and 12. It will be understood that all such auxiliary windings remain entirely idle in synchronous operation and it is, therefore, clear that the space utilization on the secondary member is best when the arrangement shown in Fig. 21 is employed, since in that case *all* of the motor windings are active at *all* starting and operating stages.

It is possible to combine the windings 9 and 12 into a single winding, producing a magnetization equal in magnitude and direction to the resultant of the magnetizations produced by 9 and by 12. Such an arrangement is shown in Fig. 41, where the winding 23 takes the place of the windings 9 and 12 of Fig. 21 and is displaced δ degrees from the brush axis b and against the direction of rotation of the primary.

While it may look like a simplification to combine the windings 9 and 12 into the single winding 23, yet it is not so in reality for it entails the use of at least one auxiliary winding which is idle during synchronous operation. Neither the shape nor the magnitude of the synchronizing torque is changed by the combination of 9 and 12, but the polyphase arrangement of secondary windings, capable of producing an induction motor torque at starting, a synchronizing torque near synchronism and unidirectional exciting and armature reaction opposing magnetizations in synchronous operation, is destroyed and the desirable conditions obtaining in Fig. 21 are not all re-established by the addition of one or more auxiliary windings such as 24 and 25. It should also be noted that the fewer auxiliary windings are added to 23 of Fig. 41 the less constant will the induction motor torque be at starting and at loads beyond the maximum load of the machine as a synchronous motor, for the reason that the element 23 of the polyphase arrangement of secondary windings is always in circuit with the commuted winding which has a considerable reactance. This condition can be bettered at starting by the use of the resistance 15, particularly, if this resistance is connected in parallel with the brushes, with the inclusion of the resistance 26 between one point of connection and brush 8. This is equivalent to connecting 15 directly in parallel to the terminals of the winding 23, *i. e.*, without the interposition of the resistance 26. In such case the resistance 26 should be a maximum at starting and the value of 15 set to cause 23 to contribute its fair share to the induction-motor starting torque. As the speed increases, 15 may be first diminished and then increased, or it can be left at its original value and then increased. When 15 is increased 26 is decreased and 15 is later entirely disconnected, preferably before the maximum asynchronous speed is reached. This resistance 15 can be used in this manner also in connection with Figs. 21 or 39. Whichever way the resistance 15 is connected, its function, insofar as the brushes and the commutator are concerned, remains the same—it protects them from an excessive current at starting.

The induction-motor torque can be made more constant under all operating conditions at the expense of maximum induction motor torque, by permanently including in each of the secondary windings contributing to the production of the induction-motor torque an impedance equal to that of the commuted winding. The addition of enough elements to the secondary member to constitute a complete polyphase winding independently of 23 also leads to better starting and operating conditions. These additional elements may constitute a squirrel cage of any desired resistance.

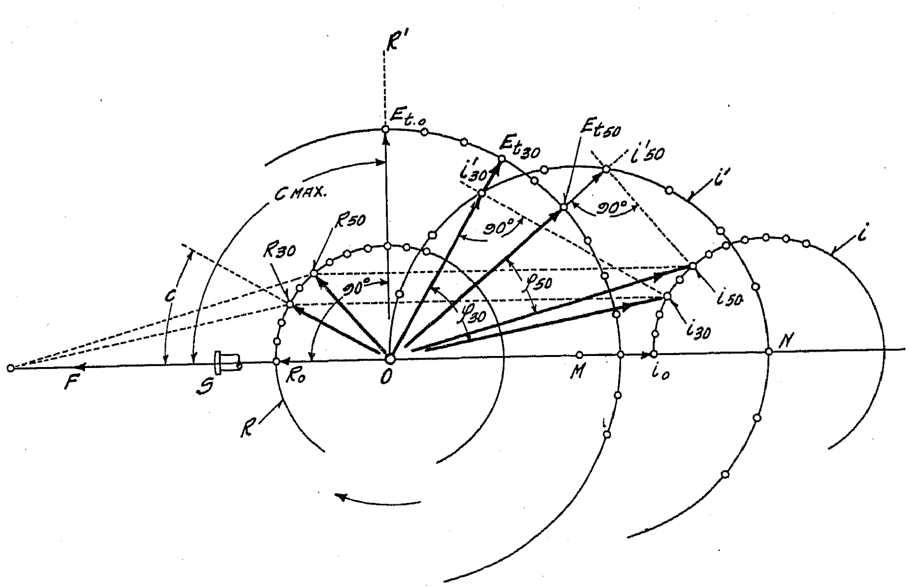


Fig. 42—Performance Diagram of a Synchronous Induction Motor with Separate and Constant Excitation

The partial or total neutralization of the armature reaction in synchronous motors or generators is by no means novel. Many of the best known electrical engineers in the United States and in Europe have repeatedly suggested and carried out different arrangements with this end in view, but the solution of this problem suggested by the writer appears to be very simple and while it may be contended that, strictly speaking, it is not a good armature reaction neutralizing device because it does not fully neutralize said reaction under all load conditions, yet it does so to a practically sufficient extent, and, in addition, permits of combinations with other elements which cooperate to

lend very desirable characteristics to a type of motor which is fast coming to the fore.

Other developments relating to synchronous-induction motors form the subject matter of a communication now in preparation and which will be published shortly.

Appendix

It is thought that a brief outline of a quick graphical method for approximately predetermining the operating characteristics of the new motor and a comparison of its performance with that of a synchronous induction motor of ordinary design will be of interest.

In order to simplify the problem, a number of partly far reaching assumptions must be made. Thus the influence of the ohmic resistance and of the reactance of the primary winding on the space position of R with respect to F and of the saturation of the magnetic circuit on the magnitude of F will be disregarded. This means that in Fig. 23, for instance, the angle c' becomes zero and the space displacement $(c - c')$ of the unidirectional magnetization F, produced by the secondary with respect to the resultant motor magnetization R, is equal to the angle c. This further means that the phase of the back e. m. f. E_r is exactly opposed to that of the terminal e. m. f. E_t. A diagram built on such assumptions will give too large a maximum primary current, too large a maximum torque and too low values for the power factor. Disregarding the effects of saturation means that at the higher values of exciting voltage the values of primary current, torque and power factor will be too high. As a first approximation and for purposes of preliminary comparisons, diagrams based on these assumptions are, however, good enough. For more accurate results the same approximate corrections must be introduced as is usual in other similar diagrams.

Fig. 42 shows the diagram for a separately excited synchronous induction motor. For no torque the magnitude of the resultant R is determined by the fact that it must, at synchronism, generate in the primary winding a back e. m. f. equal and opposed to the terminal voltage. The phase of R must lag 90 deg. behind that of E_t. If the excitation is constant and produces the unidirectional magnetization F then, in this extreme case, R must coincide with F. This position of the vector R is designated as R_o. At such time the motor will take a current i_o leading E_t by 90 deg. and producing ampere-turns equal to the difference between the ampere-turns responsible for F and for R. There will be no torque because the primary current is at right angles to the terminal and therefore to the back e. m. f. and has no component in phase opposition to the latter. The vectors $O\,F$, $O\,R$ and $O\,i$ may respresent magnetic flux, ampere-turns or current. It is merely a matter of scale.

To produce torque R must be displaced from F. For a displacement of 30 deg. the position of this resultant motor flux is designated by R_{30}. Since R is constant the locus for the end of the vector $O\,R$ is the circle R about O. The corresponding primary current i_{30} is found as one side of a parallelogram of which F is the other side and R the resultant. A similar construction yields the current i_{50}, or any other value of the total primary current. It is seen that the locus of i is a circle about N, where $O\,N = O\,F$, and with a raduis equal to $(O\,F - O\,R)$.

Since E_t is constant and always leads R by 90 deg., its locus is also a circle about O. For a 30 deg. displacement of R the terminal voltage is at $E_{t\cdot 30}$ and the torque producing component i'_{30} of i_{30} is found as the projection of i_{30} on $E_{t\cdot 30}$. The same is true of i_{50} or any other value of the primary current. The vector $O\,i'$ is proportional to the torque and the locus for i' is a circle about M with a diameter equal to F.

Maximum primary current and torque occur for $c = 90$ deg. and the power factor at the motor terminals for any value of c is the cosine of the angle φ between the corresponding i and E_t vectors.

In Fig. 43 will be found the corresponding diagram for the new motor shown in Fig. 21. In this

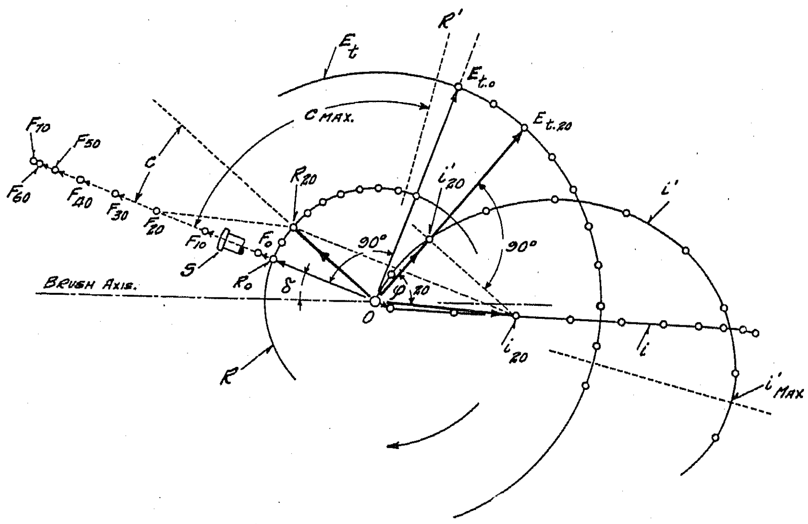


Fig. 43—Performance Diagram of New Motor

machine the ratio of ampere-turns in the windings 9 and 12 is as 1:2.75, which makes $\delta = 20$ deg. and the resistance and number of turns of the two windings are so chosen that the maximum unidirectional excitation F which they produce is equal to the constant excitation F of Fig. 42. The normal or full-load torque of the machine occurs for $c = 37$ deg. It is just about the same as or even somewhat in excess of the full-load torque of the corresponding slip-ring induction motor. At this load that component of F which is coaxial with the brush axis is in excess of the armature reaction component i. This means that the winding 12 of Fig. 21 then produces more ampere-turns than those produced by the primary or armature reaction at that load or torque. The construction is, in principle, the same as that used in connection with Fig. 42. The loci for R and E_t are circles about O, as before, but the loci for i and i' are curves. For $\delta = 20$ deg. the i curve is almost a straight line, its curvature increases as δ increases. The shape of i' does not vary so much with varying δ but approaches a circle as δ diminishes. The main difference between Figs. 42 and 43 lies in the fact that the magnitude of F changes as the angle between R and the brush axis changes, reaching a maximum when R is perpendicular to said brush axis. The exciting voltage e_b changes with this angle and is proportional to $R \times sine\,[\delta + (c - c')]$ or, in this special case, to $R \times sine\,(\delta + c)$. The value of this exciting voltage or of the corresponding unidirectional exciting ampere-turns or of the corresponding unidirectional exciting flux is given in Fig. 43 for each position of R with respect to the axis of F. Thus F_{20} corresponds to the value of e_b when the resultant is in the position R_{20} and so forth. The line current is found as one side of a parallelogram of which R is the resultant and the *corresponding* F is the other side.

The maximum torque is exerted for c equal to about 84 deg. As δ increases c decreases to some extent.

It is to be noted in Fig. 43 that e_b and therefore F first increases and then diminishes. This variation is, of course, of paramount interest in connection with the compounding characteristic or torque-power factor curve of the motor. In Fig. 44 are plotted a number of curves showing just how e_b does vary with varying torque for different values of δ and a different number of turns or a different resistance of the windings 9 and 12 of Fig. 21.

Fig. 45 shows the synchronous performance curves of the new synchronous induction motor of Figs. 21, 39, 40 or 41 and the corresponding curves of a separately excited synchronous induction motor with same maximum synchronous torque but constant excitation. The curves i_1 and $\cos\varphi_1$ show the line current and power

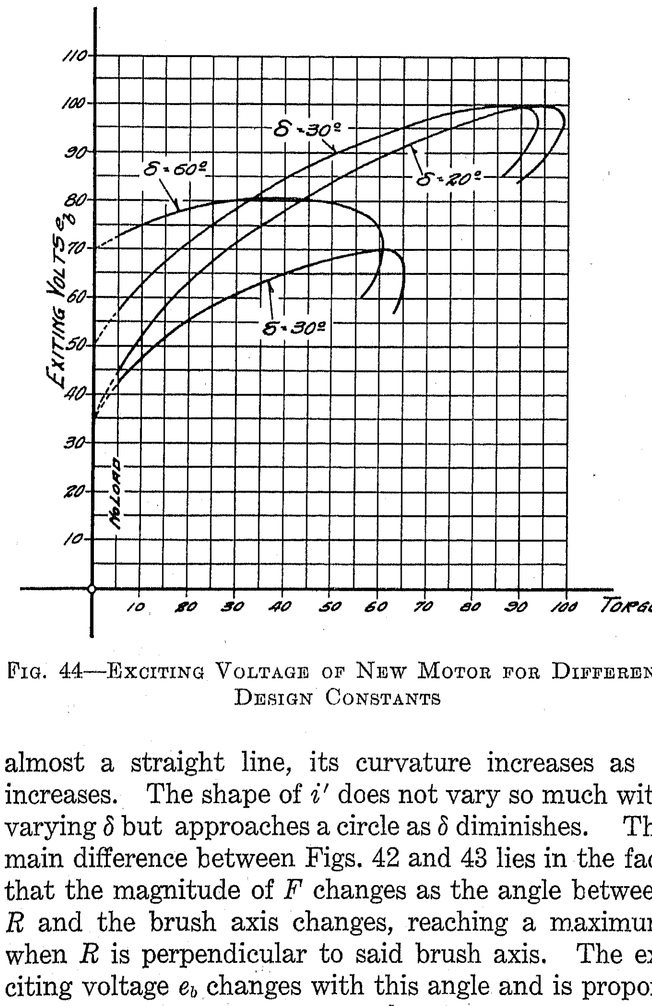


Fig. 44—Exciting Voltage of New Motor for Different Design Constants

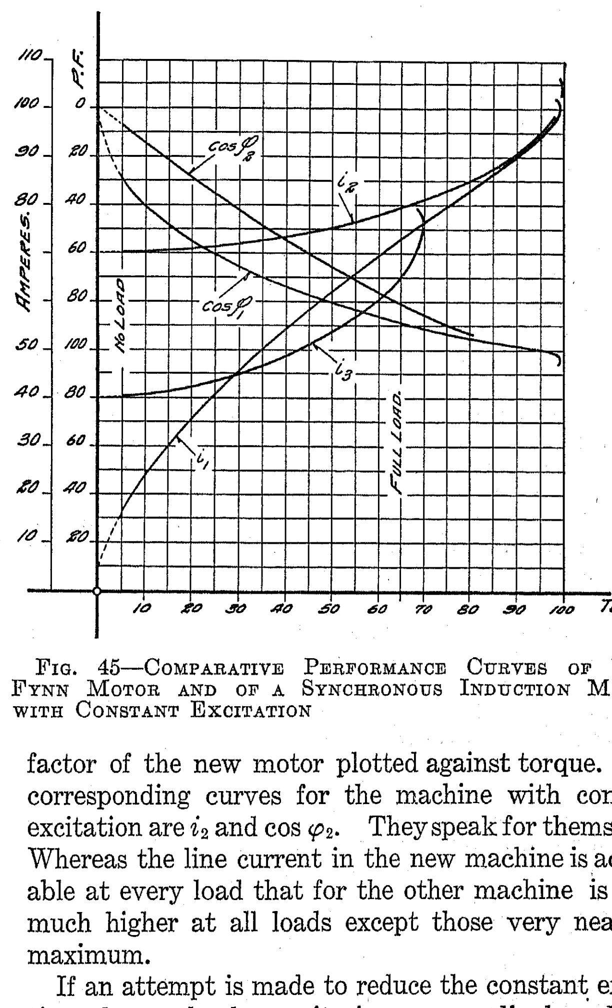


Fig. 45—Comparative Performance Curves of New Fynn Motor and of a Synchronous Induction Motor with Constant Excitation

factor of the new motor plotted against torque. The corresponding curves for the machine with constant excitation are i_2 and $\cos\varphi_2$. They speak for themselves. Whereas the line current in the new machine is acceptable at every load that for the other machine is very much higher at all loads except those very near the maximum.

If an attempt is made to reduce the constant excitation, the overload capacity is correspondingly reduced. The line current curve i_3 shows the performance with 70 per cent of the excitation corresponding to the line current curve i_2. The power factor curve for the lower constant excitation has not been plotted.

In Fig. 45 the synchronous overload capacity is 52 per cent. This is very ample in view of the fact that the non-synchronous overload capacity of the machine

is quite equal to that of a corresponding slip-ring motor and if the torque demanded exceeds the maximum synchronous capacity, the motor will slip into asynchronism without creating any disturbance whatsoever and go back to synchronism as soon as the overload has been removed.

Discussion

S. R. Bergman: I think this new motor is a courageous attempt to deal with a difficult and complex problem, namely, the problem of power-factor correction.

This motor is of the synchronous type and therefore has a constant speed over the running range. This feature, however, is of no advantage since experience has shown that the small amount of slip in the standard induction motor gives an excellent speed characteristic.

In judging a new type of motor the main question is, of course, one of economy. The motor which shows the lowest yearly operating cost to the user is the best motor. In dealing with the yearly operating cost of any electrical apparatus there are certain items which require consideration: 1st: Cost of power; 2nd: Interest and depreciation; 3rd: Maintenance; and 4th: Losses caused by interruption of service due to needed repairs.

When considering the cost of power of a motor we meet at once with difficulty due to the fact that the rates are not uniform for the whole country. Not only do different systems charge different amounts per kilowatt, but the charge due to low power factor is one that is causing a great deal of uncertainty, since no definite policy has yet been adopted. At present a number of systems have established penalizing clauses which, however, are often not enforced. Until some definite rules are established it becomes rather difficult to determine the advantage of this new motor, which possesses inherent power-factor compensation.

While this motor can be adjusted for unity power factor, or even for a leading current, it strikes me that the construction of the motor is such that it has inherently a lower efficiency than the standard induction motor. I do not hesitate to state that in my opinion, if this motor is adjusted for unity power factor the yearly cost of operation will be higher than that of a standard induction motor. This unfavorable condition is mainly due to the additional losses caused by brush losses, excitation losses, etc.

When this motor is adjusted for leading power factor, it probably has a better chance of utility since mixed with ordinary induction motors it will correct the power factor. On the other hand, there are other ways of correcting the power factor which are being successfully utilized. I mean the use of standard synchronous motors, rotary condensers and static condensers. Synchronous motors of the standard type and rotary condensers are quite advantageous in large units and the static condensers in smaller units. By a proper application of such apparatus, excellent results have been obtained. The natural evolution taking place in the electrical industry will of course, determine which of these methods is the most economical to employ to correct the power factor.

The third item referred to above was the cost of maintenance. This new machine possesses a commutator as well as slip rings and therefore, requires two sets of brushes. I also notice that it requires a starting resistance. As I understand it from the description, the armature carries two windings and the field also contains two windings. It seems to me the conclusion is obvious that this motor has a cost of maintenance far in excess of that of the standard induction motor.

Then we have the question of interruption of service caused by possible repairs. It seems that this new motor with its complicated structure would not be as reliable as the simple induction motor and therefore, would cause more lost time due to repairs which is a very serious matter to the user.

The motor industry as we know, is developing very rapidly as new water-power projects and interconnected systems are being developed in the country. The motor business and the electrical business as a whole doubles every fourth or fifth year. If the electrical industry is going to continue to be successful it is absolutely necessary that it develop along sound lines, that is, along lines which are simple so as to secure continuity of service. When judging any new piece of apparatus this point of view is of fundamental importance and with respect to this new motor its complicated structure is therefore a matter to be given serious consideration.

W. F. Dawson: I am not going to speak much about the

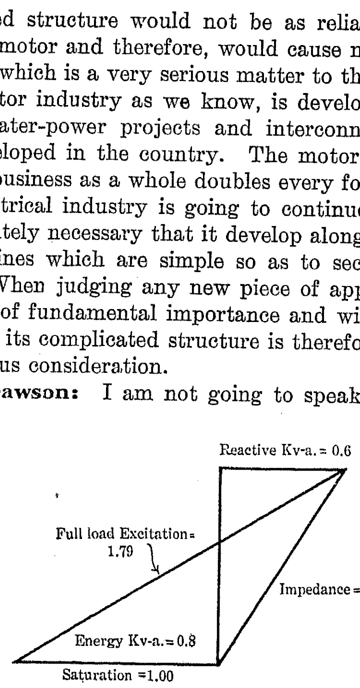


FIG. 1—EXCITATION DIAGRAM. 1.00 KV. 0.8 P. F.—0.8 KW.

motor; I think Mr. Fynn can amply defend it, but I will say a word about the effect of power-factor correction on the main generating outfit. I speak with particular feeling for the turbine alternator.

I will use the simple diagram representing the excitation of turbine alternators. Our conventional design will call for saturation 1.00 and impedance approximately equal, 1.00; 80 per cent power factor, which means 60 per cent reactive component. That gives a full-load excitation of 1.79. If the generator were operating without any reactive component and the same kw. the excitation would be 1.28; or if at unity power factor and the full kv-a. rating of the unit, the excitation would be 1.41.

The power companies pay for all that reactive component.

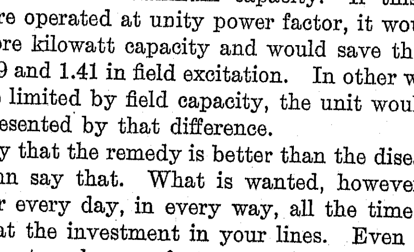


FIG. 2—EXCITATION DIAGRAM. 0.8 KW. AND 1.00 KW. 1.00. P.F.

It is the field excitation of turbine alternators, in most cases, that determines their maximum capacity. If this particular example were operated at unity power factor, it would have 25 per cent more kilowatt capacity and would save the difference between 1.79 and 1.41 in field excitation. In other words, if the output were limited by field capacity, the unit would have the margin represented by that difference.

I don't say that the remedy is better than the disease; we will let Mr. Fynn say that. What is wanted, however, is better power factor every day, in every way, all the time. Furthermore, look at the investment in your lines. Even though the turbine alternators do manufacture a 60 per cent reactive component in addition to the 80 per cent energy component, the lines do not have to carry the arithmetical sum but they do have to carry the vectorial addition.

Condensers, static or synchronous, might be placed in the power houses with the generators and thus help out the generator fields, but that would not relieve the lines of the extra load due to the reactive component. The proper place for condensers is at the point of final distribution where the power is used.

F. J. Rudd: I am particularly interested in so-called small motors, say 50 h. p. and below. To-day where individual drive is becoming more and more common, the standard mechanical construction is not always applicable. Specialization is required in many cases and it appears from the construction of this particular motor that many difficulties would be encountered when trying to fit it to various applications.

From the standpoint of certain atmospheric conditions, in many cases standard open squirrel-cage motors are satisfactory; whereas, a motor with commutator and collector rings would have to be totally enclosed. This in itself would materially increase the frame size and hence the cost for a given rating.

Another point is that where the primary voltage is higher than the normal of 550, say, for example, 2200, the problem of insulating the revolving primary winding would be much more difficult and expensive than would be required for a stationary winding, such as is employed on the usual squirrel-cage type of machine.

V. A. Fynn: The discussion brought out a point on which I have perhaps not laid sufficient stress in my paper. I was particularly interested in showing you just what theory is involved in the solution of the various problems connected with this type of motor. I think I did mention in the paper and also in my presentation that this machine could readily be inverted. This is, of course, quite simple, and when you do so, you get away from all insulating difficulties because the primary is then located on the stator. With the primary on the stator, revolving brushes must be used so long as the machine is self-excited. This revolving-brush problem has been attacked and it appears that it is capable of quite a simple solution.

But even when that is done, all of Mr. Bergman's objections are not met. I am not presenting this as a universal solution for the difficulties about which we all know, and which were so ably emphasized by Mr. Dawson. This is only one step towards a complete solution. This machine cannot compare with the squirrel-cage motor. The squirrel cage is supreme in its field, but it has a poor power factor, and the only remedy that I know of, in so far as the squirrel-cage motor is concerned, is the use of static condensers. Static condensers, theoretically, are simple enough, but they are still very expensive and have their drawbacks. If you connect static transformers to your motors, you soon run into difficulties such as have been met in Europe. Condensers are responsive to all frequencies and for that reason often exaggerate high-frequency disturbances.

The principles which I have outlined in my paper are also applicable to large separately excited machines; not only to machines of 50, 60 or 100 horse power, but to any size of machine. Just how this is done I shall have the pleasure of telling you in another paper, but in principle the solution is much the same.

While I do not claim this new motor to be a perfect machine, yet it is better than any other now available and when built with the primary on the stator and with the revolving secondary carrying the unidirectional excitation, it provides a reasonable solution in many cases.

The 65,000-Kv-a. Generator of the Niagara Falls Power Company

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Review of the Subject:—The initial hydroelectric development of the Niagara Falls Power Company is referred to and comparisons are made with the latest development, which involves three units, each with a 65,000-kv-a. generator. The amount of material in a generator as affected by speed, periodicity and efficiency aimed at, is discussed. The electrical characteristics of the first of the 65,000-kv-a. generators are outlined. The mechanical construction is described in considerable detail with the aid of cross-sectional views of certain parts and of the generator as a whole.

A special feature of this unit is a direct-connected 650-kv-a., 25-cycle generator for furnishing excitation through a motor-generator set and for driving auxiliaries, such as pumps. Some weight data are added, showing this generator to contain over one and one-half million pounds of material—undoubtedly the heaviest electrical machine ever built.

At the close of the 19th century a notable event occurred at Niagara Falls,—the development of a plant for the utilization electrically of Niagara power. Then, as now, Niagara Falls was recognized as the finest water power in the world, a cataract of great height, fed from the Great Lakes, constituting a water storage without a rival. It was in keeping with the situation that the first plant was developed with mammoth units, much larger than had been dreamed of up to that time. They were 5000 horse power each, or, translated into electrical terms, 3750 kw., eleven of these units under one roof.

Almost exactly twenty-five years later a generator with nominal rating of 65,000 kv-a. or one capable of 50 per cent greater output than the entire row of units in the original power house, was put into service. This latest generator is the product of the experience gained during the intervening years. It is three-phase, instead of two-phase; it is 12,000 volts, instead of 2200; it is 80 per cent power factor, instead of unity-power factor; it has high internal reactance, instead of low; it has losses at full-rated load equal to approximately 2 per cent of its output, whereas the original had from 7 per cent to 8 per cent; it is of the "conventional," internal-revolving field type, whereas the original had its field revolving outside of the armature; it supports on its stator, by means of thrust bearing mounted at the top, the entire weight of its own rotor and the runner of the turbine, whereas the original had its revolving part supported from an oil-pressure step bearing located underneath; it is located so close to the turbine that it has no lower guide bearing of its own, whereas the original was some 150 ft. above the turbine with several guide bearings intervening; it is equipped with brakes to bring it to rest quickly, whereas the original had no means provided for bringing it to rest, except the shutting off of the water to the turbine; it contains approximately twenty-three pounds of material per kv-a. output, whereas the original contained nearly fifty pounds.

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-11, 1924.

Although the quantity of material in this large generator is less than half that of the original per kv-a., it is very large as compared with a modern 60-cycle generator of large capacity at the higher speeds that are now common in hydraulic developments. There are three reasons for this generator having such great weight as one and one-half million pounds of material; first, the fact that it is low speed; second, it is low periodicity; third, it was designed for the highest economic efficiency.

Regarding the effect of rotative speed on weight, it may be said, that for the same electrical characteristics the lower the peripheral speed, the greater the weight of magnetic material and copper, and that the lower rotative speed always requires a lower peripheral speed to obtain the proper adjustment between the material that must be used for mechanical structure and that which must be used for the electrical parts. In the case of this generator, the peripheral speed is only 8200 ft. per minute, whereas many 50 and 60-cycle hydraulic generators that have been built at speeds from 200 to 600 rev. per min. have peripheral speeds of 12,000 ft. per min., or higher,—some of them as high as 15,000 ft. per min.

With regard to the effect of periodicity on quantity of material; assuming same output, same characteristics, same rotative speed and same peripheral velocity, the total magnetic flux in the air-gap must be the same at all periodicities, but the lower periodicity machine has fewer poles and the flux linked through armature from pole to pole is in inverse ratio to the periodicity, a 25-cycle machine having 2.4 times that of a 60-cycle; hence, a cross section of armature core that many times greater must be provided. Formerly, the lower periodicity permitted of higher magnetic densities in teeth and core of armature, but silicon steels have been developed with such high qualities in the matter of hysteresis losses that the lower periodicity no longer has any advantage in this respect. All periodicities up to 60 cycles are worked at as high saturation as permeability allows. Again, the smaller number of poles in the 25-cycle machine requires a

much greater radial depth of pole for heat dissipation reasons, and greatly increased cross section of the copper on the pole. For these reasons, the total quantity of magnetic material in the poles and the total amount of copper in the field-winding are greater than in the corresponding 60-cycle generator.

With reference to increase in material that was introduced in order to obtain the highest economic efficiency, it may be said, that as far as temperatures were concerned, the amount of copper in both armature and field could have been reduced at least 20 per cent, and the amount of magnetic material as much as 10 per cent. In order to obtain extremely high efficiency, it was necessary to reduce several or all of the various kinds of losses, windage, hysteresis and eddy current, $I^2 R$ and load losses. The most important factor in the windage losses is peripheral speed; hence, it is generally best to select smaller diameter, although it results in increased magnetic material and copper. Lower hysteresis and eddy losses may be obtained by working at lower densities; consequently, greater amount of material must be used in the magnetic parts. Lower $I^2 R$ losses can be obtained by increasing the

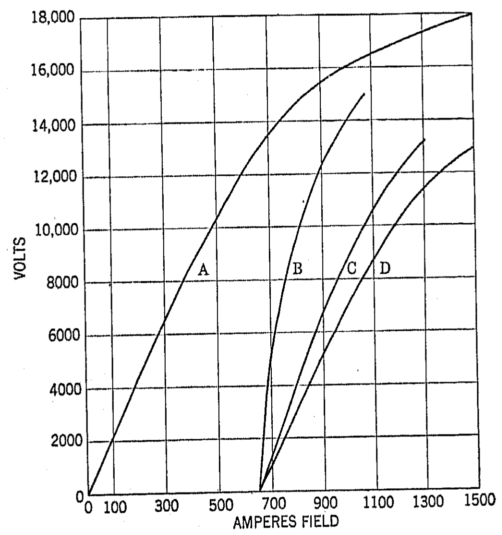

FIG. 1—THE 65,000 KV-A. GENERATOR OF THE NIAGARA FALLS POWER CO.

Saturation Curves
A—Open Circuit
B—Full Load Current 100 per cent Power Factor
C— " " " 80 " " " "
D— " " " zero " "

quantity of copper. Load losses can be kept lower by conservative design in the matter of armature reaction, but the size of the machine is increased by reason of the lower armature reaction.

The electrical characteristics of this generator are in accord with what is regarded as best for power-producing purposes in large systems. The ampere turns at no load, 12,000 volts, are almost identically the same as required for rated current on short-circuit.

The calculated armature reactance is 26 per cent.

Unusual features in the electrical design are the low point on saturation curve at rated voltage, made necessary by the requirement of operating continuously at 13,200 volts, 68,250 kv-a., 80 per cent power factor, and the very low-current densities at which copper is working, approximately 1300 amperes per square inch in armature and 1100 in field, in order to obtain the very high efficiency.

Fig. 1 contains Curves of no load, 100 per cent and 80 per cent power factor full-load saturation.

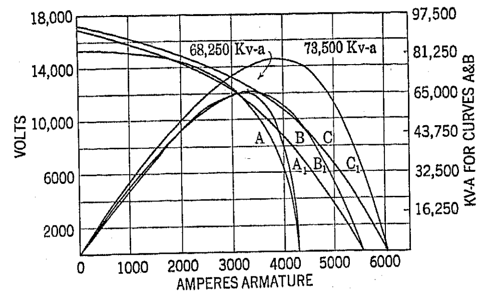

FIG. 2—FIELD CHARACTERISTIC CURVES

A—A_1 100 per cent Power Factor 12,000 Volts 65,000 kv-a.
B—B_1 80 " " " " 12,000 " 65,000 kv-a.
C—C_1 80 " " " " 13,200 " 68,250 kv-a.

Fig. 2 shows Curves of Field Characteristics for three conditions, *viz.*, 100 per cent power factor, 65,000 kv-a., 12,000 volts; 80 per cent power factor, 65,000 kv-a., 12,000 volts and 80 per cent power factor, 68,250 kv-a., 13,200 volts.

The fact that this generator is, for the time being, the largest ever built, would in itself be sufficient justification for describing it, but in addition, it contains a few features that are entirely new, as far as the writers of this paper know. Many large vertical hydraulic units have already been built, due to the ever increasing demand for power and to the greater simplicity and economy in power houses and auxiliary apparatus thus obtained. The principal parts of such large generators may be stated as stator frame; stator core; stator winding; shaft; rotor spider; poles; field coils; upper bearings' spider; bearings; oiling system; collector rings. There are numerous detail parts in connection with every one of these major parts that are of extreme importance and worthy of description. Before taking up the major parts in order, we call attention to Fig. 3, which shows the general arrangement of the generator, the stator supported by a continuous base ring, thrust bearing carrying the total weight of the generator and water-wheel rotating elements, including water thrust, mounted on the upper bearing bracket. This bracket also carries the generator guide bearing. The water-wheel guide bearing is located directly above the turbine runner. The proximity of generator rotor to turbine runner eliminates a third, or middle guide bearing, which is often placed immediately underneath the generator.

The novel features in this generator are shown in Fig. 3, in the placing of an excitation generator with

rating 650-kv-a., the stationary part suspended from the upper bearings' bracket and the revolving part mounted on the arms of the revolving spider of main generator. The collector slip rings for supplying exciting current to this small generator, as also those for main generator, are shown mounted immediately above the thrust bearing. At the extreme top is mounted a speed limiting device. These parts are protected by a housing of pleasing design. This new generator has the same lines as the 32,500 kv-a. generators, installed in the same station about three years ago and designed in accordance with the ideas of the engineers of the Niagara Falls Power Company. Fig. 4 shows the close agreement in outlines of the two generators, the one exactly double the other in capacity, and gives at the same time a clear idea of the relative space above floor, required by the respective machines.

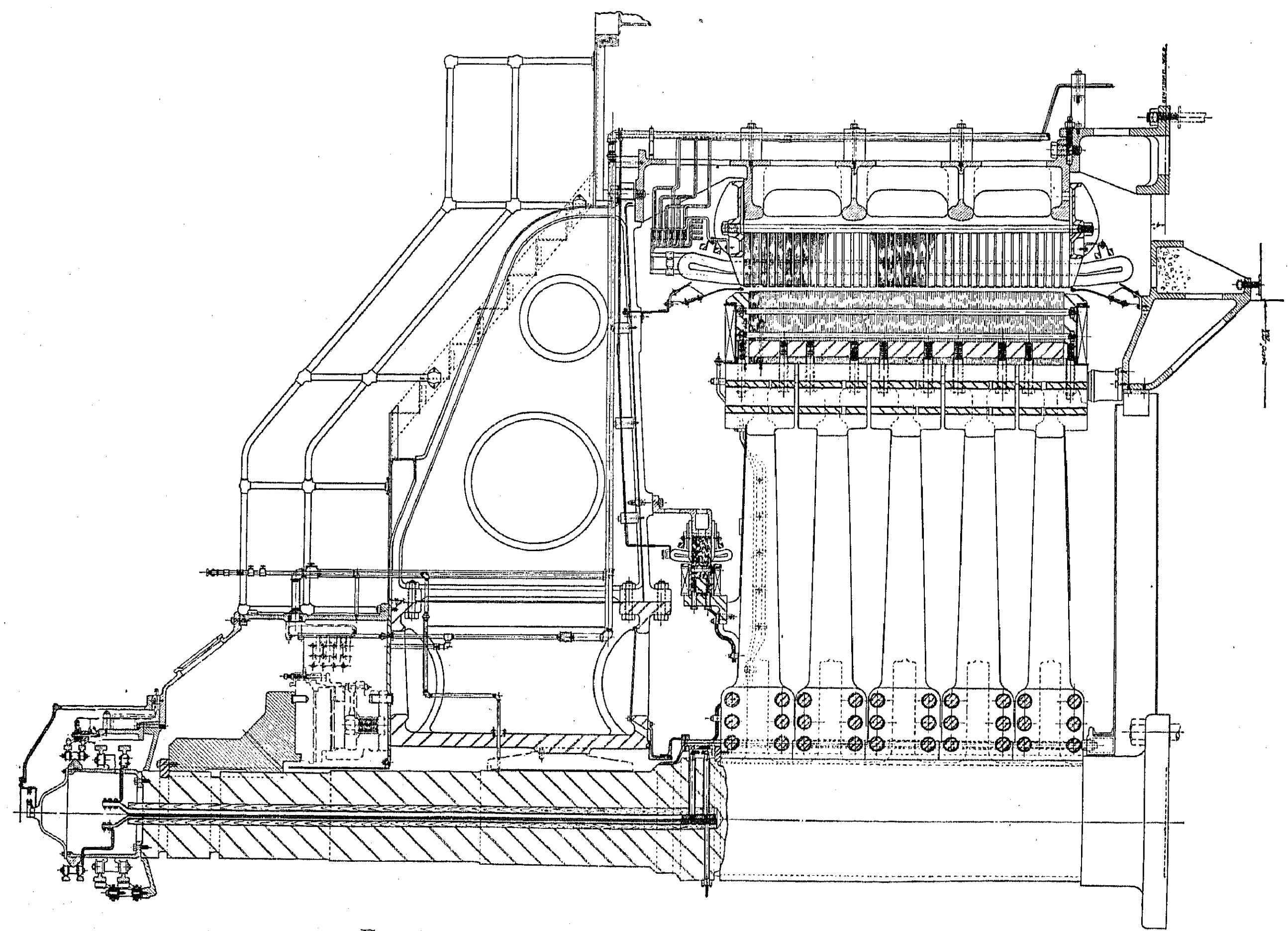

Fig. 3—Cross Section of Assembled Generator

Stator

The cast iron stator frame is made in four sections, for purposes of casting, machining, handling in the factory, transporting and erecting at power house. Each section measured 22 ft. across the arc and 10 ft. in height. The sections are keyed, doweled and put together with sufficient bolts to withstand maximum short-circuit strains. Similarly, the complete stator frame is rigidly doweled and bolted to the base ring. The final adjustments for alinment are made by means of adjusting screws in the base ring. At the top of the stator frame is a projecting flange for the attachment of ventilation housing that surrounds the machine, for the purpose of carrying away the warm air.

Stator Core

The laminations of core are assembled on keys attached to stator frame ribs by screws. The core is built up in numerous sections separated from one another by "I" beam-space blocks, so as to provide ventilating ducts. The core, by reason of its immense size, is clamped together by extra heavy cast-steel clamping flanges at top and bottom with large bolts running the entire length of the core and located between core and stator frame. By clamping the core with through bolts in this manner, the stator frame is relieved from strains which might exist if the clamping flanges were attached by bolts screwed into the stator frame, as is customary in smaller machines. Shims are provided underneath clamping flanges, in order that any looseness of core may be taken up, in case such looseness should ever develop. The core was assembled at the power house as a complete circle so that the laminations are staggered everywhere and there are

no joints in the core, which are sometimes responsible for noise, due to vibrations of the edges of the laminations immediately at the joint.

A special device for assembling and pressing the core was made for this particular installation and is shown in Fig. 5.

The laminations are of the best grade of silicon steel, of same thickness and with every sheet enamelled on both sides, in same manner and with same care as for cores of the largest and most important 60-cycle generators.

Stator Winding

The armature winding consists of 360 coils, three turns each of rectangular wire, 36 strands in multiple, spanning 70 per cent of the pole arc. The number of coils per pole is fractional, 12 $^6/_7$, to eliminate higher harmonics. The coils are connected up in four circuits per phase. All connections from coil to coil in the various phase belts, as well as the pole connections and the phase leads at beginning and endings, are made at the top of the machine. Both ends of all phase windings are carried to the busses outside frame for connecting in the differential relay protection apparatus.

The weight of one insulated coil is 186 lb. The weight of the completely insulated and connected winding is approximately 70,000 lb. The windings at heads of machine are laced back to insulated supporting bands with blocking between the coils, while the end connections are arranged in a neat and systematic manner and similarly blocked. See Fig. 6.

The insulation on turns, as also the external insula-

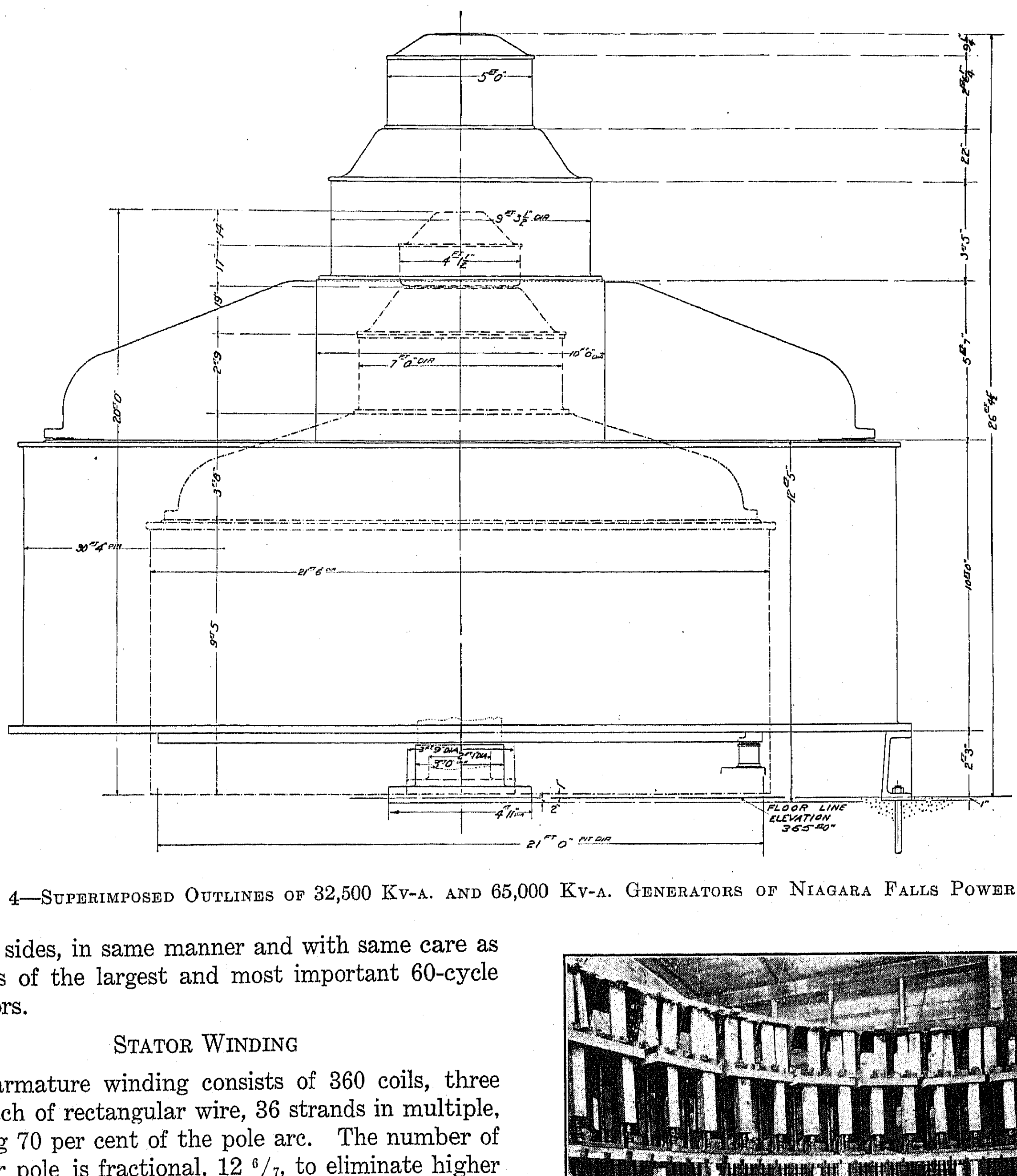

Fig. 4—Superimposed Outlines of 32,500 Kv-a. and 65,000 Kv-a. Generators of Niagara Falls Power Co.

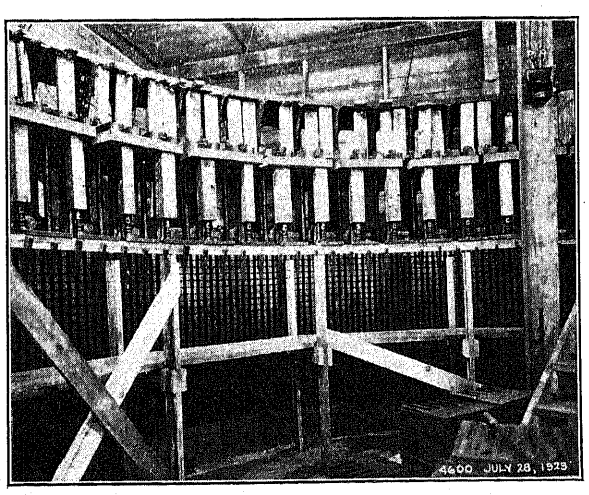

Fig. 5—Stator Core Assembling and Pressing Device

tion, consists of mica, put on in the form of tape with a specially developed varnish as sticker. The coils were subjected to vacuum treatment and moulded to size at different stages of the process. A cross section through the slot, Fig. 7, gives a clear idea of the composition of the conductor and the relative thickness of the insulation.

The insulation of such large coils has been developed with certain desirable characteristics in view, such as the elimination of all pockets for air, or other gases, to avoid internal corona and the introduction of sufficient flexibility to permit bending or yielding when coils are being assembled in deep radial slots or being removed in case repairs are ever made.

A certain amount of heat should be applied before coils are assembled, as also when taking coils out of slots. For the winding of armature of the Niagara generator in the power house a special heating oven was made—Fig. 8.

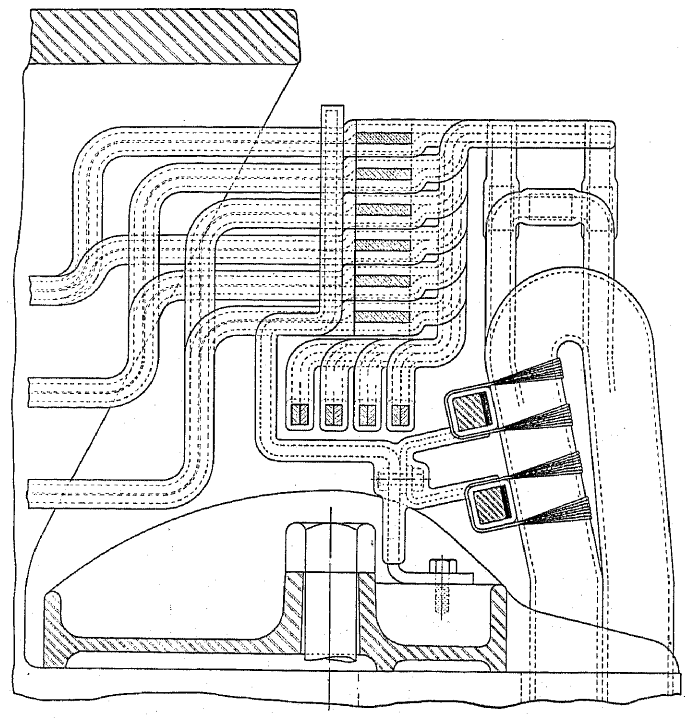

Fig. 6—Supports of Stator Winding and Connections

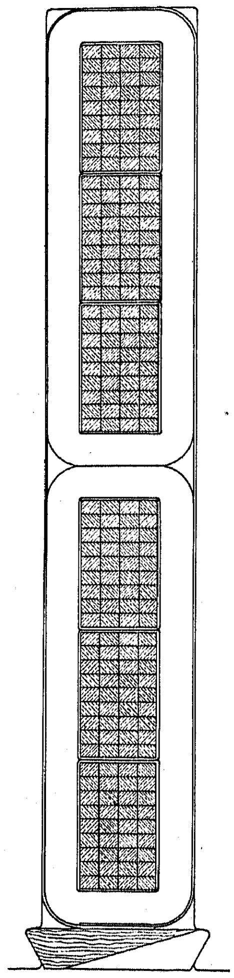

Fig. 7—Cross Section of Stator Coils in Slot

Rotor Spider

As a rule, less time is required to design the revolving field of a high-speed large-capacity generator than a low speed generator, for the reason that a laminated or plate center with dovetailed and keyed-on poles is invariably found to be the proper construction for the small-diameter, high-speed rotor. However, for a 107 rev. per min. generator of large capacity, several alternatives present themselves, requiring much study to decide which is best. Rotor spider may have laminated or plate rim, or it may be made of steel castings throughout or of steel and iron, and in various ways. Poles may be dovetailed or bolted, etc.

The rotor spider of the 65,000-kv-a. generator is cast steel throughout, made up of five sections or wheels, every wheel containing two identical castings, half wheels—Fig. 9—fastened together at each joint of the rim by two nickel steel shrink keys extending across the full breadth of rim. The half hubs are rabbeted and fastened together by numerous large nickel-steel shrink bolts. Two shaft keys, diametrically opposite and extending entire length of hub, prevent rotor turning on shaft against torque. Rotor hub at lower end rests upon a shoulder or projection of the shaft and at upper

end is held in place and kept from lifting, when brakes are applied to rim underneath, by means of a circular split ring key set into shaft and bolted to hub. This multiple-wheel type of rotor spider, with circumferential spaces at the rim between adjacent wheels, insures a supply of air for removing heat from field coils. The arms of the top and bottom wheels are

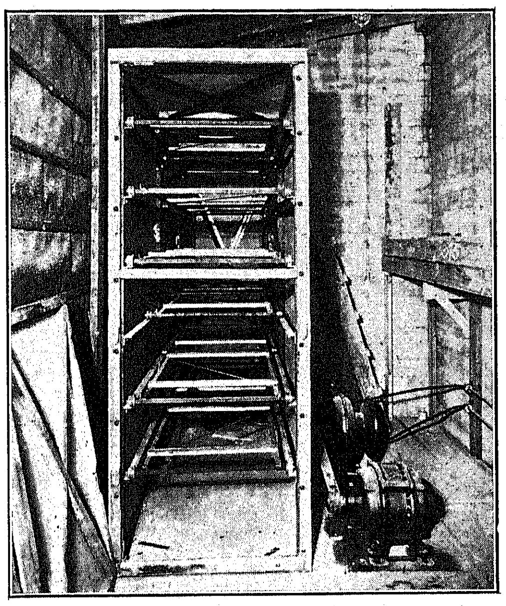

Fig. 8—Oven for Heating Armature Coils when Assembling

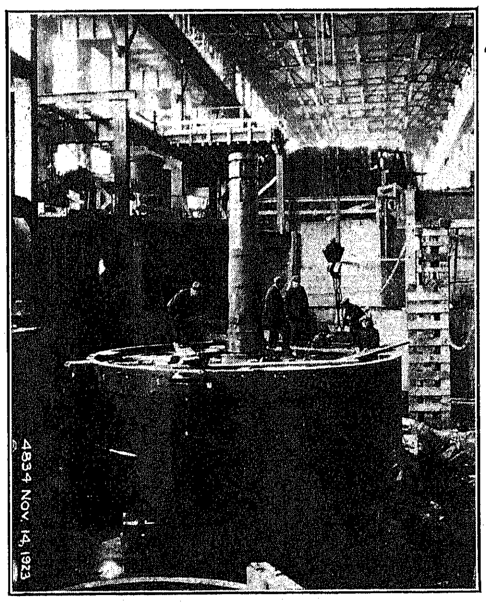

Fig. 9—Rotor Under Construction

partly covered with steel plates to reduce windage losses and increases the flow of air through the rim.

To make sure that all castings had the proper physical properties, four test coupons were cast on the rim of every half wheel. Test pieces were cut from these coupons, turned to standard size and pulled in the mechanical laboratory. The results of the tests on the forty samples from the ten castings that were used in the rotor of the first generator, were within the limit of the contract specifications.

Poles and Field Coils

The pole laminations are $\frac{1}{8}$ in. thick sheet steel, clamped between heavy cast-steel end plates and securely held by large through bolts, instead of the usual rivets. Two nickel-steel keys, running the entire length of pole, including end plates, are set into pole body and tapped to receive the bolts attaching pole to rim.

A novel feature of this design is the method of fastening pole pieces to rotor spider. Instead of the usual dovetail, keyed type of pole, twenty nickel-steel tap bolts per pole are provided, ten the length of the pole, two in parallel, or four bolts per section of rotor rim, making a total of five hundred and sixty holes drilled radially through the rotor rim. It was thought that rotor rim casting could better be "explored" for shrink holes and checks, by drilling these radial holes as against the dovetail slot, which does not cut the rim radially. Incidentally, not one shrink hole or check was discovered when drilling the five hundred and sixty holes in all ten sections.

Field Coils

Field coils are wound with copper strap 0.43 in. by 2.625 in. in cross section, which is probably the heaviest strap that has ever been used in field coils. It required from ten to twelve reels of copper for each coil, hence, about eleven brazings on an average per coil. The ordinary machines for winding copper strap on edge could not be used. Fortunately, a much heavier machine, used for forming field coils of the largest steam turbine generators, was available, although certain modifications were necessary. After forming coil, a special heating and clamping device was used to mould into shape for assembly on poles. The weight of copper in each coil is 2800 lb.

Rotor coil supporting brackets, insulated from coils are provided between each pair of poles, to prevent bulging of coils due to side thrust. On account of the size and weight of each pole, a special lifting device was made for lifting the pole into a vertical position when assembling. See Fig. 9. The revolving field, is guaranteed to withstand a runaway overspeed or speed test of 214 rev. per min., or twice normal, and such a test was made in the factory on each section of rotor spider and witnessed by customers' representatives. Special poles, each representing the weight of that part of pole over rotor spider section, were made up of laminations and end plates and attached to each section of rotor spider. Laminations, end plates and pole piece bolts, used in making test, were afterwards used in the assembly of the complete pole. All five sections of rotor spider passed double speed test successfully.

Upper Bearing Bracket

The upper bearing bracket, on which rests the thrust bearing carrying an estimated load of 1,200,000 lb.

is of the central-hub and separate-arm type. The central hub is a cylindrical steel casting, to which ten radial arms of cast steel are rabbeted, doweled and bolted, the whole when bolted together having a beveled turn lip at the ends of arm, fitting accurately into the stator frame for perfect mechanical alinment of bracket with stator frame, and securely fastened to the frame by large steel bolts. Bracket and bolts are insulated from stator frame to prevent "pitting" of the guide bearing, due to stray currents. The calculated deflection of 0.055 in. checks very closely with actual deflection of 0.063 in., obtained after generator was installed. Covers are provided between arms and are partially perforated, to allow cooling air to be taken in at top of generator. Two hand holes are provided in outer portion of each cover for the inspection of stator winding and connections at any point. A platform, supported from bearing bracket for inspection of thrust bearing, is provided, and a stairway leads to this platform from upper floor of power station.

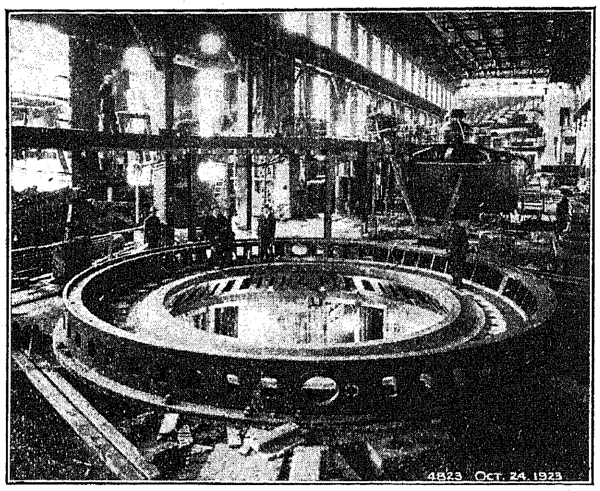

Fig. 10—Base Ring (Outer) and Brake Support (inner)

Guide and Thrust Bearing

The upper guide bearing is centrally supported, instead of having straight seat fit, as is usual in this type of generator.

Thrust bearing is of the usual Kingsbury type, with water-cooling coils, provided for cooling the oil in the bearing housing. This bearing is designed to carry a maximum load of 1,250,000 lb.

Base Ring and Brake Support

Another unusual feature of design is the segregation of the base ring, supporting the stator from the brake-supporting ring. See Fig. 10. Both rings are cast in four sections each, bolted together and separately grouted directly into the foundation. The brake supporting ring designed to take total weight of revolving elements, when jacks are applied under rotor rim to relieve weight on thrust bearing, besides supporting the brakes when used for shutting down. The base ring is provided with convenient man-holes for inspection of stator winding at bottom of generator.

Collector Rings and Connections

At the upper end of the shaft a heavy cast-iron supporting shell is bolted for the two pairs of collector rings, those of main generator and those of excitation generator. The electrical connections between rings and fields are made by means of rectangular copper strips running down from rings through a hole in shaft to a point above hub of rotor, where they are joined to cables running along arms of rotor spider to field terminals. All collector connections can be made or broken inside the supporting shell. The collector rings thus mounted with supporting shell, can be easily removed and replaced. The shell also supports the speed-limiting device, at the top of which a tachometer may be placed for taking readings of speed, if required.

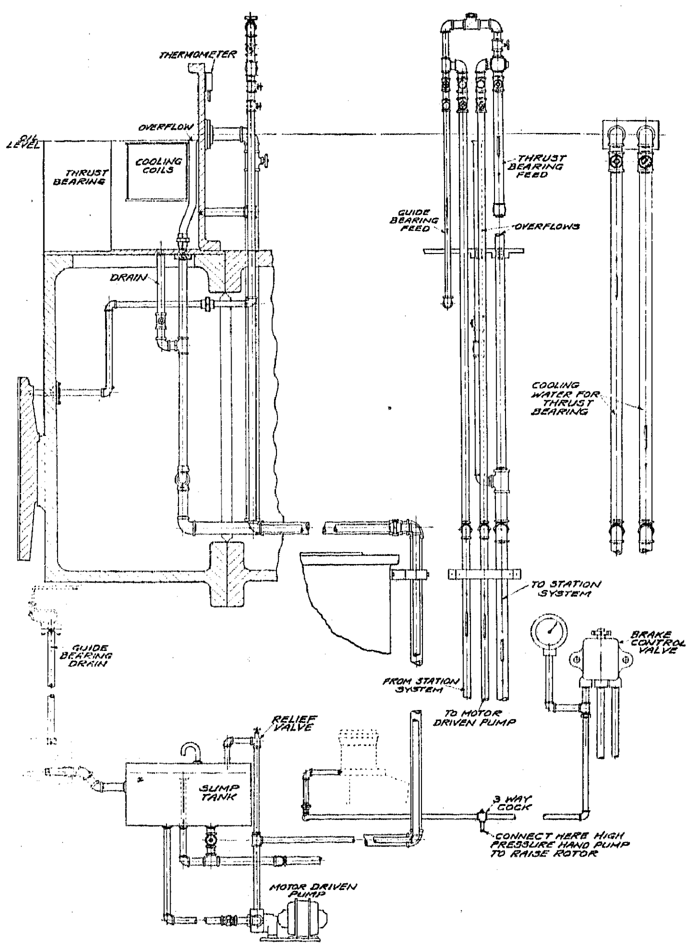

Fig. 11—Diagrammatic Plan of Piping

Oiling System

Fig. 11 shows diagrammatic arrangement of the complete piping for each generator. A feature of design in this arrangement is that generator may be supplied directly from station system, or by separate motor-driven oil pump, acting independent of the station system. Another feature is the use of the same piping

for two purposes, the application of the brakes to rotor rim to shut down (when compressed air from station system at a pressure of 150 to 200 lb. is used) and the lifting of rotor to take weight entirely off thrust bearing (when oil at a pressure of 1400 lb. per sq. in. is used). For jacking up, oil at pressure of 1400 lb. per sq. in. is supplied by a Watson-Stillman hand pump. This operation requires only a short time, and is very effective as against the more laborious method of backing off the bolts on the thrust bearing to relieve the load on bearing, when inspecting or disassembling unit.

Excitation Generators

A-c. generators, both steam and hydraulic driven, have of late been frequently provided with direct-connected exciters. In the case of the vertical units, the exciter is regularly mounted immediately above the thrust bearing at the top of the unit. In several of the largest and best known hydraulic plants excitation has been provided by individual motor-driven exciters, the motors being driven from a house generator or from the station system. As far as the writers know, this system of motor-driven exciters was first introduced many years ago into the plant of the Ontario Water & Power Co., Canadian Niagara Falls, at the suggestion of Mr. J. A. Johnson, who is now the electrical engineer of the Niagara Falls Power Co. Hence, it should not be considered strange that a new idea has been incorporated in this new 65,000 kv-a. generator by Mr. Johnson. A direct-connected alternator is used both for the purpose of driving the motor-generator set that supplies excitation, and also for supplying power to motor-driven auxiliaries.

Outside of the usual mounting of this generator, the stator being supported or "hung" from the bearing bracket arms and the rotor bolted directly to the rotor arms, no unusual problems of design are involved.

Workmanship and Installation

As a general rule, smaller generators of this type are completely assembled in the factory before shipment, to see that all parts fit together properly, and running tests are made to determine if generator comes within guarantees. Outside of assembling the rotor spider sections, without shaft, and the fitting of one or two pole pieces to determine if pole piece bolt-holes lined up with holes in rotor spider and the assembly of stairway and gallery to bearing bracket, no attempt was made to assemble this generator. Notwithstanding the fact that bearing bracket must fit stator frame accurately, that all bolt holes must line up with corresponding holes in connecting parts, that rotor spider keyways must line up accurately with shaft keyways, and all without being fitted together, when generator was finally assembled in power house for the first time, so accurately was all machine work done that, to use the words of the erecting engineer, "all parts fitted perfectly and not even one bolt had to be changed to make the assembly complete."

Weights

Stator Frame, Laminations, Windings, Flanges, etc.	470,000 lb.
Stator Base Ring	52,500 "
Brake Supporting Ring	56,500 "
Upper Bearing Bracket	119,150 "
Complete Rotor with Shaft	783,110 "
Shaft	59,432 "
One Rotor Pole & Winding	10,900 "
Miscellaneous	32,540 "
Complete Total Weight of Main Generator	1,513,800 "
" " " " Excitation	24,592 "
" " " " Unit	1,538,392 "

The $W R^2$ is 65,000,000—figures easy to remember in connection with a 65,000 kv-a. generator that has excitation auxiliary of 650 kv-a.

Discussion

R. B. Williamson (by letter): This paper is of special interest to me because I happened to be responsible for the design of one of the 32,500-kv-a. units installed in this station in 1919, and also have had in charge the design of a 65,000-kv-a. unit which will be put in the same station alongside the unit described. At the time the 32,500-kv-a. units were started, they were the largest of their kind but they had been in operation but a short time before plans were made by The Niagara Falls Power Company to install three additional units of 65,000-kv-a. output. The three 32,500-kv-a. units have now been in operation for four years or more and have given excellent service. Further, these units showed a very high efficiency, 97.5 to 98 per cent, and the larger unit was not adopted with the expectation of securing any material gain in efficiency. Neither does the large generator cost appreciably less per kv-a. output, since the speed is lower and the weight per kv-a. somewhat higher. It may be repeated here, that the 32,500-kv-a. units operate at 150 rev. per min. as against 107 rev. per min. in the case of the 65,000-kv-a. machine.

Referring to Fig. 4, it will be noted that the floor space occupied by the large generator is approximately 725 sq. ft. as compared with 365 sq. ft. for the smaller unit. Thus the floor space is almost double so that space per kv-a. is about the same in either case, so far as the generators alone are concerned. However, when all other features, such as necessary space between units, space for auxiliaries, penstocks, etc., are taken into account, it is evident that three of these large units can be put into a shorter power house than six units of 32,500 kv-a. each, and as the space available for the power house was strictly limited in this case, the larger units were chosen. In other words, it was necessary to place the largest possible generating capacity in a limited space and the 65,000 kv-a. generators in this respect were superior.

So far as the construction of the stator is concerned, it is essentially the same as that of the smaller units, though, of course, the machine is larger in diameter and higher. The chief structural differences are in the rotor which, on account of its large diameter, had to be sectionalized to a greater extent. In the unit described, the rotor spider is in ten sections, each consisting of a half wheel with the rim cast integral with the arms and half hub. The five wheels assembled on the shaft form the complete spider to which the poles are bolted. As stated in the paper, the design to be used for such a large spider is one that involves a number of considerations. Not the least of these is the ability of the steel-founder to produce satisfactory castings. Such a design has to be worked out in consultation with those who make the castings and very often the design has to be modified to conform to their requirements. Thus, designs that are quite different, may result and yet each be entirely satisfactory. In the design with which the writer has had to do, the rotor spider is in two wheels, each of which is made of seven sectors having two arms per section. The two hubs are separate castings and

the arms are bolted to heavy flanges on the hub by means of reamed bolts in the same manner as has been used for many years for large segmental engine fly wheels. The sectors are joined at the rim by heavy mild-steel tongues and dowels, and the poles are bolted to the rim. Test bars from all the rotor castings were taken from coupons and also from samples drilled from the body of the rim by means of hollow drills.

As stated in the paper, the problem of bending the field copper on edge for a field coil of this size is quite a difficult one but it has been successfully accomplished and the coils present a good even appearance. This is only one of the many problems that have to be solved in the construction and erection of such a large unit and the authors are to be congratulated on the results obtained. The writer had the pleasure of witnessing the starting of the first of these units and notwithstanding the enormous size of the machine, its operation is smooth and quiet. The third unit is now being installed and I hope at a later date to present a detailed description of it to the Institute. In external appearance and dimensions, it differs very little from the one described in the paper; in fact the engineers concerned, cooperated in every way possible to make the units present a uniform appearance and the differences are mainly in details of design which I hope to describe later.

W. I. Slichter: This paper is an important addition to the very few we have had on the subject of the design of large generators. In the PROCEEDINGS of the Institute, there was in 1914 a description of a 7500 kv-a. machine; in 1922 a description of a 32,500 kv-a. machine; in 1923, of a 45,000 kv-a. machine and this year, 1924, of a 65,000 kv-a. machine. This enumeration shows the rapid increase in capacity of water-wheel generators, as each machine described was the largest of its time. It is interesting to note that in this latest machine efficiency was given more consideration than heating, showing the effort for very high efficiency. In most large machines, if the heating is satisfactory the efficiency will also probably be satisfactory. It is well known to designers that in small machines it is difficult to get a good efficiency and if this is obtained no difficulty is experienced with heating. In large machines, particularly turbo generators, the heating is the more difficult problem and if this is satisfactory the efficiency is also satisfactory. This machine therefore marks a still further step in the development of our designs, in which the conditions have reversed themselves as the capacity increases above a certain very large figure.

The use of a fractional number of slots per pole ($12^6/_7$ slots per pole) to eliminate higher harmonics is a matter of interest as this practise, which has only been introduced in the last few years, is quite a conundrum to the engineer without experience in design, and yet it is quite a natural and desirable advance in the art of making windings to give a good electro-motive force wave-shape.

In order to compare the weight economy of this machine with others it is necessary to take into account many factors and this is best done, for purposes of comparison, by a formula, which has been applied to many lines of machines. This formula is

$$W = K \sqrt{\frac{P}{R.\ P.\ M.}}$$

in which W is the weight of the machine in pounds; P, the kv-a. rating of the machine; and $R.\ P.\ M.$, the revolutions per minute. K is a characteristic constant of a given line of machines. Thus in well known lines of sixty-cycle turbo-generators, K would run from 40,000 to 50,000 and in a line of similar machines for twenty-five cycles, K would run from 50,000 to 60,000. In the machine under discussion, K has a value of 58,000, so that this is an example of a water-wheel-driven machine of 187 rev. per min. and 8200 ft. per min. peripheral velocity which has a weight factor constant very close to that of turbo-generators which are generally understood to have a very good weight economy.

L. W. W. Morrow: One of the interesting features of this machine is the fact it has been installed at Niagara Falls; the development of hydroelectric machinery at this location has influenced the art in all parts of the world. The design of machines for this installation is unusual in that designers are given freedom to design the best possible machines and are not limited by cost values to the degree found in other locations where the units are small and the firm power is less in amount. Niagara Falls has a large and constant amount of available power which is sold largely near the development site. A fraction of a per cent gain in efficiency or in reliability on such large units and under the conditions found at this location can be capitalized for a very large sum and efficient production outweighs first cost many times.

L. P. Ferris: Mr. Foster mentioned the fact that the large 65,000-kv-a. machine described in the paper by himself and Mr. Glass has a fractional number of coils per pole, for the purpose of eliminating the higher harmonics. This is a very welcome step, from the point of view of the telephone engineer who has to deal with the effects of these higher harmonics on parallel circuits.

W. J. Foster: Mr. Dawson has asked the question about how the various losses have been determined. They have not been determined yet on the 65,000-kv-a. machine, by actual test, but they will probably be determined in about two months as soon as the second generator has been in service for a short time. The method that will be employed is the retardation method, the same as used in connection with the testing of the three 32,500-kv-a. generators. That gave most excellent and consistent results, as applied at Niagara Falls under the direction of Mr. Johnson. The retardation tests will be made on open circuit without any excitation, in order to determine the combined friction and windage losses; then under different degrees of excitation, to a considerable over-excitation. With all those curves, there isn't a shadow of a doubt as to the proper determination of the friction and windage, and of the core losses on open circuit.

Then the load losses will be determined by making similar retardation tests with the armature short-circuited, with an excitation that will give the full-load current carried during the retardation, at the 25-cycle point.

Those tests, as applied to the first generators, agreed very closely with the calculated values, showing lower figures in the case of the core losses, and practically the same as used in the calculation of efficiency for the remaining losses. But we already know from the 65,000-kv-a. machine that is operating, what its copper losses are, since we have the resistance of the armature and have determined the excitation required, which is a little lower than allowed in the calculations. So we have not much doubt that the efficiency mentioned here will be attained.

There are one or two features of that generator, that have not been brought out in the paper, and I didn't bring them out in the introductory remarks. One of these is the ventilation at the heads of the armature. The customer is very much concerned that every precaution against damage or of possible burning out of an armature coil should be taken so as to minimize the consequences of such a burn-out, and for the first time in connection with any generators that I have anything to do with, a very determined effort has been made to stop completely the swirl of air in the projecting windings of the stator. That swirl of air, especially in high-speed machines like turbine generators, once it catches fire, results as a rule in the destruction of the entire winding. In the case of slower speed machines, water-wheel machines, the damage often spreads around a considerable distance. But this particular generator, after a great deal of thought and numerous designs, has been equipped with a series of vanes at top and bottom, these vanes being made of an insulating material and also of fireproof, fire-resisting material.

Harmonics Due to Slot Openings

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Review of the Subject.—*This paper represents a brief graphical-analytical exposition, followed by a mathematical development of the harmonics due to slot openings These were proved to be even and odd multiples of the number of slots plus and minus one. The modifications upon the torque speed curves of these harmonics have been discussed. A simple Fourier analysis underlies the whole phenomena.*

* * * * *

THE object of this paper is to give in Part I a graphical-analytical demonstration and in Part II a mathematical demonstration of the harmonics caused by slot openings. The description applies particularly to the field form of a rotating field motor such as the induction motor.

Part I.

Tooth harmonics affect the performance of the motor in several ways, the more important, from a practical point of view, being the low-speed counter-torques causing cusps in the speed-torque characteristic and noise.

The depth of the cusp in the speed-torque characteristic depends upon the magnitude of the low-speed counter-torque, which in turn depends upon the intensity of the harmonic field. It is, therefore, both interesting and important that the order and magnitude of the harmonics be determined for the practical case. The solid line Fig. 1 shows the speed-torque curve of an induction motor with the effect

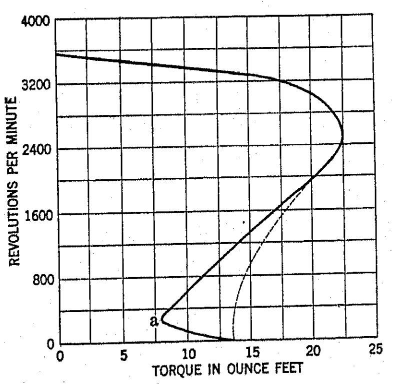


FIG. 1

of tooth harmonics shown at *a*. The dotted portion of the curve indicates the shape of the speed-torque curve if tooth harmonics were eliminated.

For the purpose of simplification, the rotating field of an induction motor only was considered and to further simplify the work involved in this anaylsis, five slots per pair of poles were assumed and the dimensions of the tooth were assumed to be in proportion to the dimensions of the teeth of a punching having the usual number of teeth. By using five teeth per pair of poles instead of the usual number in a standard motor, a large amount of unnecessary labor

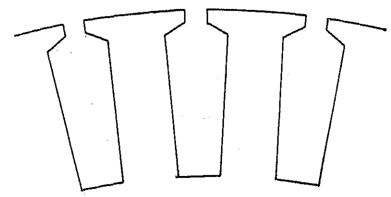


FIG. 2

was avoided. By making the shape of the teeth of the five-tooth rotor in proportion to the usual number of teeth in a standard motor, the equivalent of practical conditions was obtained. These assumptions produced a tooth of proportions shown in Fig. 2.

The air gap was of the usual size for the smaller sizes of induction motors and in this case 0.015 in. on one side.

After obtaining the normal size of tooth and slot opening of a five slot per pair of poles element, and assuming the air gap to be 0.015 in., the tooth and air gap were laid to a scale ten times normal size in order to draw in the tooth fringing as shown in Fig. 3.

The center line of the tooth was taken as the starting point and the distance from center line of slot to center line of tooth was divided into 36 equal parts and the flux paths measured at these points along the fringing lines shown in Fig. 3. Since the field strength is proportional to the reciprocals of the lengths of the paths, the reciprocals of the measured lengths were calculated and the data in the first two columns of Table I was prepared for plotting the reciprocal curve shown in Fig. 4. Since the maximum field strength is a direct function of the largest reciprocal, Column 3 of Table I was prepared on the basis of putting the per cent maximum flux equal to the largest reciprocal and other values in direct proportion. Fig. 5 was then plotted from data in Columns 1 and 3 in Table I. This curve, Fig. 5 has the same shape as Fig. 4, but the ordinates are given in per cent maximum flux instead of reciprocals which is more convenient. Curves, Figs.

*I am indebted to Mr. R. E. Hellmund for valuable suggestions.

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-11, 1924.

4 and 5 illustrate the effect of the slot opening on a uniform field.

Since the field in the air gap of an induction motor is assumed to be sinusoidal, the curve shown in Fig. 5

TABLE I

Position in Degrees	Length from Drawing	Reciprocals of Lengths
0-29.7	0.15	6.67
30	0.3	3.33
31	0.75	1.33
32	1.2	0.833
33	1.65	0.606
34	2.1	0.476
35	2.5	0.40
36	2.97	0.337
37	2.5	0.40
38	2.1	0.476
39	1.65	0.606
40	1.2	0.833
41	0.75	1.33
42	0.3	3.33
42.3-72	0.15	6.67

was multiplied by the sine curve and the instantaneous field obtained as shown in Fig. 6. Two points, one at a and the other at b are calculated below to illustrate

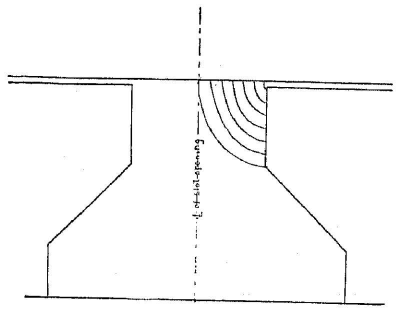


Fig. 3

the method more clearly. a is at 20 deg. on curve Fig. 5 and has an ordinate value of 100 per cent. This value is multiplied by the sine of 20 deg. to obtain the

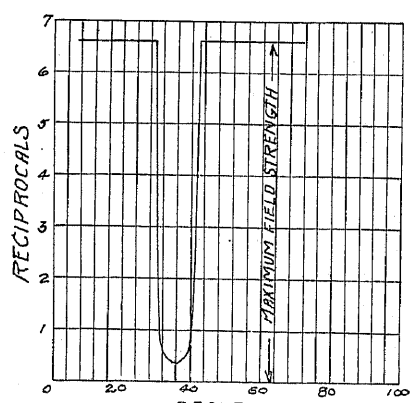


Fig. 4

corresponding ordinate for the sinusoidal field Fig. 6.

$$100 \text{ per cent} \times \sin 20 \text{ deg.} = 34.2$$

which is plotted at a Fig. 6. Likewise b at 36 deg. on Fig. 5 has a value of 5.05 per cent which, when multiplied by the sine of 36 deg., gives the corresponding ordinate for the sinusoidal field Fig. 6.

$$5.05 \text{ per cent} \times \sin 36 \text{ deg.} = 2.97$$

which is plotted at b Fig. 6.

Fig. 6 illustrates the effect of the slot openings on the

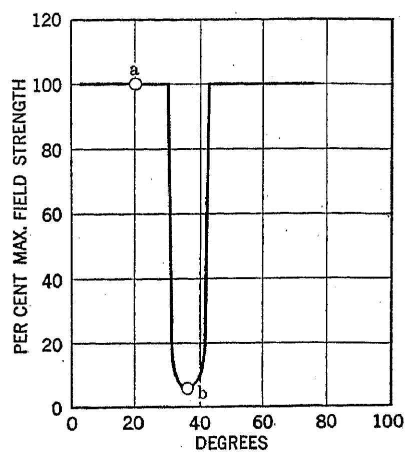


Fig. 5

field form of an induction motor. It should be noted that the indentations caused by the slot openings do not reach zero value. This, it will be understood, is due to the fringing and is what would be expected.

Since the direction of rotation of the harmonics is equally important as the kind of harmonics, it was necessary to find means for being certain of the direction of rotation of the harmonic fields as well as their order and magnitude, as compared with the direction of rotation of the fundamental. This was done by con-

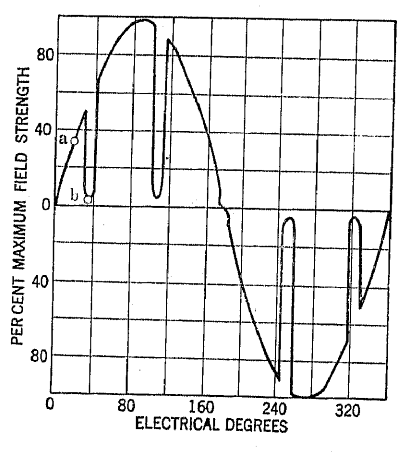


Fig. 6

sidering the fundamental as having advanced 18 electrical deg. clockwise with respect to the slotted element, which gives a new position of the indentations caused by the slot openings on the fundamental. To obtain the field form with the fundamental advanced 18 electrical deg. with respect to its former position, curve 5 with zero degrees taken at the 18 deg. point was mul-

tiplied by the sine curve in the same manner as previously explained and Fig. 7 obtained. If these two instantaneous field forms, Figs. 6 and 7 are analyzed for the various harmonics, the positions of each harmonic with respect to the fundamental in each case will give the rotation of the harmonics. This will be illustrated more fully later.

It is, of course, clear that the indentations caused

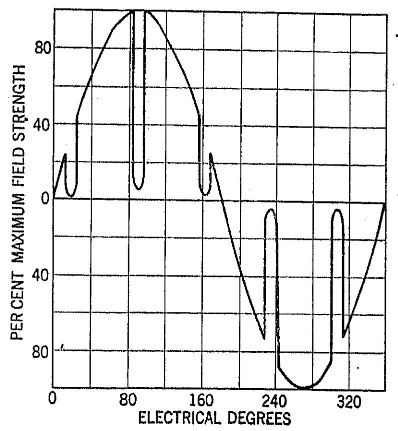


Fig. 7

by the slot openings will successively occur at all points in the fundamental in cyclical order. The task of analyzing the two instantaneous field forms, Figs. 6 and 7 for the higher harmonics was too tedious by mathematical methods and it was decided to re-plot these two instantaneous field forms on polar co-ordinates and analyze them on an available harmonic analyzer.

Figs. 8 and 9 show the instantaneous field forms Figs. 6 and 7 on polar co-ordinates. The analysis of

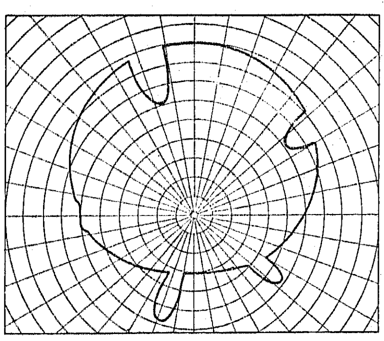


Fig. 8

these flux waves as obtained from the harmonic analyzer are shown in Table II.

An inspection of the values of the various harmonics shown in the Table brings out the fact that there are harmonics of appreciable magnitude as follows: 4th and 6th, 9th and 11th, 14th and 16th, 19th etc. . . . and that all other harmonics are zero or of negligible value. This fact may be represented mathematically as follows:

$$\text{Order of Harmonics} = k\,n \pm 1$$

where k equals any integer and n equals the number of slot openings per pair of poles.

The direction of rotation of the harmonics with respect to the fundamental was determined in the case of the 4th and 6th harmonics by plotting them on the instantaneous fields Figs. 6 and 7 from the values given in Table II as shown in Figs. 10 and 11. It will

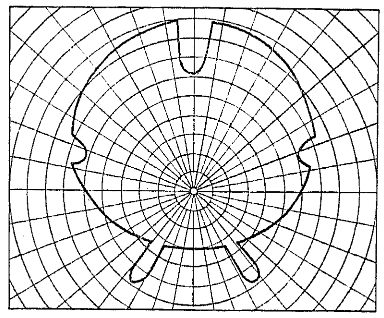


Fig. 9

be remembered that in the case of Fig. 7 the fundamental was advanced with respect to the slot openings 18 electrical deg. clockwise from its position in Fig. 6. Likewise Fig. 11 shows the conditions for the case of the fundamental having moved clockwise 18 electrical deg. from the position shown in Fig. 10. It is therefore evident that the positions of the 4th and 6th harmonics in Figs. 10 and 11 will indicate the direction of rotation of these harmonics.

In order to determine the direction of rotation of the 4th and 6th harmonics, the two instantaneous fields, Figs. 10 and 11 showing the 4th and 6th harmonics, were compared.

In the case of the 4th harmonic its maximum value occurs at 22½ deg. in the case of Fig. 10, whereas in

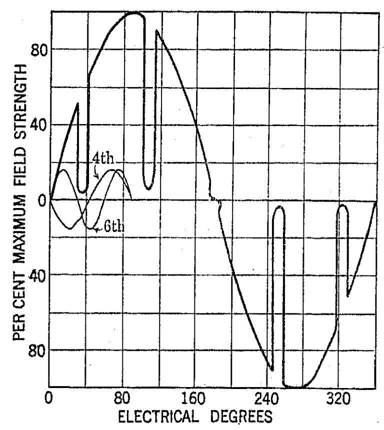


Fig. 10

the case of Fig. 11 it occurs at zero degrees. Now since the fundamental has advanced 18 electrical deg., the maximum of the 4th harmonic should be at plus 4½ deg. on Fig. 11, if the 4th harmonic did not move at all, that is, if the 4th harmonic were stationary in space. Since the maximum of the 4th harmonic on

Fig. 11 actually occurs at zero degrees it must have moved 4½ deg. contrary to the fundamental.

In the case of the 6th harmonic, its maximum value occurs at 15 deg. in the case of Fig. 10, whereas in the case of Fig. 11 it occurs at zero degrees. Since the fundamental has advanced 18 electrical deg., the maximum of the 6th harmonic should be at minus

TABLE II

	Fundamental 1st. Position		*Fundamental 2d. Position	
N	Sine	Cosine	Sine	Cosine
1	+ 93.8	0	94.1	0
2	0	− 1.1		
4	− 16.5	0	− 1.0	− 15.1
5	0	− 1.7		
6	+ 16.2	0	0	+ 16
8	0	− .8	0	− .7
9	+ 14.0	0	− 13.4	+ .6
10	0	+ .7	− .9	− .6
11	− 13.7	+ 1.1	+ 13.6	− .8
12	0	+ .8	− .6	0
14	− 10.7	− .8	+ .8	+ 10.3
15	0	0	0	+ 1.1
16	+ 10.7	− .7	− 2.0	− 10.3
17	+ .5	0	0	0
18	0	0	0	− 2.9
19	+ 7.3	0	7.5	0
20	0	0	+ .9	0

*In the second position the fundamental has moved to the right 18 electrical deg.

3 deg. on Fig. 11 if the 6th harmonic did not move at all, that is, if the 6th harmonic were stationary in space. Since the maximum of the 6th harmonic on Fig. 11 actually occurs at zero degrees, it has advanced 3 deg. with the fundamental.

Therefore, since we have assumed the fundamental to have clockwise rotation, the 4th harmonic will

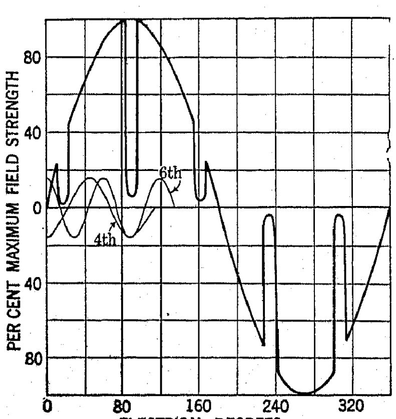


FIG. 11

rotate counter clockwise and the 6th harmonic will rotate clockwise.

The results of this graphical-analytical analysis shows that tooth harmonics are inherent and that tooth cusps are to be expected and that the tooth harmonics are of the order of an integer multiplied by the number of slots then increased and decreased by one.

After completing this investigation it occurred to the author that there ought to be a purely mathematical proof of this fact

$$\text{Order of Harmonics} = k\,n \pm 1$$

which could be obtained by a development of the series representing a rectangular wave to obtain the effect of slot openings.

This could be done by putting in place of α in

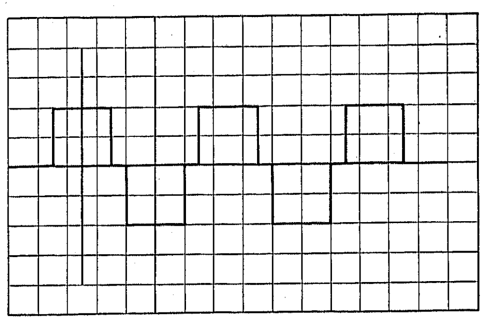


FIG. 12

Fourier's Series of the rectangular wave α plus θ and α minus θ and then adding like terms of the two series. This gives a wave as shown in Fig. 12, which can be changed to approach the form of the reciprocal wave shown in Fig. 5 by squaring the series representing this wave (Fig. 12) which gives the wave shown in Fig. 13 and which, it will be noted, is quite similar to Fig. 5. The difference between Fig. 5 and Fig. 13 is due to the

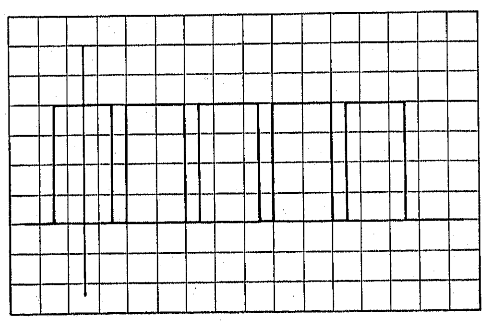


FIG. 13

fringing taken into consideration in the case of Fig. 5 which is the practical case. It will also be noted that the flux does not reduce to zero as in the case of mathematical demonstration. This difference between the mathematical case and the actual case would not affect this problem from a practical point of view.

This method of attacking the problem from a mathematical standpoint was proposed to Professor F. W. Lee, and his mathematical analysis follows in Part II of the complete paper.

Part II

While the performance of the induction motor has been discussed by Arnold, Karapetoff and Hansen[1] primarily from the point of view of the stator, little has been said upon the equally important modifications caused by the rotor. Arnold, Fritze[2], Stiel u. Ottenstein[3] have done some work upon the effect of the rotor slots in an induction motor. Now it is the purpose of this part of the paper to derive in a simple way the relations modifying the flux distribution caused by the rotor. In this exposition only the fundamental component of the static flux will be considered, since in a trapezoidal wave, under the most favorable conditions, the third harmonic is but 1/9 of the amplitude of the first, the fifth 1/25 and so forth.

From a mathematical viewpoint the flux distribution,

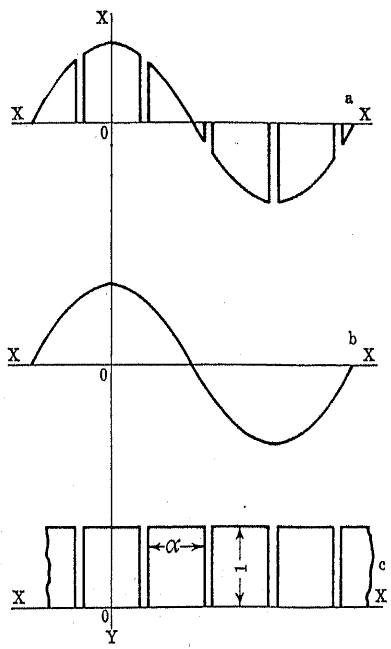


FIG. 14

as shown in a of Fig. 14, can be obtained by the product of the two curves shown in b and c.

The function of b can be considered as that of a simple cosine of unit amplitude where x is measured in electrical degrees.

$$y = \cos x \quad (1)$$

The function of c may be considered as a composite function having a rectangular Fourier series as a basis. In the previous consideration there were five fingers as indicated, and here it will be desirable to express this composite function c in terms of the angle as measured in b. For a simple rectangular wave of unit amplitude it would be

$$y = 4/\pi\,[\sin (5/2)\,x + 1/3 \sin (15/2)\,x + \text{etc.}] \quad (2)$$

1. Hansen, A. I. E. E., December 1922.
2. Fritze, Archiv für Electrotechnik, 1922.
3. Stiel u. Ottenstein, E. T. Z., 1919, page 590.

Because of symmetry it is best to choose the Y axis for this development as is shown in Fig. 15, part a, where y_1 expresses the rectangular wave of height $h/2$, and α is the tooth width in electrical degrees.

$$y_1 = (1/2)\,h\,4/\pi\,[\sin (5/2)\,(x - \alpha) + \ldots 1/n \sin^n (5/2)\,(x - \alpha)\,] \quad (3)$$

In b of Fig. 15, the same series is represented as in a except that it is displaced by $\alpha/2$ degrees to the left. Here y_2 expresses the rectangular wave

$$y_2 = (1/2)\,h\,4/\pi\,[\sin (5/2)\,(x + \alpha) + \ldots 1/n \sin^n (5/2)\,(x + \alpha)\,] \quad (4)$$

In both of these equations, n is always odd.

By subtracting the two series as is seen in c of Fig. 15, a curve is obtained which is represented by the relation.

$$\begin{aligned} y_2 - y_1 &= (1/2)\,h 4/\pi\,[\sin (5/2)\,(x + \alpha) - \sin (5/2)\,(x - \alpha) \\ &\quad + \ldots 1/n\,\{\sin (5/2)\,n\,(x + \alpha) - \sin (5/2)\,n\,(x - \alpha)\}\,] \\ &= h\,4/\pi\,[\sin 5/2\,\alpha \cos 5/2\,x \\ &\quad + \ldots 1/n \sin (5/2)\,n\,\alpha \cos (5/2)\,n\,x] \end{aligned} \quad (5)$$

If this function is squared and h is chosen unity as per part d of Fig. 15

$$(y_2 - y_1)^2 = (16/\pi^2)\,\Sigma\,\Sigma\,\frac{2}{m\,n} \cos (5/2)\,m\,x \cos (5/2)\,n\,x \sin (5/2)\,m\,x \sin (5/2)\,n\,x \quad (6)$$

This is a double manifold series, in which n has all odd values from 1 to ∞ and for each of these values of n, m has all odd values and is restricted only by the condition that $m \lesseqgtr n$.

For ease in computation of the constants, it may be expanded as the cosine function of the sum and difference of integer angles.

$$\begin{aligned} (y_2 - y_1)^2 &= (16/\pi^2)\,[\Sigma\,\Sigma\,\frac{1}{m\,n} \sin (5/2)\,n\,\alpha \sin (5/2)\,m\,\alpha\,(\cos (m + n)\,(5/2)\,x + \cos (n - m)\,(5/2)\,x)\,] \\ &= 16/\pi_2\,[A_1^1 \cos 5\,x + A_2^1 \cos 10\,x + A_3^1 \cos 15\,x + \text{etc.} \end{aligned} \quad (7)$$

By substituting values for m and n and remembering that $n - m$ is always 0 or positive, the coefficients are computed in the usual manner. Since the sum of m and n is always an even number and the difference is also always an even number, one-half of these values will give odd and even harmonics.

From the above, the coefficients enter as sine functions of odd multiples of 5/2. If now $5/2\,\alpha = 180$ electrical deg. were chosen, it would be possible to eliminate all of these coefficients. In the above case $\alpha = 2/5$ of $180 = 72$ deg. which is impossible, since it leaves no slot opening.

This indicates that from this point of view the slots should be as small as possible. However, a specific harmonic may be eliminated in the above case by choosing $5/2\,n\,\alpha = 180$, making $n\,\alpha = 72$ deg. For example, the third harmonic would make $\alpha = 24$ deg. thereby leaving a slot opening of

$$\frac{360 \text{ deg.} - 5 \times 24 \text{ deg.}}{5} = 48 \text{ deg.}$$

This would be impracticable because of the high saturation required in the teeth. It should be stated here that the starting winding which is displaced 90 deg. from the running winding will eliminate one half of the even harmonics in starting a single-phase motor where the even harmonic divided by two is odd. The respective odd harmonics on the other hand will have a cumulative action.

The function $(y_2 - y_1)^2$ may be further simplified by introducing

$$A_1 = A_1{}^1 \times 16/\pi^2,\ A_2 = A_2{}^1 \times 16/\pi^2 \text{ etc.}$$

$$(y_2 - y_1{}^2) = A_1 \cos 5\,x + A_2 \cos 10\,x + A_3 \cos 15\,x + \text{etc.} \quad (8)$$

If the flux distribution in the air gap of a machine is a simple cosine function as

$\Phi = \Phi_{max} \cos x$, if no slots were present in rotor (9)

Φ_{max} is expressed as flux per radian.

Then the resultant flux is given by

$$\Phi = \Phi_{max}\,(y_2 - y_1)^2 \cos x, \text{ if slots are present in rotor}$$
$$= \Phi_{max}\,[A_1 \cos 5\,x \cos x + A_2 \cos 10\,x \cos x + \text{etc.} \quad (10)$$

In general for k slots per 360 electrical degrees.

$$\Phi = \Phi_{max}\,[A_1 \cos k\,x \cos x + A_2 \cos 2\,k\,x \cos x + \ldots A_n \cos n\,k\,x \cos x]$$
$$= 1/2\,\Phi_{max}\,[A_1 \cos (k+1)\,x + A_1 \cos (k-1)\,x + \ldots A_n \cos (n\,k+1)\,x + A_n \cos (n\,k-1)\,x\,] \quad (11)$$

where $n = 1, 2, 3$, etc. This proves that the harmonics existing in the resultant wave are given by the relation

$$\text{Order of Harmonics} = k\,n \pm 1 \quad (12)$$

This was demonstrated by C. A. M. Weber in the first part of this paper from direct measurements of a rotor.

The above relations should be modified since the flux in the tooth slots does not reach zero but has a value $\delta\,\Phi_{max}$ as a minimum. This is shown by Fig. 16 where δ is a percentage of the flux under consideration. The flux modified by the rotor teeth is $\beta\,\Phi_{max}$. Here β is the percentage left after α has been deducted, or $\alpha + \beta = 100$ per cent. By skewing the rotor β becomes small and δ large.

With this premise the resultant flux wave in a motor is given by

$$\Phi = [\phi_{max}\,\delta + \phi_{max}\,\beta\,(y_2 - y_1)^2\,] \cos x \quad (13)$$

The modifying factor now has the curve as indicated in Fig. 17.

In a single-phase induction motor the flux varies harmonically with the time, and the equation (13) reduces to the relation

$$\Phi = [\phi_{max}\,\delta + \phi_{max}\,\beta\,(y_2 - y_1)^2\,] \cos x \cos \omega_1 t \quad (14)$$

Where ω_1 = angular velocity in electrical degrees of the supply.

If there is another winding 90 deg. displaced in space and having currents 90 deg. displaced in time the flux distribution of this winding will be given by the relation

$$\Phi = [\phi_{max}\delta + \phi_{max}\,\beta\,(y_2 - y_1)^2] \cos (x + 90) \cos(\omega_1 t + 90) = [\phi_{max}\,\delta + \phi_{max}\,\beta\,(y_2 - y_1)^2] \sin x \sin \omega_1 t \quad (15)$$

This may represent a two-phase motor or a single-phase motor with a starter winding. It is primarily the object to discuss the reaction of one winding; other windings may be added as indicated in equation (15).

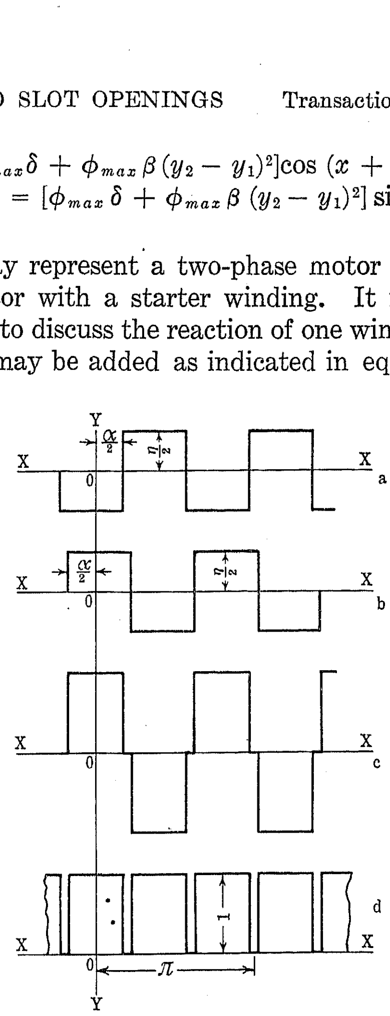


FIG. 15

Equation (14) may be expanded by introducing the sum and difference of two angles

$$\Phi = \underbrace{\phi_{max}/2\,\delta \cos (x + \omega_1 t) + \phi_{max}/2\,\delta \cos (x - \omega_1 t)}_{\text{Usual induction motor}}$$

$$\underbrace{+ (1/2)\ \phi_{max}\,\beta\,[\sum_{n=n}^{n=1} A_n \cos (n\,k+1)\,x \cos \omega_1 t + \sum_{n=n}^{n=1} A_n \cos (n\,k-1) \cos \omega_1 t]}_{\text{Modifying Factors}} \quad (16)$$

As the usual induction motor equations involve no special consideration, only the modifying factors remain

$$\sum_{n=m}^{n=1} A_n \cos (n\,k+1)\,x \cos \omega_1 t = \Sigma\,1/2\,[A_n \cos \{\,(n\,k+1)\,x + \omega_1 t\} + A_n \cos \{\,(n\,k+1)\,x - \omega_1 t\}\,]$$
$$\sum_{n=m}^{n=1} A_n \cos (n\,k-1)\,x \cos \omega_1 t = \Sigma\,(1/2)\,[\,A_n \cos \{(n\,k-1)\,x + \omega_1 t\} + A_n \cos \{(n\,k-1)\,x - \omega_1 t\}\,] \quad (17)$$

This leads to a brief examination of functions having the form

$\cos [\,(n\,k+1) + \omega_1 t]$ which may be written

$$\cos \left(\frac{2\,\pi}{\lambda}\,x + \frac{2\,\pi}{T}\,t \right) \text{ which is a traveling wave} \quad (18)$$

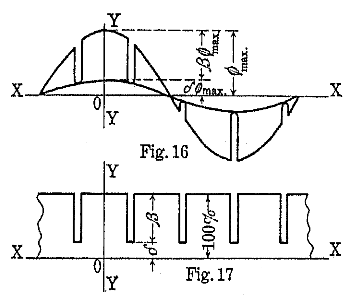
in the circumference of the air gap.

The velocity is given by $\dfrac{\dfrac{2\pi}{T}}{\dfrac{2\pi}{\lambda}} = \lambda/T$ and in the modifying factor there are the following modifying revolving fields having velocities

$$\frac{\omega_1}{n\,k+1} \text{ and } \frac{\omega_1}{n\,k-1}$$

$$-\frac{\omega_1}{n\,k+1} \text{ and } -\frac{\omega_1}{n\,k-1} \qquad (19)$$

This interpreted, means that there is a revolving flux in both directions having velocities $\dfrac{\omega_1}{n\,k+1}$ and $\dfrac{\omega_1}{n\,k-1}$ when the motor is at standstill or starting.



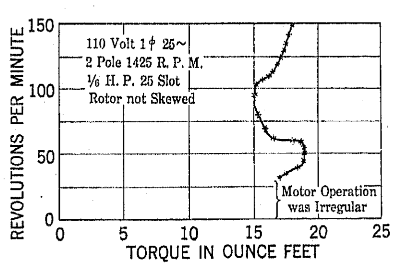


FIG. 16

Since each is exactly equal and opposite there is no tendency to start in either direction.

If the rotor rotates, the modifying function

$$(y_2 - y_1)^2 = A_1 \cos 5\,x + A_2 \cos 10\,x + A_3 \cos 15\,x \ldots A_n \cos n\,k\,x$$

rotates with it. If the angular velocity is ω_2 in electrical degrees, then this fact may be expressed in the form

$$(y_2 - y_1)^2 = A_1 \cos 5\,(x - \omega_2\,t) + A_2 \cos 10\,(x - \omega_2\,t) + \ldots A_n \cos k\,n\,(x - \omega_2\,t) \qquad (20)$$

Notice that each harmonic must have an angular velocity ω_2. The equation for the modified flux becomes

$$\sum_{n=n}^{n=1} A_n \{\cos n\,k\,(x - \omega_2\,t) \cos x \cos \omega_1\,t\} =$$
$$\sum_{n=n}^{n=1} (1/2)\,[A_n \cos \{\,(n\,k+1)\,x - n\,k\,\omega_2\,t\} \cos \omega_1\,t] +$$
$$\sum_{n=n}^{n=1} (1/2)\,[A_n \cos \{\,(n\,k-1)\,x - n\,k\,\omega_2\,t\} \cos \omega_1\,t] =$$
$$\sum_{n=n}^{n=1} (1/4)\,[A_n \cos \{\,(n\,k+1)\,x - (n\,k\,\omega_2 + \omega_1)\,t\,] +$$
$$\sum_{n=n}^{n=1} (1/4)\,[A_n \cos \{\,(n\,k+1)\,x - (n\,k\,\omega_2 - \omega_1)\,t\,] +$$
$$\sum_{n=n}^{n=1} (1/4)\,[A_n \cos \{\,(n\,k-1)\,x - (n\,k\,\omega_2 + \omega_1)\,t] +$$
$$\sum_{n=n}^{n=1} (1/4)\,[A_n \cos \{\,(n\,k-1)\,x - (n\,k\,\omega_2 - \omega_1)\,t\,]. \qquad (21)$$

Equation (21) indicates that there are for each harmonic four distinct flux waves in an induction motor. The angular velocity of these waves is

$$\left.\begin{array}{c} \dfrac{n\,k\,\omega_2 + \omega_1}{n\,k+1} \\[2ex] \dfrac{n\,k\,\omega_2 + \omega_1}{n\,k-1} \end{array}\right\} \text{ in direction of rotation} \qquad (22)$$

$$\left.\begin{array}{c} \dfrac{\omega_1 - n\,k\,\omega_2}{n\,k+1} \\[2ex] \dfrac{\omega_1 - n\,k\,\omega_2}{n\,k-1} \end{array}\right\} \text{ in opposition to rotation} \qquad (22)$$

These flux waves induce their respective voltages, and the currents due to these voltages will have torque components proportional to the cosine of the angle between the induced voltage and the current. Since the secondary reactance of a single-phase motor is high, the frequency of the currents induced by the flux in the direction of rotation is also high. The torque added because of these may be considered negligible because of the large-phase angle. The direction of these waves is always the same, irrespective of ω_2.

The last two flux waves in equation (22) are of low frequency except at the condition of starting when $\omega_2 = 0$. As soon as $n\,k\,\omega_2$ has an appreciable value relative to ω_1 the frequency rapidly decreases and the torque component of the current becomes greater until $\omega_1 = n\,k\,\omega_2$. Here the torque for this harmonic is zero because there is no induced voltage. After that the torque reverses, since the direction of the rotating flux reverses. As ω_2 becomes larger, the frequency becomes higher and after a short-speed interval its frequency will be too high to be effective for producing torque. From the equation (22) it is seen that this phenomenon usually takes place at relatively very low speeds. A typical curve which was known to be unfavorable is shown in Fig. 18, which was taken by C. A. M. Weber.

Discussion

L. P. Ferris: The paper by Messrs. Weber and Lee shows that, from the point of view of the power company, or power user, there is some advantage in the elimination of the higher harmonics which, as you all know, have a detrimental effect upon neighboring communication circuits under some conditions of association.

Some years ago, in connection with the work of the Institute Standards Committee, an effort was made to determine what effect these harmonics might have from the standpoint of the power user. This did not arouse much interest, and I have no doubt that the problem is not of large magnitude from the power man's standpoint. But it is interesting to find that, under some conditions, these harmonics—even the slot harmonics—do produce effects which it is worth while, to get rid of from this standpoint. This gives an additional incentive to the designers for a high degree of purity of wave form.

This paper comes at a time when there is being instituted a joint research, looking toward the determination of ways and means whereby the effect of harmonics may be determined and an examination made as to the practicability of improving the wave shape of machines. The manufacturers and the power companies are joining with the telephone engineers in this work, and I hope that the methods of analysis given in this paper may lend themselves to this study, in the direction of making it easier to find ways and means of improving the wave shape I have no doubt that the designers are alive to this question.

22,000-kv-a. Transformers for Niagara Falls Development

BY F. F. BRAND
Member, A. I. E. E.
General Electric Co., Pittsfield, Mass.

During the past year the Niagara Falls Power Company has put into service the first unit of an extension which will contain the three largest water wheels and generators at present in the world, each generator having a maximum rating of 65,000 kv-a. at 12,000 volts and 25 cycles.

The limited width of the right-of-way from generating station, actually only the space over an existing intake canal, necessitated stepping up to 66,000 volts to transmit 195,000 kv-a. from the complete extension.

Transformers were therefore installed on an extension of the forebay wall. These are single-phase oil-immersed water-cooled 22,000-kv-a. connected delta on the 12,000-volt side and in Y on the 39,500-volt side to give 68,500 Y.

The principal interest in these transformers is, of course, in their large size and in the construction used to obtain high efficiency. The only single-phase transformers which exceed these in output are some 60-cycle 23,600-kv-a. units of the Duquesne Light & Power Co., Pittsburg, but since the Niagara transformers are 25 cycles, the parts, if used for a 60-cycle design, would give far greater output than any existing transformers.

In these transformers it was desired to obtain the maximum efficiency at ⅞ full load, at which point the transformers would probably be loaded most of the time, as it corresponded to maximum water-wheel efficiency.

With the maximum permissible magnetic density in present-day silicon steel, to keep the magnetizing current and core temperature within reasonable limits, the loss at 25 cycles in the steel is about 1 watt per pound, while the loss in the copper in the winding is from 8 to 16 watts per pound. To obtain highest efficiency at ⅞ load it was necessary to use fewer turns and a correspondingly greater core section than would ordinarily be used and this accounts for the type of core construction used, *i. e.*, with the winding on the central leg of the core.

To meet certain specified conditions of line regulation, it was necessary that the product of per cent magnetizing current and impedance should not exceed a certain constant value. This naturally resulted in a design being used giving the minimum effective area of the leakage flux path between high-tension and low-tension windings. The spacing between high-tension and low-tension windings is ordinarily that required for insulating these windings from one another, and this distance and a fractional part of the coils themselves constitute the effective width of the leakage flux path. The magneto mechanical forces between high-tension and low-tension windings in 25-cycle transformers are usually higher than for higher frequencies. A design having a very large core area and flux and minimum area of leakage flux path would therefore be subject to higher magneto mechanical forces, both at normal load and under short-circuit conditions.

This can readily be understood when we consider that the mechanical force varies as the square of the leakage flux density between high-tension and low-tension windings and that with concentric windings the area of the leakage flux path varies as the diameter, whereas the area of the core varies as the square of its diameter, and that under short circuit conditions with sustained voltage the whole of the flux ordinarily in the core must pass between the high-tension and low-tension windings. The magneto mechanical forces would, therefore, vary approximately as the square of the core diameter.

The type of construction used is that known as th

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-11, 1924.

concentric circular coil type, in which the low-voltage winding consists of a single circular helix, adjacent to the central core member, and the high-voltage winding a series of circular disc coils arranged in cylindrical assembly outside of and concentric with the low-voltage winding.

In this type of winding the principal magneto mechanical forces between high-tension and low-tension windings are radial. The circular coil is self-sustaining

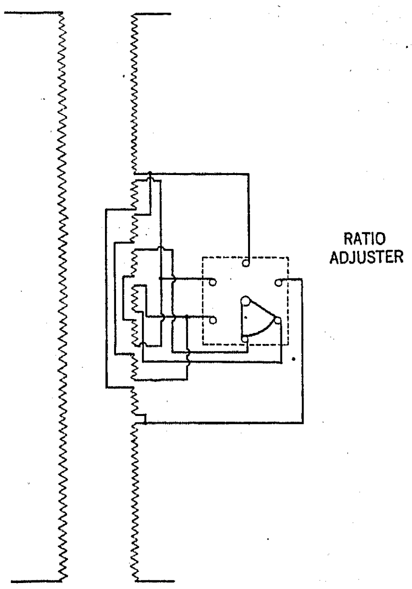


FIG. 1

against such forces, provided their value is not sufficient to exceed the elastic limit of the copper or insulation in tension or compression. If these values are not exceeded, it is only the unbalanced forces, due to the coils being assembled with the magnetic centers not co-incident, that must be considered seriously, as the coils are not self-sustaining against such axial forces.

The usual method is to provide bracing against the axial force, calculated by assuming certain definite but very conservative axial displacement of the coils to allow for variations in assembly and to add to this any displacement caused by taps which are taken off center.

To reduce the axial short-circuit forces as much as possible, the tap coils were so connected that an equal number of turns were tapped out each side of the center of the coil stack as shown in Fig. 1. Thus, the forces could only be those due to variations in assembly, which are quite small. The design calculations were based on an arbitrary displacement much greater than would be expected in practise.

Due to the self sustaining feature of circular coils against radial forces and the easy provision of bracing against axial forces, the circular-coil type of transformer can be readily designed to resist short circuit forces in all directions. In this it differs from types of transformers with non-circular coils, in which the coils tend to assume a circular shape during short circuits and which are difficult to brace against such coil distortion. In fact, this problem is so difficult that the forces, due to dissymmetry of the coils or variations in assembly, are usually neglected and no adequate bracing used to prevent distortion of this kind.

Each bank of three transformers is connected directly to one generator, so that for short circuits on the high-voltage side the generator reactance would be effective in limiting the short-circuit current to a lower value than that which would flow, if the transformer reactance alone were the limiting feature. In case of a short circuit on the low-voltage side, which is rather a remote possibility, power would be fed back from the rest of the system and the short-circuit current would be higher. To sustain the voltage under a short circuit would require a power flow of 660,000 kv-a. through one transformer bank having 10 per cent reactance.

The present system is not large enough to cause this power flow, as the voltage would fall and the short-circuit power flow be less. Conditions are, however, changing rapidly, power systems growing and it is not

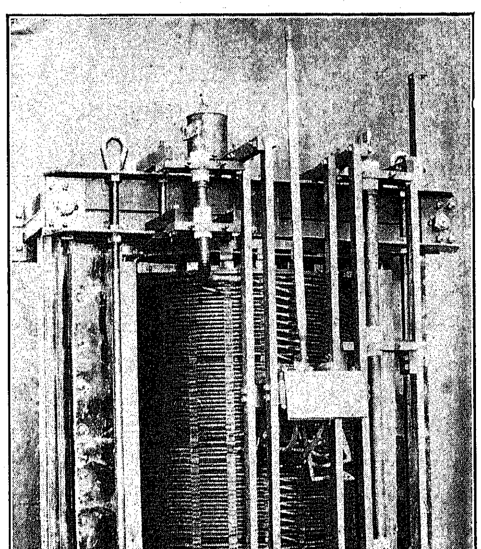


FIG. 2

good engineering to consider existing conditions as giving the limiting conditions to be met in the expected life of the transformer. No allowance, therefore, was made for any external reactance in the short-circuit calculation, the transformer being designed to withstand short circuit with fully sustained voltage.

In building transformers to stand short circuits, it is essential to consider the tremendous vibration of the

coils set up by the magneto mechanical forces, especially at 25 cycles. Tests made at Schenectady on the special short-circuit generator have shown the absolute necessity of designing the coil bracing, particularly the coil spacers, so that they will not fall out of place and thus out of alignment with each other.

A description of the spacing arrangements may therefore, be of interest. The essentials of coil bracing are:

1. To support every turn at such intervals that the short-circuit forces will not exceed the tension of compression limits on the conductor and insulation.

2. To support the turns in such a manner that a minimum amount of conductor is covered in the direction of heat flow, otherwise, the conductor covered by the spacers may heat up so much that the insulation may be damaged or the tensile strength decreased long before the conductors exposed to the cooling medium reach such limits.

3. To lock every spacer in place securely against movement by vibration.

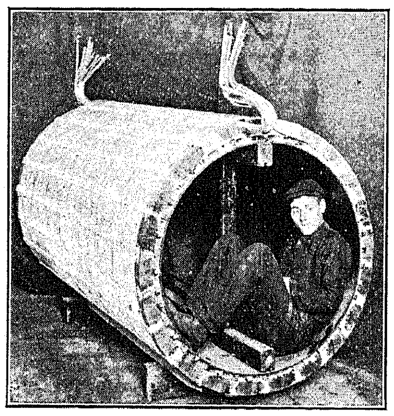


Fig. 3

4. To arrange the spacers so that they are easily inspected during and after the coils are assembled.

The coil spacers in these circular coil transformers and in others, built by the same manufacturer, are arranged radially across the face of the disc coils and the turns of the helical coils. This provides support for all conductors at approximately equal intervals.

Since the spacers are relatively narrow, two in. (5 cm.) wide as a maximum, the maximum distance of heat flow from that part of the conductor, covered by a spacer to the cooling oil, is one inch. This precludes a great difference of temperature between the part of the conductors covered by the spacers and that exposed to the oil.

The spacers between turns on the low voltage helical coil are radial. They are dovetailed onto spacers arranged axially inside the coil, which are mounted on the insulating cylinder between coil and core on which the coil is wound. The axial spacers provide a definite oil duct in a vertical direction inside the coil.

The radial spacers project out beyond the turns and are covered and held in alinment by channel pieces extending axially the complete length of the coil. The spacer extension, beyond the coil and the covering channels, provides an oil duct outside the coil.

The insulating cylinder between high voltage and low voltage is slipped over the projecting spacers and their channel pieces.

The high-voltage coil spacers are also arranged radially. Each one consists of a hair-pin type spacer placed around each coil with the open ends projecting outside the coil. It is formed of a number of parts all locked together, with the base serving to space the coil from the cylinder over which it is assembled.

The spacing of coils from cylinders is such as to provide the necessary extension of insulating diaphragms, placed between each coil to prevent creepage between adjacent coils.

The number of radial spacers in high and low-voltage coils is proportioned to the short-circuit stresses and the strength of the conductors and insulation.

The bracing against axial short-circuit forces of the low-voltage coil consists of a complete ring of bakelized insulating material of the same diameter as the coil, which abuts against the structural steel end clamps at each core yoke.

For the high-voltage coil stack similar rings are used, but in this case fibrous material is not solely depended upon. Engaging with the bakelized end rings at each end of the coil stack are square porcelain blocks, one for each line of spacers. These blocks have holes through them to contain a porcelain locking-pin, which enters the bakelized ring and into bakelized segments, placed between the porcelain blocks and the metal clamps. Adjustment of the clamps is provided by sliding one angle member on the other parts of the clamps, the pressure being maintained by adjustable studs on each clamp and tie rods between the top and bottom clamps. Adjustment in the core window is by means of a bakelized wedge.

The practise is followed in these transformers of bolting the core laminations together rigidly with re-enforcing plates on the outside of the built-up core leg and punching a large square notch at the end of each outer core leg. This notch holds an insulated square bolt which engages with the clamps and thus the core serves also as a tie rod to withstand a part of the short-circuit stresses.

INSULATION

In such moderate-voltage transformers the problem of insulating between winding or to ground is not at all difficult. The concentric-winding transformer, of course, lends itself to simplicity of insulation more than any other type. The insulation consists of cylinders of paper, treated with gums and rolled under pressure and at high temperature, forming a dense tough material which has high mechanical strength and a very high dielectric strength, both hot and cold.

These cylinders are arranged with suitable oil spaces to form the complete insulation from low voltage to core and from high to low voltage. The insulation of the high-voltage coil stack to the outer core legs consists of sheet insulation with oil spaces. The insulation from the ends of the coil stack has been described under the bracing for short circuit forces.

To reduce the potential gradient on the end high-voltage coil an electrostatic shield, connected to the line terminal of the end, is used between this coil and the core yoke. This shield and the end coil are taped to reduce the potential gradient on the oil surface and to reduce the danger from creepage to the core.

The insulation on the high-voltage conductors consists of a number of thin treated-paper tapes, having extremely high dielectric strength, and it is graded at the end coils so that the end coil will withstand line voltage between turns momentarily. This is for protection against transient voltages. The electrostatic shield connected to the line terminal modifies the distribution of transient voltages in the coil stack and reduces the concentration of voltage on the end coil to from one-quarter to one-third the value it might otherwise attain under conditions of impact of an almost vertically-fronted wave at the transformer terminals.

The electrostatic shields consist of metallic ribbons of high-resistance, wound on an insulating form of slightly less radial width than the adjacent coil. They are taped to give the same insulated radial width. The method of winding is non-inductive to the flow of the capacitance charging current. High-resistance material is used to avoid appreciable eddy-current loss in the shields, due to the high density of the leakage flux between windings.

Between each high-voltage coil is a pressboard diaphragm with an oil space between it and the coils, as protection against transient voltages. The oil spaces alone are much more than necessary to withstand the normal voltage existing between coils, due to the large number of coils and the few turns in each coil.

The pressboard diaphragm is used instead of merely increasing the oil space, due to the greater energy required to puncture such solid insulation or the time lag of the insulation against puncture by transient voltages.

The taping on electrostatic shield and line coils is additional protection against breakdown between them or the adjacent coils, due to high transient voltages, in addition to the heavier turn insulation on end coils.

The dielectric strength for one minute at normal frequency is equal to approximately line voltage between coils. Under transient stress it is much higher. Between end coils, due to the extra taping, it is much higher yet.

In the low-voltage helical winding the insulation on the conductors is principally to prevent eddy currents between the strands in parallel. The oil space between each turn serves, of course, as a cooling duct and for insulation between turns.

Cooling Arrangements

Special care was used in designing the core to arrange that all surfaces were exposed to the cooling oil. The central core leg is divided into two distinct flux paths with a large duct between them, in addition to which, there are a number of cooling ducts distributed throughout the laminations. The clamps are separated from the core by large ducts to allow the cooling oil access to all surfaces of the core yokes.

The oil for cooling the low-voltage coil is free to pass through these ducts between clamps and core yoke and to pass between the porcelain blocks used to support the high-voltage winding.

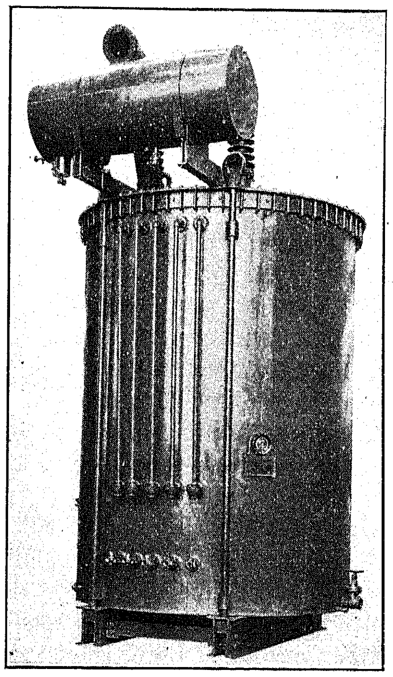


Fig. 4

The high-voltage winding is exposed directly to the oil in the tank. All coils have an oil duct on both sides and the inside edge. To provide uniform distribution between conductors in parallel, the position of these conductors is transposed. The arrangement of the tap coils in the high-voltage winding is such that this transposition is effective in them.

Due to the large number of conductors in parallel in the low-voltage winding and to the space taken up by the transpositions, it is not possible to obtain perfect equalization of current in all the strands in parallel. This fact is recognized and the average temperature rise of the low-voltage coil, as measured by resistance, is made lower than that of the high-voltage coil so that no strands will exceed the guaranteed temperature rise.

The high-voltage windings were designed to give

full kv-a. capacity continuously at any voltage from 36,000/62,300 Y to 39,500/68,500 Y, without exceeding the guaranteed temperature rise of 55 deg. cent. The rated high-voltage current was thus 611 amperes. The low-voltage winding was designed to carry a current corresponding to 611 amperes on the high voltage on any high voltage connection up to 37,600/65,120. Its rated current was therefore 1915 amperes. At 37,600/65,120 Y-volts the transformer kv-a. rating is therefore really 23,000 kv-a. The transformers are also designed to withstand 13,200 volts impressed on the 12,000-volt winding at full load.

Since the dielectric strength of fibrous insulation decreases with increasing temperature, and it is desired to obtain the maximum dielectric strength between end turns and coils, it is essential that these end coils do not attain a higher temperature than the remainder of the windings.

The cross section of the conductors in the more heavily insulated end coils of the high-voltage winding is much greater than that used in the remainder of the winding, to allow for the increased thermal drop through the extra insulation. Actually the end coils are cooler than the remaining coils.

The temperature rises of a large number of points in the coil, core and clamping structure were calculated and these calculations checked during test by insertion of thermo couples, wherever possible. These tests demonstrated that this type of design is very satisfactory for very large transformers from a thermal standpoint, as well as from a consideration of ability to resist short circuit stresses and to give great insulation strength.

It was mentioned in the early part of this paper that the design was required to give high efficiency at ⅞ full load. Actually, the efficiency at ¾ and ½ load was found to be slightly higher, as the loss per pound in the core was appreciably lower than the calculated value.

The efficiencies at various loads, based on losses measured by wattmeter at 75 deg. cent. are as follows:

Full load	99.041
7/8 full load	99.083
3/4 " "	99.11
1/2 " "	99.09
1/4 " "	98.67

The average magnetizing current at normal voltage is 2.59 per cent.

The high-voltage taps are connected to a ratio adjuster, operated from an operating handle through an oil tight stuffing box on the cover.

An interesting fact about the construction is the almost complete use of rolled steel. The tank is of cylindrical shape, of steel plate with welded seams with a flat bottom welded in place. It rests upon a structural steel truck with wheels, so that it can be rolled on a standard-gage track. The cast-steel wheels and the manhole covers are the only castings of ferrous metal. The tank cover is a pressed steel plate, with flanges for mounting of bushings, etc., extruded from it.

A cylindrical oil-conservator tank is supported from the tank cover by steel angle supports. The pressure-relief pipe is of welded steel construction and carries a relief diaphragm of glass at its upper end, designed to rupture if the pressure in the tank reaches 10 lb. per square inch.

The dimensions and weights of the transformers are as follows:

Height over Bushings	20 ft. 1 5/16 in.
Height over Oil Conservator	16 ft. 11 5/8 in.
Height over Cover	15 ft. 7/16 in.
Inside Diameter of Tank	120 in.
Floor Space over Fittings	12 ft. 10 1/4 in. by 11 ft.
Gallons of Oil	5675
Weight Complete, less Oil	117,800 lb.
Weight of Core and Coils, etc., to be Lifted from Tank	99,000 lb.
Weight of Oil	41,200 lb.
Complete Weight Oil filled	159,000 lb.

Notes on Mine Hoisting

BY F. L. STONE and F. R. GRANT
Associate, A. I. E. E. Non-Member
Both of General Electric Co., Schenectady, N. Y.

ELECTRICALLY driven mine hoists have at last come into their own. For many years manufacturers labored with prospective buyers with little results. Long and laborious calculations were made, ratings determined with great accuracy, detailed specifications drawn and presented to the customer. He usually purchased a steam hoist. The good judgment of such a decision could hardly be questioned. The steam hoists had given good and dependable service for many years. The electric hoist was a new and untried device. Power lines were not as reliable as they are today. The mine shaft is the neck of the bottle and once this becomes plugged from any cause whatsoever, the output stops and the usual trouble ensues. Operators felt they had trouble enough with most shafts and would not consider a propositon, which to them seemed to have great possibilities for trouble, the remedying of which was beyond their control.

Some small electric hoists, however, were installed inside on comparatively unimportant slopes and considering the type of equipment, gave a very good account of themselves.

These hoists were equipped with series motors usually of the railway type. The control was very crude but fairly rugged. The first electric mine hoist of which there is any record was installed at Aspen, Colorado in July, 1888 and in 1895 at the same mine a 100-kw. generator was put on a hoist and run as a motor, this equipment being (so far as can be learned) the largest of its kind in this country and perhaps in the world.

Up to the year of 1910, there are records of the installing of only a few equipments of larger than 200 horse power for mine hoists. Following the year of 1910, the sales of electric mine hoists have had a very steady and healthy growth.

Many interesting and instructive papers have been published on the calculation of mine-hoist cycles. The general method of attack is the same; *i. e.* moments of the up and down loads per turn plotted and net moments or torque derived from the algebraic sum of these moments. The friction (assumed) and the acceleration and retardation moments are added with their respective signs and the final summation moment curve changed into horse power by substitution in the well known formula: horse power $= \frac{2 \times \pi \times \text{r.p.s.}}{550} \times M$

It is not by any means the object of this short paper to enter into calculations of the cycle, but rather to show some results that have been obtained from actual installations which have been designed to meet such calculated cycles.

Many records are obtainable where the kilowatt hours per trip have checked accurately with the estimate figure given by the electric manufacturer at the time of sale. It is often very difficult to get data wherewith to check this figure as many hoists are run for other purposes than hoisting ore. In coal mines, rock is often hoisted and men and material are handled in the main shaft. So kilowatt hours taken by the hoist include much other work than actually hoisting coal.

The Lehigh Coal & Navigation Company have a water hoist which was installed some years ago. The bucket lifts 30,000 lb. of water per trip through about 865 ft. at a rope speed of 1100 ft. per min. The winder is driven by a 1200-horse power a-c. motor. An accurate record is kept of the number of buckets hoisted and nothing else is handled on this hoist.

Over a year's run, the kilowatt hours per trip on this hoist checked exactly with the figures given by the electric manufacturer, the actual figures on this hoist being 15.5 kw. hr. per trip or 1.2 kw-hr. per 1000 ton ft. This is an exceptionally good figure, better than the average. On short, fast cycles, the kw-hr. per 1000 ton ft. may run as high as 1.9.

Incidentally, it is impossible for the electric manufacturer to give a customer any positive guarantee as to the kilowatt hours per trip or per ton. This figure depends upon a number of conditions beyond the electric manufacturer's control.

The mechanical efficiency of the winding engine proper is somewhat of a guess. The rope and guide friction are variable quantities. In fast hoisting, windage must play a small part at least, and just as the sum of these quantities differs from its assumed value, so will the final result differ from the calculated.

During the last ten or fifteen years, a large number of very fast electric mine hoists have been installed, using the well-known Ward-Leonard control with or without flywheel equalization as the conditions indicate. Many of these hoists have cylindro-conical drums with very steep pitches up the cone. The duty cycles for these equipments have been calculated in the usual manner and have shown in many cases quite excessive acceleration and retardation peaks after the rotating parts had reached full speed, due to the acceleration of the up-load up the cone and the retardation of the down-load down the cone. These peaks many times produce a cycle with very jagged and irregular shape. It was, of course, fully realized that these peaks in actual practise must smooth themselves off because

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the system, due to the flexibility of the ropes, is far from rigid.

So far as the authors know, no reliable test had ever been taken on a Ward-Leonard control system using very rapid recording instruments, such as an oscillograph so that the actual shape of the duty cycle was more or less a guess, though it was felt that the area could not differ very greatly from the area of the calculated cycle. In view of the absence of such data, it was decided to run a complete test on a fast Ward-Leonard equipment and determine how close the calculated cycle and the actual cycles were checking.

The Old Ben Coal Corporation, of Chicago, have in their Southern-Illinois mines several Ward-Leonard flywheel equipments, some of which are operating on very fast cycles. It was decided that if this Company would permit inconvenience to the extent of making a test on one of their equipments, all the data necessary would be obtained. The matter was taken up with the Old Ben officials and their hearty cooperation was very gratifying, they extending every possible courtesy and assistance in the preparation for the tests. Their No. 18 mine was selected as being a very fair example of fast hoisting.

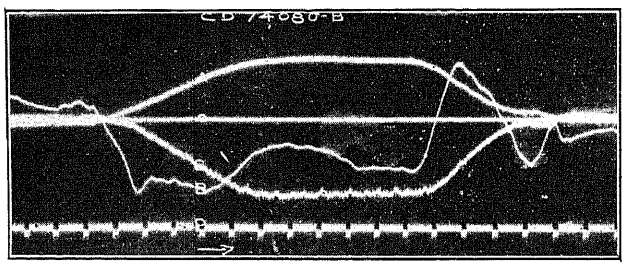

Fig. 1

The equipment at No. 18 Old Ben mine consists of the following:

1 d-c. hoist motor rated 950-horse power 40-deg. 65-rev. per min., 400-volts, direct connected to a cylindro-conical hoist drum having a small diameter of 7 ft. and a large diameter of 11 ft. with four turns up the cone.

1 Flywheel motor-generator set consisting of:

1 750-kw. 900-rev. per min., 400-volt generator

1 700 horse power 900 rev. per min. 2200-volt 3-phase 60-cycle induction motor.

1 19 kw. 900-rev. per min. 250-volt, compound-wound exciter and a steel plate flywheel, 92 in. in diameter, weighing 19,300 lb.

1 Control equipment consisting of:

1 Slip regulator with torque motor for controlling the speed and input to the flywheel set.

1 Drum controller and field rheostat for controlling the speed and direction of rotation of the hoist motor by means of varying the generator voltage.

This equipment was designed to operate under the following conditions:

Weight of coal per trip	10,000 lb.
Total distance hoisted	335 ft.
Weight of cage	15,000 lb.
Type of cage—self dumping	
Weight of mine car (open end type)	4,300 lb.
Rope diameter	1½ in.
Weight of rope per foot	3.55 lb.
Turns on small drum diameter	2.71
Turns up cone	4
Turns on large diameter	4.7
Time for acceleration	5 sec.
Time at full speed	5.5 sec.
Time for retardation	5 sec.
Rest period	4½ sec.
Trips per minute	3

From observation it was found that in adjusting the ropes on the drum, the active turns on the small diameter did not check exactly with the turns originally decided upon, the active turns on the small diameter being found to be 1.9. This, of course, materially affects the shape of the duty cycle, since the rotary acceleration and the acceleration due to climbing the cone would overlap each other slightly. Fig. 6 shows this clearly.

Using the conditions as they exist, a duty cycle was calculated in the usual manner and this cycle compared with the cycles obtained from the test. In addition to the conditions of winding the rope on the drum being slightly different from that originally calculated, the coal loads, as is always the case, differed somewhat from the calculated load in that no particular car during the actual test carried exactly 10,000 lb. of coal. The nearest approach to this, from a long series of tests, was a car containing 9,500 lb. and this value and the number of observed turns as above stated were used in making up the calculated duty cycle for checking the test results. For the purpose of simplicity, this single cycle will be discussed.

During the test, the following readings were taken:

Direct current between hoist motor and generator (oscillograph)

Voltage at motor terminals (oscillograph)

Motor speed in rev. per min. (oscillograph)

Kilowatt input to motor generator set (graphic wattmeter)

Speed of the motor-generator set (graphic volt-meter)

Time in seconds (oscillograph and timer on both graphic charts)

Weight of coal hoisted.

Fig. 1 shows the oscillograph curves as above described. The rev. per min.-curve C in Fig. 1, was obtained by a pilot generator with constant excitation belted to the hoist. The saw-tooth effect shown in this curve is undoubtedly due to the commutator bars passing under the brushes. The current curve B and

the voltage curve A were read directly on the oscillograph in the usual manner.

Fig. 2 shows the input to the motor-generator set during the cycle under discussion.

The speed curve of the motor-generator set for this particular cycle is not available, as the pilot generator belted to the motor generator set gave trouble at this particular moment. Other tests, however, showed that the slip of the motor generator set during the hoisting cycle is slightly less than that originally calculated.

From the above test readings by the application of the proper constants, it is not difficult to make up the actual horse power curve showing the horse power input to the hoist motor. Also as described early in the paper, a duty cycle was calculated, using the turns and weights as they existed. These two curves are plotted and shown in Fig. 3. The dotted line is the calculated duty cycle, and the heavy line the actual test results. more or less smoothed out in actual practise. This undoubtedly is because of the stretch in the rope. The most noticeable variations in the test cycle from the calculated cycle occur during acceleration and retardation. This may be due to quite a number of causes. The drums and rotating parts may be slightly heavier than assumed in the original calculation; and as will be seen from the speed curves, the rate of acceleration changes while the calculated duty cycle assumes a constant rate of acceleration of the rotating parts.

Fig. 4 shows the calculated torque as compared with the torque obtained from test. Here again, we find some slight differences, the most interesting among which is undoubtedly the fact that it takes a very appreciable time for the current to build up to the necessary accelerating value; whereas in the calculations, it is assumed that this current goes instantly

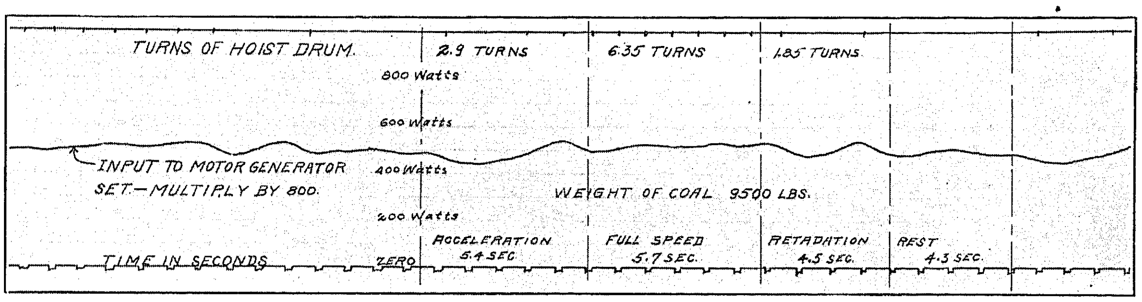

Fig. 2

The areas of the two curves check surprisingly well, the difference being less than 2 per cent.

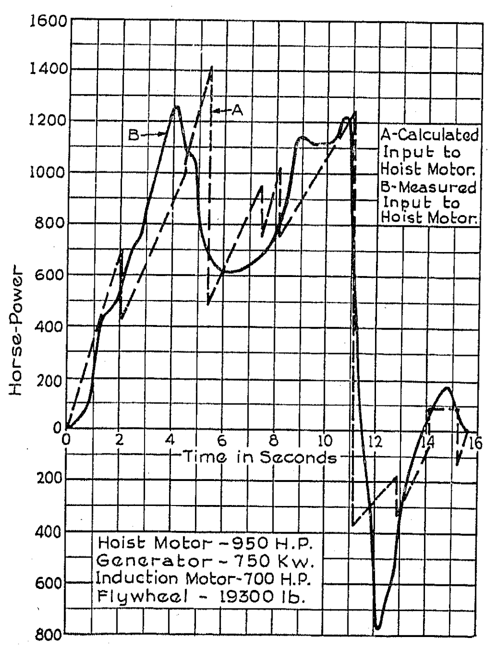

Fig. 3

It is interesting to note how the acceleration and retardation peaks, due to the effect of the cone, are to the value necessary to produce the required torque.

Fig. 5 shows speed curve of drum in rev. per min. against time and also total number of turns of drum

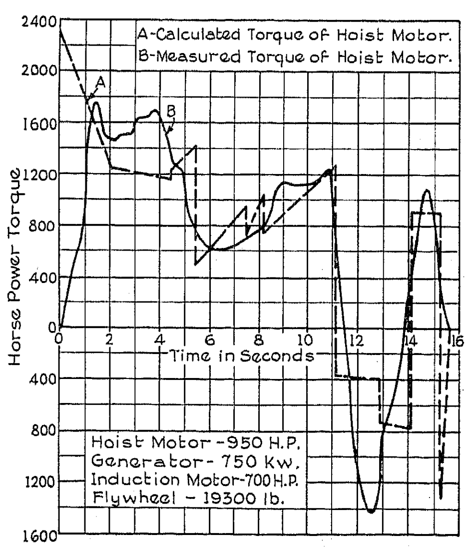

Fig. 4

against time. It is interesting to note how the rate of retardation changes, first being very high, then falling to zero and running at practically constant speed for about a second and finally retarding to rest at a high

rate. This retardation curve is not determined by any automatic devices, but is the result of the operator's experience in handling the equipment. Incidentally, curves were taken with different operators handling the hoist and in each case, the same general result is observable.

Somewhat the same general characteristics are observable during acceleration. The acceleration rate

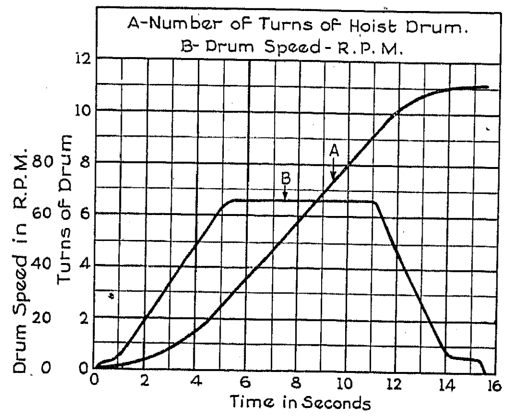

FIG. 5

in the beginning is low, then increases and remains at a practically constant rate for about four sec., then falls off somewhat gradually until constant speed is reached.

These curves are undoubtedly very interesting to a manufacturer, but what the customer is particularly interested in is the input from his line which represents to him actual expenditure. The kilowatt hours consumed in hoisting 9500 lb. through 335 ft., as indicated on the graphic wattmeter chart, amounted to 2.25

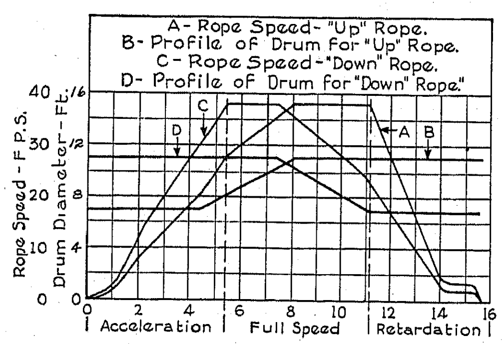

FIG. 6

kw-hr. per total trip, including the rest period or 1.41 kw-hr. per 1000-ton ft. The calculated input to the induction motor in kilowatts indicated 412 while the average reading from the graphic chart showed 402.

The calculated overall efficiency figured from actual foot pounds of energy consumed in the shaft to input to the flywheel set showed 52.4 per cent while the actual efficiency from the test showed 53.4 per cent.

Discussion

Carl Lee: The close agreement between the calculated cycle and the actual cycle of operation is very interesting and illustrates the high accuracy to which the design of electric hoists has been developed.

The smoothing out of the various curves has been noted on several hoists which have been in operation several yeras. The changes of the rope from the cone to the cylindrical part of the drum do not produce sharp changes in the load but rather gradual changes.

The efficiency of an electric hoist apparently remains very constant over a period of years as borne out by the attached figures which show the power consumption on two Ilgner Ward-Leonard hoists which have been in operation for over nine years.

This constant efficiency will be interesting to operators because with a steam hoist there is no doubt a falling off of the efficiency of the boiler plant, steam line, and steam engine after a few years use under average conditions.

W. I. Slichter: There are four important points brought out in these papers which seem to me warrant particular emphasis, as they indicate a general trend in the electrical engineering of the day.

The first point is the scientific calculation of the drum of the hoist by which the rate of acceleration is so controlled that the maximum demand on the hoist motor is limited to a reasonable peak and the load made nearly constant. This makes for economy in the size of the motor and the first cost.

The second is the use of the fly-wheel in the motor-generator set which still further reduces the maximum peak demanded of the transmission line, so that this peak is about ten per cent greater than the average. This makes for economy in the size of the transmission line and of the generator, again saving capital.

The third is the introduction of automatic control in the stations, making possible a notable reduction in the number of men required, thereby making for economy in the item of operating expense.

Finally, there is an effort to show consideration for the men, eliminating the danger of accident and enabling the men to live under more humane conditions, which illustrates the social and humane work of the engineer of the present day.

G. H. Finks: We have in Alabama quite a few electrified hoists, but no shaft hoists. The general data presented in the excellent paper of Mr. Stone and Mr. Grant does not apply in general to Alabama conditions, but the figures show that the calculated values for hoisting agree very closely with actual values obtained on tests. We have made some studies in the Alabama Power Company in connection with that, and find that that is true.

Most of the hoists in Alabama are slope hoists and they vary in length from a few hundred feet to some eight thousand. The grades vary from possibly 60 per cent down to very light grades. Also the grades found in the average mines vary.

In connection with the slope of the long-haul hoist, the trip is unbalanced in the pulling of cars. The return of the empty trip into the mine is usually done with a break. I would be interested in a discussion on the question of regeneration in such cases. We have found that such regeneration is possible and it returns appreciable percentages of power to the line.

In connection with the conduction of peaks on hoists, that becomes a matter usually to be determined between the consumer and the power company, if it happens to be purchased power, or in the case of a small power plant used to operate the hoist, a matter of not overloading a small plant; but particularly in the case of a power company, the demand applies, that is, a time limit, or in the case of almost all Alabama mines, a fifteen-minute demand period. You will have a certain number of

cycles during that period, and the momentary peaks do not usually influence the total demand.

F. L. Stone: I think I may be able to throw some light on the question of regeneration. It is entirely feasible to regenerate with an induction-motor hoist, provided you let it go a little above synchronism. Many slope hoists are operating under those conditions, but strange to say, many power companies put a ratchet on the meter.

G. H. Finks: In connection with the regeneration, you say the power company might put a ratchet on the meter. Well, that is true. A mine usually has additional load to carry which can be carried on a hoist and the meter will tend to come to a standstill as the mine load is picked up by the regenerated hoist. We have noted that a meter will not reverse until the total load is carried on the hoist. Most mines have a large amount of pumping, fans, and stuff of that kind, which has to be carried before energy can be put back into power lines, but the energy that is used for operating ventilating fans and pumps must be paid for and puchased from the power company.

ILGNER WARD-LEONARD COAL MINE HOISTS.
KW-HR. PER TON

	Mine No. 1		Mine No. 2		Mine No. 3	
Feet Hoist	395		412		444	
	Tons	Kw-hr. per Ton	Tons	Kw-hr. per Ton	Tons	Kw-hr. per Ton
1915	785,492	.63	401,550	.70		
1916	836,948	.64	550,225	.68		
1917	724,552	.64	582,110	.58		
1918	761,929	.64	674,547	.69		
1919	594,416	.67	592,677	.65		
1920	745,330	.64	781,595	.55	169,583	1.04
1921	818,400	.57	767,852	.63	585,286	.76
1922	647,348	.60	673,074	.64	532,418	.71
1923	710,667	.59	802,274	.67	772,340	.75

Automatic Substations for Industrial Plants

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THE automatic electric power station is a new development in the art. It was conceived by a central station engineer and developed by a railway engineer. It is natural, therefore, for automatic stations to be more widely used in these branches of the electric art than in industrial power applications. Nevertheless, the most promising field for automatic substations seems to be for the transformation and supply of electric power for all sorts of industrial purposes.

Electricity supply for factories, mills, mines shops and other industrial enterprises usually lends itself quite readily to automatic operation. The kind of automatic switching equipment and its application, however, depends to a large extent on the particular requirements of the industry served.

Generally, some form of power-transforming equipment is utilized for these enterprises. It may be rotating equipment such as motor generators, synchronous converters or synchronous condensers, or it may be static equipment such as transformers, static condensers or mercury tube converters. All of these have been equipped with automatic switching and control for central station and railway service and can readily be adapted to electrified industrials.

Primary power plants where water is the source are particularly well-suited for automatic control, but steam-operated stations have not yet been added to the rapidly growing list of automatic station installations.

Synchronous condensers form one of the simplest classes of rotating machines to which automatic station control equipment has been applied. They also form a class of power conservers which industrial organizations seem hesitant to adopt. It may be that heretofore the need to provide them with an operator has retarded their location at strategic points on the power network, or forced their location in existing attended stations which were remote from the preferred point for best performance. Whatever the reason may have been, the successful automatic operation of these units has very widely extended their field of application, particularly to electrified industries.

The first automatic synchronous condenser was a 3000-kv-a. unit installed by the Interstate Light & Heat Company at Hazel Green, Wisconsin. It was started in March, 1917, and is still in successful operation. It is located at a point on the power company's transmission lines where regulation had previously been quite poor. It more than paid for its cost in the first two years' operation. The latest automatic synchronous condenser is a 7500-kv-a. unit being installed by the New England Power Company at the Worcester, Mass. mills of the American Steel and Wire Company. In the meantime there have been a number of installations of automatic synchronous condensers but invariably by the power companies, although in most cases the condensers have been utilized for the correction of power-factor conditions caused by the industrial enterprises using the electricity.

The automatic control of a synchronous condenser is relatively simple. The starting impulse is given by any one of several familiar devices. A voltage relay, a power-factor relay, a time clock or simple tumbler switch may be used to start the set. The synchronous condenser is usually provided with an amortisseur winding and is brought up to speed as an induction motor. A compensator or Y-delta starting scheme may be used, either being readily adapted to automatic control. Usually during the period of acceleration, the synchro-

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nous condenser field remains disconnected from the source of exciting current and is bridged by a field-discharge resistor. This aids in giving the required starting and pull-in torque. After its field is excited, the unit has been connected to the line and full pressure applied to its windings, it begins to function as a power-factor corrector. A power-factor regulator or a voltage regulator which forms part of the automatic switching equipment adjusts the field current automatically to maintain either fixed power factor or fixed pressure within the limits of operation of the unit. Automatic synchronous condensers are provided with the usual automatic station protective devices. The bearings are equipped with bearing temperature relays which will cause the set to be shut down if the bearings tend to overheat and before damage results. The machine armature is provided with current-operated thermal relays having a time-temperature characteristic similar to that of the condenser. These automatically shut the machine down if the armature current exceeds a safe value for a given time. They also permit the unit to re-start automatically after a shut-down of a sufficient time duration to cool the windings. These thermal devices are of the integrating type and are self-adjusting for ambient temperatures. A grounding protective relay is also provided which causes the set to shut down in case of any leakage to ground in excess of 50 amperes. This cares for any insulation failures

Fig. 1—United Railways of St. Louis, Franklin Ave. Substation, St. Louis, Mo. for Synchronous Converter 600-Volts, 2-Kw. 25-Cycle 6-Phase and Portion of 6 Stub End and Multiple D-C. Reclosing Feeders 600-Volts 2000-Amperes for Reclosing from Either Side

and prevents undue damage. Single-phase starting, harmful single-phase running, reverse phase, loss of field, too low line pressure, short time a-c. overload, etc., are also cared for by the usual automatic station relays.

Synchronous motor generators form the next general class of rotating electrical machines which have been successfully equipped with automatic switching. Units of 150 kw. in capacity are now operating in many sections of the coal mining district and are typical of the smaller sizes. A 2000-kw. set is in operation on the 666-volt d-c. electrification on the main line of the New York Central Railroad and is typical of the larger sizes applied to very severe operating conditions. These units may be started on load demand or at a predetermined time or by a tumbler switch or its equivalent. The automatic switching equipment proceeds to function immediately the starting impulse is given. First, it checks the line pressure to be sure it is ample to start and run the set. Next, it checks the supply to be sure all the phases are intact and are of the

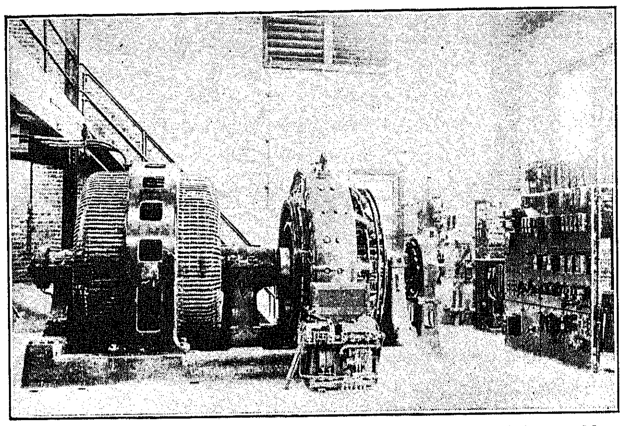

Fig. 2—Automatic Railway Substation, New York Central Railroad Co. 110th St. General View of Station Showing 2000-Kw. Motor-Generator, High Speed Circuit Breaker and Automatic Control Panels

correct sequence. Then, it checks the bearings, machine windings, and ambient temperatures, to be sure these will permit the machines to run if and when started. Next, it connects the motor to the source of power, either through a compensator or to taps on the transformer, or through the transformers direct if Y-delta starting is used. Then the set starts to rotate and increases in speed. At synchronous speed, the automatic switching equipment again takes hold and connects the field circuit to the source of the exciting current. When field current has been established, the set is transferred from the starting combination to the running combination. During the accelerating period the motor field remains disconnected from the source of exciting current and is bridged by a resistor in the same manner as is the synchronous condenser during its starting cycle. This permits individual adjustment of each motor so as to obtain a complete start under practically all conditions which can be found in service.

In the larger sets, a low value of field is applied while on the starting tap, and the final value after the machine is transferred to the running connections. In the smaller sets, the final value is applied in one step while on the starting tap. The smaller sets having 250-volt or 275-volt generators are usually excited directly from the generators. Those rated for higher pressures or arranged with voltage regulators are usually fed from an exciter either separately operated or direct connected.

The synchronous motor has the usual automatic

station protection to prevent runaway, operation without field, harmful single-phase operation, leakage to ground, short-time and long-time continued overload, bearing temperature, etc. Some of these devices, such as the bearing temperature relays, are hand reset where continued operation of the machines under any circumstances might lead to their destruction. The number of such devices, however, is a minimum so as to insure continuity of service under all conditions, excepting where the machine might fail and thus interrupt such continuity for a long time. Most of the other protective devices, such as the long-time delay overload relays, phase-checking relays, etc., are automatically reset and permit the equipment to resume normal operation as soon as the unusual conditions have been corrected.

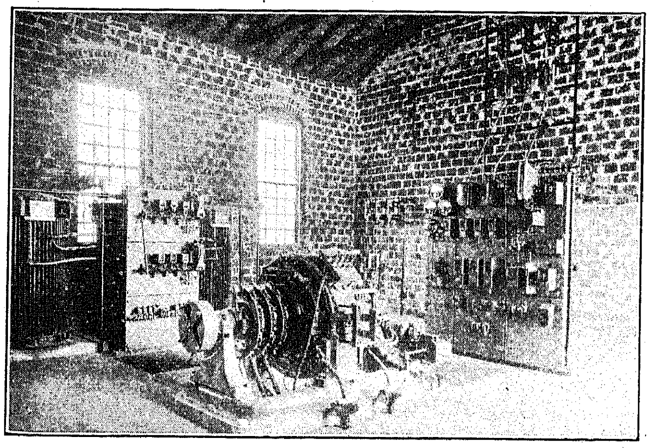

Fig. 3—Clearfield Bituminous Coal Corp., Grass Flat, Pa., Pleasant Hill Mine. Interior View of Station Showing Automatic Switching Equipment and Synchronous Converter 275-Volts 200-Kw. 6-Phase 60-Cycles and Combined Stub End and Multiple D-C. Reclosing Feeder 275-Volts 1090-Amperes

The d-c. generator is provided with protection and adjustment, depending upon the service requirements to which it will be subjected. The simpler equipments, which may be shunt or compound wound, are usually operated with a fixed shunt-field setting. This setting is chosen to give the desired terminal pressure, and in the case of multiple-unit sets, must be carefully adjusted to give the correct loading of the machines. Multiple-unit stations also require careful compounding of the units to permit the successful multiple operation of "hot" and "cold" machines.

The d-c. generators of larger sized sets are frequently provided with automatic load regulation. One type is provided with a shunt-field bucking scheme. This has been successfully applied to shunt-wound generators for multiple-pressure booster service. The shunt-field current is normally furnished by the generator itself. The current is then passed through the armature of a tiny motor-generator continuously rotating. This set, by a suitable raising or lowering of its field current, bucks or boosts the shunt-field current of the generator the correct amount and gives the desired regulation. In one example, using this method of control, load swings of from 300 per cent normal to 200 per cent reversal are successfully handled.

Another modification is a compound-wound d-c. generator with the series field connected to oppose the shunt field. This type of generator is usually operated with the series field shunted by a circuit breaker which allows only a fraction of normal current to flow through the field when it is closed. Under heavy overload conditions, the circuit breaker is automatically opened and the series field carries the full line current. Under this condition it tends to oppose the shunt field. Now by suitably proportioning the shunt and series fields, the d-c. generator can be made practically a constant-current machine and service may be maintained at a reduced pressure during emergency load conditions. As soon as the overload condition ceases, the circuit breaker automatically recloses and shunts the series field. A number of such equipments is operating on the Edison three-wire networks of many of the large electricity supply corporations, and their use is being very rapidly extended. Their judicious application to electrified industrial establishments would insure continuity of service under the most severe conditions short of complete and sustained interruption of primary electricity power supply.

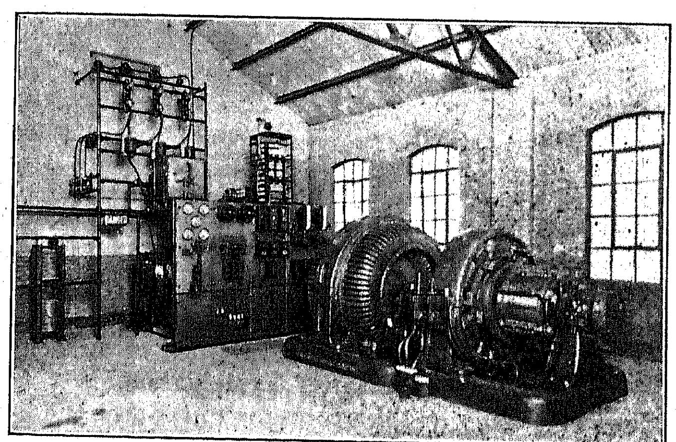

Fig. 4—300-Kw. 250/275-Volt D-C. 6000-Volt 3-Phase 60-Cycle Synchronous Motor-Generator with Automatic Control Equipment, Clearfield Bituminous Coal Corporation, Rossiter, Pa.

Synchronous converters form without exception the largest single class of rotating electrical machines which have been successfully provided with automatic switching equipment. In fact, they were the first electrical machines to be so equipped, notwithstanding the fact that with a single exception they are considered the most difficult electrical machines to operate. This is principally because of the almost universal application of synchronous converters for interurban railway service where automatic stations were first widely adopted.

The automatic switching and control of synchronous converters assumes the a-c. supply to be available

before any starting operations are performed. Consequently, all automatically-controlled synchronous converters are a-c. starting. Also, with one or two minor exceptions, the starting is accomplished with the aid of an amortisseur winding on the field.

The starting indication for an automatic synchronous converter may be any one of the usual devices, such as a pressure relay, a time switch, a tumbler switch, or their equivalent. Immediately the starting impulse has been completed, the synchronous converter is connected to the starting taps on the power transformer. One-third taps or one-half taps, depending on the design of the synchronous converter and transformers, will usually furnish sufficient torque to start the converter armature and pull it into step. Then the converter is automatically transferred from the starting taps to the running taps.

During the starting operation, the shunt field is usually opened. After the machine has reached synchronous speed, its field is either flashed with the correct

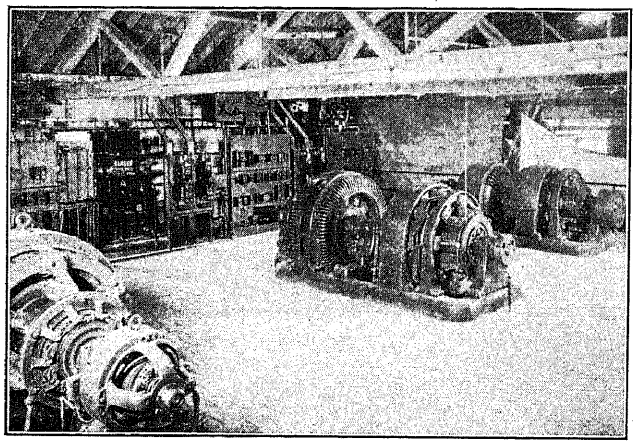

Fig. 5—Automatic Station Control Equipment, Mobile Electric Company, Mobile, Alabama. Synchronous Motor Generator Set and Control Equipment

polarity before it is made self-exciting, or its field is reversed until correct polarity is established.

The next step in the process is to connect the synchronous converter to the d-c. network or feeder, and here the inherent characteristics of the converter require a different series of operations than do motor-generator sets. The d-c. terminal pressure of a synchronous converter is regulated largely by the a-c. pressure-supply. Only a relatively narrow range of d-c. terminal-pressure adjustment is possible by field adjustment. This then requires some arrangement for safely connecting the synchronous converter to the d-c. network, even though that network may be operating at a pressure quite different from the terminal pressure of the synchronous converter. This condition is most usually encountered in all sorts of electric haulage installations, since here the wide swings in power demand are very large compared with the feeding capacity of the a-c. supply. For electric railways and commercial electric power service generally, and sometimes for industrial electric power supply, synchronous converters are equipped with load-limiting resistors. These are connected between the d-c. terminals of the synchronous converter and the d-c. network or feeder after the synchronous converter has been placed in complete running condition in so far as the a-c. side is concerned. These resistors are then usually shunted out in two or three

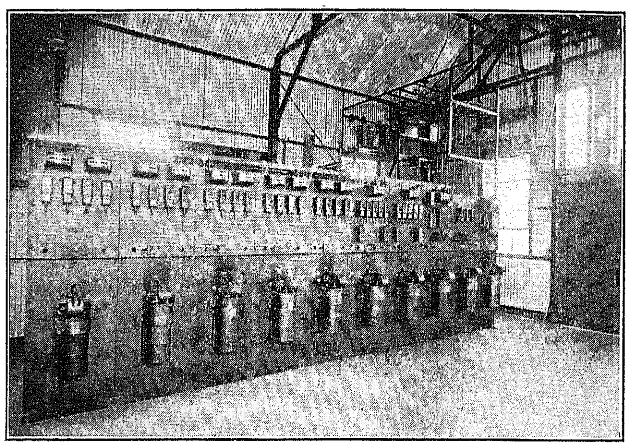

Fig. 6—Automatic Switching Equipment, Long Island Lighting Co., Port Washington, L. I. For Five 2-Phase and Two Single-Phase, 2200-Volt, 60-Cycle Reclosing Feeders and Automatic Transfer Apparatus for Two Incoming Lines

steps thus gradually loading the synchronous converter from the d-c. network. The shunting contactors or circuit breakers used for this service are usually provided with current relays having interlocks so that the synchronous converter cannot be loaded beyond its

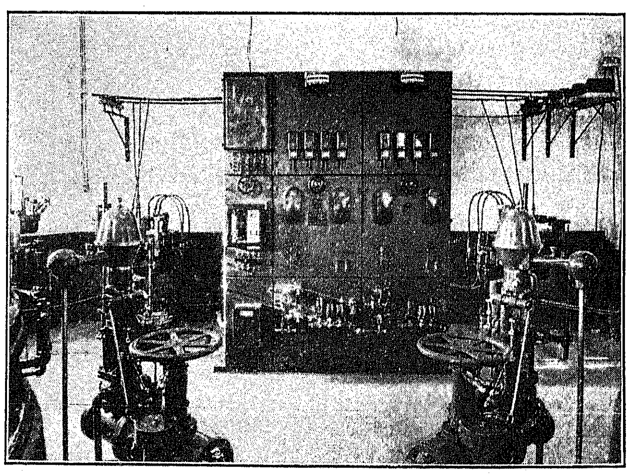

Fig. 7—Automatic Generating Station Control Board and Generator. Fairbury Mill and Elevator Company. Two 40 150-Kv-a. 180 Rev. Per Min. 2300-Volt Generators

thermal capacity. Other devices such as thermostats, located near the load-limiting resistors, permit the synchronous converter to supply current to the d-c. system within safe heating limits until the load is reduced or the resistors and converter tend to dangerously overheat. Under these conditions, the converter

is automatically shut down until it and the resistors cool. After the equipment has cooled sufficiently, a fresh start is automatically made. In some cases the load-limiting resistors are shunted out in only one step where economy in design and manufacture permit *Y*-delta starting. With this combination and with suitable transformer design, the *Y*-delta arrangement may be used to take the place of one step of the load-limiting resistors.

The load-limiting combination available for, and used with, synchronous converters permits their wide application to all sorts of electrified industries as well as to electric railway and electric power supply systems. It permits the use of the economical synchronous converter in many places where motor-generator sets were formerly considered supreme. For certain few applications, they were excelled only by specially designed motor-generators with automatic load-regulating features.

Synchronous converters for automatic operation are provided with the usual automatic station protective features such as bearing and resistor temperature relays, short and long-time continued a-c. overload relays, single-phase starting preventive and harmful continued single-phase running preventive relays, low a-c. pressure starting or operating preventive relays, overspeed switches in combination with a shunt trip circuit breaker, leakage to ground relays, etc.

Other electric power-transforming machines, such as balancers, mercury tube converters, and static transformers have been made completely automatic and are now in successful operation. Their design usually takes pattern after the best manual operation scheme, except that the time for the switching is generally reduced from a matter of minutes to a matter of seconds.

The automatic hydroelectric plant is one of the outstanding applications of automatic switching equipment to industrial enterprises. Consider, for example, the many streams of moderate size but relatively uniform flow which are found in the upper Middle West. They have been used for many generations to drive grist mills of quite good size, as well as performing many other industrial functions of varied natures. The hydro equipment which has heretofore been used is fast becoming antiquated and the output of the mills is suffering in consequence. The recent installation of automatic hydroelectric equipments in a number of these mills has wrought an important change in this section of the country. The same streams now permit not only a greater mill output in combination with the electrification of the mills, but also provide an inexpensive power supply to adjacent towns and villages. More important even is the fact that the electrified mill can now be connected to the large electric power supply system covering the country, and can either take power from this system during times of low water or can feed power into the network during times of high water.

Automatic hydroelectric stations almost invariably use synchronous generators since they can be provided with their own exciters. They almost always have a waterwheel equipped with either an oil pressure governor, a servo-motor, or motor-operated gates. The synchronous generator is usually provided with an amortisseur winding to assist in pulling the machine into step. The starting impulse is given by any one of the familiar relays or by a push button, or may be given by making alive the station feeder from an a-c. network. Immediately after the starting impulse is completed, the waterwheel gates are opened and the waterwheel started. Then the gates are shut until only enough water passes through to bring the machine up to about 95 per cent normal speed. At this point the armature is connected to the a-c. network, either directly or through transformers and immediately thereafter the field is excited. This brings the generator into phase with the network and places load on the machine.

Automatic hydroelectric stations are usually provided with automatic voltage regulators and these, in combination with the hydraulic regulating equipment, adjust and maintain the desired load on the equipment.

Automatic hydroelectric stations are provided with only such protective features as are needed for the quite simple machinery involved. Excess speed, excess electric pressure, insufficient oil pressure, bearing and machine heating, short and long-time, continued overloads, together with loss of field, each has its protective devices, which make the automatic hydroelectric station economical and safe in sizes up to 8,000 or 10,000 kw.

Feeders for a-c. and d-c. networks and supply are now not only automatic tripping on overload, but also automatic reclosing. Many forms are available, each being particularly designed for its individual application.

Discussion

C. S. Butcher: One of the earliest installations of automatic equipment in industrial plants was in the motor car industry in Detroit, where two 1,500 kw. motor generators are used to augment the power supplied by a steam plant. The steam plant is located at one end of the manufacturing aisle, which is some 600 ft. or so long, and a very heavy bus runs through a tunnel from the power house to the load, which is distributed along the bus quite uniformly. It was found, in making a study of the power plant, that the steam costs were very high, however, in the wintertime the exhaust steam is used for heating, and in working out the economics of the problem it was found that if substations were located along the bus, the steam plant, during the summer months could be shut down and considerable saving realized by purchasing power.

The first machine was installed in the summer of 1920 and consists of a shunt generator provided with a constant-voltage regulator. This very materially improved the voltage on the bus at that point, since it was more nearly in the center of distribution. As the plant grew, the next year a similar substation was added, and in that case we ran into our first difficulty of parallel operation. The machines, while being built for shunt

operation, were, as I say, equipped with constant-voltage regulators and it was quite difficult for a time to get the machines to share the load. This was done by rather an ingenious device attached to each regulator. By the use of this device, the machines were given somewhat the characteristic of a straight shunt machine not provided with voltage regulation. The installation has been entirely successful and has since been followed by a great number of installations in the industrial field generally.

Mr. Lichtenberg starts out with the power plant, quite naturally, in the development of our natural resources. However, I don't like to think that we are going to limit automatic operation to the smaller sites; in fact, we haven't done so. We now have about ready to go into operation an 8000-kv-a. hydroelectric station, completely automatically operated, equipped with two 4000-kv-a. generators. That is not large in a sense, but in proportion to the capacity of that system, that really represents quite a large operation.

Previous to that, on the same system we installed a generating station equipped with two 2000-kv-a. water-wheel generators. On that particular property 4000-kv-a. would represent about 25 per cent of the total system capacity. However, inasmuch as all of that was not connected to the load, there were times when the automatic station would represent perhaps 50 per cent or better. of the total connected capacity.

The machines are of the older type and are not equipped with damper windings, and it was necessary to find some means for synchronizing the automatic machines with those already connected with the load. Most of you are familiar with the automatic synchronizers which were first used with the old engine-driven units. With the machines of larger capacity, no one liked to depend on the automatic synchronizer. However, with that as a foundation, we started out to make an automatic synchronizer. With that, we combined an automatic speed-adjusting device, which consists of nothing more than two little synchronous motors, one connected to the line and the other connected to the incoming machine; the two motors driving together a differential which operates to regulate the speed of the incoming unit by the operation of the speed adjustor on the governor mechanism. That, in combination with a modification of the old synchronizer, worked out beautifully, and that station has now been in operation for six or eight months and has been entirely successful.

In connection with hydroelectric plants we are accustomed to thinking of large developments. Large developments often involve the condemnation of considerable portions of land in order to create the necessary storage capacity and to get the necessary head. That is done for the reason that by concentrating our power at any one point, operating cost is kept down to a minimum. If we eliminate the operating cost; that is, the labor of attendants, and develop that same head at a number of smaller sites, the same can be all automatically operated and connected to a common distribution system and operated with considerably greater economy.

Passing from the automatic hydroelectric station, we of course come to the automatic operation of a-c. feeder circuits. That equipment also is practically standardized and practically ordered by style number. The periodic reclosing equipment is perhaps the most common that is used. There are other special applications, but I will not mention those. The duty cycle on the breaker for periodic reclosing is more severe than the standard duty cycle; even the new one that has been adopted. For example, the power companies ordinarily apply this type of equipment to radial-type feeders and require that the first reclosing following an outage shall be on the order of thirty seconds. In case the feeder does not remain closed, it is closed again at the end of perhaps a period of a minute; the third reclosing taking place at the end of a period of approximately two minutes. At the end of that time, if the circuit is not closed definitely, the feeder is locked out of service. If, following any of these reclosures previous to the last one, service remains normal for a predetermined length of time, the equipment is automatically reset to zero. Therefore, the equipment is restored to complete service.

Coming down to the application of automatic substations for supplying direct-current load, we have the railway, light and power, and industrial. They perhaps pretty well cover the industrial; but just taking the railway, which has perhaps seen the greatest development in automatic switching equipment. That is where we started with the synchronous converter, which is probably the most difficult to control. I think most of us are quite familiar with some of the earlier developments which were applied particularly to interurban railways. A great many people at that time said, "Well, that's all right for small interurban roads where continuity of service is not required, but you will never see it on the larger systems." Well, it hasn't worked out that way.

At first we were forced to show a considerable saving in order to justify the installation of automatic switching, but it has been definitely proven that the operation of automatic substations is so much more satisfactory than the operation of manual stations, that automatic switching equipment is being installed now where continuity of service is of prime importance. The application of automatic substations to heavy networks, whether they are railway or light and power, is made not so much on what can be saved in operators' wages and the like, but what can be saved in distribution losses.

I have in mind our first application to Edison's load which was made in the City of St. Paul in 1920. There, a large theatre and office building was built on the very edge of the Edison three-wire, 250-volt network. To run cables to this point would have meant a large investment in ducts and in copper. It was absolutely essential, however, that they have good voltage regulation at this building, so they compared the cost of cable and ducts with the cost of an automatic substation. The economics were decidedly in favor of the automatic substation, but they were not quite sure that it would give them just the service they wanted. However, it was tried out. The inspection has been weekly, with a man visiting the station approximately twenty minutes each day, and all of the little things which might have caused a shut-down have been caught on inspection. So that station has never caused an interruption of service to that building. Several times the network has been lost and the station has automatically isolated itself from the system and supplied the isolated load.

F. M. Nash: In our concern we had occasion once to consider an automatic outdoor generating station, and I would like to ask if there are any here who know of, or can give any account of, the installation of automatic outdoor generating stations. In this particular instance we had to consider the replacing of three 1000-kw. machines with one automatic generator, outdoor type. The change was not carried out, partly because of the cost and partly because of the uncertainty of such an undertaking.

C. A. Butcher: So far no automatic switching equipment; that is, complete automatic switching equipment, has been applied to an outdoor generator, and to my knowledge there has been built only one generator for that type of service. I am pretty sure that the Allis-Chalmers Company built that.

That problem came up a number of years ago and a study was made to determine whether or not it was feasible. It was determined quite definitely that it was feasible and also practical, but uneconomical, for the reason the protection which must be afforded the generator and the generator housing would have to be of special design. The fact that there are very few applications for this class of machine, thus necessitating placing all the development costs on very few, made it an uneconomical proposition on the very few that we considered.

Effect of Certain Impurities in Storage Battery Electrolytes[1]

BY G. W. VINAL, and F. W. ALTRUP

Member, A. I. E. E.

Both of the Bureau of Standards

Review of the Subject.—*A new method for measuring the rate of sulphation of storage battery plates was recently devised at the Bureau of Standards by Vinal and Ritchie (Technologic Paper 225). This consisted of periodic weighings of plates suspended in electrolyte. The present paper is an extension of this work. Detrimental impurities when present in the solution may (1) corrode the plate, (2) accelerate the formation of lead sulphate, or (3) be deposited in the pores of the plate. In any case, the weight of the plate changes and this affords the most sensitive and exact means which we have for estimating the extent and nature of the reaction. A physical meaning can be given to the rather vague term "local action." It was found that electrolyte, containing only one part of platinum in ten million parts of the solution, increased the local action at the negative plates by 50 per cent. Copper, like platinum, deposits on the negative plates, but produces less effect. Iron is of unusual interest because it greatly accelerates the formation of lead sulphate at the negative plate. The reaction of the positive plates is slower, which mitigates its detrimental effects to some extent. Manganese is particularly destructive to the positive plates. The results of our experiments indicate that the reactions of manganese compounds in the battery are somewhat different from the previously accepted theories. Manganese in the form of manganese dioxide is deposited on the positive plates covering the active material, closing the pores and causing a part of the charging current to be wasted. The work is being extended to include the effect of other impurities.*

CONTENTS

INTRODUCTION

THE importance of obtaining exact information about the effect of impurities in storage battery electrolytes arises from the detrimental effects which many of them produce on the operating characteristics and life of the storage battery, and such information is necessary as a basis for the preparation of specifications for sulphuric acid to be used in the batteries. Engineers have recognized for a long time the necessity for maintaining a high standard of purity in the electrolyte, but within recent years millions of small batteries have passed into the hands of non-technical users who must depend upon the manufacturer and his subsidiaries for satisfactory service. The information hitherto available on this subject has been fragmentary and in some cases contradictory.

The most recent systematic research on this subject is that of Helen C. Gillette.[2] Her experiments were carried out by the method used by several previous experimenters, *i. e.*, by poisoning the cells after their electrical characteristics had been determined. This method, while valuable from the standpoint of operation does not permit us to determine quantitatively the effects produced on the positive and negative plates or to determine definitely the chemical reactions which take place.

A new method for measuring the rate of sulphation of storage battery plates was recently devised at the Bureau of Standards. A description of this method with experimental results for measurements on positive and negative plates suspended in pure sulphuric acid solutions has been published.[3] The same method and apparatus have been employed in the present investigation but with some modifications necessitated by the conditions of our experiments. In this paper the effects of small amounts of iron, maganese, platinum and copper are described.

II. PRINCIPLE AND METHOD

The discharge of either the positive or negative plates in sulphuric acid solutions results primarily in the formation of lead sulphate. A certain amount of lead sulphate is formed as the result of local action when the plates are immersed in even the purest acid solutions obtainable. Detrimental inpurities may (1) corrode the plate, (2) accelerate the formation of lead sulphate, or (3) be deposited in the pores of the plate. In any case the weight of the plate changes and this change affords the most sensitive and exact means which we have for estimating the extent of the reaction. In order to obtain comparable results it is necessary that the temperature be maintained at a constant value. This was usually accomplished by immersing the glass jars containing the electrolytes in a large water bath thermostatically controlled at 25 deg. cent. (77 deg. fahr.).

1. Prepared under the Auspices, and published with the approval of the Bureau of Standards.

2. TRANSACTIONS American Electrochemical Society 41, p. 217, 1922.

Presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., April 7-11, 1924.

3. Vinal and Ritchie, Bureau of Standards, Tech. Paper 225. *Chem. & Met.*, v. 27, p. 1116; *Electrical World*, v. 80, No. 26, p. 1383.

This temperature was maintained constant to within about 0.01 deg. cent. Two positive plates or two negative plates, suspended on glass hooks were placed in each jar. As a preliminary step the plates were given several cycles of charge and discharge following which they were fully charged and then submerged in the electrolytes to be tested. Each jar contained a solution of chemically pure sulphuric acid having a specific gravity of 1.250. The electrolytes were saturated with lead sulphate because the previous work showed this to be necessary.

A sensitive balance mounted on a marble slab above the thermostat bath was used for weighing the plates while they were immersed. Any plate could be brought

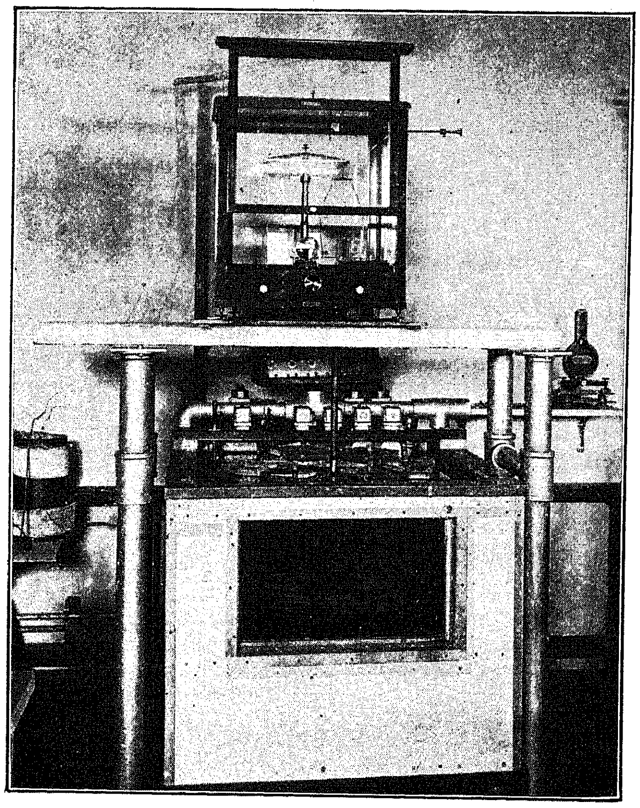

FIG. 1—APPARATUS FOR TESTING BATTERIES

directly under the arm of the balance, as the jars containing them were carried on a revolving frame. The arrangement of the apparatus is shown in Fig. 1.

A carefully measured quantity of the impurity was added to the electrolyte in each case before the plates were immersed. Simultaneously with the tests to determine the effect of the impurities, measurements were made of the rate of sulphation of the plates in pure solutions. The results of the experiments are shown by means of curves relating the total change in weight of the plates to the time in hours. Weighings of the plates were taken immediately after placing them in the solutions and they were weighed at frequent intervals during the first day because the rate of the reaction was generally the greatest at this time. As the reaction slowed down weighings were made at less frequent intervals.

A distinction must be made between the rate and the total extent of the reactions produced by the impurities. In the case of impurities such as platinum and copper, the rate of the reaction is of interest because the reaction, if allowed to proceed indefinitely, will be terminated only by the complete exhaustion of the plates. On the other hand with an impurity such as iron the total extent of the reactions is limited by the amount of the impurity added and any spontaneous oxidation or reduction which may occur. In such a case, therefore, the total reaction is of more interest than the rate of the reaction since it permits us to verify the chemical equations for the reactions that occur.

The concentrations of the impurities are expressed as percentages by weight.

III. EFFECT OF COPPER

Copper was added to the solutions of sulphuric acid saturated with lead sulphate in the concentrations shown in Table I. The solutions initially had the characteristic blue color of copper sulphate. When the two negative plates were immersed in the solutions the copper began to deposit upon the plates and the solutions gradually lost color. An analysis of the solutions after the experiment was completed showed only a small trace of copper. This shows that practically all the copper had been deposited upon the plates. Since the amount of solution in which they were immersed greatly exceeded the amount of solution that would ordinarily be present in a battery, the amount of copper deposited on each plate was in excess of that which would be found in ordinary practise. The quantity of solution was approximately ten times the amount that would be contained for the same number of plates, in a starting and lighting battery. To obtain equivalent conditions, therefore, in the case of those impurities which deposit upon the plates, the concentrations should be multiplied by a factor of 10.

TABLE I

LOCAL ACTION PRODUCED BY COPPER AT NEGATIVE PLATE

Concentrations of copper are shown in the box headings. Results are calculated as ampere-hours of equivalent discharge for one plate.

Time in Hours	Copper Concentration. 0.4	0.08	Per Cent Pure Acid
10	0.8	0.1	0.0
20	2.1	0.2	0.1
30	2.9	0.3	0.2
40	3.6	0.5	0.3
50	4.1	0.6	0.4
75	5.2	0.9	0.5
100	6.0	1.2	0.7
150	7.4	1.7	1.0
200	8.7	2.2	1.4
300	10.9	3.4	2.0
400	13.1	4.5	2.6

The deposition of the copper upon the negative plates could be easily seen. An illustration of one of these

plates, together with a similar plate taken from a pure acid solution, is shown in Fig. 2.

The local action which occurred at the plate as the result of the deposition of copper was accompanied by the evolution of hydrogen. The changes in weight of the plate during the process are shown in the curves of Fig. 3, and from these has been computed the equivalent of the local action in ampere-hours as the average value per plate. The results of the calculation are

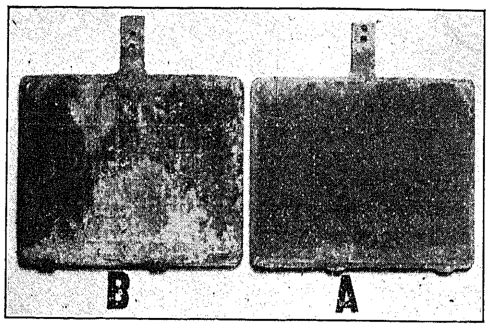

Fig. 2—Negative Plate (B) Contaminated with Copper —Plate (A) was Taken from a Pure Solution

given in Table I. The ultimate capacity of the plate has been calculated from the amount of active material contained in the plate allowing for 60 per cent sulphation. Since the plate contained 148 grams of the active negative material the ultimate capacity is calculated to be 22.9 ampere-hours.

Table I shows that during the first part of the experiment the extent of the local action produced by the

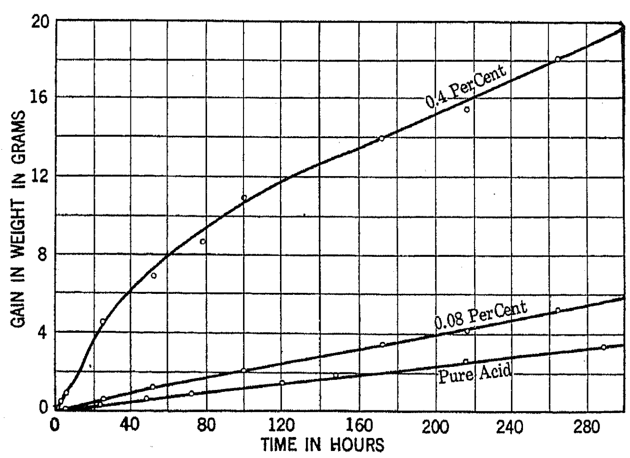

Fig. 3—Effect of Copper on Negative Plates

copper is not as great as might be expected. This is probably due to the rather high overvoltage for hydrogen on the surface of copper. The solution which contained 0.08 per cent of copper produced approximately double the amount of local action that was observed in the case of the plates immersed in the pure solution. The total number of ampere-hours of equivalent discharge at the end of 400 hours, however, was only one-fifth of the ultimate capacity of the plates. The solution containing 0.40 per cent copper produced an equivalent discharge of 13 ampere-hours per plate in a period of 400 hours. This is about half of the ultimate capacity of the plates.

IV. Effect of Platinum

Platinum has always been considered one of the most deleterious impurities. The results of this investigation amply justify this conclusion. Platinum, however, is not as common an impurity at the present time as it has been in the past, because comparatively little sulphuric acid is now concentrated in platinum vessels during the process of manufacture.

Platinum was added to the sulphuric acid solutions, as in the case of copper, in the percentages shown in Table II. The experimental results are given in Fig. 4.

The platinum evidently began to deposit upon the negative plates very quickly as they began to gas almost

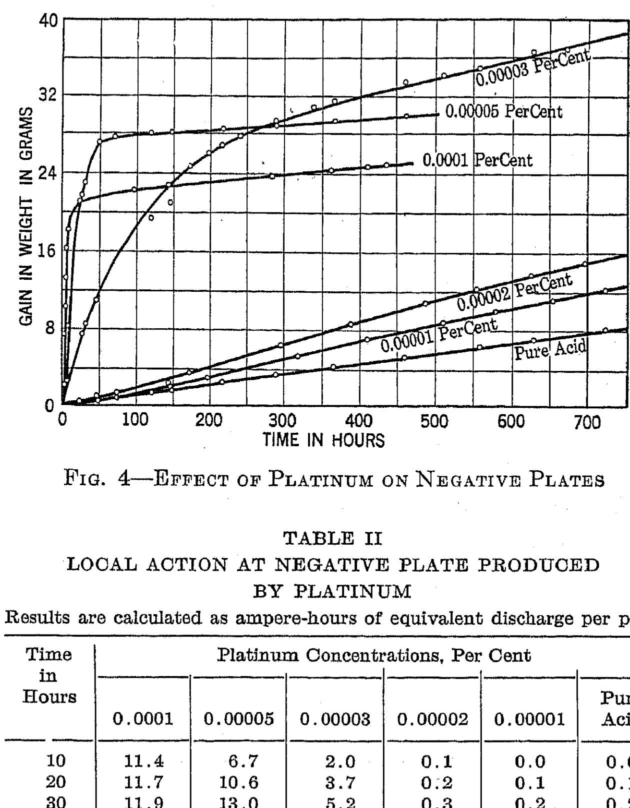

Fig. 4—Effect of Platinum on Negative Plates

TABLE II
LOCAL ACTION AT NEGATIVE PLATE PRODUCED BY PLATINUM

Results are calculated as ampere-hours of equivalent discharge per plate

Time in Hours	Platinum Concentrations, Per Cent					
	0.0001	0.00005	0.00003	0.00002	0.00001	Pure Acid
10	11.4	6.7	2.0	0.1	0.0	0.0
20	11.7	10.6	3.7	0.2	0.1	0.1
30	11.9	13.0	5.2	0.3	0.2	0.2
40	12.0	14.5	6.4	0.4	0.3	0.3
50	12.0	15.3	7.4	0.5	0.4	0.4
75	12.2	15.6	9.3	0.8	0.6	0.5
100	12.4	15.7	10.8	1.1	0.8	0.7
150	12.8	15.9	13.1	1.7	1.2	1.0
200	13.1	16.1	14.6	2.3	1.7	1.4
300	13.5	16.3	16.7	3.6	2.7	2.0
400	13.9	16.4	18.1	5.0	3.8	2.6
500		16.5	19.1	6.2	4.7	3.2
600			19.8	7.3	5.7	3.7
700			20.5	8.3	6.6	4.3
800			21.0	9.3	7.4	4.8
900			21.4	10.2	8.2	5.4
1000			21.8			5.9

immediately. The plates in solutions containing the greatest amount of platinum gassed violently and increased in weight very rapidly. The gassing was so

violent that the surface of the plates was apparently blasted off and most of the platinum thereby eliminated. With the highest concentration the plates became about 60 per cent discharged within eight hours, but after this the rate of their discharge was comparatively slow. The reaction was not quite so violent in the solutions containing the next lower percentage of platinum but it is significant that the reaction proceeded farther. At the end of 48 hours the plates were 72 per cent discharged. The solution containing 0.00003 per cent of platinum produced a reaction which was much slower but produced 95 per cent of the ultimate discharge in 1000 hours. The lower concentrations produced smaller effects but the gain in weight was appreciably greater than with pure acid. One part in ten million, or 0.00001 per cent platinum produced an increase of 50 per cent in the local action over that in pure acid solutions, which shows that the effect of extremely small amounts of platinum may be detected by this method. Chemically, it is possible to determine platinum in solutions to the extent of about one part in two million, but it is estimated that by this method the effect of as little as one part in fifty million may be determined. The local action calculated as ampere-hours of equivalent discharge for the solutions containing platinum is contained in Table II.

The curves of Fig. 4 make possible an estimate of the local action in terms of the equivalent current that would be discharged by the plate normally for the same rate of sulphation. The equivalent currents are proportional to the slopes of the lines. The curve for plates in pure acid shows the average equivalent current of the local action to be 0.0059 amperes. With this as a basis the equivalent currents during the first part of the experiment for the other curves have been calculated to be as follows:

Platinum Concentration	Current Equivalent of Local Action
Per Cent	Amperes
0.00001	0.0093
0.00002	0.0113
0.00003	0.107
0.00005	0.345
0.00010	1.71

By this calculation a physical meaning is given to the rather vague term "local action."

The plates which gassed most vigorously and from which the original lead surface was blasted off were black for the most part but showed portions covered by the white lead sulphate. The black coloration was probably due to the lamp black which was mixed with the paste when the plates were made. An illustration of one of these plates as compared with one immersed in a pure sulphuric acid solution is shown in Fig. 5.

V. Effect of Iron

Iron may exist in the sulphuric acid solutions in two states of oxidation. Iron in the ferric condition is reduced to the ferrous condition at the negative plate and then in turn oxidized to the ferric condition at the positive plate, and also to some extent by the air.

(a) *Effect on positive plates.* When iron in the ferrous condition is added to the solution, it is oxidized by the active material of the positive plates to ferric sulphate accompanied by the formation of lead sulphate and water. Assuming the following equation for the reaction

$$Pb\,O_2 + 2\,Fe\,S\,O_4 + 2\,H_2\,S\,O_4 \rightarrow Pb\,S\,O_4 + Fe_2\,(S\,O_4)_3 + 2\,H_2\,O \qquad (1)$$

The lead sulphate which is formed permits an accurate calculation to be made of the extent of the reaction from the gain in weight of the plates. The gain in weight of the positive plates must, however, be calculated as $Pb\,S\,O_4$: $(Pb\,S\,O_4 - Pb\,O_2)$, because the plate gains the sulphate radical $S\,O_4$ as the result of the reaction but loses simultaneously two oxygen atoms for each

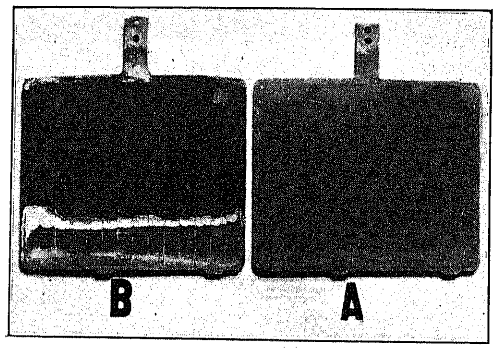

Fig. 5—Negative Plate (B) Contaminated with Platinum —Plate (A) was Taken from a Pure Solution

molecule of lead sulphate which is formed. The reaction expressed by equation (1) proceeds to completion if sufficient time is allowed. That is to say, all of the ferrous sulphate is oxidized to the ferric condition and beyond this point the rate of formation of lead sulphate is essentially the same as for plates in pure acid solutions. The results of these experiments are given in Fig. 6, which shows that the curves representing data obtained from the iron solutions become parallel to those for the pure acid solutions toward the end of the experiment. We may, therefore, calculate the amount of lead sulphate which should be formed and compare it with the amount determined by the weighings. Such a comparison is made in Table III.

The agreement is as good as can be expected. The curves shown in Fig. 6 show the average gain in weight per plate and the figures given in Table III represent the sum total of the lead sulphate formed on both plates in each jar.

Since the reaction expressed in equation (1) came to

TABLE III
COMPARISON OF CALCULATED AND OBSERVED VALUES FOR POSITIVE PLATES IN SOLUTIONS CONTAINING IRON

Amount of Iron Added		Equivalent Ferrous Sulphate	Calculated Equivalent Lead Sulphate	Observed Amount of Lead Sulphate
Per Cent	Grams	Grams	Grams	Grams
0.4	22.5	61.2	61.2	60.2
0.08	4.5	12.2	12.2	11.8
0.012	0.675	2.7	2.7	3.4

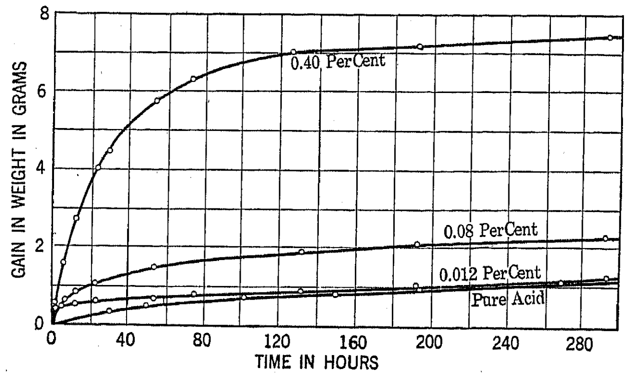

Fig. 6—Effect of Iron on Positive Plates

a definite termination this afforded an excellent opportunity to determine what the effect of introducing negative plates into the solution would be. This case represents the condition of a battery containing both positive and negative plates. One charged negative plate was immersed in each solution at the conclusion of 360 hours. These plates were not in electrical contact with the positive plates. The reduction of the iron to the ferrous condition began immediately and the

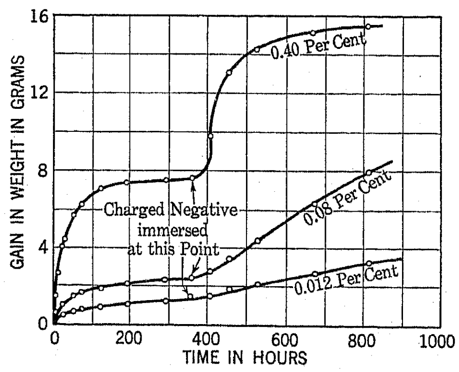

Fig. 7—Gain in Weight of Positive Plates in Solutions Containing Iron

product in turn was reoxidized by the positive plates accompanied by a further discharge. Curves showing the observations which were made are given in Fig. 7, the experiment being continued until 820 hours had elapsed.

(b) *Effect on negative plates.* The action of iron on the negative plates is much more pronounced than on the positives and the local action produced is in excess of the amount which would be calculated from the reduction of the ferric sulphate. The effect is probably the result of two simultaneous reactions that may be represented by the following equations:

$$Pb + Fe_2\,(S\,O_4)_3 \rightarrow Pb\,S\,O_4 + 2\,Fe\,S\,O_4 \qquad \textbf{(2a)}$$

$$Pb + H_2\,S\,O_4 \rightarrow Pb\,S\,O_4 + H_2 \qquad \textbf{(2b)}$$

The amounts of iron added to the solutions were 4.5 g. and 0.675 g. These are equivalent to 16.1 g. and 2.4 g. of ferric sulphate. On the basis of equation (2a) these amounts will account for 13.4 and 2.0 grams of lead sulphate respectively.

The curves, Fig. 8, show the gain in weight of the plates in terms of the sulphate $S\,O_4$ taken from the electrolyte. This gain in weight calculated to lead sulphate $P\,b\,S\,O_4$ is greatly in excess of what may be accounted for by the reduction of the iron salt from the ferric to the ferrous condition. The curves show also that the gain in weight of the plates in the solutions to which iron had been added is in excess of the sulphation of similar plates in the pure acid. This indicates clearly that the presence of iron accelerates the action

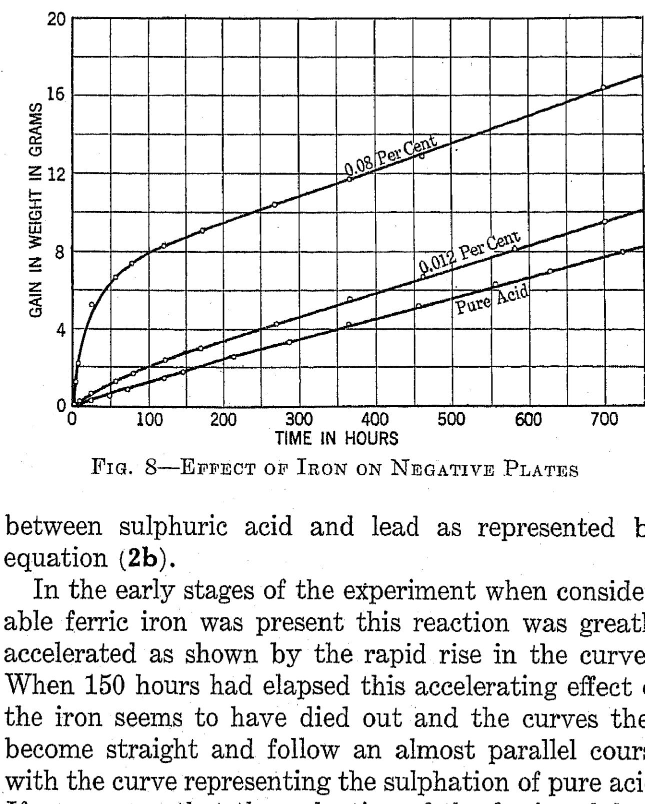

Fig. 8—Effect of Iron on Negative Plates

between sulphuric acid and lead as represented by equation (**2b**).

In the early stages of the experiment when considerable ferric iron was present this reaction was greatly accelerated as shown by the rapid rise in the curves. When 150 hours had elapsed this accelerating effect of the iron seems to have died out and the curves then become straight and follow an almost parallel course with the curve representing the sulphation of pure acid. If we assume that the reduction of the ferric sulphate according to equation (**2a**) has become complete at this point, we may calculate an average value for the amount by which the reaction between lead and sulphuric acid has been accelerated. In the solution containing the greater concentration of iron the increase over the normal sulphation in pure acid is about 7 times and in the other solution about 3 times.

After 150 hours the curves shown in Fig. 8 are all approximately straight, but they diverge slightly which shows that sulphate was being formed at a slightly

greater rate on the plates in the solutions to which iron was added than in the pure acid. This effect is probably to be accounted for by the well known slow spontaneous reoxidation of the ferrous sulphate by the air and its subsequent reduction by the negative plates during the long time that the experiment lasted.

Table IV shows the gain in weight of the plates in the solutions to which iron had been added in comparison with the gain in weight of similar plates in pure acid solutions. There were two plates in each jar and the total gain of each pair of plates is shown in the table. Fig. 8 shows the average values.

TABLE IV
EFFECT OF IRON ON NEGATIVE PLATES
The weights are the total gain of the two plates in each case, expressed in grams

Hours	Iron Concentration, Per Cent 0.08	0.012	In Pure Acid
10	4.36	0.60	0.20
20	8.80	1.10	0.48
30	10.80	1.58	0.76
40	12.00	2.00	1.00
50	12.94	2.40	1.24
75	14.26	3.26	1.94
100	15.80	4.04	2.50
150	17.60	5.48	3.64
200	19.00	6.86	4.72
300	21.70	9.58	7.12
400	24.30	12.00	9.30
500	27.00	14.24	11.44
600	29.64	16.62	13.46
700	32.40	18.80	15.40
800	35.00	20.84	17.26

In addition to the above experiments a third concentration of the iron, 0.004 per cent, was used. The results were about the same as for 0.012 per cent and as the curves would lie so near together only one of them has been plotted in the figure. The reason that the smaller amount of iron produced the same amount of sulphation as the larger amount is probably to be attributed to the spontaneous reoxidation of the ferrous sulphate.

The results of this experiment show that the presence of iron is more detrimental to negative plates than to positives because of the acceleration of the reaction between lead and sulphuric acid. Further, the ferrous sulphate which is formed suffers a slight reoxidation by air, in addition to that which would be caused by the positive plates present in a battery.

VI. Effect of Manganese

(a) *Effect on negative plates.* The experimental results obtained when negative plates were immersed in solutions containing manganese are shown in Fig. 9. The lowest curve represents the sulphation of the plates in pure acid solutions. The three curves above this are for similar solutions to which were added respectively 0.04 per cent, 0.08 per cent, and 0.40 per cent of manganese as potassium permanganate.

It is at once apparent that the effects produced are not proportional to the amounts of manganese added. The reason for this is a reaction between the sulphuric acid and the potassium permanganate which is independent of the reaction at the plates. The reaction between the permanganate and the 1.250 sp. gr. acid may be expressed by the equation

$$4\, K Mn O_4 + 6\, H_2 S O_4 \longrightarrow 2\, K_2 S O_4 + 4\, Mn S O_4 + 6\, H_2 O + 5\, O_2 \qquad (3)$$

This is a slow reaction that may be demonstrated by a simple laboratory test, several hours being required to collect enough of the oxygen to make a satisfactory test. During the experiments the gas (oxygen) given off appeared in small amounts over the entire surface of the liquid. It was not localized at the plates.

The reactions which take place at the plates in contradiction to the above reaction, which occurs whether the plates are present or not, results in decolorizing the permanganate and in the formation of lead sulphate and manganese dioxide. The reactions are not fully understood, but the following equation, which

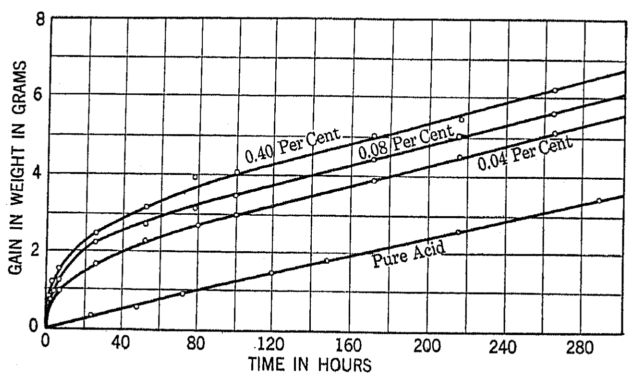

Fig. 9—Effect of Manganese on Negative Plates

is in accordance with the observed facts is believed to represent the reaction:

$$2\, K Mn O_4 + 4\, H_2 S O_4 + 3\, Pb \longrightarrow K_2 S O_4 + 2\, Mn O_2 + 3\, Pb S O_4 + 4\, H_2 O \qquad (4)$$

There are probably several intermediate reactions which take place before the end products are reached. A sludge fell to the bottom of the jar which was tested and found to be hydrated manganese dioxide. Possibly some of the manganese was converted into manganous sulphate. No gassing was visible at the plates. The equation indicates that three molecules of lead sulphate result from the reduction of 2 molecules of permanganate. The data given by the curves in Fig. 9 do not account for the amount of lead sulphate that would be expected. This is because of the spontaneous reaction expressed by equation (3) above. The permanganate in our experiments was added 18 hours before the plates were put in since we did not anticipate this reaction. The curves show a rapid sulphation of the plates for about 50 hours. Then the solution became decolorized and the curves became parallel to the curve representing the sulphation of the plates in

pure acid. The curves continued parallel during the remaining 250 hours of the experiment. The parallelism of these curves indicates that the reaction has become complete and that the manganese has been reduced to manganese dioxide and precipitated.

(b) *Positives in solutions containing manganese.* The manganese was added to the solutions as manganous sulphate, $M\,n\,S\,O_4$. The solutions were initially colorless but began to show a purple coloration almost

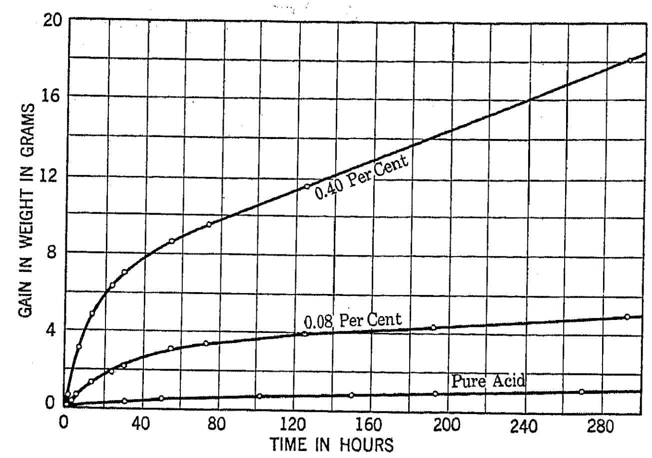

FIG. 10—EFFECT OF MANGANESE ON POSITIVE PLATES

immediately after the positive plates were immersed. This indicates the formation of permanganic acid, $H\,Mn\,O_4$, probably according to the equation

$$5\,Pb\,O_2 + 2\,Mn\,S\,O_4 + 3\,H_2\,S\,O_4 \rightarrow 2\,H\,Mn\,O_4 + 5\,Pb\,S\,O_4 + 2\,H_2\,O \quad (5)$$

The solutions containing 0.4 per cent of manganese became so dark within one hour after the plates were immersed that the plates were no longer visible.

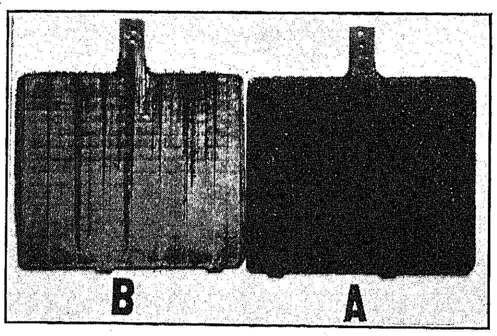

FIG. 11—POSITIVE PLATES SHOWING DEPOSIT OF MANGANESE DIOXIDE

A rapid gain in weight of the plates was observed as shown in Fig. 10. Comparing the results for both positives and negatives obtained in solutions with 0.08 per cent of manganese it was found that the gain of the positives was much larger as a result of the oxidation of the manganous sulphate than the gain of the negatives resulting from the reduction of permanganate in the same length of time. After the rapid gain during the first 50 hours, the plates continued to gain in weight at a rate somewhat more rapid than for similar plates in the pure acid solution. The rate of the gain, however, steadily decreased indicating that the reactions were becoming complete.

A deposit was formed on the plates, the sides of the glass jar, and to some extent on the surface of the liquid. The deposit on the plates after drying was a dull sooty black which filled the pores of the plates and the excess formed a rough spongy layer over most of the plate surface also. Two plates are shown in Fig. 11. Chemical analysis showed that this deposit was manganese dioxide, $Mn\,O_2$. It seems probable that the gain in weight of the plates during the first period of perhaps 50 hours is largely the lead sulphate formed as a result of oxidizing the manganous sulphate, but an important part of the total gain in weight is due to manganese

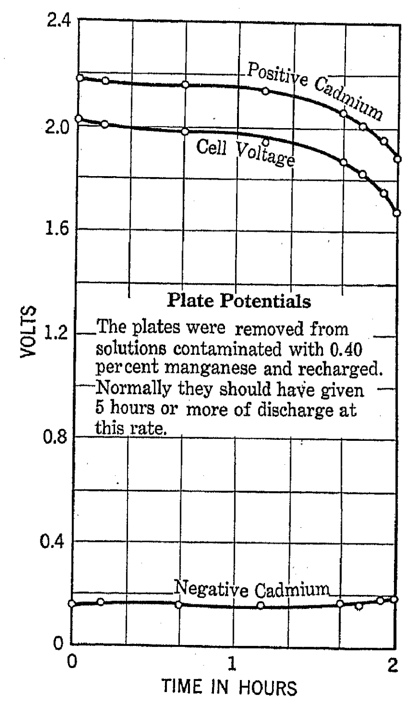

FIG. 12

dioxide deposited in the pores and on the surface of the plate. Our experiments do not permit us to estimate the relative amounts of each.

Since the tendency of the negative plates is to throw the manganese out as a sludge, it might appear that the effects of manganese would gradually disappear. This, however, is not the case as the experiments of Helen C. Gillette[4] have shown. The reason for the destructive effects of manganese are to be found in the reactions at the positive plates.

The manganese dioxide deposited upon the positive plates fills the pores of the active material and covers the plate with a non-conducting hard film in a highly oxidized state. The authors took two of the plates, a positive and a negative upon which weighings had been made

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and immersed them in pure acid solution of specific gravity 1.250 and passed a charging current through them beginning with a rate somewhat below the normal finishing rate of charge. The positive began to gas immediately. Raising the charging rate to the normal value increased the gassing and the electrolyte, initially clear, began to show a purple coloration indicating the re-formation of permanganic acid. The reaction at the positive during charge might be expressed by the equation

$$2\,Mn\,O_2 + 3\,O + H_2\,O \rightarrow 2\,H\,Mn\,O_4 \qquad (8)$$

This reaction at the positive occurred more rapidly than the reduction at the negative as shown by the increasing coloration of the electrolyte due to permanganic acid. With the re-formation of permanganic acid by the charging process the battery is again put in condition for a repetition of the reactions represented by the equation (5). It should be noted that the equation for charging makes no provision for the oxidation of the lead sulphate at the positive plate. Plates that are heavily coated with $Mn\,O_2$ can receive but little charge. The pores are stopped and part of the energy is expended in the reaction on the manganese dioxide and part in the liberation of excess oxygen which normally would oxidize the lead sulphate.

After the plates had been charged, a discharge of them was made using a cadmium electrode to measure the plate potentials. These are shown in Fig. 12. These plates were discharged at the normal 5-hour rate. The positive plates, however, gave out at the conclusion of two hours. It is quite evident from an examination of the plates that the pores were stopped and that the plates could not be fully charged.

VII. Conclusion

In the case of four impurities, copper, platinum, iron, and manganese, it has been possible to determine quantitatively the effects produced by various concentrations of the impurities and to indicate to a fair degree of certainty the nature of the chemical reactions produced. In interpreting these data, particularly with reference to the drafting of specifications for sulphuric acid, two factors must be borne in mind. One of these relates to the extent of the deleterious effects produced by the impurities and the other to the amounts of the impurities which are present in the acid as produced under normal manufacturing conditions.

In the case of platinum it was shown that the presence of one part in ten million increases the local action 50 per cent over that which is found in the case of pure acid. This amount is below that which can be determined by ordinary chemical methods. The obvious conclusion, therefore, is that no platinum whatever should be allowed by the specifications. This condition can be met by the acid manufacturers at the present time.

Copper by itself produced less effect than was anticipated. It is not desirable, however, that the amount of copper to be permitted by the specifications should be raised to as large a figure as the results of these experiments might indicate to be permissible; because (1) such an amount would be unnecessarily large and in excess of the amount found in acid of good quality, (2) because copper in combination with certain other impurities is believed to be very detrimental.

Iron presents unusual interest because of its accelerating effects on the reactions at the negative plate. These experiments show that the presence of iron affects the negative plate more severely than the positive. Iron will, therefore, tend to exist in the ferrous condition in the battery and its activity is limited by the rather slow rate at which it is oxidized to the ferric condition by the positive plate and by air. Since the negative plates ordinarily exceed the positive plates in capacity the effect of iron may be minimized. Iron is, therefore, in the same category with copper in that the maximum amount to be permitted by the specifications should be determined by manufacturing conditions rather than by the effects which it produces in the battery.

Manganese deposits upon the positive plates and produces serious effects. It covers the active material of the plate, closes the pores and causes a large amount of charging current to be wasted as gas. Although the manganese is deposited on the positive plate it does not remain there, since during the charging process part of it is thrown back into the solution as permanganic acid. The results show that manganese in the electrolyte must be kept at the lowest possible figure.

This work is being extended to include the effect of other impurities. We are indebted to Drs. Blum and Lundell of the Bureau of Standards for valuable suggestions about the chemical reactions involved in this work.

Discussion

H. M. Gassman: I would like to ask a question as to whether any experiments, including the effect of uranium salts in electrolyte, have been made?

G. W. Vinal: We have not as yet made any experiments with uranium salts. We have made experiments with some of the unusual impurities, such as tungsten. We hope to include quite a wide variety of these impurities later.

H. M. Gassman: The reason I asked that question was because I understand that is the active component of some of these revivifying solutions of electrolyte.

Mr. Vinal: I have seen some of them that are supposed to contain uranium, but I don't know that uranium really does much.

These advertised solutions may really be classified under three heads, so far as our experience goes. The first is ordinary sulphuric acid, to which is added some coloring matter. The second class contains substances which are supposed to retard sulphation. Some people still have the idea that sulphation is an unmitigated evil. As a matter of fact, if the battery didn't sulphate, there would be no discharge. The third class of impurities are those which are active corrosive agents and eat into the plate and grids.

Repeated Thermal Expansions and Contractions
Their Effect on Long Armature Coil Insulations

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THERE has been considerable discussion among engineers interested in the operation of large turbo-generators as to the possible effects of expansion and contraction on armature coil insulation. During the past ten years, generators have increased in core length from less than 100 in. to 160 in. and generators approaching 200 in. in core length are being discussed. With increasing length of cores, it has become more important to settle this question and the tests described in this paper were undertaken three years ago with this object in view. Preliminary tests were started on May 5, 1921, in order to work out some of the details of the procedure. The investigations herein described were actually started on June 2, 1922, the test coils having been prepared just previous to that time.

That some differential effect may reasonably be expected in long-core generators arises from the fact that the coefficient of thermal expansion of built-up mica and paper insulation is practically the same as that for iron[1] while the coefficient of copper is roughly 50 per cent larger. With a core length of 150 in. and an average temperature difference between copper and core teeth of 40 deg.—a usual value for high voltage windings—the difference in linear expansion between core and insulation, on the one hand, and the copper of the coils, on the other hand, will be roughly 1/32 of an inch.

Description of Apparatus and Procedure

A set of four slots was made up from punchings such as is shown in the upper part of Fig. 1. The slots were of sufficient depth to permit two insulated coils to be placed and wedged in each slot. The punchings were assembled in two sections each 54¼ in. in length including the two ⅛ in. end plates. These two sections were then placed end-on and bolted tightly together so as to constitute four slots 109¾ in. long and having the imitation vent ducts as indicated in the lower sketch of Fig. 1. There were six ½-in. imitation vents at the one end of the assembled slots; another six ½-in. vents near the middle, and three 1¼ in. vents near the middle. The positions of these vents are given on Fig. 1.

The experimental coils each had 8 square brass tubes ¼ in. by ¼ in. for conductors. Six sets of these 8 tubes were assembled into coils and insulations for a length of 118 in. with standard Westinghouse insulation for 13,200 volts according to the standard Westinghouse practise. Slight variations were made in applying the insulation in certain cases, in that the quantity of mica was varied and also in impregnating the assembled conductors before applying the insulation so as to produce a sort of slip joint. The other two coils were insulated with mica tape instead of with the regular turbo-coil insulation.

Eight thermocouples were placed with their junctions adjacent to the various conductors on each coil. There was one couple placed on each tube. There were two of these couples placed at the mid-point of the length of the coil and others at intervals of 17 in. in either direction from the middle towards the ends. These couples were

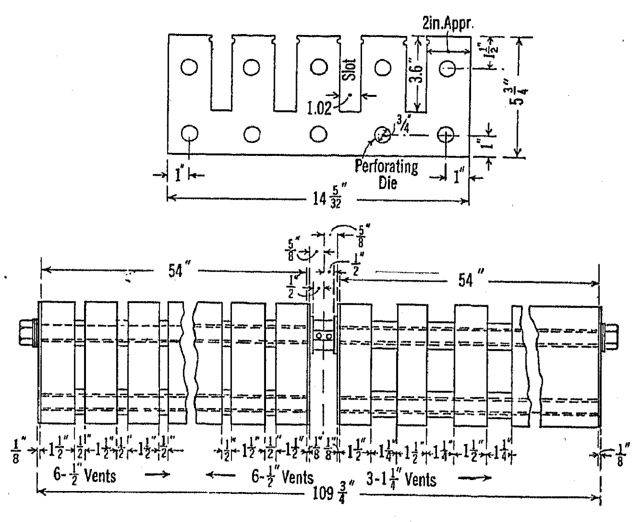

Fig. 1—Experimental Punchings for Expansion Test

made of 0.005 in. advance and copper ribbon being insulated with mica tape.

The coils after being insulated were inserted into the slots and wedged into place by the standard methods. Two coils, one over the other, were placed in each slot. The conductors of the coils were joined electrically in series. They were also joined by means of rubber tubes in parallel to air reservoirs, which were connected in series with a blower and valve. It was possible by means of the blower and valve to cool the coils by air circulation and to reverse the direction of the air through the tubes during the cooling period. The general set up is shown in Fig. 2. Some of the auxiliary apparatus used is indicated by letters on Fig. 2 and noted on the description below the figure.

The heating of the coils was produced by means of alternating current supplied by a suitable transformer.

1. This coefficient was determined by O. H. Gish of the Westinghouse Research Laboratory for Westinghouse micarta folium insulation during the early part of 1921.

Presented at the Northeastern District Regional Meeting of the A. I. E. E., Worcester, Mass., June 4-5, 1924.

The time the current was on, the starting of the blower, the reversals of the air stream or operation of the air valve, and the stopping of the blower were all controlled by means of a time contact-making device. This was so adjusted that one complete heating and cooling cycle occurred in one hour in the early experiments. In the latter part of the tests this cycle was completed each 50 min. This was the case as a result of the characteristics of the contacting device. In the early part of the tests the length of the times of heating, and cooling, and the magnitude of the heating current were so adjusted that the temperature of the conductors would reach 150 deg. cent. at their mid-points during the heating cycle, and would be cooled to approximately 100 deg. cent. during the cooling part of the cycle. The temperature of the iron stampings constituting the slot formation was so adjusted that it was approximately 100 deg. cent. This was accomplished by means of placing insulating materials over the form. Two sheets

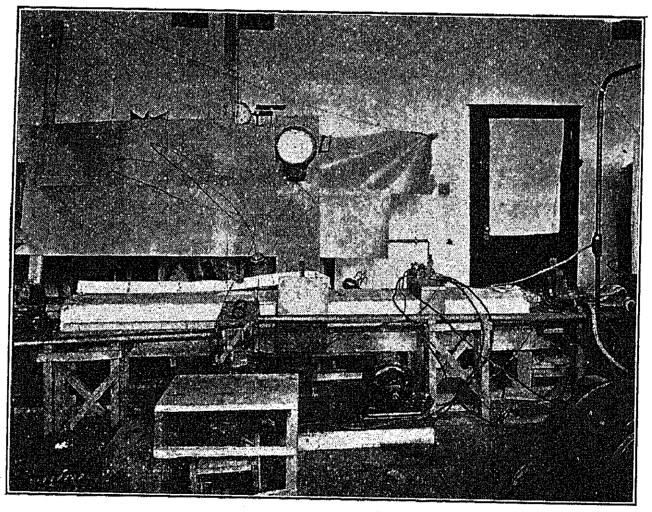

FIG. 2—LONG COIL INSULATION EXPANSION CONTRACTION DESTRUCTIVE TESTS

A —Blower
B-C—Air Valve for Reversing Direction of Air Flow
D —Recording Thermometer Used to Check Air Reversals

of regular thin sheet asbestos were found to be sufficient for this purpose.

The temperature distributions along the stampings and along the conductors before and after cooling are shown in Fig. 3. This curve shows an average temperature drop of the conductor during the cooling cycle of approximately 50 deg. cent.

The heating current was then adjusted so that this average drop in temperature of the conductors was 75 deg. cent. This particular change of temperature was such as to produce as much differential change in expansion and contraction as would be produced when a machine having coils of the length used in these tests would cool from maximum operating temperature to room temperature. The average change of 75 deg. cent. in temperature of the conductors of the coils was continued for approximately 7800 heating and cooling cycles. The insulations of the coils then still having withstood an insulation voltage test for one minute of 23,000 volts, the heating current and cycle were so changed as to obtain an average temperature change of 100 deg. cent. in the temperature of the conductors. This temperature change was continued for approximately 2500 more cycles, thus making a total of 10,300 heating and cooling cycles since the beginning of the

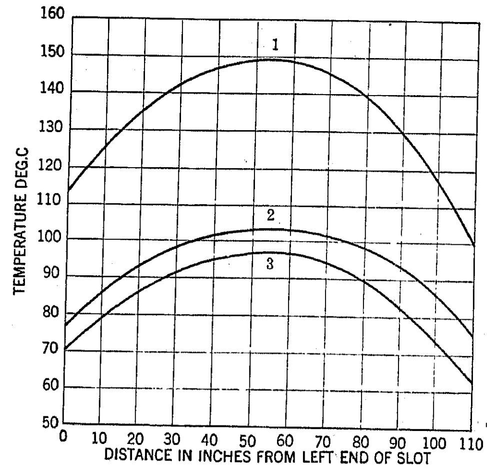

FIG. 3—TEMPERATURE DISTRIBUTION CURVES

1—Along Conductor before Cooling
2—Along Slot or Iron Stampings
3—Along Conductor after Cooling

test. The insulations still standing a voltage test of 23,000 volts for one minute, it was decided to still increase the temperature change of the conductors during the cooling cycle. To do this it was necessary to substitute water-cooling in the conductor tubes in place of the air-cooling. For this substitution, a magnetically-operated water valve was substituted for the magnetically-controlled air valve used up to this time.

Conditions of heating and cooling were so adjusted

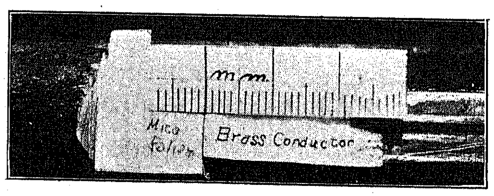

FIG. 4—RELATIVE POSITIONS OF CONDUCTOR AND INSULATION AT CLOSE OF COOLING CYCLE

that an average temperature change of the conductors of 130 deg. cent. was produced during the cooling part of the cycle. Such a change in conductor temperature was continued for approximately 825 cycles. The range of temperature change of conductors was then further increased to approximately 160 deg. cent. and was continued for approximately 400 cycles. At this point in the tests, the transformer furnishing the heating current failed on Feb. 8, 1924. The coils were tested for breakdown and after removing several inches of the

stampings from the ends of the form constituting the slots, in order to prevent sparking over the ends, it was impossible to break down the insulation of any one of the 8 coils with a voltage of 37,000. So much difficulty was encountered in preventing sparking over the ends for any higher voltage, that 37,000 was as high as was used in testing the insulation at the completion of the test.

An idea of the relative change in length of the coil insulation and the conductors during a heating and cooling cycle can be obtained by referring to Figs. 4 and 5. Fig. 4 shows the positions of the brass conduct-

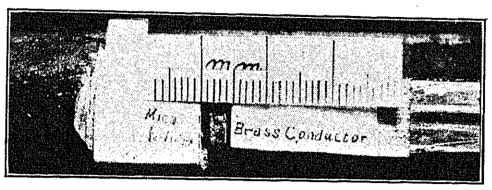

Fig. 5—Relative Positions of Conductor and Insulation at Close of Heating Cycle

ors and insulation at the close of a cooling cycle, and Fig. 5 at the close of the heating part of the cycle. These show a relative change in length of conductor and insulation of almost 5 millimeters (0.197 in.) at each end of the coils. These illustrations were made during the latter change of 160 deg. cent. of the conductors during the cooling cycle.

The following summary gives the number of cycles, the corresponding temperatures of coils at end of heating periods of each cycle, and the temperature change in the conductors during the cooling part of the cycle,

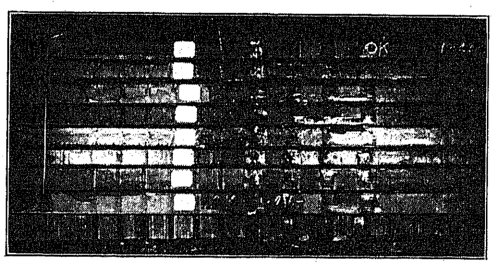

Fig. 6—Illustration of Middle Portion of Coils After Completion of Tests and Removal from Slots. Also Showing Imitation Vent Ducts in Stamping Form Constituting Slots

as well as the temperature of the iron stampings constituting the slots.

Tem. of Coil at end of heating Cycle	Temp. Drop during Cooling of Conductors	Tem. of Stampings	No. of Cycles for Temp. Range	Total No. of Cycles
150 to 160 deg. cent.	75 deg. approx.	100 deg. approx.	7800	7,800
160 to 170 deg. cent.	100 deg. "	100 deg. "	2512	10,312
150 deg.	130 deg. "	100 deg. "	825	11,137
180 deg.	160 deg. "	100 deg. "	400	11,537

After the completion of the 11,537 heating and cooling cycles and the failure of the transformer supplying the low-voltage heating current, the coils were removed from the slots and examined. Fig. 6 shows the eight coils after their removal from the slot. It is quite evident from this illustration that the paper cells which were put around the coils are completely carbonized at the middle portion of all the coils. The two coils insulated with mica tape were naturally more difficult to remove from the slots. In fact, the others were removed with considerable ease as they only bound slightly at the imitation vent ducts where the insulation had swollen slightly.

Discussion of Results

The eight samples of long coil insulation have successfully withstood the severe tests resulting from long continued heating and repeated heatings and coolings of the conductors. The range of temperature change was at first made such as would be equivalent to the differential effect produced as a result of the different coefficients of expansion of iron stampings, insulation, and brass conductors. This range of temperature change was continued for 7800 cycles, which would be as many such changes as the machine would possibly have in practise by warming up and cooling down once each day for almost 22 years. The coils were subjected to a one-third greater relative change for an equivalent of some 7 years possible heating and cooling changes. The cooling by means of water for 825 cycles and 130 deg. cent. change in conductor temperature, and for 400 cycles and 150 deg. cent. was extremely severe treatment, in that the relative changes were approximately twice as much as would be experienced in practise. These tests having extended over a period of about 20 months and the coils having been subjected to such severe treatment without failure even at 37,000 volts at the completion of the tests, indicate that all the samples of insulation were of excellent quality from both a mechanical and an electrical standpoint. Inspection of the coils after removal from the tests disclosed no places where the insulation had been sufficiently bent or folded at any vent duct so as to produce any cracking of the insulation. If such had been the case, a voltage of 37,000 would have broken down the insulation at any such points.

It might be well to mention the fact that these samples of insulation were even subjected to more severe treatment on several occasions than is to be inferred from the discussion thus far. These cases arose when the contacting device failed to operate properly at some 8 or 10 different times and thus permitted the heating current to be left on the circuit continuously and no cooling to be produced for some six or eight hours before it was discovered. At such times the temperatures of the conductors may have reached values as high as 250 deg. cent. and the iron stampings a temperature of 135 deg. cent. By looking at the coils,

it is apparent that quite high temperatures have been reached at times on account of the darkened, in fact blackened, condition of the paper, in particular at the vents in the external part of the wrappers. The discoloration was much more pronounced naturally at the middle point of the coils than at the ends. The discoloring seemed to be more pronounced on the coils which were in the lower portion of the slots as might reasonably be expected. While the paper in the insulation had very little mechanical strength left, the insulation as such was sufficiently strong mechanically to retain its form about the conductor and suffer no apparent additional destructive effect during the removing of the coils. The deterioration was very much more marked at imitation vent ducts than where it was in contact with the stampings. The paper was only just browned instead of blackened where the coil was adjacent to the stampings. The deterioration was much more pronounced also at the 1¼-in. vent ducts than at the ½-in. ducts. This is what might be expected since the wider the duct, the more readily could the air reach the paper and thus produce the effect.

Short-Circuits of A-C. Generators-II

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Review of the Subject.—*At the mid-winter convention of the A. I. E. E. in February, 1924, the writer presented a paper on the maximum instantaneous values of the currents delivered by alternating-current generators under different short-circuit conditions. A summarized table was given for the maximum values of the short-circuit currents for different winding combinations of an ideal generator on the basis of a constant field flux, and that the short-circuit occurs at the instant the axis of the field winding coincides with the axis of the particular portion of the armature winding under consideration.*

In the present paper, a similar analysis is given for the magnitude and variation of magnitude of the short-circuit currents during the transient period, that is, during the time the currents vary from the maximum to the sustained values.

Magnitude of Short-Circuit Currents During the Transient Period

In the previous analysis of the maximum instantaneous values of the short-circuit currents delivered by a-c. generators, the shape and maximum value of the armature and field current-waves were determined, first, on the assumption that the respective circuits had zero resistance; and finally it was shown that resistance in the electric circuits of either member tends to decrease the instantaneous values of the currents, for resistance in the electric circuits necessitates a change in the flux interlinkages of the respective circuits, in order to produce at any instant electromotive forces equal to the corresponding resistance drops. The physical interpretation of the effect of resistance on the rate of decay of the short-circuit currents can be greatly facilitated by assuming that:

(1) The resistance of the armature circuit is appreciable but that of the field circuit is negligible;

(2) The resistance of the field circuit is appreciable but that of the armature circuit is zero; and

(3) The resistances of both circuits are appreciable.

Effect of armature resistance:—If only the armature circuit has appreciable resistance, the net flux interlinkages of the field circuit will be maintained constant at its initial value, but the flux interlinkages of the armature circuit must change a sufficient amount at each instant to generate an electromotive force equal to its resistance drop. In considering the case in which the armature short-circuit current wave has the maximum displacement from the zero reference axis, the total change in the initial flux interlinkages of the armature circuit will be proportional to $\int_0^{t_0} i_a \, r_a \, d\, t = K\, i_{a\,max}\, r_a\, t_0$, when the armature current has decreased to zero value, and the rotor is in the angular position $(2\,\pi - \alpha_1)$ where,

i_a = instantaneous value of the armature current;

$i_{a\,max}$ = maximum value of the armature current.

r_a = resistance of the armature circuit;

t_o = time interval in seconds corresponding to an angular change of the rotor position from 0 to $2\,\pi - \alpha_1$, *i. e.* $t_0 = K^1\,(2\,\pi - \alpha_1)$; and,

K = ratio of average to maximum value of the armature current over the portion of the wave under consideration.

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Since the armature current is zero at this rotor position, the flux interlinkages of the armature circuit, due to the normal field flux will be $I_1\, M_{1a} \cos\,(2\,\pi - \alpha_1)$ or $I_1\, M_{1a} \cos \alpha_1$, and must be equal to the initial flux interlinkages of the armature circuit, minus total change required to overcome the resistance drops for this portion of the wave; that is,

$$I_1\, M_{1a} \cos \alpha_1 = I_1\, M_{1a} - \int_0^{t_0} r_a\, i_a\, d\, t = I_1\, M_{1a} - K\, i_{a\,max}\, r_a\, t_0$$

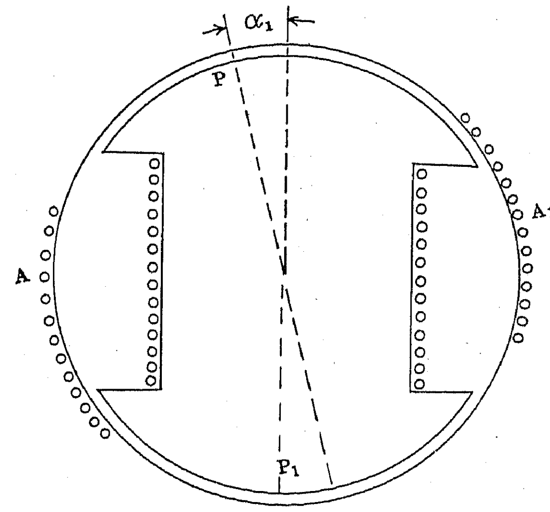

Fig. 40 a

This equation determines the approximate angular position of the rotor at the instant the armature current reaches zero value; and starting from this position, the magnitude and polarity or direction of flow of the armature current will be determined on the basis of initial flux interlinkages of $I_1\, M_{1a} \cos \alpha_1$, instead of $I_1\, M_{1a}$. As the rotor moves toward the angular position of $2\,\pi$, the field winding tends to increase the flux interlinkages of the armature circuit from $I_1\, M_{1a} \cos \alpha_1$ to $I_1\, M_{1a}$, consequently a current of opposite or negative polarity will flow in the armature circuit and resist any change in its flux interlinkages. When the rotor reaches the angular position, $2\,\pi + \alpha_1$, the mutual

flux interlinkages of the armature circuit from the field circuit are $I_1 M_{1a} \cos \alpha_1$, consequently, the armature current will again be zero. Further rotation of the rotor tends to decrease the flux interlinkages of the armature circuit and causes a current of positive polarity to flow in it. Hence, in order to offset the resistance drop, it is evident that the net flux interlinkages of the armature circuit must decrease during the positive loop, and increase during the negative loop of the current wave for each complete cycle.

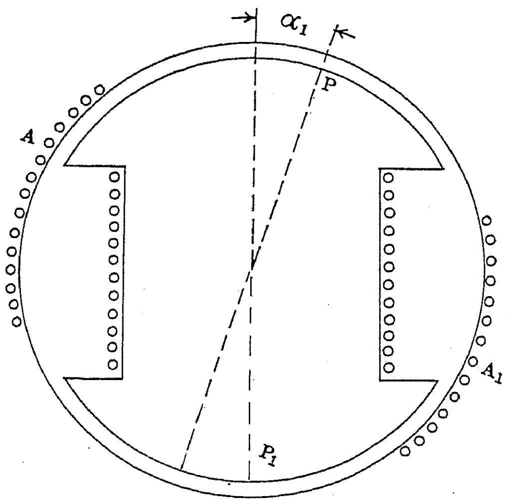

FIG. 40 B

Since the integrated flux-interlinkage change of the armature circuit is greater for the positive loops than for the negative loops, the net flux interlinkages of the armature circuit will continue to be reduced in magnitude at the end of each cycle, until the wave becomes symmetrical with respect to the zero reference axis. After the symmetrical conditions are reached, the zero values of the armature current will occur at rotor positions which correspond to odd multiples of $\frac{\pi}{2} + \theta_1$, at the instants; that is, the axis of the field winding is approximately at right angles to the axis of the armature winding. Since the initial flux interlinkages of the field winding are maintained constant, the sum of the maximum values of the positive and negative crests will be constant for any complete cycle of the current wave. When the armature current wave becomes symmetrical, the maximum values of the positive and negative loops will be one-half of the initial value that was reached at the end of the first half-cycle, and the same as the initial peak value that was reached when the short circuit occurred at the instant the axis of the field winding was at right angles to the armature winding.

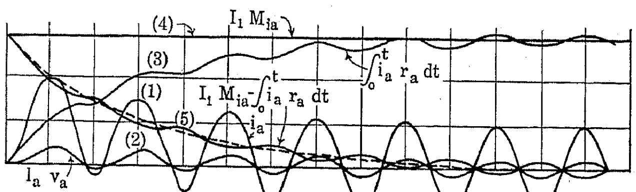

FIG. 41

In Fig. 41, the variation of the respective current and flux interlinkage quantities with respect to time are shown graphically by the following curves:—

(1) Instantaneous value of the armature current.

(2) Instantaneous value of the resistance drops in the armature circuit.

(3) Total integrated change in the armature flux interlinkages due to the resistance drops.

(4) Initial flux interlinkages of the armature circuit.

(5) Instantaneous value of the flux interlinkages of the armature circuit.

The pulsation or indentations in Curve (5) which represents the instantaneous values of the flux interlinkage of the armature circuit are of practical importance only in so far as they assist in the physical interpretation of this phenomenon. However, the average curve drawn through the points which represent the armature flux interlinkages at the beginning of each cycle is of prime importance, for it represents the rate of decrease of the flux interlinkages of the armature circuit sufficiently close for all practical purposes. This average curve does not decrease according to a straight line law, but varies according to an exponential law on account of the fact that the total integrated flux-interlinkage change is not constant but decreases for each successive cycle.

Since the sum of the peak values of the positive and negative crests of the armature current wave is constant for any cycle, it is convenient from an analytical standpoint to consider the armature current wave as being composed of a constant alternating wave super-imposed on a d-c. component, which decreases in value according to an exponential law. At the end of the first-half cycle, the d-c. component is equal to the maximum value of the alternating component and from one-quarter to one-half second later the d-c. component totally disappears. The d-c. component can be considered as produced by the rate of decay of the initial flux interlinkages of the armature circuit. Then from Helmholtz's Law, the flux interlinkages and d-c. component of the armature current will decrease according to the exponential function $E^{-\frac{r_a}{N_a} t}$ where

r_a = resistance of circuit;

N_a = equivalent leakage induction coefficient of the circuit with respect to adjacent closed circuits; and,

t = time in seconds after closure.

Effect of resistance in the field circuit:—If only the field circuit has appreciable resistance, it is evident from the previous analysis, that its total flux interlinkages at any time t_1 after the short circuit occurs, will be

$I_1 L_1 - K \int_0^t i_1 r_1 \, d t$ and in the case of polyphase alternators will continue to decrease until its mutual flux interlinkages with the armature circuit are equal to the leakage flux interlinkages of the armature circuit. Hence, for any angular position of the rotor, the change (due to rotation) in the normal field flux interlinkages will also be decreased and the maximum values of the armature current, which occur at rotor positions that are odd multiples of π, will decrease in the same ratio. When the rotor is in the angular position that are even multiples of π, its mutual flux interlinkages with the armature circuit will continually decrease in value. Since the resistance of the armature circuit is zero, currents of positive value will flow in the armature circuit at these rotor positions, in order to maintain its flux interlinkages constant at the initial value. The curves in Fig. 42 show:

(1) Initial value of field flux interlinkages.

(2) Instantaneous value of the flux interlinkages of the field circuit.

(3) Instantaneous values of the circulating current in the field circuit.

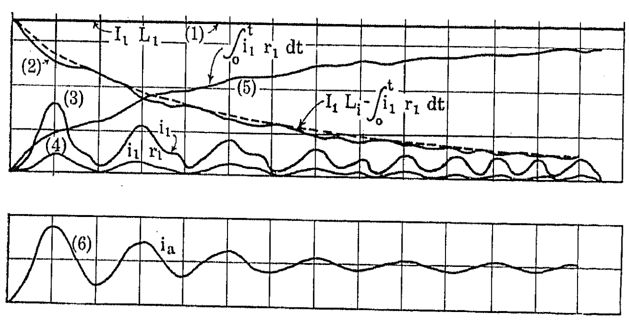

FIG. 42

(4) Instantaneous values of the resistance drops in the field winding.

(5) Instantaneous value of $\int_0^{t_1} i_1 r_1 \, d t.$;

(6) Instantaneous values of the armature current.

Then from Helmholtz's Law, the average curve which shows the flux interlinkages of the field circuit at the beginning of each cycle varies according to $I_1 K (1 + A E^{-\frac{r_1}{N_1} t})$ where

$I_1 L_1 = I_1 K (1 + A)$ = initial flux interlinkages of the field circuit.

$I_1 K_1$ = final flux interlinkages of the field circuit.

Y_1 = resistance of the field circuit.

N_1 = equivalent induction coefficient of the field circuit.

In this case the armature current can be considered as being composed of an alternating component, which decreases in value, super-imposed on a constant d-c. component. The alternating component, which has a maximum value $i_{a max} = I_a (1 + A)$, at the end of the first half-cycle after the short-circuit occurs, decreases according to the above exponential function, $(1 + A E^{-\frac{r_1}{N_1} t})$, until it reaches a final or sustained value, I_a, a few seconds later. The d-c. component remains constant, for there is no change in its initial flux interlinkages.

Effect of resistance in armature and field circuits:— If both armature and field circuits have appreciable resistance, the flux interlinkages of the armature circuit will decrease from their initial value at the instant of the change from short-circuit to zero, according to the exponential law $E^{-\frac{r_a}{N_a} t}$, and the field flux interlinkages will decrease from the maximum value before the short circuit occurs, to a final or sustained value according to the exponential law $(1 + A E^{-\frac{r_1}{N_1} t})$. Then at any instant, the d-c. component of the armature current can be represented by $I_{dc} E^{-\frac{r_a}{N_a} t}$; the peak values of the alternating component will be on the curve determined by the exponential function $I_{acs} (1 + A E^{-\frac{r_1}{N_1} t})$ and the total peak values of the armature current will lie on the curve, which is determined by the sum of the two exponential functions, $I_{dc} E^{-\frac{r_a}{N_a} t} + I_{acs} (1 + A E^{-\frac{r_1}{N_1} t})$ where,

I_{dc} = maximum value of the direct current component.

I_{acs} = maximum value of the armature current under sustained conditions.

$I_{acs} (1 + A)$ = maximum value of the alternating current component at the end of the first half cycle; and all other symbols are as previously defined.

If the instantaneous values of the alternating component vary sinusoidally with respect to time, the r. m. s. value of the armature current for any given cycle will be

$$I_{r.m.s.} = \left\{ \frac{1}{2 \pi} \int_0^{2\pi} [I_{dc} E^{-\frac{r_a}{N_a} t} + I_{acs} (1 + A E^{-\frac{r_1}{N_1} t}) \cos \omega t] \, d t \right\}^{1/2}$$

$$= \left\{ [I_{dc} E^{-\frac{r_a}{N_a} t}]^2 + \left[\frac{I_{acs} (1 + A E^{-\frac{r_1}{N_1} t})}{2} \right]^2 \right\}^{1/2}, \text{ approximately.}$$

Hence, for any given initial short-circuit conditions, instantaneous peak and r. m. s. values of the armature currents can be calculated, provided the resistance and equivalent-leakage induction coefficients of the respective circuits are known. In the preceding analysis, which covered the maximum possible value of the short-circuit currents, it was shown that the combined equivalent-leakage inductance coefficient of the armature circuit was approximately constant and equal to

$L_a - \dfrac{M_{1a}^2}{L_1}$ when the rotor was provided with a damper winding, but this induction coefficient pulsated at double frequency between maximum and minimum values,

L_a, and $L_a - \dfrac{M_{1a}^2}{L_1}$ respectively, if the rotating element did not have a damper winding. It is obvious, from analogy, that the similar induction coefficient of the field winding will be approximately constant and equal

to $L_1 - \dfrac{M_{1a}^2}{L_a}$ when the stator member is provided with polyphase windings, but will pulsate at double frequency between maximum and minimum values, L_1, and

$L_1 - \dfrac{M_{1a}^2}{L_a}$ respectively, for an a-c. generator with a single-phase armature winding. It can be shown mathematically that, so far as the rate of decrease of the short-circuit currents is concerned, the double-frequency pulsating equivalent-leakage inductance coefficients for either the armature or field circuits can be replaced by an effective coefficient which is constant

and equal to $K\left(L_{1 or a} - \dfrac{M_{1a}^2}{L_{a or 1}}\right)$ where the value of

the constant K is between 1 and 2[1]. Table II shows the relative values of the time constants for the armature and field circuits for polyphase and single-phase generators when the rotors are constructed with and without damper windings.

In the above table the actual value of the constant K depends on the effectiveness of the material and construction of the rotor or stator as a partial damper winding. It is obvious that the time constants for the different electric circuits of actual generators can be calculated with only an approximate degree of accuracy. However, the most important and definite conclusions to be drawn from the above data are:

(1) In the case of either single-phase or polyphase alternators, the addition or presence of the damper winding on the excitation element increases the rate of decrease of the d-c. component of the armature current.

(2) The presence of the damper winding does not affect the rate of decay of the alternating component of the armature short-circuit current for polyphase alternators.

(3) The rate of decrease of the a-c. component of the armature short-circuit current delivered by a single-phase alternator either with or without a damper winding is lower than that of a polyphase alternator.

TABLE II

Armature Winding	Rotor	Time Constant of: Armature Winding	Time Constant of: Field Winding
Two-phase	With damper winding....................	$\dfrac{r_{a_2\phi}}{L_{a_2\phi} - \dfrac{M_{1a}^2}{L_1}}$	$\dfrac{r_1}{L_1 - \dfrac{M^2_{1a_2\phi}}{L_{a_2\phi}}}$
Two-phase	Without damper winding.................	$\dfrac{r_{a_2\phi}}{K_{2\phi}\left(L_{a_2\phi} - \dfrac{M^2_{1a_2\phi}}{L_1}\right)}$	$\dfrac{r_1}{L_1 - \dfrac{M^2_{1a_2\phi}}{L_{a_2\phi}}}$
Three-phase	With damper winding....................	$\dfrac{r_{a_3\phi}}{L_{a_3\phi} - \dfrac{M^2_{1a_3\phi}}{L_1}}$	$\dfrac{r_1}{L_1 - \dfrac{M^2_{1a_3\phi}}{L_{a_3\phi}}}$
Three-phase	Without damper winding.................	$\dfrac{r_{a_3\phi}}{K_{3\phi}\left(L_{a_3\phi} - \dfrac{M^2_{1a_3\phi}}{L_1}\right)}$	$\dfrac{r_1}{L_1 - \dfrac{M^2_{1a_3\phi}}{L_{a_3\phi}}}$
Single-phase	With damper winding....................	$\dfrac{r_{a_1\phi}}{L_{a_1\phi} - \dfrac{M^2_{1a_1\phi}}{L_1}}$	$\dfrac{r_1}{K_{1\phi}\left(L_1 - \dfrac{M^2_{1a_1\phi}}{L_{a_1\phi}}\right)}$
Single-phase	Without damper winding.................	$\dfrac{r_{a_1\phi}}{K^1_{1\phi}\left(L_{a_1\phi} - \dfrac{M^2_{1a_1\phi}}{L_1}\right)}$	$\dfrac{r_1}{K_{1\phi}''\left(L_1 - \dfrac{M^2_{1a_1\phi}}{L_{a_1\phi}}\right)}$

1. See paper by P. Boucherot.

Permanent or Sustained Value of the Short-Circuit Current

Sustained Short-Circuit Current of Polyphase Alternators. In the case of polyphase alternators the armature currents reach final or sustained values a few seconds after the short circuit occurs and the excess circulating current in the field winding totally disappears. The sustained armature current waves are symmetrical with respect to a zero reference axis and are equal in amplitude, but displaced in phase with respect to each other. Since the resistance of the armature circuit is very small in comparison to the reactance, the armature currents lag approximately 90 deg. behind the induced voltages and the resultant armature m. m. f., which is approximately constant in shape and magnitude, is directly opposed to the m. m. f. of the field circuit. Consequently, the net field flux that interlinks the armature windings is just sufficient to neutralize the leakage flux interlinkages of the armature circuits. Then the magnitude of the sustained armature currents will be such that the total flux interlinkages of the armature circuits are equal to the total mutual flux interlinkages supplied to them from the field circuit.

In the case of a two-phase alternator the magnitude of the resultant armature m. m. f. wave is approximately equal to the m. m. f. of one phase acting alone when the current in this phase is at its maximum value. When the rotor is in such angular position that the axes of the field winding and phase A of the armature winding coincide, the induced voltage in phase A is zero and the current flowing in it is at its maximum value. Similarly, the current in phase B is zero and the voltage is at its maximum value. Consequently, the flux interlinkages of phase A due to ${}_{2\phi}I_a$ must equal the total mutual flux interlinkages that are supplied to it by the field winding. That is,

$${}_{2\phi}I_a\, L_{a_{2\phi}} = I_1\, M_{1a}\,, \text{ or}$$

$${}_{2\phi}I_a = \frac{I_1\, M_{1a}}{L_{a_{2\phi}}} \qquad (47)$$

If the reluctance of the magnetic circuit does not change for different rotor positions and the m. m. f. waves of the armature circuits are sinusoidal with respect to space and time, the r. m. s. value of the armature current is

$${}_{2\phi}I_a = \frac{I_1\, M_{1a}}{\sqrt{2}\, L_{a_{3\phi}}}\,.$$

In the case of a three-phase alternator, the sustained short-circuit current is a maximum in phase A, and at one-half of the maximum value in phases B and C when the axis of the field winding coincides with the axis of phase A of the armature winding. The mutual flux interlinkages of the armature windings, due to the field circuit, are $I_1 M_{1a}$ for phase A, and $\frac{1}{2} I_1 M_{1a}$ for phases B and C respectively or a combined value of $2\, I_1 M_{1a}$ for all three phases. If the maximum value of the current in phase A is ${}_{3\phi}I_a$, the total flux interlinkages of the armature winding, due to their own currents, are approximately ${}_{3\phi}I_a\,(2\, L_{a_{3\phi}} + 2\, M_{ab})$ where $L_{a_{3\phi}}$ is the total inductance coefficient of one leg of the armature winding and M_{ab} is the mutual inductance coefficient between any two legs of the armature winding. Since the total flux interlinkages of the armature windings must be equal to the total mutual flux interlinkages from the field circuit,

$${}_{3\phi}I_a\, 2\,(L_{a_{3\phi}} + M_{ab}) = 2\, I_1\, M_{1a}\,, \quad \text{or}$$

$${}_{3\phi}I_a = \frac{I_1\, M_{1a}}{L_{a_{3\phi}} + M_{ab}}$$

But $M_{ab} = \frac{1}{2} L_{a_{3\phi}}$, approximately and then

$${}_{3\phi}I_a = 0.667\, \frac{I_1\, M_{1a}}{L_{a_{3\phi}}} \text{ approximately} \qquad (48)$$

$${}_{3\phi}I_a = \frac{0.667}{\sqrt{2}}\; \frac{I_1\, M_{1a}}{L_{3\phi}} \text{ approximately} \qquad (48a)$$

In comparing the ratios of the sustained short-circuit currents to the full-load currents for two and three-phase generators which have the same rating, terminal voltage, and constant field flux per pole, the armature conductors per inch for the two-phase winding must be 122.5 per cent of those for the three-phase winding. If $I_1 M_{1a}$ = maximum mutual flux interlinkages of armature phase A due to the field current,

$\sqrt{3}\, I_1 M_{1a}$ = maximum mutual flux interlinkages of two legs of the three phase winding connected in series for the star type of winding.

$\sqrt{3}\, I_1 M_{1a}$ = magnitude of mutual flux interlinkages of one phase of two-phase winding in order to give the same voltage as a three-phase star type of winding.

If both windings had the same armature conductors per inch, the total inductance coefficient of one leg of the two-phase winding would be twice that of one leg of the three-phase winding, as shown by Curve (1) of Fig. 19. Since the conductors per inch for the two-phase winding are 122.5 per cent of those for a three-phase winding, the total inductance coefficients of one leg of the two-phase winding will be three times that of one leg of the three-phase winding. That is,

$$L_{a_{2\phi}} = \overline{1.225}^2 \times 2\, L_{a_{3\phi}} = 3\, L_{a_{3\phi}}$$

Then from equations (47) and (48) the sustained short-circuit currents will be

$${}_{3\phi}I_a = 0.667\, \frac{I_1\, M_{1a}}{L_{a_{3\phi}}}$$

for the three-phase winding, and

$${}_{2\phi}I_a = \frac{\sqrt{3}\, I_1\, M_{1a}}{3\, L_{a_{3\phi}}} = 0.577\, \frac{I_1\, M_{1a}}{L_{a_{3\phi}}}$$

for the two-phase winding. The maximum values of the full-load currents for the two windings have the following relation,

$${}_{2\phi}I_{aF.L.} = 0.866\; {}_{3\phi}I_{aF.L.}$$

Then the ratios of the sustained short-circuit currents to the corresponding full-load currents for the two windings will be,

$$\frac{{}_{3\phi}I_a}{{}_{3\phi}I_{aF.L.}} = \frac{0.667\,\dfrac{I_1\,M_{1a}}{L_{a_{3\phi}}}}{K\,I_1\,M_{1a}} = \frac{0.667}{K\,L_{a_{3\phi}}}$$

for the three-phase winding,

$$\frac{{}_{2\phi}I_a}{{}_{2\phi}I_{aF.L.}} = \frac{0.577\,\dfrac{I_1\,M_{1a}}{L_{a_{3\phi}}}}{0.866\,K\,I_1\,M_{1a}} = \frac{0.667}{K\,L_{a_{3\phi}}}$$

for the two-phase winding.

In the construction of actual machines, it is very improbable that the two and three-phase generators, designed to deliver the same terminal voltage, would be identical in every respect except armature conductors per inch, consequently, it is to be expected that ratios of sustained to full-load current will be slightly different for the two cases.

Sustained Short-Circuit Current of Single Phase Alternators. The magnitude of the sustained short-circuit current is more difficult to determine analytically for a single-phase generator than for a polyphase generator, on account of the fact that the leakage and mutual induction coefficients of the armature winding are neither constant in magnitude nor position with respect to the field winding. If the rotor of the single-phase generator is provided with a polyphase damper winding, then under either stable load or sustained short-circuit conditions, double-frequency currents will flow in the damper winding, the armature current wave will be practically sinusoidal in shape, and there will be no appreciable variations in the field flux or pulsations in the field current. A simple physical and quantitative analysis can be obtained by making the usual assumption that alternating flux, flux interlinkages and m. m. f. of the single-phase armature winding can be resolved into two components which are of constant shape and equal magnitude, but rotate in opposite directions at synchronous speeds. The component of the armature m. m. f. which has the same direction of rotation as the rotor, maintains a fixed space relation with respect to the field winding and produces a demagnetizing action on the field circuit just as in the case of a polyphase alternator under similar load conditions. The backward rotating component of the armature m. m. f. moves at double speed with respect to the rotor and consequently induces double-frequency currents in the damper winding. The m. m. f. produced by the double-frequency currents in the damper winding rotates synchronously with and directly opposes the counter-rotating component of the armature m. m. f. and the resultant flux interlinkages are the equivalent leakage flux interlinkages of the two windings, due to the two corresponding and opposing m. m. f's. Hence, under sustained short-circuit conditions, the magnitude of the armature current will be such that the total flux interlinkages of the forward rotating component, plus the total equivalent combined leakage flux interlinkages of the armature and damper windings, due to the backward rotating components, will be equal to the mutual flux interlinkages supplied by the field circuit.

When the rotor is in such angular position that the axes of the armature and field windings coincide, the armature current will be at its maximum value and can be readily calculated, if the proper induction coefficients are known for this rotor position.

If,

I_a = maximum value of the sustained single-phase short-circuit current;

L_a = total self-induction coefficient of the armature circuit;

M_{1a} = mutual induction coefficient of the armature and field circuits;

L_b = total self-induction coefficient of the damper winding;

M_{ab} = mutual induction coefficient of armature and damper circuits;

$L_a - \dfrac{M_{ab}^2}{L_b}$ = total equivalent leakage induction coefficient of the armature and damper circuits;

$I_1\,M_{1a}$ = total flux interlinkages of the armature circuit supplied by the field circuit;

$I_a\,L_a$ = total flux interlinkages of the single-phase armature winding;

$\tfrac{1}{2}\,I_a\,L_a$ = total flux interlinkages of the armature circuit due to the forward rotating component;

$\tfrac{1}{2}\,I_a\left(L_a - \dfrac{M_{ad}^2}{L_d}\right)$ = total combined leakage flux interlinkages of the armature circuit, due to the backward rotating component;

Then,

$$\tfrac{1}{2}\,I_a\left[\,L_a + \left(L_a - \frac{M_{ad}^2}{L_d}\right)\right] = I_1\,M_{1a}$$

$$I_a = \frac{2\,I_1\,M_{1a}}{L_a + \left(L_a - \dfrac{M_{ad}^2}{L_d}\right)} \tag{49}$$

$${}_{1\phi}I_{aW.D.} = \frac{1}{\sqrt{2}}\;\frac{2\,I_1\,M_{1a}}{L_a + \left(L_a - \dfrac{M_{ad}^2}{L_d}\right)}$$

If the rotating element of a single-phase generator is not provided with a damper winding and the construction of the rotor is such that the rotor material does not act as a squirrel cage winding, the field winding will act as a single-phase damper winding. Oscillographic records show that the odd numbered harmonics are very pronounced in the armature current wave and the

even numbered ones in the field current, when operating under either stable load or sustained short-circuit condition. A qualitative analysis of the phenomenon can be obtained by making the assumption as in the previous case, that the single-phase alternating armature flux and m. m. f. waves can be replaced by two equal and oppositely rotating waves of constant shape and magnitude. The forward-rotating component maintains a fixed space relation with respect to the field winding and reacts on it as in the case of a polyphase winding. The backward rotating component produces double-frequency currents in the field winding. This double-frequency flux or m. m. f. wave of the field winding can in turn be considered as being produced by two equal waves of constant magnitude which rotate in opposite directions at twice synchronous speed. The backward rotating wave of the field winding maintains a fixed space relation with the backward rotating wave of the armature and the resultant flux interlinkages are the combined leakage flux interlinkages of the two windings, due to the two opposing m. m. f. waves. The forward rotating wave of the field winding moves at three times synchronous speed with respect to the armature winding and consequently, produces triple-frequency currents in it. The triple-frequency alternating flux and m. m. f. waves of the armature winding can be considered as produced by equal waves which rotate in the opposite direction at three times synchronous speed. The forward rotating component maintains a fixed relation with the forward-rotating double-synchronous-speed wave of the field winding and exerts a polyphase reaction on it. The corresponding backward-rotating component of the armature triple-frequency wave moves at four times synchronous speed with respect to the rotor and produces a quadruple-frequency current in the field winding. It is evident from the foregoing analysis that all of the odd harmonics will be reflected into the armature circuit and the even harmonics into the field circuit, but the amplitude of the harmonics of both circuits rapidly decreases as the frequency increases.

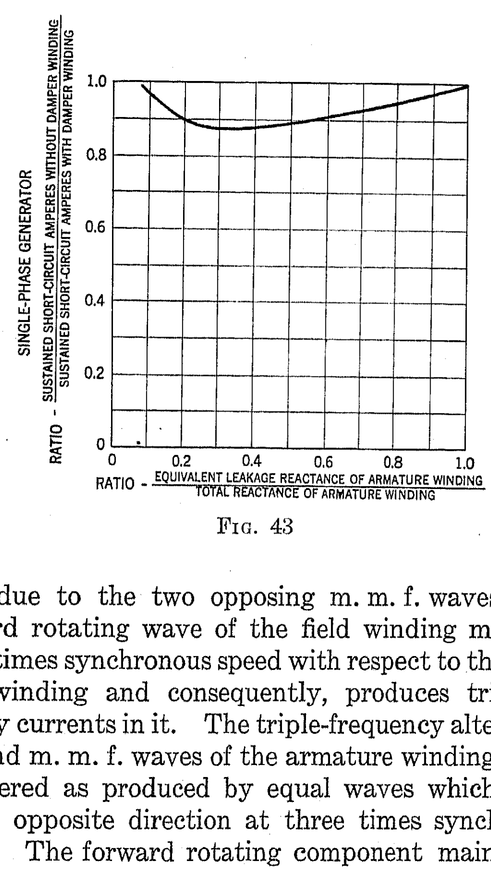

FIG. 43

A less involved quantitative analysis of the magnitude of the currents can be obtained by solving the two simultaneous equations of the flux interlinkages for the two circuits.

$$\left.\begin{aligned} I_a{}^1 L_a + (l_1 + I_1)\, M_{1a} &= 0, \text{ for the armature circuit,} \\ I_a{}^1 M_{1a} + (i_1 + I_1)\, L_1 &= I_1 L_1 K, \text{ for the field circuit} \end{aligned}\right\} \qquad (50)$$

The solution of these equations and the determination of the value of the quantity K gives[2]

$$I_a{}' = \frac{I_1 M_{1a}}{L_a} \sqrt{\frac{L_a}{L_a - \dfrac{M_{1a}{}^2}{L_1}}}$$

for the maximum value of the armature current,

$$_{1\phi}I'_{aN.D.} = \frac{I_1 M_{1a}}{\sqrt{2}\, L_a} \sqrt[4]{\frac{L_a}{L_a - \dfrac{M_{1a}{}^2}{L_1}}}$$

for the effective value of the armature current. The curve in Fig. 43 shows, graphically, the variation of the sustained current ratio $\dfrac{_{1\phi}I_{aN.D.}}{_{1\phi}I_{aW.D.}}$ as a function of

$$\frac{L_a - \dfrac{M_{1a}{}^2}{L_1}}{L_a}$$ where,

$_{1\phi}I_{aN.D.}$ = the calculated effective value of the sustained current for a single-phase generator without a damper winding;

$_{1\phi}I_{aW.D.}$ = the calculated effective value of the sustained current for a single-phase generator with a damper winding;

$L_a - \dfrac{M_{1a}{}^2}{L_1}$ = equivalent leakage induction coefficient of the armature and field windings;

L_a = total self-induction coefficient of the armature winding;

$L_a - \dfrac{M_{1a}{}^2}{L_1} = L_a - \dfrac{M_{ad}{}^2}{L_1}$, assumed relation used in this comparison.

The value of the sustained current of the single-phase generator without a damper winding is only 12½ per cent less than that of a generator with a damper winding, at the point of widest variation over a wide working range, 0.1 to 0.5, of the inductance ratios. In actual manufacturing practise it is practically impossible to construct a generator with a perfect damper winding or one without partial damping action, consequently there should be only a small expected difference in the sustained r. m. s. current values of

2. See paper on Electromagnetic Phenomena resulting from the sudden short-circuiting of an Alternator by P. Boucherot, 1912 *Atti del Congresso Internationale delle Applicazioni Elettriche.*

similar single-phase generators, when equipped either with or without a definite damper winding. Hence, in estimating the r. m. s. values of the sustained short-circuit current for a single-phase generator, it is sufficiently accurate for all practical purposes to use the formula which was developed for the care of the generator with a damper winding, namely:

$$ {}_{1\phi}I_{aW.D.} = \frac{1}{\sqrt{2}} \times \frac{2\, I_1\, M_{1a}}{L_a + L_a - \dfrac{M_{ad}^2}{L_d}} $$

$$ = \frac{\sqrt{2}\, I_1\, M_{1a}}{L_a\,(1 + K)}\ , \text{ Where } K = \frac{L_a - \dfrac{M_{ad}^2}{L_d}}{L_a} $$

It is apparent from the data given on the curves of Figs. 18 and 19 that the single-phase sustained short-

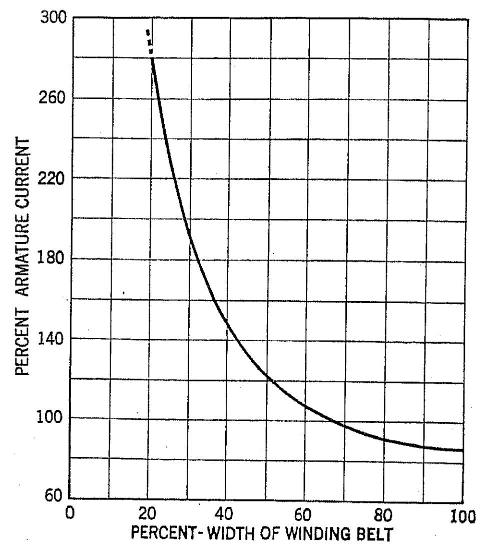

FIG. 44—CALCULATED SUSTAINED CURRENT FOR SINGLE-PHASE GENERATOR

circuit current between terminal and neutral of a three-phase generator is

$$ {}_{1\phi}I_{aT\text{-}N} = \frac{\sqrt{2}\, I_1\, M_{1a}}{L_{aT\text{-}N}\,(1 + K)} \quad \text{and} \qquad (51) $$

$$ {}_{1\phi}I_{aT\text{-}T} = \frac{\sqrt{2}\,\sqrt{3}\, I_1\, M_{1a}}{3\, L_{aT\text{-}N}\,(1 + K)} \qquad (52) $$

where the single-phase short circuit occurs between two line terminals of a three-phase star-connected winding. Then

$$ \frac{{}_{1\phi}I_{aT\text{-}N}}{{}_{1\phi}I_{aT\text{-}T}} = \frac{\dfrac{\sqrt{2}\, I_1\, M_{1a}}{L_{aT\text{-}N}\,(1 + K)}}{\dfrac{\sqrt{2}\,\sqrt{3}\, I_1\, M_{1a}}{3\, L_{aT\text{-}N}\,(1 + K)}} = \sqrt{3}. $$

That is, the sustained value of the single-phase current between terminal and neutral is $\sqrt{3}$ times that between two line terminals of a star-connected three-phase generator. The curve in Fig. 44 shows the relative magnitude of the sustained currents for different circumferential widths of the single-phase winding belt on the basis of a constant value of armature conductors per inch of the armature periphery.

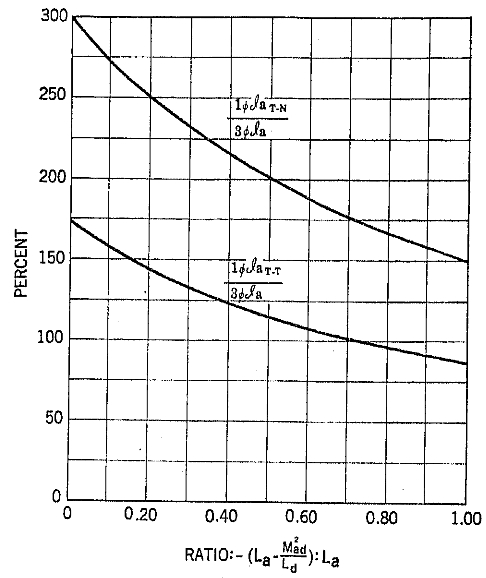

FIG. 45

Test Values of Sustained Short-Circuit Currents. The curves in Fig. 45 show the ratios of the single-phase to the three-phase sustained short-circuit currents as calculated from equations (**48a**), (**51**) and (**52**), for different values of K, where K

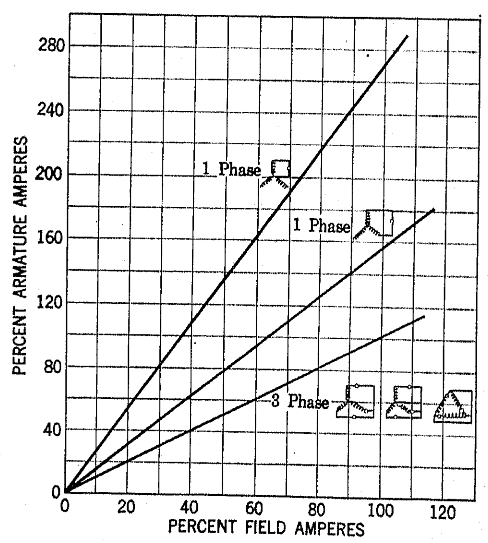

FIG. 46—SUSTAINED SHORT-CIRCUIT CURRENTS FOR 6250 KV-A TURBO-GENERATOR 3-PHASE 2-POLE 3600-REV. PER MIN.

$$ = \frac{\left(L_a - \dfrac{M_{ad}^2}{L_d}\right)}{L_a} $$ (that is the ratio of the equivalent leakage induction coefficient of one leg of the arma-

ture winding and the damper winding when the axes of the two windings coincide, to the total self-induction coefficient of one leg of the armature winding). The curves in Fig. 46 show the test values of the sustained armature winding currents and the corresponding field currents for a 6250-kv-a., three-phase, 60-cycle, 3600-rev. per min. turbo-generator when operating under the following short-circuit conditions:

(a) Single-phase short circuit between
  (1) Terminal and neutral
  (2) Terminal and terminal.

(b) Three-phase short circuits between
  (1) Terminals with the winding star-connected
  (2) Terminals and neutral with the winding star-connected
  (3) Terminals with the winding delta-connected

The equivalent leakage induction coefficient of this particular generator is approximately 15 per cent of the total self-induction coefficient of one leg of the armature winding. Using this approximate value of 15 per cent, the calculated values from the curves in Fig. 19, and the test values of the ratios of the sustained currents for the different short-circuit conditions are as follows:

	Test	Calculated
(1) Single-phase terminal and neutral to terminal and terminal	1.72	1.732
(2) Single-phase terminal and neutral to three-phase star or delta	2.69	2.61
(3) Single-phase terminal and terminal to three-phase star or delta	1.56	1.51

Single-Phase Motor-Torque Pulsations

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Review of the Subject.—*This article gives results of a study of the vibrations of small single-phase induction motors, with a view to determining their causes and eliminating the noise which they produce. The possible sources of such vibrations were analyzed, and an experimental study was made in which the principal cause was definitely determined to be the double-frequency variation of the electromagnetic torque developed by the motor. This variation of torque is fairly obvious when it is considered that the power input to the motor is pulsating and the output is uniform, but it was found that the torque variation at no-load was also very pronounced, a fact which does not seem to have been previously described. A description of the experiments made, a theoretical analysis of the torque variations, and a comparison of test with calculated results are given.*

I. Introduction

THE small single-phase induction motor has recently come to be widely used in household and office appliances, where noise is particularly objectionable. It has been found that these motors are noisy when placed upon a table or other support but are comparatively quiet when held suspended in the air. Since different opinions were held as to the causes and remedies for this noise, the writers undertook an experimental and theoretical study of the matter. In this investigation, it was satisfactorily proved that the principal vibration was torsional in character, of double the line frequency, and electromagnetic in its origin. In other words, the vibration results from a torque which pulsates through a wide amplitude, even when the motor is running light. This torque pulsation reacts upon the stator and causes it to oscillate through a small angle about the motor shaft as an axis, thus transmitting vibrations to the base upon which the motor is mounted.

II. Experimental Study

In order to distinguish between the vibrations caused by the mechanical reactions of an imperfectly balanced rotor and those caused by electromagnetic phenomena, the following test was made. A 25-cycle ¼-h. p. single-phase induction motor was placed on a light board base resting on several layers of thick felt. On running the motor, the board base readily took up the vibrations of the motor stator, because of the small inertia of the board compared with that of the motor, and because of the yielding quality of the felt upon which it rested. At normal voltage the motor vibrated very noticeably. On raising the voltage the vibrations of the motor became much more intense. When the voltage was reduced to a low value, it was found that a certain amount of vibration still remained and that no change in this amount resulted immediately after opening the motor circuit.

Since the no-load speed of rotation of an induction motor is practically constant, independent of the voltage, any vibration due to mechanical unbalance should be unaffected by voltage variations. Thus, the tests showed that the motor vibration was due in part to mechanical unbalance, which persisted down to the lowest voltage, and in part to electromagnetic forces which increased with the voltage.

As the frequency of a vibration due to mechanical unbalance is known to be equal to the number of revolutions per second, the above conclusions were verified by measuring the vibration frequencies. A sheet of paper was placed on the light baseboard with two parallel lines drawn upon it about four inches apart. A long pencil with a sharp point was held lightly in the fingers and drawn across the paper so that it passed over the space between the lines in about a second's time, describing at the same time a series of dots. Since each dot was produced by a single vibration of the base, the number of dots between the two parallel lines gave approximately the number of vibrations per second. From this method of measurement it was found that the frequency of the vibration remaining at low voltage was closely equal to the frequency of rotation. Furthermore, it was found that when a rotor was given a very careful running balance, this type of vibration could be practically eliminated, thus definitely proving it to be due to mechanical unbalance.

After obtaining a good running balance, so that the motor was almost free from vibration at very low voltages, a further study of the electromagnetic vibrations was made. A 25-cycle motor was chosen, because of its great vibration amplitude. Placing it upon a light board resting on felt as before, the vibration frequency was found to be approximately twice line frequency, or 50 cycles per sec., using the dotted line method previously described. Also, preliminary measurements made with a vibration amplitude indicator gave amplitudes approximately proportional to the square of the voltage. This would be expected of a vibration of electromagnetic origin, since the magnetic forces vary approximately as the voltage squared, below magnetic saturation.

The motor was then suspended in such a way that it could vibrate almost perfectly freely in space, without even so small a resistance as that offered by the felt. This was accomplished by placing the motor in a cradle

1. The authors wish to express their appreciation of the many suggestions and cordial cooperation of A. F. Welch and E. B. George in carrying out the work on which this paper is based.

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as shown in Fig. 1, so designed as to allow any sort of a translational or rotational motion without appreciable restraint. Its freedom of motion was facilitated by suspending it from cords attached to points P and Q which were nearly at the height of the center of gravity of the motor, and hanging these cords from the end of a flexible cantilever.

When a rotor in good static balance was placed on the cradle as shown in Fig. 1 and tested at very low voltage, the center line A B of the cantilever was found to be perfectly still, since the center of gravity of the motor remained fixed in space. However, in many such cases, the cantilever took up a torsional vibration with A B as a nodal line, the right and left arms vibrating up and down. This was evidently due to dynamic unbalance, so called because it can only be detected by a test in which the motor is actually rotating. It is manifested by a tilting or wobbling motion of the shaft, the center of gravity of the rotor remaining fixed.

After a rotor was given complete running balance,

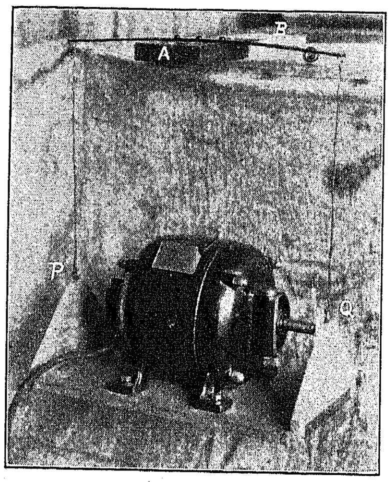

FIG. 1—MOTOR ON CRADLE SUSPENSION

it was found that all vibration of the cantilever disappeared. In short, by this method of suspension mechanical unbalance could be detected as to character and approximate magnitude. When a rotor was given no careful balance, both an up and down and a torsional motion of the cantilever were observed.

When a very carefully balanced 25-cycle motor was suspended in the cradle, the vibrations appearing as the voltage was raised were found to be always characterized by the appearance of a nodal line along the baseboard upon which no vibration could be felt. However, when the motor was placed in the cradle, this line of no vibration was found to lie centrally under the shaft, while elsewhere vibrations occurred of severity proportionate to the voltage, which increased in direct ratio with the radial distance from this nodal line. This indicated the presence of a torsional vibration through a small angle about an axis parallel to the shaft, and since a rigid body performing such a vibration must oscillate about its center of gravity, it can immediately be concluded that this axis coincides with the shaft itself. The vibration could be shown to be purely torsional with no transverse components, by observing that even with high voltages producing a very strong vibration of the base, no vibration of the cantilever could be detected with the motor placed with its axis at right angles to the cantilever as shown in Fig. 1.

When the motor was turned 90 deg. so that its axis was crosswise, a node of zero up and down vibration of the board appeared as before along the line directly under the motor shaft. The ends of the board, however, vibrated up and down, communicating their motion to the suspending cords and setting the cantilever in torsional vibration about A B as an axis.

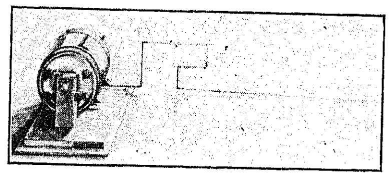

FIG. 2—MOTOR IN TRUNION SUPPORTS

Having obtained this experimental proof of the existence of a torsional vibration of the motor stator about the shaft as an axis, the next step was to find the magnitude of the vibration angle and how this angle varied with the applied voltage.

Bearing surfaces, concentric with the shaft, were turned on the outsides of the bearing housings of the motor M (Figs. 2 and 3), and the stator was then supported by these bearings in a pair of trunnions T,

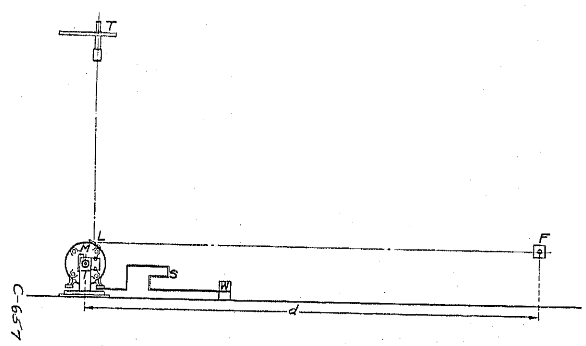

FIG. 3—MOTOR IN TRUNION SUPPORTS SHOWING METHOD OF VIBRATION MEASUREMENT

so that the entire motor could oscillate freely about the center of the shaft. Thus, angular vibrations of the stator about this axis could take place without restraint except for the spring S, whose end was held down by a weight W. This spring S yielded readily to small vibrations but kept the stator in an average position suitable for observations. On now observing the image of the lamp filament F, by means of the telescope T, after its reflection from the mirror L fixed onto the stator by means of sealing wax, a tilting of the motor caused a motion of the image of the filament in the field

of view. An angular vibration caused the filament to môve rapidly up and down so that when placed in a horizontal position and suitably illuminated it broadened out into a ribbon whose width could easily be measured on a scale in the eye piece of the telescope. Knowing the distance and the width of the ribbon, the angular amplitude of vibration was determined.

With this apparatus the angular vibration of a single-phase ¼-h. p. 25-cycle 110-volt motor was measured, using an available line frequency of 40 cycles. In Fig. 4, Curve I shows the half amplitude of the angular vibration in radians, plotted against voltage. The voltage was carried up beyond 200 volts, which could easily be done at the frequency used without overheating the motor. The triangles on Curve I show the amplitude measured as before, but

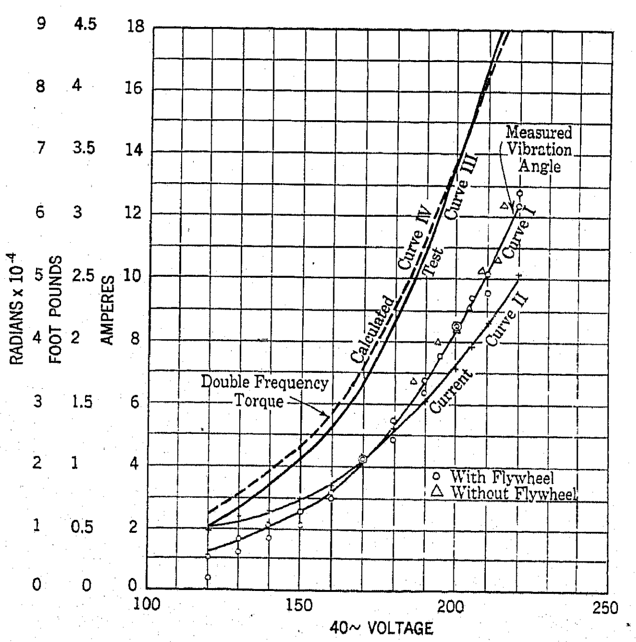

FIG. 4—NO-LOAD TESTS ON 110-VOLT MOTOR

$$T = \frac{0.00293\, I\, (V - 2.2\, I)^3}{5.8\, I^2 + (V - 0.6\, I)^2}$$

with the rotor carrying a flywheel which increased its moment of inertia over tenfold. With no flywheel, the moment of inertia of the stator was some fifteen times greater than that of the rotor. With the flywheel it was about the same. The object of this part of the test was to decrease the amplitude of the speed pulsations of the rotor, and to find if this decrease produced an appreciable increase in the torque pulsations. Comparing the circles with the triangles on Curve I, it is seen that practically no difference in the torque pulsation results when the flywheel is used. This shows that the electromagnetic effect of the variation in speed of the rotor is not great, even though the rotor have only 1/15 of the inertia of the stator.

Since the amplitude of the angular vibration of the stator is known, the amplitude of the torque pulsations may be calculated from the formula

$$T = \bar{I}\, \omega^2\, \theta_0 \qquad (1)$$

where

T = torque

$\bar{I}$ = moment of inertia of stator about axis of rotor

$$= \frac{W r^2}{g}$$

ω = $2\,\pi \times$ oscillation frequency

θ_0 = half amplitude of the vibration angle.

This equation may be derived as follows: During the oscillation the angular position of the stator varies according to the sine law, or

$$\theta = \theta_0 \sin \omega\, t \qquad (2)$$

where θ = angular position of stator at any time.

The angular velocity of motion during vibration

$$= \frac{d\,\theta}{d\,t} = \omega\, \theta_0 \cos \omega\, t \qquad (3)$$

The angular acceleration

$$= \frac{d^2\,\theta}{d\,t^2} = -\,\omega^2\, \theta_0 \sin \omega\, t \qquad (4)$$

$$\text{Therefore torque} = \bar{I}\,\frac{d^2\,\theta}{d\,t^2} = -\,\bar{I}\,\omega^2\,\theta_0 \sin\,\omega\,t$$

where the minus sign means the torque is negative, tending to restore the stator to its zero position. The amplitude of the torque variation is seen to be

$$\bar{I}\,\omega^2\,\theta_0 \qquad (5)$$

which is the required expression. In using this formula it should be noted that ω corresponds to the torque-pulsation frequency which is twice the line frequency.

Curve III shows the amplitude of torque pulsation for the motor tested as obtained from equation (5) using Curve I for values of the vibration half amplitude θ_0. This curve is seen to be derived directly from the experimental test by the use of the laws of mechanics of rotating bodies.

Curve IV shows the values of torque pulsation calculated by means of equations (24) and (28), as derived in the latter part of this paper. By these equations, the maximum half amplitude of the double-frequency torque in foot pounds is:

$$T_0 = \frac{7.04}{\text{R. P. M.}} \left[\frac{e^2\,(X_M + X_2)}{R_2^2 + (X_M + 2\,X_2)^2} \right]$$

If R_2 and X_2 are assumed negligibly small in comparison to X_M, this becomes simply $T_0 = \dfrac{7.04\, V\, \bar{I}}{\text{R. P. M.}}$ or in this case of a 2-pole motor run at 40 cycles, $T_0 = 0.00293\, E\, \bar{I}$. No-load tests on the motor gave values of 2.4 for R_1, 2.2 for X_1, 1.6 for X_2, and 1.4 for R_2. Since at no-load the numerical value of e is nearly

equal to $V - \bar{I}_1 X_1$, X_M is nearly equal to $\dfrac{(e - \bar{I}_1 X_2)}{\bar{I}_1}$

and the speed is 2400 rev. per min., the torque equation for this particular motor at no-load may be reduced to:

$$T_0 = \frac{0.00293\, \bar{I}_1 (V - 2.2\, \bar{I}_1)^3}{5.8\, \bar{I}_1^2 + (V - 0.6\, \bar{I}_1)^2} \text{ ft. lb. approximately.}$$

The actual values plotted for Curve IV were found by this equation. The agreement between Curves III and IV is sufficiently close to show the general validity of the theory.

III. Theoretical Analysis

A. On Power Flow in General. The power input to a single-phase circuit is necessarily of a pulsating character, as each time the incoming current or the voltage passes through zero, the power does likewise. No matter what the power-factor is, a single-phase a-c. circuit always draws from the line a double frequency, alternating power, equal to the full volt amperes. The "active" power is the average power input and is, of course, proportional to the

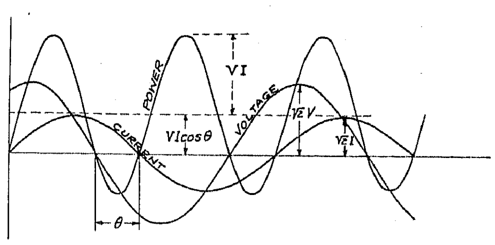

Fig. 5—Voltage and Power in a Single-Phase Circuit

power-factor. The maximum power flow is the sum of the active (or average) power, and the alternating power, as Fig. 5 shows. All this may be made clearer by analytic expression:

Suppose:

$\sqrt{2}\, V \sin \omega\, t.$ = voltage impressed on a single-phase circuit.

$\sqrt{2}\, \bar{I} \sin (\omega\, t. - \theta)$ = current flowing in a single phase.

θ = angle of lag of current behind voltage or "power-factor angle."

Then, at any instant the power flowing in the line is: $2\, V\, \bar{I} \sin \omega\, t. \sin (\omega\, t. - \theta) = V\, \bar{I}\, [\cos. \theta - \cos (2\, \omega\, t. - \theta)]$ which consists of two parts:

$V I \cos \theta$ is a uniform "active" power flowing always in one direction, and equal in magnitude to r. m. s. volts $\times$ r. m. s. amperes $\times$ power factor. $V I \cos (2\, \omega\, t. - \theta)$ is an alternating power which flows to and fro at double frequency, and whose maximum amplitude is equal to r. m. s. volts $\times$ r. m. s. amperes, independent of the power-factor. The sum of the two gives a total power which pulsates at double frequency from a maximum value in the positive direction equal to (1 + power-factor) $\times$ r. m. s. volts $\times$ r. m. s. amperes to a maximum value in the negative direction equal to (1-power-factor) $\times$ r. m. s. volts $\times$ r. m. s. amperes; as indicated in Fig. 5.

In a balanced polyphase circuit, each separate phase draws both active and alternating power as above described, but at each instant the sum of the alternating powers of the phases is zero, so that the only *net* power flowing in the circuit is the active power, distributed equally among the phases. From this point of view it is evident that the ordinary conception of reactive power as the square root of the difference of the squares of the volt amperes and the active power, has no physical reality. There is no *net* alternating power flowing in the lines, although each particular line carries an alternating power equal to its full volt amperes.

Some background of physical conceptions such as these is required if operations involving power determination are to be correctly carried through with the use of complex algebra. For, if a current is represented as $a + j\, b$ and a voltage as $c + j\, d$, referred to a common reference axis the product $(a\, c - b\, d) + j\, (b\, c + a\, d)$ is equal to the single-phase *double-frequency* alternating power and not to the (active power) $+ j$ (reactive power) as might be supposed. Being of double frequency, the phase angle of this product with reference to the current or voltage has no useful meaning, as will be seen from the fact that it changes with each change in the direction of the reference axis. To obtain the true, or active, power by multiplication, it is necessary to first reverse the sign of the j term of either voltage or current, when the product becomes $(a\, c + b\, d) \pm j\, (b\, c - a\, d)$. Here the real term represents the active power $V\, I \cos \theta$ and the j term the so-called reactive power, $V\, I \sin \theta$.

When the sign of one j term is reversed, the resulting vector product is at an angle equal to the difference of the angles of the voltage and current, and when the sign is not reversed, the angle of the vector product is equal to the sum of the voltage and current angles. As angles in this vector representation of a. c. quantities represent angular velocities times time, the former product is evidently a zero-frequency power, and the latter is a double-frequency alternating power.

B. The Energy Flow in a Single-Phase Induction Motor

Since the power inflow to a single-phase motor is pulsating, whereas the power taken by the load is uniform, the incoming energy must be alternately stored during the peaks and supplemented by released energy during the depressions. Thus, a considerable capacity for energy storage is required, and the energy stored must increase with the load. A brief consideration of the possibilities shows that this energy is stored as kinetic energy in the revolving parts, so that the torque exerted by the single-phase motor is of a pulsating character corresponding to the electrical energy input,

and the rotor speed continually fluctuates up and down in response to the torque variations, as indicated in Fig. 6.

Furthermore, even at no-load, and supposing the

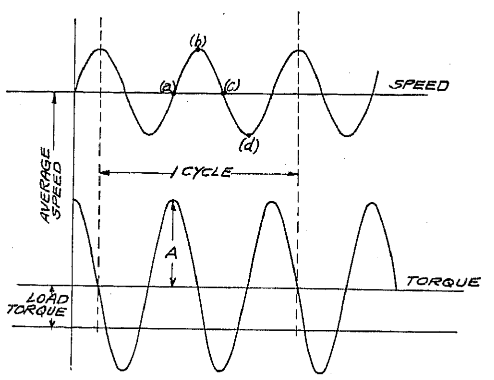

FIG. 6—DOUBLE FREQUENCY TORQUE AND SPEED VARIATIONS OF A SINGLE-PHASE MOTOR

friction torque negligible, the motor torque pulsates at double frequency, for the reason that the potential energy of the magnetic field in the cross axis is supplied entirely by the torque, since there is no direct mutual

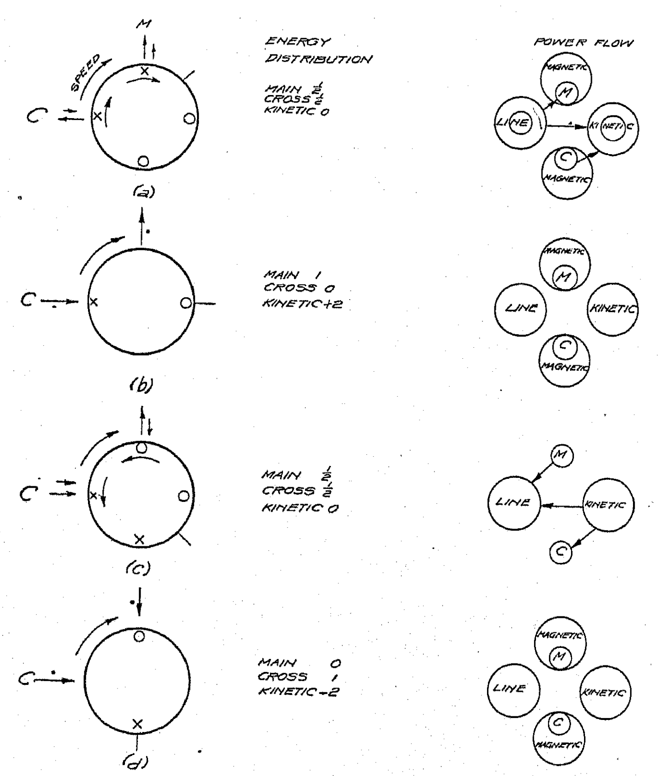

FIG. 7—ENERGY AND POWER IN A SINGLE-PHASE INDUCTION MOTOR

induction between the main and cross axis. This magnetic energy is alternately stored and released twice in each cycle and so constitutes an important cause of torque (and speed) variations entirely independent of the variations due to the load. The ratio between the double-frequency torque due to the load and that due to the cross field, is proportional to the ratio of the watts output to the magnetizing volt amperes. In very small single-phase motors the no-load torque variation is large and the additional variations due to the load are not very important, while in a very large machine the reverse is true.

In order to obtain a clear visual perception of the phenomena of torque variation at no-load it is helpful to follow through this analysis:—

Suppose that a single-phase 2-pole induction motor has zero losses and is running light at normal frequency and voltage; its speed being, therefore, exactly synchronous. Let the four circles at the left of Fig. 7 represent the rotor at four consecutive instants ⅛ cycle apart. Then, between successive instants the rotor will have advanced ⅛ revolution, as indicated by the markers at the right hand side of the circles. Also, the magnetic field will have revolved at the same rate as the rotor, so that the amplitudes of the fluxes

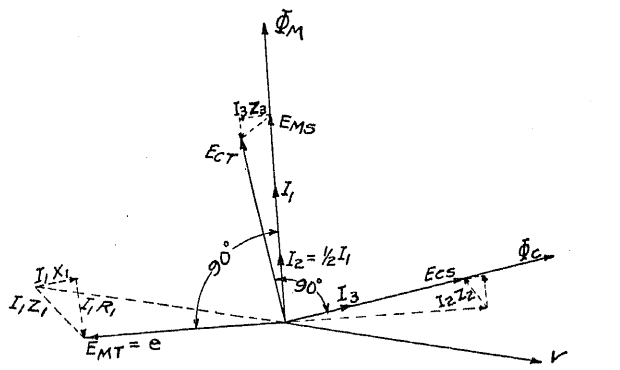

FIG. 8—NO-LOAD VECTOR DIAGRAM OF SINGLE-PHASE INDUCTION MOTOR CROSS FIELD THEORY

in the main and cross axes, respectively, will have varied in the manner shown by the arrows at the top and the left sides of the circles. The small arrows immediately beside the large flux vectors represent the rates of change of the fluxes. Thus, at *a*, the main flux is increasing slowly, at *b* it is a maximum, at *c* it is decreasing, and at *d* it is zero and is decreasing with maximum rapidity.

To determine the rotor currents at each position, it is most convenient to refer to the vector diagram, Fig. 8, and note that the cross field current, $\bar{I}_3$, is simply the magnetizing current for the cross flux, and so is always in time-phase with the latter; while the main-field rotor current $\bar{I}_2$ is in time-phase with the primary current $\bar{I}_1$, and therefore is in phase with the main flux. The *O* and *X* marks on each diagram indicate the directions of the rotor currents as thus determined. It is not convenient to determine the rotor currents by the ordinary method of dividing the resultant of the speed and transformer voltages in each axis by the rotor resistance, since under the hypothesis of zero losses, no load, the speed and transformer voltages in each axis

exactly neutralize and the current is equal to $\frac{0}{0}$, or is indeterminable.

Knowing now the currents and fluxes in each axis, the torques are readily determinable. The curved arrows inside the rotor circles indicate the directions of these torques at each instant. At *a*, the torque in each axis accelerates the rotor, at *b* both torques are zero, at *c* both torques decelerate the rotor, and at *d* both are again zero. It is, therefore, evident that the torque pulsates at double frequency, but has a zero average value. As a result of the torque variations, the rotor speed evidently pulsates also at double frequency, being a maximum at *b* and a minimum at *d*.

The exact amount that the speed fluctuates must depend on the moment of inertia of the rotor and upon the restoring torque set up as a result of the departure of the speed from synchronism. Evidently, if the speed varies 1 per cent from synchronous, the flux will cut the rotor bars at a frequency 1 per cent of line frequency, slip-frequency currents will flow in the bars, and a torque will be created equal to the load torque that would normally produce 1 per cent slip. This load torque is always in such a direction as to restore the speed to synchronous, and is proportional to the slip, so that it is always 90 deg. out of time-phase with the resultant of the other torques acting on the rotor.

The problem of calculating this resisting torque and its effects is exactly the same as that of calculating the torque reaction of a polyphase motor driving a pulsating load. This problem has been carefully studied for cases where the frequency of load pulsation was a small fraction of the line frequency, in connection with the application of motors to reciprocating compressors. The present case where the load pulsations are of double-line frequency has not been studied, and it will not be considered here. It is interesting to note, however, that for the production of the double-frequency torque of such a load a third harmonic must be drawn from the line, and so the general conclusion is reached that all single-phase motors must draw a certain amount of third harmonic current from the line, regardless of the existence of magnetic saturation. The amount of this third harmonic current should be greater, the less the normal slip of the motor and the less the inertia of the rotor. It is believed that the third harmonic current of this character drawn by commercial single-phase motors is of very small amplitude.

The experimental comparison of the fluctuations in torque with and without a flywheel on the rotor, described in part II, shows that for the motor tested, at least, the amount of this restoring torque, due to variations of the speed from synchronous, is small.

Evidently the amplitude of motion of the stator under the influence of the torque variations is inversely proportional to its moment of inertia about the shaft as an axis. At first sight, it might appear that the use of a flywheel on the rotor would increase the stator vibration, and the use of a very heavy stator would increase the fluctuations in speed of the rotor; just as a charge of powder imparts velocities to gun and projectile in inverse proportion to their weights. In this case, however, the stator and rotor vibrations are independent of each other, since the force acting is practically independent of the relative displacements; while in the case of the gun, the force between gun and projectile depends on the velocities of each.

This section of the paper has given a physical foundation upon which the abstract representation of the phenomena required for quantitative results can be based. The quantitative analysis of the phenomena is given in the succeeding sections, both the cross field and the revolving field theories being presented.

C. Analysis by the Cross-Field Theory

The cross-field theory treats the single-phase induction motor as two transformers with perpendicular axes, using the same magnetic structure. At standstill the two transformers are independent of each other, but when running, the flux of each induces in the circuit of the other a voltage proportional to the speed. The

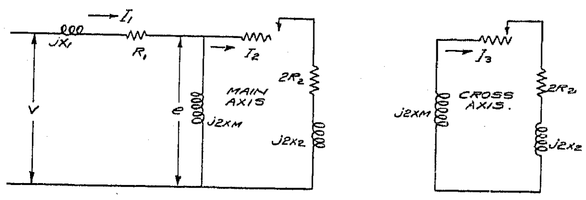

Fig. 9—Circuit Diagram. Cross Field Theory

circuit diagram and the no-load vector diagram are illustrated in Figs. 9 and 8 respectively.

Referring to Fig. 9, the operation of the motor is as follows:—an impressed voltage V creates a primary current I_1 which flows through the primary impedance, $R_1 + j X_1$ and then divides, a part, I_2, flowing through the main axis secondary winding of impedance $2(R_2 + J X_2)$, and the remainder through the magnetizing impedance, $j\,2\,X_M$. At standstill, I_1 and I_2, are nearly equal, and are both large, while there is no current in the cross axis whatever. When the rotor is started by some external means, the main flux induces a speed voltage in the cross axis equal to $-j\,(1 - s)\,e$, and this voltage produces a current in this axis; I_3. As the impedance of the cross field circuit consists almost entirely of reactance, I_3 lags nearly 90 deg. behind the speed voltage producing it. I_3 flows through the cross-field magnetizing reactance, $j\,2\,X_M$, which is equal to that of the main field, and so produces a cross flux and a proportional speed voltage in the main axis, equal to $2\,I_3\,X_M\,(1 - s)$. This last voltage is nearly in phase opposition to e, and so, as the speed rises, the available voltage in the main axis falls, until at full speed, no load, I_2 is practically equal to only $\frac{1}{2}\,I_1$. I_3

is always small, being a maximum at no-load, full-speed, and is then nearly equal to I_2.

In the circuit of Fig. 9, the constants $2\,R_2$, $2\,X_2$ and $2\,X_M$, of the cross field are, by symmetry, equal to the corresponding constants of the main field, and both are equal to the constants of the secondary as directly obtained from standstill impedance tests. Thus $2\,R_2$, $2\,X_2$ and $2\,X_M$, are each equal to the corresponding constants for one phase of a two-phase motor, or to twice the corresponding values for the forward revolving field of a single-phase motor, and this explains the insertion of the factor 2 in all quantities. In the revolving field theory, which is discussed in the next section, R_2, X_2, and X_M will be used directly.

A simple way of visualizing the reason for the existence of this factor 2 is to remember that when a squirrel cage secondary winding is subjected to an alternating flux along one axis only, each pair of bars placed symmetrically with respect to this axis will link a different amount of flux. Assuming the flux to be sinusoidally distributed, in space as well as in time, the induced currents in the rotor will be very small in the bars nearest the axis and, at any instant will vary sinusoidally thence to a maximum in the bars furthest from the axis. The rotor currents are, therefore, concentrated in the outer bars, and the total copper loss is twice as great as if all the bars had the full voltage induced in them and all carried equal currents.

Proceeding now to the actual analysis, by applying Kirchoff's Laws to the circuits of Fig. 9, we can obtain expressions for the currents and voltages as follows:

$$I_3 = \frac{-j\,(1-s)\,(e - j\,2\,I_2\,X_2)}{2\,(R_2 + j\,X_2 + j\,X_M)} \tag{6}$$

$$I_2 = \frac{e + 2\,I_3\,(X_M + X_2)\,(1-s)}{2\,(R_2 + j\,X_2)} \tag{7}$$

By solving (6) and (7) simultaneously, we find:

$$I_3 = \frac{-j\,(1-s)\,e\,R_2}{2\,[\,(R_2 + j\,X_2)\,(R_2 + j\,X_2 + j\,X_M) + (1-s)^2\,X_2\,(X_M + X_2)\,]} \tag{8}$$

$$I_2 = \frac{e\,[R_2 + j\,X_2 + j\,X_M - j\,(1-s)^2\,(X_M + X_2)\,]}{2\,[\,(R_2 + j\,X_2)\,(R_2 + j\,X_2 + j\,X_M) + (1-s)^2\,X_2\,(X_M + X_2)\,]} \tag{9}$$

Also, $I_1 = I_2 + \dfrac{e}{2\,j\,X_M} =$

$$\frac{-j\,e\,[R_2^2 + 2\,j\,R_2\,(X_M + X_2) - s\,(2-s)\,(X_M + X_2)^2]}{2\,X_M\,[R_2^2 + j\,R_2\,(X_M + 2\,X_2) - s\,(2-s)\,(X_M + X_2)]} \tag{10}$$

$$e = V - I_1\,(R_1 + j\,X_1) \tag{11}$$

The output in watts produced by the torque in each axis is equal to the product of the speed voltage induced by it times the current in the other axis. The outputs in the two axes are, accordingly:

$$W_M = I_3\,[j\,(1-s)\,(e - j\,2\,I_2\,X_2)\,] \tag{12}$$

$$W_c = I_2\,[-\,2\,(1-s)\,I_3\,(X_M + X_2)\,] \tag{13}$$

As has already been pointed out in part *I*, a single-phase power equation such as (12) consists of two distinct parts, the double-frequency alternating power, and the zero-frequency, constant power. These two parts must be treated separately. The steady power is found by reversing the signs of the j terms of the current (or voltage) and taking the real part of the resulting product; and the alternating power is the total value of the product of current and voltage. We will first find the active power, by reversing the signs of the j terms of I_3 in (12) and I_2 in (13) and performing the multiplications indicated.

$$W_M = -\,2\,(R_2 + j\,X_2 + j\,X_M)\,I_3^2 = \frac{-\,(1-s)^2\,e^2\,R_2^2\,(R_2 + j\,X_2 + j\,X_M)}{2\,[R_2^4 + R_2^2\,(X_M + 2\,X_2)^2 - 2\,s\,(2-s)\,R_2^2\,X_2\,(X_M + X_2) + s^2\,(2-s)^2\,X_2^2\,(X_M + X_2)^2\,]} \tag{14}$$

and from (7), (9) and (13):

$$W_c = I_2\,[e - 2\,I_2\,(R_2 + j\,X_2)\,] = \frac{e^2[s(2-s)(1-s)^2R_2(X_M+X_2)^2 + j(1-s)^2R_2^2(X_M+X_2)]}{2\,[R_2^4 + R_2^2\,(X_M + 2\,X_2)^2 - 2\,s\,(2-s)\,R_2^2\,X_2\,(X_M + X_2) + s^2\,(2-s)^2\,X_2^2\,(X_M + X_2)^2\,]} \tag{15}$$

The real part of (14) is evidently simply equal to the copper loss of the cross field magnetizing current. This is a decelerating torque which must be subtracted from the real part of (15) to obtain the net watts output. The j parts of (14) and (15) have no interest for us here. Adding the real parts of (14) and (15) algebraically, therefore, we obtain the net watts output:

$$\overset{\text{real}}{W_c + W_M} = \frac{(1-s)^2\,e^2\,R_2\,[s\,(2-s)\,(X_M + X_2)^2 - R_2^2.]}{2\,[R_2^4 + R_2^2\,(X_M + 2\,X_2^2)^2 - 2\,s\,(2-s)\,R_2^2\,X_2\,(X_M + X_2) + s^2\,(2-s)^2\,X_2^2\,(X_M + X_2)^2\,]} \tag{16}$$

The synchronous torque is obtained by dividing the output by $(1-s)$, since the speed of the rotor is $(1-s)$ times the synchronous speed.

By making various approximations, equation (16) can be put into several useful forms. Evidently, the output is zero at standstill, when $s = 1$. If R_2 is negligibly small in comparison with X_2 and X_M, (16) reduces to:

$$\text{Output} = \frac{R_2}{2\,X_2^2}\;\frac{(1-s)^2}{[1 - (1-s)^2]}\,,\ R_2 \text{ very small}$$

At no load, the output is zero, neglecting the friction and windage losses, and (16) reduces to

$$S = \frac{R_2^2}{2\,(X_M + X_2)^2}\ \ldots \text{ at no load} \tag{17}$$

The maximum output under the assumption of constant e (constant terminal voltage and negligible primary impedance) occurs when (16) is a maximum with respect to s, or when:

$$(1-s)^4 X_2 (X_M + X_2)^2 [X_2 (X_M + X_2)^2 - 2 R_2^2 (X_M + X_2) - 2 R_2^2 X_2] - (1-s)^2 (X_M + X_2)^2 [2 (R_2^2 + X_2^2)(X_M + X_2)^2 R_2^2 (2R_2^2 - X_2^2)] + (R_2^2 + X_2^2)[(X_M + X_2)^4 - R_2^4] = 0 \quad (18)$$

Equation (18) is of the 2nd degree in $(1-s)^2$, so that there are in all 4 values of s that give maximum points. Two of these are for forward and 2 for backward rotation, corresponding to maximum output as a motor and as a generator. To the first approximation, (13) may be solved by assuming X_M to be very large, when:

$$(1-s)^2 = 1 + \frac{R_2^2}{X_2^2}\left(1 + \frac{2 X_2}{X_M} + \ldots\right) \pm \frac{R_2 Z_2}{X_2^2}\left(1 + \frac{X_2}{X_M} + \ldots\right) \quad (19)$$

the upper signs corresponding to generator, and the lower to motor operation.

On the same basis of X_M infinite and e constant, the substitution of (9) in (16) gives for the maximum output as a motor:

$$W_{max.} = \frac{e^2 X_M}{4 (R_2 + \sqrt{R_2^2 + X_2^2})(X_M + X_2)} \quad (20)$$

The corresponding value of maximum output for one phase of a two-phase motor is obtained by making X_M infinite in equation (20), since this is the same thing as making I_3 = zero, and hence making the motor the same as one half of a two-phase motor.

Turning our attention now to the alternating power, which is equal to the numerical product of volts times amperes in each axis, the first question to settle is the relative time phase of the alternating torques in the two axes. Since equations (12) and (13) have I_3 as a common factor, the relative time phases are determined by the other factors in the equations alone. Thus, the total alternating power delivered to the rotor is given by the sum of (12) and (13), or:

$$A = \text{Alternating Power} = I_3(1-s)(-2 I_2 X_M + j e) \quad (21)$$

Or, by (8) and (9):

$$A = \frac{(1-s)^2 e^2 R_2 [(R_2^2 + 2 j R_2 (X_M + X_2) - s(2-s)(X_M + X_2)^2]}{2 [R_2^2 + j R_2 (X_M + 2 X_2) - s(2-s) X_2 (X_M + X_2)]^2} \quad (22)$$

In this case, we desire to find the total power only, and do not care about its time phase, so that we must find the square root of the sum of the squares of the real and j parts of (22):

$$A = \frac{(1-s)^2 e^2 R_2 \sqrt{R_2^4 + [s^2 + (2-s)^2] R_2^2}}{2 [R_2^4 + R_2^2 (X_M + X_2)^2 - 2 s (2-s) R_2^2 X_2} \; \frac{\overline{(X_M + X_2)^2 + s^2 (2-s)^2 (X_M + X_2)^4}}{(X_M + X_2) + s^2 (2-s)^2 X_2^2 (X_M + X_2)^2]} \quad (23)$$

At standstill, when $s = 1$, (23) reduces to zero, so that the starting torque of a single-phase motor is zero at every instant, as well as having a zero average value.

At no load, the slip is given by (17), and substituting this value for s in (23), we have, very closely:

$$A \text{ at no load} = \frac{e^2 (X_M + X_2)}{R_2^2 + (X_M + 2 X_2)^2} \quad (24)$$

As the slip increases, A also increases to a maximum value at approximately the same slip that gives maximum useful torque, and thence decreases to zero at standstill.

By dividing (17) by (11) the ratio of alternating power to real power is found to be:

$$\frac{A}{\text{Output}} = 1 - \frac{2 j R_2 (X_M + X_2)}{s(2-s)(X_M + X_2)^2 - R_2^2} \quad (25)$$

Thus the maximum instantaneous alternating power is never less than the output.

We are now ready to proceed further with the physical interpretation of these results. All the values of alternating power given in equations (26) to (24) represent maximum instantaneous values of double-frequency power. This power is expended in alternately accelerating and decelerating the revolving parts. The maximum speed to which the rotor is accelerated is such that the total energy supplied by the reactive power in one half cycle is stored in the increased kinetic energy. Thus, if R is the radius of gyration in feet, and W is the weight of the revolving parts in pounds, the ratio of the maximum rotor speed to the average is:

$$\frac{\text{R. P. M. max.}}{\text{R. P. M. av.}} = \sqrt{1 + \frac{689 A}{f (R P M)^2 W R^2}} \quad (26)$$

where A is the maximum alternating power in watts from (23) or (24), and f is the line frequency in cycles per second. If the rev. per min. are assumed to be synchronous, (26) may be expressed in terms of the number of poles of the motor, as follows:

$$\frac{\text{R. P. M. max.}}{\text{R. P. M. av.}} = \sqrt{1 + \frac{.0478 A \text{ (poles)}^2}{f^3 W R^2}} \quad (27)$$

Since the total variation in speed is very small, equations (26) and (27) may also be expressed as follows:

$$(\text{R. P. M. max.} - \text{R. P. M. av.}) = \frac{345 A}{f (\text{R. P. M.})^2 (W R^2)} \quad (26)$$

$$= \frac{.0239 A \text{ (poles)}^2}{f^3 W R^2} \quad (27)$$

The maximum torque exerted on the rotor in addition to the average, or load, torque, is:

$$\text{Max. Alternating Torque} = \frac{7.04 A}{\text{R. P. M.}} \text{ foot lbs.} \quad (28)$$

As pointed out in the introduction to this chapter, as

soon as the rotor departs from its normal speed, a restoring torque is set up, tending to restore the motor to its average speed, as a result of the change in the fundamental torque due to the changed slip. This restoring torque must be of double frequency and therefore, as the impressed voltage is only of single frequency, the line current must contain third and other harmonics in addition to the fundamental. The problem of calculating the fundamental restoring torque of a motor for impressed oscillations of low frequency has been quite completely solved, but this case of double-frequency impressed oscillations is a more difficult one that has not yet been considered.

By considering equation (**27**), it may be seen that for a single-phase motor of given electrical characteristics (A fixed), the pulsations of speed are greater, the lower the frequency, the lower the value of $W R^2$, and the greater the number of motor poles. The value of A is also greater the greater the number of poles for a given output, so that all low-speed motors of a given frequency are more subject to vibration than high-speed motors.

D. Analysis by the Revolving Field Theory. In the revolving field theory, the single-axis alternating flux set up by the primary current is resolved into two equal and oppositely revolving components. The forward component cuts the rotor conductors at slip frequency and induces in them useful currents similar to those in a polyphase induction-motor secondary. The backward field cuts the rotor conductors at a frequency $(2 - s)$ times line frequency and so induces in them large currents which practically neutralize the primary current and so reduce the net backward flux to a very small value. As the two fields are produced by the same primary current, their circuits are connected in series. The only difference in their constants is due to the difference in secondary frequency, making the apparent secondary resistance $\frac{R_2}{s}$ for the forward field and $\frac{R_2}{2 - s}$ for the backward field.

Referring to the circuit diagram of Fig. 10, the equations are:

$$I_4 = \frac{j\, I_1\, X_M}{R_2/_s + j\,(X_M + X_2)} \qquad \textbf{(29)}$$

$$I_5 = \frac{j\, I_1\, X_M}{\frac{R_2}{2 - s} + j\,(X_M + X_2)} \qquad \textbf{(30)}$$

$$e = j\, X_M\,(2\, I_1 - I_4 - I_5), \text{ or} \qquad \textbf{(31)}$$

$$e = j\, 2\, I_1\, X_M \left[\frac{R_2^2 + j\, R_2(X_M + 2\, X_2) - s\,(2 - s)\, X_2\,(X_M + X_2)}{R_2^2 + 2\, j\, R_2(X_M + X_2) - s\,(2 - s)\,(X_M + X_2)_2} \right] \qquad \textbf{(32)}$$

and $e = V - I_1\,(R_1 + j\, X_1)$.

Equation (**32**) is identical with the corresponding equation (**10**) found by the cross-field theory.

Since the forward and the backward fields exist in a common magnetic structure, each flux makes a torque with each current, or there are in all four separate torques.

The torque produced by the forward revolving flux with I_4 is of constant magnitude, independent of the time; as is also the torque produced by the backward field with I_5. This is true for the reason that we are now considering fluxes of constant magnitude in time but moving in space, whereas in the cross-field theory we considered the flux at one particular position in space, but varying with time. Thus the meaning of j in the torque equations is a rotation of 90 deg. in space, instead of in time.

The principal, or load, torque is that due to the action of the forward revolving flux and I_4. The watts developed by this torque at synchronous speed are equal to:

$$T_{44} = j\,(I_1 - I_4)\, X_M\, I_4 = I_4^2 \left(\frac{R_2}{s} + j\, X_2 \right) \qquad \textbf{(33)}$$

As noted above, the j terms here only measure the

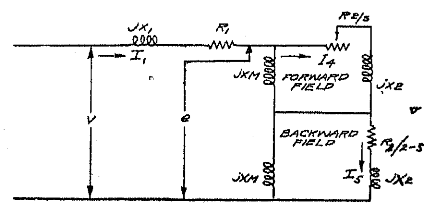

Fig. 10—Cihcuit Diagram. Revolving Field Theory

phase displacement in space, and so may be dropped without error. Therefore, the steady synchronous torque is, from (**29**) and (**33**):

$$T_{44} = \frac{e^2\, R_2\, s\,[R_2^2 + (2 - s)^2\,(X_M + X_2)^2]}{4[R_2^4 + R_2^2(X_M + 2X_2)^2 - 2s(2-s)R_2^2X_2(X_M + X_2) + s^2\,(2 - s)^2\, X_2^2\,(X_M + X_2)^2\,]} \qquad \textbf{(34)}$$

At standstill, when $s = 1$, (**34**) reduces to:

$$T_{44} = \frac{e^2\, R_2}{4\,(R_2^2 + X_2^2)} \text{ at standstill} \qquad \textbf{(35)}$$

Similarly, the synchronous torque due to the backward field and I_5 is:

$$T_{55} = j\,(I_1 - I_5)\, X_M\, I_5 = I_5^2 \left(\frac{R_2}{2 - s} + j\, X_2 \right) \qquad \textbf{(36)}$$

Dropping the j terms in (**36**), and eliminating I_5 by (**30**) and (**32**):

$$T_{55} = \frac{e^2\, R_2\,(2 - s)\,[R_2^2 + s^2\,(X_M + X_2)^2\,]}{4[R_2^4 + R_2^2(X_M + 2X_2)^2 - 2s(2-s)R_2^2X_2(X_M + X_2) + s^2\,(2 - s)^2\, X_2^2\,(X_M + X_2)^2\,]} \qquad \textbf{(37)}$$

When $s = 1$, (**37**) also reduces to (**35**), so that the backward and forward field torques are equal and opposite at standstill. As T_{44} and T_{55} are both invariant in time, the conclusion is reached that the standstill torque is zero at every instant, as well as having a zero average value, as already found by the cross-field theory.

The net constant torque in synchronous watts is equal to the difference of (**34**) and (**37**), or to:

$$T_{44} - T_{55} = \frac{e^2 R_2 (1 - s) [s (2 - s) (X_M + X_2)^2 - R_2^2]}{2[R_2^4 + R_2^2 (X_M + 2X_2)^2 - 2s(2-s) R_2^2 X_2 (X_M + X_2) + s^2 (2 - s)^2 X_2^2 (X_M + X_2)^2]} \qquad (38)$$

Equation (**38**) has the same value for any particular value of s as it has for the same value of $2 - s$, though the sign is reversed, so that the motor-torque curve is symmetrical about the point of zero speed. The output is equal to $(1 - s)$ times (**38**), which checks equation (**16**) previously found.

At no load, or running light, (**38**) is equal to zero, neglecting the friction and windage losses, so that

$$S = \frac{R_2^2}{2 (X_M + X_2)^2} \text{ at no load} \qquad (17)$$

The maximum output is given by (**20**), of the cross-field theory solution, and it occurs at the slip given by equation (**19**).

To calculate the alternating power, we must take the sum of the products of the backward field voltage by I_4 and the forward voltage by I_5. Since it is now only a question of alternating power, the phase relations of I_4 and I_5 with the fluxes they act upon are of no importance, but the phase relations of the two torques they make are important.

The voltage representing the forward flux is, from (**31**) and (**32**):

$$e_4 = j X_M (I_1 - I_4) = I_4 \left(\frac{R_2}{s} + j X_2 \right) \qquad (39)$$

and that representing the backward field is:

$$e_5 = j X_M (I_1 - I_5) = I_5 \left(\frac{R_2}{2 - s} + j X_2 \right) \qquad (40)$$

Since the relative speeds of I_4 and e_5 and of I_5 and e_4 are twice synchronous speed, the torques made by their interaction are double-frequency torques. The synchronous double-frequency torque is:

$$A = T_{45} - T_{54} = e_4 I_5 - e_5 I_4 = \frac{2 I_4 I_5 (1 - s) R_2}{s (2 - s)} \qquad (41)$$

These alternating torques are subtractive rather than additive for the same reason that the active torques are, as may be seen by comparing equations (**33**) and (**36**) with (**39**) and (**40**). This conclusion is confirmed by substituting $s = 1$ in (**41**), when it reduces to zero, as it should. Evaluating (**41**) by means of (**29**), (**30**) and (**32**), it becomes

$$A = \frac{(1 - s)\, e^2 R_2 [R_2^2 + 2 j R_2 (X_M + X_2) - s (2 - s) (X_M + X_2)^2]}{2[R_2^2 + j R_2 (X_M + 2X_2) - s(2-s) X_2 (X_M + X_2)]^2} \qquad (42)$$

Equation (**42**) is identical with (**22**) except for the factor $(1 - s)$, which appears in (**22**) but not in (**42**), as the former is actual watts output available for accelerating and decelerating the rotor, while the latter is the *synchronous* torque, or the watts output that would exist if the electrical conditions were the same and the speed was synchronous.

Starting from (**42**), then, equations (**23**) to (**28**) may be derived just as was done in the last chapter, and the same conclusions as to the effect of rotor inertia, line frequency, etc., on the torque pulsations will be reached.

IV. Conclusions

Since this double-frequency torque variation is characteristic of the single-phase motor, the only way to avoid noise in operation is to so mount the stator as to allow free torsional vibration about the shaft as an axis, without imparting the vibration to the supporting structure. The ideal mounting consists of trunnions supporting the stator through bearings concentric with the shaft. Such a mounting is quite expensive, however, and it is difficult to satisfactorily restrain the normal torque of the motor load without communicating the torque variations to the base. In general, therefore, it is best to mount the stator firmly on a support which is sufficiently flexible to take up the motion of the stator without itself producing noise.

A good example of such a mounting is a long thin board of the same width as the motor base. If the motor is affixed to this board with the shaft lengthwise, quiet operation is secured, as the board twists readily with the motion of the stator and yet does not transmit any motion to its end supports. If the motor is placed with its shaft crosswise of the board, a considerable amount of noise is produced, as the motor alternately lifts up and pushes down on the ends of the board, thus transmitting vibration through the whole structure.

Similarly, a flexible coupling on the shaft is required if gears are used, as otherwise the variations in torque and in rotor speed cause the gears to rattle. Common sense applied to each particular application in the light of a knowledge of the cause of the trouble will usually yield an easy solution of the difficulty.

A Generation of the American Institute of Electrical Engineers---1884-1924

President's Address

BY HARRIS J. RYAN

Leland Stanford, Jr., University, California

THIS is the fortieth year of the Institute. A charter member who was twenty-five when it was organized is now sixty-five and his work has been substantially completed. While the American Institute of Electrical Engineers must endure as long as the Nation, no succeeding generation can encounter the unique experiences of its charter members. They were young with a wilderness of options for action before them. They had the mentality of early manhood, boundless energy and enthusiasm and a wonderful unity of purpose.

When the Institute began, the land telegraph had been in use forty years, the Europe-to-America submarine telegraph cable eighteen years, the direct-current generator twelve, the telephone eight, the arc light about five, and the incandescent lamp had been used three years. The direct-current motor had just arrived, and the alternator, transformer, synchronous and induction motors were soon to follow. New and wonderful electrical expediencies were arriving at a rate that was bewildering for the "man in the street." This acceleration of electrical progress took a strong hold on the imagination of our people. The boys who visited the Centennial Exposition at Philadelphia in 1876 reacted powerfully as boys always have and always will in regard to new things. They went home to play with batteries, magnets, telegraphs and telephones and a little later constructed direct-current dynamos and motors as the boys of today build radio equipment.

Some of the boys of 1880 quickly mastered an understanding of the critical speed of a direct-current shunt generator that had cost Edison and Hopkinson hours of intense mental effort. For the most part they were the men who, four years later, organized the Institute. Their wonderful work is going on today and always will. In the great turning movement that education is now making, the young men and women of 1884 are today helping their children to give the grandchildren, who are the boys and girls of today, a greater opportunity to see for themselves the things of highest interest and greatest worth-whileness for knowing and doing.

The "why not" spirit of the youth who founded the Institute is splendidly reflected in the frontispiece of Volume I of the TRANSACTIONS. It is lithographed in colors and labeled "American Institute of Electrical Engineers, September 1884. The Scientific Street as Applied to Broadway, New York."

It is recorded in the beginning of Volume I of the TRANSACTIONS that "The first steps toward the formation of the Society were taken in April, 1884," when a circular was prepared and published by Dr. Nathaniel S. Keith of New York City, inviting signatures to the proposal to organize "The American Institute of Electrical Engineers." The subscribers held a meeting in the rooms of the American Society of Civil Engineers in New York on May 13, 1884, adopted rules of procedure and elected officers for the ensuing year. Thus the American Institute of Electrical Engineers began forty years ago. The membership rapidly increased to 1000 and thereafter grew steadily to more than 15,000 today.

The first communication received and printed in the TRANSACTIONS was a letter from Mr. C. J. Kintner, Examiner, Class Electricity, U. S. Patent Office dated at Washington, D. C., May 12, 1884. He expressed his regret that he would not be able to attend the organization meeting of the Electrical Engineers and begged them to come to the rescue of the Patent Office which was in a deplorable state of neglect on the part of Congress. He invited helpful attention to the contents of an attached copy of an official report on the situation which revealed that his division was undermanned and housed in inadequate quarters poorly ventilated and lighted. Of his own office room he said there was no gas and that he was obliged to use kerosene oil which seriously vitiated the atmosphere. This was the state of things when inventors had paid for the maintenance of the patent office $2,500,000 in excess of operating expenses.

Thus, in the first hour of its existence, the American Institute of Electrical Engineers was forcefully made to take note of the fact that this is an ever practical world and that vital things are never what they should be if the men most concerned fail to make them so.

Without patents properly provided for, The American Institute of Electrical Engineers could not have been maintained as a forum for the reception of new ideas as to facts, principles and processes. The only alternative would have been procedure of individuals, or of small groups of individuals, bound to secrecy for the protection of their equities and rights.

The history of the ceramic arts is a specific illustration. Porcelain has been known for centuries, nevertheless the porcelain arts have advanced much more in the few years in which ceramists have abandoned secrecy and have depended upon the United States patent system for their protection, and upon their own cooperation through the American Ceramic Society.

The longing of the people of our Country to have an

opportunity to come more intimately into contact with vast numbers of new things electrical in 1883, was clearly understood by the management of the Franklin Institute of Philadelphia. The home of the renowned Franklin had been in Philadelphia. In 1883 this city was also the home of two high school teachers who were becoming known throughout the world for their electrical achievements. In the circumstances, it was decided to hold the International Electrical Exposition of 1883 in Philadelphia under the auspices of the Franklin Institute.

The determination of the young men engaged in the electrical sciences and industries to organize a national society of electrical engineers was due in no small measure to this action of the Franklin Institute as may be learned by the following quotation from the circular that called the organization meeting: "An International Electrical Exhibition is to be held in Philadelphia next autumn, to which many of the foreign electrical savants, engineers and manufacturers will be visitors; and it would be a lasting disgrace to American electricians, if no American national electrical society was in existence to receive them with the honors due them from their co-laborers in the United States." Nothing was more natural, therefore, than to hold the first meeting of the Institute in Philadelphia for the presentation and discussion of papers in response to a cordial invitation from the Franklin Institute as recorded also in the early pages of Volume I of the TRANSACTIONS, as follows: "The Directors of the International Electrical Exhibition, Philadelphia, held under the auspices of the Franklin Institute, having kindly tendered to the Institute the free use of rooms in the exhibition building, so long as it should remain open, the offer was accepted by the Council, and the secretary was authorized to make the necessary preparations and to be present during the Exhibition as the representative of the Institute.

"The rooms were accordingly opened to members by Mr. Keith on September second, and remained open until October eleventh, during which time they were visited by several foreign electricians. On October seventh and eighth a meeting was held, in accordance with the rules, at the Continental Hotel and the Exhibition rooms, where the papers forming this volume were read and discussed. On the invitation of the Franklin Institute, the Exhibition was also inspected by the members in a body."

To Professor Edwin J. Houston belongs the never-to-be-forgotten honor of presenting to the Institute the first paper. It bore the modest title, "Some Notes on Incandescent Lamps" and dealt with the now universally known and highly valued "Edison effect." Referring to "the peculiar high vacuum phenomenon observed by Mr. Edison in some of his incandescent lamps, Prof. Houston said: "I wish to bring it before the Society for the purpose of having you puzzle over it." What a glorious result has come of "the puzzling." Little could those present have appreciated on the memorable morning of October seventh, 1884, in Philadelphia, the fact that some of them during this fortieth anniversary would witness through applications of the Edison effect the transmission of the human voice from airplane to airplane in flight and over land and ocean to the uttermost parts of the earth; that such Edison effect was due to a reaction at last of the long-sought-for electricity, that the effect was a characteristic reaction of matter within its own structural make-up and that it would be the means of producing an endless succession of most helpful facilities for research and progress.

Sir William Preece of England was present at the meeting and joined in the discussion of the Edison effect. He brought forward the fact that Dr. Oliver Lodge had recently demonstrated that fogs resulted from the formation of moisture around electrical nuclei. "He took a large glass globe, apparently chemically clean, exhausted the air, but with moisture in it. It remained perfectly transparent so long as no matter was admitted, but the moment a spark from an induction coil was passed between the two platinum electrodes in this apparently clean globe at once a cloud was formed throughout the whole globe."

It was nearly thirty years before important results were developed out of the Edison effect. The fortieth year is now passing since the fog forming nuclei were reported to the Institute and almost nothing has been done about it. What wonderful uses may yet be made of the fog effect in bringing water from the skies and in the control of fogs in places where they render traffic hazardous.

The overhead conductors to provide for the rapidly growing telephone and telegraph service in the business districts of our cities began to obscure the sky and to make city dwellers positively unhappy, besides being in fear of their lives because of the difficulties the conductors caused the fire department in fighting fires. An intense longing to get rid of overhead conductors possessed all city residents. It is natural, therefore, to find that the second paper presented to the Institute at the Philadelphia convention bore the title "Underground Wires," by W. M. Calender.

In 1884 many were living who had witnessed the almost desperate struggle, and final failure of Professor Morse, Alfred Vail and Ezra Cornell to install underground the first land telegraph between Baltimore and Washington. The enterprise was finally saved in 1884 by mounting the telegraph conductor in the air upon insulators supported on wood poles substantially as is the practise of today. These people had also witnessed the heart-breaking difficulties encountered by Cyrus W. Field and his co-workers in the Atlantic telegraph cable laying operations of 1858 and 1865, that finally culminated in complete success in the summer of 1866. As a result of such pioneer experiences there has existed almost to this day a mental momentum that has continued to place conductors overhead when they more

properly belonged underground. The technical and economic difficulties have always been many. In telephonic as well as telegraphic communication on land the technical difficulties have now been overcome and the corresponding economic boundaries have been located with the result that at a comparatively early date all our great cities will be inter-connected with underground cables in the more densely populated areas of the United States and will no longer have their telephonic and telegraphic communications broken by severe winter storms.

Interconnection for power service is encountering much the same sort of progress in relation to underground conductors. Economic boundaries in the power service for the City of Cleveland and its environs have determined the use within and without the city of huge coal burning steam-power generating stations. The problem of a heavy power connection between such stations within the city limits has been solved by the use of an 8-mile section of 66,000 volt, three-phase underground cable.

Synchronism was the title of the third paper received by the Institute at its first convention. The value and remarkable possibilities of synchronism in communication was stressed and carefully defined. The expedients available in 1884 for maintaining synchronism at distant points in the electric circuit and of obtaining valuable results by means thereof were reported upon. It is of genuine interest today to note that after forty years, advances in physical chemistry and assiduous study of talented men well organized, have so improved and extended these synchronizing facilities in their application to telephotography that extraordinary success has been obtained. Because the telephotography of today when speeded up a few thousand times will result in "seeing at a distance" the usual person entertains the belief that "teleopto" will soon be an accomplished fact.

Other papers presented at the first convention carried the titles:

The Scientific Street,

Experimental Method for Testing a Dynamo Machine,

The Earth as an Electric Circuit Completer,

Telegraphy without Wires,

Chemistry of the Carbon Filament,

The Patent Office.

Of these, The Scientific Street and The Earth as an Electric Circuit Completer have surely made the least progress during the forty years that have past. A recent electrical engineer-graduate from one of our universities, was employed in municipal power service and assigned the job of planning a power feeder to run from the main receiving substation to a heavy demand district of a new and rapidly growing city. He encountered many trouble-making factors, among which were the heating limits of the underground cables that he would have to use and the heat liberation limitations of the ducts and adjacent earth in which such cables would have to be placed. He had never seen Volume I of the TRANSACTIONS, and had never heard of The Scientific Street nor of anything that had ever been done about it, nor had he ever heard of such a thing as organization red tape. What he knew was that years before the Chief of the municipal water supply had run a huge water main through the center of the street in which the power feeder would have to go. To the young man's mind the obvious thing to do was to place his power feeder just as close to the Chief's water main as possible, thermally connecting thereto in a 100 per cent fashion if possible, so that water in the main would carry away the heat from the power cable, a plan that would surely do away with the heating difficulty.

The young man had already discovered that most things in this world are "nailed down" and assumed, therefore, that he would have to get permission from somebody to discharge the heat from the power cable into the water main. It never occured to him that he would have to go to his immediate superior in the power bureau who would say "no" or in turn would pass the matter up to the chief of the power bureau who would in turn say "no," or broach the matter delicately to the chief of the water supply bureau who might say "no" or who would designate someone in his organization to look into the matter and report with recommendations. It occurred to the young man forthwith that he should be able to agree with the Chief of the water bureau upon the highly desirable plan in ten minutes and it should not be necessary to encounter any further "fuss and feathers." He called on the Chief of the water bureau, made a direct and immediate presentation of the proposition and thereby started a series of tremors in the two bureaus that have not altogether damped out to this day; and one must hope they never will, for when things tremble they slide more easily and that is often a helpful state on the road of progress. The young man demonstrated that the incoming generations can always be counted on to renew the "why not" spirit of the charter members of the Institute unless China's mistake is made by their successors of keeping the young people so occupied by the mental activities prescribed by their ancestors that no opportunity is permitted for activity on their own initiative.

Of the subjects brought to the attention of the members of the American Institute of Electrical Engineers at its first convention "The Earth as an Electric Circuit Completer" by Thomas D. Lockwood was the most important and has progressed the least through the intervening years. Pupin has recently taught us beautifully to have high regard for all terrestrial electrical phenomena.[1]

Modern radio and all related phenomena are interpreted in terms of the electromagnetic theory of light as formulated by Maxwell. Yet Maxwell never knew of the existence of electrons. Because it is now known

1. From Immigrant to Inventor, by Michael Pupin, 1923, pp. 301-302.

that every electric field is an aggregate of the elemental electric field fragments attached to their corresponding electrons, and that every electric field in motion is thereby also a magnetic field, it follows that there can be no electromagnetic activity without a corresponding activity of the electrons to which the electric field in action is attached. It is no wonder then that Pupin, the renowned physicist and radio pioneer, had to discover through Marconi's actual trial the importance of the ground connection in radio transmission and reception, and that he was not led thereto by Maxwell's theory.

In a classical experiment Michelson determined the velocity of light in the direction that the earth is moving around the sun and again in the opposite direction and found the two velocities to be identical. To account for this unexpected result some able persons have resorted to certain extraordinary assumptions. Pending the outcome of the far-flung attack of modern physics upon the problem of the fundamental character of light it should be far more helpful to remember the never failing presence of electrons in the earth's crust and in the atmosphere upon which may terminate an ample electric field required for the transmission of any light that originates on the earth, or that arrives from outer cosmic space. If such is the case, the earth must carry with it through space its own portion of the luminiferous electromagnetic medium in which the local transmission of light occurs much as a railway passenger coach carries with it a complement of air through which the passengers hear with no change in pitch due to the velocity of the train.

The fundamental lines along which our civilization has been advancing for a century will be broken utterly if the subterranean supplies of metals, coal, petroleum and chemicals can no longer be located in abundance as heretofore. It is a disturbing fact that the more these supplies are mined the more arduous the task of locating their new reserves. Aside from the slow and expensive prospector's drill, the only means available for "sounding" the contents of the superficial strata of the earth are those afforded by mechanical vibrations over a wide range of frequencies and by electric currents in every conceivable time relation, *viz.*, continuous current, audio and radio frequency currents and electromagnetic radio waves.

It is a problem for the engineer to define and attack and not for the scientist for the reason that it involves a close study of natural phenomena for the immediate purpose of promulgating practical results through the economic expenditure of capital.

Because the application of mechanical energy for subterranean reconnaissance can only be accomplished by means of vibrations, *i. e.*, by means of sound at audio or radio frequencies, and because the best facilities for the projection and echo-reception of such sounds are electrical, the solution of the problem will remain a job for the electrical engineer. Eventually, therefore, the electrical engineer will have to assume most of the burden of locating subterraneous treasures by mechanical or electrical means. The electrical engineers of the incoming generation are surely going to be called on to study the characteristic manner in which the earth transmits mechanical and electrical energy as affected by subterranean contents for the purpose of location of water, ore and fuel.

The first ten-year period of the Institute, 1884-1894, witnessed the initial arrival of the alternating-current facilities substantially as they are known today, the practical beginning of electric traction and a great extension of electric lighting.

This period occupied from the Philadelphia Electrical Exposition to the epoch making electrical exhibit of the World's Fair, Chicago, 1893. The second decade occupied the interval between the Chicago Exposition and the St. Louis Exposition of 1904. In this period the electric lighting industry was consolidated. Electric traction, communication and power transmission made large gains.

A thoroughgoing understanding of alternating-current phenomena as encountered in the transformer, electrical machinery and transmission lines was arrived at and a successful attack was begun upon transients. The new knowledge of alternating-current phenomena made possible the second cycle of evolution of the continuous-current generator, wherein the control of the troublesome field distortion and perfect commutation were finally accomplished. It was during this period that Mr. Andrew Carnegie made his generous offer to build a home for the American Engineering Societies and the United Engineering Society was organized to make the acceptance of Mr. Carnegie's offer practicable by undertaking to build and to maintain the Engineering Societies Building at 33 West 39th Street, New York City.

Toward the close of the period, students in electrical engineering in Universities and Colleges were accorded the benefits of the Institute as "enrolled students."

The third decade extended substantially from the St. Louis to the San Francisco Exposition in 1915, and witnessed spectacular advances in incandescent lighting and transmission of power, and the advent of the electron tube for conductor and radio communication. The period that extends substantially from the San Francisco Exposition to the present constitutes the fourth and last decade of the Institute's first generation. This decade has witnessed a magnificent consolidation of the activities of the electrical engineers within and without the Institute.

In a communication research laboratory where 3000 men were at work, the cathode ray oscillograph was being forced into its second round of evolution; it had encountered its first round about twenty years earlier. The problem developed of discovering better means for focussing the cathode ray, which henceforth should be known as the electron jet. It was carefully defined

and submitted to all in the laboratory who were known to have an authoritative understanding of vacuum tube phenomena with requests for suggestions. Among the replies was the unique suggestion that a residual gas be used which would be ionized by collision of the electrons in the jet; the additional electrons thus liberated from the atoms of the gas would be dispersed and would be of no further consequence, while the heavy, slow moving positively charged atoms would remain in the path of the electrons drawing them centrally by their electric attraction to a closely restricted path or focus. This expedient, though never conceived before, was tried and worked beautifully. It would be difficult to find a better illustration of the huge benefits to be derived from the consolidation that has been going on during the recent decade.

Being the last decade of the first generation, it is necessarily the first decade of the Institute's mature membership, motivated by the mental characteristics of young and old, by the abundant working energy of the young under the cooperation and helpful guidance of the older men of profound knowledge and wide experience. It is promulgating those fine powerful drives for consolidation now engaged in by the older members with their splendid conception of what the Institute should be and with their invaluable contributions of time, effort and funds. They are mining the low grade ores and from them are refining the hard and tough metals with which they are reinforcing the working structure of the Institute, so that without failure it may carry its full load of duty for the maintenance of electrical progress for the benefit of the nation. To them the Institute exists for the advancement of electrical engineering in the highest sense. They know that in the end human character is everything, they know that it is the high privilege of the electrical engineer to understand mother nature and her ways; they know the work of the electrical engineers has required wide-spread cooperation at every turn; they have learned as no other group of men have learned the interrelation and interdependence of all things in nature, in the sciences, the arts and the social verities. They know that no group of men can achieve the amazing advances for the nation that the electrical engineers have without encountering an understanding of what should be done in the new order that their work has brought about, so that the gains shall be maximum and the losses minimum. They are aware of the higher duties thus revealed and that they may count upon the prestige and resources of the Institute to support them in their endeavors.

In a single generation and to an amazing degree the work of the electrical engineer is the cause of a vision of a new democracy and a new religion, the democracy of health and happiness and the religion that has an enhanced reverence for the present and a determination to make the future what it should be here on our earth where it has pleased God to put us and where he has asked us to be happy in the undertaking to make the best of life forever.

Application of Automatic Substations to Central Station Service in Metropolitan Districts

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Review of the Subject.—The present trend to interconnection and the installation of large capacity stations, with the changing character of city load, make relief of the operating crew of a system of the greatest importance. This can best be done by the use of automatically controlled substations in the various types of service. Equipment for each class of service is in successful operation showing great dependability and any particular problem of the operating companies can be attached, keeping in mind certain fundamentals of design, construction and application.

* * * * *

THE increase in use of electric power in the last few years and the very common interconnection between systems have greatly changed the situation in metropolitan districts with regard to power supply. The increase in power consumption has led to complications in design and has introduced a great many problems which were not present a few years ago. The industry has followed in its development work the necessity of greater use of power in a given district and the availability of equipment has in turn made possible the going to still greater density of power consumption. The natural effect of this growth has been that individual pieces of equipment have become exceedingly large, their importance in the system proportionately greater, and incidentally the first cost of each has increased.

As a side issue to this increase has come the difficulty of operators to grasp their entire situation, especially during times of stress and trouble. This has led to a load dispatcher system for each of the districts. The load dispatcher has nothing to do with the actual physical operation of the circuits, but merely directs the operation. The added complexity of the systems has made it necessary for him to have every facility, both to know what is going on and to be able to issue instructions promptly. Most of the larger cities and the surroundings have about reached the point where the human limitation will interfere unduly with continuity of service. Practically all systems are taking advantage of the higher-capacity, highly efficient, present-day generators; are installing very large generating stations to get the advantage of this higher efficiency; are going to high steam pressures, and using duplicate or ring busses in the station, bringing in more sources of energy, using heavy switching equipment, and doing all these things to increase the available power with safety and to insure the continuity of the service in a given district.

Practically every large city is throwing a loop around the city, connecting more sources of supply to this loop and feeding the load from it at advantageous points. This follows the ring bus in individual stations.

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All of this means that the operating crew must be given advantages to offset the greater complexity and responsibility thrown on them by these facilities and precautions in the equipment line. The most effective way to ease up on the operating force is to make as many of the operations follow conditions rather than instructions from the load dispatcher, aiming at all times at continuity of service to the consumer.

The application of parts and principles used in automatic control to the various types of operation necessary has been successfully made and a consequent improvement in overall service and relieving of the operating crew during times of trouble obtained. This application has and will allow greater density of load economically as conditions demand, and a review of these conditions and the way the conditions are met may be of decided advantage.

In practically all of the metropolitan districts the load has been growing around centers of commercial activity with intervening spaces of about the previous normal load density. If you will think over the large cities, you will recognize this situation as common, and also that these localized heavy load centers are not always at the most convenient point with regard to power supply. For instance, practically all cities have had a time of growth of apartment hotels or large apartment buildings in certain districts. This has caused regions of high load density with perfectly defined peaks, whose area is usually quite localized and does not change the load density in a closely adjoining territory. This region is usually well away from the generating station and points where heavy manufacturing load have been previously supplied.

It is such a growth or similar growth that makes the application of automatic control for the substations to supply the load most advantageous.

This type of growth resulted in the installations of quite a number of large and important automatic switching stations which have proven very successful in their operation.

Another type of growth that is taken off of this loop surrounding the city is the picking up of suburban towns from laterals running from this loop where small automatic switching stations give better service to these

towns and less overhead than would be occasioned by any other type of service.

The downtown districts of practically all of our larger cities are furnished with electrical service by direct current laid out on the Edison 3-wire system. These various cities were licensed to use this system of distribution in the early days as electrical experience goes, and have expanded around this system as a nucleus. At first the system was rather light weight, covering a comparatively large territory and was supplied to a great extent by belt-driven Edison Bipolar machines. As the city grew and certain sections acquired taller buildings, larger engine-driven units were installed in nearby steam stations and the copper underground increased to carry this load to the consumers. During this period, the cost of service and the relative degrees of experience by building managers and system managers, led to a great many of the larger buildings being put up to install isolated service, and run the generating system for each building, or small groups of buildings under common management. By the time skyscrapers became common the steam turbine came into common use, and as a consequence, the main power supply of cities became a-c. and converting equipment furnished for transforming this a-c. to d-c. for the Edison system.

As this process went on, the isolated plants were absorbed by the central station companies and power supplied from the Edison system. Thus the magnitude of the present Edison System was a matter of gradual growth in all cases, and at no time has it appeared expedient to do away with d-c. in the downtown districts. The operating companies have had their capital investments in duct lines and heavy underground copper which would not be entirely suitable without expensive changes for a-c. operation. Customers have d-c. equipments in their buildings and sentiment indicated that the application of a-c. to elevator motors would not meet the conditions.

With the increase in the density of the load in the downtown districts, a reserve which could be called upon instantly in case of trouble or supply interruption could be obtained by installing storage batteries in the immediate vicinity of the load. All of the large cities have such storage battery reserves and have a large amount of money tied up in this part of their capital account. These storage batteries seemed necessary on account of the importance of the load from both the risk and revenue standpoint. All of these conditions have led the various utility companies to treat their downtown load entirely different from their residential load. They made it more steady and gave much more careful attention to this service than to an equal kilowatt capacity or load in the residential or manufacturing section. In most cases they get practically no better revenue per kilowatt-hour from the downtown load than from the residential load. It has as bad, or a worse, and also an uncertain load factor, and from all appearances the investment per kilowatt-hour of load is higher and the overall efficiency lower.

It would seem as if the d-c. load of the city is always held in greater respect than any other section of a central station company's business. This service is considered as almost sacred by the operators, whether from awe, admiration or fear is not exactly plain. From the troubles that occur when the district is shut down I believe this feeling is due to fear, and the feeling is similar to "devil-worship" among certain tribes. This has caused most of the central station companies to curtail the d-c. district, lopping off all of the edges of the district possible, thus confining the d-c. district to its original or a restricted territory.

When the nature of the load is considered, one can hardly see how this business can be carried on if the tendency for substituting twenty-story buildings for three-story buildings continues in the d-c. area. When a cloud over the sky can change the load of a d-c. district by tens of thousands of kilowatts, it is evident that this load is rightfully one which requires most careful consideration. The companies have installed large rotary-converter or motor-generator sets as close to the centers of the main system as possible. Large rotaries are installed under sidewalks and in basements three stories down. In times of trouble on the Edison System the whole tension of the Company's organization is given to keeping the machines going so that the batteries will not become exhausted and the system voltage lost. The result has been a duplication of supply and equalizer cables between substations, enormously heavy connections in the streets, as well as an attempt to keep sufficient capacity in converting apparatus. However, once in a great while the d-c. voltage is lost, due to a series of accidents and the problem of picking up the system without burning up important sections of cable or breaking up the system is enormous. Various methods have been devised in different parts of the country at considerable expense. Certain systems have control from a certain point which will allow the equivalent of all the feeder circuits being thrown on machines in motion and in operating condition at the same instant. Other systems have schedules arranged and duties assigned to various employees of opening and reclosing feeder circuits in certain sequence, transferring from bus to bus as the schedule demands. Other systems have considered splitting up the mains in the streets so that trouble would only involve one small portion and not communicate to surrounding zones. This would mean expensive construction in manholes, basements or space adjoining intersections.

The application of automatic stations to such service has relieved the extreme anxiety of the operation of such a system, especially in attempting to pick it up after an interruption. It has also led to a decrease in the chances of interruption, in that protective features have been furnished so that machines would not feed

out into the system more than their safe carrying capacity. This protection has allowed the substation equipment to stay with the system, both during time of extreme load from the system and shakeup in the a-c. supply. It has also allowed the substations to be located where they will be of greatest advantage, where the feeder losses can be kept down to a minimum and in locations where manual operation of stations would be difficult, if not impossible.

The application has shown very marked saving in previous losses absorbed by the central station company; has changed these losses into revenue load and has made it safe to put the Edison systems on the same basis as the a-c. section of the Company's business as regarding continuity of service.

This knowledge that the Edison System will be maintained just as long as there is a-c. supply and after an interruption will be restored as soon as this supply is available and not burn up cables or machines in the process, avoids the necessity of taking all the former precautions to prevent the Edison system

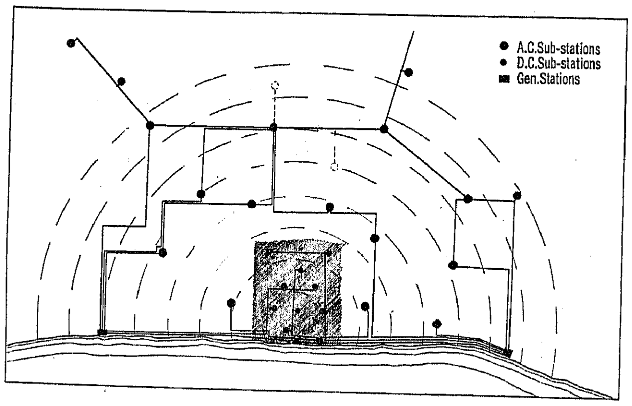

Fig. 1

being lost; in other words, the taking of sufficient precautions, but of a different and more economical form.

Another application of automatic control which has proven of particular advantage is in the taking over of load previously carried by the Edison System on a-c., doing a wholesale business in power to large users in buildings in the old d-c. district. This makes it possible to continue serving a district by the original or equivalent d-c. equipment which, if fully a d-c. section, would be very heavy, but now interspersed with an a-c. system on the basis of wholesale a-c. power.

Now let us see what has been used to do these various things.

Fig. 1 shows what can be considered a one-line diagram of a high-tension distribution in a typical metropolitan district.

The water front is either the river or lake on which the city is located. The shaded portion is the d-c. district extending back from this water front with the heavy load about one-third back from the water.

You can make your own modification by putting in a little more or taking out some of the radial feeds.

On the outer fringe you will note the small a-c. stations. These are like Figs. 2, 3, 4, 5 and 6. Just inside the loop are the larger a-c. stations. These take the building form of Figs. 7 or 8, and the control equipment of Figs. 9, 10, 11 and 12.

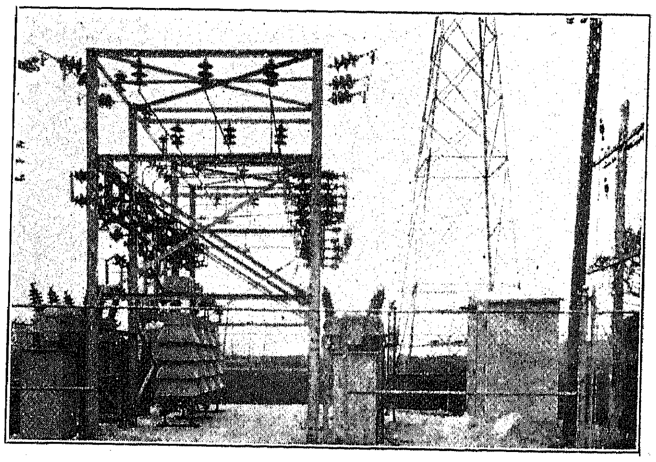

Fig. 2—Outdoor Incoming Line Equipment and Automatic A-C. Reclosing Feeder Station

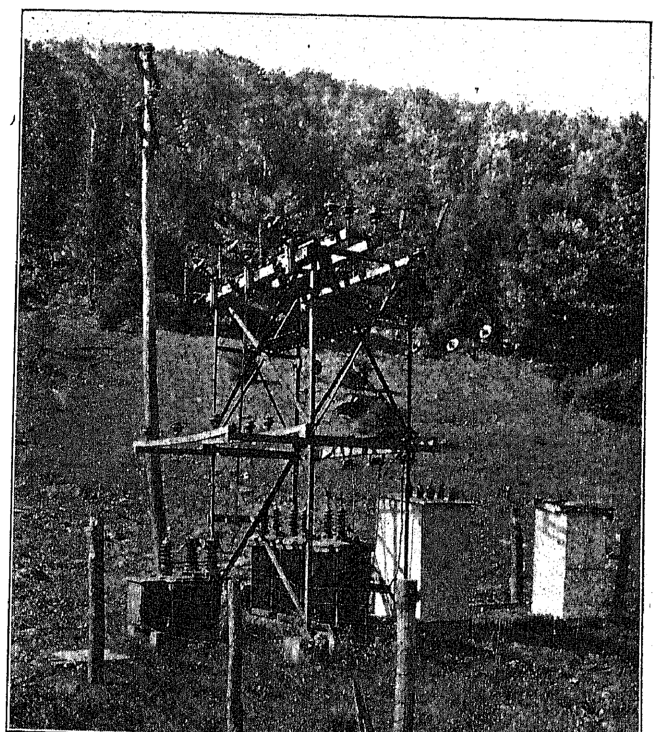

Fig. 3—Outdoor Automatic A-C. Reclosing Feeder Station of the West Branch Light and Power Co., Stamford, N. Y., Prattsville Substation

The method of increasing the capacity of a station during the peak, decreasing it during the light load and supplying the substation, varies in different localities. Certain cities increase capacity by turning on water in combination self and water-cooled transformers.

Some throw on additional transformers to the main bus and others throw on transformers and segregate the bus in sections to keep loads of a certain magnitude radial for the supply.

These various methods of handling the a-c. loads are in successful operation and these and similar ways of going at the substation problem can be used to solve any of the distribution problems to meet the particular requirements of the operating company. The main precaution to be taken is that too many operations

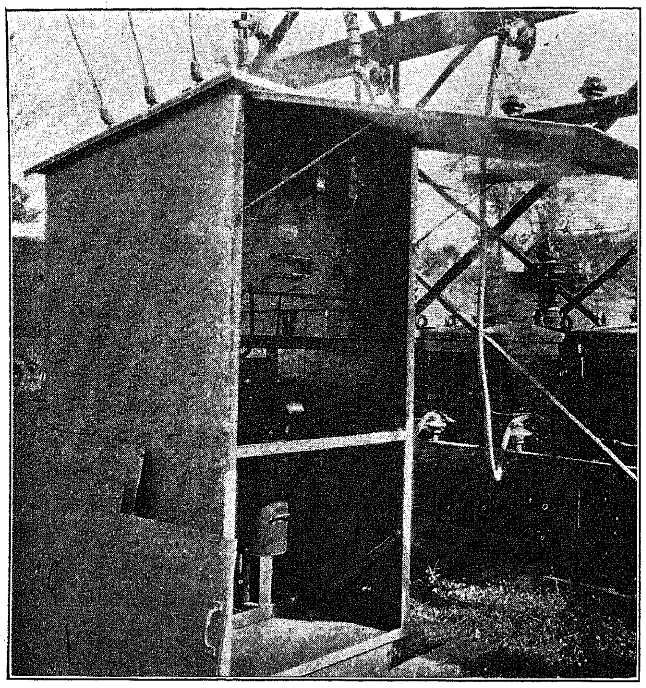

FIG. 4—WEST BRANCH LIGHT AND POWER CO., STAMFORD N. Y. PRATTSVILLE SUBSTATION

Interior view of outdoor switch house showing automatic a-c. reclosing feeder, 6600-volt 3-phase 60-cycle (Front view).

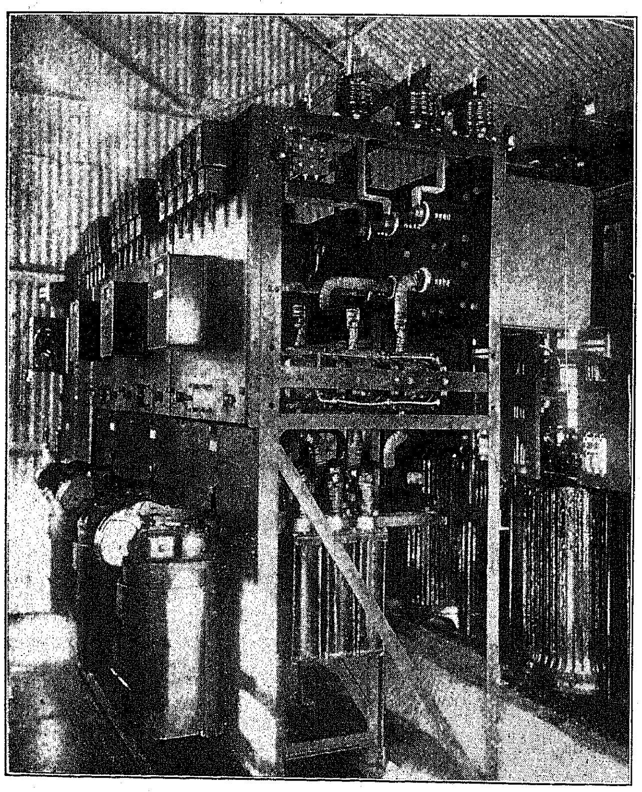

FIG. 5—UPPER HUDSON ELECTRIC AND RAILROAD CO., TANNERSVILLE, N. Y.

Interior view of station showing automatic switching equipment for three a-c. reclosing feeders, each 4000 volts, 3-phase, 60-cycles.

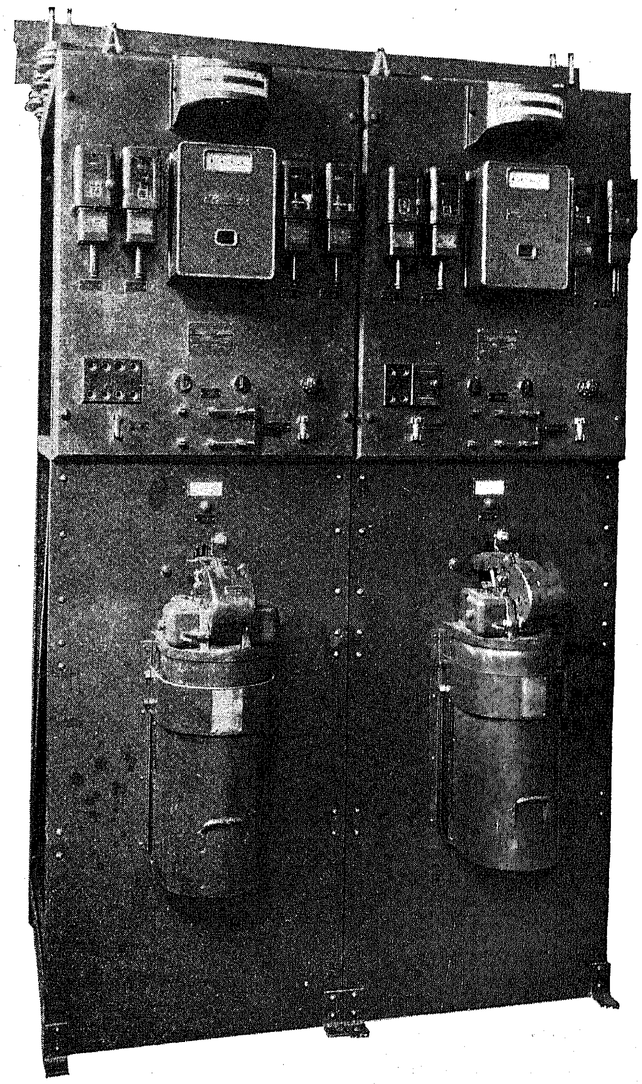

FIG. 6—AUTOMATIC SWITCHING EQUIMENT EDISON ELECTRIC COMPANY, LANCASTER, PA.

Two 4600-volt a-c. feeders 300-amperes 3-phase, 60-cycles. (Front view)

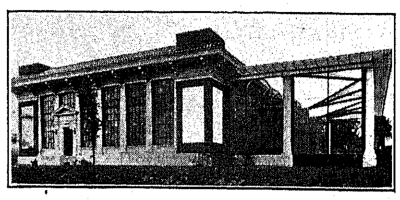

FIG. 7—AUTOMATIC STATION CONTROL EQUIPMENT, KANSAS CIY POWER AND LIGHT CO., KANSAS CITY, MO.

Automatic substation R, 6000-kv-a. 13,000/4150 2400 volts 60-cycles. (Exterior view).

should not be undertaken in providing against conditions which may not happen, or only happen occasionally. Fundamentally, the station should have its control so arranged that it will pick up the service from

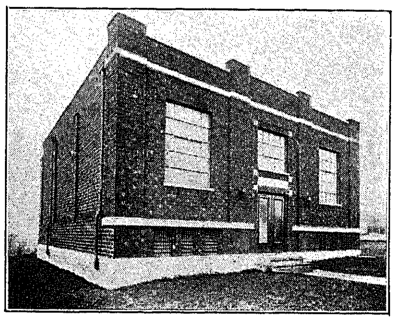

FIG. 8

any available source and provide a sufficient station capacity to carry the load at the time. However, the station should not be allowed to become over-compli-

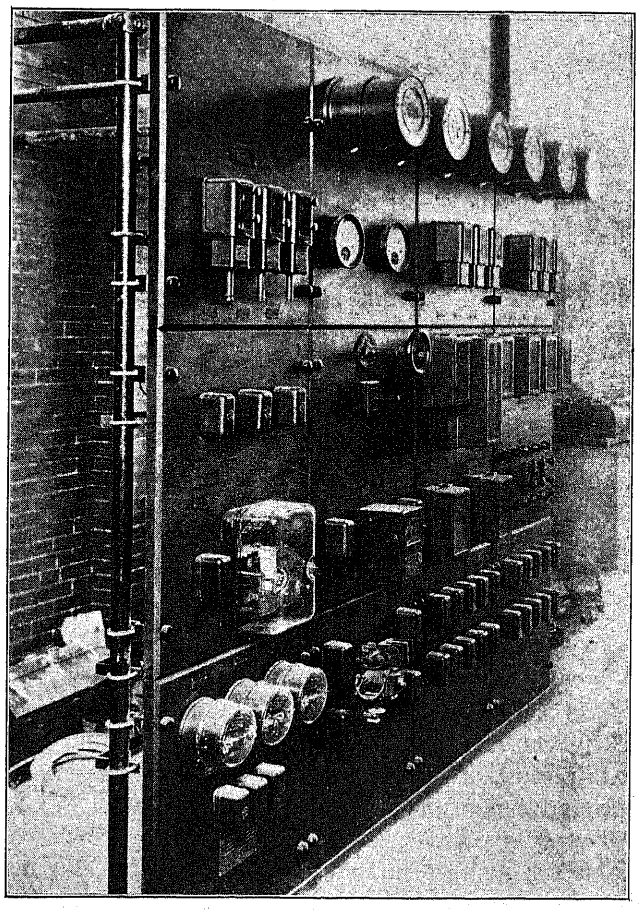

FIG. 9—AUTOMATIC STATION CONTROL EQUIPMENT STATION R, KANSAS CITY POWER AND LIGHT CO., KANSAS CITY, MO. FRONT VIEW OF MAIN RELAY PANELS AND MASTER SWITCHES

cated by too much reserve switching arrangements. While it is possible to lay out, build and properly install almost any combination of switching which can be logically defined, the complication should be avoided for the benefit of the man who must maintain the station and find troubles. These may occur, due to maintenance conditions or to operations performed manually in order to get a chance to work on particular

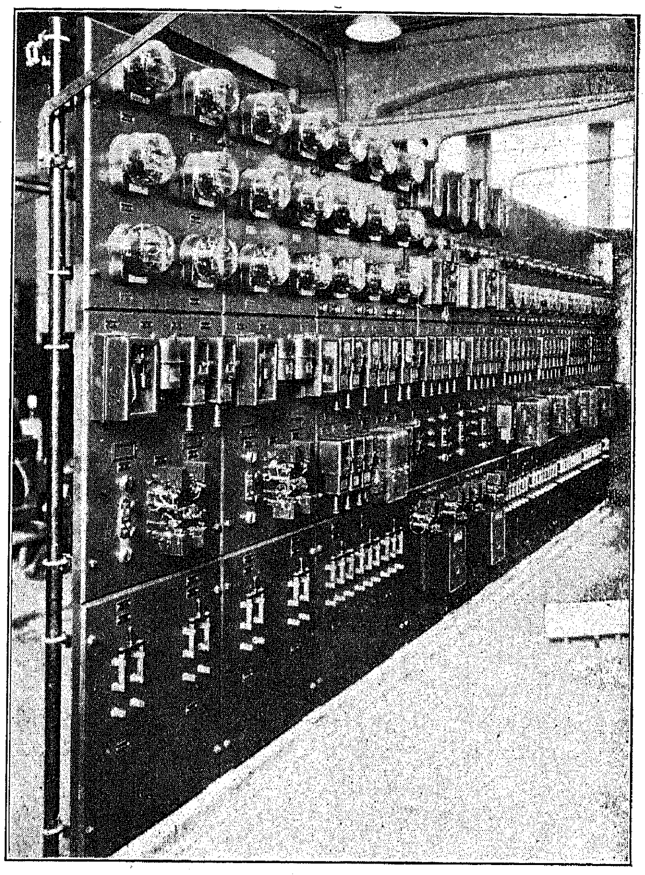

FIG. 10—AUTOMATIC SWITCHING EQUIPMENT, MILWAUKEE ELECTRIC RWY. AND LIGHT CO., MILWAUKEE, WISC., NORTH MILWAUKEE STATION

For incoming lines 26,400 volts and 13,200 volts and a-c. reclosing feeders, 4000 and 2300 volts (Front view).

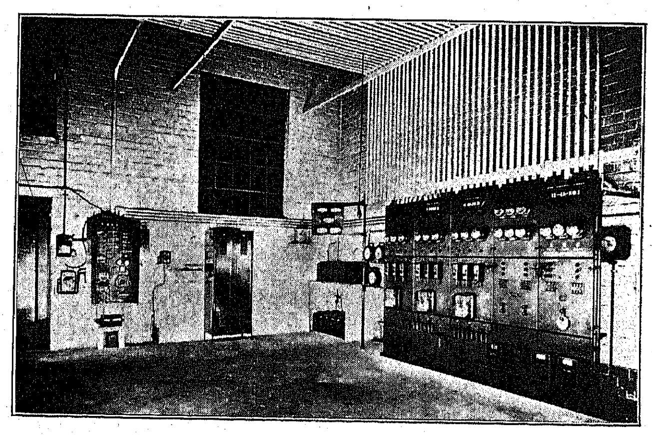

FIG. 11

circuits or transformers. In other words, the effort should be made not to transfer the operating difficulties, due to complexity and magnitude of the system, from the operating crew to the maintenance crew.

With automatic control of such stations the control equipment can be located at points close to the parts to be controlled and thus the size of building can be cut down, conduit and secondary wiring shortened and instruments required for hand control of such a station omitted. All this leads to economy in installation, which, to a certain extent, compensates for the added cost of the automatic control features. It usually works out that, overhead and operating expense being considered, the automatic station can be justified by the smaller annual charge. However, this saving is not the main factor, but rather the item of continuity of service and the establishing of sections of the city which will take care of themselves without direction from the load dispatcher, just as long as he can give them a-c. supply.

The obtaining of a location for an a-c. automatic station in a residential district is not difficult. The stations are easily made of an attractive appearance and are a credit to any neighborhood. They are not noisy and after they are completed there are no objectionable features.

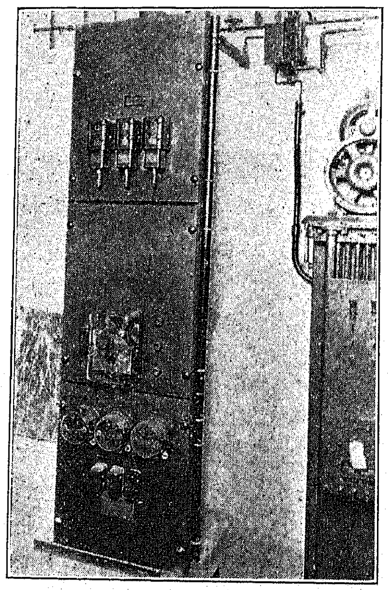

FIG. 12—AUTOMATIC STATION CONTROL EQUIPMENT STATION R, KANSAS CITY POWER AND LIGHT CO., KANSAS CITY, MO. FRONT VIEW OF TYPICAL GROUP RELAY PANEL

In the d-c. Edison 3-wire district, the application of automatic control has taken one of four forms:

1. The equipping of booster-type rotaries already installed for automatic control.
2. The equipping of motor-generator sets, either for control by external load limit or by modifying the set to approximate the heavy duty differentially-wound motor-generator set.
3. The installation of new motor-generator sets differentially-wound.
4. The installation of shunt rotaries with external load-limiting features.

All of these are in successful commercial operation, so it is possible at the present time to attack any substation problem, knowing that it has been done before in practically the form which may be demanded. The stations which are in operation are performing very creditably. They have proven that they can pick up the Edison load after an outage in a very definite time after the a-c. source has been restored. They have also proven that they will stay with the system during severe a-c. disturbance. If one station happens to get shaken off, due to its being electrically near the center of the disturbance, it will come back into the load as soon as a starting voltage is supplied. In the meantime, stations that remain with the load will not suffer, due to the absence of the one shaken off, but will adjust themselves to give out the limit of their current capacity and allow the voltage to slip to keep the load within the current capacity. We can thus with assurance attack the automatic operation of existing machines.

The principles, which should guide the operating company in selecting additions to their substation equipment for the Edison system or replacement of older or less efficient equipment, are as follows:

A motor-generator set with current-limit ability should be selected when the addition is to be made at a location where the machine will have to operate only during the peak or after a disturbance when the slightly poorer efficiency will not be the important factor, or where power-factor correction is important. These machines will not shake off of the system during any disturbance and will immediately restart on the restoration of supply.

A shunt-wound rotary should be installed where it is to operate over long periods, when it is close to the load so that the voltage regulation range will be within the regulating ability of the machine, and where the overall efficiency is of importance.

From my point of view the motor-generator set should be installed where it is to be added to existing stations, or in a particular location where there is a periodic load of short duration, when the load at other times of the day can be economically maintained from the existing network.

The shunt-wound rotary should be installed where it will get a good heavy load during all but the off-peak period, or where it replaced antiquated or under-sized equipment in the midst of a heavy load.

The 60-cycle shunt-wound rotary has proven very stable for such service and in maintaining a flat man-hole voltage, picking up the load after failure of a-c. supply through its current-limiting equipment and operating on a commercial a-c. power supply.

The main precaution to be taken in such stations is that sufficient ventilating capacity be provided. With motor-generator stations, the natural thing, on account of the smaller physical dimensions of the set, and the fact that a man does not have to be present during its

operations, is to put it in a small space. The greater losses naturally require greater ventilation and difficulty is usually encountered in providing it.

When adding a shunt-wound rotary to the system, the precautions to be taken include the supplying of clean air for ventilation, but in smaller quantities than for motor-generator set stations.

In applying automatic control to stations supplying an Edison System, it should be remembered that where machines come on, due to a droop in voltage, and go off on loss or decrease of load to a certain point, the regulation of the d-c. system should be such that the manhole voltage which brings on the added machines can decrease enough to cause the machines to come on. After they are on, the regulation attempted should be coarse enough to allow the machines to stay on and carry load. In other words, the feeders to the Edison System from automatic stations should be interleaved as between stations and the buses in the streets should not be so heavy as to avoid voltage drop which will bring on the necessary machine capacity.

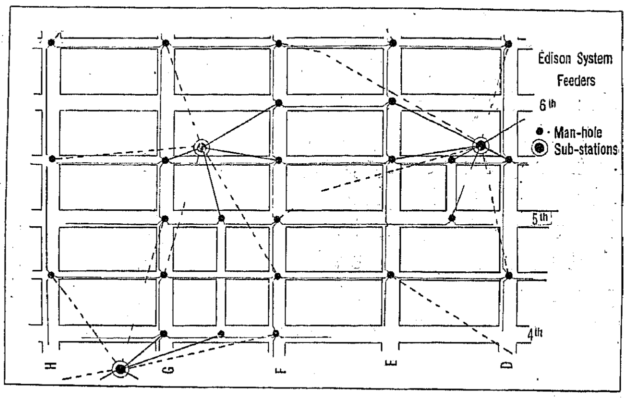

Fig. 13

Fig. 13 will illustrate what is meant by the interleaving of feeder cables.

No doubt difficulty will be experienced in practically every city in locating automatic stations in the Edison district. The selection of locations is assisted by remembering that the noise of the machines is of a nature which does not reach other parts of the building, except through the air intake and outlet; also by remembering that these can be baffled to avoid this noise being noticeable; that in an automatic station all the precautions are taken to prevent accident to the machine; that there is no operator to desert the machine when he is most needed; that the parts of the control can be located where there is physical space for them

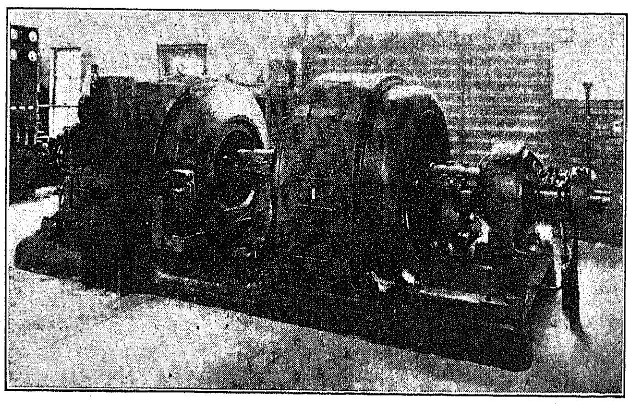

Fig. 14—New Orleans Public Service Co., New Orleans, La., Bourbon St. Station

Interior view showing automatically controlled synchronous motor-generator set. Motor 900 rev. per min. 6600 volts, 60-cycles. Generator 130/260 volts.

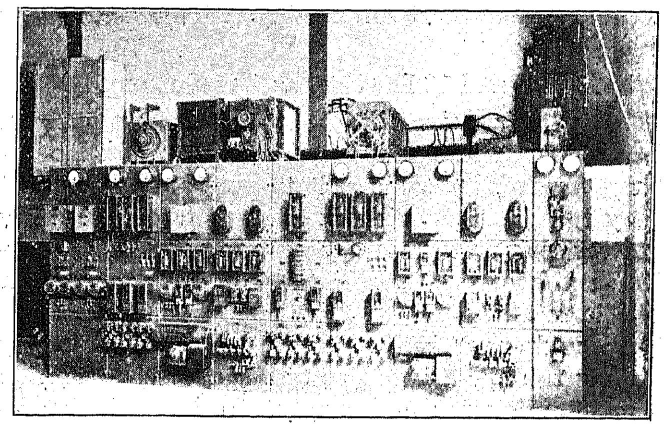

Fig. 15—New Orleans Public Service Co., New Orleans, La., Bourbon St. Station

Interior view showing automatic switching equipment for two synchronous motor generator sets, one 130/260 volts d-c. 1000 kw. and the other 260 volts d-c. 1200 kw.

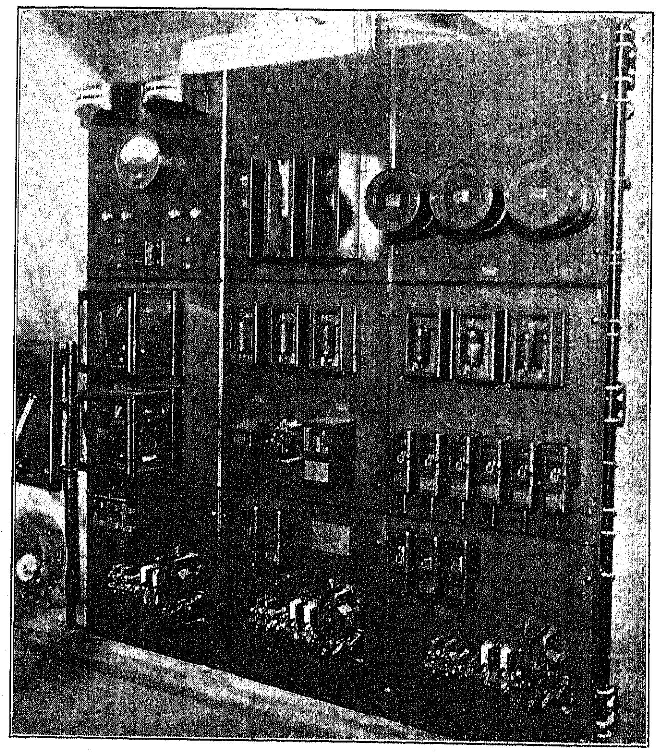

Fig. 16—Automatic Station Control Equipment, Kansas City Power and Light Co., Kansas City, Mo.

Instrument and relay panels for 1500-kw. 250-volt 3-wire synchronous converter (Front view).

and possibility of access for the inspector, even if the parts are distributed where hand operation would be cumbersome, if not impossible.

In locating such stations there is usually quite a wide latitude of location, since the station is being located in the midst of the load and the voltage drop to the feed points in the d-c. network will be low, the maximum length of run in most cases being at the most, two or three blocks.

Figs. 14, 15 and 16 illustrate such equipment.

The operation of automatic stations in metropolitan districts has, without doubt, increased the reliability of the service, by taking all the precautions against injury to apparatus without causing it to keep away from the legitimate load; by restoring interrupted service at the earliest possible moment; by making it possible to cut down losses by more frequent stations than would otherwise be economically possible and by allowing the operating force to devote their attention to maintaining the a-c. supply. It has proven economically possible to get this protection and to afford these facilities, because in so doing the reserve capacity in various branches of the equipment has been dispensed with, and equal or less money value put in protective features. The service has been improved by avoiding misunderstood instructions, experimenting by operators; mistakes in operation or in the interpretation of conditions. It has changed the work of a number of employees from the uninteresting work of waiting for something to happen in substations to the much more interesting work of maintaining such control and equipment.

Without a question, the use of automatic substations will decrease greatly the labor turnover and the difficulties of the management from such causes. Above all, it will allow the management of a property to honestly say that every precaution which is now known has been taken to give continuity of service, and if accidents do happen, to restore the service at the earliest possible moment without depending on a series of people who may understand instructions properly, but get confused at the wrong time.

Discussion

For discussion of this paper see page 781.

The Cleveland Heights Substation
of the Cleveland Electric Illuminating Company

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Review of the Subject.*—Originally designed for railway service in small units, the use of the non-attended automatic substations has gradually been extended to substations feeding d-c. networks and a-c. lighting and power distributions of increased capacity and importance.*

They are justified economically up to the point at which the fixed charges and operating costs on the additional equipment required over and above a manually-operated station, equal the costs of attendance.

A description of an a-c. station having an ultimate capacity of 12,000 kw. follows, emphasis being laid upon the more interesting features of design.

An automatic (self-restoring circuit) a-c. substation for the supply of service in a high grade residential suburb of Cleveland, was put in commission in the latter part of 1920 as a manually-operated station, and upon the completion of the installation of the necessary relays, the operators were withdrawn and its operation as a full automatic substation begun toward the latter part of January, 1921.

The operating results obtained at this station have been most gratifying and other stations of this type will be installed in such locations as may warrant them.

Because of the character of the neighborhood in which the substation site was located, it was essential to combine sightliness with utility. To this end, a reinforced concrete building of one story and basement was designed along pleasing architectural lines, and

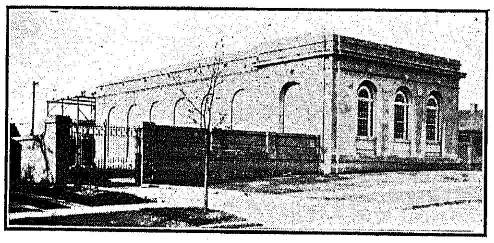

FIG. 1—VIEW OF SUBSTATION BUILDING

the outside finished in ornamental stucco. A wall of similar finish was put around the property and ornamental gates installed for the entrance of vehicles and pedestrians. Care was taken to preserve a large elm in front of the station. Fig. 1 shows the exterior of this building and a glimpse of the outdoor transformer installation.

The electrical lay-out is quite simple. 3000-kv-a. transformer banks installed out-of-doors at the rear of the buildings are fed each by an 11-kv. cable circuit of the same capacity. The cable terminates in a three-phase pothead from which the leads are led to the transformer units through an air-break disconnecting switch, the three poles of which are mechanically interconnected and operated simultaneously by manual control from below. Three such banks totalling 9000-kv-a. are now installed. Fig. 2 shows a one-line diagram of connections.

The low-tension sides of the transformers are connected for 2300 volts delta and are led through single-conductor lead-covered cables to oil breakers located in a masonry structure in the rear of the building, which structure also houses the necessary disconnecting switches, current and potential transformers.

Connection is made through these oil switches to each section of the distribution board, of which there are now three; ultimately four will be installed.

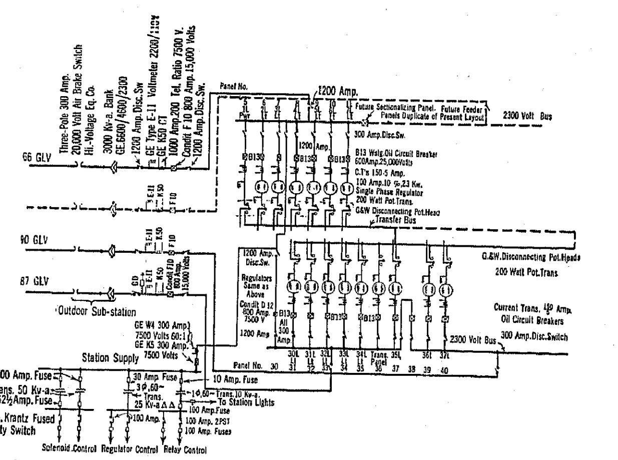

FIG. 2—GENERAL WIRING DIAGRAM

The transformers are equipped with watthour meters, wattmeters and maximum demand indicators.

Protection is obtained by means of overload induction type relays on the sending end of the transmission cables and reverse-current relays on the low-tension side of the transformers, thus cutting out a cable and bank as a unit in case of failure of either.

This arrangement has led to the practical elimination of 11-kv. busses, incoming line breakers and breakers on the high side of the transformer banks, reducing space and equipment requirements to a minimum.

Each section of the distribution board has, in addition to a tie panel connecting it to the adjacent section, seven feeder panels which normally have outgoing circuits connected to them. There is, however, a

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duplicate spare panel to which any feeder may be transferred when it is desired to inspect the oil breaker or work on any equipment. This transfer is effected by means of a duplicate 4/0 wire bus and single-phase potheads at each panel. Any equipped panel not connected to an outgoing line may be so used, however.

The feeder breakers are 600-ampere, 25,000-volt with

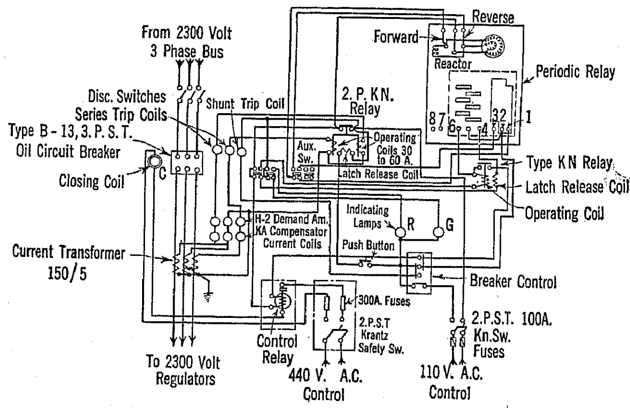

FIG. 3—WIRING DIAGRAM OF SELF-RESTORING EQUIPMENT

a rated interrupting capacity of 25,500 arc amperes at 2500 volts. The maximum ultimate short-circuit current has been computed at 28,100 amperes with all sections tied together and a dead short-circuit on the bus.

The timing relay is adjusted for three reclosures at seven-second intervals. In addition, there is a lock-out relay set for 30 amperes (900 amperes at 2200 volts)

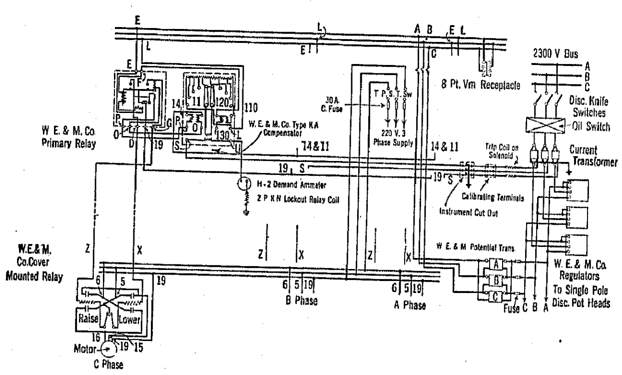

FIG. 4—WIRING DIAGRAM OF FEEDER REGULATOR CONTROL

which prevents the reclosing of the feeder breaker upon a short circuit of this or greater magnitude.

Feeders are equipped with compensators controlling motor-operated induction regulators, three single-phase units being used in each circuit. The spare panel also has regulators as part of its permanent equipment.

Each phase of the outgoing circuits is equipped with a thermal demand ammeter to indicate the maximum load carried on each phase, so that overloading of equipment may be guarded against and care taken of any necessary circuit balancing. Fig. 3 shows the wiring diagram for the self-restoring control and Fig. 4 the regulator control wiring. A view of the switchboard is shown in Fig. 5.

The regulators and station supply transformers are located in the basement and housed in cells. These cells are provided with a drain to an oil sump and with a knock-out curb which will prevent flooding the floor with oil, in the event of a regulator case being split in two under a violent short circuit.

Fig. 6 is an illustration of this portion of the installation.

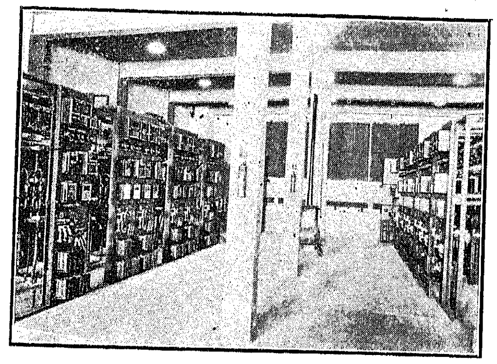

FIG. 5—GENERAL VIEW OF 2300-VOLT SWITCHBOARD

All circuits leave the building underground, the cables being trained in vaults on each side of the building which connect to a large manhole in the front.

The station has been designed for a total capacity of 12,000 kv-a., but space is available for a fifth transformer bank, should the load requirement demand it. This would permit an average peak loading of 500 kv-a. each on the 24 feeders eventually to be installed,

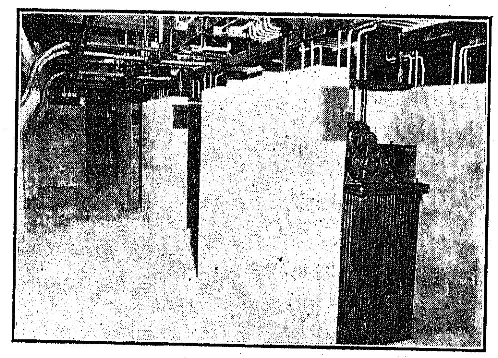

FIG. 6—VIEW OF BASEMENT SHOWING REGULATORS IN CELLS

with the fifth bank in reserve. Since the 100-ampere regulators can easily carry a two-hour lighting peak of 125 amperes, no change in equipment will be required to carry 12,000 kv-a. should it be necessary to install the fifth bank referred to previously.

The balance between the cost of operators' wages and the additional fixed charges and maintenance expense, due to the additional equipment installed over that required for a manually operated station, will be reached with the number of circuits planned and will lie between 9000 and 12,000 kv-a. of peak load, depending upon the average peak load carried per feeder.

Discussion

For discussion of this paper see page 781.

Automatic Substations for Supplying 1500 Volts Direct Current to Suburban Railways

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Review of the Subject.—*The engineer in applying automatic substations to an existing electrified traction property is often forced to make the most of a poorly designed or antiquated distribution system. By comparison, the problem of designing the distribution system for the electrification of a steam railroad is often very simple. The choice of proper converting equipment is perhaps not so easily made. The author brings attention to a number of important points which should be given careful consideration by those contemplating electrification.*

* * * * *

IN Europe and in one instance in America, 1500 volts direct current has been selected for the electrification of the suburban traffic of steam railroads operating in a number of the larger metropolitan areas.

The operators of these steam railways have set and maintain very high standards of operation. If electrification is to be justified, it must be on the basis of even better service and greater economy of operation.

The steam locomotive is an extremely reliable isolated power plant. The percentage of train delays due to locomotive failure is low. The failure of one locomotive on a steam-operated suburban road would probably delay traffic by an amount insignificant by comparison to the delay which might result from failure of power supply to an electrified system.

Perhaps one of the most important links in the power system is the substation. If the substations are designed and equipped with the same degree of care as our large modern steam-electric and hydroelectric generating stations, they will be equally reliable in operation and so leave little doubt that continuity of power supply will be comparable with steam locomotive operation.

Considering substation requirements, very careful thought must be given to the following:

1. Present and future total power requirements.
2. Location of substations.
3. Capacity of substations.
4. Selection of converting equipment.
5. Voltage and frequency of a-c. supply lines.
6. Number of a-c. supply lines to each station.
7. Selection of switching equipment.
8. Scheme of control for the operation of d-c. feeder circuits.

Power Requirements

The present power requirements of any given system may be readily calculated from an analysis of train schedules, train weights and a profile of the routes followed. Future power requirements may ordinarily be anticipated by an analysis of records to determine the average rate of increase in traffic.

Power requirements with reference to the entire system are made up of three principal elements, *viz;* total energy required, power at peak load and load factor.

The characteristics of a suburban railway are ordinarily such that the characteristics of each section of the system are similar. This will be noted from a study of train sheets and load curves. Therefore, the power requirements of the average substation which supplies some given section, will ordinarily be quite similar to those of the system as a whole. However, by studying the requirements of separate sections individually, those elements which enter for special consideration can be readily taken care of in laying out a substation or a group of substations for each section. For example, the load curve of a terminal substation will differ somewhat from that of an intermediate station. Also, holiday loads which come infrequently and which may be peculiar to some particular section of the system, may have a very decided influence on the required substation layout.

Substation Locations

The proper location for a substation is sometimes quite obvious. Also, physical limitations may often determine the substation location. The number of substations will ordinarily be determined by an economic study to determine both the fixed and the variable operating charges.

Capacity of Substations

The capacity of the substations should be sufficient so that any one substation or any unit in any substation being out of service would not interfere seriously with the maintenance of regular train schedule.

The number of units as well as the type and nominal rating of the units required in each station will be determined by a study of the average load, peak load and load factor. Since the load factor of the average suburban railway is probably less than 35 per cent, the substations will be equipped with more and smaller units than would be used if the load factor was higher. This permits of operating the machines in service on light loads at the most efficient loading. Increase in load is taken care of by putting additional machines into service.

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The average suburban train under acceleration requires from 300 to 500 kw. per car as compared to an average power requirement of perhaps 100 kw. per car. Should the train schedule be such that eight car trains, for example, are operated on a 15-minute headway, the substations would be designed to meet the peak requirements rather than average requirements. In other words, the commutating capacity required will determine the size of the substation unit.

Selection of Converting Equipment

In cases where the operation of synchronous converters is known to be entirely satisfactory, there is ordinarily little reason for considering motor-generators. If converters have not been operated under the conditions contemplated or known to exist, consideration must be given, especially to the probability of the frequent occurrence of such disturbances as those to which converters are inherently sensitive. Motor-generators should be used instead of converters on the end of long transmission lines where the voltage is subject to sudden fluctuations, such as surges resulting from switching operations or sudden changes in load. If synchronous condensers are used to regulate the voltage, the operation of synchronous converters will be quite satisfactory. If the resistance drop in the transmission line is excessive, the operation of synchronous apparatus in general will not be satisfactory.

Where the a-c. supply is to be at 13,200 volts or less, the comparison with converter must be made with converter and necessary transformers considered as a unit. Above 13,200 volts, transformers are required with motor-generators which gives decided advantages to the converter in the items of efficiency, space requirements and weight. In the smaller sizes, however, up to and including 1500 kw. at 13,200 volts, the installations of synchronous converters with automatic switching are about on a parity with the installations of synchronous motor-generators with automatic switching. With the transformers on the same floor level, there is little difference in space required. There is little difference in the first cost for equipment in capacities from 1000 to 1500 kw. The overall efficiency is in favor of the converters.

The restricted capabilities and less favorable design characteristics of the 50 and 60-cycle converter may logically be expected to show up when the maintenance expense is compared with that of the motor-generator. The great majority of the maintenance expense items is incident to the operation of the current-collecting parts of the machine. A converter, with its added collector rings, will require maintenance expense for both current-collecting elements. Due to less favorable commutating characteristics and closer spacing of all brush parts, the life of the brushes will be very much less than that of the motor-generator. Flashovers are very severe on the brush life and the most favorable unit in this respect has a great advantage from the standpoint of maintenance expense. The relative length of time that each type of unit is available for service is also an excellent guide as to the relative maintenance expense that is to be expected.

Every synchronous machine has a definite pull-out load. The synchronous machine carries load by virtue of its rotor "dropping back" in phase position sufficiently to pass the necessary load current through the impedance of its internal circuit. On large 60-cycle converters, approximately seven times full load causes the rotor to drop back sufficiently to pull out of step or slip a pole if the load is not removed in a sufficiently short time interval. Obviously, pull-out will occur at much lower load values as the a-c. supply voltage is reduced. A synchronous commutator-type machine cannot slip a pole under full voltage at the commutator without very serious flashing. To prevent flashing under these conditions, a quick-acting circuit breaker must operate to relieve the converter of its excessive load before it can drop back to its pull-out position. If the resistance of the short circuit path, including the resistance of the converter windings, is such as to limit the current to less than this figure, high-speed breaker protection is not required. Such equipment or such circuit resistance is essential with any arrangement of converter, if interruption to service from short circuit is to be prevented. In a 1500-volt system, the short-circuit current will not ordinarily be limited sufficiently by the resistance of the short-circuit path so that the high-speed circuit breaker protection must be considered as absolutely essential. There is always a contingency that the protective device may be inoperative, that the short circuit may occur inside of the protective device, or that the flashover may be caused from disturbances in the high-tension supply. In any case, the converter must be removed from service until repairs are completed. The insurance of continuity of service on converters is obviously less than with motor-generators.

The motor-generator, as has been proven by the installations on the electrified sections of a number of steam railroads, has the inherent capability of withstanding without serious injury, such short-circuit conditions as are met with in service without any special protective devices. The motor-generator, because of more conservative design characteristics, when compared with standard railway converters, may be relied upon to carry 300 per cent load in daily duty cycle as against 200 per cent peak load on 50 and 60-cycle converters. If this short-period peak-load requirement met with in the acceleration of heavy trains, determines the size of substation required, it is evident that 50 per cent more converter capacity will be required to meet the service conditions than if motor-generators are used. This difference in the momentary rating of the two types of equipment should be given serious consideration in making any comparisons in the sizes of substation equipment required for

this service. The motor-generator also has sufficient mass in its rotor to prevent it being pulled out of step by a short circuit which is relieved by any circuit breaker of ordinary speed.

Shunt and Compound Wound Converters

In applying synchronous converters to electric railway service, the characteristics of both shunt and compound-wound machines should be considered. The compound-wound machine, due to its flat voltage characteristic, has generally been considered best for all classes of electric railway service. In the case of automatic substations supplying power to a system designed for heavy service and with inherently low ohmic resistance per unit length of circuit, shunt-wound converters have decided advantages. These machines, when used with transformers of from 5 to 7 per cent impedance, instead of 15 per cent as used with compound-wound converters, have a drooping voltage characteristic, the regulation being on the order of 5 per cent. In multiple unit stations, no equalizer is required, and for this reason the automatic control is considerably simplified. If, due to congestion of traffic, one station is overloaded, the resultant drooping voltage causes adjacent stations to pick up the load in proportion to the trolley bus voltage. A congestion at one point means light load at adjacent points, and therefore, capacity required for the necessary assistance to the overloaded station is available. In stations where compound-wound converters are used, this drooping characteristic has been effected by limiting resistors being inserted in the circuit by the opening of shunting contactors controlled through the contacts of overload relays. These resistors must be of low ohmic value and of high thermal capacity and must be shunted normally by contactors capable of carrying current equal to one and one half times the nominal full load of the converters for the guaranteed overload period of the machine. In stations supplying heavy suburban service, at least two steps of resistance for each machine are required. The cost of such equipment is obviously high. In addition, the cutting in of the resistance causes a sudden drop of load. If a shunt-wound converter is used, but one step of current-limiting resistance together with its shunting contactor is necessary to provide against a very severe overload. Normally, the inherent voltage drop is sufficient to prevent overloading beyond the short-time overload capacity of the unit. The drooping characteristic of the shunt machine may be compared with that of the compound machine and limiting resistance shunted by contactors in an infinite number of steps so controlled as to increase the value of effective resistance to prevent increase in load above a predetermined maximum. The resistors for use in automatic substations and especially with compound-wound machines must be very carefully designed in order to prevent the shifting of an excessive amount of load to adjacent stations, so as to set up a condition under which the machine might be first underloaded and then overloaded, due to the opening and closing of the resistance shunting contactors. This "pumping" of the load back and forth between stations is impossible with shunt-wound machines, each of which at all times, due to the drooping characteristics, tends to shed the load.

Voltage and Frequency of A-C. Supply Lines

There is often no choice of the frequency of the power supply available, since this is usually fixed by other factors over which the railway companies will have no control. In general, 25-cycle synchronous converters operate more satisfactorily than 60-cycle converters. However, the 60-cycle converter has been so developed as to give highly satisfactory performance. 25-cycle transformers and converters are somewhat higher in first cost and occupy more space per kw.

The high-tension a-c. supply voltage to be used is determined in most cases by the distance over which the power must be transmitted. However, in comparing the cost of a number of voltages which may be used, serious consideration must be given to the fact that the cost of high-tension switch gear, lightning protective apparatus, etc., required for the substations will be greater with the higher voltages.

Number of A-C. Supply Lines to each Station

Each substation of an important suburban railway should be supplied by no less than two a-c. circuits, if the feeders are of the radial type or by a loop system on which power may be supplied from either or both directions normally, or from one direction or the other in case of failure of some part of the loop line. A single feeder may be of insufficient capacity to supply a large station, in which case two or more would be required. If two feeders are required normally, it is quite obvious that at least three supply lines should be installed if continuity of service is to be assured.

Selection of Switching Equipment

A great deal has been written regarding the economies effected by automatic stations. These are mostly concerned with savings in attendance charges and reduction in power losses. The operating records of automatic stations, show that by comparison with purely manually-operated stations, power failures are greatly reduced. Consider for example, a failure of the a-c. power supply. Within less than a minute following restoration of power, an automatic substation is completely restored to operation. Depending on the capacity of the units, the agility and alertness of the operator, and the general layout of the equipments, a manually operated station could be restored to service only after a comparatively much longer period.

The automatic switching equipment of a railway substation is designed so as to keep the machine on the

load when it is needed. Should the station be subtected to heavy overloads, either the drooping characjeristic of shunt machines or current-limiting resistors, which are automatically inserted in the circuit between the d-c. terminal of the machine and the feeder bus, cause a reduction in bus voltage to force adjacent stations to pick up a portion of the load. Thus, where a manually operated unit, not equipped with the limiting features, would be "knocked off" the load by the tripping of the machine breakers, thus causing an interruption of power, an automatically-controlled unit would merely back away from the excess overload and "stick" to the limit of its capacity.

It has been demonstrated that this feature of "wiping off" the load peaks permits the use of smaller units to handle service characterized by periodically recurring peak loads. Thus, a converter or a motor-generator in an automatic railway substation may be applied more nearly on the basis of its nominal rating than in a purely manually-operated plant which would necessarily be designed to take the entire peak-load swings.

The larger units for 1500-volt d-c. railway substations, whether motor-generators or converters, will for the most part, be made up of two 750-volt machines connected in series. This is especially true of 50 and 60-cycle synchronous converters.

Three-Phase Incoming Lines

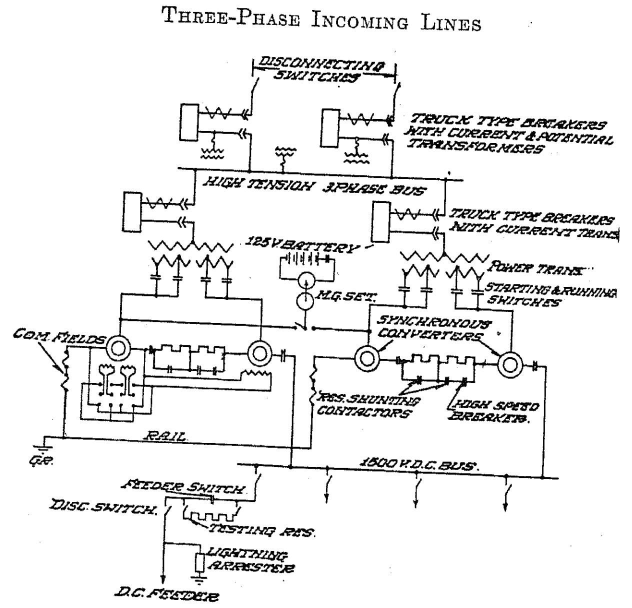

Fig. 1—Diagram of Connections for 1500-Volt D-C. Synchronous Converter Substation

Fig. 1 is a schematic diagram of a substation equipped with two 1500-volt 50-cycle synchronous converter sets. Each set is made up of two 750-volt converters connected in series. Power is supplied from two high-tension lines through oil breakers. Selective protection is provided by induction-type reverse-power relays. Each transformer is constructed with double low-tension windings, one being for each converter of a set.

Each converter of the set shown in Fig. 2 is rated at 1000 kw., 50 cycles, 750 volts, d-c. In order to reduce the space requirements, both converters were mounted on a common bedplate with the center bearing pedestal common to the a-c. ends of both machines. The converters are not mounted on the same shaft. Each rotates in a clockwise direction as viewed from the commutator end. The converters are compound-wound and connected in series to deliver 1500 volts d-c.

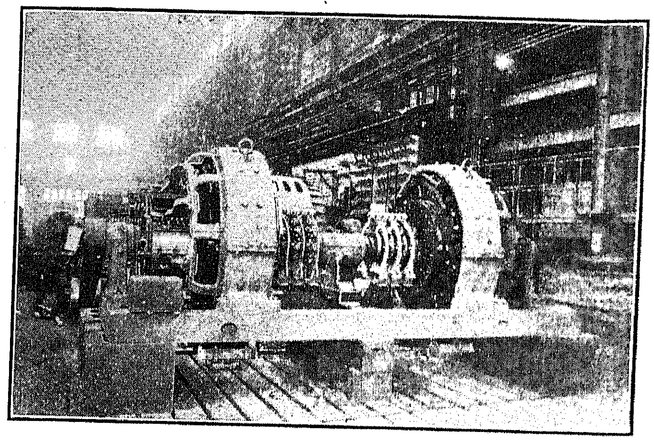

Fig. 2—2000-Kw. 50-Cycle 1500-Volt D-C. Synchronous Converter Set

The current-limiting resistor, one portion of which is shunted by a high-speed circuit breaker, is connected in series between the two converters. The shunt field of the converter connected to the trolley side of the system is permanently excited from the terminals of the machine, the negative side of which is connected to the rail return.

The lead-off set in the substation is started by means of a contact making voltage relay, the operating coil of which is connected permanently between trolley and rail. Both converters are a-c. self-starting through automatically controlled a-c. starting panels. The machine connected to the trolley side of the line is the first to start. When this converter has reached synchronous speed as is indicated by a drop in the starting current, an accelerating relay functions to start the machine connected to the negative or rail side of the system. Correct polarity is automatically established on the "low" machine, which, acting as an exciter for the "high" machine, establishes correct polarity on it also. The correct polarity thus automatically established on both machines, the transition from the a-c. starting voltage to full running voltage is made simultaneously on both machines.

After the closing of the a-c. running switches, the machines are controlled electrically as a single unit in exactly the same manner as the equipment in 600-volt railway substations, which are now so well known and so nearly standardized for this class of service.

The means for starting the second set are the same

as those supplied for multiple-unit stations operated at the lower voltages. The second set is started by the functioning of a relay, which, in effect, measures the temperature of the first machine. In case the first set is subjected to sudden overload, causing current-limiting resistance to be cut into the circuit or the voltage to drop to a predetermined limit for several seconds, the second set is started. The machines shut down on light load in the inverse order of starting.

The protective features such as bearing thermostats, overload relays, voltage protection, etc., are the same as those concerning which a great many articles have appeared in the trade journals.

Scheme of Control for the Operation of D-C. Feeder Circuits

The d-c. feeder equipment shown on the right hand of Fig. 3 is for supplying heavy suburban traffic and is known as the short-circuit detector type equipped with automatic reclosing features. This type of d-c. feeder

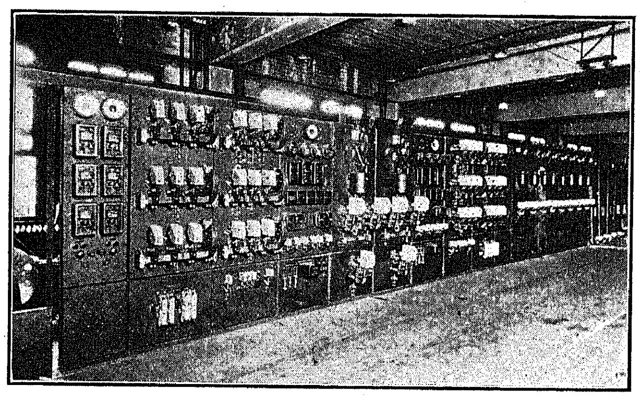

Fig. 3—Automatic Switchboard for Controlling a 4000-Kw. 1500-Volt D-C. Railway Substation Equipped with Two Sets Each, a Duplicate of the One Shown in Fig. 2. The Panels at the Right Control Eight 2000-Ampere D-C. Feeder Circuits

equipment differentiates between legitimate overload and short circuit, opening only to clear a short circuit. It has become a standard for automatic railway substations.

Any short-circuit impulse will, through separate and independent means, cause the high-speed breaker in the machine circuit and the breaker in the faulty feeder circuit to open. Due, however, to the characteristics of the two types of breakers, the high-speed breaker will open ahead of the feeder breaker and insert a block of load-limiting resistance. The value of this resistance is so adjusted as to limit the current on short circuit to the commutating capacity of the machine. As soon as the faulty feeder has been isolated by the opening of the feeder breaker, automatic reclosing devices function to re-establish full voltage by reclosing the quick-acting breaker to shunt the load-limiting resistor. The opening of the breaker in the faulty feeder circuit also sets up a testing bridge resistance circuit so arranged as to measure the resistance of the external circuit. When the trouble has been cleared and the resistance increased to approximately normal, a circuit is set up and the feeder switch automatically reclosed.

During any load incident to normal operation, such as the simultaneous acceleration of heavy trains, the feeder switch remains closed and supplies power at normal voltage, so long as the demand on the machine is not sufficient to cut in the current-limiting resistors through the operation of the overload relays. This type of feeder protection permits taking advantage of the short-time overload capacity of the d-c. feeder system under all normal operating conditions, even though the current may momentarily reach values in excess of those under some short-circuit conditions.

The circuit breakers on the feeder circuits are mechanically latched and are not dependent upon the d-c. bus voltage to hold them closed, therefore, they will remain closed regardless of the value of that voltage. Closing energy is taken from a storage battery. Due to the inherent characteristics of quick-acting circuit breakers, it is necessary, when used for feeder protection, that the holding-coil circuits be energized from the substation bus, unless an excessively large control battery is provided for this purpose. If this breaker is to properly select between short circuit and legitimate overload, it must be set to open on voltage reductions which may occur in normal operation, due to the functioning of the load-limiting devices which are arranged to reduce the bus voltage to a value which will protect the station against continued overload by transferring the load to adjacent stations. It is very evident, therefore, that such breakers, if set for selective action between short circuit and overload, are very liable to open, due to drop in voltage, when it is most important that they should remain closed.

Inasmuch as the high-speed characteristic of the breaker is essentially for machine protection, the breaker should be connected in the machine circuit and operated from voltage supplied by the machine. Thus, while the breaker is closed, it is supplied with a source of constant potential which makes it possible to accurately adjust the tripping mechanism. This tripping value will be such as to protect but one unit and, therefore, the high-speed breaker protection applied independently to each machine will be constant and independent of the number of units in operation.

Automatic switching for the control of motor-generators supplying 1500 volts d-c. follows the same scheme as that used for stations supplying the lower railway voltages, except where two generators are connected in series, in which case the control of the d-c. end of the set is very similar to that for the two converters in series, as already described.

Discussion

For discussion of this paper see page 781.

Automatic Edison Substation
of the Indianapolis Light and Heat Company

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Review of the Subject.—*This paper describes a two-unit automatic station which supplies power to a 3-wire, 250-volt, d-c. Edison network. It covers a general description of the machines and of some of the devices, of the scheme of automatic operation employed in controlling these machines, and of the method of protecting against emergency conditions.*

Although the paper itself deals with this specific installation only, the fundamentals involved are the same for the general class of equipments employed in this service. However, there are generally certain important differences caused by special operating requirements of the individual installation or system. The character of the load determines whether the station is to be in service continuously, or is to be placed in service in response to load demand, by remote control or by some other means. The capacity and regulation of the a-c. system determine whether Y-delta, compensator or reactor starting is to be employed. The location of the station with respect to the load and the layout of the d-c. system determine whether only one bus, or both high and low-voltage buses are required. These and other factors must be taken into consideration so that the equipment will meet the requirements of the individual installation or system.

Hence, each installation is to a certain extent its own problem, for it must maintain the required character of service under all conditions no matter what special operating requirements are present.

ONE of the most interesting automatic station installations on a d-c. Edison network has recently been placed in operation at Substation No. 1 of the Indianapolis Light & Heat Company of Indianapolis, Indiana.

The substation is located on the first floor of the company's main office building in the heart of the Indianapolis business district. It is adjacent to the monument circle, which is the center of the company's "mile square" d-c. distribution section of the city. Due to its being practically at the center of distribution for this section, the location is ideal for testing the operation and reliability of the automatic station for this class of service.

Previous to the installation of the present machines and the automatic control, the station contained one 1000-kw. motor-generator set, manually controlled, which had become too small for the load. This had resulted in a gradual reduction of the voltage of 240 volts, at peak loads, and even less, at its center of distribution. Hence, it was imperative that additional station capacity be installed in the immediate vicinity, to bring the voltage up to normal of 250 volts at this point, and also to relieve the nearest substation, No. 2, of a certain amount of load that it was carrying. The distance to this station is only 800 ft. and the available machine capacity in it is 2100 kw.

It was, therefore, decided to install 3000 kw. of additional station capacity in Substation No. 1, consisting of two 1500-kw. motor-generator sets, which were expected to be sufficient to allow the voltage at the center of distribution to be held at the normal value and also to take care of the load for some time to come.

The object in making the station double unit was a measure to insure a part of the station capacity being available at all times, and to save part of the running light losses during the light-load periods.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

The policy of the company in the past has been to use motor-generator sets only on its d-c. system. The flexibility of the motor-generator set as regards voltage and load control and the inherent ability of the design here adopted to protect itself against severe overload, either suddenly or gradually applied, also were further deciding factors in this case.

It might be here mentioned that the company has at present a total d-c. machine capacity in excess of 13,000 kw. installed in its six substations. There are further installed in each of substations No. 1, 2, 3, and 4 batteries each of 10,000-ampere discharge capacity for 40 min., making a total battery capacity of 40,000 amperes available for this length of time; or, on the basis of 8000 amperes each for one hour, making a total of 32,000 amperes available for one hour. Hence, it can be seen that extensive precautions are being taken against the possibility of loss of direct current in the city. In fact, The Indianapolis Light & Heat Company has the enviable record that its d-c. system has not been down once in over 31 years.

Central station operation is very exacting in its requirements,—continuity and reliability of service being absolutely essential. These features are possessed by the present day automatic station to a very high degree. The gradual replacement of the human element by mechanical and electrical devices designed to perform their own individual duties as nearly faultlessly as possible, and the saving effected by reducing operating costs, were the real deciding factors in making substation No. 1 of the Indianapolis Light & Heat Company completely automatic.

DESIGN OF MACHINES[1]

The design of the machines adopted is such as to be particularly adapted for central station work.

[1]More detailed description of this type of machines may be found in the June 1923 issue of the *G. E. Review* in articles by O. E. Shirley and by T. F. Barton and C. M. Fulk.

The rating of the a-c. motor is type *A T I*-10 poles, 2000-kv-a., 720 rev. per min. at 0.85-power factor, 4150-volts, 278-amperes, 60-cycles with 50 deg. cent. rise of the stator and 60 deg. cent. rise of the rotor. The method of starting in Y-delta with a current-limiting reactor used in series with the Y connection. This reactor is also inserted in series with the delta connection in times of emergency operation when the occasion demands.

One feature of this method of starting employed is the smoothness of acceleration and the freedom from any "bump," either at the closing of the starting or of the running breakers. The starting current is approximately 500 amperes.

The rotor is provided with a special low resistance, amortisseur winding to give a high synchronizing and pull-in torque. This has resulted in giving the motor the characteristic, Fig. 1, of being able to carry practically full-rated load as an induction motor for a given period, as well as to be able to resynchronize itself under load.

The excitation of the motor field is 250 volts and is supplied by the d-c. generator.

The rating of the d-c. generator is type *M C F*—8 poles—1500 kw.—720 rev. per min.—250-volts, 6000-amperes, with 50 deg. cent. rise. It is provided with commutating, compensating and differential-series fields. The latter is of such strength as to protect the machine properly in case of sudden d-c. overload or short circuit. It is normally shunted by a circuit breaker, so that it is only partly effective in normal operation, but

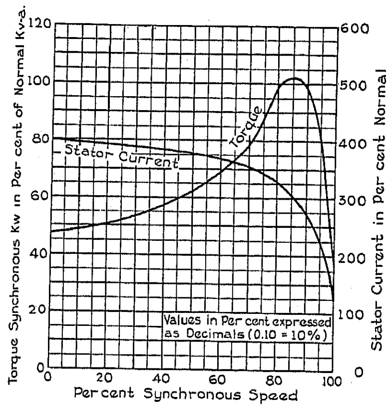


FIG. 1—TORQUE AND STATOR CURRENT CURVES OF 2000-KV-A. 60-CYCLE, 4150-VOLT A-C. MOTOR

under abnormal load conditions this circuit breaker is opened and the direct current output, or input, of the machine is very definitely limited.

As shown by the compounding curves, Fig. 2, with full load of 6000 amperes on the machine, and a dead short circuit applied, the final value of the line current with the series field shunting breaker open is 7000 amperes. It will be seen that the machine becomes practically a constant-current generator at full load current, due to the action of the differential series field and to the action of that amount of separate excitation furnished by the exciter.

The generator shunt field excitation is varied to hold normal voltage or normal current, as the case may be, by means of a motor-operated field rheostat. The generator is two-wire, the neutral being taken from balancer sets in several of the other substations. The installation of an automatically-controlled balancer set in this section is contemplated very shortly, however, to take care of the neutral current at this point.

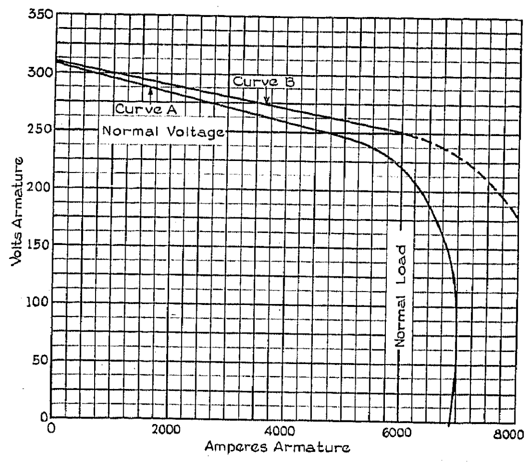


FIG. 2—COMPOUNDING CURVES OF 1500-KW., 250-VOLT D-C. GENERATORS

Curve *A*, Differential Series Field Shunting Breaker Open
Curve *B*, Differential Series Field Shunting Breaker Closed.

The rating of the direct-connected exciter is type *E C* 3 4 poles, 5-kw., 720-rev. per min., 125-volts, 40-amperes. In order to make the generator stable at low bus voltages, and to obtain the "constant current" characteristic at full-load current, the shunt field excitation is obtained from the generator and exciter armatures in series. One third of the excitation is being supplied by the exciter when the generator is operating at normal d-c. voltage. This means that the generator is essentially a separately excited machine at low bus voltages and so is perfectly stable in operation, even down to zero bus voltage.

CIRCUIT BREAKERS

The type of circuit breaker used for overload duty in the a-c. line for each machine is type *F H* 3, 15,000-volts, 500-amperes. The starting, running and reactor shunting breakers are type *F K*-4 7500-volts 300-amperes. The d-c. line breakers are type *C K*-3, 650-volts, 6000-amperes. The differential series field shunting breaker is type *C K*-3 650-volts 3000-amperes.

Control power for the various d-c. operated devices and circuit breakers is furnished by a 125-volt control battery which is kept in condition by a small trickle charge from the station bus. Control power for the a-c. operated devices is furnished by a type *H*, 5-kv-a. transformer on the incoming line for each machine.

A-C. Incoming Lines

There are at present a total of four 4100-volt 60-cycle, a-c. lines feeding into Substation No. 1 from the company's Mill Street and Kentucky Avenue generating stations. They are one for each of the two automatically-controlled machines, one emergency line, and one line for the 1000-kw. manually-controlled machine. By the use of special disconnecting potheads, either one or both of the automatically-controlled machines can be fed from any one of the first three lines.

Automatic Operation

The automatic control for these equipments in Edison network service is the same in its fundamentals as those in the industrial and railway fields. That is, the automatic control of these two machines by means of relays functions essentially the same as the manual control of similar machines. Each machine, when it is required, is started automatically from the a-c. end and balanced on the d-c. bus, exactly the same as if an operator were performing this operation. Each step in the sequence is controlled by one or more different relays which can function only when the correct electrical condition to proceed with the next step of the sequence is indicated. Voltage and current are held by varying the generator field, just exactly as if an operator were present at the switchboard, kept his attention fixed on the d-c. instruments and used his best judgment and skill in regulating the voltage and in limiting the current of the d-c. generator—only the electrical devices do it more promptly and faithfully. Upon cessation of the load demand, each machine is disconnected from the a-c. and d-c. buses, and shut down in just as logical a manner as it is started.

Protection by relays is embodied in the equipment against all ordinary emergencies and disturbances met with in central-station operation.

One unit in this installation is run leading and the other trailing. The former is the first to start upon load demand and is the second to shut down. The latter is the second to start and is the first to shut down. Either unit may be operated as the leading or trailing unit. The former always retains control of the bus voltage, the latter always tends to share its proportion of the station load.

Starting of Leading Unit

When there is demand for power, as indicated by low d-c. voltage at the center of distribution (240 volts or less), for a short time, the sequence of operations to start the leading machine and parallel it with d-c. bus is begun immediately.

The first step in the starting sequence is the checking of the bearing and machine winding temperatures, to be sure that the machine is in proper condition to be started. The a-c. line voltage is then checked to be sure that it is ample to start the machine, and to be sure that all three phases are present for this purpose. The motor-operated generator field rheostat must also be in the maximum resistance position before the starting sequence can begin.

The starting oil circuit breaker is then closed, applying full voltage to the Y-connected motor windings with the reactor in series.

At 95 per cent synchronous speed, providing exciter voltage and generator field current are present, field is applied to the synchronous motor and the oil circuit breaker short-circuiting the reactor is closed, thus placing full line voltage on the motor windings connected in Y.

The application of motor field causes the starting breaker to be opened after a time delay and the running breaker, connecting the motor windings in delta, to be closed.

This completes the final step in the starting sequence of the a-c. motor and permits the operation of paralleling the generator with the d-c. bus to begin.

Providing the polarity of the generator is correct, its voltage is increased gradually by the motor-operated field rheostat raising the d-c. terminal pressure to a value just slightly higher than that of the d-c. bus. Then the two d-c. line circuit breakers are closed. If the voltage is above 200 volts, the differential series field breaker is closed next, thus shunting about half of the differential series field current. This places the generator directly on the d-c. bus and allows it to take its load.

The d-c. voltage is then raised to normal of 250 volts and held there, providing the current output does not tend to exceed the generator rating of 6000 amperes.

Starting of Trailing Unit

When the current output of the station reaches a value of 5800 amperes, which is just slightly below the current rating of one generator, and remains at or above this value for a short time, the starting sequence of the trailing machine is begun.

This sequence takes place exactly the same as for the leading machine, the generator is paralleled with the d-c. bus in the same manner, and the differential series field breaker also is closed at 200 volts or above.

The current on this unit is then increased and that on the leading unit decreased, thus dividing the total station load between the two. The leading machine, however, retains the control of the voltage, whereas, the trailing machine merely tends to hold its share of the total value of current.

As is the case for a single unit operating alone, the operation of the two units in parallel is to hold the voltage at 250 volts up to their combined rated current output, and then with still further increasing station load to hold this rated current output at the expense of reduced voltage.

Shutting Down of Trailing Unit

On decrease in station load for a certain length of time to a value capable of being carried by one machine alone, the indication is given to shut down the trailing unit. The load is then gradually removed from this unit by the motor-operated field rheostat and is transferred to the leading unit, which holds the d-c. voltage constant. When the load has been entirely transferred, and a slight value of d-c. reversal is obtained (just below the running light current of the set), the machine is disconnected from the a-c. end and d-c. buses and is shut down.

Shutting Down of Leading Unit

Upon further decrease of station load for a certain length of time to a value such that the voltage at the center of distribution can be held practically at 250 volts by the surrounding stations, the indication is given to shut down the leading unit. The machine is then disconnected from the a-c. and d-c. buses and shut down.

Normally, the leading unit is in operation continuously, both day and night. The trailing unit usually starts on load demand at about 8 a. m. and shuts down on underload at about 10 p. m.

Automatic Station Relays

The voltage-regulating relay used in this equipment operates the generator shunt field rheostat of the leading machine to hold constant d-c. voltage. It is the same type of relay which has been used so successfully in automatic induction regulator installations, for holding constant voltage on a-c. feeders. It is of the single coil and plunger type and is actuated by the d-c. voltage at the center of distribution. It is provided with two sets of contacts, one for the "raise" and the other for the "lower" of the rheostat, and adjusted with the necessary floating range to hold the voltage within one per cent of normal.

The voltage equalizing relay, which is used to parallel the generator with the d-c. bus, consists of two coils operating a pivoted armature containing two sets of contacts. One coil is across the generator voltage, and the other coil across the bus voltage. At the proper time in the starting sequence, and as long as the bus voltage is higher than that of the generator, one set of contacts causes the "raising" circuit of the motor-operated field rehostat of the generator to be made, and thereby increases its shunt field excitation and hence its terminal voltage. As soon as the generator voltage becomes slightly higher than the bus voltage (about 1 per cent at normal bus voltage) the armature snaps over, opening this first set of contacts and closing the second set. This energizes the closing circuits of the d-c. line circuit breakers, thus connecting the generator to the bus.

The current balance relay is used to divide the total current properly between two machines while they both are in operation. It is of the same type as the voltage-equalizing relay, except that it has current coils instead of voltage coils, and that the contacts are set to float over a range of approximately 5 per cent at normal load, instead of being set with snap action. One coil is operated from a shunt in the line circuit of one machine and the other coil from a shunt in the line circuit of the other machine. The contacts are connected to operate the generator shunt field rheostat of the trailing machine. The "raising" contact is made when the current of the leading machine is the greater and the "lowering" contact when that of the trailing machine is the greater, thus balancing the load between the two units.

The differential frequency relay used is of the pivoted double-plunger armature type, with a total of four coils, two acting upon each plunger. One coil for each plunger is operated from the 220-volt 60-cycle control bus and the remaining coil for each is operated from the voltage (approximately 95 volts, 30 cycles at synchronous speed) obtained from a pair of slip rings on the direct-connected exciter. By means of suitable values of resistance and reactance connected in series with alternate coils of the relay, it is possible to calibrate it to operate on a given percentage difference in frequency between the two a-c. sources on its coils. Hence, the relay is adjusted to close its contacts when the machine is up to 95 per cent synchronous speed, and open them at 90 per cent synchronous speed. This gives the desired characteristics for the relay to allow motor field to be applied at the proper time during the starting sequence. It also functions to remove the motor field during certain system disturbances and again apply it when the disturbance has passed, as described later.

Protection

Protection is provided for contingencies which are of sufficiently serious nature to demand an immediate shut-down of the unit affected and to prevent its starting or restarting, until the station is given manual attention by an inspector, as follows:

1. Overspeed of 15 per cent above normal, as indicated by a hand-reset speed-limit switch on the end of the shaft.

2. Severe a-c. overload, as indicated by hand-reset plunger-type overload relays in the current transformer circuit.

3. Excessive bearing temperature rise, as indicated by hand-reset sylphone-type bearing-temperature relays, whose bulbs are inserted in the machine bearings.

4. Failure to complete the starting sequence within the normal time, as indicated by a time-delay relay set slightly longer than the time normally required to complete the entire a-c. and d-c. starting sequence.

5. Failure of insulation of either a-c. or d-c. machine to ground, or flashover of generator to ground, as indicated by a hand-reset current relay, whose coil

forms a low-resistance path from the machine frame to ground.

6. Loss of generator field, as indicated by a current relay in series with the generator field circuit.

7. Loss of motor field, as indicated by a balance relay, one coil of which is a current coil in series with the field circuit, and the other of which is a potential coil

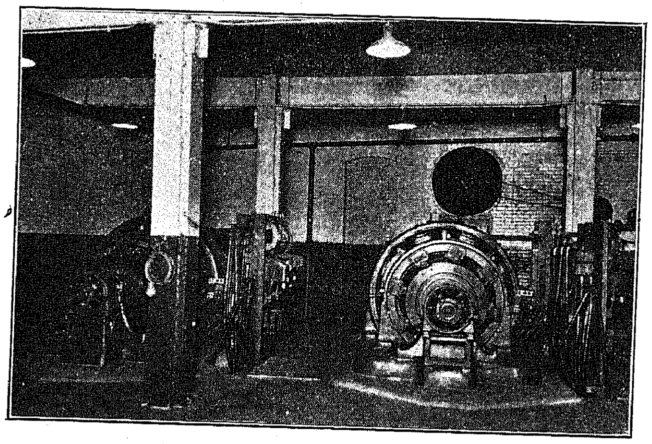


FIG. 3—TWO 1500-KW., 60-CYCLE, 4150-VOLT A-C., 250-VOLT D-C. AUTOMATICALLY CONTROLLED MOTOR-GENERATOR

across the generator voltage. The relay operates only in case of loss of field current while generator voltage is present.

8. Reverse polarity of generator during the starting sequence, as indicated by a polarized relay whose potential coil is energized by d-c. generator voltage.

Protection is also provided for cases of a nature such that it is necessary to suspend normal operation tem-

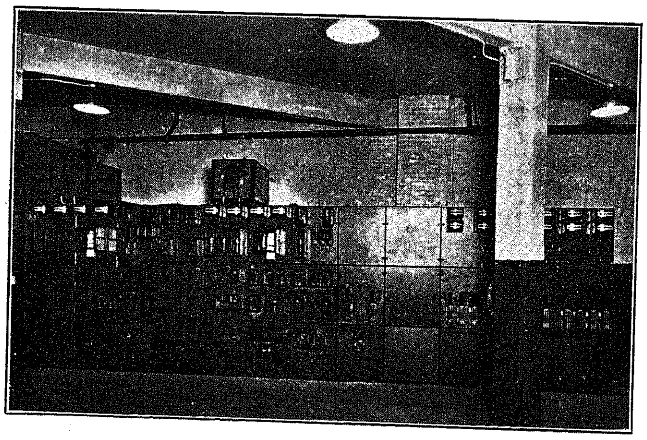


FIG. 4—AUTOMATIC SWITCHBOARD FOR CONTROLLING TWO 1500-KW. MOTOR GENERATORS

porarily during the emergency, but allow its immediate resumption when the emergency has disappeared, as follows:

1. A-c. undervoltage during the starting sequence, by means of a sensitive potential relay across the control power transformer.

2. Single-phase starting, by means of a potential relay across a potential transformer, whose primary is on a different phase than that of the a-c. undervoltage relay.

3. Excessive temperature rise of motor windings due to continued moderate overload on one or all phases, by means of current-operated thermal relays having a time-temperature characteristic similar to that of the motor.

4. D-c. reverse power, by means of a sensitive d-c. reverse-power relay, set to operate on a value of current less than the running light current of the set.

5. D-c. overload, by means of a potential relay, set to operate at 200 volts d-c. connected across the d-c. generator armature.

OPERATING FEATURES

Upon d-c. reverse power as indicated by the d-c. reverse-power and underload relay:

FIG. 5—INTERIOR OF STATION SHOWING AUTOMATIC SWITCHBOARD AND TWO 1500-KW. MOTOR GENERATORS

1. The generator differential series field, which becomes cumulative compound for motor operation, is inserted full strength by opening the differential series field shunting breaker;

2. The shunt field excitation of the generator is prevented from being decreased by opening the "lowering" circuit of the motor-operated field rheostat;

3. The shunt field excitation of the generator is also held at a fixed minimum value by shunting a sufficient amount of the field rheostat to give practically no-load running light field current.

The above operations limit the amount of reverse current that can be taken from the d-c. system.

If this condition continues for a certain length of time, the set is shut down on d-c. underload. If it disappears before the expiration of this time, normal operation is resumed immediately.

Upon d-c. short circuit or overload, as indicated by the d-c. undervoltage relay which operates on a reduc-

tion in d-c. bus voltage to 200 volts or less, the generator differential series field is inserted full strength, thus definitely limiting the output of the machine. Upon increase of the bus voltage, normal operation is again resumed.

In case of a-c. or d-c. system disturbances sufficiently severe to pull the motor out of step, provision is made in the control to allow it to pull back into step if possible without disconnecting the unit from either the a-c. or d-c. buses.

At approximately 90 per cent synchronous speed, as indicated by the differential frequency relay:

1. The motor field is removed by opening the motor-field contactor;
2. The reactor is inserted in series with the motor windings, by opening the reactor short-circuiting breaker;
3. The generator differential series field is inserted full strength, by opening the differential series field shunting breaker;
4. The shunt field excitation of the generator is gradually decreased by "lowering" the motor-operated field rheostat.

The result of the above operations, which cause the motor to function as an induction motor, limit the current drawn from the a-c. system and reduce the load on the generator, is to give the motor an opportunity to pull back into step by virtue of its high resynchronizing and pull-in torque. Fig. 6 is an oscillogram of a similar machine showing resynchronizing from 90 per cent speed caused by loss of the a-c. supply.

Protection is afforded against overheating of the pole face winding by causing the set to be disconnected from the a-c. and d-c. buses within a predetermined time (40 sec.), in case load or voltage conditions do not permit synchronizing.

Also, the set is shut down if the speed drops to a value of approximately 70 per cent of normal, below which the pull-in torque of the motor begins to fall off rapidly. In both of the above cases a restart takes place immediately if proper starting conditions obtain.

If the a-c. supply is interrupted, provision is made to run the set from the d-c. end for a given time so that it may be in readiness to pick up its load immediately upon return of a-c. power.

Upon interruption of the a-c. supply as indicated by the d-c. reverse power and underload relay and the a-c. undercurrent relay:

1. The generator differential series field is inserted full strength by opening the differential series field shunting breaker.
2. The shunt-field excitation of the generator is prevented from being decreased by opening the "lowering" circuit of the motor-operated field rheostat.
3. The shunt-field excitation of the generator is also held at a fixed minimum value by shunting a sufficient amount of the field rheostat to give practically no-load running light field current.
4. The motor field is removed by opening the motor-field contactor.
5. The reactor is inserted in series with the motor

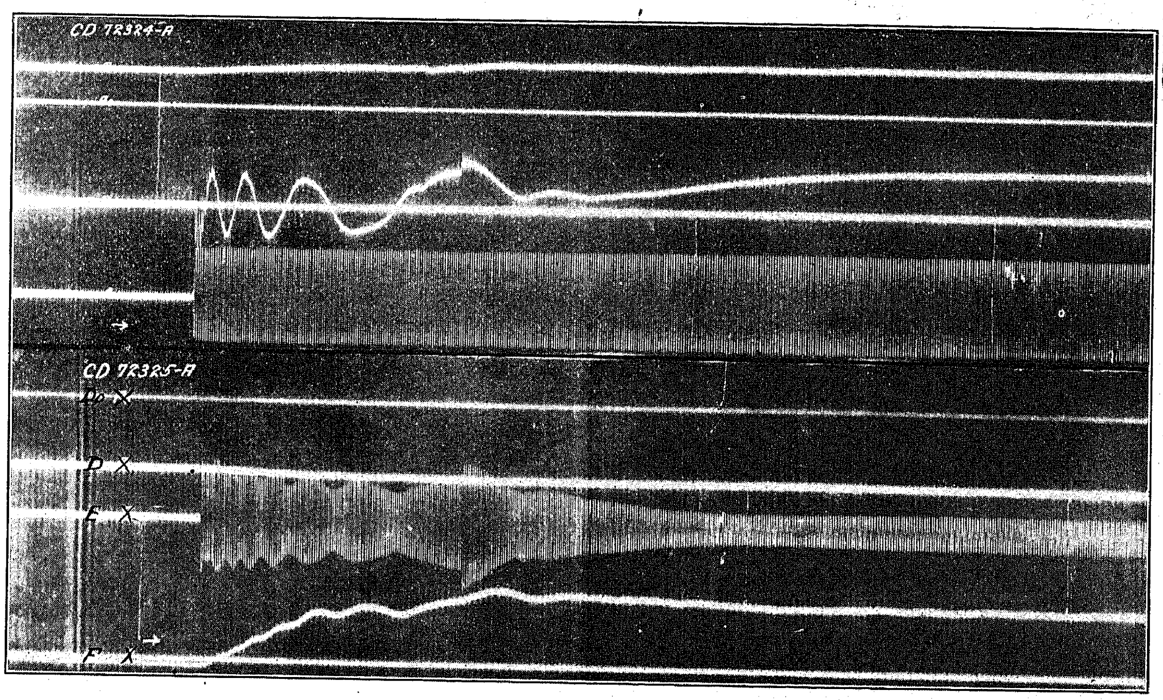


Fig. 6—Oscillograms of Resynchronizing Test from 90 Per Cent Speed, Motor Connected to Line Through 10 Per Cent Series Reactors

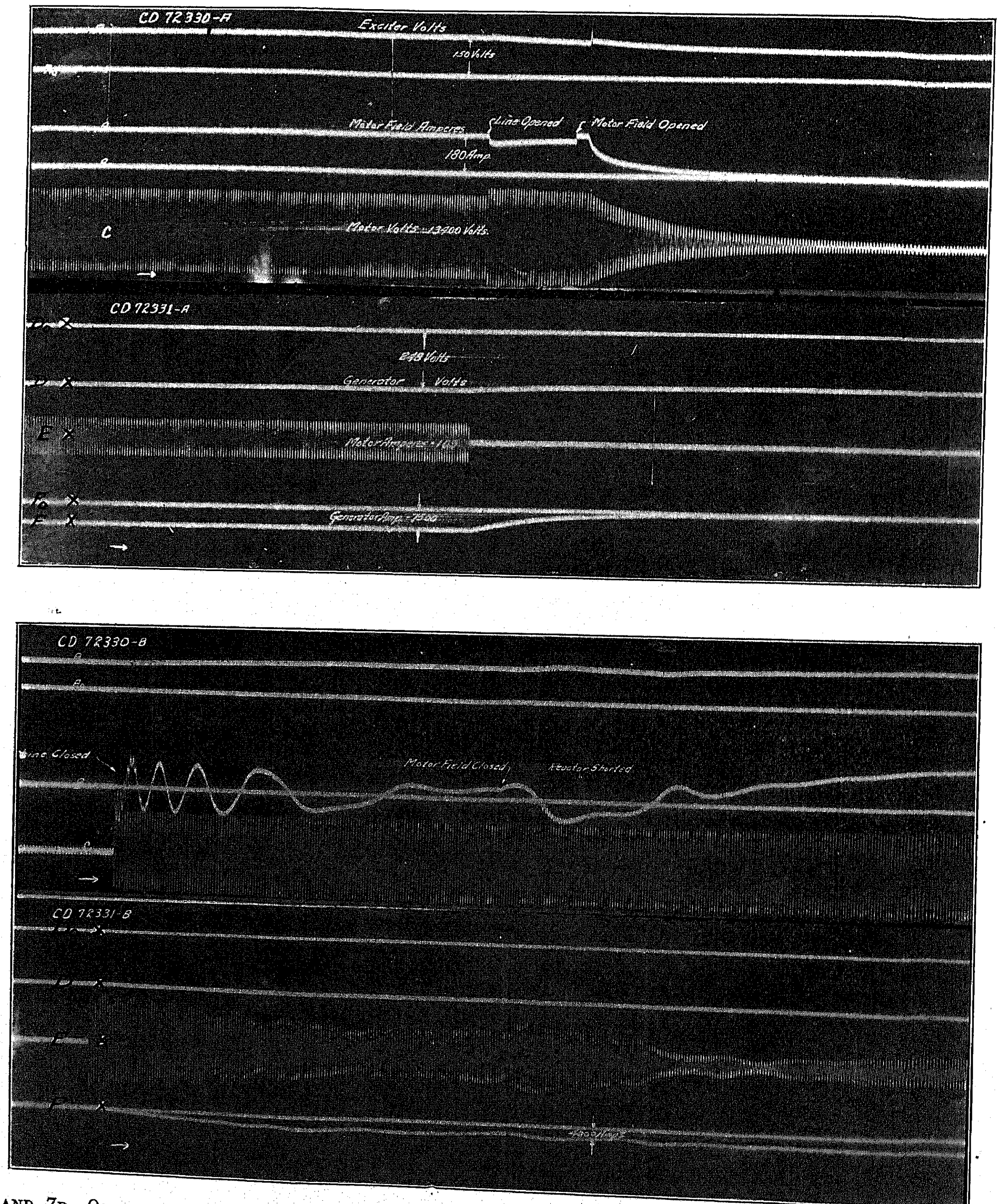
Curve *A*, Exciter Voltage; *B*, Motor Field Current; *C*, A-C. Line Voltage; *D*, D-C. Generator Terminal Voltage; *E*, A-C. Line Current; and *F*, D-C. Line Current.

windings by opening the reactor short circuiting breaker.

Operations 2 and 3 prevent the possibility of over-speed while running from the d-c. end, and operations 1, 4 and 5 place the set in readiness for the return of a-c. power.

Upon return of a-c. power with the motor out of step, the control is in the proper position to allow the motor to pull itself back into step and pick up its load. Fig. 7 is an oscillogram showing the sequence of operations caused by the interruption of the a-c. supply for a period of 3.8 seconds.

If a-c. power does not return within a given time, or if the speed drops below 70 per cent of normal, the set is shut down and is allowed to restart immediately if proper starting conditions are present.



FIGS. 7A AND 7B—OSCILLOGRAMS OF INTERRUPTION OF POWER SUPPLY AND RESYNCHRONIZING TEST. MOTOR CONNECTED TO LINE THROUGH 10 PER CENT SERIES REACTORS

Curve *A*, Exciter Voltage; *B*, Motor Field Current; *C*, A-C. Motor Voltage; *D*, D-C. Generator Terminal Voltage; *E*, A-C. Line Current; and *F*, D-C. Line Current.

In case of a-c. line short circuits, causing a drop in a-c. voltage down to 30 per cent of normal, or less, for a period of several seconds, the same sequence of operations takes place as upon interruption of the a-c. supply. The only difference is, that the device which indicates this condition is the a-c. undervoltage relay set at the proper value instead of the a-c. undercurrent relay.

Double Unit Operation

Either machine can be made leading or trailing by means of throwing a multi-pole change-over switch in one of two positions. This change-over also can be made without interruption of service whether one or both machines are in operation at the time.

However, in case the normally leading machine is running alone and shuts down, due to the functioning of a protective device, the trailing machine is started immediately and automatically takes the place of the leading machine, by means of an emergency starting relay provided for this purpose.

If both machines are operating, and the leading machine shuts down due to the functioning of a protective device, the trailing machine automatically takes the place of the leading machine by means of the same relay, without interruption of service.

In either case, when the normally leading machine is restarted, it is paralleled with the d-c. bus, as in normal operation, and the control functions to restore automatically the normal status of leading and trailing machines. If the load is sufficient to warrant the operation of both machines, they both remain connected to the bus. If the load is not sufficient to warrant the operation of more than one machine, the load is shifted gradually from the trailing to the leading machine and when completely shifted over, the trailing machine is shut down.

It is the intention very shortly to run the two machines from separate incoming lines, one from Mill Street station and one from Kentucky Avenue station instead of from the one line from Mill Street Station alone. In the latter case, it is quite important on power failure on one line, to have the machine on the remaining line automatically become the leading machine and function as such in case of sufficient load demand. This is accomplished in the same manner as failure of the normally leading machine, due to the functioning of a protective device, as already explained, and so as long as power is present on one of the lines, the machine on it will respond to load demand as the leading machine. When power returns on the other line, each machine is again returned to its normal status without interruption of service.

Application

Although this particular installation is one which occupies a most important position with respect to the total d-c. load of the system, the type of machines and control here used are being applied to outlying stations with equal success. Neither is the application limited to d-c. Edison systems alone, but it can be extended to other fields in which fairly steady loads are met and where uninterrupted service is of prime importance. The inherent ability of the machine to protect themselves against severe overloads in either direction, without the use of any complicated regulating equipment, is a decided advantage which should warrant favorable consideration.

Discussion

For discussion of this paper see page 781.

Operating Experience with Automatic Equipment on an Edison System

BY F. D. WYATT

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Review of the Subject.—*The paper describes the conversion equipment serving the Edison System in Cincinnati and the application of automatic machines and controls to that system. During two years' operating experience with automatic equipment, it was found to be very reliable and to operate much quicker than manually-controlled equipment. Tests on the automatic equipment show it to be very valuable for service restoration work.*

* * * * *

I. Description of System

THE Edison System in Cincinnati supplies an area of approximately 2.2 sq. mi. including all of the down-town business district of wholesale and retail stores, hotels, theatres, small factories, and warehouses. The load center is in a high-priced district which necessitates substations of as small proportions as possible. The service requirements are very exacting as to outages, which means that the most reliable conversion equipment must be used so as to limit the size of stand-by batteries.

The peak load on this system in 1923 was 25,300 kw. (185,000 amperes) which was 22.6 per cent of the total system peak kw. load. The average peak load on the Edison system throughout the year is 150,000 amperes to 165,000 amperes at a load factor of 50 per cent.

There are at present six substations supplying the d-c. system as follows: (1) 4th Street Battery, (2) Plum St. Station, (3) Eighth Street, (4) Gano, (5) Peebles and (6) Jackson; the first four of which are manually operated stations and the last two are automatic motor-generator stations.

The synchronous motor-generator sets at 4th Street Battery, Gano Substation and Plum St. Station are fed from the 4325-volt bus at Plum St. Station.

The five 13,000-ampere synchronous converters at Plum Street Station, 8th St. Substation and Gano Substation and the two 7500-ampere automatic motor-generator sets at Peebles and Jackson Substations are fed from the 13,200-volt cable system radiating from West End generating station.

The conversion and storage battery capacity of the stations is shown by Table I.

It can be seen that there is a total installed conversion capacity of 99,660 amperes per side or 199,320 amperes at 150 volts. However, in addition to the conversion capacity there are three standby batteries having a combined six-minute rating of 84,850 amperes per side or 169,700 amperes total at a limiting voltage of 82 volts per side. The six-minute rating is the maximum current that can safely be taken from the batteries without serious injury to the plates. The discharge rate is controlled by motor-operated end cell switches.

It is evident that reliable conversion equipment can assist materially in saving the stand-by batteries during outages caused by system disturbances.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

TABLE I

Station No.	Name	Conversion Equipment: M-G. Sets: No.	M-G. Sets: Rating amps. per side	Syn. Converters: No.	Syn. Converters: Rating amps. per side	Stand-by Battery: Rating 6-min. amperes
1	4th St. Battery	2	4,000	..	..	13,050
2	Plum St. Station	1	3,330	1	13,000	..
3	Eighth St.			2	13,000	37,200
4	Gano	1	3,330	2	13,000	34,600
		1	5,000			
5	Peebles	1	7,500			
6	Jackson St.	1	7,500			
Total		7	34,660	5	65,000	84,850

II. Problems of Operation

On account of the requirements of incandescent lighting and elevator load it is imperative to keep the voltage within close limits and to maintain the voltage balance.

The load conditions require conversion equipment near the load centers so as to cut down on the size of copper necessary for low losses and close regulation. The unbalanced load necessitates three wire equipment to maintain balanced voltage. For unbalanced conditions two generator motor-generator sets, and 3-wire synchronous converters are used.

Another problem in connection with an Edison System is that of securing skilled and efficient operators to handle the rotating equipment. The cost of training and developing operators with the ability to handle rotating equipment rapidly during a disturbance is quite large.

Following an a-c. shut down of sufficient duration to exhaust the stand-by batteries, equipment and methods must be available to properly restore service on the Edison System without undue delay or damage to apparatus.

III. Influence of Automatic Equipment on Distribution of Conversion Apparatus

The use of automatic stations permits the economical use of smaller units located closer to the load centers, thus reducing the distribution losses. Also, one

operator can take care of the operation and inspection of several stations thereby cutting down the operating expenses. Having a larger number of small automatic substations on a system aids greatly in restoring service after a complete shut-down. The automatic equipment can be put on the line and will pick up its full load current, thereby raising the voltage on the whole system to a point where the other conversion equipment can be brought in and be safely loaded up, until normal voltage is again obtained. This process restores service to the Edison System in the minimum time with the least strain on the machines and system.

IV. Disturbance Conditions

Disturbances on the a-c. system which cause a dip in voltage will cause a corresponding reduction in the synchronous converter output voltage. When the synchronous converter operates in parallel with motor-generators and stand-by batteries, or carries a considerable amount of motor load, the reversal of current through the synchronous converter, resulting from a reduction of voltage, may be of such magnitude as to cause damage or flashover to the synchronous converter. It is necessary, therefore, to limit the amount of reverse current that will flow or take the converter off the line. As the regulation of a synchronous converter is usually low, a reduction of a-c. voltage of less than 10 per cent will ordinarily cause a reversal of output current.

A severe reduction of a-c. voltage will cause a synchronous motor-generator set to drop out of step, due to the reduction of synchronizing torque at the lower voltage.

In the case of a complete failure of the a-c. supply, all synchronous converters and motor-generator sets will be shut down and the Edison load carried by the stand-by batteries for an interval until conversion equipment is again placed in service or the batteries become exhausted. The sooner conversion machines can be restored to service after an a-c. outage, the less will the capacity of the batteries need to be. A number of tests of the starting period for the various type machines gave the following results:

TABLE II

Equipment	Method of Starting	Time to Start
Manually-operated synchronous converter....	D-C.	4 min.
" " " motor-generator	D-C.	4½ min.
" " " converter....	A-C.	1¼ min.
" " " motor-generator	A-C.	2 min.
Automatic synchronous motor-generator	A-C.	40 sec.

From these figures, it is apparent that the automatic motor-generator sets will begin to carry load in much less time than other conversion equipment. These machines will start up immediately on return of the a-c. supply while the manually-operated machines must be started by the station operator, which may involve additional delays.

The automatic synchronous motor-generators are of considerable value in reducing the length of discharge of the stand-by batteries.

V. Selection of Automatic Conversion Equipment

In the selection of automatic conversion equipment, both motor-generators and rotary converters were considered in regard to the following three main items: (1) stability, (2) simplicity and (3) efficiency. It was found that the synchronous motor-generator set was much more stable than the synchronous converter under all kinds of disturbance on both load and supply circuits. The control equipment for an automatic synchronous motor-generator set is much simpler than the automatic synchronous converter control and is less liable to cause trouble. The synchronous converter has somewhat better efficiency than the synchronous motor-generator set. On account of its better commutating characteristics, greater stability, load-limiting characteristic and flexibility of voltage control, the automatic synchronous motor-generator set was chosen. The load-limiting characteristic of the motor-generator is such that it will limit the output current to not more than full rating from zero to full rated voltage. It is, therefore, of considerable value for service restoration, following the total shut-down of the Edison System.

These automatic motor-generator sets will begin supplying full-load current from practically zero voltage and thus gradually build up the potential on the entire net-work to a point where the synchronous converters can safely take their share of the load. By this method service can be restored to the Edison System in the minimum of time with the least ill effects.

The automatic synchronous motor-generator sets and the semi-automatic rotary converters will be next described and their performance records given under various disturbances.

VI. Peebles Automatic Synchronous Motor-Generator Set

Description. The set consists of three units, a synchronous motor, a d-c. generator, and an exciter, all connected together on the same shaft, Fig. 1. The field of the motor is furnished by the exciter, the field of the generator is across bus voltage in series with the exciter voltage, and the exciter is self-excited.

Motor. The synchronous motor is designed to give sufficient torque to synchronize with approximately full-load on the generator, provided the supply voltage is normal. Series reactance is connected in the motor circuit, across which full voltage may be impressed.

The motor torque on starting is furnished by the amortisseur winding. The motor field is energized by a slip relay at 95 per cent of synchronous speed, which furnishes sufficient torque to pull the motor up

to synchronous speed. For more than 5 per cent slip the torque would be reduced, if the field was excited. The use of the slip relay, adjusted to open the field circuit at a predetermined slip and to close it at a slightly lower value, makes available the maximum torque at all speeds. The rating of the motor is 2500 kv-a. 60-cycles 600-rev. per min.

Generator. The generator is of the commutating pole, compound-wound type with shunt and series differential windings on the main poles. The shunt

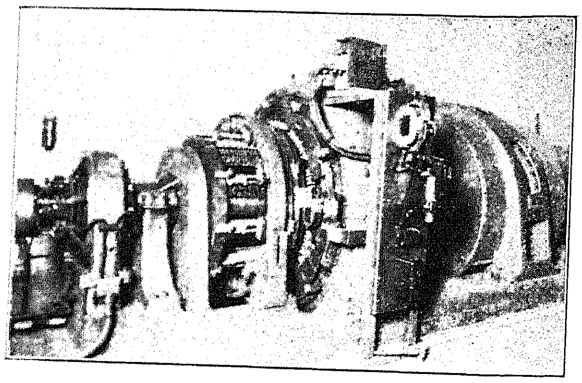

FIG. 1—A 7500-AMPERE 250-VOLT AUTOMATIC SYNCHRONOUS MOTOR-GENERATOR SET

winding is excited at 375 volts from a series connection of the exciter, (125 volts) and the generator, (250 volts). The series differential winding is normally short-circuited by a circuit breaker with sufficient resistance in its connections to allow approximately 20 per cent differential excitation for the generator under normal conditions. Under short-circuit conditions the above-mentioned circuit breaker opens, giving full differential protection to the generator and allowing only full-load current to flow with the generator short-circuited. The rating of the generator is 7500 amp. 600 r. p. m. 250 volts no-load and full-load 1875 kw. continuous 55 deg. cent. rise.

Exciter. The exciter is a commutating pole, compound-wound generator with shunt windings self-excited. The magnetic design is such as to permit stable operation over a wide range by the exciter field rheostat control. The cross section of the pole is reduced at one point so that that portion of the iron is saturated even when the exciter voltage is as low as 80 volts. This permits the omission of the synchronous motor field rheostat. The exciter is rated as follows: d-c. generator, volts no-load 125; full-load 125; 28 kw., continuous 50 deg. cent. rise.

Control Equipment. The control equipment Fig. 2 operates to bring the set up to speed and put it on load when the supply cable is energized.

The set is fully protected by relays against the following conditions:

1. Overheating of machines.
2. Overheating of bearings.
3. Grounding of machine windings.
4. Overspeed.
5. Flashover of generator or exciter.
6. Excessive current output or input on d-c. end.
7. Heating of amortisseur winding.
8. Reversed polarity on generator.
9. Loss of field on synchronous motor.
10. Single-phase starting.
11. Phase reversal.
12. A-c. overload.
13. Delayed starting through any cause.
14. Interference of air supply.

Performance. The sequence of operations is as follows:

The switch is closed at the cable source which energizes the potential and power transformers. The master starting element and the single-phase and reverse-phase protecting relay operate, providing all three-phases are all right, phase rotation is correct and the voltage is above 70 per cent. The master starting relay will not close below 70 per cent normal voltage, but will remain closed down to 20 per cent normal voltage. This insures the machine staying on the line through all ordinary surges.

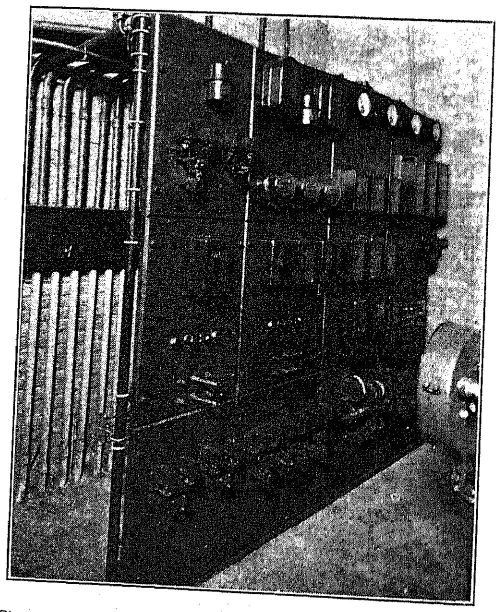

FIG. 2—CONTROL EQUIPMENT FOR A 7500-AMPERE 250-VOLT AUTOMATIC SYNCHRONOUS MOTOR-GENERATOR SET

The control current contactor closes next, providing all the protective devices are closed, causing the starting oil circuit breaker to go in. The machine then comes up to speed with the starting reactors in series. At 95 per cent speed the slip relay, connected across slip rings on the exciter, closes, allowing the motor field contactor to close and excite the motor. The main running switch which short-circuits the reactors then goes in and the motor is on full-line voltage. With the motor on the line, a contact-making voltmeter balances

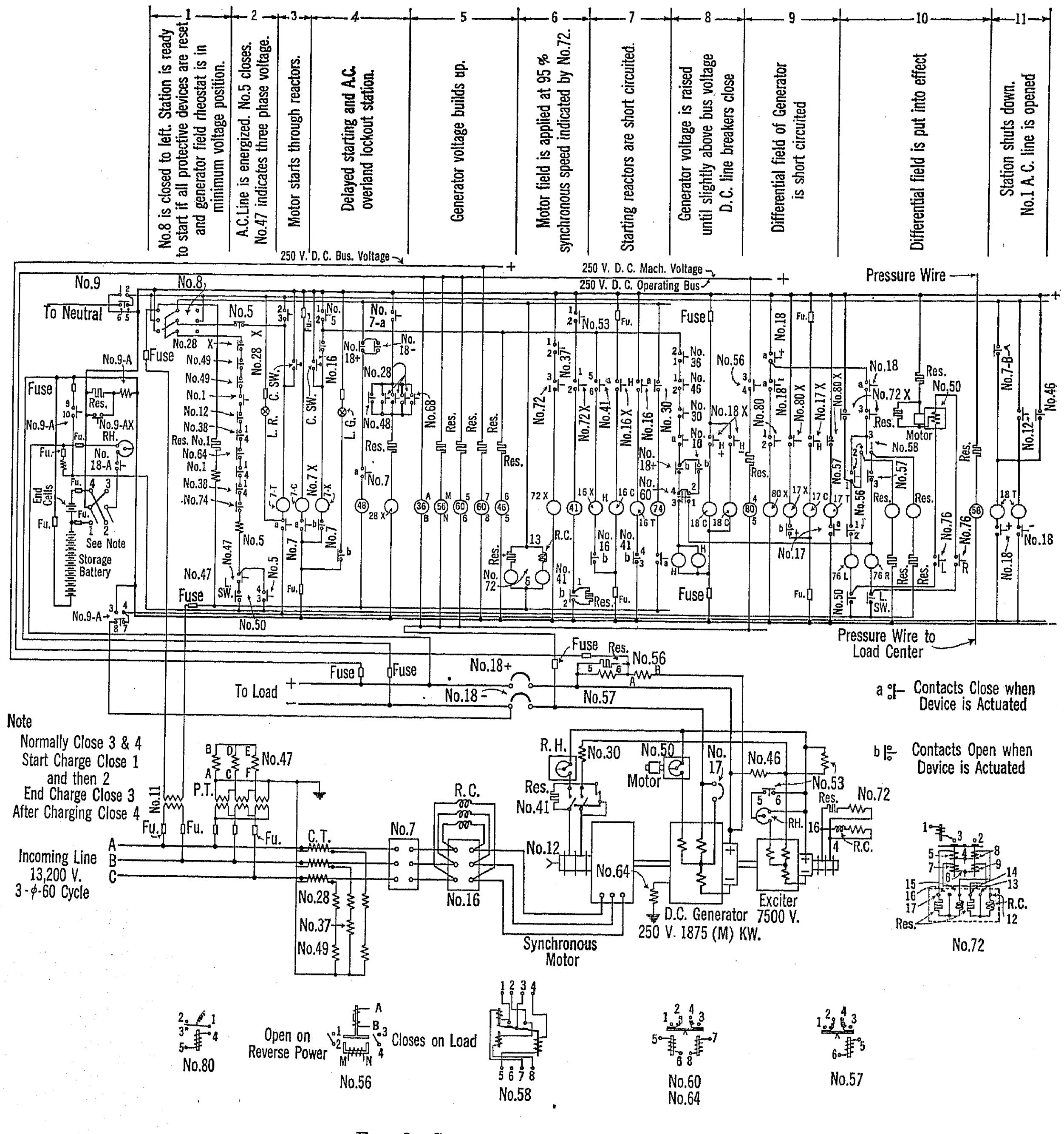

FIG. 3—SEQUENCE CONTROL DIAGRAM.

1.	Master starting element.
5.	Control current contactor.
7.	Main oil circuit breaker and mechanism.
8.	Control power switch.
11.	Control power transformer.
12.	Overspeed device—hand reset.
16.	Running oil circuit breaker and mechanism.
16X.	Hesitating closing control relay for 16.
17.	Series field breaker.
17X.	Auxiliary closing control relay for 17.
18 +.	Positive breaker and mechanism.
18X +.	Auxiliary closing control relay for 18 +.
18—.	Negative breaker and mechanism.
+18XH.	Hesitating closing control relay for 18 + and 18 —.
28.	A-c. Overload time delay relay.
28X.	Auxiliary closing control relay for 28, 48 and 68.
30.	A-c. machine field relay.
36.	Polarized relay.
37.	Underload relay (A-c.)
37X.	Auxiliary closing control relay for 37.
38.	Bearing-temperature relay—hand reset.
41.	Synchronous motor field contactor.
46.	Generator relay.
47.	Potential reverse-phase relay.
48.	Starting-protective relay.
49.	A-c. machine-temperature relay.
50.	Motor-operated field rheostat.
53.	Exciter relay.
56.	D-c. reverse-power relay.
57.	Current-regulating relay.
58.	Voltage-regulating relay.
60.	D-c. voltage equalizing relay.
64.	D-c. grounding-protective relay—hand reset.
68.	D-c. temperature relay.
72.	Slip relay.
72X.	Auxiliary closing control relay for 72.
74.	Time delay tripping relay.
76.	Rheostat control relay.
80.	D-c. undervoltage relay.
80X.	Auxiliary closing control relay for 80.
9a.	Battery failure contactor.
9aX.	Auxiliary push button switch for 9a.

the voltage of the generator with the bus and when balance is reached the d-c. circuit breakers close, providing the generator has correct polarity. Should all the above operations take too much time through any cause whatsoever, a timing relay, which starts functioning upon the closing of the starting switch and times the interval until the d-c. breakers close, will take the set off the line and lock it out. As the machine picks up load, the differential field is short circuited. However, there is enough resistance in the breaker and contacts to give about 20 per cent differential effect under normal conditions.

With the machine under load, the contact making voltmeter regulates the output so as to keep constant voltage at some predetermined load center. When full-load is reached, the current-control relay keeps the current constant down to nearly zero voltage, by lowering the voltage. When the voltage has dropped to 75 per cent of normal or 180 volts, the series field breaker opens and there is a momentary drop of voltage and current. The contact-making ammeter counteracts this effect and raises the current to the full-load value of the machine again. Thus on a d-c. short circuit when all the synchronous converter equipment is lost, the automatic motor-generator set will continue to put full-load current into the system until the trouble clears itself. Then the voltage will be gradually raised until conditions are again normal. In case of trouble on the a-c. system, the series field short-circuiting breaker opens, due to reverse generator current and strengthens the generator field, thereby reducing its speed and effectively preventing the set from pumping power into the a-c. system.

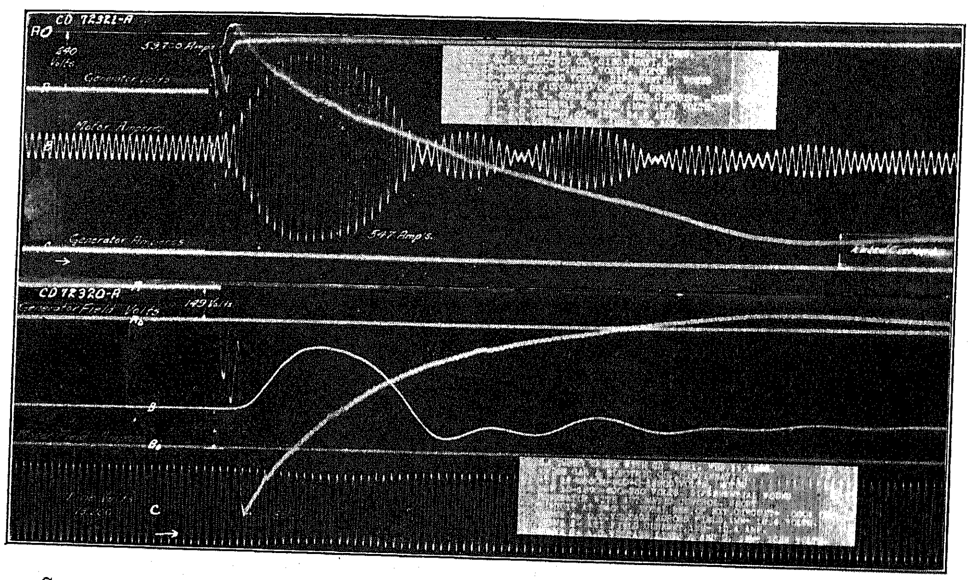

Fig. 4—Oscillogram Showing the Effect of a D-C. Short Circuit on a 7500-Ampere 250-Volt Synchronous Motor-Generator Set

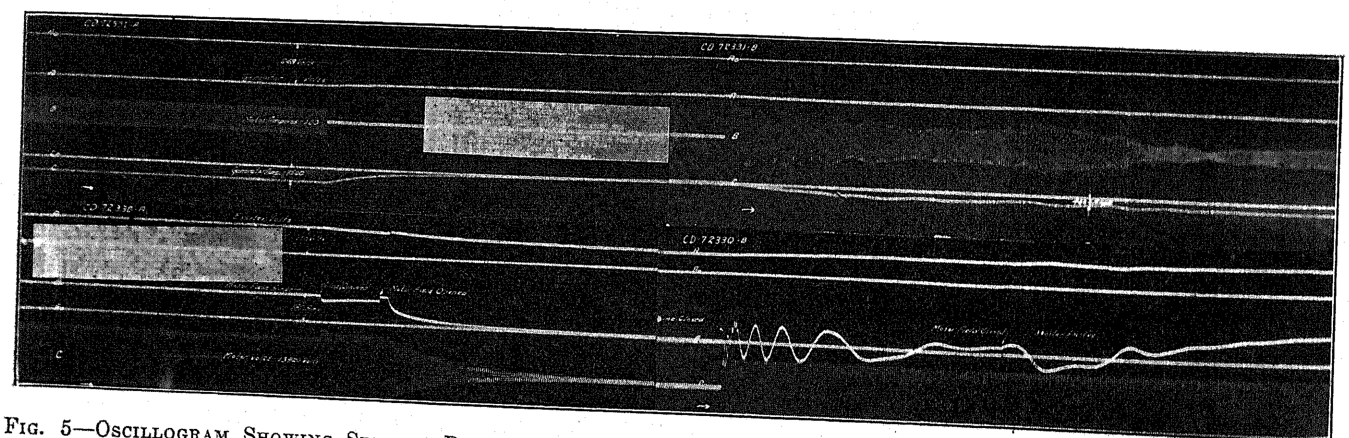

Fig. 5—Oscillogram Showing Service Restoration after an A-C. Failure on a 7500-Ampere 250-Volt Automatic Synchronous Motor-Generator Set

D-C. Short Circuit. Tests were made to determine the behavior of the motor-generator set under a full metallic short circuit on the bus bars. The performance is shown in Fig. 4. The short circuit was caused by closing an air circuit breaker which had main, auxiliary and carbon contacts. The successive closing of these contacts is shown on the oscillogram. The

generator current reached a maximum value of 59,700 amperes in the total time of four cycles. The main contact of the breakers closed in two and one-half cycles, after the closing operation started. The generator voltage had been reestablished to normal so that the short-circuit current actually reached its maximum value in one and one-half cycles. This overload current was reduced to rated value of the machine (7500 amperes) in 1.1 sec.

The generator successfully operated under this very heavy momentary load without undue flashing of the commutator.

The motor was knocked out of step by the high torque required by the generator, but successfully pulled back into step within 0.75 sec. This is the most severe d-c. short circuit it is possible to obtain, as the total value of the external resistance from the machine terminals for this short circuit was only 0.00041 ohms.

The duty of the generator in closing in on a dead Edison System would be very much less, as the generator voltage would be reduced to a value of 110 volts and considerable feeder resistance would be interposed in the circuit.

Performance of Machine during A-C. Disturbance. With the set running as in normal operation, carrying a rated load of 7500 amperes, the a-c. cable feeding the station was opened up and reclosed after an interval of 4.56 sec. The operations which took place on the machine were as follows: the set continuing to run on the d-c. side at reduced speed. The operations show very clearly in Fig. 5.

The short-circuiting breaker across the differential series field opened up, which further increased the field strength of the generator, reducing the speed still further.

Due to the operation of the a-c. under current relays, the motor field was disconnected from the exciter and short-circuited and the oil circuit breaker short-circuiting the starting reactors was opened up.

When the a-c. supply was closed in again, the motor speeded up, the induction motor starting current being limited by the starting reactors. When the motor attained 95 per cent of normal speed, the synchronous motor field was closed. Fifteen cycles later the starting reactors were short-circuited and the motor pulled into step. In the meantime, the generator began carrying load as soon as the a-c. supply was re-established. The generator was carrying a load of 4000 amperes when the synchronous motor pulled in step. The generator current was then increased to the rated value by the control relay equipment.

The conditions surrounding this test are somewhat more severe than the usual a-c. disturbance which reduces the system voltage to 50 per cent or in an extreme case to 30 per cent of normal.

VII. Semi-Automatic Synchronous Converter

Tests were performed on semi-automatic booster-type synchronous converters to determine their performance under various disturbances on the a-c. system. The synchronous converters were modern booster-type machines rated 13,000 amperes 240/300 volts.

Sequence of Operations. The control board for the semi-automatic rotary converter is shown in Fig. 6 and the sequence diagram in Fig. 7.

The rotary is started as any manually-controlled machine equipped for star-delta a-c. starting, voltage and polarity checked by the operator and the negative d-c. breaker closed.

There are two breakers on the positive lead of the converter. A 4000-ampere breaker in series with a resistance stack of 0.008 ohms (resistance breaker) and the main 16,000-ampere breaker.

After the resistance breaker is closed and the machine picks up load in the proper direction, the main positive breaker will close automatically. After the main positive breaker closes the neutral breaker will close

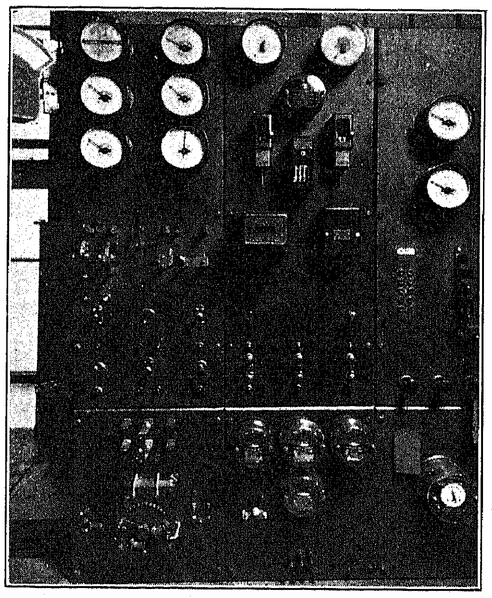

Fig. 6—Control Board for a 13,000-Ampere 240/300-Volt Semi-Automatic Synchronous Converter

automatically. The neutral breaker may be held open or closed at will by a manually-operated interlock switch.

The neutral breaker opens when either negative or positive breaker opens. The a-c. line and all d-c. breakers are tripped simultaneously when the protective circuit opens, due to any one of the following faults:

1. Overspeed.
2. A-c. overload.
3. A-c. undervoltage (80 per cent with time delay).
4. Machine ground.
5. Transformer ground high side.

Automatic Operation. At times of reduction of a-c. voltage, due to disturbance sufficient to cause reversal of current in the synchronous converter, contacts in the d-c. power directional relay open up, causing the opening of the main positive d-c. breaker

and the neutral breaker leaving the machine tied to the d-c. busses through the current-limiting resistor.

The reversal of current from the d-c. busses through the machine is held within safe values until normal a-c. voltage is reestablished, at which time the proper flow of current occurs and the power directional relay closes its contacts, causing the main positive d-c. breaker to close and short-circuit the current-limiting resistance.

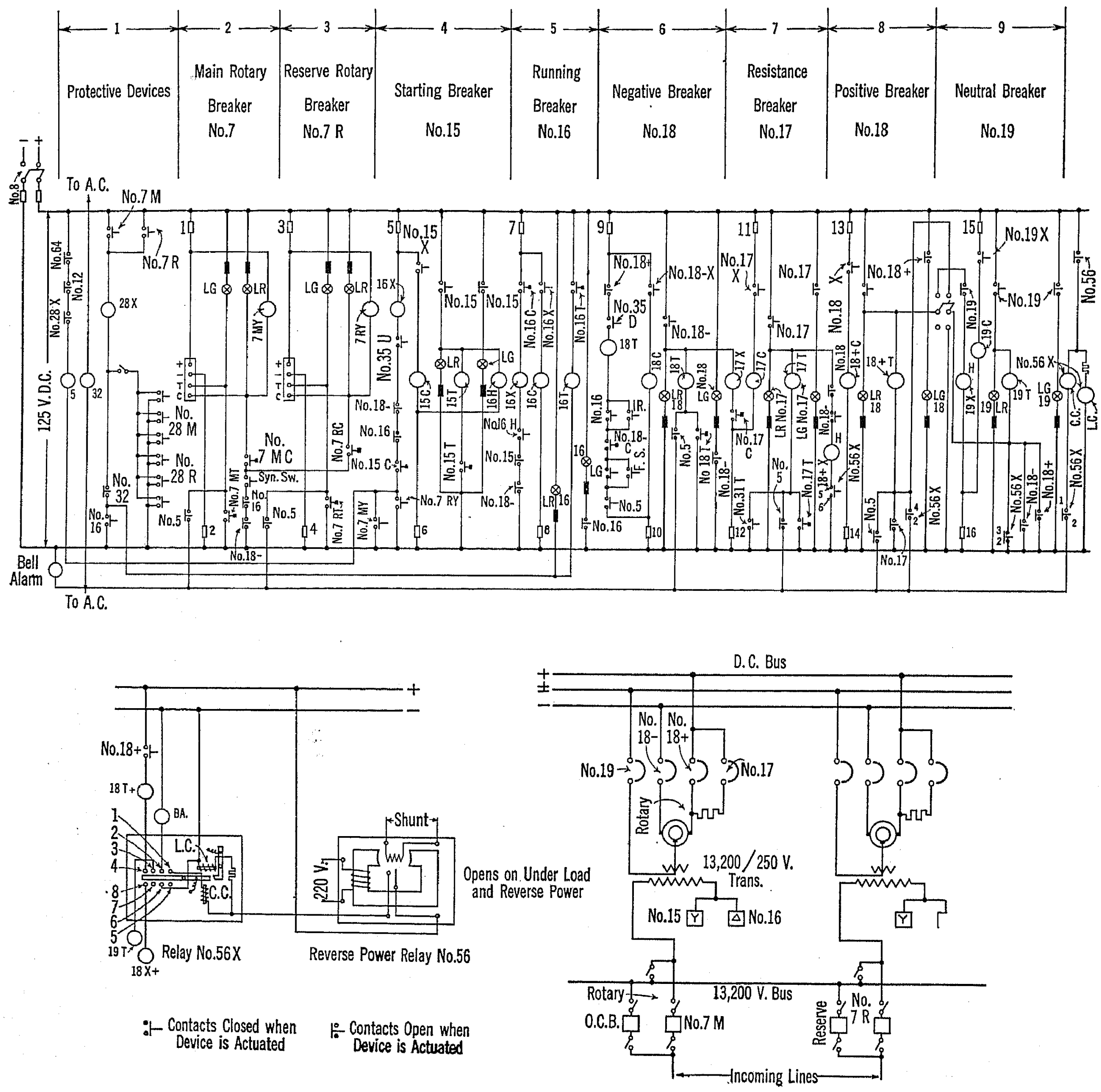

FIG. 7—SEQUENCE CONTROL DIAGRAM.

- 5. Protective contactor.
- 7M. Main rotary breaker and mechanism.
- 7R. Reserve rotary breaker and mechanism.
- 7MY. Auxiliary interlock for 7M.
- 7RY. Auxiliary interlock for 7R.
- 8. Control power switch.
- 12. Overspeed device—hand reset.
- 15. Starting breaker and mechanism.
- 15X. Closing control relay for 15.
- 16. Running breaker and mechanism.
- 16X Auxiliary relay for 16H.
- 16H. Hestitating closing control relay for 16.
- 17. Resistance breaker and mechanism.
- 17X Closing control relay for 17.
- 18 + Positive breaker and mechanism.
- +18XH. Hestitaing closing control relay for 18 +
- 18. Negative breaker and mechanism.
- 18X. Closing control relay for 18 –.
- 19. Neutral breaker and mechanism.
- 19X. Auxiliary closing control relay for 19.
- 28M. A-c. overload time delay relay for 7M.
- 28R. Overload relays for 7R.
- 28X. Auxiliary closing control relay for 28—hand reset.
- 31. Compensator thermostat.
- 32. Under-voltage relay.
- 35U. Brush lowering device—closed when brushes are up.
- 35D. Brush lifting device—closed when brushes are down.
- 38. Bearing temperature relays—hand reset.
- 56. Reverse power relay.
- 56X. Auxiliary closing control relay for 56.
- 64. Flashing protective relay—hand reset.
- B. A. Bell alarm—hand reset.
- L. R. Red Lamp.
- L. G. Green lamp.
- T. Trip.
- C. Close.
- T. C. Trip coil.
- C. C. Closing coil.

In case the reversal of current continues for an undue length of time, the machine will be disconnected from the positive d-c. bus by the operation of a thermostatic relay mounted on the current-limiting resistor.

The curves shown in Fig. 8 give the operation of this

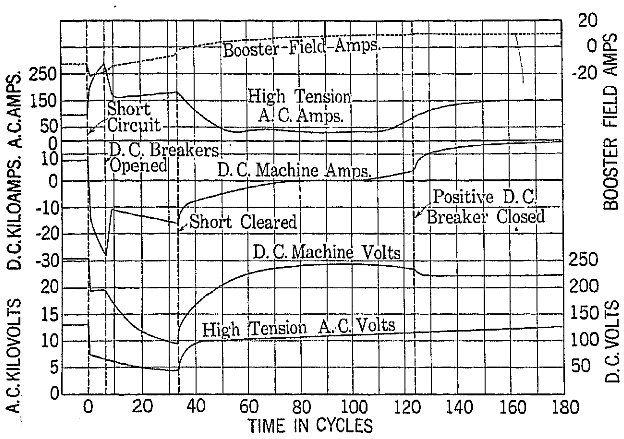

Fig. 8—Performance of a 13,000-Ampere 240/300-Volt Semi-Automatic Synchronous Converter under a 13,200-Volt 3-Phase Short Circuit

equipment under a three-phase short circuit on the 13,200-volt system.

This curve shows a change of current in the synchronous converter armature from 8000 amperes in the normal direction to 27,500 amperes reversed in six and one-half cycles, at which time the main positive breaker opened, allowing the series resistance to limit the reversed current to 11,000 amperes. After the a-c. voltage was restored to normal, the positive d-c. breaker closed and the machine continued to carry load.

It was found during these tests that the synchronous converter, when separately excited, took less reverse current and regained its load somewhat quicker than when self-excited.

This scheme of protection has been used on synchronous converters for a period of 18 months and a large number of successful operations of the automatic equipment have been observed.

Conclusion

1. The automatic synchronous motor-generator sets have been found to be equally reliable to the manually-operated machines under normal conditions and much more reliable under disturbance conditions.

2. The automatic motor-generator set functions very satisfactorily for service restoration.

3. The semi-automatic control applied to synchronous converters reduces the durations of the resultant disturbances on the d-c. system caused from dips in a-c. voltage.

4. Both kinds of automatic conversion equipment described in the article reduce the duty of stand-by batteries materially.

Discussion

For discussion of this paper see page 781.

Present Practise in the Automatic Operation of Hydroelectric Generating Stations

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Review of the Subject.—Present practise in the design of automatic hydroelectric generating stations is reviewed. Various methods of starting and controlling are listed together with their limitations. Certain points in dispute, such as the W R^2 required for speed regulation and brakes for bearing protection are presented in the hope that discussion will aid in clarifying the situation to such a degree as to permit of standardization of practise. A brief description of the sequence of operations in a modern automatic switching equipment, is given.

* * * * *

The advantages to be gained by the automatic operation of hydroelectric generating units are now so well known that their recapitulation is unnecessary at this time. It is the purpose of this paper to present a brief but comprehensive outline of present day practise as applied to such units.

Hydraulic Development

The elimination of the cost of manual operation makes possible a much different treatment of many hydraulic developments. As the cost of the hydraulic development is usually a large item in the total cost of the water power station, this item should be considered carefully in the light of possible savings incident to the employment of automatic control. By splitting up the total available head into two or more smaller heads, a considerable saving in the purchase of flowage rights is possible. Against this must be balanced the higher cost of wheel and generator per kilowatt. As the cost of the actual dam rises rapidly with increasing height there is some saving in this item. A careful balance of the cost items entering into construction of any given development should always be made to see that no further economies are possible before any given distribution of the available head is finally adopted. Careful consideration will many times enable the production of kilowatt hours at a lower figure than possible with the old method of maximum head for a few plants. The modern method involves the use of many small units with lower heads for each and with automatic control.

Types of Wheels

Most of the automatic water power stations that have been supplied have been of the low head variety, the working heads varying from a minimum of 9 ft. to 75 ft. They are the reaction or Francis type and the propeller type, of which the Nagler is a good example. The latter is noted especially for the high specific speeds possible on very low heads with a consequent economy in generator cost. However, efficiencies of the two wheels at the average loads expected should be carefully compared, as a reduction of a few kilowatt hours in the station output will soon nullify any saving in the first cost. As the efficiency curves for the two types are not of similar shapes for varying combinations of head and size, no general rule can be adopted. A careful analysis of each project should be made before a definite type of wheel is selected.

The foregoing remarks apply to vertical wheels which are almost universally used for low heads. There are a few engineers who yet cling to the horizontal reaction wheel. Correct application of this type of wheel to low head installations are rare. Efficiencies equivalent to those obtainable with the vertical type are difficult to secure. Construction of the building and penstock will be more expensive while the generator will cost somewhat less.

For small units and high heads the impulse wheel is used exclusively. It is customary to build the generator with extra large bearings and mount the runner outboard on the generator shaft.

Operation with Respect to Remaining System

There are several possible methods of operation and control, the selection of any specific type depending upon the conditions of the system. The more usual methods are listed below.

(a) Fixed gate opening.

(b) Regulation of load with respect to head of water available.

(c) Regulation of speed by centrifugal governor.

Form (a) is used where a relatively small unit is closely tied to a large system which can absorb all the power generated at all times and which has other units under control of speed governors. Two forms of automatic water control are available. The head gates or the wicket gates may be opened wide by an electric motor supplied with energy from a control transformer or from the station battery if there be such. An oil pressure device may be used with a construction similar to the speed governor but without a centrifugal element. A small solenoid valve admits oil to the servo-motor and causes the wicket gates to open, a float switch is used to start the wheel when excess water is available and to stop it when the forebay is lowered to the desired limit. No intermediate steps are possible.

Form (b) is used where surplus water is used from a

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source in which a specified head must be maintained at all times. If oil-operated gates are used, a float switch is caused to actuate a pilot valve, which in turn actuates the servo-motor in such a manner as to draw a predetermined amount of water for a given head. If electrically operated gates are used, the float switch is caused to actuate a pilot valve, which in turn actuates the servo-motor in such a manner as to draw a predetermined amount of water for a given head. If electrically operated gates are used, the float switch actuates a contact device, which in turn energizes the motor on the gates to increase or decrease their opening. An antihunting device is mechanically connected to the gate mechanism and the contact device.

Method (c) is that most widely used and is to be recommended for most applications. Units equipped with speed governors may be safely operated on any size or type of system. Water-level control may be added to the speed control if desired. Most modern governors consist of an oil pressure tank, a centrifugal device, a valve mechanism and a servo-motor or oil cylinder for the actual mechanical operation of the wicket gates. Past practise has been to drive the centrifugal element from the main generator shaft by a belt. This drive is rapidly being replaced by one employing a small synchronous motor energized from potential transformers connected to the generator. If belt drive is used in an automatic station, an automatic trip should be connected to the idler pulley so that breakage of the belt will take the unit out of service. In an automatic station where a self synchronizing generator is used, the synchronous driving motor is not energized until after the oil switch is closed. There may be some overspeeding as a result of such operation, but as the modern wheel and generator are built to stand the maximum speed obtainable with gates open wide and no load on the generator, no trouble need be anticipated from such excess speed.

Oil pressure for the operation of the governor is obtained from a pump and stored in a closed reservoir against air pressure. The pump may be belt-driven from the generator shaft or motor-driven by a polyphase a-c. motor supplied from transformers connected to the a-c. bus. The latter method is much to be preferred. If a station battery be available an emergency pump may be operated with a d-c. motor. It is customary to provide an oil pressure relay set to cut off the station while there is yet sufficient oil pressure in the reservoir to close the gates.

The governor is equipped with one or more control solenoids which operate small auxiliary oil valves, which in turn open the main valves to the servo-motor. In one form of governor there is but one solenoid which opens the governor wide, but under the retarding influence of a dash pot. With the control methods in general use there is some overspeeding until the generator is connected to the line, as the synchronous motor driving the centrifugal element is not energized until that time. A second form uses one solenoid to open the gates just enough to accelerate the wheel to synchronous speed, the second solenoid being energized after the generator is connected to the line. This second solenoid causes the governor to open wide but under control of the speed governor, which is now operative because the synchronous driving motor is energized. Both forms of control keep the solenoids energized throughout the period of operation. To stop the wheel the solenoid circuit is opened. The gates immediately close and the wheel stops.

As the gate movement is necessarily slow and the response of the wheel to the governor is therefore slow, a certain amount of flywheel effect is necessary to stabilize the generator during rapid changes in load. The importance of this effect is greatly over-estimated as a large system is a very effective flywheel in itself. The amount of inertia required is expressed by the well-known formula WR^2. The amount required is specified by the waterwheel manufacturer, but supplied by the generator builder either in the form of additional weight in the rotor or by the use of a separate flywheel. Since the wheel builder is not penalized by his inertia requirements he usually specifies rather more than less than enough. Much money is wasted in following these excessive requirements. Engineers with small hydro projects to execute would do well to investigate the available flywheel effect of their systems before purchasing a lot of useless cast iron attached to their generators.

Brakes

There is some dispute regarding the necessity for automatically-operated brakes to stop the wheel when rubbish or erosion causes enough leakage to operate the wheel at low speeds. If the thrust bearing operates at low speeds it will not carry enough oil between the bearing surfaces to maintain separation. If the wheel continues to rotate, the bearing will be destroyed. Some engineers insist that brakes be applied to definitely stop rotation when the gates are closed. Other engineers maintain that when only enough water is passed to rotate the wheel slowly with oil between the surfaces, there will be an enormous increase in friction when the oil film is squeezed out. Such increased friction will result in stoppage of the wheel before damage is done. Since a difference of opinion exists, it is suggested that this point be stressed in discussion so that the practise regarding the application of brakes may be standardized.

Where brakes are used, they may be operated by oil pressure from the governor reservoir or by a separate source of air pressure. While the amount of oil required is not large, this item should not be overlooked when purchasing the governor. Oil pressure may be applied through the medium of magnet valves or through valves mechanically actuated by the movement of the water-wheel gates. Air pressure may be applied

through magnet valves of the kind developed for electro-pneumatic railway control.

Brakes of the oil pressure variety seem slightly less objectionable than the air-operated type because of the necessity of an automatically operated compressor for the latter. As any automatic station is improved by simplification, it would be highly desirable to eliminate automatically controlled brakes, if the designers of thrust bearings could agree to this as standard practise. Hand-operated brakes should be furnished for holding the wheel when the unit is being cleaned or repaired, if no automatic brakes are used.

Bearing Lubrication and Protection

The usual circulating oil system with self-contained pump, driven from the generator shaft, is customarily used in automatic stations for lubrication of the thrust and guide bearings. No automatic devices have been worked out for the lubrication of the wicket bearings, although some inexpensive simple system would seem warranted. Even the present grease-cup method is not entirely adequate because there is usually no method of insuring equal distribution of grease between the upper and lower wicket bearings. This is a problem the waterwheel manufacturers should take steps to solve.

Thermostat protection should always be provided for all main bearings. The guide bearings of vertical machines and the main bearings of horizontal machines should have the active element in actual contact with the babbit. In the horizontal type this contact should always be established at the center of the bottom of the bearing, as this is the point of greatest pressure and consequent heating. The thrust-bearings of vertical machines are customarily operated under oil. The temperature of the oil is therefore a good clue to that of the bearing. The active element of the thermostat is immersed in the oil near the thrust surfaces. Most of the thermal devices used for bearing protection are of the vapor-expansion type, which insures ample energy for operation of contacts. These devices are made suitable for the bearing to be protected and are not provided with adjustments for changing the temperature at which operation of the contacts is effected.

Generator Characteristics and Equipment

The generators used in modern unattended stations should be of the self-synchronizing type. To prevent disturbance to the system at time of connection to the line, these generators should have as high a reactance as is feasible, to limit the rush of current when the oil breaker is closed. Heavy damper windings are necessary to quickly pull the generator in step with the line. These generators are usually accelerated to approximate synchronous speed and then connected to the line before excitation is applied. Immediately after the closing of the breaker, the field switch is closed. The normal time element introduced by the high inductance of the field winding is sufficient to prevent excessive mechanical stresses in the windings.

Generators for unattended stations with automatic control should always be equipped with direct connected exciters. There are a few exceptions to this rule, such as excitation drawn from d-c. lighting net work or from an excitation bus in an adjoining station. Belted exciters should be avoided even though the reduction in price makes this type apparently desirable. Troubles may be expected with the belt and with the idler pulley.

Two forms of speed relays are used for indicating approximation to synchronous speed. A centrifugal

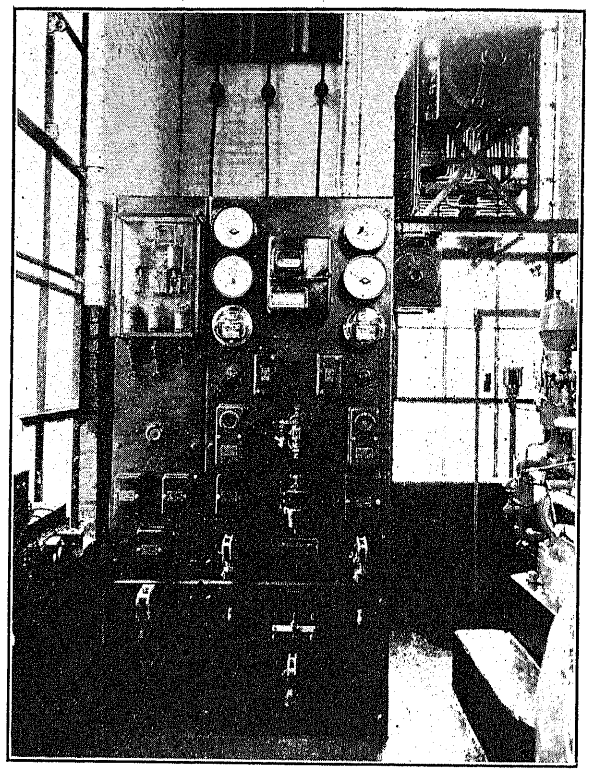


Fig. 1—Small Automatic Hydroelectric Generating Station with Vertical Self-Synchronizing Generator. This is a Typical Installation

switch may be attached to the end of the generator shaft or a specially designed magneto generator may be geared to the shaft, its voltage, as indicated by a relay, constituting the desired indication.

A second element in the centrifugal switch or a second voltage relay on the magneto is used to indicate excessive speed.

Functions of Control Equipment

The initial starting indication for the automatic control may be obtained in any one of several ways. The following list covers the majority of the methods which have been used or discussed.

(A) Water level.
(B) System frequency.
(C) Remote control by pilot wire.
(D) Supervisory control over the telephone circuit.

(E) Manual operation of control switch in station.
(F) Control by manipulation of the a-c. line.
(G) Control by time switch.

Combinations of one or more of the above have been used in several installations. Where the station is controlled entirely by (A) the indication is obtained from a float switch which makes contact when the water in the forebay rises above a predetermined point. Some small systems are using the method (B) to call additional capacity into service when the drop in system frequency indicates an overload on the generating equipment in service. This method has a very limited application to systems fed principally from small water plants. If any steam turbine stations are used, the overload capacity of the turbine will maintain the normal frequency and defeat the purpose of the automatic station. If several automatic stations are used on the same system with this form of control, there is no suitable method of selection, therefore several stations may be called into service when only one is wanted. If differential timing is attempted, the time required to start the last stations is prohibitive since it must exceed all the time required for the first stations to start and connect to the line.

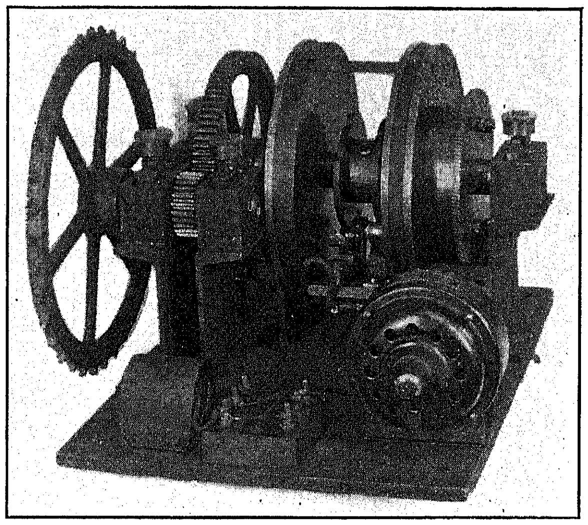


FIG. 2—CONTACT DEVICE WITH ELECTRICALLY OPERATED ANTIHUNTING MECHANISM FOR AUTOMATIC CONTROL OF WATER LEVEL IN FOREBAY. THIS DEVICE IS LOCATED ABOUT 2000 FT. FROM THE STATION IT CONTROLS WHICH CONTAINS A 1000-KV-A. GENERATOR AND IMPULSE WHEEL OPERATION ON 450-FT. HEAD

Method (C) is suitable where the distance to an adjoining station with manual operation is not excessive. Method (D) is being used to a large extent in the stations being installed at the present time. There are several types of equipment available at a wide range in price. By the use of some of these it is possible to control the starting and stopping of the unit, the load, the voltage, and the outgoing feeders. The same equipment enables the distant operator to ascertain the available head of water, the gate opening, the load in kilowatts, the condition of the feeders and any other indications that may be desired. All these functions may be obtained with the various forms of equipment over circuits of from two to four wires, which wires may be used for magneto or local battery telephones. None of these systems will work on the same circuit with common battery telephones.

Where natural conditions and the size of the plant warrant the expense, it is customary to have one resident maintenance man who starts and stops the station with a control-switch method (E) as instructed by the load dispatcher. This man is not expected to remain in the station but is subject to call by the dispatcher whenever required. A siren on the station roof is frequently used for this purpose.

Method (F) is occasionally employed where the automatic station is on an independent line which may be switched from another station. The station is started by closing the oil switch on the line in the manual station. An a-c. voltage relay then causes the unit to be started. Upon opening the oil switch the machine will be tripped by overspeed, due to the sudden reduction in load, or by a contact on the waterwheel gate which is closed in the no-load position, which position the governor will promptly cause the gates to assume, if the overspeed trip does not have a chance to operate.

Method (G) may be employed where hydroelectric units are used to supply energy to industrial plants having fixed working hours or to switch on additional capacity in advance of known periods of peak load.

In addition to the automatic means described above, there are two methods of partial automatic starting, one by means of a multi-wire control and actual synchronizing by hand, the other by actual hand starting with protective features so that the station may be permitted to operate without an attendant.

Current for the operation may be drawn from a transformer connected to the a-c. line or from a small storage battery maintained in a charged condition by automatic devices with current from the exciter. If the first method is used, the unit cannot be put into service unless the line voltage is above 80 per cent normal. If the second method is used, the unit may be put into service regardless of the line condition.

SEQUENCE OF OPERATIONS

The usual sequence of operations after the initial starting signal is given by any one of the automatic indications is mentioned below.

(1) The master relay is energized.

At this time the exciter-field rheostat is short-circuited to aid the exciter to build up to normal voltage as rapidly as possible. The generator-field contactor is in the de-energized position, its back contact shorting the field through a discharge resistance.

(2) The governor solenoid is energized.

The gates are thus caused to open and the unit accelerates. The exciter builds up rapidly.

(3) At an approximation of synchronous speed the speed-indicating device closes the control for the oil switch. The closing energy for this switch is obtained from the exciter. If the voltage has not built up to the minimum operating point for the breaker, there will be a slight delay in closing until the voltage does reach this point.

(4) The oil switch closes, connecting the unexcited generator to the a-c. line. The short circuit is also removed from the exciter field rheostat. The circuit of the field-switch operating coil is energized.

(5) The field switch closes, connecting full excitation to the generator field. The generator pulls into step with the line and as the gates continue to open, picks up its load.

The normal sequence of operations provides for the shutting down of the unit without disturbance by first de-energizing the governor solenoid to close the gates and then tripping the oil breaker with a contact on the

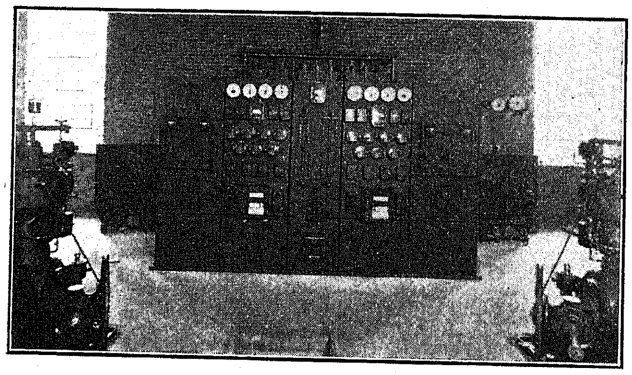


Fig. 3—Switchboard for the Automatic Operation of Two-2200-Kv-A. Waterwheel Generators. The Large Square Case in the Center of Either End Panel is the Speed Matching Device. Above is the Automatic Synchronizer and Below is the Control Element of the High Speed Rheostatic Regulator

gate mechanism at the position corresponding to no load. The field switch is then opened and the brakes, if any, are applied.

Regulation of Voltage

There are several methods of operation with regard to voltage control. Those in general use are as follows:

(1) Fixed excitation. This is only applicable where the automatic unit is electrically close to a regulated generator and where it carries a considerable proportion of its full load at all times.

(2) Regulation by the standard vibrating regulator.

(3) Regulation by a contact-making voltmeter and an ordinary motor-operated field rheostat in the generator circuit.

(4) Regulation by the high-speed rheostatic regulator.

With any one of the three types of regulation it is necessary that some form of wattless compensation be employed, so that the automatic unit will not carry an undue share of the wattless load of the system.

Protective Features

As the automatic stations must operate under all conditions without human attention, it is necessary that the generator be protected against almost every imaginable operating contingency. It is manifestly impossible to anticipate all forms of trouble and it is manifestly impracticable for financial reasons to furnish protective devices for the extremely remote contingencies; therefore, it is customary to disregard those troubles which require the coincidence of two or more improbable events to damage the machine.

All larger generators are equipped with balanced relay protection to open the breaker and field switch should there be an insulation failure. The cost of this protection is considerable and an arbitrary lower limit of 500 kv-a. has been set, below which it is not customary to furnish such protection.

Two thermal relays are connected in the secondary circuits of the current transformers to measure the heating effect of the output of the generator. Should the safe limit of heating be approached, these relays will shut down the unit until cooled down, after which restarting is permitted.

Field failure is caught by a relay in series with the generator field with a time element sufficient to bridge over any ordinary surge.

Short circuits external to the station are caught by two different means. The first employs the usual overload relays and shuts down the unit until the rise in a-c. line voltage, fed from other stations, indicates that the trouble has been remedied. The second method uses the line voltage as an indication of trouble and allows the machine to deliver as much current as it is able, until the reduction in voltage operates a line voltage relay or until a thermal relay operates. The machine is reconnected on rise in line voltage as in the first case.

It is advisable to provide protective devices against excessive rise in either exciter or generator voltage. Relays which form a crude vibrating regulator insert large amounts of resistance in the exciter field under either of such emergencies.

Any high-tension equipment should be located outside the building. The high-tension construction should be as simple as possible, without any high-tension circuit breaker if the unit be under 500 kv-a. The arrester should be of the non-fluid type which require no charging or attention. A separate set of disconnecting switches should be provided for the arresters.

Special Forms of Control

When a generator forms too great a percentage of the total generation to permit of self-synchronizing, or if the generator be difficult or expensive to equip with damper windings, an automatic speed matcher and synchronizer is available. The speed matcher consists of two small synchronous motors, one for the machine and one for the bus. These are connected to a me-

chanical differential, which in turn is connected to contacts which control the speed-adjusting motor on the waterwheel governor. Any difference in speed of the two motors results in a movement of the differential and consequent operation of the adjusting motor. The speed of the wheel is thus adjusted as though in an ordinary manual station. The automatic synchronizer awaits the first favorable point and closes the breaker. The governor is then opened wide to pick up the load.

It is also possible to ease the shock of self synchronizing by the use of current-limiting reactors which are connected between the machine and the bus until synchronous operation is attained. The reactors are then shorted by a second breaker. The same result may be obtained if step-up transformers are used for the output by synchronizing with a low voltage tap, after which the generator is transferred to full voltage just as a tap-started synchronous motor is controlled.

High head plants offer additional problems in control, as the velocity of the water is so great that it must be handled with great care. Load control is usually had by opening or closing the needle valve with a motor. The deflector which is used for speed control should be interconnected with the needle-valve control so that there will be no unnecessary waste of water off the deflector. Thus if the wheel is carrying 90 per cent load and system conditions cause the wheel to drop to 70 per cent load, the deflector will be wasting 20 per cent of the water. The automatic control should promptly close the needle valve sufficiently so that this 20 per cent of water may be saved. Hydraulic engineers will doubtless question the accuracy of these figures but the percentages are intended only for illustration.

In order that the small automatic generating stations may prove to be a financial success, it is obvious that construction costs must be minimized. Past manual practise in building design and equipment should be forgotten. It should be remembered that extreme simplicity of construction, small units located at frequent intervals and entire absence of emergency switching or reserve capacity in any one station, will deliver more kilowatt hours at less expense than a few large units with expensive hydraulic development, elaborate emergency equipment and fine buildings.

Discussion

PAPERS ON AUTOMATIC STATIONS

(Place, Wallau, Butcher, Bany, Wyatt, Wensley)
Chicago, Ill., June 25, 1924

R. H. Earle: In the development of the hydraulic division of automatic control systems for hydroelectric units, it has been the experience of the writer that the major problem is to obtain the desired action of the turbine gates during the periods of starting and stopping the machine. In response to electrical signals the turbine gates must be opened or closed and the gate opening must be controlled in such a manner as to prevent violent speed changes and to prevent unnecessary shocks to the machine itself, the water conduits, and the power system. As stated by Mr. Wensley this action is secured by modifying the usual governor system, by using motor-operated gates, or by using a special electrically controlled service motor to move the gates.

The operations of starting and stopping may be accomplished with varying degrees of refinement depending upon how much disturbance is allowable in the particular installation. In general the limiting feature is the amount of disturbance which the electrical power system or the penstocks can withstand, as the machine is sufficiently strong to withstand any shocks which are likely to occur. It follows, therefore, that when a small low-head machine is connected into a large power system, a comparatively crude type of control may be used satisfactorily, and as the machine becomes larger in comparison with the remainder of the power system, or where long penstocks are encountered, more precise methods of starting and stopping must be used.

Besides the main problem of starting and stopping, there is the question of holding the turbine gates closed when the machine is shut down and a number of protective features which fall in the hydraulic section of the control system.

If a governor is used means should be provided to close the gates in case of breakage of the governor belt or failure of power for the flyball motor.

In the governor oil-pressure system a pressure-operated switch is arranged to stop the turbine in case the pressure falls below a certain value. In some of the more elaborate systems the oil level is automatically maintained at the correct height in the pressure tank. Oil pressure is conserved in the pressure tank when the machine is shut down by an automatic valve in the pipe line between the pressure tank and governor. This valve closes automatically when the machine is stopped and thus prevents oil leakage. In addition, it is desirable that the oil pump be motor driven, the motor being controlled by a pressure regulator in such a manner as to maintain normal oil pressure in the pressure tank.

Except for bearing protection, explained by Mr. Wensley, the protective features just mentioned include all which are usually necessary.

It is interesting to observe the growing tendency to apply remote and automatic methods to large turbines in stations where there are operators in order to centralize the control at the switchboard and to obtain increased safety of the machinery.

One of the most common examples is an automatic stopping system which is being included as a safety feature on nearly all the large installations. By the opening of a single switch the switchboard operator is enabled to close the turbine gates quickly, disconnect the generator from the line and apply the brakes, all without the assistance of the second or third operator. Any number of such switches may be used, located as desired throughout the station.

These features will be recognized as the emergency stopping means found in the automatic station. in which case the opening of the control circuit is accomplished by the bearing-temperature relays or other protective devices.

This system is being adopted rapidly on the largest machines, not only because it affords means for stopping the unit in the minimum possible time, but because in case the operator becomes confused, he has to do but one thing to stop the machine. This system has prevented at least one disastrous accident to a large unit in the knowledge of the writer.

The most common remote-control features used in manually controlled stations are the switchboard control of the speed-setting and load-setting adjustments of the governor, together

with an indication at the switchboard of the gate opening at which the machine is running. This equipment is designed to be used with large units where it is especially desirable to have the machine under the direct control of the switchboard operator. With this equipment the switchboard operator can start and stop the machine with no assistance from the second operator instead of the usual custom of having the second operator start the machine and bring it up to speed by hand.

We have explained these applications to manually controlled stations to show how the developments in automatic control are not only influencing the smallest water power developments but are being adopted on the largest units just as fast as they prove to be as dependable as the human operator.

We would like to bring out the relation between the Types *C* and *D* control mentioned in Mr. Wensley's paper, Type *C* being the remote control of the distant station by pilot wire and Type *D* being supervisory control over the telephone circuit. The latter is, of course, only a special case of the Type *C* and is the method used by one manufacturer. The general form of the pilot wire control includes a few wires between the master and automatic stations together with suitable selectors by which these wires may be connected into various local circuits at the two stations. In this way various functions can be accomplished without having separate pilot wires for each. Different manufacturers accomplish these results by various forms of selectors.

As to general hydraulic practise, we received the impression from Mr. Wensley's statements regarding Francis and Nagler type runners that the efficiency of the runner alone should govern the choice between the two and that the Nagler type should be adopted with some caution.

We wish to point out that it is not the runner efficiency but the overall plant efficiency which determines the choice between the two types of runners. At the very lowest heads especially for very large units, the Francis wheel will not allow commercial speeds so that the Nagler type is practically the only solution. At a considerably higher range of heads and for smaller capacities the Nagler wheel is less suited because the gain from speed is insufficient to neutralize the lesser part gate efficiency and the Francis type should be used down to the limit of prohibitive clogging. Between these extremes is a region in which either type is practical. In making the choice here, the following points should be considered in addition to the first cost:

The efficiencies of the Nagler and Francis runners will usually be about the same at the normal load, but the Francis type may be better at fractional loads.

The speed of the Nagler wheel will be higher allowing a more efficient generator.

The Nagler runner will maintain its original efficiency and power whereas the passage of trash may clog the smaller passages of the Francis runner, reducing the efficiency and power if not necessitating a complete shut-down of the machine for cleaning.

The ability of the Nagler runner to pass large objects without clogging allows a wide spacing of the trash racks with a consequent smaller loss of head through the racks.

The resultant of all these conditions favors the Nagler type of wheel more and more as the head becomes less, particularly if the generator is connected into a large power system or in case there are several units in the station so that the turbine can run continuously at the most efficient load.

Considering motor-driven flyballs, we believe that this type is not superseding the older type to the extent that Mr. Wensley states: Belt-driven or direct-connected flyballs have certain inherent advantages over the motor-driven types as built at present in that they register the speed of the machine at all times independently of the electrical conditions of the generator. The older types are often preferred for this reason.

The advantage of the motor drive is, of course, that it is very flexible in its location, is neat and simple in appearance, and may be somewhat cheaper than the belt drive.

Mr. Wensley suggests a reduction in the WR^2 required in a hydroelectric unit, and states that the purpose of the flywheel effect is to stabilize the generator during changes of load while the governor is moving the gates. We wish to add that the purpose of the WR^2 is not only to smooth out load changes, but even more important to absorb changes in torque. Contrary to popular understanding a hydraulic turbine is not a constant-torque machine particularly under low heads which includes most of the automatic stations. In low-head machines the water velocities are high compared with the head and a large part of the total head is draft head. As a result the water flows at high velocities and low pressure and flow conditions are disturbed. The resulting torque on the wheel is not only variable but is likely to have sudden pulsations which must be ironed out by the flywheel.

A unit having too small a WR^2 is not only difficult to govern during synchronizing but does not deliver a constant amount of power. Even though the latter is not objectionable in itself, the varying output causes a continuuus re-adjusting of the gates by the governor with consequent wear on the regulating machinery. Although machines having too small a WR^2 are rare, they occur often enough to establish quite definitely what is the minimum flywheel effect required.

As for the necessity of brakes, the writer knows of some machines which have been in operation for a period of years which do not come to a dead stop but drift very slowly without damage to the thrust bearing. The real advantage of brakes appears when it is desired to stop quickly in emergency and we favor their use in automatic stations when the cost is not prohibitive.

H. A. S. Howarth (by letter): Abundant operating and test data show that Kingsbury thrust bearings do not require brakes for their protection. Enforced running at very low speed actually improves the bearing surfaces. (See *Transactions* A. S. M. E. 1919, Slow-Speed and Other Tests of Kingsbury Thrust Gearings). As a hydroelectric unit slows down complete separation of the bearing surfaces is maintained almost to the point of stopping. The better the fit of the surfaces the slower the bearing will rub before metallic contact is noticeable. From about one rev. per min. to rest the friction coefficient rises markedly and assists in bringing the unit to rest. When the unit is kept turning slowly by water leakage, no harm will result to the thrust bearings, but a brake will be required for stopping the wheel.

W. H. Millan: In connection with Mr. Bany's paper, it appears that the synchronous motor field is excited from the terminals of the generator. This is wrong because we would expect the set to stay on the line irrespective of the generator voltage. A situation may arise where it is necessary for this machine to operate possibly an hour at a voltage in the neighborhood of twenty-five or fifty and I do not believe that the motor would stay on the line as an induction motor (with the field open) without locking out on temperature. Also I do not quite understand how the set is held on the d-c. bus as a d-c. motor when a-c. power has disappeared (waiting for power return) when the d-c. reverse-power relay is set to take the machine off at a reverse value less than the running-light current of the set.

In connection with storage-battery trickle charge. we had quite a discussion in our subcommittee which brought out the fact that installing a storage battery in an automatic station and charging it continuously at 0.2 or 0.3 ampere did not satisfy conditions. Some of us have had experiences with pasted-plate batteries where 0.3 ampere applied continuously disintegrated the grids and at the end of a year and a half we suddenly found ourselves without a battery in an important automatic station.

Referring to the emergency lockout relay which Mr. Wallau mentions in connection with his a-c. reclosing feeder equipment

which prevents a feeder reclosing if the short circuit has been in excess of 900 amperes, I believe this an excellent idea but feel that on heavy or important circuits this limit should be increased. I have seen circuits burned clear only after three reclosures when the behavior of the switch gear indicated a short-circuit current many times this limit.

J. L. Woodbridge: I think the statement in this paper that "the 6-minute rating is the maximum current that can safely be taken from the batteries without serious injury to the plates" may be misleading. A battery may discharge at any rate that the cells can give for the length of time they can give that discharge, without injury to the battery. I call this point to attention because the paper as it now stands might be construed to mean that there is some high rate of discharge that is injurious to battery plates and this is not true.

There is, of course, a practical limit to the useful output of the battery both from the standpoint of voltage drop and from the standpoint of duration and the 6-minute rate of the battery is approaching this practical limit. In other words, at higher rates than the 6-minute rate, the voltage drop becomes excessive and the output of the battery is less useful on that account. Furthermore, the time limit is so short at higher rates that in many cases it would not cover the duration of the interruption. The 6-minute rate is, therefore, usually considered the practical limit in determining the size of the battery for a given service, although in special cases higher rates have been provided for.

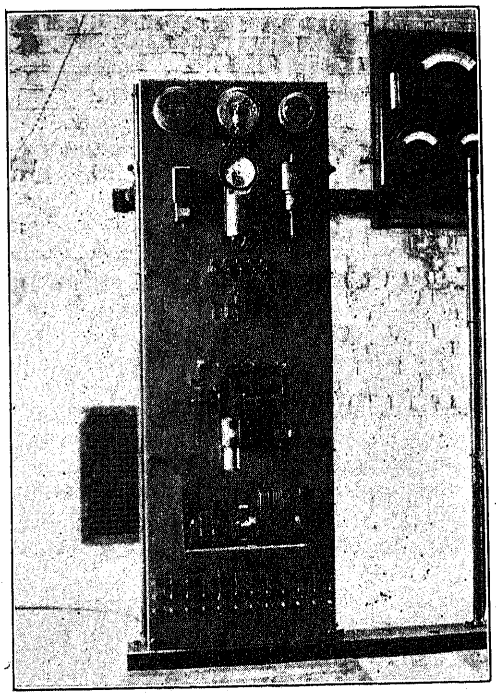


FIG. 1—AUTOMATIC FEEDER OR RAILWAY SWITCHBOARD

It should be carefully noted, however, that any limit thus fixed has nothing to do with any possible injury to the plates.

V. E. Thelin: We have developed a simplified type of substation in Chicago which is fully automatic, being operated in response to time. This station, which has been in operation for approximately five years, is just completing three years of operation without any maintenance or inspection being given and in that time we have had two interruptions. One was due to a finger on a relay being bent out of shape and the other due to the clock stopping on account of dirt which was cleaned out with gasoline and is running satisfactorily to date.

We are using Bullock compound-wound rotaries, which have the series field short-circuited and are, therefore, operating as shunt-wound rotaries. We have great confidence in the shunt-wound rotary due to the fact that on extreme overloads the voltage drops, thus transferring the load to surrounding substations.

We are using automatic reclosing circuit breakers on the feeders, without any resistance whatsoever in series with them, and if a feeder becomes short-circuited, it is cut off and the breaker closes again automatically when trouble is cleared.

The perfect operation of this equipment is due to its simplicity, the entire control panel being approximately 20 in. wide and 70 in. high. A brief description of the operation of this equipment is as follows:

Time clock No. 1 in the upper portion of the slide closes the master relay No. 2 which in turn closes the half-tap contactor No. 3 which connects the rotary to the half-voltage taps on

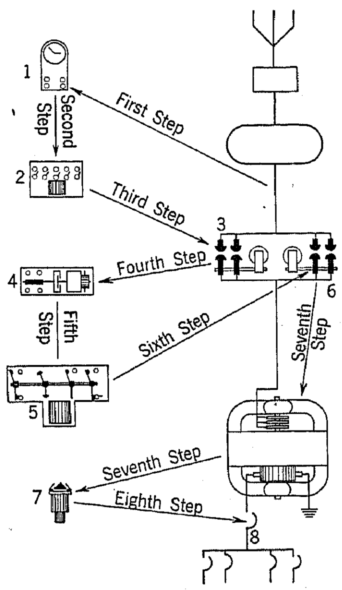


FIG. 2—DIAGRAM SHOWING SEQUENCE OF STARTING OPERATIONS

the transformer. The converter then starts up to synchronous speed with the field excited from the substation bus and the rotary, therefore, comes up with correct polarity. In case it should come up incorrect, the polarized relay will shut the station down, and after a definite time, it will start again, slip a pole and thus correct the polarity. Assuming that it comes up correctly, the polarized relay No. 4 then operates and closes the combination changeover switch No. 5 which opens the half-tap contactor No. 3, closes the full-tap contactor No. 6, changes the connection to the shunt-field windings from the bus to its own self excitation, after which relay No. 7 operates, closing the automatic reclosing circuit breaker No. 8 which picks up the station load.

Further details of the operation of this station can be found in the A. I. E. E. PROCEEDINGS for December 1922.

David K. Blake: The practise of applying small automatic substations to d-c. networks approaches a-c. distribution. As I see it, there are three factors which have hindered the use of alternating current in this territory. The first has been the battery, the second the elevator and the third the customer's equipment. The improved reliability of supply to substations has eliminated the necessity of the battery, at least for most companies. A-c. elevator equipment is now available to meet most requirements. The remaining factor is the customer's equipment which is by no means a small factor.

The practise of supplying small automatic a-c. substations in the surrounding territory approaches the higher voltage distribution with a consequent increase in distribution economy and improvement in voltage regulation. Other advantages of

this practise are that "all of the eggs are not in one basket" and the short-circuit currents on the distribution circuits are considerably reduced. This reduction in short-circuit currents has numerous advantages, such as the reduction in duty on oil circuit breakers and other substation equipment as well as the duty of the primary cutouts located on the distribution feeders.

G. I. Wright: While there has been a very rapid development in the automatic substation field, the methods of overload and short-circuit protection and arrangement of switching apparatus are far from standardized and substations made up of two 2000-kw. 1500-volt conversion sets, as described by Mr. Butcher, are about the largest so far and these are still under construction. When it is realized that a large suburban 1500-volt electrification will necessitate substations of 10,000 or 12,000 kw., which means either four, five, or six units, depending upon the size used, it is doubtful if full automatic operation is advisable. Partial automatic operation for such stations, such as the use of automatic feeder reclosing equipments, and push button control of starting the machines, is thought to be very advantageous.

The systems of supervisory control, which have recently been developed, give an excellent means of controlling circuit breakers at switching and tie stations from a remote point and thus isolate the affected portions of the distribution system in case of trouble.

The author, in general, states that with 60-cycle supply for 1500-volt service, the motor-generator is superior to the converter in reliability, maintenance cost, and the amount of installed capacity necessary to carry short time overloads. Where the machines are protected against injurious flashovers by modern high-speed circuit breaker, it is not thought that the maintenance of a two-unit synchronous converter should be any greater than that of a three-unit motor-generator set. There is considerable difference in opinion as to the amount of installed capacity necessary for a given load in this class of service. The author states that where the capacity of the substation is determined by the short-time or commutating capacity of the machines, it is necessary to install 50 per cent more converter capacity than if motor generators were used. The substations which fall in this classification are those in the outlying sections of a suburban system where the maximum loads are those caused by simultaneously starting of long trains during the rush-hour periods, and which do not occur more than a very few times each day. Due to the infrequency of these loads, the ability of the stations to shift overloads to adjoining stations and the fact that it is considered desirable to install a complete spare unit in each station, which could be operated for these short periods if necessary, it is not thought essential to install appreciably more capacity if converters rather than motor-generators are used.

Even if the maintenance of the converters is decreased by not loading them up to their rated capacity, the investments in extra capacity to reduce this maintenance would be out of all proportion to the savings which could be expected. This can be appreciated when it is realized that one 2000-kw. unit installed complete represents an investment of possibly $80,000.

In regard to the relative efficiency of conversion of the two types of apparatus, all-day efficiencies were carefully calculated for a large suburban steam-railroad electrification, using 2000-kw. 1500-volt conversion sets, and a difference of 3¾ per cent in efficiency was found in favor of the synchronous converter.

A. M. Garrett: Mr. Butcher's paper deals principally with future installations, because the first installation of such capacity in this country is yet on paper. In comparing the efficiencies of the converter and the motor generator, I agree with Mr. Wright in his deductions that there is a pronounced difference in the amount of efficiency when the study is made in economies in regard to 24-hour load. It is gratifying to the operating companies whenever a railway company decides to make its traction system 1500 volts direct-current as this is in line with the operating company's policy of one frequency in generation. It will give greater flexibility to the operating company and therefore better service to the customer.

The paper shows the manufacturers are apparently alive to the design and the problems that will come up with the 1500-volt direct-current service. They are designing machines of a size that will apply to steam-road electrification where the substations will have capacities of 8000, 12,000, possibly 16,000 kw. They are not only providing machines to take care of the higher current and voltage, but they are providing the necessary application for automatic operation.

In the case of automatic operation with either a converter or motor generator, the necessary protection should be taken to prevent certain lockout features from coming into action with every abnormal condition. A flashover on a commutator by no means indicates the unit is disabled or even requires inspection, and precautions should be taken to prevent, under such circumstances, the withdrawal of the machine from service until inspected.

A. C. Grayson: The Public Service Electric Company of New Jersey has recently placed in operation three noiseproof automatic substations, which are known as the Miller Street, Norfolk Street and Race Street Substations. The Miller and Norfolk Street Substations are located in Newark, N. J., within a mile radius of the center of the city, and the Race Street Substation is located in Bloomfield, N. J., about 4 miles from Newark.

The Miller Street Substation is probably one of the largest combined commercial and railway automatic substations in the country. The present capacity consists of two 13,200-volt supply feeders, one bank of three 1000-kv-a. one-phase, transformers: one bank of three 2000 kv-a. one-phase transformers, two 1500-kw., 600-volt d-c. synchronous motor-generator sets, and three 4150-volt, three-phase feeders. The ultimate capacity will consist of five 13,200-volt feeders, two banks of transformers having a capacity of 10,000 kv-a. each, two- 1500-kw. motor-generator sets and twelve 4150-volt, three-phase feeders. The entire equipment is fully automatic with the exception of the 13200-volt supply feeders, which are manually controlled and equipped with reverse-power relay protection.

The transformer banks have automatic control so arranged that they are connected or disconnected from the bus depending upon the amount of load carried. In addition to this they are equipped with the usual protective features such as overload, differential and temperature relays.

The commercial 4150-volt, three-phase feeders are automatic d-c. controlled, with three reclosures on overload before lock-out.

Either of the two motor-generator sets (depending upon which one is leading), starts up automatically on low voltage and the second machine cuts in when the load exceeds the capacity of the first. Shut-down is of course accomplished in the inverse order. Starting voltage is supplied from 30 per cent starting taps on the power transformers, thus eliminating the use of starting compensators.

A double railway bus is provided, one operating at 550 and the other at 600 volts, direct-current. By means of double-throw switches the railway feeders may be connected to either of the two buses. Usually the local heavy feeders operate at 550 volts and the long feeders at 600 volts. The two buses are connected by a bus tie breaker which is closed when only one machine is in operation, the bus voltage being 550 volts. When the first machine is overloaded the second machine cuts in at 550 volts; the bus tie breaker then opens and the second machine builds up its voltage to 600. Each railway feeder is protected by a high-speed circuit breaker which cuts in a load indicating resistance when the breaker opens on short circuit. There is a spare equipment connected to a transfer bus so that it can replace any equipment taken out of service for inspection or repairs.

The substation building is 60 ft. x 100 ft. x 30 ft. high of two story construction. It is divided into four separate rooms for the high-tension equipment, transformers, commercial equipment and motor generators.

On account of the location and the noisy operation of motor-generator sets, sound-proof construction was used throughout with special sound-proofing in the motor-generator room.

Forced ventilation is supplied to the motor generator by means of two 20,000 cu. ft. per min. blowers, one for each unit. Air is taken at the top of the rear of the building, and is drawn through a series of baffles to the blowers in the basement which discharge the air to the pits under each machine. Each blower is started and stopped automatically with its machine. After the air leaves the machines it escapes through another series of baffles to the outside. The baffles are so arranged as to give alternate contraction and expansion of the air without causing an appreciable drop in static pressure.

The walls and ceiling of the motor-generator room are covered with a one-half inch thickness of flaxilinum with a one inch air space between the flaxilinum and the brick wall. The flaxilinum is then covered with plaster to give a finished appearance. As a precaution against transmission of vibrations, the motor-generator foundations are completely isolated from the building. The foundation block is cast separately and rests on what is known as "anti-vibro" block. The vertical spaces between the foundation and the floor are filled with a plastic compound. There is only one door leading from the motor-generator room to the outside and this is a double refrigerator type.

With both machines running the noise cannot be detected when standing on the outside of the building, and for all practicable purposes it is a 100 per cent noise-proof installation.

No sound-proofing is used in the other rooms of the substation except that which is afforded by the solid brick walls.

The substation has been operating automatically about four months and has proved very satisfactory.

The Norfolk St. substation is of the same design as Miller St. except that the present installed capacity consists of two 13200-volt feeder equipments, three 2000-kv-a. transformers, two 4150-volt three-phase feeders and one 1500-kw. synchronous motor-generator set. On account of its location, the architecture is not quite as elaborate as Miller Street but the same methods of sound-proofing were used. This substation has been operating satisfactorily for about three months.

The Race Street Substation is a single unit, automatic railway substation with one 100-kw. rotary converter. The building is of sound-proof construction with solid brick walls, but no sound-proofing is used on the inside of the walls. This precaution was not thought necessary because of its location, and the fact that the converter is less noisy than a motor generator.

No forced ventilation is used, the converter drawing the air from the outside of the building and discharging it through the roof.

This substation has also been in satisfactory operation for about three months.

The Public Service Electric Company have now in the process of design or construction, fourteen new automatic substations. These are divided into three classes: (1) Commercial (4150 Volt, a-c.); (2) Railway; (3) Combined commercial and railway.

In addition to these, a number of the older substations are being changed over from 2400 volts, two-phase, to 4150 volts, three-phase and the new three-phase feeders will be made automatic reclosing even though an operator is required for the remaining apparatus in the substation. It is thought that the satisfactory operation resulting from automatic reclosing feeders justifies the slight increase in expense as compared to the manually operated.

A study is now being made on the use of supervisory control and it is very probable that it will be adopted for these substations in the near future.

H. T. Porter: An important advantage of automatic operation of hydroelectric plants is the increased amount of kilowatt hours obtainable as compared with manually operated plants. The average operator knows little and cares less about efficient utilization of water and in the latter type of station he is to a large extent in control of the manner in which water is used. In the automatic type proper consideration by the engineers of the best way to operate the plant first determines the type of control to be adopted and secondly the method of operation. Many variable factors are eliminated from operation and the plant may be so designed as to shut down under unfavorable conditions and await more favorable conditions of load, or water, provided of course, that some storage, or pondage, is available and the station is tied into a fairly large system. This point is brought out because it has proved an important factor in several recent cases in the decision to install automatic equipment.

Mr. Wensley's paper clearly explains the methods of operation, but does not state how the turbine gates are held closed. In practically all types of automatic control the governor is provided with a solenoid-operated oil-cylinder brake which holds the turbine gates closed when the unit is not in operation. Further, along the line of governor, a good practise is to provide for starting the oil-pressure pump when oil pressure is admitted to governor. Coincident with these occurrences the brake on the turbine-gate shaft is released and the turbine swing gates partially open allowing the unit to come up to synchronizing speed and under governor control. Proper design of the turbine gates prevents their swinging fully open, except under governor control and upon a heavy demand for load, since they are generally designed to balance at from ¼ to ¾ of full open position. This prevents the unit from ever obtaining full runaway speed, and also precludes possibility of the gates slamming shut or opening violently. The gates can, of course, be designed to balance at any predetermined opening.

The motor-driven governor flyball has thoroughly demonstrated its superiority over the mechanically belt-driven element. Its use eliminates belts, pulleys, belt tighteners and attendance, and replaces a heretofore source of weakness, especially in the automatic plant.

Where automatic stations are a part of, and subordinate to, a comparatively large system and main stations, their generator flywheel effect may be considerably reduced. In other words, if the system is previously provided with sufficient inertia to hold the system frequency to satisfactory limits the automatic plant need as a minimum require only enough inertia to allow of synchronizing and placing the unit on the line. As a rule, however, the generator flywheel effect should be such as to give a constant of at least 2,500,000 in the formula $C \frac{W^2 \times R.P.M.^2 R.}{B\,H\,P}$

In instances where a plant is required to do considerable regulating, this value should be in the neighborhood of 5,000,000 and in the case of a plant designed to take care of regulation of the major part of a system this value may be as high as 10,000,000.

Engineers usually specify $W\,R^2$ as computed from the formula

$W\,R^2 \frac{810{,}000 \times \text{h. p.} \times T}{(R\,P\,M)^2 \times d}$. Where h. p. is load applied, T is time of governor action, and d is percentage of speed rise. As a matter of convenience full-load conditions are generally assumed. This application should always be made with a view towards what will occur when load changes smaller than ½ load occur, since full and very large load changes are infrequent and of little consequence. Under approximately open-water conditions the governor should ordinarily be set to operate the gates through their full stroke in not less than 1½ seconds and through a stroke corresponding to about $^1/_{10}$ load change in ¾ to 1 second. Very large units require longer time of governor action to avoid racking the operating mechanism. For practi-

cal purposes proportional load changes may be assumed to require a directly proportional time of governor action. Further, a $^1/_{10}$-load change should not cause a speed variation of more than 2½ per cent from normal and a good value is 1½ per cent. This may be applied to the above formula to determine proper $W R^2$. This load change is commercial and the engineer must use his judgment and knowledge of the system to determine permissible speed or frequency variation. Also it should be kept in mind that the above formula is for a water-rheostat load and does not take into account the effect of the rest of the electrical system in steadying the frequency. If closed conduits supply water to the unit, the governor time element must be such as to limit pressure variations to safe values and the $W R^2$ must be increased. Pressure variations in themselves affect regulation approximately in direct proportion to the 3/2 power of the momentary increase or decrease in effective head. Regulation is dependent to a large extent upon local conditions and it is almost impossible to set forth any definite methods for determination of $W R^2$ required which will be applicable to all cases.

Automatically oil-operated brakes in the speaker's opinion should not usually be required. They introduce an extra item of control with attendant complications and cost. Oil is usually taken from the governor system for their operation and the piping is liable to develop leaks. When the turbine gates are closed conditions can be made and should be such that the unit will stop. Hand-operated mechanical brakes should be provided to stop the wheel if the gates should get clogged and to hold the unit when under inspection, etc. When the turbine-gate mechanism becomes so worn that the wheel continues to rotate, adjustments or repairs as found necessary should be made.

The lubrication of turbine bearings is more or less difficult because water under pressure containing foreign matter is always introduced and it is a question of lubricating each bearing individually. The grease cup is practically a thing of the past, since it is hard to put grease under sufficient pressure, to lubricate properly some bearings without having grease squeeze out through the threads. Pressure fittings of various types such as the "alemite" have been used with good success. The grease compressor may be of the portable type for small plants or the stationary type for large plants with a flexible hose connection. This lubrication need only be periodic and can be taken care of by the visiting attendant. If the grease compressor is portable it should have sufficient capacity to grease all of the turbine bearings with one filling. The main turbine bearing is always of the water-lubricating lignum-vitae type, or oil-lubricated babbitted type. The latter is gaining some favor for large vertical units. Its principal disadvantages are inaccessibility on vertical units and possibility of mixture of water and oil. A motor-driven oil pump and auxiliary belt-driven oil pump should be supplied so that one pump will always be available. Thermostat protection is usually provided for this type. With the lignum-vitae type an electrical alarm attachment should be provided to indicate failure of lubricating water supply to bearing. Water-supply piping should be suitably lagged to prevent danger of water freezing in winter in an unheated automatic plant.

There are some plants being installed from which governor has been omitted and a mechanical hand and electrically operated control supplied. The only reason is to effect a saving in first cost, and this saving is small if properly designed equipment is supplied. Usually a governor is justified since it allows greater flexibility of operation and makes the unit more independent of the rest of the system in case of trouble.

F. R. George: Mr. Wensley issued somewhat of a challenge to the operating man. The operating man in California deals with the steam plant as a standby, and, therefore, his hydro becomes the base load of his system and requires more attention than the steam plants. However, the efficiencies in the steam plant, as Mr. Wensley stated, are more carefully looked after. The hydro plants are not looked after in the same manner, I think perhaps because of a lack of satisfactory unit of efficiency comparable to so many kilowatts per barrel of oil.

We have a great deal of difficulty in keeping ahead of the demand, and, therefore, when it comes to a question of going to our boards of directors and asking them for appropriations to put in an automatic plant or to revamp an old plant to automatic operation, the first question asked us is: "About how many kilowatts shall we get out of the plant? Naturally the automatic plant lends itself only (I say so advisedly) to small capacities. When we have our plans all worked up and find that it is practical and perfectly possible to convert an old plant and we reach the point where we are asked the amount of kilowatt hours we are going to get from that plant, and it is a comparably insignificant amount, the Board of Directors say: "No, we must devote our time and money at present to the development of new power in order to keep abreast of the demand" and, therefore, the installation of the automatic control device is again delayed or postponed.

The California situation at the present time lends itself very well to the thought and to the entertainment of the application of automatic devices. We are now facing one of the driest seasons in our history, meaning that our operating costs are going to be quite excessive due to the necessity for reversing our operation and placing dependence upon our steam generating equipment to offset the shortage in the hydro-power plants due to the drought, and therefore any economies which we could effect in the operation of our hydro plants would help in a measure to offset the unusual high operating costs which will maintain due to the drought.

E. E. Woodward: There seems to be a great deal of misunderstanding in regard to the synchronous-motor governor drive. Our company was probably the pioneer in using it to any extent, and we have been using it for about three years, and we have no complaints so far.

While called a synchronous-motor drive, the motor is not synchronous at all but is squirrel-cage induction three-or two-phase motor. This motor will start at 25 per cent of full voltage, and while it is not synchronous at this voltage it is very nearly so at 50 per cent voltage, and at full voltage the lag is so small that it cannot easily be detected.

Mr. Porter brought out the matter of brakes being needed to hold the gates closed on an automatic plant. That was one of the first things we discovered, and we have a very simple brake on the governor which is operated by oil pressure. It is really operated by a spring. We have an automatic throttle valve which is closed so as to keep the pressure in the tank during the period of rest. By this means we are able to hold the pressure for two or three weeks, if necessary, and have enough to start with. When the oil pressure is shut off from the governor, the spring sets the brake and the gates are locked closed. As soon as the oil pressure is thrown into the governor, the brake is released.

In the matter of generator brakes, we don't undertake to say whether they should be used or not, but if they are and if they use oil from the governor, we want that oil pressure to be shut off after the wheel is stopped, otherwise we might lose much oil through the brake cylinders and leakage elsewhere. We are now developing a brake valve that will apply the brakes and release them after an interval of five, ten or fifteen minutes, as may be desired.

L. F. Harza: I am not fully in accord with Mr. Wensley's belief that automatic operation will result in dividing up an available head into several smaller heads for reduction in flowage damages and saving in the cost of dams. The saving in cost of dams is doubtful when we consider the resulting multiplication of cofferdams, control gates and machinery. The storage obtained usually justifies the higher dam. Likewise the higher dam is not crippled in time of flood while the several substitute low head projects might cease entirely to operate under flood condi-

tions. However, this is a question which can be settled only for each individual river.

It is our belief that many of the features developed for the automatic plant can well be applied to a manual station of any size, regardless of how well its size may justify the expense of operators. We are all familiar with mistakes made by operators in emergencies. Cool headed action by a human operator can be least expected when most needed. Protective relays may at times fail to function but should be more reliable under emergencies than human operators.

We are also of the opinion that self synchronizing either by damper windings or synchronous motors and mechanical differential will ultimately replace manual synchronizing even in manual stations. It would eliminate the human element in one more operation where a mistake might be disastrous. All the operator would then need to do would be to close a master switch which would start the cycle of operation necessary to put a unit into service.

We have developed several automatic stations, equipped with hand brakes only. I regret to advise that the units do not all stop rotating when tripped out of service, even though the heads are low, in two cases as low as 8.5 ft. No harm has resulted as yet from failure to stop but the slow speed of rotation suggests the danger of failure of the oil film and injury to the bearing. Moreover, if tripped out by the bearing thermostats failure, to stop rotating would defeat the purpose of these thermostats. I shall be inclined for future installations to favor automatic brakes. Automatic brakes can be operated from the governor tank by air as well if not better than by oil as far as interfering with the normal functions of the governor is concerned. Connection to the governor tank should be closed automatically as soon as the unit stops, to prevent continued loss of air or oil. Most units when once stopped will not start again after releasing the brakes.

I am not convinced that engineers usually waste money on flywheel effect. Where an installation is a small part of the generating capacity of a system operating in parallel with a system of steam turbo-generators, a small flywheel effect can be used and the mistake of over-weight of rotor may at times be made. More often however, the case is that the manufacturer has a standard line of patterns which he tries to adapt to vertical or horizontal machines, water wheel or engine-driven by the least possible adjustments. The manufacturer should have a vertical line of machines developed for water-wheel service.

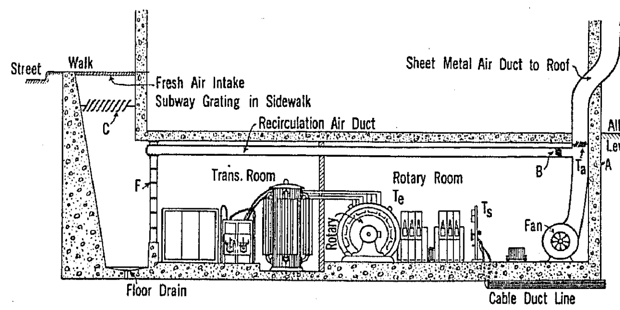
The writer is convinced by experience with the development of several automatic stations that they have a great future. Automatic means are now available for meeting almost any operating condition or emergency with a degree of success and reliability, dependent chiefly upon the relays. Different manufacturers, to avoid patent infringement, accomplish the same purpose with relays often operating upon widely different principles and sometimes with widely different degrees of success. It behooves the purchaser to examine carefully into the relays offered with any installation, as to the extent of their previous application and success.

F. C. Hanker: There are one or two points I would like to bring out in connection with the automatic generator station. Mr. Harza has mentioned the effect of the method of starting on the system operation. If you take the small, low-speed machines, the current they will draw when thrown on the line at full voltage will not exceed three times the rating of the unit. But when you consider the larger, high-speed machines, that current becomes more of a factor in the system operation and must be recognized in the method of operation or the method of control that is selected.

The experience in manual operation with the starting of large synchronous condensers shows conclusively that those machines can be started on a system of comparatively small size without disturbance by using proper precautions in the starting. But when you come to a high-speed large-capacity unit relative to the size of the system, then it is necessary to take the starting current into account. That is one place where the automatic synchronizer is quite important.

Another phase of the problem involves manually operated stations. In the Northwest, and I presume in other systems, there has been trouble experienced following an interruption to service in getting the stations back on the line and restoring service in a minimum of time. In some cases recent failures have shown that the time to restore units to service is about fifteen minutes. A good deal of that time is spent by the operators in getting the various water wheels and hydraulic elements stabilized, so that if they had automatic synchronizers, those automatic synchronizers could be applied to the main units and the operators could devote their entire time to the operation of the hydraulic parts, governors and water wheels, and get those stabilized.

The automatic control equipment of itself has been well developed in the synchronous-converter, motor-generator and the synchronous-generator application. No equipment is perfect, but the experience we have had makes us very hopeful that this equipment can be developed to a point where its efficiency and reliability will more than equal the hand-operated station.



T_a	(Thermostat)	Closes A and C Opens B	When Temperature falls below 85 deg. Fahr.
T_e	"	Prevents fan running when temperature is lower than 70 deg. Fahr.	
T_s	"	Turns steam into radiators when temperature is below 80 deg. Fahr.	
F	Dry type filter		

S. E. Bettes (by letter): Where automatic substations are located in building basements a special ventilation system is required to insure an adequate supply of clean air to keep the operating temperature below the danger point. This system must adopt itself to all the requirements of the automatic station operation and as in the case of the station itself change its own operation to suit the conditions existing.

The following outline of operation is used in our automatic rotary-converter substations and is proving very satisfactory. The accompanying sketch shows the ventilation schematically.

All air entering the station is drawn through the sidewalk grating and through the dampers C. After passing through the dry-type air filters which remove the dirt, the clean air goes through the transformer and high-tension switch room into the rotary-converter room. Self-closing fire doors are provided in this wall to prevent fire from passing from one room to the other.

The fans in the rear of the station draw the hot air out of the station and force it up the duct to the discharge which is above the roof of the building. There are two fans, each capable of supplying sufficient air to cool the station.

The automatic features provided are:

The fans are automatically started when the rotary is put on the line unless the temperature is lower than 70 deg. Fahr. When the air coming from the rotary is warmer than 70 deg. the

thermostat *T e* closes the automatic starter and the fan starts. If the air passing the thermostat *T a* is above 85 deg. fahr., all the air goes up the duct to the exhaust. If the air is just less than 85 deg. fahr., the dampers at *A* and *C* start to close and the damper *B* starts to open. This allows part of the air to return to the intake end of the station and tempers the cold air coming in. A further reduction in temperature of the air passing thermostat *T a* causes *A* and *C* to close entirely and all the air goes back through the station.

To provide a source of heat when no machines are operating, steam radiators are placed so as to warm parts likely to be damaged by cold. The thermostat *T s* turns on the steam if the temperature goes below 60 deg. fahr.

The failure of the damper control will cause the *A* and *C* damper to open and the *B* damper to close so that the machines will get the maximum amount of cooling air.

Any draft action caused by warm air up the exhaust duct after the machines are shut down will remove warm air from the station until the temperature reaches 85 deg. when the thermostats will close *C* and *A* and prevent further cooling. This is important during the winter months.

Rain coming through the sidewalk grating cannot enter the station and is disposed of through the drains.

H. Bany: In answer to Mr. Millan's question regarding whether the generator is disconnected from the d-c. bus upon reverse power, nothing further occurs than the three operations mentioned under operating features on the fifth page of the paper. The d-c. line breakers do not open and the set merely runs idle from the d-c. end. If any other operations do occur, they must be caused by some other abnormal condition, such as, a-c. undervoltage or a-c. power failure.

It does not seem that there is any sacrifice being made by having the motor field excited directly from the generator as Mr. Millan questions, instead of from the exciter. Since normal excitation on the motor gives about 0.8 leading power factor at full load, if the load and excitation are both reduced proportionally, the motor will not pull out of step at the lower d-c. bus voltages. If the a-c. voltage should be low so that the motor does pull out of step, the time delay of 40 seconds mentioned in the first paragraph of the second column, page 765, will shut down the set and a restart will not be allowed until the a-c. line voltage is again 80 per cent of normal or above.

C. A. Butcher: With reference to remarks by Messrs. Wright and Garrett, concerning comparative efficiencies of converters and motor-generators, the difference in all-day efficiency is less in favor of converters, if it is established that smaller motor generators may be used to supply the peak loads incident to infrequently scheduled heavy trains, or in other words, if the substation capacity is based on commutating capacity, rather than the continuous rating of the machines. Mr. Wright states that this may apply to the outlying substations; however, it would seem that during periods of light traffic, such as is the case during the off-peak load, the same applies to all stations.

The largest 1500-volt automatic railway substation soon to be placed in operation has a total capacity of 8000 kw., made up of four 2000-kw. converter sets.

R. J. Wensley: It is my feeling that the purpose of this paper has been accomplished. It was intentionally written in such a manner as to bring out in discussion some of the disputed points in the operation of Automatic Hydroelectric Generating Equipments. The discussions by Mr. R. H. Earle and Mr. H. T. Porter are both distinct contributions to the general subject. The discussion of the desirable amount of flywheel effect as given by Mr. Porter is of especial value.

C. W. Place: I wish to point out that the method of operation as outlined by the previous speaker, is not the one that will most nearly meet conditions in the middle western territory. The most economical operation will be that of using the steam plants as base-load plants and the hydroelectric stations for carrying peaks. The introduction of more and more interconnection of systems and the building of base-load steam plants, makes this use of hydroelectric stations particularly valuable.

Selective Circuits and Static Interference

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Review of the Subject.—*The present paper has its inception in the need of a correct understanding of the behavior of selective circuits when subjected to irregular and random inteference, and of devising a practically useful figure of merit for comparing circuits designed to reduce the effects of this type of interference. The problem is essentially a statistical one and the results must be expressed in terms of mean values. The mathematical theory is developed from the idea of the spectrum of the interference and the response of the selective circuit is expressed in terms of the mean square current and mean power absorbed. The application of the formulas deduced to the case of static interference is discussed and it is shown that deductions of practical value are possible in spite of meager information regarding the precise nature and origin of static interference.*

The outstanding deductions of practical value may be summarized as follows:

1. *Even with absolutely ideal selective circuits, an irreducible minimum of interference will be absorbed, and this minimum increases linearly with the frequency range necessary for signaling.*

2. *The wave-filter, when properly designed, approximates quite closely to the ideal selective circuit, and little, if any, improvement over its present form may be expected as regards static interference.*

3. *As regards static or random interference, it is quite useless to employ extremely high selectivity. The gain, as compared with circuits of only moderate selectivity, is very small, and is inevitably accompanied by disadvantages such as sluggishness of response with consequent slowing down of the possible speed of signaling.*

4. *A formula is developed, which, together with relatively simple experimental data, provides for the accurate determination of the spectrum of static interference.*

5. *An application of the theory and formulas of the paper to representative circuit arrangements and schemes designed to reduce static interference, shows that they are incapable of reducing, in any substantial degree, the mean interference, as compared with what can be done with simple filters and tuned circuits. The underlying reason lies in the nature of the interference itself.*

* * * * *

I

The selective circuit is an extremely important element of every radio receiving set, and on its efficient design and operation depends the economical use of the available frequency range. The theory and design of selective circuits, particularly of their most conspicuous and important type, the electric wave filter, have been highly developed, and it is now possible to communicate simultaneously without undue interference on neighboring channels with a quite small frequency separation. On the other hand too much has been expected of the selective circuit in the way of eliminating types of interference which inherently do not admit of elimination by any form of selective circuit. I refer to the large amount of inventive thought devoted to devising ingenious and complicated circuit arrangements designed to eliminate *static interference.* Work on this problem has been for the most part futile, on account of the lack of a clear analysis of the problem and a failure to perceive inherent limitations on its solutions by means of selective circuits.

The object of this paper is twofold: (1) To develop the mathematical theory of the behavior of selective circuits when subjected to random, irregular disturbances, hereinafter defined and designated as *random interference.* This will include a formula which is proposed as a measure of the *figure of merit of selective circuits with respect to random interference.* (2) On the basis of the theory to examine the problem of *static interference* with particular reference to the question of its elimination by means of selective circuits. The mathematical theory shows, as might be expected, that the complete solution of this problem requires experimental data regarding the frequency distribution of static interference which is now lacking. On the other hand, it throws a great deal of light on the whole problem and supplies a formula which furnishes the theoretical basis for an actual determination of the spectrum of static. Furthermore, on the basis of a certain mild and physically reasonable assumption, it makes possible general deductions of practical value which are certainly qualitatively correct and are believed to involve no quantitatively serious error. These conclusions, it may be stated, are in general agreement with the large, though unsystematized, body of information regarding the behavior of selective circuits to static interference, and with the meager data available regarding the wave form of elementary static disturbances.

The outstanding conclusions of practical value of the present study may be summarized as follows:

(1) Even with absolutely ideal selective circuits, an irreducible minimum of interference will be absorbed, and this minimum increases linearly with the frequency range necessary for signaling.

(2) The wave-filter, when properly designed, approximates quite closely to the ideal selective circuit, and little, if any, improvement over its present form may be expected as regards static interference.

(3) As regards static or random interference, it is quite useless to employ extremely high selectivity. The gain, as compared with circuits of only moderate selectivity, is very small, and is inevitably accompanied by disadvantages such as sluggishness of response with consequent slowing down of the possible speed of signaling.

(4) By aid of a simple, easily computed formula, it

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should be possible to determine experimentally the frequency spectrum of static.

(5) Formulas given below for comparing the relative efficiencies of selective circuits on the basis of signal-to-interference energy ratio are believed to have considerable practical value in estimating the relative utility of selective circuits as regards static interference.

II

Discrimination between signal and interference by means of selective circuits depends on taking advantage of differences in their wave forms, and hence on differences in their *frequency spectra*. It is, therefore, the function of the selective circuit to respond effectively to the range of frequencies essential to the signal while discriminating against all other frequencies.

Interference in radio and wire communication may be broadly classified as *systematic* and *random*, although no absolutely hard and fast distinctions are possible. *Systematic interference* includes those disturbances which are predominantly steady-state or those whose energy is almost all contained in a relatively narrow band of the frequency range. For example, interference from individual radio-telephone and slow-speed radio telegraph stations is to be classified as systematic. *Random interference*, which is discussed in detail later, may be provisionally defined as the aggregate of a large number of elementary disturbances which originate in a large number of unrelated sources, vary in an irregular, arbitrary manner, and are characterized statistically by no sharply predominate frequency. An intermediate type of interference, which may be termed either *quasi-systematic* or *quasi-random*, depending on the point of view, is the aggregate of a large number of individual disturbances, all of the same wave form, but having an irregular or random time distribution.

In the present paper we shall be largely concerned with random interference, as defined above, because it is believed that it represents more or less closely the general character of *static* interference. This question may be left for the present, however, with the remark that the subsequent analysis shows that, as regards important practical applications and deductions, a knowledge of the exact nature and frequency distribution of static interference is not necessary.

Now when dealing with random disturbance, as defined above, no information whatsoever is furnished as regards instantaneous values. In its essence, therefore, the problem is a statistical one and the conclusions must be expressed in terms of mean values. In the present paper formulas will be derived for the *mean energy* and *mean square current* absorbed by selective circuits from random interference, and their applications to the static problem and the protection afforded by selective networks against static will be discussed.

The analysis takes its start with certain general formulas given by the writer in a recent paper[1], which may be stated as follows:

Suppose that a selective network is subjected to an impressed force $\phi(t)$. We shall suppose that this force exists only in the time interval, or epoch, $o \leqq t \leqq T$, during which it is everywhere finite and has only a finite number of discontinuities and a finite number of maxima and minima. It is then representable by the Fourier Integral

$$\phi(t) = 1/\pi \int_0^\infty |f(\omega)| \,.\, \cos[\omega t + \theta(\omega)]\, d\omega \tag{1}$$

where

$$|f(\omega)|^2 = \left[\int_0^\infty \phi(t) \cos \omega t \, dt\right]^2 + \left[\int_0^\infty \phi(t) \sin \omega t \, dt\right]^2 \tag{2}$$

Now let this force $\phi(t)$ be applied to the network in the *driving* branch and let the resulting current in the *receiving* branch be denoted by $I(t)$. Let $Z(i\omega)$ denote the steady-state *transfer* impedance of the network at frequency $\omega/2\pi$: that is the ratio of e. m. f. in *driving* branch to current in *receiving* branch. Further let $z(i\omega)$ and $\cos \alpha(\omega)$ denote the corresponding impedance and power factor of the receiving branch. It may then be shown that

$$\int_0^\infty [I(t)]^2 \, dt = 1/\pi \int_0^\infty \frac{|f(\omega)|^2}{|Z(i\omega)|^2} \, d\omega \tag{3}$$

and that the total energy W absorbed by the receiving branch is given by

$$W = 1/\pi \int_0^\infty \frac{|f(\omega)|^2}{|Z(i\omega)|^2} |z(i\omega)| \,.\cos \alpha(\omega) \,.\, d\omega \tag{4}$$

To apply the formulas given above to the problem of random interference, consider a time interval, or epoch, say from $t = o$ to $t = T$, during which the network is subjected to a disturbance made up of a large number of unrelated elementary disturbances or forces, $\phi_1(t), \phi_2(t) \ldots \phi_n(t)$.

If we write

$$\Phi(t) = \phi_1(t) + \phi_2(t) + \ldots + \phi_n(t)$$

then by (1), $\Phi(t)$ can be represented as

$$\Phi(t) = 1/\pi \int_0^\infty |F(\omega)| \,.\, \cos[\omega t + \theta(\omega)]\, d\omega$$

and

$$\int_0^\infty [I(t)]^2 \, dt = 1/\pi \int_0^\infty \frac{|F(\omega)|^2}{|Z(i\omega)|^2} \, d\omega \tag{3}$$

We now introduce the function $R(\omega)$, which will be termed the *energy spectrum* of the random interference, and which is analytically defined by the equation

$$R(\omega) = 1/T\, |F(\omega)|^2 \tag{5}$$

1. Transient Oscillations in Electric Wave Filters, Carson and Zobel, *Bell System Technical Journal*, July 1923.

Dividing both sides of (3) and (4) by T we get

$$\bar{I}^2 = 1/\pi \int_0^\infty \frac{R(\omega)}{|Z(i\omega)|^2} d\omega \qquad (6)$$

$$\bar{P} = 1/\pi \int_0^\infty \frac{R(\omega)}{|Z(i\omega)|^2} |z(i\omega)| . \cos\alpha(\omega) . d\omega \qquad (7)$$

$\bar{I}^2$, $\bar{P}$ and $R(\omega)$ become independent of the T provided the epoch is made sufficiently great. *$\bar{I}^2$ is the mean square current and $\bar{P}$ the mean power absorbed by the receiving branch from the random interference.*

In the applications of the foregoing formulas to the problem under discussion, the mean square current $\bar{I}^2$ of the formula (6) will be taken as the relative measure of interference instead of the mean power $\bar{P}$ of formula (7). The reason for this is the superior simplicity, both as regards interpretation and computation, of formula (6). The adoption of $\bar{I}^2$ as the criterion of interference may be justified as follows:

(1) In a great many important cases, including in particular experimental arrangements for the measurement of the static energy spectrum, the receiving device is substantially a pure resistance. In such cases multiplication of $\bar{I}^2$ by a constant gives the actual mean power $\bar{P}$.

(2) It is often convenient and desirable in comparing selective networks to have a standard termination and receiving device. A three-element vacuum tube with a pure resistance output impedance suggests itself, and for this arrangement formulas (6) and (7) are equal within a constant.

(3) We are usually concerned with relative amounts of energy absorbed from static as compared with that absorbed from signal. Variation of the receiver impedance from a pure constant resistance would only in the extreme cases affect this ratio to any great extent. In other words, the ratio calculated from formula (6) would not differ greatly from the ratio calculated from (7).

(4) While the interference actually apperceived either visually or by ear will certainly depend upon and increase with the energy absorbed from static, it is not at all certain that it increases linearly therewith. Consequently, it is believed that the additional refinement of formula (7) as compared with formula (6) is not justified by our present knowledge and that the representation of the receiving device as a pure constant resistance is sufficiently accurate for present purposes. It will be understood, however, that throughout the following argument and formulas, $\bar{P}$ of formula (7) may be substituted for $\bar{I}^2$ of (6), when the additional refinement seems justified. The theory is in no sense limited to the idea of a pure constant resistance receiver, although the simplicity of the formulas and their ease of computation is considerably enhanced thereby.

The problem of random interference, as formulated by equations (6) and (7) was briefly discussed by the writer in "Transient Oscillations in Electric Wave Filters"[2] and a number of general conclusions reached. That discussion will be briefly summarized, after which a more detailed analysis of the problem will be given.

Referring to formula (6), since both numerator and denominator of the integrand are everywhere $\geq o$, it follows from the mean value theorem that a value $\bar{\omega}$ of ω exists such that

$$\bar{I}^2 = \frac{R(\bar{\omega})}{\pi} \int_0^\infty \frac{d\omega}{|Z(i\omega)|^2} \qquad (8)$$

The approximate location of $\bar{\omega}$ on the frequency scale is based on the following considerations:

(a) In the case of efficient selective circuits designed to select a continuous finite range of frequencies in the interval $\omega_1 \leq \omega \leq \omega_2$, the important contributions to the integral (6) are confined to a finite continuous range of frequencies which includes, but is not greatly in excess of, the range which the circuit is designed to select. This fact is a consequence of the impedance characteristics of selective circuits, and the following properties of the spectrum $R(\omega)$ of random interference, which are discussed in detail subsequently.

(b) $R(\omega)$ is a continuous finite function of ω which converges to zero at infinity and is everywhere positive. It possesses no sharp maxima or minima, and its variation with respect to ω, where it exists, is relatively slow.

On the basis of these considerations it will be assumed that $\bar{\omega}$ lies within the band $\omega_1 \leq \omega \leq \omega_2$ and that without serious error it may be taken as the mid-frequency ω_m of the band which may be defined either as $(\omega_1 + \omega_2)/2$ or as $\sqrt{\omega_1 \omega_2}$. Consequently

$$\bar{I}^2 = \frac{R(\omega_m)}{\pi} \int_0^\infty \frac{d\omega}{|Z(i\omega)|^2} \qquad (9)$$

From (9) it follows that the mean square current $\bar{I}^2$, due to random interference, is made up of two factors: one $R(\omega_m)$ which is proportional to the energy level of the interference spectrum at mid-frequency $\omega_m/2\pi$: and, second, the integral

$$\rho = 1/\pi \int_0^\infty \frac{d\omega}{|Z(i\omega)|^2} \qquad (10)$$

which is independent of the character and intensity of the interference. Thus

$$\bar{I}^2 = \rho R(\omega_m) \qquad (11)$$

Formula (11) is of considerable practical importance, because by its aid the spectral energy level $R(\omega)$ can be determined, once $\bar{I}^2$ is experimentally measured and the frequency characteristics of the receiving network specified or measured. It is approximate, as discussed

2. L. c.

above, but can be made as accurate as desired by employing a sufficiently sharply selective network.

The formula for the *figure of merit of a selective circuit with respect to random interference* is constructed as follows:

Let the signaling energy be supposed to be spread continuously and uniformly over the frequency interval corresponding to $\omega_1 \leqq \omega \leqq \omega_2$. Then the mean square signal current is given by

$$\frac{E^2}{\pi} \int_{\omega_1}^{\omega_2} \frac{d\,\omega}{|Z\,(i\,\omega)|^2}$$

or, rather, on the basis of the same transmitted energy to

$$E^2 \frac{1}{\pi\,(\omega_2 - \omega_1)} \int_{\omega_1}^{\omega_2} \frac{d\,\omega}{|Z\,(i\,\omega)|^2} = E^2\,\sigma \qquad (12)$$

The ratio of the mean square currents, due to signal and to interference, is

$$\frac{E^2}{R\,(\omega_m)} \cdot \sigma/\rho \qquad (13)$$

The first factor $\frac{E^2}{R\,(\omega_m)}$ depends only on the signal and interference energy levels, and does not involve the properties of the network. The second factor σ/ρ depends only on the network and measures the efficiency with which it excludes energy outside the signaling range. It will, therefore, be termed *the figure of merit of the selective circuit* and denoted by S, thus

$$S = \sigma/\rho = \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} \frac{d\,\omega}{Z\,(i\,\omega)^2} \div \int_0^\infty \frac{d\,\omega}{|Z\,(i\,\omega)|^2} \qquad (14)$$

Stated in words, *the figure of merit of a selective circuit with respect to random interference is equal to the ratio of the mean square signal and interference currents in the receiver, divided by the corresponding ratio in an ideal band filter which transmits without loss all currents in a "unit" band* ($\omega_2 - \omega_1 = 1$) *and absolutely extinguishes currents outside this band.*

III

Before taking up practical applications of the foregoing formulas, further consideration will be given to the hypothesis, fundamental to the argument, that over the frequency range which includes the important contributions to the integral $\int_0^\infty \frac{d\,\omega}{|Z\,(i\,\omega)|^2}$ the spectrum $R\,(\omega)$ has negligible fluctuations so that the integral

$$\int_0^\infty \frac{R\,(\omega)}{|Z\,(i\,\omega)|^2}\,d\,\omega$$

may, without appreciable error, be replaced by

$$R\,(\omega_m) \int_0^\infty \frac{d\,\omega}{|Z\,(i\,\omega)|^2}$$

where $\omega_m/2\,\pi$ is the "mid-frequency" of the selective circuit.

The original argument in support of this hypothesis was to the effect that, since the interference is made up of a large number of unrelated elementary disturbances distributed at random in time, any sharp maxima or minima in the spectrum of the individual disturbances would be smoothed out in the spectrum of the aggregate disturbance. This argument is still believed to be quite sound: the importance of the question, however, certainly calls for the more detailed analysis which follows:

Let
$$\Phi\,(t) = \sum_1^N \phi_r\,(t - t_r) \qquad (15)$$

where t_r denotes the time of incidence of the r^{th} disturbance $\phi_r\,(t)$. The elementary disturbances $\phi_1, \phi_2 \ldots \phi_N$ are all perfectly arbitrary, so that $\Phi\,(t)$ as defined by **(15)** is the most general type of disturbance possible. The only assumption made as yet is that the instants of incidence $t_1 \ldots t_N$ are distributed at random over the epoch $o \leqq t \leqq T$; an assumption which is clearly in accordance with the facts in the case of static interference. If we write

$$C_r\,(\omega) = \int_0^\infty \phi_r\,(t) \cos \omega\,t\,d\,t$$

$$S_r\,(\omega) = \int_0^\infty \phi_r\,(t) \sin \omega\,t\,d\,t \qquad (16)$$

it follows from **(2)** and **(15)**, after some easy rearrangements that

$$|F\,(\omega)|^2 = \sum_{r=1}^N \sum_{s=1}^N \cos \omega\,(t_r - t_s)\,[C_r\,(\omega)\,C_s\,(\omega) + S_r\,(\omega)\,S_s\,(\omega)] = \sum C_r^2\,(\omega) + S_r^2\,(\omega) \qquad (17)$$
$$+ \sum_{r \neq s} \sum \cos \omega\,(t_r - t_s)\,[C_r\,(\omega)\,C_s\,(\omega) + S_r\,(\omega)\,S_s\,(\omega)]$$

The first summation is simply $\sum |f_r(\omega)|^2$. The double summation involves the factor $\cos \omega\,(t_r - t_s)$. Now by virtue of the assumption of random time distribution of the elementary disturbances, it follows that t_r and t_s, which are independent, may each lie anywhere in the epoch $o \leqq t \leqq T$ with all values equally likely. The mean value of $|F\,(\omega)|^2$ is therefore gotten by

averaging[3] with respect to t_r and t_s over all possible values, whence

$$|F(\omega)|^2 = \sum |f_r(\omega)|^2 + 2/T^2 \frac{1-\cos\omega T}{\omega^2} \times$$
$$\sum\sum [C_r(\omega) C_s(\omega) + S_r(\omega) S_s(\omega)] \quad (18)$$

and

$$\bar{I}^2 = \frac{1}{\pi T} \sum \int_0^\infty \frac{|f_r(\omega)|^2}{|Z(i\omega)|^2} d\omega$$
$$+ \frac{2}{\pi T^2} \sum\sum \int_0^\infty \frac{1-\cos\omega T}{\omega^2 T} [C_r(\omega) C_s(\omega)$$
$$+ S_r(\omega) S_s(\omega)] \frac{d\omega}{|Z(i\omega)|^2}$$

Now in the double summation if the epoch T is made sufficiently great, the factor $\frac{(1-\cos\omega T)}{\omega^2 T}$ vanishes everywhere except in the neighborhood of $\omega = o$. Consequently, the double summation can be written as

$$\frac{2}{\pi T^2} \int_0^\infty \frac{1-\cos\omega T}{\omega^2 T^2} d\omega T \cdot \sum\sum \frac{C_r(o) C_s(o)}{|Z(o)|^2}$$
$$= \frac{1}{T^2} \sum\sum \frac{C_r(o) C_s(o)}{|Z(o)|^2}$$

Finally, if we write $N/T = n$ = average number of disturbances per unit time, and make use of formula (**2**), we get

$$\bar{I}^2 = \frac{n}{N} \sum 1/\pi \int_0^\infty \frac{|f_r(\omega)|^2}{|Z(i\omega)|^2} d\omega \quad (19)$$
$$+ \frac{n^2}{N^2} \cdot \frac{1}{|Z(o)|^2} \cdot \sum\sum \int_0^\infty \phi_r(t)\, dt \cdot \int_0^\infty \phi_s(t)\, dt$$

which can also be written as

$$\bar{I}^2 = \frac{n}{N} \sum \int_0^\infty i_r^2\, dt$$
$$+ \frac{n^2}{N^2} \sum\sum \int_0^\infty i_r\, dt \cdot \int_0^\infty i_s\, dt \quad (20)$$

when $i_r = i_r(t)$ is the current due to the r^{th} disturbance $\phi_r(t)$.

Now the double summation vanishes when, due to the presence of a condenser or transformer, the circuit does not transmit direct current to the receiving branch. Furthermore, if the disturbances are oscillatory or alternate in sign at random, it will be negligibly small compared with the single summation. Consequently, it is of negligible significance in the practical applications contemplated, and will, therefore, be omitted except in special cases. Therefore, disregarding the double summation, the foregoing analysis may be summarized as follows:

$$R(\omega) = \frac{n}{N} \sum |f_r(\omega)|^2 = n . r(\omega) \quad (21)$$

$$\bar{I}^2 = \frac{n}{N} \sum 1/\pi \int_0^\infty \frac{|f_r(\omega)|^2}{|Z(i\omega)|^2} d\omega \quad (22)$$

$$= \frac{n}{N} \sum \int_0^\infty i_r^2\, dt = n \int_0^\infty \bar{i}^2\, dt \quad (23)$$

$$\bar{P} = \frac{n}{N} \int_0^\infty \frac{r(\omega)}{|Z(i\omega)|^2} |z(i\omega)| \cdot \cos\alpha(\omega) \cdot d\omega \quad (24)$$

$$= \frac{n}{N} \sum w_r = n . \bar{w} \quad (25)$$

In these formulas n denotes the average number of elementary disturbances per unit time, w_r the energy absorbed from the r^{th} disturbance $\phi_r(t)$, and $\bar{P}$ the mean power absorbed from the aggregate disturbance. $r(\omega)$ is defined by formula (**20**) and is the mean spectrum of the aggregate disturbance, thus

$$r(\omega) = 1/N \sum |f_r(\omega)|^2 = R(\omega)/N \quad (26)$$

We are now in a position to discuss more precisely the approximations, fundamental to formulas (**9**)-(**14**),

$$\int_0^\infty \frac{R(\omega)}{|Z(i\omega)|^2} d\omega = R(\omega_m) \int_0^\infty \frac{d\omega}{|Z(i\omega)|^2} \quad (27)$$

The approximation involved in this formula consists in identifying $\omega_m/2\pi$ with the "mid-frequency" of the selective circuit, and is based on the hypothesis that over the range of frequencies, which includes the important contribution to the integral (**22**), the fluctuation of $R(\omega)$ may be ignored.

Now it is evident from formulas (**21**)-(**25**) that the theoretically complete solution of the problem requires that $R(\omega)$ be specified over the entire frequency range from $\omega = o$ to $\omega = \infty$. Obviously, the required information cannot be deduced without making some additional hypothesis regarding the character of the interference or the mechanism in which it originates. On the other hand, the mere assumption that the individual elementary disturbances $\phi_1 \ldots \phi_N$ differ among themselves substantially in wave form and duration, or that the maxima of the corresponding spectra $|f_r(\omega)|$ are distributed over a considerable frequency range, is sufficient to establish the conclusion that the individual fluctuations are smoothed out in the aggregate and that consequently $r(\omega)$ and hence

3. The averaging process with respect to the parameters t_r and t_s employed above logically applies to the average result in a very large number of epochs during which the system is exposed to the same set of disturbances with different but random time distributions. Otherwise stated, the averaging process gives the mean value corresponding to all possible equally likely times of incidence of the elementary disturbances. The assumption is, therefore, that if the epoch is made sufficiently large, the actual effect of the unrelated elementary disturbances will in the long run be the same as the average effect of all possible and equally likely distributions of the elementary disturbances.

$R(\omega)$ would have negligible fluctuations, or curvature with respect to ω, over any limited range of frequencies comparable to a signaling range.

It is admitted, of course, that the foregoing statements are purely qualitative, as they must be in the absence of any precise information regarding the wave forms of the elementary disturbances constituting random interference. On the other hand, the fact that static is encountered at all frequencies without any sharp changes in its intensity as the frequency is varied, and that the assumption of a systematic wave form for the elementary disturbances would be physically unreasonable, constitute strong inferential support of the hypothesis underlying equation (**27**). Watt and Appleton (*Proc. Roy. Soc.* April 3, 1923) supply the only experimental data regarding the wave forms of the elementary disturbances which they found to be classifiable under general types with rather widely variable amplitudes and durations. Rough calculations of $r(\omega)$, based on their results, are in support of the hypothesis made in this paper, at least in the radio frequency range. In addition, the writer has made calculations based on a number of reasonable assumptions regarding variations of wave form among the individual disturbances, all of which resulted in a spectrum $R(\omega)$ of negligible fluctuations over a frequency range necessary to justify equation (**27**) for efficient selective circuits. However, the problem is not theoretically solvable by pure mathematical analysis, so that the rigorous verification of the theory of selectivity developed in this paper must be based on experimental evidence. On the other hand, it is submitted that the hypothesis introduced regarding static interference is not such as to vitiate the conclusions, qualitatively considered, or in general to introduce serious quantitative errors. Furthermore, even if it were admitted for the sake of argument that the figure of merit S was not an accurate measure of the ratio of mean square signal to interference current, nevertheless, it is a true measure of the excellence of the circuit in excluding interference energy outside the necessary frequency range.

IV

The practical applications of the foregoing analysis depend upon the formulas

$$\bar{I}^2 = \frac{R(\omega_m)}{\pi} \int_0^\infty \frac{d\,\omega}{|Z(i\,\omega)|^2} = \rho \cdot R(\omega_m) \qquad (11)$$

and

$$S = \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} \frac{d\,\omega}{|Z(i\,\omega)|^2} \div \int_0^\infty \frac{d\,\omega}{|Z(i\,\omega)|^2}$$

$$= \frac{1}{\omega_2 - \omega_1}\,\sigma/\rho \qquad (14)$$

which contain all the information which it is possible to deduce in the case of purely random interference. They are based on the principle that the effect of the interference on the signaling system is measured by the mean square interference current in the receiving branch, and that the efficiency of the selective circuit is measured by the ratio of the mean square signal and interference currents. As stated above, in the case of random interference results must be expressed in terms of mean values, and it is clear that either the mean square current or the mean energy is a fundamental and logical criterion.

Referring to formula (**11**), the following important proposition is deducible.

If the signaling system requires the transmissions of a band of frequencies corresponding to the interval $\omega_2 - \omega_1$, and if the selective circuit is efficiently designed to this end, then the mean square interference current is proportional to the frequency band width $\dfrac{(\omega_2 - \omega_1)}{2\,\pi}$.

This follows from the fact that, in the case of efficiently designed band-filters, designed to select the frequency range $\dfrac{(\omega_2 - \omega_1)}{2\,\pi}$ and exclude other frequencies, the integral $\int_0^\infty \dfrac{d\,\omega}{|Z(i\,\omega)|^2}$ is proportional to $\omega_2 - \omega_1$ to a high degree of approximation.

The practical consequences of these propositions are important and immediate. It follows that as the signaling speed is increased, the amount of interference inevitably increases practically linearly and that this increase is inherent. Again, it shows the advantage of single vs. double side-band transmission in carrier telephony, as pointed out by the writer in a recent paper.[4] It should be noted that the increased interference with increased signaling band width is not due to any failure of the selective circuit to exclude energy outside the signaling range, but to the inherent necessity of absorbing the interference energy lying inside this range. The only way in which the interference can be reduced, assuming an efficiently designed band-filter and a prescribed frequency range $\dfrac{(\omega_2 - \omega_1)}{2\,\pi}$, is to select a carrier frequency, at which the energy spectrum $R(\omega)$ of the interference is low.

Formula (**11**) *provides the theoretical basis for an actual determination of the static spectrum.* Measurement of $\bar{I}^2$ over a sufficiently long interval, together with the measured or calculated data for evaluating the integral $\int_0^\infty \dfrac{d\,\omega}{|Z(i\,\omega)|^2}$, determines $R(\omega_m)$ and this determination can be made as accurate as desired by employing a sufficiently sharply tuned circuit or a sufficiently narrow band filter. It is suggested that

4. Signal-to-Static-Interference Ratio in Radio Telephony, *Proc. I. R. E. E.*, June 1923.

the experimental data could be gotten without great difficulty, and that the resulting information regarding the statistical frequency distribution of static would be of large practical value.

The selective figure of merit S as defined by (**14**) is made up of two factors, $\dfrac{1}{(\omega_2 - \omega_1)}$ which is inversely proportional to the required signaling frequency range; and the ratio of the integrals σ/ρ. This ratio is unity for an ideally designed selective circuit, and can actually be made to approximate closely to unity with correctly designed band-filters. Formula (**14**) is believed to have very considerable value in comparing various circuits designed to eliminate interference, and is easily computed graphically when the frequency characteristics of the selective circuit are specified.

The general propositions deducible from it may be briefly listed and discussed as follows:

With a signaling frequency range $\dfrac{(\omega_2 - \omega_1)}{2\pi}$ *specified, the upper limiting value of* S *with a theoretically ideal selective circuit is* $\dfrac{1}{(\omega_2 - \omega_1)}$, *and the excellence of the actual circuit is measured by the closeness with which its figure of merit approaches this limiting value.*

Formula (**14**) for the figure of merit S has been applied to the study of the optimum design of selective circuits and to an analysis of a large number of arrangements designed to eliminate or reduce static interference. The outstanding conclusions from this study may be briefly reviewed and summarized as follows:

The form of the integrals σ and ρ, taking into account the signaling requirements, shows that the optimum selective circuit, as measured by S, is one which has a constant transfer impedance over the signaling frequency range $\dfrac{(\omega_2 - \omega_1)}{2\pi}$, and attenuates as sharply as possible currents of all frequencies outside this range. Now this is precisely the ideal to which the band filter, when properly designed and terminated, closely approximates, and leads to the inference that *the wave filter is the best possible form of selective circuit, as regards random interference.* Its superiority from the steady-state point of view has, of course, long been known.

An investigation of the effect of securing extremely high selectivity by means of filters of a large number of sections was made, and led to the following conclusion:

In the case of an efficiently designed band-filter, terminated in the proper resistance to substantially eliminate reflection losses, the figure of merit is given to a good approximation by the equation

$$S = \frac{1}{\omega_2 - \omega_1} \, \frac{1}{1 + 1/16\, n^2}$$

where n is the number of filter sections and $\dfrac{(\omega_2 - \omega_1)}{2\pi}$ the transmission band. It follows that *the selective figure of merit increases inappreciably with an increase in the number of filter sections beyond 2, and that the band filter of a few sections can be designed to have a figure of merit closely approximating the ideal limiting value,* $\dfrac{1}{(\omega_2 - \omega_1)}$.

This proposition is merely a special case of the general principle that, as regards static interference, it is useless to employ extremely high selectivity. The gain obtainable, as compared with only a moderate amount of selectivity is slight and is inherently accompanied by an increased sluggishness of the circuit. That is to say, as the selectivity is increased, the time required for the signals to build up is increased, with a reduction in quality and possible signaling speed.

Another circuit of practical interest, which has been proposed as a solution of the "static" problem in radio-communication consists of a series of sharply tuned oscillation circuits, unilaterally coupled through amplifiers.[5] This circuit is designed to receive only a single frequency to which all the individual oscillation circuits are tuned. The figure of merit of this circuit is approximately

$$S = L/R \, \frac{2^{2n-2}\,(n-1)!^2}{(2n-2)!}$$

where n denotes the number of sections or stages, and L and R are the inductance and resistance of the individual oscillation circuits. The outstanding fact in this formula is the slow rate of increase of S with the number of stages. For example, if the number of stages is increased from 1 to 5, the figure of merit increases only by the factor 3.66, while for a further increase in n the gain is very slow. This gain, furthermore, is accompanied by a serious increase in the "sluggishness" of the circuit: That is, in the particular example cited, by an increase of 5 to 1 in the time required for signals to build up to their steady state.

The analysis of a number of representative schemes, such as the introduction of resistance to damp out disturbances, balancing schemes designed to neutralize static without affecting the signal, detuning to change the natural oscillation frequency of the circuit, demodulation through several frequency stages, etc. has shown that they are one and all without value in increasing the ratio of mean square signal to interference current. In the light of the general theory, the reason for this is clear and the limitation imposed

5. See U. S. Patent No. 1173079 to Alexanderson.

6. When the number of stages n is fairly large, the selective figure of merit becomes proportional to $\sqrt{n}$ and the building-up time to n.

on the solution of the static problem by means of selective circuits is seen to be inherent in the nature of the interference itself.

Discussion

L. W. W. Morrow: I would just like to ask the question, if we are faced with the proposition of always suffering from static interference and is there no remedy?

J. R. Carson: The question of static interference is the same problem we come across in telephone transmission, that is, when the signal comes so weak that the interference energy level becomes comparable with the signal, we have got to either face interference or we have got to raise our power.

To my mind, the attempt to actually eliminate static by any form of selective circuit would be comparable with the attempt to find a perpetual motion machine. It is there and we have certainly got to receive a frequency range for signaling and from the very nature of the thing, must receive the interference in that range. We have all recognized that principle more or less in telephone engineering: in engineering long lines we must put in repeaters at intervals, depending upon the amount of interference or the interference energy level.

J. D. Robinson: Mr. Carson modifies his statement in a manner which possibly might be a little bit indeterminate for most of us. He says that he has no hopes for the selective circuit. Now that bears out an unknown number of things. Just why that modification?

J. R. Carson: There are, of course, possibilities other than frequency selection. For example, we all know that the wave antenna has been successful to a certain extent in eliminating interference, and I don't wish to imply that that form will not be successful in reducing it by a substantial amount, but that is a different basis, that is not frequency discrimination, that is discrimination with respect to direction in which the signal interference comes. That is, the interference comes in from all directions, the signal from one direction. We have here, therefore, a means of excluding part of the interference, and in some cases a very large part without any frequency discrimination whatsoever. Here again, however, we encounter an irreducible minimum of interference, namely the interference arriving from substantially the same direction as the signal.

L. J. Peters: About a year ago I carried on some studies, some of those carried on by Mr. Carson, using about the same methods, although not quite so elaborate.

On the second basis, that is the basis of average power over a period of time or the average grouping square current, I arrived at about the same conclusions, but my own dissatisfaction with the method of average power was the uncertainty as to whether you could compute the actual interference by using average power, that is, over a period of time. I would like to know Mr. Carson's idea on that subject. You might have a condition that would produce considerable noise in your receiver, and yet the average power that you got from that might figure out to be fairly low. I was hoping that I could arrive at some means by which you could actually estimate the maximum amount of noise you might get in your receivers.

Mr. Carson: The question, as I understand it, really comes down to how close a measure of interference the mean square current furnishes, I mentioned in the paper that that is a question that can't be answered, exactly, except by experiment.

On the other hand, suppose we have two circuits and they absorb the same amount of power. In the first place since we don't know what the individual form of disturbance is, the only basis which we can define our circuit to is as shown in the paper, on the other hand it is reasonably certain that they will be substantially the same in the long run though at any particular instant and for some particular type of interference, one circuit might be superior to the other and vice-versa. In the case of random disturbances, however, it is impossible to attempt to deal with the particular wave form of the instantaneous interference.

I have also examined the question as to whether, for example, a wave-filter of a large number of sections as compared with the wave-filter of a small number of sections, having practically the same mean current, might reduce the peak, and found that it would depend upon the relative frequency and form of the individual disturbances. In one case you might find that in a long wave-filter the peak would actually be larger than in the short wave-filter, but owing to the absolute random character of the interference, nothing in the way of design can be done on that basis, and as regards the actual effect in reducing intelligibility, that would have to be subject to experiment. That is, if we have two filters which absorb the same mean interference, it is purely a question for experiment as to whether we can distinguish in the two circuits any difference in the effects of the interference on the intelligibility of the signal, with which we are primarily concerned.

J. Slepian (by letter): Every observer of radio progress must be bewildered by the great variety of circuits, most plausibly and ingeniously contrived for eliminating the great bugbear of radio, static interference. Now, Mr. Carson sheds a great light on this subject by showing that practically all those various schemes are inherently alike; that when properly designed they are capable of reducing the static signal ratio just so far, and no farther. This is a very great achievement, and if it could be popularized, and made widely known, it would save a tremendous amount of useless work on the part of not only amateurs, but many professional men. Also, much of this effort might be diverted into paths which are beyond the assumptions of Mr. Carson's analysis, and may therefore have a chance of success. It is with this last in mind, that I would like to ask some questions, with the hope of specifying more precisely the types of circuit or apparatus to which Mr. Carson's analysis applies.

Apparently, the selective circuits of Mr. Carson's paper exercise only a frequency discrimination. That is, they have impedances which depend only on the frequency, when steady state is reached with a sinusoidal input. Circuits made up only of elements with linear characteristics will always have this property.

Now, frequency is not an instantaneous property of a voltage or current. A voltage or current must be watched for a finite length of time before periodicity can be noted, and a frequency assigned to it. Thus determining a frequency or a component frequency, must involve a remembering or integrating process, by which the past is tied up with the present. It is only natural therefore to find that Mr. Carson's analysis is based on the properties of certain integrals. Mr. Carson finds that the integrating processes, which determine the response of the selective circuits, when applied to random disturbances such as static, give residues, which can not be reduced below a certain figure.

Now the question I wish to raise, is whether the introduction of elements with non-linear characteristics does not get outside the limitations which Mr. Carson shows inherently exist for linear elements. The response of such elements cannot be described only in terms of frequency, but such instantaneous quantities as amplitude must be considered. For example, I am told, that considerable improvement in static ratio may be obtained by limiting devices, which prevent response to inputs exceeding a certain amplitude. Such a device has no selectivity whatsoever, being independent of frequency, and the formulas of Mr. Carson, if blindly applied, would indicate that it would be entirely ineffective; yet the device does actually reduce static.

I had an opportunity to mention these points to Mr. Carson, and he told me he thought it possible to take into account non-

linear elements by considering relations between combination frequencies. I would be very pleased if he would outline how this might be done. When dealing with elements which depend on instantaneous quantities such as amplitude, will it not be necessary to consider the whole infinite series of combination frequencies?

J. R. Carson: Dr. Slepian, in his interesting discussion has very properly called attention to the fact that the selective circuits dealt with in my paper are linear networks and function solely by *frequency* discrimination. He has also raised the important question as to what can be done in the way of eliminating static interference by *non-linear* devices which function on the base of *amplitude* discrimination. This is a very complicated question, quite outside the scope of the paper itself.—I am rather afraid that in our conversation referred to by Dr. Slepian, I gave him a too optimistic impression. At any rate, while I have given the problem considerable study, I am not prepared as yet, to make any extensive discussion of it, certainly not an analytical one. There are, however, a few features of the problem which are of interest, and which indicate that a great deal more experimental data regarding the physical and psychological phenomena of audition must be accumulated before anything like a satisfactory handling of the problem is possible.

If the receiving circuit includes a non-linear circuit element, then when interference and signal are simultaneously present, *the signal is modulated by the interference.* If the non-linear characteristic is given analytically, it is possible to calculate the form of the modulated or distorted signal, but it is not, as yet, possible to predict the effect of such modulation in the intelligibility of the signal received. That, however, this effect and the possible benefits to be expected from the non-linear device must be dependent on the intensity and frequency of the interfering disturbances can be inferred from the following ideal limiting case of the non-linear device.

Suppose that we have an ideal device which automatically destroys both signal and interference during the intervals of time when the interference exceeds a specified tolerable limit, but does not affect the signal except during these intervals. It will be clear, I think, that such a device may prove either beneficial or injurious, depending on the duration and frequency of these intervals and the relative intensity of the interference. For occasional bursts of intense interference of short duration its effect ought to be beneficial. For frequent bursts of interference of duration, it would probably completely kill the intelligibility. Consideration of this ideal case does not permit us to make any general inferences; it does, however, I think, show how the whole problem is bound up with the phenomena of audition, and that it depends strictly on the character of the interference. In other words no such general statements can be made regarding non-linear devices for reducing static interference, as can be made in the case of frequency selection.

The Transmission Unit
and Telephone Transmission Reference Systems

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Review of the Subject.—*Consideration is given to the method of determining and expressing the transmission efficiencies of telephone circuits and apparatus, and of the desirable qualifications for a unit in which to express these efficiencies. The "transmission unit" described in this paper has been selected as being much more suitable for this purpose under present conditions than the "mile of standard cable" which has been generally used in the past.*

* * * * *

THE "mile of standard cable" has been used in telephone engineering in this country for over twenty years, and during that time has been adopted in other countries, as the unit for expressing the transmission efficiency of telephone circuits and apparatus. In the present state of the telephone art, this unit has been found, however, to be not entirely suitable and it has recently been replaced in the Bell System by another unit which for the present, at least, has been called simply the "transmission unit." Before considering the reasons for such a fundamental change and the relative merits of the two units, it may be well to review briefly the general method of determining the efficiency of such circuits and the apparatus associated with them.

The function of a telephone circuit is to reproduce at one terminal the speech sounds which are impressed upon it at the other terminal. The input and output of the circuit are in the form of sound and its efficiency as a transmission system may be expressed as the ratio of the sound power output to the sound power input. For commercial circuits, this ratio may be of the order of 0.01 to 0.001.

In the operation of the system, the sound power input is converted by the transmitter into electrical power, which is transmitted over the line to the receiver and there reconverted into sound power. The effect of inserting a section of line or piece of apparatus or of making any change in the circuit can be determined in terms of the variation which it produces in the ratio of the sound power output to the sound power input, or, if this latter is kept constant, in terms of the ratio of sound power output after the change to that obtained before the change was made. It should be noted particularly that the change in the output power of the system is the real measure of the effect of any part of the circuit on the efficiency of the system and that the ratio of the power leaving any part to that entering it is not necessarily the measure of this effect. For example, a pure reactance placed in series between the transmitter and the line, may change the power delivered to the line by the transmitter and hence the output of the receiver—the magnitude and direction of the change being determined by the impedance

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relations at the point of insertion. The ratio of the power leaving the reactance to that entering it is, of course, unity, as no power is dissipated in a pure reactance. In other words, the transmission efficiency of any part of a circuit cannot be considered solely from the standpoint of the ratio of output to input power for that part, or the power dissipated in that part, but must be defined in terms of its effect on the ratio of output to input power for the whole system.

By determining the effect of separately inserting the many pieces of apparatus that may form parts of typical telephone circuits, an index can be established for each of these parts of its effect on the efficiency of the circuit for the conditions of which the circuit tested is typical. Similarly, the power dissipated in unit lengths of the various types of line can be determined by noting the change in power output of the receiver caused by increasing any line by a unit length. Such indices of the transmission efficiencies of the various parts of a circuit obviously have many applications in designing and engineering telephone circuits. These indices could be taken as the ratios expressing the change in the output power of the system. This, however, has certain disadvantages. For example, the

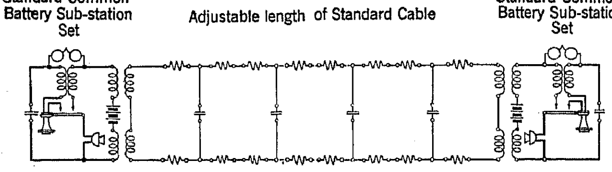

FIG. 1

combined effect of a number of parts would then be expressed as a product of a number of ratios. Likewise, for the case of a number of parts n of the same type in series, such as a line n miles in length, the effect would be expressed as the ratio for one part or one mile of the line, raised to the nth power. In many cases, these ratios and the powers to which they would need to be raised would be such as to make their handling cumbersome. If, however, these indices are expressed in terms of a logarithmic function of a ratio selected as a unit, the sum of any number of such indices for the parts of a circuit is the corresponding index for the power ratio giving the effect of the combination of these parts.

The "mile of standard cable" is such a logarithmic function of a power ratio. The new unit also meets this important requirement.

DEFINITION OF THE TRANSMISSION UNIT

The "transmission unit" (abbreviated $T\,U$) has been chosen so that two amounts of power differ by one transmission unit when they are in the ratio of $10^{0.1}$ and any two amounts of power differ by N units when they are in the ratio of $10^{N(0.1)}$. The number of transmission units corresponding to the ratio of any two powers P_1 and P_2, is then the common logarithm (logarithm to the base 10) of the ratio P_1/P_2, divided by 0.1. This may be written $N = 10 \log_{10} P_1/P_2$. Since N is a logarithmic function of the power ratio, any two numbers of units, N_1 and N_2, corresponding respectively to two ratios, P_a/P_b and P_c/P_d, may be added and the result $N_1 + N_2$, will correspond to the product of the ratios, $P_a/P_b \times P_c/P_d$.

From the above it is seen that the measure in transmission units of the ratio of two amounts of power P_1 and P_2 is N, where

$$N = \frac{\log \frac{P_1}{P_2}}{\log 10^{0.1}}$$

In other words, the transmission unit is a logarithmic measure of power ratio and is numerically equal to $\log 10^{0.1}$.

The reasons for the selection of this unit and the method of applying it, can probably be best brought out by a consideration of the practise which has been followed in determining and expressing the efficiencies of telephone circuits and apparatus in terms of "miles of standard cable."

STANDARD REFERENCE CIRCUIT

Fig. 1 shows what has been designated the "standard reference circuit." It consists of two common battery telephone sets of the type standard in the Bell System at the time this circuit was adopted, connected through repeating coils or transformers to a variable length of "standard cable." This cable is an artificial line having a resistance of 88 ohms and a capacity of 0.054 microfarad per loop mile which is representative of the type of telephone cable then generally used in this country.

For a given loudness of speech sounds entering the transmitter at one end of the circuit, the loudness of the reproduced sounds given out by the receiver at the other end can be varied by changing the amount of standard cable in the circuit. Also, the amount of cable in the circuit can be used to express the ratio of the power of the reproduced sounds to that of the impressed sounds. Due to the dissipation of electrical power in the cable, this ratio and consequently the loudness of the reproduced sounds become less as the amount of cable is increased and greater as the length of cable is decreased.

This circuit then became the measuring or reference system for engineering the telephone plant and the "mile of standard cable" became the unit in which the measurements were expressed. This circuit was used to set the service standards in designing and laying out the telephone plant. Thus, the reproduction obtained over this circuit with a length of cable of about twenty miles was found suitable and practicable for local exchange, that is, intracity service, and that corresponding to about thirty miles for toll or intercity service.

Any telephone circuit was rated by its comparison with the standard circuit. This comparison was on the basis of a speaker talking alternately over the circuit to be measured and the standard circuit and a listener switching similarly at the receiving ends, the amount of cable in the standard circuit being adjusted until the listener judged the volume of the sounds reproduced by the two systems to be equal. The number of miles of cable in the standard circuit was then used as the "transmission equivalent" of the circuit under test. The effect of any change in the circuit under test on the efficiency of that circuit could then be measured by determining the variation in the amount of standard cable required to make the sounds reproduced by the two systems again equal and the number of miles of standard cable required to compensate for this change was used as the index of this effect. In this way the relative efficiencies of two transmitters or receivers could be determined. Likewise, the power dissipation per unit length or the attenuation, of the trunk in the circuit under test could be equated to miles of standard cable. Since in each case, the standard cable is used to adjust the volume of the reproduced sound, "the mile of standard cable" corresponds to the ratio of two amounts of sound power, or as this change in sound power is produced by changing the power delivered to the telephone receiver, to a ratio of two amounts of electrical power.

If the addition of a mile of standard cable to a long trunk of the standard circuit causes the power reaching the end of the trunk to decrease by a ratio r, then the insertion of two miles will decrease the received power by a ratio of r^2 of that obtained before the two miles were inserted. A number of miles of cable, n, inserted will reduce the received power to a ratio r^n. Thus the power ratio corresponding to any given number of miles of cable is an exponential function of the ratio corresponding to one mile, the exponent being the length in miles. The length in miles is, therefore, a logarithmic function of the power ratio.

In an infinite length of uniform line having resistance, inductance, capacity and conductance of R, L, C and G per unit length, the attenuation a per unit length, of a current of frequency f flowing along the line can be shown to be equal to the real part of the expression

$$a + j\,b = \sqrt{(R + j\,2\,\pi\,f\,L)\;(G + j\,2\,\pi\,f\,C)}$$

For the standard cable line, since L and G are zero

$$a = \sqrt{\pi\,f\,R\,C}$$

and since $R = 88$ ohms and $C = 0.054$ microfarad per mile the current attenuation per mile of standard cable is

$$a = 0.00386\,\sqrt{f}$$

If I_1' and I_2' are the currents, respectively, at the beginning and end of a mile of line, then

$$I_1'/I_2' = e^a \text{ or } a = \log_e I_1'/I_2'$$

Similarly if I_1 and I_2 are the currents, at points 1 and 2, respectively, at the beginning and end of a section of l miles

$$I_1/I_2 = e^{La} \text{ and } l\,a = \log_e I_1/I_2$$

For this case, the effect of inserting the section of l miles into the line on the current at point 2, or at any point beyond 2, is that the currents at the point before and after the insertion are in the same ratio as I_1/I_2. Furthermore, since the impedance of the line looking toward the receiving end is the same at points 1 and 2 (and at any other points), then the ratio of the powers at the two points is equal to the square of the current ratio.

Thus the power attenuation is represented by

$$P_1/P_2 = (I_1/I_2)^2 = e^{2la}$$

Similarly for a line, terminated in a fixed impedance which may be different from the characteristic impedance of the line, the ratio of the powers received before and after a change in the length of the line is equal to the square of the ratio of the corresponding currents. On the basis of this relation, and because it is in general more convenient to measure or compute currents than powers, the current ratio has often been used in determining the equivalent of any piece of apparatus or line in terms of standard cable. It should be noted, however, that such a current ratio can be properly used as an index of the transmission efficiency of a part of a circuit only when it is equal to the square root of the ratio of the corresponding powers. Also, of course, the voltage ratio can be similarly used when it meets the same requirement.

Limitations in Use of Standard Cable Unit

As shown above, the attenuation, either of current or power, corresponding to the mile of standard cable is directly proportional to the square root of the frequency of the current under consideration. This means that the standard cable mile corresponds not only to a certain volume change in the reproduced speech sounds, but also to a distortion change. For comparisons between the standard cable circuit and commercial circuits with talking tests and as long as most of the commercial circuits had distortion comparable to that of standard cable, this two-fold effect of standard cable was desirable. At present, however, many types of circuits are being used which have much less distortion than standard cable. Also, the use of voice testing has been largely given up in the plant and it is now the general practise to determine the efficiency of circuits and apparatus on the basis of measurements and computations for single-frequency currents, a correlation having been established between these latter results and those of voice tests. These factors have made it desirable to have a unit for expressing transmission efficiencies which is distortionless, that is, not a function of frequency.

Qualifications of a New Unit

The consideration of a new unit for measuring transmission efficiency brought out the following desirable qualifications:

1. *Logarithmic in Character*. Some of the reasons for this have already been discussed. In addition, the

application of such a unit in measurements of sound make a logarithmic unit desirable, since the sensation of loudness in the ear is a logarithmic function of the energy of the sound.

2. *Distortionless.* The advantages of a unit which is independent of frequency have been referred to above. In expressing the efficiency of the transmission of the high frequencies involved in carrier and radio circuits, such a unit is particularly desirable.

3. *Based on Power Ratio.* This is desirable because the power ratio is the real measure of transmission efficiency. As pointed out above, the current ratio can be used only when it is equal to the square root of the power ratio. Having the unit based on a power ratio does not, of course, require that measurements or computations be made on a power basis.

In considering the conversions between sound and electrical energy, it is obviously advantageous to have a unit based directly on a power ratio.

4. *Based on Some Simple Relation.* This is desirable in connection with the matter of getting a unit which may be widely used and may find applications in several fields.

5. *Approximately Equal in Effect on Volume to a "Mile of Standard Cable."* One reason for this is the practical one of avoiding material changes in the conceptions which have been built up regarding the magnitude of such things as transmission service standards. Also, the sound power changes which can be detected by the ear are of the order of that corresponding to a mile of standard cable. In measuring telephone lines and apparatus with single-frequency currents, it has been found that an accuracy of about one-tenth of a mile can be obtained readily and is sufficient practically.

6. *Convenient for Computations.* This refers to the matter of changing from computed or measured current or power ratios to transmission units or vice versa.

Properties of the Transmission Unit

A consideration of the above qualifications and of the various units suggested, led to the adoption of the power ratio of $10^{0.1}$ as the most suitable ratio on which to base the unit of transmission efficiency. The transmission unit is logarithmic, distortionless, is based on a power ratio and its relation to that ratio is a simple one. Its effect on the transmission of telephonic power corresponding to speech sounds is about 6 per cent less than that of one mile of standard cable. Regarding its use in computations, it has the advantage that the number of units corresponding to any power ratio, or current ratio, can be determined from a table of common logarithms.

For a power ratio of 2, the logarithm is 0.301 and the corresponding number of units is, therefore, this logarithm multiplied by 10, which is 3.01 *T U.* For a power ratio of 0.5, the logarithm is $9.699-10 = -0.301$ and the number of units is -3.01 *T U.* A power ratio of 2 represents a gain of 3.01 units, and a power ratio of 0.5 corresponds to a loss of 3.01 units. If the above ratios were for current, the logarithms would be multiplied by 20. Thus a current ratio of 2 corresponds to a gain of 6.02 units and a current ratio of 0.5 corresponds to a loss of 6.02 units.

It will be noted that the *T U* is based on the same ratio $10^{0.1}$ as the series of preferred numbers which has been used in some European countries and has been proposed here as the basis for size standardization in manufactured articles.[1] In common with this series, the *T U* has the advantage that many of the whole numbers of units correspond approximately to easily remembered ratios as shown in the following table.

APPROXIMATE POWER RATIO

Transmission Units	For Losses Fractional	For Losses Decimal	For Gains Decimal
1	4/5	0.8	1.25
2	2/3	0.63	1.6
3	1/2	0.5	2.
4	2/5	0.4	2.5
5	1/3	0.32	3.2
6	1/4	0.25	4.
7	1/5	0.2	5.
8	1/6	0.16	6.
9	1/8	0.125	8.
10	1/10	0.1	10.
20	1/100	0.01	100.
30	1/1000	0.001	1000.

It will be seen that the ratio for a gain of a given number of *T U,* is the reciprocal of the ratio for a loss of the same number of units. Also for an increase of 3 in the number of units, the loss ratio is approximately halved and the gain ratio doubled. If the approximate loss ratios corresponding to 1, 2 and 3 units are remembered, the others can be easily obtained.

From this consideration of the properties of the transmission unit, it is evident that there is much to commend its use in telephone transmission work. Furthermore, since its advantages are not peculiar to this work, such a unit may find applications in other fields. It is now being used in some of the work on sound.

New Telephone Transmission Reference System

With the standardization of the distortionless unit of transmission it is desirable also to adopt for a transmission reference system a telephone circuit which will be distortionless from sound input to the transmitter to sound output from the receiver. This system will consist of three elements, a transmitter, a line and a receiver. Each will be designed to be practically distortionless and the operation of each will be capable of being defined in definite physical units so that it can be reproduced from these physical values. Thus, the transmitter element will be specified in terms of the ratio, over the frequency range, of the electrical power output to the sound power input, this ratio being expressed in transmission units. The receiver element

1. Size Standardization by Preferred Numbers, C. F. Hirshfeld and C. H. Berry, Mechanical Engineering. December, 1922.

will be specified likewise in terms of the ratio of sound power output to electrical power input. The output impedance of the transmitter and the input impedance of the receiver elements will be 600 ohms resistance. The line will be distortionless with adjustments calibrated in transmission units and will have a characteristic impedance of 600 ohms resistance.

Such a reference system is now being constructed. The transmitter element consists of a condenser type transmitter and multi-stage vacuum tube amplifier. The receiver element consists of an amplifier and specially damped receiver. Each element is adjusted to give only negligible distortion over the frequency range.

It is proposed when this system is completed and adjusted that it will be adopted as the Transmission Reference System for telephone transmission work. Other secondary reference systems, employing commercial-type apparatus will be calibrated in terms of the primary system and used for field or laboratory tests when such commercial type systems are needed.

Discussion

A. M. Wilson: Of course, the application of preferred numbers offers a very reasonable solution for a number of problems that are arising more and more with different sizes of equipment and materials; but the use of the distortionless circuit should be of service in connection with problems in inductive interference.

I should like to know if there is any possibility of using this type of circuit in investigations of inductive interference, instead of, or in conjunction with, the telephone-interference-factor meter.

The type of meter is connected to a power circuit, for the purpose of indicating what may be going on in a neighboring telephone circuit. It would seem more logical, and very desirable, to have an instrument connected directly in the telephone circuit to indicate the effects produced in the telephone circuit.

W. H. Martin: Referring to Mr. Wilson's question, a distortionless circuit of the type referred to has been used in a fundamental investigation in the laboratory of the effects of loudness, distortion and the presence of noise on the intelligibility of reproduced speech sounds. For this purpose, the circuit was arranged so that all three of these factors could be varied, either separately or in combination. In this way, the circuit is of service in connection with the consideration of the noise produced in telephone circuits by induction from electrical power systems.

The telephone-interference-factor meter was designed to provide a ready and portable means for rating the wave shapes of power systems and apparatus from the standpoint of producing interference to telephone circuits. As described in the paper by H. S. Osborne in the 1919 TRANSACTIONS of the A. I. E. E., it is based on the interfering effects of extraneous single-frequency currents in a commercial telephone receiver on the intelligibility of speech sounds given out by the receiver and also on the general relation that the current induced in telephone circuits by a given voltage or current in a power circuit is approximately proportional to the frequency. It, therefore, takes into account both the inductive effect between the power and telephone circuits and the interfering effect of the currents in the telephone circuit. The distortionless circuit which has been described in the paper on the transmission unit does not provide a means for measuring interference which can be used instead of, or in conjunction with, the telephone-interference-factor meter.

Sensitive Radio-Frequency Relay

BY GEORGE LEWIS
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Review of the Subject.—*The present paper has been prepared with a view of describing the theoretical properties, mechanical construction, and possible application of a new electron relay, invented and developed by Samuel Ruben, a physicist of New York. This relay has been developed to control currents as great as 5 amperes by means of extremely small operating currents of any frequency, and its use is suggested, therefore, in radio and carrier-current systems, and where similar requirements are to be met.*

* * * * *

PRINCIPLES OF OPERATION

THE operation of this relay is primarily based upon the employment of the kinetic energy of the thermionic discharges brought about by a suitable emission element, such as a filament, when directed upon a sensitively responsive anode. The impact of the electron stream upon the anode results in the expenditure of the kinetic energy of the bombarding stream, which is translated as;

1. A slight and negligible mechanical effect,
2. A negligible radiation of low-wave length and
3. A temperature rise of the anode.

Various tubes have been constructed by Mr. Ruben, in which he has employed different anodes for the utilization of the thermionic bombardment principle. As the temperature rise of the anode is the major effect in this type of tube, amounting to practically the entire translation of the kinetic energy of the electron stream, the anode adopted as the most serviceable is the thermo-sensitive type. (Figs. 1 and 2).

In the type of tube described in this paper, the translated energy is employed as a means for operating the contacts of a relay located within the tube.

The rapidity of the action of this device is dependent on the rate of heating and the rate of cooling of the anode, which, in turn, are dependent upon factors as follows:

Rate of heating—dependent on:

(a) Power of bombarding electrons—this should be as great as possible.

(b) Heat capacity of anode—this should be as small as possible.

Rate of cooling—dependent on:

(a) Ability of anode to radiate power.

(b) Heat capacity of anode—this should be as small as possible.

In the device as now developed, the anode is composed of a blackened strip of nichrome which is initially maintained at a high temperature, in the neighborhood of 600 deg. cent. The strip form gives a large radiating surface with a small mass and therefore small heat capacity. The high initial temperature gives the anode increased ability to radiate power, since this depends on the fourth power of the temperature.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chiacgo, Ill., June 23-27, 1924.

This relay comprises four main elements:

1. A source of electron emission—a cathode.
2. A means for controlling the electron emission flow,
3. A sensitively responsive anode, and
4. A movable contact arm and a stationary contact for the control of a high-density external circuit.

The cathode element employed in this relay is a platinum-iridium strip having a newly-developed oxide coating which gives the cathode a high thermionic

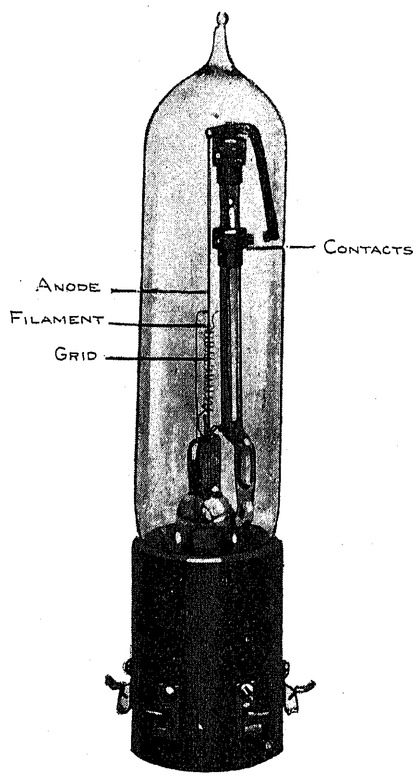

FIG. 1

efficiency and a long life. The strip is suspended parallel to the sensitively responsive anode, which is composed of nichrome, attached at one end to the movable contact arm of the make-and-break device, controlled by the movement of the anode responding to the impact of the electron stream upon it. The electron discharge from the filament is electrostatically controlled by a grid element interposed between the filament and anode, so that in operation the grid

controls the anode movement by virtue of its control of the electron stream.

The anode is maintained by a local circuit at a temperature at which the movable contact is normally at a point very minutely spaced from the stationary contact. Such close contact spacing, without any tendency to arc or spark, even when the external circuit carries a current of considerable potential, is made possible by the degree of evacuation to which the tube is subjected before the sealing-off process is completed.

As the necessary slight movement of the contact arm ot the make-and-break device is controlled by the thermo-expansive anode of the described characteris-

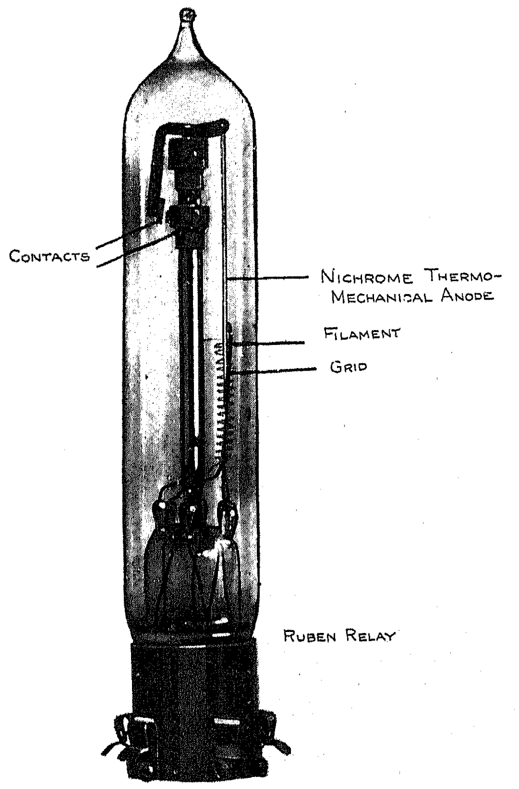

Fig. 2

tics, the operating speed of the device is high compared with usual thermal devices.

To avoid any deformation of the elements of this device from the repeated application of high temperatures, especially in the process of evacuation, great care has been taken in their composition and design. The result is a notable stability of adjustment under operating conditions.

Nichrome was selected for the anode as it was found to possess the following desirable electrical and physical characteristics:

a. Operating constancy over long and interrupted periods of use,

b. High and constant values of electrical resistance at various operating temperatures,

c. Suitable coefficient of expansion and

d. Adequate mechanical strength at operating temperatures.

The contacts of the circuit control device are composed of tungsten, one being affixed to a lever supported for the proper ratio of its arms. The lever and stationary contact support are composed of nickel-manganese, an alloy found suitable for vacuous devices. One end of the lever carries the tungsten contact, and the other is attached to the thermo-expansive anode. As the anode expands and contracts in response to the action of the controlling electron stream, the movable

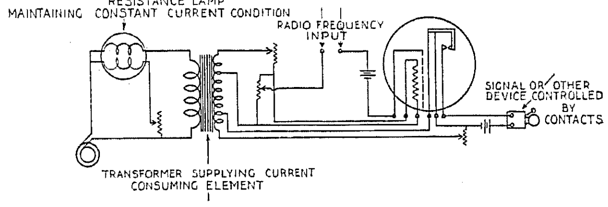

Fig. 3

contact moves towards and away from the stationary contact, thus serving to close and open the external circuit.

The normal position of the lever can be made practically independent of the position of the apparatus itself, which can be mounted in a horizontal or vertical plane.

The actual energy necessary to control the external circuit of high density is of very small magnitude, being only that necessary to electrostatically charge the grid element sufficiently to modulate the impacting electron stream, as in the present thermionic devices. The

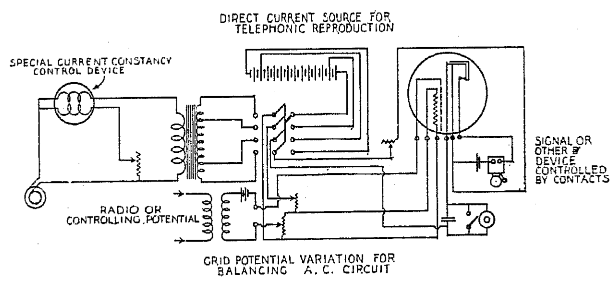

Fig. 4

time required for the actuation of the lever is controlled by the heating current normally discharged through the anode.

Connection of Relay in Circuits

Fig. 3 shows a typical connection of such a relay into a circuit. In the circuit as shown the relay is operated by incoming radio signals, and controls a signaling device indicated as a call bell. The anode is so connected that its energizing circuit is partially short-circuited when the contact of the relay circuit is

closed, thus causing the anode to cool by radiation, and causing the circuit to open. By this rapid heating and cooling of the anode element the relay contact opens and closes the external circuit at a rate of about 20 times per second.

The relay can be employed in this manner as a call device for radio or carrier-wave circuits for either telegraphy or telephony. When used in this manner, all of its circuits can be supplied with energy from a single transformer as indicated.

To keep the adjustment constant and independent of line variations, a ballast lamp specially developed for this relay has been supplied. The lamp comprises two resistance elements in a hydrogen filled tube, one element being iron, which controls a current flowing to the primary of the transformer, and operated at about 500 deg. cent. at which temperature iron has an exceedingly large resistance temperature coefficient. The other element is nichrome wire which is directly connected across the line; its temperature is adjusted by means of an external resistance to bring the controlling iron resistance element to a proper temperature. When adjusted, large variations in the line potential are compensated and the relay adjustment is stable.

Fig. 4 shows the same circuit with a change-over switch to allow the use of telephone receivers in the plate circuit, by which a call-signal relay and an audio-frequency relay in one tube are obtained.

This relay can be connected in a circuit as a reflex, a regeneration or an amplification tube, or in any connection in which the usual three-element tube is applicable, with the added advantage of the local circuit control of current of high density or high potential.

In conclusion, it may be stated that the Ruben relay has been designed to obtain a relay which could be controlled by minute energies in the form of currents of any frequency, either radio or otherwise, could be operated at adequate speeds for certain telegraph applications, and could control circuits of considerable current and potential. Its application is suggested for any conditions where these requirements are to be fulfilled, such as radio and carrier calling and recording systems, as a telegraph recorder and repeater, either line or cable, and for similar purposes.

Discussion

J. Slepian: The device described by Mr. Lewis has several distinct elementary parts performing rather distinct functions; there are the filament and grid, giving the usual detector action, there is a thermally responsive element and there is a contact which opens and closes.

A pertinent question seems to me to be whether anything gained beyond compactness in this rather ingenious combination of all these elements, and whether it is necessary to sacrifice the best performance of some of the individual elements for the sake of getting this compactness.

Now it does seem as if the detector action of the tube has to be modified. A standard plate surrounding the filament in the usual way cannot be used because that would give too large a volume for the thermally responsive part. Instead a very small plate must be used and thus a detector element of higher impedance than the standard tube results. There appears then to be some disadvantage here. On the other hand, if one tries to separate the two functions, and use a standard thermionic tube to deliver electrical power into a thermally responsive element, great difficulty will be encountered in constructing a thermally responsive element with sufficiently high resistance to work with an ordinary tube.

By having the michrome ribbon heated directly by electron bombardment, a thermal element results which has a high impedance. There is, therefore, considerable advantage in the combination.

I would like to call attention to some points with respect to the operation of contacts in high vacuum. At first sight it would seem that a very great advantage is to be obtained by working contacts in a vacuum. In a high vacuum an arc is impossible, and since the principal trouble with contacts which handle any considerable power is the destructive arcing by putting the contacts into a vacuum, one would expect to eliminate the arc and to be able to handle tremendous power for an indefinite time.

This argument seemed so plausible to me that a number of years ago I actually carried on a considerable number of tests on contacts opened and closed in vacuum. These contacts were opened and closed by magnetic means, not by the method shown here. The results I obtained were very disappointing. After working on it quite a bit I arrived at the following understanding as to why better results were not to be expected.

As a contact is opened, the area of contact reduces very rapidly from a finite area down to a very small value. Just at the last moment of break, the resistance of the contact goes up enormously, and the voltage of the circuit begins to concentrate on it, so that whatever the voltage of the circuit is it will all act on this last contact.

It is not very difficult to calculate the temperature rise at a point of contact between two materials when a voltage is applied to it. As I remember the figures, the formula comes out $\frac{E^2}{33\,K\,\rho}$, where E is the voltage, K the heat conductivity and ρ the electrical resistivity. Taking the heat conductivity of the metal as unity, and 10^{-6} as the resistivity, for 1 volt, we get 3000 degrees. That is, the last point of contact, for one volt in a circuit, will be necessarily raised to 3000 degrees. Hence, there must be melting of the very last point of contact, and for the heavier currents that I dealt with in my own experiments I found that the contact soon roughed and became useless after a few operations, since it was impracticable to apply heavy mechanical pressure in the vacuum. Furthermore, the volatilization of metal spoiled the vacuum and permitted arcs to flow.

The Transient Visualizer

BY H. M. TURNER
Member, A. I. E. E.
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THE "transient visualizer" which is described in this paper was developed to make possible a more satisfactory study of transients by means of the oscillograph. It is, of course, well known that the oscillograph has been an effective instrument for many years for studying a-c. phenomena where the successive cycles are identical and are repeated synchronously, but in the case of transient phenomena which are not recurrent, there have been limitations to its usefulness.

It has not been possible to make visual observations of transients, because the trace produced by a single sweep of the light spot across the viewing screen is insufficient to make an impression on the eye. As a consequence, only photographic records were obtainable and satisfactory results were difficult to obtain for two reasons. In the first place, the amplitude of the curves could not be determined beforehand, and in the second place it was difficult to place the curves properly on the film.

In order to overcome some of these limitations the transient visualizer was developed. The function of the device is to repeat the transient phenomena periodically so that they may be studied in the same way as the a-c. phenomena have always been studied. As a result of periodic repetition, stationary curves of the transients may be obtained on the oscillograph screen. Furthermore, as the amplitude and position of these curves may be controlled, coordinated photographic records may be secured.

The transient visualizer consists essentially of a synchronous switch in the form of a contact-making drum or commutator, driven through helical gears by a synchronous motor. By means of interchangeable gears this drum may be driven at various desired speeds. The speed of the drum bears a definite relation to the speed of the rotating mirror in the oscillograph. As this commutator opens or closes the test circuit at definite periods, the transient is repeated. The curve of the transient is similarly repeated in the oscillograph and, due to the fixed relative speeds of drum and mirror, the light spot follows the same path at each repetition and the result is a visible stationary curve.

While observing the curve, adjustments may be made to regulate the amplitude of the curve as well as its exact position on the screen. Thus, the effect of changed conditions may be studied and, furthermore, accurately located photographic records may be taken. If photographic records of several different curves are to be taken on the same film, their positions relative to each other may be adjusted. This is possible whether (1) the curves are to be taken at the same time by different elements of the oscillograph or (2) they are to be taken at different times on the same film. Several different circuits may be connected so that their respective phenomena may be studied (or recorded) in succession.

The main part of the visualizer drum consists of six circular sections mounted on a common shaft, as shown in Fig. 1. This shaft is driven through gears by a self-starting synchronous motor. The circumference of five of the drum sections is divided into two segments, one a conductor and the other an insulator. The conducting segments are of various lengths, for use with transients of different duration, and their relative angular spacing around the circumference of the drum is such as to give maximum flexibility in its use. The sixth section is one continuous conductor and is electrically connected to the five others.

Brushes mounted on a common arm make contact

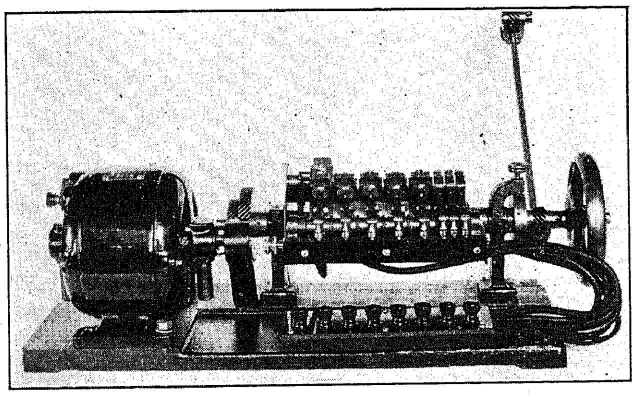

FIG. 1

with these sections. This brush arm is rotatable through a large angle and may be locked in any desired position by a setscrew. Connections are made to the brushes through flexible leads terminating in binding posts to which the circuits under test and the oscillograph are connected. At the extreme left is an additional brush carried by a disk which may be rotated independently of the main arm, or locked so as to turn with it. This brush rests on the same section as one of the other brushes which is more clearly visible in the illustration. It is used for making special adjustments which will be mentioned later. Each brush is provided with a convenient means for lifting when idle. The small geared shaft at the right is for the purpose of driving the photographic film drum of the oscillograph.

When the transient visualizer is used for visual study of curves, either with the projection oscillograph or the usual laboratory type, it may be mounted at any convenient place. Fig. 2 shows it being used with

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

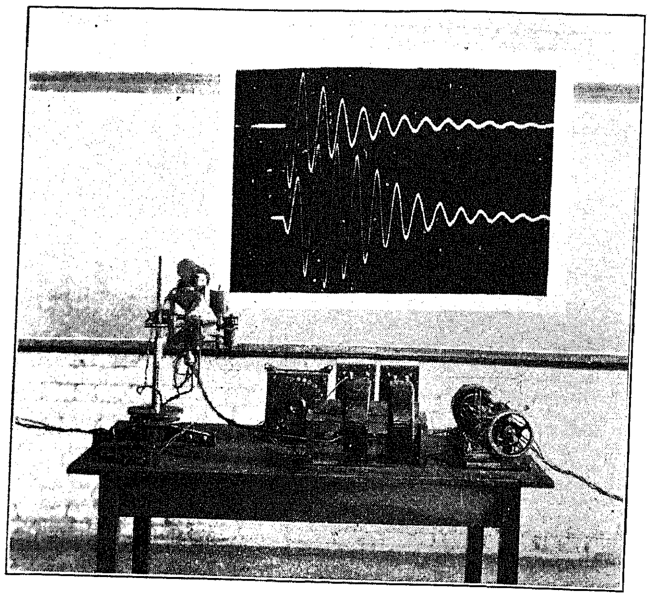

Fig. 2

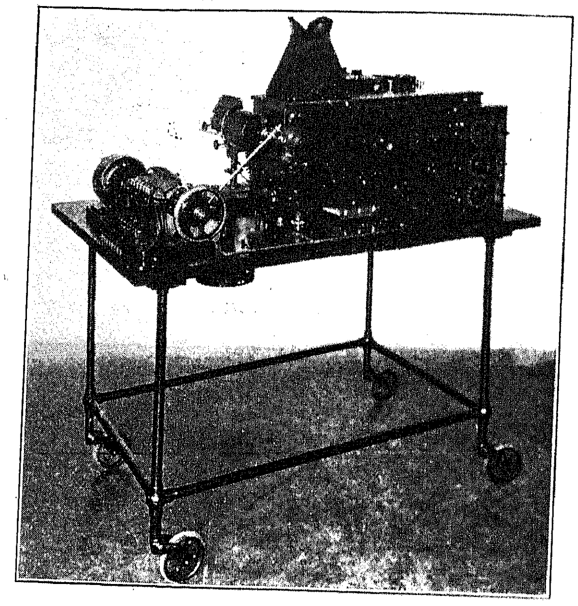

Fig. 3

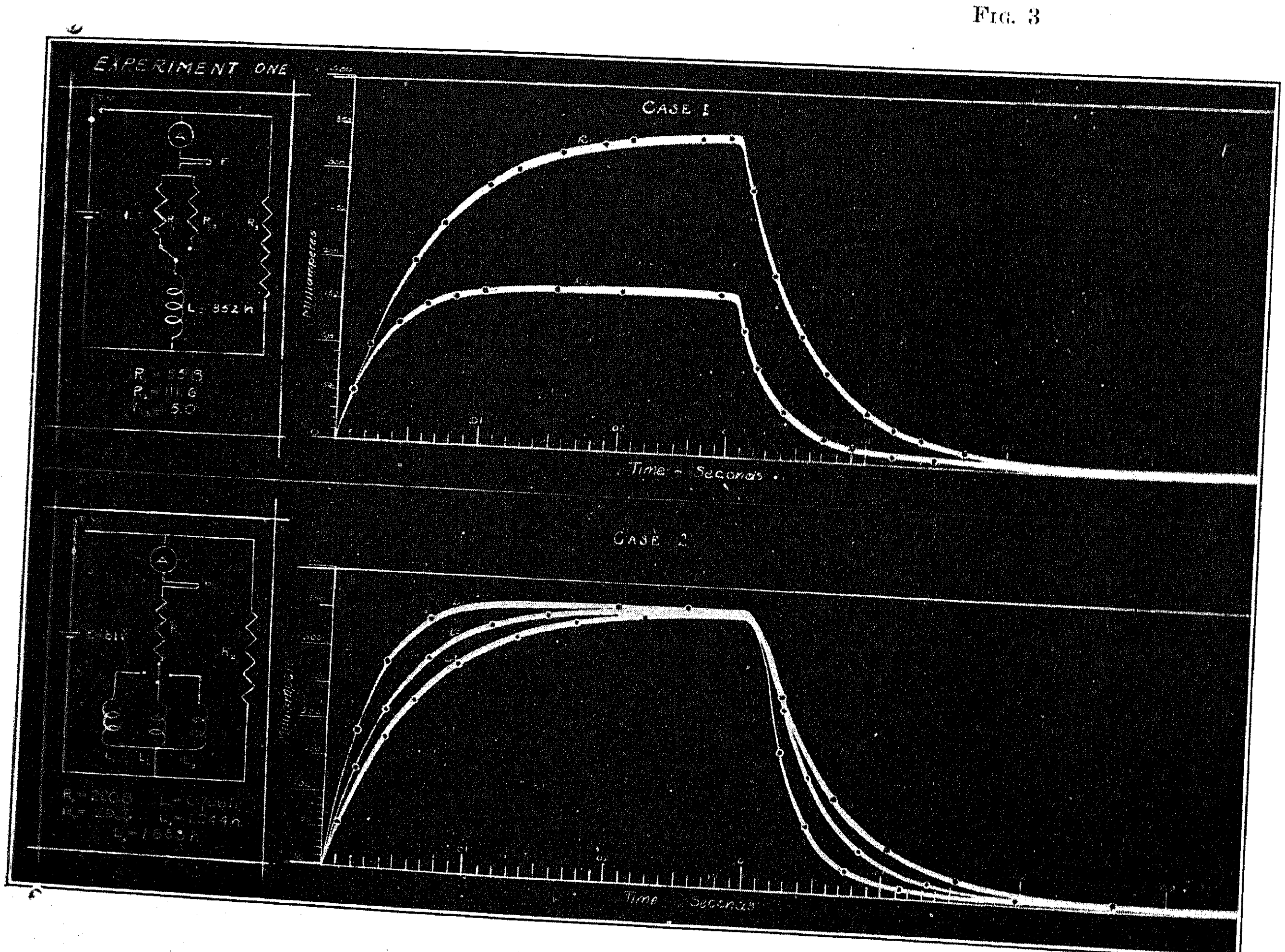

Fig. 4

the Crane projection oscillograph for studying coupled circuits. When the spacing between the coils is varied, the curves give immediate evidence of the change in coupling. Or with the secondary open, the effect of a change in the primary resistance, inductance or capacity upon the amplitude, frequency and decrement may be instantly observed. A study of this kind has a fascination for the observer that stimulates interest in the mathematical analysis. With a few known values of inductance and capacity the relation $f = 1/2\,\pi\,\sqrt{L\,C}$ may be experimentally verified by counting the number of cycles on a convenient length

of the screen and plotting the results on logarithmic paper.

In the case of d-c. transients the position of the curves on the screen may be changed either by restart-

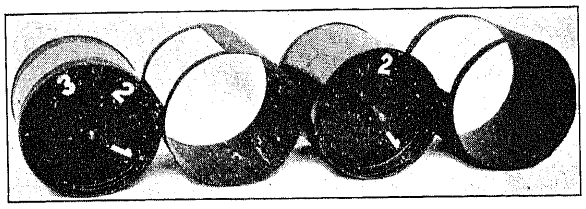

Fig. 5

simple matter to start the curve at any desired point of the film. First, with the transient visualizer motor running, rotate the brush arm until the curve appears on the screen so as to make sure of the correct amplitude; second, stop the transient visualizer motor and rotate the commutator by means of the handwheel until the brushes are at the point of closing the circuit, then loosen the setscrew that holds the gear to the film drum shaft and then rotate the drum to the desired angular position.

Since an oscillogram can be made to start at a definite point on the film it is possible to make multiple exposures. In Fig. 4, the upper film was exposed twice

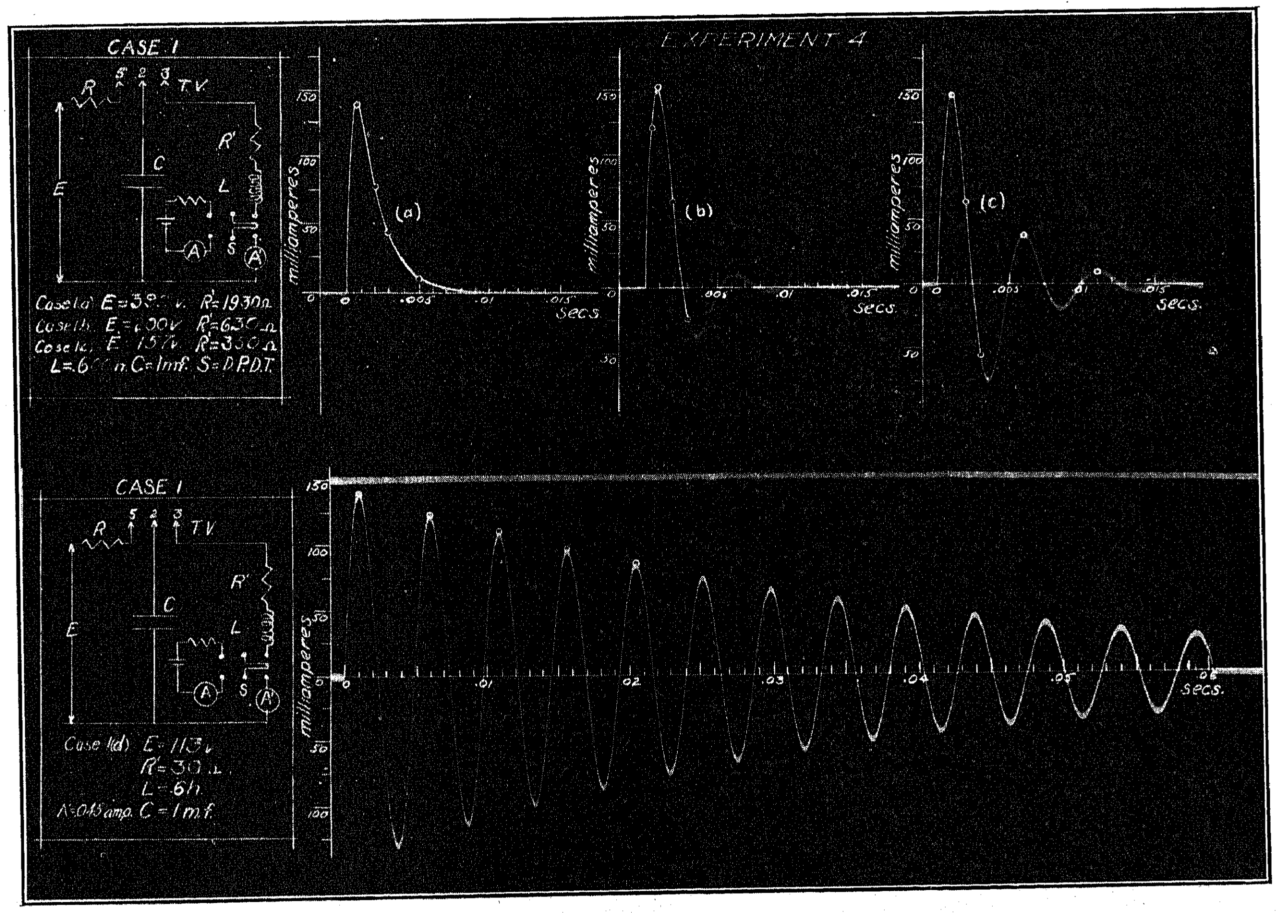

Fig. 6

ing the transient visualizer motor until the correct position is obtained, or by rotating the brush arm.

For photographic purposes, the transient visualizer should be mounted in front of the oscillograph cabinet at a distance of about 8 or 9 in. and connected directly to the film drum by means of two sets of helical gears, as shown in Fig. 3. Since the mechanical relation between these parts can be adjusted, it is possible to start the oscillogram at any predetermined point of the film and by the selection of suitable gear ratios to control the spread of the curves, thus saving time and films and improving the appearance of the oscillograms. When investigating d-c. transients, it is a

and shows the effect of a change of resistance upon the building-up of the current in a circuit of constant inductance. The lower film was exposed three times and shows the effect of inductance upon the building-up of current in a circuit of constant resistance. The same oscillograph element and field strength were used for the curves of a given set, therefore the comparison is direct and shows most advantageously the effect of a change in circuit conditions. The small circles indicate calculated values.

Frequently, it is desirable to take a number of exposures on a film where each set of curves occupies but a portion of the length of the film. This is an

advantage when the phenomena being studied normally occupy but a small part of the film and where it is desirable to bring several curves together for the purpose of comparison. Two points are involved; first, the ability to select the point on the film at which each exposure is to be started; second, to shield all of the film except the portion on which the exposure is to be made. Special shields have been devised for this purpose which are shown in Fig. 5. If reasonable care is used in slipping these shields over the drum, there is little likelihood of scratching the film. In shifting the shield to expose different portions of the film, it is simply rotated on the end rims of the drum, instead of removing, to further reduce the possibility of scratching. The value of the transient visualizer in work involving multiple exposure is clearly illustrated by Fig. 6 which shows the effect of resistance upon the discharge of a condenser through an inductance.

In the study of a-c. phenomena (transient and periodic) an important feature is the constant cycle length. By this is meant that the length of a cycle on the film is independent of frequency changes of the power source, since any change in angular velocity of the film is exactly balanced by the change in the period of the cycle. This constant cycle length is extremely useful in comparing wave forms. It is not even necessary that the curves to be compared be taken the same day or on the same machine. Fig. 7, film 1, which was taken recently, shows the impressed voltage (110) and the exciting current for: (a) 144 turns giving a low value of saturation for the first exposure and (b) 84 turns giving a high saturation for the second exposure. Film 2, taken two years ago, shows the impressed electromotive force, the secondary induced e. m. f. and the e. m. f. induced in the tertiary winding around the magnetic shunt. The shunt path has two air gaps, and therefore its reluctance will be practically constant and the induced e. m. f. in tertiary winding will vary approximately as the rate of change of the exciting current. Since the cycle length is the same for the two films, they may be lined up in the proper phase, and the relation between the current and the induced e. m. f. made clear.

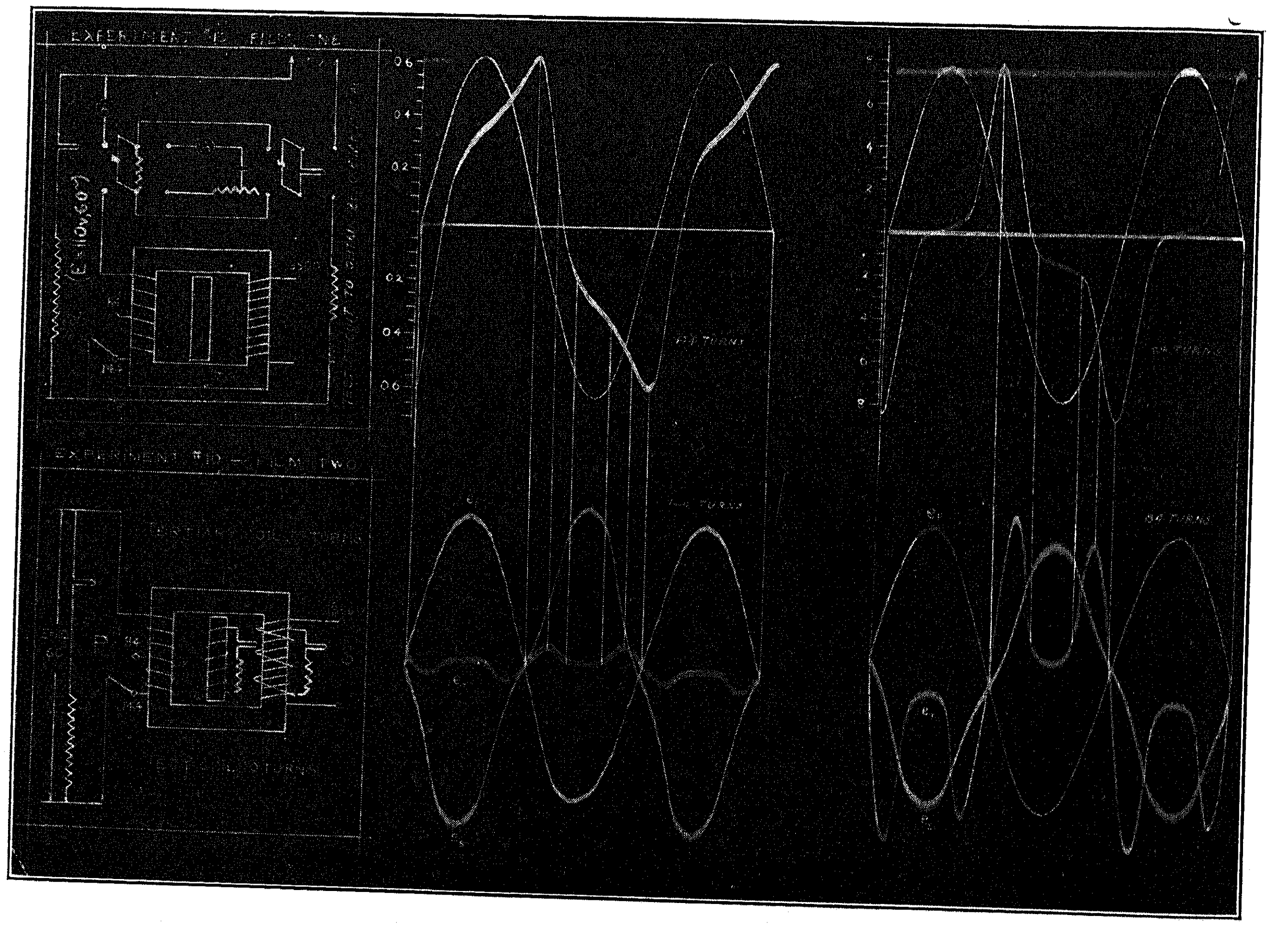

Fig. 7

Occasionally it is desirable to show the components of a mathematical solution in order to better understand the physical significance of the problem. If an alternating e. m. f. is impressed upon a circuit of resistance and inductance in series the solution is

$$i = I \sin (w\,t - \phi) - I'\,\epsilon^{-Rt_0/L}$$

transient current = permanent component − logarithmic component. I is the maximum of the permanent current, ϕ is the angle of lag, I' is the value the current

would normally have for the e. m. f. at the point of the wave where the circuit was closed and t_0 is the time counted from the instant of closing the switch. The use of the transient visualizer for showing the various components will be made clear by referring to Case 1, Fig. 8. At the first exposure the e. m. f. and the transient current were taken, at the second exposure (on the upper film) the sinusoidal component or permanent alternating current and on the third exposure the transient component. All of the current curves are to the same scale and in their proper phase relation.

Since a-c. transients depend upon the point of the e. m. f. wave at which the circuit is closed, it is desirable to be able to close the circuit at any predetermined point. This may be accomplished by rotating the brush arm while making observations on the screen. Fig. 8, Case 1, shows the circuit closed when the e. m. f. was passing through zero, and Case 2 shows the effect produced by closing the circuit at different points of the e. m. f. wave.

A convenient test circuit for determining the zero point of the wave is an air-cored inductance whose power factor is near zero. With a suitable d-c. ammeter in series with the a-c. line, adjust the transient visualizer brush arm until the ammeter gives a maximum deflection. This feature is especially valuable where two or more oscillograms of alternating currents are to be taken on different parts of the same film, as shown in Fig. 7. This is a matter that needs going into more in detail, otherwise the synchronous motors pulling in at different phase relations will cause the inexperienced operator considerable trouble. Assume that the two periodic curves shown in Fig. 7 are to be taken and that for the sake of appearance of the oscillogram the exposures are to be started at zero e. m. f. Use a shield with an opening one and a half cycles in length and arrange the shield stops so that the second exposure begins exactly two cycles from the first. First, with the switch shown in the diagram thrown to the test circuit on the right, start the transient visualizer motor and adjust the brush arm until the circuit is closed at the zero e. m. f. and the transient curve is seen on the screen (this will be called the reference condition). Second, with the oscillograph motor still running, stop the transient visualizer motor and slowly rotate the commutator by means of the handwheel until the test circuit is just closed, as indicated by the current curve appearing on the screen, when the film arm may be adjusted to start the first exposure at the correct point. Third, restart the transient visualizer motor two or three times, if necessary, or until the reference condition is re-established. Fourth, stop the oscillograph motor and throw both switches to the left so the element will be in series with the primary of

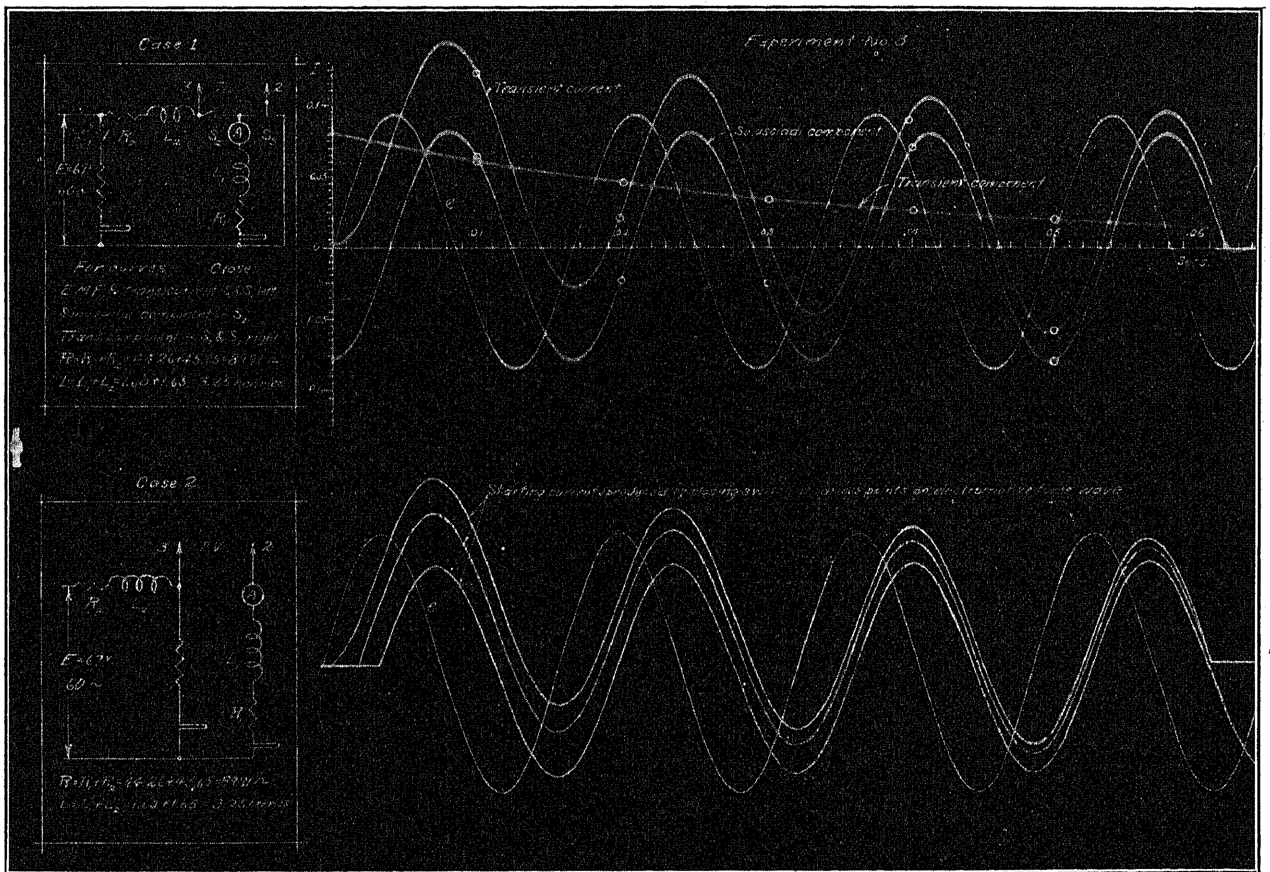

Fig. 8

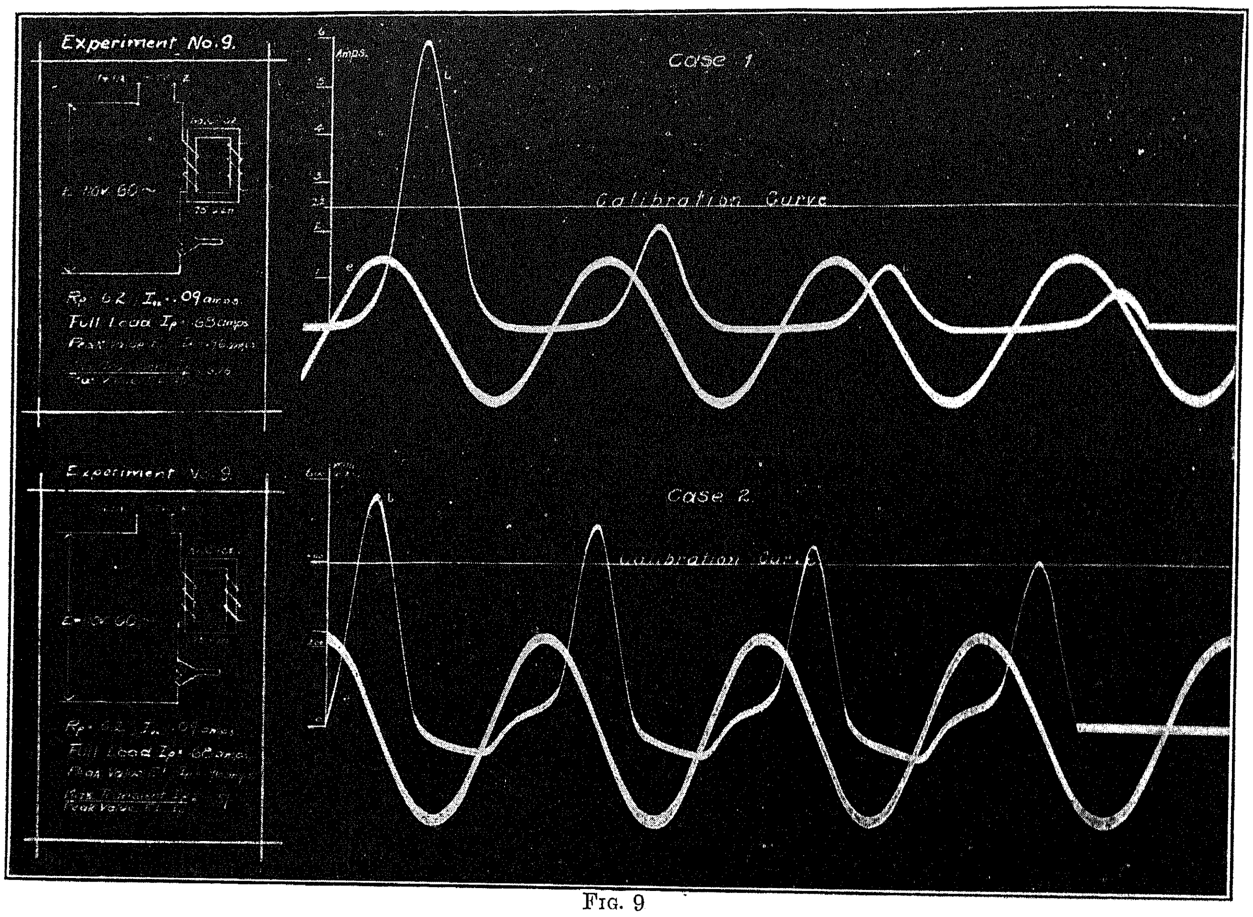
Experiment No. 9.
Case 1
Calibration Curve
Amps.
Full Load
Experiment No. 9.
Case 2.
Calibration Curve
Full Load

FIG. 9

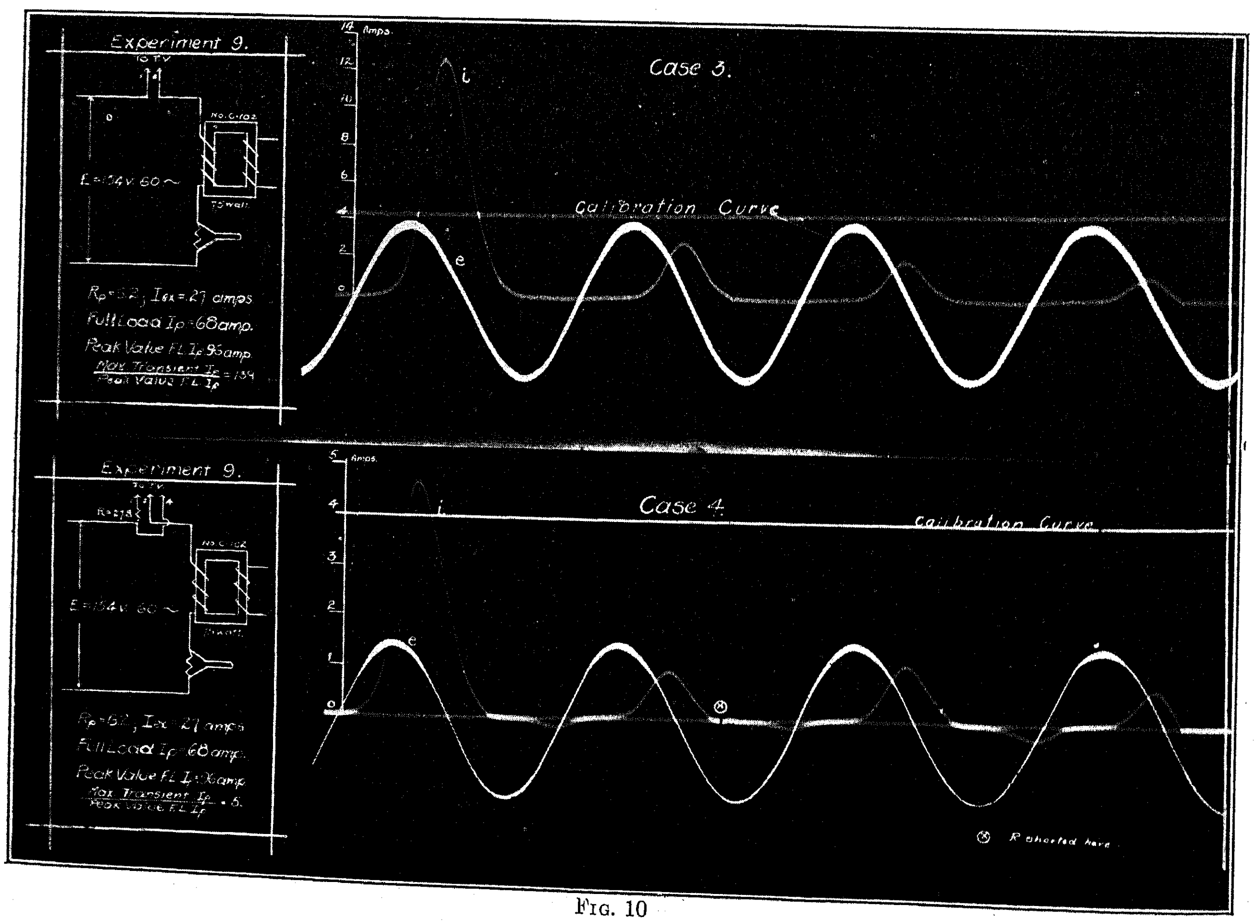
Experiment 9.
Case 3.
Calibration Curve
E=114V 60~
Full Load Ip=68amp.
Experiment 9.
Case 4.
Calibration Curve
R shorted here

FIG. 10

the transformer and make the exposure. Fifth, with the test circuit again connected, restart the oscillograph motor and until the reference condition is restored, stop the transient visualizer motor and change the film shield to position 2. Sixth, replace the film drum and restart the transient visualizer motor until the reference condition is restored, stop the oscillograph motor and make the exposure. After a little experience it is surprising how easy it is to perform these operations and how quickly it can be done. Where successive exposures are to be taken on a film it is necessary to keep one motor running in order to re-establish the reference condition. It should be pointed out that after the brush arm and film arm have once been adjusted, any number of oscillograms may be taken without further attention.

When studying the starting current in transformers, the requirements are rather severe. It is necessary to be able to start the oscillogram at any desired point of the film, to close the circuit at any desired point on the e. m. f. wave, to control at will the amount and direction of the residual magnetism, and in some cases to introduce a series resistance for limiting the initial starting current and later to short-circuit it after the disturbance is somewhat reduced. The control of the residual magnetism is accomplished by the use of a micrometer adjustment of the auxiliary brush, previously referred to, by means of which the point of opening the circuit is under the control of the operator. It is interesting to note that where a maximum residual is to be left in the transformer, the circuit may be opened without sparking at the brushes by adjusting the auxiliary brush so the circuit is opened just as the current reaches zero, decreasing from a maximum value. If the current is decreasing from a positive maximum, a positive residual is left in the transformer ready for the next cycle of events. By referring to the curves of Figs. 9 and 10, it will be seen that there is absolutely no sign of sparking at the brushes. In Fig. 9, Case 1, the circuit was closed where the e. m. f. was passing through zero increasing in a positive direction and with a positive residual. Case 2 is the same except that the circuit was closed when the e. m. f. was a positive maximum. Fig. 10, Case 2, is the same as Fig. 9, Case 1, except that the impressed e. m. f. was increased 40 per cent and due to the higher saturation the current increased 225 per cent. Case 4, is the same as Case 3 except that a series resistance equal to about four times the primary copper resistance was introduced to limit the initial current. The current was decreased to approximately one-third that of Case 3. This resistance was short-circuited after about one and a half cycles, and as a result, the third peak is slightly higher than the second. On account of the fact that the initial current may be many times full-load value, these experiments were performed upon small transformers in order to avoid serious line disturbances.

By means of the transient visualizer all of the transient curves reproduced in this paper may be studied on the screen of an oscillograph in exactly the same way as a-c. curves.

Two oscillatory circuits, tuned to the same frequency when coupled, no longer oscillate at their natural frequency. The change in frequency is shown by Fig. 11 where Curve a is the primary current when the two circuits are coupled, and Curve b shows the free period of the primary.

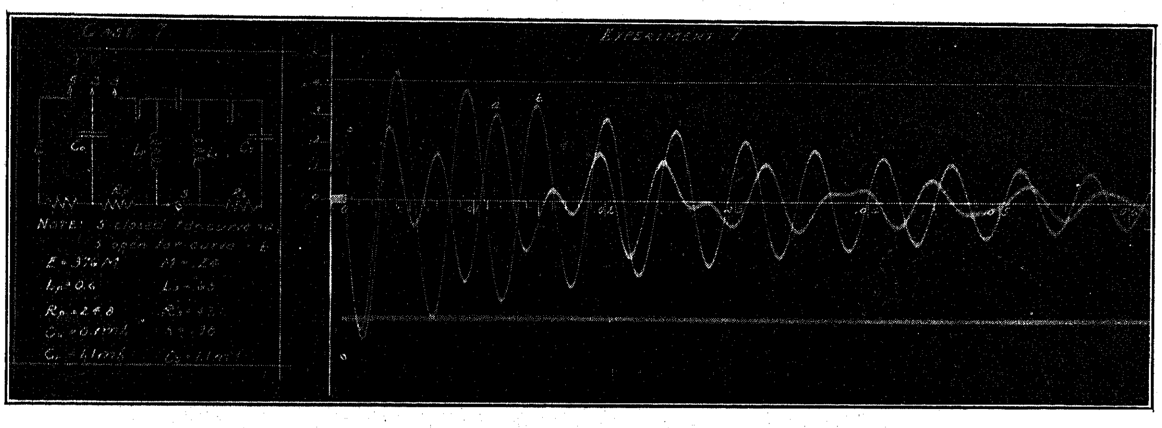

Fig. 11

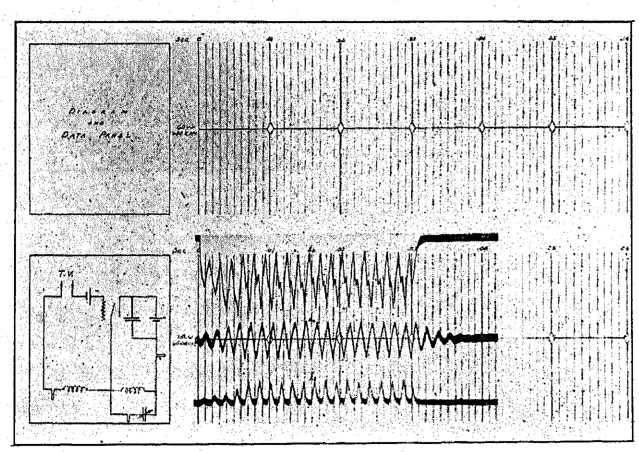

Fig. 12

By means of an opaque paper shield an unexposed space on the film may be conveniently reserved for a diagram of connections and essential data. This is an important feature, for it makes the record complete in itself and eliminates the possibility of losing the data. The curves mean much more when studied in connection with the circuit diagram.

Usually one element of the oscillograph is required for a timing curve. With the transient visualizer, a more satisfactory and convenient way is as follows: The synchronous motor, and therefore the film drum, runs at a speed that is determined by the frequency of the power source, which is quite constant over considerable periods of time, so if the speed is measured by a precision tachometer either before or after the oscillogram is taken, an accurate time scale is possible. Knowing the diameter of the film drum and the speed range to be covered, a series of charts for different speeds may be made up and the time marked off in thousandths of a second as indicated in Fig. 12. Then all that is necessary is to place the film upon the proper chart and mark off with a pen the desired units of time and trace the panel as shown in the lower half of Fig. 12 which shows a negative of the starting currents in a vacuum tube oscillator when the grid circuit is completed by the transient visualizer.

The use of the transient visualizer in making a progressive study of periodic phenomena is well illustrated by Fig. 13 which shows curves of kenotron and S-tube rectifiers for half-wave and full-wave rectification without filters and also with four different kinds of filters. For the convenience of comparison all current curves were taken with one oscillograph element. Including ground lines, each film was exposed a total of 13 times. All of these curves are brought together in order that they may be studied to the best advantage.

In a short article there is time for only a few hints as to the possibilities of the transient visualizer, but experience will suggest many other applications. Many uses are found for it in connection with the study of transmission line problems and three-phase transients. Practically the whole field of transient phenomena is brought within the understanding of students and engineers. It helps the mathematically inclined to visualize the physical relations, and it enables a person with limited technical training to obtain an understanding of complex phenomena. It largely eliminates the element of chance from the experimental study of transients and puts it on a scientific basis.

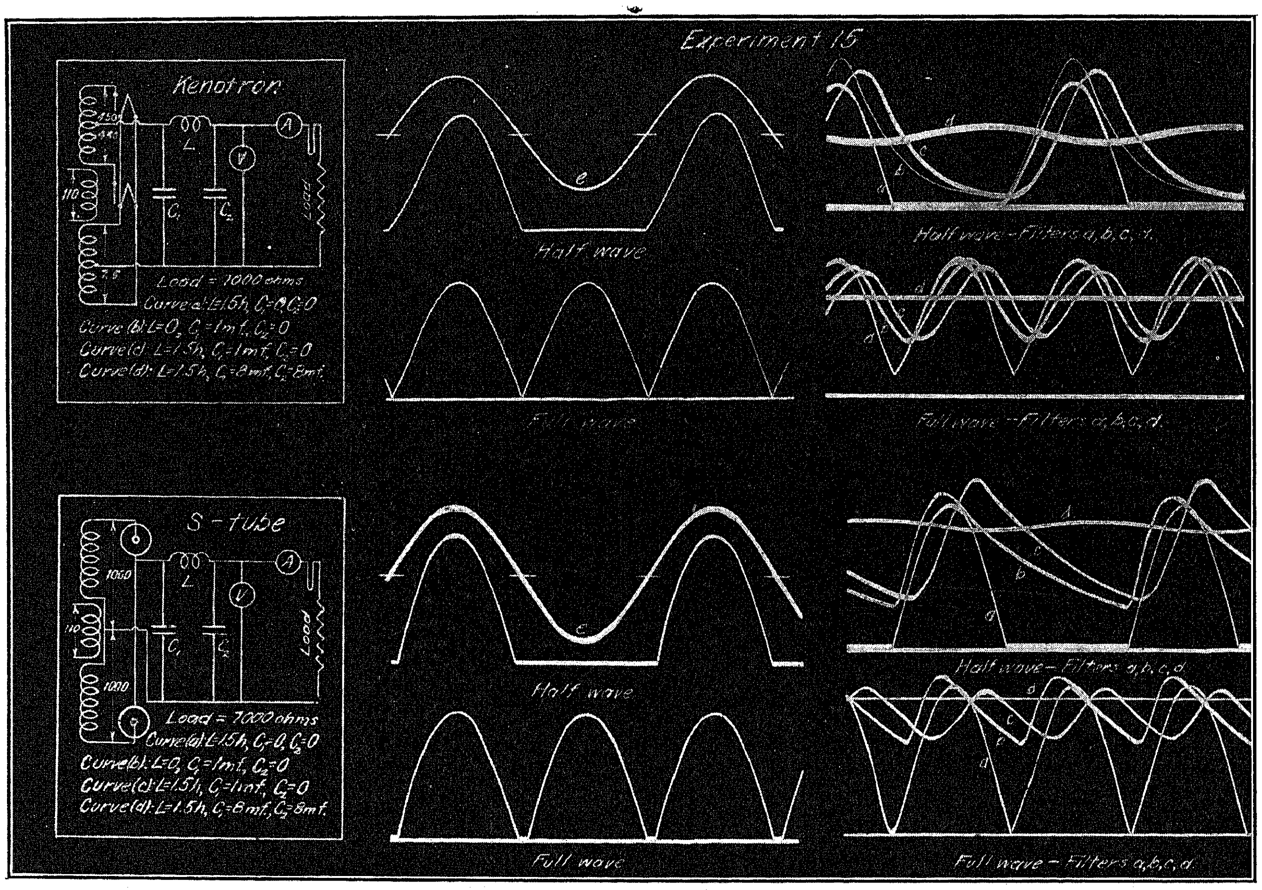

Fig. 13

The oscillograms reproduced in this paper were taken by graduate students in their regular laboratory work in Communication Engineering. The author is greatly indebted to A. J. Ralph, expert technician of the Department of Electrical Engineering, Yale University

for his excellent suggestions regarding mechanical details and for his painstaking care and expert workmanship in the construction of the five machines used in our laboratory.

Discussion

J. R. Craighead: This device serves the purpose of repeating a transient so that it can be properly studied by the eye or where there is any question of suitable illumination can be better photographed because of repetition. This should lend itself to the use of higher velocities and consequently larger and more easily readible records on the oscillograph than have been obtained. The possible limit of speed on the oscillograph in the direction of the motion of the film is chiefly the photographic effect. This device should enable the photographic effect to be much improved, provided the nature of the switch is such that duplication can be assured in the transient at excessively high speeds. Can Mr. Palmer furnish any statement regarding the practicable limits?

Another point which enters into the successful duplication of a transient is the effect of varying contact in a switch. In attempting to get single transients at definite points in the e. m. f. wave, I have had occasion to work with somewhat larger currents and the results have not been successful. That is, it is possible to construct a switch for small currents or for potentials which carry practically no current at which duplication can be made quite accurately.

When we get to apparatus the size of ordinary oil switches and large circuits, this kind of duplication is not practicable because although the actual duplicating relay can be made to operate accurately, the oil switch itself has a sufficient variation in time so that it is not practicable to secure a duplication of the particular point on the wave that is desired.

In regard to the separation of components of wave forms, as mentioned on page 5, the method of separation is not very clearly stated. I understand it to be the use of the transient visualizer to maintain a constant zero, and alteration of the circuit connections is necessary to obtain the oscillogram in each particular case. I should like to ask Mr. Palmer whether this solution is that which is intended.

H. M. Turner: Mr. Craighead raised the question regarding duplication at high film speed. I can say that we have experienced no difficulty with retracing when operating at a film speed of 1200 ft. per min. Usually, however, we are operating at 600 ft. per min. and the retracing is practically perfect. It should be pointed out that a small constant friction load on the photographic drum shaft is necessary to prevent oscillations.

So far as current-carrying capacity is concerned there is no reason why it cannot be greatly increased by using wider brushes. In demonstrating fundamental principles there is no particular advantage in using large currents. In general it is convenient to use currents of 15 amperes or less. However, when studying the performance of particular pieces of equipment where several hundred amperes, or in extreme cases, probably several thousand are involved a special design would be required. In the case of alternating current, advantage may be taken of the fact that the circuit may be opened when the current is at or near the zero value.

With reference to the method of separating the components, the transient visualizer makes it possible to start all components from the same point on the film. First, the transient current is taken by closing the circuit at the desired point at the electromotive force wave. Second, the transient visualizer is short-circuited and the permanent component is taken. Third, with the permanent component flowing through the oscillograph element in the reversed direction the line is short circuited by means of the transient visualizer and the transient component is taken in proper relation. In order to avoid tripping the circuit breaker, two identical circuits are connected in series and the short circuit is applied to one of them.

Temperature Rise of Stationary Electrical Apparatus as Influenced by Radiation, Convection and Altitude

BY V. M. MONTSINGER and W. H. COONEY
Member, A. I. E. E. Associate, A. I. E. E.
Both of General Electric Co., Pittsfield, Mass.

Review of the Subject.—Part I of the paper takes up first the division of losses by radiation and convection from tall vertical planes. It is shown that the loss by convection from tall planes can be expressed by the formula

$$W_c = 0.0014\, A\, \theta^{5/4}$$

in which W_c is the watts, A the square inches of surface and θ the temperature rise in degrees centigrade.

It is then shown that the division of losses from a black plane is, for temperature rises up to 100 deg. cent., approximately 45 per cent convection and 55 per cent radiation. Based upon this division, a simple formula is given for determining the ratio of losses for an irregular surface where the greater part of the loss is by convection.

The effect of various colors on the temperature rise of transformers having both plain and corrugated tanks is discussed.

In Part II it is shown that altitude does not affect radiation. The general average of the results obtained by various investigators shows that convection varies as the square root of the air density. Based on this, it is shown that for a constant loss by convection the temperature rise increases about 4.6 per cent for each 1000 meters increase in altitude. For transformers where a part of the loss is by radiation (unaffected by air density) the effect is reduced by the ratio of the convection loss to the total loss. Also the effect on the winding rise over the ambient is further reduced by the fact that the winding rise over the oil is not affected by altitude. The effect of the copper loss, however, is to increase the effect of altitude because the resistance is increased by temperature.

Based upon the above facts it is shown that the temperature rise and rating of the two main classes of oil-immersed self-cooled transformers are affected by altitude as follows:

	Per cent increase in Copper Rise per 1000 Meters Increase in Altitude	*Per cent decrease in Kv-a. per 1000 Meters Increase in Altitude*
Self-cooled transformers.........		
(a) with plain tanks..........	*1.75*	*1.35*
(b) with corrugated, tubular and radiator tanks............	*3.0*	*2.3*

INTRODUCTION

WHILE the main purpose of this paper (which is a companion paper of the one by Doherty and Carter[1]) is to give the effect of altitude on the heating of stationary apparatus cooled by radiation and free convection, it is very essential to first determine how the cooling is effected. For instance, we must know the relative parts radiation and convection play in cooling the different shapes of surfaces used, especially for self-cooled oil-immersed transformers, because (1) the ratio of losses by these two modes of cooling varies, depending on the shape and color of the surface, etc., and (2) these two modes of cooling are not affected alike by altitude. The paper, therefore, falls into two logical divisions, namely:—

1. The temperature rise of various shaped surfaces as influenced by radiation and free convection for a given altitude and
2. Effect of altitude on temperature rise of apparatus having various shapes of surfaces.

One of the writers gave a paper[2] before the Institute in June 1916 in which was shown by tests how the heating of three transformer tanks, each having different shapes of surfaces, were affected by altitude.

Since that time considerable experimental work has been carried on to establish, first, the correct formula for free convection; and second, a method for calculating the cooling capacity of various shapes of surfaces under a constant air pressure where both radiation and convection enter into the cooling, especially where convection plays such an important part as it does for tank surfaces having different widths and depths of corrugations. So far as literature shows, no method has ever been given by which to predetermine with any degree of accuracy the thermal efficiency of corrugations.

Part I gives the results of tests made on a large plate to determine the formula for free convection. Also the results of tests and a discussion are given on the effect of different colored cases on the temperature rise of transformers in the shade and sunshine.

Part II deals with the effect of altitude on the temperature rise of transformers having different shapes of tank surfaces, of rheostats, bus bars, reactors, etc.

Part I

TEMPERATURE RISE OF VARIOUS SHAPED SURFACES AS INFLUENCED BY RADIATION AND CONVECTION FOR A GIVEN ALTITUDE

A. Radiation. The accepted law of radiation, known as the Stefan-Boltzman law, is of the form:

$$W_r/A = K\, e\, (T_2^4 - T_1^4) \qquad (1)$$

where W_r/A is the watts dissipated per unit surface, K is a constant, e is the emissivity factor depending on color, being 1.0 for a lamp black surface, and T_2 and T_1 are the hot body and ambient temperatures in absolute degrees centigrade respectively. If expressed

1. Effect of Altitude on Temperature Rise by R. E. Doherty and E. S. Carter.

2. Effect of Barometric Pressure on Temperature Rise of Self-Cooled Stationary Induction Apparatus" by V. M. Montsinger.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27 1924.

in watts per square inch, according to the latest accepted value, $K = 3.68 \times 10^{-11}$.

Fig. 1 shows the radiation of heat in watts per square inch plotted against temperature rises over various ambient temperatures.

B. Convection. Several years ago, Dr. Irving Langmuir developed and published[3] the film theory which assumes that heat loss by convection is dissipated by first passing through an adhering film of gas, where most of the temperature drop occurs, and then is carried away by convection air currents.

While the film theory formula checked tests made at high temperatures, it did not check tests made on tall planes at low temperatures. According to the formula, the loss was approximately proportional to the temperature rise up to 100 deg. cent., whereas numerous tests made on various kinds of tank surfaces showed that convection loss varied as the temperature rise raised to the 5/4 power. This was pointed out in the paper presented in 1916.

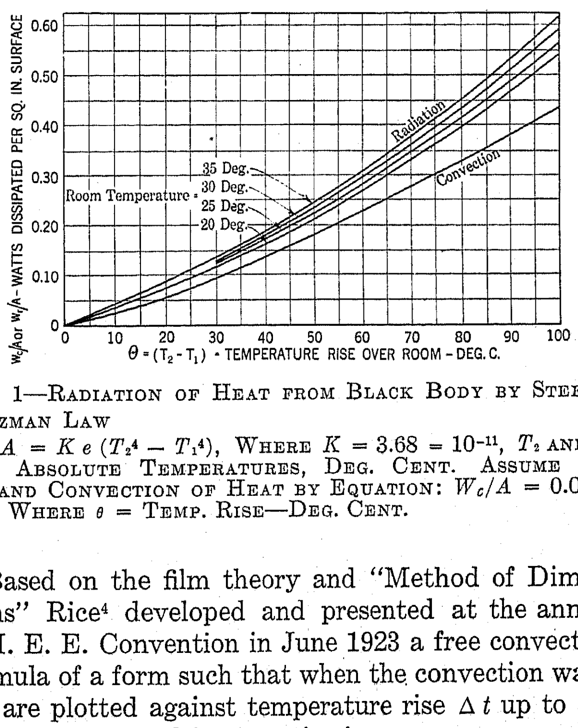

FIG. 1—RADIATION OF HEAT FROM BLACK BODY BY STEFAN-BOLZMAN LAW

$W_r/A = K\,e\,(T_2^4 - T_1^4)$, WHERE $K = 3.68 = 10^{-11}$, T_2 AND T_1 ARE ABSOLUTE TEMPERATURES, DEG. CENT. ASSUME $e = 0.9$ AND CONVECTION OF HEAT BY EQUATION: $W_c/A = 0.0014\ \theta^{1.25}$, WHERE θ = TEMP. RISE—DEG. CENT.

Based on the film theory and "Method of Dimensions" Rice[4] developed and presented at the annual A. I. E. E. Convention in June 1923 a free convection formula of a form such that when the convection watts W_c are plotted against temperature rise Δt up to 100 deg. cent. the resulting equation is:

$$W_c = K\,A\,\Delta\,t^{1.06} \tag{2}$$

in which K is a constant, A is the area.

But from 100 to 500 deg. cent. rise, the exponent in Rice's formula is approximately 1.25.

Since the temperature rises of most electrical apparatus do not exceed 100 deg. cent. it appeared to be of sufficient importance to make accurate laboratory tests to determine (1) the correct value of the exponent and (2) the constant for vertical surfaces representing tall tanks. As Rice's formula was known by the writers well in advance of its publication, there was sufficient time to complete these tests and give, in a condensed form, the results in a discussion of his paper. In this discussion it was shown that free convection from tall planes can be expressed for temperature rises up to 300 deg. cent. by the formula:

$$W_c = K\,A\,\theta^{5/4} \tag{3}$$

in which W_c = convection watts,
K = 0.0014 for a plane 31.5″ tall
A = area in sq. in.
θ = temperature rise in degrees centigrade

Fig. 1, also shows values of W_c/A plotted against θ.

Further details as to how the formula was derived are given later under "Experimental Observations on Vertical Plate."

Recently Rice[5] has, by introducing the temperature coefficient of density of the air in his previous formula, developed one for large vertical plane surfaces in air which agrees in substance with equation (3). It is of the form:

$$W_c = 0.0078\,A\,(1/H)^{1/4}\,P^{1/2}\,(1/T_{avg})^{.123}\,\Delta\,t^{5/4} \tag{4}$$

in which W_c = Loss in watts
A = Area of surface
H = Height of plane
P = Absolute air pressure in atmospheres
T_{avg} = Average of hot body and ambient absolute temperatures degree K.
Δt = Temperature rise of hot body in deg. cent.

For a rise of approximately 50 deg. cent. in a 25 deg. cent. room and for a plane 31.5 in. (80 cm.) tall, equation (4) reduces to

$$W_c = 0.001285\,A\,P^{1/2}\,\Delta\,t^{5/4} \tag{5}$$

Rice has, therefore, cleared up the disagreement between the *rate* of *loss* with *temperature rise* which has existed between the film theory formula and results of tests made at low temperature differences. We now have for the first time a true conception of the physics of convection for both high and low temperatures differences.

C—Radiation and Convection from Various Shaped Surfaces

(*a*) *Large Vertical Planes.* Table I shows the calculated division of losses from a plain black surface, using the convection formula $W_c = 0.0014\,A\,\theta^{1.25}$ and the radiation formula $W_r = 3.68 \times 10^{-11}\,A\,e\,(T_2^4 - T_1^4)$, in which $e = 0.9$ and $T_1 = 298$ deg. K (25 deg. cent.)

For practical purposes we can say that for vertical planes the losses from, say, 20 to 100 deg. cent. rise, are 55 per cent radiation and 45 per cent convection.

(*b*) *Large Vertical Corrugated Surfaces.* One of the most universal methods of increasing the area of a trans-

3. PROC. A. I. E. E. Feb. 1913.

4. Free Convection of Heat in Gases and Liquids by C. W. Rice. Presented at A. I. E. E. Annual Convention in Swampscott, Mass., June 1923.

5. Free Convection of Heat in Gases and Liquids by C. W. Rice. Presented at A. I. E. E. Midwinter Convention in Philadelphia, February, 1924.

TABLE I
DIVISION OF LOSSES FROM LARGE PLAIN, BLACK VERTICAL SURFACE IN 25 DEG. CENT. ROOM AT SEA LEVEL

Temperature Rise Deg. Cent.	Watts per sq. in. by:		Per cent of Total Loss by:	
	Radiation	Convection	Radiation	Convection
10	0.039	0.025	61	39
20	0.0765	0.057	57.5	42.5
30	0.121	0.095	56	44
40	0.171	0.137	55.5	44.5
50	0.224	0.182	55	45
60	0.28	0.230	55	45
70	0.344	0.279	55	45
80	0.415	0.329	56	44
90	0.492	0.382	56	44
100	0.57	0.439	56.5	43.5

former tank without increasing its floor space is to corrugate the surface. Since there are so many possible variations in the pitch depth and height of corrugations, it is very essential to be able to predetermine the cooling capacity, *i. e.* the loss per unit of area of the developed surface for various temperature rises.

(*c*) *Surface of Corrugations Effective for Radiation and Convection Radiation:* Referring to Fig. 2 the surface A represents the envelope of a single corrugation

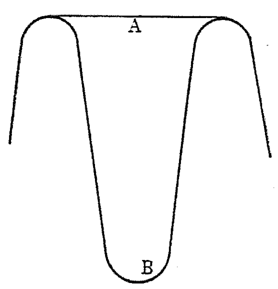

FIG. 2—SHOWING SURFACES OF A CORRUGATION EFFECTIVE FOR RADIATION AND CONVECTION

B. It is obvious that both sides of A will radiate equal amounts of heat. Now B will radiate the same amount of heat to A as A radiates to B; otherwise one body would gain heat at the expense of the other. Since A is the envelope area of B and, furthermore, since A and B have equal radiating capacities, the true radiating surface of a corrugation (or any irregular surface) is its outer envelope. (A mathematical proof of this which is rather long, is given by Rud. Kuchler in *Electrotechnische Zeitschrift,* Jan. 1923, Vol. 44).

Convection. The surface effective for convections is of course the total developed area, providing the air currents are allowed to circulate freely over the entire surface.

(*d*) *Total Cooling Capacity of Plain and Corrugated Surfaces*

(*1*) *Actual Losses:* The actual losses for any given maximum oil rise of either a plain or corrugated tank are difficult of accurate predetermination for the following reasons: (1) There are certain losses from the bottom and cover whose temperature rises are seldom accurately known; (2) the vertical tank gradient must be known; and (3) there exists a certain temperature drop from the oil to the tank, which drop must be known. However, in practise it is just as important and very convenient to be able to make comparisons of the heating of tanks having different shapes of surface with other tanks having similar vertical gradients. For instance, if we know what a plain tank will do for any given oil rise, it is convenient to know what some other tank with a corrugated, surface will do for the same oil rise.

(*2*) *Relative Losses:* For any given top oil rise (providing the vertical gradients of the tank are similar) the ratio of the loss from the average shape of corrugation used in practise to the loss from a plain tank can therefore be expressed by the following equation:

$$L = \frac{55\, E\, e \times 45\, D}{100\, D} \tag{6}$$

in which L = ratio of total loss per unit of developed area to loss per unit of plain area, having the same temperature rise of the oil.

E = Outside envelope area.

e = Emissivity factor = 1.00 for ordinary black paint in this case.[6]

D = Developed area of surface.

There are, of course, limitations as to depth and width of the air space in the corrugation for which equation (**6**) holds. Experience has shown that it holds for a ratio of depth divided by width of approximately 4. However, as this ratio increases, it would be expected that the air circulation would become more and more restricted until the loss by convection was materially reduced. For instance, if the width of air space in a corrugation, 6 in. depth was only 1/4 in., there is no question but that the air would be so restricted that the convection loss would be reduced enormously.

If the entire surface of a tank consists of similar corrugations we can let E equal the pitch, and D the developed length of a single corrugation and by substituting these values in equation (**6**) determine L. For instance if the pitch is 2 in. and the developed length is 6.6 in., the value of

$$L = \frac{55 \times 2 \times 1 + 45 \times 6.6}{100 \times 6.6} = 0.62$$

In other words one sq. in. of the corrugated area will dissipate 0.62 as much loss as one sq. in. of a plain area.

Now, since this tank has 6.6 in. of surface to each 2 in. of envelope, it has 3.3 times the total area of a plain tank of the same outside diameter. Therefore, the total loss dissipated by this corrugated tank will be $3.3 \times 0.62 = 2.04$ times that of the plain tank.

If equation (**6**) is expressed in terms of total loss L_1 per unit of envelope area instead of loss per unit of developed area (L), the equation reduces to

$$L_1 = L\, D/E$$

$$= \frac{55\, E\, e + 45\, D}{100\, E} \tag{7}$$

6. See Table II for values of e for Various Colors.

Effect of Height. Carefully made tests have shown that the effect of height is very small, the loss being only about 3 per cent more for a corrugated surface 36 in. in height than one 72 in. in height. In fact, experience has shown that the effect of height is so small that for practical purposes it can be neglected for all heights greater than about 2 or 3 ft.

D. Effect of Color on Temperature Rise. Table II gives the emissivity factors for various colors of paints often used on transformer tanks. These values were obtained from tests made on a plate 20 in. by 20 in. in a vertical position.

TABLE II
EFFECT OF COLOR ON EMISSIVITY

Color of Paint	Emissivity Compared with Black Surface (e = 0.9)
Black	1.00
Dark Green	0.97
Dull Red	0.91
" Grey	0.81
White	0.76
Copper (busbars)	0.65
Aluminum	0.62

The effect of color on radiation raises the question of how the temperature rise of a transformer is affected by various colors of paint when in the shade, or how it is affected by absorption of heat when exposed to the rays of the sun.

Since the division of losses from a plain black tank is approximately 55 per cent radiation and 45 per cent convection, the effect of other colors on the temperature rise may be easily calculated for any shape of surface in the shade. Table III shows this for a plain surface.

TABLE III
EFFECT OF COLOR ON TEMPERATURE RISE OF PLAIN TANKS

Color of Paint	Surface Rise in per cent of that for Black Surface
Black	100
Dark Green	101
Dull Red	104
" Grey	110
White	117
Aluminum	120

TABLE IV
EFFECT OF COLOR ON TEMPERATURE RISE OF TANKS HAVING APPROXIMATELY 75 PER CENT OF LOSS BY CONVECTION AND 25 PER CENT BY RADIATION

Color of Paint	Percentage Rise
Black	100
Dark Green	100.8
Dull Red	101.5
" Gray	104
White	105.8
Aluminum	109

It is thus apparent that a tank with an irregular surface is not appreciably affected by color.

IN SUNSHINE

(*a*) *Plain Tank.* A plain tank will probably be affected the most. The results of tests made by Messrs. Moore and Moulton[7] of the San Joaquin Light and Power Corporation to determine the effect on the temperature rise of both black and gray plain tanks in the sunshine, are summarized by them as follows:

"The number of successive days during which tests were made and the consistency of the results leave little doubt as to the accuracy of the full load tests. The fact that a gray paint will not reduce the oil temperature more than 3 or 4 deg. fahr. or 1 or 2 deg. cent. during the extremely hot weather encountered in the San Joaquin Valley seems established. This was a disappointment in view of the seemingly prevalent belief that a much larger reduction for gray paint would be found."

It is apparent that the increased temperature, due to the lighter color (and reduced radiation) is approximately counteracted by the reduced absorption of heat from the rays of the sun.

(*b*) *Tanks with Irregular Surfaces.* Since heat absorbed by radiation is a function of the envelope surface only, there is every reason to believe that a tank of irregular shape will not be affected as much by the sun's rays, for a given color, as a plain tank.

It should be understood, of course, that the temperature rise will be greater in the sunshine than in the shade, regardless of color of tanks, the difference depending on several factors, such as direction of the sun's rays, the area of the tank exposed to the sun, and the intensity of the rays. Unfortunately, no reliable data appears to be available on this point.

E. Experimental Observations on Vertical Plate. The plate was of cast iron, (Figs. 3 and 4) 31.5 in. (80 cm.) high, 13.1 in. (33.3 cm.) wide, by 1 1/16 in. (2.7 cm.) in thickness, and had imbedded in it sheath wire units of equal resistance about 2 in. apart. This produced a uniform temperature over the whole area, including both sides, and eliminated the necessity of making appreciable stray losses corrections which would have been necessary if one side of the plate had been blanketed, as is sometimes done in investigations of this kind. In some of the tests the terminals and

TABLE V
LAMP BLACK SURFACE

W_t/A = total watts per square inch.
W_r/A = watts per sq. in. radiation (calculated)
$W_c/A = W_t/A - W_r/A$ = watts per sq. in. convection.
Area dissipating heat = 917.4 sq. in.

Test No.	Ambient		Average TempRise Over Ambient	W_t/A	W_r/A (e = 0.9)	W_c/A
	Air	Wall				
1	30	29	12.6	0.08095	0.04948	0.03147
2	32.7	31.4	22.5	0.1508	0.0954	0.0554
3	25.6	23.6	29.6	0.2145	0.1206	0.0939
4	28.5	27.8	38.7	0.3075	0.1705	0.137
5	31	30	51.5	0.4358	0.2465	0.1893
6	29.7	29	60.9	0.5450	0.3006	0.2444
7	29.7	28.4	72.1	0.6562	0.3735	0.2827

7. Effects of Various Colored Cases on Oil Temperatures of Distribution Transformers by L. J. Moore and J. H. Moulton in *Journal of Electricity and Western Industry*, June 1923.

TABLE VI
NICKEL PLATED SURFACE
Area Dissipating Heat = 931.2 square in. e = .07

Test No.	Ambient Air	Average Temp. Rise Over Ambient	W_t/A	W_r/A (e = .07)	W_c/A
8	23.2	13.7	0.03921	0.00394	0.0353
9	28.5	15.4	0.04779	0.00468	0.0431
10	30	21	0.06892	0.00669	0.06223
11	29.5	33.6	0.113	0.0113	0.1017
12	33.5	50.1	0.192	0.01898	0.173
13	28.2	64.2	0.2637	0.02476	0.2389
14	26.3	80.3	0.3561	0.03289	0.3232
15	23.8	91.6	0.4349	0.03867	0.3962
16	25.0	93.8	0.4349	0.04048	0.3944
17	27.0	101.8	0.4877	0.04635	0.4413
18	24.3	110.1	0.5647	0.05095	0.5137
19	25.6	118.6	0.6142	0.05749	0.5567
20	25.2	127.2	0.6828	0.06492	0.6179
21	28.3	143.5	0.7965	0.07975	0.7167
22	22.6	152.5	0.8635	0.08425	0.7792
23	28.2	60.8	0.2427	0.02294	0.2198
24	25.3	31.3	0.1141	0.00998	0.1041
25	24.8	60.1	0.2465	0.02194	0.2246
26	24.7	42.7	0.1602	0.01438	0.1458
27	28.7	309.8	2.615	0.3388	2.2762
28	30.7	241.3	1.699	0.2055	1.4935
29	26.2	178.8	1.043	0.1140	0.929
30	28.5	242	1.7615	0.2035	1.558

TABLE VII
PARTLY OXIDIZED SURFACE
Area Dissipating Heat = 836.5 sq. in.
Emissivity Factor e = 0.52

Test No.	Ambient		Average Temp. Rice °C Over Ambient	W_t/A	W_r/A (e = .52)	W_c/A
	Air	Wall				
31	34	36.1	279.5	3.653	2.096	1.557
32	33.6	35.5	250.4	3.046	1.676	1.370
33	33.5	34.8	193.5	1.983	1.027	0.956
34	33	34.5	214	2.507	1.23	1.277
35	33.8	35.0	227.4	2.609	1.393	1.216
36	34.8	36.8	263.4	3.343	1.863	1.48
37	33.3	34.4	199.2	2.132	1.078	1.054
38	32.7	33.7	170.8	1.741	0.821	0.920
39	31.2	31.5	145.8	1.418	0.621	0.797
40	33.1	33.4	118.3	0.9865	0.4534	0.5331
41	32.6	32.6	69.4	0.5347	0.2116	0.3231
42	32.2	32.1	55.6	0.3569	0.1592	0.1977
43	31	31.1	55	0.3537	0.1549	0.2024
44	31.2	31.4	38.9	0.2363	0.1014	0.1349
45	31.2	31.5	29.8	0.1766	0.07435	0.10225
46	31.2	31.5	20.3	0.1126	0.04845	0.06415
47	31.3	31.6	14.4	0.07534	0.03364	0.0417
48	31.3	32.3	206.6	2.272	1.132	1.14
49	32.1	32.5	149.8	1.383	0.6545	0.7285
50	31.8	32.0	102.5	0.83	0.362	0.468
51	31	31.0	79	0.5908	0.2487	0.3421
52	32.7	32.6	47	0.2944	0.1292	0.1652
53	32.7	32.5	25.6	0.1474	0.06372	0.08368
54	32.5	32.3	14.5	0.07628	0.03422	0.04206
55	31.5	31.4	67.5	0.4693	0.202	0.2673

edges of the plate were blanketed. This accounts for the slight variation in the areas given in Tables V, VI, and VII.

The plate was suspended in a vertical position in the air in an open room having a constant temperature. Fifteen thermocouples were soldered in holes in the surface on one side and five on the other side of the plate, the five being used merely as a check to see if both sides were at the same temperature. The plate temperature was taken as the average of all thermo-couples. Direct current was used to supply the loss. All readings of volts and amperes and thermc-couples were taken with a potentiometer. For convection, the air was used as the ambient. For radiation, the temperature of the walls of the room about 10 ft. distant was used for the ambient, although the air and wall temperatures were usually the same.

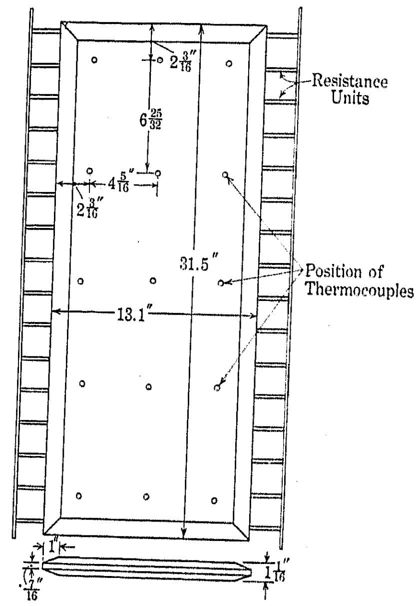

Fig. 3—Dimensions of Vertical Plate Used to Determine Convection Formula

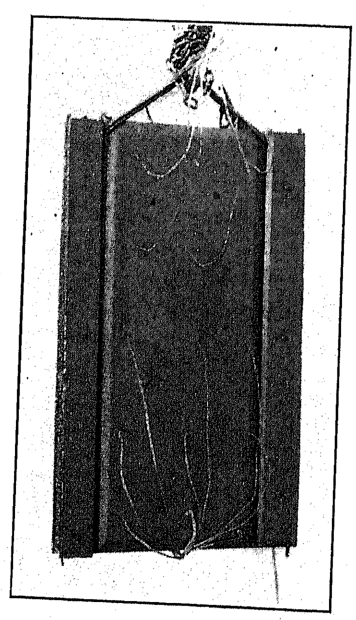

Fig. 4—Front View of Plate Used to Determine Convection Formula Blanketing on Terminals

Three series of runs were made with both sides of the hot plate under the same conditions, namely:

(1) painted a lamp black.
(2) nickel-plated and polished, and
(3) with the surfaces partly oxidized, before starting the tests.

For the first condition, the paint began to scale off when the temperature reached approximately 100 deg. cent. or about 72 deg. rise, and the test had to be discontinued. Up to this point, the loss by convection was taken as the difference between the total loss and the loss calculated by the standard radiation law, assuming that the emissivity factor was 0.9 of that for a perfect black body, as shown in Fig. 1.

The convection loss points fell practically on a straight line on double-log paper, the equation of the line being $W_c/A = 0.0014\ \theta^{1.25}$ where W_c/A is the watts per square inch of surface and θ is the temperature rise in degrees centigrade. Watts per square inch are plotted against θ in Fig. 5. The test data are given in Table V.

For the other two conditions of the plate surface, the emissivity factor for radiation was found by plotting the total loss in watts per square inch against temperature rise. The difference between this and the convection loss curve found from the black plate divided by the theoretical radiation loss where $e = 1.0$ gives the new emissivity factor. For instance, with the nickel plated surface, the total watts per square inch, when plotted in curve form, are 0.200 for a 50 deg. cent. rise over a 30 deg. cent ambient. Subtracting

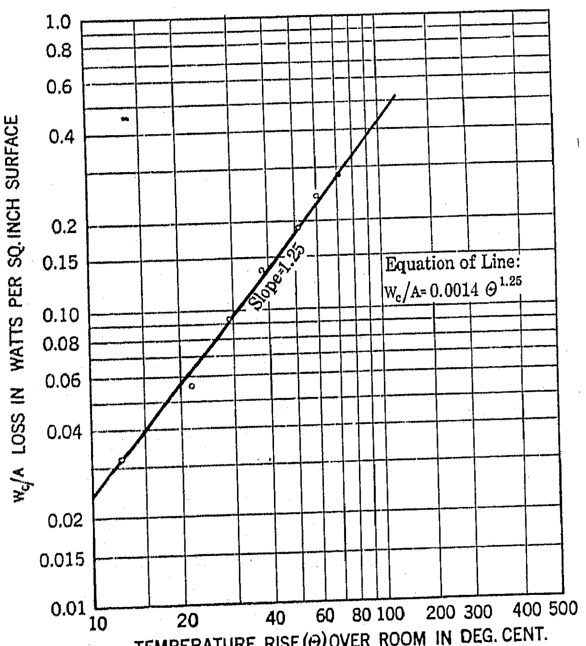

Fig. 5—Heat Loss by Free Convection from a Vertical Plane 31.5 in. in Height and 13.1 in. in Width, Painted Lamp Black

the convection value of 0.182, as shown on Fig. 1, gives a radiation loss of 0.018. But W_r/A, (for $e = 1$) is 0.261, so the emissivity factor $e = 0.018/0.261 = 0.069$ or approximately 0.07 This value of emissivity checks fairly well with that obtained by other investigators. The results obtained with the nickel-plated surface are shown in Table VI.

Up to about 150 deg. cent rise, the convection loss points fell in a straight line on double-log paper, Fig. 6, the slope of the line being about 1.25. From 150 to 200 deg. rise the points gradually drew away from the straight line and at 300 deg. rise the loss was about 25 per cent higher than the straight line.

At first it was thought that this departure from a straight line might be due to a change in the law, but as will be seen later where tests made with the surfaces

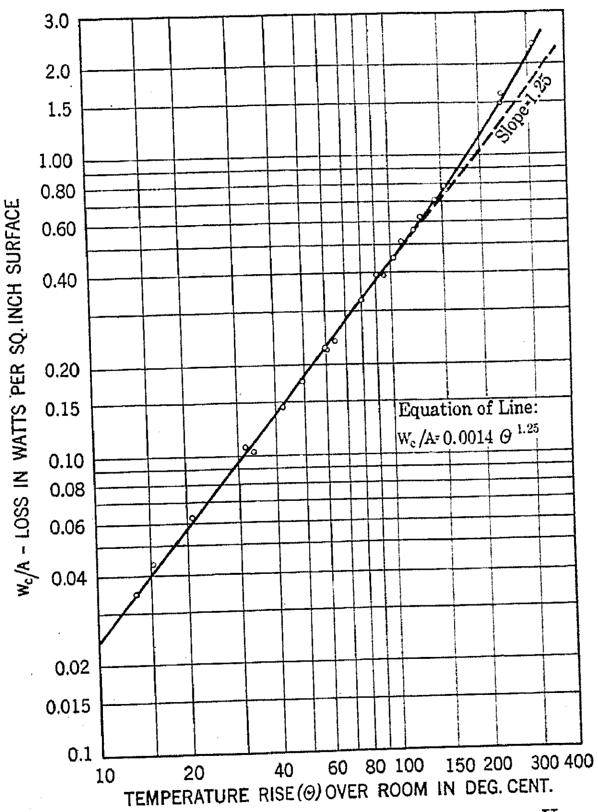

Fig. 6—Heat Loss by Free Convection from a Vertical Plane 31.5 In. in Height and 13.1 In. in Width, Surfaces Nickel-Plated and Polished

partly oxidized did not show this departure, it was apparently due to a gradual oxidation of the surface which at first was not discernible to the eye but which gradually increased the emissivity factor. At 280 deg. cent. rise, the surface became so tarnished that the tests were discontinued.

For the last set of tests the surfaces were painted black and then subjected to a temperature of 325 deg. cent. for about a day to let it get "set" before starting the tests. Some of the paint came off. What remained turned a brownish color.

Tests were taken first with the temperature decreasing and second with increasing temperatures. The emissivity factor, determined as described before, was 0.52. A tabulation of the data obtained with the partly oxidized surface is given in Table VII.

The convection loss plotted vs. temperature rise (Fig. 7) on double-log paper fell practically on a straight line up to 280 deg. cent. rise— as far as the temperature was taken—with a slope of the line of 1.25.

The point to be emphasized is that for a large vertical surface and for the temperature rises used in most electrical apparatus, the loss by convection can be expressed by an exponential equation in which the loss varies as the 5/4 power of the temperature rise. This

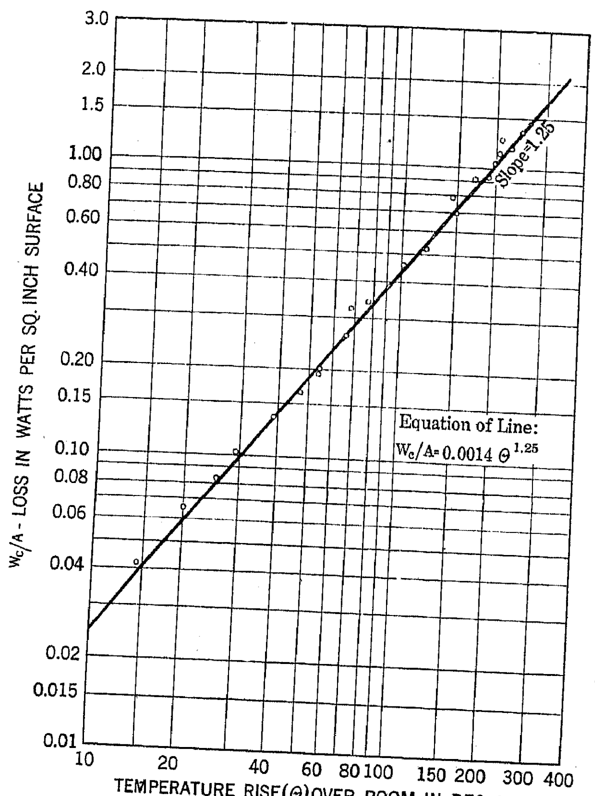

Fig. 7—Heat Loss by Free Convection from Vertical Plane 31.5 In. in Height and 13.1 In. in Width, Surfaces Partly Oxidized before Tests

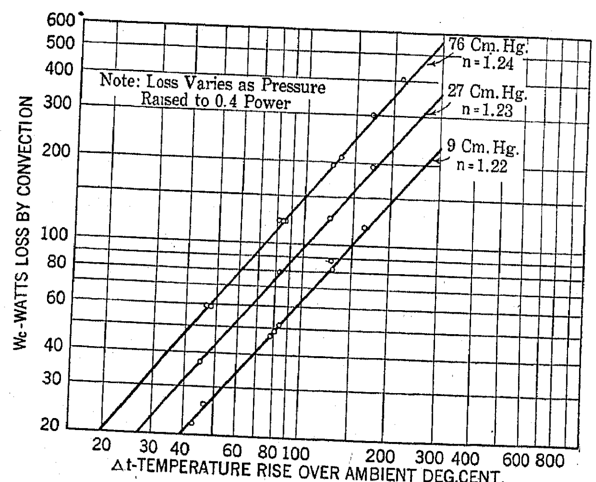

Fig. 8—Effect of Pressure on Loss by Convection, from Data Obtained by C. W. Rice on a Cylinder 1.68 In. in Diameter and 48 In. long

agrees approximately with tests made by Messrs. Ezer Griffiths and A. H. Davis, conducted under the auspices of the Department of Scientific and Industrial Research of England and shown in their Special Report No. 9 issued in 1922 on "The Transmission of Heat by Radiation and Convection." They tested vertical planes of various heights for rises up to 100 deg. cent. For heights of 69 in. and 104 in. they found the value of the exponent to be 1.3 and 1.34 respectively, while for heights less than 69 in. the exponent was 5/4. The constant in their formula was the same for all heights above approximately 12 in. (30 cm.). This checks the writers' experience with various heights of tank surfaces, excepting that the value of the constant found from the tests on the vertical plate was 1.4×10^{-3} when expressed in watts per square inch of surface, or about 9.5 per cent higher than Griffith's and Davis' value (1.28×10^{-3}) for the corresponding height. However, they found that the constant increased quite rapidly for heights less than about 12 in. (30 cm.) as will be seen from the following table.

Height in inches	Constant
11.8 (30 cm.)	0.00128
7.87 (20 ")	0.00175
5.9 (15 ")	0.002
3.94 (10 ")	0.0023
1.97 (5 ")	0.0035

Part II

Effect of Altitude on Temperature Rise of Self-Cooled Transformers and Other Apparatus Having Various Shapes of Surfaces

Self-Cooled Transformers

(*1*) *Radiation.* Altitude, of course, does not affect radiation of heat.

It was shown in Part I that the maximum part radiations plays in practise is approximately 55 per cent at or near sea level for a plain black surface, and that for certain types of complicated tank surfaces, the loss by radiation may be as low as 15 per cent of the total.

(*2*) *Convection.* The general results obtained by various investigators indicate that the loss by convection varies approximately as the square root of the air density. While Doherty and Carter's tests show that the exponent is 0.542, Rice's data[8] obtained on horizontal cylinders tested with the same apparatus show that the exponent lies below 0.5. For example, the data from Rice's Table III have been plotted on double-log paper and are shown in Fig. 8. (Before plotting these results the losses were all corrected for the three nearest pressures, namely, 9, 27, and 76 cm. of mercury.) These results are quite consistent and show that for the cylinder 1.68 in. (4.28 cm.) in diameter and 48 in. (122 cm.) long, the loss varies as the pressure raised to the 0.4 power. The data shown in his Table II for the largest cylinder 4.5 in. (11.42 cm.) diameter by 60 in. (152.5 cm.) long, while not so complete and not replotted, show that the loss varies as the pressure raised to the 0.45 power.

No single test or set of tests in an investigation of

8. Free Convection of Heat in Gases and Liquids, II A. I. E. E. Journal, February, 1924, by C. W. Rice.

this nature is conclusive and the best thing to do in drawing a conclusion is to take a general average of all the available data. In reviewing the results of various investigators, it has been found that the value of the exponent ranges anywhere from 0.4 to about 2/3. Rice's[9] final formula, *i. e.* equation (4), shows that for tall vertical planes, the loss is proportional to the square root of the pressure. The writer has found, as will be shown later, that the square root law agrees best with his experimental observations made in 1915 at three different altitudes on various shapes of tank surfaces.

Based on the square root law, the formula for free convection is:

$$W_c = K\,A\,\theta^{5/4}\,P^{1/2} \qquad (8)$$

in which W_c = Convection loss in watts
A = Area in square inches.
K = Constant = 0.0014 at or near sea level.
θ = Temperature rise in deg. cent.
P = Air pressure in atmospheres.

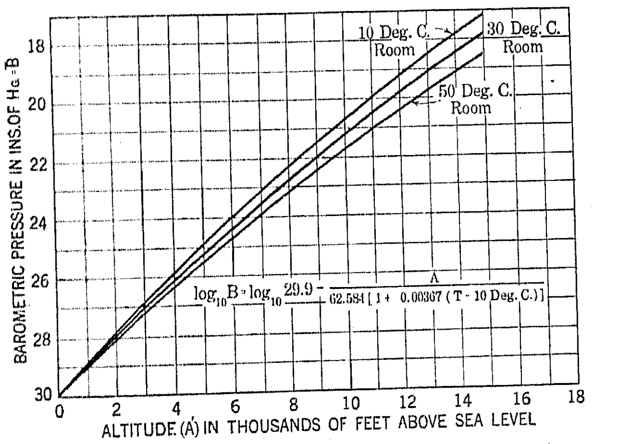

FIG. 9—VARIATION OF BAROMETRIC PRESSURE WITH ALTITUDE

Expressed in the reverse order, equation (8) becomes:

$$\theta = \frac{1}{P^{.4}}\left(\frac{W_c}{A\,K}\right)^{.8} \qquad (9)$$

For a constant loss dissipated by free convection the temperature rise therefore varies inversely as the air pressure raised to the 0.4 power.

Fig. 9 gives curves of barometric pressure in inches of mercury vs. altitude in thousands of feet above sea level for three different room temperatures. These curves were calculated by the Smithsonian Institute's formula:

$$\mathrm{Log}_{10} P = \log_{10} 29.9 - \left[\frac{A'}{62.58\,(1 + 0.00367\,(T - 10°\,C)}\right] \qquad (10)$$

in which P = Pressure in inches of mercury.
T = Temperature in deg. cent.
A' = Altitude in 1000's of feet above sea level.

9. Free Convection of Heat in Gases and Liquids, II A. I. E. E. JOURNAL, February, 1924, by C. W. Rice.

Substituting the values of P for various values of altitude (taken from 30 deg. cent. room temperature curve in Fig. 9) in equation (9), we find the following relation between altitude and temperature rise where all loss is by convection:

Altitude in thousands of feet	2	4	6	8	10	12	14
Per cent increase in temperature rise...........	2.9	5.8	8.5	11.8	14.5	17.9	21.2

It will be noted that the rate of increase in temperature is about 1.5 per cent per 1000 ft. increase in altitude.

Since air density does not affect radiation, the correction is smaller than the above for tanks where part of the loss is dissipated by radiation. For example, for a constant loss the following equation holds approximately for any shape of surface:

$$\theta_1 = \theta + \frac{1.5\,A'\,\theta}{100}\left[\frac{W_c}{W_c + W_r}\right] \qquad (11)$$

in which θ_1 = temperature rise in deg. cent. at higher altitude.
θ = temperature rise at lower altitude.
A' = altitude in thousands of feet above sea level.
W_c = loss by convection at the average altitude considered.
W_r = loss by radiation at the average altitude considered.

Table VIII gives the comparison between calculated

TABLE VIII

COMPARISON OF ESTIMATED AND TESTED TANK TEMPERATURE RISES OBTAINED IN 1915 AT VARIOUS ALTITUDES

Location....................			Pittsfield		Boulder		Leadville	
Observed inches of mercury,..			28.7		24.3		20.7	
Altitude in feet for 30 deg. cent. room..............			1200.		6000.		10,700.	
Depth of Corrug.	Approx. Average Ratio $\frac{W_c}{W_c + W_r}$		Max. Temperature Rise deg. cent.					
			Oil	Tank	Oil	Tank	Oil	Tank
9.35 in.	0.85	Test	50.7	47.5†	54.3	51.5	57.3	54.2
		Calc.*	—	—	54.6	51.2	56.8	53.2
3.5 in.	0.65	Test	56.5	52.4†	57.5	53.9	61.7	55.9
		Calc.	—	—	59.1	54.8	61.7	57.2
0 in. (Plain)	0.45	Test	60.	48.5	58.2	46.3	62.8	51.4
		Calc.	—	—	62.	50.0	63.8	51.6

*By equa. (11) and based on rise at Pittsfield.
†By thermometer in inner bend of corrugation.

and tested values of maximum oil and tank rises for three styles of tanks, each of which was heated with a constant loss at Pittsfield, Mass., Boulder and Leadville, Colorado.[10] In fact, all conditions excepting altitude were as nearly the same as it was possible to make them, by using the same temporary housing, meters, thermometers, etc. The tank thermometers, after being

10. See paper by V. M. Montsinger, Effect of Barometric Pressure on Temperature Rise of Self-Cooled Stationary Induction Apparatus. A. I. E. E. JOURNAL, June, 1916.

placed on the surface in Pittsfield, were not disturbed until after the tests were completed at Leadville.

It will be noted that with the exception of the test on the plain tank at Boulder, the tests and calculated values (based on square root law) agree as well as could be expected. The other heat runs with lower rises given in the June 1916 A. I. E. E. paper show approximately the same agreement with the calculated values.

So far, we have discussed only conditions where the loss is constant; *i. e.*, the loss is unaffected by the increased temperature. In transformers where a part (copper loss) of the total loss is increased with temperature, the effect is to increase the percentage somewhat greater than 1.5 per cent for each 1000 ft. increase in altitude. For instance, according to equation (**15**) given in the paper by V. M. Montsinger of June, 1916: to which we previously referred:

$$\frac{\theta}{\theta_s} = (1 + \phi_2/100)\left(1 + \frac{\dfrac{\phi_2\, n\, a\, \theta_s}{100}}{234 + \theta_0 + a\,\theta_s\,(1 - n)}\right) \tag{12}$$

in which

θ = Temperature rise at higher altitude

θ_s = Temperature rise at lower altitude

ϕ_2 = Per cent increase in temperature rise per 1000 ft. in going from lower to higher altitude with a constant loss.

n = 0.8 = reciprocal of exponent in convection formula

a = Ratio of copper loss to total (iron plus copper) loss.

θ_0 = Room temperature in deg. cent.

The following shows the effect of the increased copper loss, due to increased temperature in increasing the rise for various ratios of copper to iron losses as calculated by equation (**12**)

—a— Ratio of Copper Loss to total loss	Increase in Temperature Rise in Per cent.	
	Per 1000 ft.	Per 1000 meters
0	1.5	4.58
0.50	1.62	4.94
0.67	1.65	5.03
1.00	1.75	5.24

The effect of temperature is to decrease the iron loss, but its effect is so small, being in the order of 1 or 2 per cent for a change of 40 to 50 deg. cent., that it can be neglected. Also it is standard practise to express the temperature increase or decrease in per cent of the guaranteed winding rise over the ambient. Since the winding rise over the tank surface is not affected by air density, the 1.5 per cent per 1000 ft. should be reduced by the ratio of the winding rise to the tank surface rise for all self-cooled oil-immersed transformers. Expressed in meters, the general equation for any type of self-cooled transformer or other type of self-cooled stationary apparatus may be expressed, close enough for practical purposes, by:

$$\phi = A''\, N\, S\, (4.6 + a) \tag{13}$$

in which ϕ = per cent increase in temperature rise

A'' = Increase in altitude in thousands of meters

N = Ratio of tank surface rise to winding rise over ambient temperature.

$S = \dfrac{W_c}{W_c + W_r}$ = ratio of convection to total loss

a = ratio of copper to total loss.

In case it is more convenient to express S in terms of the ratio of surfaces effective for convection and radiation *i. e.* the developed and envelope surfaces, we may put

$$S = \frac{45\,D}{45\,D + 55\,Ee}$$

$$= \frac{D}{D + 1.22\,Ee}$$

in which S = shape factor

D = total developed area

E = outside envelope area

e = emissivity factor

If the difference in altitude considered is large, the ratio of 45 to 55 no longer exactly holds because for a given rise the loss by convection decreases as the altitude increases, whereas the loss by radiation remains the same. For example, in going from sea level to 5000 ft. above sea level, the convection loss for a given temperature rise over a 30 deg. cent. ambient is decreased by the factor $\left(\dfrac{25.2}{29.9}\right)^{0.5} = 0.915$. This means that the 45 per cent factor has been reduced to 41.2 per cent. During this time, the temperature rise has slightly increased but the ratio of 42.2 to 55 still holds. For altitude differences of, say, 10,000 ft. (3000 m.) and over, it should be satisfactory for practical purposes to use the ratio corresponding to an average altitude of 5000 ft. and write

$$S = \frac{D}{D + 1.33\,Ee}$$

Summary of Parts I and II

1. The loss by free convection is proportional to the temperature rise raised to the 5/4 power.

2. For a vertical tall plane at sea level (and where the emissivity = 0.9) the ratio of losses by free convection and radiation are approximately 45 to 55 respectively.

3. For a given temperature rise, the loss by convection is proportional to the square root of the barometric pressure.

TABLE IX
EFFECT OF ALTITUDE ON TEMPERATURE RISE

Type of apparatus	N	S	a	ϕ
(a) Oil Immersed self-cooled transformers	—	—	—	Per cent in copper rise per 1000 m. (approx.)
(1) with plain tanks	40/55	0.45	2/3	1.75
(2) with corrugated, tubular and radiator tanks	40/55	0.80	2/3	3.0
(b) Air-cooled Transformers of Dry Type				
(1) Core wound coils	40/55	0.60	2/3	2.3
(2) Form wound coils	1.0	1.0	1.0	5.25
(c) Air Cooled Reactors	1.0	1.0	1.0	5.25
(d) Rheostats, Relays, etc.	1.0	0.80	1.0	4.5
(e) Bus bars	1.0	0.6	1.0	3.5
(f) Water-cooled transformers				0.0

4. For a given loss, the temperature rise (when the loss is by free convection) is inversely proportional to the barometric pressure raised to the 0.4 power.

5. For a constant loss dissipated by free convection, the surface temperature rise increases about 4.6 per cent for each 1000 m. increase in altitude.

6. By assigning practical values to the factors in the formula: $\phi = A'' N S (4.6 + a)$ Table IX gives the effect of altitude on the temperature rise of different types of stationary apparatus.

7. Considered from the standpoint of effect of altitude on change in kv-a. for a given temperature rise, Table X shows the per cent variation for representative cases of the two classes of transformers given under 6 (a) in which it is assumed (1) that the oil rise varies as the total loss raised to the 0.8 power, (2) that the winding rise over the oil is proportional to the square of the load, (3) that the ratio of copper to iron loss at sea level is 2:1 respectively, and (4) that the ratio of the copper to the oil rise is 55 to 40 respectively.

TABLE X
EFFECT OF ALTITUDE ON KV-A. OF SELF-COOLED TRANSFORMERS

Transformer with:	Per cent Decrease in kv-a. per 1000 m.
(a) Plain tanks	1.35
(b) Corrugated, tubular and radiator tank	2.3

Discussion

For discussion of this paper see page 839.

Effect of Altitude on Temperature Rise

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Review of the Subject.—*The problem is to determine the effect of altitude in increasing the temperature rise of electrical apparatus.*

Stated in detail, heat is dissipated from the various surfaces of apparatus principally by convection and radiation. The former is a function both of temperature rise and of air density, that is, of altitude; the latter, of temperature rise only. The energy loss in the apparatus, that is, the heat to be dissipated, is not changed significantly by altitude. Thus, lower air density at higher altitude means decreased convection. Hence, higher temperature, which increases both convection and radiation, is required to carry off the same heat. Specifically, then, the problem is to relate these several factors so that quantitative calculations can be made for various types of apparatus.

An investigation was made in 1921 to determine how dissipation by convection varies with temperature, air density, and also with respect to the air movement—i. e., whether it is a blast or merely the natural movement created by temperature difference: in other words, whether it is "forced" or "free" convection. The results check reasonably with those obtained by Montsinger[1], Rice[2] and others.

The law of radiation, of course, has long since been well known.

The tested apparatus, consisting of electrically heated plates, was enclosed in a large wooden tank or drum, in which the air pressure could be controlled.

The constants from these tests were used in setting up the general equation (26), *and equations,* (30) *and* (32), *which give the increase in temperature rise occasioned by lower air pressure, for free and forced convection respectively. These are plotted in Figs. 16 and 17. Reasonable assumptions simplify these equations for practical calculations for various kinds of apparatus.*

To study altitude effect on actual rotating apparatus, a 7½ kv-a. synchronous motor was tested in the drum at various air pressures. The actual increase in temperature rise agrees well with the values calculated by the above equations. Also tests were made on a 2550-h. p. high-speed synchronous motor, first at Lynn, Mass. (sea level), *and later at Cerro De Pasco, South America, at an altitude of 3700 m. These tests afforded an opportunity to check calculations against actual data on rotating apparatus.*

Table I gives the increase in temperature rise which, on the above bases, would be expected in various types of electrical apparatus.

Introduction

THE effect of altitude on temperature rise has been a live engineering question ever since temperature guarantees have been made on electrical apparatus for operation at high altitudes. The answer has been evolved piecemeal. While a number of investigations[3] bearing on the subject preceded Montsinger's[1] paper in 1916, yet the latter appears to have been the first real attempt to make a rational calculation of altitude effect for a specific type of apparatus, *i. e.*, transformers. Later investigations by Rice[2] and also by Davis[4] afforded valuable, additional data regarding the dissipation of heat from surfaces. In none of these, however, has any attempt been made to calculate the effect of altitude for apparatus involving "forced convection,"[5] such as rotating apparatus and air-blast transformers. It is the purpose of the present paper to cover this ground, as well as to confirm Montsinger's results on stationary apparatus.

Plan of the Paper

Before the effect of altitude could be calculated, it was necessary to establish how dissipation by convection is related to temperature, air density, and air movement. This was determined by tests made in 1921. Dissipation by radiation is well known.[6] Thus, the paper is divided as follows:

(A) Tests on heated plates, to determine the variation of convection with temperature, air density and air movement.

(B) Comparison of results with those of other investigators.

(C) Final equations, which are applicable to various types of electrical apparatus, and the premises from which they were derived.

(D) Comparison of calculations with test results.
- a. Plates tested in the drum.
- b. 7½ kv-a. synchronous motor tested in the drum.
- c. 2250-h. p. synchronous motor tested at Lynn, Mass. and then at Cerro De Pasco, S. A., altitude 12,000 ft.

(E) Calculation and tabulation of temperature corrections for various types of apparatus.

(F) Acknowledgments.

(G) Notation.

A. Tests on Heated Plates

In order to determine the effect of atmospheric pressure on temperature rise, electrically heated plates, Figs. 3 and 4, were tested in a large drum, Fig. 1, in which the pressure was controlled by a vacuum pump, and the temperature by water pipes, Fig. 2. In spite of the pipes, practically uniform wall temperatures were secured by surrounding the plates by a cylindrical card-board lining, Fig. 2.

1. V. M. Montsinger, A. I. E. E. TRANS., Vol. XXXV, p. 559, 1916.

2. C. W. Rice, A. I. E. E. JOURNAL, Dec. 1923, p. 1288.
C. W. Rice, Mid-Winter Meeting A. I. E. E., Feb. 1924.

3. See (B) Comparison of Results with those of Other Investigators.

4. A. H. Davis, *Phil. Mag.*, Vol. 40, p. 692, 1920; Vol. 41, p. 899, 1921; Vol. 43. p. 324, 1922; Vol. 44, p. 420, 1922.

5. High velocity of cooling air relative to the velocity in "free convection," that is, air movement produced by temperature difference alone.

6. Stefan-Boltzman Law: $W_r = 5.7\, e\, (T_2^4 - T_1^4) \times 10^{-12}$ watt sper sq. cm. where T_2 and T_1 are respectively the absolute temperatures of the surface radiating, and of the surface receiving, the heat, and e is the constant of emissivitv.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

Temperatures were measured by thermo-couples:

(a) Partially embedded at a number of points at the surface of the card-board;

(b) Suspended in ambient air;

(c) Soldered on the surface of the plate.

Tests were made with and without fan to determine the effect of air movement—that is, to determine the relative effects of "free" and "forced" convection. The tests are thus divided into groups: (1) Free Convection; (2) Forced Convection.

1. *Free Convection.* A number of preliminary tests were made on the "single plate," shown in Fig. 3. However, slight uncertainties regarding dissipation from the sides and bottom,[7] precluded these particular data from influencing final conclusions regarding free convection. It is worth noting, however, that the results checked well with subsequent, more accurate data.

To avoid such uncertainties a "double plate," Fig. 4, was used. It was two steel plates with a heating element, between them, thus affording practically uniform surface and temperature. The plate surfaces were vertical.

Tests were made at four different pressures, namely 30, 22, 15 and 7 in. (absolute) of mercury. At each pressure a number of points was taken, giving curves of temperature rise vs. watts input, Fig. 5.

7. Although these were heavily insulated from the heated surface.

The computed zero pressure curve, Fig. 5, gives the watts dissipated by radiation. It is important to establish this curve accurately, since conclusions regarding convection rest upon it. It was arrived at in two different ways: analytically, from the Stefan-Boltzman Law,[8] and graphically, by extrapolating the watts-pressure curves, Fig. 7, to zero pressure. The curves in Fig. 7 were derived from Fig. 5.

The following method was used in analyzing the test curves given in Fig. 5. Since convection is a function of pressure, whereas radiation is not, it follows that the difference between any two curves as measured at constant temperature rise, must be the difference in convection produced by the two different pressures. Therefore, for any curve, Fig. 5, the abscissas, as reckoned from the radiation curve, instead of from zero watts, give the dissipation by convection. In this way, the curves in Fig. 8 were derived showing the variation of dissipation by pure convection, with temperatue rise and pressure.

Next, the plate was given a coat of aluminum paint, and tests under otherwise similar conditions were repeated. The amount of convection, as found by subtracting the radiation loss from the total watts input, should be practically unchanged by the change in paint; and it was so found. The radiation curve with aluminum paint was obtained by using the

8. Assuming $e = 0.9$ for the blackened surface.

TABLE I

CALCULATED PER CENT CORRECTION IN TEMPERATURE RISE FOR EACH 1000 METERS ALTITUDE, CALCULATED IN SECTION E

NOTE: These values are not proposed for A. I. E. E. rules. They might form the basis for rules; but, in this table, they merely represent the final results of the paper, as calculated under the assumptions in Section E.

Apparatus	Element	Per cent Temp. Corr.* per 1000 Meters x_m as applied to surface temperature rise	as applied to copper temperature rise
Turbine generators (enclosed ventilation)	armature coil	10.7	6.25
	field (rotor)	11.0	7.15
Enclosed type generator and motor			
(a) enclosed ventilation	armature coil	10.7	6.25
	field coil		
	insulated	10.7	7.75
	bare copper strip	10.7	10.7
(b) Open type (generators and motors)	armature coil	9.0	6.5
	field coil		
	insulated	9.0	6.5
	bare cu. strip	9.0	9.0
Totally enclosed motors. No ventilation	armature or field	5.35	4.6
Transformers	tank surface		
(a) Water-cooled		0	0
(b) Air-blast†		11.2	11.2
(c) Natural-draft†		4.6	4.6
(d) Oil-immersed, self-cooled			
(1) plain tank		2.2	1.6
(2) corrugated, tubular or radiator tank		4.2	3.06
Reactors and Bus Bars		4.7	4.7
Rheostats, Meters, Relays and Miscellaneous Stationary Apparatus		4.2	4.2

*The difference between these columns is due merely to the different percentage base: surface rise in one, copper rise in the other.

†Hot spot surface temperature may be as high as the *average* copper temperature as measured by resistance.

emissivity e for aluminum given by Langmuir.[9] The close agreement thus found, was an additional check on the radiation curve. See Fig. 12.

It is assumed that the rate of dissipation by convection, W_c, is the following function of temperature rise θ and pressure p:

$$W_c = k\,\theta^l\,p^m \tag{1}$$

Plotting the data given in Fig. 8 on log-log paper, Figs, 9 and 10, the following values were found: $l = 1.3$. $m = 0.54$.

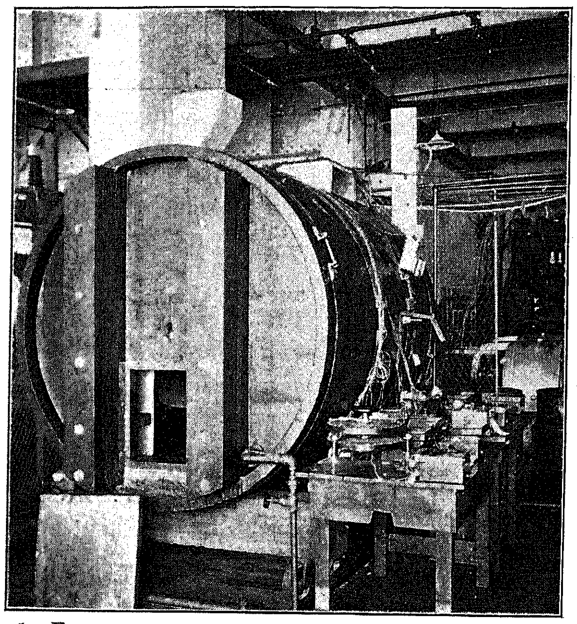


FIG. 1—EXTERNAL VIEW OF DRUM USED IN ALTITUDE TESTS

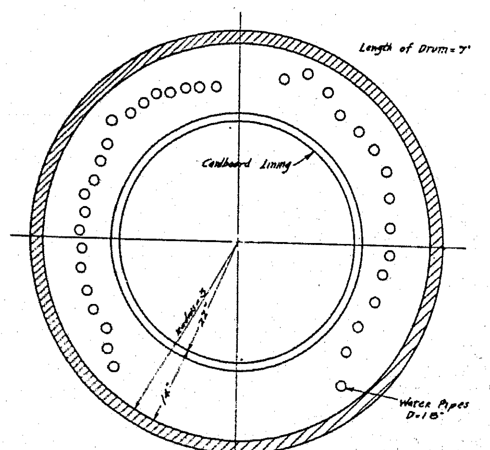


FIG. 2—CROSS SECTION OF DRUM SHOWING WATER PIPES AND CARDBOARD LINING

The data in Figs. 5 and 8 were plotted in another form, Fig. 11, convenient for the present study, showing the increase in temperature rise $\dfrac{\Delta\,\theta}{\theta_s}$ vs. the pressure p, for the same watts dissipated. One of these curves gives this increase for 47 per cent[10] convection and the other for 100 per cent convection. Since the present investigation is only concerned with the range[11] 20 in. to 30 in., the curves have been approximated by straight lines, Fig. 11, cutting the curves at 23 in.

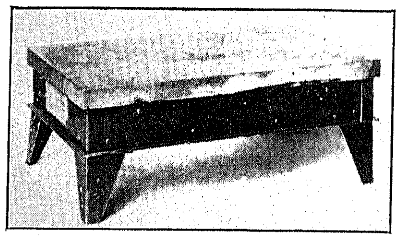


FIG. 3—SINGLE PLATE USED IN ALTITUDE TESTS

2. *Forced Convection.* Tests were made on the single plate[12] using a 16 in. fan for ventilation. First, the fan blast was directed across the plate, parallel with the surface; then, at an angle of 45 deg. The plate surface was horizontal. The fan speed was

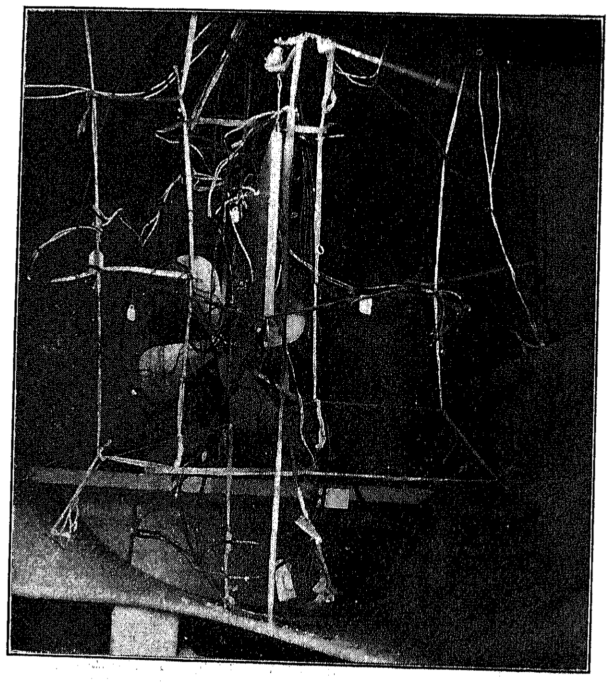


FIG. 4—DOUBLE PLATE MOUNTED IN DRUM, AS USED IN ALTITUDE TESTS

9. *Trans. Am. Chem. Soc.*, Vol. XXIII, p. 229, 1891, e = 0.67 at 325 deg. K, e = 0.60 at 350 deg. K, e = 0.53 at 365 deg. K, e = 0.45 at 400 deg. K.

10. 47 per cent convection, 53 per cent radiation, as found by comparing the radiation curve with total watts curve at 30 in. in Fig. 5.

11. This corresponds to altitudes up to about 12,000 ft. = 3700 meters.

12. Although these tests were subject to slight uncertainties, as discussed under "Free Convection," yet the errors, which were small in the latter case, are negligible in this case, where the blast across the heated surface carried off practically all of the heat. Moreover, this investigation is concerned with comparative, not absolute results. Forced convection was tried on the double plate, but the edge of the plates split the stream of air, caused eddies and therefore non-uniform temperatures over the surfaces.

kept constant under all conditions, giving a maximum air velocity of 1000 ft. per min.

The results are shown in Figs. 13 and 14. These were analyzed, in the same manner as the "free convec-

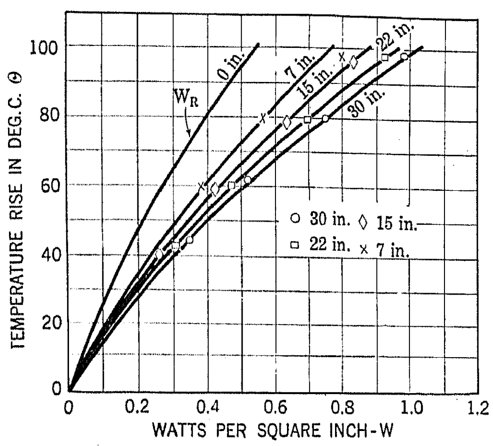


FIG. 5—TEST CURVES ON DOUBLE PLATE—FREE CONVECTION

tion" tests and the average results given for forced convection,

$$W_c \sim \theta^{1.00}\, p^{0.73}$$

3. *Summary of Results, Plate Tests.*

Free Convection, $W_c \sim \theta^{1.3}\, p^{0.5}$

Forced Convection, $W_c \sim \theta^{1.0}\, p^{0.73}$

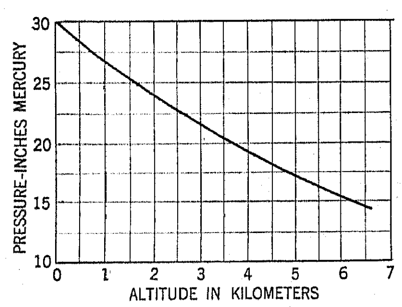


FIG. 6—ATMOSPHERIC PRESSURE—ALTITUDE CURVE

The per cent increase in temperature rise y_m per 1000 m. altitude as determined by the straight line approximation, Fig. 11 is given in Table II.

TABLE II

Convection W_c	Free or Forced Convection	y_n
47 per cent	Free	2.20 per cent
100 per cent	Free	4.80 per cent
74 per cent	Forced	5.50 per cent
100 per cent	Forced	8.5 per cent

B. COMPARISON OF RESULTS WITH THOSE OF OTHER INVESTIGATORS

1. *Free Convection.* The results of previous investigations can be put in the form,

$$W_c \sim \theta^l\, p^m$$

They are:

	l	m
Dulong & Pitt (1817)	1.23	n (n varies from 0.38 to 0.517 with nature of gas.)
Lorenz[13]	1.25	0.5
Kennelly & Sanborn[14]	0.5	—
Compan[15]	1.23	0.45
Montsinger[1]	1.25	—
Davis[4]	1.23	0.466
Griffiths & Davis[16]	1.25	—
Rice[17]	$1.0 + \frac{n}{2}$	n

The results found by the authors for free convection are: $l = 1.3$, $m = 0.54$.

The agreement is good with the values listed above, there being only a slight difference in the exponent l.

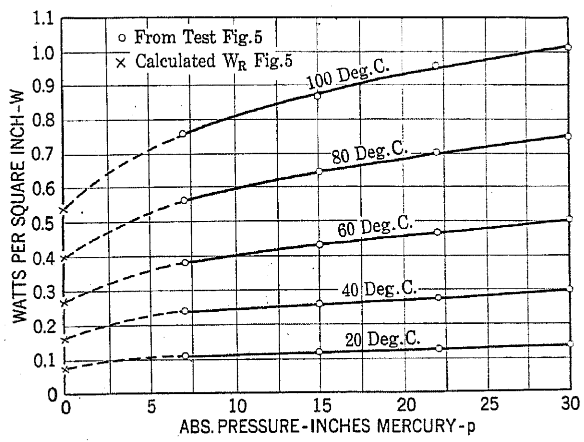


FIG. 7—DOUBLE PLATE TESTS—FREE CONVECTION: WATTS DISSIPATED VS. ABSOLUTE PRESSURE

The following average results will be used in equations for free convection:

$$W_c \sim \theta^{1.25}\, p^{0.5}$$

2. *Forced Convection.* Results of other investigations are:

Rice[18] $l = 1.0$ $m =$ (see footnote)

Davis[4] $l = 1.0$

The authors found for forced convection,

$$W_c \sim \theta^{1.0}\, p^{0.73}$$

Data regarding W_c as function of p for forced convection are not available to as satisfactory extent as

13. L. Lorenz, *Ann d'Phys.*, Vol. XIII, p. 582, 1881.

14. A. E. Kennelly & H. S. Sanborn, *Proc. Am. Phil. Soc.*, Vol. LM, p. 55, 1914.

15. Paul Compan, *Ann d'Phys.*, Vol. 26, p. 488, 1902.

16. Dept. of Scientific & Industrial Research, Food Investigation Board, Sp. Report 9—H. M. Stationary Office London, 1922.

17. C. W. Rice, Mid-Winter Meeting A. I. E. E., Feb. 1924. Rice finds that n varies with type of surface from ½ to 2/3, but that for most surfaces, $n \simeq$ ½. This gives $n = 1.25$, $m = 0.5$.

18. C. W. Rice, A. I. E. E. JOURNAL, Dec. 1923, p. 1288. Rice found that m varies from ½ to 1 depending on the shape and nature of surface, and on the degree of turbulence.

for free convection; but, Rice's results check reasonably with those found by the authors.

Thus, the following relation will be used in equations for forced convection in various apparatus.

$$W_c \sim \theta^{1.0} p^{0.75}$$

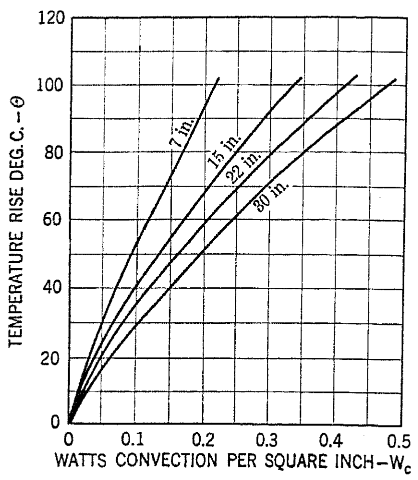


FIG. 8—DOUBLE PLATE TESTS—FREE CONVECTION—100% W_c CURVES. DERIVED FROM FIG. 5

In this, one may have confidence in the exponent of θ. There may be slight question as to the possible variation of the exponent of p with character of surface, air velocity, etc.; but, it is the authors' opinion that the value $m = 0.75$ is a reasonable average for the various conditions in electrical apparatus, to which the results have been applied in this paper.

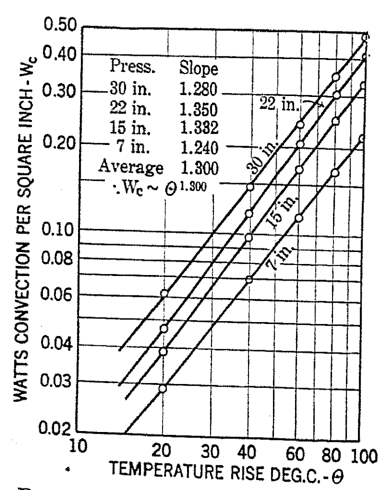


FIG. 9—DOUBLE PLATE TESTS—FREE CONVECTION LOG W_c VS. LOG θ

C. FINAL EQUATIONS AND PREMISES FROM WHICH THEY WERE DERIVED

1. *Premises.* (a) From (B), the dissipation by convection is:

$$W_c \sim \theta^l p^m \quad \text{(general)}$$
$$W_c \sim \theta^{1.25} p^{0.5} \quad \text{(free convection)}$$
$$W_c \sim \theta^{1.0} p^{0.75} \quad \text{(forced convection)}$$

(b) Dissipation by radiation,[10]

$$W_r = 5.7\, e\, (T_2^4 - T_1^4) \cong k\, (T_2 - T_1)^{1.25} = k\, \theta^{1.25} = k\, \theta^n$$

(c) Relation between altitude and pressure, as shown in Fig. 6.

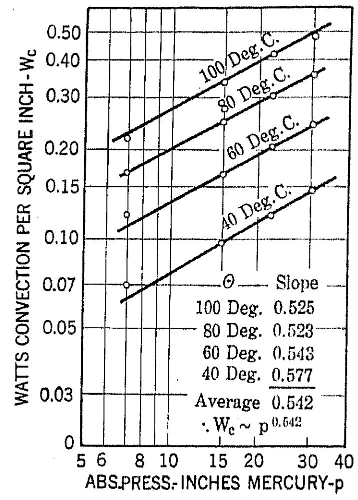


FIG. 10—DOUBLE PLATE TESTS—FREE CONVECTION LOG W_c VS. LOG p

(d) At constant speed of rotation the velocity of cooling air is the same at all pressures.

(e) Temperature rise of a given volume of air, for a given increment of heat, is inversely as the pressure.

(f) The total losses of electrical apparatus are

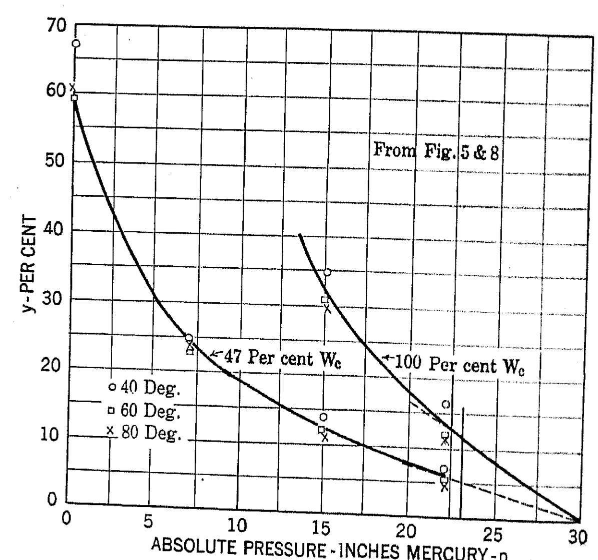


FIG. 11—DOUBLE PLATE TESTS—FREE CONVECTION

Test curves showing the general character of the increase in temperature rise above rise at 30 in. vs. absolute pressure inches mercury.

dissipated in two ways: convection and radiation. Thus, the total dissipation is,

$$W = W_c + W_r \qquad (1)$$

19. In the range up to a temperature rise of 100 deg. cent. this approximate relation holds. This is shown in Fig. 15, by plotting the following relations on log paper:

$$(T_2^4 - T_1^4) \text{ vs. } (T_2 - T_1)$$

It is assumed that T_1 (which, strictly, is the temperature of the surface receiving the radiation) is equal to the temperature of the cooling air.

(g) In electrical apparatus, the total heat dissipated from a surface comes from three possible sources:

1. Copper losses ($I^2 R$) which increase with altitude, due to the higher temperature and, therefore, higher resistance.

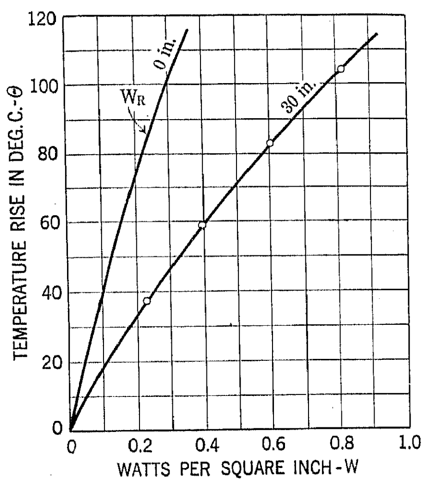


FIG. 12—TEST CURVE ON DOUBLE PLATE, COATED WITH ALUMINUM PAINT—FREE CONVECTION

2. Windage losses, which decrease with altitude, being inversely as the density of the air.
3. Core loss and other losses which are not affected by altitude.

(h) The total temperature difference between the copper and ambient air, comprises the drop through insulation, which drop is not affected by altitude; drop from the exposed surface to the cooling air; and the temperature difference due to the rise of the cooling air as it passes through the machine. The latter two are each affected by altitude.

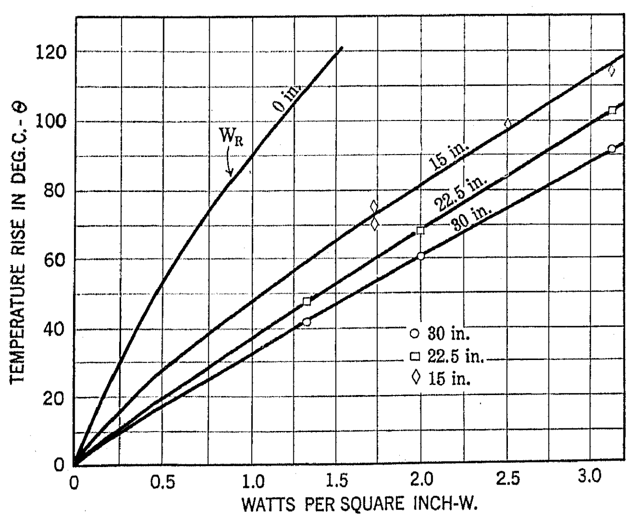


FIG. 13—TEST CURVES ON SINGLE PLATE, FORCED CONVECTION PLATE HORIZONTAL AND FAN DIRECTED HORIZONTALLY

(i) Free convection applies to stationary apparatus; forced convection, to all rotating apparatus.

(j) Sea-level pressure taken as 30 in. of mercury.

2. *Final Equations.* From the above premises, the following equations[20] are derived in Appendix A:

(*a*) *General.*

$$y = \frac{\Delta\theta}{\theta_s} = \frac{w_c D - k_1 \dfrac{\Delta p}{p_s} + k_2 \dfrac{\Delta p}{p\,a}}{w_c\,[1\,(1-D) - n] + n - k_3} \qquad (18)$$

where:

$$D = 1 - \left(\frac{p_a}{p_s}\right)^m ; \quad k_1 = b\,\frac{234 + T_s}{T_c}$$

$$k_2 = \frac{a\,\theta_s'}{T_c} ; \qquad k_3 = \frac{a\,\theta_s}{T_c}$$

$$T_c = 234 + T_s + a\,\theta_c^s$$

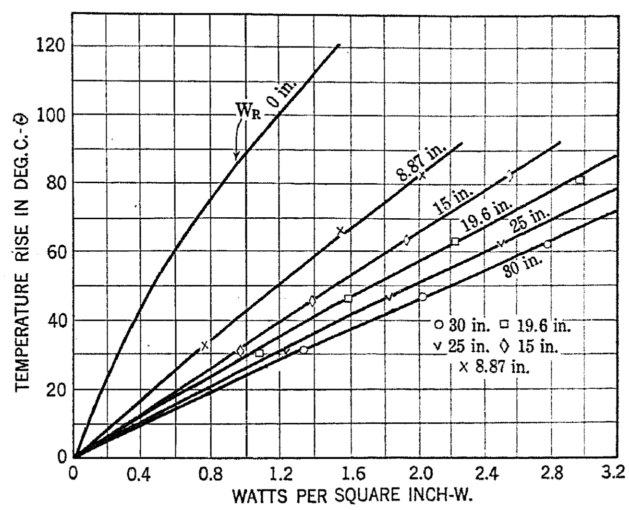


FIG. 14—TEST CURVES ON SINGLE PLATE, FORCED CONVECTION PLATE HORIZONTAL AND FAN POINTING DOWNWARD AT 50 DEG.

and where ($\Delta\,\theta$) is the increase (due to altitude) in *surface* temperature rise above the rise (θ_s) at sea level.

$$z = \frac{\Delta\,\theta'}{\theta_s} = \frac{\Delta\,p}{p_a} \qquad (20)$$

where ($\Delta\,\theta'$) is the increase, due to altitude, in temperature rise of the cooling air above its rise (θ_s') at sea level.

also

$$x = \frac{\Delta\,\theta + \Delta\,\theta'}{\theta_s + \theta_s'} = y\,\frac{\theta_s'}{\theta_s + \theta_s'}\,(z - y) \qquad (24)$$

(*b*) *Special Cases and Approximations.*[21]

1. *Stationary Apparatus—Free Convection.*

$$y_m = y/A = \frac{w_c\,D}{A\left[\,1.25\,(1 - w_c\,D) - \dfrac{a\,\theta_s}{T_c}\,\right]}$$

(as fraction) (26)

20. See "NOTATION" for definitions of symbols.
21. See Appendix A.

or, for altitude less than 3500 m., and assuming $\theta_s' = \theta_s$

$$y_m \cong y/A = w_c\left(4.8 + \frac{a\,\theta_s}{60}\right) \text{ in per cent} \qquad (28)$$

Eqs. (26) and (28) are plotted in Fig. 16.

(2) *Stationary Apparatus—Forced Convection.*

$$y_m = y/A = \frac{w_c D + \dfrac{a\,\theta_s'}{T_c}\,\dfrac{\Delta p}{P_a} - k_1 \dfrac{\Delta p}{P_s}}{A\left[1 - w_c D + 0.25\, w_r - \dfrac{a\,\theta_s}{T_c}\right]}$$

(as fraction) **(30)**

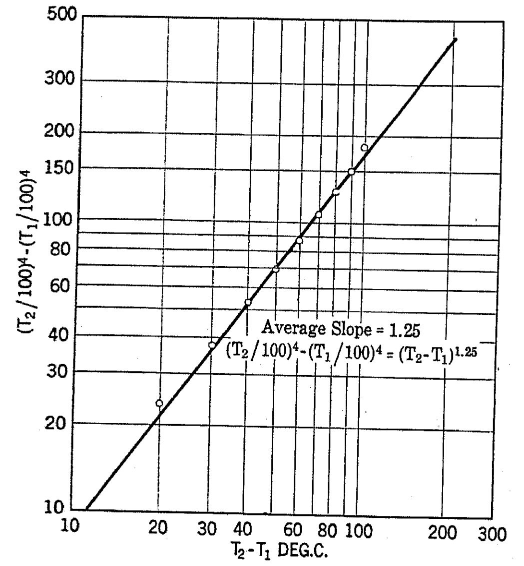


Fig. 15—Log $(T_2/100)^4 - (T_1/100)^4$ vs. Log $(T_2 - T_1)$

or, for altitude less than 3500 meters,

$$y_m \cong 14\, w_c + \frac{a\,\theta_s}{16} - 4.7 \text{ in per cent} \qquad (31)$$

Equations (29) and (31) are plotted in Fig. 17.

The above corrections y apply to *surface* temperature rise with respect to the cooling air. With respect to the ingoing air temperature, the correction is,

$$x_m \cong 11.6\, w_c \qquad (34)$$

(3) *Rotating Apparatus—Forced Convection.*

$$y_m \cong y/A = \frac{w_c D - 0.09\,\dfrac{\Delta p}{p_s} + \dfrac{a\,\theta_s'}{T_c}\,\dfrac{\Delta p}{p_a}}{A\left[1 - w_c D + 0.25\, w_r - \dfrac{a\,\theta_s}{T_c}\right]}$$

(as fraction) (35)

or, for altitudes less than 3500 m.,

$$y_m \cong 14\, w_c - 4 \quad \text{in per cent} \qquad (36)$$

Also

$$x_m \cong 10.7\, w_c \quad \text{in per cent} \qquad (37)$$

D. Comparison of Calculations with Test Results

1. *Plate Tests.* To properly appraise the value of this comparison, remember that in the analysis of the test data, convection and radiation were segregated into separate and different functional relations with respect to pressure and temperature rise. These relations were incorporated with other premises, Section C, into the general equations (**18**) and (**24**), applicable to conditions in various classes of apparatus.

Applying equation (**18**) to the conditions of the plate test, equations[22] (**38**) and (**39**) are obtained for free and forced convection respectively. These relations are represented in Figs. 16 and 17 by the curves $a\,\theta_s = 0$; they are also plotted in Fig. 20 with test points shown for comparison.

The test data referred to these curves comprise only four points:

Single plate:

$w_c = 1.0$ for forced convection
$w_c = 0.74$ " " "

Double plate:

$w_c = 1.0$ for free convection
$w_c = 0.47$ " " "

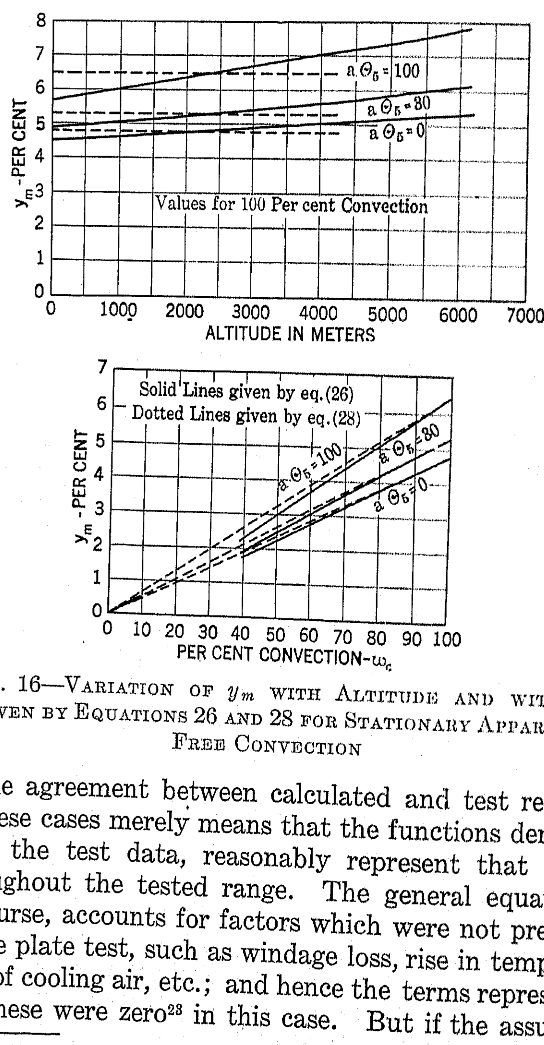


Fig. 16—Variation of y_m with Altitude and with w_c as Given by Equations 26 and 28 for Stationary Apparatus, Free Convection

The agreement between calculated and test results in these cases merely means that the functions derived from the test data, reasonably represent that data throughout the tested range. The general equation, of course, accounts for factors which were not present in the plate test, such as windage loss, rise in temperature of cooling air, etc.; and hence the terms representing these were zero[23] in this case. But if the assump-

22. In eq. (38) and (39), pressure is expressed in terms of both altitude and inches of mercury. The relation between these is shown in Fig. 6.

23. Since the test values of y were taken at constant watts dissipation, $a = 0$; and since the rise in temperature of the cooling air was negligible, $\theta'_s = 0$.

tions regarding these latter factors are sound, and if the tests show, as they do, that the former check with facts, one may have reasonable confidence in applying the general equation to various apparatus.

2. *Motor Tests.* In order to determine whether the calculations would give reliable results for more complicated commercial apparatus, a 7½-kv-a. synchronous motor, Fig. 21, was placed in the drum and tested at various pressures, namely, 30 in., 22 ½ in., 15 in. and

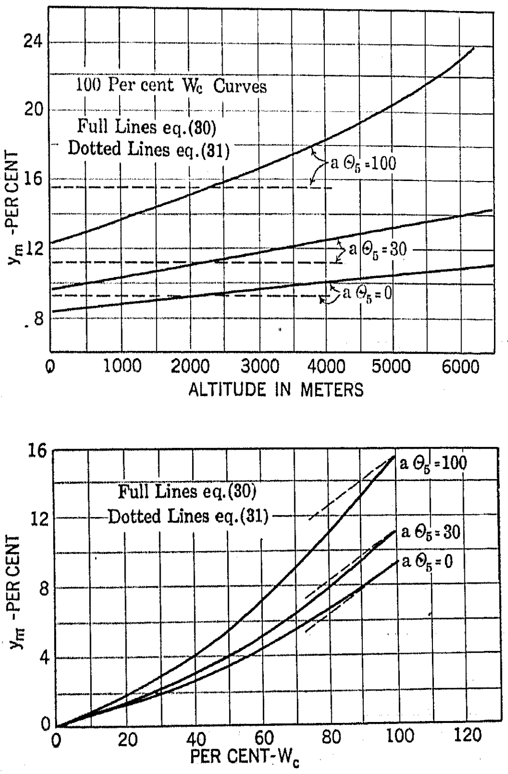


FIG. 17—VARIATION OF y_m WITH ALTITUDE AS GIVEN BY EQUATIONS 30 AND 31. THESE CURVES APPLY TO STATIONARY APPARATUS WITH FORCED CONVECTION

7 in. of mercury, absolute. The temperature of the armature was taken at several points by detectors placed in the slots *between coils*; and the temperature of the field copper was taken by resistance. Figs. 22 and 23 show the temperature rise of the armature and field respectively (above ingoing air temperature) vs. line current. A curve is drawn through the 30 in. points giving the temperature rises found on the machine by test at sea level conditions. The ordinates of this curve were increased by calculation with the equations derived in the paper. The new curves, with reference to the test points, show the agreement.

The curves were arrived at as follows: It is very roughly estimated that an open-type motor such as this would have about 80 per cent convection; average values[24] $a\,\theta_s = 30$ and $\theta_s'/(\theta_s + \theta_s') = 1/4$. From Fig. 6, 22½ in. corresponds to 2560 m. and 15 in. corresponds to 6180 m.

24. $a = 0.75$ and $\theta_s = 40$ deg. under the conditions of test.

Eq. (**34**)[25] gives the correction x_m which should be applied to the surface temperature rise at 30 in., to obtain the rise at 22½ in. Thus, at 22½ in., $x = A\,x_m = 11.6\,a\,w_c = 11.6 \times 2.56 \times 0.80 = 23.8$ per cent.

For 15 in., the general equation (**32**), plotted in Fig. 18, gives $x_m = 14.7$ per cent; and for 7 in., direct calculation gives the total correction $x = 193$ per cent. Thus, for 15 in. $x = 14.7 \times 6.18 \times 0.80 = 72.5$ per cent and for 7 in. $x = 193$ per cent.

These corrections apply to surface temperatures. The plotted points in Fig. 22 were very nearly[26] equal to the copper temperature rise θ_c. It is estimated that the surface temperature was 5 deg. lower[27] than that shown by the detector, at an armature current of 50 amperes, and this temperature difference is reduced as the square of current for other values. The corrections x given above were applied to the surface temperatures thus estimated, and then the insulation drop was again added to obtain the curves as shown in Fig. 22.

The temperature rises of the field winding for various air pressures were similarly analyzed. Here, the average temperature of the copper was taken by resistance, and it is assumed that at the load corresponding to 50 amperes-line, (3.1 amp. field), the surface tempera-

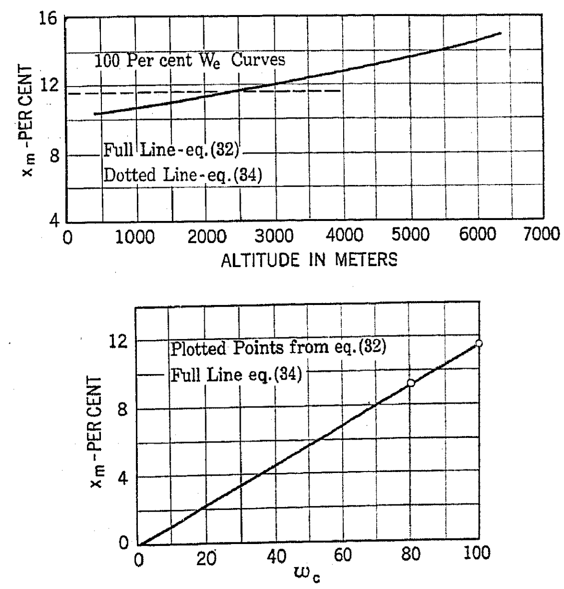


FIG. 18—VARIATION OF x_m WITH ALTITUDE AND WITH w_c AS GIVEN BY EQUATIONS 32 AND 34. THESE CURVES APPLY TO STATIONARY APPARATUS WITH FORCED CONVECTION AND ARE BASED ON $a\,\theta_s = 30$ AND $\theta_s'/\theta_s = 1/4$

ture rise is 3/4 of that of the copper. From these assumptions and following a similar procedure as in the armature test data, the curves Fig. 23 were drawn.

In considering these tests, it is worth noting that the

25. This is the approximate equation which gives practically the same results in the range 20 in. to 30 in., as the general eq. (24).

26. The temperature detectors were placed between the coils, the coils being insulated for low voltage—220 volts.

27. Insulation drop, which is not affected by altitude.

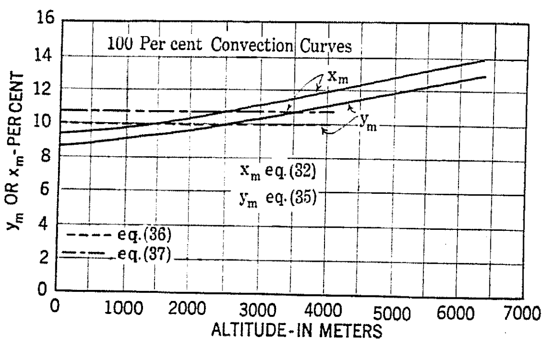
7-in. pressure corresponds to an altitude of about 43,000 ft.

3. *Cerro De Pasco Synchronous Motor.* Rated ATI-2-2550-h. p. 2000-kw.—3600-rev. per min.—2300-volt.

Tests at Aroya Plant of Cerro De Pasco Corp., taken by A. Giese.

This motor was installed at an altitude of 12,200 ft. (3700 m.). It had been given very careful tests at Lynn, Mass. (sea level) and these were repeated at the altitude. However, the latter were very unsatisfactory for the following reasons:

1. The temperature indicator was out of order; hence the only temperatures taken were surface temperatures, which were read immediately after the shut-down.



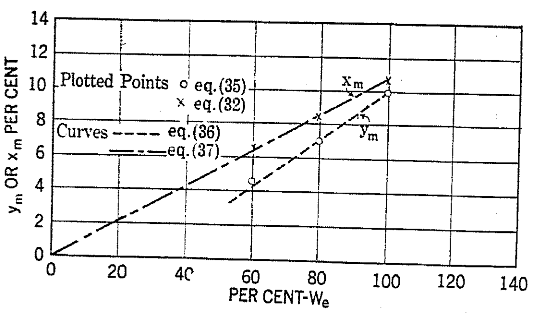


FIG. 19—VARIATION OF y_m WITH ALTITUDE AND w_c AS GIVEN BY EQUATIONS 35 AND 36. VARIATION OF x_m WITH ALTITUDE AS GIVEN BY EQUATIONS 32 AND 37

These curves apply to rotating apparatus, forced convection and are based on $a\,\theta_s = 30$, $\theta_s'/\theta_s = 1/4$ and $b = 0.10$.

2. Full load could not quite be obtained on the motor.

3. The power factor was not reported.

4. The field resistance was taken only after shut-down.

5. The field current was more, the armature voltage less, than at Lynn.

On the other hand, a small amount of information was given as follows:

1. Accurate temperatures were taken for the rise in temperature of the air passing through the machine.

2. The shut-down temperatures allow a rough sort of comparison for the field temperature rise.

3. Shut-down temperatures at Lynn and Cerro De Pasco can be compared.

Barometer Reading		Volts line	Amperes line	Amperes field	Kw.
Lynn	29.6 in.	2300	498	93.5	1990
Cerro De. Pasco	19.4 in.	2240	495	100.0	1900

Difference is 10.2 in., corresponding to $A - 3.7$.

$$z = \frac{\Delta p}{p_a} = \frac{10.2}{19.4} = 52.5 \text{ per cent} \quad x_m = 11.5 \text{ per cent}$$

(Fig. 18)

$$x = 11.5\,A\,w_c = 11.5 \times 3.7 \times 1.0 = 42.5 \text{ per cent.}$$

COOLING AIR

Lynn						Cerro de Pasco		
Test at 1990-kw. load Temp. deg. cent.			Lynn rise corrected for 1900 kw.	z	Calc. rise	Test at 1900-kw. load Temp. deg. cent.		
In	Out	Rise				In	Out	Rise
28.7	45.7	17.0	16.0	52.5	24.4	20.7	44.5	23.8

FIELD TEMPERATURE

—LYNN—

Ambient	Field copper	rise	Temp. Rise corrected for 1900 kw.	Estimated * surface Temp. rise
28.2	69.8	41.6	40	20

CERRO DE PASCO

x	Calc. Surface Rise	Calc. Copper Temp. Rise	Ambient Temp. (test)	Shut down Temp.	
				Calc.	Test
42.5	28.5	48.5	19.7	68.2	65.4

*This is merely a rough guess. These field temp. data are not of great value because the surface temperature is not definitely known.

The following shows a comparison of the maximum readings of thermometers placed on motor after shut-down:

LYNN

	Temp.	Room Temp.	Rise θ	x
Arm. core avg	52.9	28.2	24.7	42.5 per cent
Arm. core max	57.0		28.8	
Vent ducts avg	54.8		26.6	
Vent ducts max	55.0		26.8	
Ribs avg	51.0		22.8	
Ribs max	52.0		23.8	

CERRO DE PASCO

	θ_a	θ_a corr. 1900 kw.	Room Temp.	Calc. Temp.	Test Temp.
Arm. core Avg	35.2	33.6	19.7	53.3	59.7
Arm. core max	41.0	39.1		58.8	62.0
Vent ducts avg	37.9	36.2		55.9	55.3
Vent ducts max	38.2	36.5		56.2	57.0
Ribs avg	32.5	31.0		50.7	49.8
Ribs max	33.9	32.4		52.1	50.0

Of the above data, those relating to air temperatures are of the greatest value as definite, carefully taken readings. They also afford the best check on calculations. The thermometer readings show as good agreement with calculations as might be expected. It should be noted that the temperature corrections are of the order of 40 to 50 per cent.

E. Calculation of Temperature Correction for Various Types of Apparatus

1. *Transformers*:

(*a*) *Oil-immersed, self-cooled, Plain tank. Free Convection.*

Assume:

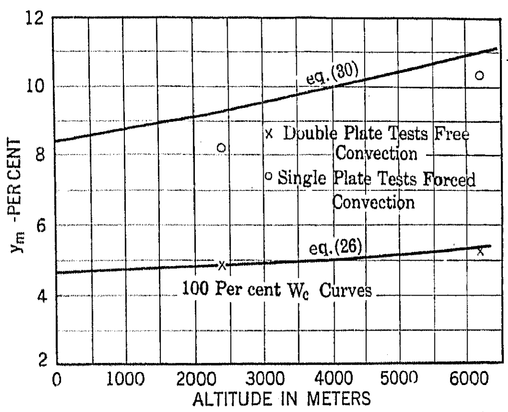
$$a = 2/3; \quad \theta_s = 40; \quad w_c = 0.45; \quad \theta_{cs} = 55$$

From Fig. 16B, taking $a\ \theta_s = 30$, $y_m = 2.2$ per cent. Based on $\theta_{cs} = 55$ deg., $y_m' = 1.6$ per cent.

Corrugated, tubular and radiation tanks. Free Convection.

Assume:

$$a = 2/3; \quad \theta_s = 40; \quad w_c = 0.8; \quad \theta_{cs} = 55$$



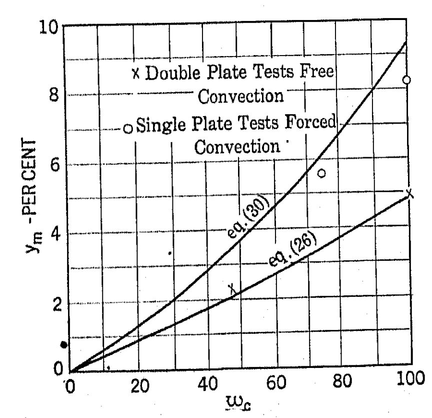


Fig. 20—Comparison of Test Points with Calculated Curves for Free and Forced Convection—Plate Tests

From Fig. 16B,

$$y_m = 4.2 \text{ per cent}; \quad y_m' = 3.06 \text{ per cent}$$

(*b*) *Water-cooled.*

Correction = 0

(*c*) *Natural Draft.*

Free Convection. Here, the hot spot temperature of the coil surface may be practically as high as the average temperature of the copper, as measured by resistance.

Thus $\theta_s = \theta_{cs} = 50$

Assume further:

$$a = 1.0; \quad w_c = 0.8$$

Question arises whether correction should be made on account of temperature rise θ_s' of cooling air above ingoing air. Eq. (**24**) for x is based on *constant* velocity of cooling air, as produced by a fan, and is

$$x = y \frac{\theta_s'}{\theta_s + \theta_s'} (z - y) \qquad (\mathbf{24})$$

But in the natural draft transformer the velocity increases with temperature rise. For constant velocity

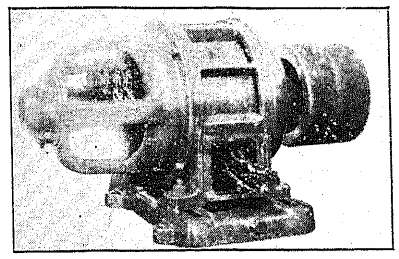


Fig. 21—Synchronous Motor which Was Tested in the Drum Fig. 1. Rated: 7.5-Kv-a. 60-Cycles 1800-Rev. per Min. 110-Volts

z is always *slightly* greater than y. Increased velocity would, of course, decrease z. In the absence of definite data, it is assumed that in this case

$$z = y.$$

Hence, the second term in (**24**) vanishes, that is, the correction in y for θ_s' is zero.

Thus, from Fig. 16B,

$$y_m = 4.6 \text{ per cent} \qquad y_m' = 4.6 \text{ per cent}$$

(*d*) *Air Blast.*

Forced Convection.

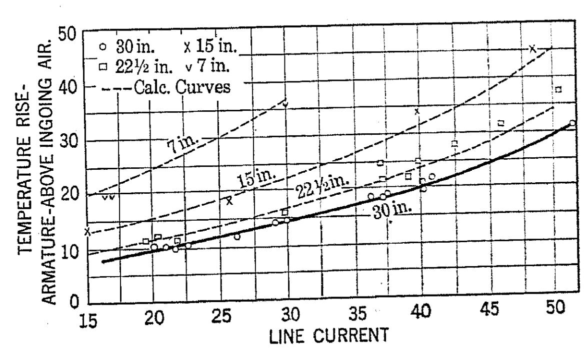


Fig. 22—Comparison Test Results with Calculated Curves. Temperature Rise of Armature Coils (as Given by Detectors in the Slots) above Ingoing Air Temperature vs. Line Current

Assume:

$$a = 1.0; \quad w_c = 1.0; \quad \theta_s + \theta_s' = 50 \text{ deg. cent.};[28] \quad \theta_{cs} = 50 \text{ deg. cent.}$$

$$T_c = 354 \text{ deg. cent.}; \quad b = 0.10; \quad T_s = 20.$$

Total temperature rise of air $0 = 25$ deg. cent. Taking half of this for θ_s' the average rise of cooling air,

$$\theta_s = 37.5; \quad \theta_s' = 12.5; \quad \frac{\theta_s'}{\theta_s + \theta_s'} = 1/4$$

28. As in natural draft transformer, the spot surface temperature may be practically equal to the copper temperature.

Applying eq. (**29**) and (**32**),[29]

$y_m = 10.7$ per cent $x_m' = x_m = 11.2$ per cent

2. *Reactors and Bus Bars*:

Free Convection.

Assume:

$a = 1.0$ $w_c = 0.8$ $\theta_s = \theta_{cs} = 60$ deg. cent.

From Fig. 16B,

$y_m = 4.7$ per cent $y_m' = 4.7$ per cent

3. *Rheostats, Meters, Relays and Miscellaneous Stationary Apparatus.*

Free Convection.

Assume:

$a = 0.5$ $\theta_s = 50$ deg. $w_c = 0.8$

From Fig. 16B,

$y_m = 4.2$ per cent

4. *Totally Enclosed Motors*:

The surface temperatures of the iron and coils are considered. The total temperature drop between these surfaces and the outside air, comprises

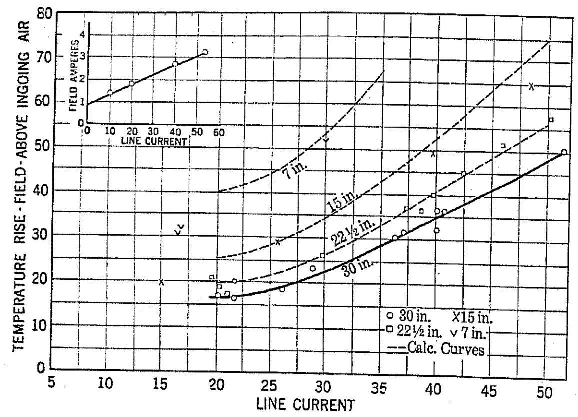


FIG. 23—COMPARISON TEST RESULTS WITH CALCULATED CURVES, TEMPERATURE RISE OF FIELD, BY RESISTANCE, ABOVE INGOING AIR TEMPERATURE VS. LINE CURRENT

(a) drop from hot surface to inside air;
(b) drop from inside air to casing;
(c) drop through casing;
(d) drop from casing to ambient air.

(a) and (b) are forced convection; (c) independent of altitude; (d) free convection. Assuming the following distribution of these at sea level,

(a) 10 deg., (b) 10 deg., (c) 5 deg., (d) 35 deg.

the following resultant correction, applicable to coil surface rise, is calculated.

$x_m = 5.35$ per cent

5. *Open Type Rotating Apparatus.*

Forced Convection.

(*a*) *Stator Coils.*

Assume:

$a = 0.7$ $\theta_s \times \theta_s' = 40$ $w_c = 0.9$ $\theta_s' = 10$ $\theta_{cs} = 55$

From eq. (**35**),

$y_m = 8$ per cent

and eq. (**32**)

$x_m = 8.94$ per cent $x_m' = 6.5$ per cent

(*b*) *Rotor Coils, Insulated.*

Same as (*a*), $x_m = 8.94$ per cent $x_m' = 6.5$ per cent

(*c*) *Rotor Coils, Bare Copper Strip.*

Assume:

$a = 1.0$ $w_c = 0.9$ $\theta_s \times \theta_s' = 55$ $\theta_{cs} = 55$ $\theta_s' = 10$

From (**35**) and (**32**)

$y_m = 9$ per cent

and

$x_m = 9.5$ per cent $x_m' = 9.5$ per cent

6. *Enclosed Type Rotating Apparatus with Directed Ventilation.*

Forced Convection.

(*a*) *Turbo-Generators*:

Armature Coils:

Assume:

$a = 0.7$ $w_c = 1.0$ $\theta_s + \theta_s' = 35$ $T_s = 20$
$\theta_{cs} = 60$ $T_c = 314$ $\theta_s' = 12$

By eq. (**35**) and (**32**)

$x_m = 10.69$ per cent $x_m' = 6.25$ per cent

Rotor:

Assume:

$a = 1.0$ $w_c = 1.0$ $\theta_s + \theta_s' = 52$ $\theta_s' = 12$
$\theta_{cs} = 80$ $T_s = 20$ $T_c = 334$

Thus,

$x_m = 11.07$ per cent $x_m' = 7.15$ per cent

(*b*) *Other Apparatus in Class* (6)

Stator Coils. Same as (*a*)

$x_m = 10.69$ per cent $x_m' = 6.25$ per cent

Rotor Coils, insulated.

Same as stator coils.

$x_m = 10.69$ per cent $x_m' = 6.25$ per cent

x_m same as for surface rise of stator coils.

$x_m = 10.69$ per cent $x_m' = 10.69$ per cent

F. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the valuable assistance of C. W. Cutler and R. F. Franklin in making tests and analyzing the data; also, of W. F. Dawson and A. Giese, in making tests on the 2250-h. p. synchronous motor.

G. NOTATION

A = altitude in thousands of meters

a = fraction of total heat dissipated from surface due to copper loss.

b = fraction of total heat dissipated from surface due to windage loss.

c = fraction of total heat dissipated from surface due to core loss and other losses.

$D = 1 - \left(\frac{P_a}{P_s}\right)^m$

e = emissivity constant in Stefan-Boltzman Law.

θ = temperature rise deg. cent. of the surface above the cooling air (cooling air may be of higher temperature than ambient air, as in machines with enclosed ventilation).

29. The unit correction, y_m = per cent correction per 1000 m. is based on values obtained for 23 in. pressure, $A = 2.4$. This approximation is made throughout the paper.

θ_s = θ at sea level.
θ_a = θ at altitude.
$\Delta\,\theta$ = $\theta_a - \theta_s$
θ' = temperature rise of the cooling air, above ambient air.
θ_s' = θ' at sea level.
θ_a' = θ' at altitude.
$\Delta\,\theta'$ = $\theta_a' - \theta_s'$
θ_{cs} = temperature rise of copper above ambient air temperature at sea level.
θ_{ca} = temperature rise of copper above ambient air temperature at altitude.
θ_i = temperature drop through insulation.
$\Delta\,\theta_c$ = $\theta_{ca} - \theta_{cs}$
p = absolute pressure, inches of mercury.
p_s = p at sea level (assumed to be 30 in.)
p_a = p at altitude.
$\Delta\,p$ = $p_s - p_a$
T_a = temperature ambient air deg. cent. at altitude.
T_c = $234 + T_s + a\,\theta_{cs}$.
T_2 = temperature heated surface deg. cent. absolute.
T_1 = temperature walls deg. cent. absolute.
T_s = " ambient air deg. cent. at sea level.
$\Delta\,T$ = $T_a - T_s$
W = total watts dissipated.
W_0 = W when copper temperature equals ambient temperature, T_s.
W_s = W at sea level.
W_a = W at altitude.
W_c = watts per square inch dissipated by convection.
W_r = " " " " " " radiation.
W_{cs} = W_c at sea level.
W_{ca} = W_c at altitude.
W_{rs} = W_r at sea level.
W_{ra} = W_r at altitude.
w_c = fraction of total watts, dissipated by convection at sea level = W_{cs}/W_s.
w_r = fraction of total watts, dissipated by radiation at sea level = W_{rs}/W_s.
$w_r + w_c = 1.0$.
x = fractional increase of surface temperature rise above ambient temperature $= \dfrac{\Delta\,\theta + \Delta\,\theta'}{\theta_s + \theta_s'}$
x_m = x per 1000 m.
$x_m' = \dfrac{\theta_s}{\theta_{cs}}\,x_m$
y = fractional increase of θ with altitude = θ/θ_s.
y_m = y per 1000 m.
$y_m' = \dfrac{\theta_s}{\theta_{cs}}$ = per cent temperature correction based on copper temperature.
z = fractional increase of θ' with altitude = θ'/θ_s'

Appendix A

1. Derivation of General Equation

The total power W from the surface considered, is dissipated by convection and radiation. Thus,

$$W = W_c + W_r \tag{1}$$

$W_0 = W$ when copper temp. = ambient air temperature T_s. Let,

a = fraction of W_0 due to copper loss, when the copper temperature = ambient temperature T_s.
b = fraction of W_0 due to windage loss.
c = " " " " " iron and other losses.

Thus,

$$a + b + c = 1.0 \tag{2}$$

The dissipated loss due to $i^2\,r$ at a copper temperature equal to the ambient temperature T_s is $a\,W_0$. At a temperature rise θ_{cs} of the copper above T_s, under operating conditions at sea level, the dissipated $i^2\,r$ loss at the same current would be[30]

$$a\,W_0\left(\frac{234 + T_s + \theta_{cs}}{234 + T_s}\right) = a\,W_0\left(1 + \frac{\theta_{cs}}{234 + T_s}\right) \tag{3}$$

At altitude, with an ambient temperature T_a, and a temperature rise θ_{ca} of the copper, the copper loss to be dissipated would be,

$$a\,W_0\left(\frac{234 + T_a + \theta_{ca}}{234 + T_s}\right) = a\,W_0\left[1 + \frac{(T_a - T_s) + \theta_{ca}}{234 + T_s}\right] \tag{4}$$

The component of W_0 due to windage loss at sea level is

$$b\,W_0 \tag{5}$$

and at altitude,

$$b\,W_0\,\frac{p_a}{p_s} = b\,W_0\left(1 - \frac{p_s - p_a}{p_s}\right) \tag{6}$$

Likewise, the component due to core loss, etc. at *any* altitude is

$$c\,W_0 \tag{7}$$

Consequently, the total dissipation at sea level is

$$W_s = a\,W_0\left(1 + \frac{\theta_{cs}}{234 + T_s}\right) + b\,W_0 + c\,W_0$$

or, by (2)

$$W_s = W_0\left[\frac{234 + T_s + a\,\theta_{cs}}{234 + T_s}\right] \tag{8}$$

Thus, the total dissipation at altitude is

$$W_a = W_0\left[\frac{234 + T_s + a\,(T_a - T_s) + a\,\theta_{ca} - b\,\dfrac{p_s - p_a}{p_s}\,(234 + T_s)}{234 + T_s}\right] \tag{9}$$

30. Greater in proportion to temperature of copper.

Dividing (8) by (9) and also adding and subtracting $a\ \theta_{cs}$ in the numerator of (9),

$$W_a = W_s\left[1 + \frac{a\ \Delta\ T + a\ \Delta\ \theta_c}{234 + T_s + a\ \theta_{cs}} - b\,\frac{\Delta\ p}{p_s}\ \frac{234 + T_s}{234 + T_s + a\ \theta_{cs}}\right] \qquad (10)$$

Equation (10) gives the total watts loss W_a dissipated at altitude, in terms of that at sea level. The next step is to deduce separate expressions for the power dissipated by convection and radiation, that is,

W_c and W_r

and to equate their sum to (10), in accordance with (1). Convection,

$$W_c \sim \theta^l\ p^m \qquad (11)$$

and radiation[31]

$$W_r \sim \theta^n$$

Thus it follows,

$$W_{ca} = W_{cs}\left(\frac{\theta_a}{\theta_s}\right)^l\left(\frac{p_a}{p_s}\right)^m$$

and

$$W_{ra} = W_{rs}\left(\frac{\theta_a}{\theta_s}\right)^n$$

Therefore,

$$W_a = W_{cs}\left(\frac{\theta_a}{\theta_s}\right)^l\left(\frac{p_a}{p_s}\right)^m + W_{rs}\left(\frac{\theta_a}{\theta_s}\right)^n \qquad (12)$$

In order to express results in terms of the fractional increase in temperature rise due to altitude, that is, in terms of

$$\frac{\theta_a - \theta_s}{\theta_s} = \frac{\Delta\ \theta}{\theta_s},$$

Expand $\left(\frac{\theta_a}{\theta_s}\right)^l$ by the binomial theorem. Thus,

$$\left(\frac{\theta_a}{\theta_s}\right)^l = \left(1 + \frac{\Delta\ \theta}{\theta_s}\right)^l = 1 + l\,\frac{\Delta\ \theta}{\theta_s} + \frac{l\,(l-1)}{2}\left(\frac{\Delta\ \theta}{\theta_s}\right)^2 + \ldots$$

or,[32] approximately,

$$\left(\frac{\theta_a}{\theta_s}\right) \cong 1 + l\,\frac{\Delta\ \theta}{\theta_s}$$

Similarly,

$$\left(\frac{\theta_a}{\theta_s}\right)^n \cong 1 + n\,\frac{\Delta\ \theta}{\theta_s} \qquad (13)$$

Substituting (13) in (12),

$$W_a = W_{cs}\left(\frac{p_a}{p_s}\right)^m\left(1 + l\,\frac{\Delta\ \theta}{\theta_s}\right) + W_{rs}\left(1 + n\,\frac{\theta}{\theta_s}\right) \qquad (14)$$

Equating (10) and (14), and substituting the following:

$$n\ w_r = n - n\ w_c$$

$$\Delta\ \theta' = z\ \theta_s' = \frac{\Delta\ p}{p_a}\ \theta_s' \quad \text{(by eq. 20)}$$

$$\Delta\ \theta_c = (\theta_a + \theta_a' + \theta_i) - (\theta_s + \theta_s' + \theta_i) = \Delta\ \theta + \Delta\ \theta' \qquad (15)$$

The drop θ_i through the insulation is not affected by altitude, and hence cancels out.

We have,

$$y = \frac{\Delta\ \theta}{\theta_s} = \frac{w_c\,D - b\,\dfrac{\Delta\ p}{p_s}\,\dfrac{234 + T_s}{T_c} + \dfrac{\Delta\ p}{p_a}\,\dfrac{a\ \theta_s'}{T_c} + \dfrac{a\ \Delta\ T}{T_c}}{w_c\,[l\,(1-D) - n] + n - \dfrac{a\ \theta_s}{T_c}} \qquad (16)$$

where

$$D = 1 - \left(\frac{p_a}{p_s}\right)^m$$

Expressing as k_s' those terms which are not functions of altitude, we get

$$y = \frac{\Delta\ \theta}{\theta_s} = \frac{w_c\,D - k_1\,\dfrac{\Delta\ p}{p_s} + k_2\,\dfrac{\Delta\ p}{p_a} + k_4}{w_c\,[l\,(1-D) - n] + n - k_3} \qquad (17)$$

where

$$k_1 = b\,\frac{234 + T_s}{T_c} \qquad k_2 = \frac{a\ \theta_s'}{T_c}$$

$$k_3 = \frac{a\ \theta_s}{T_c} \qquad k_4 = \frac{a\ \Delta\ T}{T_c}$$

If the ambient temperature at altitude and at sea level is the same, then

$$k_4 = 0$$

Then the general equation of the increase, y, in the *surface* temperature rise *above the cooling air is*,

$$y = \frac{\Delta\ \theta}{\theta_s} = \frac{w_c\,D - k_1\,\dfrac{\Delta\ p}{p_s} + k_2\,\dfrac{\Delta\ p}{p_a}}{w_c\,[l\,(1-D) - n] + n - k_3} \qquad (18)$$

2. Correction for Temperature Rise of Cooling Air above Temperature of Ingoing Air

A fan running at constant speed develops a pressure proportional to the inlet density of the air. The pressure drop due to the rate of flow of a given volume of air through given passages, is also proportional to the air density. The volume of air forced through the machine by fans would, therefore, not be changed significantly

31. See Fig. 15, in which it is shown that, in the range 0 to 100 deg. cent., n is approximately 1.25.

32. The error in neglecting the third term is as follows: if l is 1.0, as in forced convection, error is zero; if l = 1.25, as in free convection, and $\frac{\Delta\ \theta}{\theta_s}$ = 0.1, error = 1.5 per cent of *second term*; when $\frac{\Delta\ \theta}{\theta_s}$ = 0.3, it is 3.7 per cent. Thus, the error is within the accuracy with which l is known.

when the density of the air decreases; but the weight of air passing will decrease directly with the pressure. Therefore, if the losses in the machine are unchanged so that the same amount of heat must be carried away by the air, temperature rise of the air will be inversely proportional to its density, or to the atmospheric pressure.

Thus,

$$\frac{\theta_a'}{\theta_s'} = \frac{p_s}{p_a} \qquad (19)$$

by definition,

$$z = \frac{\theta_a' - \theta_s'}{\theta_s'} = \frac{p_s}{p_a} - 1 = \frac{\Delta p}{p_a} \qquad (20)$$

The correction y for rise above cooling air temperature is given by equation (18). It is now necessary to combine y and z so as to give x, the increase in the surface temperature rise above ingoing air. By definition,

$$x = \frac{\Delta \theta + \Delta \theta}{\theta_s + \theta_s'} \qquad (21)$$

$$\left.\begin{aligned} \theta_a &= (1 + y)\,\theta_s \\ \theta_a' &= (1 + z)\,\theta_s' \end{aligned}\right\} \qquad (22)$$

$$\theta_a + \theta_a' = (1 + x)\,(\theta_s + \theta_s') \qquad (23)$$

Substituing (22) in (23)

$$x = y\frac{\theta_s}{\theta_s + \theta_s'} + z\frac{\theta_s'}{\theta_s + \theta_s'} = y + \frac{\theta_s'}{\theta_s + \theta_s'}(z - y) \qquad (24)$$

3. Special Cases

(a) *Stationary Apparatus—free convection.*

$m = 1/2 \quad l = 1.25 \quad n = 1.25 \quad b = 0 \quad \theta_s'^{33} = 0$

Substituting these values in (18),

$$y = \frac{\Delta \theta}{\theta_s} = \frac{w_c\, D}{1.25\,(1 - w_c\, D) - k_3}$$

$$= \frac{w_c\, D}{1.25\,(1 - w_c\, D) - \dfrac{a\,\theta_s}{T_c}} \qquad (25)$$

In terms of y_m,

$$y_m = y/A = \frac{w_c\, D}{A\left[1.25\,(1 - w_c\, D) - \dfrac{a\,\theta_s}{T_c}\right]} \text{ as fraction} \qquad (26)$$

Equation (26) is plotted in full lines in Figs. 16. Fig. 16A is for 100 per cent convection, that is, $w_c = 1.0$, with $a\,\theta_s$ as parameter, and A as abscissa. The relation between A and p is given on Fig. 6. It will be noted that these curves can be closely approximated within the range $A = 0$ and $A = 3.5$, *i. e.*, 3500 meters, by the dotted lines, which cut the curves at 2400 m. = 23 in. pressure. This, of course, covers the practical range. The dotted lines are expressed by,

$$y_m \cong 4.8 + \frac{a\,\theta_s}{60} \text{ in per cent} \qquad (27)$$

In 16B, equation (26) is plotted, y_m vs. w_c, with $a\,\theta_s$ as parameter. Here the 2400-m. point, established in 16A, is taken as the basis for approximation. It is seen that the full line curves may be approximated by the dotted lines, which are expressed by

$$y_m \cong w_c\left(4.8 + \frac{a\,\theta_s}{60}\right) \text{ in per cent} \qquad (28)$$

This is the final, approximate equation for the special case of stationary apparatus with free convection.

(b) *Stationary Apparatus—forced convection.*

$m = 3/4 \quad l = 1.0 \quad n = 1.25 \quad b = 0$

Substituting these values in (18),

$$y = \frac{\Delta \theta}{\theta_s} = \frac{w_c\, D + \dfrac{a\,\theta_s'}{T_c}\,\dfrac{\Delta p}{p_a} - k_1\,\dfrac{\Delta p}{p_s}}{1 - w_c\, D + 0.25\, w_r - \dfrac{a\,\theta_s}{T_c}}$$

and

$$y_m = y/A$$

$$= \frac{w_c\, D + \dfrac{a\,\theta_s'}{T_c}\,\dfrac{\Delta p}{p_a} - b\,\dfrac{234 + T_s}{T_c}\,\dfrac{\Delta p}{p_s}}{A\left[1 - w_c\, D + 0.25\, w_r - \dfrac{a\,\theta_s}{T_c}\right]} \qquad (29)$$

If θ_s' and b are negligible,

$$y_m = \frac{w_c\, D}{A\left[1 - w_c\, D + 0.25\, w_r - \dfrac{a\,\theta_s}{T_c}\right]} \text{ as fraction} \qquad (30)$$

Plotting, by equation (29), y_m vs. A, and also vs. w_c, as in the case of free convection, the full line curves on Figs. 17A and 17B are obtained. These curves are approximated, *within the range* $w_c > 0.75$, by the dotted lines, which are expressed by

$$y_m \cong 14\, w_c + \frac{a\,\theta_s}{16} - 4.7 \text{ in per cent} \qquad (31)$$

The above corrections apply to the *surface* temperature rise. If there is an appreciable rise in temperature of the cooling air, as in apparatus with enclosed ventilating systems, then further correction is necessary. Thus the total correction x of surface temperature rise above the temperature of ingoing air is, by eq. (24)

$$x = y + \frac{\theta_s'}{\theta + \theta_s'}(z - y)$$

and

$$x_m = x/A = y_m + \frac{\theta_s'}{\theta_s + \theta_s'}(z/A - y_m) \qquad (32)$$

33. θ_s' modifies the k_2 term in (18). Since T is of the order of 275 deg., each degree of θ_s' would change the value of

$$k_2\,\frac{\Delta p}{p_a} = \frac{a\,\theta_s'}{T_c}\,\frac{\Delta p}{p_a}$$

by about ⅓ of a per cent; and this term is small compared with the $w_c\, D$ term. Therefore, since θ_s' is small in any case except, perhaps, corrugated surfaces, or natural draft transformers, it will be neglected except in these two cases, which will be discussed in section E of the paper.

In order to obtain approximate numerical values of x for apparatus with forced cooling, such as air blast transformers, assume:

(1) total temperature rise of air = 25 deg. cent.
(2) surface rise 50 deg. cent.
(3) $a = 0.8$
(4) $w_c = 1.0$

Thus

$$\theta_s' = 25/2 = 12.5 \text{ deg. cent.}$$
$$\theta_s = 50 - 12.5 = 37\tfrac{1}{2} \text{ deg. cent.}$$
$$a\,\theta_s = 30$$

$$\frac{\theta_s'}{\theta_s + \theta_s'} = \tfrac{1}{4}$$

As in previous cases, basing the approximate correction y_m on the value at $p_a = 23$ in., $A = 2.4$, the full-line curves, Figs. 18A and 18B for x_m are obtained by substituting the above data in (29) and (32).

For altitudes up to 3500 m. the full-line curve Fig. 18A for $w_c = 1.0$ can be roughly approximated by the constant value

$$x_m \cong 11.6 \tag{33}$$

and Fig. 18B, almost exactly by

$$x_m \cong 11.6\, w_c \tag{34}$$

(c) *Rotating Apparatus—forced convection.*

Assuming that the air velocity in any rotating apparatus is high enough to be regarded as forced convection, the constants which apply in this case follow:

$$m = 3/4 \quad l = 1.0 \quad n = 1.25 \quad b = 0.1$$[34]

Assume

$$T_s = 20 \text{ deg. cent.} \qquad T_c = 275 \text{ deg. cent.}$$

Substituting in (18),

$$y_m = y/A = \frac{w_c D - 0.09\dfrac{\Delta p}{p_s} + \dfrac{a\,\theta_s'}{T_c}\,\dfrac{\Delta p}{p_a}}{A\left[1 - w_c D + 0.25\, w_r - \dfrac{a\,\theta_s}{T_c}\right]} \quad \text{as fraction} \tag{35}$$

Using values of $a\,\theta_s'$, θ_s', etc. assumed for (b), thus obtaining y_m from (35) and then x_m from (32), Fig. 19 is plotted. Fig. 19A shows the increase in temperature correction due to increased temperature rise of cooling air.

These curves are practically straight lines, and hence can be closely approximated by

$$y_m \cong 14\, w_c - 4 \text{ in per cent} \tag{36}$$

and

$$x_m = 10.7\, w_c \text{ in per cent} \tag{37}$$

(d) *Plate Tests.*

For free convection tests, equation (26) applies. Since temperature rise θ at *constant watts* is considered at different pressures, $a = 0$. Thus,

$$y_m = \frac{w_c D}{A\,[1.25\,(1 - w_c D)\,]} \tag{38}$$

This, of course, is the same relation as shown in Fig. 16 for $a\,\theta_s = 0$.

For forced convection, (30) applies, with $a = 0$. Thus,

$$y_m = \frac{w_c D}{A\,[1 - w_c D + 0.25\, w_r]} \tag{39}$$

This is the same relation as shown in Fig. 17 for $a\,\theta_s = 0$.

Appendix B

Description of Apparatus

Drum. Dimensions and the general arrangement are shown in Figs. 1 and 2.

The drum was exhausted by motor driven pump; the pressure was measured by the *U*-tube shown in Fig. 1; the air temperature inside of the drum was controlled by water pipes; the wires were brought out at the top; drum was kept sealed by paraffin.

Plates. The "single plate" is shown in Fig. 3. Surface 10 in. by 15 in. and thickness 7/8 in. Thermo-couples were soldered to the surface. The latter was coated with lamp black.

The "double plate" is shown in position in Fig. 4. On account of the near, end-view, the illustration gives undue prominence to the wires and tape surrounding the plate. Thus, a view directly toward the plate surface, would have shown much less of these than appear in Fig. 1.

The cardboard lining is also shown in Fig. 1.

The plate dimensions, surface 10 in. by 15 in., and thickness, ½ in.

Surface temperatures were measured by thermo-couples soldered to surface. The latter was coated first with lamp black and, for subsequent tests, with aluminum paint.

The plate was suspended by cord to practically eliminate conduction of heat through the supports.

Some conduction through thermo-couple leads was, of course, inevitable; but this was practically negligible, being reduced by using small leads, heavily insulated.

34. Average value too low for high speed, and too high for low speed.

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Discussion

TEMPERATURE RISE OF STATIONARY ELECTRICAL APPARATUS AS INFLUENCED BY RADIATION, CONVECTION AND ALTITUDE

(Montsinger and Cooney)

EFFECT OF ALTITUDE ON TEMPERATURE RISE

(Doherty and Carter)

Chicago, Illinois, June 25, 1924

W. J. Foster: For a great many years I have been a party to the effort to obtain correction factors for rotating machinery from data collected at high altitudes and at sea level, thus deriving empirical formula, but I must confess that it seems a hopeless task.

I think Messrs. Doherty and Carter have pursued the only feasible method, that is, establish the conditions of air density by exhausting the air in something similar to the drum they have used. After all this is done, there remains the difficulty of classifying rotating apparatus. There is the wide open machine where no attempt whatever is made to give direction to the movement of air which is the cooling medium, and on the other hand, the totally inclosed machine where practically all the heat is carried off by the air passing through. Between those two there are all kinds of partially inclosed machines, and the effect on temperature rise is often very accidental. That is why it is so difficult to determine the proper correction even though you use a large number of cases between machines that have been tested at approximately sea level and after installation at high altitudes.

One can understand this statement if he is familiar with what often happens on the testing floor of the manufacturer. If for any reason a machine is changed in its position during test (I am now speaking of a machine that is largely open or only partially inclosed) the resultant temperatures with the same conditions of load will vary a few degrees, due to the movement of the air, to the amount of air passing through and to the other machines that happen to be running in the vicinity.

I have seen the same thing in machines installed outside, *e. g.*, in the case of a 5000 kw. direct-connected steam-engine-driven generator at low speed, where temperatures were known at certain times to be considerably higher than at other times. After careful examination it was found due to the accident of the direction of the wind outside the building when the machine was started up.

I have watched Mr. Montsinger with considerable interest during the last ten years in his efforts to obtain proper correction factors for stationary apparatus. I think he has the problem quite well solved, and that Mr. Doherty has attacked the problem in the correct manner.

When I first studied heat many years ago in the secondary schools, we had the dissipation of heat as due to radiation, conduction and convection. I still like to think of the conduction factor and in a totally inclosed machine of the cooling medium, the air which passes through as a carrier is quite similar to what we would have if water or some liquid were used which would have greater capacity for heat than the air—that makes the problem simpler in totally inclosed machines.

When it comes to a matter of establishing standards, I hope to see this whole subject put on the basis of sea level and the formula for convection applied from sea level. It seems to me that is the only scientific way for a body of engineers to proceed in standardization.

E. B. Paxton: The present A. I. E. E. Standards Rule (Paragraph 2215) recommends that the temperature rise of electrical machinery intended for operation at high altitudes shall be reduced at the rate of 10 per cent for each 1000 meters by which the altitude exceeds 1000 meters. A comparison of the values given in the first column of Table I of the paper by Messrs. Doherty and Carter shows this value to be in close agreement with the results given there for rotating machinery, namely, the values: 9 per cent, 10.7 per cent and 11 per cent.

The present A. I. E. E. rule makes no distinction between methods of temperature measurement when applying the altitude correction, whereas strictly speaking a lesser percentage should be employed when correcting for temperature rise measured by embedded temperature detectors than when correcting for temperature rise measured by thermometer on the surface due to the increased percentage base. A graphical interpretation of the present A. I. E. E. rule applied to the limiting temperature rises allowed for Class A and B insulation, which shows the correction in degrees instead of per cent, is given in Fig. 1.

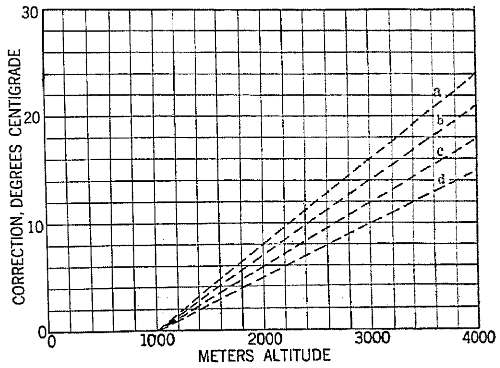

Fig. 1—Present A. I. E. E. Recommendation for Temperature-Rise Correction

This covers machines intended to operate at altitudes above 1000 meters. The curves (*a*) (*b*) (*c*) and (*d*) show the value of the correction in degrees centigrade, which amounts to 10 per cent of the standard temperature rise, applied to machines having:

(*a*)	Class B insulation,	80	deg.	standard rise by	embedded detector.
(*b*)	" B "	70	"	" " "	thermometer.
(*c*)	" A "	60	"	" " "	embedded detector.
(*d*)	" A "	50	"	" " "	thermometer.

Following this rule it is evident that a machine with Class A insulation specially rated for operation at 3000 meters will be given a rating occasioned by:

(1) 50 deg. − (10% × 2 × 50) = 40 deg. rise.
at sea level if the temperature is measured by surface thermometer; or

(2) 60 deg. − (10% × 2 × 60) = 48 deg. rise
at sea level if the temperature is measured by embedded temperature detector.

The correction is 10 deg. in the first case and 12 deg. in the second case. Obviously for a given load the drop from the copper to the surface is the same regardless of the altutude so that the number of degrees correction should be the same in

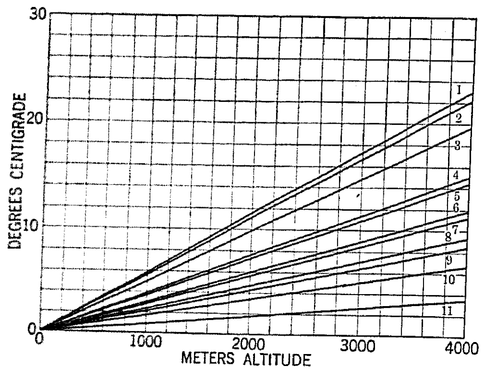

FIG. 2—INCREASE IN TEMPERATURE RISE OF VARIOUS MACHINES WITH ALTITUDE

The increase is given in degrees C. The values are computed from the values in the Doherty and Carter paper.

(1) Turbine generator fields (rotors), enclosed ventilation. 11 per cent of 52-deg. surface rise or 7.15 per cent of 80-deg. internal rise.

(2) Air-blast transformers. 11 per cent of 50-deg. rise, surface or internal.

(3) Generator and motor bare-copper-strip fields, open type. 9 per cent of 55 deg. rise.

(4) Turbo-generator armatures, enclosed ventilation, and generators and motors, enclosed type. 10.7 per cent of 35-deg. surface rise or 6.25 per cent of 60-deg. internal rise.

(5) Generators and motors with insulated fields and armatures, open type. 9 per cent of 40-deg. surface rise or 6.5 per cent of 55-deg. internal rise.

(6) Totally enclosed motors. 5.35 per cent of 55-deg. rise.

(7) Reactors and bus bars. 4.7 per cent of 60-deg. rise.

(8) Natural-draft transformers. 4.6 per cent of 50-deg. rise.

(9) Rheostats, meters and miscellaneous stationary apparatus. 4.2 per cent of 50-deg. surface rise.

(10) Oil-insulated self-cooled corrugated-tank transformers. 4.2 per cent of 40-deg. surface rise or 3.06 per cent of 55-deg. internal rise.

(11) Oil-insulated self-cooled, plain-tank transformers. 2.2 per cent of 40-deg. surface rise or 1.6 per cent of 55-deg. internal rise.

Based on 10 per cent and 4 per cent of A. I. E. E. Standard temperature rise.

either case. The method of expressing the correction in per cent based on a single percentage base is incorrect to this extent.

Mr. Doherty has recognized this fact in his paper by establishing a lower percentage correction when internal temperature is considered as shown by the second column of values in Table I. Had the corrections been expressed in degrees correction rather than in percentage of rise correction, one set of values would replace the two sets of percentages given for internal and surface measurements for any one class of insulation. For example, the percentage established for Turbine Generator Fields with enclosed ventilation is 11 per cent of 52 deg. surface temperature rise or 7.15 per cent of 80 deg. internal temperature rise = approximately 6 deg. correction per 1000 meters.

Fig. 2 shows the values given by Mr. Doherty expressed in degrees instead of percentages. He arrived at these values by selecting typical cases of temperature rise for a given type of machine and applying the percentage correction. Consideration of the first column of percentages in Table I of the paper or of Fig. 2 shows that all apparatus falls roughly into one of two groups to which the correction percentages of 10 per cent and 4 per cent per 1000 meters for surface temperature may be fairly definitely applied. These two groups are as follows:

Rotating Machines (Except Totally Enclosed Motors) Air Blast Transformers	10%
Transformers (Except Air-Blast Transformers) Reactors and Busbars Rheostats, Meters, Relays and Miscellaneous Stationary apparatus Totally Enclosed Motors	4%

The dotted lines in Fig. 3 show the degrees correction obtained by applying the above percentages to the standard A. I. E. E. temperature rises for Class A and Class B insulation.

The solid lines show a compromise between the dotted lines which should be sufficiently accurate for standardization purposes.

Fig. 4 shows a superposition of the curves in Fig. 1, Fig. 2 and Fig. 3, namely:

The preceding part of this discussion has served to correlate the data at hand and enables a proposition for the numerical values of a standard correction to be made. The remaining problem is one of application in the A. I. E. E. Standards.

The present A. I. E. E. Rule, par. 2215 reads as follows:

"2215 **Altitude.**—*Increased altitude has the effect of increasing the temperature rise of some types of machinery. In the absence of information in regard to the height above sea level at which a machine is intended to work in ordinary service, this height is assumed not to exceed 1000 meters (3300 ft.). For machinery operating at an altitude of 1000 meters or less, a test at any altitude less than 1000 meters is satisfactory, and no correction shall be applied to the observed temperatures. Machines intended for operation at higher altitudes shall be regarded as special. It is recommended that when a machine is intended for service at altitudes above 1000 meters (3300 ft.), the permissible temperature rise at sea level shall be reduced by 1 per cent for each 100 meters (330 ft.) by which the altitude exceeds 1000 meters."

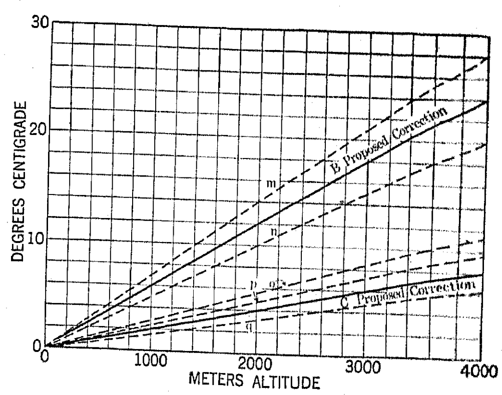

FIG. 3—INCREASE IN DEGREES TEMPERATURE RISE WITH ALTITUDE

Curve	Per Cent	Surface Rise
(m)	10	70
(n)	10	50
(o)	4	70
(p)	4	60
(q)	4	40

The proposed corrections are shown by curves (B) and (C). (B) is 10 per cent of 60-deg. surface rise. (C) is 4 per cent of 50-deg. surface rise.

There are two objections to the rule as stated:

(1) The rule is loose in that a machine tested anywhere under 1000 meters is considered suitable for operation anywhere under 1000 meters. This may lead to a difference in temperature rise of about 6 degrees, thus causing the temperature of a machine to exceed the allowable temperature limit by that amount when operating at 1000 meters in a 40 deg. ambient temperature, if it is tested near sea level at the limiting standard temperature rise, as many machines now are.

(2) The rule states that all machines for operation above 1000 meters shall be regarded as special.

In the interests of economy it is advisable to reduce in so far as possible the special applications of machines. Fundamentally there is no more reason why there should be a multiplicity of ratings for different altitudes than for different temperatures of the air in which a machine must work due to different climatic conditions. The effect of increased temperature, due to increase in temperature rise, of a machine operating at abnormal altitudes is of no more consequence than the same effect, due to an average correspondingly great difference in ambient air temperatures, existing in different localities. Yet in the former instance it has been considered necessary to give the machine a rating based on an entirely different temperature rise. Weather Bureau reports show a difference of 11 deg. cent. for the month of July between the normal surface temperatures of the northern and southern portions of the United States, a difference in temperature which corresponds to the extreme correction necessary for an increase in altitude of 2000 meters, namely that for turbine-generator fields. For this difference of 11 deg., which occurs in innumerably more instances than a similar difference for altitude, there is no provision made in the A. I. E. E. Standards—nor should there be. The mean daily maximum temperature for the month of July for same regions differ by about 17 deg. cent. which corresponds to the extreme correction for an increase in altitude of nearly 3000 meters.

There is also probably the less reason for these different ratings for different altitudes because generally the effect of increased altitudes is offset by lower ambient temperature conditions. A report published by the weather bureau of the United States makes the statement that:

"The observed decrease in temperature with elevation is, on the average about 1 degree Fahrenheit for 330 ft. (5. 5 deg. cent. per 1000 meters): it is more rapid in summer than in winter."

If credence can be attached to the above statement there will be only a very occasional occurrence of such unusual conditions

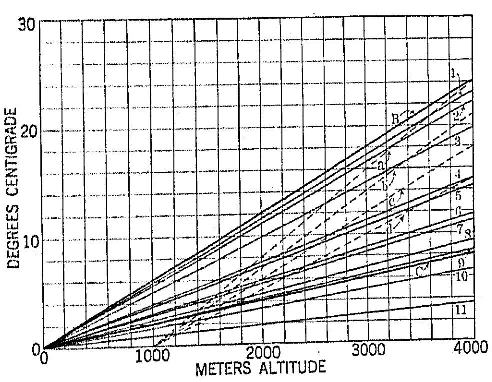

Fig. 4—How Proposed Corrections Compare Numerically with Present A. I. E. E. Recommendation and Experimental Data

This chart is a combination of Figs. 1, 2 and 3. Curves (1) to (11) are calculated from experimental data on various machines as listed under Fig. 1. Curves (*a*), (*b*), (*c*) and (*d*) are the present A. I. E. E. recommendations as shown in Fig. 2. Curve (*B*) is the proposed temperature correction for the kinds of apparatus shown by curves (1) to (5). Curve (*C*) is the proposed correction for kinds of machines shown by Curves (6) to (11). These two curves are shown also in Fig. 3.

of service as to warrant special consideration. It should be borne in mind, however, that in many instances machines are operated in buildings the inside temperatures of which may not correspond closely to the outside temperatures.

The matter of application of machines for high altitudes could be greatly simplified by considering altitude as one of the service conditions that affect the heating of electrical machinery in operation. The correction could be made most simply and effectively by considering it in connection with the limiting ambient temperature in which a machine may safely carry its rated load.

Fig. 5 shows the limiting ambient temperature at various altitudes in which a standard apparatus, rated on a basis of sea level, may be expected to carry its rated load without exceeding the maximum limiting temperature established by the A. I. E. E. Standards. It will be clearly seen that these curves have been

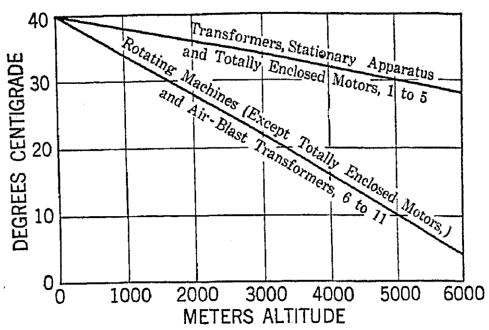

Fig. 5—Proposed Rule Limiting Cooling-Air Temperature at High Altitudes

This chart illustrates the proposed rule which designates the limiting ambient temperature in which standard apparatus rated on a basis of sea level, may be expected to carry rated load without exceeding the maximum limiting temperature.

arrived at by subtracting from a 40-deg. ambient the resultant correction given in Fig. 3.

Expressed in words the proposed guide would be:— Rotating machines (except totally enclosed motors) and air-blast transformers.

These types of apparatus shall be suitable for carrying their rated load provided the ambient temperature does not exceed a value of 40 deg., less 6 deg. for each 1000 meters by which the altitude exceeds sea level.

Transformers (except air-blast transformers)
Reactors and bus bars
Rheostats, meters, relays and
Miscellaneous stationary apparatus
Totally enclosed motors.

These types of apparatus shall be suitable for carrying their rated load provided the ambient temperature does not exceed a value of 40 deg., less 2 deg. for each 1000 meters by which the altitude exceeds sea level.

The effect of the proposed method of applying the rule is to make the correction definite over all altitudes including altitudes less than 1000 meters. But it should be noted as important that a standard apparatus rated on a basis of sea level would be on the proposed basis suitable for operation up to at least 1000 meters under usual conditions of service and that the size or rating of machines applied under these conditions will not be changed. Furthermore, the effect is to regard the application of apparatus at higher altitudes as special either by de-rating or using a larger machine, only when the higher altitude is coupled with unfavorable values of ambient temperatures, as given by the proposed rule.

L. B. Cherry: I wish to ask Messrs. Montsinger and Doherty if, in their research, due consideration was given to the possible effects that varying degrees of humidity might have on the rise of temperature. It is readily seen that even at high altitudes, equipment might be installed in basements or even under-ground enclosures where, with contributing sources of moisture, and under conditions of poor ventilation, sea-level conditions of humidity migh prevail, and thus effect heat radiation. Do varying degrees of humidity seriously effect radiation?

H. M. Hobart: I want to speak in favor of Mr. Paxton's proposition. This is a subject on which we have been working in the Standards Committee for a good many years but we have had to wait for Mr. Paxton to show us the simple and satisfactory way to deal with it.

I don't think that from Mr. Paxton's brief presentation we would all be sufficiently impressed with its merits and I hope that it will be given the most careful study.

There is one little matter that ought to be taken into account in actually working out the way in which Mr. Paxton's plan should be assimilated into the standardization scheme, if it meets with the approval with which I hope it will meet, and that is that all these little refinements when we are able to figure them out to a nicety, result in showing people how to work closer to the so-called approved limit. It comes within the ability of the operating engineers to load their machines for a larger part of the time up to those values that have been described as "approved" limits. Now when these limits were established, they were associated with conditions which practically ensured that the average temperature reached was 10, 15 or more degrees below the established limit. But when people are encouraged to take up this very interesting practise of adjusting the load, putting on more and more load until a limiting value is reached or approached, the deterioration of insulation is bound to be more rapid. The deterioration may still be quite slow, the machine may have a satisfactorily long life, but even though the life may be satisfactory, it is bound to be shorter than if it were running at a lower temperature. Maybe 105 deg. is too conservative or maybe it is just right, but whichever it is, the machine will have a shorter life with this new knowledge that we are getting, tempting us to approach limits, than when we kept further away from those limits, regarding them as sorts of danger signals. I don't say there is any danger in approaching and staying at those limits; that is a point to be investigated but certainly it is a new and comparatively untried thing to operate machines in accordance with this new conception of the limiting values and it is bound to shorten their life. The life may still be entirely satisfactory but it can't help being shorter as far as relates to insulation deterioration. I don't believe insulation deterioration is often a factor determining the life of a machine. Other factors enter in determining when a machine should be thrown out and a new machine put in. Nevertheless, it is a fact if you have a scheme of things that makes you run 5 or 10 deg. higher temperature than you used to run, as far as the insulation is concerned, the life of the machine will be shorter, and that ought to be taken into account in working this scheme into the plan of things. If 105 deg. is the right limit with the old ideas, it ought to be reduced to 100; if 105 is already too conservative on the old ideas, it will probably be about right for the new basis.

J. R. Craighead: I want to put in one little plea for simplicity in this rule. The feature I think that has been most satisfactory in the previous rules has been the fact that virtually there was no rule up to 1000 meters. That allowance was made as representing the general conditions under which apparatus satisfactory at sea level would work reasonably well.

Now Mr. Paxton's rule as shown in Fig. 5 would require that for every foot advanced from sea level there is a definite change in the condition under which a customer can operate his apparatus. I think that is really impracticable. Whether the rules are made this way or not, it will be necessary for the operator, and to some extent the manufacturer, to base their work on the idea that there is a flat step at the start of the curve, a rise over which apparatus for sea level can be used without special investigation or separate temperature limit. And it would seem to me a very desirable thing that the Institute rules should be so worded or arranged that this flat step is taken care of in the rules. At the present time this is up to 1000 meters; possibly that is not the correct value for it, but some flat first step should be assumed.

E. B. Paxton: I would like to say in regard to the point Mr. Craighead made, that the correction up to 1000 meters would generally be so small it might be regarded as negligible in operating the machine, that is, no one who operates the machine is going to worry about whether ambient temperature is 36 deg. or 40 deg.

What I am proposing is that this correction be applied as a matter of operating, as one of the service conditions which will affect the way the machine may be operated rather than to make it apply to the rating of the machine.

There is one further point I would like to make in that same connection and that is that there is a natural tendency for the temperature of the cooling air to be less at the higher altitudes. The weather bureau made the report that the temperature would probably decrease 5½ deg. for every 1000 meters of altitude. Now just how much credence should be given to that for the reason that machines are often operated inside, is a question, but I think very likely the inside temperature drop may be of a corresponding degree.

N. S. Diamant (by letter): In Doherty and Carter's paper the statement appears that the heat transmission by convection was practically unchanged when the surface was painted with aluminum. It would appear as if they imply free convection. If so, their results and their theory would seem to be in contradiction to the tests submitted several years ago by the late John R. Allen of the University of Michigan to the Society of Heating and Ventilating Engineers. Allen found that with aluminum paint the heat transmission by free convection of ordinary house radiators was reduced from 100 per cent for a bare unpainted cast-iron radiator to 75 per cent. The effect of other paints seemed to be on the same side, namely, there was a decrease in free convection for most paints that were tried.

The tests I made several years ago on the effect of paint on forced convection agree with that of the authors, namely, that the paint has no effect on forced convection.

In connection with this subject it may be added that it has been found by Allen and other investigators that it is the last coat of paint that effects heat transmission by free convection.

With reference to the effect of altitude on the cooling of dynamo electric machines of the rotating type or machines cooled by forced convection, some very interesting material will be found in the excellent work done by the Bureau of Standards on the cooling of airplane engines when flying in various altitudes.

F. D. Newbury: I desire to discuss these two papers from the stand-point of their relation to the A. I. E. E. Standards. I have had the privilege of reading Mr. Paxton's valuable discussion, and wish to endorse his suggestion that the altitude correction in the Standards apply to the limiting value of cooling-air temperature rather than to temperature rise. The adoption of this suggestion would completely avoid the necessity for special machines designed for special temperature rises and places this question of altitude correction among service conditions where it properly belongs.

The present Institute Standards, paragraph 2215, provide that no correction for altitude shall be made for altitudes of 1000 meters and less. The data presented by Messrs. Doherty and Carter indicate that for rotating machines the correction at 1000 meters would be in the neighborhood of 6 deg. with proportionately smaller corrections for lower altitudes. In spite of this maximum difference of about 6 deg., I believe the present provision that no correction for altitude be made except for altitudes above 1000 meters should be retained. This suggestion is made in the interest of simplicity, and because this rule has been well-established, not only in this country, but in practically all of the important manufacturing countries. The I. E. C. Rules for Electrical Machinery contain the following paragraph:

"Altitude.—In the absence of any information in regard to the height above sea level at which the machines is intended to work in ordinary service, this height is assumed not to exceed

1000 meters. If the machine is intended to work at an altitude above 1000 meters, a correction to the temperature rise should be applied. The value for this correction has not yet been fixed by the I. E. C."

As an example of British practice, the British Standard Specification for Electrical Performance of Industrial Motors and Generators (No. 168 of 1923 maybe quoted:—

"The Standard Ratings provided in this Specification are suitable for machines operating at altitudes not exceeding 3300 ft. above sea level. Machines for use at higher altitudes are dealt with in Clause 44. Machines intended for service at altitudes above 10,000 ft. are not considered standard."

(Clause 44.) "When a machine intended for service at high altitudes is tested near sea level, the limits of temperature rise given in Table I shall be reduced $1\frac{1}{2}$ per cent for each 1000 ft. above sea level at which the machine is intended to work in service. The correction shall not be appled for altitudes below 3300 ft."

The German Rules (V. D. E. of 1923) apply to "Generators operating at an altitude of 1000 meters or less. Special agreement is necessary for higher altitudes."

As an example of French practice, the French Standards for Electrical Machinery produced, by the heavy-machinery manufacturers and approved by l'Union des Syndicats de l'Electricite, July 22, 1920, contain a paragraph to the same effect as paragraph 2215 of the A. I. E. E. Standards.

It will be observed that the rule providing for no correction up to and including 1000 meters has been adopted by the I. E. C. and by four of the most important manufacturing countries. Any change of the A. I. E. E. Standards in this particular would indoubtedly be difficult to establish internationally, and we should differ from established international standards only when the importance of the matter is very great.

From the standpoint of our every-day work, I believe the present rule providing for a correction only above 1000 meters should be retained. The great bulk of electrical apparatus is operated in locations having an altitude less than 2000 ft. (600 meters). All of the eastern half of the United States (east of 100 deg. longitude, passing through Texas, Oklahoma, Kansas, Nebraska, and the Dakotas), is less than 2000 ft. altitude, except for the higher ridges of the eastern mountains. The experimental data furnished by these two papers show that the correction for 1000 meters amounts to a reduction of 4 deg. to 6 deg. for various parts and types of rotating machines, applied either to the guaranteed temperature rise, or, as proposed by Mr. Paxton, applied to the 40 deg. cooling-air temperature. Probably more than 95 per cent of the rotating machinery built in the United States is installed in locations having altitudes less than 2000 ft., for which the permissible cooling-air temperature would be reduced, according to the papers under discussion, only 2 or 3 deg. For transformers, the reduction in permissible air temperature for altitudes less than 1000 meters would be less than 1 deg. If the proposal for a correction starting at sea-level were adopted, I am confident it would be a dead letter for the great majority of electrical apparatus tested.

The data presented in these papers indicate that some change in the present correction should be made for altitudes sufficiently high to make the correction worth while. As I have indicated, I believe we should retain the present plan of no correction until 1000 meters is exceeded. I believe also that we should adopt Mr. Paxton's suggestion, and apply the altitude correction to the permissible cooling-air temperature, instead of to the guaranteed temperature rise, as now provided for in the A. I. E. E Standards. In order to retain the present rule of no correction until 1000 meters is exceeded, and at the same time make the corrections for higher altitudes agree with the experimental data that has been presented, and further, to avoid odd and fractional values of limiting air temperatures, I suggest that the Standards contain a table of definite limiting cooling-air temperatures for various altitudes, instead of a general rule expressed either by a formula or by a curve.

The following statement is suggested:

SERVICE CONDITIONS

Usual Service Conditions.

Temperature and Altitude.—Machines conforming with these standards shall be suitable for carrying their rated load, when the temperature of the cooling air does not exceed the values, at the corresponding altitudes, given below:

Up to and including 1000 meters (3300 ft.) . .	40 deg. cent.
Above 1000 meters to and including 1500 meters. . . . . (5000 ft.).	35 deg. cent.
Above 1500 meters to and including 2000 meters. . . . . (6600 ft.).	30 deg. cent.
Above 2000 meters to and including 3000 meters. . . . . (10,000 ft.)	25 deg. cent.
Above 3000 meters.	20 deg. cent.

The corresponding values for self-cooled transformers would be:

Up to and including 1000 meters (3300 ft.)	40 deg. cent.
Above 1000 meters to and including 3000 meters. . . . . (10,000 ft.).	35 deg. cent.
Above 3000 meters.	30 deg. cent.

V. M. Montsinger: From the nature of these discussions it is not necessary to comment on the various methods proposed for incorporating the altitude correction in the Standards. This must be considered at some future meeting of the Standards Committee.

In reference to Mr. Cherry's question about the effect of humidity on the temperature rise, the following are the approximate facts.

Humidity of course does not affect the loss of heat by radiation. The loss of heat by free convection is practically unaffected by the moisture unless the air is fog laden. The amount of water ordinarily in the air is so small even though the humidity is high that the effect is in the order of one per cent or less. The question of humidity has been considered but the conclusion was that for practical purposes it can be neglected.

R. E. Doherty: The authors are gratified that the results outlined in their paper may prove useful as a basis for the revision of the A. I. E. E. Rules relating to the subject.

The principle on which Mr. Paxton's plan is based is fundatally sound, and the details of his plan are, in our opinion, the most promising that have been proposed.

Mr. Montsinger has answered Mr. Cherry's inquiry about humidity.

Underground Alternating-Current Network Distribution for Central Station Systems

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Review of the Subject.—*1. Certain experimental arrangements for an underground alternating current distribution system are covered in the paper.*

2. *Low costs and characteristics satisfactory for general utilization of alternating current are contrasted with difficulty of obtaining reliability. The advantage of adopting system characteristics which will make each service suitable for lighting, appliances, and motors is indicated. There is described experience with a combined light and power network on one set of mains. Effect of voltage variation on incandescent lamp illumination is discussed. An appendix covers tests on this subject.*

3. *Reliability in service of standard distribution materials is considered. Tests on cables are described which indicate that for underground distribution conditions, low-voltage cables will eliminate arcing faults while high-voltage cables will not do so. An appendix of arcing tests is attached.*

4. Certain factors are discussed relative to size of mains and location of transformers necessary to make the low voltage network clear its own faults.

5. *There is a description of an experimental system consisting of several radial high-voltage feeders, the distribution transformers of all these being connected on the low-voltage side to a common low-voltage cable network.*

6. The reactance of the transformer circuits is almost three times that of standard distribution transformers.

7. The only protective devices used are automatic circuit breakers installed in the low-voltage cables between the distribution transformers and the network. The switches open on a reversal of energy from the network and close when the transformers are in a condition to supply power to the network. The devices are sensitive enough to open on transformer magnetization energy.

8. Great reliability obtained for the network as a switch failure on a high-voltage fault is the equivalent of a short-circuit on the secondary network.

9. Ability to switch automatically all transformers by controlling the supply end of the feeder to which they are connected, allows a higher all-day efficiency due to the saving of iron loss at periods of light load. This feature also makes it easy to work on high-voltage equipment.

10. A description of the equipment used in this experimental installation and operating results of switch equipment from date of installation in April 1922 to March 1924 are given.

11. It is believed that with low-voltage a-c. underground networks arranged as described, the reliability is the same as the reliability of the sources supplying the various radial high-voltage feeders. The possibility in the future of a protected network arrangement of the supply system is mentioned.

IT is the purpose of this paper to describe the design and operating results of certain experimental distribution arrangements[1] on the 60-cycle system of the United Electric Light and Power Company in Manhattan.

Alternating current distribution has had general use on overhead service because of its economy in first cost and low operating expense, and the underground service has increased rapidly in the past few years because of the above conditions. In general, this system's characteristics for utilization of the service has been satisfactory. The disadvantage found in variable speed motor applications are compensated for to a large extent, by the extremely simple constant speed motors. Other characteristics in most instances have been good.

Reliability of supply and the adoption of system characteristics for services which will make it possible to apply universally lamps, appliances and motors at any service, are the principal subjects considered in this paper.

The practise in this country has been for most central stations to supply general power service at twice the voltage used for lighting or general appliances. In a-c. distribution this resulted in having separate power services for polyphase motors. There are many advantages in having any service capable of supplying all kinds of service. The common problem, in considering the possibility of combining light and power on one set of mains and services, is the permissible variation in illumination of incandescent lamps, due to changes in voltage. The principal cause of voltage variation is the starting current required by motors. The writer was unable to discover in 1921 any published information concerning variation in voltage which was possible without having a visible change in illumination for standard incandescent lamps. Results of tests, made shortly after (described in Appendix A), indicate that two per cent instantaneous variation in voltage cannot be detected and that three per cent is the maximum possible for the highest grade service. Changes of five per cent, however, will not be observed in most conditions found in practise.

A light and power polyphase network on a single set of mains was placed in service with 300-kw. installed transformer capacity feeding from five locations and having 15 elevator motors, as well as a typical city load of apartment lighting, appliances and miscellaneous motors. The calculated maximum lighting voltage variation at the services was three per cent. Experience of 12 months' operation is that such a system will give illumination without flicker at a lower cost and

1. In an endeavor to describe briefly the designs adopted, common terms to explain new conditions have been used. Some of these have broader meanings than are applied herein. It is believed however that the subject matter will not be misleading if these limits are realized.

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that its services will be of the so-called universal type, in that lamps, appliances, or motors can be connected to any of them.

Reliability of supply is of paramount importance to any successful system. It is also desirable in designing a distribution system to use as much standard equipment as possible. Our studies of service reliability contemplated distribution from substations or other sources, with available voltages in excess of 2000 volts, and stepping down to values which were suitable for standard lamps, appliances, and motors.

Service equipment has been developed to a high state of reliability. Edison systems (d-c. network distribution) had a large share in bringing this about. Present apparatus is considered suitable for a highly reliable system.

The other elements entering into an underground a-c. distribution system are—

(a) Cables—(1) for low-voltage and (2) for high-voltage.

(b) Distribution transformers

(c) Protective devices

For reliability in operation standard cables were examined to determine their performance in case of fault. Tests were made (described in Appendix B) which indicate that if an arcing fault develops between copper and sheath, on a 60-cycle single-conductor cable of less than 220 volts, the fault will burn itself off. If an arc develops on a similar high-voltage cable, the fault will continue if the restoring voltage on rupturing the arc is in excess of 1000 volts between conductor and sheath, providing the fault current is at least 500 amperes. At 2000 volts, fault currents as low as 100 amperes will continue to arc. It is thus possible to use a network or grid of low-voltage cables which will rid itself of arcing faults. High-voltage cables, however, must be equipped with protective devices to eliminate faults on them.

If a low-voltage network is designed so that any of the conductors which become short-circuited with a low resistance path will receive enough current from the network to heat the copper of the faulty section causing an arcing fault to be set up, then the network will be able to protect itself against any faults which occur on it. With standard cables, it is necessary to have a minimum voltage gradient in the cable of at least a six volt drop per 100 ft. to melt solder in the joints or damage the insulation, and about four times this amount to melt the copper. These relations set a maximum size of network cable for any given capacity and make it necessary to have certain limiting relations for size and spacing of transformers. (Appendix C deals with solid short circuits on a low-voltage network).

With a low-voltage network which will eliminate its own faults and with high-voltage cables requiring protective equipment to handle faults which may develop, it is possible to feed from the high-voltage sources to the low-voltage network—(a) radially with a single-feeder, (b) from several radial feeders supplied from one or more synchronized sources, or (c) from a high-voltage network which in turn is fed from several feeders from one or more synchronized sources. Obviously, a reliable supply cannot be obtained for a network from a single-radial feeder. As a high-voltage network requires a quantity of expensive protective equipment and extra cable is required to obtain the slight advantage of having somewhat smaller capacity in distribution transformers, the proper arrangement appears to be a network fed in multiple from the secondaries of distribution transformers, a number of each being connected to any one of several radial high-voltage feeders.

With the use of a low-voltage a-c. network, fed from several radial feeders, the distribution of loads between different transformers made it appear desirable to increase the reactance of the standard transformers in order to more nearly balance the loads on transformers, and provide relatively high reactance paths in case a network is fed from two or more separate synchronized sources. Reactance of from two to three times the values now in use on distribution transformers is not considered excessive for such service.

Distribution transformers and, as set up previously, high-voltage cables are liable to develop faults which will persist. In providing protective equipment for the above, it was considered that by installing a circuit breaker in the low-voltage cables between each distribution transformer and the secondary network which which would open on a reversal of energy, any high-voltage fault would be disconnected from the network. The reliability of the system would be much greater than the reliability of any switch, as a failure of one device would never result in more than a shortcircuit on the secondary cables, which can be burned off without interruption to the network. In practise this is accomplished by having high-capacity low-voltage fuses on the network side of the switch, which melt in case of switch failure before the cables feeding to the network are damaged. No other protective devices are used except the customary circuit breakers on the supply end of the various high-voltage cables feeding the network.

Since it is possible to get reverse energy from the network in the form of iron loss to distribution transformers in case the high-voltage supply to any feeder is disconnected, the switches were made sensitive enough to function on these values of reverse energy and were made so that they would close automatically in case the high-voltage feeder was made alive, and circuit conditions were such that the transformers would supply energy to the network after the switches close. With this arrangement it is customary to disconnect high-voltage feeders at the supply end and the switches at each of the distribution transformers on this feeder will open automatically. By doing this at light load periods, the remaining feeders connected to the net-

work are loaded up to their maximum efficiency point, or in other words, most of the usual iron loss of the distribution transformers and regulators now out of service is saved. This also facilitates maintenance work on the high-voltage cables and transformers. It is estimated that the improvement in all-day efficiency on most systems receiving power from steam generating stations will offset a large part of the cost of the protective equipment. The elimination of all the usual protective equipment installed at distribution transformers or at sectionalizing points on the high-voltage cable is also a distinct saving.

The first installation of equipment in Manhattan was made with 29 switches supplied from four high-voltage feeders. Standard distribution transformers were equipped with external reactance, making the total reactance 8.7 per cent on this installation. (It is proposed to use 10 per cent total reactance on additions). The system was started on April 12, 1922 and has been in continuous service up to April 1st of this year.

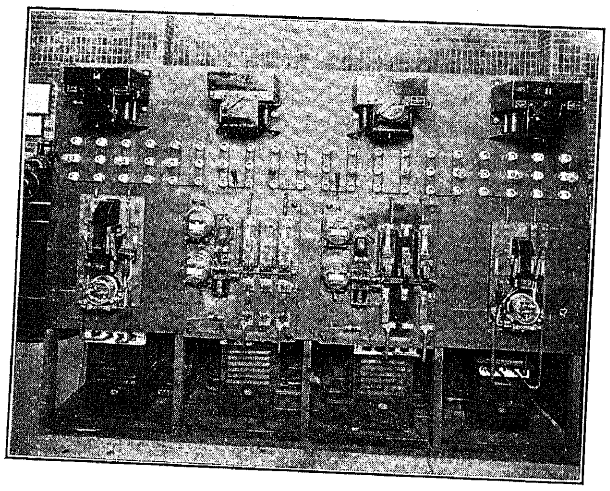

FIG. 1

The automatic switch equipment used consists of a carbon circuit breaker provided with a closing coil, and a latch which is tripped by the armature of a low voltage auxiliary relay. The circuit of the auxiliary relay is controlled by the main or master relay and as its electrical source is from the transformer secondary side of the switch, it is also used to actuate the contacts which make and break the closing coil circuit. The closing coil is thus prevented from receiving energy unless the potential is sufficient to cause the switch to close. The main relay is of the induction disc type and has two current circuits and one potential circuit, so that with the switch open one circuit is shunted across the open contacts of the switch in series with a resistance of the ballast type (tungsten lamps are generally used). The potential coil of the relay is connected across the network and the relay thus closes its contacts only if the transformer is capable of supplying energy to the network. When the switch is closed, this coil is short-circuited by the switch and the second current coil actuates the relay. This is connected to the terminals of a magnetic shunt in the circuit between the distribution transformer and the switch. It receives a large component of the current from the transformer to the network for currents of 5 per cent of switch rating. For larger current values, the magnetic shunt saturates and only allows a slight

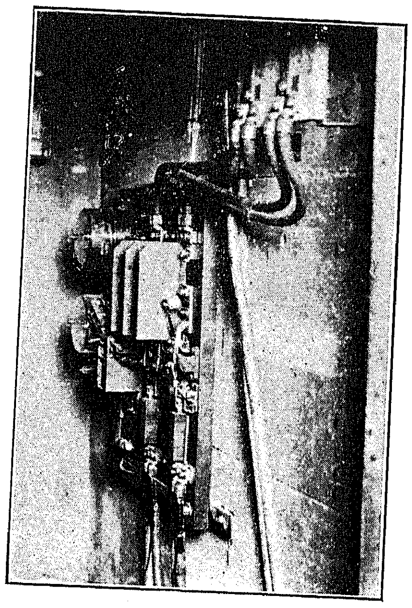

FIG. 2

increase of current in the main relay coil. This coil with the potential coil across the network makes the relay function as an energy directional device and its contacts open the low-voltage auxiliary relay circuit in case of a reversal of energy, even of the magnitude of magnetization energy of the transformer. The relay

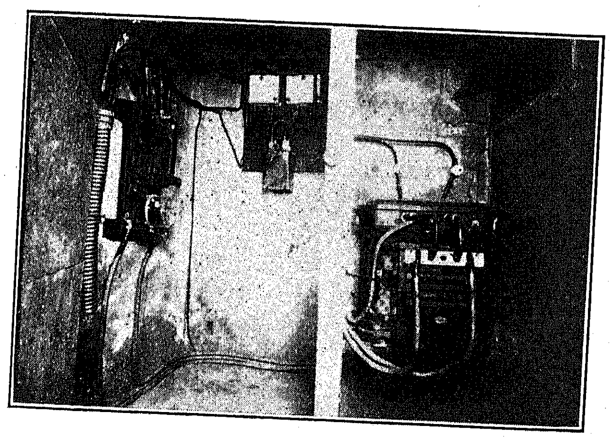

FIG. 3

has spring adjustment to keep closed its contacts with the network dead, and auxiliary electrical circuits to compensate for the spring adjustment when the network is alive and give a positive range in adjustment for either opening or closing the switch.

An auxiliary contact is provided to close a circuit to a resistance across the transformer side of the switch

when the main switch contacts are open. This resistance is used to melt a wax indicator in case the transformer secondary is made alive for several minutes without having the switch close its contacts. A failure of a network switch to automatically close is thus indicated on inspection.

Fig. 1 shows four switches connected for a single-phase demonstration. The end switches on either side are of the first type produced, while the two center switches are of the later three-pole type for three-phase

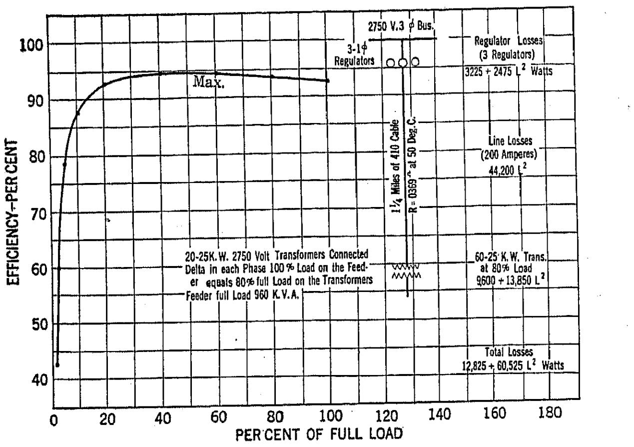

Fig. 4—Distribution Feeder Efficiency from Substation Bus to Distribution Transformer Secondary

service. Connection with the distribution transformers is made at the bottom. (In this case with external reactors in series). The network connections are at the top of the switch through a link fuse. Most of the essential elements are more clearly shown in this figure than in Figs. 2 and 3 which are illustrations of actual installations. The switch has been tested for manhole operation. All of the switches in service to date, however, have been in waterproof vaults.

It is the practise to cut out one or more of the four feeders supplying this service at different times during the day, depending upon the loading conditions. It is not possible to obtain full economy owing to the limited size of the present installation and the necessity of keeping at least two feeders always in service.

Fig. 4 shows the calculated efficiency curve of a radial feeder and its distribution transformers. Dielectric losses and secondary network copper losses are not included. It is evident that maximum efficiency point is about one-half full load kv-a. As long as there are feeders out of service it is desirable to keep the load on feeders in service below the maximum efficiency load indicated, for shorter network cable feeds and to have available capacity in service for reliability of supply.

Fig. 5 is the substation duration curve of the load in kilowatts. It shows the number of hours per year that the load exceeds any given percentage of the peak load. If the capacity of feeders and distribution transformers is assumed to be 20 per cent in excess of the peak (this amount can only be realized in practise on a network) and the kv-a. duration curve of the particular network is assumed to be like Fig. 5, then there is less than 1500 hrs. per year when the feeder loadings will exceed 50 per cent of their capacity, which is assumed above to be the maximum efficiency point. The possible improvement in all-day efficiency is dependent upon the number of feeders supplying the network, but in practise this can fall between eight and twelve on the basis of having two in service at low-load periods.

In the installation of this experimental automatic multiple-feed net-work supply, feeder capacity and amount and arrangement of distribution capacity were made liberal to be certain that all unusual conditions would be met. It was found that with high-reactance transformers it is unnecessary to space distribution transformers so that any particular district is supplied from all the sources of supply, it being sufficient that several sources be adjacent to the district.

The practise of disconnecting feeders at their sources during light load periods has given operating experience and has developed equipment faults which otherwise would have required a long time to develop. Since the principal purpose of this equipment is to give reliability to the supply, it is advantageous to know any switch which will not function on a reversal of energy. When the source is disconnected, the switches are subjected to a relatively small amount of back feed and in case of failure to open, the network will feed back to the source and the particular switch failure can be located on inspection.

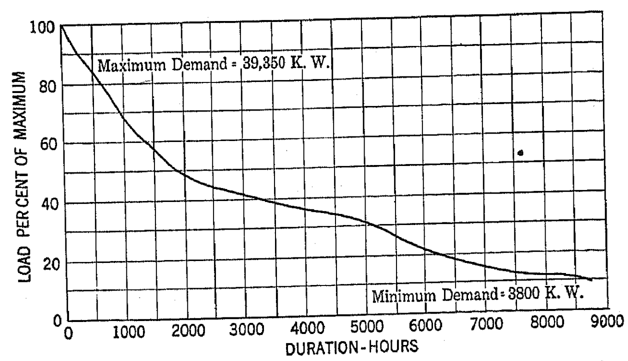

Fig. 5—Duration Curve for U. E. L. & P. Co. 60-Cycle Substations 1923. Data for Wattmeter Readings Taken Each Half Hour Throughout the Year

Since April 12, 1922, the date the network was placed in service, there have been no failures of any kind that affected the service on the network. Three transformers grounded and in each case the feeders were disconnected from their source, which immediately cleared the transformer from the network. Several single and polyphase short-circuit tests were made on the high-voltage feeders in conjunction with tests on some oil circuit breakers. In every case the network supply was continuous and satisfactory. The failures of switches to function with 48,232 total opera-

tions has been 440 (see Table I). Many of the failures are not failures to open, as one-half the total operations are those of closing the switches automatically. Failures to open are in all cases with magnetization of transformers as the load. It is probable that some of these would have functioned if the occasion had been a high-voltage fault. With one or two hand-made sample switches of a new design, much better results have been obtained. A quantity of new switches is being manufactured at present and should be in service early this summer. Many of the faults now occurring will be remedied when it is possible to replace existing equipment on the system and make necessary changes to eliminate the failures which have developed. We expect

TABLE I.
NETWORK SWITCH OPERATIONS
Number and Type of Failures

Cause of Failure	1922 Apr. 12 to Dec. 31	1923 Jan. 1 to Dec. 31	1924 Jan. 1 to Mar. 22	Total
Sticking of no-voltage relay core...	73	53	2	128
Poor contacts on no-voltage relay switch....................	5	5	2	12
Latch failure....................	8	3	2	13
Failure of pallet switch............	14	7	0	21
Burnt switch leaves.............	3	0	0	3
Master relay out of adjustment....	3	11	2	16
Friction in master relay...........	27	15	4	46
Dirt on master relay contacts......	2	2	0	4
Dirt on relay disc................	1	0	0	1
Burnt out closing coil (cause unknown).................	4	3	1	8
Burnt out closing coil (caused by faulty no-voltage relay switch)	3	0	0	3
Burnt out closing coil (caused by switch chattering due to faulty latch)....................	1	0	0	1
Switch chattering................	5	0	0	5
Lead to closing coil burnt out......	1	0	0	1
Tripping mechanism out of adjustment......................	0	24	2	26
Causes—undetermined*..........	130	17	0	147
Causes—Miscellaneous...........	1	4	0	5
Total failures............	281	144	15	440
Total operations.........	26,451	19,602	2,179	48,232
Per cent failures..............	1.062	0.734	0.688	0.912

*Includes switch failures from undetermined causes and back feeds on undetermined switches.

this class of equipment to give operating results ultimately which will compare with modern railway signal equipment. At present they are inspected once every two weeks. It is believed that except at heavily loaded periods these will be extended to periods of six weeks. The new switches on order and those now in service when reconstructed will equip to capacity 12 radial high-voltage feeders and place in service about six times the present number of switches.

Our experience indicates that with this design of underground a-c. low-voltage network, fed from several different sources, the reliability will be the same as the reliability of the sources supplying the various radial high-voltage feeders.

With the increased capacity in low-voltage networks, it will be possible to supply the feeders from two or more separate bus sections so that all the sources will not be affected by faults on one section. A development of this arrangement of operating separate bus sections in multiple makes the supply system take on the character of a network, in that it is possible to transmit the entire capacity of the generators to the low-voltage network, yet the capacity concentrated in any part of the system is a relatively small part of the total capacity. It will, of course, be necessary to equip the generating system with the usual protective devices, although of reduced capacity, compared to that required for a system where paralleling of as many supply points as possible on each bus has the effect of increasing the capacity available at that point.

Appendix A

Voltage Variation Effect on Incandescent Lamp Illumination

Tests were made during 1921 and 1922 of the effect of voltage variation on incandescent lamp illumination to determine possible amount and rate of variation without noticeable flicker. The results depend upon the particular observer, but as the object was to obtain the limits, the observers used for final results were the ones that indicated low values consistently.

A slide wire resistance was used to vary the voltage on the lamps. For instantaneous rates the resistance was cut-in and cut-out of the lamp circuit by means of a knife switch. For other rates of change, apparatus was constructed to move the slider on the resistance at various rates of speed. By measuring the volt drop per inch of resistance, the rate of change in volts per second on the lamp was determined. In order to eliminate the effect of starting and stopping, short-circuiting sleeves were put on the resistance so that only definite changes in voltage could be made from start to finish. The observers were located in a dark room with no extraneous source of light remote from the voltage varying device.

Complete sets of tests were made with 25-watt lamps and with 100-watt lamps.

Tests were made with one lamp 18 in. from the observer's paper.

Another set of readings was taken with two lamps 18 in. apart and 18 in. from the observer's paper. One lamp being flickered and the other being held at constant potential.

Similar readings were taken with the lamp source 8 ft. from the floor with the observer sitting under it in a chair reading.

Tests were made with two lamps 8 ft. from the floor, located 18 in. apart, one being flickered and the other held constant.

Under all of the above conditions, one set of tests was made with decreasing voltage, a second with increasing voltage, and a third with decreasing voltage and then restoring the same to original value.

In Fig. 6, there are four curves showing change in voltage necessary before flicker is noticed in respect to rate of change in voltage per second.

The summary of results of above tests is as follows:

1. That within limits the slower the rate of change, the greater the total voltage change necessary before flicker is noticed.

2. That after a certain point the voltage variation is noticed, no matter how slowly the change is made.

3. That at high intensities the candlepower does not make an appreciable difference, but that at low intensities the flicker is less noticeable, the lower the intensity.

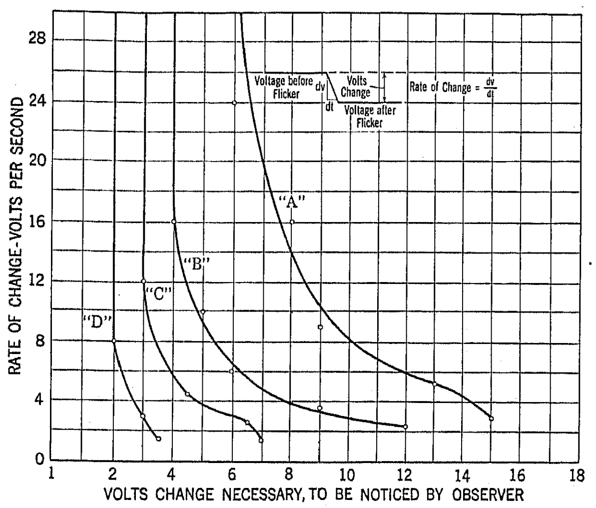

Fig. 6—Incandescent Lamp Flicker Tests, Observer Reading Newspaper

Curve *A*—Two Lamps, 8 ft. from paper and 18 in. apart voltage on one lamp varied.
Curve *B*—Two Lamps, 18 in. from paper and 18 in. apart voltage one lamp varied.
Curve *C*—One Lamp 8 ft. from paper.
Curve *D*—One Lamp 18 in. from paper.
(Mazda Type *B* 115-Volt Lamps were used. The same results were obtained with 25 and 100-Watt Lamps and with increasing and decreasing voltage).

4. The greater the distance to the light source, the larger may be the total voltage change before flicker is noticed.

5. That when there is an additional source of light of constant intensity, a greater voltage change is necessary before flicker is noticed.

6. That the same variations are noticed whether the voltage is increasing or decreasing.

7. That when the drop is made instantaneously the same variations are noticed, if the volts are dropped and allowed to remain at the lower value, as when the voltage is dropped and immediately restored.

8. That at rapid rates of change the maximum allowable variations are from two to six volts and at very slow rates of change from four to eighteen volts, depending upon the condition of illumination.

The above results have been found to be in general agreement with the operating experience of several engineers in using lamps on the same circuits with motors. A recent paper "Voltage Fluctuation and Its Effect upon Lighting" by C. A. Williams, before the Pennsylvania Electric Association in Philadelphia, contains considerable data on the subject.

Appendix B

Alternating Current Arc Tests

Single-conductor cables were tested by removing a section of lead and insulation and setting up an arc between conductor and sheath with a fuse wire short circuit. The circuit arrangements are shown in Fig. 7, resistance being utilized to limit the arc current for all except the larger values of current used. Results of tests are plotted in Fig. 8 to 13 inclusive, and characteristic oscillograph results are also shown on following pages.

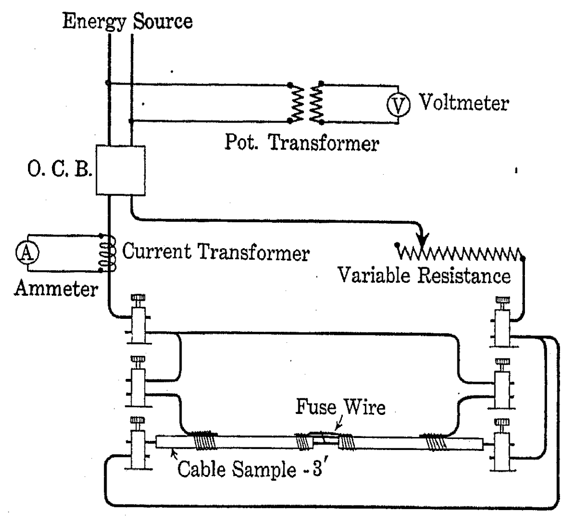

Fig. 7—A-C. Arc Tests. 60-Cycle 800-Kv-a. Motor-Generator Set Direct or with 100-Kv-a. 2750/220-Volt Transformers Connected Parallel-Series

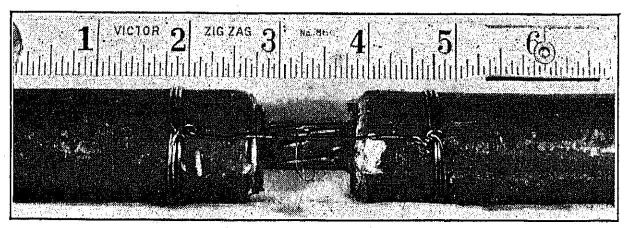

Fig. 8

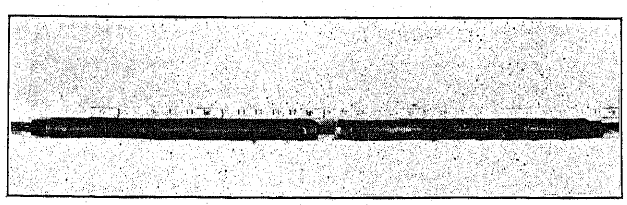

Fig. 9

Comparatively few tests were made at points where

arcs continued, as it was attempted to define the highest values where they would not persist beyond the first few cycles. One set of tests was made on d-c., which showed similar characteristics, although the voltage values were considerably reduced. (Fig. 14).

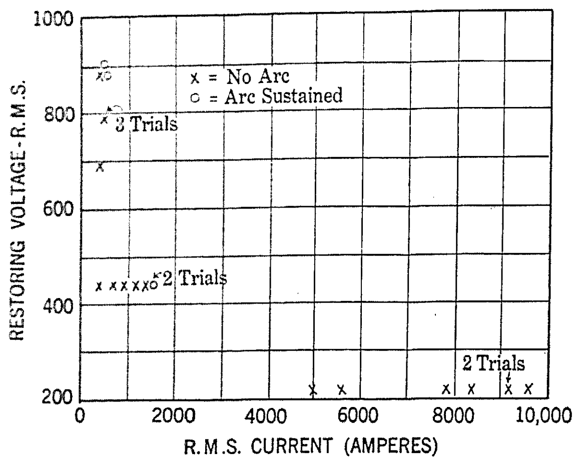

FIG. 10—A-C. ARC DATA. 4/0 CABLE 6/32 IN. RUBBER 1/8 IN. LEAD

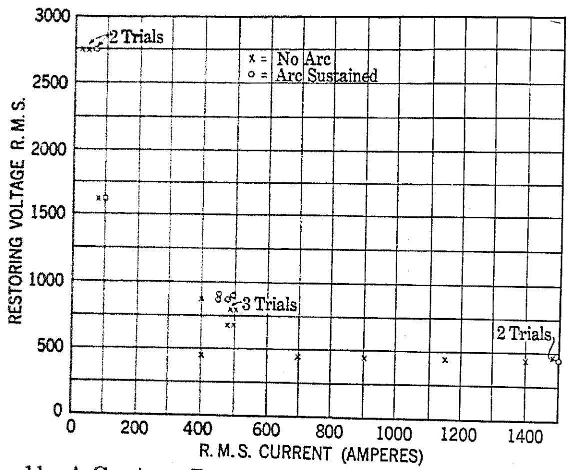

FIG. 11—A-C. ARC DATA. 4/0 CABLE 6/32 IN. RUBBER 1/8 IN. LEAD

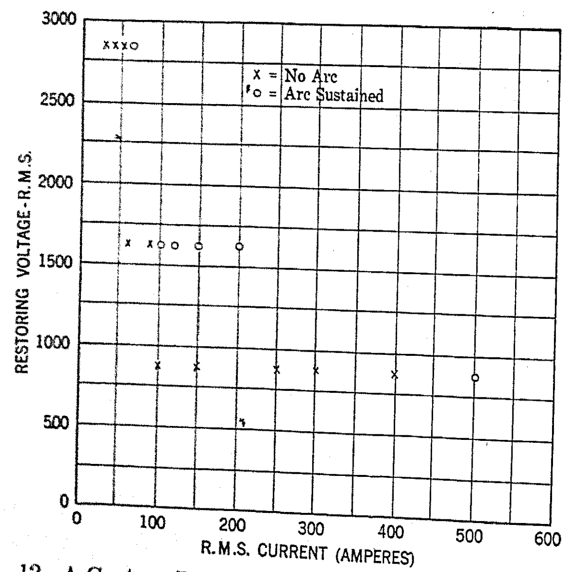

FIG. 12—A-C. ARC DATA. No. 3 CABLE 6/32 IN. RUBBER 1/8 IN. LEAD

Appendix C

SELF PROTECTING A. C. NETWORK

When cables are subject to solid short circuits they heat up to an ultimate limit where the copper melts, provided there is a sufficient potential drop maintained across the shorted conductors. This potential drop is a function of time, which, for fusing in less than one minute, is dependent upon time only, irrespective of copper cross section; while for time of fusing of over one minute, insulation thickness and overall diameter of

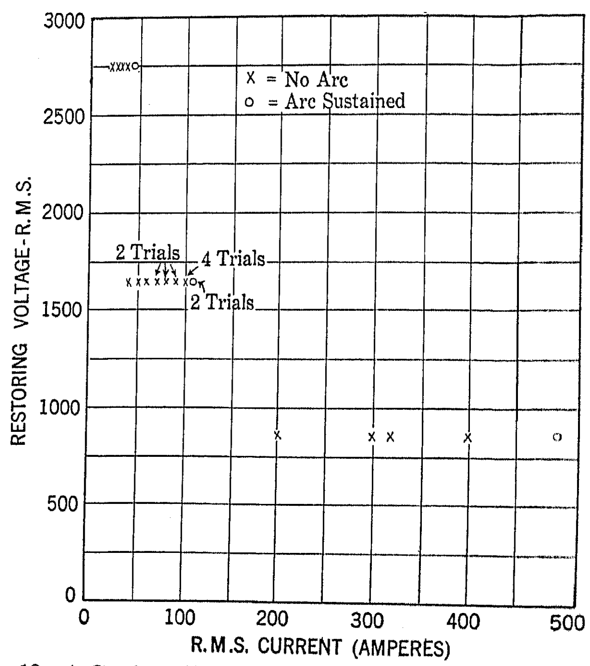

FIG. 13—A-C. ARC DATA. No. 3 CABLE 9/32 IN. PAPER 1/8 IN. LEAD

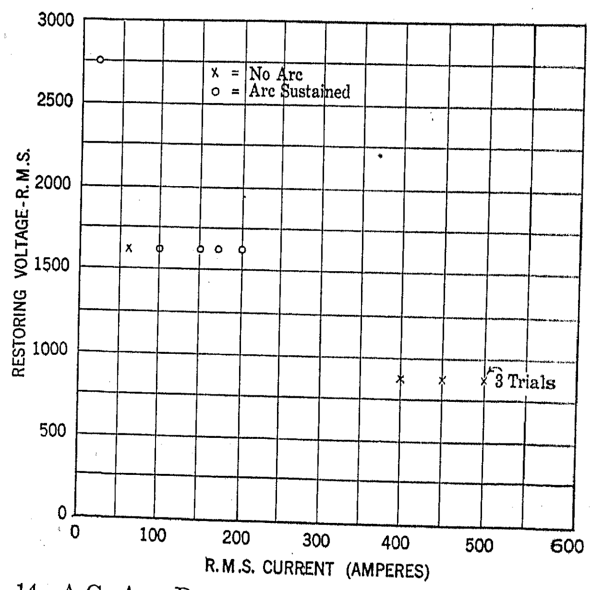

FIG. 14—A-C. ARC DATA. 80,000 CIR. MIL CABLE 3/32 IN. RUBBER 3/32 IN. LEAD

OSCILLOGRAMS—Source of Supply, 800-Kv-a., 60-Cycle Motor Generator Set

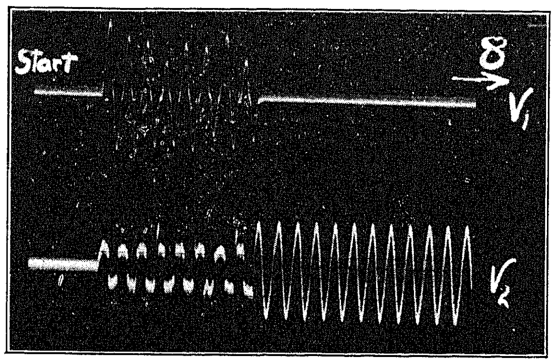

FIG. 15—4/0 CABLE—6/32 IN. RUBBER AND 1/8 IN. LEAD

Short-circuit on 1-100 kv-a. transformer. 3 per cent internal and 3 per cent external impedance. Open circuit voltage 220 r. m. s., 292 max. arc voltage 133 max. arc held 7½ cycles. At beginning of third, fifth and seventh cycles and middle of fifth cycle, current does not flow until voltage is near maximum value and does not reach full value as voltage rapidly diminishes. Current is in phase with voltage. Max. value of current 7100 amperes.

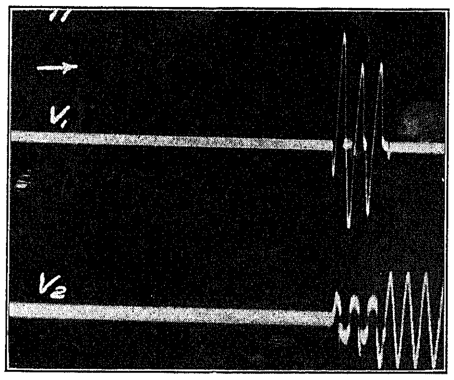

FIG. 16—4/0 CABLE—6/32 IN. RUBBER AND 1/8 IN. LEAD

Short circuit on one 100-kv-a. transformer, 3 per cent internal impedance. Open circuit voltage 220 r. m. s., 293 max. Arc voltage 117 max. arc held 3 cycles and tried to reestablish on second half of fourth cycle. Max. value of current 11,100 amperes.

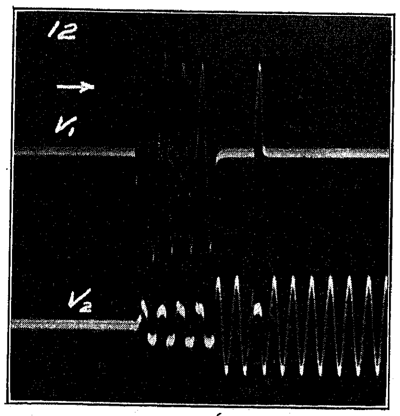

FIG. 17—4/0 CABLE—6/32 IN. RUBBER AND 1/8 IN. LEAD; COVERED WITH 2-FT. PIECE OF 3-IN. FIBRE CONDUIT AND RAGS STUFFED IN ENDS

Short circuit on one 100-kv-a. transformer, 3 per cent internal impedance. Open circuit voltage 220 r. m. s., 300 max. arc voltage 117 max. arc held 4½ cycles and tried to reestablish on seventh cycle. Max. value of current 11,850 amperes. Probably the attempt to reestablish was due to being in conduit and took place before same was fully cleared of gas by the explosive force.

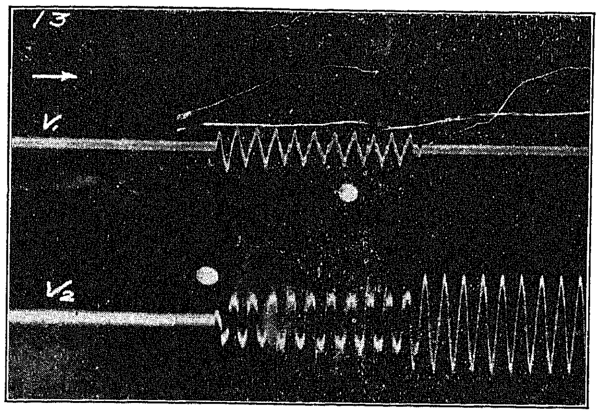

FIG. 18—4/0 CABLE, 6/32 IN. RUBBER AND 1/8 IN. LEAD

Short circuit on two 100-kv-a. transformers in parallel; 3 per cent internal impedance. Open circuit voltage 220 r. m. s., 300 max. arc voltage 150 max. arc held 11 cycles. Max. value of current 13,100 amperes. Shows the cessation of current at zero potential until voltage has risen enough to restore arc.

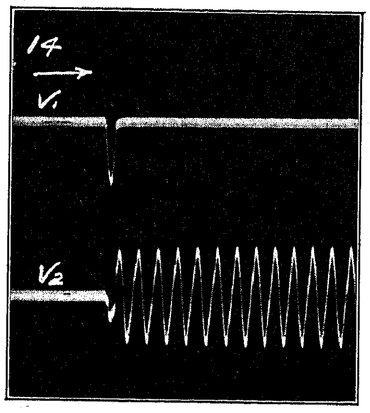

FIG. 19—SAME AS FIG. 18 EXCEPT CABLE SAMPLE WAS COVERED WITH CONDUIT AS FOR FIG. 16. EXPLOSIVE FORCE CLEARED CONDUIT ALMOST INSTANTLY AND ARC HELD 1/2 CYCLE

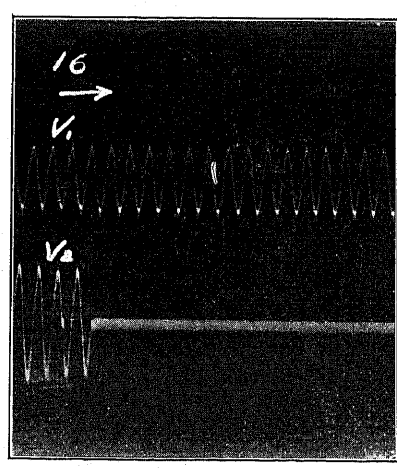

FIG. 20—4/0 CABLE—6/32 IN. RUBBER AND 1/8 IN. LEAD

Open circuit voltage 900 r. m. s., 1300 max. Resistance to limit current to 450 amperes r. m. s. Max. current when arcing 782 amperes. Arc held until switch was opened and film shows end of current wave, which is fairly smooth and indicates that arc was well sustained.

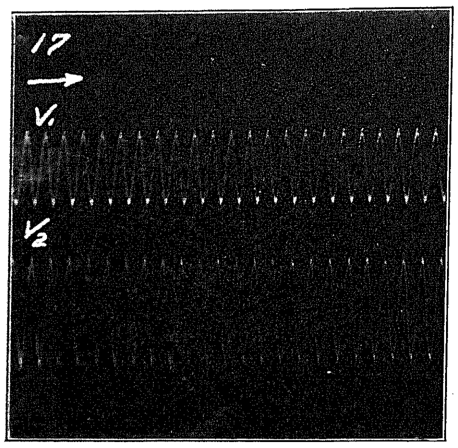

Fig. 21—4/0 Cable—6/32 In. Rubber and 1/8 In. Lead

Open circuit voltage 787 r. m. s., 1215 max. current-limiting resistance to limit current to 480 amperes r. m. s. Max. current when arcing 748 amperes. This film was taken in middle of arc time which was extinguished by opening switch. Arc probably would have died out, as near the end of the film the current stays at zero value for part of the cycle (on several cycles).

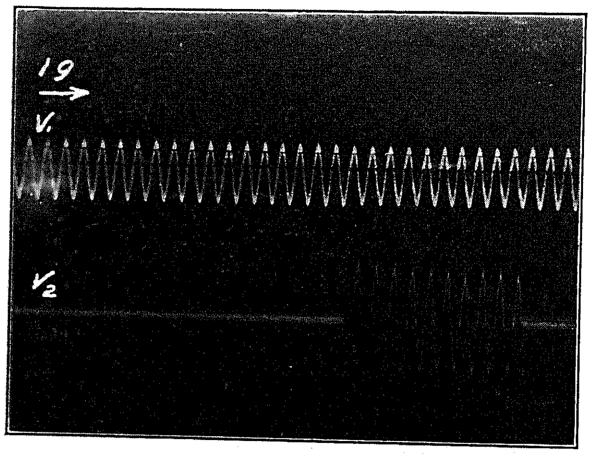

Fig. 22—4/0 Cable—6/32 In. Rubber and 1/8 In. Lead

Open circuit voltage 687 r. m. s., 1000 max. resistance to limit current to 490 amperes r. m. s. Max. current when arcing 765 amperes. Arc held 10 cycles under these conditions.

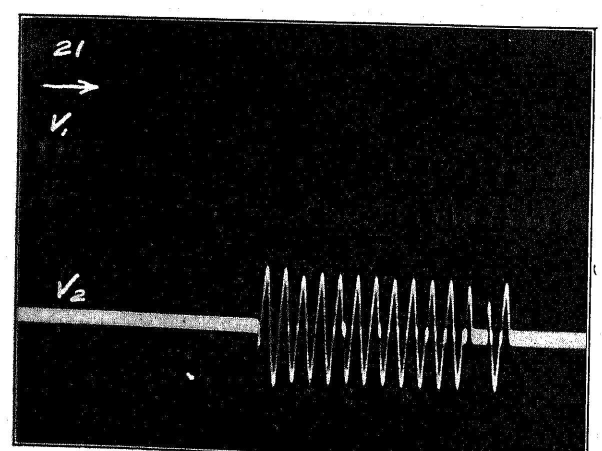

Fig. 23—4/0 Cable—6/32 In. Rubber and 1/8 In. Lead

Open circuit voltage 780 r. m. s., 1100 max. Resistance to limit current to 500 amperes r. m. s. max. current when arcing 803 amperes. Shows arc attempting to break at zero value of current and voltage. Gaps of more than ½ cycle are indicated.

cable become additional factors. These two latter factors show little divergence, for the general cable

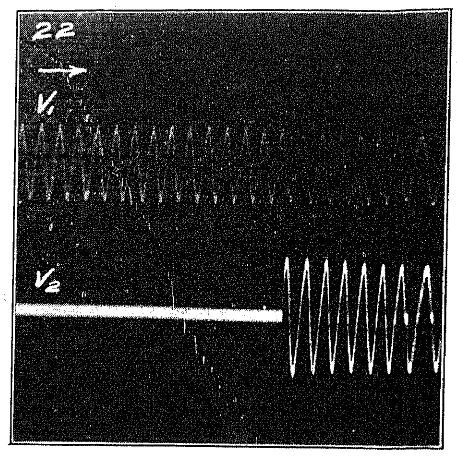

Fig. 24—4/0 Cable—6/32 In. Rubber and 1/8 In. Lead

Open circuit voltage 880 r. m. s., 1210 max. resistance to limit current to 480 amperes r. m. s. max. current when arcing 765 amperes. This arc held and switch was opened finally. Shows sputtering of arc and attempts to break. Voltage was sufficient to keep on restoring arc. Indentation in voltage waves are where arc restores suddenly.

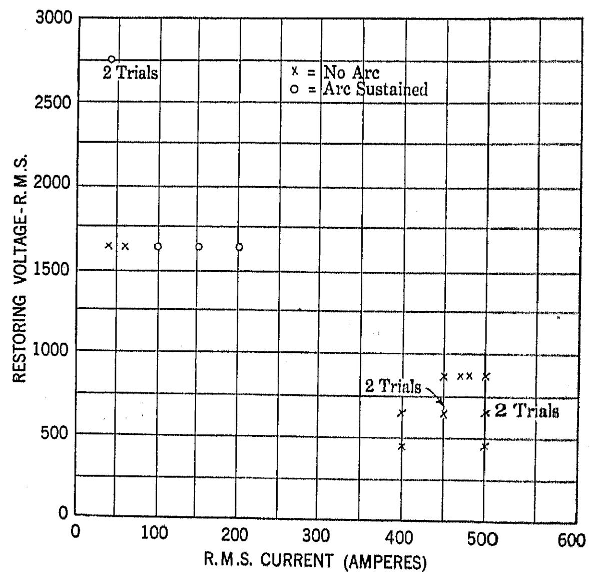

Fig. 25—A-C. Arc Data. 40,000 Cir. Mil Cable 3/32 In. Rubber 3/32 In. Lead

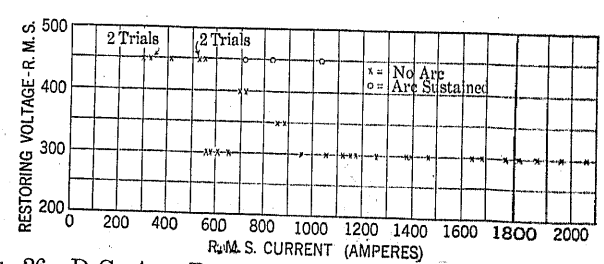

Fig. 26—D-C. Arc Data No. 4/0 Cable 6/32 In. Rubber 1/8 In. Lead

sizes which would be used in networks. Thermal capacity curves for this cable are shown in Fig. 27.

Since a minimum limit of six volts per 100 ft. is required to melt soldered joints and break down the

chemical structure of ordinary insulating material, this fixes the maximum transformer spacing. The choice of minimum voltage gradient in the designing of a self-clearing network will lie between the above value and 24 volts per 100 ft., which is the minimum voltage gradient required to fuse the copper conductors.

The maximum transformer spacing for a simple net-

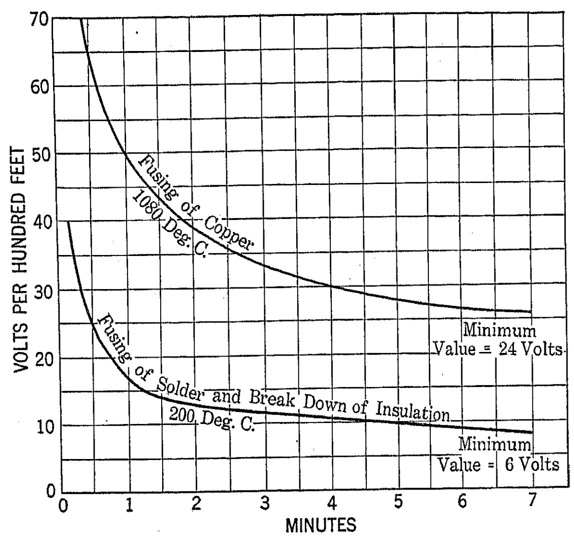

FIG. 27—VOLTAGE REQUIRED TO CLEAR SHORT CIRCUITS ON 200,000-500,000 CIR. MIL CABLES

work is numerically equal to the phase voltage of the network, divided by the chosen voltage gradient.[1] This spacing may be increased by making the network more complex, for example, the transformer spacing may be doubled by employing parallel mains tied together midway between transformer points. The configuration of the network grid is fixed usually by the arrangement of streets in the district. It is desirable to have transformer spacing and junction points of the grid coincide for the purpose of economy and regulation.

Determination of the maximum transformer size can be made from the load density and maximum spacing.

In order that destruction of copper under network short circuit may occur in the cable and not in the transformer, it is desirable that fusing current of the cable shall not be more than twice the current which can be carried safely for the required time by the adjacent transformer bank.

The maximum limits of transformer size, spacing and cable size are thus established for a self-protecting network.

Cross section of the cables may then be reduced to a size which will give the allowable regulation with the maximum instantaneous increment of current likely to be obtained. Transformer size may then be reduced, with the accompanying reduction in spacing and cable size until an economic balance is reached between transformers and cable, keeping in mind the ultimate growth in the district. Any reduction made in transformer size and spacing or in cable size will not affect the ability of the network to burn off short circuits.

For the district of Manhattan where the experimental network installation was installed, the system contemplated 200,000-c. m. single-conductor cables on both sides of all streets, transformers located at each street intersection, and a maximum grid spacing of 880 ft. in one direction and 280 ft. in the other. It was found that with 10 per cent reactance transformers, at least 25 kv-a. per phase would be required to give sufficient current to maintain more than the required six-volt drop for the extreme case of secondary short-circuit. This is considerably lower than the load density of the district requires.

Discussion

For discussion of this paper see page 869.

1. On the basis of maintaining constant secondary voltage at transformers.

General Light and Power Supply of Chicago

BY G. M. ARMBRUST AND J. B. JACKSON
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Commonwealth Edison Company, Chicago, Ill

***Review of the Subject.**—The development of a distribution system is largely determined by the load density and its rate of change. The following is a discussion of these factors and their influence on the Chicago distribution system:*

The d-c. system which supplies the central part of the city includes an area of about one sq. mi. in which the load is expected to reach 200,000 kw. in 20 years. This would economically require substation supply of about 10 substations of 25,000 kw. each.

Surrounding the small d-c. area the general light and power supply over the city is by means of 60-cycle, 4,000-volt circuits, except for the larger industrial loads which are supplied from 12,000-volt lines. The load density of the greater part of the 4,000-volt system is about 4,000 kv-a. per square mile, and the economical supply would be from 7,000 kv-a. remote controlled substations spaced about 1.3 miles. The maximum density of load on this system is 10,000 kv-a. which would require 10,000-kv-a. substations.

Calculations indicate that, with increasing load densities, the economy of this intermediate distribution voltage disappears, and in the ultimate development higher distribution voltages are necessary.

THE design of a distribution system for a city or community is naturally influenced by local conditions, such as the civic and architectural character, arrangement of streets, nature of load, construction costs, etc. Under the conditions existing, the development of a distribution system is then determined by the load density of a considerable area and its rate of change. The purpose of the following discussion is to describe the load distribution and density which influence the development of distribution systems in Chicago. The system discussed is for supply to general light and power and does not include that for railway or large units of power, which require special supply from the generating stations.

Energy is distributed by 4000 volt, three-phase, 60-cycle circuits over all of the area of the city for units of load up to 300 or 400 kv-a., except the "downtown" portion, which is supplied by the low-tension, direct-current network. Large single demands are supplied through individual transformer installations directly from the 12,000-volt, 60-cycle system.

Direct Current Distribution

The direct-current system at present covers eight square miles and distributes about 150,000 kw. maximum. About five square miles of the outer area of this system is being transferred to the a-c. system for better distribution economy, and to restrict the growth of the 25-cycle generating system which carries most of this load. The area to be cut over comprises about 20 per cent of the d-c. load and the density is about that of the more heavily loaded 60-cycle districts. The remaining area of three square miles in the congested business district will continue to be supplied from the d-c. net work, because of the great investment in the existing equipment and the desire of some of us for battery reserve.

The design of the present d-c. distribution system, particularly in the congested areas, is limited by physical conditions, such as space available for distribution cables, sites, and capacities of substations available, etc., necessary to supply the demand. Of the total present load on this system, about 75,000 kw. is concentrated in about one square mile in the downtown loop district. Studies have been made of the demands, load, characteristics of the classes of business and their probable growth, to determine the future requirements of the distribution system.

About one-sixth of the area of the congested business district is covered by large retail stores, the average load of which, when developed with modern buildings, will ultimately be about 2500 kw. per square block. Most of the district is covered with large office buildings, hotels, and theatres. The modern office building load is about 2800 kw. per square block, and under the present permitted height of buildings may reach 6000 kw. per square block in certain locations where exterior flood lighting is also used. The modern large hotels average about 2800 kw. per square block, and the theatres about 1500 kw. Railroads and terminals now occupying considerable space in the downtown district will, after electrification, have high buildings above them, probably of the office building type with loads of 2800 kw. per square block. A small portion of the area is occupied by wholesale stores, warehouses, small manufacturing, etc., which will not average over 500 kw. per block. These demands for various classes of buildings with their diversities, indicate a possible load of 200,000 kw. in this square mile in the next twenty years. If such a system was being laid out new, with sufficient space available for substations and distribution feeders, the most economical supply would be from 8 or 10 substations of 25,000 kw. capacity, each distributing over a radius of two blocks. However, as there is not sufficient space in the average downtown street for the number of feeder cables required for the larger substations, it would probably be necessary to install 20 or more substations of smaller capacity. With this arrangement, 10 or more would probably be automatically-operated or remote-controlled from the remaining manually-operated substations.

60-Cycle Distribution

The 60-cycle, 4000-volt general light and power supply covers an area of about 150 sq. mi. and supplies 330,000 kv-a. over about 375 circuits of 1000-kv-a. capacity each. These circuits are operated radially, but are provided with emergency switching centers for the transfer of load between circuits. Customers whose load is of a special nature are being provided with supply from two circuits with oil switch throw-over. Feeders are practically all underground and

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primary and secondary distribution principally overhead from 22 attended substations of 10,000 to 20,000 kv-a. capacity and 19 remote control substations of 3000 to 6000 kv-a. capacity. The remote-control substations are supplied and controlled from the attended substations, and it is planned to make all future substations of this type. The rate of increase of load supplied by this system is about 18 per cent per year, and the detailed connections for its supply are continually undergoing changes in design necessary to provide for increased capacity and to improve economy.

The average load density is about 2000 kv-a. per sq. mi. and if uniformly distributed, the economical size of substation would be about 5000 kv-a. The maximum present load density is 10,000 kv-a. per sq. mi. and occurs in residence district solidly built up with high-grade apartment buildings, hotels, theatres, etc. The most economical size of substation to supply 4000-volt distribution for this density of load is about 10,000 kv-a. spaced about one mile apart. This capacity is also about the maximum unit of capacity of the present transmission cables. While at present only a few spots of comparatively small area have such dense loads, the present rate of increase and the development of high-class residence and hotel districts indicate an area of some 10 sq. mi. will have reached such density within a few years. Measurements made of loads of test blocks of various typical areas in the city over a period of years indicate the load density has, by no means, reached saturation.

The density of a very large part of the area served from this system is about 4000 kv-a. per sq. mi., and in general, the economical design of distribution circuits for this loading would determine the standard size and arrangement of circuits. Calculations show the economical size of substation is 7000 kv-a., spaced about 1.3 mi., and the economical size of feeder is 350,000 cm. This is influenced also by the decrease in the amount of feeder regulation required for the shorter and more uniform length of feeders, as compared with the original standard size of 1/0.

As the density increases, the economy of the 4000-volt intermediate system between the 12,000-volt transmission and the secondary distribution voltage is lessening. In the area of present maximum density of the system, there is a number of square blocks having loads of from 500 to 2000 kv-a. which could be supplied from a single transformer installation and secondary mains. The increasing number of such cases in these more densely loaded areas make it desirable to extend the 12,000-volt system into these districts, taking over the existing load in certain heavily loaded blocks, and as the load of the district, as a whole, increases, ultimately eliminating the 4000-volt system in the district.

Details of schemes for supplying secondary low-tension mains direct from the 12,000-volt system have been worked out and it is proposed to install 12,000-volt feeders of about 4000-kv-a. capacity, supplying transformers to be located in the vaults. These feeders are to be from the 12,000-volt attended substations and two lines brought into each vault with provision for automatically disconnecting a faulty line.

The cost of regulation is materially decreased by this scheme, as compared with 4000-volt distribution, where the entire load is regulated on practically every feeder. Regulation can be provided in the transformer vaults and usually would be necessary on only a small part of its total load.

Industrial power loads of 400-kv-a. or more are supplied at 12,000 volts from lines subsidiary to the 12,000-volt transmission system. These lines, where possible, are arranged in loops usually of 4000-kv-a. capacity from attended substations. There are a total of about 125 such installations with an average maximum load of between 500 and 600 kv-a. The loop arrangements offer a convenient and economical scheme where the customers are grouped, as they usually are, and provide means of protecting the general transmission system from interruption due to trouble on loops or in the customer's vaults, and at the same time providing the customer with two sources of supply, and automatic reserve in case of line failure. Current-limiting reactors are installed in the supply to the loops, to prevent an excessive rush of current into damaged equipment, and sections of the loops are controlled by relays, which automatically sectionalize or cut out a damaged link without interruption.

Another arrangement under consideration provides for two or more tap feeders to a group of customers, each one being provided with taps on two feeders with throw-over oil switches, automatically-operated. This scheme would require less switching equipment and often a more economical cable arrangement.

In general, the rapid increase in loads of central station companies during the past few years has made necessary radical changes in the distribution systems to meet the demands with the best possible economy. The problems of redesigning are often complicated by the large amount of investments already made and which must be used to the best advantage. The great increase in investment required for distribution makes necessary a continual and thorough study of the characteristics and rate of change of loads for some years ahead.

In accordance with the request of the Meetings and Papers Committee to shorten papers as much as possible the above is presented as some conclusions from detailed calculations. It is hoped that an opportunity will be given in the discussion to present the data on which they are based.

We appreciate that distribution systems in Chicago are in a transient stage and much study and a long sight ahead is necessary for their development. We recognize also with the increasing loads the tendency for higher distribution voltages.

Discussion

For discussion of this paper see page 869.

A Study of Underground Distribution Systems
for the City of New Orleans

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Review of the Subject.—Recent developments in alternating current low-voltage networks have aroused a considerable amount of interest as to the application of a-c. distribution to the service requirements of business centers of large cities. Therefore, when the New Orleans Public Service Inc. was recently confronted with the necessity for rehabilitating its underground distribution system, it was decided that a thorough investigation should be made to determine the economics and advantages of various a-c. systems as compared with d-c. distribution. This paper gives the results of the investigation which was made, and describes the system which was selected to form the basis for serving the future growth of load in the underground district of New Orleans and for eventually displacing the present distribution system.

* * * * *

Part I

COMPARISON OF DIFFERENT TYPES OF SYSTEMS AND GENERAL DISCUSSION OF THE PREFERRED SYSTEM

THE distribution of electrical energy to business districts of large cities, or other similar districts having exceptionally high-load densities, involves service of the highest order, and presents conditions and requirements which differ widely from those of electrical distribution in the far more numerous and extended territories having moderate-load densities. The three-wire Edison d-c. system during many years of operation has established for itself an excellent record of reliability of service. Furthermore, the load characteristics of metropolitan districts are more favorable to the economics of d-c. distribution than those of other territories. This system has therefore continued since the early beginnings of the industry to serve the large business centers, while a-c. distribution has obtained a firm foothold in nearly all other portions of the central station's domain.

Within recent months, however, important developments have been made in the adaption of a-c. distribution to the peculiar requirements of heavily loaded business districts, and an unusual amount of interest in the possibilities opened up by these developments has been aroused. In consequence of this, when it was recently found necessary to completely rehabilitate a large underground low-voltage distribution system serving the business district of New Orleans—namely, that of the New Orleans Public Service Inc.—It was decided that a thorough study should be made of both present systems and new developments in underground distribution before selecting the type of system which would form the basis of the proposed rehabilitation. An investigation was therefore made of a number of distribution systems throughout the country, and a study as to economy and practicability was worked out for typical systems in their application to New Orleans conditions. The purpose of this study was to determine what combination of the features of systems now in use, both old and new, would be the most suitable for reliably and economically serving the business district of New Orleans.

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The system selected as a result of this study contains several of the more radical features of certain existing systems in different parts of the country. It is not yet in operation, but when installed will perhaps be the first system to utilize all these features in combination. However, no one feature is proposed which has not been tried in actual service. Briefly described, the system which was adopted comprises the following main features which may be better understood by referring to the diagram Fig. 1:

1. Direct transmission of energy from generating station to distribution transformers at 13,200 volts. The distribution substation is eliminated, and only one transformation is used, the voltage of each feeder being controlled by separate automatic induction regulators located in the generating station.

2. Use of an interconnected network of a-c. low-voltage mains served by a multiplicity of distribution transformers and by a number of 13,200-volt feeders as mentioned above. All protective devices external to the generating station are eliminated except one—namely, a reverse energy opening, automatic reclosing network switch, one unit being installed in the secondary leads of each distribution transformer bank.

3. Use of one system of secondary mains for both lighting and power service. This combination is effected by the selection of a three-phase four-wire Y-connected low-voltage system with 115-volt service between each live leg and neutral, giving 200-volts as the delta voltage.

The purpose of this paper is to set forth the results of the above-mentioned study and to discuss the various factors which entered into the selection of this particular system. Before going into details, however, it will be well to give a brief description of the existing systems which now serve the underground district of New Orleans.

Brief Description of the Present System. The underground district of New Orleans is shown on the map of Fig. 2. While somewhat irregular in shape, it is roughly twenty city blocks long and ten blocks wide.

The total present load in this district is served almost entirely by two 120/240 volt d-c. systems which parallel each other. Of these we shall consider only the system operated by the New Orleans Public Service Inc. This system is served by three substations, all within the underground district and reasonably well located with respect to the distribution of load. The center of the underground district is some two miles from the main generating station of the New Orleans Public Service Inc. While the present transmission feeders consist of 6600-volt underground cable, a 13,200-volt bus will be established in the generating station in the near future and a portion of the transmission system will be changed to this voltage.

The d-c. system of the New Orleans Public Service Inc. is now partly overhead and partly underground. The overhead portions are located at the two extreme ends of the underground district, and while they represent a considerable portion of the territory served by the d-c. system, nevertheless the load carried by these overhead portions is small. The underground portion of the d-c. system consists of the familiar network of mains tapped at various points by feeders. The feeders consist largely of 1,000,000-cm. single-conductor cable and are tied into the mains through a novel type of junction box. This junction box has the appearance of a large fire plug and is usually located at street intersections. It was designed to overcome difficulties, due to lack of drainage of duct systems which, however, have been largely overcome. Both feeders and mains are fused. The mains consist largely of 500,000-cm. cable for the outside legs, the neutral being of a smaller cross section in most cases. Load characteristics and other similar features of the system will be described in another part of the paper.

Brief Description of Systems Considered in the Study. It was assumed throughout the entire study that with the introduction of any new system an extended period of time will be required in order to entirely displace the system now in service. For purposes of calculation, a 30-year period was assumed—that is, that it would require 30 years to completely replace the existing system with a new system, the latter in the meantime being extended so as to parallel the old system throughout the district. The calculations cover the first 10 years of

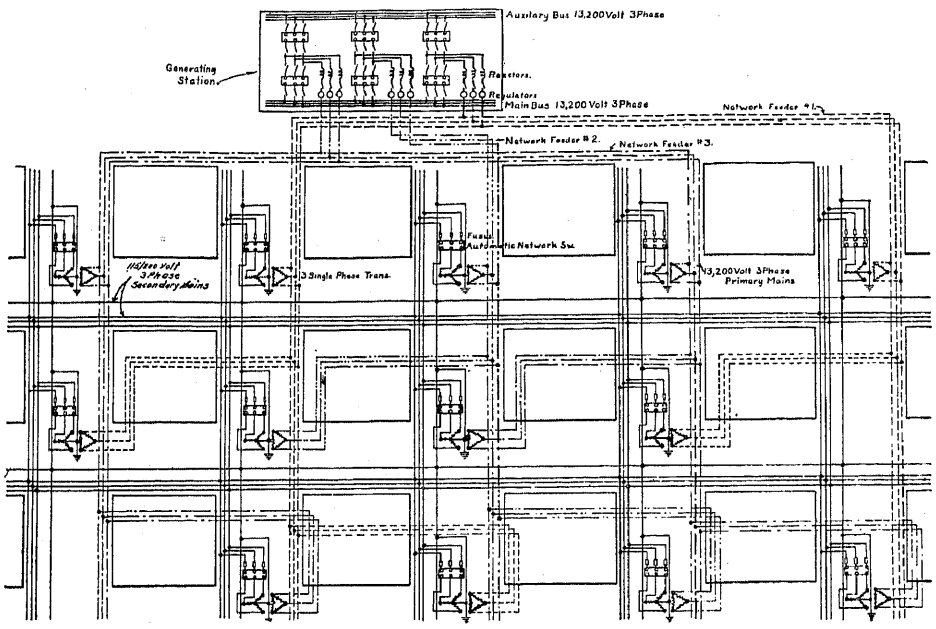


Fig. 1—Elementary Diagram Showing General Arrangement of Cable and Apparatus in the Type of Distribution System Selected for Serving the Underground District of New Orleans

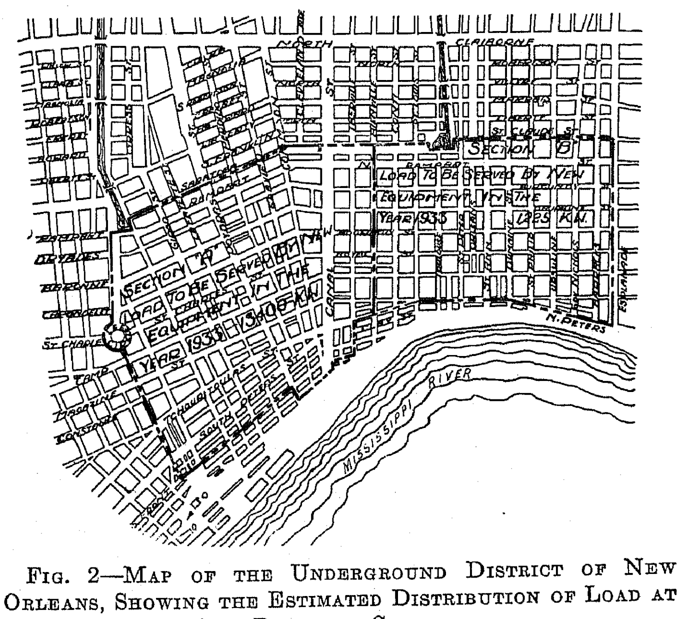


Fig. 2—Map of the Underground District of New Orleans, Showing the Estimated Distribution of Load at the End of a 10-Year Period of Growth

this period and are therefore based on a load equivalent to one-third of the present load of the underground district plus the total growth for the 10 year period.

Before presenting the tabulated cost data, a brief description will be given in the following of each of the systems which were considered. Five types of distribution systems were included in the study and these may be briefly described as follows:

1. *13,200-volt primary with network secondary.*

This is the system which was selected for New Orleans and shows the greatest economy of all those considered. No details, additional to those given above, need be included at this point. In this system, as in all the other a-c. systems considered, the service was assumed to be 3-phase 4-wire 115/200 volt.

2. *13,200-volt radial.*

This system differs from the first in that each transformer bank feeds an isolated section of secondary mains and no attempt is made to operate the transformers in parallel.

3. *4,000-volt primary with network secondary.*

This system is similar to the first-mentioned system, except that the distribution is at 4000 volt instead of 13,200 volt necessitating in this case, the usual step-down transformer substation to convert from 13,200 volt to 4000 volts.

4. *4000-volt Radial.*

This system is the familiar 4000-volt 4-wire grounded neutral system. Each 4000-volt feeder serves an individual district and each distribution transformer bank serves an isolated section of secondary mains.

5. *Combined d-c. and a-c.*

In connection with a previous study of distribution in New Orleans, the engineering features of the problem of maintaining d-c. service in the main business district were thoroughly investigated. This investigation, however, contemplated the change-over of the outlying sections from d-c. to a-c. service. For purposes of comparison with the above systems, which contemplate eliminating d-c. entirely, the economics of the combined d-c. and a-c. system are included. This, of course, presents the d-c. system under the most favorable conditions. Furthermore, the presence of an existing d-c. network eliminates a large item of investment in this case, tending still further to favor the economics of d-c. service. The division of territory between a-c. and d-c. service is shown in Fig. 2, together with the estimated load densities. The a-c. territory comprises a portion of the old French Quarter designated as Section B on the map. The a-c. system in this case is the same as No. 4 above—namely, 4000 volts radial.

Tabulated Summary of Results. The calculations are all based on cable and apparatus necessary to serve a total load equivalent to the growth of the load in the underground district for ten years plus one-third of the existing d-c. load in Section A (See Fig. 2) and all the existing d-c. load in Section B. This is equivalent to assuming that in ten years, with no renewals and replacements, one-third of the present d-c. equipment in Section A will be unfit for service, and that in the case of the a-c. systems a corresponding amount of load must be transferred to the new a-c. equipment. The investment figures for the d-c. system, therefore, include what might properly be called renewals and replacements for the present system, but which for convenience have been combined with the investment in new equipment to obtain the investment totals shown in the tables.

Before working out the detailed analysis of the four a-c. systems a preliminary study was made of the economics of transformer spacing and sizes of secondary mains for each of the four systems. This was done in order to have the comparison based on the most economical equipment for each system. The load densities in the two sections shown in Fig. 2 are so greatly different that the equipment for each of these sections was considered independently, and the transformer spacing and sizes of secondary mains were worked out separately for each of these two districts.

The figures for total investment, overall efficiencies, etc., include all of the distribution system between generating station feeder circuit breakers and the point of connection of the service laterals to secondary mains. The service laterals and service equipment were not included in calculations, since these in general would be the same for all five systems except for certain differences in the cost of meters, etc. between a-c. and d-c. service, and these differences would be in favor of the a-c. systems.

It should be noted particularly that throughout the calculations the investment figures were intentionally made liberal for the more novel systems, thus placing a handicap on these systems and favoring the others.

The totals shown in Table I are intended to provide a complete economic comparison between the various types of systems. They include fixed charges on new investment, energy and demand charges for electrical losses in the system, substation operators' salaries for the systems which have manually-operated substations, and maintenance of automatic switches for the network systems, all being based on the characteristics of the new equipment at the end of a ten-year period of growth. To facilitate the comparison, figures showing the differences between the annual charges of particular types of systems have been added.

Table II gives the estimated aggregate new investment to the end of the ten-year period, itemized to show individual totals for each of the various kinds of equipment. To facilitate the comparison, figures showing the differences between the total new investments of particular types of systems have been added. The investment figures are based on equipment which would be necessary if there were no losses in the system. The investment in additional equipment made necessary by the losses is included separately in the table showing "Demand Charges for Losses" which also includes some generating station charges.

TABLE I

SUMMARY OF TOTAL ANNUAL CHARGES

	Network 13,200 Volts	Radial 13,200 Volts	Network 4000 Volts	Radial 4000 Volts	Combined A-C. and D-C.
Charges on New Investment at 14 per cent	$193,000	$233,000	$277,500	$306,500	$335,300
Energy Charges for Losses	18,920	22,730	37,680	40,440	103,206
Demand Charges for Losses	32,310	27,490	61,170	52,050	123,980
Substation Operators' Salaries			11,900	11,900	Automatic
Maintenance of Network Switches	1,780		2,560		
Total Annual Charges	$246,010	$283,220	$390,810	$410,890	$562,486

SAVINGS IN TOTAL ANNUAL CHARGES

	Savings in Dollars	Savings in Per Cent of Total for Network 13,200-Volt System
Network 13,200 Volts vs. Combined d-c. and a-c.	$316,476	128 per cent
Network 13,200 volts vs. Network 4000 volts	144,800	59 per cent
Radial 13,200 volts vs. Radial 4000 volts	127,670	52 per cent
Network 13,200 volts vs. Radial 13,200 volts	37,210	15 per cent
Network 4000 volts vs. Radial 4000 volts	20,080	8 per cent

TABLE II

SUMMARY OF NEW INVESTMENT FOR VARIOUS PARTS OF THE SYSTEM

	Network 13,200 Volts	Radial 13,200 Volts	Network 4000 Volts	Radial 4000 Volts	Combined A-C. and D-C.
Secondary Mains and Ducts	$596,000	$596,000	$492,000	$492,000	$262,000
Secondary Junction Boxes		24,000		32,100	102,300
Network Switches	48,000		64,200		
Distribution Transformers	206,500	266,000	215,500	287,000	35,800
Transformer Manholes	67,200	112,700	87,300	142,400	36,500
Transformer Vaults	22,900	22,900	15,200	15,200	
Primary Switches and Fuses		172,500		85,800	18,400
Primary Automatic					
Double Throw Switches		12,700		6,300	
Distribution Primary Cables	117,600	139,000	112,300	133,000	30,800
Distribution Feeders	180,000	180,000	105,400	105,400	767,000
Regulators	116,300	116,300	87,600	87,600	5,600
Space for Regulators in Generating Station	24,200	24,200			
Substation exclusive of Regulators			645,000	645,000	1,003,000
Transmission Lines and Ducts			157,800	157,800	133,400
Total New Investment	$1,378,700	$1,666,300	$1,982,300	$2,189,600	$2,394,800

	Savings in Dollars	Savings in Per Cent of Total for Network 13,200-Volt System
Network 13,200 volts vs. Combined d-c. and a-c.	$1,016,100	74 per cent
Network 13,200 volts vs. Network 4000 volts	603,600	44 per cent
Radial 13,200 volts vs. Radial 4000 volts	523,300	38 per cent
Network 13,200 volts vs. Radial 13,200 volts	287,600	21 per cent
Network 4000 volts vs. Radial 4000 volts	207,300	15 per cent

TABLE III

SUMMARY OF ANNUAL ENERGY (KILOWATT-HOUR) CHARGES FOR LOSSES IN VARIOUS PARTS OF SYSTEM

Losses In	Network 13,200 Volts	Radial 13,200 Volts	Network 4,000 Volts	Radial 4,000 Volts	Combined A-C. and D-C.
Secondary Mains	$3,230	$2,130	$1,930	$1,390	$320
Network Switches	520		680		
Distribution Transformers	10,500	15,800	9,000	13,000	1,130
Distribution Primary Cable	20	20	50	50	6
Distribution Feeders	2,570	2,360	2,050	1,750	9,900
Regulators	2,080	2,420	1,570	1,850	350
Substations exclusive of Regulators			19,700	19,700	89,100
Transmission Lines			2,700	2,700	2,400
Total Energy Charges	$18,920	$22,730	$37,680	$40,440	$103,206

Table III is intended to show the value of generated energy which is wasted, due to losses in the system. The charges are made up of certain generating station costs, mainly coal. The details of the distribution of energy costs are given in the latter part of this paper and will not be taken up here.

TABLE IV
DISTRIBUTION IN UNDERGROUND DISTRICT OF NEW ORLEANS
SUMMARY OF ANNUAL DEMAND CHARGES FOR LOSSES IN VARIOUS PARTS OF SYSTEM

Losses In	Network 13,200 Volts	Radial 13,200 Volts	Network 4000 Volts	Radial 4000 Volts	Combined A-C. and D-C.
Secondary Mains	$5,300	$2,700	$3,940	$2,090	$250
Network Switches	460		670		
Distribution Transformers	18,220	16,460	22,580	15,980	1,020
Distribution Primary Cable	70	70	160	160	40
Distribution Feeders	5,680	5,680	5,920	5,920	31,350
Regulators	2,580	2,580	5,520	5,520	150
Substations exclusive of Regulators			16,230	16,230	85,710
Transmission Lines			6,150	6,150	5,460
Total Demand Charges	$32,310	$27,490	$61,170	$52,050	$123,980

TABLE V
ANNUAL ENERGY LOSSES IN THOUSANDS OF KILOWATT-HOURS AND OVERALL EFFICIENCIES

	Network 13,200 Volts	Radial 13,200 Volts	Network 4,000 Volts	Radial 4,000 Volts	Combined A-C. and D-C.
	Multiply by 1,000	to obtain kw-hr.			
Total M kwh. delivered at Sec. Mains	56,300	56,300	56,300	56,300	56,300
Losses in Sec. Mains	538	354	322	233	54
Losses in Network Switches	87		114		
Losses in Distribution Transformers	1,760	2,639	1,498	2,182	188
Losses in Distribution Primary Cable	4	4	9	9	1
Losses in Distribution Feeders	429	393	341	292	1,650
Losses in Regulators	346	403	263	309	21
Losses in Substation exclusive of Regulators			3,290	3,290	14,850
Losses in Transmission Lines			451	451	400
Total kwh. delivered at Generating Station	59,464	60,093	62,588	63,046	73,464
Overall (Annual) Efficiency	94.7	93.7	90.0	89.3	76.7

TABLE VI
LOSSES IN KILOWATTS AND EFFICIENCIES AT SYSTEM PEAK LOAD

	Network 13,200 Volts	Radial 13,200 Volts	Network 4,000 Volts	Radial 4,000 Volts	Combined A-C. and D-C.
Kw. Load at Sec. Mains	14,625	14,625	14,625	14,625	14,625
Losses in Sec. Mains	123	56	76	38	5
Losses in Network Switches	13		17		
Losses in Distribution Transformers	396	343	345	293	26
Losses in Distribution Primary Cable	2	2	4	4	1
Losses in Distribution Feeders	178	178	141	141	750
Losses in Regulators	86	86	59	59	4
Losses in Substations exclusive of Regulators			483	483	2,740
Losses in Transmission Lines			205	205	182
Kw. Load at Generating Station	15,423	15,290	15,955	15,848	18,333
Efficiency at System Peak Load	94.8	95.6	91.7	92.2	79.7

TABLE VII
DEMAND IN KILOVOLT-AMPERES AND GENERATING STATION POWER FACTORS AT SYSTEM PEAK LOAD

	Network 13,200 Volts	Radial 13,200 Volts	Network 4,000 Volts	Radial 4,000 Volts	Combined A-C. and D-C.
Kv-a. Load at Sec. Mains	17,200	17,200	17,200	17,200	14,625
Kv-a. Increase in Sec. Mains	253	118	137	69	5
Kv-a. Increase in Network Switches	11		14		
Kv-a. Increase in Distribution Transformers	1,776	1,649	1,353	782	26
Kv-a. Increase in Distribution Primary Cables	3	3	5	5	1
Kv-a. Increase in Distribution Feeders	302	302	233	233	750
Kv-a. Increase in Regulators	458	458	576	576	4
Kv-a. Increase in Substations exclusive of Regulator			1,355	1,355	2,740
Kv-a. Increase in Transmission Lines			408	408	182
Kv-a. Load at Generating Station	20,003	19,730	21,281	20,628	18,333
Power Factor at Generating Station	77.1	77.5	74.9	76.8	100.0*

*Based on the assumption that rotating machines will be operated with sufficient leading current to produce unity power factor at the generating station.

The figures given in Table IV represent fixed charges on investment in equipment in various parts of the system, which investment is made necessary by the losses (including wattless kilovolt-ampere due to reduced power factor) at the peak load of the system. Generating station investment is included in these charges, and also certain portions of generating station operating charges, as explained in the latter part of this paper.

Table V shows the estimated yearly losses in

TABLE VIII—PHYSICAL CHARACTERISTICS OF VARIOUS DISTRIBUTION SYSTEMS CONSIDERED IN THE STUDY

	Network 13,200 Volts	Radial 13,200 Volts	Network 4000 Volts	Radial 4000 Volts	Combined D-C. and A-C. System
Secondary Mains	Both Sections, 3-phase, 4-wire, 115/199-volt, single-conductor cable Section A, 500,000-cm., Section B 4/0-A. w. g. (The above voltages are those at lamp socket and motor terminals).	Both Sections, 3-phase, 4-wire, 115/199-volt, single-conductor cable, Section A, 500,000-cm., Section B 4/0-A. w. g. The above voltages are those at lamp socket and motor terminals.	Both Sections, 3-phase, 4-wire, 115/199-volt, single-conductor cable, Section A, 350,000-cm., Section B 2/0-A. w. g. The above voltages are those at lamp socket and motor terminals.	Both Sections, 3-phase, 4-wire, 115/199-volt, single-conductor cable, Section A, 350,000-cm., Section B 2/0-A. w. g. The above voltages are those at lamp socket and motor terminals.	Section A 3-wire, 120/240 volt d-c. single-conductor cable, Section B, 3-phase, 4-wire, 115/199 - volt, single-conductor cable 2/0-A. w. g. The above voltages are those at lamp socket and motor terminals.
Secondary Junction Boxes	None	Both Sections, 3-pole, 5-way, sizes to suit secondary mains.	None	Both Sections, 3-pole, 5-way, sizes to suit secondary mains.	Section A, ornamental casting, street type similar to existing, Section B, 3-pole 5-way, sizes to suit secondary mains.
Automatic Network Switches	Both Sections, Westinghouse, reverse energy opening, automatic reclosing with no voltage release 3 - pole, 115/199-volt, Section A, 1000-ampere, Section B, 500-amperes.	None	Both Sections, Westinghouse, reverse energy opening, automatic reclosing with no voltage release 3 - pole, 115/199-volt, Section A, 750-amperes, Section B, 250-amperes.	None	None
Distribution Transformers	Both Sections, single-phase, subway type, 13,200/115 - volt, 10 per cent reactance, 3 per bank, Section A, 100-kv-a. each, Section B, 50-kv-a. each.	Both Sections, single-phase, subway type, 13,200/115 - volt, standard reactance, 3 per bank, Section A, 100-kv-a. each, Section B, 50-kv-a. each.	Both Sections, single-phase, subway type, 2300/115 - volt, 10 per cent reactance, 3 per bank, Section A, 75-kv-a. each, Section B, 25-kv-a. each.	Both Sections, single-phase, subway type, 2300 / 115 - volt, standard reactance, 3 per bank, Section A, 75-kv-a. each, Section B, 25-kv-a. each.	Section B, single-phase, subway type, 2300/115 - volt standard reactance, 3 per bank 25 kv-a. each.
Primary Junction Boxes and Fused Cutouts	None	Both Sections, 3-pole, 2-way, 13,200 - volt, junction boxes for sectionalizing primary mains and special 13,200-volt fused cutouts in transformer primary leads.	None	Both Sections, 3-pole, 2-way, 4000 - volt, junction boxes for sectionalizing primary mains and single-pole fused cutouts in transformer primary leads.	Section B, 3-pole, 2-way, 4000 - volt junction boxes for sectionalizing primary mains and single pole fused cutouts in transformer primary leads.
Primary Mains	Both Sections, 3-phase, 4 - wire, 13,200 - volt, single-conductor cable 4/0 and 2/0 A. w. g.	Both Sections, 3-phase, 4 - wire, 13,200 - volt, single-conductor cable 4/0 and 2/0 A. w. g.	Both Sections, 3-phase, 4 - wire, 2300 - volt, single-conductor cable 4/0 and 2/0 A. w. g.	Both Sections, 3-phase, 4 - wire, 2300 - volt, single-conductor cable 4/0 and 2/0 A. w. g.	Section B, 3-phase, 4-wire, 2300-volt, single-conductor cable 4/0 and 2/0 A. w. g.
Primary Automatic Double Throw Switches	None	Special 3-pole, 13,200 volt, automatic double-throw switches with no voltage control and including the necessary potential transformers.	None	Westinghouse, 3 - pole, 4000-volt, automatic double-throw switches with no voltage control and including the necessary potential transformers.	None
Distribution Feeders	Both Sections, 3-phase, 13,200 volt, 3-conductor cable 4/0 A. w. g.	Both Sections, 3-phase, 13,200 - volt, 3-conductor cable 4/0 A. w. g.	Both Sections, 3-phase, 4 - wire, 4000-volt, 3 - conductor cable 300,000 - cm. (secondary neutral used).	Both Sections, 3-phase, 4 - wire, 4000-volt, 3 - conductor cable 300,000 - cm. (secondary neutral used).	Section A, 240 volt d-c., 1,000,000 - cm. single-conductor cable. Section B, 3-phase, 4-wire, 4000 - volt, 3-conductor cable 300,000-cm. (secondary neutral used).
Regulators	Single - phase, 13,200-volt, 150-amperes, 10 per cent, buck or boost, 3 regulators per feeder.	Single - phase, 13,200-volt, 150-amperes, 10 per cent buck or boost, 3 regulators per feeder.	Single - phase, 2300-volt, 200-amperes, 10 per cent buck or boost, 3 regulators per feeder.	Single - phase, 2300-volt, 200-amperes, 10 per cent buck or boost, 3 regulators per feeder.	Single -phase, 2300-volt, 200-amperes, 10 per cent buck or boost, 3 regulators per feeder.
Substations	None	None	13,200/4000 volt step-down transformer substations with necessary switching equipment and auxiliary apparatus.	13,200/4000-volt step-down transformer substations with necessary switching equipment and auxiliary-apparatus.	13,200-volt, a-c., to 120/240-volt d-c. substations including motor generator sets and apparatus for automatic operation. Also equipment necessary to supply small amount of 4000-volt a-c. load.
Transmission Feeders	None	None	3 - phase, 13,200 volt, 3 - conductor cable, 350,000-cm. each conductor.	3 - phase, 13,200 - volt, 3 - conductor cable, 350,000-cm. each conductor.	3 - phase, 13,200 - volt, 3 - conductor cable, 350,000-cm. each conductor.

Note: The above is based on a load in the underground district corresponding to ten years' growth plus 1/3 the present d-c. load.
Section A is the section of the underground district uptown from Bienville St.
Section B is the section of the underground district downtown from Bienville St.

thousands of kilovolt-hours in the various parts of the system. The overall efficiency of each type of system as calculated from these losses is also shown.

Table VI shows the estimated losses in kw. at the annual system peak load, and also the peak efficiency of each type of system as calculated from these losses.

Table VII shows the kilovolt-ampere peak load at the customer's service laterals and the increase in kilovolt-ampere as this load passes through the various parts of the system. The generating station power-factor at the peak load is also shown for each type of system. The kilovolt-ampere increases were all worked out vectorially and the individual figures represent the arithmetical differences between kilovolt-ampere input and kilovolt-ampere output for the particular parts of the system.

Table VIII is included in order to provide a ready reference to the cable sizes, apparatus ratings, etc. for the five types of systems.

TABLE IX
NUMBER OF UNITS IN MOST IMPORTANT PARTS OF SYSTEM

	Net-work 13,200 Volts	Radial 13,200 Volts	Net-work 4000 Volts	Radial 4000 Volts	Com-bined A-c. and D-c.
Transformer Banks......	74.1	104.0	106.5	154.7	32.0
Distribution Feeders.....	6.28	6.28	15.6	15.6	150.0
Substations............			2.0	2.0	4.0
Transmission Feeders....			3.59	3.59	

Note: The above fractional values were used rather than the nearest whole number in order not to penalize any particular system for the particular values of load which were taken to represent the ten years' growth.

Table IX gives the number of transformer banks, number of distribution feeders, etc., which were used as a basis for the calculations.

A. C. Versus D. C. Distribution. It is to be expected that certain difficulties will be encountered in making the change from d-c. to a-c. service in New Orleans, and one of the chief questions that has been raised in this connection concerns the difficulty of obtaining space underneath the streets for the location of transformer manholes. It is intended in the adopted system, in so far as practical, to locate the transformers in vaults on the customer's premises, thus reducing the cost and at the same time placing the transformers near the load to be served, as only large buildings, of course, will be selected for this arrangement. Unfortunately, however, there will be comparatively few locations in New Orleans where this type of construction can be used, due to the small number of buildings which have basements. A great deal of discussion has been devoted to the matter of transformer manholes in congested districts, and it will undoubtedly be difficult in some cases to find clear space sufficiently large for the purpose underneath the street. However, no fear is entertained as to the possibility of ultimately obtaining space for all the necessary manholes by resorting to various combinations of design. Fig. 3 shows roughly the dimensions of the proposed typical transformer manhole which were used for the calculations. These dimensions are very liberal, and undoubtedly the actual manholes as installed will be somewhat smaller.

The theoretically ideal location for a transformer manhole is at a street intersection, due to the possibility of feeding into the mains in four directions. However, the actual location in average cases will probably be near the intersection but removed several feet back from the corner so as to avoid the multiplicity of subsurface structures. In some cases it may be necessary to excavate to a sufficient depth to bring the ceiling below certain types of subsurface structures, and in a few cases it may possibly be necessary to construct three small manholes instead of one, each manhole housing one of the three transformers. It is very doubtful, however, whether it will be necessary in any ordinary case to use this particular construction.

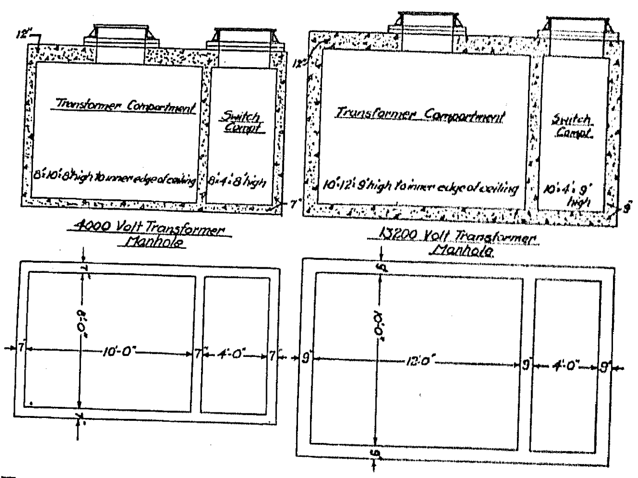


FIG. 3—TYPICAL TRANSFORMER MANHOLES FOR 4000-VOLT AND 13,200-VOLT DISTRIBUTION, SHOWING DIMENSIONS USED IN ESTIMATING THE COST OF CONSTRUCTION

Certain applications of power have heretofore been considered as inherently requiring d-c. service. One of the principal cases of this kind in connection with service to business districts is that of high-speed elevators. That is, there has heretofore been a real difficulty in obtaining satisfactory high-speed elevators designed for a-c. service. A great deal of progress has been made in a-c. elevator design in recent years, however, and satisfactory a-c. equipment is now available in speeds equal to the highest speeds now used in d-c. equipment. Probably the most commonly used type of a-c. high-speed equipment is that utilizing variable voltage control. This type of equipment consists of individual motor-generator sets for each elevator supplying variable voltage d-c. to the elevator motors. The simplicity of control and economy of operation of this type of equipment are now fairly well known. There are also medium high-speed installations in service supply-

ing a-c. directly to the elevator motor that have been successful.

It may be stated in general that a-c. equipment can now be supplied for practically all the needs of modern business centers and no difficulty is expected in lining up new business to conform to the new type of service. Existing lighting services in a large number of cases can be transferred directly from the d-c. system to the new a-c. mains. Existing d-c. services which involves large amounts of motor load need not be transferred to the a-c. system at once. It is intended to gradually reduce this type of load and ultimately eliminate it by extending the process over a period of years sufficient to allow for 100 per cent depreciation of existing equipment. That is, it is expected that in the process of years of growth d-c. motor customers can be made to gradually disappear coincidently with the depreciation of existing d-c. motors and equipment. To facilitate this process, the present underground d-c. system will be completely paralleled by the new a-c. system, and the transfer of service connections, aside from changes on the customer's premises, will be merely a matter of cutting and splicing.

A certain amount of actual change over of customer's equipment will, of course, be desirable from time to time, and the expense of doing this must be borne by the company. This will apply particularly to the sections which are now served overhead. Fortunately, however, the motor load in these sections is small.

13,200-Volt Distribution. A central station company on the west coast has for a number of years been operating an 11,000-volt grounded "Y" underground distribution system, the characteristics of which are briefly as follows:

For single-phase service standard 6600-volt subway-type transformers are used between line and neutral. For 3-phase service single units are connected in "Y" on the high-tension side with neutral omitted, this being the same practise as the standard on the 4000-volt system. Paper insulated 15,000-volt cable is used exclusively. A single-pole combined switch and fuse developed by the central station company is used for sectionalizing and protective purposes. A record of excellent service is claimed for this system, notwithstanding the fact that the higher voltage may introduce a somewhat greater possibility of cable and transformer failure, and also that a high-voltage cable failure causes an interruption to comparatively large area. The economic advantage of the use of a higher voltage for systems such as that in New Orleans is immediately apparent. By using distribution transformers having primary suitable for 13,200 volts, the usual step-down transformer substation is eliminated, with the corresponding saving in investment, increase in efficiency and elimination of the usual operating charges for substation personnel. The reliability of service would presumably be somewhat impaired by the use of the higher voltage on a straight radial distribution system. However, by the use of the interconnected secondary network this problem is solved and the chief objection to the higher voltage is eliminated.

The system adopted for New Orleans contemplates the use of 3-conductor, 13,200-volt feeders to operate at about 150 amperes per leg, the conductor size being No. 4/0 a. w. g. Three single-phase regulators located in the generating station will be used for each feeder, these regulators being wound on the primary side for full delta voltage and having a capacity of 150-amperes line current. The size of feeders selected is a compromise between good economy and reliability. Considering the value of the total load for the underground district, it did not seem desirable to concentrate a greater amount of power in single units than that corresponding to this size of feeder.

It will be noted by referring to Fig. 1 that no primary fuses or disconnecting switches whatever will be used. These devices are considered unnecessary in connection with the interconnected secondary network. The failure of a distribution transformer will, therefore, have the same status as a primary cable failure, and will mean the disconnection from the system of an entire primary feeder unit. This, however, should not involve interruption to service. The question has been raised as to whether internal distribution transformer short circuit between turns or on the low-voltage side might not draw such a small current as to fail to open the generating station switch. No difficulty from this source is expected since experience has shown that minor transformer short circuits very rapidly develop into short circuits of sufficient magnitude to operate the station protective equipment.

The distribution transformers will be connected delta-Y as shown in Fig. 1, and the primaries will therefore be wound for 13,200 volts. These transformers will all be single-phase of the subway type and will have 10 per cent internal reactance for purposes of load distribution in the secondary network.

It was originally proposed to use a 3-phase 4-wire system for the primary with grounded neutral, designing the distribution transformers for 7630 primary and connecting them "Y-Y" but the relative merits of this plan as against the "Delta-Y" connection are somewhat open to question. In view of certain disadvantages of the grounded neutral system, such as the necessity of maintaining a neutral connection between the generating station and distribution transformers, the necessity of permanently grounding the generating station 13,200-volts bus, etc., the "Delta-Y" connection was selected.

Interconnected A-C. Low Voltage Network System. The low voltage a-c. network system proposed for New Orleans is similar to the system which has been in operation in New York for a number of months. This system makes use of automatic network switches which insure reliability of service and also provide several features of economy. This particular system is de-

scribed in another paper and it is not within the scope of this paper to go into details. However, a brief description will be given.

The secondary mains of a given territory are interconnected to form a network similar to that of the familiar d-c. low-voltage system. This network is fed by a comparatively large number of distribution transformers. These transformers are divided into several groups, each group being served by the distributors of an individual high-voltage feeder and the transformers of each group being located at points distributed throughout the territory served by the network. In the secondary leads of each distribution transformer bank is inserted one of the network switches mentioned above. In the case of the transformer or primary cable failure the automatic switches of all transformers connected to the primary unit in question will open and this whole unit, including the feeder, distributors and the transformers, will be disconnected from the system, leaving the service throughout the secondary network intact, since the other primary feeders and transformers which serve the network will not be affected. This operation is accomplished in much the same manner that a transmission feeder is disconnected from a substation bus in case of short circuit. The reliability of service is, therefore, practically equivalent to that of the substation itself, or in the case of the 13,200-volt system, equivalent to that of the generating station bus. In case one or more of the network switches should fail to open, these switches will be disconnected from the system by fuses of somewhat higher capacity than that of usual practise. After the trouble is cleared and voltage is restored to the faulty feeder, the automatic switches will reclose, reconnecting the transformers to the system. If the cables should become crossed in repairing the fault, the switches will remain open until the proper relations are restored by reconnecting the cables. The switches will also remain open until the voltage in the transformer secondaries is somewhat above that of the network, so that the flow of energy will be from the transformers into the network after the switches are closed.

By opening the feeder switches in the substation (or generating station in case of the 13,200-volt distribution) the automatic switches can be caused to open, thereby eliminating the iron losses of some of the transformers during light-load periods. As the load comes on, the automatic switches can be caused to reclose by closing the substation or generating station switch.

This system has the advantage over other systems which lack the automatic reclosing feature, in that it provides increased efficiency due to the reduction of iron losses during light load periods. This saving, however, is comparatively small in this case, due partly to the high-load factor in the New Orleans underground district, the load factor for the year 1922 being approximately 44 per cent at the d-c. substation.

However, there are a good many advantages in the ability to open and close the network switches at will by manipulation in the generating station or substation and in the ability to quickly reclose these switches after having them open. In view of this and also in view of the fact that the cost of the full automatic switch is very nearly the same as that of present types of switches or other devices which are automatic on short circuit only, the type having the automatic reclosing features was selected for the New Orleans system.

Aside from the saving in iron losses mentioned above, and aside from the features of reliability of service, the use of an interconnected low-voltage network of the type selected for New Orleans provides certain inherent economies. Chief of these is the possibility of reducing installed transformer capacity by taking advantage of diversity factor. Another large item of saving lies in the possibility of eliminating primary protective devices. In the case of the 13,200-volt system this is of great importance, since there are no such devices for this voltage and for underground service available on the market. As mentioned previously, the distribution transformers in the New Orleans system will be so located in relation to the primary feeders serving them that the transformers corresponding to any one primary feeder will be scattered throughout the district served by the network. In addition to this, the transformers will have 10 per cent internal reactance. As a result, it is to be expected that the total load of the district will be very evenly divided between the various transformer banks, both under conditions when all high-tension feeders are in service and when one or more of these feeders is out of service. The high reactance will have a tendency to cause larger variations in voltage on the secondary mains than would otherwise be expected, but since the secondary mains themselves will be of comparatively large size, the effect of the transformer reactance on voltage fluctuations should be confined mainly to the changes in the entire load of the network rather than to local load fluctuations. That is, local fluctuations of load will be distributed between a number of transformer banks and the effects will not be as great as might be supposed from consideration of the increased reactance alone without relation to the impedance constants of the entire system.

3-Phase 4-Wire Secondary Service. Considerable attention has been given lately to ways and means of combining a-c. lighting and power service so as to supply both from the same transformer banks and secondary mains. This matter has assumed added importance in connection with the development of secondary a-c. networks, since two complete sets of secondary mains are required for network systems, if power and lighting are served separately.

One of the systems which has proved to be most promising in filling the requirements for a combined power and lighting service, and the one which was selected for New Orleans is the 3-phase 4-wire Y-connected system, the lighting loads being connected be-

tween each outside wire and neutral and 3-phase power loads being served from the three outside wires. In this system it is impossible to maintain both the lighting voltage and the power voltage of the secondary mains standard. A number of different voltage combinations has been proposed in order to compromise between a lighting voltage higher than standard and a power voltage lower than standard. The voltages selected for the New Orleans system are 115 volts (at the lamp socket) between each outside wire and neutral, giving 200 volts between outside wires. This selection was made in order to provide standard voltage for the most important service involving the largest number of customers—namely, lighting; and to allow any penalty to fall on the service involving the least number of customers and pieces of apparatus—namely, power. In order to insure satisfactory service for the motor load in connection with this non-standard voltage, there are several possibilities which may be utilized.

First, it is expected that possibly 90 per cent of all standard motors which are operated on this system will perform in a satisfactory manner. Reduced voltage has the same effect as increasing the percentage loading of the motor or conversely decreasing the nominal load rating. Since the majority of all motors in operation throughout the country are being operated at loads well below nameplate rating, the reduction in voltage should have generally a beneficial effect on the system in serving to increase the power factor.

Second, where cases are encountered in which standard motors will not operate at the lower voltage, the situation can be remedied by the use of small auto transformers, stepping up the voltage to the proper amount.

Third, as the system expands and is better understood by the customers, it should be possible to purchase motors designed for the lower voltage. In the majority of cases this will mean only slight changes in the winding, all other parts of the motor remaining standard.

The question of metering a system of this kind is important but presents no insurmountable difficulties. Since 3-wire lighting services in such a system will require polyphase meters, it will be desirable to require the installation of 2-wire services for all small lighting loads. A move in this direction has already been made in connection with the present tendency to use 10 ampere 2-wire meters instead of 5-ampere 3-wire meters. Where power is served at a different rate than lighting, two meters are necessary and these can be standard meters. Where power and lighting are served under the same rate, the usual two-element meter designed for 3-phase 4-wire service can be used.

The question of flickering of lights caused by the starting of motors has frequently been raised in connection with proposals for combined power and lighting systems. However, practical experience has shown that systems of this kind can be operated without difficulty, if properly designed. A 3-phase 4-wire secondary network has been in operation in Memphis for a number of years and it is largely the experience gained from the operating of this system that has furnished a basis for predictions of performance of the system described in this paper. In the Memphis system standard 220-volt motors have been found to operate satisfactorily in the majority of cases at 200 volts. The metering has been taken care of satisfactorily and flickering of lights has been found to be of negligible proportions if the proper design is used.

Reliability of Service. In laying out the protective system of a distribution network of the type adopted for New Orleans, the secondary mains must necessarily be considered as the starting point. The chief consideration which was followed in this case was that of building up a solid and extensive network which will be capable of burning off low-voltage short circuits, and which will, in general, by virtue of its very magnitude, be enabled to maintain service regardless of all ordinary low-voltage failures.

In carrying out this principle, it is proposed to tie the secondary mains into a solid network, and eliminate sectionalizing fuses. This is in accordance with standard practise in certain large d-c. systems, and has proven its effectiveness during many years of operation.

As an additional precaution, however, against failure due to short circuit of the secondary mains, it is proposed to separate the secondary network into two or more sections, connected by manually-operated switches so that in case of extreme emergency the entire network can be divided into a small number of units. In this connection, voltage wires will also be provided running from some convenient point in each of the sections to a convenient central point, where the voltage conditions can be watched. By this method serious and continued trouble, due to low-voltage short circuits can be located quickly by observing the voltages in the various portions of the network, and the section in trouble can be quickly disconnected from the remainder of the system.

Primary short circuits will, of course, be cleared from the system by the network switches and the station switch, all other protective devices, except high-capacity fuses at the network switch terminals, being eliminated.

The simplicity of the system should, therefore, in itself constitute a reasonable amount of insurance against failure. That is, the fact that multiplicity of apparatus such as substation rotating apparatus, transformers, numerous circuit breakers, etc., is avoided, should greatly reduce the number of apparatus failures. Furthermore, in considering the network switches themselves in connection with the reliability of the system as a whole, particular note should be made of the fact that full reliance is placed in no one switch and that the failure of one or more switches to function will not necessarily mean service interruption, for in this case the secondary fuses backing up a faulty switch or switches will be blown and will eliminate the faulty

apparatus from the system. Reliance is, therefore, placed in numbers rather than in individual pieces of apparatus and it is this feature, together with extreme simplicity, that should recommend the system from the standpoint of continuity of service.

It was contemplated in laying out the system that sufficient primary feeder and transformer capacity will be provided to carry the maximum yearly load with one feeder and its corresponding transformers out of service, and without loading any unit above normal capacity under this condition. This is, of course, in accordance with conservative design and corresponds with the usual practise as to transmission feeders under conditions where continuity of service is of primary importance.

In making the calculations, reliability of service was also carefully considered in connection with the radial systems and these were made as nearly comparable with the network systems as was practicable. For this purpose, a certain number of automatic double-throw primary switches were included for large customers and the radial systems were laid out for quick sectionalizing for eliminating trouble by the familiar cut and try method by which service is restored to the circuit, one portion at a time after an outage has occurred.

Part II

Methods of Calculation

Load Characteristics. The characteristics of the present d-c. load can for convenience be divided into three classes of data:

1. Geographical Distribution
2. Yearly growth
3. Hourly variations

Power-factor and diversity characteristics of the future a-c. load are treated under a separate heading.

In order to obtain the geographical distribution of the total load for the underground district, data collected in connection with a former survey of the system was used. These data were received in the form of a map showing the total kw-hr. reading of meters in each block in the underground district divided by the number of days over which these readings were taken. These data were studied so as to obtain, for purposes of calculation, a small number of districts in which the average load would be approximately the same for each block in the district. Fig. 4 shows this subdivision into three districts and the total existing load in each district is indicated. These figures were obtained by calculation from the kw-hr. data by applying the proportion of daily kw-hr. in each district to the present peak load for the whole district.

In determining the yearly growth and the corresponding load distribution for ten years hence, a number of factors had to be considered. Data showing the previous peaks of the two d-c. systems were available. However, these data were not of a great deal of value in determining the growth for the coming ten-year period. The reason for this is that records of the exact boundaries of territories served by either of the two d-c. systems were not available for past years. Probably, the most definite indication of the amount of growth which can be expected was given by the growth of the yearly peak of the generating station. This, of course, indicates in a general way the growth of the whole city. Therefore, the latter data were given particular weight in estimating future growth. By combining those various classes of data and using a certain amount of judgment as to future growth, the peak load to be served by the new apparatus for the whole district at the end of the ten-year period was estimated to be approximately 14,625 kw.

The matter of determining the geographical distribution of this load was even more difficult. However, by studying the existing distribution and the general trend of expansion, a reasonably accurate estimate was obtained. There is a very definite

Fig. 4—Map of the Underground District of New Orleans, Showing the Distribution of the Present D-C. Load

trend of growth toward Section III and away from the French Quarter. This is due in part to the development of the better class of residential districts in that direction and in part to the campaign against modernizing the French Quarter which is being continually carried on. Taking these various factors into consideration, it was thought that the sections designated as II and III in Fig. 4 could be combined to form a single district which will have a load, to be taken by new equipment, about equally distributed over the whole area at the end of ten years. This division of territory was therefore made for the calculations, as shown on the previous Fig. 2 and described in the preceding part of this paper.

In order to provide an accurate method of calculating losses of each of the four proposed systems, a set of data was worked out which gives the number of hours

per year, during which the actual load is equal in amount to given fractions of the yearly peak load. The method of obtaining these data were as follows: Typical curves were selected for each month of the year 1922. Three daily curves were included for each month, one for a typical Sunday, one for a typical Wednesday and one for Saturday. For these various days, the curves of all the substations in the underground district were combined in order to obtain the total load of the district. Then for all the daily curves, the number of hours during which the load was a certain fraction of the yearly peak was counted. These figures were then multiplied by the total number of days in the month representing that particular type of curve. That is, the figures for the Sunday curves were multiplied by the number of Sundays in the month. Figures for the Saturday curves were multiplied by the number of Saturdays in the month, and the figures for the Wednesday curves were multiplied by the number of all days in the month other than Saturdays and Sundays. The total figures obtained represent very nearly the number of hours during which each fraction of the peak load existed for the whole year. As a matter of general interest, these data have been included in this report in the form of a curve. (See Fig. 5). For convenience, the peak load is divided into eighths, and the various points of the curves at one eighth, two eighths, three eighths, etc., represent the number of hours during which these particular fractions of the peak load represented the actual load during the year. It is understood, of course, that the points designated as one eighth, two eighths, three eighths, etc., do not represent an exact fraction of the peak load but rather a band of loads covering all loads up to 1/16 of the peak load above and down to 1/16 of the peak load below the horizontal line on the load curve which represents the particular fraction of the load designated.

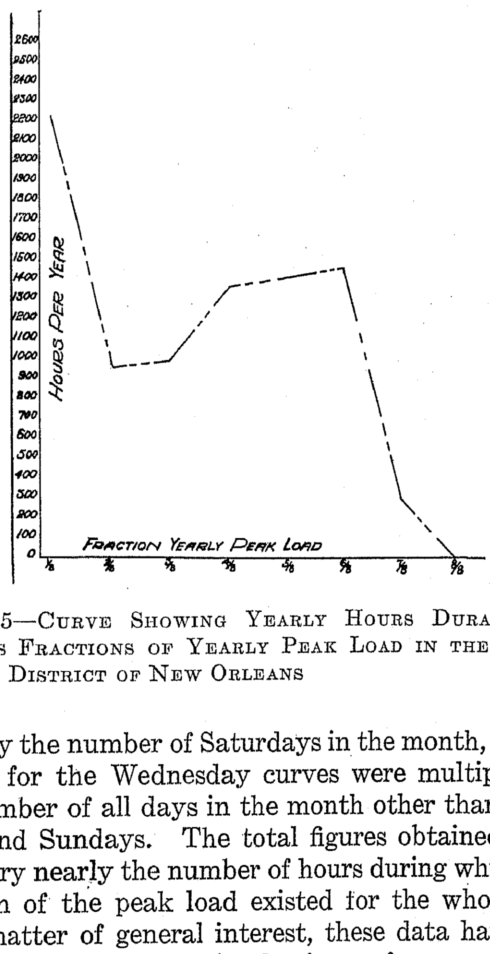


Fig. 5—Curve Showing Yearly Hours Duration of Various Fractions of Yearly Peak Load in the Underground District of New Orleans

Losses. To facilitate the calculation of yearly losses, factors were worked out from the load duration data, which factors could be used as multipliers of the peak losses to obtain the annual losses in kilowatt-hours. These factors may be called loss factors and may be expressed as the equivalent number of hours per year during which the peak losses should exist in order to produce yearly losses equal in amount to the actual losses. The factors for the copper losses of the various parts of the radial systems were obtained by taking the summation of the products of the squares of the various fractions of peak load times the corresponding number of hours per year, as shown on the duration curves. In the radial systems the various annual core losses were, of course, obtained by multiplying the particular core loss in kilowatts by the total number of hours in a year. In the network system the derivation of the duration factors was somewhat more complicated. It was necessary in this case to assume a certain schedule for cutting circuits out of service, based on the load duration curves. The amount of copper in service at different times in accordance with such a schedule was then taken into account. The loss factor for core loss in this case is less than that for the radial systems, while the loss factor for copper loss is more.

The actual loss factors which were obtained in the above manner are as follows:

Type of Losses	System and Apparatus	Loss Factor
Copper	Transformers and Regulators of Network System	2422
"	Secondary Mains of Network System	3022
"	Transformers, Regulators and Secondary Mains of Radial System	2207
Iron	Transformers and Regulators of Network System	6346
"	Transformers and Regulators of Radial System	8760

In calculating losses in secondary mains, an arbitrary symmetrical spacing of transformers was first assumed and the losses calculated for this case. The losses for the desired transformer spacing were then calculated by proportion under the assumption that these losses vary inversely as the square of the number of transformer banks (or d-c. feeders) keeping the size of the banks constant. This relation depends upon placing all transformer banks at street intersections, and may not strictly apply where they are placed indiscriminately.

It is interesting to note in this connection that for a given secondary network and given capacity of trans-

former banks or d-c. feeders, the losses of secondary mains are independent of the amount of load which is supplied to the district. In other words, applying this to the specific problem in hand, the d-c. mains could carry the total load of the district at the end of the ten-year period with no increase of losses in the mains over those existing at the present time, provided that the same sizes of d-c. feeders were used and that they were loaded with the same values of current with respect to their capacity, as at present. This assumes a uniform distribution of load throughout the district. This fact was considered in calculating the losses of the d-c. system.

Investment. The investment figures were for the most part based on actual prices of apparatus and material, plus the cost of installation. Full allowance was made for such incidental items as construction equipment, engineering, interest during construction, etc. In the case of substations, however, the estimates were based on data covering existing stations and those now under construction. Data covering the investment in substations of companies other than the Public Service Inc. were also studied.

Power Factor and Transformer Demand Factor. Due to the fact that the entire service of the underground district of New Orleans is now d-c., it was very difficult to obtain any definite idea of the power-factors and transformer demand factors which would exist in the new a-c. system, without making a very complete and laborious survey of the connected load in the district. In order to avoid the necessity of such a survey, it was thought to be sufficient to use as the basis of any estimate, figures of diversity and power-factor which could be obtained from existing a-c. systems in similar districts of other cities. For this purpose data were obtained from several sources and these together with known characteristics of the New Orleans system, were used to arrive at the estimated factors. In addition to this, a number of typical load curves of building services for the New Orleans underground district were obtained by actual test and these data were also studied.

The actual factors that were used are as follows:

Average power factor at the customers' services .... 85 per cent

Ratio of station peak load to connected transformer capacity for the radial systems .... 60 per cent

Ratio of station peak load to connected transformer capacity for the network systems .... 83 per cent

Energy Cost. Energy cost plays a very important part in any economic study of various systems of electrical distribution. It is therefore of considerable importance that the proper method be used in determining the energy cost. This is a subject that offers an exceedingly large number of possibilities of refinement. Therefore, an enormous amount of time might be spent in carrying out calculations of energy cost to the last degree of refinement. This might be time well spent, if the basic figures of energy costs were exact and unvarying. On the other hand, it is, of course, useless to employ a very high degree of refinement in the calculations, if there is a possibility of large variations in the basic cost figures. Furthermore, in the systems which have been considered in this paper, the variations in efficiency are, in general, opposite to the variations in investment. That is, the systems involving the least investment have the greatest efficiencies. Therefore, inaccuracies in the methods of calculating energy costs would affect to some extent the total amounts of the economy figures, but would not, generally speaking, affect the final conclusions as to the most economical system.

Nevertheless, for the New Orleans calculations, in order to obtain figures of energy cost having an accuracy which would be as little open to question as possible, considerable time was devoted to a study of the basic principles underlying energy costs, and to the actual calculations of the costs of losses in the proposed systems.

In working out the costs for the losses in the New Orleans systems, the following points were observed:

(1) Subdivision of unit cost into a demand charge and a kilowatt-hour charge.

(2) Subdivision of the demand charge between the various classes of apparatus through which the losses pass.

(3) Inclusion in the demand charge of a charge for wattless kilovolt-amperes produced by the a-c. distribution apparatus.

The demand charge was taken to represent fixed charges on investment in the increment (necessary to supply the losses) in the capacity of the various types of apparatus through which the losses must pass. In addition to these fixed charges on investment, certain portions of the operating expense of the generating station were included in the demand charge.

The output or energy charges consist solely of the remaining operating charges of the generating station. The subdivision of generating station operating charges between energy and demand charges was taken as follows:

	Demand Per Cent	Energy Kw-hr. Per Cent
Superintendence and Wages....	90	10
Fuel..........................	10	90
Water.........................	25	75
Lubricants....................	25	75
Station Supplies..............	100	..

This is admittedly an arbitrary subdivision.

Fixed charges were assumed to be 14 per cent of investment in each case, this percentage being divided as follows:

Interest	6.25 per cent
Taxes	3.10 per cent
Depreciation	4.15 per cent
Insurance	0.50 per cent
	14.00 per cent

The demand charges, other than those of the generating station, were based on actual investment in apparatus as calculated for the various systems, this being reduced to figures of unit investment per kilovolt ampere peak load. These charges were, therefore, different for the different systems. The charges for generating station investment were based on existing investment reduced to the unit value per kw. peak load.

There was a definite reason for basing the generating station demand charges upon kilowatts rather than kilovolt-amperes. This was the fact that there is very little variation of generating station investment per kilowatt with varying power-factor of the load. The steam end of the station is, of course, unaffected by power-factor, and the generating units are affected very little. The cost of switching equipment is mainly dependent upon short-circuit rupturing capacity and is, therefore, little affected by the load power-factor.

In view of this, the demand charges for losses in various parts of the system were divided into two items one for the generating station based on kilowatt of the losses, and the other for the remainder of the system, between the generating station and the location of the losses, based on kilovolt-ampere increase. The calculation of kilovolt-ampere increase through the various types of equipment was done vectorially, starting with 85 per cent power-factor at the customer's load.

Operating charges for the substations were omitted from the loss charges as being too small to materially affect the results.

It is, of course, understood that the generating station demand charges should not be applied against the underground system peak losses, unless the latter peak is coincident with the generating station peak. Such is approximately the case, however, under the load conditions which exist in New Orleans.

Network Switch Maintenance

For the majority of apparatus the value of 14 per cent for fixed charges was taken to include an ample amount for maintenance. However, in order that no question might be raised as to this item for the automatic switches, due to their somewhat complicated construction, an amount was added to cover maintenance of these switches for each of the two network systems in addition to the usual 14 per cent charge on investment.

Conclusion

Data have been presented, showing that considerable saving can be effected both in investment and in operating charges during a 10-year period of growth by introducing a special type of a-c. system, paralleling the present d-c. system in the underground district of a particular large city. The a-c. system selected in this case consists of 13,200-volt feeders, feeding directly into distribution transformers which in turn serve an interconnected low-voltage 3-phase 4-wire system through automatic network switches. This system is one of several which were investigated and it is the most economical under the particular conditions encountered of all those considered.

The practicability of the proposed system has been thoroughly investigated, and while certain difficulties in establishing such a system are recognized and experience may bring about some modifications of detail, methods of overcoming the outstanding difficulties are available in all cases.

Special consideration has been given to reliability of service, and the selected system contains features which provide a high degree of reliability, comparable with that of the best of modern d-c. systems, if stand-by storage battery equipment be eliminated from consideration.

Discussion

UNDERGROUND ALTERNATING-CURRENT NETWORK DISTRIBUTION FOR CENTRAL STATION SYSTEMS

(Kehoe)

GENERAL LIGHT AND POWER SUPPLY OF CHICAGO

(Armbrust and Jackson)

A STUDY OF UNDERGROUND DISTRIBUTION SYSTEMS FOR THE CITY OF NEW ORLEANS

(Bullard)

Chicago, Ill., June 26, 1924

E. R. Thomas: I would like to discuss a few things on the flicker problem which came up in Mr. Kehoe's paper. In Fig. 6 of his paper, there is a plot of some data that were collected on the flicker of incandescent lamps. This subject of lamp flicker dates back to the old problem of 25 versus 60 cycles, or what minimum frequency can be used for lighting. In the study of that subject we are indebted primarily, I believe, to Messrs. Kennelly and Whiting for their thorough research on critical frequency, which was made with a specially arranged Bunsen photometer.[1]

Fig. 1 herewith shows the results of their data and was primarily intended as discussion of frequency in regard to flicker. Now, we are probably more interested in the so-called dip or wink, that is if we are going to put both power and light on the same set of mains, what will the increments of starting currents do to our light?

In connection with this, Messrs. Kennelly and Whiting have obtained some data of various ranges of flicker and plotted this against light intensities. These are shown by the curves, *D*, *E*, *F*, and *G*, that is 44, 33, 7.5 and 3.5 per cent flicker in light intensities.

Dr. H. E. Ives at the Bureau of Standards obtained some interesting data on flicker.[2] He used a rather unique method in studying voltage dip. This consisted in employing an alternator whose voltage and frequency could be varied, and superimposing

1. The Frequency of Flicker at which Variation of Illumination Vanish, Kennelly and Whiting, *Trans.* N. E. L. A., 1907, pp. 327.

2. Allowable Amplitudes and Frequencies of Voltage Fluctuation in Incandescent Lamp Works, H. E. Ives, *Trans.* llumination Engineering Society, 1909, pp. 709.

this a-c. voltage on a d-c. voltage. The proportion of a-c. voltage to d-c. voltage was varied to give a range from pure d-c. voltage to pure a-c. voltage.

Figure 2 is a replot on semi-log coordinates of Dr. Ives' data on 25-watt tungsten lamps. You will notice that it gives a rather straight-line plot through the points. To this I have added a plot of the data collected by Mr. Kehoe, which appears in the lower left-hand corner. I believe that the reversal of

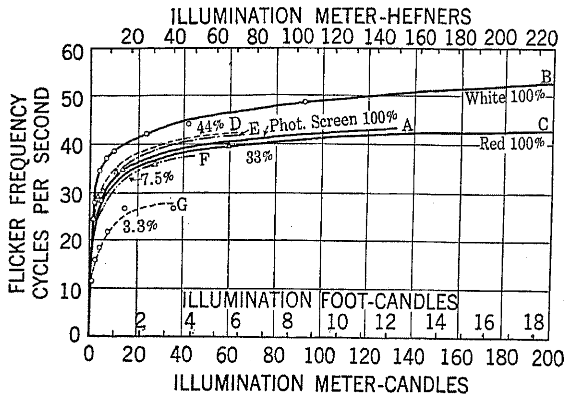


Fig. 1—Curve of Frequencies at which Flickering in Different Intensities of Illumination, Appeared to Vanish

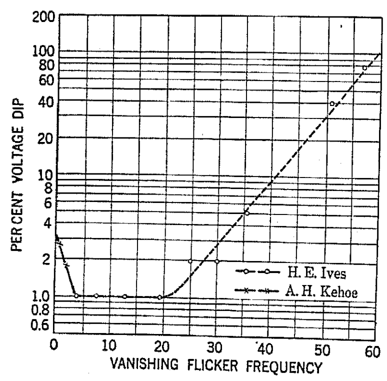


Fig. 2—Voltage Variation which Causes Flicker Phenomena 25 Watt Tungsten Lamp

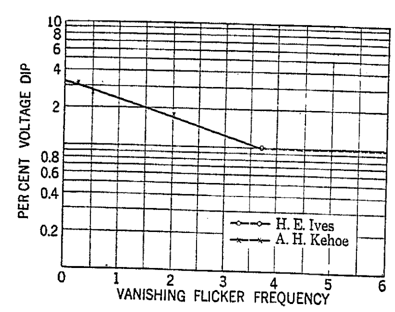


Fig. 3—Voltage Variations which Cause Flicker Phenomena 25 Watt Tungsten Lamp

these curves may probably be explained as depending on the physical reactions on the pupil of the eye attempting to adapt itself to a varying intensity of light.

Figure 3 is merely a repetition of Fig. 2 showing some of Mr. Kehoe's data plotted to a larger scale which bears out the same fundamental laws laid down by Messrs. Kennelly and Whiting. Some of the points brought out were:

The smallest range of flicker which could be recognized with certainty was about 1.4 per cent variation in illumination. This would correspond to about 0.5 per cent variation in voltage, on an ordinary Mazda lamp, and is somewhat lower than any point which we were able to observe.

Another point of interest from their data is that the most sensitive flicker frequency for small ranges of flicker is in the neighborhood of 2½ cycles. Our data shows this occurring at about 3½ cycles.

Messrs. Kennelly and Whiting have stated that this flicker frequency ceases to be objectionable at less than 7.5 per cent. Their figures are all given in terms of illumination, while ours are in per cent of voltage fluctuation. The 7.5 per cent change in illumination corresponds to about 2.4 per cent voltage variation on an ordinary Mazda lamp, thus checking our research fairly closely.

J. E. B. Stuart, Jr.: The remarks I want to make concern a continuation of some of the arcing data taken in connection with Mr. Kehoe's paper.

All of the data presented in the paper were taken with the idea of establishing a critical-point curve, or something like that, but we soon decided that the results were too inconsistent for plotting curves of any accuracy, so after the paper was handed in we did a lot of test work, where it was known that the arc would persist. We didn't try to fix the persisting point accurately but just ran through a number of tests where we were either sure of getting a good arc or none at all, and thus obtained roughly upper and lower limits. I may say that as far as our results showed, the plot of the voltage and current seems to be a hyperbolic function, but we have not done enough work to make sure of it.

All of the tests taken in the beginning were made with the arc from conductor to sheath, which is the condition usually met, especially where a single-conductor cable is used. We noticed a very peculiar action after a while. The lead sheath burned back much faster than either the conductor or the insulation and eventually this action tended to extinguish the arc, so we tried a few tests with copper-to-copper arcs. I won't describe the hook-up. It showed plainly that it is a much more persistent effect and much more dangerous when an arc is once started. The burning back is uniform. The arc is very uniform. The oscillograms show it too.

In all we tested about two-hundred samples, and took over seventy oscillograms. I have picked out several of them as they are rather interesting and they are reproduced in the accompanying illustration.

F. C. Hanker: Mr. Kehoe's paper presented some interesting data on distribution systems, and it has been very reassuring as to the reliability of the network. There is one point in connection with the so-called universal system described, particularly in the voltages that may be selected. There has been considerable work on standardization of utilization voltages, as it is called, in which the voltage at the lamp or utilization device has been standardized at 115 volts with recognized departures at 110 and 120. That standardization was satisfactory under past conditions as affecting motors and control as higher voltages were even multiples of the lamp voltage.

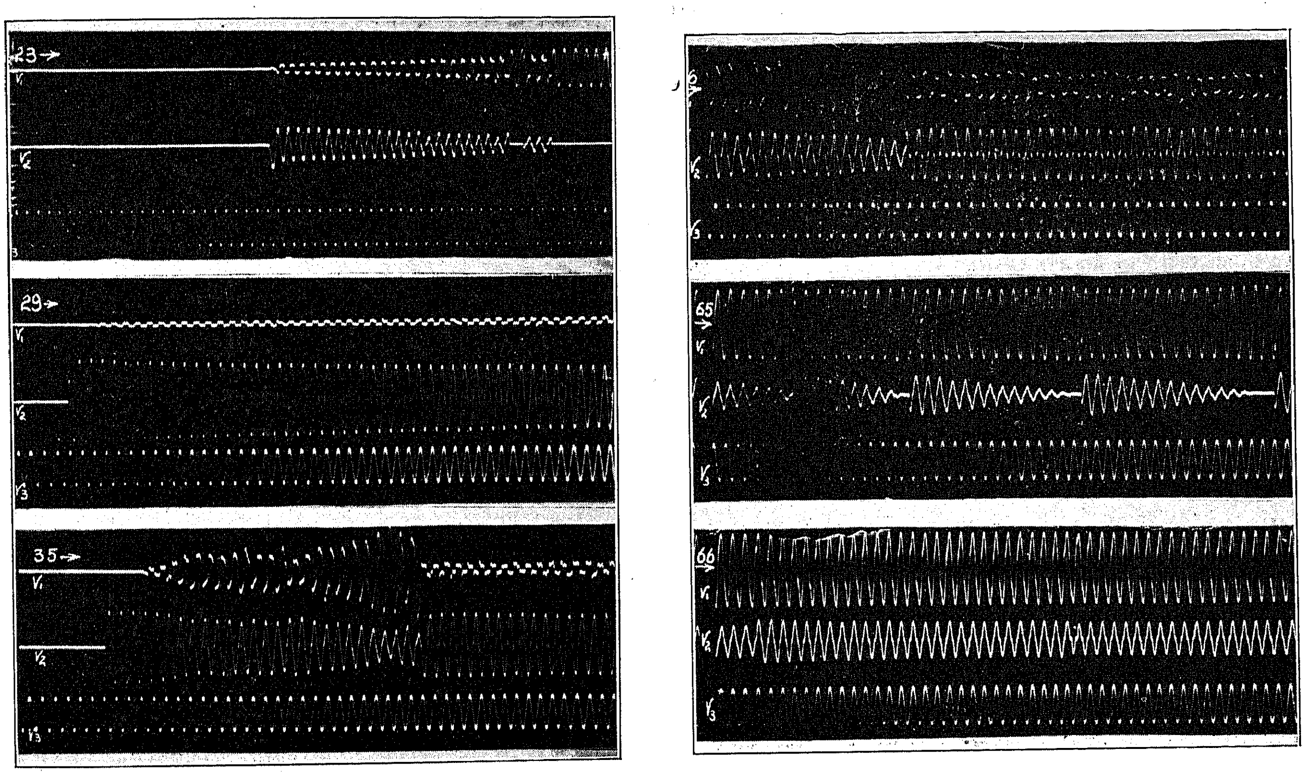
With the three-phase, four-wire system, which is apparently the one most generally considered on the network, we have a factor which causes trouble on the motor and the control voltage in case the two lower voltages of the utilization device on single-phase is adopted. If you take the 110-volt standard the three-phase potential will be 190 volts, which is considerably less than the tolerance allowable on standard motors at say 220 volts or the control of the same voltage range or multiple. The 115-volt standard corresponds to 198 volts and the 120-volt to 208 volts, which is more nearly the standard of the control and motor devices.

The other difficulty is in connection with the single-phase utilization devices. In case you adopt 220 volts or slightly

higher, it would be necessary to go to at least 125 volts, line to neutral to secure 220 volts on the delta.

Any development or system that is adopted should take into consideration the total cost to the industry, because if you take an odd voltage, such as would result in the using of 110 volts or 115 volts in a number of cases, there will be the additional expense of development and the possibility of trouble with existing devices. There probably would not be that difficulty where you come to systems that have conditions like New York, or some of the larger systems, where it is necessary that they maintain very close voltage regulation. If the system is generally used that condition would not always exist and we will have voltage drops that will bring us much below the guaranteed stove and that class, are designed so that they have quite a wide voltage variation and still are satisfactory. Others, like the electric iron, waffle iron, and devices of that general type require rather a close voltage regulation, and at the present time it is necessary that we have two standards to meet the range that we encounter in different parts of the country.

The lamp situation prior to the present standardization, I think, is sufficient warning to see the necessity for careful standardization and careful study of the voltages that will be used on the network system. Undoubtedly there is no question as to the economy from the distribution system itself. But when you take into consideration the other factors, it is quite necessary that considerable thought be given to voltage standardization.



Arcing Tests on Lead-Covered Cable

These oscillograms show the voltage and current in arcs from sheath to copper of single-conductor cables. The conductor and the sheath were short-circuited by copper wire (or, in case of Film No. 23, by a nail driven through the sheath). Sixty-cycle Voltage was then applied between sheath and conductor. The voltage across the arc is shown by V_1. The current through the arc is shown by V_2. V_3 shows the 60-cycle timing voltage. Further details on the respective tests are given in the following:

Film No. 23—4/0 cable, 6/32 in. rubber. Nail driven in the center of cable. Voltage (440 volts) applied at one end of copper and sheath. Maximum current, 11,000 amperes. Voltage wave increases gradually and current wave decreases gradually until the arc breaks.

Film No. 29—4/0 cable, 6/32 in. rubber. Arc started at center with three strands of No. 18 copper. Voltage (1600 volts) applied at both ends. Maximum current 555 amperes. The arc held twelve seconds. The duration of the arc at this voltage is not consistent. In this test the dying out of current and the consequential building up of voltage are clearly shown.

Film No. 35—500,000 cir. mil conductor, 20/32 in. paper. Arc started with two strands of No. 32 copper at one end. Voltage (2750 volts) applied at other end. Maximum current, 294 amperes. A rather unstable condition, which is characteristic of a number of the arcs, is shown, but this arc held for twelve seconds.

Films Nos. 6, 65, and 66—Three stages of an arc on 4/0, 6/32 in. rubber cable. The arc was started with two strands of No. 32 copper, at one end. Voltage (2750 volts) was applied at the other end. Maximum current, 635 amperes. The arc held until the switch was opened at the end of two and one-half minutes. Film No. 6 shows the sputtering effect which is characteristic of sheath to copper arcs. They go out sometimes, remain out a few cycles and then reappear. Film No. 65 was taken a minute after the start of the arc. The arc appears to go out but then revives again. Film No. 66 was taken two minutes after the start and the arc is still persisting strongly.

satisfactory operating condition on the standard equipment.

Before there is a too general use of the new system, if it is generally adopted, there should be some agreement as to the voltages to be adopted in order to minimize as much as possible the additional expense that may be involved in the development of the new equipment for the different conditions.

There are a great number of utilization devices in service now, and a great deal of difficulty caused in shifting from one town to another, or, in a number of cases, from one part of the city to another, in that the utilization devices are not generally applicable to the wide voltage range. Some devices, such as the toaster

A. H. Sweetnam: In connection with Mr. Kehoe's paper there has been considerable comment outside the meeting regarding the relative current demand of a standard three-phase motor wound for, we will say 220 volts, when operated at 220 volts and at some lower voltage, such as 208 or 199 volts, supplied by a low-tension network.

It follows, naturally, that a motor wound for 220 volts and operated at a lower voltage, other conditions being the same, will draw less excitation and greater load current. Tests which I have had made on several small three-phase squirrel-cage motors indicate an appreciable improvement in power-factor

when operated below rated voltage, but at the expense of lowered efficiency from approximately one-fourth to full load. I do not know that this condition will prevail with motors having characteristics differing from those tested, but it would appear to me that this question should be very carefully determined before giving consideration to the adoption of such a network scheme. The commercial aspect seems to have a great deal of bearing since, under the conditions stated, customers supplied from a network would be paying a premium by reason of the lowered efficiency of the equipment.

R. A. Paine: I think Mr. Kehoe has taken a big step in advance in enabling central-station companies who have been burdened with a rather expensive d-c. system in order to maintain the highest reliability of service, to decrease this expense, and I want to agree with one of the comments that Mr. Bullard made, *i. e.*, if you leave out the storage battery, I think we have a fair promise of getting as reliable service from the a-c. system as from the d-c. system.

In connection with the arcing tests made by Mr. Kehoe, I think they are fundamentally sound and in agreement with theory. I think this is due to two reasons. One is that in the a-c. system, of course, the voltage goes through a zero point, and the other is that if you take a circuit and use d-c. on that circuit and then also use a-c. on the same circuit, you have, in addition to the resistance when you use a-c., reactance which will tend to lower the voltage across the gap and, therefore, tend to extinguish the arc. One of the serious questions that has confronted most distribution engineers is: Is the a-c. 220-volt system going to be self-clearing as our d-c. systems always have been?

With radial a-c. systems at the present time we have something less than 0.1 per cent interruptions, and any device which will take care of that 0.1 per cent interruption must be very strong. I do not believe that we have as yet had enough experience to tell what devices will be suitable for manhole use, as I believe all of Mr. Kehoe's installations are made in vaults inside of buildings and from that point supply the outside underground system.

As to the matter of the operation of 220-volt motors on say 200 volts or 208 volts, I believe that the gain in power factor of operating these 220-volt motors at somewhat reduced voltage will in a large degree offset any loss of efficiency, as most motor installations are considerably overmotored.

There is one thing, though, that must be done. We must *maintain* at the customer's service the somewhat reduced voltage. We can not have any long shoestring secondary.

Mr. Kehoe discussed the question of high-reactance transformers. It is absolutely essential in a network, using different sized transformers, that the reactance of the present distribution transformer be increased over its present 3 or 4 per cent, and it would seem somewhere between 7 and 10 per cent is the proper figure.

However, if any company wishes to try an experimental network, we have recently made some studies which would indicate that if you use an identical size of transformer throughout your network system, let us say you adopt a 50, 75 or 100 kv-a. as your standard, using all of one size, you can operate a network satisfactorily. Of course, that would be only a transient condition, because you can not operate continually a growing network with only one size of transformer.

E. P. Peck: Mr. Kehoe's paper, I think, brings out the first solution of what now appears to be the coming method of supplying a-c. underground load. The handicap that the a-c. system had for years was that it could not stand on the same footing with the d-c. system in reliability.

The United Company was, to my best knowledge, the first to put the a-c. subway system on a very reliable basis. The system that they described, as well as the systems described in the next two papers, appealed to me very much, because they have done a thing which has been done time and time again in history in a number of lines. They have simplified the mechanism. Almost any development starts out with a complicated mechanism, which gets more complicated by adding devices to take care of different troubles that come up. Finally some one comes out with a very simple thing that either does not have all the troubles or else takes care of them in a very simple way. This applies, I think, particularly to the system described by Mr. Bullard for New Orleans.

One of the questions that occurred to me in connection with the New Orleans system particularly, although it applies also to the system proposed for Chicago, if I understand it correctly, in which you have high-voltage feeders, 11,000, 12,000, or 13,200 volts supplying high-reactance transformers directly and from there into a network, is: What will the regulation be all over that network? This applies also to Mr. Kehoe's paper where the voltage is lower. The 10 per cent reactance transformer has quite a high drop, particularly where it carries motor load. The primary voltage will undoubtedly be well sustained, but how will the secondary voltage behave under all conditions of starting motors and other load variations?

The self-protection feature which is referred to in Mr. Kehoe's paper is, as far as I can recall, the first analysis of a thing that a good many have recognized in a general way. Personally, I will have to admit that I was very much surprised to find that arcs were self-extinguishing on voltages as high as this paper shows, particularly with quite high currents. However, there is still a question as to just what will happen in case of the high concentrations of energy that will occur on these large systems, also a question as to what will happen in terminal boxes, at lugs and things of that kind.

H. W. Smith: These papers contain a valuable contribution to the art of electrical distribution. During the past five or ten years, there has been intensive development in the field of large generating stations, high-voltage transmission, and the use of automatic switching equipment for substations. The best talent in the industry has been employed on these problems with the result that tremendous strides have been made. There is, however, a growing realization of the fact that the total investment in the distribution system is greater than in generating and substations, and that great economies can be obtained by applying the same grade of engineering talent to study the problems of distribution.

In many large cities, the Edison three-wire system has been the accepted standard for distribution in the underground section. The Edison system, backed up as it generally is with large storage-battery reserve, has proved extremely reliable. However, the cost of distribution is much greater than for alternating current, and there is a general tendency in most cities towards the restriction of the Edison d-c. load, and in some cases studies have been made towards replacement of the Edison d-c. by an a-c. system.

In studying the a-c. distribution system with a view to making it more reliable, a very valuable contribution has been made by Mr. Kehoe, and his paper clearly indicates the lines under which he has been working.

In considering this subject, I wish to call attention to some valuable papers which have been written on this subject. At the 14st Convention of the Association of Edison Illuminating Companies in October 1922, Mr. W. C. Eglin, Vice President and Chief Engineer of the Philadelphia Electric Company, presented a paper on the "Future Development of Distribution Systems." This paper deals with the general economics of the distribution problem, and recommends a three-phase, four-wire, secondary distribution system. It is pointed out that the ideal system for general distribution at lighting voltage must comply with several requirements:

(1) It must be polyphase to permit the operation of motors.

(2) It must be a symmetrical polyphase system.

(3) The voltages available must be suitable for both light and power requirements.

(4) One conductor must be a neutral which can be grounded in such a way as to limit the voltage of all other conductors to ground to a value below 150 volts.

A three-phase, four-wire system is recommended, the only drawback being its adaptation to the present normal voltages of lighting and power apparatus.

In studying the general system, the economics of the whole industry should be considered, and any system which covers special apparatus should, if possible, be restricted. If it is necessary to carry two lines of motors or appliances, it will place a burden on the industry as a whole.

The use of a three-phase, four-wire secondary system with grounded neutral will require changes in standard control equipment as three overload relays will be necessary to give complete protection. Also in the case of low voltages, certain changes may be necessary in the design of holding coils of contactors.

The distribution system should be laid out so that the standard squirrel-cage motors can be used as far as possible with standard control equipment.

The automatic network switch mentioned in these two papers will be built in the following sizes—250, 500, 800 and 1200 amperes.

It will be of great advantage to the industry if in the development of the a-c. network, the requirements can be met with as standard apparatus as possible. In other parts of the world, there are three-phase, four-wire systems, 200-350, 220-380 and 240-415 volts, and it is to be hoped that this condition will not obtain here.

Another contribution to this subject was made in a paper by Mr. M. T. Crawford, entitled "Alternating Current Underground Distribution" presented at the 41st Convention of the Association of Edison Illuminating Companies, 1922. This system which is used in Seattle, parallels all distribution transformers on the secondary mains with primary connections on alternate feeders and with oil circuit breakers installed in the secondary of each transformer, which are equipped with power directional relays operative on a reversal of power flow equal to or greater than 150 per cent of the transformer capacity. These switches are not automatically reclosing.

H. A. Stanley: I want to speak more particularly about Mr. Bullard's paper. He has a fine paper and is starting out to accomplish a very desirable end, but I would like to indicate two or three points where I think he may perhaps get into trouble; two or three difficulties, perhaps, inherent in the proposed system.

If you distribute lighting and power from the same set of mains, you must provide regulating capacity for the total power load as well as the lighting load, which introduces somewhat of an economic waste. I think I foresee possible difficulties in rate making. Rates are ordinarily differentiated in regard to investment, but you have here the same investment for both power and lighting and you have only the load factor differential left to take into account in making a power rate below the lighting schedule.

Mr. Sweetnam has noted a point in reference to power-factor —improved power factor of the motor load benefiting the central station but the customer paying for it. Speaking from the viewpoint of a smaller company, and this perhaps would not apply to a large company; it seems to me the lower voltage on the motors is going to lead to a further difficulty. It would be difficult for the smaller company to maintain (already too low) constant voltage. Two hundred volts at no load leaves no leeway for voltage fluctuations with load. The comment of the motor manufacturers on this point should be of interest. If you are starting from zero and building a new company and your customer has not become accustomed to good voltage, that is one thing, but the customer who has become accustomed to it will not take readily to the lower voltage.

It seems to me the system lends itself somewhat to the theft of current and more than ordinary precautions will have to be taken in metering.

Now, there may be a few possible difficulties in apparatus details. It strikes me that the network protective device does not lend itself readily to manhole installation, at least in the present state of the art; there are some major difficulties to be overcome in perfecting the device for that purpose.

Another point I think is important if I correctly interpret Mr. Bullard's paper. To work on a 13,000-volt feeder, it obviously must be dead. The theory appears to be to operate the feeder switch at the substation and the automatic devices will function at the far end of the feeder. On a feeder of this size there will be from ten to twenty-five network switches. How are you going to protect the cable splicer? On the usual theory of disconnecting switches it would be necessary to have two breaks in series at each possible point where the cable could be energized. I should hardly suppose you could depend on the network switch for protection of human life. Are disconnecting switches in the program and will you send a man around to open them at twenty or thirty-points before doing maintenance work?

W. L. Abbott: I remember conventions thirty years ago where the lion and the lamb of a-c. and d-c. did not concert so harmoniously as they have in this meeting. It was conceded as a great victory for the a-c. school when d-c. systems were fed from a-c. generators in transmission lines.

With that combination of the two, it was thought the ideal system had been evolved, but now take a glance at the situation here in the Loop in Chicago, where Mr. Armbrust pointed out we have now a load of 75,000 kw. per square mile, and are contemplating a load of 200,000 kw. within twenty years, and, of course, it won't stop at the end of twenty years.

How are we going to distribute that load? In fact, how are we going to transform it? Substations are not altogether desirable, and space for substations is not readily available, and it is a growing problem to find locations where these substations may be placed sufficiently near together in such a congested territory as the Chicago Loop district.

Then, there is the other problem of distributing this power through the streets. If you could see a cross-section of the downtown streets of Chicago, you would wonder where the next main is going to get in at all. The street is full from the surface ten feet down. True, more space could be made available if all of these underground structures could be taken out and rearranged, but they are crisscrossed and with huge manholes and sewer openings which have just about reached the limit of capacity. Then, again, the heat evolved by these d-c. distribution lines is very considerable, and any greater congestion of those lines is going to be a serious matter.

We will have to get more power and have more power distributed in the Loop than we have now. It can not be done much longer by adding more copper in our distribution. It will have to be done at higher voltage, and the last transformation must be made in the building which is to be served.

That is the problem which is approaching.

R. B. Mateer (by letter): In many of our cities accepted primary distribution voltages are 2400 and 4000, while transmission potentials over cables linking generating and distributing substation range from 6000 to 13,200 volts, and any step forward which will limit the investment in or eliminate the use of distributing substation is commendable.

It is suggested, however, that in all interconnected network the transformers should be of a uniform or standard size and as far as possible, evenly spaced. Such practice will not only be a step forward in our efforts at standardization, but will materially assist in uniform loading of the step-down equipment.

A. H. Kehoe: The discussions have indicated the importance of keeping standard equipment throughout the entire electrical supply system, including the utilization devices. The adoption of three-phase, four-wire combined light and power services

having a reduced voltage for motor operation, is sometimes considered as making it necessary to adopt an additional line of motors having a lower voltage rating than is at present the standard. On a large number of electrical supply systems, motors are operating at reduced voltage due to the low power factor current supplied through small sizes of isolated transformers. The adoption of a three-phase, four-wire supply is for the purpose of supplying lighting service from combined mains, and there is no reason to use this system, except at locations where satisfactory lighting service can be obtained. Therefore, polyphase voltage when used for motors will not be below the amount now regularly found in practise.

I doubt if for typical motors the average loading condition makes for lower efficiency. Generally there is an improvement in efficiency due to the reduced voltage, up to about three-quarters load on the motor, which is probably above the average loading of motors. Because of the large number of motors now operating at reduced voltage, I doubt if the efficiency will be changed by the adoption of three-phase, four-wire systems.

Regulation due to transformer reactance is accomplished with automatic regulators in each supply feeder. The division of current is so nearly uniform that the regulation between any two points in the network depends upon the drop in the secondary mains, as the automatic regulating equipment will maintain a constant voltage at the low-voltage supply points to the mains.

It is important in designing a reliable network to omit any type of equipment in which faults may develop which will not clear themselves. The concentration of current at any point where trouble can occur is usually well below the values indicated in my paper, and there should be no necessity of exceeding these to meet any conditions which are likely to arise in the next few years.

In presenting distribution-system designs, uniform and standard connections of network cables and locations for distribution transformers are assumed. In actual service however, it is inevitable in many locations to have concentrated loads which are as large as the entire remaining load in the immediate vicinity. In such cases the center of distribution of the particular district is not at the existing intersections of the mains, but is at the particular concentrated load. The economical design will then not allow the adoption of a single size of distribution transformer and a standard transformer location throughout the entire system. This condition results in the network of street mains being of smaller capacity than would otherwise be the case to supply load. There results a lack of uniformity and standardization of equipment which however does not make it any more difficult to obtain uniform loading of distribution transformers, if indicated design is used.

Concerning protection of cable splicers, in many districts one non-automatic break is considered sufficient, and two breaks in series have been used with one—an automatic oil circuit breaker. With these network units open, and the high-voltage supply grounded, it is impossible to have the switch function automatically as its closing energy is obtained from the supply side of the switch. If one of these switches is closed manually with the high-voltage cable grounded in the immediate vicinity of workmen, the condition will be similar to closing a knife disconnecting switch on a grounded feeder, except that the fuse protection at the network switch will clear the backfeed.

W. R. Bullard: The question of the selection of service voltage of the three-phase four-wire system, so as to satisfy as nearly as possible existing voltage standards, is one that has received a lot of attention by various engineers. One of the schemes which was given some consideration for use in New Orleans would utilize 400 volts as the delta voltage giving 230 volts between each line and neutral. In this case autotransformers would be connected between line and neutral at lighting services in order to obtain a neutral for the usual three-wire 115/230-volt system. Existing motors, which are connected delta, could be reconnected *Y* and operated without further change on the 400-volt system. This proposal has the advantage of a saving in copper of the low-voltage mains due to the higher voltage. However, in this case it was found that the cost of the autotransformers and losses therein would more than offset any saving in losses and cost of the secondary mains.

In this connection, I would refer to a point which was brought out in my paper; namely, that once the size of the secondary mains and the corresponding capacity of distribution transformer installations is established, these secondary mains will continue to serve the load indefinitely with practically the same copper losses, regardless of what the growth of load may be. Low-voltage mains therefore continue in a more or less fixed physical state after they are once installed. On the other hand the connected capacity of autotransformers in a 400-volt system would continue to grow with the growth of load and their cost would some day out-strip the cost of the low-voltage mains, regardless of any economy which might be secured by the use of a higher voltage system at the start. Furthermore, I believe it has been brought out in Mr. Kehoe's paper that the 400-volt system would not be able to burn off short circuits with the same facility that the 200-volt system will.

It is somewhat unfortunate for the three-phase four-wire system that the present standards for lighting and power voltage are just what they are, but in selecting the voltages to use on this system it would seem that an out-and-out compromise effecting both types of service would have some serious disadvantages. The use of 120 volts for lighting, for instance, would give a somewhat higher voltage for power than the use of 115 volts, but the resulting voltage would still be non-standard for motors and the lighting voltage in this case would not be the preferred standard. Time alone will tell just what the trend of standardization will be. However, in the meantime we have selected what seemed to us to be the most desirable arrangement in view of the present standards.

I might also mention that while we expect to supply the customers at the start with 115/200 volts, we are buying transformers rated at 120 volts. This will reduce the duty on regulators at heavy loads and will enable a change to 120 volts to be made without difficulty if it is ever found to be desirable.

In regard to Mr. Stanley's point concerning the difficulty of adjusting rates of power and lighting service to meet the combined power and lighting system, I feel that the rates should be treated as secondary to the economics of the problem rather than modify the economics by using a more costly system in order to satisfy existing rates. Furthermore, common facilities for the power and lighting loads are usually provided in the generating station and are also provided from that point at least to some point in the substation. It is therefore rather difficult to see why the same principle should not be extended to the remainder of the distribution system as far as rates are concerned.

Referring to Mr. Stanley's point concerning the additional cost of regulator capacity to regulate the power load, I feel that the case must be an exceptional one indeed where it can be shown to be more economical to provide separate facilities and regulate only the lighting load. If the reference is to station regulators, this type of equipment represents a small part of the cost of the whole system and any saving of regulator capacity will, I believe, at least for New Orleans, be much more than offset by the additional cost for duplicate primary cables which would be necessary to serve unregulated power loads.

I failed to mention in my paper the method of protection to workmen which we expect to use in case work is to be done on primary cables or on transformers. The procedure in this case will be first,—to kill the primary feeder by opening the switch at the generating station and

Second,—to ground and short circuit the primary cables in the generating station and also at one or two points external to the station, points being selected for this purpose which will offer most effective protection, and permanent facilities for grounding and short circuiting being provided at these points.

Equivalent Single-Phase Networks
for Calculating Short-Circuit Currents Due to Grounds on Three-Phase Star Grounded Systems

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Review of the Subject.—*This paper presents a method for calculating the steady state value of the short-circuit current in a fault to ground on a power system operated with grounded neutral, and the distribution of this current throughout the system. Constant impedances and electromotive forces in the system, and electrically short lines, are assumed, and line capacitance is neglected.*

If, at the time the fault to ground occurs, the distribution of the load current in the system is known, the total current in any portion of the system under the short circuit condition may be calculated by means of this method. By "total current" is meant here the sum of that part of the fault current which appears in the branch considered, and the normal current in the branch due to the loads. The latter current, of course, does not appear in the fault.

Formulas and equivalent circuits for the usual three-phase transformer and generator connections used in practise, are given. The use of such circuits permits the calculation of the fault current and its distribution in the power system from an equivalent single-phase network. Since currents in a three-phase network under balanced conditions may also be calculated from a single-phase network, it is accordingly possible to calculate, entirely on a single-phase two-wire basis, the total current in any branch of a star grounded network for a ground on any phase.

The setting up of equivalent two-wire single-phase networks similar to those for the three-phase case is not generally possible where the number of phases exceeds three.

The value of the method lies in its enabling one to calculate on a single-phase two-wire basis the short circuit current (steady state) due to a ground on a three-phase grounded neutral system, as regards both magnitude and distribution and taking into account all system loads. In the usual approximate method of making short-circuit calculations, a single-phase-to-neutral network is substituted for the actual network. While this method involves less labor than that proposed in the paper, the results obtained by it are inexact, the effect of non-grounded loads being usually ignored. The method of the paper involves much less work than that required by three-phase calculations giving equal accuracy.

An illustrative example is given.

INTRODUCTION

THE determination of the magnitude and distribution of currents in power lines under short-circuit conditions[1] is of large interest to all engineers engaged in power transmission problems. It is of interest also to telephone engineers, who are concerned with the inductive effects of these currents. It was from the latter standpoint that the studies on which this paper is based were undertaken. The results, however, are submitted here largely on account of their more general interest and utility.

Calculations of the magnitude and distribution of the fault or residual current, due to a ground on a non-grounded power system, are not difficult according to the methods worked out in Technical Report No. 52 of the Joint Committee on Inductive Interference of California.[2] The calculations are much more complex and difficult in the case of star-grounded power systems as in the case considered in this paper. Approximate calculations can, of course, be made by the well-known "equivalent single-phase to neutral" method. However, where substantial loads, either concentrated or distributed along the line, are located at points not remote from the position of the fault, the neglect of the residual current in the sound phases through these loads may lead to considerable errors in estimates of induced voltages. To the power engineer interested in accurate relay settings the additional accuracy of the proposed method may be of importance.

Considering a three-phase system for example, it is, of course, possible to take the loads into account by working out the currents on a three-phase basis. This is a laborious proceeding and the time involved is likely to be prohibitive if the network is at all complicated. By taking account of the symmetry, with respect to the faulty phase, presented by a normally balanced three-phase system, and by making use of a fictitious generator[3] in the fault in the place of all sources of e. m. f. in the power network, it is possible to simplify the calculation and to replace the three-phase network with a single-phase two-wire network that takes account of all loads. The method of doing this is described below.

In the case of a system of more than three phases it is, of course, still possible to replace all electromotive forces in the network by one at the fault. While this reduces the calculations to a single-phase basis, it is not generally possible to complete the simplification by replacing the network by a two-wire network.

Methods in common use for the approximate calculation of these fault currents are based on what amounts to an assumption of steady-state conditions. The same assumption underlies the methods to be discussed

1. O. R. Schurig, JOURNAL A. I. E. E., June 1923, p. 605. This article contains a bibliography of the literature of the subject.

2. See "Inductive Interference," *California Railroad Commission*, 1919, pp. 353-376.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

3. The possibility of using a fictitious generator in connection with problems similar to that under discussion here is referred to by L. P. Ferris (TRANS. A. I. E. E., Vol. XLI, p. 90, 1922), in a discussion of a paper by Conwell and Evans.

in this paper. The determination of transient short-circuit currents is, of course, desirable. While methods have been devised for determining low-frequency transients by measurements in a model of the actual network under study,[4] it seems unlikely that the general calculation of transients can be simplified to such a point as to be of practical value to the operating engineer. Allowance is often made for the fact that maximum instantaneous values may be larger than the sustained values by the use of the so-called transient impedances. This can likewise be done in the application of the methods described in this paper.

An essential feature of the method here described is the reduction of the number of metallic conductors in the circuit to two. With a three-phase system, this can always be done in the case of a single fault to ground, assuming the system symmetrical. With certain other types of fault, this reduction cannot be readily effected, but some simplification is still usually possible by the replacement of the system electromotive forces by single electromotive forces at the fault or faults.

The magnitude and distribution of the short-circuit currents are the principal factors of interest in the power system, so far as induction into telephone circuits under abnormal conditions on a grounded neutral power system is concerned. The potential differences between any two points in the network may also be of interest in certain cases, however, and these may be readily determined by the method under discussion.

The following two well-known network theorems are the basis of the method to be described:

(a) Superposition Theorem: In any network with an arbitrary distribution of n e. m. fs. in its branches the current in any branch is the sum of the n currents which would be produced in this branch by the n electromotive forces taken separately in their respective positions in the network. (In applying this theorem, it is evident that instead of considering all the e. m. fs. separately they may be combined into groups in any desired way, subject, of course, to the condition that in the net result each shall have been used once and only once).

(b) In any network of impedances having an arbitrary distribution of e. m. fs. in its branches, the current in any branch is equal to the negative of the voltage across an open circuit in the branch, divided by the impedance of the network looking into this open circuit with all e. m. fs. removed.

Theorems (a) and (b) are true only providing there are no unilateral impedances in the networks.

The proof of theorem (b) follows at once from theorem (a). Referring to Fig. 1, the current I in the branch 1, 2 is the same as would be produced in that branch jointly by two superposed systems of electromotive forces, *viz.*:

(1) The n existing e. m. fs. plus an e. m. f. V_0 in the branch 1, 2 of such magnitude and phase that the current due to the n other e. m. fs. is exactly cancelled in branch 1, 2.

(2) A single e. m. f. $-V_0$ in the branch 1, 2.

Evidently the e. m. f. referred to in (1) is numerically equal to the open circuit voltage of branch 1, 2. Z being, as indicated, the impedance looking into the network from terminals 1, 2, the current in branch 1, 2 in case (2) is $-V_0/Z$, and it must be also I, since the total current in branch 1, 2 in case (1) is zero.

In connection with the application of this theorem to be made presently, it should be pointed out that the "existing e. m. fs." assumed to be removed, include not only e. m. fs. due to prime movers, but also those maintained by mechanical loads, in virtue of which they normally receive energy from the electrical system. It will be apparent from this that the method under discussion automatically takes into account the momentary return under short-circuit conditions of energy to the system from machines which are normally receivers.

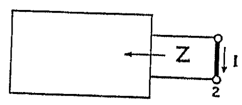

FIG. 1

METHOD OF CALCULATING SHORT-CIRCUIT CURRENT—THREE-PHASE SYSTEM

The following discussion describes by means of an example the manner in which the magnitude of the short circuit current through the fault may be calculated. Considering the network shown in Fig. 2, G is a star-connected generator; T, a bank of transformers connected in delta on the station side and star-grounded on the line side; S, a bank of (non-grounded) step-down transformers at the substation; and L is the substation load. The line has become accidentally grounded at E as shown. It is required to determine the short-circuit current to ground.

Let Z_g[5] = impedance of each generator winding

E_g = magnitude of induced generator voltage in each winding (Y-induced voltage)

Z_T = impedance of each transformer of bank T (low-tension basis)

U_T = ratio of transformation of each transformer of bank T

Z_s = impedance of each transformer of bank S (low-tension basis)

U_s = ratio of transformation of each transformer of bank S

Z_L[5] = impedance of each leg of the load

4. O. R. Schurig, JOURNAL A. I. E. E., October 1923, p. 1033.

5. This is the actual impedance, *i. e.*, the impedance in the absence of induced e. m. fs. representing the delivery of energy to or from sources external to the electrical system.

z'' = self impedance per mile of transmission line circuit composed of the two ungrounded conductors in parallel and the third conductor as a return.

z' = self impedance per mile of transmission line circuit composed of the two ungrounded conductors in parallel and earth return

z = self impedance per mile of transmission line circuit composed of the grounded conductor and earth return.

l = distance in miles from T to E

a = distance in miles from power house to substation.

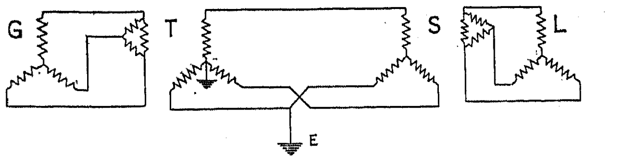

Fig. 2

The short-circuit current to ground may be obtained by the use of theorem (b), whose application to this case is illustrated in Fig. 3. In this figure, V_0 is the open circuit voltage at the fault, *i. e.*, the voltage to neutral of the system. The short-circuit current is $I + I_1$, I being the current in the faulty conductor directly to the neutral ground at T, and I_1 the current reaching the same point over a parallel path, E to S and the ungrounded conductors to T.

Applying Kirchhoff's voltage equation around the path taken by current I, and neglecting for the present the mutual impedance between the line circuits whose self-impedances are designated, z, z',

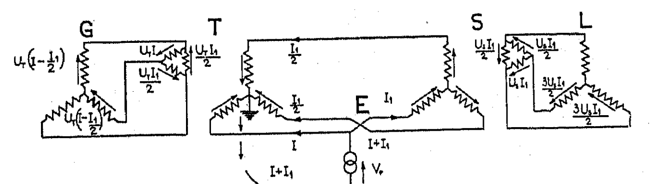

Fig. 3

$$V_0 - l z I - [\,(U_T I)\, Z_T + 2 Z_g\, U_T (I - I_1/2)\,]\, U_T = 0$$

or $$[l z + U_T^2 (Z_T + 2 Z_g)\,]\, I - U_T^2 Z_g I_1 = V_0 \qquad (1)$$

Similarly around the path taken by the current I_1

$$V_0 - (a - l)\, z'' I_1 - \left[\, 2 Z_L \left(\frac{3\, U_s}{2} \right) I_1 + Z_s U_s I_1 \right] U_s - \left[\, Z_L \left(\frac{3\, U_s}{2} \right) I_1 + Z_s U_s I_1/2 \right] U_s - l z' I_1 - [\, - Z_g U_T (I - I_1/2) + Z_T (U_T I_1/2)\,]\, U_T = 0$$

or $$- (U_T^2 Z_g)\, I + [\,(a - l)\, z'' + l z' + (3/2)\, U_s^2 (3 Z_L + Z_s) + (U_T^2/2)\,(Z_T + Z_g)\,]\, I_1 = V_0 \quad (2)$$

From (1) and (2)

$$I/I_1 = \frac{(a - l)\, z'' + l z' + (3/2)\, U_s^2 (3 Z_L + Z_s) + (U_T^2/2)\,(Z_T + 3 Z_g)}{l z + U_T^2 (Z_T + 3 Z_g)} \qquad (3)$$

From equation (1) and (2) $I + I_1$ may be obtained.

When the mutual impedance, which may be denoted by z_m, between the circuits whose self-impedances are designated z and z', is taken into account, equations (1) and (2) become

$$[l z + U_T^2 (Z_T + 2 Z_g)\,]\, I - [\,(U_T^2 Z_g - l z_m)\,]\, I_1 = V_0 \qquad (4)$$

$$(- U_T^2 Z_g + l z_m)\, I + [\,(a - l)\, z'' + l z'^{\,2} + (3/2)\, U_s^2 (3 Z_L + Z_s) + (U_T^2/2)\,(Z_T + Z_g)\,]\, I_1 = V_0 \qquad (5)$$

In calculations of short-circuit currents z_m is frequently disregarded. In some cases rather large errors may result from disregarding it, and where accuracy is desired, it should be included unless it can be shown that its effect is unimportant.

Distribution of Currents

The magnitude of the short-circuit current *through the fault* may thus be obtained as outlined above. It

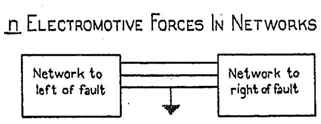

Fig. 4

is usually necessary, however, to determine the current *in some specified branch* of the network other than the fault itself. The *total* current in any branch may be considered as made up of two currents, namely, the portion of the fault current in the branch, and the normal current due to the system loads. The latter current, of course, does not appear in the fault. These currents may be determined as follows: Figs. 4, 5 and 6 represent the power network under three conditions, namely: (1) the actual network (Fig. 4) with the fault as shown; (2) the actual network after removing all sources of e. m. f. (but not the impedances of the windings in which they are generated), and with the fictitious generator of voltage V_0 in the fault (Fig. 5); and, (3) the actual network (Fig. 6) with the fictitious generator. By the superposition theorem, the difference of the currents in any branch for the conditions exhibited by Figs. 5 and 6 gives the current for the condition of Fig. 4 and this is the desired result, namely, the total current in that branch under the conditions of fault. The current in the branch due to the arrangement of Fig. 5 is (aside from a 180-deg. phase displacement) the portion of the *fault current*

in the branch, and may be calculated by means of the formulas given in the preceding section. Since V_0 is equal to the open circuit voltage at the fault, the currents obtained with Fig. 6 are those in the power network with the fault removed, *i. e.*, the load currents. Consequently, the total current in any branch of the network is obtained by means of the method of analysis above outlined.

Although theorems (a) and (b) have thus far been applied to a particular network, it is apparent that they may be applied in a similar manner to other types of network as well. The treatment of a large number of three-phase network elements commonly found in high-tension transmission systems by this method is given in the next two sections.

In most cases of inductive interference with communication circuits, due to grounds on star-grounded power systems, the fault or residual currents are many times more important as regards inductive effects than the normal-load currents (which do not appear in the fault). Consequently it is sufficiently accurate

n Electromotive Forces In Networks Removed

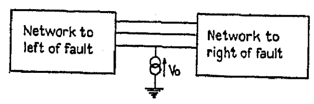

Fig. 5

n Electromotive Forces In Networks

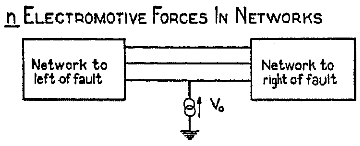

Fig. 6

in such cases to consider only the magnitude and distribution of the fault current. In such cases it is necessary to consider merely the network typified by Fig. 5.

Equivalent Two Terminal Networks for Three-Phase Systems

The preceding discussion has shown how to replace the three-phase system with a single-phase system. In applying the results, it is convenient to have equivalent two-terminal impedances to replace the three-terminal impedances presented by power apparatus at generating and substations.

Returning to the arrangement shown in Figs. 2 and 3, and assuming symmetry in the three phases, it is evident that the two sound conductors may be replaced by a single conductor as $F\,D$ in Fig. 7. To determine equivalent two-terminal impedances for the generating and substations for use with the resulting two-wire system, we may further assume impedances Z, Y and X with mutual impedance M between Z and Y, as indicated in Fig. 7. These impedances are then to be determined in terms of the impedances of the generator, load and transformers. The impedances of the branches, $A\,B$ and $E\,F$, Fig. 7, will evidently be z and z' ohms per mile and that of $B\,C$, with $D\,E$ as return, z'' ohms per (loop) mile, by the preceding work. (These ignore mutual impedance between the circuits whose self-impedances are z and z'. If this cannot be ignored, it is necessary to assume a mutual impedance z_m between $A\,B$ and $F\,E$.) Then

$$V_0 = l\,z\,I + Y\,I - M\,I_1$$

$$V_0 = [\,(a - l)\,z'' + X + l\,z' + Z]\,I_1 - M\,I$$

By comparison with the equations (1) and (2)

$$X = (3/2)\,U_s^2\,(Z_s + 3\,Z_L)$$

$$Y = U_T^2\,(Z_T + 2\,Z_g)$$

$$Z = (U_T^2/2)\,(Z_T + Z_g)$$

$M = U_T^2\,Z_g$, and represents the voltage *drop* from G to F per ampere from A to G or the voltage drop from G to A per ampere from F to G

By similar processes, the equivalent networks for the various transformer connections exhibited in the diagrams of Figs. A and B may be obtained.

In these diagrams the symbols U, U' and U'' refer to transformer ratios from line side to station side and the impedances Z_T, Z_g, Z_T', Z_g', etc. to transformer

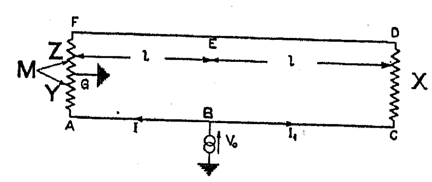

Fig. 7

or machine winding impedances on the station-side voltage basis. Diagrams 13 to 18 inclusive deal with 3-winding transformers.[6] It is to be noted that in the diagrams for the open delta (19 and 20) and Scott (21, 22 and 23) connections the accidental ground is assumed to exist on the phase, with respect to which the other two phases are symmetrical. Diagrams 24 and 25 illustrate combinations of some of the network elements with line circuits.

The scheme of reduction of the three-phase system to a single-phase system which has been described, assumes electrical symmetry, not only in apparatus but also in lines. In practical cases, dissymmetry in lines when untransposed is probably the only dissymmetry which it might be necessary to consider and even here, it is probable that if mean self and mutual impedances are used, the error from assuming symmetry would be small.

Systems of More than Three Phases

The setting up of equivalent two-wire single-phase network elements similar to those shown in the diagrams of Figs. A and B is not generally possible where the number of phases exceeds three. Where only

6. For theory of the 3-winding transformer see Peters and Skinner, Trans. A. I. E. E., Vol. XL, pp. 1181-1199, 1921. Also Boyajian, A. I. E. E. Journal, April 1924, pp. 345-355.

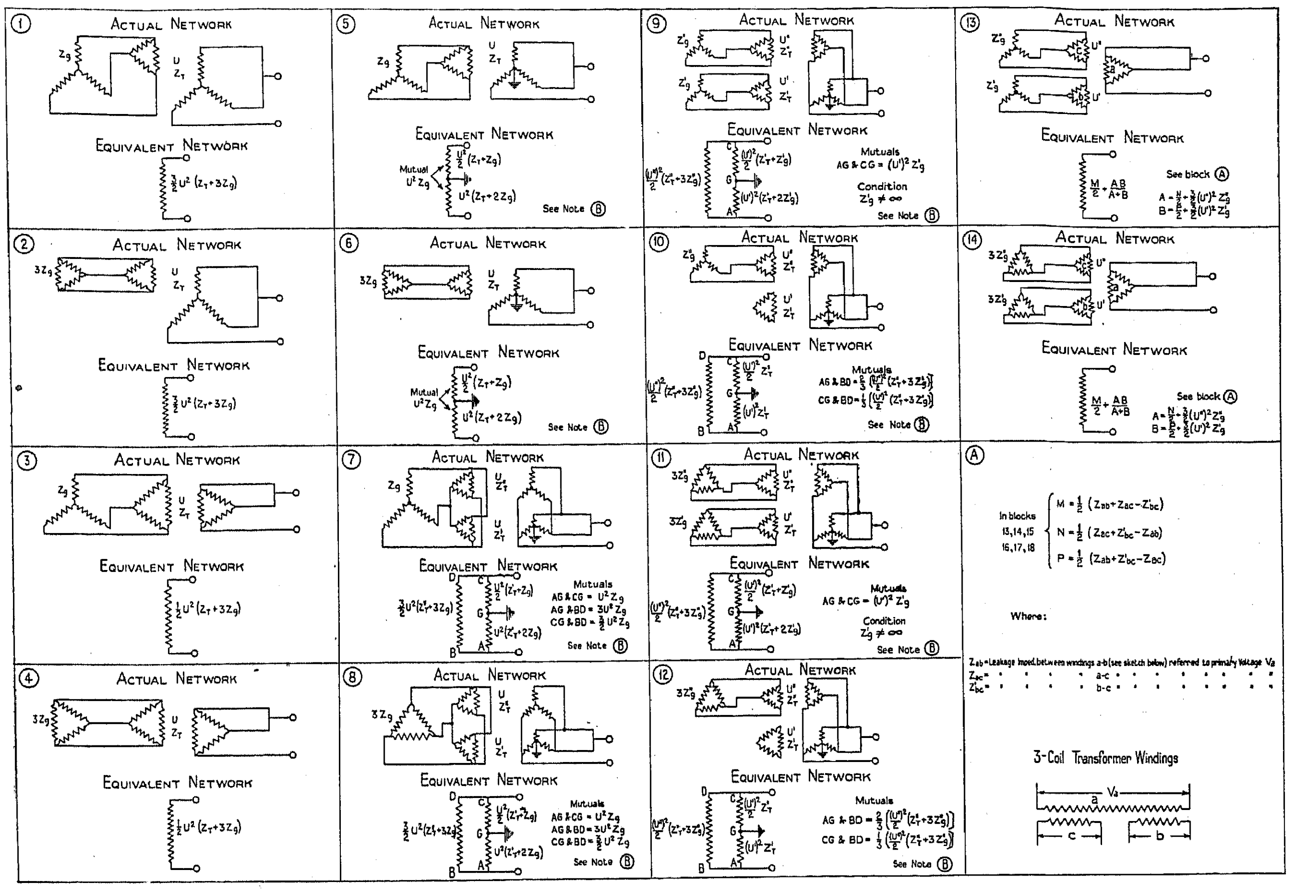
Actual Network
Equivalent Network
See Note (B)
Mutuals
Condition
See block (A)
(A)
In blocks 13, 14, 15, 16, 17, 18
M = ½ (Zab + Zac − Z'bc)
N = ½ (Zac + Z'bc − Zab)
P = ½ (Zab + Z'bc − Zac)
Where:
Zab = Leakage Imped. between windings a-b (see sketch below) referred to primary Voltage Va
Zac = " " " " a-c " " " " " " " "
Z'bc = " " " " b-c " " " " " " " "
3-Coil Transformer Windings

Fig. A

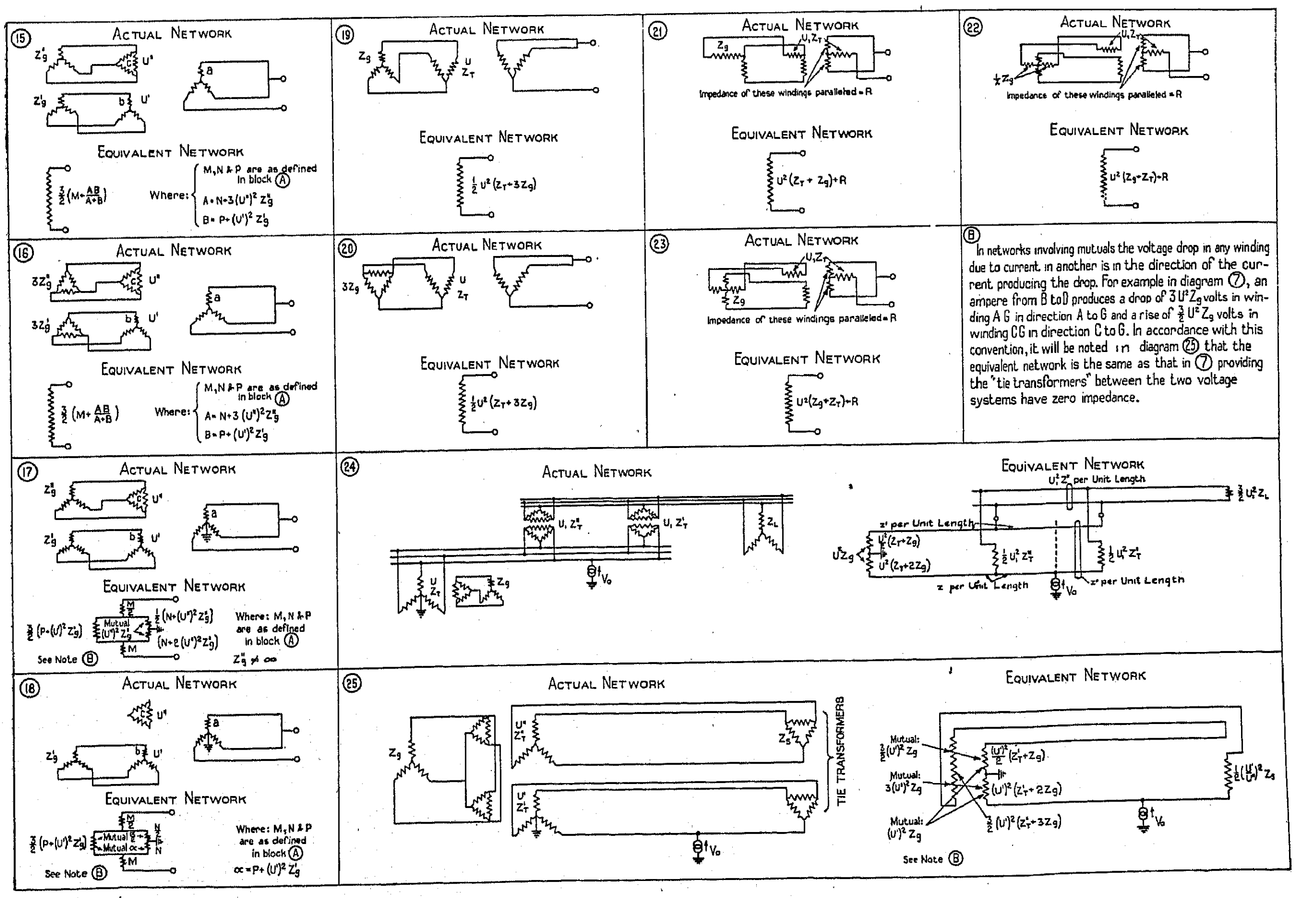
Actual Network
Equivalent Network
Impedance of these windings paralleled = R
See Note (B)
(B)
In networks involving mutuals the voltage drop in any winding due to current in another is in the direction of the current producing the drop. For example in diagram (7), an ampere from B to D produces a drop of 3 U²Zg volts in winding A G in direction A to G and a rise of 3/2 U² Zg volts in winding CG in direction C to G. In accordance with this convention, it will be noted in diagram (25) that the equivalent network is the same as that in (7) providing the "tie transformers" between the two voltage systems have zero impedance.
per Unit Length
TIE TRANSFORMERS

Fig. B

three phases are involved, the two sound phases are symmetrical with respect to the faulty phase (assuming, of course, equality in line and apparatus constants for the three phases). With four or more phases, however, except in very special cases, the three or more sound phases do not form a system having complete symmetry with respect to the faulty phase, and they therefore cannot be combined with the faulty phase, by formulas of general validity, to form a single-phase two-wire network.

ILLUSTRATIVE EXAMPLE

The network of Fig. 8 consists of 44-kv. and 110-kv. lines interconnected at three places, fed by four generating stations and supplying one large load. The kilovolt-ampere capacities of all machines and transformers, the connections of the transformers, and the lengths of the several transmission circuits, with sizes of conductors, are indicated in the figure. The method developed above will be applied to the calculation of the residual current in the various branches of this network due to a ground on one phase of the 44-kv. line at a point 28 mi. to the right of station B.

The equivalent single-phase two-wire network is shown in Fig. 9. Diagrams 1, 3, 5, 24 and 25, of Figs. A and B are used to obtain the equivalent networks. For the condition assumed, grounded apparatus on the 110-kv. lines behaves as though ungrounded.

In determining the impedances of the various branches, the resistance components have, for simplicity been neglected. The percentage reactances of all transformers and generators or loads have been taken at 5 and 10 per cent respectively. The reactances

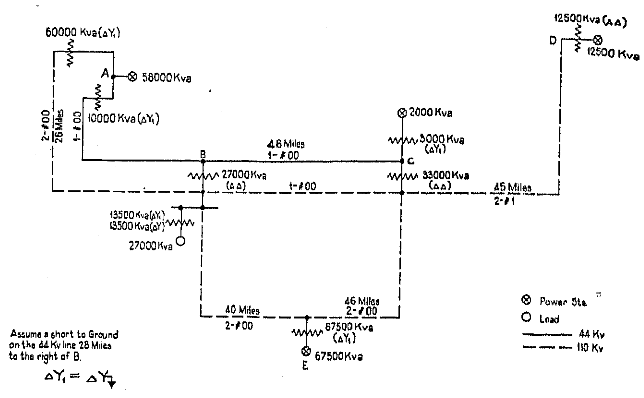

FIG. 8

in ohms of the machines and transformers are found from the percentage reactances and kilovolt-ampere capacities in the usual way, and the numerical values of the impedances of the various equivalent networks may then be found at once. Thus, for the case of generating station A, using the notation of diagram 25, Fig. B,

$$Z_o = (10/100.)\frac{V^2}{1000 \text{ kv-a.}}$$

$$(U')^2 Z_o = (10/100).\frac{V^2\left[\dfrac{44{,}000}{\sqrt{3}\,(\sqrt{3}\,V)}\right]^2}{1000\,(58{,}000/3)} = 1.11$$

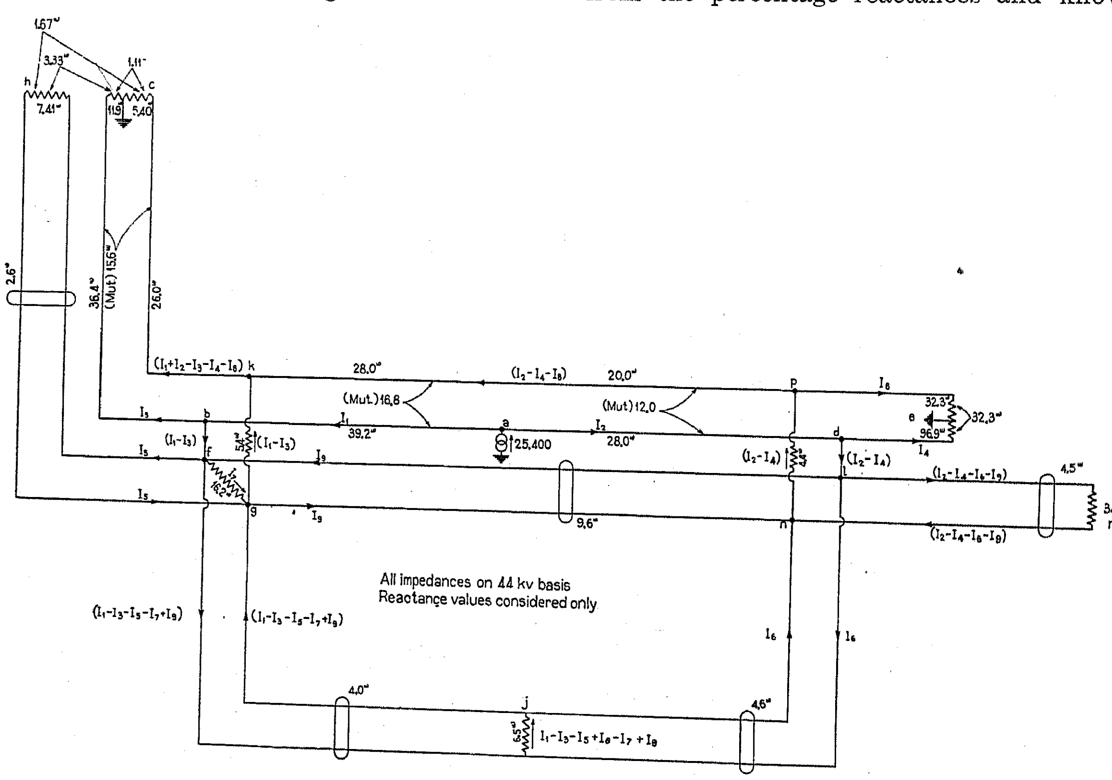

FIG. 9

and similarly

$$(U')^2 Z_T' = \frac{5\,V^2 \left[\dfrac{44{,}000}{\sqrt{3}\,V} \right]^2}{100\,(1000)\,(10{,}000/3)} = 9.68,$$

$(U')^2 Z_T'' = 9.68/6 = 1.61;$

Consequently

$$\frac{(U')^2}{2}\,(Z_T' + Z_o) = 5.40;\ (3/2)\,(U')^2 Z_o = 1.67$$

$$(U')^2\,(Z_T' + 2\,Z_o) = 11.9;\ 3\,(U')^2 Z_o = 3.33$$

$$(3/2)\,(U')^2\,(Z_T'' + 3\,Z_o) = 7.41;\ (U')^2 Z_o = 1.11$$

The equivalent impedances of the other generating and load networks may be similarly determined and are given in Fig. 9. In all cases they are on a 44-kv. basis.

As regards the transmission lines, the reactances of circuits composed of one and two (similar) wires with ground return are given approximately by the following formulas:

$$x = 0.00465\,f \log_{10} \frac{2\,h}{r} \text{ ohms/mile (one wire)}$$

$$x = 0.00233\,f \log_{10} \frac{(2\,h)^2}{r\,D} \text{ ohms/mile (two wires)}$$

where f is the frequency in cycles per second, r is the radius of, and D the distance between, the wires, and h is the depth of the equivalent ground plane.[7]

Substituting $f = 60$, $r = 0.182$ inches (00 wire), $D = 8$ ft. and $h = 500$ ft., it is found that

Reactance of one wire with earth return equals 1.4 ohms per mile.

Reactance of two wires (8 ft. spacing) with earth return equals 1.0 ohms per mile.

Also, the reactance of a metallic circuit composed of one wire as one side and two wires in parallel as the other is about 1.2 ohms per mile; and of a circuit composed of two wires as one side and four wires as the other, about 0.6 ohms per mile.

The mutual impedance z_m between two circuits having the ground as a common side and one and two wires respectively as the other sides may be computed from the formula

$$z_m = 0.00465\,f \log_{10} \left(\frac{2\,h}{D} \right) \text{ ohms/mile}$$

where f is the frequency in cycles per second, h is the depth of the equivalent ground plane and D is the distance between the wires of the two circuits. From this, using the same values for f, h and D as before

$$z_m = 0.6 \text{ ohms/mile}$$

The impedances (neglecting resistances) of the various transmission line circuits resulting from these figures are given in Fig. 9 (all on a 44-kv. basis).

The remainder of the work consists in solving the network for the currents whose magnitudes are desired, using Kirchhoff's laws. For the case under discussion, the currents (for notation refer to Fig. 9) are found to be

$I_1 = 395$ $I_4 = 222$ $I_7 = 105$

$I_2 = 557$ $I_5 = 177$ $I_8 = 380$

$I_3 = 149$ $I_6 = 198$ $I_9 = 75$

These, of course, do not include the normal load currents.

It is interesting to note the difference in the calculated residual currents as determined by the single-phase-to-neutral method and by the method given in this paper. The results based on the same line and apparatus impedances for the two methods are as follows:

Section	Ground Currents	
	Single-Phase-to-Neutral Method	Method given in this Paper
Fault to Station B	290	350
Station B to Station A	290	350
Fault to Station C	203	602

The large difference in the two currents designated "Fault to Station C" is due to the fact that the impedances between points designated d and e and p and e (Fig. 9) are relatively large, while those from p to n and in the 110-kv. network are small. Consequently, current finds a relatively easy path from d through the 110-kv. network to the two sound phases of the 44 kv. circuit thence to the ground at C, which path is ignored in the single-phase-to-neutral method. The difference in the other currents is only 21 per cent as based on the values obtained by the single-phase-to-neutral method.

In conclusion, I wish to express my appreciation of many valuable suggestions received from H. M. Trueblood. I am also indebted to C. M. Hebbert for checking the mathematical deductions.

7. For four wires with ground return $x = 0.00058\,f \log_{10} \dfrac{(2\,h)^8}{12\,r^2\,D^6}$ ohms/mile. See Technical Report No. 64 in "Inductive Interference." (*California Railroad Commission*, 1919), pp. 653, 654. Also see page 171 where curves and formulas showing the inductance of circuits with ground return are given.

Discussion

H. M. Trueblood: The question of the degree of precision worth while in estimating short-circuit currents in power networks presents itself as of immediate importance as soon as consideration of the problem of the inductive effects of these currents in exposed communication circuits is undertaken; for it is necessary only to glance over a single-line diagram of a large modern power system to realize that an exact calculation of the distribution of residual current due to an arbitrarily located fault to ground will be a lengthy, difficult and expensive proceeding.

Certain factors tend toward the conclusion that a comparatively low order of precision is all that can be justified. Among these are the matter of the impedance of the fault itself, and the uncertainty introduced into the calculated induction due to

lack of knowledge of the distribution of current in the earth. As to the first of these, it is usually essential to know what maximum effects may be expected. From this point of view, the impedance of the fault may be neglected, because on practically any system faults to ground can occur having impedances negligible compared to the other impedances involved.

Regarding the distribution of current in the earth, it is known from tests that this is very different in different localities. Where there is no information on this point, a large uncertainty is introduced into the calculated induction for parallels of moderate or wide separations, although the uncertainty regarding the magnitude of the short-circuit current itself due to this cause is of a much smaller order, probably never over 15 per cent. However, I do not think this uncertainty is of much importance as an argument for limiting an otherwise desirable precision in estimates of short-circuit current. In important cases, the effect of the distribution of the earth current can always be determined by tests, and it is reasonable to expect that as time goes on, general information regarding this important factor in different localities will be collected. Cases occur, of course, in which estimates of induction of a rather rough character suffice, as when it is known that the induced voltages will be small. However, in cases in which the induced voltages will be large, even where they are well beyond tolerable magnitudes, fairly precise estimates are usually necessary, since it is generally desirable to determine what reductions in the magnitude of the

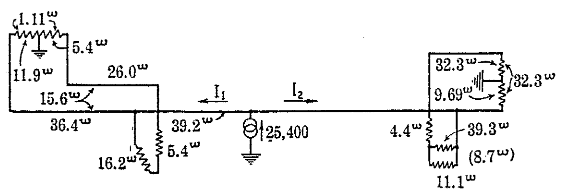

FIG. 1

residual current might be effected by possible changes in the power system.

From such considerations as these, it has seemed desirable to some of us who have been studying the problem of low-frequency induction in telephone circuits, to have a method of estimating power-system short-circuit currents, accurate, let us say, to 10 or 15 per cent., and capable of application to fairly complicated networks. Fortunately for us, the problem is one of so much importance in the operation of power systems that it has received a great deal of attention from power engineers, and our work upon it has benefited greatly from a study of the valuable and ingenious methods of attack which have been devised by these engineers and which are referred to in Mr. Shetzline's paper.

While the time and labor for a rigorous solution are greatly reduced by the methods suggested by Mr. Shetzline (he has stated, for instance, that only about 10 hours' work was necessary to obtain the solution of the illustrative example given in his paper), the amount of computation necessary would, nevertheless, in some cases still be excessive. Two general possibilities for bringing the labor involved in such cases within bounds have received attention. The first of these involves a judicious simplification of the network, and this usually means also a combination of the method of the paper and the "equivalent single-phase" method, an eye being kept throughout the work on the matter of what short circuit locations are important, what currents it is necessary to know accurately and what currents are unimportant. These questions, of course, are directly related to the location of exposures.

In the accompanying diagrams and in the tabulated numerical results, Mr. Shetzline's illustrative example is carried further, in order to illustrate the application of this process of simplification. It is assumed that only two currents are desired—those to the right (I_2) and the left (I_1) of the fault. Fig. 1 herewith is the simplified network, derived from Fig. 9 of Mr. Shetzline's paper by consolidating the 6.5-ohm generating station and the 4.6-ohm line to the right of it into the single 11.1-ohm impedance shown

	I_1	I_2
Exact Method................	350	602
Method of Fig. 1..............	355	575
Method of Fig. 2..............	290	203

at the right in Fig. 1, and by suppressing certain branches of the network completely. In a process of this sort, one is guided by a consideration of the relative magnitudes of the self and mutual impedances, and their positions in the network with reference

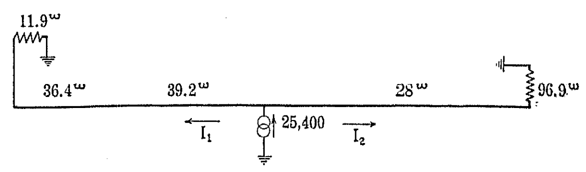

FIG. 2

to the branch carrying the desired current. Fig. 2 illustrates the "equivalent single-phase method." As the table shows, the simplified method gives results within 4 or 5 per cent of the correct values, whereas the single-phase method gives one result which is only about one-third of the correct value. The work involved in applying the method of Fig. 1 is only a little greater than that of the single-phase method.

The solution just discussed is satisfactory for the purpose of

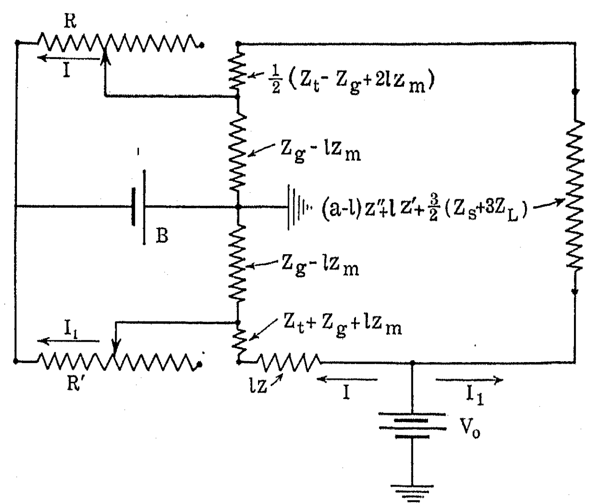

FIG. 3

determining the currents I_1 and I_2, which alone we have supposed to be of interest in this case. It would not, however, be adequate to the demands of an engineer whose job it is to determine relay settings for the system. This brings me to the second possibility, which is that of reproducing in miniature the equivalent network of the system worked out according to Mr. Shetzline's paper, and obtaining a desired current distribution by direct readings on ammeters. This scheme, of course, suggests itself at once to any one acquainted with the use of calculating tables by power companies and with the papers by power engineers on this subject. The difficulties in setting up a miniature network, using direct current (and thus assuming a common phase angle for all impedances), are encountered in connection with the mutual impedances which it is necessary to

use at certain places in Mr. Shetzline's equivalent networks. Aside from this, the network is, of course, simpler than the three-wire network, since it has only two wires. I am not prepared to say that in all cases it will be found possible to represent the mutual impedances correctly, although so far in our consideration of the question, we have found no cases in which it could not be done.

The adjoining Figs. 3 and 4 illustrate arrangements for representing the mutual impedance M of Fig. 7 of Mr. Shetzline's paper. The transformer ratio is taken as unity, for simplicity. The resistance R and R' in Fig. 3 are adjusted until the two currents marked I are equal, and until the two currents marked I_1 are equal. The battery B may have any suitable voltage. When this adjustment has been made, and the other impedances have the indicated values, the d-c. miniature network correctly reproduces the network of Mr. Shetzline's Figures 2 and 3, of course, under the assumption that all impedances have a common

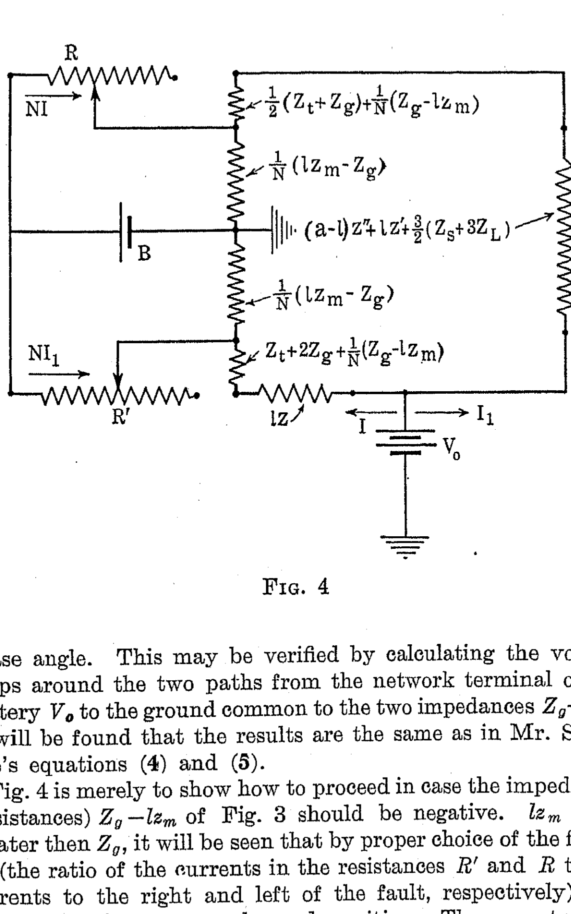

FIG. 4

phase angle. This may be verified by calculating the voltage drops around the two paths from the network terminal of the battery V_0 to the ground common to the two impedances $Z_g - lz_m$. It will be found that the results are the same as in Mr. Shetzline's equations (4) and (5).

Fig. 4 is merely to show how to proceed in case the impedances (resistances) $Z_g - lz_m$ of Fig. 3 should be negative. lz_m being greater then Z_g, it will be seen that by proper choice of the factor N (the ratio of the currents in the resistances R' and R to the currents to the right and left of the fault, respectively), the remaining resistances may be made positive. The correctness of the construction may be verified as in the other case.

R. D. Evans (by letter): Mr. Shetzline in his paper has presented an ingenious solution of a complicated problem and without doubt his solution is essentially correct.

In reviewing the paper I do not find that different effective impedances of machines when carrying single-phase loads and when carrying the balanced polyphase loads have been assumed. However, the method presented in the paper is a general one and may readily be modified to meet this condition, it being necessary only to determine new values for machine impedances.

There appears to be a typographical error in connection with the illustrative example. One would expect that the 13,500-kv-a. transformer at Station B would supply a relatively large residual current. For this reason it would appear that the residual current flowing from Station B to Station A should be considerably less than the value of 350 amperes given in the last table.

It has been our practice to solve single-phase short-circuit problems by the use of "phase-sequence quantities." Our method is an application of the general method developed by C. L. Fortescue[2] for the solution of unbalanced-polyphase-circuit problems. This method has been developed particularly for the solution of the various power-circuit problems and possesses a number of advantages, some of which will be mentioned. The phase-sequence method of calculation may be arranged so that steps used in the solution of line-to-ground short circuits may also be used to solve the other types of short circuits, namely, the three-phase short circuit and the single-phase line-to-line short circuit. A further advantage of this method is in the ease of application to calculating boards, because the complications of inductive coupling between branches, mentioned during the discussion, are avoided. It is my intention to publish, in the near future, an article describing this method and when this has been done, the relative merits of the two methods for the calculation of the various problems may be determined. In closing, I wish to emphasize the great advantage which, in my opinion, resides in the phase-sequence method from the analytical point of view in giving adequate conceptions of unbalanced-circuit conditions.

R. A. Shetzline: Mr. Evans has correctly pointed out that the impedances designated Z_g and Z_L should be those for the proper current distribution in the machines, and also that this matter may be taken care of without difficulty and without impairment of the generality of the method.

Mr. Evans' reference to a typographical error is probably due to a misunderstanding regarding the line to which the 13,500-kv-a. transformers at station B are connected. From his statements, I presume that he believes them to be directly connected to the 44-kv. line. However, the transformers are metallically separated from the 44-kv. line by the 27,000-kv-a. bank of delta-delta transformers. Accordingly, the residual current in the section fault to station B will also be that in the section station B to station A.

Mr. Evans has stated that the phase-sequence method of calculation may be used to determine three-phase short-circuit and single-phase line-to-line short-circuit currents, in addition to single-phase line-to-ground short-circuit currents. Three-phase short-circuit currents are readily determined on a single-phase basis by well known methods and present no difficulties. As regards the calculation of single-phase line-to-line short-circuit currents, I wish to state that the substitution of equivalent networks and of a single fictitious generator is applicable to this case also, with corresponding results as regards simplicity of solution.

2. "The Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks" by C. L. Fortescue, TRANS. A. I. E. E., Vol. XXXVII, pp. 1027-1115, 1921.

Standardization in Construction and Operation
as Applied to Light and Power Companies

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Reveiw of the Subject.—*Standardization is a subject that is very much discussed. The advantages of standardization have long been realized by industrial and manufacturing companies. The electric light and power companies seems to be slow in adopting standards of construction and this article relates the experience of one company who tried it out.*

Definition and Object

A STANDARD is ordinarily defined as something which is set up as a unit of reference, something accepted as more or less perfect which is used for comparison of objects of the same kind.

Standardization is the adoption of a standard. In engineering however, this is not necessarily so. Here a certain object may be accomplished in a number of different but of equally good ways. The adoption of one of these or the limitation of the number to a selected few would be standardization in a true sense.

The objects sought by standardization in engineering are:

1. To provide a standard so that a given job can be carried out in a satisfactory and workmanlike manner.
2. To accomplish this in a minimum of time with a minimum of effort at a minimum cost.
3. To insure continuity of service either of the apparatus or structure in question itself or of the apparatus to which it is auxiliary, which service is a prime requisite in the art and business of generating and supplying electric light and power.

Accomplishment

For the past five years the electric light and power properties under the control of the American Gas and Electric Company have been following a system of standardized construction in the distribution and substation divisions and the beneficial results obtained have been so clearly marked that we have been encouraged to push the idea of standardization in other directions.

While the results of standardizing have been shown to be most satisfactory in both substation and distribution work, at the same time, the problems involved are of a different nature and they will, therefore, be discussed separately.

Distribution Work

The problem encountered was to try and sell standards of construction to a group of men from different parts of the country, who for years had been accustomed to do certain work in a certain fashion, each man having his own ideas and in most cases at variance with the ideas of the others. It also involved the problem of human engineering as well.

A preliminary set of standards was prepared and several meetings of all interested parties held. As a result of these meetings, a compromise series of construction drawings has been prepared, covering the various types of structures normally encountered in distribution work. These are complete including as they do a full bill of material. They have been

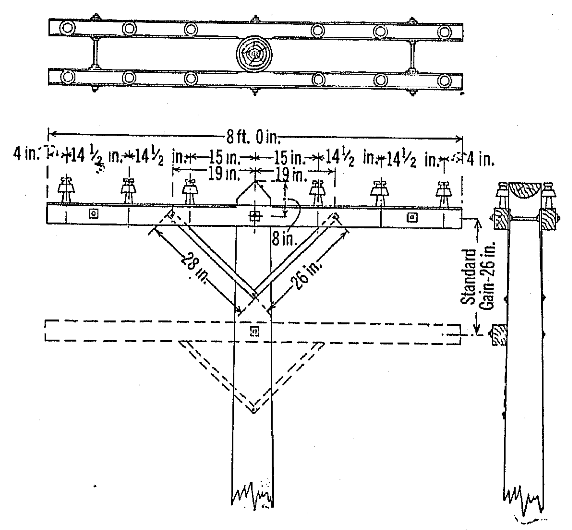

FIG. 1—STANDARD 4000-VOLT LINE CONSTRUCTION FOR SINGLE AND DOUBLE ARM

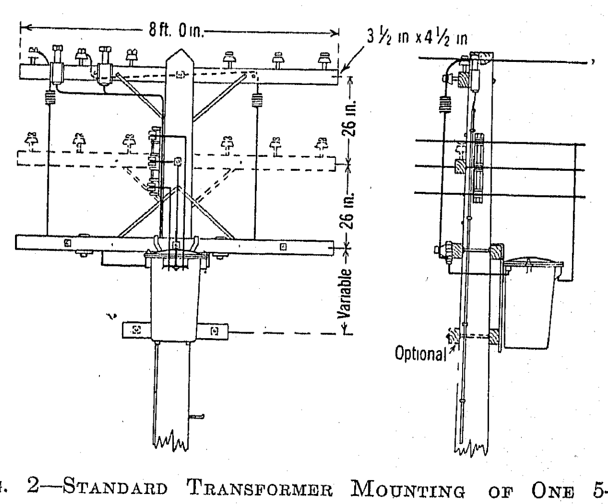

FIG. 2—STANDARD TRANSFORMER MOUNTING OF ONE 5-15 KV-A. UNIT

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

printed and are furnished to the distribution departments of the various properties who are expected to adhere to the construction shown in the standard drawings and to employ only approved line material. Figs. 1, 2, 3 and 4 are typical of such standards and the illustrations shown in Figs. 5, 6, 7, 8 and 9 are illustra-

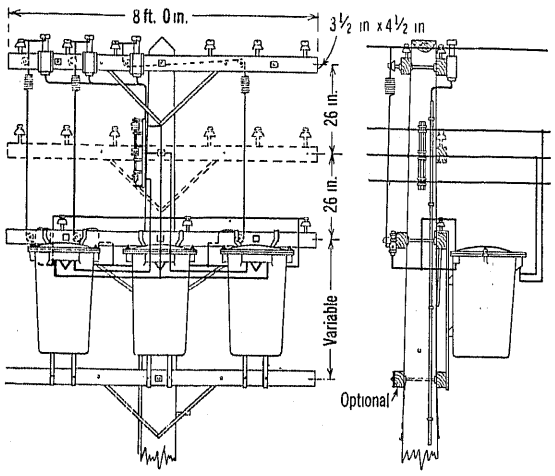

FIG. 3—STANDARD TRANSFORMER MOUNTING OF THREE UNITS 5-15 KV-A.

tive of the installations carried out in accordance with the above drawings.

It is quite evident from the illustrations cited that if the field is provided with a standard, that it can confidently be expected to produce a workmanlike job. Left

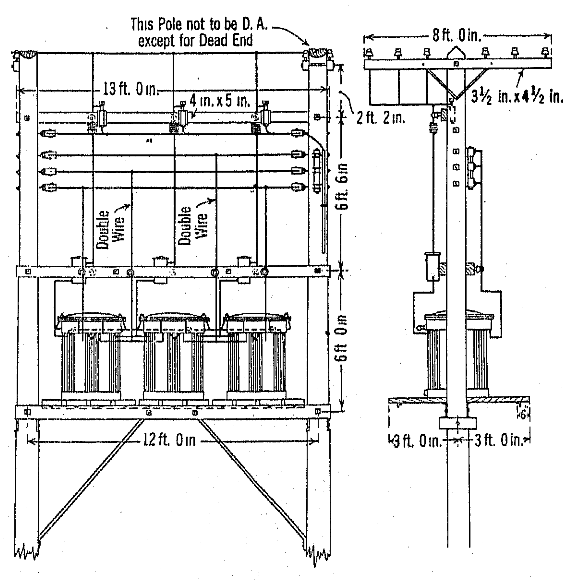

FIG. 4—STANDARD PLATFORM MOUNTING OF THREE TRANSFORMERS MAX. 3-75 KV-A. UNITS

however, to itself, the field is quite likely to produce something like Figs. 10, 11 and 12 which, while bad, are not by far the worst that could be shown.

This method of carrying out construction is far superior to the individualistic method. That it does introduce economies not available otherwise can be shown by the following:

1. It makes it possible to purchase the material involved more reasonably.

2. The material can be transferred from one district to another or from one storeroom to another and will naturally fit the corresponding job equally well in the new location.

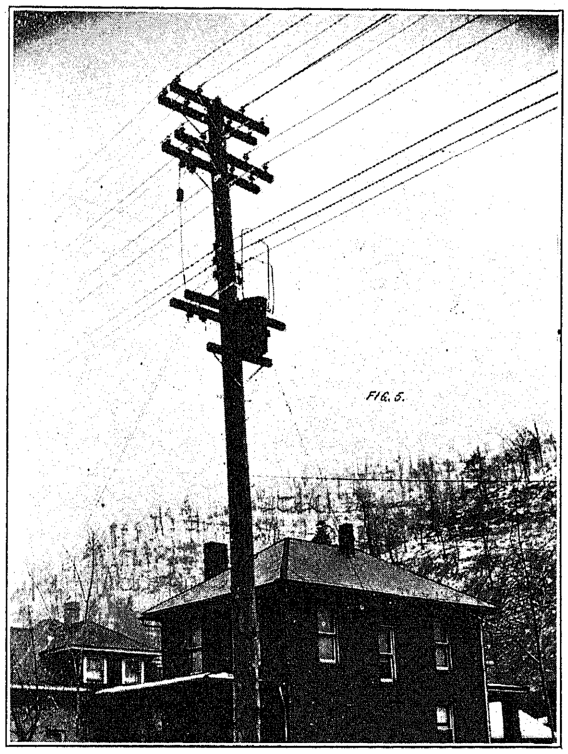

FIG. 5

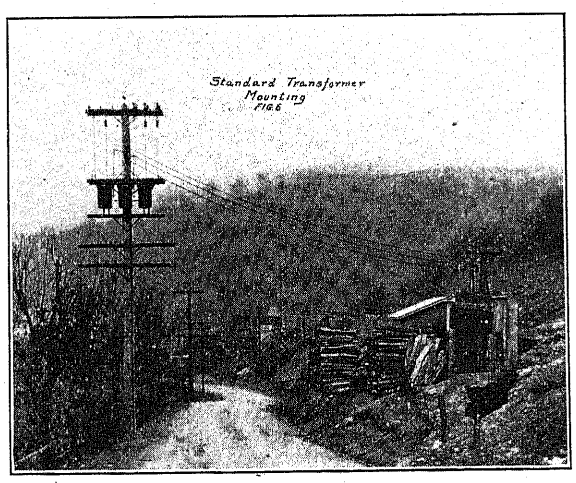

FIG. 6—STANDARD TRANSFORMER MOUNTING

3. The accumulation of obsolete material in the various storerooms is eliminated. If a standard is to be changed and a particular storeroom has an over-supply of a given item of material, it is rapidly distributed and used up without delaying materially the adoption of the new standard.

The above points can be elaborated, but it is not believed necessary; the main point is that the use of standardized form of construction makes it possible in

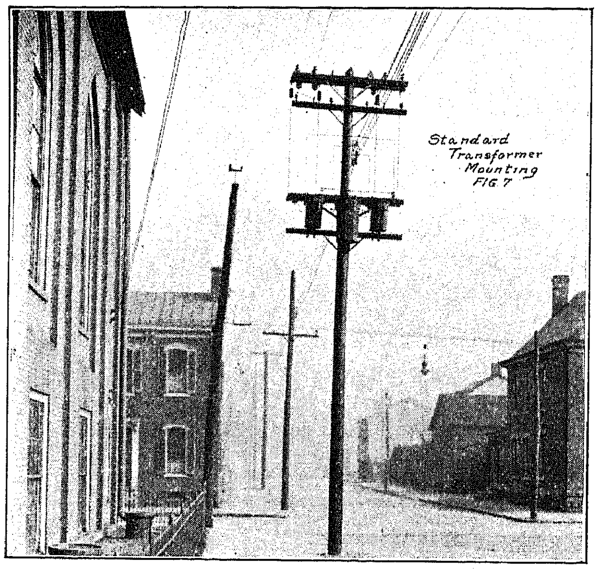

FIG. 7—STANDARD TRANSFORMER MOUNTING

the long run to do work cheaper than would be possible otherwise. Further, as has been shown, a more workmanlike job invariably results from the use of this form

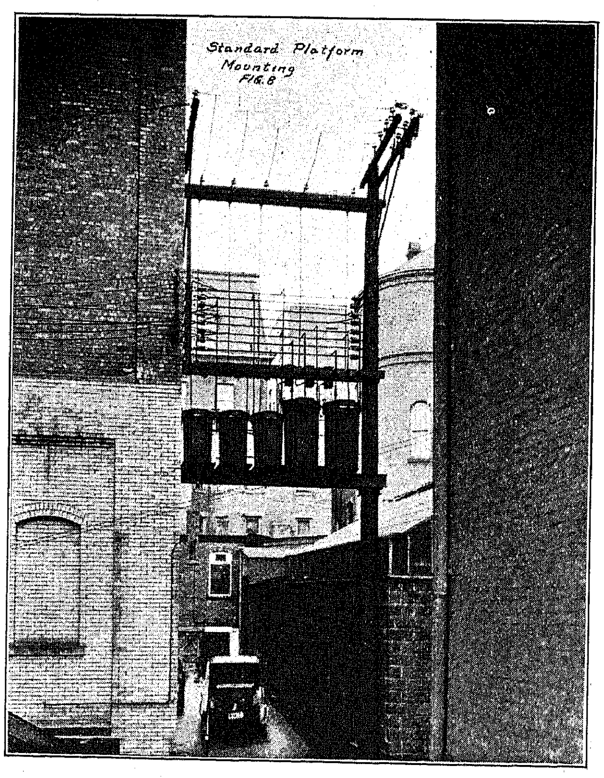

FIG. 8—STANDARD PLATFORM MOUNTING

of construction and a more workmanlike job will mean better service.

Standardized construction results in training a group of men at each property to do a certain type of work in exactly the same manner and employing the same material as at any other property of a given group. This is particularly valuable in case of unreasonable

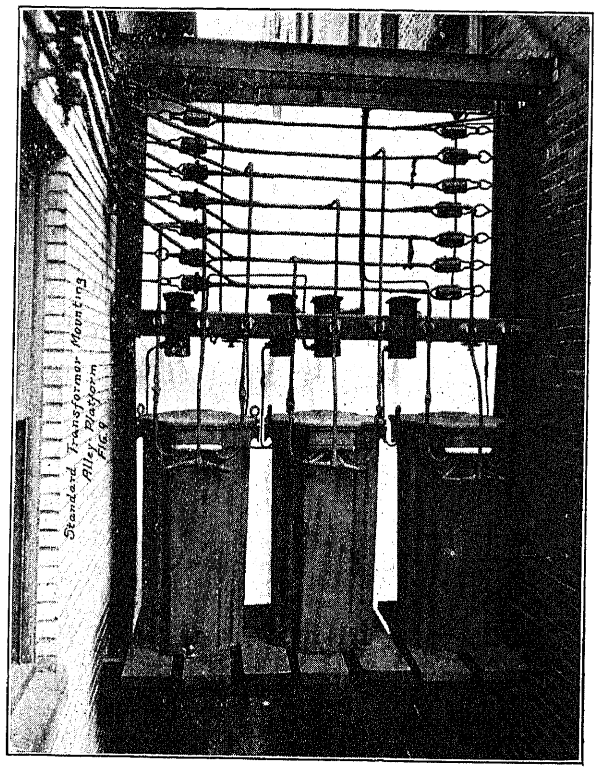

FIG. 9—STANDARD TRANSFORMER MOUNTING ALLEY PLATFORM

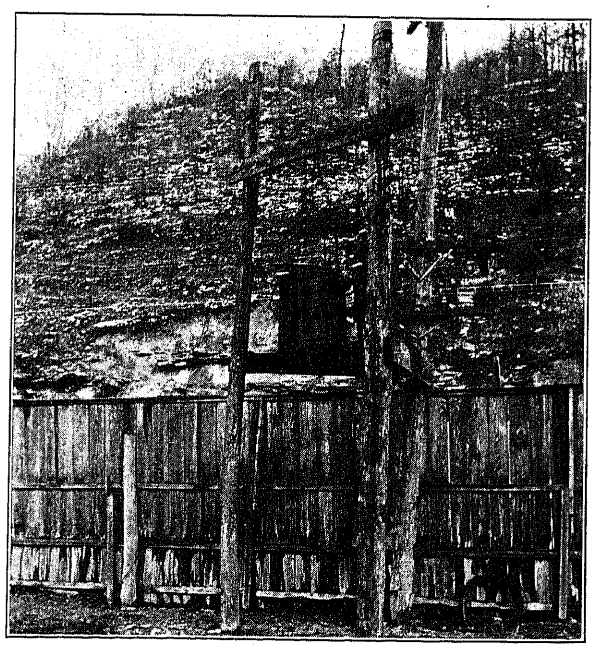

FIG. 10

labor demands. The ability in case of labor trouble to bring in a number of men from separate districts who are thoroughly familiar with the work and who can,

from the start, jump into the work and carry it along the same lines as customary elsewhere, not only makes it possible to render better service but also tends to produce a more reasonable attitude on the part of labor.

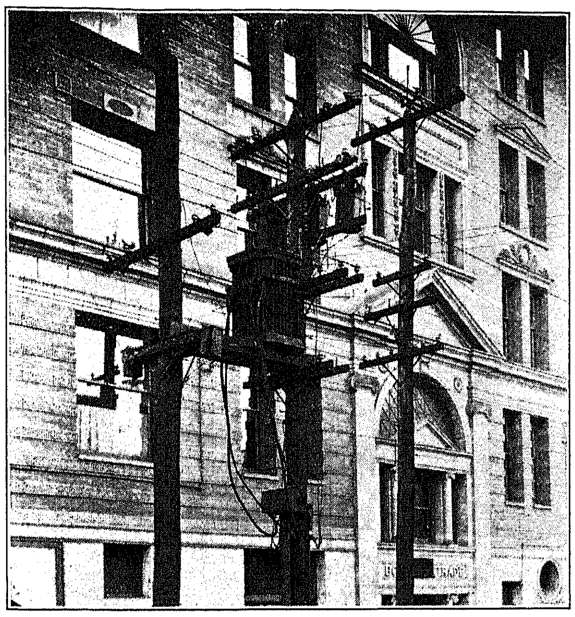

Fig. 11—Obsolete Construction

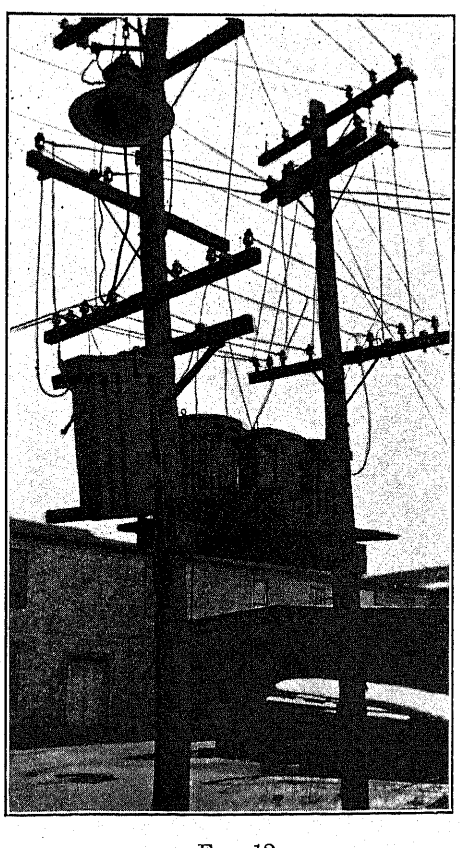

Fig. 12

Again, in the case of the holding company extending and taking over new properties where individualistic methods of construction have been going on, a problem frequently encountered is that of arresting the expansion of the existing system with an organization used to nothing else. The availability of a standardized system simplifies this problem considerably, as it makes it

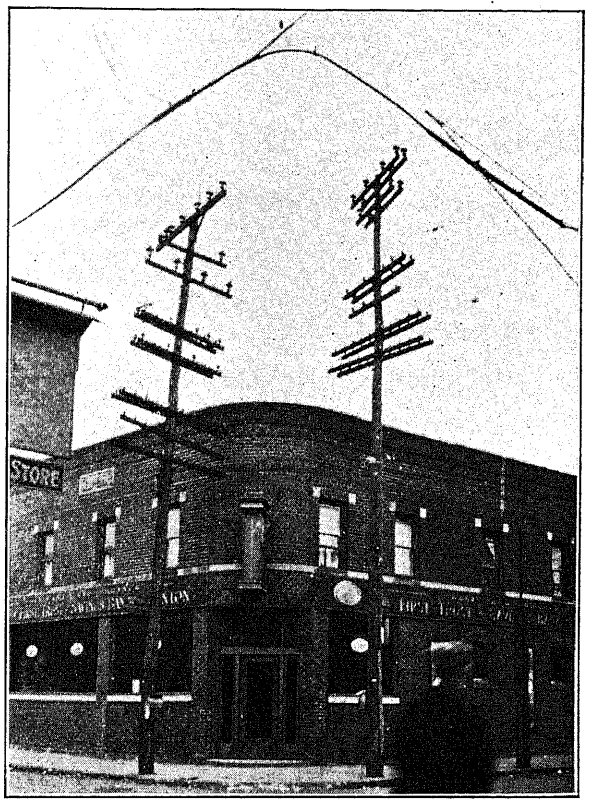

Fig. 13

Fig. 14

possible to select a few men from the older group, all accustomed to the one given method and to rebuild the new organization around these as nuclei.

There is still another benefit derived from the adop-

tion of standardized construction, which seems to us of great importance, and that is the question of appearances as affecting the relations between the utility and the municipality and more specifically the agitation for underground service. There is no doubt that in the past the question of appearance disturbed the distribution construction men very little. Especially true has this been in cities of moderate size population, say from 30 to 100,000. It was cheapest or it appeared cheapest to neglect appearances. But, with the growth of the city and the development of a civic pride, these formerly neglected eyesores, illustration of which is shown in Fig. 11 and again in Figs. 12, 13 and 14, are taken up and agitation for their removal is started.

With the example of the larger cities before them, the removal of overhead distribution and the substitution of underground is generally quickly enacted into an ordinance. The burden of establishing the unreasonableness of any such legislation falls on the utility, a burden that is doubly burdensome since it involves not only expensive litigation but results also in loss of public goodwill. A good deal of this can be forestalled, however, if the utility will set its house in order in time and standardization is a great accelerator toward that end.

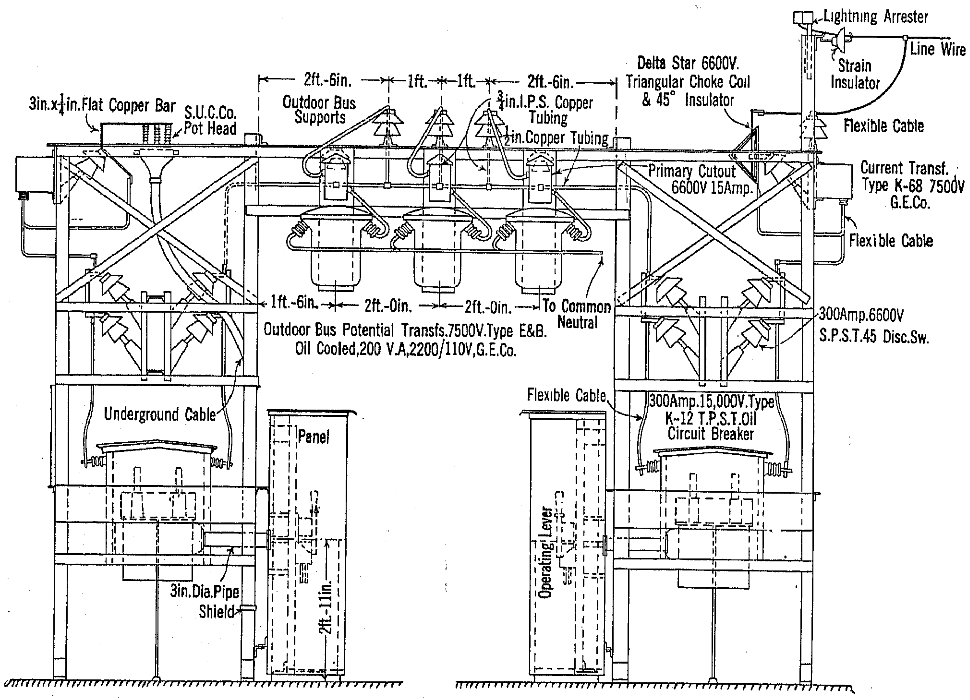

FIG. 15

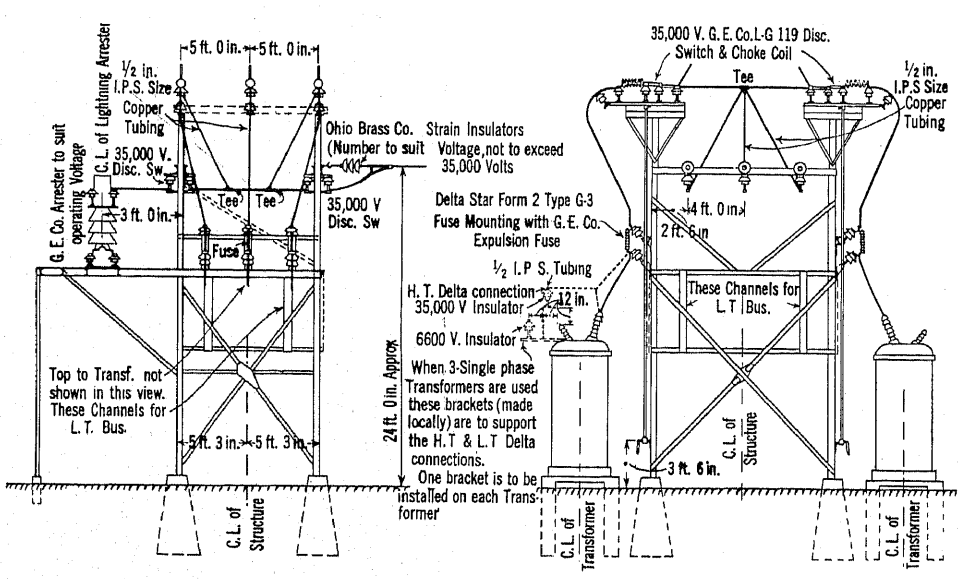

FIG. 16

Possibilities and Limits

Of the possibilities and limitations to standardization, as far as overhead distribution work is concerned, little need be said except that the logical limit here is the point where almost the entire field is covered. There is no reason why almost every type of construction problem encountered should not be covered by a standard, and in so far as it has been possible to anticipate them, they have actually been covered. New conditions and situations arise from time to time and as fast as a satisfactory solution is reached it is incorporated as a standard. The results achieved have been most satisfactory from every angle.

There is one pitfall in the path of standardization that we have tried to avoid and that is the danger of automatizing the various distribution departments and destroying their initiative. We believe successful care has been taken of this problem by bringing the various responsible heads of the distribution departments together each year for a period of 3 or 4 days and going over in open discussion all the standards; dissecting each one from a construction standpoint and from a standpoint of material entering into it. The result is not only a set of standards that is alive, and by alive is meant incorporating the best engineering ideas and the latest developments in the line of apparatus, but the maintenance of a critical attitude on the part of the field men toward the standards, an attitude of questioning the infallibility of any standards, no matter who the authors or how late their adoption. So much for distribution work.

The case of substations is somewhat different from that of distribution work, as the field men dealt with are perhaps of an entirely different type. In general, however, the objects sought are the same. To re-state them, however, they are:

1. To provide a standard so that a given job can be carried out in a satisfactory and workmanlike manner.

2. To accomplish this in a minimum of time with a minimum of effort and at a minimum cost.

3. To insure continuity of service either of the apparatus or structure in question or of the apparatus to which it is auxiliary. This service, is a prime requisite in the art and business of generating and supplying electric light and power.

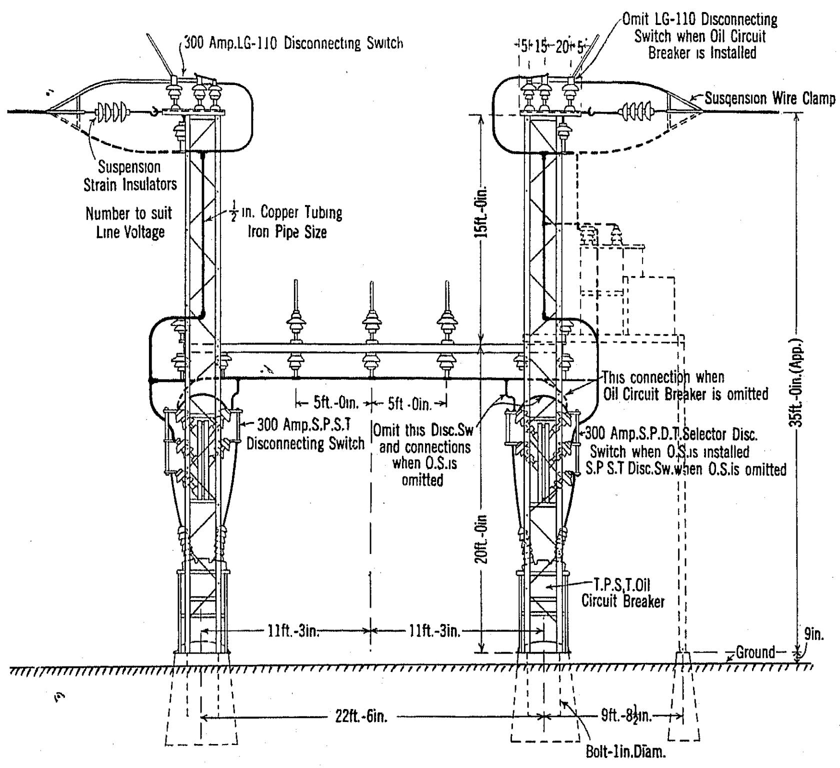

Fig. 17

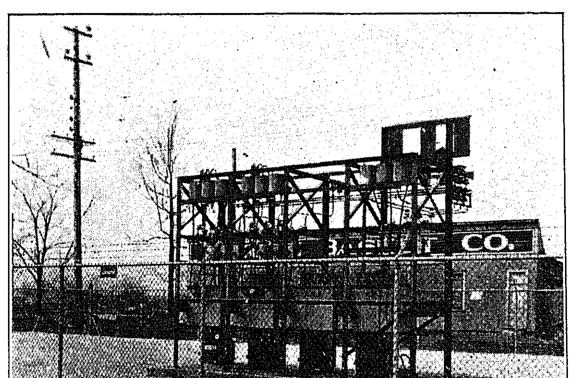

Fig. 18

That these results are possible of achievement by standardization in the case of substation work will be evident from the citations of things aaccomplished given below:

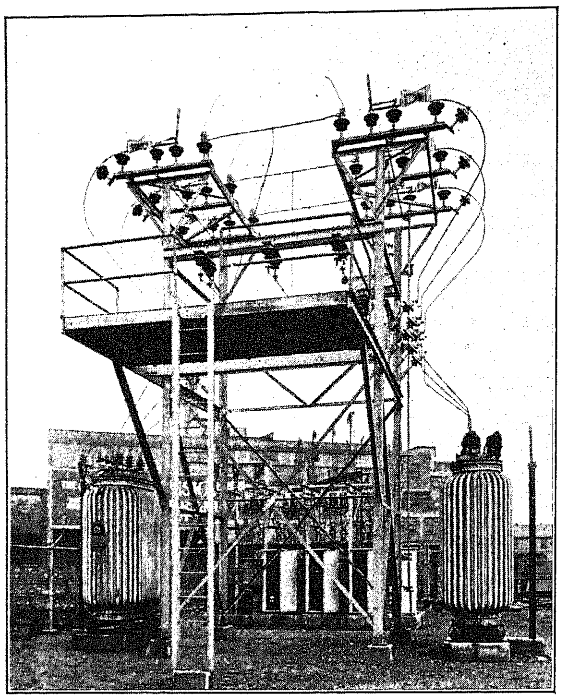

FIG. 19

SUBSTATIONS

As in the case of distribution work, a series of complete construction drawings has been prepared, covering a variety of substations ranging from 2300 volts up to 66,000 volts. Typical types of substations covered

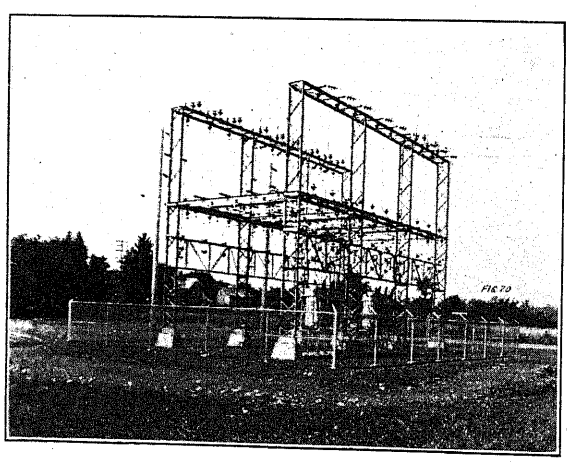

FIG. 20

are the tapoff transformer station, the sectionalizing station, the low-tension distribution station and the manifold combinations of these. Examples of these are shown in Figs. 15, 16 and 17 while Figs. 18, 19 and 20 illustrate installations made from the standards shown in Figs. 15, 16 and 17, respectively.

When a certain substation is found necessary and the field is in possession of a set of standard substations, it can easily pick a suitable one, and having the drawings before it, estimating is made comparatively simple. If later, estimates and requisitions are approved, the central engineering office need only check the selection of the standard and if that is concurred in, it is a matter of only a day or two before complete working drawings and bills of material are sent to the field. The fact that substations have previously been built from similar bills

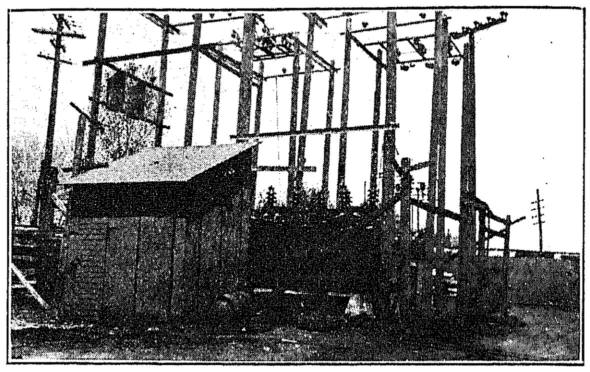

FIG. 21

of material insures the presence of all the necessary material on the ground when all that has been ordered has arrived and further insures the perfect assembly of material. This results, of course, in economical construction. Further, quantity buying of certain items of material results in saving from that direction, though the main saving is in the engineering expense and in the time involved in getting construction plans out to the field.

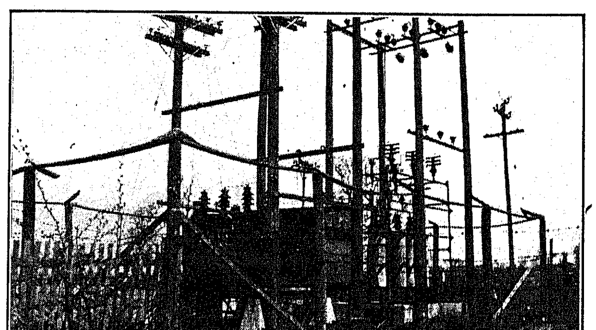

FIG. 22

It might be argued that the same results could be accomplished without standardization and this is perhaps true for the larger utilities but our experience with many of the moderate size ones has convinced us that if left to themselves, the results will almost always be deplorable. Thus Figs. 21 and 22 are examples of such substations that were in operation the early part of 1924. No elaborate arguments are necessary to prove that the construction shown in Fig. 19 is bound to

result in a much better grade of service than could be expected from a construction shown in Fig. 21.

Possibilities and Limits

In the case of substations as in the case of distribution work, the possibilities are again very great, that is, most substation installations can be taken care of by standards but the limitations are even greater. Unless care is exercised in the application of standards it is quite conceivable that all the economies will be more than counterbalanced by the unwarranted extra expenditures resulting from the misapplication of the standard. That, therefore, is a point that has to be watched by the organization responsible for the issuance of the standards. In other words, it is up to them to see that where a case demands special engineering treatment, that case receives it, but this particular limitation applies not only to standardization as regards the substations but to other forms of standardization as well. Summed up in another way, while standardization can eliminate a vast amount of routine drafting and engineering, it is a mistake to try and make it serve for engineering brains.

Reference has been made both in the case of distribution and substation work to the idea of insurance of continuity of service. This is attained in these two cases indirectly only and only through the fact that standardization invariably results in a more workmanlike installation. Continuity of service, however, cannot be obtained as a result merely of a workmanlike initial installation. A necessary adjunct to this is a never-ceasing system of maintenance and inspection of every piece of apparatus and equipment from the turbine to the distribution transformer. Our experience with standardization in other channels has led us to the conclusion that standardization could be applied to advantage in this work also. Accordingly, we have worked out a maintenance and inspection procedure for every piece of apparatus in the power link and this was made the same regardless of location of the pieces of apparatus in question. The frequency of inspection has been made different for different sizes and types of stations, thus giving weight to the importance of a particular station in maintaining service.

This system has only recently been worked out and will be put into effect within the next four weeks. It is impossible, therefore, to give any results of actual experience. It is expected, as a result of the installation of the system, to decrease the number of shut-downs of stations, substations and interconnecting lines; to maintain our equipment in better physical condition, and further, to know at all times what to expect as regards performance of the apparatus and equipment.

Summary

An attempt was made to show that standardization in construction, as applied to light and power companies, results in a better type of construction, and further, in the possibility and probability of more reliable service; that it results in definite economies in construction and engineering; that it requires continual alertness to preclude the possibility of misapplication of standards, and finally, that the idea can be developed further into maintenance and operation with equally beneficial results.

Discussion

E. P. Peck: In connection with Mr. Sindeband's paper, the plan, as a whole, is unquestionably right. The details will vary as time goes on. The main thing in a plan of that kind is to be sure that your standards are workable and that there are no unworkable theoretical ideas that some one happens to have put into them. I foresee that he is going to have a good deal of trouble with specialty salesmen. As soon as you standardize and decide to use one man's equipment, every manufacturer of specialties in the country will concentrate on you to have you change over and use his specialty.

Current-Limiting Reactor Characteristics

BY S. I. OESTERREICHER

Member, A. I. E. E.

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Review of the Subject.—*To prevent in large generating and distribution systems the possible concentration of enormous amounts of electrical energies at certain critical points, makes the extensive use of the current-limiting reactor a necessity.*

Connected reactor installations of 30,000 to 50,000-kv-a. capacities are not unusual any more.

Aside from the many important issues involved in the generating system as a whole by the reactor equipment, the reactor itself became a very important factor.

After the paramount issue of short-circuit protection is disposed of, a number of minor features is to be considered in the reactor design.

The general characteristics of the various winding types, the circuit connections affecting the true reactance values, current distributions, throughout a multilayer reactor and conductor efficiencies are considered.

Creeping due to heating of the conductor is evaluated.

Attention is called to the fact, that the attraction or repulsion of the turns or layers within a reactor depends upon the layer or turn interconnections.

The mechanical forces upon the conductor during short-circuit stresses are investigated and an approximate expression is given for their calculation.

In view of the important functions performed by the air core type current-limiting reactors in large generating and distributing systems, it might be of interest, perhaps, to direct attention to a few of their inherent characteristics, as influenced by their designs and the usually encountered installations.

Current-limiting reactors are connected into the circuits of electric generating and distribution systems with one particular object in view. Namely, to relieve the system of the abnormal and dangerous stresses, caused by a number of operating disturbances. However, circuit disturbances are comparatively few and far apart and as a further coincidence, do not always reoccur upon the same circuits. Thus, it seems that while reactors are primarily installed to meet transients only, in reality they meet two distinctly unrelated operating conditions.

In view of this and the additional fact that single reactor installations of 30,000 kv-a. connected rated capacities are not unusual any more, it must be conceded that the reactor characteristics during normal operating conditions are also important, and well worth considering.

Types of Reactor Windings

An air core type current-limiting reactor is the simplest conceivable electrical apparatus. It consists of a number of convolutions of a copper conductor. The convolutions are held into a rigid predetermined interrelation by some suitable arrangement like clamps, braces, arms, etc.

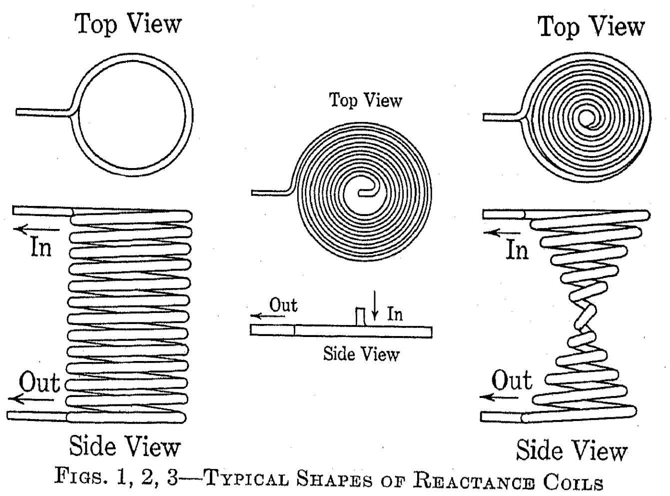
The inherent general characteristics of the reactor are somewhat influenced by the type of winding method used in its construction and after installation, by the relation and the circuit connection of the reactor in regard to other reactors placed in its immediate vicinity.

There are two principal types of winding shapes used in the design of current-limiting reactors, viz:

(1) Concentric multiple turns or concentric coaxial multiple turns. The turns might be connected either in series or in parallel, or both, in series—in parallel. This winding type is also called a drum winding. (Fig. 1).

(2) A spiral, wound either in single or multiple layers, connected also either in series or in parallel. This type of a winding is known as a disk or pancake type winding (Fig. 2).

As a modification of the concentric coaxial winding and for the sake of records only, the hour glass type winding is mentioned here and shown diagramatically in Fig. 3. Its use, however, is mostly restricted to



Figs. 1, 2, 3—Typical Shapes of Reactance Coils

lightning arrester choke coils. Due to difficulties in manufacture when designed for larger current ratings and the uneven stress distribution in different parts of the winding during short circuits, makes this a most undesirable winding form for current-limiting reactors.

The superiority of the disk winding over the drum winding is so overwhelming that for current-limiting reactors the first type is used almost exclusively. The summarized advantages are as follows:

1. For a given winding volume and turn spacing, a disk-wound coil will have a greater inductance value than a drum-wound coil.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

2. A disk-wound multilayer coil can be braced much more securely and conveniently than a drum-wound coil.

3. The voltage gradient between turns and layers and the electrostatic field distribution across the entire winding is much more uniform. This assumes that proper care has been taken for an even magnetic field distribution, by properly located conductor transpositions of a multilayer winding.

4. During short-circuit stresses, the magnetic field of a multilayer disk-winding attracts all the various layers toward the center layer. Thus, the predominant force is that of compression.

The only advantage of a drum winding is that if a reactor of adjustable inductance values has to be designed, it lends itself somewhat more convenient to sub-divisions and interconnections than a multilayer disk-winding.

The reactance x of an air core solenoid is given by the expression

$$X = a\,N^2\,Q\,10^{-6} \qquad (1)$$

where a = the radius of the solenoid,
N = number of turns, and
Q = a constant depending upon the frequency

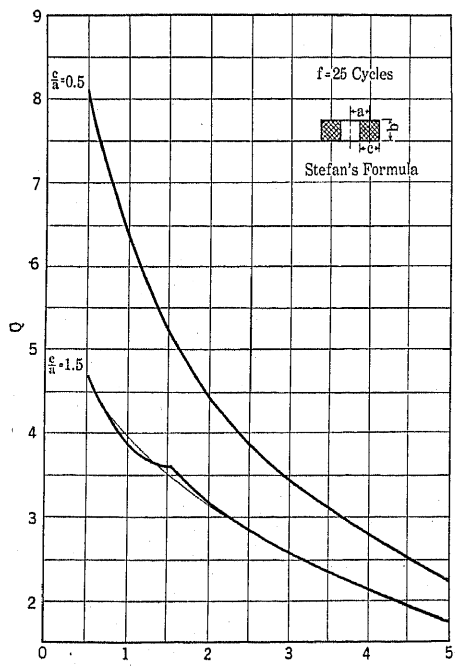


FIG. 4

f and the shape factor of the winding. The shape factor is determined by the relation of the length b and thickness c over the radius a of the coil.

The value of Q being primarily a function of the inductance of the winding, or still more general, the characteristic of the solenoid, is better expressed by

$$Q = 2\,\pi\,f\,L \quad \times \text{ constant} \qquad (2)$$

For L a great number of formulas have been derived and published. Those best adopted for the calculation of current-limiting reactors are the Stefan formula for multilayer disk coils and the Nagaoka formula for concentric coaxial multilayer coils. In both formulas auxiliary units determine the shape factor of the coil. A characteristic curve as to how the shape of the coil influences the value of Q is shown in Fig. 4. The curves are plotted from the Stefan formula.

In connection with the numerous available inductance formulas, it is, perhaps, proper here to mention the fact that most of the formulas were developed for the solution of a particular problem in view. However, all formulas assume an even current sheet throughout the entire winding area. This is an unobtainable condition in large air-spaced windings, the type of which is used almost exclusively in current-limiting reactors. It is therefore evident that a very painstaking exact calculation method is liable to be just as much in error as a simple one.

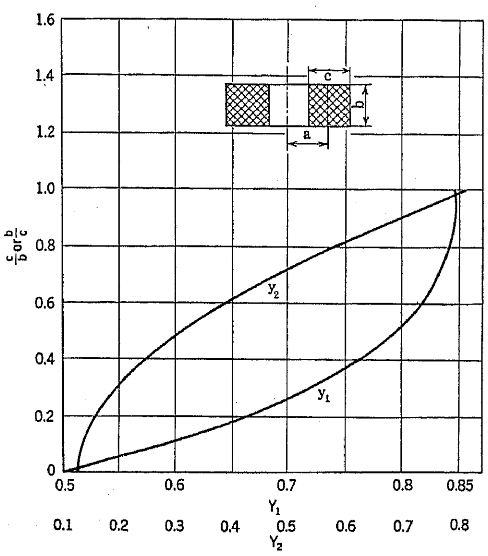


FIG. 5—VALUES OF y_1 AND y_2 CANNOT EXTEND BEYOND THE CURVES SHOWN BELOW
PLOTTED FROM TABLE ON PAGE 196
U. S. BUREAU STANDARDS BULLETIN
STEFAN'S FORMULA

For this reason, some of the simpler inductance formulas are much more desirable for everyday use than some of the more exact ones. In most of the formulas, using one or the other type of compiled auxiliary units, the expressions for the calculations of the auxiliary units are also given. Evidently for the purpose to enable the user to go beyond the already compiled tables. If such values are calculated beyond the tables, they should be used rather cautiously. For instance, if the y_1, and y_2 values of the Stefan formula are plotted against each other, it will be found that the formula will become unreliable for any computation beyond the intersection of the two curves, that is, beyond a certain shape factor, for which the Stefan formula was developed. (Fig. 5).

Inasmuch as part of the design of a reactor is to find

the shortest conductor length which has the desired inductance L, naturally the most efficient ratios of c. to .a and b. to .a were quickly evaluated and are usually stated as

$$a : b : c = 1.5 : 1.2 : 1.$$

However, it seems that undue importance is given to these most efficient ratios, especially as far as current-limiting reactors are concerned.

While it is true, of course, that the aim in the reactor design is also to have the relation of minimum conductor length and maximum inductance as close to the most efficient ratios as possible, this aim is usually offset by the fact that in many installations no space is available for such ideal coils. Another extenuating circumstance for the partial neglect of the theoretical ratios lies in the fact that during short-circuit stresses, it is of much greater importance to have a certain copper volume V in the coil, which is capable of withstanding the heating caused by the short-circuit currents for a certain specified time, than to have an ideal conductor length.

Furthermore, from equation (**1**), it is readily seen that the reactance x increases as a square function of the number of turns. Thus a small additional conductor length in the form of a few turns quickly offsets a certain variation in the coil dimension ratios.

In many previous papers about reactors, much stress has been laid upon small conductor area. No doubt, on account of the fact that with increasing conductor areas, the eddy current losses increase as a square function of the conductor diameter. The eddy current losses again, by increasing the effective resistance of the conductor, would considerably increase the total $I^2 R$ (a-c.) losses of the coil. For this reason, we often read not only about the most economical conductor length, but also about the most economical conductor area.

Due to such theoretical considerations in the past, a number of reactor failures occurred where the failures had to be attributed to the mechanical weakness of the conductor. The conductors ruptured on account of the intense vibrations caused by the magnetic field of the coil during short circuits. Thus, a compromise had to be made between electrical theory and mechanical strength. In this respect, the prevailing opinion at present is that the winding as a whole should have sufficient heat radiating area to safely conduct the heat losses of the conductor into the surrounding medium without any undue increase in the temperature rise of the reactor and within the prescribed limits of the materials used in the coil construction. However, the conductor should be strong enough to withstand any force, regardless of whether caused by thermal or magnetic origin or whether the eddy current losses increase or not.

To fulfill this condition in both directions and to enable the convenient winding of the conductor into a certain form, the total conductor area is subdivided into as few parts as absolutely essential for a conservative design. In many reactors the current rating is such that to provide an even heating of the entire conductor area, the conductor must be split up into more parts, otherwise the crowding of the current in certain parts of the winding will cause excessive temperature rises.

When such a split multiple conductor is wound into a certain winding space and at both ends connected in parallel, it is cut, of course, with an unequal magnetic field intensity. This is due to the fact that the various layers are placed into various magnetic planes of the

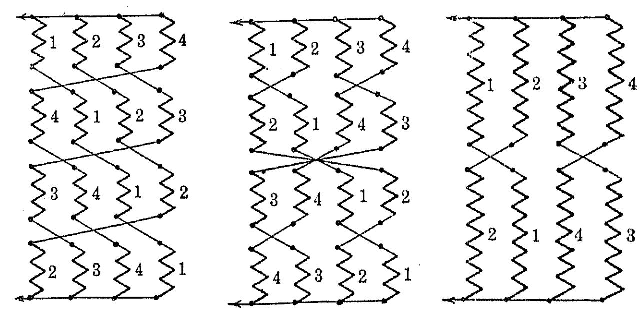


FIG. 6—FOR DISK WOUND TYPE, NUMBER OF TURNS IN ALL LAYERS EQUAL

FIG. 7—FOR DISK WOUND TYPE, NUMBER OF TURNS IN ALL LAYERS EQUAL

FIG. 8—FOR CONCENTRIC COAXIAL TYPE, NUMBER OF TURNS IN EACH TWO SUCCESSIVE LAYERS EQUAL

SHOWING VARIOUS TRANSPOSITION METHODS IN MULTILAYER WINDINGS TO OBTAIN EQUAL CURRENT DISTRIBUTION THROUGHOUT EACH LAYER.

coil. To remedy this shortcoming of the arrangement, layer transpositions become necessary. In Figs. 6, 7 and 8 are shown diagramatically the most frequently used transposition methods. All the transpositions are so arranged that the turns in series of any one layer are carried through a uniform magnetic field throughout the coil.

Another advantage derived from these transpositions is in an electrostatic direction and is due to the reduction of the dielectric stresses in as many

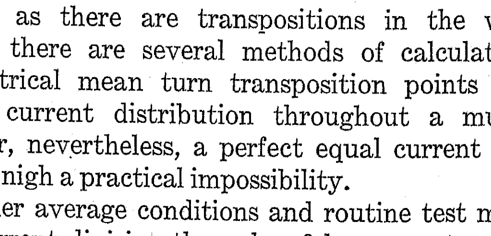


FIG. 9—CURRENT DISTRIBUTION THROUGH 9 MULTILAYER CURRENT-LIMITING REACTOR. TOTAL CURRENT 341 AMPERES. AVERAGE CURRENT 37.8 AMPERES = 100 PER CENT.

points as there are transpositions in the winding. While there are several methods of calculating the geometrical mean turn transposition points to give equal current distribution throughout a multilayer reactor, nevertheless, a perfect equal current division is well nigh a practical impossibility.

Under average conditions and routine test methods, the current division through a 9-layer reactor winding will be as indicated by the values in Fig. 9.

Inasmuch as every installed reactor is a cause of a certain voltage drop in its respective circuit, it seems paradoxical to speak about the most economical or the most efficient winding. However, as soon as the adding of artificial reactance into a generating system becomes a necessity, the resulting benefits usually offset certain inherent reactor disadvantages.

The apparent efficiency of a reactor is given by the expression:

$$\text{Efficiency} = 1 - \frac{I^2 R_{eff.}}{E I} \tag{3}$$

where I = full-load current in amperes, R_{eff} = the effective (a-c.) resistance and E = reactive volts across the reactor.

If maximum short-circuit protection would not be the main object in the reactor design, the most econom-

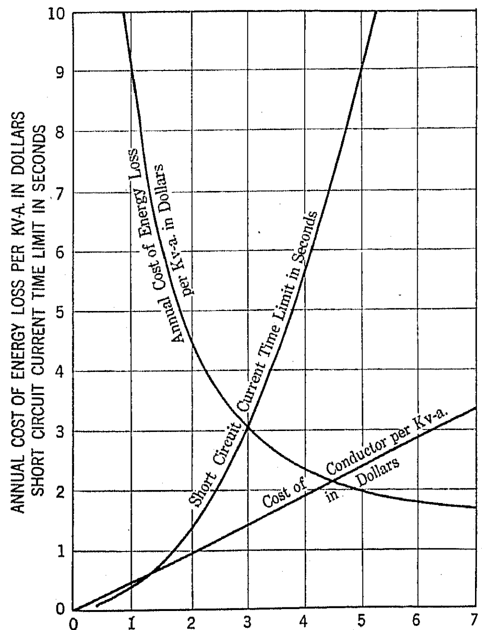


FIG. 10—TYPICAL RELATION OF CONDUCTOR COST, ECONOMY AND SHORT CIRCUIT TIME LIMIT DURATION OF REACTANCE COILS

ical conductor area would evidently follow Kelvin's law, which is given by

$$A = F I \sqrt{\frac{C e}{C c C a}} \tag{4}$$

Where A = cross section of conductor, I = square root of annual mean current squares.

F = a constant

$C e$ = cost of energy loss per kilowatt-hour in dollars

$C c$ = cost of copper per unit weight in dollars

$C a$ = annual charge in per cent on the conductor cost.

According to the units selected and the maximum temperature rise, the factor F in this equation has to be determined for the various types or makes of reactors.

A typical interrelation of economical copper volume, annual losses and short-circuit protection of a current-limiting reactor conductor is given in Fig. 10. The curves are based on a 65 deg. cent. temperature rise, 10 per cent annual charge upon the conductor and two cents per kw-hr. energy cost.

However, in many instances, especially upon 60-cycle reactors (2 to 3 per cent reactance) such interrelation cannot be followed.

To be able to offer a sufficient safety factor in the design of the reactor, it is best to neglect the heat radiation of the conductor during the short-circuit stress. That is, assume that no heat is lost through radiation in such short time interval as a short-circuit duration.

With such an assumption, the following relation exists between the conductor volume, short-circuit current, and effective (a-c.) resistance:

$I_{sc}^2 \times R_{eff.} \times t$ = gram calories $\times$ spec. heat of conductor $\times$ deg. cent. temperature rise.

Hence, the maximum short-circuit time limit in seconds for a reactor with a copper conductor, equals:

$$t = \frac{\text{Weight of copper in lb.} \times 180 \times \text{deg. cent. temperature rise}}{I_{sc}^2 \times R_{eff.}} \tag{5}$$

For the temperature rise during the short circuit, the temperature limitations of the respective materials used in the reactor construction have to be considered. Assuming that the normal continuous full-load temperature of a reactor reaches 65 deg. cent., assuming further, the ambient room temperature at 40 deg. cent., the various designs allow from 40 to 150 deg. cent. ultimate temperature rise during the short-circuit time lapse.

Inasmuch as the short-circuit time limit is usually a predetermined value, equation (5) can conveniently be converted to give the required cross sectional area of the conductor.

From (1) $x = a N^2 Q 10^{-6}$

The length l of the winding (in ft.)

$$1 = 0.524 \times a \times N \tag{6}$$

from which, the weight of the winding in lb.

$$W = 0.524 \times a \times N \times 0.32 \times \text{circ. mils} \times 10^{-3} \tag{7}$$

thus, area of conductor in circular mils

$$A = \frac{I_{sc}^2 \times R_{eff.} \times t \times 33.3}{a \times N \times x^\circ C \,\text{temperature rise}} \tag{8}$$

INFLUENCE OF MUTUAL INDUCTANCES BETWEEN REACTORS

In three-phase generating or distribution systems, a set of three neighboring reactors is usually connected into one generator, bus tie, station tie or feeder circuit. The three reactors may be installed either adjacent or super-imposed. In both arrangements the coils might be mounted with their axis either parallel or coaxial. As far as the consecutive phase rotation of the entire system is concerned, this also might be arranged in either a vertical or a horizontal plane. In both elec-

trical groupings, the three phases might consecutively rotate like 1-2-3——1-2-3———or the phases might be arranged 1-1-1———2-2-2———3-3-3——— These different arrangements and connections must necessarily influence the instantaneous magnetic flux directions between adjacent or superimposed reactors. Inasmuch as the instantaneous flux directions determine the average additive or subtractive values of the

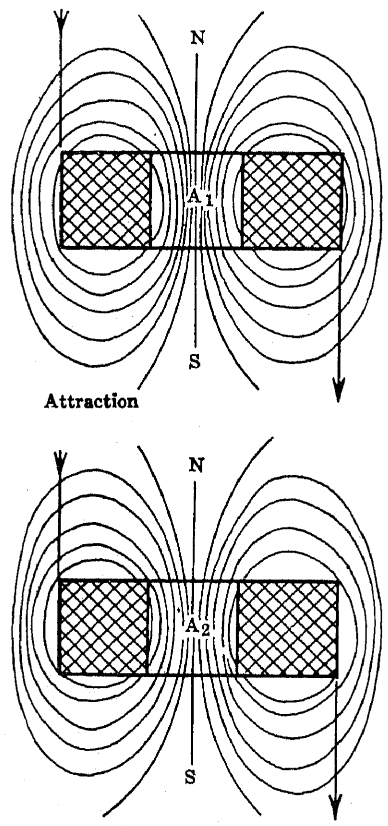


FIG. 11

mutual inductances between reactors, it follows, that the total inductance of each reactor will be influenced by the respective reactor arrangement.

For a simple illustration of the above, in Fig. 11, two windings have been superimposed in a coaxial relation. The current and winding directions are assumed to be in the same directions. The fluxes between reactors are attracting each other. The mutual inductances of the coils are additive. The total inductance L_t of coils A_1 and A_2 if connected in series equals:

$$L_t = L_{A1} + L_{A2} + 2_M \qquad (9)$$

Without changing any other features of this group, except turning the axis of these windings in a parallel relation (Fig. 12) the magnetic fluxes between the coils will now repeal each other. The mutual inductance (M) will be subtractive and L_t in this case is

$$L_t = L_{A1} + L_{A2} - 2_M \qquad (10)$$

Assuming that for a certain reactor intercenter spacing, the mutual inductance is only 3 per cent of that of the self inductance of any one coil; the fact that their axial arrangement is different is sufficient to reduce the total reactance of the circuit with as much as 12 per cent from the desired values.

Another important fact shown by Figs. 11 and 12 is that during short-circuit conditions, in the first case the coils must be braced against compression, whereas in the second case the same braces are subjected to tensile stresses.

While in principal the above claims hold true for any circuit, when applying the same to polyphase systems, due to the phase relations of the interlinked system, certain modifications have to be made.

Let three magnetic fluxes, 1, 2, 3, displaced by 120 deg. from each other, periodically vary in an interlinked system as shown in Fig. 13. The instantaneous polarities and their values through three air core solenoids 1-2-3 in C, are indicated at the zero instant of 1 and tabulated for one half period in Table I. Averaging the fluxes which attract each other between any two solenoids throughout the half cycle, the approximate average values of 1.05, 1.17 and 1.05 will be found to

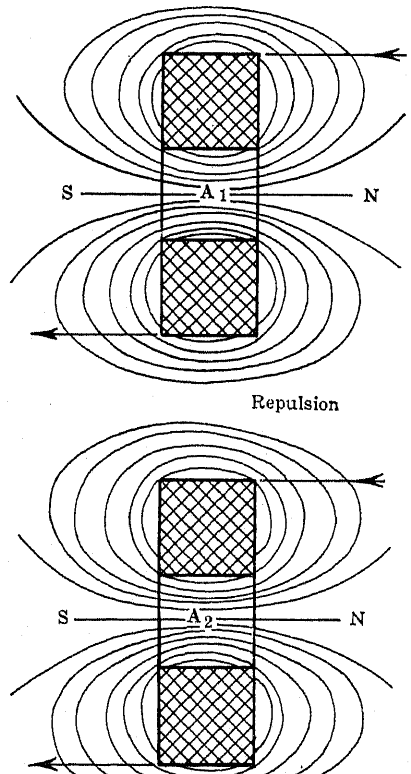
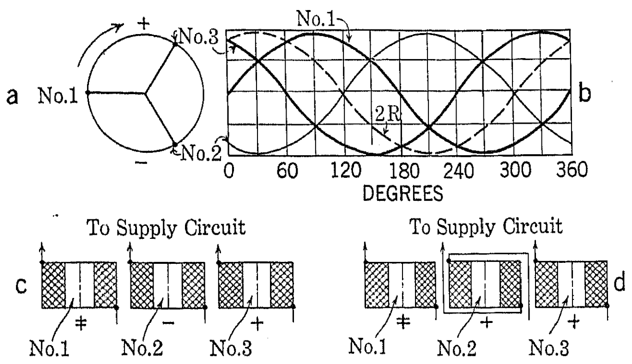


FIG. 12

exist between coil 1 and 2, 2 and 3, 3 and 1. Reversing the connection in coil 2, as indicated in Fig. 13d, will also reverse the instantaneous magnetic polarity of this coil. To receive a clear conception as to the instantaneous relation of this 2 R magnetic polarity in regard to coil 1 and 3, the reversed polarity 2 R is plotted in its reversed direction between curves 1 and 3 as 2 R (Fig. 13b). If these reversed values are again tabu-

lated, it will be found, as shown in Table II, that the average values between coils 1 and 2 and 2 and 3 are only about one half the average values of Table I. The average values between coils 1 and 3 are the same in both cases.



INSTANTANEOUS POLARITY AT 0 DEG.

TABLE I
INST. POLARITY CURVES, 1, 2 AND 3

0	1	1—2	2	2—3	3	3—1
0	0	0.87	−0.87	1.73	+0.87	0.87
30	+0.5	1.5	−1	1.5	+0.5	0
60	+0.87	1.73	−0.87	0.87	0	0.87
90	+1.	1.5	−0.5	0	−0.5	1.5
120	+0.87	0.87	0	0.87	−0.87	1.73
150	+0.5	0	+0.5	1.5	−1.	1.5
180	0	0.87	+0.87	1.73	−0.87	0.87
Average.........		1.05		1.17		1.05

TABLE II
INST. POLARITY CURVES 1, 2_r AND 3

0	1	1—2_r	2_r	2_r—3	3	3—1
0	+0	0.87	+0.87	0	+0.87	0.87
30	+0.5	0.5	+1.	0.5	+0.5	0
60	+0.87	0	+0.87	0.87	0	0.87
90	+1.	0.5	+0.5	1.	−0.5	1.5
120	+0.87	0.87	0	0.87	−0.87	1.73
150	+0.5	1.	−0.5	0.5	−1.	1.5
180	0	0.87	−0.87	0	−.087	0.87
Average	....	0.66		0.53		1.05

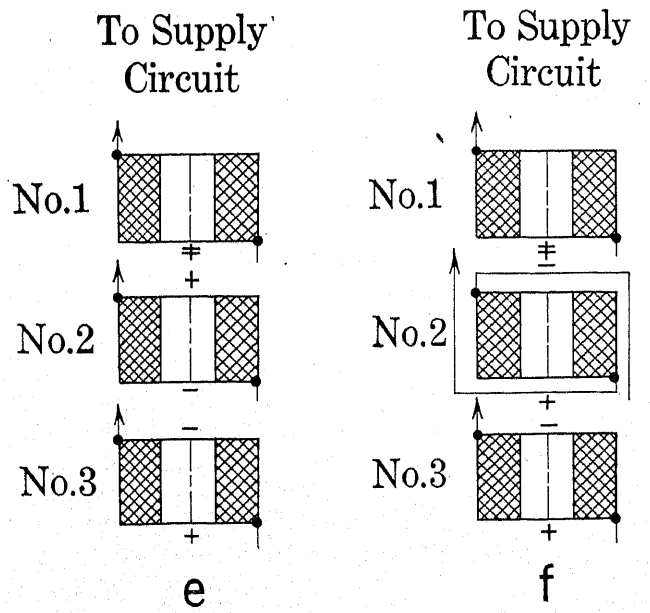


FIG. 13

Thus, by reversing the middle coil connection, or what amounts to the same, by reversing the middle coil winding direction of three coils supplied from a three phase circuit, the average mutual inductance effects between such coils are greatly reduced. Although the average mutual inductances between the two outside coils of either arrangement are the same, nevertheless, the usual double center spacing distance makes the effect less appreciable. The difference between coaxial and parallel axis mounted coils is only in the instan-

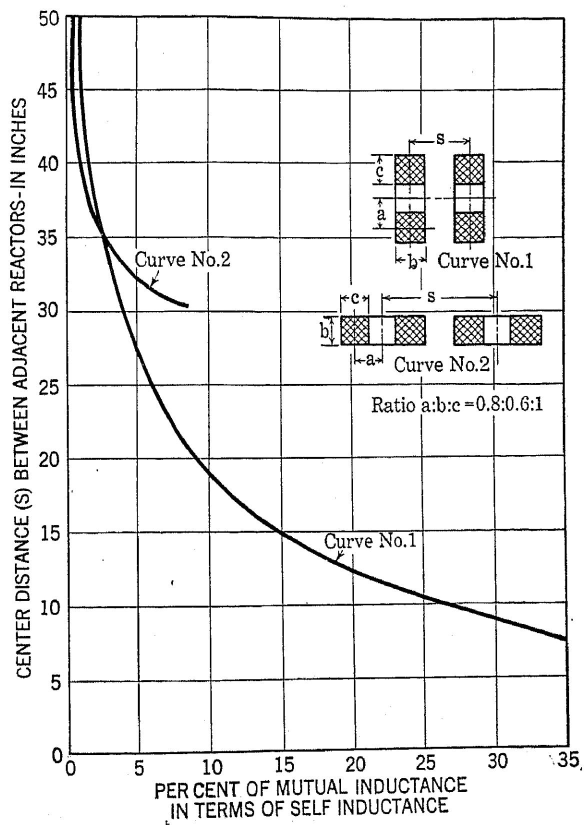


FIG. 14—VARIATION OF MUTUAL INDUCTANCE, IN TERMS OF SELF-INDUCTANCE, BETWEEN PARALLEL OR COAXIALLY MOUNTED CURRENT-LIMITING REACTORS

taneous polarities; the average values, however, remain the same for both arrangements.

An interesting point of such a connection reversal on the middle coils of a common three-phase group is that the sum of the three magnetic fluxes between the three windings is zero only at one point of a half cycle. Aside from this, the mutual inductances between two reactors is influenced partly by the shape factor and partly by the number of turns of each coil, both of which have usually the same numerical values for two coils connected into the same circuit. By far the greatest influence upon the mutual inductance is the spacing distance S between a set of coils.

Fig. 14 shows the characteristic relations of the

mutual inductances between adjacent coils of both principal mounting methods and throughout the usually encountered spacing ranges. The mutual inductance M for the different spacings is expressed in terms of the self-inductances of the respective coils. 100 per cent mutual inductance, corresponding to the total self-inductance L of the coil under consideration. The selected coil proportions are typical to the ones used in the designs of current-limiting feeder reactors, built to fit into the usually allotted power station compartments.

The preceding paragraphs may be summarized as follows:

(1) The type of winding used in a current-limiting reactor affects the electrical characteristics of the coil.

(2) The most economical conductor does not necessarily give the desired short-circuit protection.

(3) The copper volume of the reactor conductor should be selected, to give the desired short-circuit time limit protection regardless of economy to the design.

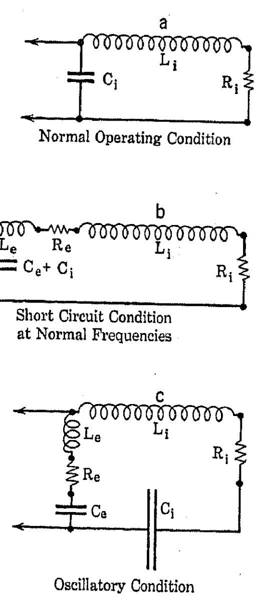


FIG. 15—EQUIVALENT CURRENT-LIMITING REACTOR CIRCUITS

(4) The influence of voltage drop between reactors, or the repelling or attracting forces may be controlled by proper intercenter spacings or by properly selected circuit connections.

While from the design standpoint or during normal operating conditions a reactance coil is comparatively a very simple apparatus, nevertheless, an equivalent electrical circuit of the same, has all the elements and combinations of the complex circuits. Some equivalent reactor circuits are given in Fig. 15. Fig. 15a shows the circuit in its simplest form and as it is usually assumed from the design standpoint. The circuit has an overwhelming amount of inductance L_i in series with an exceedingly small amount of resistance R_i, (usually of a value of about $1\frac{1}{2}$ to 4 per cent of X). Both inductance and resistance are shunted by a slight capacity C_i.

For normal conditions, the resistance as well as the capacity are usually neglected. During short circuits at normal frequencies, the external inductance L_e and external resistance R_e are in series with the internal inductance L_i and resistance R_i. Both the internal and external capacity $C_e + C_i$ shunt the circuit (Fig. 15b).

During normal frequencies, the external capacity

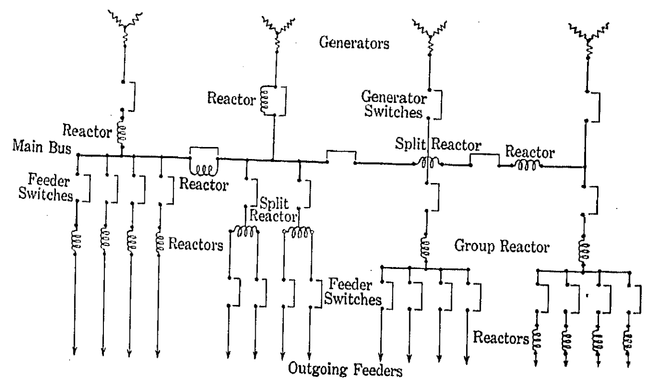


FIG. 16—ONE-LINE DIAGRAM OF VARIOUS GENERATOR, BUS AND FEEDER REACTOR CIRCUITS

of the circuit added to the internal capacity of the coil very seldom reaches the critical or oscillatory case. Under certain conditions, a reactor might oscillate upon itself. Such an oscillating circuit is equivalent to the one shown in Fig. 15c.

Certain distinctions are made between the various reactors, in relation to their particular location in a generating or distribution system. Thus we speak of

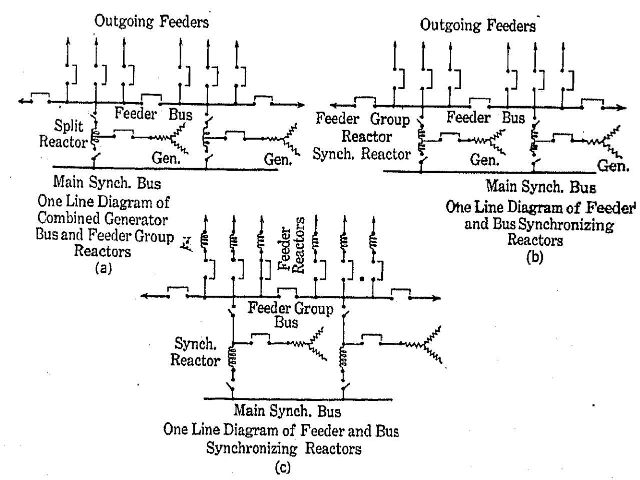


FIG. 17

generators, feeder, bus tie, station tie, synchronizing, equalizing reactors, etc., as the case might be. The refinement might be carried on still further, in the form of some of the various combinations of the above-named reactors, Figs. 16 and 17 indicate some of the most frequently used reactor connections. However, all these combinations affect only the numerical values of the reactors as to ratings, current-carrying capacities,

annual efficiencies, investment costs, short-circuit stresses, etc., and have nothing to do with the typical characteristics of the installations or that of the reactors.

Short Circuit Stresses

In three-phase systems a number of short circuits may occur, from which several may stress the reactor

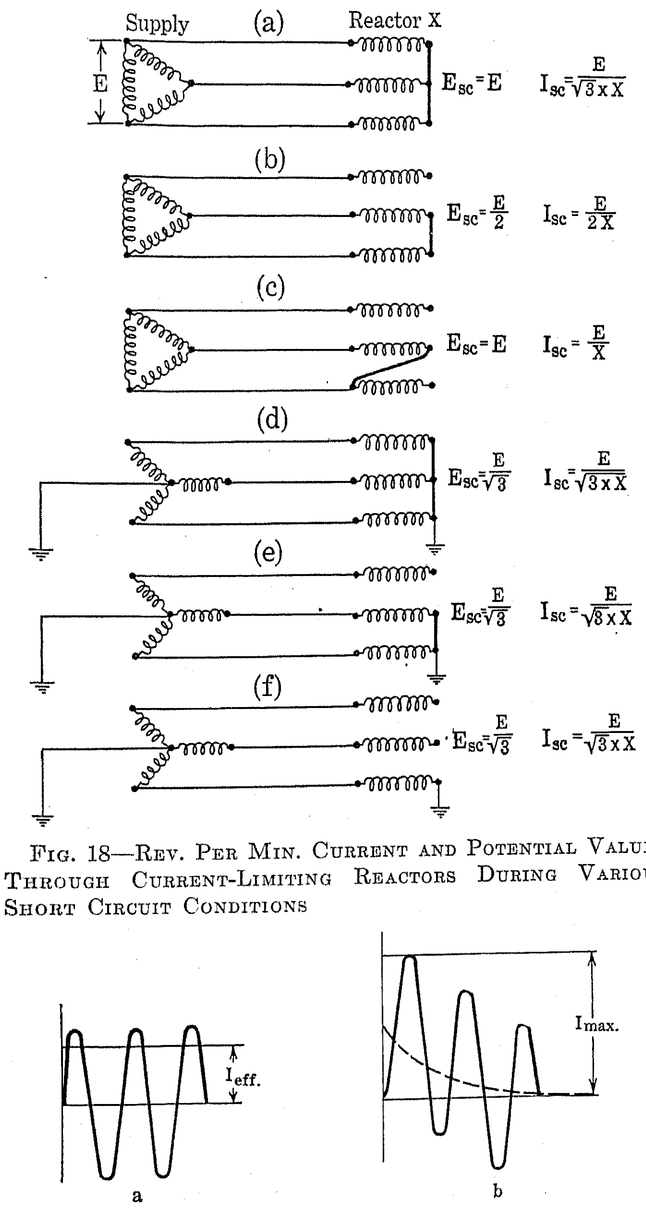


Fig. 18—Rev. Per Min. Current and Potential Values Through Current-Limiting Reactors During Various Short Circuit Conditions

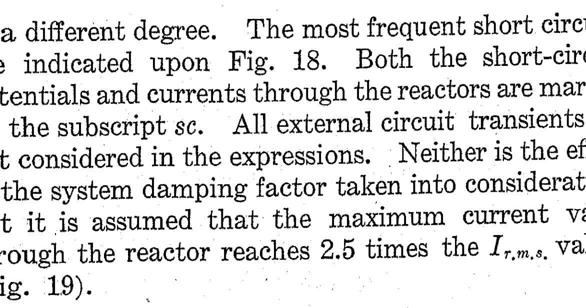


Fig. 19

to a different degree. The most frequent short circuits are indicated upon Fig. 18. Both the short-circuit potentials and currents through the reactors are marked by the subscript *sc*. All external circuit transients are not considered in the expressions. Neither is the effect of the system damping factor taken into consideration, but it is assumed that the maximum current value through the reactor reaches 2.5 times the $I_{r.m.s.}$ values (Fig. 19).

During short circuits there are three different stresses acting upon a reactor, namely:

(a) Thermal
(b) Mechanical (Electromagnetic)
(c) Electrical (Electrostatic)

The above three stresses may act simultaneously. Any one of these may be the predominating factor and thus affect the severity of the combined stresses upon the reactor.

(*a*) *Thermal Stresses.* Due to the inherent conductor resistance of the reactors, there is a heat generation in the coil, which forces the conductor continually to expand or to contract. This shape change takes place, of course, both in axial as well as in radial direction of the conductor cross section. During normal operation, this heat generation expressed in kilowatts is usually very small, hardly more than two to four per cent of the reactor rating. However, during short-circuit stresses, this heat generation increases as the squares of the current ratios increase. Expressing this as a function of the per cent of the coil reactance

$$W_{sc} = (2.5\, I\, 100/\%)^2\, R_{eff}. \tag{11}$$

where I = normal full load current (r. m. s.)

R_{eff} = resistance of the conductor just prior to the short circuit, (a-c. resistance).

Thus during a short circuit with five per cent reactance, the heat generation in a reactor will be about 400 times the normal $I^2 R$ value, with three per cent reactance 1110 times the normal value, etc. This heat generation causes the conductor to expand rapidly. Assuming that a reactor conductor at 15 deg. cent. has a length of 425 ft., and limiting the maximum ultimate short-circuit temperature to 150 deg. cent., the length of this conductor, according to the Stephan values, will be

$$l\,t = 425\left(\frac{1 + 135 \times 161}{10^7} + \frac{135^2 \times 403}{10}\right) = 426.24 \text{ ft.}$$

that is, during a two-second short-circuit time lapse, the conductor length increases about 14¾ in. Unless all conductor supporting arms yield evenly under such a sudden creeping stress, it might happen that between two adjacent supports or braces, there will be an accumulation of the linear expansion.

This seems to be a rather reasonable anticipation in time and is caused usually by the repeated heating and cooling of the conductor upon the supporting anchorages which loose their grip upon the conductor. Such a defect will cause considerable buckling of the conductor between two or more supports. The buckling alone, perhaps, would not be serious enough to endanger the proper functioning of the reactor, providing the conductor spacing is large enough or the insulation sufficient to prevent internal short circuits between layers or turns. However, such a buckled turn section is also under the influence of the magnetic attraction or repulsion of the layers and turns of its surrounding. The magnetic stress manifests itself during each half cycle

from zero to maximum in the form of a vehement vibration of the conductor. Furthermore, during the short-circuit time lapse, the conductor temperature increases rapidly. At or around 300 deg. cent. the strength of the copper decreases some 20 to 30 per cent. The elongation shrinks from 55 per cent at 0 deg. cent. to 40 per cent at 300 deg. cent. and the metal becomes brittle. Thus a condition will be easily created and a failure caused which has to be attributed to the combination of the thermal and magnetic stresses at a critical point in the reactor winding.

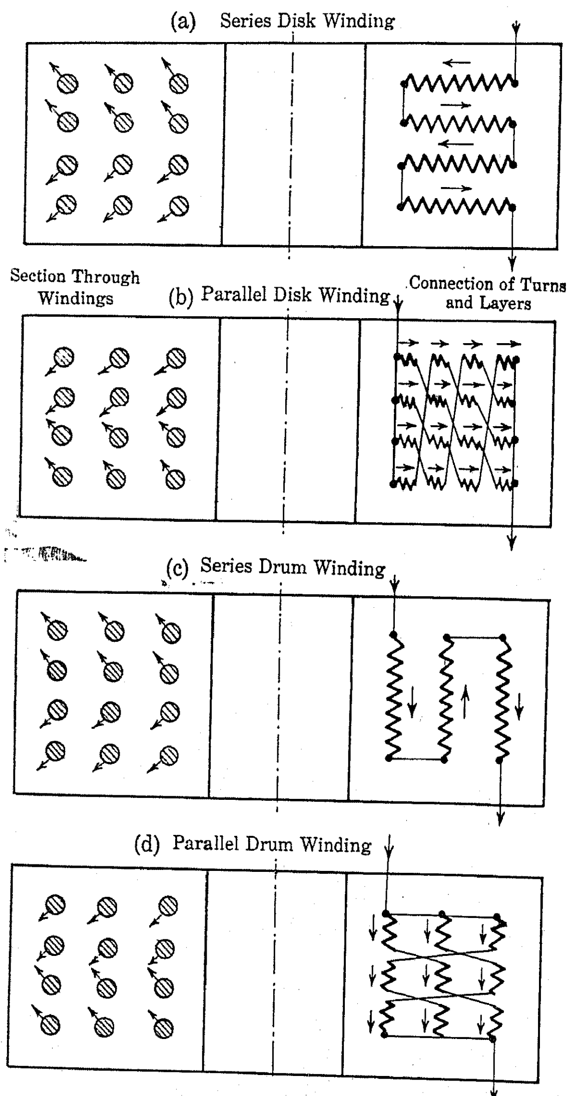


Fig. 20—Mechanical Forces Acting Upon a Reactor Conductor Due to the Main and Leakage Flux

(b) *Mechanical Stresses.* Under normal operating conditions the mechanical forces acting within a reactor winding are negligible. During short circuits, however, these forces increase to extremely high values. The internal forces act in a radial as well as axial direction of the winding. The radial forces are due to the tendency of each turn to place itself into the neutral flux zone, that is, farthest from its own magnetic center. The force subjects the conductor turn to a tensile stress. The axial forces within the reactor are due to the attraction or repulsion between the various layers, turns and the unsymmetrical end effects. Whether the forces are attractive or repulsive, depends upon the interconnections between turns or layers and upon the current distribution throughout the winding.

In Fig. 20, four different connection methods are shown with their force directions indicated by the arrows. There are several other possible connections for which the resultant forces may vary somewhat. Thus the winding supporting structure must be designed to suit in some cases tensile stresses which tend to disrupt the arrangement, whereas in other cases the predominant forces subject the housing to compression. An exact mathematical solution for the various reactor internal stresses is not available. Several approximations have been offered for the solution of the problem. Such a method derived from the combinations of several familiar expressions is offered below. The formula is based upon the assumption, that the entire inductance of the winding is concentrated in one center turn, having a as its mean diameter. The constant in the formula assumes a shape factor a: b: c: = 8: 6: 1 and if used within the Stefan formula, y_1, and y_2, constant ranges gives results about eight per cent high.

If x = total reactance of reactor in ohms
f = frequency
a = mean radius in inches
I = short-circuit current in amperes.

Then force F in lb. $= \dfrac{1.2 \times X}{f \times a} \times I^2$ (12)

Assuming an internal reactor connection method as per Fig. 20B, a maximum short-circuit current of 14,000 amperes and a current-limiting reactor of the following characteristics:

11 layers in parallel
61 turns in series
0.5 ohms reactance on 25 cycles.
11.5″ mean radius

Then F per layer $= \dfrac{1.2 \times 0.5}{25 \times 11.5} \times (14{,}000^2/11) = 3370$ lb.

If the maximum unsupported conductor length is 8 in. and the turn section modulus in a radial direction is 0.0013, then the tensile stress, per square inch for each turn section

$$= \frac{3370 \times 8}{0.0013 \times 61 \times 72.2} = 4692 \text{ lb.}$$

(c) *Electrical Stresses.* During normal frequencies the electrostatic stresses, due to the small internal capacity of the reactor, are negligible. If the frequency is considerably increased, it might happen that the air core reactor, which is an almost ideal inductance, becomes a pure condenser and discharges between turns and layers. The simplest case of this phenomena is a

single-layer series winding. In such a winding, each turn acts in regard to the next one as a condenser. The successive charges gradually increase toward the end turns. If anywhere in the arrangement the dielectric strength between turns varies, if the potential is high enough or the wave steep enough at that point, a discharge may occur.

In multilayer series or parallel windings the electrostatic field distribution is not so simple. In such a coil, besides each turn, each layer has also a charge in regard to its neighbor. Their values are different between different layers and between layers and ground. As in all high-frequency phenomena, the reactor is no exception in its inconsistent actions. Much research has to be done, before entirely reliable practical data will be available. In the meanwhile, such high-frequency oscillations like lightning should be deflected from a system before it can do damage anywhere. If a lightning arrester cannot do this, a reactor certainly will not do it.

Nevertheless a number of well understood conditions can be created within the reactor windings, by which the unequal electrostatic stresses are considerably reduced or equalized. Well selected transpositions lower considerably the critical points between layers. Dielectrics with even and permanent characteristics which are used between turns or layers, will invariably reduce the stresses. High-potential high-frequency tests, faithfully made, will always detect any weakness in the design or in the coil construction.

Discussion

For discussion of this paper see page 940.

Current-Limiting Reactors

Their Design, Installation and Operation

BY F. H. KIERSTEAD and H. O. STEPHENS
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Review of the Subject.—*The essential features in the design, installation and operation of current-limiting reactors are pointed out. The application of reactors to different circuits is not treated because a number of good papers dealing with this subject has been presented at Institute meetings.*

Design.—*A current-limiting reactor should have low reactance at low currents and high reactance at high currents. Such a reactor has been developed and is known as the "saturated core" reactor. It has not been used as yet for current-limiting protection to any extent. A reactor having an iron core and only an a-c. winding gives a drooping voltage-ampere characteristic and, therefore, is not suitable for current-limiting protection. Reactors with air cores have straight line volt-ampere characteristics and, therefore, are well adapted for this protection.*

Since the only function of current-limiting reactors is to limit the current during short circuits to safe values, they should be capable of performing this function when other apparatus is being destroyed, due to excessive current. For this reason, heat resisting materials should be used for holding and insulating the conductor of a reactor. A useful formula is given for determining short-circuit temperature rises.

A current-limiting reactor functions as such only when there are short circuits on the system. Short circuits are almost sure to be preceded or accompanied by voltage disturbances of abnormal value. Therefore, the insulation factor of safety in a reactor should be relatively high.

Reactors with shunting resistors give to the system in which they are placed, the protection from over-voltage that the resistors afford and are, therefore, to be recommended.

Reactors are subjected to internal magnetic forces due to the current in the reactor itself and to external magnetic force due to the field between adjacent reactors. A method is given for calculating the magnitude and direction of the internal forces. Tests show that the peak magnetic force should be used in calculating the strength of the conductor.

The direction of the external forces is discussed and reference is made to H. B. Dwight's formulas for calculating their magnitude. Tests show that the stresses on the members which support the reactor against the external forces depend in a decisive way upon the ratio of the mechanical frequency to the electrical frequency.

Installation.—*The installation of reactors is considered from the following view points.*

I. Arrangement.
II. Compartments.
III. Bracing and Securing in Place.
IV. Support of the Leads.
V. Ventilation.

Operation.—*In general, reactors require very little attention in service, but no effort should be spared to keep them free from foreign conducting material. Loose magnetic material such as nails, etc., are particularly dangerous because the magnetic field of the reactor during a short circuit will pick up nails at quite a distance from the reactor and draw them into the winding. Tests show that such foreign material lodged in the winding causes the reactor to instantly arc over during a short circuit.*

It is the purpose of this paper to point out the essential features in the design, installation and operation of current-limiting reactors. The application of reactors to different circuits is not treated, since a number of good papers dealing with this subject have been presented at Institute meetings.

Design

Under this heading the fundamental conditions in the design of reactors for satisfactory and reliable service will be considered. These conditions are as follows:

I —Straight Line Volt-Ampere Characteristics
II —Liberal Thermal Capacity
III—Insulation Strength Should be High Since a Reactor is a Protective Device
IV—High Mechanical Strength
V —Most Economical Losses

In order to show why these characteristics are essential, we have considered the design of reactors from each of these five points of view.

I—Straight Line Volt-Ampere Characteristics

High efficiency and good regulation require current-limiting reactors to have as low reactive drop as possible when normal current is flowing. Good short-circuit current control, on the other hand, demands the maximum reactive drop possible.

A reactor having low reactance at low currents and high reactance at high currents would meet these conditions. Such a reactor has been developed and is known as the "saturated core" reactor.[1] It has an a-c. and a d-c. winding on an iron core. Normally the d-c. ampere turns are so large compared with the a-c. that the core is saturated with a d-c. flux and the normal a-c. ampere turns have no appreciable effect in changing the core density. For this reason, the reactance at all currents up to normal rated current is negligible. At high currents the a-c. ampere turns begin to have an appreciable effect on the core density and the reactance of the reactor increases very rapidly. Such reactors, however, are in the developmental stage, and as yet have not been used for current-limiting protection to any extent.

A reactor having an iron magnetic circuit and only an a-c. winding gives a drooping volt-ampere character-

· *Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.*

1. See A. I. E. E. paper by Boyajian on "Theory of D-C. Excitation of Reactors and Regulators" and A. I. E. E. paper by Blake on "Application of Saturated Core Reactors and Regulators."

istic due to saturation of the iron and, therefore, is not suitable for current-limiting protection. For example, the magnetic flux in a 5 per cent reactor must increase 20 times normal at short circuit, if the flux wave is symmetrical, and 40 times if it is assymmetrical. The flux wave will be assymmetrical if the short circuit occurs when the voltage wave is passing through zero. If the flux density in the core was 500 lines per sq. cm. at normal current (a ridiculously low value), then at short circuit with assymmetrical current the flux density would have to increase to 20,000 lines per sq. cm. Therefore, the reactance in ohms introduced in the circuit by an iron core reactor would be very much lower at short-circuit current than at normal current and to limit the short-circuit current to a particular value would require a much greater reactance at normal current than a reactor having a straight line characteristic. For this reason, iron core reactors have not been used to any extent for short-circuit protection.

Reactors with air magnetic circuits have constant reactance for any current and for this reason are well adapted for current limiting protection.

II. Liberal Thermal Capacity

Since the only function of current-limiting reactors

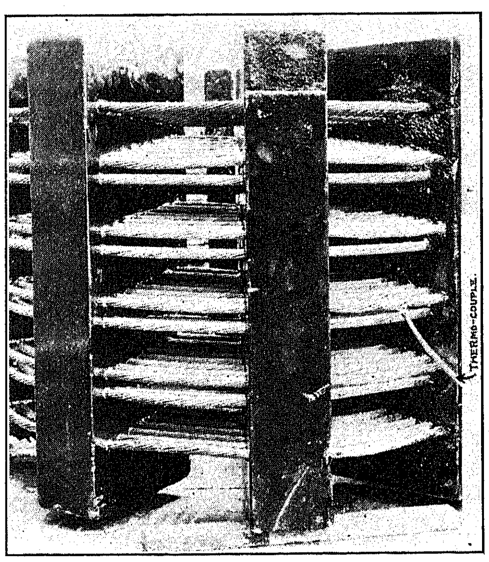

Fig. 1—Reactor after Having Been Subjected to a Temperature of 725 Deg. Cent. Resulting from Short Circuit.

is to limit the current during short circuits to safe values, they should be capable of satisfactorily performing this function when all the other apparatus in the circuit is being destroyed due to the excessive current. As long as the reactor is operative, the short circuit is limited to a single circuit, but if the reactor fails due to a prolonged short circuit, the whole station may become involved. Of course, it would be impracticable to build all reactors with sufficient thermal capacity to withstand a short circuit for an indefinite length of time and it is the duty of the operator to clear the circuit within a reasonable time. On the other hand every effort should be made to use heat-resisting material for supporting and insulating the conductor of the reactor. If this is done, the temperature of the conductor may be carried nearly to the melting point during short circuits without failure. Fig. 1 shows a reactor on which a short-circuit test was continued until the conductor reached a temperature of 725 deg. cent. The reactor was uninjured by this test and was fully

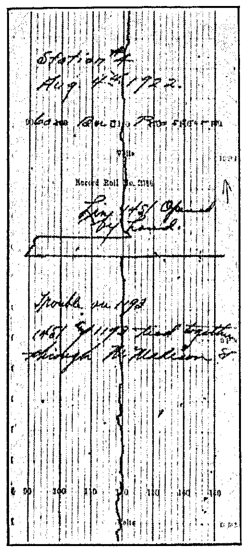

Fig. 2—Record of Bus Bar Voltage Showing 6-Min. Short Circuit

operative. This shows the value of heat-resisting supports.

A number of reports of long continued short circuits have been received. Among these reports is one from a large central station describing a six-minute short circuit on a 0.5-ohm reactor in a 12,000-volt feeder. Fig. 2 gives the record of the station bus voltage during this interval. The duration of the drop in voltage indicates the time during which the feeder was short-circuited. This reactor is still in service. Other reports simply give additional instances where the heat-resisting supports had enabled the reactor to function through a long continued short circuit. Actual experience with reactors in service for many years has proven beyond doubt that the supports which hold the conductor should be heat-resisting.

The size of the conductor to be used in a reactor may depend on the heating at normal load or during short circuit, or again it may be determined by the maximum permissible losses.

The temperature limits depend on the materials in contact with the conductor. With organic material there is always the danger of carbonization at the hot inaccessible spots during operation and during short circuits this danger becomes more imminent. Furthermore, there are usually no indications of such carboni-

zation until it has caused a failure. With inorganic material this danger does not exist and higher temperatures may be permitted.

In 1914 when the first cast-in-concrete reactors were being built, there was some doubt as to what should be the limiting temperatures. Tests indicated that very high temperatures would be safe but the conservative practise of 125 deg. cent. for the limiting temperature at normal current and 350 deg. cent. at short circuit was adopted. In determining short-circuit temperatures, it is assumed that the full voltage is maintained across the reactor and that all of the heat is stored in the conductor during the specified duration of the short circuit. Since that time there have been many indications that these reactors could be safely operated at higher temperatures and no indications that these temperatures are too high. However, these conservative limits are being maintained.

The following is a useful formula for determining the short-circuit temperature rise, assuming that all of the heat is stored in the conductor.

$$T = \frac{1}{\dfrac{92 \times A^2 \times 10^6}{K\,(1 + \theta \times .00427)\,t\,I^2} - \dfrac{1}{2\,(234.5 + \theta)}}$$

where,

T is the short-circuit temperature rise in degrees cent.
A is the cross section of conductor in sq. in.

K is the eddy current factor $= \dfrac{\text{Total copper losses}}{I^2 R}$

θ is the initial temperature
t is the time in seconds
I is the short-circuit current in amperes

The maximum anticipated time required to clear a short circuit should be given by the purchaser in his specifications, since he has the necessary data on the time settings of the relays which open the circuit. Where these data are not given, it is customary to design the reactor to stand full short-circuit current for at least three seconds.

III. Insulation Strength Should be High Since a Reactor is a Protective Device

A current-limiting reactor functions as such only when there are short circuits on the system. Short circuits are almost sure to be preceded or accompanied by voltage disturbances of abnormal value. Therefore, the insulation factor of safety in the reactor should be relatively high.

The reactor is a point of reflection for steep front waves and thus doubles any such wave that strikes it. It is, therefore, to be expected that nearly double line voltage will appear between the reactor and ground and across the reactor terminals during sudden changes in the system as when switching or during a disturbance such as a flashover of an insulator. Voltages of this order are an every-day occurrence. On more rare occasions voltages of more than double line voltage may be reflected from the reactor, as for instance, when there has been a disturbance which has resulted in a wave of more than line voltage being sent out toward the reactor. If, when this wave reaches the reactor its value is still more than line voltage, then it will be reflected with a value of more than double line voltage. Tests have proven that steep front impulses are rapidly damped out, so that unless such a disturbance happens near a reactor its wave will be considerably reduced before it reaches the reactor. This is especially true of underground cables since their insulation losses very rapidly damp out impulses. Therefore, it should be exceptional for voltage impulses higher than double line voltage to be reflected from reactors in underground cable systems.

On overhead lines lightning disturbances may cause impulse voltages on the lines as high as the line insulators or the lightning arresters will permit. Such disturbances may be reflected by the reactor at double their initial value. During such disturbances the voltage from the reactor to ground and across its terminals is liable to reach any value which the line insulators or the lightning arresters will permit.

Resonance between two circuits, one containing high electrostatic capacity and low inductance and the other containing low electrostatic capacity and high inductance, may cause high voltages to be built up not only across reactors but also from the line to ground. Authorities differ as to the magnitude of these voltages. In laboratory tests high voltages are obtainable. In tests on a large underground cable system 36 per cent over-voltage was observed with circuits considerably out of resonance. On account of operating conditions the investigation had to be discontinued before tests could be made with the circuits in resonance. This subject should be investigated more thoroughly because of its bearing on central station disturbances.

Operating experience with cast-in concrete reactors with liberal spacing and low voltage between adjacent layers and turns has indicated that voltages high enough to cause flash-over have not been obtained. On the other hand, a very high voltage is required to flash over these reactors. For instance, a reactor for a 13,200-volt circuit requires 60,000 volts to arc over the insulators to ground and 100,000 volts at high frequency to flash over the winding. In other words, this experience has simply indicated that voltages higher than the above have not been experienced.

Formerly, it was believed that some of the early failures of reactors were due to over-voltage, but later investigations using an especially constructed short-circuit testing generator proved conclusively that these failures were due to magnetic forces causing adjacent turns to touch. These tests practically eliminated any apprehension of reactor trouble due to excessive voltages.

The voltage concentration at the end turns and end layers in reactors due to steep front waves is very much

less than in transformers, because the series capacity (that is, the capacity between turns and layers) is so much greater than the capacity of the reactor to ground.[2] In tests on reactors with steep front waves 29 per cent of the voltage of the initial wave has been observed between the first and second layers of the reactor and 10 per cent between the first and second turns.[3]

The conclusions to be drawn from these investigations of the transient voltages that reactors may have to stand, is that these voltages may be as high as the weakest insulation or the lightning arrester setting will permit, and the reactor should have a reasonable factor of safety above these limits.

Reactors with shunting resistors give to the system in which they are placed the protection from over-voltage that the resistors afford and are, therefore, to be recommended. Shunting resistors also reduce the transient voltages on the reactor and conceivably it would be permissible to reduce the insulation of such reactors, but this practise has not been recommended.

The reasons for the use of resistors shunting reactors are two-fold. First, resistors protect a circuit by absorbing the energy which would cause high voltages. Such energy is always of higher frequency than the generated frequency and, therefore, piles up high voltages across reactors. If these reactors are shunted by resistors this voltage is applied to the resistors and the high frequency energy is absorbed. On the other hand, under normal conditions the voltage across the reactor, and therefore across the resistor, is very small and little energy is absorbed. A resistor shunting a reactor is, therefore, selective in its operation, absorbing a negligible amount of useful energy but absorbing a high percentage of destructive energy. Second, when reactors are placed in all the feeders from a central station, any disturbance originating in the station (as in switching) is prevented by the reactors from passing freely out in the lines where it may be dissipated, but if each reactor is shunted by a resistor, each furnishes a path for the escape of the disturbance into the feeders.

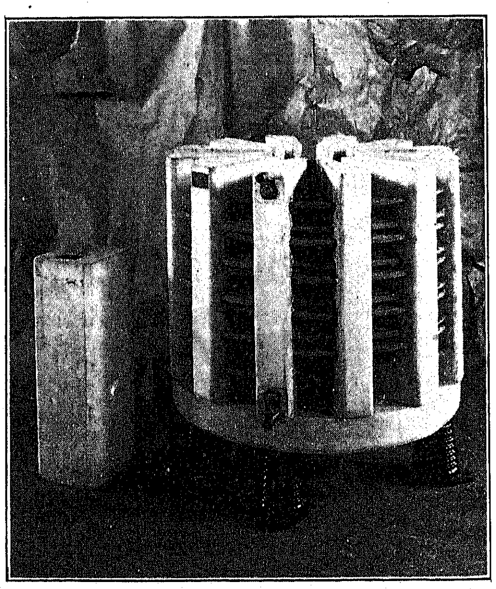

Fig. 3—Reactor with Resistor Removed

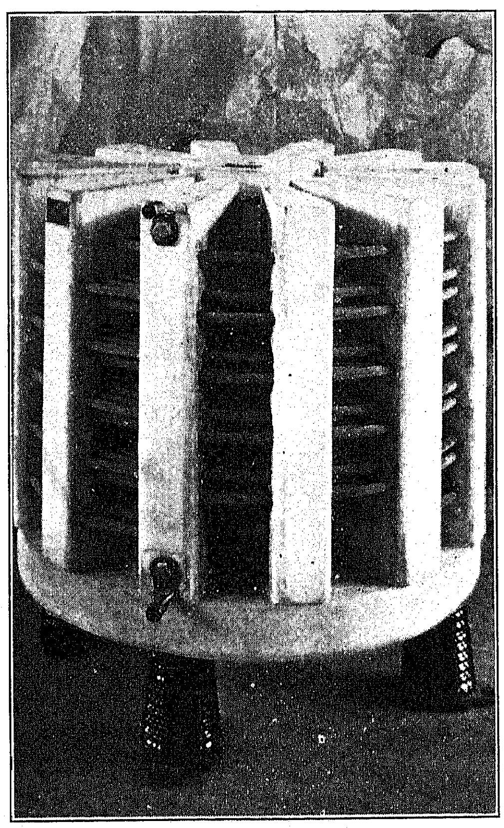

Fig. 4—Reactor with Resistor Assembled

The resistor is assembled in the center of the reactor so that no additional space is required. Fig. 3 shows a reactor and its resistor and Fig. 4 shows the same reactor with its resistor assembled.

IV. High Mechanical Strength

Reactors are subjected to magnetic forces due to the current in the reactor itself, which we will call internal forces, and to magnetic forces due to the field between adjacent reactors, which we will call external forces.

a. Internal Forces. The direction and relative magnitude of the internal forces are shown in Fig. 5. It will be noted that they are in a direction to place the supports for the conductor in compression and most of the turns of the conductor itself in tension.

In order to calculate the strength of a reactor to withstand these internal forces, it is necessary to determine the turn that is most liable to fail, which we will call the critical turn. The magnetic force on this turn

2. A paper by Blume and Boyajian entitled "Abnormal Voltages within Transformers" published in the A. I. E. E. Trans. 1919, page 577 gives the reasons.

3. See paper by Kierstead and Meeker entitled "Voltage Stresses in Reactors," A. I. E. E. Trans. 1920, page 1289.

and its ability to withstand this force should be calculated. The force on the critical turn is determined by summing up the forces of each individual turn upon it. This is most conveniently done by determining separately the arithmetical sum of the axial and radial forces and combining these results vectorially, which gives the resultant force on the critical turn both in magnitude and direction. Where the turns are all equally and symmetrically placed, this summation can be obtained by integration, but in the type of reactor shown in Fig. 5, this integration becomes too involved and it is more convenient to calculate the forces separately and record the results in curves and tabulations suitable for the particular type of reactor. The following formulas, derived from Maxwell's equation for the mutual inductances between circles, are convenient and accurate for calculating the axial and radial forces between two turns

$$P_1 = 4.5 \times 10^{-8} \times \frac{Z}{A} \times \frac{1}{\sqrt{(a+A)^2 + Z^2}} \times \left[\frac{a^2 + A^2 + Z^2}{(a-A)^2 + Z^2} E - F \right]$$

$$P_2 = 4.5 \times 10^{-8} \times \frac{1}{\sqrt{(a+A)^2 + Z^2}} \times \left[\frac{a^2 - A^2 - Z^2}{(a-A)^2 + Z^2} E + F \right]$$

where,

P_1 and P_2 are respectively the axial and radial forces in pounds per ampere per inch length of the conductor.

A is the radius of the larger circle in inches

a is the radius of the smaller circle in inches

Z is the distance between the planes of the circles in inches.

F and E are the complete elliptic integrals of the first and second kind, respectively, to modulus

$$k = \frac{2\sqrt{aA}}{\sqrt{(a+A)^2 + Z^2}}.$$

Their value may be obtained from Table XII and XIII in the Appendix of Bulletin of the Bureau of Standards, Vol. 8, No. 1 (Scientific Papers No. 169).

Referring to Fig. 5, it will be noted that the inner turns are subjected to the greatest forces. However, they are not the critical turns because the span of these turns between supports is much less than that of the outside turns, and the force is in such a direction as to place the conductor in tension. While the force on an outside turn is less, the greater span between supports, coupled with the fact that the direction of the force is such as to cause the turn to slacken, makes the turn much less capable of resisting it. If all the turns were symmetrically placed, the outside top and bottom turns would be the critical turns, but in the type of reactor shown in Fig. 5, due to the converging of the layers of turns, the outside turn in the second layer from the top and the second layer from the bottom are the critical turns. Fig. 6 shows the critical turn of a reactor just starting to bend under a short-circuit test to determine the effect of these forces.

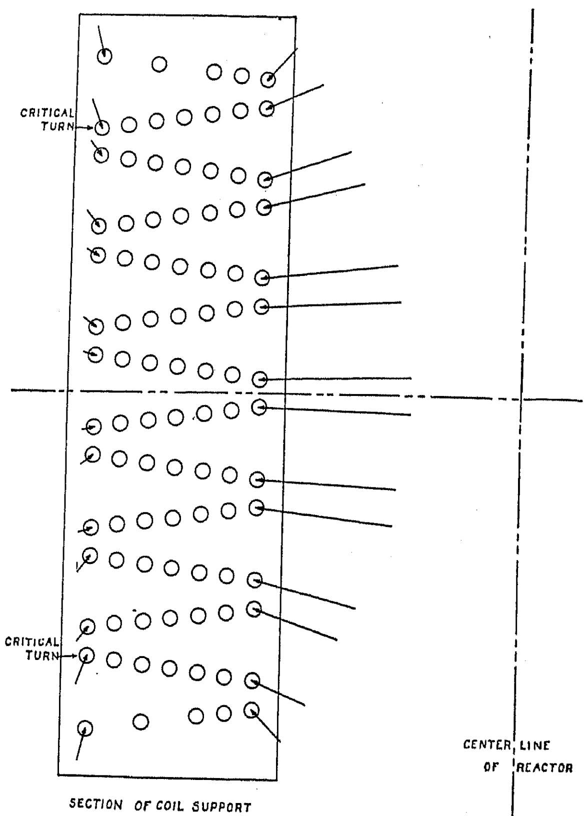

Fig. 5—Direction and Relative Magnitude of Internal Forces in a Reactor

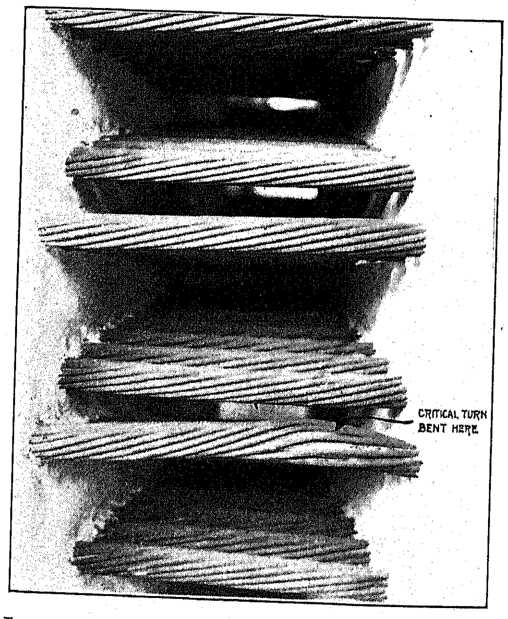

Fig. 6—Illustrating Bending of Critical Turn of Reactor by Magnetic Force during Short Circuit

Investigation of these forces has shown that the force on the critical turn in some ratings of reactors

may be as great as 300 lb. per inch of length of conductor. Such a reactor with a critical turn of 32 in. diameter would have a total force on the turn of 30,000 lb. When it is realized that this is only the force on one turn of a reactor, and at that, not the turn with the maximum force, an idea is obtained of the tremendous forces that reactor supports must withstand.

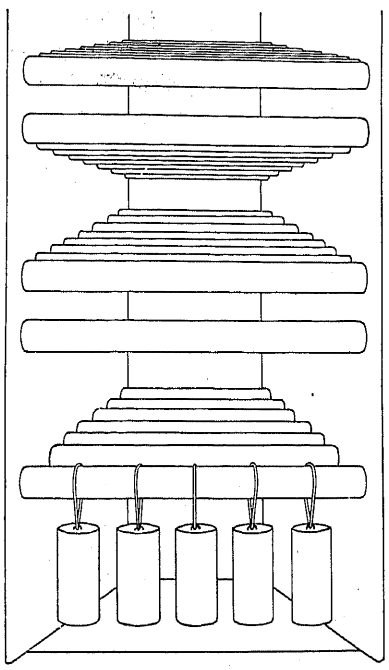

FIG. 7—METHOD OF APPLYING FORCE TO TEST STRENGTH OF TURNS

In order to measure the strength of the conductor to withstand magnetic forces, weights were applied to the conductor as shown in Fig. 7 and the deflection was measured for different weights.

With an accurate method of calculating the forces and with data for determining the strength of the turns,

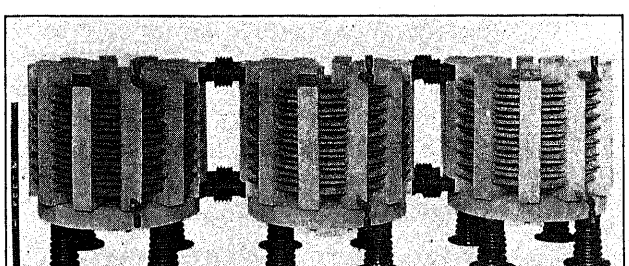

FIG. 8—SET OF THREE-PHASE REACTORS PLACED WITH AXES PARALLEL WITH BRACES BETWEEN PHASES

the ability of a reactor to withstand a steady magnetic force could be determined. It was necessary, however, to test reactors under short circuits up to destruction to determine their strength under the suddenly applied force of a short circuit. The results of these tests proved that the peak magnetic force and not the average should be used in calculating the strength of the conductors.

b. External Force. When reactors which are wound in the same direction are placed with their axes parallel as in Fig. 8, the magnetic forces will be attractive if the current in the one flows in the opposite direction to the current in the other, and repulsive if the currents are in the same direction. If they are mounted coaxially as in Fig. 9, the direction of the forces will be just the reverse. Therefore, the direction of the forces between similar reactors with their axes parallel will be attractive if the reactors are all connected symmetrically, and repulsive if the connection to the middle phase is reversed to that of the other phases. If the reactors are placed coaxially, the direction of the forces will be repulsive if symmetrically connected, and attractive if the middle phase is reversed.

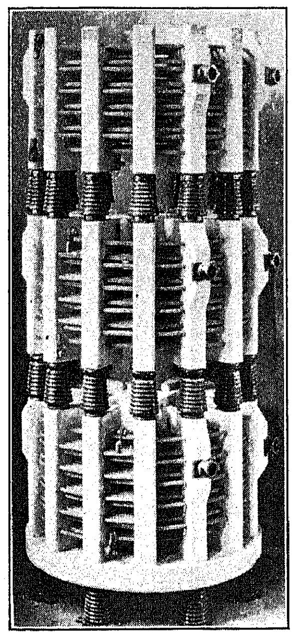

FIG. 9—SET OF THREE-PHASE REACTORS PLACED COAXIALLY WITH PROVISION FOR BRACING AGAINST WALL OF COMPARTMENT

The magnitude of the force between adjacent reactors of a three-phase circuit is greater during single-phase short circuits than it is during three-phase short circuits, (although the single-phase short-circuit current is only 86.6 per cent of the three-phase current) and, therefore, stresses caused by these forces should be based on single-phase short circuits. Forces due to single-phase short circuits are higher because the fluxes from adjacent reactors are in phase or 180 deg. out of phase during single-phase short circuits, while they are 120 deg. or 60 deg. out of phase during three-phase short circuits.

Forces of considerable magnitude may occur between adjacent reactors in different circuits as well as between adjacent reactors in the same circuit. A good example of this is where two feeders in parallel feed the same substation. A failure of either feeder results in a short

circuit on both, and if the failure occurs near the substation, then the current flowing in the two feeders will be approximately equal. If the reactors in these two feeders are adjacent they will exert a magnetic force upon each other. These forces will be greater if the adjacent reactors are connected to the same phase, since the currents will be in phase during either three-phase or single-phase short circuits. If the adjacent reactors are connected to different phases, the forces will have

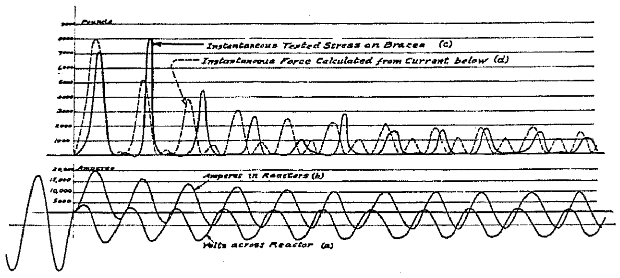

FIG. 10—OSCILLOGRAPHIC RECORD OF SHORT-CIRCUIT TEST TO DETERMINE FORCES BETWEEN ADJACENT REACTORS AND STRESSES ON BRACES

the same value as is the case of reactors connected in the same circuit. The resulting forces will be 33 per cent. greater if the reactors are in the same phase than if they are in different phases. It must be remembered, however, that with two feeders in parallel short-circuited, the current in each reactor will not be as great as it would be if only one feeder was short-circuited, since the station bus voltage will drop more with the

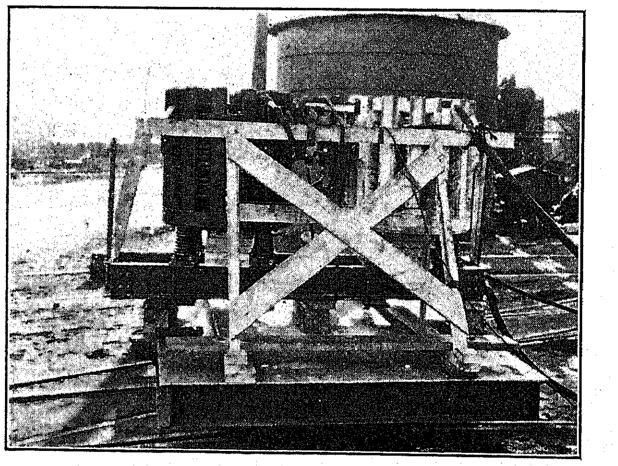

FIG. 11—SET-UP OF REACTORS FOR SHORT-CIRCUIT TESTS RECORDED IN FIG. 10

two feeders short-circuited than with one. The reactance of the feeders out to a substation would further reduce the current. Therefore, it is probable that the forces between adjacent reactors in different feeders never will be as great as those between adjacent reactors in the same circuit.

Formulas for the calculation of the forces between reactors placed coaxially[4] and with their axes parallel[5] have been given by H. B. Dwight. These formulas have been checked and found to give good agreement with tests and, therefore, the method of calculation of these forces is not being given here.

With the formulas developed, the calculation of these forces is a simple problem. However, it is a more difficult problem to determine the stresses on the members which support the reactor against this force, for the reason that it varies with time after short-circuit in a very complicated manner, as shown on Curve c, Fig. 10. Curves a, b, and c have been reproduced from an oscillogram taken during a short-circuit test in an investigation to determine these stresses, while Curve d is a calculated curve. Fig. 11 shows the set-up of the apparatus for these tests. The tests consisted in exciting a 25-cycle 27,000-kv-a. generator for 13,200 volts, then throwing a switch which short-circuited two reactors connected in series across two of the terminals of the generator and measuring the voltage across the reactors, current in the reactors and the forces on the

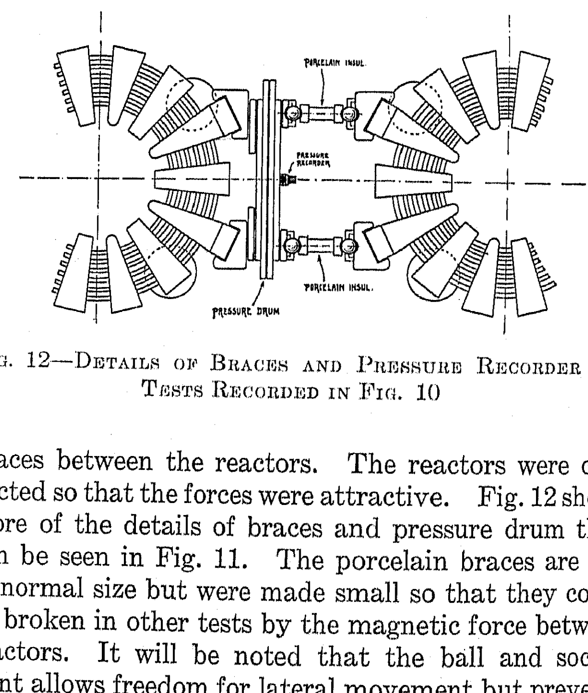

FIG. 12—DETAILS OF BRACES AND PRESSURE RECORDER FOR TESTS RECORDED IN FIG. 10

braces between the reactors. The reactors were connected so that the forces were attractive. Fig. 12 shows more of the details of braces and pressure drum than can be seen in Fig. 11. The porcelain braces are not of normal size but were made small so that they could be broken in other tests by the magnetic force between reactors. It will be noted that the ball and socket joint allows freedom for lateral movement but prevents lengthwise motion. The wooden structure around the reactors was intended to prevent the reactor falling when tests were made to determine the magnetic force necessary to break the porcelain supports and braces.

Curve a Fig. 10 gives the voltage across the reactors, Curve b gives the current in the reactors and Curve c gives the pressure exerted upon the braces by the magnetic force. Curve d was obtained from Curve b by multiplying the square of the current at any instant by the force constant. This curve thus gives the mag-

4. "Repulsion and Mutual Inductance of Reactance Coils with the Same Axis" by H. B. Dwight, *El. Journal* 1918, page 166.

5. "Some new Formulas for Reactance Coils" by H. B. Dwight, TRANS. A. I. E. E. 1919, page 1675.

netic force exerted by the reactor at any instant while Curve *c* gives the stress placed upon the braces by this force. Attention is called to the fact that the instrument which measured the pressure on the braces was not equipped to measure any tension that may have been placed upon them by the oscillation of the reactors.

It will be noted that while the first peak of pressure on the braces is almost as high as the first peak of the magnetic force, the second peak of pressure is higher than the highest peak of force. Thus, it is shown that not only is it not safe to use the average magnetic force in calculating the stresses on the braces but it is not even safe to use the peak force.

In order to determine the stresses on the supporting insulators with no braces, similar short-circuit tests were made with the supporting insulators bolted down to the frame work but with no braces between the reactors. The tests were started at low current and the current was gradually increased until the insulators were broken. The tests were repeated until sufficient insulators were broken to obtain a good average of the force required to break them. It was then desired to compare this with a steady force applied at the same points of the reactors. Therefore, a calibrated spring was placed between the reactors in such a manner that by screwing up bolts a known force tending to draw the reactors together was applied. The set-up for these tests is shown in Fig. 13. Sufficient insulators were broken so that a good average of the steady force required to break them was obtained. The result of these tests showed that the peak magnetic force (calculated from the observed current) was 2.5 times the steady force required to break the insulators and, therefore, indicates that the average magnetic force is more nearly the correct force to use in calculating the stresses on the supporting insulators when no braces are used.

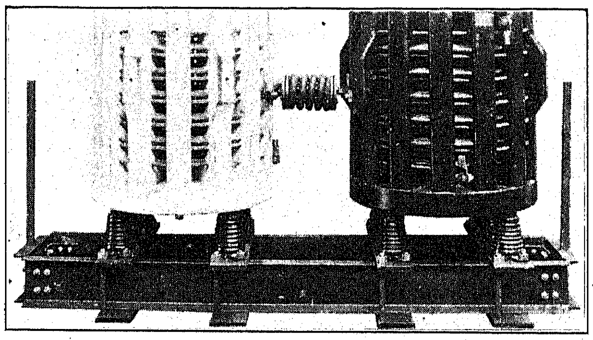

Fig. 13—Method of Applying a Steady Force between Reactors to Determine the Strength of the Supporting Insulators. The Spring was Calibrated and a Known Force Applied Tending to Draw the Reactors Together

The difference between these two sets of tests consists chiefly in the fact that the mechanical frequency of the reactors was changed. In the first set of tests the mechanical frequency was close enough to the electrical frequency so that partial resonance occurred and the pressure on the braces was greater than the peak force. In the second set of tests, the mechanical frequency was naturally very much reduced by the removal of the braces and the inertia of the reactor effectively smoothed out the sharp peaks of the magnetic force thereby materially decreasing the stress on the supporting insulators.

In a paper[6] presented at the Spring Convention last year by Doherty and Kierstead entitled "Short-Circuit Forces on Reactor Supports" the following statement was made. "If a reactor is not held rigidly against all motion, the maximum force on the holding device will depend in a decisive way upon the ratio of the frequency n of natural oscillation to the electrical frequency f." Although the investigation is not completed the results of these tests are given here because they are experimental proof of the accuracy of the conclusions reached in the paper mentioned.

The following data on the reactors tested are given for the benefit of those who may be interested in checking up their mathematical calculations of forces with these tests.

Weight of Reactors	1550 lb.
Force between reactors at one ampere	2×10^{-5} lb.
Force required to decrease distance between reactors 0.0625 in.	
(a) with braces assembled	20,000 lb.
(b) without braces	1100 lb.
Electrical frequency	25 cycles
Pressure between reactors with no magnetic force applied	0

Since the forces between reactors vary with their shape, dimensions, distance between centers and their short-circuit ampere turns, it follows that these forces vary tremendously with different reactors. Peak forces as high as 33,000 lb. have come to the authors' attention. In the average feeder reactor this force is not over 10,000 lbs.

Because of the great force placed upon reactors, both internal and external, they require a construction of very high mechanical strength.

V. Most Economical Losses

It is generally possible to reduce materially the losses in a given reactor by using larger conductors or more conductors in multiple. While this results in greater first cost, the ultimate cost may be less on account of the reduced operating expense. Fig. 14 indicates how the most economical value of losses may be reached. In order for the manufacturer to determine this value, the following data should be available.

1. Cost per kw. hour of supplying losses.
2. The root mean square current in the circuit for an entire year.
3. The investment charge in per cent.

From the first two items the total yearly cost of supplying the losses in a given reactor can be calcu-

6. See Journal A. I. E. E., August, 1923.

lated and the operating expense curve obtained. From the last item, the investment expense curve can be calculated. The total expense curve is obtained from the sum of the ordinates of the other two. The most economical reactor for the given case can be selected from this curve.

The eddy current losses in reactors may be made comparatively small by properly stranding the conductor and by the use of properly placed conductors in multiple. In good designs these losses are rarely over

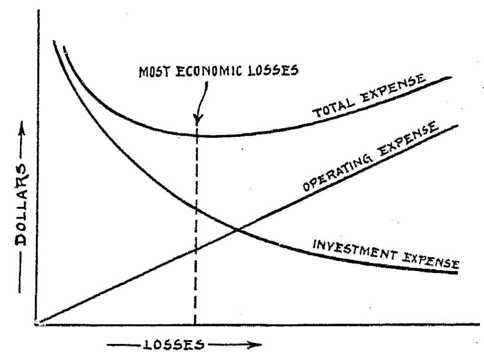

FIG. 14—CURVES SHOWING HOW THE MOST ECONOMICAL VALUE OF LOSSES CAN BE ARRIVED AT

20 per cent of the $I^2 R$ loss and usually not more than 10 per cent.

Structural steel work and other magnetic material if located too close to the reactors will cause additional losses. If a clearance from the winding of the reactor to these materials of one-half the diameter of the winding is maintained, the additional loss from this source is usually negligible.

If the magnetic field of a reactor links a loop of conducting material, as for instance, structural steel or pipe frame-work, current will be induced in the loop and will result in additional losses.

Since the power factor of reactors is very small, their losses cannot be accurately measured by ordinary wattmeters and especially designed low power-factor wattmeters are required.

Installation

The installation of reactors will be considered from the following points of view:

I—Arrangement
II—Compartments
III—Bracing and Securing in Place
IV—Support of the Leads
V—Ventilation

I. Arrangement

Reactors are usually arranged in a station as close to the circuit breakers as possible. A three-phase set of reactors may be placed side by side in the same compartment as is shown in Fig. 8, or one above the other as in Fig. 9. A much better practise is to place each reactor in a separate compartment as is shown in Fig. 15. The great advantage of this is that if anything should happen to cause the failure of a reactor in one phase, the arc will not be communicated to the other phases. This not only saves the other reactors from damage, but what is worth more, it gives to the system the protection afforded by the other two reactors, which is a very substantial percentage of the protection afforded by all three. When reactors are installed in separate compartments, the compartments are usually arranged side by side and often with three tiers of compartments so that the reactors of one circuit are placed one above the other. When reactors in different circuits are placed side by side the forces between them can be reduced 33 per cent if care is taken not to connect adjacent reactors to the same phase. One of the best arrangements for reactors is the so-called phase isolation arrangement where the reactors in different phases are placed on different floors of the station.

II. Compartments

There is a decided tendency to make the compartments for reactors too small for a liberally designed reactor. It should be realized that a reactor is a protective apparatus and conditions should be made as favorable as possible for it to give this protection.

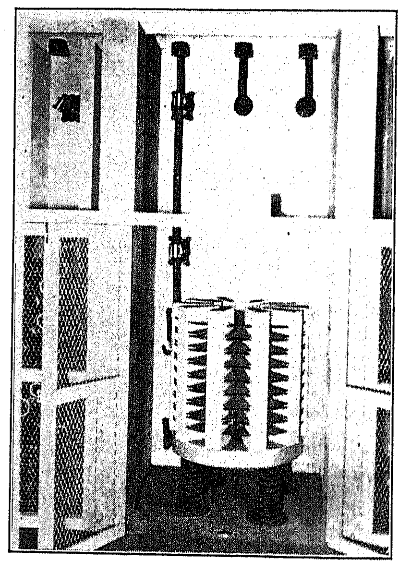

FIG. 15—REACTOR INSTALLED IN A SEPARATE COMPARTMENT

a. Electrical Clearance. In keeping with the principle that the insulation of reactors should be higher than that of other apparatus in the circuit, electrical clearances for reactors should be greater than those allowed other apparatus in the same station.

b. Magnetic Clearance. Clearances are required between reactors and magnetic materials for the reason that reactors have a strong stray magnetic field which will cause magnetic material to heat, and such heating which may or may not be objectionable from a tem-

perature standpoint will result in higher losses in the reactor.

Reinforcing rods in concrete walls and floors while magnetic, usually are not large enough to cause appreciable heating or extra losses. An idea of the size and shapes of iron bodies which are objectionable can be obtained from an investigation made many years ago. The reactor used in the test and the location of the magnetic material are shown in Fig. 16. The reactor was rated 25 cycles, 400 kv-a., 229 volts, 1750 amperes, and was carrying 25-cycle current of 1750 amperes when the tests were made.

A description of the materials of which temperatures were measured is as follows: The Part numbers refer to the numbers shown in Fig. 16.

Part 1—Iron plate ¼ in. by 12 in. by 10 ft.
Part 2—Galvanized iron wire screen 3 ft. by 10 ft.
Part 3—Fine Brass netting 3 ft. by 10 ft.
Part 4—Rolled Steel I-Beam 4 in. by 4¼ ft.
Parts 5, 6, and 7—Rolled steel angles 3 in. by 3 in. by ½ in. by 2 ft.

The following table gives the temperature rises of the above materials and their distances from the winding of the reactor:

TEMPERATURE RISES

Distance from the Reactor	Iron Plate Part 1			Iron Screen Part 2	Brass Screen Part 3
	Top	Middle	Bottom	Middle	Middle
4 in.	54 deg.	84 deg.	36 deg.	4 deg.	5 deg.
8 in.	36 deg.	61 deg.	24 deg.		
12 in.	29 deg.	51 deg.	20 deg.		

Distance from top of Winding	Steel I-Beam Part 4		Distance from Coil	Steel Angles Part No.		
	End	Middle		5	6	7*
14 in.	16 deg.	39 deg.	1 in.	11 deg.	33 deg.	20 deg.
18 in.	13 deg.	25 deg.	8 in.	9 deg.	13 deg.	7 deg.
22 in.	11 deg.	16 deg.	12 in.	9 deg.	9 deg.	5 deg.

*Part No. 7 was 20 in. below winding.

It will be noted that the highest observed temperature rise was on the iron plate and the next highest on the eye-beam. These parts should be expected to get the hottest for two reasons: first, they are the longest parts used, and second, they are each placed with their long dimensions approximately parallel to the path of the flux.

The effect of the magnetic field is enormously different for different ratings of reactors and these rises are not given as a guide to the magnitudes which might be expected with other reactors, but are given to show how the shape and position of the magnetic parts affect the heating.

Special care should be taken to avoid long magnetic bodies with their axes parallel to the field of the reactor. Experience has shown that the clearance between large magnetic parts and any point of the winding should not be less than one-half of the overall diameter of the winding.

Care should be taken to avoid any loops or turns formed by structural steel near a reactor where its field can link them. Plans for a recent installation contemplated reactors on a frame-work similar to that shown in Fig. 17. The legs of this frame-work were to be cast in concrete so that it would be a strong enough mechanically. With this arrangement, the flux from the reactor would pass through the square turn made by the top of the frame-work and cause a circulating current to flow. A temporary set-up was made as shown in Fig. 17 and tests gave a temperature rise of 125 deg.

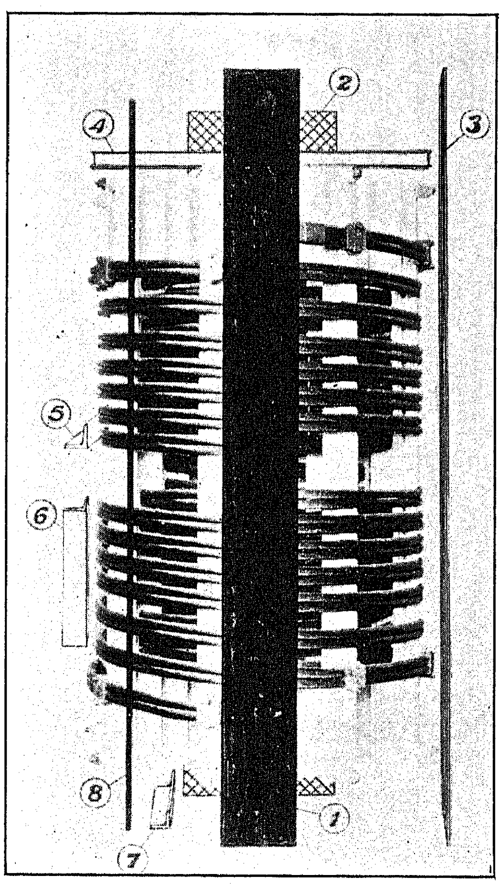

FIG. 16—DISPOSITION OF MATERIALS ABOUT CURRENT-LIMITING REACTOR IN HEATING INVESTIGATION

cent., but when one corner of the frame-work was insulated so as to open the turn, the temperature rise dropped to 56 deg. cent.

A rectangular copper bar loop as shown in Fig. 18 was placed over the structural steel supporting base with the steel loop complete as in the first test. The copper loop effectively shielded the steel structure and with the same current in the reactor a rise of 56 deg. cent. was obtained in the copper loop. The loss and heating of the short-circuited shielding loop can be controlled by varying its cross section. This method of reducing stray losses may prove useful in special cases.

III. Bracing and Securing in Place

Many reactors can be sufficiently secured against the magnetic force of the next adjacent reactor by bolting the supporting insulators to the floor and in such cases they should be rigidly bolted.

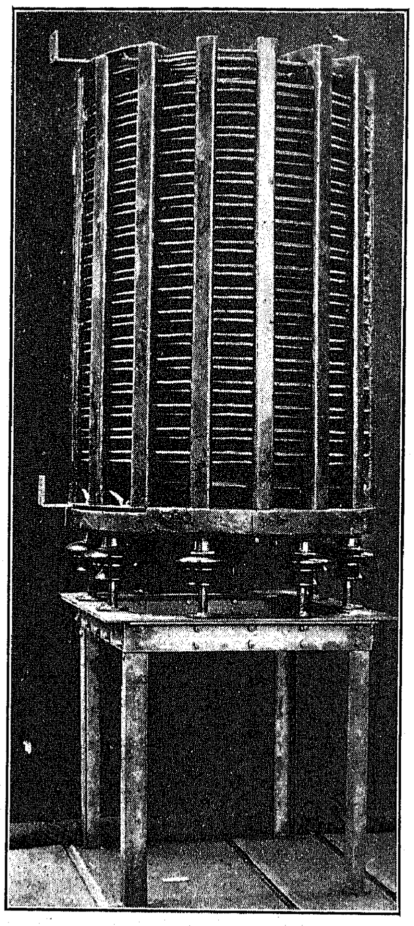

Fig. 17—Set-Up for Measuring Heating in Structural Steel Frame Work Caused by Magnetic Field of Reactor

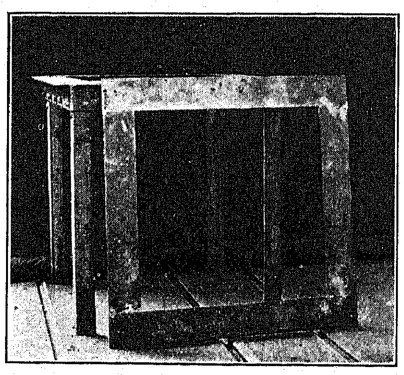

Fig. 18—Rectangular Turn of Bar Copper for Shielding Structural Steel Frame Work from Magnetic Field of Reactor Shown in Fig. 17

Where the magnetic force is large, braces must be used to brace the reactor. Since these braces are generally porcelain, which is many times stronger in compression than in tension, the reactors should be connected so that the magnetic force places compression on the insulators. In the first part of this paper, the direction of the forces for different connections was given. The supporting insulators should be bolted to the floor whether or not braces between reactors are used.

When reactors are placed in separate cells, the distance between adjacent reactors may be such that braces are required and in this case, the reactors must be braced against the cell walls or to the ceiling above, if the lifting force due to the reactor above is sufficient.

IV. Support of the Leads

The reactor exerts a force upon the leads which

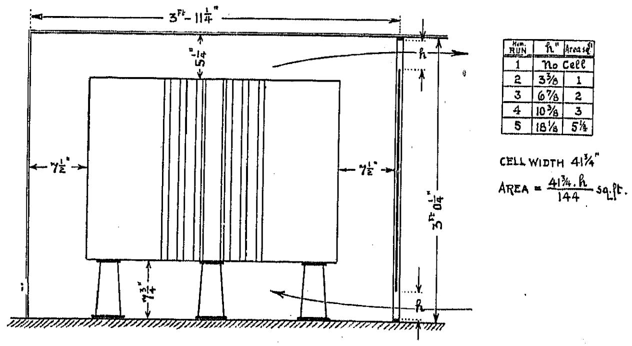

Fig. 19—Arrangement of Reactor in Cell for Tests to Determine Size of Ventilator Openings

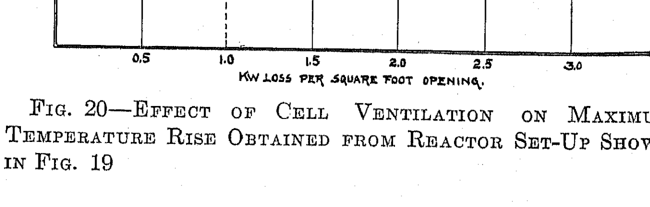

Fig. 20—Effect of Cell Ventilation on Maximum Temperature Rise Obtained from Reactor Set-Up Shown in Fig. 19

connect it in the circuit and the leads should be securely supported, but flexible enough to allow a slight movement. In general, the supports for the leads should be within a foot from the terminal and the direction of the lead as it approaches the reactor should be radial if possible. Potheads in general have been shown to be vulnerable points in a circuit and they should be located where a failure of a pothead will not communicate to the reactor.

V. Ventilation

Reactor cells should be adequately ventilated so as not to increase the temperature rise appreciably. Fig.

19 shows how tests were made with varying dimensions of openings at the top and bottom of one wall of the cell, and Fig. 20 gives the results of these tests. The conclusion reached is that the ventilation should be not less than one square foot of opening at both the top and bottom of the cell for each kilowatt of loss.

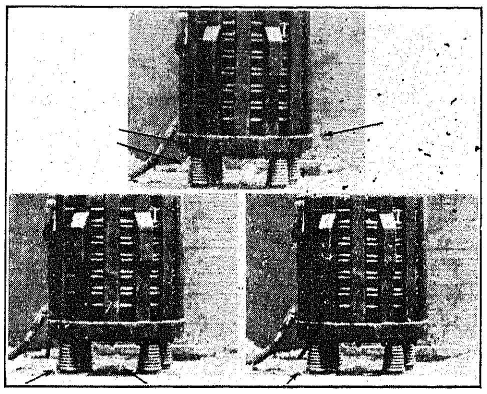

FIG. 21—REPRODUCTION OF SECTION OF MOTION PICTURE FILM SHOWING NAILS (PAINTED WHITE) BEING DRAWN TOWARD REACTOR BY ITS MAGNETIC FIELD. ARROWS INDICATE NAILS.

Operation

In general, reactors require extremely little attention in service but no effort should be spared to keep them free from foreign conducting material. Loose magnetic material, such as nails, etc., are particularly dangerous because the magnetic field of the reactor during short

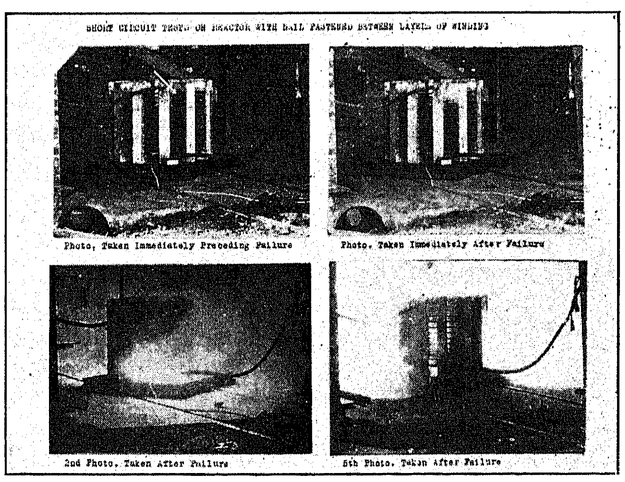

FIG. 22—REPRODUCTION OF SECTION OF MOTION PICTURE FILM SHOWING ARCING UNDER SHORT CIRCUIT CAUSED BY NAIL PLACED ACROSS TWO LAYERS OF TURNS. ARROW INDICATES NAIL. EXPOSURES 16 PER SECOND.

circuits will pick up nails at quite a distance from the reactor and draw them into the winding.

Short-circuit tests were made with nails strewn along the floor under a reactor but tied to strings so that they could not reach the winding. When the short circuit occurred the nails were drawn up from the floor and as close to the reactor winding as the strings would allow. Motion pictures were taken during this test and Fig. 21 gives enlargements of three of these pictures showing the nails (which are painted white) being drawn up from the floor.

A test was also made with a nail tied so as to bridge between the first and second layers of turns at the top. Fig. 22 shows enlargement of motion pictures taken during this test. The first picture was taken immediately before the failure, the second when the arc was just starting at the nail, and the third (one sixteenth second later) shows the reactor subjected to a complete flash-over. This indicates how rapidly an incipient arc may be spread magnetically throughout a reactor under short circuit.

These tests show that it is necessary not only to keep the reactor itself free from foreign conducting material but also to remove all unsecured magnetic material from its vicinity. This is about the only precaution that need be observed in the operation of current-limiting reactors.

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Discussion

For discussion of this paper see page 940.

Current-Limiting Reactors

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Review of the Subject,—*Current-limiting reactors are desirable only insofar as they are strictly reliable protective devices. In this paper general considerations of the factors affecting reliability are outlined, and some of the early weaknesses and the means taken to eliminate them are enumerated. Modern reactors are considered to be reliable and on large systems are considered practically indispensable.*

The thermal duty of a current-limiting reactor is a consideration affecting the reliability which has been the subject of considerable discussion. Various opinions have been expressed ranging from the idea that a reactor should fail first and open the circuit in the event of short circuit, to the idea that it should have enough thermal capacity to withstand short-circuit currents for long periods of time. A middle ground is suggested in this paper where recognition is given both to the protective function of the reactor and to the practical consideration of dimensions and cost.

GOOD voltage regulation was the aim of the electrical engineer in the early days when generating units and systems were small compared with those to which we are now accustomed. Low power factor was the great influence which operated against this ideal. Reactance was the element that produced low power factor. Consequently, reactance was a thing to be eliminated as far as possible.

When generating units and transmission systems became larger and more extensive, voltage regulation began to be of somewhat less importance. The heavy currents developed at times of short circuit became a matter of concern. Gradually reactance began to be recognized as the solution for preventing destructive short-circuit currents and the present extensive use of the current-limiting reactor is a natural accompaniment of the growth of the modern power system.

The reactor is a desirable thing only in so far as it is a strictly reliable protective device. Its cost and its energy losses are undesirable and it takes up valuable space. Moreover, the very magnetic field which is depended upon for it to function may be the cause of troubles in operation. Service experiences, however, in the past few years have enabled manufacturers to offer reactors that are entirely reliable and in large generating systems they are practically indispensable.

Early Installations

The first example of current-limiting reactors, or "choke coils," placed in the circuit to guard against the destructive effects of heavy short-circuit currents, was in the Cos Cob Station of the New York, New Haven & Hartford Railroad, as far as the author's recollection goes. These coils were built by the Westinghouse Electric & Mfg. Co. and were put into operation in 1908 to protect the generating units. Little or nothing was known at that time about the practical design of air-core reactors, while the reactance determination of generators was very well understood. As a result, these first coils were made with an iron core and with very large air gaps which again were similar in general form to the small air gaps of a generator. The punchings were made up with slots to receive the coils, which were very similar to those used in a generator armature. The iron part of the magnetic circuit was worked at a very low normal density, in order to keep the density in the core below the saturation point under short-circuit conditions. As a result, these reactors gave a practically straight line voltage characteristic from no-load current up to the short-circuit current. The general construction of the reactor is shown in Fig. 1.

The second installation of current-limiting reactors, as the author remembers it, was made by the Commonwealth Edison Co. of Chicago. These coils were built by the General Electric Company. No magnetic material was used in their construction. They were made up of bare stranded copper cable wound in cylindrical form on a concrete core with the layers spaced by means of treated wooden strips.

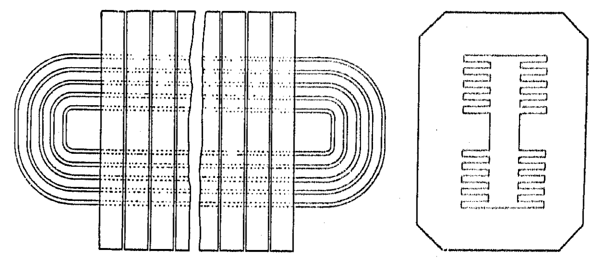

Fig. 1—Iron-Core Current-Limiting Reactor Used by the New York, New Haven & Hartford Railroad in the Cos Cob Station

General Considerations

When engineers first turned their attention to the problem of providing inductance for current-limiting purposes, it was realized that the reactor would be subjected to high electrical and mechanical stresses when short circuits occurred. It was appreciated that on account of its concentrated inductance, high-frequency disturbances would throw great voltage stresses on the reactor, especially on its end turns, but the real magnitude of these stresses was more or less unknown.

It was also recognized that the mechanical stresses

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in a current-limiting reactor would be very large, on account of the extremely large number of concentrated ampere turns under short-circuit conditions. Mechanical stresses vary with the square of the ampere turns and they depend upon the degree to which these ampere-turns are concentrated or broken up into groups. In a generator, the ampere-turns are distributed over the entire periphery of the armature. In a transformer they are broken up into a number of groups and in the primary and secondary groups these ampere-turns cancel each other to a very large extent, since the currents are in opposite directions in the high-tension and low-tension windings. In a current-limiting reactor, however, the current flows in only one direction and the ampere-turns are very highly concentrated, resulting in mechanical stresses of enormous magnitude.

These forces are in such a direction that the tendency is to make the coil enclose a greater total flux, that is, they tend to increase the diameter of the coil and shorten its axial length. The forces tending to increase the diameter produce a tensile stress in the conductors of the coil. The tendency to shorten the axial length creates a compressive stress on the spacers between the conductors and bending stresses in the conductors themselves.

The strong magnetic field that permeates the coil, due to the large number of turns in the reactor, would produce prohibitively large losses within the conductors if special precautions were not taken to guard against them. A solid conductor, for instance, would give rise to very great eddy current losses within it and these losses would multiply rapidly if the diameter of the conductor were to be increased. For these reasons, a finely stranded cable is the obvious conductor for a current-limiting reactor and the smaller this cable can be made practicably, the better is the conductor from the point of view of low eddy current loss.

Using a small cable requires that a number of them be employed in parallel for currents greater than the capacity of a single conductor. The danger of paralleling circuits in the presence of a strong magnetic field, varying in intensity at different points, will immediately be apparent to one having experience in such matters. In order to avoid unequal division of current or circulating current in the cables, the Westinghouse Company uses a special method of winding for paralleled cables, whereby each conductor passes through a series of positions with respect to the magnetic field which are exactly duplicated for every other conductor in parallel with it. This is accomplished by transpositions in the slots when winding.

As an illustration of this method of winding with two cables in parallel, one cable is carried around one-half the circumference of the inner row of slots in the first layer, and at that point transposed to the next row of slots. The second cable is started at this point in the inner row of slots and is carried around through the second half of the circumference. Consequently, in the circumference of the inner row of slots the two cables pass through positions in the structure which are identical with respect to the magnetic field and they are exposed to the same integrated magnetic conditions. This method of easy transpositions from one row of slots to the next is carried out through the entire structure and the condition of equal resistances and equal inductances in the parallel conductors is accomplished. Unequal division of current in the paralleled paths is in this way effectively eliminated. This method of winding may be employed for any reasonable number of cables by making the number of columns of cleats a multiple of the number of cables used.

The strong magnetic field also causes magnetic forces between adjacent conductors in the winding, especially those close to the top and bottom of the coil where the accumulative effects are greatest. These forces are guarded against by placing the spacer supports between conductors close enough together to keep the maximum forces within the strength of the conductors. Shortening the length of the span between supports must not be carried too far, however, for the greater the number of supports, the less is the radiating surface of the conductors and the higher the operating temperature. The size of the conductors and the arrangement of the structure used must be selected to give full consideration to both of these conflicting factors.

A fairly high operating temperature is permissible, considering the fireproof construction of the modern current-limiting reactor. However, the operating temperature at normal current is often of minor importance compared to the temperature reached during short-circuit conditions. When a very heavy current flows for a short time, the copper must store up the vastly increased energy loss and it appears as a rapidly increasing temperature. The structure in supporting the cables must for this reason provide for the expansion of the conductors at the time of short circuit.

The thermal duty of a reactor can best be appreciated by considering a specific case, for instance, a coil having a reactance of three per cent under normal conditions. Under short-circuit conditions, assuming no other reactance in the system, this coil would carry $33\frac{1}{3}$ times its normal current. The thermal results, like the mechanical stresses, vary with the square of this current, and the temperature will immediately start toward the point corresponding to something over 1000 times the normal heat units. While this rapid increase of temperature lasts over only a comparatively short period, the temperature reached at the end of the period is often the determining point in designing a reactor from the point of view of heating. The 5 per cent reactor would be designed with a higher normal operating temperature than the three per cent coil, for the heat units developed in such a coil at the time of short circuit, again assuming no other reactance

in the system, would be 400 times normal rather than 1000 times. Consequently, it is reasonable to neglect the normal operating temperature in writing specifications for current-limiting reactors, except in cases of reactors for a high value of inductance where the short-circuit current is not a great many times the normal current. The designer's chief interest in the normal operating temperature relates usually to its effects on the temperatures reached under short-circuit conditions.

There are various viewpoints among engineers as to the length of time during which a current-limiting reactor should carry short-circuit current without failure due to excessive temperature.

The extreme point of view in one direction would be that the reactor should be the weak link in the chain and, like a fuse, should fail and open the circuit before the other apparatus can be affected. Most engineers will agree that the premeditated interruption of a heavy alternating current by means of the failure of a highly inductive device like a reactor would be an undesirable thing.

An extreme view in the other direction is that the reactor should have thermal capacity in such abundance that it can withstand its short-circuit current for a period as long as ten minutes. The point of view for such an opinion is based on disturbances of record which have lasted even longer than ten minutes before the source of the trouble has been found. Reactors designed with such thermal capacity would be so expensive and would occupy so much space that they probably would not be found practicable.

Between these two viewpoints is a middle ground where recognition can be given both to the protective function of the reactor and to the practical considerations of dimensions and cost. The maximum current that can flow through a reactor is that which would obtain if full line voltage were applied across its terminals with no other impedance in the circuit. If a reactor were required to withstand such a current for a period of five seconds, it would represent a fair compromise between the cost and dimensions of a reactor having greater thermal capacity and the protection which such a period of time would provide for the system.

The First Westinghouse Reactors

The reactors illustrated in Fig. 2 are typical of the first of the air-core current-limiting reactors built by the Westinghouse Electric & Mfg. Co. in 1913. They were wound in the form of a cylindrical coil with pancake layers of bare stranded copper cable laid in grooved spacers of treated hard wood. The columns formed by these spacers were bolted together with brass tie rods. Insulating supports or feet at the bottom and a terminal board at the top completed the structure. Ordinary transformer practise was followed in designing the terminal board, except that greater spacing was provided between the terminals than used in transformer practise. The conductors were soldered into the terminals which were clamped between nuts at the lower end of the studs shown in the illustration. Fig. 3 shows the coil partly wound.

It is clear now that the type of construction used in these original designs was fully strong enough to withstand the mechanical shocks of short circuit. Rather elaborate factory tests were made shortly after the first coils were built in an effort to find out whether the limit of mechanical strength of these coils could be reached.[1] A 15,000-kv-a. generator was short-circuited a number of times with increasing excitations with a single reactor in circuit and again with a bank of three reactors, and tests made to the limit of the generator excitation failed to find any weaknesses in the design. Large short-circuit currents were produced during these tests and the magnetic forces within the reactor were correspondingly high. For determining the fitness of the design for service on a very large system, such tests are necessarily inconclusive, but service records since that time have abundantly

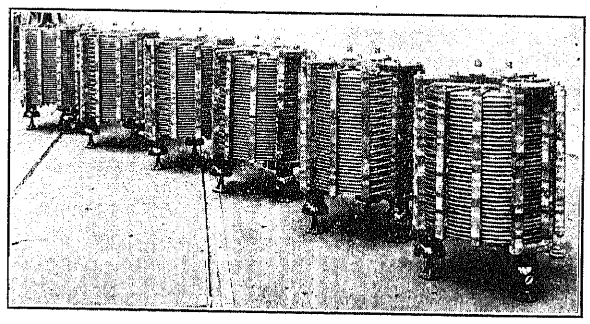

Fig. 2—The First Westinghouse Air-Core Current-Limiting Reactors

proven that the design is adequate, at least for short-circuit currents as we know them up to the present time. The same general arrangement of windings and structure has been used ever since by the Westinghouse Company.

Troubles that have occurred in service have been due mainly to an under-estimate of the excessively high transient voltages that are set up by disturbances in the system and of their effects when concentrated upon the reactor. It was very soon found that these voltages must have been as high as 50,000 volts across the reactor when connected in a 11,000-volt circuit, judging from the distances over which they jumped. This voltage manifested itself principally across the end turns of the coil and across the terminals when they were placed together at one end of the reactor, as in the original coils, Fig. 2.

Another trouble early encountered was found to be due to the metallic tie rods used to clamp the columns of spacers. Surge voltages found a path from con-

1. *Electrical Journal*, Vol. 11, April 1914, pages 188 to 207.

ductors at one end to conductors at the other end across the spacers and through these metallic rods, thus short-circuiting the coil.

A few cases of mechanical weakness in the insulating feet and of flashover to ground of insulators that were inadequate also occurred. Trouble of a different kind developed in one or two instances due to soldered

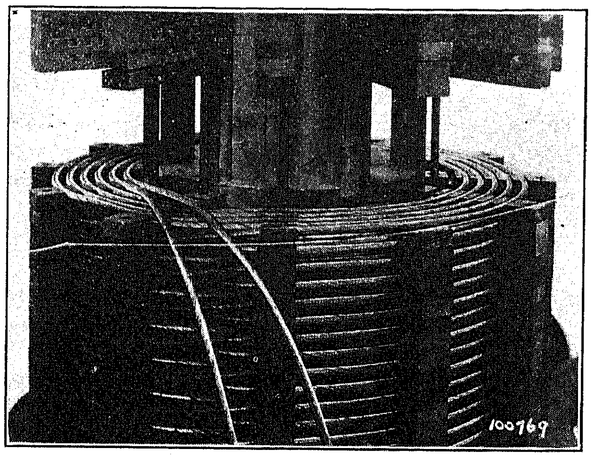

FIG. 3—SHOWING METHOD OF WINDING THE REACTORS OF FIG. 2

terminals which failed and opened up the circuit within the reactor, causing considerable burning due to arcing.

One by one all of these weaknesses have been eliminated. Through the troubles which have occurred in service, the modern reactor has been developed to the point where it is a protective device having the me-

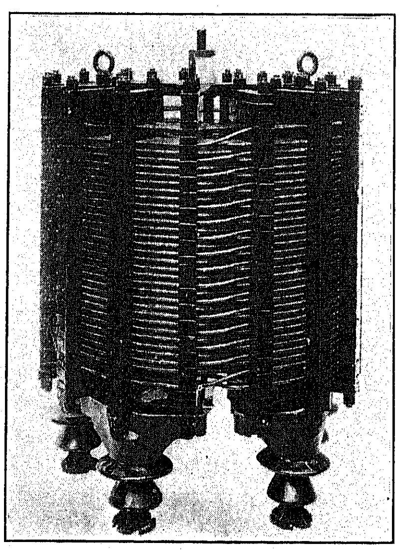

FIG. 4—A MODERN WESTINGHOUSE CURRENT-LIMITING REACTOR

chanical sturdiness and the electrical strength to carry out its function successfully and to justify its use.

SUMMARY OF ELEMENTS OF DESIGN OF THE MODERN WESTINGHOUSE REACTOR

The principal elements that contribute to the strength and efficiency of the modern reactor may be briefly summarized as follows:

1. Finely stranded cables of small diameter keep to a minimum the stray losses within the conductors due to

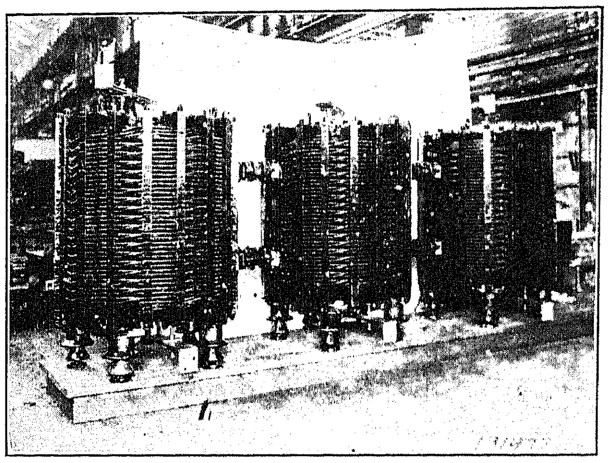

FIG. 5—SINGLE-PHASE REACTORS IN A THREE-PHASE BANK, BRACED TO WITHSTAND FORCES OF ATTRACTION BETWEEN UNITS

the magnetic field. Paralleled sections of windings are used where necessary to carry the current and retain this feature. By reason of the special method of winding, these paralleled lengths of cables have the same resistances and the same inductances, and extra

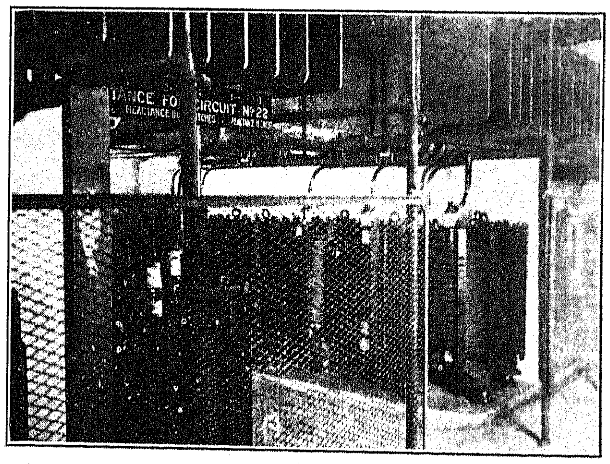

FIG. 6—AN INSTALLATION OF REACTORS

losses, due to unequal division of current, are eliminated. As a final finish, the complete reactor is dipped and baked a number of times in a fire-resisting enamel. This enamel permeates the strands of the cable and helps further to reduce the stray losses. It also serves to stiffen the conductors and give them greater mechanical strength.

2. Extra spacing of end turns eliminates danger of flashover, due to high-frequency transient voltages at times of disturbances on the system.

3. Fire proof composition spacers molded under

great pressure form the supporting structure. The slots into which the cables are laid in the winding operation provide sufficient clearance to permit expansion of the copper, due to the rapid heating at the time of short circuit. Wooden rods of treated straight-grained hickory are used to clamp the columns of spacers together. Being entirely enclosed in the fire-proof spacers, these wooden rods do not form an inflammable element.

4. One terminal is placed at one end of the coil and the other at the opposite end to eliminate the danger of flashing across terminals.

5. Normal current densities are set low enough to keep temperatures within safe limits when the sudden heating, due to heavy short-circuit currents, develops.

6. Solderless clamped connectors are used to connect the ends of the winding to the terminals. Contact nuts, made with a lug having a radial saw-cut through it, are arranged so that they can be bolted after being tightened. Once set, they are permanently locked in position. Brass tubes placed over the leads between the winding and the point of connection to the terminal studs give the necessary stiffness to these leads to eliminate troubles due to mechanical forces exerted on them when heavy short-circuit currents flow.

7. Extra heavy pin-type or pedestal-type insulating supports give the necessary mechanical and electrical strength to eliminate troubles due to mechanical breakage or to flashover to ground.

Figs. 4, 5 and 6 illustrate examples of modern Westinghouse current-limiting reactors.

Discussion

For discussion of this paper see page 940.

Theory of D-C. Excited Iron-Core Reactors and Regulators

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***Review of the Subject.**—In the usual electrical machinery the magnetic saturation characteristic of iron is a handicap and in a-c. circuits gives rise to such undesirable characteristics as wave distortion and variable reactance, decreasing with increasing load. Furthermore, these characteristics are inherent in the material and can not be controlled by design to suit individual requirements or changing load conditions. Control of these characteristics can, however, be accomplished if the mean magnetic density in the core be controlled by means of a suitable d-c. excitation, in which event, the saturation characteristics of the iron can be put to some useful applications.*

(1) APPLICATION AS A FREQUENCY MULTIPLIER

The second and third harmonics of magnetization have been utilized in radio to multiply the supply frequency. Using a sufficient number of stages, theoretically any desired radio frequency may be obtained from commercial frequencies. However, the efficiency and regulation of the scheme are very poor, and this application has therefore become almost obsolete by the advent of the vacuum tube oscillator and the high-frequency alternator. When used at all, the number of stages would be kept to a minimum.

(2) APPLICATION AS A SERIES CURRENT-LIMITING REACTOR

The ordinary iron-core reactor is saturated at increasing values of load, and thus having a much lower reactance at short circuit than at normal load, is undesirable as a current-limiting reactor. However, its characteristic is inverted over a wide and useful portion of the curve when excited by means of direct current, so that it shows an increasing reactance with increasing load. By suitable design, the maximum reactance may be made to coincide with the short-circuit condition without departing from the usual economical values of flux and current densities and from the economical ratios of core and copper. The ratio of the short-circuit reactance to normal-load reactance may be as high as three to one. This characteristic of the d-c. excited reactor does not mean that the d-c. excitation increases the permeability of the core and the reactance of the winding, but just this, that the fixed d-c. excitation lowers the effective permeability and reactance at normal loads many times more than at short-circuit, and thus the short-circuit reactance becomes much higher than the normal reactance. This highly desirable rising-reactance characteristic of the d-c. excited reactor is at the expense of higher losses and cost, and therefore, unless there is a very particular need for the rising-reactance characteristic, the use of this type of a reactor is not recommended.

(3) APPLICATION AS A FEEDER VOLTAGE REGULATOR

A pair of auto transformers of suitable design (equipped with suitable windings for d-c. excitation) may be utilized as feeder-voltage regulators. A number of such sets have been built and tested and some have been put into service. This type of a voltage regulator has a number of advantages over the other types in that, (a) it has no moving parts, (b) can be designed for any voltage on account of its transformer construction, (c) is four or five times or more as fast in operation as some other types and, (d) can be built mechanically much stronger to withstand short-circuit forces. Its disadvantages are high losses and higher cost in all instances where the induction regulator can be designed for the line voltage without transformers. This type of a voltage regulator is therefore desirable only when high speed of operation is essential or when the circuit voltage is very high.

(4) APPLICATION AS A SHUNT-REACTOR FOR POWER-FACTOR CONTROL

A d-c. excited reactor may be used also as a shunt-reactor constituting a variable reactive load on the lines to control power-factor by varying the d-c. excitation. A possible application is to the neutralization of charging current of high voltage transmission lines and control of line regulation. In this application the wave-shape of the reactor current becomes of importance. Many oscillograms taken in various three-phase connections are given, which seem to indicate that although very pure sine wave of current can not be obtained economically, yet the harmonics may be reduced to practically harmless proportions.

RATIO OF CONTROLLING AND CONTROLLED KV-A.

The control of the output of a reactor may be by alternating current as well as by direct current, the frequency of the controlling current being independent of the frequency of the controlled current. To a first approximation, and ignoring the copper losses, the ratio of the kv-a. of the controlling current to the kv-a. of the controlled current is that of the ratio of the respective frequencies. This principle has been taken advantage of in Alexanderson's "Magnetic Amplifier" in the modulator circuit of his system of radio transmission. In d-c. control, the necessary control kv-a. is just the copper loss approximately corresponding to the controlled kv-a.

Introduction

THE superposition of d-c. excitation on an iron-core static alternating current apparatus, such as a reactor, or transformer or autotransformer, modifies its characteristics very profoundly and leads to new problems and a variety of applications. Some of the more important applications are[1]:

1. Frequency converter, or high-frequency generator, suitable for use in radio.
2. Short-circuit current-limiting reactor having inherently low normal reactance and inherently high short-circuit reactance.
3. Feeder voltage regulator with no moving parts, for central station and laboratory use.
4. Variable series reactor with no moving parts, with d-c. control of its reactance to regulate load current or voltage.
5. Variable shunt-reactor with no moving parts, with d-c. control of its a-c. exciting current, constituting an adjustable reactive load, for use, for instance, for the neutralization of the charging current of high-voltage transmission lines and also thereby for the control of their regulation.

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1. See also the companion paper by Mr. D. K. Blake, on "Applications of Saturated Core Reactors and Regulators."

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

General Theory

Three characteristics of iron-core reactors (without d-c.) are commonly known: *viz.*

(a) The reactance of an iron-core reactor depends on the core-density on account of the variable permeability of iron.

(b) Due to this same cause, the current taken by such a reactor at sine wave voltage (or the voltage consumed at sine wave current) is distorted; and

(c) Such a reactor is entirely unsuited for short-circuit protection because, due to saturation, its short-circuit reactance is much lower than its normal reactance, while the requirements of a desirable protective reactor are that if possible it should have a low reactance at normal loads and a high reactance at short-circuit.

From the first characteristic mentioned above, it follows that if the mean density of the core be controlled by the superposition of a d-c. excitation, the reactance of the apparatus can be controlled thereby. This is the basis of the applications 3, 4 and 5 mentioned above.

From the second characteristic mentioned above, it follows that by biasing the core density by means of d-c. excitation, more wave distortion may result. This feature is taken advantage of in the application for the generation of high frequency for radio. This application, however, has been rendered obsolete by the advent of the vacuum-tube oscillator and will, therefore, not be discussed in this paper. In other applications, wave distortion and harmonics are objectionable and must be eliminated or minimized. While at first sight it might appear that wave distortion will be so bad that no applications other than the production of harmonics could be practicable, yet experience shows the reverse to be true, so that while wave distortion can be minimized so as not to be a limitation to most applications, it can not be accentuated and loaded for the production of harmonics to as great a degree as one would expect.

The characteristics that have been indicated so far do not as yet give any clue as to the suitability of a d-c-a-c. excited reactor for short-circuit protection. This application results from the fact that the d-c. excitation accentuates very markedly the lower bend of the magnetization curve of iron, as will be discussed in detail under the heading of "Volt-Ampere Characteristics of D-C-A-C. Excited Core."

Schemes for the Application of D-C. Excitation

In applying d-c. excitation to an a-c. apparatus, provision must be made for protection against three possible disturbances, *viz*:

1. The d-c. supply circuit must be protected from induced alternating voltages, and, if possible, also from alternating currents.

2. For all applications, except for the generation of high frequency for radio, the line currents and voltages should be symmetrical and of as good shape as possible.

Both of these conditions can be satisfied by using two subunits balanced against each other as shown in Figs. 1, 2, 3 and 4 instead of using a single-core single-unit. In these figures the arrows show relative directions of a-c. and d-c. fluxes, a consideration of which will show that normally no a-c. voltage will appear between the d-c. terminals. For instance, considering Fig. 1, it will be evident that the a-c. voltages induced in the d-c.

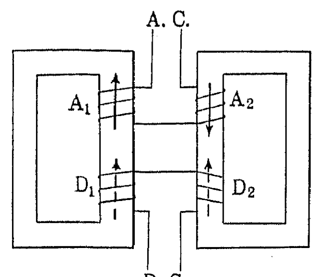

Fig. 1—Connection of Two Single-Phase Transformers as a Reactor Suitable for D-C. Excitation. Solid Arrows Represent A-C. Flux, Dotted Arrows, D-C. Flux.

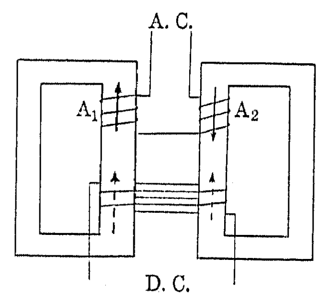

Fig. 2—An Improvement over the Arrangement Shown in Fig. 1. The D-C. Coil Surrounds both Units at Once, and Therefore no A-C. Voltage of Fundamental Frequency can Build up in it. Solid Arrows Represent A-C. Flux, Dotted Arrows D-C. Flux

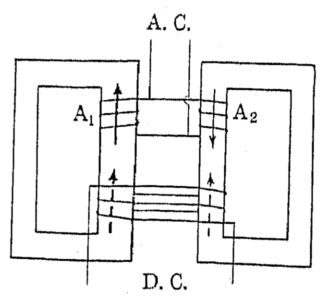

Fig. 3—An Improvement over the Arrangements Shown in Figs. 1 and 2. A Second Harmonic Current Circulates in the Parallel Branches of the A-C. Circuit, and no Appreciable Second Harmonic Voltage Appears Anywhere

coils D_1 and D_2 will neutralize each other, and no -c. voltage of normal frequency will appear between the d-c. line terminals. However, in this case there will be an a-c. voltage to ground from these terminals, and this high potential stress will be impressed on the d-c. supply system. Usually the d-c. winding has many times the turns of the a-c. windings, and this voltage would amount to a great deal. This disadvantage may be

practically obviated by grounding the d-c. supply system or one of the terminals, but, even then, the d-c. winding will require insulation corresponding to its maximum induced voltage. The difficulty is completely obviated by making the d-c. winding to enclose the two cores at once as shown in Fig. 2. Here the induced voltage is neutralized in each turn and therefore no large a-c. voltage can appear in any part of the d-c. circuit, regardless of the number of turns of the d-c. coil.

Although the normal frequency voltages induced in the d-c. winding neutralize each other in every turn in the design of Fig. 2, yet, whenever there is wave distortion (and there always is some wave distortion) the even harmonics of the induced voltages do not cancel each other in each turn of the d-c. coil but add to each other, and if the impedance of the d-c. supply circuit is high enough not to short-circuit this voltage (as for instance in the case in which the current of the d-c. coil represents full load to the d-c. generator), destructive even harmonic voltages may appear across the d-c. circuit. This difficulty is practically completely eliminated by connecting the two a-c. coils A_1 and A_2 in parallel as shown in Figs. 3 and 4, instead of in series as in Fig. 2. The even harmonics are then short-circuited and circulated in the a-c. coils without appearing anywhere in the outside circuits. This is a desirable feature, not only on account of the elimination of avoidable voltage stresses, but also on account of the additional advantages that by this means, (a), the wave shapes of currents and voltages in the external lines are improved, (b), the effectiveness of the d-c. excitation in controlling the a-c. reactance is increased, and (c), the time constant of the apparatus is reduced and its speed of response to changes in the controlling d-c. excitation is increased.

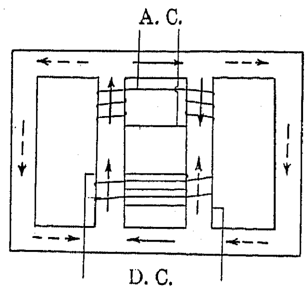

FIG. 4—SHOWING THE CORE CONSTRUCTION WHICH IS PREFERRED AS ELIMINATING THE STRAYING OF A-C. FLUX AND REDUCING LOSSES

VOLT-AMPERE CHARACTERISTICS

The volt-ampere characteristics of a device like that of Fig. 3 is shown in Figs. 5 and 6, the former taken with sine-wave voltage, the latter with sine-wave current. Before discussing their differences, we might discuss the features common to both, as in general they are alike. A number of characteristics may be observed as follows:

1. By applying sufficient d-c. excitation, the a-c. current taken by the device at a given voltage (or the voltage consumed at a given current) may be controlled over a very wide range. The lowest limit to this voltage consumed, or the highest limit to the current taken, is fixed by the impedance which the device would have without the iron core. That is, at increasing relative values of d-c. excitation, the volt-ampere characteristics of the device is as though it had no iron core.

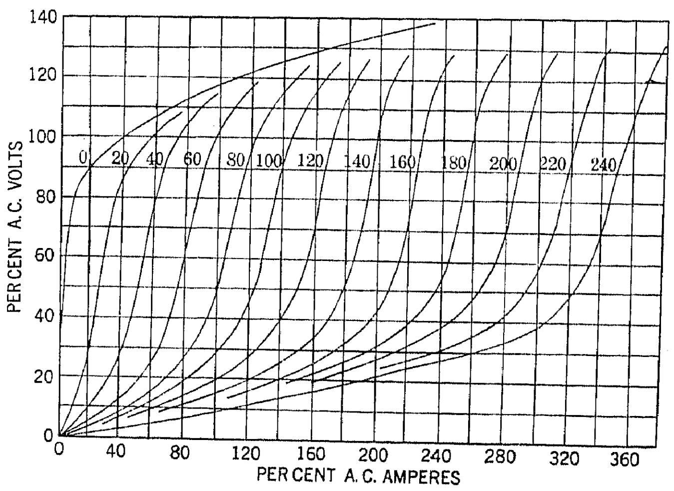

FIG. 5—VOLT-AMPERE CHARACTERISTIC CURVES OF A CORE WITH SIMULTANEOUS D-C. AND A-C. EXCITATION. NUMBERS ON THE CURVES REFER TO THE D-C. AMPERES. TURN RATIO OF D-C. AND A-C. WINDINGS 1:1. TESTS MADE WITH APPROXIMATELY SINE-WAVE VOLTAGE

2. There is an economic limit to the value of d-c. excitation and range of control, because beyond a certain zone the addition of more d-c. excitation does not produce a proportionate change in the a-c. current or voltage. For instance, at 100 amperes a-c., the application of 100 amperes d-c. cuts down the voltage from 118 to 29. If the d-c. excitation be doubled, the

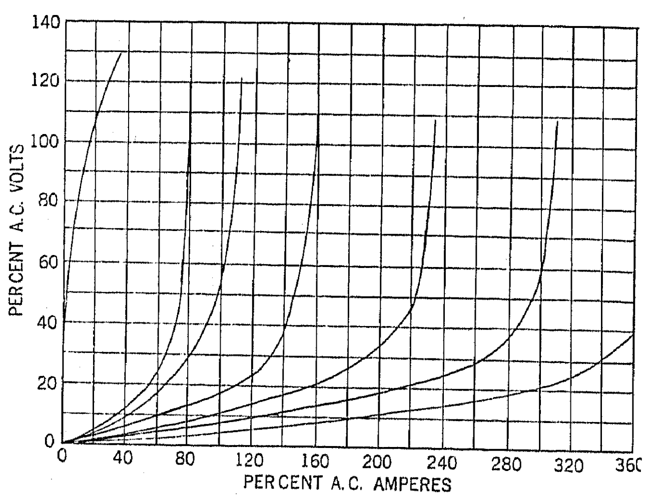

FIG. 6—SIMILAR TO FIG. 5, EXCEPT THAT THE TESTS WERE MADE WITH APPROXIMATELY SINE-WAVE CURRENT. NOTE THE GREATER STEEPNESS OF THESE CURVES

voltage is reduced to 12; if it be doubled again, the voltage is reduced to about 7.

3. The volt-ampere curve for combined d-c-a-c. excitation has two bends—one at the lower densities, the other at the higher densities. The volt-ampere curve for pure a-c. has also two such bends, but just because the lower bend occurs at extremely low core densities and small values of current, it does not show in the usual curves drawn for the working range of currents and voltages. If the initial portion of the a-c. volt-ampere curve for zero d-c. is drawn to a large scale, it looks as shown in Fig. 7, exposing the lower bend.

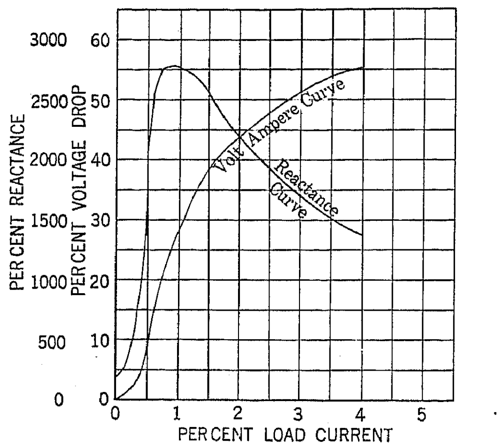

Fig. 7—These Curves show that on a Certain Commercial Size of an Economical Transformer or Reactor Design, the Useful Range of the Core Reactance Corresponds to a Current which is Less than 1 Per Cent of the Rated Capacity of the Winding

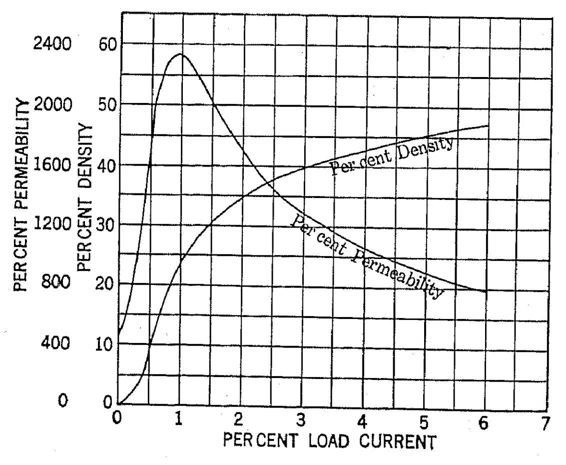

Fig. 8—Similar to Fig. 7 Except that the Characteristics are Given in Terms of Permeability and Core Density Instead of Reactance and Voltage

This lower bend is not merely a characteristic of the a-c. curves, but also is characteristic of the d-c. magnetization curve of iron shown in Fig. 8. In the light of these remarks, and by again referring to Figs. 5 and 6, it will be evident that the addition of d-c. excitation does not introduce a new bend to the curve but shifts the two bends of the a-c. volt-ampere curve far towards the higher values of current and voltage, and this fact makes it possible to make commercial use of the lower bend as in an automatic protective reactor having low inherent normal reactance and high inherent short-circuit reactance.

That the volt-ampere curve with superposed d-c. must have two bends or knees may also be seen with the aid of the following theoretical considerations.

Since the d-c. excitation is kept constant (though of different value for different curves) and only the a-c. current and voltage are varied in these volt-ampere curves, it will be evident that at the initial parts of the curves (near the origin) the d-c. current will be many times the a-c. current, and the reactance of the unit will be completely determined by the d-c. density in the core. The initial part of the volt-ampere curve is therefore straight and has a slope very much smaller than that of the zero-d-c. volt ampere curve. That is, the reactance or voltage drop at this point of the curve is very low as compared with the zero-d-c. curve. This fixes the general character of the initial end of the volt-ampere curve.

Considering the final end of the volt-ampere curve, it will be evident that as we go higher and higher in the a-c. flux density and current, the fixed d-c. excitation will be a smaller and smaller percentage of the total current and will therefore less and less influence the a-c. volt-ampere characteristic. That is, the final end of the volt-ampere curve with d-c. excitation will approach asymptotically the volt-ampere curve without direct-current.

Having thus determined the character of the initial and final parts of the curve, it will be evident that the smoothest or simplest curve that will join these two parts will require two bends or knees.

The place where the mean value of the a-c. current equals the value of the d-c. current would be approximately the inflexion point of the curve, that is, below this point the d-c. will be dominant and the curve will be convex downward; above this point the a-c. will be dominant and the curve will be convex upward.

Examining Fig. 5, it is found that this is approximately true. Assuming sine wave a-c., the inflexion point corresponds to an a-c. of which the mean value is only ten to fifteen per cent higher than the d-c. This is a good qualitative check, as the analysis given above is not intended to be anything but qualitative on account of the varying permeability of the core along the cycle.

Meaning of the Two Bends of the Volt-Ampere Curve

The meaning of the two bends in the d-c. saturation curve is that the iron core has its highest permeability between the two bends,—at the inflexion point of the curve. Above the upper bend and below the lower

bend the permeability of the core is lower. In the a-c. volt-ampere characteristic, the meaning of the two bends is that the reactor has its highest reactance in the zone between these two bends and that above the upper bend and below the lower bend its reactance is lower.

To be more precise in the discussion of permeability (or permeance) and reactance, we might make the observation here that this matter may be considered in two different ways, giving somewhat different results. Thus, if we consider the permeability or reactance as the slope of the curve, it corresponds to that characteristic which is effective for small changes of current at given densities (mathematically proportional to $d\,B/d\,H$, or $d\,\phi/d\,H$, or $d\,E/d\,I$). This view is useful for a number of applications. The author does not know a good name for it, but it has been sometimes called "instantaneous" permeability or reactance, and could probably be called more appropriately "differential" permeability or reactance in contrast with the second point of view which makes the permeability or reactance the ratio of the total B or E to the total H or I, and which may therefore be called the *mean* or integral permeability or reactance. The maximum integral permeability or reactance occurs at the upper bend, not midway between the two bends, as will be shown below. This second definition is the one that is of more importance in the applications discussed in this paper.

Theory of the Application of the D-C-A-C. Excited Reactor as a Protective Impedance

The most desirable characteristic of a protective reactor is that it should have a low inherent reactance for the normal load current and a high inherent reactance for short-circuit. An iron-core reactor (without air gap), that can be commercially or economically designed, fails to meet this condition, because increasing currents saturate the core and reduce the reactance. This statement has to be qualified as applying to "commercial or economical" designs, because if the core be operated normally at densities below the lower bend, the desired characteristics may be secured, although at a very prohibitive cost.

The a-c. volt-ampere curve (with zero d-c.) of Fig. 5 is plotted to a large scale in Fig. 7. In order that the economic significance of the currents and voltages may be easily grasped, the currents are given in per cent of what would correspond to the normal current of a commercially economical transformer of the same proportions as the reactor under consideration and the voltages are given in per cent of the normal rated voltage of the equivalent transformer. The normal density or voltage and the normal current are, of course, somewhat arbitrarily fixed, but inasmuch as these curves are to be used for comparative purposes and not for their absolute values, a very great deal is gained in intelligibility by specifying per cent voltage and per cent current representative of economical practise even though somewhat arbitrary. From the volt-ampere curve, a reactance curve has also been calculated and plotted against the per cent current.

Referring to the reactance curve of Fig. 7, the following characteristics will be observed:

The maximum reactance of the unit, which is desired to occur at short circuit, occurs at about 1 per cent of its normal rated current. To be used as a protective reactor, then, the unit must be operated normally at a small fraction of one per cent capacity so that the short circuit will be about 1 per cent load to it. This is indicative of the degree of its economic merit. The larger the capacity of the unit, the smaller will be the per cent load at which its maximum reactance occurs if no d-c. excitation is used.

The addition of direct current, however, shifts the two bends of the volt-ampere curve and the maximum of the reactance curve towards higher values of current and voltage. By using sufficient d-c. excitation, the

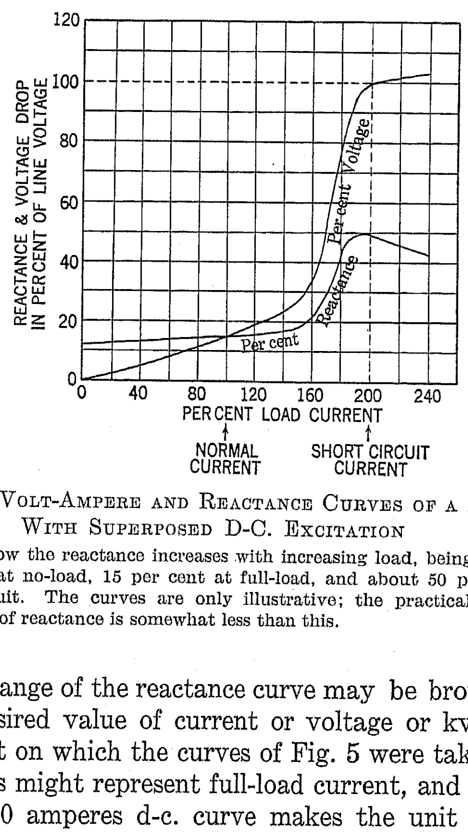

Fig. 9—Volt-Ampere and Reactance Curves of a Reactor With Superposed D-C. Excitation

Note how the reactance increases with increasing load, being about 12 per cent at no-load, 15 per cent at full-load, and about 50 per cent at short-circuit. The curves are only illustrative; the practical range of variation of reactance is somewhat less than this.

useful range of the reactance curve may be brought to any desired value of current or voltage or kv-a. In the unit on which the curves of Fig. 5 were taken, 200 amperes might represent full-load current, and we find that 240 amperes d-c. curve makes the unit a most desirable protective reactor. This curve is plotted in Fig. 9 in terms of per cent load current, per cent voltage drop and per cent reactance. We find that at normal load current the device consumes about 15 per cent voltage, that is, it has 15 per cent reactance; while at short circuit it has 50 per cent reactance, making the short-circuit current only twice normal. While practically in all the other apparatus, the short-circuit reactance is either the same as, or smaller than, the normal reactance, in a properly designed d-c-a-c. reactor, the short-circuit reactance may thus be about three times the normal reactance.

The value of d-c. excitation that brings the useful

range of the reactance curve to the normal working load of the unit is of the order of 120 per cent—160 per cent of the a-c. excitation (effective values). This holds true for apparatus sizes. Theoretically, in "peanut" sizes the d-c. excitation might be dispensed with, since the smaller the unit, the lower the economical core density would be and the nearer its magnetizing current would approach the value of its rated load current.

In a protective reactor of this type, it is not necessary to remove the d-c. excitation on short-circuit. Referring to Fig. 5, it will be seen that above the upper bends of the volt-ampere curves, there is very little difference in the voltage consumed with or without d-c. excitation. This is due to the fact that at this point the d-c. ampere turns, being much smaller than the a-c. ampere turns, have very little effect on reactance. How the ratio of d-c. to a-c. influences reactances is clearly brought out in Fig. 10.

The curve of Fig. 10 is an average for various densities and sizes, and is therefore useful only qualitatively. It

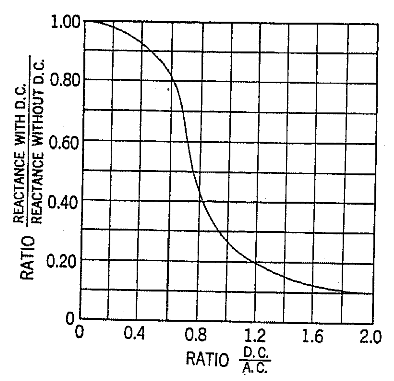

FIG. 10—CURVE SHOWS HOW REACTANCE VARIES WITH RATIO OF D-C. EXCITATION TO A-C. EXCITATION, AND IS ONLY ILLUSTRATIVE, REPRESENTING AVERAGE CHARACTERISTICS OVER COMMERCIAL WORKING RANGES

shows that the ratio of d-c. excitation to a-c. excitation at normal load ought to be in the range of 1.2 to 1.6. It also shows that when this ratio falls below 0.8, the reactance increases very fast. The higher ratio of d-c. is advantageous in leading to smaller core loss and better wave shape.

The d-c. to a-c. excitation ratio must not be misinterpreted as though it were a power ratio, for it is merely an ampere-turn ratio within the reactor, while the ratio of d-c. power to a-c. power or volt-amperes is only of the order of one or two per cent, that is, the d-c. power that is necessary to operate such a device is that required to furnish the copper loss and is, therefore, only a couple of a per cent of the a-c. volt-ampere or less, depending on the kv-a. rating of the unit.

The consideration of the ratio of the power of controlling current (in this case d-c.) to the controlled current (a-c.) has an interesting bearing on the cases where both the controlling and controlled currents may be alternating. Ignoring the resistance losses, we could say, as a general rule, that the ratio of the volt-amperes in the controlling and controlled currents is the same as the ratio of their frequencies. This principle has been utilized by Mr. Alexanderson in radio in what has been called a "magnetic amplifier," being a reactor of this type in which the controlling current is of telegraph key or audio frequency while the controlled current is of radio frequency.

DIFFERENCE IN THE VOLT-AMPERE CHARACTERISTICS ON SINE-WAVE VOLTAGES AND SINE-WAVE CURRENTS

The volt-ampere curve of an iron-core reactor (without d-c. excitation), if taken on a sine-wave voltage, shows distinctly a saturation effect as is commonly known. However, if the tests be conducted on sine-wave current, saturation effect is only slightly marked, the curve being quite steep even far beyond the upper bend. This difference and the reason thereof has already been masterfully discussed by Steinmetz in his book on the "Theory and Calculation of Electric Circuits," in the chapter on the shaping of waves by magnetic saturation, to which the interested reader should refer. (Ordinarily sine-wave current condition is practically approached by connecting in series with the iron-core device a constant impedance of a value considerably higher than that of the former. The higher this ratio, the nearer is the approach to sine-wave current.)

What is true of the relative characteristics of an iron-core reactor on sine-wave current and sine-wave voltage without d-c. excitation, is also true when d-c. excitation is superposed, as shown in Figs. 5 and 6. The latter set of curves is much higher and steeper, that is, it indicates a higher reactance and very much less saturation effect.

The normal operation and characteristics of a series reactor are those for sine-wave current, its short-circuit operation and characteristics are those for sine-wave voltage. At intermediate conditions of load, the characteristics are intermediate. In most applications, the conditions change between sine-wave current and sine-wave voltage, depending on the load conditions, with corresponding changes in characteristics. It is very important, therefore, to bear in mind that many of the characteristic curves given in this paper are for purposes of illustration and can not be used directly for design purposes. In any particular problem, characteristic curves may be obtained and plotted as explained.

WAVE-SHAPE

In considering the wave-shape characteristics of d-c-a-c. induction devices, it should be borne in mind that in most applications the voltage or current of the device is only a small fraction of the total voltage or current of the circuit, and that therefore, a first-order factor in such a device is only a second-order factor for the lines, and a second-order factor for such a device is entirely

negligible for the lines. For instance, if a series reactor consuming 15 per cent voltage has a 30 per cent third harmonic in its voltage wave-shape, this will appear only as a 5 per cent third harmonic in the line or load voltage. For this reason, considerable wave distortion in the voltage or current of the d-c-a-c. device is ordinarily permissible. Not to make the picture too optimistic, however, it might be said here that it is conceivable that some rare circuit constants or connections may lead to an accentuation of these harmonics, in which cases, then, of course, the application of such a device would be limited.

Oscillograms

The following oscillograms were taken on small laboratory models but apply to large units just as well.

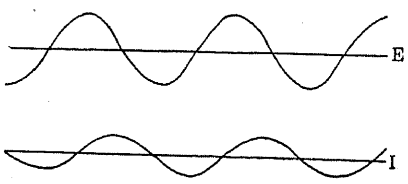

Fig. 11—Wave-Shapes of Current I and Voltage E at Full Load Observed in and Across a Series Reactor Having D-C. Excitation Considerably Larger than the A-C.

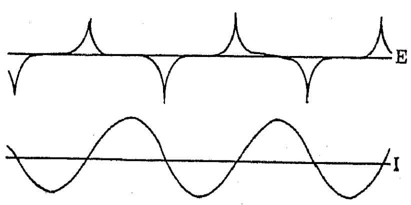

Fig. 12—Wave-Shapes of Current I and Voltage E in and Across a Series Reactor in Which the D-C. Excitation Was About One-Third of the A-C.

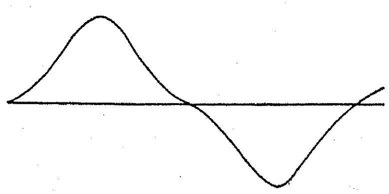

Fig. 13—Wave-Shape of Short-Circuit Current Through D-C. Excited Reactor

Note that it is better than the usual wave-shape of the exciting current of a transformer

Fig. 11 corresponds to the normal load condition of a series reactor, with d-c. excitation well in excess of a-c. excitation. A large constant series impedance was used to insure good wave shape for the currents. It is seen that the wave-shape of both the current and the voltage drop are excellent.

Fig. 12 corresponds to a large overload between normal load and short-circuit, the d-c-a-c. ratio being about ⅓. The current is still of fairly good shape but the voltage drop through the reactor is considerably peaked.

Fig. 13 corresponds to the short-circuit condition. The voltage drop, being the same as the generator voltage, has fairly good wave-shape, but the current is very much distorted. Wave distortion under short-circuit conditions is practically of no consequence in this application.

Advantages and Limitations of the Series Protective Reactor

The main advantages of this type of a current-limiting reactor are: (1) as already explained, it has a much higher reactance at short-circuit than at normal load; and, (2) its reactance is entirely inherent and automatic without requiring any relays, switches or other control equipment.

The main limitations of such a reactor are its cost and losses. (1) The cost of such a reactor may be estimated as follows: the reactor must be good for the normal load current and the line voltage, hence it will have a kv-a. rating of the same order as the load kv-a. Since the reactor has two windings, one for a-c. and one for d-c., it is equivalent to a transformer of the same kv-a. rating as the load. The d-c. ampere-turns are

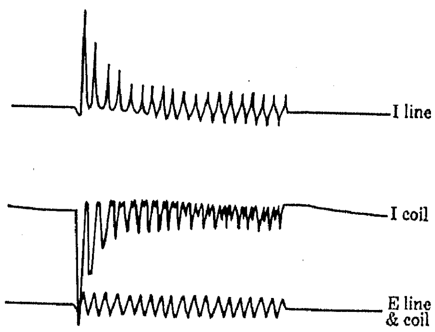

Fig. 14—Transient Rush of Current Due to Sudden Switching of a D-C. Excited Reactor

from 20 per cent to 60 per cent greater than the a-c. ampere-turns, but this is partially offset by the fact that the core density for the short-circuit condition may be designed for a much higher value than that of a normal transformer. That the complete reactor has to consist of two subunits tends to increase its cost somewhat above its equivalent transformer kv-a. rating would indicate. In concrete terms, then, such a reactor for a 1000-kv-a. circuit will have a cost somewhat greater than that of a 1000-kv-a. transformer. This fact naturally limits its very extensive application. If it should be desired to maintain the short-circuit current indefinitely on the ground so that it is only two or three times normal current, the rating and cost of the reactor is naturally further increased.

(2) The losses of such a reactor are somewhat larger than those of the equivalent transformer. Although normally the a-c. flux density of the reactor is between 10 per cent and 20 per cent of the normal a-c. density of the equivalent transformer, yet, due to the superposition of the large d-c. excitation, the core loss of the former will equal or exceed the core loss of the

latter. Those with any electrical design experience will appreciate further that saturation and resultant stray fluxes will lead to considerable stray losses. This matter of losses will be further discussed at a later point.

A further limitation of the series reactor that might be mentioned is, if the short-circuit starts with a large transient flux component, the protective or current-limiting ability of the reactor is temporarily paralyzed, and the current is limited largely by the air-core reactance of the windings, until the transient has practically died out. This transient characteristic is well shown in Fig. 14. Note the initial rush of current at sudden short-circuit. This oscillogram was taken on a connection similar to Fig. 1 and therefore shows a prominent normal second harmonic plus a transient fundamental in the d-c. circuit. This second harmonic would be almost entirely absent in the d-c. circuit if there were a multiple connection in the a-c. circuit as shown in Figs. 3 or 4.

Feeder Voltage Regulator Application

Out of a number of ways in which the d-c-a-c. principle can be applied to voltage regulation, the one

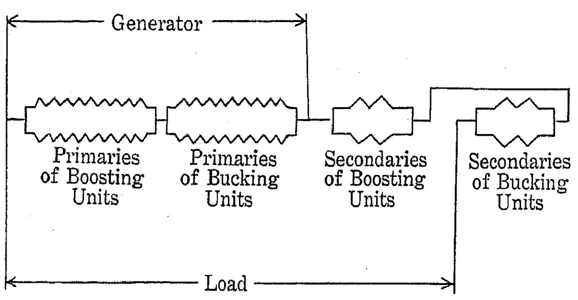

Fig. 15—Connection of Two Auto-Transformers as a Voltage Regulator, One Tending to Boost the Line Voltage, the Other Tending to Lower it. Their Relative Value and Resultant Effect is Controlled by Varying their D-C. Excitation

that has been found to give the best results is as follows: Suppose it is desired to regulate the voltage 10 per cent above and 10 per cent below the normal voltage. Two autotransformers with 10 per cent secondaries and each equipped with a d-c. winding are connected into the circuit as shown in Fig. 15, that is, the primaries of the two autotransformers are connected in series with each other and across the supply lines, and the secondaries are connected in series with each other and in series with the load or feeder lines. One of the secondaries is reversed, so that one unit tends to raise the feeder voltage, the other tends to lower it. Each autotransformer unit consists of two subunits and has therefore two primary windings and two secondary windings, and one d-c. winding enclosing both subunits as shown in Figs. 2 to 4. If one pair of a-c. windings, as for instance the two secondaries of one unit, is connected in parallel, the other pair, that is, the two primaries of the same unit, may be connected in series if desired.

The operation of such an equipment is as follows:

(*a*) *No-load operation.*

Referring to Fig. 15 it is seen that unit No. 1 tends to raise the load voltage and unit No. 2 tends to lower it. As the two units may be assumed duplicates, therefore, normally with equal or no d-c. excitation in either unit, they balance each other and the load voltage is neither raised nor lowered. Suppose the d-c. coil of No. 2 is fully excited, and that of No. 1 is unexcited. Then, unit No. 2 will be saturated and will have a negligible reactance and No. 1 will take up the full-line voltage, and thus the line voltage instead of dividing equally between P_1 and P_2, will be concentrated practically entirely across P_1. If the voltage across P_2 is negligible, that induced in S_2 will also be negligible, and the voltage across S_1 will be practically 10 per cent of the line voltage and will boost the feeder voltage by this percentage. This gives us the maximum boosting condition.

If now the d-c. winding of No. 1, the "boosting" unit, is fully excited, and that of No. 2, the "bucking" unit, is left unexcited, the "boosting" unit will be saturated and will therefore have negligible reactance and negligible voltage drops across its primary and secondary, while the "bucking" unit takes up the full line voltage and its secondary "bucks" the load voltage by 10 per cent. This gives us the maximum "bucking" condition.

Intermediate values of load voltage between maximum "boost" and maximum "buck" are obtained by intermediate values of d-c. excitation in the two units.

(*b*) *Full-load operation.*

Under load conditions the general operation of the device is exactly as described for the no-load condition, except that the amount of regulation is somewhat modified by the reactive drop through the device. The power transformation through the device being proportional to the secondary voltage, the magnitude of the primary current is also proportional thereto.

Thus, with 10 per cent total resultant secondary voltage, the primary-load current is also 10 per cent of the external-load current, and with zero total resultant secondary voltage, the primary-load current is zero. Furthermore, the phase angle of the primary-load current is also controlled by the phase angle of the total resultant secondary voltage, and therefore the primary currents for the "bucking" conditions are reversed in direction with respect to the primary currents corresponding to the "boosting" conditions.

It follows, therefore, that at maximum and minimum load voltages, one unit acts as a transformer, the other as a reactor. At intermediate values of voltages, the units act partially as transformers and partially as reactors.

When a unit is acting purely as a transformer (that is, as autotransformer), its primary and secondary ampere-turns are equal and opposite, exactly as in any transformer, neutralizing each other's reactance so far

as the core is concerned, and producing a reactive drop only through leakage.

When a unit is acting entirely as a reactor, its primary and secondary ampere-turns are again equal, but in phase with each other, and, therefore, instead of neutralizing each other, their resultant produces a reactive drop in virtue of the core. Of course, when a unit is acting as a pure reactor, its core is saturated by a large d-c. excitation, and the reactance of the unit is reduced to a very small fraction of what it would otherwise be. In practise, this reactance is found to be of the order of three or four per cent, effective at the load terminals. This may appear as very small but it is about twice the reactance that an induction regulator of similar capacity introduces into the circuit, and causes considerable regulation at low power-factor loads. Regulation introduced by this reactive drop has to be taken care of by overwinding the secondary of the "boosting" unit if load is dominantly lagging, and overwinding the secondary of the "bucking" unit if the load is dominantly leading, or overwinding both secondaries for interchangeable units and for general application.

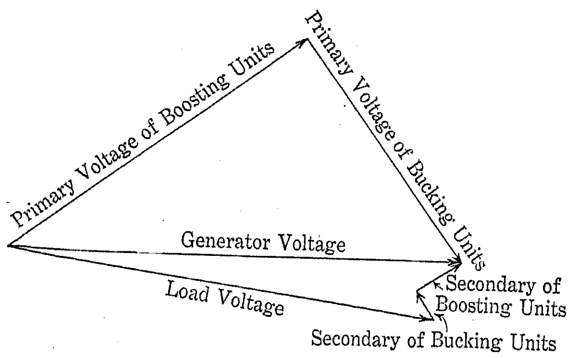

FIG. 16—VOLTAGE DIAGRAM OF REGULATOR AT NEUTRAL POINT WITH HIGH POWER-FACTOR LOAD

At intermediate values of load voltages between maximum and minimum, the primary and secondary load ampere-turns are not equal in either unit, the per cent difference between the primary and the secondary ampere-turns changing gradually (in the inverse order in the two units) as operation changes from maximum "boost" to maximum "buck," and produces a reactive drop. This reactive drop at intermediate values of load voltage does not do any harm of primary importance since any regulation that it causes may be corrected by raising the voltage to the desired degree by means of the d-c. control. The effect of reactance and d-c. excitation at those intermediate points is observed more in the exciting current and losses.

With unity power-factor loads, the reactive drop is in quadrature with the line voltage and, being rather large at the neutral point of voltage-regulation, raises the voltages across the units considerably as shown in the vector diagram of Fig. 16. This is mentioned to explain the vector relationship of the reactive drops which, however, are of no very particular practical harm or utility.

The influence of reactive drop on exciting current is this, that with the regulator running "bucking" at lagging power-factor loads, or "boosting" on leading power-factor loads, the reactive component of the

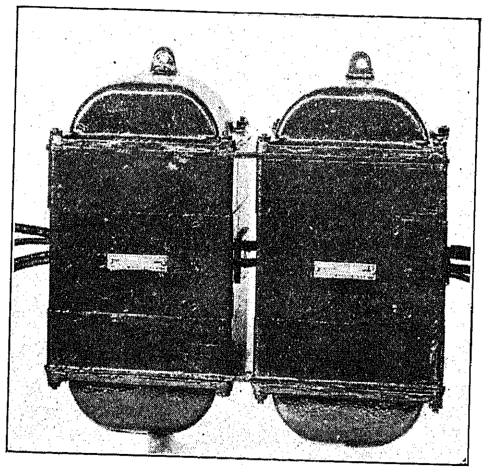

FIG. 17—A SMALL REGULATOR DESIGNED FOR CONTROL BY D-C. EXCITATION

secondary current acts as magnetizing current and therefore the primary current is reduced considerably; while at lagging power-factor loads with the regulator operating "boosting," or at leading power-factor loads with the regulator operating "bucking," the reactive component of the load current demagnetizes the core

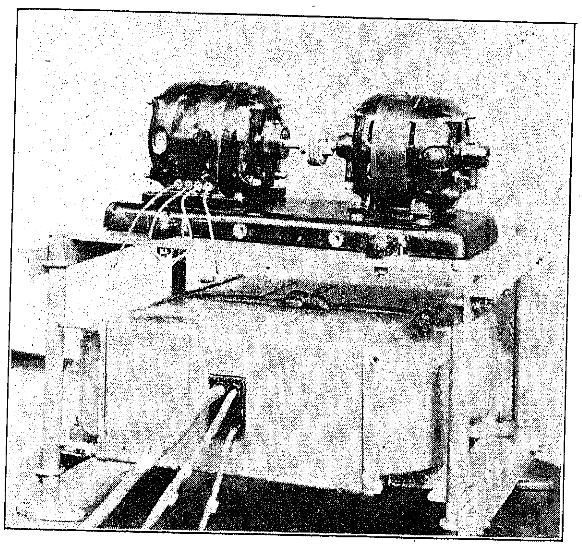

FIG. 18—A SMALL REGULATOR AND ITS MOTOR-GENERATOR SET FOR CONTROL OF REGULATION BY DIRECT CURRENT

and increases the exciting current and total current in the primary.

A number of such regulators have been built and tested in sizes from a few kv-a. up to 460-kv-a. feeders. Some of these have been put into service and are giving perfect satisfaction. Two small regulators are shown

in Figs. 17-19, and a large regulator for (460-kv-a. feeder) is shown in Figs. 20 and 21. The two units constituting the regulators are clearly seen in these figures. In Fig. 20 one is mounted on top of the other, although the subunits constituting each autotransformer unit are not easy to distinguish, because the outside coil is the d-c. coil and encloses both subunits at once. The top view of this is shown in Fig. 21. Fig. 22 is the voltage time curve on a feeder with and without the small regulator shown in Figs. 18-19.

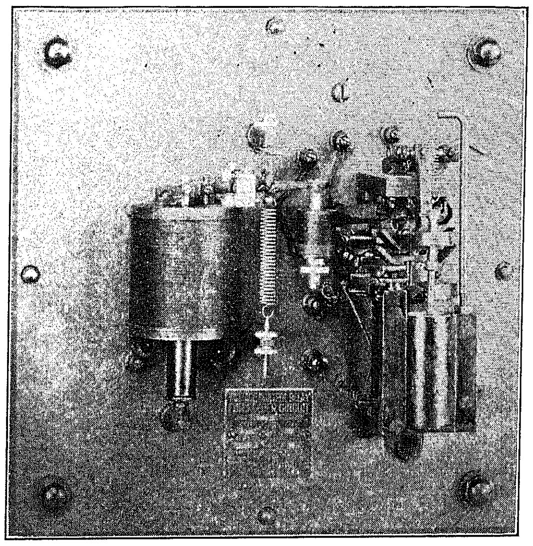

FIG. 19—AUTOMATIC CONTACT MAKING VOLTMETER FOR REGULATOR

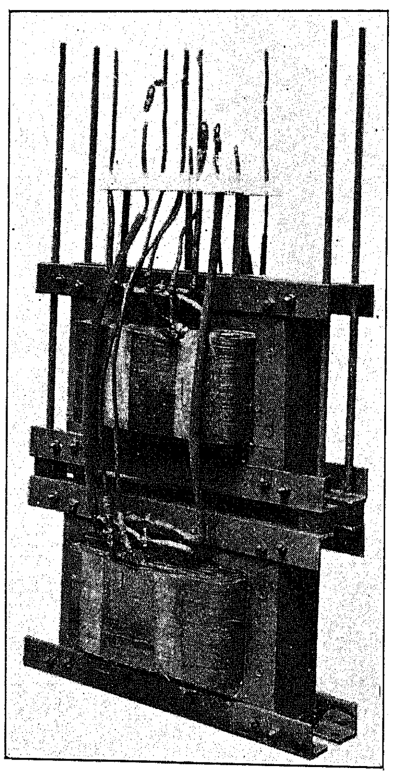

FIG. 20—CORE AND COILS OF A 46-KV-A. REGULATOR (460-KV-A. OUTPUT) FOR 60-CYCLE, 2300-VOLT CIRCUITS

AUTOMATIC CONTROL OF D-C. EXCITATION

Since a commercial voltage regulator must be automatic, it is necessary that the control of d-c. excitation be automatic and be governed from a contact-making voltmeter. This can be accomplished in a number of ways, one simple method being as follows: Referring to Fig. 23, No. 1 and No. 2 are the d-c. coils of the "boosting" and "bucking" units, and A and B are small generators capable of furnishing the $I^2 R$ losses in these d-c. windings of the regulator. Generator A is continuously excited, while the field of B is controlled by a contact-making voltmeter somewhat similar to those used for generator field control. If the field of B is open, A will excite winding No. 1 fully, the current flowing through the armature of B. Winding No. 2 having a large resistance, as compared with the armature of B, little or no current will flow through it. On the other hand, if the field of B is closed, the voltage of B opposes and neutralizes the voltage of A in the circuit of winding No. 1 and no current flows in it, while winding No. 2 is fully excited from B. By varying the duration of closing and opening of the field of B, or by cutting resistance in and out at suitably fast intervals of proper duration, any desired ratio of d-c. excitation in the windings No. 1 and No. 2 may be obtained. D-c. generators and automatic control equipment are shown in Figs. 18 and 19.

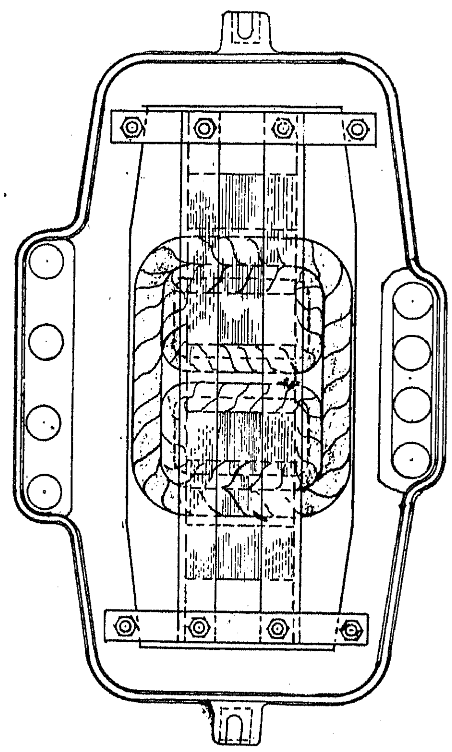

FIG. 21

WAVE-SHAPE

A large number of oscillograms were taken on some of these regulators, and those which are representative of various conditions of operation are here reproduced.

No-Load Oscillograms

Fig. 24 represents the maximum "boosting" condition. Excellent wave shape is observed in the voltages of generators, feeder, secondary of "boosting" unit, and in the total secondary voltage. The voltage of the "bucking" unit (the saturated unit in this case) is considerably distorted, but its effective value is only about one fifth of one per cent, and therefore does not sensibly affect the feeder voltage.

Fig. 25 represents the maximum "bucking" condition. Again, those voltages which are of any importance have excellent wave-shape. Inasmuch as it is the "boosting" unit which is saturated in this case, the distorted voltage belongs to it.

Fig. 26 represents the "neutral" condition. In this case the secondary voltages of the two units are opposed to each other and practically neutralize each other, except for difference in their wave shape which leaves a residue of peculiar wave-shape as the total resultant secondary voltage. The effective value of this was immeasurably small.

Full-Load Tests

Fig. 27 represents the maximum "boosting" condition on a water box load. Both generator and feeder voltages are seen to be excellent. Maximum distortion naturally occurs in the saturated "bucking" unit: The distortion is small and the voltage is negligible.

Fig. 28 represents the maximum "bucking" condition and was also taken on a water box load. Wave-shape is similar to the preceding case. Of particular interest are the current waves shown in this figure. The line current is symmetrical and of good wave-shape; but the coil-current, *i. e.*, the current in one branch of the multiple paths, is unsymmetrical, showing the circulating second harmonic current very prominently by the succession of peaked and flat half-cycles.

Fig. 29 represents the "neutral" condition. The high reactance and high exciting current ocurring at this condition have impressed on the load voltage and current a visible mark of the general wave-shape of exciting current of a transformer.

Effect of Low Power Factor and Dynamic Loads

To secure data on the performance of the regulator on low power factors and motor loads, a large series of tests was made on a synchronous motor load.

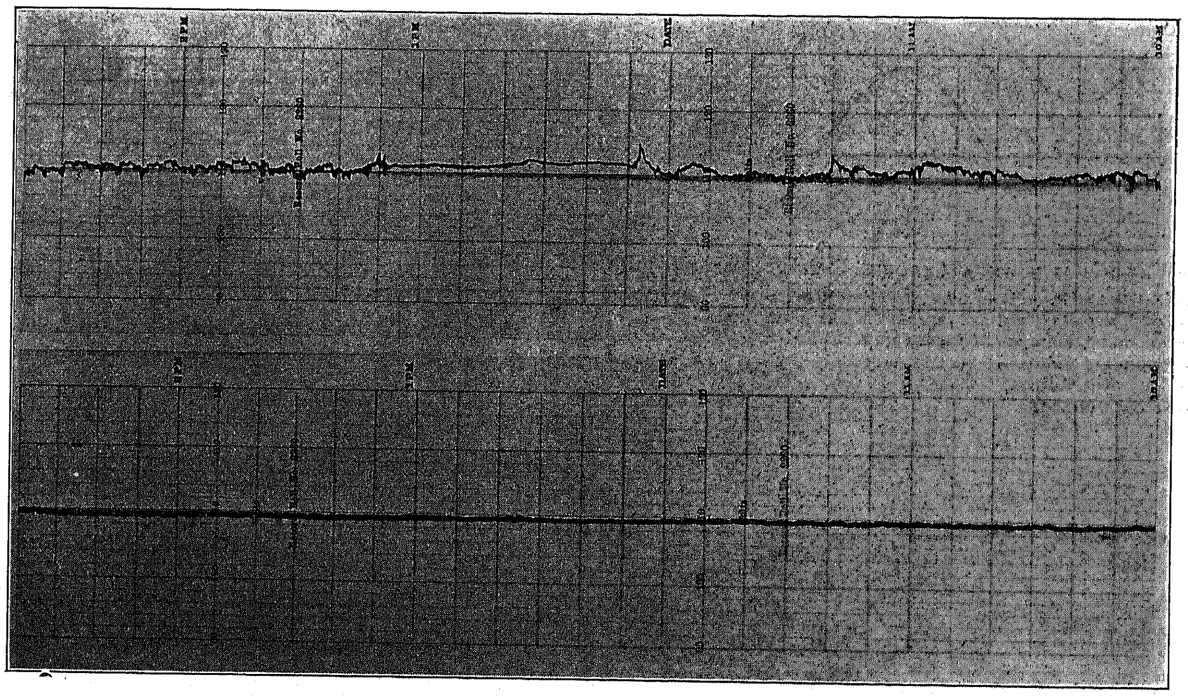

Fig. 22—Regulation Chart of a Circuit with and without the Regulator

Fig. 30 represents unity power-factor load on a synchronous motor at maximum "boosting" condition and differs in no essential respect from those for a water-box load, except that feeder and generator voltages are purer in shape on account of the fact that a motor of

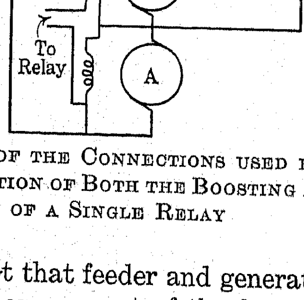

Fig. 23—One of the Connections used for Controlling the D-C. Excitation of Both the Boosting and the Bucking Units by Means of a Single Relay

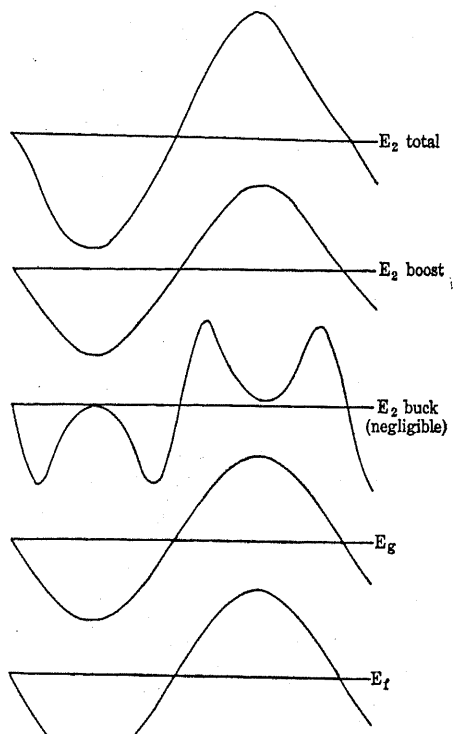

FIG. 24—WAVE-SHAPES IN A CIRCUIT CONTROLLED BY D-C. EXCITED REGULATOR. NO-LOAD, MAXIMUM BOOST CONDITION

E_2 total = Total secondary voltage (across the series windings)
E_2 boost = Secondary voltage of boosting unit
E_2 buck = Secondary voltage of bucking unit
E_g = Generator voltage.
E_f = Feeder voltage.

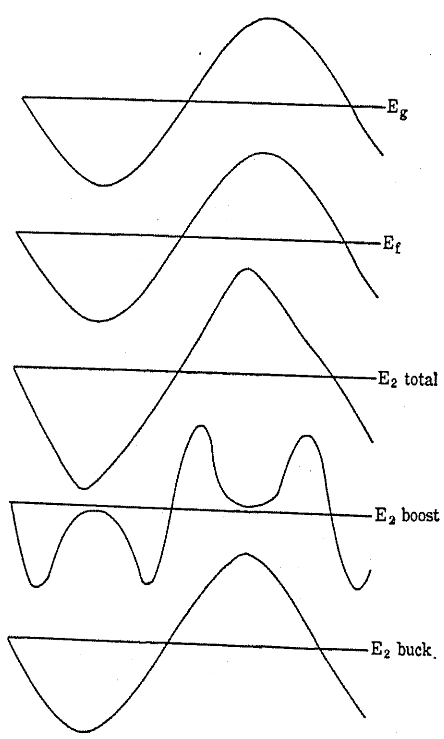

FIG. 25—WAVE-SHAPES UNDER NO-LOAD, MAXIMUM BUCK CONDITION

proper design tends to maintain the line voltage in pure form. However, the current taken by a motor tends to be considerably distorted if there is an appreciable distortion in the line voltage, due to the fact that the motor short-circuits those harmonics which do not exist in its own e. m. f. The distortion in the total series secondary voltage is negligible as a percentage of the load voltage.

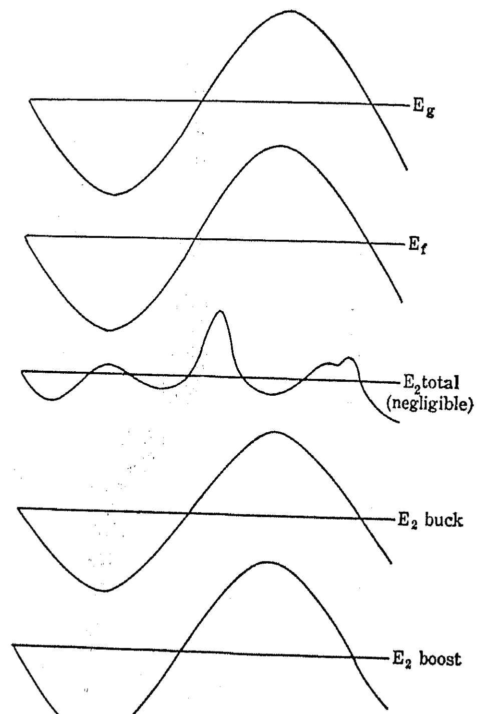

FIG. 26—WAVE-SHAPE AT NO-LOAD, NEUTRAL POSITION

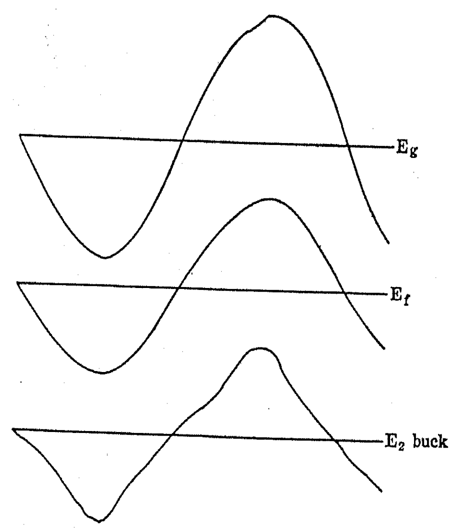

FIG. 27—WAVE-SHAPES AT UNITY POWER FACTOR, FULL-LOAD, MAXIMUM BOOST

Fig. 31 represents the case of 84 per cent leading power-factor at the maximum "boosting" condition. The load voltage is still pure, but the generator voltage shows some distortion. It may not be amiss to emphasize here the fact that a certain amount of wave distortion at full loads of various power-factors is a

common characteristic of many commercial generators of medium and small sizes and therefore it must not be assumed that all wave distortions observed in these tests were due to the regulator.

Fig. 32 represents the case of 70 per cent lagging power-factor on synchronous motor. All the important quantities have good wave-shape. The distortion in

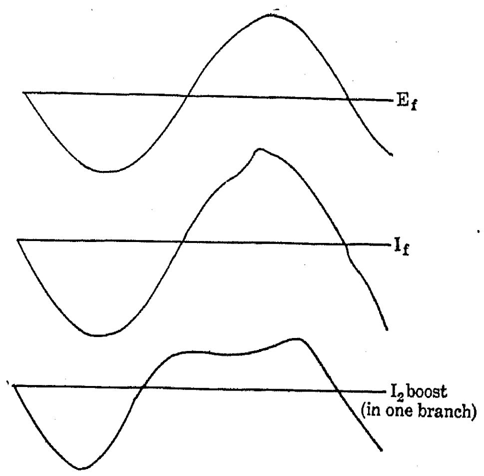

Fig. 28—Wave-Shapes at Unity Power-Factor, Full Load, Maximum Buck

Note that the currents in the two branches of the series windings of the boosting unit are unsymmetrical due to large circulating second harmonic component, but their resultant, which is the feeder current (I_f), is very symmetrical.

the secondary voltage is not distinguishable in the feeder voltage. The coil current in the saturated ("bucking") unit shows the circulating second harmonic. The current in the primary coil betrays the wave shape of excitation current.

Switching Oscillograms

Fig. 33 represents switching the d-c. excitation so as

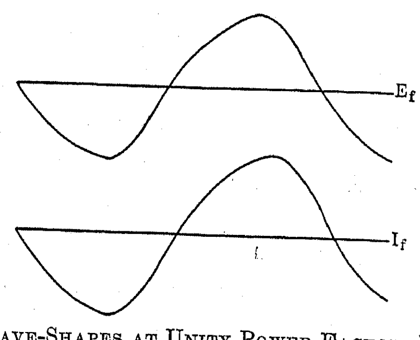

Fig. 29—Wave-Shapes at Unity Power-Factor, Full-Load, Neutral Position

to change the operation from maximum "boosting" to maximum "bucking" at no load. It takes a good many cycles before the change in the direct current shows itself in the secondary resultant voltage. This latter, which "boosts" the load voltage, gradually dies down to zero and then builds up in reverse phase angle to "buck" the load voltage.

Fig. 34 represents a similar switching test but made under full load. Here the secondary resultant voltage does not die down to zero during the transition but

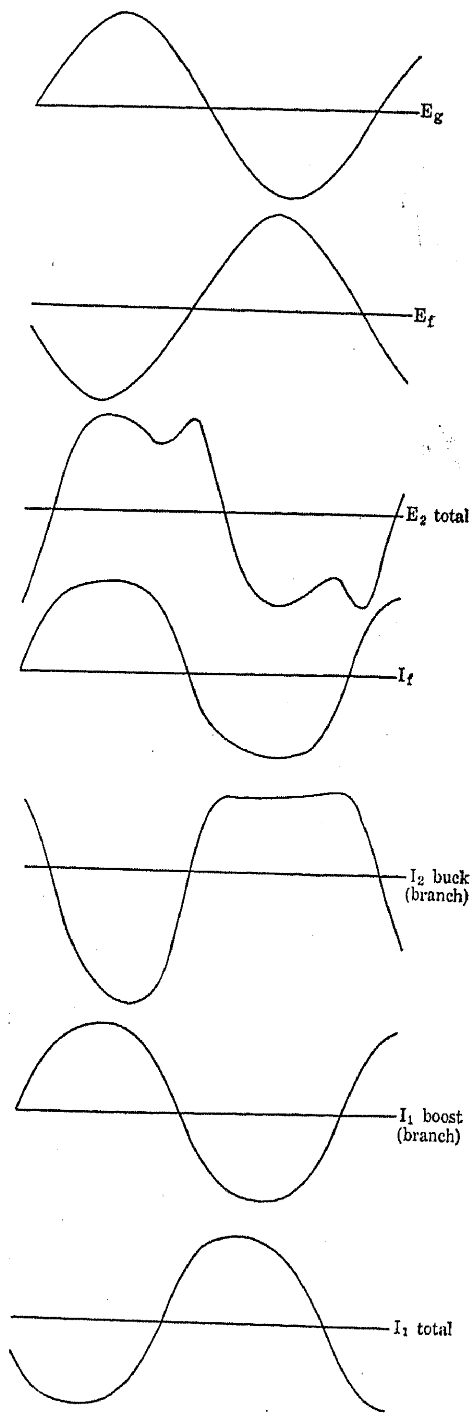

Fig. 30—Wave-Shapes at Unity Power-Factor, Synchronous Motor Load, Maximum Boost

Note that the voltage wave-shapes are very much improved but the current wave-shapes are not so good. The effect of circulating second harmonic current is seen in the branch current of the secondary of the saturated bucking unit, "I_2 buck." No circulating second harmonic is found in the unsaturated boosting unit.

builds up to about double value and then comes down to normal value, reversing its phase angle gradually during the transition. The total secondary voltage does not

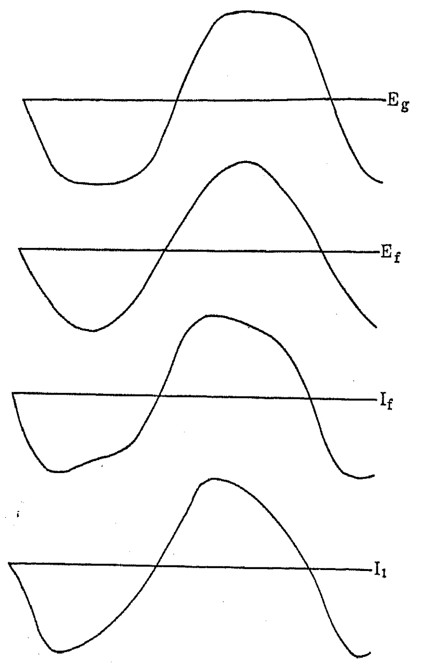

FIG. 31—WAVE-SHAPES UNDER CONDITIONS SIMILAR TO THOSE OF FIG. 30 EXCEPT THAT THE POWER FACTOR IS ABOUT 85 PER CENT LEADING

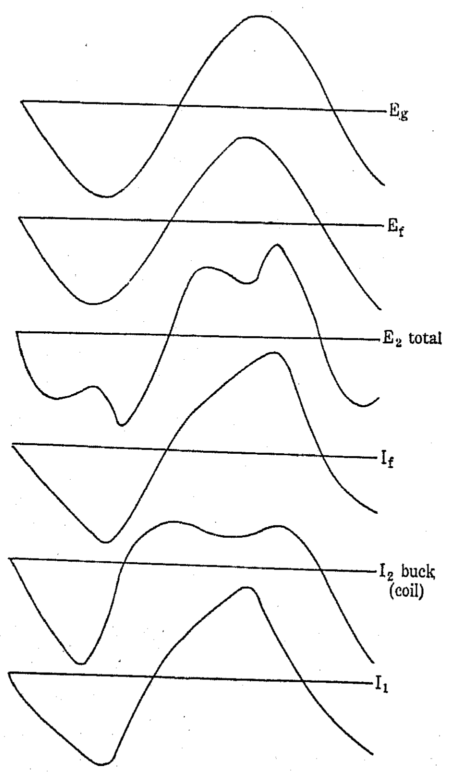

FIG. 32—CONDITIONS SIMILAR TO THOSE OF FIGS. 30 AND 31, EXCEPT THAT THE POWER FACTOR IS LAGGING, AND THE VOLTAGE IS ONLY PARTIALLY BOOSTING

come down to zero in this case because the large load-current flowing through it produces a large reactive drop.

ADVANTAGES AND LIMITATIONS OF THIS TYPE OF REGULATOR

A. *The main advantages are:*

1. *Speed of Operation.* While the average induction regulator requires 8 to 12 seconds to go from maximum "boost" to maximum "buck," this type of a voltage regulator requires only between one and two seconds for a similar change. The time required is the combination of the time constants of the regulator, d-c. generator, and control equipment.

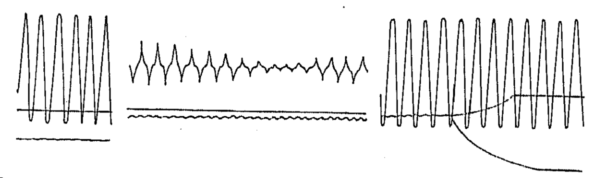

FIG. 33—SWITCHING TEST AT NO-LOAD, CHANGING FROM MAXIMUM BOOST TO MAXIMUM BUCK. FILM CUT AND SHORTENED AT TWO POINTS. TOP WAVE: TOTAL SECONDARY SERIES VOLTAGE

Note how it decreases to zero and then builds up in reversed phase, so as to change from "boost" to "buck." Middle and bottom waves: the D-C. currents in the two units one building up, the other dying down

2. *Suitability for High Voltages.* Since the construction of this regulator is that of a transformer, it can be designed for any voltage desired, while induction type regulators may be said to have an upper limit of 13,000 volts. For circuits above that voltage, an induction regulator requires two transformers, one for its primary, and one for its secondary.

3. *Greater Mechanical Strength.* Since this type of a regulator has no moving parts, it can be built much stronger than movable or rotatable regulators to withstand the short-circuit stresses which are very severe in

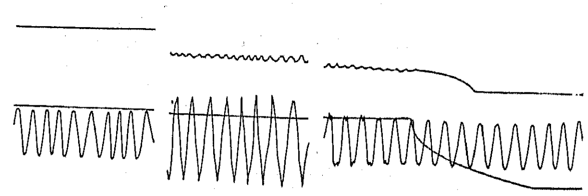

FIG. 34—SWITCHING TEST AT FULL-LOAD, CHANGING FROM MAXIMUM BOOST TO MAXIMUM BUCK. BOTTOM WAVE: TOTAL SECONDARY SERIES VOLTAGE

Note that it does not diminish but actually increases during the transition period, the change from *boost* to *buck* being accomplished by a gradual reversal in the phase of the total secondary (series) voltage

all regulators on account of their autotransformer connection. The ability of this static regulator to withstand short circuits is further improved by its higher inherent reactance. This advantage probably is not as great as one might expect on account of the fact that on short circuit all autotransformers are severely overexcited and hence the saturation of the cores must be reckoned with.

B. *The limitations of this type of a regulator are:*

1. *Higher cost for low voltages.* At 6600 volts or

lower, this type of a regulator costs somewhat more than an induction regulator of the same capacity. This is due to the fact that the equivalent transformer rating of such a regulator is about four times the maximum kv-a. which it transforms. Thus, first, the fact that a "boosting" and a "bucking" unit must be provided, doubles the size of the apparatus. Second, the fact that d-c. ampere-turns of sufficient capacity to neutralize the total primary and secondary a-c. ampere-turns must be provided for each unit, at least doubles the size of the apparatus again, and thus we have a regulator of which the equivalent transformer capacity is about four times the maximum kv-a. which it transforms.

However, at those high voltages for which an induction regulator would need insulating transformers, this type of regulator may prove more economical.

2. *Higher Losses.* Both on account of the duplication and multiplicity of windings, which tends to increase the copper losses, and on account of the superposed d-c-a-c. excitation of the core, which tends to increase the core loss, this type of regulator has higher losses than an equivalent induction regulator.

On account of these limitations, this regulator is as yet considered in a developmental stage and is not available for general commercial use.

Historical Note

Attempts to make use of the saturation characteristics of iron for purposes of voltage regulation date as far back as 1900. A. R. Everest, W. T. Williams and William Stanley experimented with various connections, in all of which, however, no direct current was used. The basic principle underlying them was the fact that a theoretically saturated reactor would not sensibly alter its voltage drop with varying loads but would alter its phase angle. On no-load, such a series reactor would be made to carry a magnetizing current load, and its drop, being in phase with the line voltage, would lower the feeder voltage. When a unity power-factor load is put on, the resultant power-factor of the current flowing through the reactor would be very high and the drop through the reactor would be shifted into approximate quadrature with the line voltage. Thus, the full-load feeder voltage would become higher than at no-load. Looking back with our present day knowledge, we can see that such schemes could not be a success, inasmuch as they yield no means for control and depend on a high power factor of load for their operation, being inoperative for low power-factor loads.

One of the earliest workers with d-c. excited reactors for regulation was Francis B. Crocker. He used an autotransformer to obtain a boost in voltage which might be neutralized by a reactive drop as desired. By putting d-c. excitation on the autotransformer, the exciting current of the latter could be altered, and this exciting current being large would produce a reactive drop and regulation through the leakage reactance between primary and secondary. The scheme was intended for application to rotary converters, with two d-c. coils on the autotransformer, one being in shunt and the other in series with the d-c. lines, so that the variations in the d-c. load would automatically control the a-c. voltage.

The regulator connection described in the present paper is the William Stanley connection to which d-c. control is added. Considerable work has been done recently by E. F. Alexanderson on the applications of d-c. excited reactors. Much inspiration has come from him in these investigations both directly and indirectly.

Shunt Reactor Application

The shunt reactor application of such a device for the neutralization of the charging current of transmission lines has been considered in a number of instances by operating and consulting engineers, but no actual application has as yet been carried out. In such service, not only would such a device reduce the charging current load on the central station, but also would control the voltage of the line. In this latter service it would take the place of an underexcited synchronous condenser, and would have a number of advantages over it, namely, (a) a static device with no rotating parts and requiring practically no attention, (b) can be designed for any voltage, and, (c) would be more economical and more efficient than a synchronous condenser outfit requiring transformers. It would, however, have three disadvantages over the synchronous condenser outfit, in that, (a) while the voltage-regulating character of the synchronous condenser is inherent and very marked, so that it could perform to quite an extent without a contact making voltmeter control on its field, the performance of the static device has to be dependent on such a control and is therefore somewhat slower. In order that the losses of this device may not be excessive, it has to be operated not much above the upper bend of the magnetization curve and hence advantage can not be taken very economically of the flat saturation portion of the volt-ampere curve which corresponds to high losses and also to greater distortion. (b) While a synchronous condenser can operate either leading or lagging, this device can operate only lagging. (The range of lagging load, minimum to maximum, could easily be made 1 to 10 or 20). (c) The static device is bound to have a certain amount of wave distortion. The harmonics, however, would be in the current not appreciably in the voltage, and would, therefore, probably not do any appreciable harm except as a small useless reactive kv-a. to be furnished from the generating system. What wave-shapes of current are to be expected may be estimated from the following oscillograms.[2] Any such application must, necessarily, be polyphase, and although there are many possible poly-

2. The author is indebted to Messrs. L. H. Junken and E. W. Hawley for the oscillograms in Figs. 35-45.

phase connections, only a few have reasonably good wave form and all-around desirability.

It is interesting to note that polyphase applications are capable of yielding much better wave-shape than single-phase applications, on account of the fact that the triple harmonic, the most prominent harmonic, can be eliminated from the lines and circulated internally in three-phase connections but not in single-phase connections.

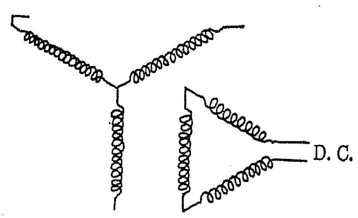

FIG. 35

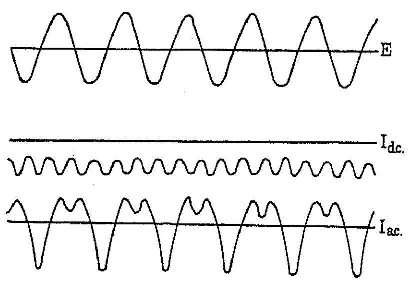

FIG. 36—WAVE-SHAPES IN THE CIRCUIT OF FIG. 35. TOP WAVE: LINE-TO-NEUTRAL VOLTAGE. MIDDLE WAVE: CURRENT IN D-C. CIRCUIT. BOTTOM WAVE: LINE CURRENT

Note the third harmonic in the D-C. circuit, and the second harmonic n the A-C. lines

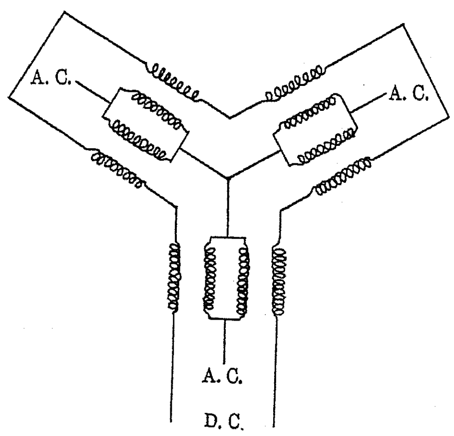

FIG. 37

OSCILLOGRAMS OF POLYPHASE SHUNT REACTORS

Y-Delta Connection of Three-Units.

In this connection (Fig. 35) the direct current is circulated in the delta. It is self-evident from theoretical considerations and may be observed in the oscillograms of Fig. 36 that this connection forces a third harmonic current into the d-c. circuit and a second harmonic into the a-c. lines, and is therefore worthless for such applications.

Y-Delta Connection with Six-Units.

In this connection (Fig. 37), the a-c. current having two parallel paths (excited by d-c. in opposite direc-

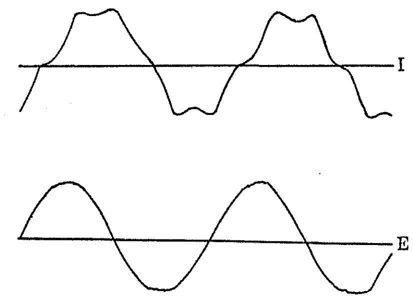

FIG. 38—WAVE-SHAPES IN THE CIRCUIT OF FIG. 37. LINE CURRENT AND LINE VOLTAGE

Note the prominent fifth harmonic in the line current, indicative of high A-C. density and low D-C. excitation

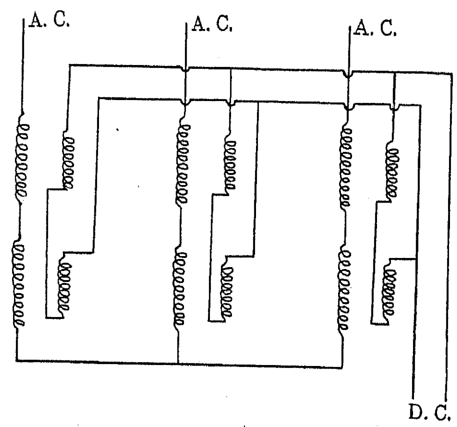

FIG. 39

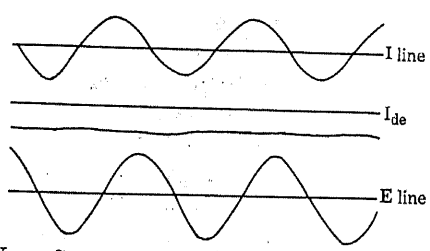

FIG. 40—WAVE-SHAPES IN THE CIRCUIT OF FIG. 39, TYPICAL OF LOW A-C. DENSITY AND HIGH D-C. EXCITATION

Note how free the D-C. current is from harmonics

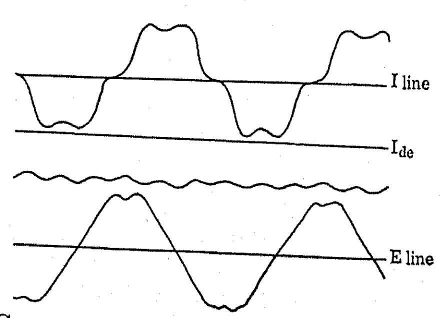

FIG. 41—CONDITIONS SIMILAR TO FIG. 40, BUT HIGH A-C. DENSITY

tions) the even harmonic circulates between the two parallel subunits and does not appear in the lines, as will be observed in Fig. 38. This connection is one of

the best. If the two parallel subunits, constituting each phase, are built as in Figs. 3 or 4, the three d-c. windings may then be connected either in series or in parallel. Another possibility is to have two d-c. windings, each enclosing three polyphase subunits at once. The two d-c. windings may then be connected either in series or in parallel.

Y-Delta Connection with Six-Units, Parallel Connection on the D-C. Side. (*Fig. 39*)

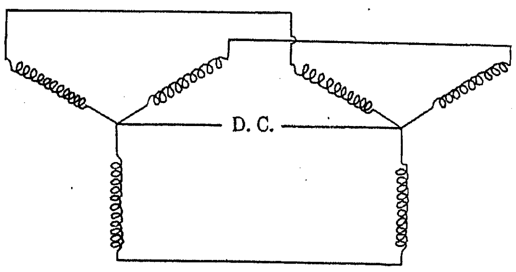

FIG. 42

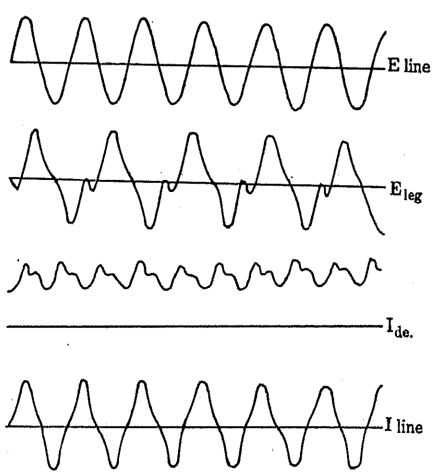

FIG. 43—WAVE-SHAPES IN THE CIRCUIT OF FIG. 42
Note the third harmonic in the leg voltage and D-C. current

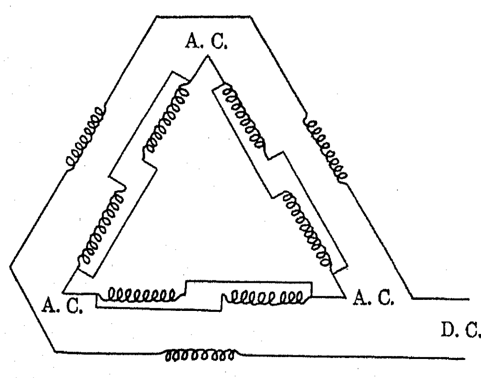

FIG. 44

In this connection the even harmonic currents circulate in the d-c. windings but do not appear in the external d-c. lines. Considerable third harmonic voltage is generated line-to-neutral. Fig. 40 shows excellent wave shape because it has been taken at only 26 per cent rated voltage. Fig. 41 is more representative, being taken at rated voltage. Fig. 40 has been given here to show how undervoltage improves wave shape, and this points to a possible solution of the problem of wave shape: to design for as low a density as is economically practicable.

Y-Y Connection. (*Fig. 42*)

This connection is economical in windings by circulating the direct current in the a-c. windings. It suffers from large third harmonic voltages line-to-neutral. If the neutral is grounded, this third harmonic voltage may be intensified to destructive values through line capacitance. If the neutral is isolated, the d-c. circuit would be subjected to destructive voltages in case of

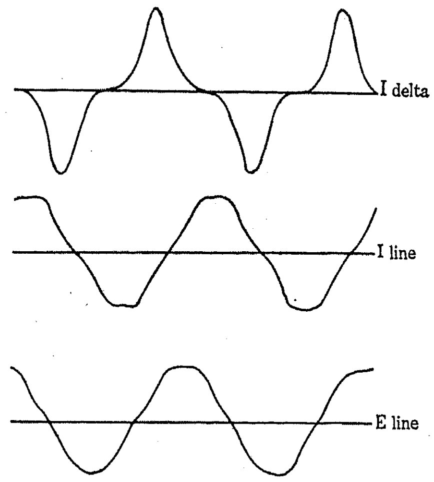

FIG. 45—WAVE-SHAPES IN THE CIRCUIT OF FIG. 44

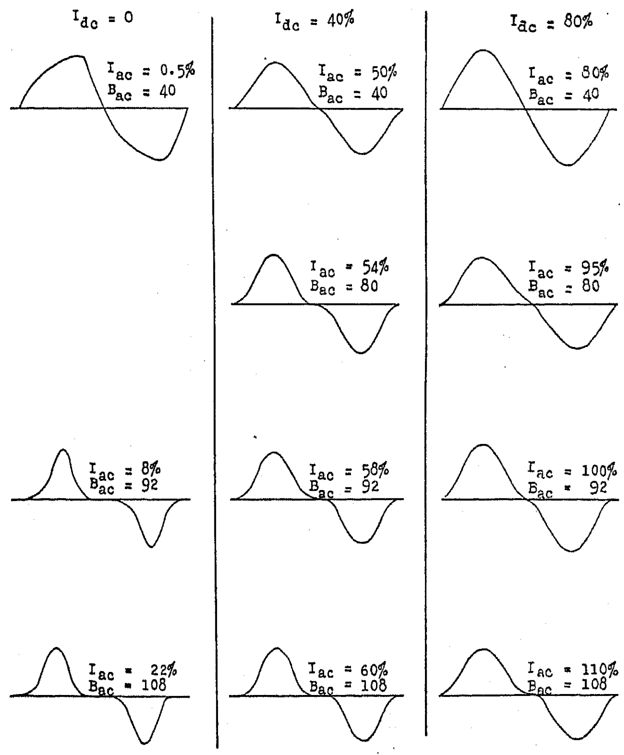

FIG. 46—OSCILLOGRAMS SHOWING THE EFFECT OF A-C. DENSITY, AND OF THE RATIO OF D-C. TO A-C. EXCITATION, ON WAVE-SHAPE OF A SHUNT REACTOR

line grounds. This connection can be used only if equipped with a delta winding to circulate the third, and the neutral solidly grounded. Fig. 43 shows the line voltage (normal rated value), line current, leg voltage and d-c. current. Note the harmonics in the leg voltage and in the neutral current.

Delta-Delta Connection. (*Fig. 44*)

This connection, although not very suitable for the highest voltages, gives a better wave-form than most others. The wave-shape of current (Fig. 45) is pretty good considering the shape of the impressed line voltage. Fifth harmonic is evident in the line current, third harmonic in the delta current.

General Remarks on Wave Shape of Shunt Reactor

Although the second harmonic is eliminated from the lines by means of a suitable parallel connection in each phase, and the third harmonic is eliminated by virtue of a symmetrical three-phase connection, no simple means seems to be available for the complete elimination of the fifth, seventh, and higher odd harmonics from the line current, and it appears, therefore, that in any shunt reactor application, these harmonics will have to be tolerated to some extent. However, some oscillograms to be discussed below show that these harmonics can be reduced to negligible proportions by designing the reactors for rather low densities.

The three-phase oscillograms discussed in the foregoing were taken by very small laboratory apparatus and are not very representative of what would be obtained by power size apparatus. They were given and discussed here so as to show the relative merits of different three-phase connections. To get an exact idea of the wave shape which would be obtained on power size apparatus, some single-phase tests were made on a reactor of suitable design loaded to as high as 500 kv-a. The oscillograms taken under various conditions are grouped in Fig. 46 in three columns. The first column shows the exciting current wave-shapes at various core densities with no d-c. excitation. The second column shows wave-shapes at various densities with 50 per cent rated d-c. excitation, and the third column shows wave-shapes with 100 per cent rated d-c. excitation. It is evident that the larger the d-c. excitation and a-c. current, the purer is the wave-shape. The current wave at 80-kiloline density in the third column is so good that it seems to leave little to be desired. Judging from these oscillograms, it appears as though the additional a-c. current which is drawn on account of the presence of the d-c. current is very much purer in wave shape than the exciting current without d-c., and that therefore the numerical value of the harmonics at moderate densities are more or less constant or increase very little with increasing a-c. current drawn. Thus, with increasing a-c. current drawn, the percentage of the harmonics rapidly diminishes and the wave shape improves.

General Discussion of Core Loss

Rather large core losses were observed in these investigations due, no doubt, to large stray losses occasioned by leakage of flux to clamps and tank, for which reason also attempts at a theoretical calculation of the losses proved futile. One point is certain, that core loss increases very little as compared with the exciting current in the presence of d-c. excitation. In the light of these considerations, it will be evident that considerable reduction of losses may be accomplished by confining the leakage flux and by limiting the portion of the core which is saturated. These two objectives are accomplished in a large measure by a core construction as shown in Fig. 4. Here the middle yoke, carrying only the a-c. flux and no d-c. flux, prevents the fringing and flaring out of the a-c. flux as it comes out from the inside core legs, and also limits the portion of the core which carries superposed d-c-a-c. flux in a saturated condition. This aspect of the investigation is not as yet completed. However, this is a matter of improvement and refinement, and does not directly affect the operation of the devices in the applications described in the foregoing.

Acknowledgment

The author acknowledges the cooperation of Messrs. W. B. Kirke, F. Dubsky and C. H. Kline in these investigations. The development of the automatic control for the regulator is due to Messrs E. J. Murphy and L. W. Thompson.

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Discussion

For discussion of this paper see page 940.

The Application of the Saturated-Core Reactor and Regulator

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Review of the Subject.—The purpose of this paper is to state briefly the use of voltage regulators and reactors in transmission and distribution systems and to mention operating requirements which might be best met by the use of saturated-core type regulators or reactors. The saturated-core type regulator and reactor consists of an iron core with two sets of coils. One set of coils is connected to a d-c. circuit and the other set to an a-c. circuit. The theory, design and construction of the saturated-core type regulator and reactor is discussed in A. Boyajian's paper.

* * * * *

Voltage Regulator

The economic operation of large transmission and distribution systems and the rendering of satisfactory electric service from these systems require voltage regulation and control in various parts of the system, owing to the change in voltage with load variations. Generator voltage regulators are generally employed to maintain constant voltage at either the generator or substation bus. Synchronous condensers are also used extensively to maintain a constant voltage at a substation bus by supplying wattless kv-a. to the system, thereby changing the power factor of the energy transmitted over the transmission lines. Induction type regulators are generally used in distribution feeder-circuits to maintain a constant voltage at the feeder load center. To a less extent they are used in industrial plants for regulating lighting circuits, electric furnace loads, welding loads, and electrolytic processes.

The tying together of two large stations or systems and the formation of networks or meshes by the interconnection of substations, generating stations or systems requires the use of voltage regulating equipment to control the voltage, the flow of reactive kv-a., and the energy component. Synchronous condensers are used to meet many of these conditions but there are some conditions where they will not accomplish all that is required. Synchronous condensers are generally used to hold constant voltage at certain points of the system by supplying reactive kv-a. to the system. The condensers have no control over the division of the reactive kv-a. flowing in two or more parallel paths between two points as this is determined by the impedance of the paths. Induction regulators and transformer tap-changing equipments are also used to hold constant voltage at certain points of the system, and where required, to determine the value and path of flow of either reactive kv-a., energy component, or both. Where it is required that the voltage adjustment shall follow a smooth curve, the tap-changing equipment alone is not applicable, as this equipment changes the voltage in steps as great as 2½ per cent and 5 per cent. The induction regulator, the combination of tap-changing equipment and induction regulator, and the synchronous booster give a smooth voltage adjustment. The induction regulator and the synchronous booster, however, cannot be built for very high voltages and it is necessary, therefore, on a high-tension line to use shunt and series transformers between the regulator or booster and the high-tension line or bus.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

The construction of the saturated core type regulator is similar to a transformer, consisting of an iron core and coils. The saturated core type regulator, therefore, can be built for as high a voltage as can the transformer. For a 220,000-volt circuit a 220,000-volt saturated-core regulator could be built. From the voltage standpoint then, in order to justify the choice of the saturated core type regulator, it must have a lower annual charge than either the induction regulator or synchronous booster with their accompanying transformer equipment. No large, high-voltage, saturated core regulators have been built so far, and therefore, the operation and exact costs are uncertain; but from a general study of the situation the indications are that the saturated core regulator would probably be cheaper for high voltage than the well-known combination of induction regulators with shunt and series transformers.

The induction regulator meets most requirements for distribution feeder-circuits. It requires about 8 to 11 sec. to go from full buck to full boost. One of the important factors in perceptible lamp flicker is the time required for the voltage to return to normal after a sudden change in value. For satisfactory service to lighting, it is important that fluctuating loads, which will cause sudden voltage changes at the feeder load-center, should not be supplied from the combined power and lighting feeder. They should be supplied from a general power feeder or from a special feeder entailing a consequent increase in distribution cost. Even general power feeders have connected to them resistance and motor loads which require close voltage regulation. If the fluctuating load is of sufficient magnitude, it would cause voltage changes at the substation which would affect the other feeders. Such voltage fluctuations would not be permissible and would prohibit connecting the load to the system.

The automatic tap-changing regulator has been used to a small extent for quick voltage adjustment. It requires about three seconds to go from full buck to full boost. It is limited in ampere capacity and voltage, it costs more than the induction regulator and has a high maintenance.

The synchronous booster is fast but expensive; it is noisy and has a high maintenance.

The saturated-core regulator can go from full buck to full boost in about one or two seconds, which is two to three times as fast as the tap-changing regulator and five to eleven times as fast as the induction regulator. While one or two seconds may not be a fast enough adjustment for some loads, it should be sufficiently fast to permit connecting most fluctuating loads, which previously required special feeders, to the regular feeders and connecting loads to the system which could not otherwise be connected. Where the tree-system of distribution feeder is used, the loads between the substation bus and the load center will still have voltage fluctuations. These fluctuations are due to the voltage drop from the substation bus to the load-center and for a given circuit the amplitude of the fluctuations depends on the proximity of the load to the substation. The load-center system of distribution eliminates this difficulty. Any loads connected to the same branch or mains as the fluctuating load are also subject to voltage fluctuations, the amplitude of which depends on the distance from the load center. Many important industrial loads which require close voltage regulation could be connected to mains other than the one supplying fluctuating loads. In other cases, it may be more economical or feasible to equip the industrial load with a saturated core regulator.

The saturated-core regulator is applicable to laboratory work where close regulation and fast operation are required to give a stable condition of voltage. A saturated-core type regulator was developed for the Mellon Institute of Industrial Research, Pittsburgh, Pa. to correct the voltage fluctuation of $\pm$ 6 volts occurring over their 110-volt supply. Very close regulation was needed in order to keep constant voltage on various heating units under tests.

The saturated-core regulator has higher losses and costs about 25 per cent to 50 per cent more than the induction regulator. It, therefore, might be used in place of the induction regulator only when fast adjustment is required or in high-voltage circuits. The saturated core regulator is still in a developmental stage and is not available for general commercial use.

Current-Limiting Reactor

The most frequent disturbances to which a transmission and distribution system is subjected are short circuits or heavy overloads which cause excessive currents, low frequency, and low voltage over the system. The effect over the system of these disturbances depends on the layout, extent, and capacity of the system and its protective equipment. The protective equipment consists first, of relays and oil circuit breakers to remove the faulty circuit from the system, and second, of series reactors or neutral grounding resistors to limit the amount of current flow into the fault.

The air-core type of current-limiting reactor is used in busses, tie lines, feeders and generator circuits, as

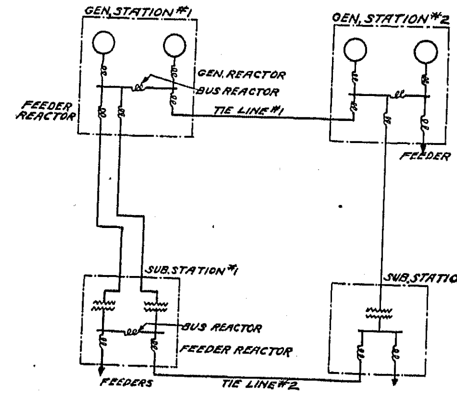


Fig. 1—Application of Current-Limiting Reactors to a Central Station System

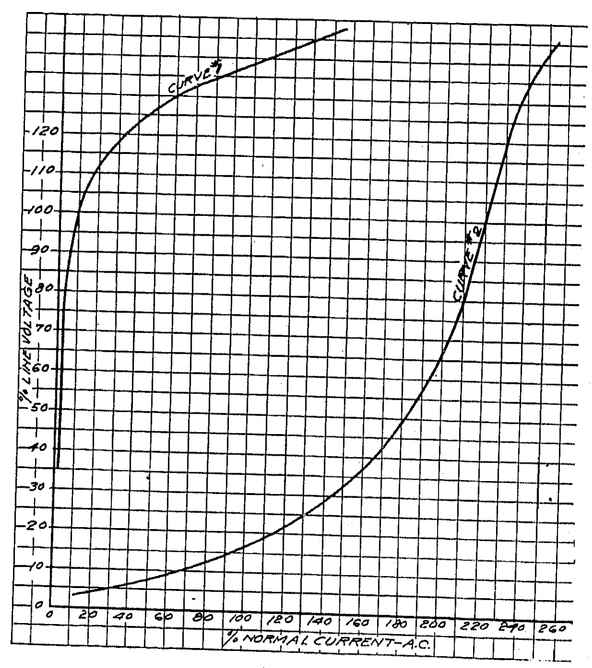


Fig. 2—Characteristic Curve of the Iron Core Reactor with and without d-c. Excitation

shown in Fig. 1. The voltage drop across the air-core reactor increases directly with the current flowing through it. The voltage drop across the iron core reactor increases directly with the current up to a certain point only, and then the voltage curve bends over as shown by Curve 1, Fig. 2. If the iron core reactor is magnetically saturated by the application of direct current, then the voltage drop across the reactor increases in a greater proportion than the cur-

rent between certain points as shown by Curve 2, Fig. 2. It is this characteristic of the saturated core type reactor which will determine its application in place of the air-core type reactor. The reactor can be designed with a ratio, of reactance under short circuit to reactance at normal load of 3 to 1, *i. e.*, if the normal load reactance is 10 per cent, the reactance under short circuit will be about 30 per cent. Three and one third times normal, then, would be the maximum current possible to force through the reactor at normal voltage of the system.

The change from normal value of reactance to short-circuit value is not instantaneous, but is similar to the time element of the synchronous reactance of an alternator under short-circuit conditions. Magnetic flux in the core cannot change instantly, but changes along an exponential curve of time. Approximately full reactance will be reached only after 0.08 to 0.16 sec. Since the reactance is not instantly available, the saturated core type reactor possesses no advantage over the air-core type reactor in protecting a system from the initial electromagnetic forces under short-circuit conditions. It does, however, reduce the current to be interrupted by an oil circuit breaker in series with the reactor, since the full protective effect is realized in a time shorter than the usual interval between the beginning of the short circuit and the parting of the oil circuit breaker contacts.

Current-limiting reactors are used in generator circuits of large modern generating stations, using the isolated phase arrangement of busses, to prevent a generator failure from being virtually a bus short circuit. They are also used in the circuits of generators of old design which will not stand short-circuit stresses. Bus reactors are used to sectionalize the main bus to limit the amount of power that can be transferred from a good bus section to a faulty bus section. Feeder reactors are used to limit the amount of power flow into a feeder fault. Limiting the flow of power into a fault by reactors reduces the duty on oil circuit breakers, the stresses and heating of busses, of cables, of current transformers and of disconnecting switches, and maintains the system voltage. Reactors in tie-lines between stations, substations and different systems are effective in limiting power transfer through the tie-lines under short-circuit conditions. They are particularly desirable where a small kv-a. capacity station, substation or system ties in with a large kv-a. capacity station, substation or system. The air-core type of current-limiting reactors successfully meets most requirements for generator, bus, feeder and tie-line reactors. In case the air-core reactor, to give the protection required, would cause too large a voltage drop under normal load, the saturated core reactor may then be substituted. In such cases, the saturated core reactor could be used giving approximately 33 per cent of the reactive drop of the air-core reactor under normal load. This type of reactor has several times the loss of the air-core type and a much greater cost, both of which will prevent its use except in special cases.

Application of the Saturated-Core Type Reactor to Station Auxiliary Bus

To improve the station economy and insure continuity of service, house alternators for the station auxiliaries are operated in parallel with the main generators. The house alternator is usually of a lower voltage than the main generators and is usually connected to the main generator bus through house transformers. The power transfer through the house transformers may be from the main generator bus to the house service bus or vice versa, depending on operating and load conditions.

Certain auxiliaries such as boiler feed water pumps, circulating water pumps and motor-driven exciters are essential to the operation of the station and they must be protected from interruption due to system disturbances.

The effect of the system disturbances on the continuous auxiliaries can be reduced by dividing the house service bus into sections with a current-limiting reactor

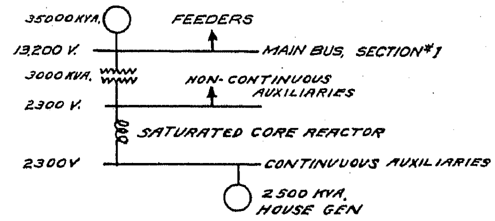


Fig. 3—Saturated-Core Current-Limiting Reactor Applied to Station Auxiliary Bus

as shown in Fig. 3. The house transformer and non-continuous auxiliaries would be connected to one section and the house alternator and continuous auxiliaries to the other section. The effectiveness of the reactor depends upon its ohmic value, the greater the reactance the less effect will the disturbance have on the house alternator and continuous auxiliaries. The reactor will not be required to carry the full kilovolt ampere output of the house generator or transformer and, therefore, may be designed for a smaller voltage drop than if designed for full output of either house transformer or generator. Even under this condition the ohmic value required will probably be so high that the voltage drop would be prohibitive. Therefore, the air-core type of reactor is undesirable for this service since its ohmic value is constant. The saturated-core type of reactor may be suitable for this service since its ohmic value increases with an increase in alternating current. The saturated core reactor would be designed for the required reactance under short-circuit conditions. The reactance under normal conditions could be about 33 per cent of the short-circuit reactance.

An installation of saturated-core current-limiting reactors connected similar to Fig. 3 is now in operation at the new Cahokia Station of the Union Electric Light & Power Co. near East St. Louis, Ill. The house transformers are rated at 3000-kv-a. and the house turbine at 2500 kv-a. The reactors are designed to carry 660 kv-a. under normal conditions at a 15 per cent reactance drop. The short-circuit reactance is approximately 38.5 per cent, which means that it is impossible to force more than 1710 kv-a. through the reactors at normal voltage. If the house generator should be carrying full load with 660 kv-a. flowing from the continuous bus through the reactors to the non-continuous bus, it would be impossible to add more than 1050 kv-a. or 42 per cent overload to the house generator in case of a disturbance on either the system, main generator bus or non-continuous bus. If it is required that the reactor shall carry the continuous auxiliaries in case of the house alternator failure, the

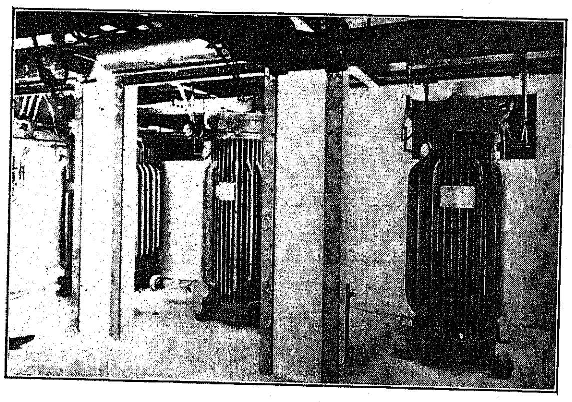


Fig. 4—Installation of Single-Phase Auxiliary Bus Saturated-Core Reactors at the Cahokia Station of the Union Electric Light & Power Company

reactor would have to be designed with less reactance but would not be so effective. This requirement may be best taken care of by oil circuit breaker and relay equipment which would shunt the reactor in case of house alternator trouble. Fig. 4 gives some idea of the general appearance and size of the reactors. Each single-phase unit requires 15.7 amperes, 125-volt d-c. excitation. The estimated total loss is approximately 7 kw. for each unit.

Variable Reactance

The saturated-core reactor may be used as a variable reactance by varying the d-c. excitation. It is being used in this manner in transoceanic radio communication to modulate the antenna current of the Alexanderson alternator and to control the speed of the induction motor driving the Alexanderson alternator.

When used to modulate the antenna current the variable reactance is known as the "magnetic amplifier." It is inductively coupled to the antenna circuit by a special combination of tuned circuits. By this method the small current in the d-c. winding of the amplifier can be made to control a very large antenna current. The d-c. circuit may be controlled by a telegraph key or amplified telephone current. Because of its fast magnetic action it is well adapted to high-speed telegraphic transmission and to telephonic transmission.

When used to control the speed of the induction motor the reactor is connected in series with the motor primary leads. The d-c. excitation of the reactor is controlled by a voltage regulator similar to a generator voltage regulator. The voltage regulator is controlled by a rectified current from a tuned circuit connected to one of the coils of the Alexanderson alternator. The tuned circuit resonates at a frequency slightly above normal. When the antenna circuit is detuned by the magnetic amplifier, the alternator drops its load. In order to reduce the power input to the motor, after losing its load, the speed tends to increase but is checked by the action of the tuned circuit, the regulator and the reactor which reduces the voltage applied to the motor. Reducing the voltage applied to the motor reduces the power input to the motor to correspond with the light load condition at normal speed. A more complete description of the magnetic amplifier and speed-regulating equipment may be found in the *General Electric Review*, October, 1920.[1]

The saturated-core reactor could be used as a variable shunt reactance to control the lagging kilovolt-ampere required to counteract the charging current of very long, high-voltage transmission lines during light load conditions, but up to the present time they have seemed unnecessary as synchronous condensers are required to supply leading kilovolt-amperes under heavy load conditions and either inherently possess or can be built with sufficient lagging kilovolt-amperes to meet the requirements at little added expense. For example, a 250-mile 220-kv. transmission line requires at no load, 31,200-kv-a. lagging at the receiver end to maintain the receiver voltage equal in value to the generator end. For a 120,000-kw. unity power factor load 16,500-kv-a. leading is required to maintain the receiver voltage equal to the generator voltage. Some condenser capacity is necessary for correcting the load to unity power-factor which would be approximately 58,000-kv-a. for a 0.90 power-factor load. Then, the total condenser capacity required is about 75,000 kv-a. Three 25,000-kv-a. condenser units of normal design could supply 55,000 lagging kv-a. which is much more than is required for the no-load condition.

Discussion

PAPERS ON CURRENT-LIMITING REACTORS

(Oesterreicher, Stephens and Kierstead, Dann, Boyajian, Blake)

Chicago, Ill., June 26, 1924

N. L. Pollard: There are only a few points which I wish to emphasize. One is the question of thermal capacity which has been causing the committee considerable worry during the last two years.

If reactors having too small a thermal capacity are used,

1. "The Transoceanic Radio Communication" by E. F. W. Alexanderson.

"The Alexanderson System for Radio Communication" by Elmer E. Bucher.

it is possible for them to fail thermally in case the relays on the circuit do not function properly. On the other hand, if reactors having a long-time thermal capacity are purchased, the cost is excessive. In order to get some idea of the increase in cost, I secured a quotation from one manufacturer for a reactor having a thermal capacity o' 13 seconds and an alternate price on another reactor of the same capacity having a thermal capacity of 22 seconds. The price of the 22-second thermal capacity reactor, if I recall correctly, was approximately 35 per cent greater. The price increases very rapidly as the thermal capacity goes up.

Some operating men are of the opinion that if a reactor has a thermal capacity slightly in excess of that of the cable, it will be sufficient.

Mr. Dann brought out in his paper the fact that there were two extreme points of view. One that the reactor might act as a fuse and be the first to fail in the circuit; the other, that the reactor should have enough thermal capacity so that it would be the last thing in the circuit to fail. The reactor is supposed to be a protective device and cuts down the amount of the short circuit so that the oil circuit breaker can safely rupture it. If it should fail, the oil circuit breaker might not be able to rupture the increased current and what was originally a feeder short circuit becomes a bus-short circuit. This, as we all know, is a very serious matter in a large generating system.

From what information the committee has been able to obtain, the failures this last year have been fewer, although there have been several serious failures which only demonstrate the fact that the reactor is not yet a perfect piece of apparatus and that improvements can be made that will make it even better than it is at present. During the last few months several serious failures have been called to my attention.

R. S. Schurig (by letter): I wish to discuss the paper by Messrs. Kierstead and Stephens and wish particularly to refer to the mechanical stresses occurring in the insulators during short circuits. The test data submitted by the authors in Fig. 10 show clearly that the observed peak stress on the braces is slightly in excess of the peak electromagnetic force. The authors thus reach the important conclusion that the peak stress on the braces is likely to exceed not only the *average* electromagnetic force but also the *peak* electromagnetic force.

The peak stress will, of course, be a maximum in the immediate vicinity of resonance.[6] Since the peak stress varies materially from a relatively low value in some cases to a relatively high value at resonance, it is desirable to know—

1. How to predict roughly whether a reactor is resonant or not and

2. How to estimate the peak stress at resonance.

These two items will be discussed in the following:

1. In order to predict whether a reactor is resonant, one must determine its natural frequency, which may roughly be calculated from the following approximate formula:

$$f = 3.13 \sqrt{\frac{S}{W + \frac{I_0}{r^2}}} \text{ cycles per sec.} \quad (1)$$

where

S is the stiffness of the reactor unit in pounds per inch, the stiffness being the force (applied at the center of gravity of the reactor and acting in the direction of the displacement) to give unit deflection. In the set-up of Fig. 8, for instance, the stiffness would be measured in a horizontal direction.

W is the weight of the reactor in pounds.

r is the distance from the base of the foot insulators to the center of gravity, assuming that the reactor when displaced rocks about an axis in the plane of the base of the foot insulators. The minimum value of r for a rigid reactor is the distance from the base to the center of gravity, not including the height of the base insulators.

I_0 is the moment of inertia of the reactor about an axis perpendicular to the vertical axis of the reactor and passing through the center of gravity. If the reactor is considered as a hollow cylinder having outer and inner radii R_1 and R_2, and a height h, measured without insulators, all in inches, I_0 is

$$I_0 = \frac{W}{4}\left(R_1^2 + R_2 + \frac{h^2}{3}\right) \text{lb.-in.}^2 \quad (2)$$

The above formula for natural frequency is based on the following assumptions:

(1) that the reactor displacement is one of translation and rotation combined. In the total absence of rotation the natural frequency becomes simply:

$$f = 3.13 \sqrt{\frac{S}{W}} \text{ cyc. per sec.} \quad (3)$$

(2) that the displacement is elastic. Insulators are, of course, not perfectly elastic, but tests of busbar insulators, for instance, have shown that their behavior, in vibration, may be closely approximated by assuming them elastic.

(3) that the damping is small.

(4) that, for the derivation of the moment of inertia I_0 as expressed in (2), the center of gravity of the reactor is at the geometrical center of the unit and that the mass is uniformly distributed within the volume of a hollow cylinder.

(5) that the reactor base insulators are bolted to a rigid, massive foundation.

Example 1. Reactor without Braces

Estimate the natural frequency, when the constants are:

W = 1550 lb.
R_1 = 16 in.
R_2 = 6 in.
h = 30 in.
r = 15 in.

$S = \frac{1000}{\frac{1}{32}} = 32{,}000$ lb. per in. assuming that a horizontal steady force of 1000 lb. applied at the center of gravity of the reactor displaces the center 1/32 in. horizontally.

Solving first for I_0:

$$I_0 = \frac{1550}{4}\left(256 + 36 + \frac{900}{3}\right) = 230{,}000 \text{ lb. in.}^2$$

Then

$$\frac{I_0}{r^2} = \frac{230{,}000}{225} = 1020 \text{ lb.}$$

It is seen that the effect of the rotation due to rocking is equivalent to adding more than 1000 lb. to the weight of the reactor. Hence the natural frequency is from (1)

$$f = 3.13 \sqrt{\frac{32{,}000}{1550 + 1020}}$$

or f = 11 cycles per sec. approximately.

If no rocking occurred, *i. e.* if the motion were pure translation, the frequency would approximately:

$$f = 3.13 \sqrt{\frac{32{,}000}{1550}}$$

or f = 14 cycles per sec.

Example 2.

Reactor similar to that of example 1, but ***equipped with braces,***

6. Resonance occurs when an electromagnetic force having a frequency equal to the natural frequency of the reactor is applied, the natural frequency being the frequency of free vibrations, such as would occur, for instance, if a heavy direct-current were suddenly applied to a pair of adjacent reactors.

such that a steady horizontal compression force of 20,000 lb. applied to two adjacent reactors at the level of their centers of gravity shortens the distance between the centers by 0.01 inch. Hence

$$S = \frac{20{,}000}{0.005} = 4{,}000{,}000 \text{ lb. per in.}$$

If the other constants are as above, the natural frequency is roughly, from (1)

$$f = 3.13 \sqrt{\frac{4{,}000{,}000}{1550 + 1020}}$$

or $f = 123$ cycles per sec.

If total absence of rotation were assumed here, the natural frequency would be

$f = 159$ cycle per sec.

It is seen then that, for a reactor of a given size, the stiffness of the base supports and braces is the chief factor determining whether or not mechanical resonance can occur. Thus, a reactor which is not resonant when set up without braces, may become resonant or near resonant when braces are used.

(2) *The resonant peak stresses* may be calculated[7] by assuming elastic supports and braces, if the following items are known:

initial value of average electromagnetic force, (defined as the electromagnetic force due to the r. m. s. initial short-circuit current).

natural frequency of reactor

mechanical damping of reactor unit

decrement of short-circuit current.

If the current decrements given by Hewlett, Mahoney and Burnham,[8] for heavy short circuits (*i.e.*, system reactance up to 30 per cent) are used, and the free-vibration decrements due to the mechanical damping of the reactor insulators are assumed to be the same as those for busbar insulators,[9] the calculated peak stresses, expressed in terms of the initial value of average electromagnetic force, are approximately as follows for 60-cycle short-circuits:

If the natural frequency is one tenth the current frequency or lower, the maximum stress is 1.0 times the initial average magnetic force.

If the natural frequency is half the current frequency, the maximum stress is 2.2 times the initial average magnetic force.

If the natural frequency is equal to the current frequency, the maximum stress is 4.5 times the initial average magnetic force.

If the natural frequency is twice the current frequency, the maximum stress is 3.2 times the initial average magnetic force.

If the natural frequency is four times the current frequency, or larger, the maximum stress is 2.5 times the initial average magnetic force.

For 25-cycle short-circuits, the peak stresses are somewhat reduced being as much as 25 per cent lower.

The above data indicate that the highest stress is due to resonance at current frequency, while the stress due to resonance at twice the current frequency is less. The high resonant stress at current frequency is due to the d-c. component of short-circuit current.

It follows from the above figures, that a low natural frequency, well below half the current frequency, gives lower stresses than any high natural frequency whether resonant or not.

A further point to which attention is to be called is the occurrence of *tension stresses in the braces*. The reactors although connected so that the average stress on the braces is compression, resonance, or near-resonance, will bring about heavy tension stresses of the order of twice the initial average electromagnetic force.

R. A. Hentz: That a reactor is a protective device pure and simple has been pointed out and this fact should be kept prominently in mind. In large metropolitan systems the reactor is second only to the oil circuit breakers in protecting the system. Furthermore, it aids the circuit breakers in performing their duty by keeping the tremendous concentrations of energy occurring at times of short circuit within safe limits.

Unlike the oil-circuit breaker, some part of the cost of which may be charged to the normal operation of the system, the current-limiting reactor must function entirely as a protective device and the entire cost is chargeable as a premium on the insurance obtained from such protection. This premium is very high, consisting as it does not only of the operating losses in the reactors but also of very considerable capital charges. These capital charges include not only the reactors themselves but also, what is a very sizeable item, the extra cost of building and compartments to house them and the leads and insulators to reach them. Obviously with such a heavy charge for protection, operating engineers want to be sure that they are actually obtaining the insurance being paid for. With this before us, it hardly seems that the matter of reliability could be stressed too emphatically and by reliability is meant not only that of the reactors themselves for which the manufacturers are responsible, but also that of the design and installation made by the operating companies.

The thermal capacity of reactors has been touched upon and this is a most important consideration affecting their reliability. Certainly, any protective device, which due to insufficient capacity, would burn up just at the time it was to justify its installation, should not be considered. The total cost including building, compartments, etc., is so large that the little extra investment for increased copper cross section to obtain a device adequate for all conditions to which it may be subjected, is well worth while. Indeed, when considering the total investment, the difference between reactors with scant copper and ones built upon safe design is a very small percentage, and this is partially returned through decreased operating losses. It would seem that a copper cross section with a thermal capacity above that of the underground cable it protects is advisable.

Reference was made in Messrs. Kierstead and Stephens' paper to shunting resistors. The effectiveness of these devices is something about which definite information is much to be desired. Some of the early troubles with the reactors, thought at first to be over-voltage but later determined to be insufficient mechanical strength, and referred to at the bottom of the third page, second column, of this paper, occurred on The Philadelphia Electric system, and a large number of resistors were added. Of a total of some 378 feeder coils supplying mainly underground cables, about one-third of these have shunting resistors. Uniformly good service has been obtained from coils with and without resistors.

A number of uses of reactors have been brought out in the several papers this morning, but one to which I personally am somewhat partial has not been touched upon, namely, their use for starting large 2300-volt motors driving generating-station auxiliaries. It is now generally considered that the best way of starting these motors is on full voltage, but in certain cases, particularly with some high-speed motors, this is not advisable, and in such cases reactor starting is a very satisfactory method.

7. The theory of the calculations is that employed in the paper by Doherty and Kierstead "Short-Circuit Forces on Reactor Supports"; A. I. E. E. JOURNAL, August 1923.

8. "Rating and Selection of Oil Circuit Breakers," A. I. E. E. TRANS. 1918, p. 123.

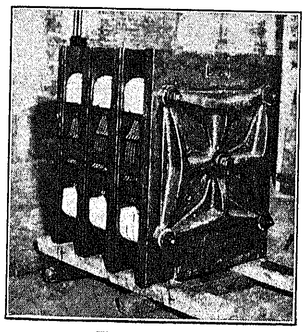
9. The amplitude of free vibrations of bus insulators was found by tests to diminish as $e^{-0.5 \frac{q}{n} t}$ diminishes with time, where q is 2π times the natural frequency of the insulator and n, the sharpness of resonance, is a constant averaging about 5 for a variety of tests of porcelain bus supports. Thus for a 20-cycle support, the decay of the free vibrations is expressed by

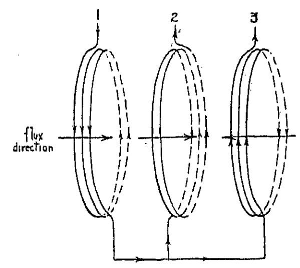
$$e^{-0.5 \frac{q}{n} t} = e^{-12.6\, t}$$

It is quite likely that the reactor insulators and busbar insulators have similar rates of decrement at corresponding natural frequencies, because in both cases the stress is absorbed by insulator units consisting of porcelain members held in metal supports.

It possesses an additional advantage where differential relay protection is used, as the currents in the line and in the motor neutrals are always equal. We have several motors driving air and condensate pumps so equipped, and they start with entire satisfaction.

W. B. Kirke: In the paper by Messrs. Kierstead and Stephens on Current-Limiting Reactors, the authors make a general statement that in three-phase reactors the mechanical forces between coils will be repulsive if symmetrically connected, and attractive if the middle phase is reversed. This statement is true only if one is considering the average force throughout a cycle after the transients of the short-circuit currents have died out. If one is considering peak forces during the transient periods there are times when repulsive forces of very great magnitude can be built up between an outside coil and the other two of a three-phase reactor, even with the middle phase reversed. This condition is shown in the accompanying diagram where the instantaneous value of the current is near the maixmum in an outside coil and returns in the other two.





Three-Phase Current-Limiting Reactor

Due to the very great space limitations in the galleries of the two Waterside stations of The New York Edison Company, we were compelled to install reactors of minimum dimensions. Three-phase insulated-winding type of coils offered the only practical possibility. In these reactors there are developed under the worst possible conditions with a short circuit at the terminals of the reactors, with full 11,000-volt connected capacity and with the short circuit occurring at a certain point on the voltage wave, repulsive forces whose values are in the neighborhood of 100 times the weight of the reactors. To withstand such forces, the reactors are equipped with non-magnetic manganese-steel headers which are designed to withstand a breaking load of 400,000 lb. They are held together by four chrome-nickel steel bolts and one center bolt of Tobin bronze. The headers are designed to eliminate, as much as is consistent with mechanical strength, circulating currents induced in the headers. This is accomplishd by placing the metal in four Y sections which gives the minimum volume of metal in the positions where the currents circulate.

Joseph Slepian: The matter of resistor-shunted reactors has received considerable discussion in the past, and Messrs. Kierstead and Stephens mention this briefly in their paper. It is not often that we get in such a brief account a clear statement as to two rather distinct actions that such a resistor may have. First of all, it may convert energy into heat, and also being a conductive shunt, it may by-pass energy. Messrs. Kierstead and Stephens point out that there will be dissipation of energy in the case of surges from the outside of the system coming into a protected station. On the other hand, they point out that the resistors will permit energy to pass out from any disturbance that occurs within a station on the line. It is clear that these actions do take place, but the action of passing energy may take place equally well when the disturbance occurs outside on the system and the resistor will permit some of this energy to pass by the reactor and into the protected station.

It is on this ground, I believe, that a good deal of theoretical opposition to shunting resistors has been based. It is evidently a quantitative matter. It is the relation between the value of the resistor and the values of the circuit constants that is going to determine what effect is predominant, and a general qualitative statement cannot, I believe, be made. The question to be determined is whether the constants that arise in practise and the resistance values that may be used are of such values as to make one action or the other predominant.

Mr. Boyajian's paper has been of very great interest to me, and on one point I believe I differ a little. In the curve for the d-c. excited reactor under variable a-c. excitation, Mr. Boyajian, as I understand it, correlates or identifies the first bend with the small inflection that one gets in the magnetization curve on direct current. It seems to me that is not at all an essential element here and that one would get the same effects that Mr. Boyajian speaks of if this inflection were not there. One must consider a complete curve for positive and negative excitations (Fig. 1 herewith). The little inflection at the origin is then hardly apparent. With zero d-c. excitation, if one super-imposes an alternating magnetomotive force the ampere turns vary between two equally spaced limits on either side, and the flux variation are determined by the curve above and below the axis. If one gives, however, a d-c. excitation that brings one to a point as in Fig. 2, then superimposes alternating magnetomotive force, the extremes of magnetomotive force then occupy points in the curve that are not symmetrical with respect to the origin. One may get only a small change of flux with one direction of current and a large change of flux with the other direction of the alternating current. It is evident that we may interpret the application of direct current as a shifting of the origin on the excitation curve, so that with direct current imposed the curve may be shifted up, or with large enough direct current it may be shifted so that the lower saturation is shifted through the origin and the curve becomes displaced.

When we have two cores, with opposite d-c. excitations, and add the two fluxes together the resultant curve will be as in Figs.3 and 4. With the alternating current applied, for small alternating current, the extremes of current give only small changes of flux, but with large alternating currents the extremes of flux begin to be large. Thus I identify the lower bend in Mr. Boyajian's curves with the saturation and not with the inflection that one gets for small magnetizations, in the usual virgin magnetization curve.

Another point of considerable interest to me is the relation that Mr. Boyajian gives between the kv-a. of controlling frequency in such apparatus as Alexanderson's modulator, and the controlled frequency. Sometime ago I carried on an analytical investigation as to what relations necessarily exist between powers, which may be exchanged between frequencies in an iron reactor, and for simplicity and shortness of time I will just state the nature of the results for a simple inductance. When an inductance has iron in it, instead of there being a straight-line relation between flux and ampere-turns, it is curved, and we may express the inductance of a coil as a power series in terms of the current. That is $L = L_0 + L_1 i + L_2 i^2 + \ldots \ldots \ldots$ The constant terms would correspond to ordinary inductance, and we are all familiar with its effects. It is the higher terms that tie the frequencies together. If we consider the second term alone

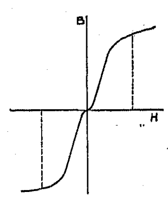


Fig. 1

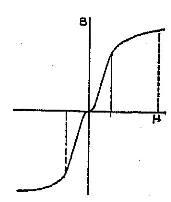


Fig. 2

by itself, we find that it operates to tie frequencies together in groups of *threes* and exchange power between these frequencies. A group of three frequencies which this ties together will necessarily be related in such a way that the sum of the frequencies with properly chosen signs will equal zero.

Then there is a further very interesting relation between the powers that are exchanged between these three frequencies. If power P_1 of frequency F_1, power P_2 of frequency F_2 and power P_3 of frequency F_2, are tied together by the term L_1 in a power exchange, then these powers are all proportional to their respect-

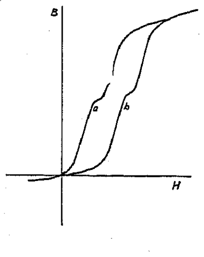


FIG. 3

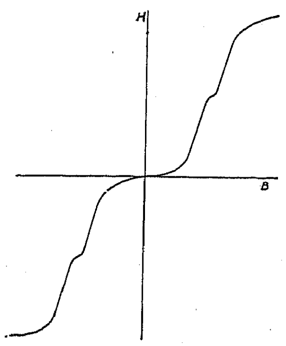


FIG. 4

ive frequencies, and the sum of the powers with the properly chosen signs must necessarily be zero.

Just as an example to make this clear, we may take the case of the Alexanderson modulator or amplifier. We may have normal radio frequency of 50,000 cycles, and a controlling frequency of 1000 cycles. Then as the result of the interaction in the iron core, some energy from the 50,000 cycles may be converted into 51,000 cycles. The first relation which I mentioned is satisfied because 50,000 + 1000 − 51,000 = 0. Furthermore, the powers corresponding to the three frequencies are proportional to the frequencies themselves, so as Mr. Boyajian points out, if you have a controlling frequency of 1000 cycles and a controlled frequency of 50,000 cycles the power of the 1000-cycle frequency is to the power of the 50,000 frequency input as one is to fifty.

V. Karapetoff: Designers of reactors would benefit by a wider use of the concept of permeance, both in testing and in computing reactors. Generally speaking, the inductance of a coil can be written in the form

$$L = n_c^2 P_c + \Sigma n_p^2 \Delta P_p \qquad (1)$$

where n is the number of turns, P is a permeance, the subscript c stands for "complete" and the subscript P for "partial."[10] The two terms on the right-hand side can be combined into one, using the so-called equivalent permeance, so that eq. (1) becomes

$$L = n_c^2 P_{eq}$$

This formula will be found useful in predetermination of the inductance of a coil from test data. Let a coil of dimensions a, b, c, and whose number of turns is N, have an inductance L, determined by an actual test. The equivalent permeance of the coil is

$$P_{eq} = L/N^2 \qquad (3)$$

Let it be required to design a reactor of the same proportions but for a different inductance L_1. Let the dimensions of the new reactor be $m\,a$, $m\,b$, $m\,c$, and the number of turns N_1. The permeance of similar magnetic circuits varies as their linear dimensions (as is also the case with the conductance of similar electric circuits). Thus, the permeance of the new circuit is $m\,P_{eq}$, and its inductance is

$$L_1 = B_1^2 P_{eq}\, m \qquad (4)$$

Knowing L_1 and P_{eq}, the designer can choose m and N_1 to satisfy this equation. Out of an infinite number of solutions he can select one which gives a minimum cost, or a minimum loss, or standard dimensions, etc.

The foregoing formulas are also useful in judging about the best proportions of a coil. It will be seen from eq. (1) that for a maximum inductance the permeance must be as great as possible, and moreover as many linkages as possible must be complete rather than partial. This means short magnetic paths of large cross-section and avoiding turns which link only with a small fraction of the total magnetic flux.

The concept of permeance is also useful in the determination of electromagnetic forces acting upon a reactor (*ibid.*, the last chapter). The derivative of the permeance in a given direction is proportional to the component of the mechanical force acting in that direction. Therefore, these forces can be estimated on the principle of virtual displacements. Moreover, mechanical forces become zero when the inductance reaches its maximum, because then its derivative becomes equal to zero. This shows the advantage of choosing coil proportions corresponding as nearly as possible to maximum inductance. Mechanical stresses are then smaller and can be more readily controlled.

So far the shape of protective reactors has been largely determined by manufacturing considerations and by space economy. It would be of interest to ascertain, both experimentally and mathematically, nature's own shapes of maximum inductance. The following experiment is suggested. Let an extremely flexible coiled conductor of a definite length be immersed in transparent insulating oil, and supported at several points say by means of floating corks, to be in an indefinite mechanical equilibrium. Let a strong d-c. current be passed through the conductor. The conductor will then tend to assume the shape of a coil of maximum inductance, and useful conclusions may be drawn from its final shape. Guides and obstacles may be placed in the tank to limit the motion in certain directions, and in this way practical shapes may be obtained, perhaps unknown at present.

In Fig. 5 of the paper by Kierstead and Stephens the mechanical forces are shown to be of the nature of compression, indicating that the reactor tends to increase its permeance to a maximum by shortening the magnetic paths and increasing their cross-section. On the other hand, in Mr. Oesterreicher's Fig. 20 some of the arrowheads seem to indicate a repulsion between the turns. While it is possible to connect turns or layers for mutual repulsion, such an arrangement would correspond to a non-inductive winding and therefore would not be suitable for a protective reactor. The sketches in these two papers should be reconciled.

Mr. Oesterreicher's Tables I and II are not clear. If a summation over half a cycle is desired, the readings corresponding to 180 deg. should be omitted because they belong to the beginning of the second half cycle.

S. I. Osterreicher: The two paramount requirements of every reactor are thermal capacity and mechanical strength. From the standpoint of the design, these two requirements go hand in hand. Large thermal capacity during short circuits demands large conductor volumes, which automatically gives the reactor mechanical strength. The limitations imposed upon these conditions are mostly economic and partly space requirements. Both are within the sole control of the operating engineer.

Regarding the conductor stranding of which Mr. Dann speaks, I believe it is well to use as large individual wires in a stranded cable as consistent with a conservative design. The advantage of large individual wires is a natural stiffness of the stranded cable. During short-circuit stresses, this cable stiffness gives the reactor additional mechanical strength.

To limit the eddy currents to reasonable values in a coarse cable stranding, our firm uses enameled wires in stranded cables. The eddy-current losses in such enameled-wire cables are equal to or less than, those in the fine stranding.

Among the many interesting points of the Kierstead and

10. See V. Karapetoff, "The Magnetic Circuit," Chap. X.

Stephens paper I find a number in which I readily agree. To a few however I wish to make some comments. One of the statements—that reactors must be made of fire-proof materials only—needs additional qualification.

It so happens that we manufacture reactors made of fire-proof materials as well as some with fibrous materials. Our experience with the fibrous-type coils is as good as with the fire-proof reactors. The limiting feature in both cases is the thermal capacity. If these limits are greatly exceeded, both types suffer equally. While I am greatly interested in the test which the authors cite about a fire-proof reactor which has been driven up to 725 deg., what I would like to ask is do the authors still consider this reactor safe for central-station operation after the test.

At 725 deg. cent. the copper loses almost 75 per cent of its original tensile strength and will rupture readily when subjected to mechanical vibrations caused by the magnetic fields surrounding the conductor.

I have data of several reactor conductor ruptures caused by what I believe is nothing else than excessive heating of the copper.

The authors also mention some flashover test made upon certain types of reactors, which showed dielectric strength up to 100,000 volts. If such is the case I can see no reason for installing auxiliary devices within the reactor to protect against over voltages. If a 13,200-volt system is subjected to over voltages anywhere near the 100,000-volt mark, such system will break down at dozens of places before the critical breakdown voltage of the reactor is reached.

The expression given by Prof. Karapetoff for the permeance factor is a great help to the reactor designer. This factor is also known as the inductance or shape factor of the winding.

I believe, Prof. Morgan Brooks in University of Illinois Bulletin No. 10 as well as the late Dr. Rosa in the Bureau of Standards Bulletin calls attention to a similar permeance or inductance factor of solenoids.

In regard to Figs. 20A and 20C of my paper, which show the winding turns in repulsion toward each other, I wish to call Prof. Karapetoff's attention to my statement, "whether the forces are attractive or repulsive, depends upon the interconnection between turns or layers and upon the current distribution throughout the winding."

In multilayer windings all layers or turns cannot be placed in the same magnetic planes, thus it must be evident that their inductances and mutual inductances cannot be the same. If this is true, it cannot be claimed that all turns are always trying to locate themselves in the common magnetic center of the solenoid. From my experience, as far as the end layers and turns of a solenoid are concerned I saw a number of windings, distorted in a general direction as indicated by the arrows.

It seems that Prof. Karapetoff, himself, thinks that the experiment which he suggests if carried out, will lead to some unknown shapes in solenoid design, which I believe, can be caused only by the uneven self and mutual inductances between layers and turns. The discrepancy in the summation of Tables 1 and 2 is due to the step-by-step method used and should not be expected to be analytically exact.

F. H. Kierstead: In his book entitled "The Magnetic Circuit," Professor Karapetoff has presented a very rational method of calculating magnetic forces, which we have used and found to be of very great value not only in predetermining the exact value of a magnetic force and its direction, but also in predetermining how changes in the circuit conditions will affect the force.

In regard to the direction of the force on a particular turn of a reactor, I wish to say that it is our conception that the turn tries to move in a direction such that the inductance of the circuit will be increased. Now if a turn of a reactor moves away from the rest of the turns, the inductance will be decreased but if it moves toward the rest of the turns the inductance will be increased. Therefore, the forces on the turns of a reactor are in such a direction as to try to cause them to move together. An exception to this is conceivable in the case where the conductor of a reactor consists of several strands in parallel and the paralleling of these strands is so poorly done that the current in one of the strands is actually reversed in direction to that in the other strands. In this case, the former strand would be repelled by the latter strands. That would mean tremendous losses in the reactor, and I don't think it occurs in any modern reactor.

In answer to Dr. Slepian's discussion in which he points out that some energy is by-passed by the resistor into the protected station, I wish to state that it is our practise to make the resistance of the resistor well above the surge impedance of the circuits to which it is connected so that the energy which it by-passes is of very much reduced voltage.

We have in our paper stated that the resistors permit a portion of the high-voltage energy of a disturbance that occurs within a station to pass out into the lines. Dr. Slepian has stated in his discussion of our paper that the resistors will equally well permit a portion of the high-voltage energy of a disturbance which occurs on the lines to pass into the station. Dr. Slepian is correct in the case where there is only one feeder connected to the bus but where there are several feeders equipped with reactors and resistors a disturbance originating within the station has several paths of escape out into the lines while a disturbance originating out on one of the lines has only one path into the station. Therefore, since there is usually more than one feeder connected to a bus, the resistor will in general by-pass considerably more high-voltage energy from a station into the lines than from the lines into the station.

In regard to whether resistors are required or not, I wish to state it is not a question of protecting a reactor from over-voltages, but rather it is a question of whether a central station company wishes to obtain the advantages of the absorption of high-voltage energy which will take place in the resistor, and thus reduce the value of over-voltages which may arise on the system.

Mr. Kirke in his discussion of our paper has pointed out that in the case of a three-phase set of reactors placed coaxially and connected with the middle phase reversed, there are some conditions under which the force for a portion of the cycle will be repulsive although the average force over a cycle will be attractive. This is true but the peak value of such repulsive forces in the most favorable case is only 1/8 the peak value of the attractive force under the most favorable conditions and the average force over a cycle under the conditions which gives the highest instantaneous repulsive force is zero. It has been our experienc that in general, the distance between adjacent phases of coaxially placed reactors has been so large that the repulsive forces were not large if the middle phase was reversed.

The value of Mr. Schurig's discussion of our paper lies chiefly in the fact that it gives a method of determining the stresses on reactors. The constants he has taken necessarily are only typical of one rating of reactors, and, therefore, cannot represent the great range of reactors which are liable to be encountered.

W. M. Dann: The discussion we have had brings out that not all engineers are in full sympathy with the shunted reactor because the inductance of the reactor is a real barrier to high-frequency disturbances and it seems like folly to add something which might by-pass some of those disturbances into the circuits that are to be protected and in that way detract from the effectiveness of the barrier which is intended to keep those disturbances on the side of the reactor where they originate.

Mr. Osterreicher referred to the Cos Cob iron-core reactors. It really was a sort of bold stroke to design those reactors and put them in the circuits. I remember that at the time all of us looked with more or less awe on those big iron-core reactors.

I think that my remarks about finely stranded cable may have given Mr. Osterreicher a wrong impression because, of course, the strands in the cable are large enough to give it mechanical strength. There have been no cases of trouble that I know of where the cable itself has not had sufficient mechanical strength to withstand the mechanical stresses.

A. Boyajian: As mentioned in my paper, Messrs. W. B. Kirke and F. Dubsky cooperated with the writer in this investigation at one time, and I was very much pleased that Mr. Kirke was here and discussed the subject. I agree with him in the discussion which he has offered.

I am pleased that Dr. Slepian has verified by profound mathematics the conclusion at which I arrive from a simple physical consideration relative to the ratio of controlling and

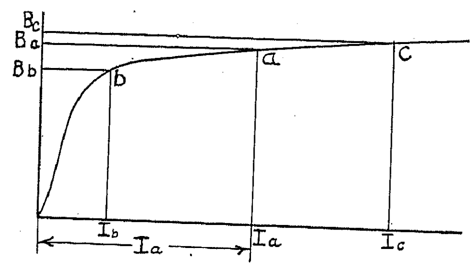


FIG. 1—D-C. MAGNETIZATION CURVE

controlled kilovolt-amperes being equal to that of respective frequencies.

As to the shape and interpretation of the volt-ampere curve, on which subject Dr. Slepian feels that he differs a little from me, I doubt if there is any essential difference. In my paper I have first stated the fact that each one of the three representative magnetization curves (*viz.*, I—the simple d-c. *B-H* curve, II—the simple a-c. volt-ampere curve, and III—the a-c. volt-ampere curve with superposed d-c. excitation) have two bends. This is a fact regardless of our interpretations, and is a satisfying parallelism to know. The bends of case III are shifted to the right as compared with those of case II, the more so the larger the d-c. excitation. The existence and position of the two bends in the case of combined d-c-a-c. excitation, I have interpreted as follows.

Referring to the accompanying Fig. 1, let the d-c. excitation correspond to the point a of the magnetization curve. Superposition of alternating current will vary the flux in the directions of b and c. In the range between b and c the alternating-current reactance will be very low and practically constant,—very low on account of the saturated condition of the core, and practically constant on account of the fact that the magnetization graph is a straight line in this zone. It follows, therefore, that for values of alternating current up to $(I_a - I_b)$, the volt-ampere curve will be straight and of small slope like the initial solid portion of Curve III (Fig. 2).

For large values of alternating current, the presence of direct current will exert only a small influence on reactance, so that Curve III must finally approach Curve II, (Fig. 2), asymptotically, never crossing it. Having the initial and final portions of Curve III, we can draw in its intermediate portion as shown dotted. At the initial part, *i. e.*, up to the first bend, wave shape is fairly free from harmonics. At the final part, that is above the second bend, wave shape is more and more that of simple a-c. excitation without d-c. Between the two bends, *i. e.*, in the part shown dotted, wave-shape has a very prominent second harmonic. The substance of this detailed explanation will be found in the text of my paper. Whether the point of view of "the shifting of the origin" offered by Dr. Slepian is any more illuminating or satisfying than the point of view of "the shifting of the two bends" (which latter, by the way, is a statement of fact) is a matter to be decided by every person for himself. In either case, recourse has to be taken to the d-c. magnetization curve for a detailed explanation and here the two explanations coincide.

I have learned from Prof. Bush that tests at M. I. T. gave smaller losses than those obtained by us, and it is hinted that the difference is due to second harmonic copper losses in the d-c. circuit. The oscillograms given in my paper indicate an entirely negligible second harmonic current in the d-c. circuit, this being accomplished by proper balanced design. The correct explanation of the differences seems to be in stray losses.

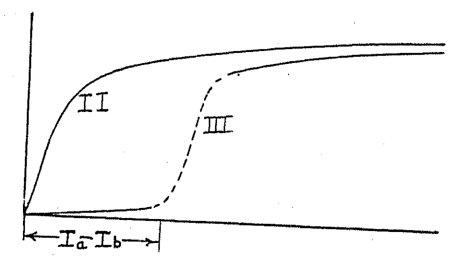


FIG. 2—II. A-C. VOLT-AMPERE CURVE WITHOUT D-C.
III. A-C. VOLT-AMPERE CURVE, D-C. SUPERPOSED

Since the writing of this paper, our units were tested with and without tank, with and without steel clamps, and considerable changes in losses were observed. Although stray losses are more or less unavoidable phenomena with large units, especially with saturated cores, it appears that the determination of their exact location and magnitude may lead to considerable improvements in losses by suitable design to reduce them to a minimum.

High-Voltage Impregnated Paper Cables

BY WM. A. DEL MAR
Fellow, A. I. E. E.

and

C. F. HANSON
Member, A. I. E. E.

Both of the Habirshaw Elec. Cable Co., Yonkers, N. Y.

Review of the Subject.—*A limit of carrying capacity was reached in high voltage cables due to high dielectric losses, and in order to overcome this limitation, science had to be called upon to introduce measurements and tests to control this quality. The exact experience gained in the development of these tests by scientific men working in cooperation with the men of practical experience enabled the cable industry to attack the next limitation that confronted it, namely, dielectric strength. The industry has already made considerable progress, and the present problem is the complete elimination of occluded air and vapor from the insulation. Air films have been regarded as causes of low dielectric strength, due to ionization of the air and consequent formation of hot spots. It is advanced, herein, that a more useful conception of the danger of air films, is that they promote internal surface leakage. It has also been generally believed that air films can be detected by the slope of the voltage power-factor characteristic. It is contended herein that such is not the case. It is pointed out that the foundation for future developments has been laid by the equipment and organization of American cable manufacturing plants for accurate quality control by continual testing of raw and process materials.*

FEW tasks are now facing the electrical engineer in which more general interest is being taken than the development of cables for higher voltages.

For many years, cables have been operating with a very fair degree of success at tensions of 19 to 25 kv. When attempts were made to make cables for somewhat higher voltages, dimensional limitations prevented the retention of the potential gradients that time had proved to be reliable and it became necessary to subject the insulation to higher stresses. It was found that at these stresses general weaknesses would develop and weak spots would be revealed in the insulation, which would not show at the lower stresses.

The first step was to reduce the dielectric loss, then to use a compound that would not be too stiff at low temperatures, and then to control the dielectric qualities of the paper. When this was accomplished, factory failures were practically eliminated and field failures greatly reduced.

Analysis of the latter showed three things: first, that at high stresses all mechanical defects, arising either in manufacture or subsequent handling, reveal themselves sooner or later; second, that air films, under certain circumstances may be the cause, and third, that operating conditions with special reference to potential surges have to be more carefully controlled.

It therefore became necessary for the manufacturers to establish a new standard of mechanical construction and to carry out elaborate researches on the elimination of air films. Much progress has been made in both of these matters during the past few years, and cables are now being made having unprecedented dielectric strengths, figures as high as 200 kv. per cm. (average stress) having been attained on single-conductor cables, while three-conductor cables for 25,000 volts and higher cannot be broken down because no way has yet been found to make the ends immune from flashing or failure at the crotch. But more important than this, these cables have unprecedented ability to withstand high stresses for long periods without deterioration.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

These results have been obtained by the careful control of materials and processes. This paper is devoted principally to a description of the principal control tests developed by an American cable manufacturer.

Four types of experimental samples were used for the research work required to ascertain the crucial properties which must be controlled and their optimum values.

1st. Impregnated sheets of paper (usually three deep) were tested between A. S. T. M. electrodes as shown in Fig. 1. Such flat samples are inexpensive, are quickly prepared and eliminate all structural complications, *i. e.*, they enable the material to be tested without introducing any element based on the mechanical make-up of a cable. Thousands of tests on samples of this character were made.

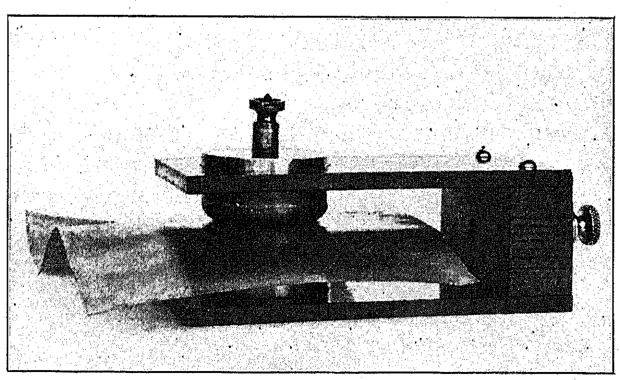


FIG. 1—A. S. T. M. ELECTRODES AS APPLIED TO THE TESTING OF FLAT SAMPLES OF TREATED PAPER

2nd. Samples of cable six in. long, the drying and impregnation of which could be carried out in a miniature apparatus are shown in Fig. 2.

3rd. Samples of cable without sheaths, about 15 in. long were treated with different kinds of impregnating compounds. Tin foil was then applied to the middle part of the sample to serve as a sheath and rolls of insulation were applied at the ends, as shown in Fig. 3 to provide a long leakage path and a barrier to prevent a flashover of the test voltage. The samples were tested with a voltage two to three times the normal

operating voltage for two hundred hours. Periodically, the samples were tested for dielectric loss or power factor.

4th. Samples of cables, with sheaths, about 15 ft. long, dried and impregnated in a small tank and tested with their ends in oil for momentary dielectric breakdown are shown in Fig. 4, or with their ends taped with asbestos for prolonged voltage applications. During the years 1922 and 1923, 526 special experimental lengths of this kind were made and tested.

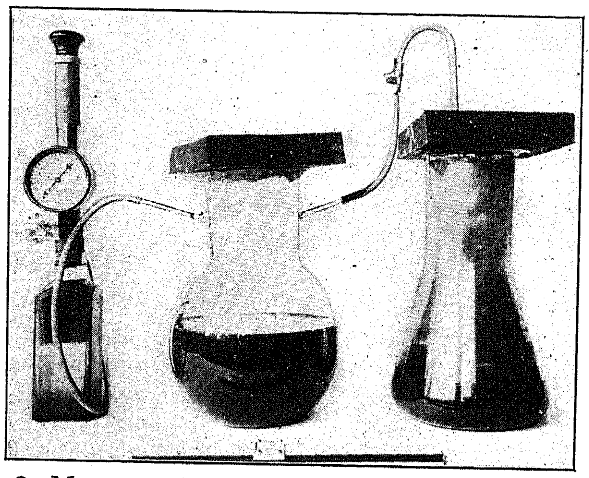


Fig. 2—Miniature Apparatus for Drying and Impregnating Cable

Some of the variables which were altered were as follows: *Oil*: Viscosity, resistivity, dielectric strength, flow point, susceptibility to oxidation, influence of rosin and other substances.

Paper: Amount of manila fiber, amount of rag stock, amount of jute stock, amount of wood stock, hydration, density, thickness, oil reluctance, tearing strength along and across grain.

Taping: Width of tape, tightness, lap.

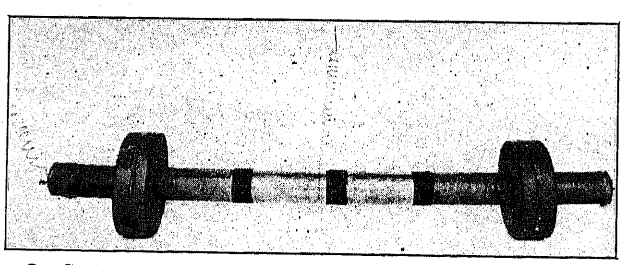


Fig. 3—Sample of Cable 15 In. Long for Artificial Aging

Drying: Time, proportion of time in air and in vacuum, temperature.

Impregnating: Time, temperature, vacuum, pressure maintenance, and freshness of oil.

Cooling: Time, final temperature, interval between tank and press, medium for cooling cable, *i. e.*, oil or air.

This list by no means exhausts all the variables and it should be clearly understood that the optimum value of each variable is not a constant, but depends upon the combination of other variables associated with it.

The experimental determination of the optimum cable, assuming five values of each variable to be tried, would therefore involve the making and testing of at least thirty-six billion kinds of experimental cables.

The reduction of this number to a workable series of tests can only be effected by the application of theory. A theoretical understanding of cable manufacture would include thermodynamics, mechanics, oil-chemistry, paper-chemistry, and electrophysics.

The 15-ft. samples were also tested or examined for some or all of the following qualities: dielectric strength, dielectric loss, ionization, freedom from air films and general appearance. Many were tested for dielectric loss both before and after an aging test at about 2½ times normal working stress.

These tests gave a large amount of valuable data but the most surprising thing learned was that most of the variables could be altered over a considerable range without essentially altering the quality of the cable. On the other hand, quality was found to be very sensitive to a few of the variables.

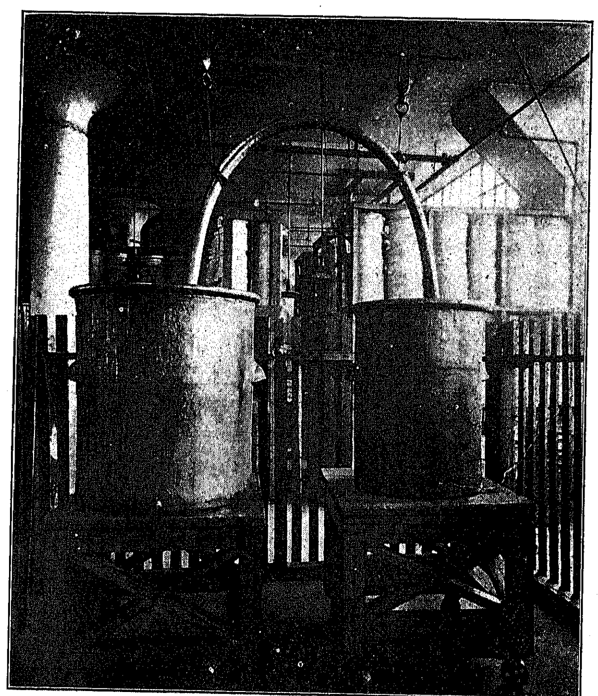


Fig. 4—Samples of Cable 15 Ft. Long Being Tested for Momentary Dielectric Breakdown

Practical advantage was taken of the information gained in these four series of tests, to establish a system of quality control based on careful regulation of those variables to which cable quality was found to be most sensitive.

The remainder of this paper is devoted principally to a description of these essential tests.

Raw Material Control

There is probably no phase of the manufacture of impregnated paper cables more important than the control of quality of the raw materials.

The essential tests on paper are as follows:

1. Dielectric strength when impregnated.
2. Tearing strength.
3. Folding endurance.
4. Air permeability, *i. e.*, compactness of fibers.
5. Oil permeability.

Dielectric strength is measured on three sheets, 10 by 16 in., dried by heating at 105 deg. cent. for 16 hr. and impregnated in standard compound for two hours at 80 deg. cent. Tests were made with a transformer at 60 cycles, using the A. S. T. M. standard electrodes shown in Fig. 1. Three sheets in series are tested at a time and 10 breakdown values are obtained on each series of sheets. A blind sample, taken from a roll of paper which was found to be very uniform, is run with each test. Errors due to differences in impregnating are eliminated by impregnating the test samples and blind sample together and expressing the results on the former in terms of those obtained on the latter. The distance between electrodes being dependent upon the thickness of both paper and oil films is, unfortunately, not strictly comparable from test to test, as it is impracticable to control the relative thickness of these two elements. Furthermore, due to the high heat conductivity of the electrodes, the breakdown voltages should be compared on the basis of the square roots of the thickness. In spite of these difficulties, the test has proven valuable for control purposes as indicated by a record for one year as shown in Fig. 5.

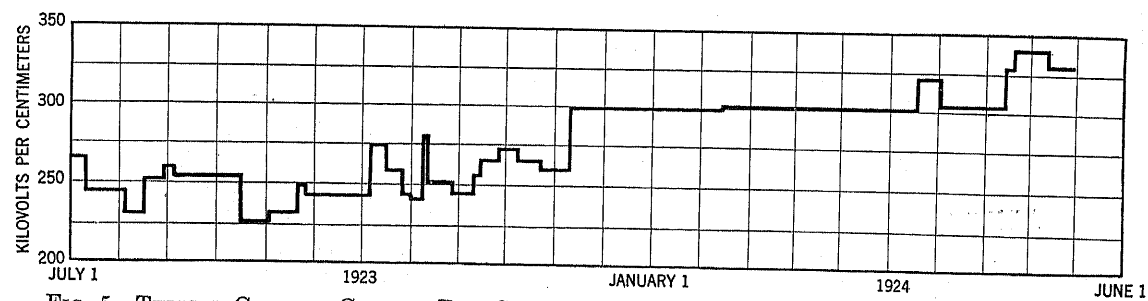


FIG. 5—TYPICAL CONTROL CHART. THIS SHOWS THE DIELECTRIC STRENGTH OF IMPREGNATED PAPER

Tearing strength is measured by an Elmendorf machine as described in the TRANSACTIONS of the A. I. E. E., 1921, Vol. XL, page 143. This instrument measures the energy required to tear a given length of paper. It is shown in Fig. 6.

Folding endurance is measured with the M. I. T. tester shown in Fig. 7. This apparatus holds a strip of paper at approximately constant tension (usually one kg-m.) and folds it over steel edges of standard curvature (0.03 in. diameter) through an angle of about 120 deg. on each side of the straight position.

Density, as it is improperly called, meaning the compactness of fibers, is measured by a Gurley Densometer, Fig. 8, an instrument that measures the number of seconds required for 100 cu. cm. of air to be forced through one sq. in. of paper under a pressure of 570 g.[1]

All physical tests on paper are made in a room kept at a standard humidity of 65 per cent by means of a Bahnson humidifier.

Oil permeability is measured by noting the time required for oil of standard viscosity to make its appearance through a single sheet. The test suggested by Dr. H. H. Brown is made by shaping the paper into a rectangle about 3 by 5 cm. with turned-up edges, and floating it on a dish of oil, as shown in Fig. 9.

The essential tests on oil are as follows:

1. Dielectric strength.
2. Resistivity.
3. Viscosity.
4. Hardness.
5. Solidifying point.
6. Some cable manufacturers test the oil for power factor.

Dielectric strength is measured between electrodes one in. in diameter and 1/10 in. apart at 85 deg. cent. using 60-cycle voltage. The apparatus is shown in Fig. 10. The results are liable to be quite erratic and the average of at least five readings must be taken for each test.

Resistivity is measured by galvanometer method using a cell, Fig. 11, with electrodes of 56 sq. cm. area and 0.5 mm. apart, as described in TRANSACTIONS of the A. I. E. E. 1922, Vol. XLI, page 569. The results vary over a wide range but if care be taken in making measurements, fairly close checks may be obtained on successive tests. Care must be taken to avoid con-

1. It was pointed out by the authors in an Institute paper that impregnated paper insulation is virtually oil insulation improved electrically by the interposition of barriers to break up circulation and ionic migration.

This conception suggested that variations of dielectric strength might be due to variations in the baffling effect of the paper and an attempt was therefore made to find a measure of this effect. Such a measure was finally found by the authors, in collaboration with Dr. H. H. Brown, in the readings of the Gurley Densometer, an apparatus which measures the compactness of the paper fibers by the permeability of the paper to air under pressure.

Control of the air permeability or density, as its reciprocal is erroneously called, led both to a general raising of the dielectric strength level and in great measure to its equalization at that level.

tamination of the specimens and apparatus by oxygen, sweat, condensed moisture and dirt.

Viscosity is measured with a Saybolt Universal Viscosimeter, Fig. 12. The specimen is placed in an

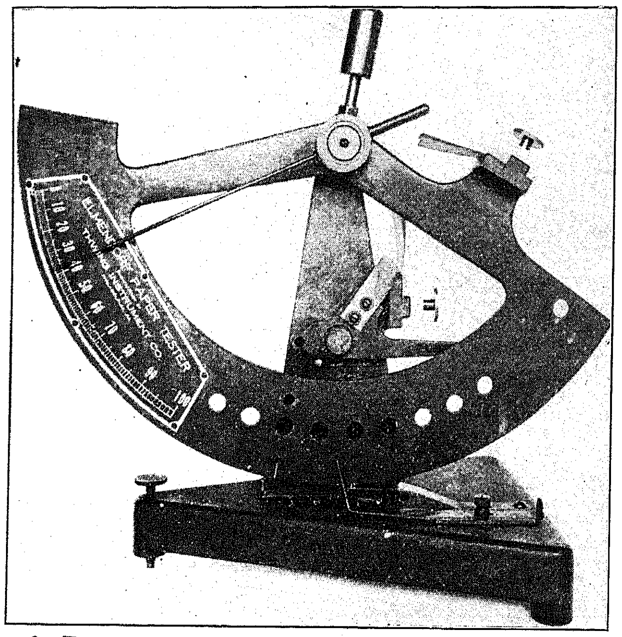


Fig. 6—Elmendorf Paper Tester Used to Test the Tearing Strength of Paper

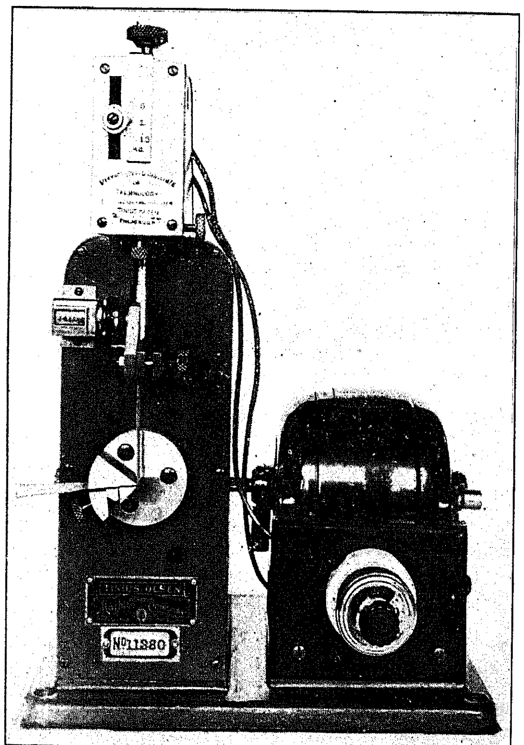


Fig. 7—M. I. T. Folding Endurance Machine

inner receptacle where it is kept at the desired temperature, usually 210 deg. fahr., (99 deg. cent.) by means of oil kept at about 216 deg. fahr. (102 deg. cent.) in the outer bath. The number of seconds required for 60 cu. cm. of compound to pass through a standard orifice is the viscosity. Measurement of hardness have not been developed to a point that justifies a description.

The solidifying point is measured by putting about 10 or 15 cu. cm. of molten compound into a porcelain crucible and stirring with a thermometer while it cools until it is just hard enough not to pour. The temperature is noted and recorded as the solidifying point.

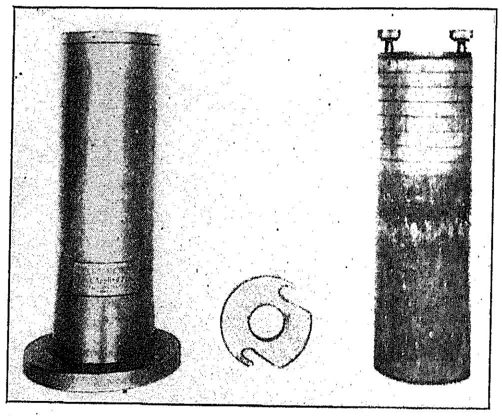


Fig. 8—The Gurley Densometer for Measuring the Air Permeability of Paper

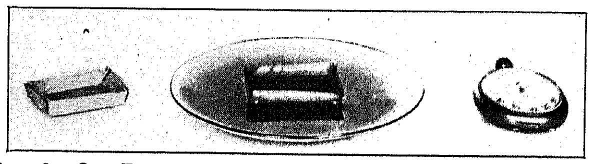


Fig. 9—Oil Permeability is Measured by Floating the Paper Boat on Oil and Noting the Time for the Oil to Penetrate through the Paper

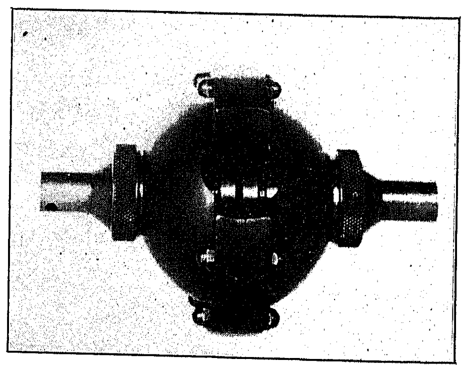


Fig. 10—A. S. T. M. Standard Gap for Testing the Dielectric Strength of Oil

Process Control

Completion of drying is ascertained either by capacity tests made with a 60-cycle a-c. bridge, a capacity meter, or by dielectric loss tests. A typical drying curve is shown in Fig. 13.

The quality of the oil is controlled by tests of resistivity, dielectric strength and viscosity, similar to those described under tests of raw materials.

The impregnated paper on or from cables is tested for

folding endurance, density, saturation and dielectric strength. Saturation is determined by extraction with carbon tetrachlorid in the syphon extraction flask shown in Fig. 14. This gives the ratio of oil to paper on a three-in. sample of cable. The test is supplemented

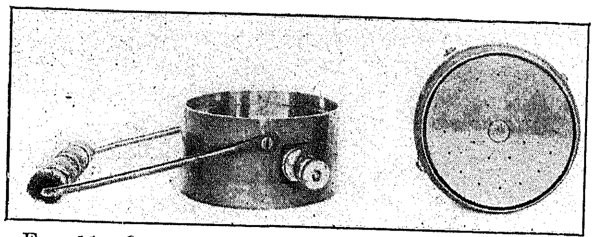


Fig. 11—Cell for Measuring Resistivity of Oil

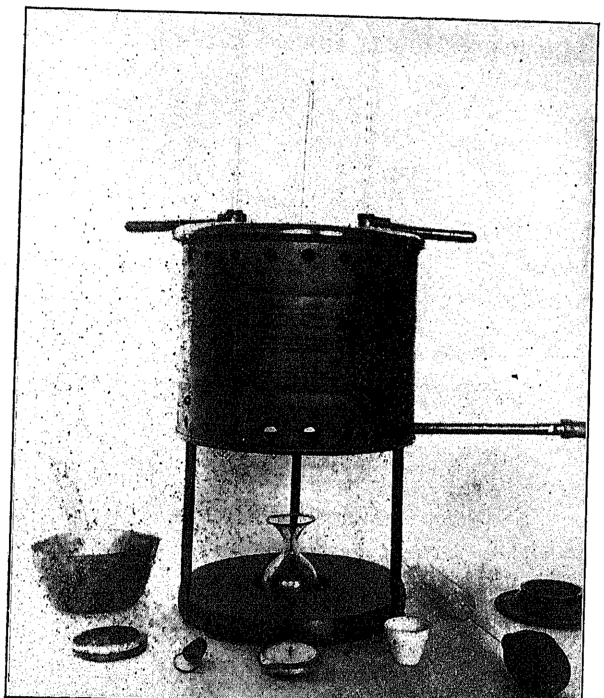


Fig. 12—Saybolt Universal Viscosimeter

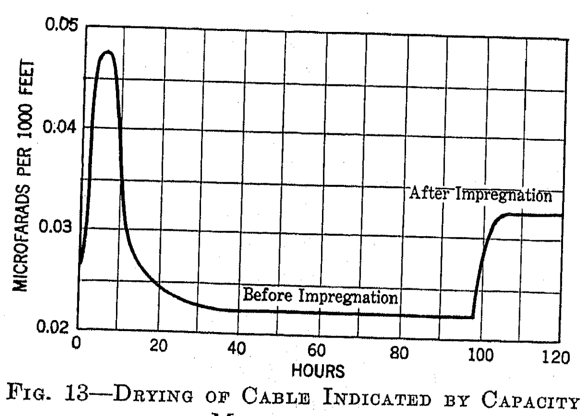


Fig. 13—Drying of Cable Indicated by Capacity Measurements

by visual inspection of samples several feet long which are stripped tape by tape and examined for surface oil throughout. Research work is under way to find a better check upon saturation.

A number of sections 15 ft. long is taken every week from commercial lengths and tested to failure with 60-cycle voltage. A considerable proportion of these lengths is tested for dielectric loss by the compensated wattmeter method (Pender's Handbook, 1922, page 865). Readings are taken at several voltages to determine the so-called ionization characteristic of the cable in order to collect data on this subject.

Periodic aging tests are made with the 15-ft. samples described above. The usual test is at 2½ times normal

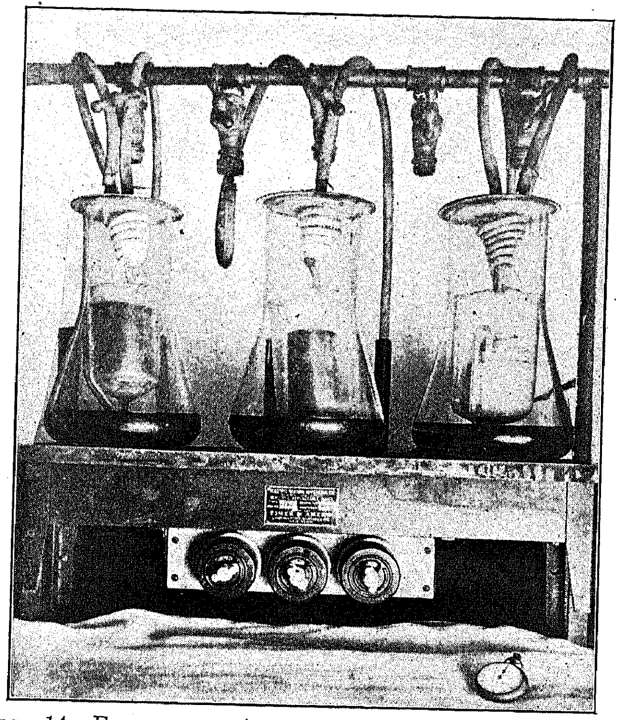


Fig. 14—Extraction Apparatus Used in Determining the Ratio of the Weight of Impregnating Compound to That of Paper

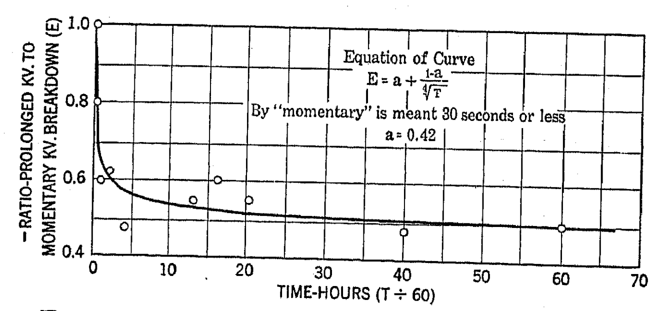


Fig. 15—Time Lag of Paper-Insulated Cables

working voltage, and the criterion is the chemical stability of the oil.

Several sections per week of the same length are submitted to the bending test, broken down with high voltage and examined for tears.[2]

A few 15-ft. sections have also been tested to destruction with prolonged voltage applications to determine

2. It is of interest to note that while European specifications often call for no torn tapes, the criterion is usually the dielectric strength and not a visual examination as in America.

the time lag of impregnated paper insulation as it applies to electric cables. The dots in Fig. 15 are the data obtained from these tests, whereas the curve is the locus of the following equation given by F. W. Peek, *G. E. Review*, November 1915, and V. M. Montsinger, JOUR. A. I. E. E., Feb. 1924, page 146.

$$E = a + \frac{1 - a}{\sqrt[4]{T}}$$

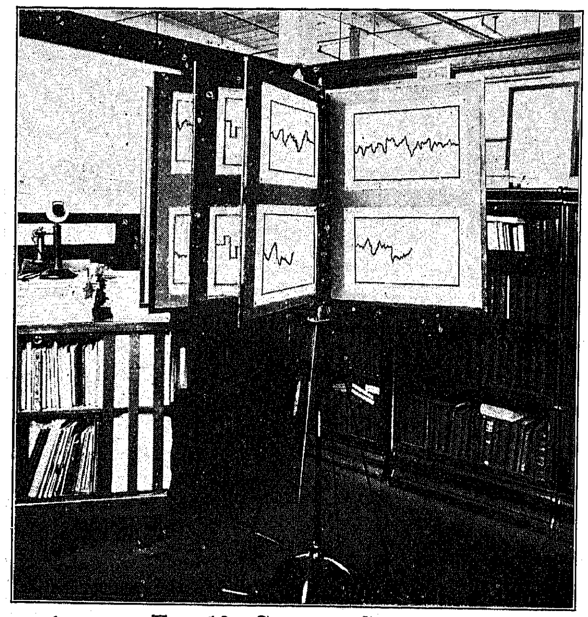


FIG. 16—CONTROL CHARTS

Where

E = the ratio of prolonged breakdown voltage to momentary breakdown voltage. The momentary breakdown voltage is that which causes breakdown in 30 sec. or less.

T = time in minutes.

a = a constant for a given type of cable.

The value of a computed from the breakdown data is 0.42 for these cables. If the value 0.42 is substituted for a in the equation and T made infinite, it would seem that these cables would withstand continuously a voltage equal to 42 per cent of the momentary breakdown voltage. We know, however, from experience that this condition cannot be fulfilled in practise. It follows, therefore, that the equation should be used only for values of T within certain limits. The reason for this limitation, is, perhaps, that other factors of a deteriorating nature enter into action as time goes on.[3]

There are other tests to control processes and materials which cannot be discussed at the present time, but which contribute materially to the results.

3. It should be noted that tests lasting a half hour bear a more or less definite relation to the standard five-min. test and are, therefore, unlikely to be of any more value. Artificial aging tests, on the other hand, are made under conditions designed to promote deterioration, and therefore reveal something not shown by the five-min. test.

The data from all these tests are plotted on charts (Fig. 16) so that the tendency of every characteristic can be noted day by day. One effect has been to reduce the number of high-tension cable failures on factory test, to a negligible number. This saving effected has paid many times over for the cost of the research work upon which it was based.

This work has furthermore, laid the foundation upon which future developments for higher voltages must depend.

Appendix I.

COMPOUNDS WITH AND WITHOUT ROSIN

The American manufacturers were led to abandon rosin largely because of its chemical instability at the temperatures at which American cables are operated. The lower temperatures at which European cables are used, made this consideration comparatively unimportant to them and they probably made their decision on the basis of the greater surface tension between paper and the cylinder oil rosin mixture.

The addition of rosin to a petrolatum or a mineral oil lowers its electric resistivity and various percentages of rosin have an interesting effect upon the resistivity of petrolatum at different temperatures, Fig. 17A

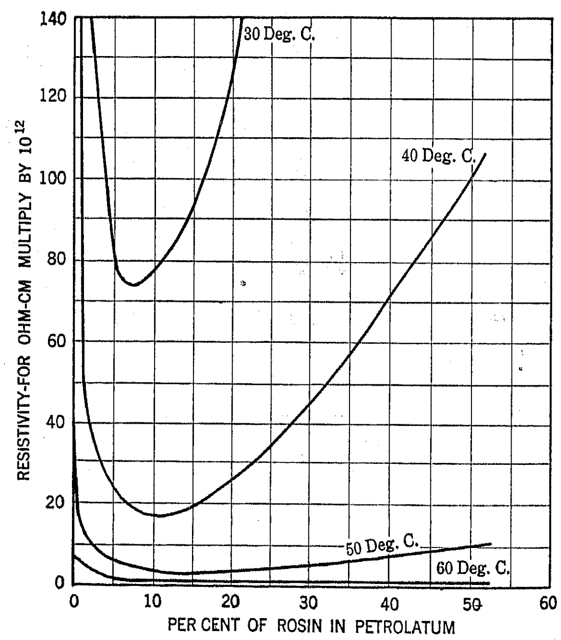


FIG. 17A—THE EFFECT OF ROSIN ON THE RESISTIVITY OF PETROLATUM

and Fig. 17B. At temperatures of 50 deg. cent. or less, the resistivity drops as rosin is added, up to 10 per cent, as shown in Fig. 17A. A further addition of rosin raises the resistivity again, but no amount of rosin added will bring the resistivity up to its original value. At higher temperatures, the addition of rosin up to 10 per cent lowers the resistivity. Further addition of rosin holds the resistivity approximately constant, as shown, for example, by the 80-deg. curve in Fig. 17B.

This reduction of resistivity of petrolatum in itself, does not necessarily have a bad effect upon the quality of the cable, but, unfortunately, the reduction in resistivity is accompanied by an increase in dielectric loss or power factor at the higher temperatures. If low power factor is obtained by the use of a large rosin content, such as 50 per cent, the compound becomes too stiff at low temperatures. In Fig. 18 are given two power-factor curves of two groups of cables.

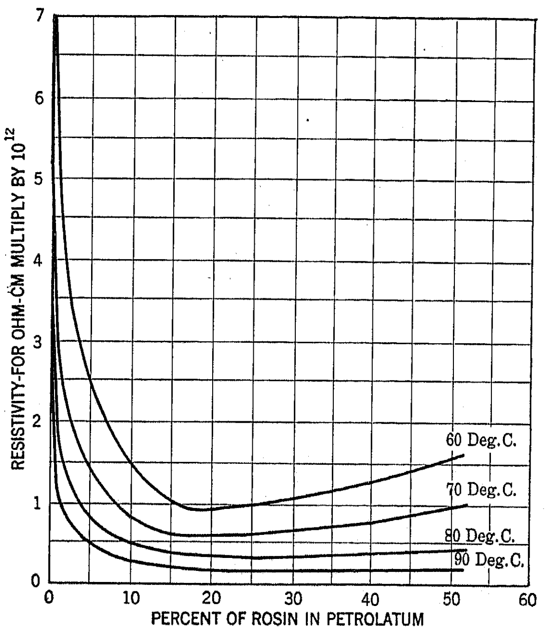


FIG. 17B—THE EFFECT OF ROSIN ON THE RESISTIVITY OF PETROLATUM

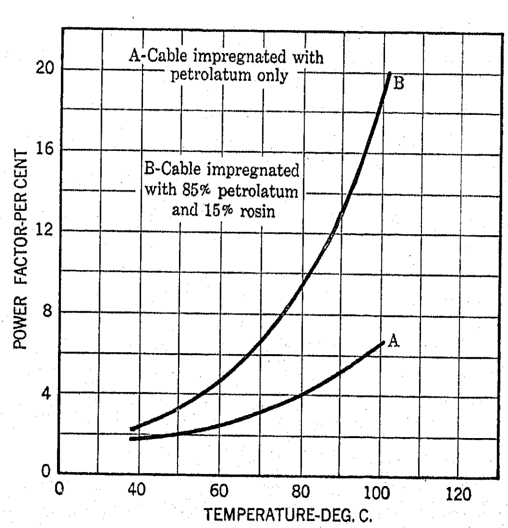


FIG. 18—RELATION BETWEEN POWER FACTOR AND TEMPERATURE OF A PAPER-INSULATED CABLE

These cables are identical, except the *B* cables contain 15 per cent of rosin, whereas the *A* cables contain no rosin. The serious effect of rosin in cables is that it lowers materially the critical temperature of cumulative heating.

The above data are presented to show that the use of rosin was stopped for definite and clearly understood reasons. This should not be construed as a final abandonment of this material, as further research work is under way which may lead to the development of methods of using it without incurring the disadvantages originally found.

Appendix II

HOT SPOTS IN CABLES

It has frequently been observed that impregnated paper-insulated cables, which have been subjected to a high dielectric stress for a considerable period, show hot spots which are regarded as incipient failures. It was recently the authors' good fortune to examine a piece of cable in which several hot spots occurred which showed practically all the characteristics which had been noted individually in previous cases.

The cable was single-conductor stranded, having a cross-sectional area of 600,000 cir. mils and insulated with 39/64ths of manila rope paper, applied with an open butt. The power factor of the insulation was about 1¾ per cent at 85 deg. cent.

This cable was subjected to 135,000 volts alternating for about twenty minutes and opened for examination.

There were 108 layers of paper, of which the outer ones were 8 mils and the inner ones 5 mils thick. The 8-mil paper was found to be entirely free from any evidence of puncture or hot spots. The same applies to the outer few layers of 5-mil paper, but upon reaching about the fiftieth paper from the conductor, evidences of hot spots became numerous.

The first evidence consisted of a tree-like design such as is characteristic of photographs of lightning and other high voltage discharges.[4] Upon examining these "tree" marks it was found that the paper fibers were charred. It was found that while the "tree" design in one layer nearly always crossed the "tree" design in the next layer, these designs were never superimposed, but each "tree" was independently formed, the general direction being across the tape. As more and more layers of paper were removed, the trunks of the "trees" were found to be more and more deeply charred until at about the 44th layer from the conductor, the paper was found to be burned right through. This continued for about fourteen layers when the charring gradually diminished until it ceased at the 18th layer from the conductor. The remaining papers down to the conductor were found to be merely punctured, each paper having a small hole with a charred edge. Other hot spots of a similar character were found which, however, did not extend to the conductor, but were confined exclusively to the intermediate layers of tape. An unpleasant odor was noticed in the vicinity of the tree patterns.

The existence of tree patterns proves the occurrence

4. F. A. S. Kleine, C. F. Proos, and J. C. van Stavern, Conference Internationale des Grands Réseaux Electriques, 1923, observed the same effect in European cable.

within cables of what is generally known as surface leakage.

Surface leakage occurs at all voltages, being merely conduction through films at low voltages, but partaking of the nature of streamers as the air ionizes at higher voltages. Hence, at low voltages, it obeys Ohm's law, but at high voltages, the volt-ampere characteristic depends upon ionic saturation.

The air ionization theory as generally understood, is that at a certain stress, the dielectric loss increases to such a high value that local cumulative heating occurs. Furthermore, according to this theory, a sudden increase of power factor with gradually increasing voltage, presages destructive ionization.

It is our opinion that this theory is not complete and requires a new interpretation; and we believe that the surface leakage phenomena described above, suggest the necessary modification.

Leakage current flows at low voltage without visible manifestation but at high voltages, ionization of the air causes it to flow as streamer discharges with a liberation of energy which varies with the degree of ionization. Failure results from charring of insulation either along the path or at the ends of the streamer.

While the surface leakage theory is based upon visual observations, it is of interest to note how it explains certain phenomena better than the orthodox ionization theory.

1. The lower specific capacity of air films should tend to divert the electrical stress to the surrounding insulation of higher specific capacity. The films, therefore, will not carry k times the ambient stress but considerably less. This would lead one to expect signs of failure around the edges of air films, but these are never found. All the signs of burning occur squarely on the surfaces exposed to the air films, as would be expected if caused by surface leakage.

2. Some cables in which there are numerous large air films in which ionization presumably occurs, have operated satisfactorily at high stresses for years without trouble. In such cases, the reason for the absence of streamers was probably low resistivity of the oil which kept the leakage in the stage of ohmic conduction, and thus restrained the local liberation of energy. Such cables, however, fail from cumulative heating when loaded.

3. It is a part of the film ionization theory that the increase of power factor with voltage is due to air ionization. If a cable be made without adequate drying of the paper and poorly impregnated, the increase of power factor with voltage should be quite marked, as the specific capacity of the solid part of the insulation will be abnormally high, thereby throwing a higher stress on the air films. Experiment shows, however, that in such a cable the power factor will be constant over a wide range of voltage. This is entirely consistent with the leakage theory, because, as suggested above, in a cable made of low resistivity, oil and paper, the leakage current will increase with the voltage according to Ohm's law over the full range.

Discussion

D. W. Roper: A point which should be more prominently set forth is that as the operating voltage of a cable increases, the quality must improve. What is needed at the present time is some method of measuring the quality of the insulation and determining the maximum operating voltage for which a given quality of cable is suited, and to which it should be limited. There are also needed some other tests to be applied to cable at the factory in order to determine whether it will be satisfactory for the service for which it is intended. Apparently some further research as well as cooperation between the manufacturing and operating companies is needed in order to develop the proper tests for this purpose.

W. F. Davidson: The authors refer to the difficulties experienced a few years ago on the use of very stiff impregnating compounds and the resultant formation of voids when the cable was bent. They imply that this difficulty has been entirely avoided by the use of lighter compounds but I am of the opinion that their statement is a little too optimistic. Some day we may reach the point where voids are not developed by bending.

The authors call attention to the fact that many of the phenomena associated with air films in cables cannot be fully explained on the simple hypothesis that ionization results from a stress in the air film K times that in the solid portions of the dielectric. This is a natural consequence of the fact that the air films are not infinite in extent but really are of comparatively small area so that creepage phenomena play an important part. A number of very interesting possibilities are suggested by the considerations of stress distributions with alternating fields and with continuous fields. Some theoretical work has already been started and experimental work has been mapped out. Until more is known of the values of the numerous physical constants it will be impossible to complete studies along this line.

A rough shop method for determining solidifying point of compound is described. Rather more consistent results can be obtained by taking the temperature-viscosity curve by means of a MacMichael viscosimeter. This instrument gives results of very satisfactory precision with a minimum of observational error. Probably in point of time it is rather faster than the method described in the paper. Studies of a considerable number of cable samples have indicated the possibility of getting the temperature-viscosity curve over a wide range from three or four points after the general characteristic has been determined.

R. A. Paine, Jr.: The authors inferred in their paper that if an air film can be maintained at low pressure, they are thus able to get a flat power-factor characteristic. I should be glad if the authors could be able to give us some data or results of any experiments that they have been able to make along that line, as it would seem that it would be almost impossible to maintain this air film at low pressure.

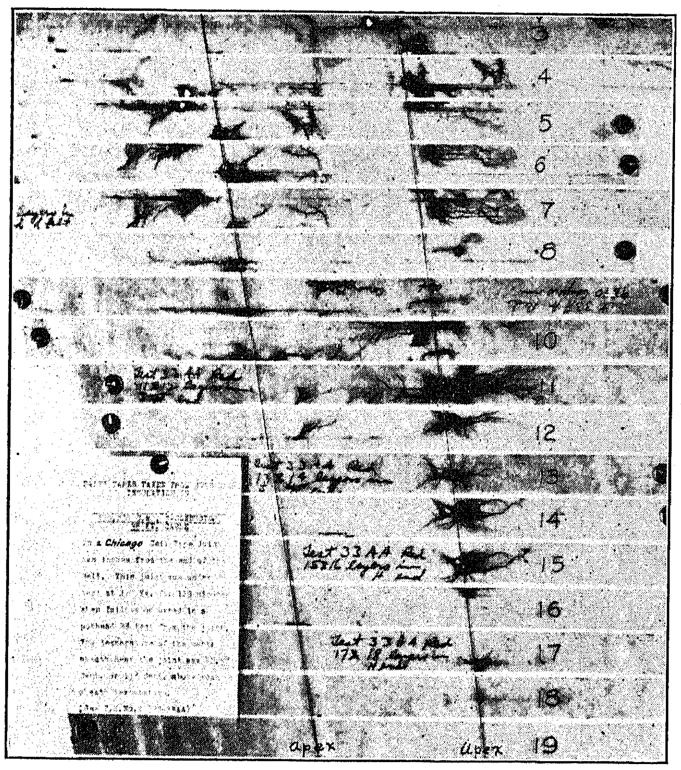
The authors speak of the ratio of oil to paper as a means of determining saturation. I am glad to note they say they are going to do some further research work along that line in order to get a better method of determining the degree of saturation. It seems to me that they are missing one point, and that is what we really want to know is if the complete volume inside of the cable is filled. What we really ought to have, I believe, is the total volume, and then take into account the total amount of paper and the total amount of oil in it rather than work on a ratio basis.

There are at least three of the American cable manufacturers now who can make cables with almost no wrinkles, practically

none, and I think within the next year we ought to be reasonably sure of getting cables that will have no trouble standing up in operation over a period of years.

H. Halperin: I wish to emphasize the remark of Mr. Del Mar, that there is a very urgent need for securing uniformly good cable insulation. In a 10-mile line of three-conductor cable with 350,000 cir. mil. sector conductors, there are about 29,000 sq. ft. of copper surface from which a failure might occur; and defects in a comparatively very small per cent of the total insulation can easily result in poor operation. In dissecting test and service failures of 13-kv. and 35-kv. cable in Chicago, the insulation immediately adjacent to the failure has frequently been found radically different in construction and impregnation from the remaining portion of the same length.

In connection with the carbonized tree patterns that were found by Mr. Del Mar, we have found that these also appeared in the endurance tests at 127-kv., three-phase on the three-conductor, 35-kv. cable. In the illustration shown herewith, these designs were taken from the conductor insulation in a joint



2 in. outside of the belt. Usually the carbonized patterns in the cable were most pronounced at or near the apex of the sector, that is, the portion pointing toward the center of the cable. The most pronounced formation of these tree patterns was found in cables which withstood the test for the longest time.

The authors state that the compound "X" has been found in the outer conductor insulation and inner belt insulation. Frequently in three-conductor, 35-kv. cable we have found it throughout the conductor insulation and in some cases the formation was most marked adjacent to the copper.

In the latter part of the paper a curve of power factor against temperature for a cable with 15 per cent rosin in its compound shows the power factor at 80 deg. cent. to be over 9 per cent; and the discussion is to the effect that the presence of rosin materially affected the critical temperature in cumulative heating. This effect is negligible with the many European cables being made and which have been made, with higher percentages of rosin than 15 per cent and power factors about half or less than those shown in the curve *B* for temperatures of 60 and 80 deg. cent.

E. R. Thomas: I would like to contribute a little discussion to the paper by Messrs. Del Mar and Hanson. We have made some tests in New York of high-potential stresses on impregnating compounds. I will give somewhat of a resumé of our tests. We used some special glass test tubes, the outer diameter of one being a few mils less than the inner diameter of the other. In the larger test tube was put a small quantity of mineral oil, heated very slightly, enough to form a liquid state, dropped the other tube inside and in that way obtained a very thin film of impregnating compound. This was subjected to a high-voltage gradient by putting a mercury electrode in the inner tube, immersing the outer tube in salt water and then applying a high potential between electrodes.

The visual phenomenon which occured when putting this under stress was a very light corona discharge in the tube. There was no occluded air in this very thin film which would be quite readily discernable by holding it to the light. After this had been on test for some two hours, some changes went on. We had sealed the tube at the top with a small rubber balloon which became inflated and thus gave evidence that some gas was given off. The noted change occurring in the oil was that it goes over to a state of solidification and from its resemblance we had termed it Swiss cheese. It changes from the oil, which is amber, to a rather yellow waxy substance with numerous void spaces in it giving a Swiss-cheese appearance.

Several batches of this were run until we had a sufficient quantity to make a breakdown test. This was melted into an oil cup, and, by the way, goes back to a waxy substance after being melted. The breakdown test showed that there was no practical difference in breakdown strength from the original compound. We also have found that by modifying a mineral oil compound with some other things this change over in substance can be somewhat lessened and I believe that there is a considerable field of research open in this line.

H. F. Randolph: The results obtained by Messrs. Del Mar and Hanson on overstressed cables are particularly interesting, since we have noticed a great many of these points in our investigation in the after-performance of underground cables in the past year.

It appears that dielectric loss in high-voltage paper cables has been reduced to a minimum and is no longer a controlling factor. It would be interesting to know if any other qualities were sacrificed in order to accomplish this.

The authors state that although very high voltages are obtained on samples of three-conductor cables, it is practically impossible to break them down in laboratory tests, but still failures occur in actual operation of these same types of cables, although operating conditions previous to, and at the time of, failure are apparently normal. We have examined samples of cable adjacent to some of these failures and have noticed a similar condition to that mentioned by Mr. Del Mar, the perforation of the layers of paper, but we have not noticed any tree designs. It would be interesting to know the approximate voltage that would be apt to cause this condition.

I would also like to ask the question as to whether a percentage of rosin-oil compound mixed with a mineral-oil compound would have any decided effect on the life of the cable.

A. H. Kehoe: I think the real value of the so-called power-factor test is not brought out in the Del Mar-Hanson paper. The test consists in the change in power factor on test samples of cables with an increase in voltage of the proper amount. It may be true, as stated in the paper, that the curve of power factor increases, diminishes, or stays constant, with a certain line of material, but after a certain kind of cable is brought through a factory and this characteristic determined, there is but one thing that changes the quality in the cable and this resolves itself into "air" in the cable. With an increase of air in a cable

the power-factor curve will be increased over the normal characteristic as soon as the ionization point of the air is arrived at on increasing the voltage for the power-factor test.

In the same paper there is described results of tests at two and one-half times normal working stress. It should be realized that the power-factor test intended to indicate the presence of air by change in power factor is not a matter of normal working stress on the cable. It is a matter rather of stressing the cable insulation up to certain definite values where air (if it be in the insulating structure) will be ionized. This becomes a matter of physical dimensions and properties of the insulation and has no definite relation to normal working stress.

G. B. Shanklin: In reference to the theory of ionization the following statement is found on the third page of the paper by Messrs. Del Mar and Hanson: "It is our opinion that this theory is not complete and requires a new interpretation; and we believe that the surface leakage phenomena described, suggest the necessary modification." The surface leakage phenomena mentioned in the above statement refer to the characteristic fern leaf discharge path between paper wrappings, frequently, if not always, found after subjection to high-potential test, that is, a stress considerably higher than normal.

The configuration of this type of discharge is typical of all high-frequency discharges, such as lightning, static discharge over insulators, etc. It is the result of a destructive groping or reaching about for the weakest path, which in this case happens to be along the paper surfaces. The high impedance at these very high frequencies is the cause of these discharges not being directional. They do not necessarily follow air or gas pockets but actually form their own pockets by generation of gas as they travel along.

Apparently, Messrs. Del Mar and Hanson have confused the above phenomena with the slow, steady glow discharge which occurs in gas pockets at lower voltage stressing and constitutes the true ionization with which cable engineers are concerned. The effects of this are much more gradual and the range of travel much more limited. The result is a slow chemical change or disintegration, not necessarily accompanied by charring. It is more to be feared than the first-mentioned phenomena, for it might occur at normal operating voltage, while the first, as far as my observation has gone, always requires considerable over stressing.

This second type of discharge did not escape their observation for they give a very accurate description of its effects. Their explanation, suggesting the formation of fatty acids, seems very reasonable and probable. There is not a sufficient quantity of nitrogen present to form nitro products.

The various methods of factory test and control they describe are now being followed by most, if not all, of the cable manufacturers. Most of these have proved reliable guides to a better quality cable. A few have not. In particular, no reliable test has yet been developed to determine the degree of impregnation. The method of extraction with carbon tetrachlorid or other solvents has not proved satisfactory. So far, the old method of visual examination and expert judgement has not been displaced.

I was very much interested in their characteristic "breakdown versus time" curve, given in Fig. 15, and fully agree with their statement that such curves are of no value in determining ultimate life at normal voltage by extrapolation, due to the fact that other factors of a deteriorating nature enter into action as time goes on.

C. F. Hanson: I should like to make a reply to Mr. Halperin's statement regarding the last part of the paper, referring to Curves A and B. We do not say it is not possible to secure lower power factors using rosin than are given by Curve A, nor do we say that we always need to get as high a power factor as given by Curve B. These curves were given simply to show the relative values when we do and do not use rosin. They are the results of three cables, all treated identically, the same oil being used, only that in one case 15 per cent of rosin was added; while in the other cases, no rosin was added.

The data were obtained some years ago when oil refineries had not developed in refining petrolatum the technique applied today. With present-day oil, lower power factors may be obtained than those shown in Curves A and B.

Wm. A. Del Mar: Mr. Roper is right in pointing out that as the operating voltage of a cable increases, the quality must improve. The term "quality," however, is a somewhat indefinite one and should not be interpreted as applying to all of the characteristics of the cable. Some discrimination is required to determine which characteristics should be improved as the voltage increases. For instance, the quality of the paper as set forth in ordinary specifications should be decreased rather than increased for higher voltages. By this I mean that more or less wood pulp should be allowed. Similarly, flexibility requirements should be reduced rather than increased in the case of high-voltage cables so as to permit the use of materials of greater dielectric strength at the expense of the mechanical qualities.

It is also quite possible that the dielectric loss should not be too low in cables for very high voltages in order to give an opportunity for the dissipation of local transients in the insulation. The main quality which requires to be improved as the voltage is raised is the saturation, with special reference to the elimination of air.

Since the paper was written, an apparatus has been developed by our laboratories, for measuring saturation as a ratio of air to insulation by volume. It consists of an inverted funnel with a graduated closed-top tube. This is filled with water, placed in a jar of air-free water with a section of cable about an inch thick under it. The jar is then evacuated and the vacuum maintained over night. This results in sucking the air out of the cable and causes it to collect in the tube of the funnel. The vacuum is then broken, causing the air in the funnel to contract to its atmospheric volume. The displacement of the cable is noted, its copper and lead removed, and their displacement noted. The volume of insulation is the difference between the displacement of the cable and that of the combined copper and lead. The saturation is expressed as the ratio by volume of air to insulation. The insulation is then examined under water to make sure that all the air has been removed. There is an error which it seems impossible to eliminate, namely that air is sucked from the strands as well as from the insulation, so that if the interstices between strands are not well filled, the insulation will be debited accordingly. This, however, has not affected the practical usefulness of the device which is now in regular use for control tests.

Mr. Thomas has described experiments which, he says, have enabled him to produce a yellow, waxy substance with numerous voids, merely by exposing mineral oil to a strong electrical field. We have repeated the experiment and obtained the same results. The material, as stated by Mr. Thomas, melts and has the same electric strength as the original compound. The reason for this is that it is the original compound, the changed appearance after the application of stress being due merely to an emulsification. Mr. Thomas's experiment did not result in producing the material referred to as X in our paper, as this latter cannot be melted. It is neither fusible nor soluble in any solvent available. Chemical analysis shows it to be free of nitrogen and to contain a very small quantity of oxygen. Research work is now under way which, it is hoped, will lead to its chemical identification.

In reply to Mr. Randolph, the tree patterns are readily formed at three-fourths of the short-period breakdown voltage. The addition of rosin-oil compound to mineral-oil compound does not improve the life of the cable.

An important difference between our leakage tree patterns and the steady glow of ionization described by Mr. Shanklin is that the former are matters of visual observation, whereas the existence of the latter is known only by theoretical deduction. Mr. Shanklin says that tree patterns are the result of destructive groping or reaching for the weakest path. We are entirely in accord with this but believe that the weakest path must be along the air pockets because air ionizes at a much lower voltage than oil.

In conclusion, I would take some exception to the general tendency of some of the speakers to assume that American cable manufacturers have not kept up with the general progress of electrical industry. It should be remembered that they have been laboring under a handicap unknown either to other branches of electrical industry or to cable manufacturers in other countries. I refer to the tendency of American cable users to put into their specifications clauses, the undue emphasis of which is entirely irrelevant to the proper performance of the cables but which forces the manufacturers into competition on unessentials rather than allowing them to concentrate their efforts on factors of major importance. Principal among these irrelevant requirements have been excessive flexibility and excessively low dielectric losses. Another requirement of the same character which is now assuming prominence is the so-called ionization test.

If the American manufacturers succeed in making cables which pass American specifications and have characteristics as good as the best European cables, they will have accomplished an infinitely more difficult task than the Europeans have done, and this goal has actually been reached by some of them.

The Direct Method of Calculation of Capacitance of Conductors

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Review of the Subject.—*The capacitance, or electrostatic capacity, of conductors of various shapes has practically always been calculated by the "inverse method" of first assuming certain charges of electricity and from them calculating equipotential lines and surfaces, which, in the more fortunate cases, agreed exactly with the shapes of the conductors. In other cases, such as that of a three-conductor sheathed cable, the agreement, and therefore the calculated capacitance, were only approximate.*

For multiconductor cables with round conductors, a "direct method" of calculation of capacitance is described in this paper, which uses the exact shape of the round conductors, and which gives accurate expressions for the irregular distribution of electricity on the conductors, and for the value of the capacitance.

Formulas and examples are given for the following cases: 1. Finite wire and infinitesimal wire. 2. Single-phase overhead line. 3. Two conductors and sheath. 4. Three conductors and sheath.

THE direct method of calculation of capacitance, as distinguished from the more usual inverse method, consists in first assuming the shape of the conductors, and from that calculating the distribution of electric charge and the capacitance. In the inverse method, one finds the shape of the conductors from the potential calculated from charges of electricity concentrated at points or uniformly distributed along lines or over surfaces. If the equipotential lines are not true circles, then the calculated capacitance is not correct for circular conductors, but is only approximate. This is the case with two-conductor and three-conductor sheathed cables.

In this paper, accurate formulas using the direct method are given for four cases: first, a finite wire and an infinitesimal wire; second, the capacitance of a single-phase overhead line; third, the capacitance of the sheath on one side and the two conductors on the other side, of a two-conductor cable; and fourth, the capacitance of the sheath on one side and the three conductors on the other side, of a three-conductor cable. For the second case an accurate standard formula is available for comparison. The formula for the third case, given in this paper, was given by the writer in a discussion in the JOURNAL of the A. I. E. E., November, 1923, page 1208.

It appears that practically all formulas for capacitance have been based on the inverse method. This is true, at any rate, of the published formulas for capacitance of multiconductor sheathed cables. It is stated by Clerk Maxwell in Chapter VII of "Electricity and Magnetism," published in 1873, that every electrical problem of which we know the solution has been constructed by the inverse process of finding the shape of the conductors from the potential due to assumed charges of electricity. He also states that the only method by which one can expect to solve a new problem is by reducing it to one of the cases in which a similar problem has been constructed by the inverse process. Maxwell's statement has been quoted in fairly recent articles, and the limitation which he gives that the inverse process must always be used and that one cannot start with an assumed shape of conductor seems to hold in the most recent calculations of capacitance. It seems evident that the direct method described in this paper, which accurately uses the direct process of starting with given shapes of conductors, should be of considerable use.

The demand for greater accuracy in this type of calculation is shown by the paper published by Mr. D. M. Simons,[1] who gave results for two, three and four-conductor cables which are more accurate than those previously available obtained by the inverse process of calculation. Mr. Simons' results were obtained by graphically correcting the length of the lines of flow pertaining to the inverse method of calculation. His results, pointed out in his paper, still contain a certain amount of approximation since the position of the lines of flow is not that corresponding to true circular conductors, and that accounts for some of the discrepancy between his results and those given in this paper.

In connection with the calculation of the numerical example of the three-conductor cable given in this paper, the writer wishes to acknowledge valuable assistance given by Mr. D. M. Simons in checking and locating errors in the preliminary work.

In the direct method of calculation of capacitance described in this paper, one first assumes a uniform electric charge on the surfaces of the round conductors and their images. From this the resulting charge at any point on the conductors can be calculated. This may be called the first additional charge, and it is in the form of a Fourier series, that is, a series involving $\cos\theta$, $\cos 2\theta$, $\cos 3\theta$, etc. The cosines often disappear when integrated around the circle. One can now calculate the second additional charge, which results from the first additional charge, and so on until the terms become small.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

1. "Cable Geometry and the Calculation of Current-Carrying Capacity," by D. M. Simons, JOURNAL A. I. E. E., May, 1923, page 525.

Finite Wire and Infinitesimal Wire

This is not a practical case, but the solution is used in the problems to follow.

Let there be a finite wire A, carrying an electric charge Q per centimeter, and let there be an infinitesimal wire B carrying an electric charge $-Q$ per centimeter. Quantities are in absolute units. Let O be the point of zero potential.

The surface density of the charge on A will not be uniform, owing to the presence of the charge on B. From the symmetry of the arrangement, the surface density $q_{(\theta)}$ at any point P can be expressed as a Fourier series, the angles being measured from the line $B\,D$. Thus

$$q_{(\theta)} = H_0 + H_1 \cos\theta + H_2 \cos 2\theta + \ldots + H_n \cos n\theta + \ldots$$

where H_1, H_2 etc. are undetermined constants. The total charge on the wire A is

$$\int_0^{2\pi} q_{(\theta)}\, a\, d\theta = 2\pi a H_0 = Q$$

Therefore, $$H_0 = \frac{Q}{2\pi a}$$

The work done against the elementary charge $q_{(\theta)}\, a\, d\theta$ at P, in carrying a unit charge from O to N is

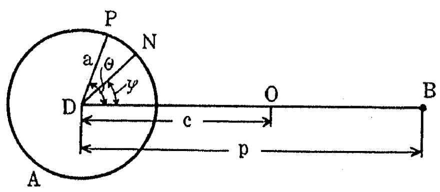

Fig 1—Finite Wire and Infinitesimal Wire

$$2\, q_{(\theta)}\, a\, d\theta \operatorname{logh} \frac{O\,P^*}{N\,P} \qquad (1)$$

The work done against the charges on both conductors is

$$-2\,Q \operatorname{logh} \frac{O\,B}{N\,B} + 2\,a \int_0^{2\pi} q_{(\theta)} \operatorname{logh} \frac{O\,P}{N\,P}\, d\theta \qquad (2)$$

The following expansions are now required:

$$\operatorname{logh} \frac{N\,B}{p} = -\left\{ \frac{a}{p}\cos\varphi + \frac{a^2}{2\,p^2}\cos 2\varphi + \ldots + \frac{a^n}{n\,p^n}\cos n\varphi + \ldots \right\} \qquad (3)$$

$$\operatorname{logh} \frac{N\,P}{a} = -\left\{ \cos(\theta-\varphi) + \frac{1}{2}\cos 2(\theta-\varphi) + \ldots + \frac{1}{n}\cos n(\theta-\varphi) + \ldots \right\} \qquad (4)$$

*Elements of the Mathematical Theory of Electricity and Magnetism, by J. J. Thomson, fourth edition, page 94.

$$\operatorname{logh} \frac{O\,P}{c} = -\left\{ \frac{a}{c}\cos\theta + \frac{a^2}{2\,c^2}\cos 2\theta + \ldots + \frac{a^n}{n\,c^n}\cos n\theta + \ldots \right\} \qquad (5)$$

The derivation of the last series, which is similar to that of the others, is as follows:

$$O\,P^2 = c^2 + a^2 - 2\,a\,c\cos\theta$$

$$\frac{O\,P^2}{c^2} = 1 - a/c\,(e^{j\theta} + e^{-j\theta}) + a^2/c^2$$

$$= (1 - a/c\; e^{j\theta})(1 - a/c\; e^{-j\theta})$$

$$2 \operatorname{logh} \frac{O\,P}{c} = -\left\{ \frac{a}{c} e^{j\theta} + \frac{a^2}{2\,c^2} e^{2j\theta} + \ldots + \frac{a^n}{n\,c^n} e^{nj\theta} + \ldots + \frac{a}{c} e^{-j\theta} + \frac{a^2}{2\,c^2} e^{-2j\theta} + \ldots + \frac{a^n}{n\,c^n} e^{-nj\theta} + \ldots \right\}$$

from which equation (**5**) follows directly. Series (**3**), (**4**) and (**5**) are used in calculations of proximity effect in conductors.

Equation (**2**) can now be written: Work in moving a unit charge from O to N

$$= 2\,Q \operatorname{logh}\left(\frac{p}{p-c}\right) - 2\,Q \left\{ \frac{a}{p}\cos\varphi + \frac{a^2}{2\,p^2}\cos 2\varphi + \ldots + \frac{a^n}{n\,p^n}\cos n\varphi + \ldots \right\}$$

$$+ 2\,a \int_0^{2\pi} \left[q_{(\theta)} \operatorname{logh} \frac{O\,P}{c} - q_{(\theta)} \operatorname{logh} \frac{N\,P}{a} + q_{(\theta)} \operatorname{logh} c/a \right] d\theta \qquad (6)$$

where $$q_{(\theta)} = \frac{Q}{2\pi a} + H_1\cos\theta + H_2\cos 2\theta + \ldots + H_n \cos n\theta + \ldots$$

Now $$\int_0^{2\pi} \cos m\theta \cos n\theta\, d\theta$$

$$= 1/2 \int_0^{2\pi} \{\cos(m+n)\theta + \cos(m-n)\theta\}\, d\theta \qquad (7)$$

This is equal to zero when m is not equal to n and it is equal to π when $m = n$. Thus, in multiplying one series by another in (**6**), most of the terms are equal to zero.

Work in moving a unit charge from O to N

$$= 2\,Q \operatorname{logh}\left(\frac{p}{p-c}\right) - 2\,Q\left\{\frac{a}{p}\cos\varphi + \frac{a^2}{2\,p^2}\cos 2\,\varphi + \ldots + \frac{a^n}{n\,p^n}\cos n\,\varphi + \ldots\right\}$$

$$+ 2\,\pi\,a\Big[-H_1\frac{a}{c} - H_2\frac{a^2}{2\,c^2} - \ldots - H_n\frac{a^n}{n\,c^n} - \ldots + H_1\cos\varphi + H_2\frac{\cos 2\,\varphi}{2} + \ldots + H_n\frac{\cos n\,\varphi}{n} + \ldots + 2\frac{Q}{2\,\pi\,a}\operatorname{logh}\frac{c}{a}\Big]$$

This is true for any value of φ, since the potential is constant for all points on the surface of the conductor, and so the coefficients of $\cos\varphi$, $\cos 2\,\varphi$, $\cos n\,\varphi$ etc. can be separately equated to zero. Therefore,

$$H_1 = \frac{Q}{\pi\,a}\,a/p$$

$$H_n = \frac{Q}{\pi\,a}\,a^n/p^n$$

$$q_{(\theta)} = \frac{Q}{2\,\pi\,a} + \frac{Q}{\pi\,a}\{\,a/p\cos\theta + a^2/p^2\cos 2\,\theta + \ldots + a^n/p^n\cos n\,\theta + \ldots\}$$

$$\text{or } q_{(\theta)} = \frac{Q}{2\,\pi\,a} + \frac{Q}{\pi\,a}\sum_{n=1}^{n=\infty}\frac{a^n}{p^n}\cos n\,\theta \qquad (8)$$

This gives the distribution of the electric charge on the finite conductor, under the influence of a charge concentrated at B. This is a quantity required to be known for calculating capacitance in the problems described in this paper.

Single-Phase Overhead Line

The formula for this case is given to provide a check on the correctness of the direct method. The standard formula, derived by the inverse method, is shorter, and gives exactly the same numerical results.

Let there be two equal wires A and B and let one carry an electric charge Q per centimeter and the other a charge $-Q$ per centimeter. Then the density at P is $\frac{Q}{2\,\pi\,a}$ and at T it is $\frac{-Q}{2\,\pi\,a}$, assuming uniform distribution.

The density at P due to the elementary charge at T,

$$-\frac{Q}{2\,\pi\,a}\,a\,d\,\gamma$$

is

$$\frac{Q\,d\,\gamma}{2\,\pi}\times\frac{1}{\pi\,a}\sum_{n=1}^{n=\infty}\frac{a^n}{d^n}\cos n\,(\theta-\beta) \qquad (9)$$

Now

$$\frac{\cos n\,\beta}{d^n} = 1/s^n\left[1+\sum_{k=1}^{k=\infty}\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\,\frac{a^k}{s^k}\cos k\,\gamma\right] \qquad (10)$$

and

$$\frac{\sin n\,\beta}{d^n} = 1/s^n\sum_{k=1}^{k=\infty}\frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\,\frac{a^k}{s^k}\sin k\,\gamma\ * \qquad (11)$$

Integrate expression (9) from $\gamma = 0$ to $2\,\pi$, keeping θ constant. Then the density at P due to the uniform charge on wire B is

$$\frac{Q}{\pi\,a}\sum_{n=1}^{n=\infty}\frac{a^n}{s^n}\cos n\,\theta \qquad (12)$$

Let $\qquad A_n = a^n/s^n \qquad (13)$

This is the first additional charge. A similar expression gives the first additional charge on wire B, namely,

$$-\frac{Q}{\pi\,a}\sum_{n=1}^{n=\infty}A_n\cos n\,\gamma$$

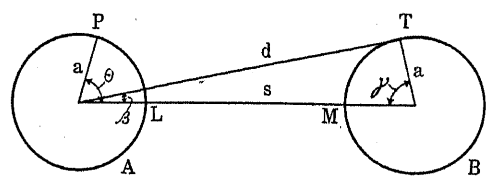

Fig 2—Single-Phase Overhead Line

The density at P due to the elementary charge at T,

$$-\frac{Q}{\pi\,a}\sum_{n=1}^{n=\infty}A_n\cos n\,\gamma\,a\,d\,\gamma$$

is $$\frac{Q\,d\,\gamma}{\pi}\times\frac{1}{\pi\,a}\sum_{n=1}^{n=\infty}A_n\cos n\,\gamma\sum_{m=1}^{m=\infty}\frac{a^m}{d^m}\cos m\,(\theta-\beta)$$

Using equations (7), (10) and (11), and integrating from $\gamma = 0$ to $2\,\pi$, we find that the density at P, due to the first additional charge on wire B is

$$\frac{Q}{\pi\,a}\,[B_1\cos\theta + B_2\cos 2\,\theta + \ldots + B_n\cos n\,\theta + \ldots] \qquad (14)$$

$$\text{where } B_n = a^n/s^n\left\{\frac{\underline{|n+1-1}}{\underline{|n-1}\,\underline{|1}}\,a/s\,A_1 + \ldots + \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}}\,a^k/s^k\,A_k + \ldots\right\} \qquad (15)$$

*"An Integration Method of Deriving the A-C. Resistance and Inductance of Conductors," by H. L. Curtis, Scientific Paper No. 374 of the Bureau of Standards, Washington D. C., April, 1920, Appendix 2.

This is the second additional charge on wire A. Similarly, the third additional charge on wire A is

$$\frac{Q}{\pi a}\,[C_1 \cos\theta + C_2 \cos 2\theta + \ldots + C_n \cos n\theta + \ldots] \quad (16)$$

$$\text{where } C_n = a^n/s^n \left\{ \frac{\lfloor n+1-1}{\lfloor n-1 \lfloor 1} a/s\, B_1 + \ldots + \frac{\lfloor n+k-1}{\lfloor n-1 \lfloor k} a^k/s^k A_k + \ldots \right\} \quad (17)$$

and so on. The total charge at P is

$$\frac{Q}{\pi a}\,[1/2 + L_1 \cos\theta + L_2 \cos 2\theta + \ldots + L_n \cos n\theta + \ldots] \quad (18)$$

$$\text{where} \quad L_n = A_n + B_n + C_n + \ldots \quad (19)$$

The capacitance C may be found by calculating the work done in moving a unit charge from M to L against the charge on wire A. This is

$$\frac{2Q}{\pi}\int_0^{2\pi} [1/2 + L_1\cos\theta + \ldots + L_n \cos n\theta + \ldots]$$

$$\left[\operatorname{logh}\left(\frac{s-a}{a}\right) + \cos\theta + \frac{1}{2}\cos 2\theta + \ldots + \frac{1}{n}\cos n\theta + \ldots - \frac{a}{s-a}\cos\theta - \frac{a^2}{2(s-a)^2}\cos 2\theta - \ldots - \frac{a^n}{n(s-a)^n}\cos n\theta - \ldots\right] d\theta$$

This is equal to $\frac{Q}{2C}$. Therefore,

$$1/C = 4\left[\operatorname{logh}\frac{s-a}{a} + L_1\left\{1 - \frac{a}{s-a}\right\} + \ldots + L_n/n\left\{1 - \frac{a^n}{(s-a)^n}\right\} + \ldots\right] \quad (20)$$

An alternative expression of different algebraical form can be obtained by finding the work done against the charges on both wires by carrying a unit charge to the surface of one wire from the neutral point midway between the two wires. This gives

$$1/C = 4\left[\operatorname{logh} s/a - \frac{a}{s}L_1 - \frac{a^2}{2s^2}L_2 - \ldots - \frac{a^n}{ns^n}L_n - \ldots\right] \quad (21)$$

Formulas (**20**) and (**21**) both give exactly the same numerical results as the standard formula:

$$1/C = 4 \operatorname{logh}\frac{s + \sqrt{s^2 - 4a^2}}{2a} = 4\cosh^{-1}\frac{s}{2a} \;* \quad (22)$$

EXAMPLE OF SINGLE-PHASE OVERHEAD LINE

Let $\quad s/a = 10$

Then $1/C = 9.16972$ by formulas (20), (21) and (22), in absolute units.

TWO-CONDUCTOR SHEATHED CABLE (CAPACITANCE OF THE SHEATH AGAINST THE TWO CONDUCTORS)

Let the radius of the conductors of the cable be a cm. and let the inside radius of the sheath be c cm. Then, as is well known in connection with calculations of capacitance, image conductors can be assumed at a distance s and of radius a', which will carry charges equal and opposite to those of the cable conductors. The inner surface of the sheath can be considered as a

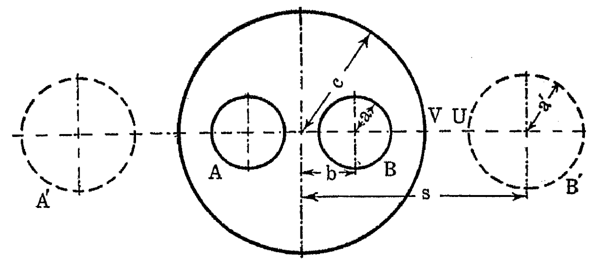

FIG. 3—TWO-CONDUCTOR SHEATHED CABLE

surface of zero potential, carrying no charges. The dimensions a' and s are given by

$$a' = \frac{a\,c^2}{b^2 - a^2} \quad (23)$$

and

$$s = \frac{b\,c^2}{b^2 - a^2} \;** \quad (24)$$

The first additional density of charge at any point of the surface of conductor B, due to uniformly distributed charges on the other three conductors may be calculated in a manner similar to that used for an overhead single-phase circuit. The angle θ is used for the conductor to the right of B and the angle $(\theta - \pi)$ is used for the conductors to the left. The first additional density on B is

$$\frac{Q}{\pi a}\sum_{n=1}^{n=\infty} A_n \cos n\theta \quad (25)$$

*For the derivation of formula (22) see "Transmission Line Formulas" by H. B. Dwight, 1st edition, Chapter XIII, or other books on alternating-current theory.

**"Elements of the Mathematical Theory of Electricity and Magnetism," by J. J. Thomson, fourth edition, pages 149 and 176.

$$\text{where } A_n = \left(\frac{a}{s-b}\right)^n - \left(\frac{-a}{2b}\right)^n + \left(\frac{-a}{s+b}\right)^n \tag{26}$$

The first additional density on B' is

$$\frac{Q}{\pi a'} \sum_{n=1}^{n=\infty} F_n \cos n\gamma \tag{27}$$

$$\text{where } F_n = -\left(\frac{a'}{s-b}\right)^n + \left(\frac{a'}{2s}\right)^n - \left(\frac{a'}{s+b}\right)^n \tag{28}$$

The second additional density on B is

$$\frac{Q}{\pi a} \sum_{n=1}^{n=\infty} B_n \cos n\theta \tag{29}$$

where $B_n =$

$$\begin{aligned} &-\left(\frac{a}{s-b}\right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{a'}{s-b}\right)^k F_k \\ &-\left(\frac{-a}{2b}\right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{-a}{2b}\right)^k A_k \\ &-\left(\frac{-a}{s+b}\right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{a'}{s+b}\right)^k F_k \end{aligned} \tag{30}$$

The second additional density on B' is

$$\frac{Q}{\pi a'} \sum_{n=1}^{n=\infty} G_n \cos n\gamma \tag{31}$$

where $G_n =$

$$\begin{aligned} &-\left(\frac{a'}{s-b}\right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{a}{s-b}\right)^k A_k \\ &-\left(\frac{a'}{2s}\right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{a'}{2s}\right)^k F_k \\ &-\left(\frac{a'}{s+b}\right)^n \sum_{k=1}^{k=\infty} \frac{\underline{|n+k-1}}{\underline{|n-1}\,\underline{|k}} \left(\frac{-a}{s+b}\right)^k A_k \end{aligned} \tag{32}$$

For C_n and H_n use the same formulas as for B_n and G_n except change A to B and F to G. Then for D_n and I_n change B to C and G to H, and so on.

The total density on B is

$$\frac{Q}{\pi a}\left[1/2 + \sum_{n=1}^{n=\infty} L_n \cos n\theta\right] \tag{33}$$

$$\text{where} \quad L_n = A_n + B_n + C_n + \ldots \tag{34}$$

The total density on B' is

$$\frac{Q}{\pi a'}\left[-1/2 + \sum_{n=1}^{n=\infty} M_n \cos n\gamma\right] \tag{35}$$

$$\text{where} \quad M_n = F_n + G_n + H_n + \ldots \tag{36}$$

It is now necessary to calculate the work done against all the charges, when carrying a unit charge from the surface of an image conductor to the surface of one of the conductors of the cable. This quantity of work is twice the potential between the grounded sheath and the two conductors which are taken connected in parallel in this calculation. Therefore,

$$\text{Work} = \frac{4Q}{C}$$

where C is the capacitance of the sheath on one side and the two conductors on the other. The amount of the work is calculated by using equation (1) of this paper, and the following result is obtained:

$$\begin{aligned} 1/C = {} & \operatorname{logh} \frac{(s-b-a')(s-b-a)(s+b-a')(s+b+a)}{a a'(2b+a)(2s-a')} \\ & + \sum_{n=1}^{n=\infty} L_n/n \left[1 - \left(\frac{a}{s-b-a'}\right)^n - \left(\frac{-a}{s+b-a'}\right)^n + \left(\frac{-a}{2b+a}\right)^n\right] - \sum_{n=1}^{n=\infty} M_n/n \Bigg[1 \\ & - \left(\frac{a'}{s-b-a}\right)^n - \left(\frac{a'}{s+b+a}\right)^n + \left(\frac{a'}{2s-a'}\right)^n\Bigg] \end{aligned} \tag{37}$$

Equation (37), in connection with (23), (24), (26), (28), (30), (32), (34) and (36), is used in calculating numerical values of capacitance.

Example, Two-Conductor Cable. $a = 1, b = 2, c = 5$ Then $s = 50/3$ and $a' = 25/3$.

This corresponds to $t/T = 1$ and $\frac{T+t}{d} = 1$ in Mr. Simons' paper previously referred to, Table I, and the calculation gives a geometric factor of 1.730 to compare with 1.718 given by the graphical method. The difference in this case is less than one per cent.

The geometric factor suggested by D. M. Simons, for capacitance of an n-conductor cable, between the sheath on one side and the n conductors on the other, is

$$G_1 = \frac{k\,n}{2C}$$

where k is the permittivity of the dielectric, and C is the capacitance per unit length for the above connection. The geometric factor can also be used in formulas for thermal resistance.

Three-Conductor Sheathed Cable, (Capacitance of the Sheath against the Three Conductors)

The size and location of the image conductors are given by the following equations which are similar to (23) and (24) of the calculation for a two-conductor cable:

$$a' = \frac{c^2 a}{b^2 - a^2} \tag{38}$$

$$s + b = \frac{c^2 b}{b^2 - a^2} \tag{39}$$

The first additional density on A, due to uniform density of charge on all the other conductors is

$$\frac{Q}{\pi a}\sum_{n=1}^{n=\infty} A_n \cos n\,\theta \qquad (40)$$

where $A_n = \dfrac{a^n}{s^n} - \dfrac{2\,a^n}{t^n}\cos n\,\delta + \dfrac{2\,a^n}{u^n}\cos n\,\sigma \qquad (41)$

where Q is the electric charge per centimeter on each conductor, and where the various dimensions and angles are indicated in Fig. 4.

The first additional density on A' is

$$\frac{Q}{\pi a'}\sum_{n=1}^{n=\infty} F_n \cos n\,\gamma \qquad (42)$$

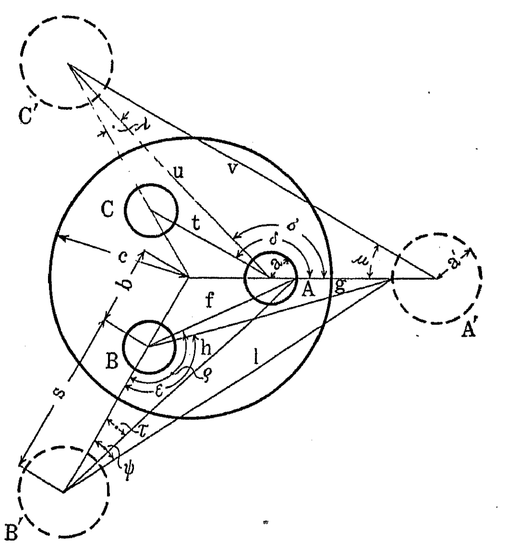

FIG. 4—THREE CONDUCTOR SHEATHED CABLE

where

$$F_n = -\frac{a'^n}{s^n} - \frac{2\,a'^n}{u^n}\cos n\,\lambda + \frac{2\,a'^n}{v^n}\cos n\,\mu \qquad (43)$$

The second additional density on A is

$$\frac{Q}{\pi a}\sum_{n=1}^{n=\infty} B_n \cos n\,\theta \qquad (44)$$

$$\text{where } B_n = -\,a^n/s^n \sum_{k=1}^{k=\infty}\frac{|\underline{n+k-1}}{|\underline{n-1}\,|\underline{k}}\,\frac{a'^k}{s^k}F_k$$

$$-\frac{2\,a^n}{t^n}\cos n\,\delta\sum_{k=1}^{k=\infty}\frac{|\underline{n+k-1}}{|\underline{n-1}\,|\underline{k}}\,\frac{a^k}{t^k}A_k\cos k\,\delta$$

$$-\frac{2\,a^n}{u^n}\cos n\,\sigma\sum_{k=1}^{k=\infty}\frac{|\underline{n+k-1}}{|\underline{n-1}\,|\underline{k}}\,\frac{a'^k}{u^k}F_k\cos k\,\lambda \qquad (45)$$

The second additional density on A' is

$$\frac{Q}{\pi a'}\sum_{n=1}^{n=\infty} G_n \cos n\,\gamma \qquad (46)$$

$$\text{where } G_n = -\,a'^n/s^n \sum_{k=1}^{k=\infty}\frac{|\underline{n+k-1}}{|\underline{n-1}\,|\underline{k}}\,\frac{a^k}{s^k}A_k$$

$$-\frac{2\,a'^n}{u^n}\cos n\,\lambda\sum_{k=1}^{k=\infty}\frac{|\underline{n+k-1}}{|\underline{n-1}\,|\underline{k}}\,\frac{a^k}{u^k}A_k\cos k\,\sigma$$

$$-\frac{2\,a'^n}{v^n}\cos n\,\mu\sum_{k=1}^{k=\infty}\frac{|\underline{n+k-1}}{|\underline{n-1}\,|\underline{k}}\,\frac{a'^k}{v^k}F_k\cos k\,\mu \qquad (47)$$

For C_n and H_n use the same formulas as for B_n and G_n except change A to B and F to G. Then for D_n and I_n change B to C and G to H, and so on.

The total density on A is

$$\frac{Q}{\pi a}\left[1/2 + \sum_{n=1}^{n=\infty} L_n \cos n\,\theta\right] \qquad (48)$$

where $\quad L_n = A_n + B_n + C_n + \ldots \qquad (49)$

The total density on A' is

$$\frac{Q}{\pi a'}\left[-\,1/2 + \sum_{n=1}^{n=\infty} M_n \cos n\,\gamma\right] \qquad (50)$$

where $\quad M_n = F_n + G_n + H_n + \ldots \qquad (51)$

The work done in carrying a unit charge from the surface of conductor A' to the surface of conductor A, along the line joining their centers, is equal to

$$\frac{6\,Q}{C}$$

where C is the capacitance of the condenser consisting of the sheath on one side and the three conductors on the other.

The result of integrating the work, due to all the charges is

$$1/C = 1/3\,\mathrm{logh}\,\frac{(s-a')\,(s-a)}{a\,a'} + 2/3\,\mathrm{logh}\,\frac{g\,h}{f\,l}$$

$$+\,1/3\sum_{n=1}^{n=\infty} L_n/n\left[1-\left(\frac{a}{s-a'}\right)^n\right.$$

$$\left.+\,2\,(a/f)^n\cos n\,\rho - 2\,(a/g)^n\cos n\,\epsilon\right]$$

$$-\,1/3\sum_{n=1}^{n=\infty} M_n/n\left[1-\left(\frac{a'}{s-a}\right)^n\right.$$

$$\left.-\,2\,(a'/h)^n\cos n\,\tau + 2\,(a'/l)^n\cos n\,\psi\right] \qquad (52)$$

EXAMPLE, THREE-CONDUCTOR CABLE

$$a = 1,\quad b = \sqrt{3},\quad c = 2+\sqrt{3}$$

This corresponds to $t/T = 1$ and $\dfrac{T+t}{d} = 1/2$ in Mr. Simons' paper previously referred to, Table I. The calculation gives a geometric factor of 1.465 to compare with 1.455 obtained by the graphical method. The difference is less than one per cent.

Discussion

D. M. Simons: I would like to emphasize Mr. Dwight's statement that while his solution is in terms of capacity, in so far as the geometric relationships are concerned, his work is immediately applicable to the calculation of thermal resistance, electrical resistance, dielectric loss, etc. His paper is especially valuable inasmuch as it makes possible a rigid calculation of thermal resistance between conductors and sheath, which calculation is believed to be the most usual calculation where the geometric relationship between the three conductors as one electrode and the sheath as the other electrode are involved.

I would like to point out also that Mr. Dwight's formula is really merely a guide to a long and difficult mathematical process. It will be found that its application is by no means similar to the use of a simple formula involving one or two slide-rule operations, but that it is really a laborious computation. It is to be hoped that someone will have an opportunity of making a large series of such computations so as to make the exact data available throughout the range of the usual proportions of three-conductor cables.

Vladimir Karapetoff: Mr. Dwight's paper is certainly a bold attempt to treat the problem in a somewhat novel manner, and I hope that the method outlined is not only correct but is the simplest method possible for this particular problem. I mean that while we cannot solve this problem by the so-called method of inversion, yet there is another method which gives splendid results for two spheres.[2] This latter method would be of no use if it were not for two facts: first, that all the fictitious charges are concentrated between the center of each sphere and its periphery on the center line; and, secondly, that all consecutive charges decrease in magnitude. Therefore, by putting an infinite number of charges along both center lines, a new electrical system is created such that, for that system, both spheres are equipotential surfaces. According to a general theory of electrostatics, if a possible distribution of electricity in equilibrium has been found, then this is the only possible distribution. Thus, having found a solution, we know that it is the solution.

It seems to me that Mr. Dwight goes to an unnecessary complication by not limiting himself to linear charges by taking charges distributed upon cylinders, and I wish to suggest that he try the same method as used with spheres.

S. E. Pero: (by letter): The final results in Mr. Dwight's paper seem to admit of simplification by a continuation of the analysis. At least in the case of the single-phase overhead-line formula which is the one I have examined particularly, the solution of the series equation by the method of successive substitutions will reduce the formula to the usual form by which Mr. Dwight checked his result numerically. This method of solution was applied by Mr. John R. Carson in his paper "Wave Propagation over Parallel Wires: The Proximity Effect," *Phil. Mag.* April, 1921, and in allied investigations. In fact, in 1920, we obtained a solution for the capacity in the case of two wires enclosed in a sheath, of which the single-phase circuit was treated as a special case.

Mr. Dwight's analysis of the latter problem may be continued in the following way. Denoting by λ, the ratio a/s where a is the radius of the conductors and s the interaxial separation, the capacity c between the wires of a single-phase overhead line, is, by formulas (13), (15), (17), (19) and (21) of the paper

$$\frac{1}{C} = 4\left[\log\frac{1}{\lambda} - \lambda L_1 - \frac{\lambda^2}{2} L_2 - \ldots \frac{\lambda^n}{n} L_n - \ldots\right] \quad (1)$$

where

$$L_n = A_n + B_n + C_n + \ldots \quad (2)$$

$$A_n = \lambda^n$$

$$B_n = \lambda^n \left[\frac{n}{1\,!}\lambda A_1 + \frac{n\,(n+1)}{2\,!}\lambda^2 A_2 + \ldots\right] \quad (3)$$

$$C_n = \lambda^n \left[\frac{n}{1\,!}\lambda B_1 + \frac{n\,(n+1)}{2\,!}\lambda^2 B_2 + \ldots\right]$$

Consequently, by equations (2) and (3),

$$L_n = \lambda^n \left[1 + \Sigma_1^k \frac{(n+k-1)\,!}{(n-1)\,!\,k\,!}\lambda^k L_k\right] \quad (4)$$

$$= \lambda^n t_n \quad (5)$$

where

$$t_n = 1 + n\,\lambda^2 t_1 + \frac{n\,(n+1)}{2\,!}\lambda^4 t_2 + \ldots \quad (6)$$

Equation (6) is identical in form with equation (35) of Mr. Carson's paper on proximity effect. The solution may be obtained as explained in that paper, or may be derived by successive substitutions. By the latter method, the solution of the set of equations (6) is given by the limit of the sequence $t_n^{(0)}\, t_n^{(1)} \ldots t_n^{(n)} \ldots$ where

$$t_n^{(0)} = 1$$

$$t_n^{(1)} = \left(\frac{1}{1-\lambda^2}\right)^n$$

$$t_n^{(2)} = \left(\frac{1}{1-\dfrac{\lambda^2}{1-\lambda^2}}\right)^n$$

or, finally, t_1 is given by the continued fraction

$$t_1 = \cfrac{1}{1-\cfrac{\lambda^2}{1-\cfrac{\lambda^2}{1-\lambda^2 \ldots}}}$$

$$= t = 2\,\frac{1-\sqrt{1-(2\,\lambda)^2}}{(2\,\lambda)^2} \quad (7)$$

and

$$t_n = t^n \quad (8)$$

From equations (5) and (8),

$$\lambda^n L_n = \lambda^{2n} t^n \quad (9)$$

and substituting this, (1) becomes

$$\frac{1}{C} = 4\left[\log\frac{1}{\lambda} + \log\,(1-\lambda^2 t)\right]$$

$$= 4\log\frac{1+\sqrt{1-4\,\lambda^2}}{2\,\lambda}$$

$$= 4\log\frac{s+\sqrt{s^2-4\,a^2}}{2\,a} \quad (10)$$

which is formula (22) of the paper, the usual expression for the capacity in this case.

While the succeeding formulas are considerably more complicated it is possible that the foregoing method is applicable to them also.

Formulas (10) and (11) are credited to Mr. Curtis, As a matter of fact, these formulas are very old and they, or their equivalents, are found in such standard works as Hobson "Plane Trigonometry" 2nd edition, p. 261, Bromwich "Theory of Infinite Series" pp. 158-159 and Webster "The Dynamics of Particles and of Rigid, Elastic and Fluid Bodies," p. 391.

H. B. Dwight: The mathematical derivation by Miss Pero of the standard formula for capacitance of two overhead wires, from series (21) of my paper, is a very interesting and useful

2 A. Russell, "Alt. Currents," Vol. I, Chap. 8

result. The simplification obtained in this way in the expression for capacitance is considerable, and if, as she suggests, a similar simplification can be made in the three-conductor formula (52), it would be a great advantage.

A check on the use of inverse surfaces or image surfaces for calculation of capacitance, as employed in my paper, can be made by using this method to calculate the capacitance of a sheathed cable with one eccentric conductor. The result is exactly the same algebraical expression as given by Alexander Russell, in "Treatise on the Theory of Alternating Currents," Edition of 1914, page 165, although his result was obtained without using images.

The fact that the three discussions of this paper suggest that it would be desirable to have a mathematical investigation of considerable length undertaken, shows that in electrical engineering, the same as in the science of physics, research cannot be carried on entirely by laboratory experiments, but a definite place must be given to research by calculation.

The JOURNAL of the A. I. E. E., by publishing papers which give the results of electrical engineering research by calculation, has done a great deal to make this type of research possible. This support should be continued. In spite of the demand which is sometimes expressed that all papers published should be interesting reading, research papers should be judged according to the criterion of whether they constitute a step in the solution of a problem in electrical engineering.

Mr. Simons is quite right about the length of the calculation in the paper which I presented. In fact, I would have hesitated to have written this up as a paper and sent it in if the cable people had not assured me (this applies particularly to the three-conductor calculation, which is extra long) that in spite of the length they wanted to use it.

There are very large opportunities, especially in this calculation of shortening the work for the people using the calculations. That is a long, tedious work but is quite worth while.

Dielectric Field in an Electric Power Cable-II.

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I. Introduction

THERE are certain properties of multiconductor cables, the solution of which up to the present time has been made mathematically in approximate form only. These problems are, however, capable of satisfactory solution experimentally. The writer presents herewith an experimental solution of some of these problems, this work being a continuation of data presented in a paper[1] before the Institute in 1919. Determination is made of dielectric stresses in three-conductor cables and of the geometric properties affecting the calculation of capacity, dielectric losses, etc., in such cables.

The nature of these problems is best illustrated by considering the case of a single-conductor cable where the determination of the capacitance, in terms of the specific inductive capacity of the material and of the dimensions, reduces to a problem of plane geometry and of simple calculus, the solution having been given by many authors. Thermal resistance or conduction and insulation resistance, etc. are also calculated by the same type of formula. Furthermore, by similar means, the electrostatic stress at any portion of the dielectric may be determined. The present investigation results in making available a considerable amount of similar information for three-conductor cables. In the 1919 work, cable models were prepared consisting of three symmetrically spaced tubes representing the three conductors, surrounded by a fourth tube representing the sheath, all immersed in an electrically conducting liquid (electrolyte). Electrical resistance measurements and difference of potential for stress measurements made on these cable models result in a solution of the geometric problem for the dimensions of the models used and the properties determined from these models are exactly those for actual cables having cross sections which are similar figures geometrically. In making this application, it is assumed that the capacities and stresses of an actual cable are but little affected by the spiralling effect, that is, the twisting or cabling together of the conductors.

During 1920 and 1921, a further series of similar experiments was performed using a different type of model. The models in this series of tests consisted of sheets of tinfoil on which were soldered three copper rings to represent the conductors, surrounded by a fourth larger ring to represent the sheath. The earlier article gave data on three-conductor cables with round conductors only. The new series covers part of the range of the earlier article, but has extreme cases in addition, and also includes the case of sector-shaped conductors. Furthermore, data are given on round duplex cables.

It seems worthwhile, therefore, to supplement the previous paper by these new data, not only to check and enlarge the previous data, but because up to the present time no data of this type have been published on sector-shaped conductors.

Furthermore, while data have been presented confirming part of those published in 1919, some data recently published are not in entire agreement with the writer's early work and it is desirable to point out that additional data obtained are confirmatory of our earlier work.

Sacchetto[1] performed a series of experiments with cable models made of electrodes in a tub of electrolyte as in our early work, and obtained curves from which the geometric relations used in the calculation of capacity, temperature rise, dielectric loss, etc., can be calculated. He did not investigate the stresses. Sacchetto's results agree very closely with the earlier work for the case of three-phase operation, but depart from it somewhat for the case of three conductors as one electrode, against the sheath as the other. Fortunately, additional data are available on this subject, since a recent paper by Simons points out that this problem is capable of graphical solution for the case of three conductors against the sheath. His results check the earlier work of the writer throughout the entire range of that work, to an accuracy of 2 per cent, which is believed to be a very satisfactory check. Furthermore, Simons showed that while exact mathematical calculation has not been possible, an approximate mathematically-derived formula of Mie gives results which are definitely in error in a certain direction. For example, the capacity calculated from this formula is always greater than the true capacity. In the range where Sacchetto's data depart materially from those of the writer, capacities still greater than those determined from Mie's formula are shown. Sacchetto's data are therefore in error at least by the amount of the difference between his data and Mie's formula, whereas the writer's data are consistent with Mie's formula and agree with Simons' graphical solution. This phase of the matter therefore seems to be sufficiently answered without further reference.

Shortly after Sachetto's paper, an article was published by Emanueli on the subject of stress in three-conductor cables, the method of measurement being also based on cable models composed of tubes in a tank of electrolyte. Emanueli's results differ from the writer's results by as much as 12 per cent.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

1. See bibliography at end of paper for all articles cited.

In an article written in 1922 by Middleton, Dawes, and Davis, dielectric strength tests of single and three-conductor cables were reported and, based on an assumption that the breakdown voltage would be dependent upon the maximum stress, a formula or method of calculation of maximum stress was suggested. The method they advocate gives values of stress more than 30 per cent higher for the cases mentioned than the values herein given.

The present paper is therefore written not only to extend the range of the data already published, but to analyze these discrepancies and to establish the true values of the stresses in three-conductor cables. It is worthy of emphasis at this point that it is no part of the present paper to discuss the effect of distribution of stress upon the breakdown voltage, or the allowable working voltage of a given cable.

II. Description of Cable Models

The cable models of the later series were prepared in the following manner. From a roll of tinfoil 15 in. wide, strips 15 in. and 4 in. were cut alternately, the square pieces being used for cable models and the 4-in. strips for resistivity measurements. The foil was 0.0035 in. thick. A copper strip 0.450 by 0.096 in. was bent around a wooden form with a 14.5-in. diameter. The two ends were held together with about ¼ in. overlap and were brazed together with silver solder. The ends were then beveled off smooth. Both edges of the ring were carefully tinned. The ring was then laid on one of the square sheets of tinfoil and a soldering iron run around the upper edge. This soldered the lower edge to the tinfoil. It took a considerable amount of preliminary work to do this accurately, but in the final models the foil was connected to the ring at all points, with very little solder extending out over the foil from the ring. This ring represents the sheath.

The round conductors were obtained by cutting bands ¾ in. wide from copper tubes of the desired diameter. These were then trued up on a lathe to about ⅝ in. wide. Three of these rings were soldered to the tinfoil in the same manner as the sheath at predetermined locations to represent the three conductors of the cable with given proportions of insulation. The sector conductors were prepared as follows: A sector cross-section was drawn in full scale on a sheet of paper, and inside of this was drawn another sector differing from the first by the thickness of the copper strip used for the sheath. A wooden form was then prepared to fit exactly the inner sector, and a copper strip was carefully bent around this form, soldered together, and soldered to the tinfoil exactly as in the case of the sheath.

To the sheath and to each of the conductors were soldered three copper leads, the three leads to each electrode being made carefully of the same length. With this means of introducing the current to each electrode at different points, the electrode surfaces approach more nearly equipotential surfaces, and indeed any practical difference of potential between different points of the surface of one electrode is eliminated.

In addition to the tinfoil cable models, seven sector cable models of the electrolyte tank type were measured for geometric properties in 1918. These data were not given in the 1919 paper, and will therefore be included here. Table I gives the dimensions of all the models used.

TABLE I
DIMENSIONS OF CABLE MODELS IN INCHES

Cable Model		Conductors			Insulation		D
No.	Type	No.	d	Shape	T	t	
10	Tinfoil	1	4.00	Round	5.25	..	14.50
11	Tinfoil	3	1.25	Round	1.89	1.95	14.60
12	Tinfoil	3	2.00	Round	1.62	1.62	14.50
15	Tinfoil	2	5.75	Round	0.56	0.35	14.40
16	Tinfoil	2	3.75	Round	1.31	0.87	14.50
17	Tinfoil	2	2.00	Round	1.96	1.34	14.56
19	Tinfoil	3	5.50	Round	0.42	0.42	14.40
20	Tinfoil	3	1.14	Round	2.43	0.61	14.35
21	Tinfoil	3	2.00	Round	2.11	0.51	14.38
23	Tinfoil	3	3.62*	Sector *A*	1.24	1.29	14.34
24	Tinfoil	3	3.62*	Sector *A*	1.68	0.38	14.38
25	Tinfoil	3	1.81*	Sector *B*	1.80	1.80	14.52
26	Tinfoil	3	1.81*	Sector *B*	2.37	0.59	14.52
27	Tinfoil	1	1.25	Round	6.63	..	14.50
28	Tinfoil	3	3.90*	Sector *C*	1.28	1.28	14.36
29	Electrolyte	3	2.29*	Sector *D*	1.39	0.00	9.90
30	Electrolyte	3	2.29*	Sector *D*	2.38	0.59	15.40
31	Electrolyte	3	2.29*	Sector *D*	2.65	0.00	15.40
32	Electrolyte	3	2.29*	Sector *D*	1.82	1.82	15.40
33	Electrolyte	3	2.29*	Sector *D*	1.83	0.00	11.80
34	Electrolyte	3	2.29*	Sector *D*	1.64	0.41	11.80
35	Electrolyte	3	2.29*	Sector *D*	1.25	1.25	11.80

d = Conductor diameter.
T = Conductor insulation thickness.
t = Belt insulation thickness.
D = Diameter under sheath.
*d for sector is diameter of round conductor of same cross-section.

Three different forms of sector were used for the tinfoil "cables," the main dimensions and shapes being given in Fig. 1. Sectors *A* and *B* are similar to those used in the cables manufactured by the company with which the writer is associated. Sector *C* was designed as an extreme sector with sharp curvature at the various points. We might add that the major and minor axes of the sectors are the following: for sector *A*, 4.75 in., 2.92 in.; for Sector *B*, 2.38 in., 1.46 in.; for Sector *C*, 6.06 in., 3.02 in. The shape of sector used with the electrolytic tank models Nos. 29 to 35 which we will call Sector *D*, is similar in general characteristics to sectors *A* and *B*. The radius of curvature at the three sharper tips was ⅝ in., and the radius of curvature of the outer surface was 4 in. The major axis was 2.97 in. and the minor axis 1.84 in. Fig. 2 shows the four sector cables of tinfoil form on which measurements of stress were made, and is offered to assist in visualizing the proportions of these cables.

Alexander Russell, in one of his articles, suggested the solution of these geometric problems by means of models using metal foil in the manner above described. We had in mind several purposes in adopting this method for a further series of measurements, in-

stead of the one previously used. The method removed all doubt as to the possibility of polarization. A method was devised by which it was possible to use, in connection with the foil models, a null method for measuring the stress at a given point. A null method was considered to have certain possible advantages from the standpoint of insuring reliability of the measurement, as compared with a deflection method. Other incidental advantages of the foil models are that it is easier to locate definitely the position of the finder

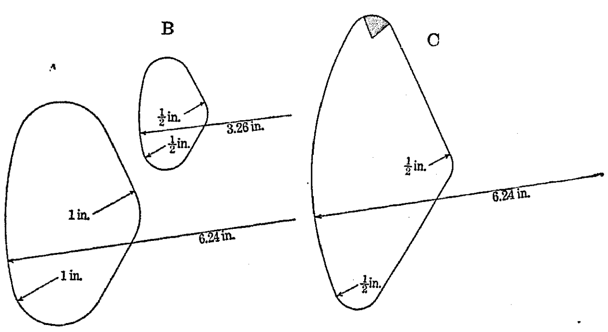


Fig. 1

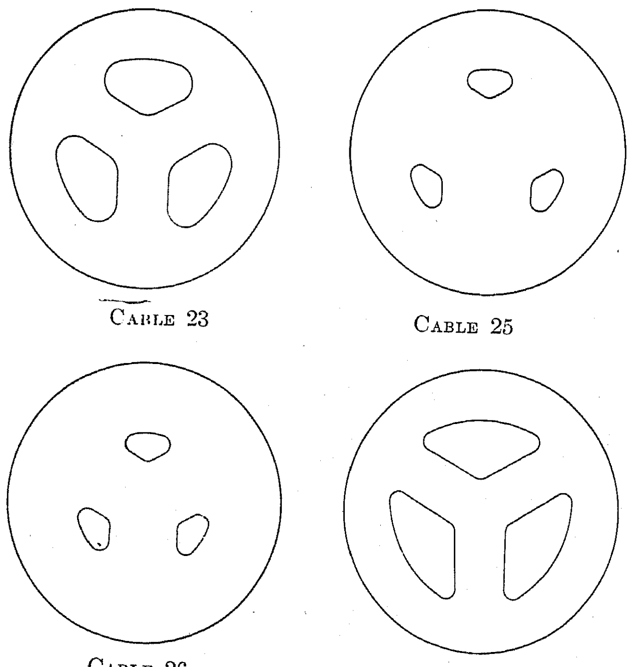


Fig. 2

in the stress measurement; the models are permanent, and after they have been prepared they are always available and check measurements can be made at a later time, both in regard to electrical measurements and measurements of the model dimensions. The electrolytic models have, of course, advantages also, which may be mentioned as follows: There is definite assurance that the medium which represents the insulation is uniform and homogeneous, while there is always danger of variations in foil thickness and the possibility of an accumulation of solder near the conductor surface in the tinfoil models; it is easier to assemble the models, though this advantage is offset by the fact that as soon as a measurement is finished and a new arrangement set up, the original model is lost. Speaking now from experience in using one method and then the other, it is believed that on the whole it is possible to obtain, with a given degree of care and skill, a higher degree of accuracy with the electrolytic type models. The particular value of the measurement with the foil models lies in the greatly added weight that is given the experimental results by the agreement between two sets of measurements carried on under such widely different conditions.

III. Geometric Factor

In the earlier work, the writer published a figure from which the charging current in any connection could be determined for three-conductor cables with round conductors, and it was pointed out how dielectric losses, temperature rise, capacity and insulation resistance could be determined from these data. Sacchetto gave his data in terms of capacity. The data might, of course, just as well have been given in terms of temperature rise or insulation resistance, and Simons has suggested that it is probably more logical to give the data, not in terms of any of these quantities, but as a numerical factor which he calls the "geometric factor" from which all of these quantities may be determined. The advantage of this method of presentation is that the formula for the various quantities is presented directly in terms of fundamentals, instead of in terms of charging current or capacity or in whatever form the data are given. The writer has adopted this notation for the new data.

The definition and meaning of the geometric factor are given by Simons and need not be repeated here, except to state that in general the geometric factor is merely a number which can be substituted for the logarithm which appears in the usual approximate formulas. He also gives formulas in terms of the geometric factor and the specific properties of the insulation, from which insulation resistance, capacity, charging current, temperature rise, and dielectric loss can be directly obtained. G_1 is defined as the geometric factor for the three conductors as one electrode against the sheath as the other, and if it is substituted in the formulas, the properties of the cable will be obtained for that connection. G_2 is the geometric factor under three-phase operation. The geometric factor for any other connection can be obtained from G_1 and G_2 from formulas given by Simons, or can be derived directly from Russell's relationships.

The method used was to measure the resistance between the rings representing the conductors and the ring representing the sheath in the desired connections, and from these measurements to calculate the geometric factors. In order to determine G_1, the resistance was

measured between the three conductors as one electrode, against the sheath as the other. G_2 could be determined by measuring the resistance to neutral with three-phase voltage impressed on the conductors. It has been shown, however, that the resistance to neutral under three-phase pressure is exactly one-half the resistance between any two conductors, if the third conductor and the sheath are insulated. The resistance between two conductors, pair by pair, was measured, and the resistance to neutral was calculated by dividing the average measured value by two. The method used with the electrolyte models has been described in the earlier paper. For the foil models, a standard Kelvin double bridge was used. The three leads connected to each electrode as previously mentioned formed one of the pairs of leads to that electrode, the potential point being separately attached by means of a clip. The measurements were made at whatever room temperature existed at the time and the resistances were corrected to a standard temperature, 25 deg. cent. being chosen, by the following formula:

$$R_{25} = R_t [1 + 0.0042 (25 - t)] \tag{1}$$

In order to calculate the geometric factors from resistance measurements, it is necessary to eliminate the foil thickness and resistivity. This was accomplished as follows: As mentioned in the description of the models, alternate strips of foil, 4 and 15 inches wide respectively, were cut from a 15-in. roll. The square piece was used for a model, and the 4-in. strips were used for a specific resistance measurement. A piece of copper strip was soldered to each end of the 4-in. pieces of foil, and the resistance measured between the strips. From this measurement it was possible to calculate the resistance between opposite edges of a square of foil. Such a resistance depends only on the foil thickness and resistivity and is independent of the length of the side of the square. The average of the two values from the strips originally on each side of a model and corrected to 25 deg. cent. is a constant, characteristic of the model, which will be called K. It may be remarked that a test was made to determine if the resistance of the foil might be different in the direction of the roll as compared with the width of the roll. For this purpose, some special samples were cut with the length parallel to the length of the roll and measurement of resistance made for comparison with the resistance measurement of the strip cut crosswise. The resistivity in the two directions was found to be the same.

The geometric factors may be calculated from the resistance measurements as follows: The insulation resistance of a cable may be expressed by the following standard formula, G being substituted for the usual logarithm:

$$R = \frac{\rho\, G}{2\, n\, \pi} \tag{2}$$

where

R = the resistance in ohms per centimeter length of cable.

ρ = resistivity of the insulation ohm-cm. units.

G = geometric factor for the connection used.

n = number of conductors in cable.

The resistance of the foil, of course, corresponds to the insulation resistance of a cable. If the foil thickness is t centimeters, then the model may be considered as a cable t centimeters long, and the resistance of the foil of the model R is equal to $\rho\, G/2\, n\, \pi\, t$ ohms. The resistance K is equal to ρ/t. If ρ from this be substituted in the equation for the resistance of the model, both ρ and t are eliminated, and we have the following final equation for the geometric factor.

$$G = \frac{2\, n\, \pi\, R}{K} \tag{3}$$

G_1 is obtained by substituting the measured resistance between three conductors and the sheath in (3), and G_2 by substituting the resistance to neutral under three-phase pressure.

For the models composed of tubes in an electrolytic tank, the length and resistivity must also be eliminated. This was done by removing the three electrodes used as the conductors and inserting one circular electrode with a diameter of 2.5 in. concentric with the sheath, and measuring the resistance between the conductor and the sheath. This is equivalent to a single-conductor cable model and is mathematically calculable. From this measurement a constant K suitable for substitution in equation (3) can be calculated. K in this case is not actually the resistance between opposite edges of a square, but is equal to the resistance between the conductor and the sheath multiplied by $2\,\pi$ and divided by $\log_e (R/r)$.

The following Table II gives the values of the resistances at 25 deg. cent. in the two connections considered, the value of K, and the values of the geometric factors as calculated by formula (3). The table also shows the geometric factors for the different cable models as determined by the data from the 1919 paper and also from the paper by Simons, and shows the per cent deviation of the new values from the older ones. For cable 11, the geometric factors are beyond the range of either data, and G_1 and G_2 are compared with values calculated by Russell's formulas, since Simons shows that these formulas are closely correct for such a large value of the ratio $(T + t)/d$. For the case of sector cables, the geometric factor is compared with its value for cables with the same insulation thicknesses, but with round conductors of the same cross-section.

The values of the geometric factor for cables with round conductors are in satisfactory agreement with the earlier determinations, the check being almost perfect in most cases. The check is particularly significant when it is considered how different the three methods are. The present data for round conductor

TABLE II
NEW VALUES OF THE GEOMETRIC FACTORS AND COMPARISON WITH THE OLD

A—Cables with Round Conductors

Model No.	Ratios		Resistance in Ohms at 25 Deg. Cent.		K	G_1				G_2		
	$\frac{T+t}{d}$	$\frac{t}{T}$	Three Conductors versus Sheath	To Neutral		New	1919	Simons	Deviation	New	1919	Deviation
10	1.31	..	0.000538	..	0.00271	1.25	1.28*	..	−2.7%	..	..	..
11	3.07	1.03	0.000460	0.000828	0.00268	3.23	3.15†	..	+2.6	5.81	5.52†	+5.2%
12	1.61	1.00	0.000360	0.000595	0.00272	2.49	2.51	2.49	−0.8	4.12	4.04	+2.1
15	0.16	0.62	0.000139	0.000176	0.00263	0.66	..	0.65	+1.5	0.82	..	..
16	0.58	0.67	0.000282	0.000361	0.00264	1.34	..	1.32	+1.5	1.72	..	..
17	1.65	0.68	0.000448	0.000617	0.00264	2.13	..	2.10	+1.5	2.94	..	..
19	0.15	1.00	0.000107	0.000156	0.00259	0.78	..	0.73	+6.6	1.13	..	..
20	2.43	0.25	0.000390	0.000815	0.00274	2.68	..	2.68	0	5.61	5.62	−0.2
21	1.31	0.24	0.000306	0.000608	0.00275	2.14	2.16	2.15	−0.9	4.17	4.20	−1.1

B—Cables with Sector Conductors

Model No.	Ratios		Resistance in Ohms at 25 Deg. Cent.		K	G_1				G_2		
	$\frac{T+t}{d}$	$\frac{t}{T}$	Three Conductors versus Sheath	To Neutral		Sector (New)	Equivalent Round 1919	Equivalent Round Simons	Correction Factor	Sector (New)	1919 Round	Correction Factor
23	0.70	1.04	0.000226	0.000340	0.00275	1.55	1.70	1.73	0.907	2.33	2.57	0.907
24	0.57	0.23	0.000190	0.000358	0.00272	1.32	1.52	1.50	0.875	2.48	2.64	0.940
25	1.99	1.00	0.000380	0.000634	0.00280	2.56	2.73	2.70	0.922	4.27	4.43	0.964
26	1.63	0.25	0.000336	0.000674	0.00283	2.24	..	2.34	0.957	4.49	4.70	0.955
28	0.66	1.00	0.000201	0.000304	0.00280	1.35	1.66	1.67	0.808	2.05	2.48	0.827
29	0.61	0.00	178	355	2657	1.26	1.54	1.52	0.835	2.52	2.95	0.855
30	1.30	0.25	423	811	3970	2.01	2.15	2.15	0.935	3.85	4.18	0.921
31	1.16	0.00	384	784	3814	1.90	2.01	2.00	0.950	3.87	4.20	0.922
32	1.59	1.00	681	1061	5390	2.38	2.49	2.48	0.960	3.71	4.01	0.926
33	0.80	0.00	443	896	5583	1.50	1.73	1.72	0.872	3.03	3.47	0.873
34	0.90	0.25	541	973	6015	1.63	1.87	1.85	0.877	3.05	3.44	0.887
35	1.09	1.00	498	755	4806	1.95	2.12	2.14	0.916	2.96	3.29	0.899

*Calculated by ordinary logarithmic formula.
†Calculated by Russell's formula.

cables may therefore be considered merely as a confirmation of the earlier data. The maximum deviation occurs in the case of cable 19. This may be explained in part at least by the fact that the graphical method is least accurate in this range of dimensions, and also that the experimental method is least accurate for extreme dimensions. Thus in cable 19, a very small absolute variation in the spacing is a large proportion of the insulation thickness, producing a correspondingly large error.

The data for sector-shaped conductors are believed to be the first published data on this subject with the exception of three short paragraphs in the writer's 1919 paper and for some data on one sector cable model mentioned by W. A. Del Mar in a discussion at the same meeting, and some experimental measurements in connection with thermal resistance published in Appendix VII of the Second Report on the Research on the Heating of Buried Cables. Simons suggested that the geometric factors for the sector cable might be obtained in approximate form by diminishing the geometric factor for the corresponding round conductor cable by 10 per cent. The present data make it possible to obtain a more accurate answer. The geometric factor for sector cables may be obtained by finding the geometric factor for a cable with round conductors with the same insulation thicknesses and the same size of conductor from any of the available data, and then decreasing this by a percentage read from Fig. 3. This method will reproduce the data of Table II quite closely. G_2 of cable 24 seems erratic. Neglecting this one value, the average deviation of Fig. 3 from the data of Table II regardless of sign is 2.3 per cent, the maximum deviation being—

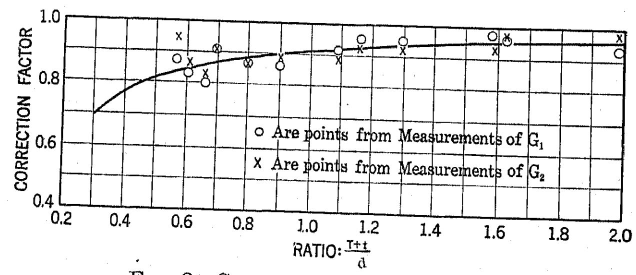


FIG. 3—SECTOR CORRECTION FACTOR

6.0 per cent for G_2 of cable 28 which has the sharp sectors. The correction curve of Fig. 3 is actually a curve whose equation is $y = 1 - 0.093/x$. For round conductor cables Simons has given formulas for G_1 and G_2 which express accurately these values as determined by previous experimental work and by his calculation. Corresponding empirical formulas for sector cables can

therefore be obtained by multiplying his values by

$$1 - \frac{0.093}{(T + t)/d}$$

It seems not improbable that if the shape of the sectors had been designed on an exactly uniform mathematical basis, a more consistent curve for the correction factor might have been obtained. Also if results were of sufficient precision, a different curve could, of course, be drawn for each value of the ratio t/T and there would be different values for G_1 and G_2. It was not deemed that the present data justified more than the single curve as shown. Results of precision will not be obtainable except for approximately a given shape of sector until added numbers of points are obtained for different shapes. We might add, however, that the accuracy which can be reached for sector cables by the use of Fig. 3 is undoubtedly greater than that usually obtained for the case of round conductor cables by means of the approximate formulas which were in ordinary use up to a few years ago.

We have mentioned above two experimental determinations of the geometric factor and one graphical solution. In addition to this work, H. B. Dwight has presented a rigid solution for G_1 for a two-conductor cable, that is, his formula contains a series of converging series, so that any required degree of accuracy can be obtained. He stated that he had also developed a formula for G_1 of a three-conductor cable with round conductors and when this is presented, this phase of the matter will be finally settled.

The Second Report on the Research on the Heating of Buried Cables contained in its appendix two more determinations, not in terms of geometric factor, but in terms of quantities from which the geometric factor can be obtained. In Appendix IV Wedmore shows a graphical solution of this problem which is entirely different in method from Simons', but we are unable to compare the results because the scale on which Wedmore's data are plotted in the published article is such that no accuracy of reading is possible.

In Appendix III of this report some measurements were shown for cable models still different from either of the types mentioned in this paper. Three models were made, each composed of three cylinders surrounded by a fourth cylinder, in the form of an air condenser. It was possible to measure the various capacities of this cable model. We are unable, however, to compare our data with the results of the measurements on these three air-insulated cable models for the following reason: While in a cable model composed of three electrodes surrounded by a fourth electrode, the three "conductors" can be located anywhere inside the "sheath," providing they are symmetrically arranged, this is not the case with an actual cable formed by applying insulation to each of the three conductors, with a belt insulation around the three, where there are certain definite limitations. For instance, half the distance between adjacent conductors is the conductor insulation thickness. The distance between conductor and sheath is the conductor insulation plus the belt insulation thickness. It is obvious that the conductors cannot be nearer the sheath than half the distance between conductors, this condition being the case of a cable with no belt. The first two air-insulated cable models violated this principle, as the conductors are considerably nearer the sheath than half the distance between conductors, and are therefore impossible practical cables. Stated differently, if the circles representing the conductors and the circle representing the sheath are drawn and it is attempted to draw circles representing the boundary of the conductor insulation, such circles will extend outside of the cable. The third model was not impossible, but was an improbable cable according to American practise, in that the electrodes were so located that it corresponded to a cable with a belt insulation two and one-half times greater than the conductor insulation. Our data do not, of course, cover any of these cases.

Electrostatic Stresses

In recent years there has been considerable discussion and by no means entire agreement on the maximum permissible stresses in cables. Whatever these limits may be and whatever the importance of the limits, they cannot be applied or determined unless the stresses can be calculated. It therefore seems well worthwhile to enlarge the scope of the previous data, to check further the accuracy of the work, and to give new data for sector cables.

The geometric factor of all cables which are similar geometric figures is exactly the same, regardless of the absolute dimensions of the cables. The stresses, however, depend not only upon the geometric configuration but upon absolute dimensions and upon the voltage applied. We may say, however, that the stresses will be the same in cables which are similar geometrically if the applied voltages are proportional to the size of the cables. What amounts to the same thing is that the ratio of maximum (or any other stress) to the average stress in a certain part of the cable is a constant for cables which are geometrically similar. The problem is therefore solved for a particular cable model, if the maximum (or other) stress and the average stress through any region are determined, and the ratio of the two is given.

In the earlier work, it was experimentally determined that the maximum stress and other stresses in the part of the cable toward the center were independent of the belt insulation thickness. The stresses vary according to the ratio of conductor insulation thickness to conductor diameter, and may be completely determined if we plot the ratio of stress to average stress as a function of the ratio of conductor insulation thickness to conductor diameter.

The stress of greatest interest is undoubtedly the

maximum stress. If the conductors are very close together, that is, if the conductor insulation is very thin, the maximum stress will occur at the conductor surface on a line joining conductor centers. This is obvious if we go to the limit and imagine the conductor insulation as approaching zero. In this case the stress between conductors will approach infinity, while all other stresses will be finite. With the more usual proportions of insulation thickness and conductor diameter, however, it was shown in the previous work that the maximum stress occurs at the surface of the conductor and towards the axis of the cable, that is, on a line joining the conductor center and the cable center. For very thin insulations, where the maximum stress is at the surface of the conductor and toward an adjacent conductor, it can be calculated by a formula given by Russell. As the insulation thickness is increased, the maximum stress is towards the center, and at an intermediate point the stress all along the surface of the conductor between these two points is approximately the same. Indeed, with the usual thicknesses of insulation, the stress at the surface of the conductor on the line joining conductor centers is very nearly as great as the stress on the surface of the conductor nearest the center of the cable. So far no one has calculated the maximum stress when it occurs in the usual position toward the cable center; hence the need of an experimental determination. The complete curve for maximum stress as a ratio to average stress will therefore consist of two parts; when the insulation is thin we may use a curve obtained by Russell's formula, and for thicker insulations, use experimental data, joining the two curves for a short intermediate section by a curve tangent to the two.

The experimental method of determining stress is the following (described in full with an example in Appendices A and C). A finder was made by fixing two needle points exactly one-quarter inch apart. Half way between the two was a pointer. The pointer is placed at any point on a tinfoil cable model, and the voltage between the two points is measured; this voltage will be a measure of the average stress throughout this quarter inch, or to within about one per cent, the actual stress at the pointer. The voltage between the two points was measured as described in the appendix, being balanced against a voltage adjustable in magnitude and phase angle using a sensitive a-c. galvanometer to detect balance.

Fig. 4 shows a drawing in terms of which the measurements can be most easily explained. The stress was measured along the 180-deg. line in order to determine the true maximum stress, as well as the stress at the center of the cable and the stress at the edge of the fillers. It was measured also at various points along the zero-deg. line, in order to determine the maximum stress towards the sheath. The stress was then measured in a radial and also a tangential direction on various circles around each conductor, in order to be able to find stresses at other points. This procedure was carried out completely except for the case of some of the sector models. By "stress" in this paragraph, we mean potentiometer readings which are proportional to the stress. Though it was possible to determine actual voltages as will be shown, this was not necessary.

We will now describe in detail how the data of the stress measurements were put into useful form. We will begin with the determination for the stress at the surface of the conductor at the 180-deg. line, that is, across the center of the cable, which is normally the point of maximum stress. As stated above, the stress was measured along the 180-deg. line at frequent intervals, starting with ¼ in. from the surface of the conductor. The stress could then be plotted as a function of the distance from the conductor surface along the 180-deg. line, from the conductor surface to the cable

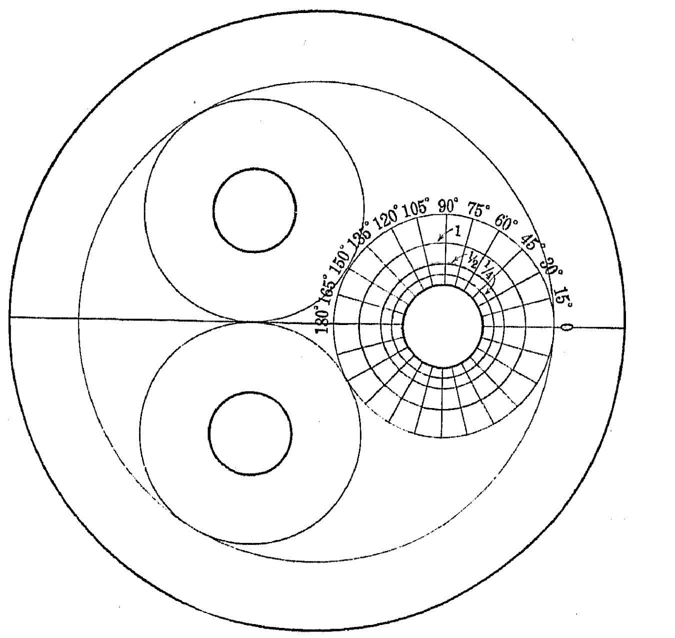


FIG. 4

center. The actual stress at the conductor surface, the maximum stress, is not measured directly, though it might be determined roughly by extrapolating this curve towards the conductor surface.

The maximum stress, however, is much more accurately determinable than this. In a single-conductor cable, the stress varies throughout the field according to a very simple law, that is, it is inversely proportional to the distance from the conductor center. In a multi-conductor cable, no matter what law of variation may be effective at points remote from the conductor surface, the law of variation of stress approaches the same simple law of inverse stress proportionality to the distance from the conductor center, as the surface of the conductor is approached. If the conductor is not round, then instead of conductor center, the statement applies to the center of curvature of the adjacent surface of the con-

ductor. This follows directly from the fact that the lines of electric stress must be normal to a conducting surface, at that surface. If for a single-conductor cable we plot the reciprocal of the stress along a line perpendicular to the surface of the conductor, we obtain a straight line which, if produced, will pass through zero at the center of the conductor. Thus, if data showing the stress at a very few points are plotted, it becomes a simple matter to determine quite accurately in this way the stress at the conductor surface. If the same method of plotting, that is, reciprocal of stress versus distance from the conductor surface, be used for a three-conductor cable, it is found that the data can be represented quite accurately by a straight line which makes extrapolation to zero distance, that is, conductor surface, relatively easy and exact. For round conductor cable, these curves remain straight lines (and pointing in the direction of zero at the conductor center) for a long distance from the conductor surface and the extrapolation is as easy and as accurate as for single-conductor cable. For sector-conductor cable the lines are straight from the nearest point to the surface measured and remain so for a long distance. At the outer tips they point to zero very nearly at the center of curvature of the arc at the tip. At the center they point to zero well beyond this point. They must therefore change direction nearer the surface. We have, however, continued them as straight lines, which means that the values given for the stress at this point are too low by a very small amount. At the surface toward the sheath the line points toward zero at a point much nearer than the center of curvature. As these also were continued as straight lines, the stress determined at this point is very slightly too high.

The maximum stress thus determined is plotted on the stress curve, thus completing the curve from the conductor surface to the cable center along the 180 deg. line. The average ordinate of this curve is the average stress along this line, and this value is found by measuring the area by a planimeter and dividing the area by the distance between conductor surface and cable center. Another way of looking at this is that since the curve has stress as ordinates and distance as abscissas, the area is the voltage between the cable center and the conductor surface in the arbitrary units. This voltage divided by the distance will, of course, give the average stress.

For purposes of ready numerical calculation, the average stress between conductor surface and cable center is awkward to calculate. It therefore seems better as a matter of convenience to find the average stress between conductors, and express the other stresses in terms of this. For this purpose, by the average stress between conductors, we mean the numerical average obtained by dividing the voltage between conductors by the distance between them.

We now have the maximum stress, the average stress between conductors, and stresses at various parts of the cable cross-section obtained by direct measurement. They are all expressed in arbitrary units. Dividing all these values, however, by the average stress expressed in the same units, we obtain a value which is a characteristic of the cable, and the final results are given in this form.

In addition to the maximum stress and various stresses in the fillers, the stress at the conductor surface and toward the sheath is sometimes of interest, that is, the stress at the conductor surface on the zero deg. line. This can be obtained by plotting the reciprocals of stress as measured along the zero deg. line against distance from conductor center exactly as the maximum stress toward the center was obtained. This stress is, of course, a function not only of the conductor insulation thickness, but also of the belt, and it must therefore be expressed not in terms of the average stress between conductors but of the average stress between a conductor and the sheath. This determination makes possible an incidental check. As stated before, the

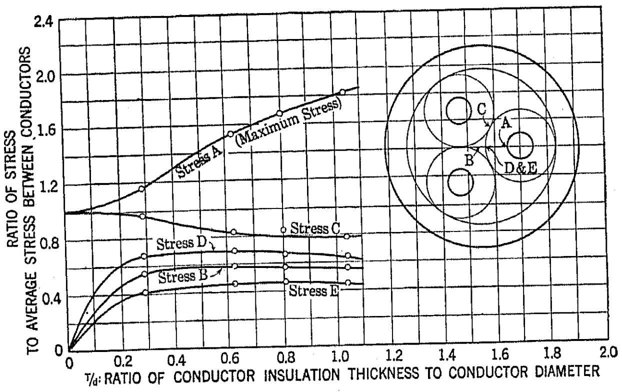


FIG. 5—STRESSES IN THREE-CONDUCTOR CABLES WITH ROUND CONDUCTORS

area of a curve of stress plotted against distance is equal to voltage. The voltage from conductor surface to cable center along the 180-deg. line and the voltage from conductor surface to sheath along the zero-deg. line are equal. The areas under the two curves, therefore, should be equal. A satisfactory check in this regard was obtained, the areas usually being equal to within a few per cent.

Fig. 5 shows the ratio of stress at certain points to average stress between conductors as a function of the ratio of conductor insulation thickness to conductor diameter, for round conductor cables. The points corresponding to the lower two values of the ratio are repeated from the 1919 data, while the other two are from cables 12 and 21, a complete curve thus being obtained and the new data being apparently entirely consistent with the early data, though covering a different range so that exact comparison is not possible.

Sector Stresses

Fig. 6 gives the same data for such sector cables for which stress measurements were made in the present series.

Since the stress was measured along various lines radiating from about the center of the conductor, as shown in Fig. 4, for the round conductor cables, it is possible to find the stress at the surface of the conductor

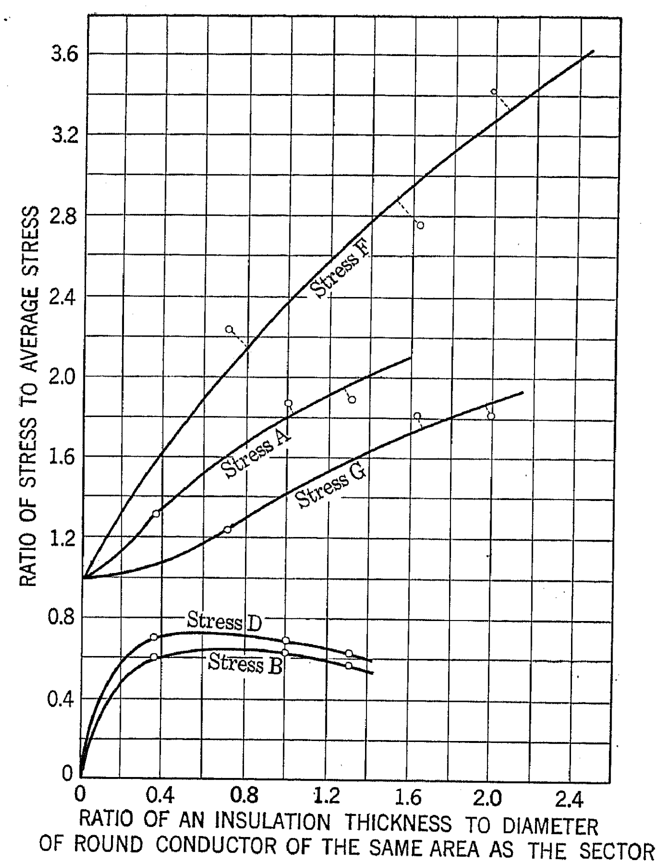


Fig. 6—Stresses in Three-Conductor Cables with Sector Conductors

The stresses are described in Table III and some of them are shown graphically in Fig. 5.

Note: The stresses A, B and D are given as a ratio in terms of the average stress between conductors, and are plotted against the ratio of conductor insulation thickness to conductor diameter.

Stresses F and G are however toward the sheath, and are therefore plotted in terms of the average stress between a conductor and the sheath; and plotted against the ratio of total insulation thickness between conductor and sheath to conductor diameter.

No direct comparison should be made on this figure between the former and the latter stresses, as the scales are not comparable.

along any of these lines by a similar method. The point of greatest interest in regard to the stresses in sector cables is that it has been found that with only one exception the maximum stress occurred not at the point toward the center of the cable but at the outer tip of the sector and in the general direction of the sheath. This maximum stress is practically constant along a short arc at the outer tips of the sector, this arc being indicated in the shaded portion of Fig. 1. It is also of interest to note that the stress toward the cable center, which is the region of maximum stress in round-conductor cables, is only very slightly greater for sector cables than for round.

The stress F next the tips of the sector conductor toward the sheath, which is often the maximum stress in the cable, is best defined in terms of the average stress toward the sheath, just as in the case of stress along the zero-deg. line. The stresses F and G are thus expressed in terms of the average stress between conductor and sheath, as shown in Fig. 6, but in Table III, they are also expressed for the particular models investigated in terms of the average stress between conductors in order that they may be compared with the other stresses.

The curve F, showing the stress next the tip of the sector conductor, is not of as general application as the curve for stress of round conductor cable or even the curve showing other stresses in the sector cables because the value of the stress depends so greatly upon the radius of curvature at the tip. For instance, the maximum of cable 28 with sharp cornered sectors is considerably above this curve. The various curves however for sector cable will make it possible to obtain a close approximation of the stress at several points including the maximum stress, where the sector conductors are of reasonably normal shape.

In the chart showing stresses in sector cable, no curves are shown for the stresses at C and E as in the case of a chart for stresses of round conductor cable, as these measurements were omitted in the measurement of the sector models.

IV. Check of Accuracy of Method

Up to the present time, no one has derived an accurate method of calculating the actual maximum stress in a three-conductor cable with round conductor, or any of the stresses where the conductors are sector-shaped. Our experimental method may however, be searchingly tested, since there are other stresses which can be calculated. To begin with, the maximum stress can be exactly calculated for single-conductor cables. This can be expressed as a ratio of maximum to average stress. The identical process above described in detail was performed with two single-conductor models, cables 10 and 27, and the ratio of maximum and minimum stress to average stress as determined experimentally can be compared with the calculated values.

Furthermore, there are two stresses in a three-conductor cable with round conductors which can be calculated accurately. Russell has given a rigid formula for the calculation of maximum and minimum stress between two eccentric cylinders. The stress at the conductor surface and at the sheath along the shortest line between conductor and sheath (*i. e.*, along the zero-deg. line) could therefore be calculated by Russell's formula if it was proved that these stresses were uninfluenced to an extent of practical importance by the presence of the

TABLE III

TABULATION OF RESULTS OF STRESS MEASUREMENTS ON THREE-CONDUCTOR CABLES

All Stresses are Expressed as a Ratio in Terms of an Average Stress

Cable Number	Conductor Shape	Stress: In Terms of Average between Conductors A	B	C	D	E	F	G	In Terms of Average between a Conductor and the Sheath F	G
*	Round	1.16	0.55	0.96	0.67	0.42	..	..	..	..
*	Round	1.54	0.59	0.83	0.69	0.46	..	..	..	..
12	Round	1.67	0.57	0.84	0.66	0.47	..	1.17	..	2.21
21	Round	1.81	0.55	0.77	0.63	0.44	..	1.68	..	1.95
23	Sector	1.31	0.60	0.47	0.70	..	1.28	0.70	2.24	1.23
25	Sector	1.87	0.63	..	0.68	..	2.10	1.10	3.44	1.81
26	Sector	1.89	0.57	..	0.64	..	2.54	1.67	2.76	1.81
28	Sector	1.44	0.64	..	0.73	..	1.60	0.67	2.77	1.17

*These are the 1919 data transformed to the present basis; the two models have ratios, T/d, of 0.288 and 0.625 respectively.

A = stress at conductor surface and toward cable center.
B = stress at cable center.
C = stress midway between conductors on line between their centers and in the direction of this line.
D = stress at outer edge of conductor insulation on line joining conductor and cable centers.
E = ditto, but normal to this line.
F = maximum stress at outer tip of sector conductor.
G = stress at conductor surface and toward the sheath.

other two conductors. This was definitely proved for cables of the forms tested and described in the 1919 article, and it may therefore be concluded that these two stresses can be determined experimentally and mathematically, and give another check of the data. Table IV gives the ratio of maximum stress to average stress for the two single-conductor cables, and the two above-mentioned stresses for the case of the two three-conductor cables with round conductors. It also gives the calculated values, and the per cent error.

TABLE IV

CHECK OF EXPERIMENTAL METHOD

For the Three-Conductor Cables, the Stresses are Along the Shortest Line from Conductor to Sheath, *i. e.*, the Zero-Deg. Line

Cable	Type	Ratio of Stress to Average Stress: Maximum Stress: Measured	Calculated	Per Cent Error	Minimum Stress: Measured	Calculated	Per Cent Error
No. 10	1-Condr.	2.04	2.04	0%	0.563	0.563	0%
27	1-Condr.	4.35	4.33	+0.4	0.375	0.373	+0.5
12	3-Condr.	2.21	2.07	+6.7	0.614	0.614	0
21	3-Condr.	1.95	1.90	+2.5	0.654	0.681	−4.1

The agreement of measured values with the calculated is well within the expected accuracy of the measurements, especially for cable 27.

The great difficulty with the method of tinfoil models is the danger of irregularities in the thickness and composition of the foil. It is believed, however, that the fact that all the stresses were measured and calculated separately for each of the three conductors, and that each of the final values is thus the average of three, would indicate that the errors are probably averaged out. We believe therefore that the present data are probably correct to within 4 per cent, and that undoubtedly a large amount of it is even more accurate.

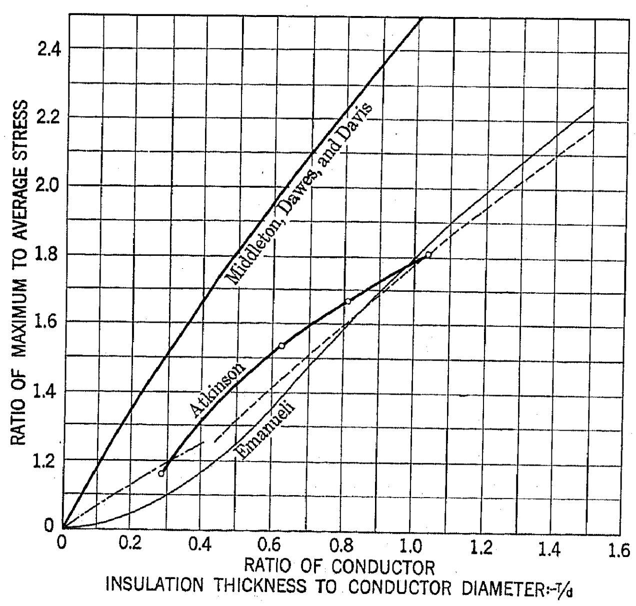
V. Comparison with Other Data

The difference between our own results and those of Emanueli will now be considered, the remarks being limited to a comparison of the values of maximum stress only. As stated before, the maximum stress in its usual location cannot be calculated. Starting with the data available as to geometric factor we have derived an equation by which the average stresses around the entire circumference of the conductor may be determined. This derivation is given in Appendix B. Obviously, the true maximum stress must be greater than the average of the stresses around the conductor surface, and from this equation we have a *lower limit* to the value of maximum stress. In the case of cables with thin belts, the stress toward the sheath is very close to the maximum stress toward the center, and the difference between average and maximum stress at the surface of the conductor is reduced to a minimum. In Fig. 7 is plotted our experimental curve for maximum stress in terms of average, Emanueli's data for the same quantity, and another curve calculated by the method advocated by Middleton, Davis and Dawes. In addition, the curve calculated by equation (7) of Appendix B is shown, this curve having been calculated on the basis of three-conductor cables with no belt, so that the stress toward the sheath would be about the same as that toward the center. We know, therefore, definitely that the true maximum stress must lie above this curve and also that it is probably not very far above it for cables with thick insulation (large T/d).

By inspection of Fig. 7, we see immediately that much of Emanueli's curve for maximum stress is lower than the average of the stresses around the conductor surface, and therefore must be too low, at least by the amount of the difference. For small values of T/d the maximum stress occurs at the surface of the conductor toward an adjacent conductor, as stated before,

and this can be calculated by Russell's formula. One can see that Emanueli's curve is incorrect for this range and by a large amount.

Emanueli gave very little of his experimental results, most of his data being shown merely in the final curve form. For that reason, there is little opportunity to analyze his results and to suggest a possible explanation for the extent of the error of parts of his curves. It seems not unlikely, however, that the explanation lies in the inherent difficulty of accurate determination by his method. He used a tank of electrolyte as did we, but in our method we have determined stresses at various points *by direct measurement of stresses* and the results should be as accurate as the measurement made. Emanueli, on the other hand did not determine stresses directly but used a single-point finder and plotted equipotentials. From this data he was able to plot



—·—·—·— from Russell's formula for maximum stress between cylinders.
- - - - - - - - from formula 6, Appendix B. Limiting minimum value.

Fig. 7—A Comparison between Various Determinations of Maximum Stress in Three-Conductor Cables

the voltage curve in various directions, and from his voltage curve he derives the stress curve. Any inaccuracy in the measurement of potentials is very greatly magnified in the determination therefrom of the stresses. The stresses are virtually the difference in potential between successive points on the potential curve, so that small errors in measurement are greatly magnified. This explanation is partly discounted by the fact that Emanueli also applied his method to the determination of stress in single-conductor cables and was able to check the calculated values to within 2 per cent, though no further details of this check were published. The difficulty with his method, which is described above, is somewhat less serious with a single-conductor cable for the same outside diameter of model than for three-conductor cables as the distances are greater and a given inaccuracy in point of distance in the setting of the finder would produce a smaller percentage error in the final result. Others have used a similar method of determining and plotting equipotential lines for the purpose of determining stresses and, while such diagrams are very instructive and help one to visualize the condition of the field, the method inherently is rather unsatisfactory for the obtaining of exact mathematical data.

A brief statement of the experimental data quoted by Middleton, Dawes, and Davis, will show that it is not a suitable basis for determination of stresses. They experimentally determined the breakdown strength of a series of triplex cables and single-conductor cables with similar insulating material. They calculated the maximum stress at breakdown of their single-conductor cables by the rigid formula for this, and of the three-conductor cables by the writer's 1919 data and also by a method of their own. They found that the maximum stress at breakdown in the three-conductor cable calculated by their method was somewhat more nearly equal to the maximum stress at breakdown in the single-conductor cables than was the value calculated by the writer's method. From this they concluded that their method was more accurate than the writer's, the difference between the two being quite large.

In view of the demonstrated accuracy of our method, their logic is not very convincing. In the first place, they assume that cables of different proportions have a constant value of maximum stress at breakdown and that three-conductor cables and single-conductor cables break down at the same value of maximum stress, both of these assumptions being unproved. We believe that the result of their experiments proves the fallacy of these assumptions for the samples tested, rather than proving a basis for calculating maximum stress.

It is worth repeating that no attempt is made here to draw conclusions as to the importance of maximum stress in determining breakdown strength, or allowable operating voltage of any given cable. A mathematical tool is furnished which will be useful in correlating experimental or practical data bearing on breakdown strength or allowable operating voltage of different cables. As far as concerns the present paper, the data contained herein may as well be used to prove, that the ratio of maximum stress to average stress is unimportant as it is to prove that the breakdown strength is dependent upon the maximum stress.

In this investigation, account has not been taken of the effect of stranding, though some data on that were given in the previous paper, and no account has been taken of the effect of spiralling or cabling of the conductors. In addition to extending the scope of the data now available, future work should be done on these things, in particular to determine the effect of stranding on the sector conductor.

TABLE V
DETERMINATION OF MAXIMUM STRESSES IN CABLE MODEL 25 ON ZERO-DEG. AND 180-DEG. LINES

Distance from Conductor	Stress toward Cable Center (180-deg. Line)						Stress toward Sheath (Zero-deg. Line)					
	Conductor 1		Conductor 2		Conductor 3		Conductor 1		Conductor 2		Conductor 3	
	S	$1/S$	S	$1/S$	S	$1/S$	S	$1/S$	S	$1/S$	S	$1/S$
0 in.	555	0.00180	555	0.00180	538	0.00186	322	0.00311	333	0.00300	329	0.00304
0.25 "	445	0.00225	442	0.00226	426	0.00235	293	0.00341	295	0.00339	290	0.00345
0.50 "	370	0.00270	375	0.00266	361	0.00277	241	0.00415	272	0.00368	266.	0.00376
1.00 "	287	0.00359	290	0.00345	284	0.00352	204	0.00490	224	0.00446	209	0.00478
2.00 "	192	...	199	...	192	...	147	...	152	...	145	...
2.14 "	187	...	188	...	182	...	...	...	...	...	...	...

VI. Conclusions

The satisfactory check between the old and new data for the geometric factor, together with their check with Simons' graphical method, leads to the belief that this phase of the matter has had a satisfactory solution, and that any of the data may be used confidently, including the new data for sector-shaped conductors.

We believe that our own determination of stress as opposed to that of Emanueli is quite completely vindicated for the following three reasons:

1. The measurement of the stresses which could also be calculated gave a very close check as indicated in Table V.

2. The new data fall directly in line with the earlier data, in spite of the fact that the earlier cable models were tanks of electrolyte and the present models foil sheets.

3. Direct measurement of stress, which is the method we used, is inherently capable of a higher degree of accuracy than a determination of stress from voltage curves.

One of the most important conclusions to be derived from the present data is believed to be the fact that the maximum stress in cables with sector conductors may occur at the outer tips of the sector rather than toward the cable center, and that this stress may be somewhat greater than that in a cable with round conductors. No generalizations of this feature can be made, however, inasmuch as the maximum stress in round conductor cables depends upon the conductor insulation thickness and the diameter only, while the belt thickness is a determinant in the case of sector stresses.

Appendix A

The electrical connections used in measuring stresses with the foil model are shown in Fig. 8. At the left of the diagram is shown the cable model with the exploring device, and above it are shown the transformers and circuits for supplying the three-phase current to the conductors of the model. In the center of the figure is shown the device used for producing an e. m. f. variable in amount and phase. At the right is shown the detecting galvanometer and related circuits.

The three-phase 220-volt supply is stepped down by three transformers A, each single-phase transformer being 220 volts primary to 1 volt secondary, the three primaries being connected in delta and the secondaries in star. Current is led from each phase of the secondary circuit to one of the condensers, through a non-inductive, zero temperature coefficient resistance of value 1/15 ohms. The neutral of the transformer is connected to the sheath of the cable model and this circuit is grounded.

Two transformers B and C, shown in the middle of the diagram, are used to furnish potential of varying amount and phase. Transformer B is 110 volts primary with 4-volt secondary with taps at one, two, and three volts. Transformer C is 95.5 volts primary to 4 volts secondary. Except for the difference in the primary turns, the two transformers are identical. Across one part of each secondary is connected a resistance R_1 of about 25 ohms. This resistance is made up of 150 turns wound on a non-metallic cylinder. A slider is arranged to connect to any one of these. This resistance serves to divide the one volt into 150 units and the resistance is low enough in amount in proportion to the 100,000-ohm resistance which is in series with the meter circuits so that the potential is not affected by the current drawn. By an arrangement of switches shown, the secondaries of transformers B and C are connected in series and any proportion of the voltage of either may be used in units of 1/150 volt. The current from this point is led through a 100,000-ohm resistance box and through a fixed resistance of 100 ohms.

On the right is shown the alternating galvanometer, a special shunt on the moving coil and means for supplying potential to the field coil. By means of two Scott-connected transformers, B and E, two secondary voltages are available, at right angles to each other in phase. With a double throw switch, the field of the galvanometer may be excited from either one or the other of these transformers. By means of the adjustable resistance shunting the condenser, the phase of the voltage across the field may be varied.

The shunt used on the moving coil circuit of the galvanometer is similar in purpose to the usual galvanometer shunt and similar in arrangement except that a special connection is attached and arrangement is

made so that a 100-ohm resistance is connected across the galvanometer terminals when the shunt is in any position except the most sensitive. In general, with an a-c. galvanometer, variation of the resistance across the moving coil will cause a shift in the electrical zero, which is decidedly inconvenient and is avoided by the adjustment shown.

The deflection of an a-c. galvanometer for a given current through its moving coil is dependent upon the phase of that current with respect to the phase of the field current. In the use of the arrangement shown, the field of the galvanometer was first adjusted so that the galvanometer would have zero sensitivity for current in phase with the potential of one of the secondaries of the transformers A. When the field is supplied from the other of its two transformers, the sensitivities for current of the same phase would conversely be a maximum.

In order to measure the stress at any point, the exploring device is put in the desired position, with the two points in contact with the surface of the foil, the switch at P is closed, and current is supplied to the cable model by closing the switch for the primary circuits of transformers A. The other two primary switches are closed and the voltage supplied from the secondaries of the transformers B and C is varied until a balance is obtained on the galvanometer, thus signifying that the voltage across the 100 ohms as produced by transformers B and C, is equal to the voltage on the exploring device, both in amount and phase. With the adjustment of the field circuit of the galvanometers already men-tioned, it will be found that the galvanometer is sensi-tive to variation in the voltage supplied by transfor-mer B when the field is excited from one source, and sensi-tive to voltage from transformer C when the field is supplied from the other source, and that a considerable amount of variation in voltage of either transformer can be made without greatly affecting the galvanometer when the source for the field is not in a position which gives the maximum sensitivity. Thus the process of balancing can be carried on quickly, usually an approxi-mate adjustment of one transformer circuit being made, then an accurate adjustment of the second trans-former, and finally an accurate adjustment of the fi

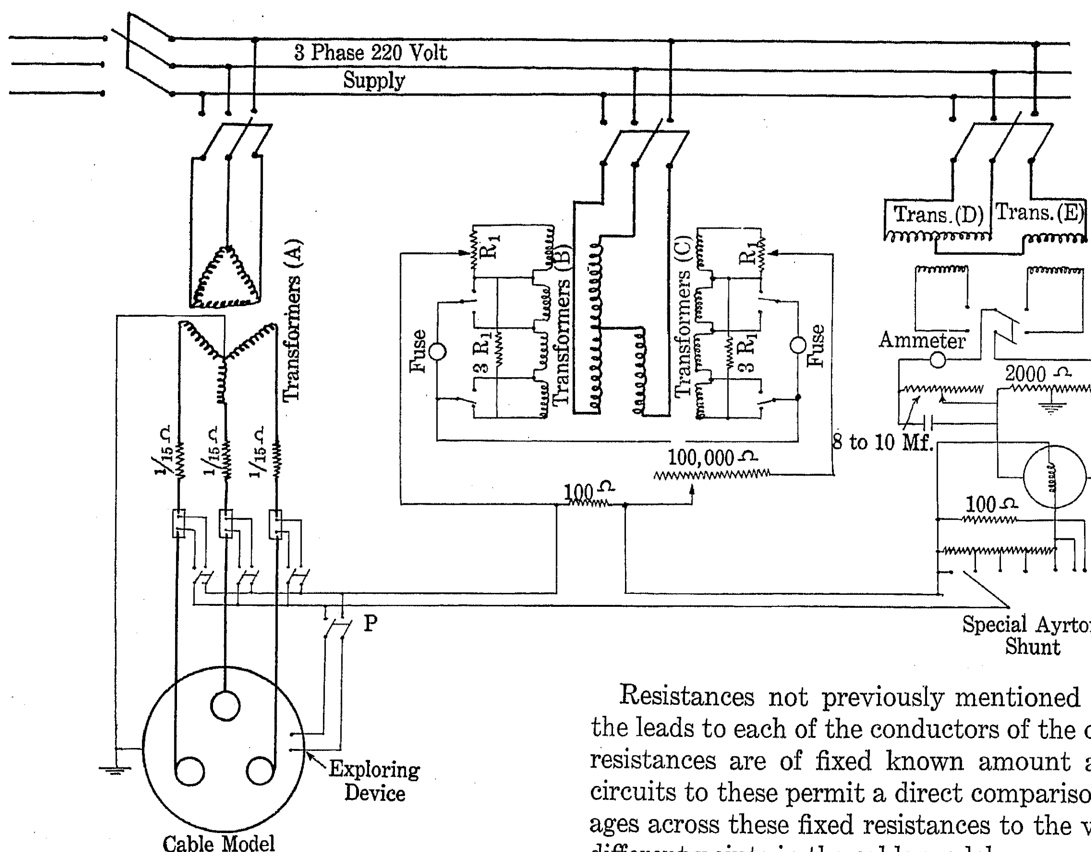


Fig. 8

Resistances not previously mentioned are shown in the leads to each of the conductors of the cable. These resistances are of fixed known amount and potential circuits to these permit a direct comparison of the volt-ages across these fixed resistances to the voltage across different points in the cable model.

The arrangement is independent of variations in primary voltage because voltage on transformers A will vary in the same ratio as voltage on transformers B and C and therefore the adjustment obtained will be the same through large variation of primary supply voltage which therefore does not need to be taken into account.

Appendix B

Derivation of Formula for Limiting Value of Maximum Stress in Triplex Cables

The experimental work with cable models has shown that while the maximum stress occurs at the surface of the conductor in a direction toward the cable axis, for small sizes of conductor and with thick insulation thick-nesses, the maximum stress is not very much greater than the stresses practically all around the circumference of a conductor. That is, the stresses at the conduct

surface are quite uniform for small conductor cables. If a cable has a thick belt, as compared with the conductor insulation, the stress toward the outside will be less than the maximum. If a cable is chosen with a thin belt or no belt, then the stress toward the outside can be made the same as the maximum stress, and in such a cable the stresses around the entire circumference of a conductor are still more uniform.

If the stresses around the surface of the conductor are absolutely uniform, then the equipotential lines near the surface of the conductor would be circles, or speaking in terms of a cable model of unit length, the equipotential surfaces would be cylinders. If the stresses are not absolutely uniform around the entire surface, the introduction of a metallic cylinder in the insulation but close to the conductor surface will make the stresses uniform, and will therefore tend to reduce the maximum stress and raise the stresses which are the lowest, the stresses in this case being very close to an average of the stresses which would exist around the conductor surface without the cylinder.

Let us consider a cable of unit length, with a belt so thin that the stress toward the sheath is practically equal to the stress at the surface of the conductor toward the center, or in other words, so that the cable will have quite uniform stresses around the surface. Let us assume that a metallic cylinder is inserted in the insulation and fairly close to the surface of the conductor.

Let C_{10} = capacity of conductor to neutral
C_{20} = capacity of cylinder to neutral
C_{12} = capacity between conductor and cylinder
Let E_{10} = voltage of conductor to neutral
E_{12} = voltage impressed between the conductor and the cylinder

Let the conductor radius equal r_1 and the cylinder radius equal r_2. We may now calculate the maximum stress at the surface of the conductor, considering only the condenser made up of the cylinder and the conductor by the ordinary single-conductor formula as follows:

$$\text{Maximum stress} = \frac{E_{12}}{r_1 \log \frac{r_2}{r_1}} \qquad (1)$$

We know that

$$C_{10} = \frac{3\,K}{2\,G_2} \qquad (2)$$

where K = the permittivity or self-inductive capacity of the insulation
and G_2 = the geometric factor

and
$$C_{12} = \frac{K}{2 \log \frac{r_2}{r_1}} \qquad (3)$$

We have expressed the maximum stress at the surface of the conductor in terms of the voltage between the conductor and the cylinder, while we, of course, desire to express it in terms of the voltage between conductor and neutral. We have, however, expressions for the capacities C_{10} and C_{12}. Inasmuch as in a series circuit voltage is inversely proportional to the capacity, we may write directly

$$\frac{E_{12}}{E_{10}} = \frac{C_{10}}{C_{12}} \qquad (4)$$

Substituting the values of C_{10} and C_{12} from (2) and (3) in (4), we obtain

$$E_{12} = E_{10} \times \frac{3 \log \frac{r_2}{r_1}}{G_2} \qquad (5)$$

If this value of E_{12} is substituted in equation (1) we obtain

$$\text{Maximum stress} = \frac{3\,E_{10}}{r_1\,G_2} \qquad (6)$$

We thus have an expression for the maximum stress in a three-conductor cable in terms of known quantities, and have eliminated the cylinder which we imagined to have been inserted in the insulation. The derivation, however, hinges on the fact that the cylinder was present, and therefore this equation gives not the true maximum stress in a given cable, but the average of the maximum stresses around the surface of the conductor. We know that the maximum stress must be close to this value for very small conductors, and that for the case of a cable with a thin belt of such thickness that the stress toward the outside is equal to the stress toward the center, the maximum stress will not be very far different from this value even for larger sizes of conductor. We know definitely that in all cases the true maximum stress will be greater than the maximum stress from this formula.

The average stress between conductors is equal to $\sqrt{3}\,E/2\,T$. We therefore have directly that the ratio of maximum stress to average stress equals

$$\frac{4\sqrt{3}}{G_2} \times \frac{T}{d} \qquad (7)$$

and the broken line curve of Fig. 8 has been plotted by this formula.

Appendix C

EXAMPLE OF ACTUAL DETERMINATION OF STRESS

As an example we will take sector cable model 25. Various potentiometer readings were obtained and these readings which are proportional to stress will be referred to as "stresses" in this appendix. The stresses measured along the zero-deg. line and the 180-deg. line are shown in Table V, while some of the more important stresses at various points in the fillers will be found in Table VI.

Referring to Table V, it will be noted that the measured stress S and its reciprocal are shown for each conductor on both the zero-deg. line and the 180-deg. line, the various stresses being taken at various dis-

tances from the conductor surface. All the stresses shown are measured values except the stress at zero distance from the conductor surface, the maximum stress along that line, which is what must be determined. This may be found as follows. The reciprocals of all the stresses are obtained and are plotted against distance from conductor surface for each conductor and for both the lines under investigation. It will be found that the reciprocals of the points nearest the conductor surface, as mentioned in the text, fall on a straight line, which line passes through the center of the conductor of a round conductor cable. No figure is given of this because the construction is so obvious. In many cases the points fell on perfectly straight lines, and in all cases there was very little doubt as to a close determination of the line. From this curve, therefore, it is possible to read the reciprocal of stress at distance zero from the conductor surface, or, in other words, the reciprocal of the maximum stress on the line. The reciprocals of stress at distance zero shown in Table V were obtained by this means, and the reciprocal of these values is, of course, the stress at the surface of the conductor.

Having obtained the stresses at the surface of the conductor by this means, a curve was drawn both for the zero-deg. and 180-deg. lines, plotting stress against distance from conductor surface. For the zero-deg. line this goes out to the sheath, and for the 180-deg. line this stress is carried out along the line from the conductor surface to the center or axis of the cable. The area under these curves is now found by means of a planimeter, and the figures being stress plotted against the distance, the areas will be proportional to voltage, or, in other words, we obtain numbers proportional to the voltage between the conductor and center and the conductor and sheath, namely, the voltage to neutral. Dividing this by the distance from conductor to center and conductor to sheath respectively, we obtain numbers proportional to the average stress between these points.

Reference will now be made to Table VI. In Table VI we show the area under the curve of stress versus distance, both toward cable center and toward sheath, and the temperature. At the right we show these areas proportional to voltage, corrected for temperature, and the average. The two averages being proportional to voltage should be equal, and it will be noted in this case that there is a 2 per cent variation. To illustrate the consistency of the results, the greatest deviation in this respect was in cable 21 where the difference was 7.8 per cent. Having obtained these various areas proportional to the voltage, we divide them by the distance, and obtain in lines 4 and 13 respectively the numbers proportional to the average stress, which are corrected for the temperature.

The temperature correction is necessary because a 1/15-ohm resistance of zero temperature coefficient was connected in series with each of the conductors of the model. In Appendix A we have shown that the effect is as though the applied voltage was strictly constant, and therefore the voltage on the cable model itself and the measured stresses depended on the distribution of voltage between the external resistance and the resistance of the model, the latter having a decided temperature coefficient; in fact the temperature coefficient shown in formula (1). This necessitates correcting all the values to a standard temperature which was taken as 25 deg. This temperature has no bearing whatsoever on practical stresses, as our final results are all in terms of ratios; it was necessary, however, to have both the stresses from which the ratio was obtained corrected to the same temperature.

The data in this paper are given, however, in terms not of average stress toward the center but of average stress between conductors, and the average of the average stresses toward the center, namely, 300, is transformed to the average stress between conductors by multiplying it by the square root of 3, and the distance from conductor surface to cable center, and dividing this by the distance between conductors, obtaining the value 288, in terms of which most of the data are to be expressed. The stress at the surface of the conductor A, as obtained as shown in Table V is again given in

TABLE VI
DETERMINATION OF AVERAGE STRESSES AND CORRECTIONS FOR TEMPERATURE AND DATA FROM WHICH FINAL STRESS RATIOS WERE OBTAINED FOR CABLE MODEL 25

	Conductor 1		Conductor 2		Conductor 3		Stress or Area at 25 Deg. Cent.			
	Area or Stress	T Deg. Cent.	Area or Stress	T Deg. Cent.	Area or Stress	T Deg. Cent.	Condr. 1	Condr. 2	Condr. 3	Average
Area to Cable Center................(1)	6.59	29.0	6.73	30.5	6.50	29.2	6.48	6.58	6.39	6.48
Area to Sheath......................(2)	6.23	29.5	6.72	30.5	6.42	28.0	6.12	6.57	6.34	6.34
Ratio of (1) to (2)..................(3)	—	—	—	—	—	—	—	—		1.022
Average Stress toward Center........(4)	—	—	—	—	—	—	302	300	299	300
Average Stress between Conductors...(5)	—	—	—	—	—	—	—	—	—	288
Stress A............................(6)	555	29.0	555	30.5	538	29.2	545	542	529	539
Stress B............................(7)	184	29.0	185	30.5	182	29.2	181	181	179	180
Stress C............................(8)	—	—	—	—	—	—	—	—	—	—
Stress D............................(9)	197	29.0	201	30.5	195	29.2	194	196	197	196
Stress E..........................(10)	—	—	—	—	—	—	—	—	—	—
Stress F..........................(11)	667	29.5	609	32.0	609	31.6	654	594	592	—
	625	29.8	606	31.5	606	30.3	612	592	592	606
Stress G..........................(12)	322	29.5	333	30.5	329	28.0	316	325	312	318
Average Stress toward Sheath.......(13)							170	182	176	176

line 6 of Table VI and is corrected for temperature, as well as averaged. The stress, as stated before, was measured in many other places of the cable cross-section, including stresses at frequent intervals all around the conductor and at various distances from the conductor surface. The stress at the cable center was obtained three times, in measuring along the 180-deg. line for each of the three conductors. These values are shown in line 7, where they are corrected for temperature and averaged. The stress C was not measured in this cable, the stress D was measured and is shown; the stress E was not measured in this cable. The stress F, namely, the stress at the surface of the conductor at the sector tips, was obtained exactly as the stresses A and B were obtained; that is, the stresses were measured on various lines around the conductor at different distances from the surface. The stresses on any radial line could be determined, and by plotting the reciprocals of stress, it is possible to obtain the reciprocal of stress at the conductor surface, and therefore obtain the actual stress at the conductor surface. By this means not only were the stresses A and G obtained as described, but stresses all the way round the surface of the conductor, which in some cases turned out to be higher at the tips of a sector and in a general direction toward the sheath. These are shown in line 11, the two values being for the two tips of each conductor, all corrected for temperature and averaged.

From the values shown as the average at 25 deg. cent., the final values of stress ratio shown in Table III were obtained directly, that is, the A, B and D stress ratios in Table III were obtained by dividing the averaged stresses 539, 180, and 196 respectively by the average stress of line 5 or 288. The stress ratios F and were obtained by dividing 606 and 318 respectively by the average stress toward the sheath from line 13, namely, 176.

It is interesting to compare the determinations of stresses for the three individual conductors with the average of the three to determine the uniformity of the models and of the foil thickness. Cable model 25 shows results possibly a little more uniform than others, but all the results were much more uniform than had been originally feared. For instance, the maximum deviation of the stress A for any one conductor from the average for the three conductors of a model is 4 per cent for model 21. Only two cases worse than that were found, namely a deviation from average of 8 per cent in stress F of cable model 25 shown in line 11 of Table VI.

Appendix D

Example of Application of Stress Data

To illustrate the use of the data on stresses in a practical way, the stresses will be calculated for the case of a three-conductor cable 350,000 cm. in cross-section, insulated with 10/32 in. on each conductor and 5/32 in. belt for 33,000 volts working pressure.

We will first consider that the conductors are round. The average stress on which Fig. 5 is based is the average stress between conductors. Ordinarily, stresses are expressed in terms of kilovolts per centimeter, and therefore the average stress must be obtained in those units. With 10/32 in. insulation on each conductor, the separation between conductors is 20/32 in. The voltage is 33 kv., and if 33 is divided by 20/32 and again divided by 2.54 to change into centimeters, we find that the average stress is 20.8 kv. per cm. In Fig. 5 the abscissas are in terms of T/d, which in this case is $0.3125/0.681 = 0.46$. Referring to Fig. 5, we can obtain a series of multipliers corresponding to the abscissa 0.46 of 1.37, 0.58, 0.88, 0.69, and 0.44, corresponding to the A, B, C, D, and E stresses. Multiplying these values by the average stress, 20.8, we obtain 28.5, 12.1, 18.3, 14.4, 9.2 kv. per cm. respectively for the above-mentioned stresses.

If the conductors of the above cable are sector in shape, our data is to be obtained from Fig. 6. Stresses A, B, and D are expressed in terms of the average stress between conductors and plotted against T/d exactly as in the case of Fig. 5. The average stress is 20.8 kv. as before and T/d has the same value. We find the following multipliers from Fig. 6; 1.41, 0.63, and 0.72, and multiplying them by the average stress 20.8, we obtain 29.3, 13.1, and 15.0 kv. per cm. for the A, B, and D stresses respectively. The F and G stresses being toward the sheath, are plotted in terms of average stress between conductor and sheath, the abscissas of the figure being $(T + t)/d$. The average stress is obtained by dividing the voltage to ground, namely, 33 divided by the square root of three, or 19.0 by the distance between conductor and sheath in centimeters, or 15/32 in. times 2.54. This gives 16.0 kv. per cm. as the average stress to ground. $(T + t)/d$ is equal to $0.469/0.681 = 0.69$. Finding the multipliers from Fig. 6 corresponding to this abscissa, namely 2.02 and 1.23, and multiplying these by the average stress 16.0, we obtain 32.3 and 19.7 kv. per cm. for the F and G stresses respectively. In this particular case the F stress is the greatest, the stress at the tips of the sector, this being 10 per cent higher than the A stress in the sector cable, or 13 per cent higher than the maximum stress in the corresponding round conductor cable.

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Discussion

For discussion of this paper see page 988.

Potential Gradient and Flux Density
Their Measurement by an Improved Method in Irregular Electrostatic and Magnetic Fields

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Review of the Subject.—*The safety of electric power cables is dependent in part upon the maximum electrostatic flux density in the dielectric. The core loss in revolving machinery is, in part, a pole-face loss, dependent on variations in the magnetic flux density. Alternators work best in parallel, and give less interference with communication circuits, if of a good wave form, of flux density. These fields of electrostatic and magnetic flux are often too irregular to lend themselves to an accurate computation, and hence experimental methods of study are needed. The writers have improved a method suggested by F. W. Carter, and the results obtained give promise of a field for useful study. The method itself may be briefly described as measuring an electric model of the field to be studied cut out of sheet metal. By the use of a contact-making device, with two contacts, held at a fixed distance apart, but movable to any part of the edge of the metal sheet, voltage gradients may be measured upon a galvanometer. From these measurements, the flux densities may be calculated. The paper describes the method in detail and also gives some results of study on the three problems mentioned. It is shown how this method of test can be used to improve pole-shoe design. The importance of pole-shoe design is pointed out. An appendix is included, showing that correct pole-shoe design is needed to eliminate the so-called "tooth ripples."*

ONE of the authors published in the A. I. E. E. PROCEEDINGS some time ago, a paper on the "Reluctance of Some Irregular Magnetic Fields."[1] Some of the results were applicable to electrostatic fields, and some of the measurements gave an indication of flux density variation. The latter, however, were crude, and lately the experiments were resumed with a view to improving this feature. These new experiments, like the old, are based on a method of test suggested by F. W. Carter.[2] This method is based upon the following principles. (1) The electric, the electrostatic, and the magnetic fields follow the same equation, that of LaPlace. (2) Models of the electrostatic and the magnetic fields can be made out of conducting material if (a) the boundaries are of the same shape, (b) if the electrodes are fastened in similar places. (3) An electric current passed through the specimen will cause effects similar to the magnetic or electrostatic fields desired to be studied. Namely, by the use of a voltmeter, the equipotential surfaces may be traced and the resistance of the field measured.[3] (4) In two-dimensional distributions of flux, the equipotential lines and the flux lines are conjugate, *i. e.*, mutually interchangeable, if the equipotential portions of the boundary are interchanged with the insulated portions. In the papers cited, this principle was not utilized; in fact, would have been impracticable. Messrs. Fortescue and Farnesworth, and Mr. Rice were studying fields which were surfaces of rotation, and not two-dimensional fields. For these no conjugate field exists. Mr. Atkinson used polyphase currents, which would have made the use of conjugate fields too difficult. So far as we know the only use of this last principle other than our own is by C. H. Smoot.[4]

Some attempt was made in the previous paper to obtain the potential gradient by taking differential readings. Thus in Fig. 1, current was passed through the specimen, and through a slide wire in parallel, and points of zero deflection were obtained on the slide wire for different points on the specimen. If the balance points on the slide wire corresponding to the points X and X' on the specimen are V and V', then the gradient at a point midway between X and X' may be taken as,

$$G = \frac{(V' - V)}{(X' - X)}$$

This method of procedure was inaccurate because: (a) the millivoltmeter was insensitive; (b) the distance X could not be read accurately; (c) slide wire reading V was inaccurate because of limited length; (d) there was lack of uniformity of the slide wire; (e) there was variation in the relative strengths of current in the wire and in the specimen. This criticism applies also to all cases where the gradient is judged from the closeness of equipotential surfaces in a map of a field, as in Mr. Rice's paper.[5]

The problem of measurement of flux density and

1. J. F. H. Douglas, PROC. *A. I. E. E.*, May, 1915.
2. F. W. Carter, *Electrical World and Engineer*, Vol. 38, page 884.
3. The above three principles have been utilized and acknowledged in at least three Institute papers besides the one cited. These are C. L. Fortescue and S. W. Farnesworth, "Air as an Insulator when in the Presence of Insulating Bodies of Higher Specific Inductive Capacity," A. I. E. E. TRANS., 1913, Vol. 32, Part 1, p. 893. C. W. Rice, "Electrostatic Problem," A. I. E. E. TRANS., 1917, Vol. 36, p. 905. R. W. Atkinson, "The Dielectric Field in an Electric Power Cable," A. I. E. E. TRANS., 1919, Vol. 38, Part 2, p. 971.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

4. *Jour.*, West. Soc. of Eng., 1905, page 500.
5. *Loc. cit.*, p. 927.

potential gradient was felt to be of considerable importance, and that improvement in the method of measurement was desirable. To mention only two such cases, we may refer to the problem of the wave form of flux density in an alternator, and the gradient in a power cable with three conductors. The specimen in Fig. 1 may be taken to represent the air-gap of an alternator. Surface 1 is the pole shoe, surface 2 is the armature surface, 3 is a neutral surface between the poles, 4 and 5 are lines of force. The flux density at X is desired as a function of X. It will be observed that in Fig. 1, the conjugate character of the potential

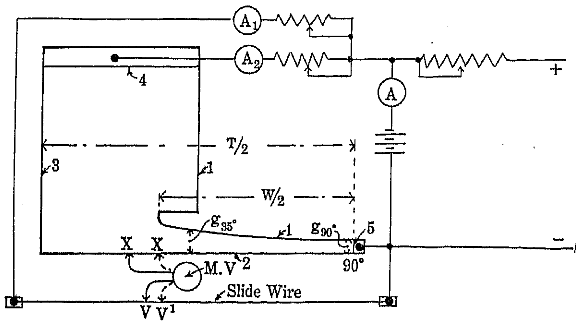


FIG. 1—FLUX-DENSITY TEST FOR ALTERNATOR

and flux lines is utilized. The flux lines 4 and 5 are made electrodes, and the magnetically equipotential surfaces 1, 2 and 3 are left insulated. By measuring the electric potential gradient at X, namely $\left(\frac{d\,V}{d\,X}\right)$ the density in the magnetically equivalent and conjugate case is obtained.

We may note here in connection with Fig. 1 some of the advantages of testing the conjugate field with a metal sheet. (1) In most cases the electrodes used may be short in length, straight or of slight curvature, easily soldered to the sheet, and easily kept at a uniform potential throughout. (2) Smaller currents were required and more sensitive indications were obtainable. (3) The places of most interest for determining the density or gradient were by this means transferred to the free edge of the sheet, where contacts were most easily made. (4) In modifying proportions of the specimen, instead of resoldering electrodes, the free edge of the sheet was trimmed. This saved a great deal of material and time.

Another case of some interest, the voltage gradient in a cable, is illustrated in Fig. 2. It seemed to us that the insulation strain was greatest at the surface of the conductors and at that instant of time when one of the three conductors was charged positively, one negatively, and one at zero potential. We made our tests accordingly. However, a more recent study of Mr. Atkinson's paper indicates that further tests corresponding to other instants of time would be desirable.[6] Only one half of the cable is shown in the figure, there being a line of symmetry along a diameter. Conductor A is positive. Conductor B, the sheath and the diameter are neutral. Owing to the convergence of the electrostatic lines of force upon the conductor, the gradient at the surface of the conductor is of most interest. The gradient is proportional to the dielectric flux density, and the latter was the quantity actually measured. The electrodes are fastened so as to make the field conjugate to the electrostatic field in the cable. The electrodes 4 and 5 are soldered to flux lines. The insulated edges 1 and 2 are the equipotential surfaces in the cable. By measuring the gradient at X the dielectric flux density in the conjugate electrostatic case is obtained.

Our improved method of test may be illustrated by Fig. 2. Current was taken from a six-volt storage battery on account of its steadiness; and, this was kept on a slight charge. The current was regulated by means of a German silver ribbon resistance capable of continuous adjustment, and kept at a constant value with a Weston ammeter. The gradient at the point X was measured by the direct deflection of a sensitive Leeds and Northrup galvanometer, which was connected to two terminals 6 and 7 located approximately one centimeter apart, but held at a constant distance by a wooden spacing member. A resistance box R was included for the purpose of regulating the maximum deflection of the voltmeter, usually to 5 or 10 cm. The contacts were of brass, the same as the test sheets, in order to reduce thermal effects. The switch S was opened after each reading, and the "zero" thus obtained was used to correct the readings. It is hoped that this procedure eliminated residual thermal effects, usually of the order of one millimeter. Contacts

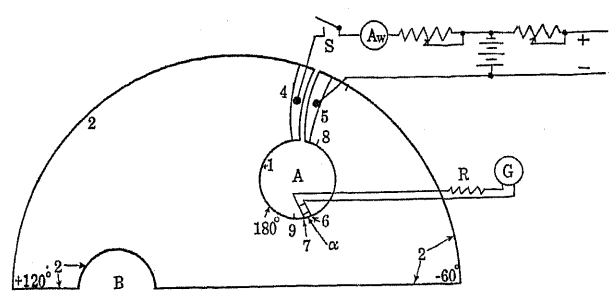


FIG. 2—FLUX-DENSITY TEST IN A THREE-PHASE CABLE

6 and 7 were pivoted to a yoke and held against the edge of the sheet by a spring. The other end of the spring was attached to a block, which was clamped to the edge of the sheet. In this manner an intimate contact was secured, which was flexible, easily adjusted, and movable from place to place. Credit must be given to R. W. Atkinson for the use of a double-contact electrode, for use in measurement of gradient with electrolytes and in the non-conjugate field, and with alternating currents.[7]

6. *Loc. cit.*, p. 991.

7. *Loc. cit.*, p. 983.

It is to be noted that when a double contact is to be used at the boundaries of a field, the electrodes should be oriented parallel to the edge of the sheet, if this is a line of flow; on the other hand, if the boundary tested is equipotential, the electrodes must be oriented perpendicular to the same. This is another advantage for the conjugate method of test, since the orienting of the electrodes is automatic. A single measurement will also suffice on the edge of the sheet. In Mr. Atkinson's paper, the gradient was measured at a point 1/16 in. from the boundary of the conductor, and by several measurements was extrapolated for the boundary itself.[8] It is no criticism of the accuracy of Mr. Atkinson's results to state that the method of

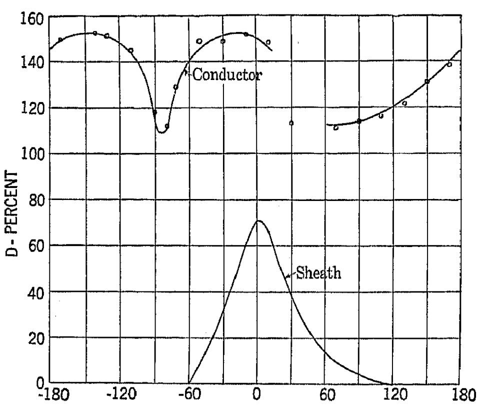


FIG. 3—GRADIENT IN A THREE-PHASE CABLE

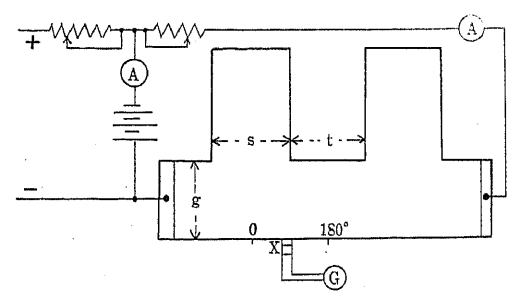


FIG. 4—FLUX-DENSITY TEST FOR A SLOTTED ARMATURE

testing the conjugate field gives superior rapidity of test at the boundary of the field. For points on the interior of the field, the electrodes should be oriented so as to secure a maximum deflection, if the true gradient at these points is desired. This is best done by trial. We understand that in most cases Mr. Atkinson oriented his contacts so as to be radial with the conductor. On this account we venture to question the accuracy of his results as far as they refer to the field on the interior of the three-phase cable. For instance, in his Fig. 8 at the point E-90 deg. the electrodes were radial. On the other hand, from Wm. A. Del Mar's discussion and Fig. 1, the field, at one instant at least, is some 45 deg. from the radial position. We do not understand, however, that Mr. Atkinson claimed entire accuracy for these results, nor do we stress it as an important point.

8. *Loc. cit.*, p. 987.

The sheets out of which our specimens were cut were of 20 or 24-gage sheet brass, one ft. in width, and of length varying from 24 to 32 in. Currents of approximately 10 amperes were used through these sheets. We calibrated our galvanometer by the use of a rectangular sheet taken from the same stock.

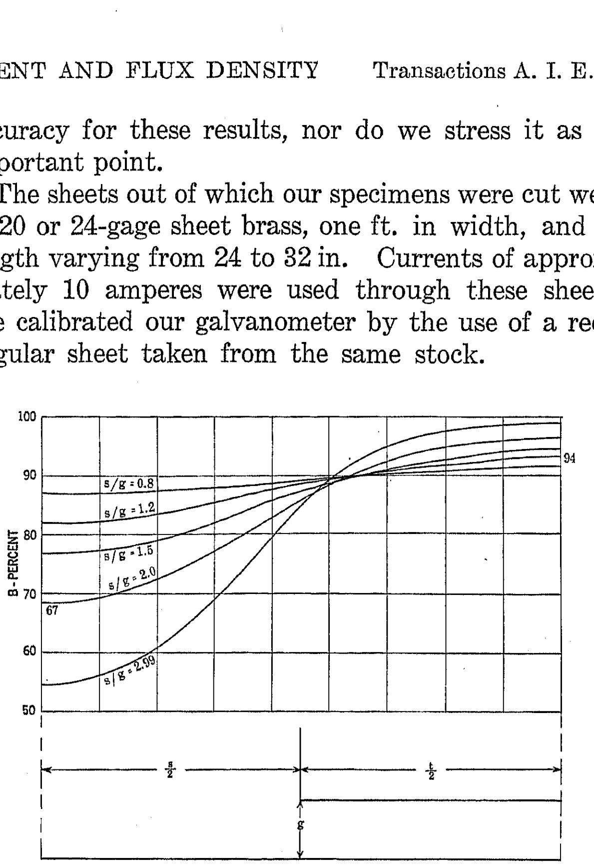


FIG. 5—POLE-SHOE DENSITY WITH SLOTTED ARMATURE

At first we attempted to use a null method; but the simplicity of the direct deflection method, the rapidity with which it was possible to get readings, and the consistency of the results (when corrected for thermal effects) convinced us that the null method was not necessary. In this we find that Mr. Atkinson supports our conclusion.[9]

Some question might be raised as to whether contacts

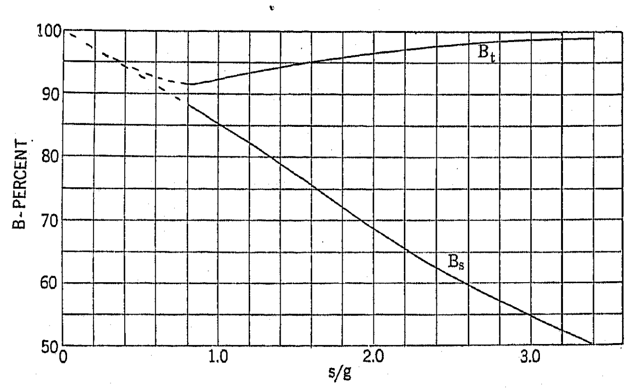


FIG. 6—MAXIMUM AND MINIMUM POLE-FACE DENSITY

one centimeter apart would give a good approximation to the actual gradient at the mid-point. We arrived at the following conclusion, with reference to the problem of alternator wave form: If the base in Fig. 1 be 90 cm., a contactor with one centimeter spacing would measure the flux density far more accurately than necessary. The 19th harmonic would be determined to ½ per cent, and the 41st harmonic to two

9. *Loc. cit.*, p. 988.

per cent, and others in proportion. We feel, therefore, that this error does not enter to any appreciable extent.

The results of our test on the cable are shown in Fig. 3. The angles θ are measured from the point where the density is the greatest, namely point 8 in Fig. 2. It will be noted that the densities are greatest at the points 8 and 9 where the conductor approaches a neutral electric surface. The gradient obtained with a uniform strip, of width equal to the minimum dielectric thickness, is taken as 100 per cent. It may be

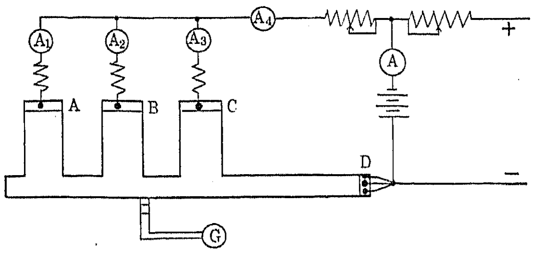


FIG. 7—FLUX-DENSITY TEST FOR TURBO-ALTERNATOR

concluded that, owing to the curvature of the lines of force, the gradient in cables is larger than the value computed on a simple volt per mil basis. This is in no respect a novel conclusion. To advance the knowledge of cable stresses our tests would have to be very considerably expanded. We feel, however, that we have shown that our method will be of considerable utility.

We suggest two series of tests. The first series would be with sheets with electrodes attached as in

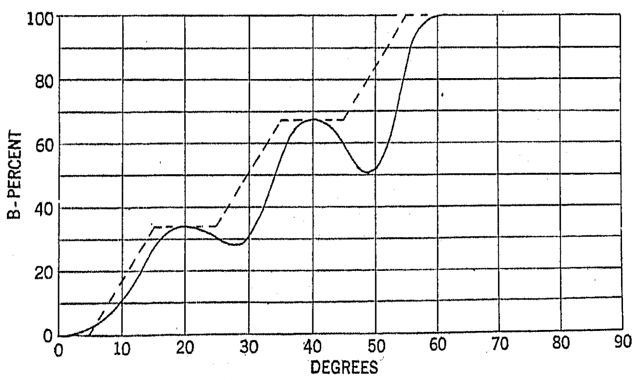


FIG. 8—FLUX-DENSITY WAVE IN TURBO-ALTERNATOR

Fig. 2; the second series would naturally be with electrodes attached to imitate conditions at 90 deg. time phase later. In each series several sheets would have to be used, each with a certain amount of belt insulation. Each sheet would require trimming and testing several times, to correspond with several conductor sizes. Professor Karapetoff, in his "Electric Circuit,"[10] gives a method of superposing fields in equilibrium, which would enable one to obtain, from these two tests, the gradient in the cable at any instant of time phase.

In Fig. 4 is shown a model of an air-gap with a slotted armature surface, the teeth and the slots being equal in width. The flux density was measured for various proportions of slot and air-gap; our results are plotted in Fig. 5. The abscissas represent positions on the pole face, the ordinates represent the flux density in the pole face, expressed as a percentage of that obtaining with a smooth core armature. The maximum and minimum values of density are plotted in Fig. 6 as a function of the ratio of the slot width S to the air-gap G. The curve B_t represents the density opposite the tooth center, the curve B_s represents the density opposite the slot center. These waves are nearly sinusoidal. The two curves of B_t and B_s indicate the amount of pole face loss that may be

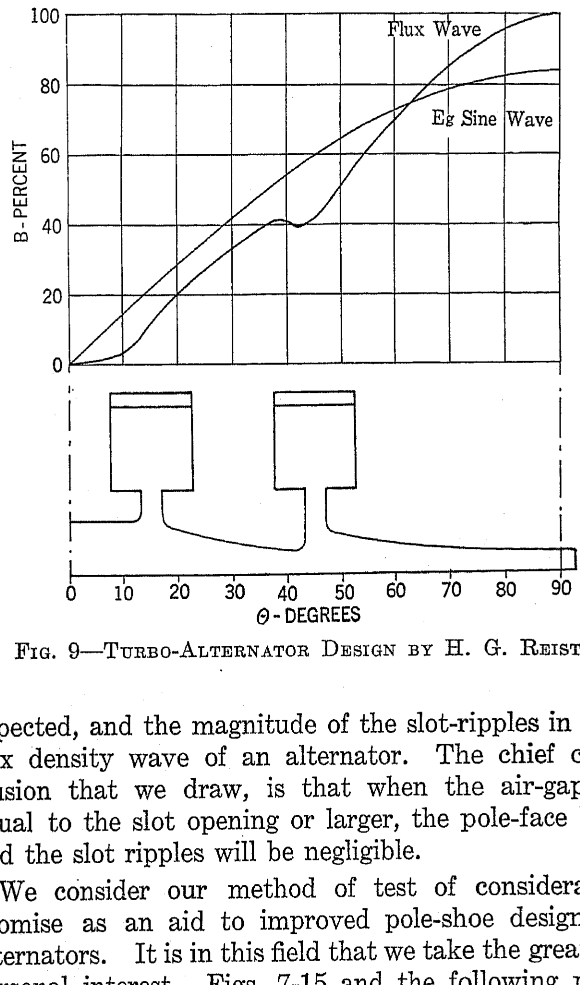


FIG. 9—TURBO-ALTERNATOR DESIGN BY H. G. REIST

expected, and the magnitude of the slot-ripples in the flux density wave of an alternator. The chief conclusion that we draw, is that when the air-gap is equal to the slot opening or larger, the pole-face loss and the slot ripples will be negligible.

We consider our method of test of considerable promise as an aid to improved pole-shoe design in alternators. It is in this field that we take the greatest personal interest. Figs. 7-15 and the following part of the paper are devoted to this one topic. In Fig. 7 is shown a specimen for testing the flux distribution in the air-gap of a turbo-alternator. The stepwise variation of the m. m. f. of the field winding was imitated, by causing the test current to enter the sheet in three different places by electrodes A, B and C, and to leave by the electrode D. These currents were adjusted for equality to correspond with the

10. Page 176.

practical case of an equal number of ampere-turns in each slot. The different electrodes were made of copper to equalize the potential, and wherever needed, as at *D*, several wires were soldered to the electrodes to achieve this end. By adjusting the relative length

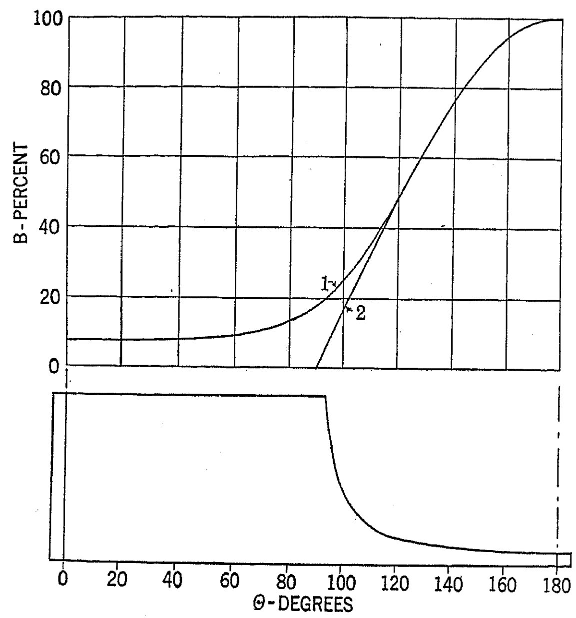


FIG. 10—INDUCTOR ALTERNATOR DESIGN BY J. F. KELLEY

of these wires, the edge of the copper electrode, adjacent to the equipotential boundary of the field, could be kept accurately in an equipotential condition. The measured flux density wave is shown in Fig. 8, together with the wave shown by Gray[11] as applying to this

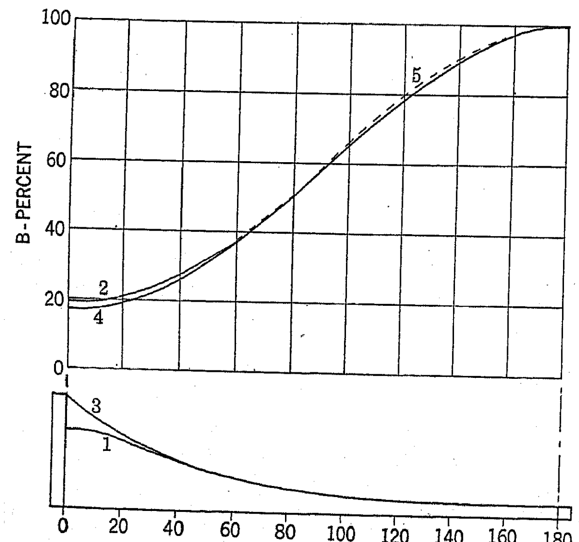


FIG. 11—IMPROVED INDUCTOR ALTERNATOR DESIGN

case. It is seen that the wave form with a uniform gap, and open slots is very poor in form, giving a large deviation and containing apparently a large 17th and 19th harmonic.

In Fig. 9 is shown a turbo-alternator pole-shoe shape given by H. G. Reist,[12] together with the results of a test made by us upon a specimen cut to this shape. It will be seen by a comparison with Fig. 8 what a great improvement in wave form was secured by using (a) a central member diverging from the armature surface, (b) side members diverging at a greater angle, and (c) unequal air-gaps on either side of the slots separating the central from the side members. Fig. 9 also shows a sine wave for the purpose of comparison. By advancing the pole shoe toward the armature surface where the flux density is too low and *vice*

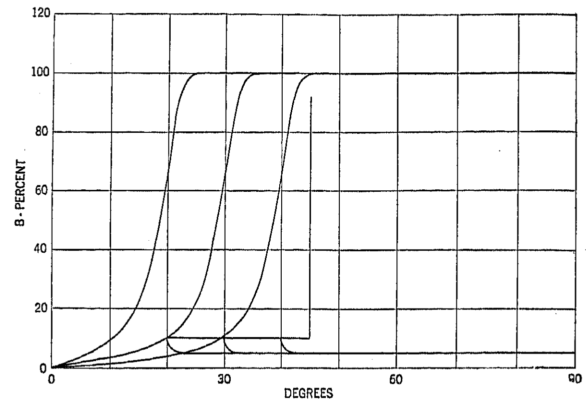


FIG. 12—FLUX-DENSITY WAVES WITH SALIENT POLE ALTERNATOR WITH UNIFORM AIR-GAP

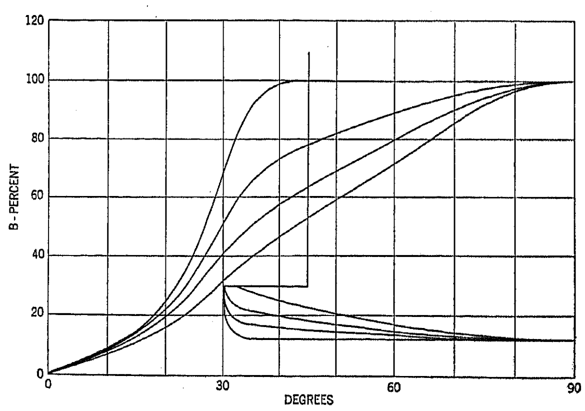


FIG. 13—FLUX-DENSITY WAVES WITH SALIENT POLE ALTERNATOR WITH 67 PER CENT POLE ENCLOSURE AND CURVED POLE FACE

versa, the wave of flux density should be further improved.

In Fig. 10 is shown the shape of the pole face of an inductor alternator disclosed by J. F. Kelley.[13] This pole face was shaped to an inverse sinusoidal curve, which theoretically should have given a good wave form of flux density. Fig. 10 also gives (1) the wave as measured by us, and (2) a wave computed on the basis of radial lines of force, *i. e.*, disregarding their curvature and convergence. It will be readily seen that a purely theoretical formula, which does not take

11. Electric Machine Design, p. 288.

12. U. S. Patent, No. 1,008,561.
13. U. S. Patent, No. 529,918.

into account these phenomena, is open to considerable error.

Fig. 11 represents an attempt upon our part to improve the shape of the pole shoe of an inductor alternator. The line 1 represents the shape of the pole face computed on the basis of radial lines of force, to produce the sine wave 5. Curve 2 represents the measured wave of flux density obtained with a specimen shaped like Curve 1. This density is somewhat too high at points where it should be lowest. A second trial was made using the shape given by Curve 3, which increased the gap where the density was too high. Curve 4 is the result of a test made on this specimen. This curve is so close to 5, the sine wave, that the latter is shown as dotted only where the two deviate from each other. We conclude that our method of test should be able to give more nearly ideal shapes of alternator pole shoes.

With reference to the wave form of salient pole alternators, with poles of alternating polarity, we have tested specimens of the general form shown in Fig. 1, but with differing proportions. One series of tests was made with a uniform air-gap 6 per cent of the pole pitch, but with various ratios of (W/T), that is, differing pole enclosures. The results of these tests are shown in Fig. 12. A second series of tests was made with specimens of 67 per cent pole enclosure, but with varying degrees of pole face curvature. The results of this series of tests is shown in Fig. 13.

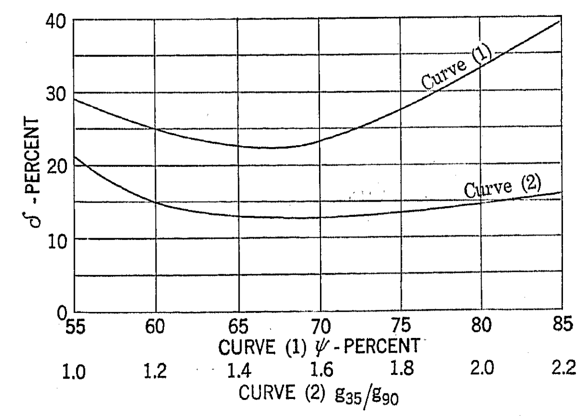


Fig. 14—Deviation Factor of Flux Wave of a Salient Pole Alternator vs. Per Cent Pole Enclosure and Pole Face Curvature

The deviation factors of all these waves were computed and are plotted in Fig. 14. Curve 1 shows the variation of the deviation factor δ with the pole enclosure ψ. Curve 2 shows the variation of δ with the pole face curvature as represented by the ratio of the air-gap at 35 deg. to the air-gap at the center of the pole. These curves show a minimum deviation from the sine wave when the pole enclosure is approximately 67 per cent and when the gap at the 35-deg. position is approximately 1.5 times the minimum gap. The results tend to justify present practise. However, further improvement can be made, since the wave forms disclosed in Figs. 12 and 13 are still far from a sine wave. The investigation reported upon here is incomplete, in that pole faces of only simple curvature were considered. The results would be more useful if harmonic analyses had been included or at least, the telephone interference factor had been shown. We hope to have further results at a later date.

We wish to state here our reasons for believing that the elimination of telephone interference and other

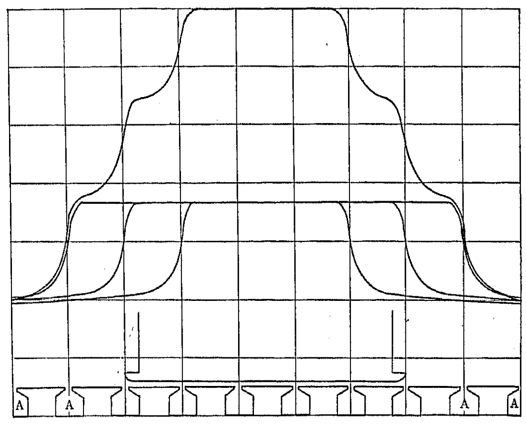


Fig. 15—Voltage Wave showing "Slot Ripples" Due to Poor Pole-Shoe Design

harmful effects of poor wave form, is in large measure dependent upon improvements in pole-shoe design. In the first place, with the alleged exception of tooth ripples, a good pole-shoe design goes to the root of the matter and removes harmonics at their source. In the second place, other remedies are palliative and partial only. For example, the use of distributed and short chord windings eliminates some harmonics in the wave of voltage, but not all. What is more, the ones that are eliminated may easily make the others more apparent. For example, in Fig. 15 is shown a wave of e. m. f. built up of three waves like one of those in Fig. 12 displaced in phase 20 deg. It corresponds to the voltage induced in one phase of a three-phase machine, with 9 slots per pole, the slots almost closed, and in which variations in air gap reluctance would not occur. The resultant wave, instead of being more nearly a sine wave, shows a pronounced 17th and 19th harmonic. The fact of the matter is the original smooth wave contained those prominent 17th and 19th harmonics, as well as many others which concealed them. However, the distribution factor of the winding is such that harmonics other than the 17th and 19th are eliminated in large measure, while these two are not eliminated and so stand revealed. It is to be noted that these harmonics are of the so-called tooth-ripple frequency, and the ripples in the wave might be attributed to variations in the air-gap reluctance caused by the teeth. It is a fact that in every case the distribution factor of a winding is such

as to eliminate partially harmonics of other than tooth-ripple frequency, but not the harmonics of tooth-ripple frequency. We conclude that harmonics of tooth-ripple frequency may not be due to the teeth at all, but may represent harmonics present in the original flux wave, because of poor pole-shoe design.

We add an appendix containing a proof that tooth ripples occur only with pole shoes giving a wave of flux density which in itself contains harmonics of tooth-ripple frequency.

Appendix

A Proof that Slot Ripples are Due to Incorrect Pole-Shoe Design

In Fig. 16, let A represent a smooth core armature surface and B the pole shoe designed so that every harmonic except the nth has been eliminated. Let the magnitude of this harmonic in terms of the fundamental be a. Let C be center lines of symmetry located midway between the poles. Let X be any point on the armature surfaces at which the flux density is desired expressed in electrical radians from the eutral axis. Let G be the equivalent air-gap from the armature at this point to the pole face; that

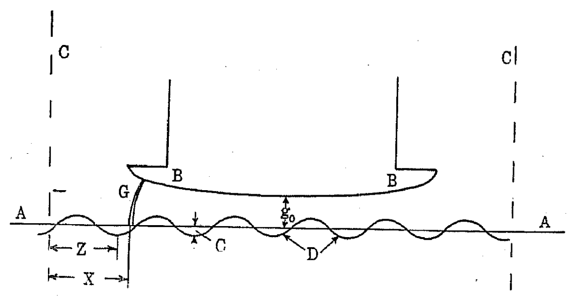


Fig. 16—Construction Used in Proof that Slot Ripples Are Due to Poor Pole-Shoe Design

is, G is the curved distance shown corrected for the tapering of the tube of force. Let G_0 be the gap at the center of the pole which would yield the peak of the fundamental sine wave. The distance G may be defined by equation,

$$G = \frac{G_0}{(\operatorname{Sin} X + a \operatorname{Sin} n X)} \tag{1}$$

Let D be the equivalent armature surface of a toothed armature in its first approximation—a sine wave. Let the point Z be the location of a slot center. Let c be the deviation of this equivalent armature surface from that of the smooth core armature. Let the ratio of (c/G_0) be designated by b. Let there be m slots per pole.

The effect of the slotted armature surface is to increase to effective gap by an amount $\operatorname{Cos}(2m)(X - Z)$. The equivalent air-gap with the slotted armature is, therefore,

$$G = \frac{G_o}{(\operatorname{Sin} X + a \operatorname{Sin} n X)} + c \operatorname{Cos}(2m)(X - Z) \tag{2}$$

The flux density in the air-gap may be expected to decrease in the ratio that the gap is increased, or if B_1 is the maximum value of the fundamental component of flux density, and B is the density at the point X in the slotted armature, we have the equation,

$$\frac{B}{B_1} = \frac{G_0}{G} = \frac{(\operatorname{Sin} X + a \operatorname{Sin} n X)}{[1 + (c/G_0) \operatorname{Sin} X \operatorname{Cos} 2m (X - Z) + a (c/G_0) \operatorname{Sin} n X \operatorname{Cos} 2m (X - Z)]} \tag{3}$$

Upon expanding this expression by polynomial division we obtain

$$\begin{aligned}(B/B_1) = {} & \operatorname{Sin} X + a \operatorname{Sin} n X \\ & - b \operatorname{Sin}^2 X \operatorname{Cos} 2m (X - Z) \\ & - 2ab \operatorname{Sin} n X \operatorname{Sin} X \operatorname{Cos} 2m (X - Z) \\ & - b^2 \operatorname{Sin}^3 X \operatorname{Cos}^2 2m (X - Z) + \ldots\end{aligned} \tag{4}$$

The total flux ϕ expressed as a fraction of the fundamental component ϕ_1 is obtained by an integration from $X = O$ to $X = P i$. That is

$$\phi/\phi_1 = (1/2) \int_0^\pi (B/B_1)\, d X \tag{5}$$

Equation (4) was substituted into equation (5) and was finally integrated be the use of trigonometric reduction formulas. In case $n \neq (2m \pm 1)$

$$\phi/\phi_1 = 1 + (a/n) + (2/3) b^2 + 3 b^2 \operatorname{Cos} 4m\ c/(16 m^2 - 9)(16 m^2 - 1)$$

The last term is extremely small and moreover of double-tooth frequency. We conclude, if the pole shoe does not contain a harmonic of tooth-ripple frequency, there will be no material fluctuation in the magnetic flux of a pulsating nature.

In case $n = (2m \pm 1)$ we get the following result (omitting the last three terms above), namely

$$\phi/\phi_1 = 1 \mp (\ /4)\, a b \operatorname{Cos} 2 m Z \tag{7}$$

This is a flux that pulsates with a frequency $2m$ and which will cause in the armature winding harmonics of frequency $(2m + 1)$ and $(2m - 1)$. This means that in addition to the harmonics of tooth ripple frequency, due to the pole shoe itself, the interaction of pole shoe and slotted armature surface tends to increase the result.

Discussion

DIELECTRIC FIELD IN AN ELECTRIC POWER CABLE—II

(Atkinson)

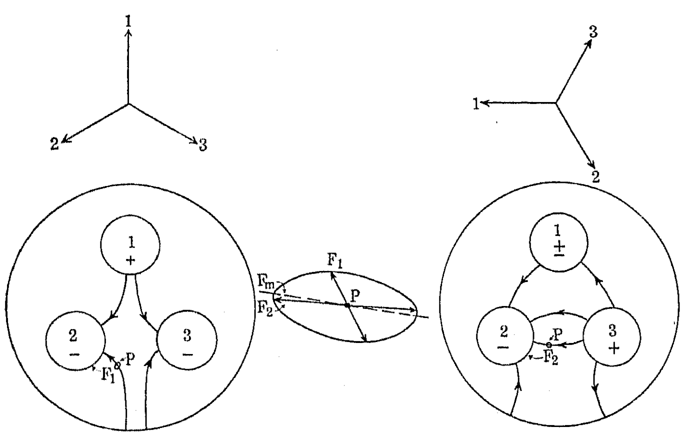
POTENTIAL GRADIENT AND FLUX DENSITY

(Douglas and Kane)

D. M. Simons: The paper by Messrs. Douglas and Kane is of great interest and I think their method can be applied to cable work advantageously.

They have possibly been more interested in the method than in the results of their test, and have so stated. One of their results in regard to cables might cause a slight confusion. Referring to Fig. 3, they mention in the text that they have discovered that the point of maximum stress in their three-conductor cable model was at the surface of the conductor and at the point nearest to the outer sheath, while most observers have found it toward the center of the cable. Mr. Atkinson has pointed out in his previous paper that the stress toward the

sheath depends not only on the conductor insulation but also on the belt insulation and that if the thickness of the belt is reduced, the maximum stress swings around from the center or axis of the cable to the direction of the sheath. As far as I can tell from rough measurements of Fig. 2, the authors have apparently dealt with a cable which had all the insulation on the conductors, but no belt insulation. Their experimental results are therefore correct, but they do not disprove, and I am sure are not intended to disprove, the general statement that in practise cables normally having an appreciable thickness of belt, the maximum stress is toward the center or axis of the cable.



H. Halperin: In regard to Mr. Atkinson's paper, it has been our experience in test and service failures that a large percentage of the failures in three-conductor, 33-kv., sector cables were at the outer corners of the sectors. According to the author, the maximum dielectric stress is at these corners.

E. W. Kane: In regard to Mr. Simons' remarks in relation to Fig. 2, he is correct in stating that in our particular test we made no belt insulation, and we agree with his conclusion; referring to Fig. 3, it may be noted that part of the curve will be lower at the right-hand side when the belt insulation is used.

In regard to Mr. Atkinson's paper on the gradient in three-phase cables, it is perhaps ungracious to attack the accuracy of his results on the basis of a single test; however, a theoretical point of some importance in involved. The electrostatic field in any three-phase cable is, in certain places a pulsating field, but in other places it is a rotating electrostatic field.

In Fig. 1 at the left, is shown the field at an instant when the charge on the conductor 1 is a maximum, the field shown at the right is ninety degrees time phase later, when the charge on conductor 1 is zero.

Considering the field at the point P at the instant of time shown in the left-hand figure, the field is upward, to the left and rather small. At the instant of time phase shown in the right-hand figure, the field is nearly horizontal and much larger. At other instants of time these components are present in different degrees. The elipse in the center shows the locus of the force F at the point P at various intervals of time. It is a rotating electrostatic field.

The electrodes measuring maximum gradient should be oriented along the axis of maximum F, that is F_m and not radially to the conductor. At the surface of the conductor, the field is always radial and there is no criticism of Mr. Atkinson's results here. We contend merely that our method has advantages in the line of speed for these conditions.

A three-phase field can be split up in two single-phase fields which can be combined geometrically at all points and algebraically where they are in the same direction, as at the conductor surface.

R. W. Atkinson: I am going to refer to the paper of Messrs. Douglas and Kane wherein they mention to some of my work. They speak of the fact that the field is not radial at all points. Just in a word, the point that they make does not affect the accuracy of the data which has been given. It is possible for stress to be greater in some other direction. The stress, though, has all been given in either radial stress at the surface of the conductor which is correct as given, or it has been given as stress at some other part of the insulation and in a certain direction. Usually the radial stress approaches very nearly the maximum stress that can exist at a point in any direction.

Some Notes on Street Lighting

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Review of the Subject.—*This paper, after discussing the criterion of street lighting effectiveness, presents comments upon certain of the variables of street lighting.*

Test data are presented upon certain sizes and types of incandescent lamps for street lighting. An attempt is made to indicate something of the present status of street lighting, including the presentation of a selected list of modern lamp posts and lighting equipments. The paper concludes with data to show the character of installations now recommended by experts for the lighting of various classes of streets, and the advance in recent years in ideas of desirable levels of street illumination.

* * * * *

A GOOD way to begin any activity is to define the purpose in view. The purposes of street lighting have been defined variously. Starting thus from different premises, writers on the subject have naturally arrived at divergent conclusions. In consequence there is uncertainty as to the principles which should guide in the design of street illumination.

The Purposes of Street Lighting. Reduced to lowest terms it is believed that the purposes of street illumination are:—first to reveal, and second to embellish.

Protection against the hazard of criminal violence and collision; security in avoiding obstacles and inequalities in roadway; facility in finding one's way about, all require that the light which is provided shall reveal what it is important to see on or about the street. No system of street lighting is effective which fails to achieve this purpose. Any system of street lighting is effective largely in proportion as it serves this purpose.

Of secondary importance, to be weighed with respect to the type of street involved, is the extent to which a street lighting system, by reason of the design and location of lamp supports and equipment, promotes the good appearance of the street in the daytime, and by virtue of these things and its characteristic of light distribution, promotes the good appearance of the street at night.

Subsidiary purposes which street lighting systems have been designed to serve are advertising of the locality, attracting trade, etc. These more properly fall in the category of advertising display, but are sometimes combined with street lighting for reasons of expediency.

Among diverse requirements for a successful street lighting system which have been put forward, the following may be noted:

"The problem of street illumination is to produce a uniform low intensity."

* * * * "I recognize that a moderate diversity (of illumination) is much more satisfactory."

* * * * "Avoidance of glare is the most important factor in street lighting."

It is submitted that any of these and other alleged major requirements to be served by street lighting may be tested fairly by ascertaining to what extent they contribute to the revealing power of the street lighting system and to the embellishment of the street. It would perhaps be desirable to sacrifice other features to secure a "uniform low intensity" if by doing so the revealing power of the lighting would be greater, but until such has been demonstrated to be the case, the writer for one is unwilling to subscribe to this statement. The test of whether uniform distribution of light along the street or a "moderate diversity" is better lies in a determination of revealing power of the two kinds of lighting. That "avoidance of glare" is not the "most important factor" is evidenced by the obvious fact that this can be accomplished by extinguishing street lamps, which would not at all contribute to the revealing power of the lighting system.

In this country the history of the street lighting art records the prevalence for a period of years of first the "2000 candlepower" criterion and subsequently the candlepower delivered 10 deg. below the horizontal as measures of street lighting effectiveness. The causes contributing to the adoption of these two criteria are now well understood. Neither could have come into use had revealing power been recognized as the principal test of street lighting.

Effectiveness in Street Lighting

There is at this time no generally accepted criterion of effectiveness in street lighting. It is customary to decide between competing street lighting systems by any means which best commend themselves to those responsible and to stipulate in contracts the lamps, equipment and service which together constitute the street lighting system. Among the means availed of in choosing a system are usually inspection, observation, and tests of the lighting units in a laboratory or on the street or both. Actual attempts to determine relative revealing power of various systems are rarely if ever, made in practise.

Effectiveness in a street lighting system like personal charm is recognized when encountered, but is difficult to define. It depends upon the right combination of several qualities, some of which are perhaps intangible. However, when a street lighting system attains to thorough effectiveness, the illuminants will be found to be located and spaced suitably, to be mounted at a

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desirable height and to be enclosed or equipped in such a way as to present to the eye a desirable combination of candlepower and brightness for the lighting unit and of distribution of light along the street and upon buildings. Suitability and desirability in these particulars sometimes hinge upon the local conditions peculiar to the installations. A few comments of general applicability may, however, be in order at this point of the discussion.

Location of Lamps. For effectiveness in illuminating streets there is no location quite so favorable as directly over the driveway. Where the lamps are few and far between and of small size, and where the pavement is of asphalt or other material which tends to take a polish from motor traffic, the advantages of locating the lamps over the driveway are greatest. Thus, guy wire suspension and mast-arm post mountings possess advantage in such installations. When the lamps are numerous and of large size, and the street illumination is of higher intensity, precise location of the lamps is of less importance since the greatest revealing effectiveness does not have to be obtained from them. This point is well illustrated in Fig. 1 in which a modern twin lamp installation supersedes an earlier mast-arm installation. The new installation is effective in spite of a location which makes for much lower effectiveness per unit of light. When a relatively great amount of light is produced, effectiveness of utilization may be sacrificed somewhat to secure improved appearance. With the lesser quantity of light afforded by the earlier installation, the location of lamps over the street was essential in order to utilize with fair efficiency the relatively small amount of light which was produced.

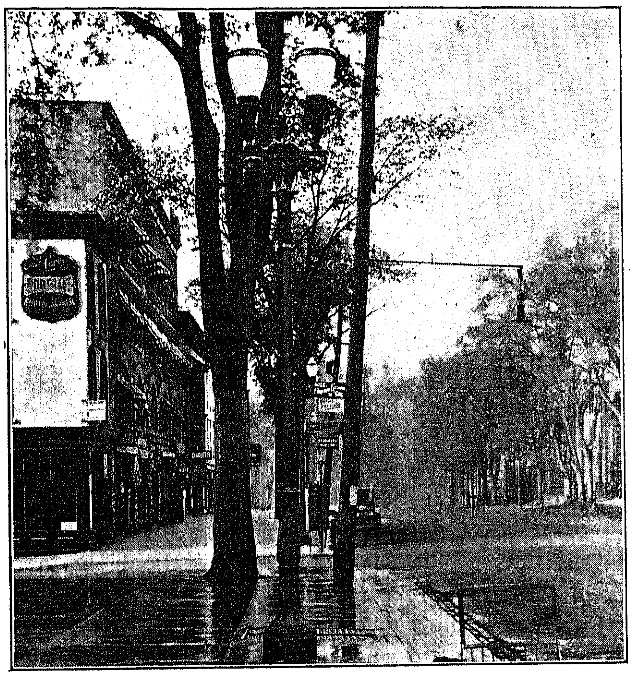

FIG. 1

In the old system, but little light being available, the lamps were located over the roadway for greatest effectiveness. The more adequate modern lighting permits location of lamps for pleasing effect.

Spacing of Lamps. Evidently spacing intervals should be small enough to avoid dark areas between lamps. On the other hand, the writer's experience has indicated that it is not desirable to incur large expense through reducing spacing intervals in an attempt to approximate uniformity of illumination along the street. Discernment in the street at night is largely dependent on contrasts of light and shadow. Studies of revealing power made for the joint street lighting committees of the National Electric Light Association and the Association of Edison Illuminating Companies 1914-1915, indicated that when uniformity is attained through multiplicity of small illuminants, contrasts are diminished through the elimination or reduction of shadows, so that objects on the street and depressions or holes in the street surface are not seen so well as they are when lighted from fewer, larger lamps. These, through failing to provide uniformity of illumination, do produce relatively strong shadows which are an aid to visibility.

Mounting Height. In connection with the control of light for street lighting purposes, much planning and design have been influenced by a desire to approximate uniformity of illumination along the street. Attention in this connection has been focused upon illumination intensity curves such as those shown in Fig. 2. Other aspects of street lighting effectiveness have sometimes been sacrificed to secure a considerable percentage increase, but a small absolute increase in

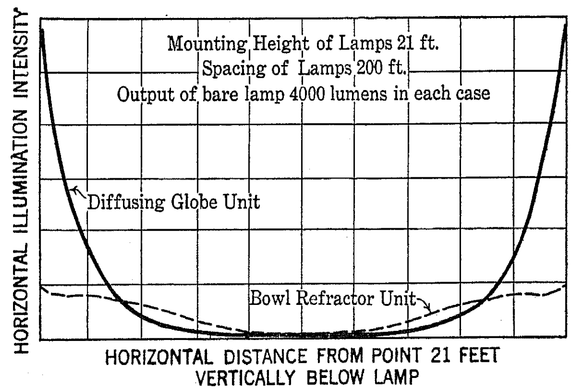

FIG. 2

The desire to approach uniformity of illumination along the street has influenced street lighting design quite generally.

illumination midway between lamps. Such improvement in midpoint illumination can be had, of course, only at the expense of largely increased candlepower just below the horizontal. To avoid glare as a consequence of such light distribution, the units have sometimes been mounted high. The resultant effect upon the illumination curve has been thought to be good.

It must not be forgotten, however, that as lamps of symmetrical horizontal distribution are mounted higher, the proportion of light flux delivered upon the street

surface is diminished. This effect is increased if the candlepower just below the horizontal is made large to reinforce the illumination midway between lamps. In Fig. 3 curves are shown to illustrate the relations that are here involved. Considering the total light produced, a smaller part is delivered below the horizontal from a diffusing globe unit than from a bowl refractor. With low mountings of the bowl refractor, the advantage in the proportion of light delivered upon the street is quite large. At 43 ft. mounting height, which, of course, is higher than may be considered practicable when lamp posts are employed, the proportion of light delivered upon the street is the same for the two types of distribution. If, however, some light is desired above the horizontal, and the lower hemispherical light flux alone is considered in this relationship, the relative proportions delivered upon the street surface are shown in the two lower curves,

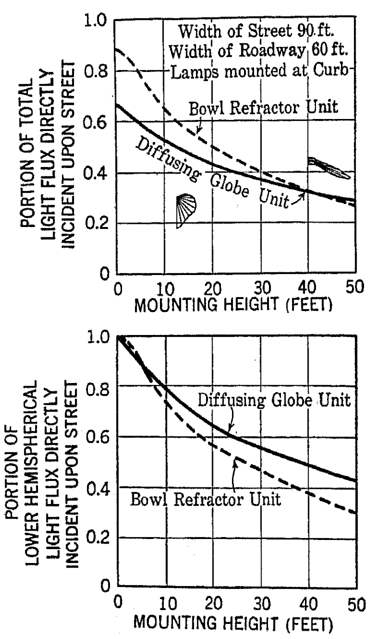

FIG. 3

With increased mounting heights, the proportion of light flux delivered directly to the street surface becomes smaller.

the bowl refractor delivering a smaller proportion of flux from all practicable mounting heights.

This consideration apparently has entered into the design of two-way and four-way refractors, described later, which are intended to deliver larger proportions of the light along the streets by utilizing the light which otherwise in some installations would be wasted or used to poor advantage in other directions.

Characteristics of Equipment and Illumination. The avoidance of undue glare is important to street lighting effectiveness. Glare depends largely upon the brightness and candlepower of the lighting unit within the field of vision and is modified by the brightness and extent of other illuminated areas likewise within the field of vision. It is modified also as the source is removed from the center of the field of vision.

The relationship between candlepower and brightness of source, and the relationship of these to location and background in street lighting are not well understood. The writer has been unable to accept the indications of his own and other attempts to measure and evaluate such effects. In none of them has the subjective element been eliminated or brought under adequate control. No way has been found to reproduce for test purposes the conditions of attention which obtain in the ordinary use of the street by drivers and pedestrians, nor have means been found to evaluate the feeling of satisfaction and contentment with one lighting system which may not be experienced with another lighting system, when the latter cannot be said to produce serious glare which can be measured in visual tests, but is still unsatisfactory as to brightness or candlepower in the direction of the eye. Considerable pioneer work has been done along these lines and street lighting systems have been designed with reference to the results of such investigations. The writer's experience in the conduct of investigations in this field and his observation of designs, based upon such investigations, lead him to feel that the answer has not been found by such means.

The problem resolves itself in the main, into a means of delivering sufficient light upon a street without producing serious glare. The usual means employed are to mount illuminants at a moderate height and employ sufficiently large globes to keep the brightness below the point which produces serious glare, or else to mount the lamps high with a view to removing them from the center of the field of vision, employing some means of directing the light downward upon the street.

An interesting comparison of these two methods has been afforded through demonstration installations in Columbus, Ohio during the spring of 1924. These installations have covered too short a length of street to make the demonstrations entirely convincing, but the contrast between the two systems was displayed in so striking a fashion as to occasion surprise that, after years of effort along the lines of improvement in street lighting, leaders in the art could entertain such diverse views as to means of accomplishing the desired ends.

One more comment on the desirability of uniformity in street illumination should be made. When lamps of moderate or large size are mounted at relatively large spacing intervals, the only way to approach uniformity of lighting is to redirect the light along the street with a large excess at angles slightly below the horizontal, in order to deliver enough light midway between lamps to render the illumination in such areas comparable with that nearer the lamps. This can be accomplished only at the expense of such high candlepower and brightness at angles slightly below the horizontal as to occasion serious glare at any practicable mounting heights. The superiority of substantially uniform illumination along the street over a moderate

diversity, resulting from more natural light distribution characteristics, is usually insufficient to compensate for the attendant condition of glare.

Uniform illumination of moonlight is excellent. But uniform illumination achieved by many small illuminants staggered along both curbs or by means of candlepower distribution curves, having an excess just below the horizontal, imposes damaging visual handicaps and, in the writer's opinion, is not be to desired. Observation and experience indicate that in street lighting moderation is generally desirable. Moderate size lamps, equipped so as to modify light distribution somewhat in the direction of uniformity of illumination, moderately bright, moderately spaced and at moderate heights, lend themselves to successful street illumination in the generality of cases. Departures toward either extreme in any particular may be justifiable and may be desirable, but in general, moderation is the best rule of practise.

It appears to be indicated that in proportion as little light is available for streets, it is important to employ equipment which is designed to direct it along the street to the exclusion of the side of the street. Conversely, when ample light is available, unnatural and dissymmetrical light distributions become unnecessary and buildings along the street may be lighted as well as the street surface with advantage to visibility conditions and to the appearance of the street.

In the interesting and useful demonstration of street lighting systems made available last winter, through the cooperation of the National Lamp Works of the General Electric Company, The Cleveland Electric Illuminating Company and the Cleveland Municipal Department of Light and Heat, comparison was made between lamps of a given size, spaced at equal intervals but mounted at respectively 16½, 21 and 26 ft. above the street level, the highest mounting being out over the street and the lowest mounting being over the curb. The lowest mounting resulted in the greatest amount of light being delivered upon the street with some appreciable glare. The highest mounting resulted in a smaller quantity of light being delivered upon the street and less glare. The writer's independent judgment favored the medium mounting height as producing the largest effectiveness under the particular conditions which prevailed, and he was advised subsequently that this judgment tallied with that of most observers. This verdict was applicable, of course, only to the particular conditions of these trial installations. But the incident illustrates the fact that the most desirable mounting height in any case can best be determined by trial in the street.

Some years ago in New York City, in order to ascertain the best location and arrangement of lamps for lighting a boulevard, lamp posts were mounted in rock ballasted barrels which were shifted about to secure best practicable locations. The result was excellent utilization of the light from the lamps. Some such method of experiment in the street is to be recommended wherever it is practicable to apply it.

Illuminants

Rating Accuracy of Tungsten Filament Lamps. Like all illuminants which are employed in street lighting, tungsten filament lamps differ individually in light output and efficiency when new, and vary somewhat as they are operated. It is not possible to present any tests of rating or of life performance which may be regarded as representative of all tungsten lamp products or of any one product at all times. In Figs. 4 and 5, however, there are shown ratings and life performance data of Mazda lamps of the street series and multiple types which are perhaps as nearly representative as any that might be chosen.

The tests of rating represented in Fig. 4 in the form of target diagrams bring together samples of various Mazda products selected over a number of months.

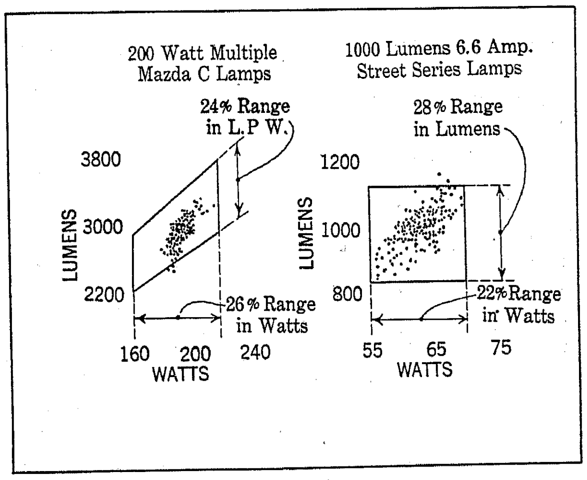

Fig. 4
Rating accuracy of incandescent lamps for street lighting.

It is to be expected that a similar number of lamps of some one product produced at one time would display somewhat more consistency than is here shown, but such a group would be less representative than the group chosen. The trapezoid and the rectangle represent the tolerances of standard lamp specifications, and each dot indicates by its location on the diagram the lumens and watts of a lamp. In the sample ratings illustrated, a range of ± 14 per cent in lumens for series lamps and of ± 12 per cent in lumens per watt for multiple lamps is indicated for new lamps by the standard specification tolerances.

It is understood that there are in existence some street lighting specifications which call for closer conformity to rated lumens on the part of individual lamps than has been attained by the most progressive lamp manufacturers. Such provisions, if complied with, occasion undue expense in the selection of lamps and accomplish little, if anything, of advantage to the public. If not complied with, they introduce possi-

bilities of trouble which are undesirable from every point of view. It is submitted that street lighting specifications ought not to prescribe closer adherence to rating on the part of the illuminants than best practise permits, unless it is understood that such closer adherence involves increased cost which must be met at public expense.

Laboratory performance throughout life with respect to light output and efficiency is indicated in Fig. 5 for samples of series and multiple Mazda lamps of the type illustrated.[1] These performance characteristics are perhaps as representative as any that might be chosen. The performance characteristics of other sizes, types and makes of lamps may be materially different from those illustrated. The better lumen maintenance of the series lamps as compared with the multiple lamps is the natural result of the increased watts expended in the lamp as the filament resistance increases throughout life. The lumen maintenance of the multiple lamp, though not so good as that of the

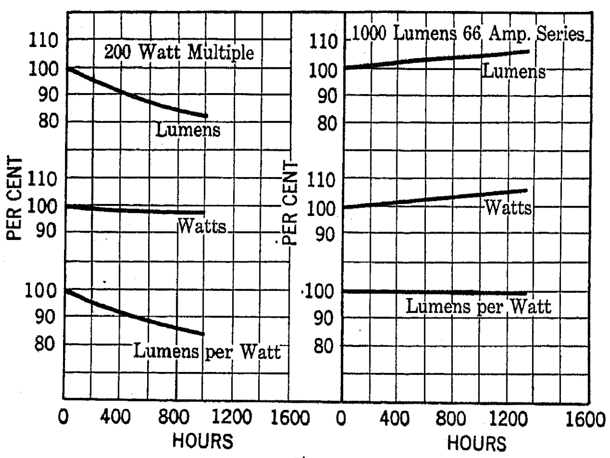

FIG. 5

Life performance of certain sizes of incandescent lamps for street lighting.

series lamp, is excellent as compared with earlier types of incandescent lamps.

These changes in light output throughout the lives of lamps do not take into account losses due to dust or discoloration of reflectors or globes.

No statistics are available to the writer to show the change in light output of modern magnetite arc lamps throughout the period of electrode life and between cleaning and trim periods.

Multiple Lamps. With the development of a successful system of remote control[2] and of reliable time switches, the multiple incandescent lamp operated from commercial service mains has come into much greater favor than it formerly enjoyed for street lighting purposes.

1. Lamp test statistics are available through the courtesy of the Lamp Committee of the Association of Edison Illuminating Companies.

2. Remote Control of Multiple Street Lighting—W. T. Dempsey, JOURNAL A. I. E. E., October 1923, page 1106.

Arc Lamps. The arc lamp has now been very largely superseded by the incandescent lamp for utilitarian lighting of secondary streets. In its more powerful form, however, employed with diffusing globes, the magnetite lamp is a very lively factor in lighting the most important and distinctive streets of many cities.

PRESENT STATUS OF STREET LIGHTING IN THIS COUNTRY

Municipal Expenditures. Cities ordinarily spend about 75 cents per capita per annum for street lighting. It seems to be the general feeling of those, who after study of the subject have expressed themselves in the technical literature, that about twice this amount should be spent on the average for street lighting, in order to provide illumination which is adequate to meet the requirements of modern congestion of high speed traffic. Differences in local conditions may double or halve the sum required in a given community to afford street lighting of the desired effectiveness.

Unity of Design. Street lighting systems in most American cities suffer from lack of uniformity and style, having been designed at different times by people whose ideas as to the requirements differed. For the most part, systems have grown from small beginnings, according to the seeming expediency of the hour and without any coherent principle as a basis upon which to develop a homogeneous system expressive of the city's character and reflecting favorably upon its management. Few, indeed, of the larger cities have attempted, as St. Louis is now doing, to introduce an entirely new street lighting system designed with unity of purpose to serve present needs and to be capable of expansion for future requirements without sacrifice of unity of design.

Lamp Posts. In lamp mountings, independent posts for one or more lamps or brackets on trolley line poles are in general use. The cluster of five lamps, more or less, in diffusing globes mounted low is passé. The columnar post for one lamp, the bracket post for lantern type of housing, and the modern multiple lamp post are illustrated by a variety of samples in Figs. 6 and 7. The inclusion of transformers in the bases of lamp posts has had an undesirable influence in making the diameter larger than is consonant with the purpose of the post, often robbing it of grace. In this respect, the posts for multiple lamps have possessed an advantage. Recent transformer types of smaller dimensions have mitigated this difficulty.

To a considerable extent the leading manufacturers of lamp posts, lamps and lamp equipment have influenced the type of street lighting installations, particularly in the smaller cities. For the most part, new street lighting systems which have been installed have been superior to existing systems. In street lighting the tendency to passing styles has not exceeded the bounds of moderation because innovations have had to justify themselves in the eyes of the purchasers

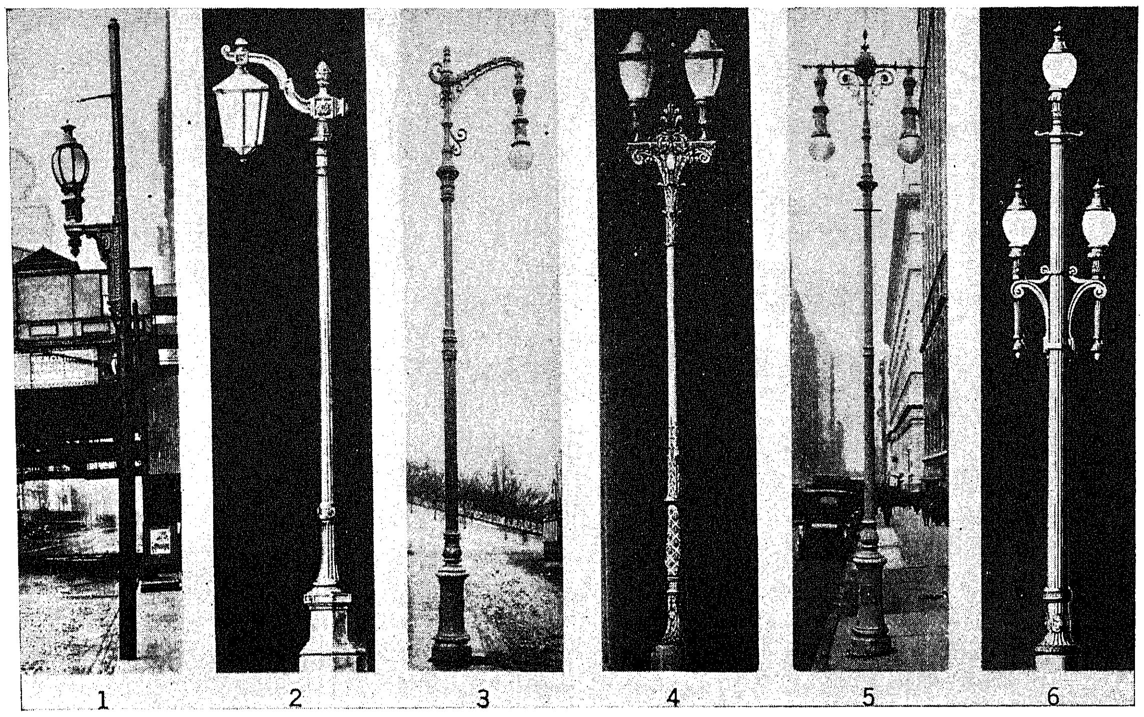

Fig. 6—Modern Street Lamp Posts

1. Chicago bracket on trolley wire post
2. Cleveland bracket
3. New York bracket
4. Saratoga twin lamp post with large and small alternate lamps in each globe
5. New York twin lamp post
6. Salt Lake City triple lamp post

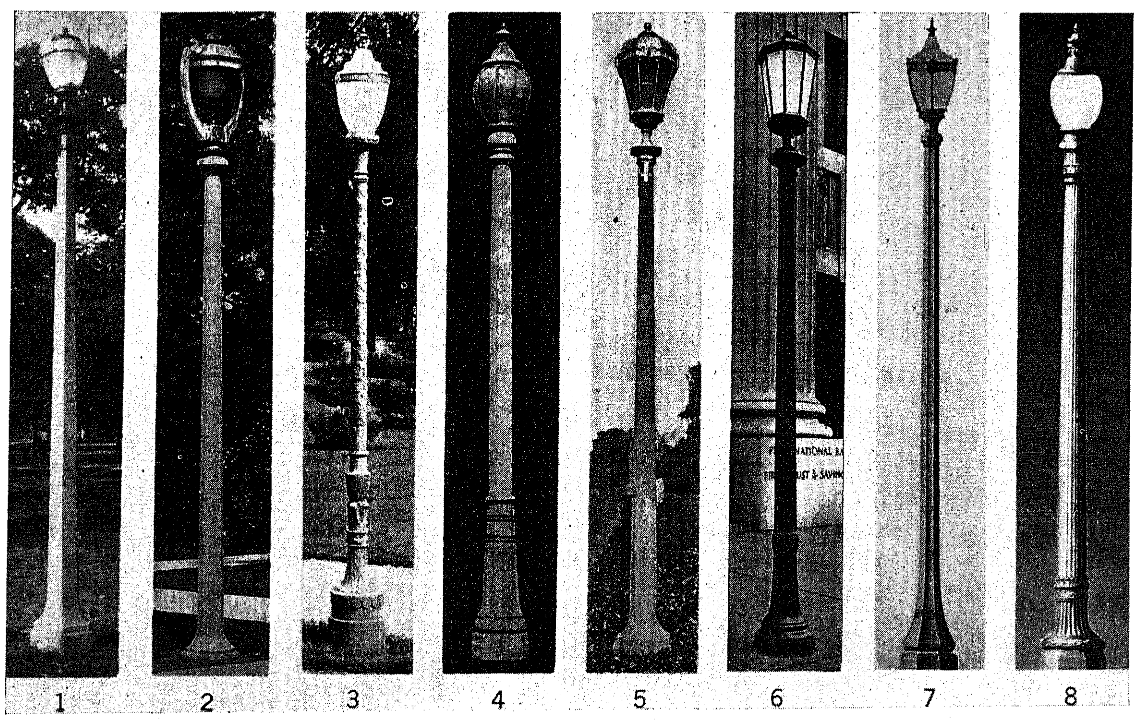

Fig. 7—Designs Exhibiting Modern Tendencies in Street Lamp Posts and Equipment. These Illustrations are not Adjusted to a Common Scale.

1. Prismatic refractor globe
2. Prismatic bowl as used in Milwaukee
3. Diffusing globe as used in Saratoga
4. Diffusing panels enclosing reflector and dome refractor
5. Prismatic glass panels as proposed for St. Louis
6. Stippled glass panels enclosing dome refractor as used in Cleveland
7. Diffusing glass globe enclosing dome or bi-lux refractor
8. Rippled glass globe with or without interior refractor as used with magnetite lamp.

and users who have generally inquired rather carefully into their merits before committing themselves to the large expenditures involved.

ATMOSPHERE AND TRADITION IN STREET LIGHTING

Economy dictates, at least for smaller installations, the choice of standard lamp posts and equipments. Such choice, however, loses distinctive qualities for the installation. There are some communities which are perhaps so characterless and colorless as to possess no traditions and to afford to the observer no distinctive features. When, however, a community is of distinctive character, it would seem that lamp post and lamp equipment design afford an excellent opportunity for the exhibition of atmosphere and traditions which ought to be availed of where practicable. Two illustrative installations occur in this connection. Some of the street lighting of Riverside, California was until recently provided from concrete posts the heads of which represented the archway of a Mission belfry, while a bell-shaped lamp shade represented the Mission Bell, the whole embodying the romantic Mission traditions of the neighborhood. Another instance is offered by the historic lighting of Independence Square, Philadelphia, in which the modern lamp posts and lanterns are designed to preserve the traditions through faithful reproduction of the oil lamp lighting of the Square which is said to have been designed by Benjamin Franklin. In neither case did the lighting achieve high effectiveness from the point of view of utilitarian street lighting. Both designs, however, are notable as worthy attempts at preservation of traditions in street lighting design and as illustrations of attempts to obtain a logically distinctive lighting system.

Lamp Equipments. Perhaps no phase of the street lighting problem is mooted so generally as is the question of equipment of the lamps. Preference for urban street lighting ranges from a diffusing ball, largely lacking in directive qualities, to optical equipments which afford maximum opportunity for directing the light as desired. Among intermediate equipments some combination of a slightly diffusing outer globe of large area with an internal refractor appears to meet with rather general approval. These combine a measure of control of the light with a reduction in brightness, achieving a satisfactory degree of effectiveness for many street lighting purposes. Lighting equipments of these types are illustrated in Fig. 8.

Notable advances have been made in recent years in the adaptation of prismatic glassware to street lighting. The prismatic refractor as a complete globe for street lighting units has proved very effective in redirecting light as desired. In its original form, limited apparently by cost considerations to relatively small size, its brightness was rather high and, in particular when used with the larger lamps, was too high for many purposes. A more recent development of the refractor in a larger size (16¾ in. upright inverted) has mitigated this difficulty.

More recently the prismatic refractor has found an interesting field of usefulness as an internal directing element of a street lighting fixture employed with a larger outer globe, usually more or less diffusive in character. Such directing refractors employ horizontal prisms to direct the light upward or downward, or vertical prisms to direct the light along the street, diverting it from the side of the street. Some illustrations of refractors employed in this way appear in Fig. 8.

Another recent development of the refractor, which is designed to form a complete lighting equipment,

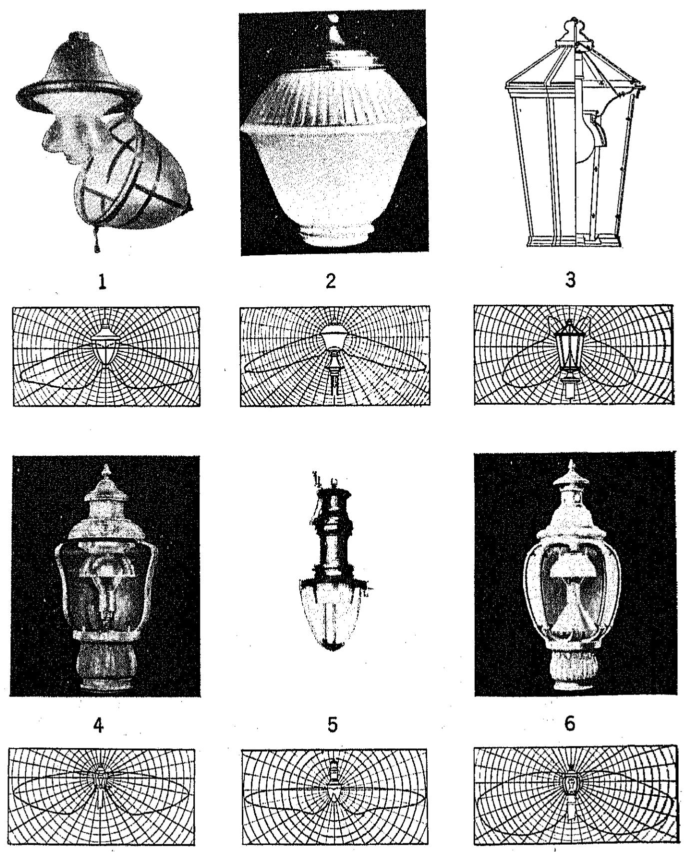

FIG. 8—PRISMATIC GLASS FOR LIGHT CONTROL

1. Equipment used at East Cleveland
2. Refractor post top
3. Lantern as used at Cleveland
4. Stippled glass globe with dome refractor
5. Magnetite lamp with inner refractor
6. Panelled stippled glass globe with dome refractor and reflector.

employs a system of prisms, designed to direct much of the light in two directions up and down the street, or in four directions at rectangular street intersections. The horizontal distribution of candlepower for such refractors shows either two or four pronounced lobes. Where accurate setting of such equipment can be assured, and where the high brightness is not objectionable, these two and four-way refractors may be found useful. They are not intended ordinarily to be applied chiefly in "ornamental" lighting systems; *i. e.*, with underground distribution and ornate posts and housings.

Still another application of prismatic glassware is in

the form of panels of a lantern type of street units as illustrated in Fig. 7.

Highway Lighting Units

A feature of modern development in street lighting is the lighting of highways. This is very desirable as a contribution toward the construction of pole lines in rural communities and toward the solution of the difficult automobile headlighting problem, in addition to the direct advantages of street lighting which apply to highways as well as to city streets. In the lighting of highways spacing intervals must of necessity be great and the design of equipment must be such as to utilize to the utmost the available light flux for delivery along a narrow strip of highway. A concentration of light up and down the road with a generous flux of light on the street below the lamp is indicated. Although some attempts in this direction were made at an earlier date, highway units were not put forward actively until the availability of the concentrated filament, gas-filled tungsten lamp made possible higher effectiveness from such devices than was formerly attainable. Three designs, with practically the same objects in view, are illustrated in Fig. 9. All accomplish more or less well the purpose of delivering the light along the highway. None is conspicuous for grace or beauty. In highway lighting units, "handsome is as handsome does."

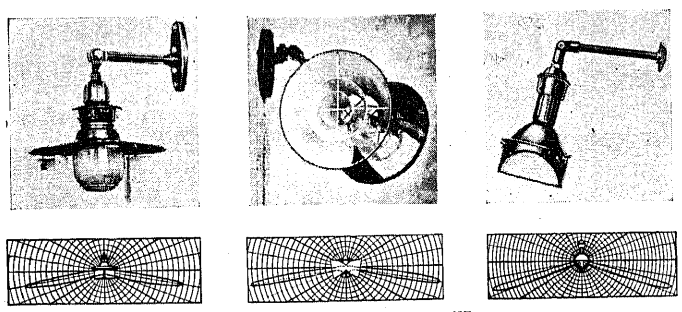

Fig. 9—Highway Lighting Units

Standards of Illumination for Various Classes of Streets

Several writers[3] on street lighting recently have classified streets and suggested ranges of illumination suitable for each. In Table I an attempt is made to combine these statements to indicate modern ideas of suitable levels of illumination. Obviously, in consolidating them certain liberties have had to be taken. The table, however, is probably fairly representative of their views as expressed. For each class of street, the range of lower limits and the range of upper limits have been shown for each particular of street lighting which has been covered by these writers.

In relating lamp lumens to illumination on the street surface, it is necessary to take into account the loss of light in the lamp equipment, the width of the street, the location and height of the lighting unit and the reflection of light from buildings.

Advancing Standards of Street Illumination

Increase in high-speed vehicular traffic and growing appreciation of the importance and value of good street lighting have brought about a rapid advance in levels of illumination advocated for street lighting. A good illustration is afforded by the comparison in Table II between illumination intensities regarded as typical of

3. C. W. Koiner, Chairman Commission on Street Lighting, Society for Municipal Improvements, *Elec. World*, Dec. 23, 1922.
Engineer Commissioner—District of Columbia, Letter, March 25, 1924.
A. F. Dickerson, General Electric Company, *Elec. World*, July 22, 1922.
R. E. Greiner, Edison Lamp Works, *Bulletin L. D.* 144.
E. A. Anderson, National Lamp Works, *Bulletin No. 46*.
N. E. L. A. Street & Highway Lighting Division, 1922.

TABLE I
RECOMMENDED STANDARDS FOR STREET LIGHTING

	Class of Street	Cp. per Unit	Mounting Height (Feet)	Spacing (Feet)	Lamp Lumens per Linear Foot of Street
Range of lower limits	Primary Business	600—1000	14—18	60—100	100—330
" "upper "		2500—5000	25	150	250—1000
Range of lower limits	Secondary Business	600—1000	14—15	80—100	50—150
" "upper "		1000—2500	16—25	125	160—500
Range of lower limits	Outlying Business	250	12	60— 80	20—125
" "upper "		600	16	80—125	100—200
Range of lower limits	Wholesale and manufacturing district	250— 400	20	125—150	20— 50
" "upper "		1000—1500	25—30	250—300	50—100
Range of lower limits	Thoroughfares	250— 400	15—20	75—150	10—100
" "upper "		600—1500	25—30	200—300	30—125
Range of lower limits	Residential	100— 250	10—14	100—150	6— 40
" "upper "		600	20—25	250—350	8— 50
Range of lower limits	Boulevards	250— 400	12—15	100—125	10— 60
" "upper "		600—1000	20—25	200—300	30— 80
Range of lower limits	Parks	250	12—14	100—125	10— 40
" "upper "		600—1000	20—25	200—300	30— 50
Range of lower limits	Outlying districts, alleys and side streets	100	14—15	100—200	2.5— 10
" "upper "		250— 600	18—20	250—400	5— 50
Range of lower limits	Highways	250	25—30	250—300	4— 8
" "upper "		400	35	400—600	8— 12.5

TABLE II

ADVANCING IDEAS OF SUITABLE LEVELS OF STREET ILLUMINATION

1916 Standards			1924 St. Louis Proposal	
Class of Streets	Avg. Horiz. Illumination Intensity	Desirable Characteristic	Zone	Average Ft.-candles
Important avenues and heavy traffic streets	0.5-1.0 ft. c.	Ample light on building	Downtown retail	1.68 ft. c.
			Intermediate Zone 1	1.20
Secondary business streets	0.1-0.2	Ample light on building	Intermediate Zone 2	0.7
			Intermediate Zone 3	0.47
			Major thoroughfares outside central zones	0.30
City residence streets	0.05-0.1	Subdued light on bldg. fronts	Residential heavy traffic	0.16
Suburban highways	0.02-0.04	Max. light on roadway		
Suburban residence streets	0.005-0.02	Very subdued light on building fronts	Residential outlying	0.09

practise in 1916[4], and levels of illumination intensity proposed for the new lighting of the streets of St. Louis.[5] The St. Louis proposed illumination levels are considerably in advance of the standards advocated in Table I. The writer does not know how representative they are of modern ideas, but if they are no more than the expression of convictions reached by municipal authorities after investigation, the contrast which they form with the levels regarded as typical in 1916 is a striking commentary on the advance of street lighting theory if not of practise.

It is not clear at this time how far this advance in standards of street lighting will be carried, but it is clear that large advances are being made with results that are beneficial to the public, the utility company and the manufacturer of equipment.

The author desires to thank representatives of the General Electric Company, the Holophane Glass Company and the Westinghouse Electric and Manufacturing Company for street lighting information supplied upon request, and to express appreciation to several others who have assisted in the preparation of this paper.

4. Lighting of Streets—P. S. Millar, I. E. S.—U. of P. Lecture Course, 1916.

5. Ralf Toensfeldt, Department of Public Utilities, St. Louis, *Electrical World*, Nov. 24, 1923, page 1065.

Discussion

L. A. S. Wood: Mr. Millar has preached the gospel of moderation, and, while I subscribe to that doctrine, it is rather difficult to determine just where moderation ends and intemperance begins.

I hope that Mr. Millar's remarks regarding low intensities with diversified illumination will not be taken as his recommendations for modern street-lighting practise. In another part of his paper he refers to the desirability of high intensities with uniform illumination, provided such can be obtained without undue glare, and this, in my opinion, is the best kind of illumination suitable for modern traffic conditions.

The tables on the last page showing the tendency of modern street-lighting practise toward higher intensities are very interesting, and it is gratifying to note that the standards set by the City of St. Louis are so very much higher now than those prevalent in 1916. I feel, however, that there is still a very wide margin for increased intensities of illumination before we get out of the range of moderation.

Mr. Millar refers to the control of light by prismatic refractors and the possibilities of directing the light flux in such a manner as to build up the illumination between units to higher intensities than could be obtained without such control. In this connection, I would like to refer to a prismatic refractor which was presented at the Illuminating Engineering Society last fall. This refractor bends part of the light flux from the direction of the property line and adds it to the light flux in the directions "up and down," the street producing unsymmetrical or what is sometimes called "asymmetrical distribution." I believe that the tendency from now on will be the direction of asymmetrical distribution as compared to symmetrical distribution for street illumination.

F. C. Caldwell: I would particularly emphasize the difficulty experienced in laying down practicable specifications for good street lighting. This is, of course, especially true where, with high-intensity lighting, the elements of aesthetic effect and of making the street a pleasant place in which to walk or ride are important factors. Probably an adequate illumination of the building fronts as well as of the pavement should be included in such specifications and perhaps information on the foot-candle intensities needed for this purpose may become available. Indeed, I hope we may make a small beginning along this line in connection with the Columbus demonstrations.

Apropos of this type of lighting, I am not quite so sure as Mr. Millar seems to be, about the place of the refractor. With him I would question whether any refractor now available is suited to this case, but might not one be made that would give just the illumination desired. While a rather uneven or spotted illumination on the pavement is not objectionable, should not the illumination on the buildings be as uniform as practicable. Would it not be in order then to design a refractor which would redirect that part of the light, perhaps 30 to 40 per cent, which now falls upon the adjacent buildings, on the same side of the street. This makes a bright spot opposite the light and is otherwise ineffectively used because of the small angle at which it strikes the walls. Again there is all the flux that escapes to the sky from a diffusing globe.

If without attempting to increase the illumination on the pavement or to make it more uniform, most of the light in these two zones, that is the flux which does not fall upon the sidewalks, the pavement or the opposite buildings, could be redirected by a refractor and thrown across the street onto the building fronts, a refractor might well justify itself even in high-intensity and ornamental lighting.

It is almost useless however to throw light onto dark-faced buildings. If the illumination of the walls as well as the floor of the street is accepted as important and the city is to spend money to accomplish it, an organized effort, probably by educa-

tion rather than ordinance, should be made to get owners to use light-colored building material and to keep it clean. Once such a practise becomes general, a builder would hesitate to place his building at the disadvantage of being invisible at night.

From another point of view, assymmetric refractors may become important in the high intensity of business streets. The movement to eliminate poles will, if carried to its logical conclusion, result in the support of the trolley wires from the fronts of the buildings and the mounting of street lights on large brackets. A downtown street without an obstruction from end to end, is a pleasant thing to think upon. Here refractors would be almost necessary.

At another point the public should take an interest in conditions accessory to street lighting. Hitherto illumination has had to take second place to the trees on the street. With changing conditions it appears to be time for this order to be reversed. I know a street where the the trees are kept, for the most part, back to the line of the house-fronts. The effect of openess and breadth is very pleasing. Perhaps however, such an arrangement of the trees as a general policy would be too much to hope for. It is not at all unreasonable however to locate the trees on the lot line instead of at the curb and this is a requirement well within the rights of the public, for safety, if for no other reason. Again, regulations for the trimming up of trees may well be made and enforced, on the same grounds.

The testing of the Columbus demonstration will be completed next week. While for the most part the results are not yet worked out, a few points seem to have been rather clearly established. One is that four 15,000-lumen lights on a 500-ft. block give more satisfactory results than ten 6000-lumen lamps, which is the spacing hitherto used. Again our experience indicates that 150 lumens per linear foot gives very satisfactory lighting for ordinary business streets and that 300 lumens makes fine "whiteway" illumination. Also 25 to 50 lumens per foot covers very nicely the usual range of residence streets and boulevards.

The important questions of incandescent versus magnetite-arc lamps and of diffusing globes vs. refractors, symmetrical and unsymmetrical, have not had final consideration, but we hope to throw some light upon them before we are through.

Mr. Millar has spoken of character in standards. There are few expenditures involving so much money that we make with more inadequate judgment than in the purchase of lamp standards. Their selection is generally left to someone who, in other matters of art, would certainly not claim to be a qualified critic. Probably it is usually the salesman, rather than the standard which decides the matter.

The increasing emphasis which is being put upon the aesthetic aspects of city planning indicates the reference of such a selection to the best practicable commission of architects and artists. In the case of large orders, a standard specially designed by a competent artist should be used and if it can embody some feature characteristic of the city so much the better.

Finally, let me advise anyone who would conduct an extended series of tests of street lighting to arrange to close up his other affairs for the time being. Our efforts in Columbus have convinced me that the most practicable place to test street illumination is anywhere else but on the street.

G. H. Stickney: We all realize that the new thing, which is creating the need for better street lighting and more light, is the automobile. Ten or fifteen years ago the principal traffic on city streets was moving about six miles per hour. Now it has attained a speed of twenty or more miles per hour.

It is, therefore, necessary to see more clearly and further if accidents are to be avoided. Street lighting, in general, has not adapted itself to this demand which the automobile has created, and greater progress in this direction is essential to the preservation of life, limb, and property.

The question of symmetrical and asymmetrical distribution of light from street-lighting units is a very live one at the present time. It is not merely a comparison of two systems, since there are many possible characteristics of asymmetrical distributions. There are several schools of thought on the subject, and more experience is necessary before the best conclusions can be expected.

Past experience has indicated, that when such new possibilities are opened up there is a tendency to go to extremes. Having made such mistakes, I am inclined to expect the best practise to be found nearer a medial point rather than at an extreme. I believe asymmetrical distribution is here to stay, but not to the exclusion of symmetrical distribution. Nor do I think it probable that the best forms of asymmetrical distribution have yet been secured.

We should expect to find ways of utilizing light more effectively, but this one feature should not be over emphasized to the exclusion of other considerations.

There seems to be a natural tendency to think that it is desirable to put as large a portion of the light on the pavement and provide uniform illumination over this entire area. However, practically all of the tests and demonstrations have indicated that when the amount of light is limited, as is usual in street lighting, better visibility is secured where sections near the unit receive more illumination than the more distant areas, provided, of course, this is not carried too far. Such illumination often appears uniform to the eye, and seems to give better results than a similar amount of light so distributed as to give measured uniformity.

This is undoubtedly one of the factors which is bringing about the tendency toward the use of higher power units, even with incandescent lamps, in spite of availability of small sizes. It is, of course, promoted also by the superior economy of the larger lamps.

D. K. Blake (by letter): Mr. Millar, in his paper (page 994) mentions three important factors in the development of street-lighting systems which should be of considerable interest to the distribution engineers. They are first, the development of remote-control systems; second, the practise of connecting multiple incandescent lamps to the commercial service mains and, third, the increase in municipal expenditures per capita for street lighting. Because of the expenditures involved and because of the relation of the street-lighting system to the distribution system these factors are additional reasons for an intensive and extensive economic study of the distribution system as a whole.

It seems to me that the highway-lighting units mentioned on page 997 should be of considerable help in solving the problem of rural electrification. Because the highway-lighting unit increases the rural load and consequently reduces the cost of central station service to the farmer, its use should be justified on the highways where otherwise it would not be advisable to spend the money for the installation of these units.

The Flashing Characteristics of Series and Compound-Wound Motors

BY RALPH E. FERRIS
Associate, A. I. E. E.
Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

Review of the Subject.—*This paper, to a certain extent, is in the nature of a log covering several years' test on a line of dynamotors. The flashing characteristics of series and compound-wound motors are discussed with special reference to these tests. The principles derived from the study of these machines have, however, been applied successfully to other motors. Machines of this type are apt to have a critical flash point, depending upon the constants of the electric and magnetic circuits. The oscillograph has been used to determine the best relation of shunt to series field for a given difference of speed between full load and no load. Methods of improving the flashing characteristics of a given machine are discussed. In some cases it is shown that a damper on a series-wound motor gives better flashing characteristics. An attempt to explain the nature of a flashover on a d-c. machine is made.*

THE writer has had considerable doubt as to the best method of presenting the material contained in the following paper. First, it could be presented in a more or less abstract way giving examples to support the theoretical conclusions; second, it could be given purely as a history of a series of tests; or third, it could be conveyed as a combination of the first and second methods.

The third method was finally decided upon because it not only contains to a certain extent a narrative interest, but at the same time gives a vehicle for presenting conclusions which in the end are borne out at least in part by the tests recorded.

Some classes of motor service require the machine running continuously under a variable load from low or no load up to maximum. An occasional second requirement is that the motor be capable of withstanding full line voltage applied at any time from standstill up to normal speed. The first requirement necessitates a shunt field, while the second calls for a series field, thus giving for the combination a compound-wound machine.

The flashing characteristics of a machine of this type under the application of full or over-voltage are more or less varied and interesting. It is the purpose of this paper to discuss these, in connection with a particular example, this example being a line of dynamotors and dynamotor compressors for use on 1200 or 1500-volt railway service. A detailed description of this type of apparatus may be found in the *Electric Journal* for October, 1913.

Briefly, the dynamotor consists of two separate and distinct armature windings on the same core, each connected to its individual commutator. The two windings have a common field and therefore the generated e. m. f. per turn is the same for each winding at any given field excitation; or the voltage at each commutator is equal to some factor times the number of turns in series in the winding.

In reality, the dynamotor is a d-c. voltage transformer. If, for example, it is desired to transform from 1200 to 600 volts, the armature windings may have the same number of turns and the two connected in series externally. Current at 600 volts may then be obtained by tapping off at the middle point between the two windings. With the introduction of high-voltage direct current for railway work, when the locomotive or car is required to operate either on 1200 or 600 volts, the dynamotor presented itself as a means of generating 600 volts direct current, for lights, control, compressor and blower motors while on the 1200-volt section. This connection is shown in Fig. 1.

The dynamotor-compressor combines the dynamotor, the compressor motor, and blower motor all in one, thus materially reducing the weight and also the space required for the outfit. This is accomplished by using the dynamotor as a motor and driving the compressor thereby through a set of gears and a multiple-disk clutch.

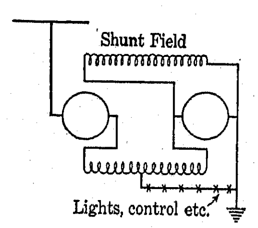

FIG. 1

If a blower is required on the car or locomotive, an extended shaft is used and the blower mounted thereon.

While the paper deals mainly with the dynamotor-compressor as an example, the fundamentals are the same for all series or compound machines and the experience obtained and principles involved have been applied to a number of other type machines with success.

The first machine built was for use on 1200 volts reducing down to 600 volts. This machine was run for several months on a very severe cycle, and appeared to be satisfactory in every way, except the capacity was somewhat low for the service to which it was to be applied. This machine will be referred to as No. 1. The next machine designed was for a commercial order, and was for use on 1500 volts reducing down to 750 volts, and will be referred to as No. 2. A large number

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of flash tests was made on No. 2 dynamotor with the compressor both on and off. The results of these tests gave evidence that a permanent resistance should be placed in series with the dynamotor. A resistance was used and from all tests made seemed to give satisfactory conditions as regards flashing. Unfortunately, however, the very point where the dynamotor was most susceptible to flashing was missed during the flash test in the factory. After being placed in service it was found that the dynamotor flashed when the trolley came off the overhead wire, or was pulled off, and then put on again after the dynamotor had slowed down to a certain speed. In other words, No. 2 dynamotor had a critical flash point, at approximately 75 per cent of maximum speed running light.

The flashing on No. 2 dynamotor was apparently prevented by the use of a pneumatic starting switch. Arrangements were made for starting with a resistance in series and in approximately 4/5 of a second the pneumatic switch was closed, short-circuiting a portion of the resistance, leaving only a small amount permanently in circuit. As was stated above, this arrangement of permanent resistance, starting resistance, and cutout switch apparently worked O. K. on the No. 2 dynamotor, and also on the No. 3 dynamotor which had been developed for 1200-600-volt service. A fourth dynamotor, which will be called No. 4, was then developed for use on 1200-600 volts, but driving a large size compressor. On both previous machines the greatest tendency to flash was found with the dynamotor running light. On the No. 4 dynamotor, however, the heavy current drawn through the starting resistance with compressor on, so lowered the voltage applied to this machine that when this resistance was short-circuited, it was practically the equivalent of throwing line voltage on the dynamotor, thus causing a flashover.

At this point, the flashovers on the No. 4 machines began to be more thoroughly investigated than on any of the preceding dynamotors. As it is our purpose to cover progressively the trouble encountered on this type of machine, we will, at this point, discuss at some length the flashovers.

There is a number of theories concerning the nature of flashing in general, one of which is a rise in voltage as the direct cause, this theory will be discussed more fully later in connection with the oscillographs. A second theory is as follows: If any sparking occurs at the brushes, the gaps between the successive bars as they leave the brush may be bridged by small arcs. If the voltage between the bars is fairly high from any cause, this arc may be maintained until the other brush is reached. This being the case, the successive bridging arcs may form a chain of arcs from one brush to the other, thus giving practically a short circuit and complete flashover. It would seem therefore, from this theory that poor commutation is an entering wedge for a flashover. As will be shown later in connection with the oscillograph, the flashover on this particular type of machine is probably a combination of rise in voltage and low resistance path.

The No. 1, 2, 3, and 4 dynamotors were wound with a compound field, the connection being made as shown in Fig. 1, the shunt and series field coils are on the same magnetic circuit, in fact, at start they form a series transformer. Assuming the normal direction of current in shunt field to be positive, the heavy rush of current through the series coils gives a negative current in the shunt field. This negative current in the shunt, of course, opposes the series field and gives therefore a weak resultant field at start. The heavy armature current in connection with this weak field gives a highly distorted resultant field and consequent sparking at the brushes. This, in connection with the high maximum volts per bar, in all probability starts the flash. It should be noted in connection with the distorting effect of the armature that the ratio of resultant field ampere turns to armature ampere-turns is not constant, being higher at high current, due to the increased magnetizing current required as the iron becomes saturated.

The foregoing does not, however, explain why the dynamotor will flash at or around some critical speed, considerably below normal, but will not flash at standing start or with a comparatively quick opening and closing of the supply circuit. The following explanation is offered: At standing start the current is excessive and gives heavy sparking under the brushes, but the bridging arcs are not carried around at sufficient speed to give a low resistance path and consequent flashover, also as before stated, the ratio of resultant field ampere-turns to armature ampere-turns is a maximum. In other words, the small arcs bridging the bars, due to sparking at one brush, die out before reaching the other brush and by the time the peripheral speed of the commutator has reached a sufficiently high value to carry the arcs around, the sparking at the brush has ceased. When the dynamotor is thrown on the line while running between 25 and 80 per cent of normal speed, the same condition as regards weak field obtains to a certain extent, and therefore to the same extent the distorted field and sparking at brushes. In this case then, the dynamotor is sparking quite freely and at the same time is rotating at a fairly high speed, thus carrying the bridging arcs around from one brush to the other causing a continuous short circuit between bars and flashover. The flashing would seem to be, therefore, a function of the sparking at the brushes and the speed.

As stated before, the concentric shunt and series field winding form in substance a series transformer, the series winding corresponding to the primary and the shunt short-circuited through the low-tension armature, to the low-tension winding, (See Fig. 1). In any transformer the primary ampere-turn may be considered as made up of 2 parts: one element which supplies the magnetizing and core loss current and another element which supplies the working current. The working

current ampere-turns are always practically equal to the secondary ampere-turns. This being true, for every rush of current in the series coils, there will be a corresponding rush in the shunt coil, modified to a certain extent by the damping effect of the magnetic circuit. Everything else being equal, the higher the resistance of the secondary, the higher will be the voltage required to establish the secondary current, this higher voltage meaning in turn a more quickly established flux. In other words, if a variable resistance is placed in series with the secondary of a series transformer, the total flux may be varied directly by changing the resistance. This means simply, of course, that the magnetizing current of a series transformer changes directly with resistance of secondary. It would seem therefore, that in the case of a compound-wound machine, the higher the volts drop per turn of shunt coil, the stronger will be the resultant field at start. A high-volt drop per turn of shunt coil may be obtained in two ways: first, by a high resistance per turn, as before stated, and second, by a low ratio of shunt to series turns. The second method, would, of course, give a heavy current in the shunt field and consequent high-volt drop per turn, so in final effect the two methods are the same. This higher-volt drop per turn would naturally require a higher rate of change in flux and consequently a more quickly established and heavier resultant field. This point will be treated more fully when the oscillographs taken on these machines are discussed.

We will now return to the No. 4 machine. From a consideration of the foregoing points and also from oscillograph tests, the following investigations were made:—The original resistance used on the No. 4 dynamotor was 15 per cent permanent and 85 per cent starting. As before stated, the dynamotor flashed under start when the starting resistance was cut out with the compressor on, but not when it was disconnected. This indicated that too much resistance was cut out by the starting switch, and tests were made to find the best combination of permanent and starting resistance without at the same time making the permanent resistance unduly high. A large number of tests were made using different combinations of resistance and different time elements on the starting switch. It was finally found that approximately one-third permanent and two-thirds starting resistance with a 4/5 sec. time-element switch gave fairly good results. It was, however, decided to make further tests of a different nature to see if it would be possible to make the dynamotor still more safe as regards flashing.

Flash tests were made with the shunt field excited before the line switch was closed, connections being made as shown in Fig. 2. Switch B was closed first and then switch A was closed. Under these conditions very little, if any, resistance was needed to keep the dynamotor from flashing. This arrangement was not considered practicable, however, for various reasons, the chief of which was the automatic closing of switch A.

The next arrangement tried was that shown in Fig. 3. In this case A is the starting resistance, which after a short time is short-circuited by switch B which is operated by compressed air, the air valve being operated by magnet C, which is connected across the permanent resistance D. The shunt field F was connected in parallel with resistance E in series. This would, in effect, increase the volt drop per turn and so give stronger field at start. The method shown in Fig. 3 actually worked somewhat better than the original but was still thought to be too unstable for use on this machine. This method will be referred to later in

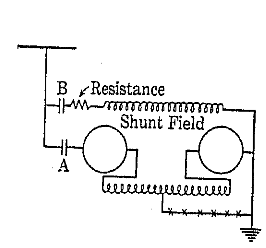

Fig. 2

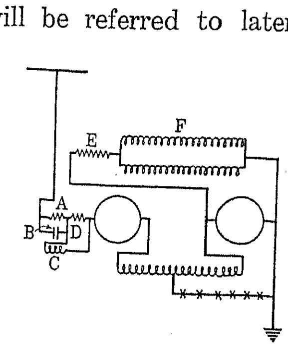

Fig. 3

connection with a machine designed for 1500-600-volt operation.

The next arrangement tried was that shown in Fig. 4. In this arrangement A is a permanent resistance, D is a switch, which not only closes the main circuit but short circuits the resistance B in series with the shunt field. The coil C was used for operating air valves to switch D. At start, therefore, the shunt field is excited before the switch D is closed. The time element of the switch D being about 4/5 sec. The above arrangement failed to work for the following reasons:—When the main circuit was opened to the dynamotor, the switch D failed to operate until the generator voltage on the machine had dropped so low that coil C would not operate the air valves.

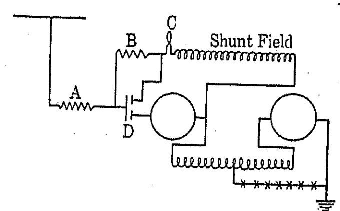

Fig. 4

After trying various other schemes in the hope to eliminate flashing and still hold the same field design, it was considered best to eliminate the shunt field on the No. 4 dynamotor entirely. On the No. 4 machine it was possible to use a straight-series winding as the equipment called for a fan to be mounted on an extended shaft, the fan giving sufficient load to hold the speed down when the dynamotor was not driving the compressor.

As the experience with this type of machine is being

given in a more or less chronological order, it will be necessary at this point to discuss a few of the oscillographs taken. Before going to this part of the subject, however, it should be stated that the flash or buck always occurred on the low-voltage armature. This is due to the fact that the winding for the low-voltage side lies in the bottom of the slot and so has a very much higher reactance voltage.

current rose to between 30 and 40 amperes, at which point the dynamotor bucked and the main current rose further to approximately 50 amperes. The oscillograph also shows that the shunt field reversed and rose to approximately 1.7 amperes. During this period, the voltage across the low-tension armature remained fairly constant up to bucking point, where it starts to fall off, due to the excess current pulled through the

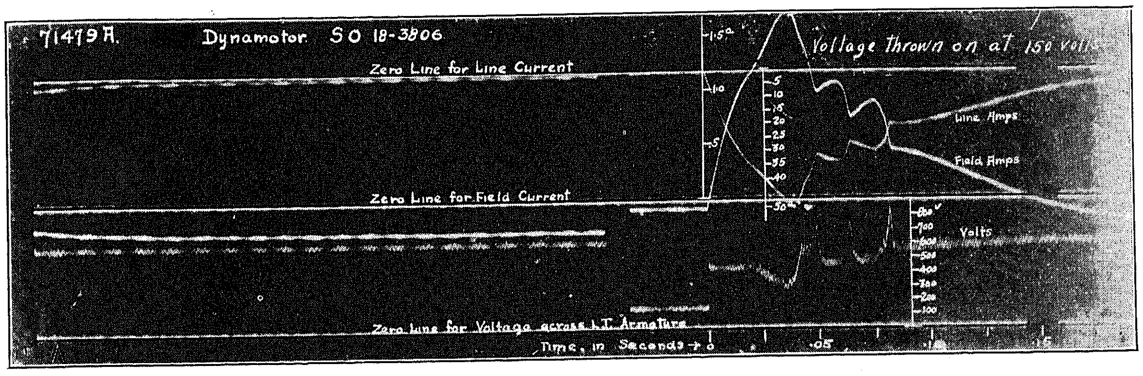

Fig. 5

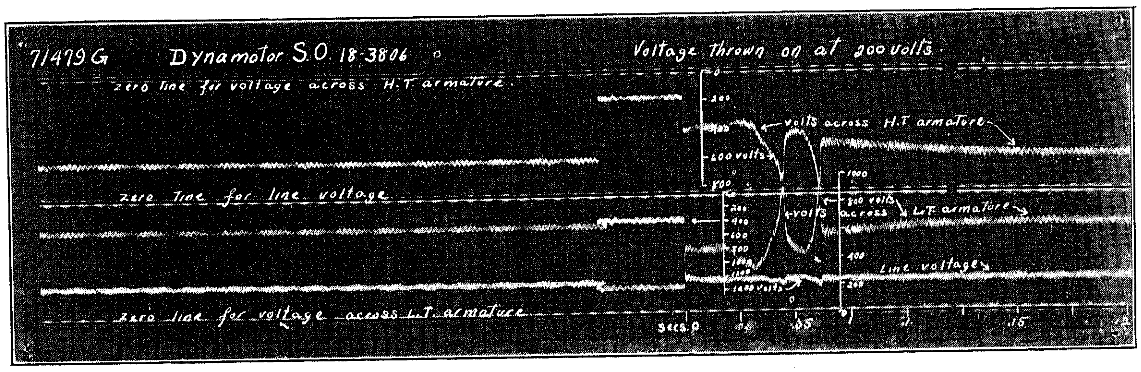

Fig. 6

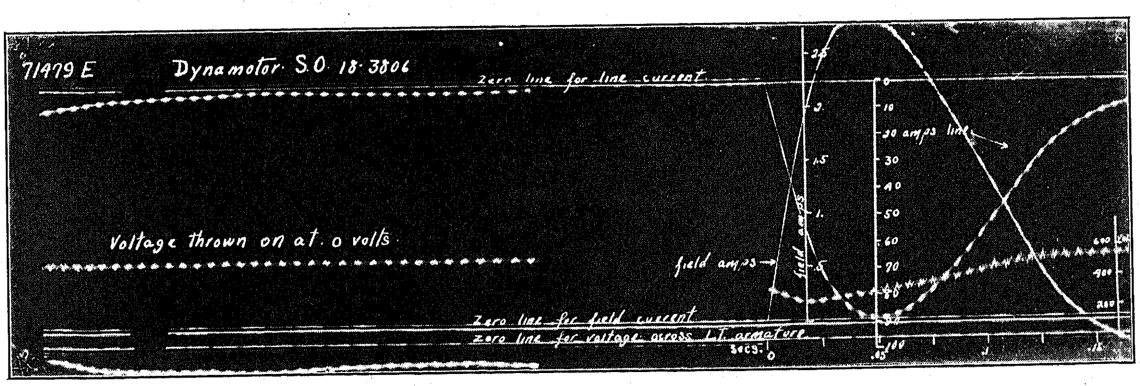

Fig. 7

The first oscillograph tests were made to determine what happened when the machine flashed. Fig. 5 shows the record made on a No. 1 dynamotor. A volt meter was placed across the low-voltage armature and the line switch opened. When the generator voltage had dropped to 150 volts, full-line voltage was thrown on again, the voltage in this case being 1200. There was no resistance in series with the machine, so the main high-tension armature and field, these two, therefore, absorbed more than their share of the line voltage. Up to this point the phenomena is fairly clear, but now the voltage across the low-tension armature starts to rise and reaches a value of approximately 800 volts, when the machine bucks again. This voltage rise is repeated three separate and distinct times, the last indicating the highest rise in voltage.

At first, it might be thought that this rise in voltage on the low-tension armature is due to its high reactance. This may be partly the case, but to a very small extent, as is clearly shown by the oscillograph of voltage on both the high-tension and low-tension armatures. See Fig. 6. This oscillograph shows the line volts as well as the volts across the high and low-voltage armatures. The maximum voltage reached on the low-voltage armature is approximately 900 volts and the maximum on the high-voltage armature is approximately 800, or a total of approximately 1700 volts, in this case 36 per cent rise over line voltage, which is approximately 1250 volts. The machine in this case bucked three times. What may be called the primary buck is due to rise in current and two secondary bucks due apparently to rise in voltage.

If reference is made to the oscillograph shown in Fig. 7, which is a record of the voltage across the low-voltage armature line amperes and shunt-field amperes when 1200 volts are thrown on the machine at standstill, it will be noted that while the main current rises to 89 amperes, twice that shown in Fig. 5, the machine does not buck. It would seem, therefore, that the theory regarding bridging sparks was probably an explanation of the primary bucks. The rise in voltage, which is the apparent cause of secondary buck, is probably due to stored energy in the field which is a result of the excess rush of current due to the primary buck.

Oscillographs were taken on the No. 2, 3 and 4 machines and in general showed the same results as those taken on No. 1.

It will be noted that in all cases the shunt field reverses from its normal direction and thus opposes the series field. As far as can be observed from the oscillographs, for example, Fig. 7, there is no observable time between the start of rise of series-field current and the reversal and rise of shunt-field current.

With the data from the foregoing tests, a method of designing compound-wound machines with given characteristics, which will have the best possible flashing characteristics, was worked out as follows: Assume for the compound-wound machine that the maximum and minimum speed limits are given, the maximum and minimum current, the maximum permissible watt loss in field, and the maximum available space for field winding. The maximum and minimum speed fixes the maximum and minimum ampere-turns with a given saturation curve.

A = Min. ampere-turns total.
I_1 = Max. load current.
I_2 = Min. load current (No load).
T = Turns of series field.
D = Ampere-turns from shunt field.
K = Ratio of max. ampere-turns to min. ampere turns

$$\therefore K A = \text{Max. ampere-turns.} \quad (1)$$

$$A = D + I_2 T \quad (2)$$

$$K A = D + I_1 T \quad (3)$$

Substituting the value for A from No. 2 in No. 3 we have

$$K (D + I_2 T) = D + I_1 T \quad (4)$$

From which

$$T = \frac{K D - D}{I_1 - K I_2} = \frac{D (K - 1)}{I_1 - K I_2} \quad (5)$$

Equation **5** gives T in terms of D as from assumption K, I_1 and I_2 are known.

The value of T so found may be substituted in terms of D in either **2** or **3** and the value of D found at once.

The value of D so found may be substituted in **2** or **3** and the value of T found. It will be noted from equation **5** that T increases directly as K, so the greater the difference between the maximum and minimun ampere-turns, the stronger the series field may be made. There are two ways of making K large, first, a large difference between the maximum and minimum speed, or second, making the machine highly saturated and working well up on the saturation curve at maximum load. As long as $K I_2$ is less than I_1 the shunt field will be accumulative with the series field, but with $K I_2$ equal to or greater than I_1 no shunt field will be required to keep the speed within the required limits. This means simply that the machine would be worked so high on the saturation curve and with so many turns on the series field that no shunt would be required to keep the machine from overspeeding, the saturation on the other hand.holding the speed up on the heavy load. Ordinarily a machine of this type, *i. e.*, with series field only, would mean an abnormal design, if at times it is required to run with no-load current only.

There is now a method of finding the maximum number of turns on series field, also the ampere-turns of shunt, but the minimum number of turns on shunt field is not yet found, and as it is desirable to have the ratio of series to shunt turns a maximum in order, as before explained, to obtain a quickly established field, the next step is to find the minimum number of shunt turns.

The minimum number of shunt turns will be limited by heating considerations only. If a shunt field with a certain number of turns and a certain resistance giving a certain number of ampere-turns is considered, the number of turns may be reduced without materially changing the number of ampere-turns, providing the mean length of turn remains approximately the same. The reason for this, is very evident, for as the number of turns is decreased, the total resistance of shunt field is reduced in the same proportion, and therefore the current is increased sufficiently to keep the value of ampere-turns approximately a constant. The heating, however, will increase, for all the heat in the shunt field is due to amperes squared times resistance, and as the resistance decreases directly, the heat will vary inversely with the number of turns.

Any theoretical method of calculating the minimum

number of shunt turns involves so many variables or assumed values that for practical purposes it is hopeless.

Perhaps the best method for the problem involved is to start with a very high value of amperes per square inch in shunt field copper and then check for heating and space. The amperes per square inch in the shunt coils of some machines designed has been run up as high as 1400 amperes per sq. in. This, of course,

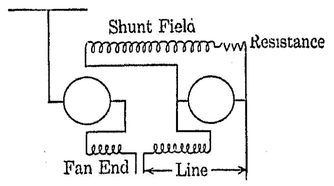

Fig. 8

depends a great deal upon type of machine, shape of coil, etc., and no rule can be given.

A No. 5 machine was designed for use on 1200 volts, driving a fan and using a series field, also a No. 6 designed in the same way, but for use on 1500 volts. The No. 5 machine operated O. K. but the No. 6 gave some little trouble after being placed in service and will be referred to later.

A compound-wound machine was now designed along the lines given and will be referred to as No. 7. The No. 7 machine was intended to replace the No. 3 machine which was not entirely satisfactory as regards flashing characteristics. The result obtained was highly satisfactory, and it was evident that a compound-wound motor could be designed to start at full voltage at any point and without having abnormal proportions. The No. 7 machine had exactly the same design as the No. 3, except for change in field and shape of air-gap. The beneficial results were practically all obtained from the changed field as the No. 3 machine was equipped with a new field and had as good flashing characteristics as the No. 7 machine. Flash tests were made on the No. 7 machine, using the connections as shown in Fig. 8. From previous discussions the object of the resistance in the shunt field will be understood. In a given machine, the introduction of outside resistance in the shunt field may or may not be beneficial as regards flashing. This will be referred to under the discussion of a later machine. The highest voltage which the machine would stand at all points was 750 on low-tension commutator. Flash tests were made on the low-tension winding because, as before stated, the inductance of this winding is much higher than the high-tension. Regular flash tests were made with both windings in series, but with no resistance in series with armature or in shunt field and 1200 volts was applied at all speeds and under all conditions, that is, with compressor on or off, or lighting load on or off, with very good results. Flash tests were also made with resistance in series with armature but no resistance in shunt field. Under these conditions 1500 volts were applied without flashing the machine.

In order to make reasonably sure that the No. 7 machine would operate satisfactorily in service, a full car equipment was mounted on a fly-wheel test, and the control, main motors, and dynamotor compressor run for some time on 1350-volt cycle test. In connection with this cycle test, mention should be made of a rather unexpected source of trouble under service conditions. When a car or locomotive crosses a section break or when the trolley for any reason looses and makes contact while the control is on, the following conditions exist with a compound-wound dynamotor. First, the dynamotor, if the compressor is not on, keeps on rotating and acts as a generator supplying current to the main motor, thus tending to drive the car; second, when the trolley comes in contact with the supply wire again the current is quickly reversed. The strain on the machine is very heavy under the first or second conditions and either may cause a flash. Tests were made on the No. 7 machine, mounted with main motors and control, to determine whether it would stand such service. On these tests the machine gave no evidence of flashing even on 1350 volts with a very heavy generating plant. Oscillographs were taken on the No. 7 machine and are shown in Figs. 9, 10 and 11.

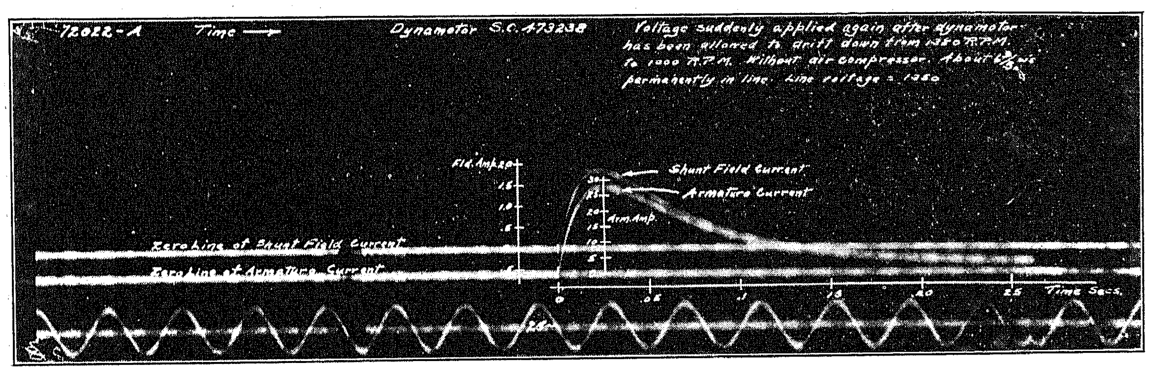

Fig. 9

The oscillograph shown in Fig. 9 was taken at the time 1350 volts were applied after the machine had drifted down to approximately 75 per cent of maximum speed, running light. Taking values from this oscillograph, the resultant ampere-turns are calculated and

plotted against time, as Curve 1 in Fig. 12. The ratio of resultant field ampere-turns per pole to armature ampere-turns per pole are also plotted against time, giving Curve 1 in Fig. 13.

The oscillograph shown in Fig. 10 was taken at the time 1350 volts were suddenly applied to machine at standstill with compressor cut in, and the oscillograph shown in Fig. 11 was taken under the same conditions, except that the compressor was not cut in.

It will be noticed that the main and shunt-field current reaches approximately the same maximum value with compressor either on or off. The maximum value of current, however, with compressor off is reached in a little shorter time than with compressor on.

From the oscillograph as shown in Fig. 11, the same values were calculated as for the oscillograph of Fig. 9 and plotted as curves 2 in Fig. 12 and 13 respectively, in order to obtain a comparison between a standing start and a start at 75 per cent of maximum speed. The resultant ampere-turns are in both cases comparatively low at 1/100 sec., after which they rise rapidly until they reach a fairly constant value, which, of course, gradually falls back to normal. The resultant turns in the case of a standing start are very much higher than is the case with the running start, but the ratio of the resultant field ampere-turns to armature ampere-turns are not greatly different in the two cases.

Considering the values shown in Fig. 12 alone would indicate that the flashing characteristics at standing start would be better than at running start, however, the current commutated at standing start is much higher than at running start, which would offset the advantages to a certain extent. The values of current are therefore plotted against time in Fig. 13 in order to show the magnitude of starting current in each case, when plotted to the same scale. Curve 4, Fig. 13 shows current for running start on machine No. 7 and Curve

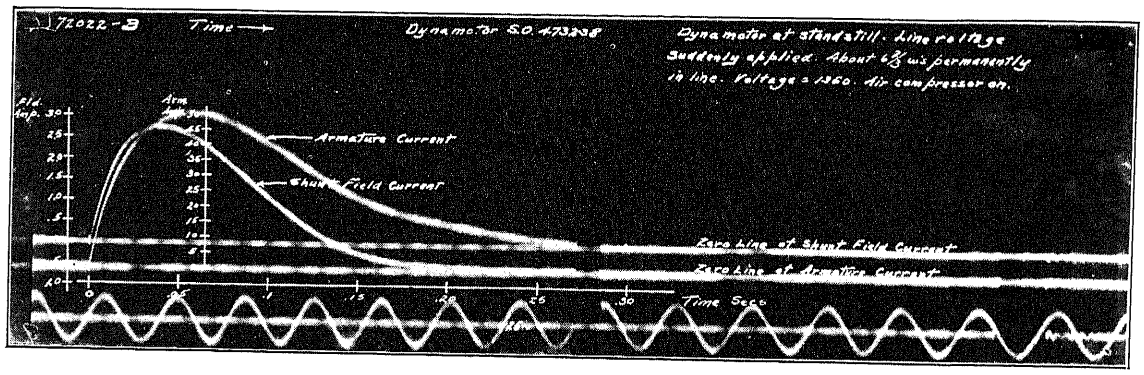

Fig. 10

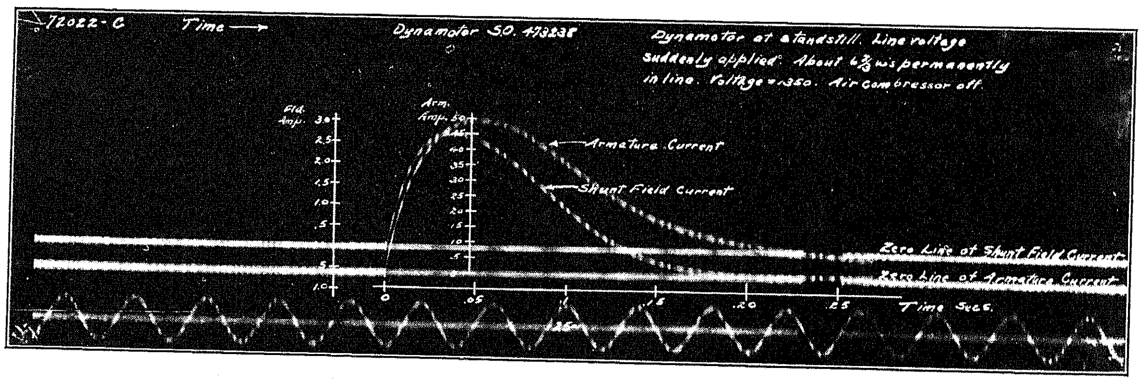

Fig. 11

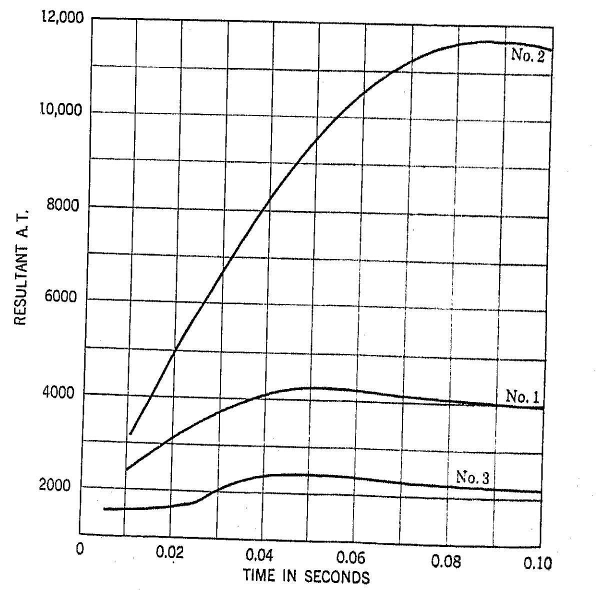

Fig. 12—Resultant Ampere Turns

Curve 1 = Resultant ampere-turns on No. 7 at start from 75 per cent maximum speed

Curve 2 = Resultant ampere-turns on No. 7 at start from standstill

Curve 3 = Resultant ampere-turns on No. 8 at start from 75 per cent maximum speed

5 for standing start. Much higher resultant ampere-turns in the case of standing start, even with the same ratio of field to armature ampere-turns as at running start, gives a better field form and consequent lower maximum volts per bar, thus reducing the tendency for bridging arcs before referred to in this paper.

In connection with the No. 7 machine, mention will be made of a specially designed dynamotor and tests on same. As brought out in the previous discussion, the shunt and series field on the same magnetic circuit

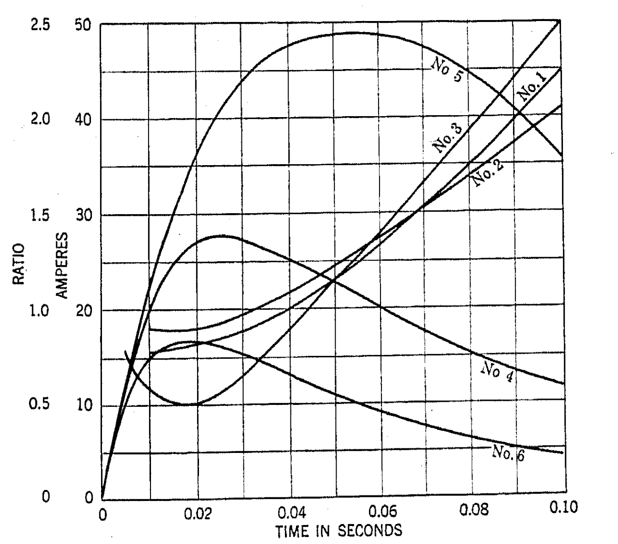

FIG. 13—RATIO OF RESULTANT FIELD AMPERE TURNS TO ARMATURE AMPERE-TURNS AND CURRENT IN ARMATURE

Curve 1 = Ratio on No. 7 at start from 75 per cent maximum speed light
Curve 2 = Ratio on No. 7 at start from standstill
Curve 3 = Ratio on No. 8 at start from 75 per cent maximum speed light
Curve 4 = Amperes No. 7 at start from 75 per cent maximum speed light
Curve 5 = Amperes No. 7 at start from standstill
Curve 6 = Amperes No. 8 at start from 75 per cent maximum speed light

form in fact a series transformer, the shunt bucking the series at start. An obvious theoretical solution of this difficulty is to place the shunt and series field on a separate magnetic circuit, but practically this solution is rather hard to apply. One method of splitting up the magnetic circuit is shown in Fig. 14. The trouble with a design such as shown in Fig. 14 is that the machine would be quite bulky. A second method would be, considering a four-pole machine, to place the shunt coils on two-poles, a north and a south, and the series coils on the other two poles. A third method is to place series coils on all four poles and shunt coils on only two poles. The series coils which are in combination with the shunt coils are to be of a fewer number of turns than those on the other pair of poles. This last method has the advantage of permitting the fields to be so designed that they are magnetically balanced under the most prevalent load conditions.

In order to test the advantages as regards flashing of this third method, the shunt coils were cut out on two poles of a No. 7 machine. With this arrangement of field, the machine stood 1300 volts under all conditions which was materially better than with the regular field. This better-flash voltage, however, might have been due to the fewer total number of shunt turns. The effect of fewer shunt turns and higher drop per turn has been discussed previously. The results obtained on this preliminary test were considered sufficiently good to warrant designing special field coils for further tests. The special coils were wound with a comparatively large number of series turns on the straight series coils and the remaining necessary series turns placed on the compound coil. Sufficient shunt ampere-turns were used to keep the maximum speed down to approximately what it was on the regular machine.

On flash tests this machine which will be called No. 7-*A* commutated very well at all running starts, but at standing start flashed to ground at a little over 1200 volts, or in other words, the regular No. 7 machine was better at standing start than the No. 7-*A*. The No. 7-*A* machine was different from all the other compound-wound machines, in that it had better flashing characteristics at running start than at standing start. The reason for this was not understood just at this time, but tests of later machines gave an explanation and solution and will be referred to later.

At about this time a dynamotor which will be called No. 8 was developed for use on 1500-600 volts, that is, with 900 volts on one commutator and 600 on the other. The winding lying in top of slot was made the high-voltage winding, as the commutation volts for a given number of turns are lower on the top winding than on the bottom winding. The compound-wound field was designed in the best possible proportions, using an external resistance in series with the shunt to give a more quickly established and stronger resultant field, as will be understood from the previous discussion. Flash tests were made on the No. 8 machine but the

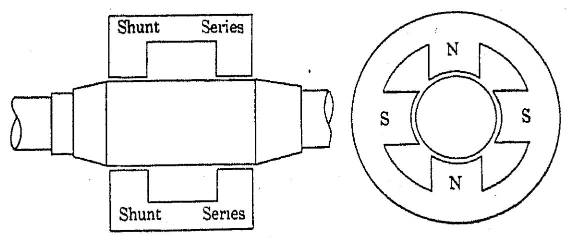

FIG. 14

results were not as good as expected, due probably to the reduced total number of commutator bars. However, with an additional series resistance over that estimated, the machine was considered safe.

A question had been raised as to the effect the so-called "lighting load" has on the flashing characteristics of a given machine, so to settle this question oscillographs shown in Figs. 15 and 16 were taken. The connections of dynamotor and oscillograph resistance are shown in Fig. 15.

The oscillograph of Fig. 15 shows the current A of high-tension winding, current B of lighting load, current C of low-tension winding. As shown by this oscillograph, the lighting load current rises at once to approximately full value and is not affected by the heavy rush of current in the dynamotor. At any given time the sum of the current in B and C should equal the current in A.

explained as follows: On all previous machines the flashovers had all occurred on the low-voltage commutator, but on the No. 8, due to the 900 volts on high-voltage commutator, the flashover seemed as likely to occur on the high-voltage as on the low-voltage commutator. On ordinary tests, but with lighting load, the high-voltage commutator commutates more current by the amount of the lighting load, and in the same

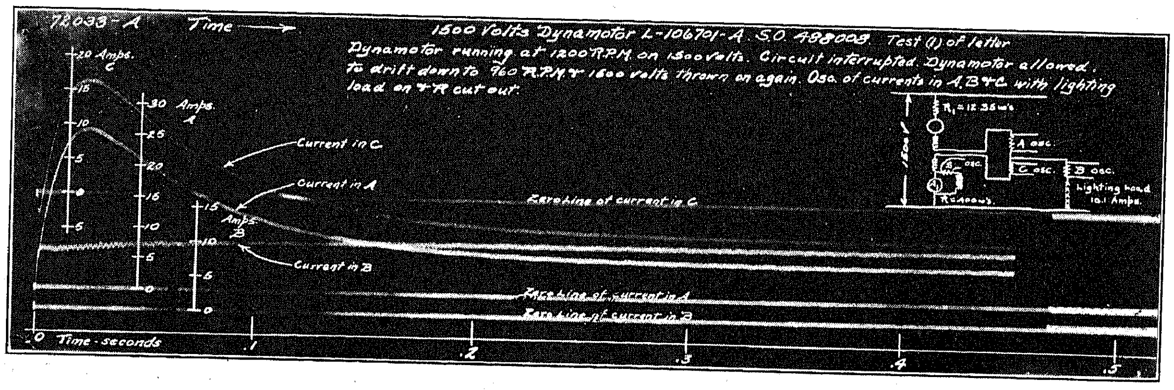

Fig. 15

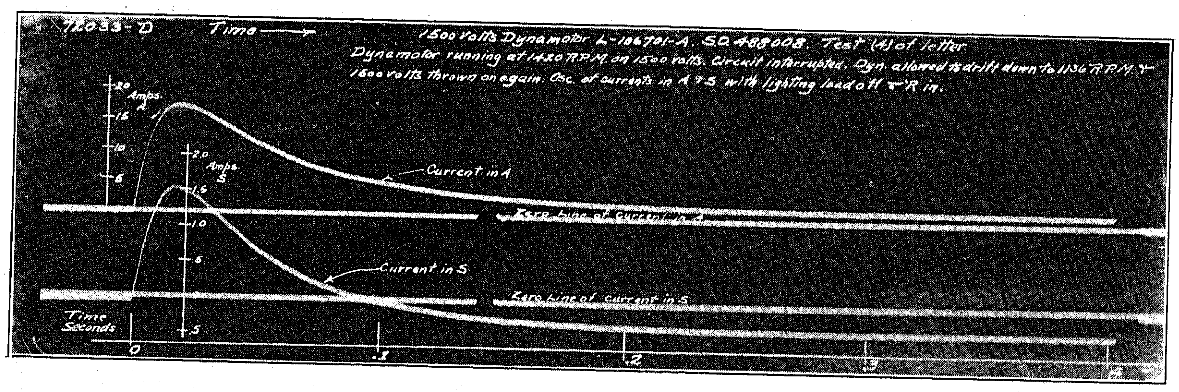

Fig. 16

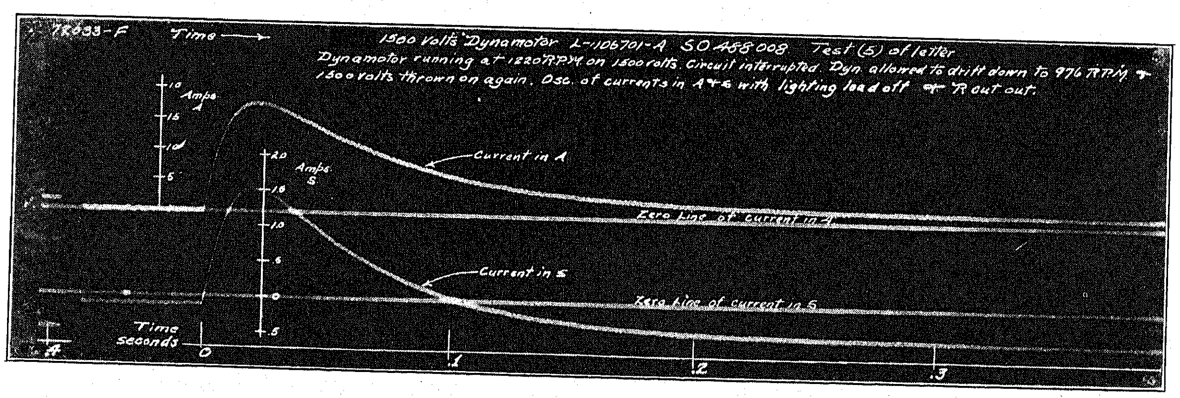

Fig. 17

Flash tests were made on the No. 8 machine at 1500 volts without lighting load and without resistance in shunt with satisfactory results. However, under the same conditions but with lighting load on, the machine flashed over on high-voltage side. The same tests as above were made with resistance in shunt with the same result. The greater susceptibility of the No. 8 machine for flashing with lighting load on may probably be

proportion lowers the flash voltage on the machine over that without lighting load.

The oscillographs shown in Figs. 16 and 17 were taken to show the effect of the external shunt resistance R. These two oscillographs show very little, if any, difference in either the maximum value of the current or shape of current wave. As a matter of fact, the use of external resistance in the shunt does not in itself better

the flashing characteristics of a given machine, but does permit the design of the shunt field with comparatively few turns without overheating. It was actually found on test that in most cases the insertion of resistance in shunt field of dynamotor, while running light, made the flashing characteristics worse. When the machine was driving a blower, however, the insertion of resistance in shunt field made the flashing gradually better, until after reaching a high point with the resistance the flashing was eliminated. These two results on the face of the matter may seem contradictory, but the following is a possible explanation. With no resistance in shunt and while running light the dynamotor speed is say, 1300. As resistance is placed in the shunt the speed runs up. Now it is found on test that the worst flashing on any particular machine is at a speed which is some certain per cent of maximum speed, say, *i. e.*, 80 per cent.

The shunt field is active as long as the armature is rotating so the counter e. m. f. dies out gradually. The highest current rush may be at standing start but the flashing is a function of the current commutated and the speed. Therefore, if at a speed of 1300, the counter e. m. f., per armature is 600, then at 80 per cent of 1300 or 1040 rev. per min. the counter e. m. f. is 480, and the current rush would reach a certain value say, 40 amperes. Now, when resistance is placed in shunt field the speed runs up to say, 1600 rev. per min., at 80 per cent of 1600 or 1280 rev. per min. the counter e. m. f. is again 480 and the current rush would again be 40 amperes or more. It is readily seen, therefore, that the machine is commutating 40 amperes at 1040 rev. per min. in one case and 40 amperes at 1280 rev. per min. in the other and would naturally expect worse flashing. If, however, the speed is held down by the series field when machine is loaded by the fan, the speed will not run up very much when resistance is placed in the shunt. As explained in another place, the resistance in shunt will help, providing it is not more than offset by high speed which would be the case where the machine was running light.

From oscillograph of Fig. 16 the resultant field ampere-turns and ratio of resultant field ampere-turns to armature ampere-turns were calculated and the results plotted against time, giving Curve 3 of Fig. 12 and Curve 3 of Fig. 13. The value of current is also plotted against time as Curve 6 in Fig. 13. In comparison to the same curves from the No. 7 machine, it will be noticed while the ratio of field to armature ampere-turns is much lower on No. 8 machine for running start, the current commutated is also much lower. Results similar to the above are not shown for any machine except the No. 7 and No. 8, as the oscillographs of preceding machines were not taken in just the same way, so comparison would show nothing.

All the No. 2 machines had been placed in service with compound field even though some were driving a fan and could therefore be operated with a series field. After these machines had been out for some time it was decided to equip one with a series field. This will be called the No. 9 machine. The series field was designed with a very large number of turns and the dynamotor was thus expected to have fine flashing characteristics, but when placed on test was found to flash at standing start at normal voltage, as had been the case with the No. 7-*A* machine. At all running starts the machine did not flash. The nature of the flashover did not seem to be the same as with the compound-wound machine, not being nearly so vicious. Apparently this flashover was due to the quickly established field, this field giving rise to high voltage between bars, due to transformer effect, which in combination with heavy starting current caused the flash. Evidently therefore, a certain amount of damping effect was desirable on this machine and in order to determine as nearly as possible the proper amount, a compound-wound field was used and resistance placed in shunt field in increasing amount until a point was found where the dynamotor did not flash, at either running or standing start. A copper damper of equivalent damping effect to the shunt field was made up and placed on the series-wound No. 9 dynamotor. The results on flash tests with this arrangement were very fine indeed, the dynamotor starting without any resistance in series with armature, at all speeds and under all conditions without signs of flashing.

The above results would seem to indicate that the flashover voltage at standing start of a series machine such as railway motors might be raised by the use of a damper. However, on railway motors high-flash voltage at running start is most essential and any damping effect apparently lowers the flashover voltage.

The No. 7-*A* machine will be referred to again in connection with the tests made on copper dampers in the above. After the results obtained on the No. 9 dynamotor by the use of dampers, the two series coils of the No. 7-*A* machine were assembled with similar dampers. On flash tests the No. 7-*A* machine equipped with dampers gave very much better results than it did without dampers, but at 1300 volts flashed from high voltage brush holder on low voltage commutator to ground. At all running starts the dynamotor gave no signs of flashing. From this it was evident that a heavier damper would give better results. A heavier damper was not tried but the brush holder which flashed to ground was insulated by taping with treated cloth, and when this was done the machine stood 1425 volts at all speeds and under all conditions without a flash.

It should be noted that unless otherwise stated all flash tests are made without external resistance in series with the armature.

It has previously been stated that further reference would be made to the No. 6 dynamotor. After being in service it was reported that this machine would flashover under some conditions at standing start. Copper dampers similar to those used on the No. 9 dynamotor

were made up and placed on the No. 6 machine. After these were installed the flashing was entirely eliminated on this machine and demonstrated thoroughly the decided benefit obtained from the damper.

The No. 4 and No. 5 machines were both series-wound but the fields were not of sufficient strength to require dampers, which shows that dampers might not in all cases prove beneficial.

After the No. 8 machine had been in service a short time, it was found that at certain points on the line and under certain conditions the dynamotor would flash on the high-voltage side. The flashing was most prevalent at the point where the 1500-volt line crossed a 1200-volt line, which involved as a matter of protection two grounded sections. A short description of the tests which located the point of trouble may be of interest. A car was equipped with a No. 8 dynamotor and variable resistance connected in series. Flash tests were made by running over dead sections and making and breaking contact with the trolley at short intervals. These tests were made with a high resistance in series with dynamotor and continued with resistance down to a fairly low value, but at no time did dynamotor flash over. The car was then run over the above-mentioned crossover which involves the following sequence: First from 1500-volt trolley to dead section, then to grounded section, then to second dead section, then to 1200-volt trolley for a distance of approximately 18 in., then back over the dead and grounded section to the 1500-volt trolley. Each of the grounded and dead sections was approximately two ft. long.

The car was first run over the crossover at a speed of 15 to 25 mi. per hr. with the series resistance at a low value, under all conditions of operation, but at no time did a flashover occur. The car speed was then reduced to approximately six mi. per hr. with the same results. It was found, however, that when running at an approximate speed of 10 mi. per hr. even with a high resistance in series that the dynamotor would flash nearly every time the car passed the crossover. The flashover appeared to occur at the time the trolley made contact with the second grounded section after passing over the 1200-volt trolley.

The above details of tests are given to show the severe service sometimes met and also the difficulties of locating the exact point of trouble. The effect on the dynamotor of the sequence of dead and grounded sections will be understood by the oscillograph on the No. 7 machine. Of course, with the ground switch to grounded section open and the control off no flash occurred at the crossover. It was decided to apply a remedy which has been mentioned in connection with tests on the compound-wound No. 4 machine, that is; to connect the shunt field in two parallel circuits and place resistance in series to keep it from overheating. When this was done the flashing on the No. 8 machine was entirely eliminated. The effect of the parallel shunt field will be understood from the discussion on the effect of number of turns in shunt and also from Figs. 1 and 3. In Fig. 1 where the shunt is connected all in series, the total transformer volts induced by rush of current in series field will be twice as great as in Fig. 3. The resistance of each parallel circuit in Fig. 3 is one half the total of Fig. 1, but in addition, the external resistance is added which cuts down the opposing current of shunt on start but permits the running strength to remain the same.

The last dynamotors designed were the No. 10 and No. 10-*A* for use on 1500-750 volts, and were intended to replace the No. 2 machine in the standard line. All refinements of design were incorporated in the No. 10 machine, and in addition, a larger number of commutator bars were used then on the No. 2 machine. The results as expected were very good, flash tests being made at 1800 volts with only a comparatively small resistance in series.

In conclusion, therefore, it can be said that a compound or series-wound machine can be designed which will give little trouble from flashing though subjected to the most severe operating conditions, if the following points are observed:

(1) Certain reasonable physical limits which have not been touched on in this article must be maintained, such as number of commutator bars, distance between brushholders and total thickness of mica between brushholders.

(2) Strong field in comparison to armature.

(3) High ratio of series-field turns to shunt-field turns. This may be accomplished in two ways with a given difference between maximum and full-load speed.

(a) By working motor well up on the saturation curve.

(b) By working shunt at a high volt drop per turn, accomplished by either using a high value of amperes per sq. in. in shunt field if heating permits, or with external resistance in series.

(4) For series-wound machines it may be found in some cases that a damper is required to prevent flashing at standing start.

(5) For some types of machines a divided magnetic circuit for shunt and series winding may be used with good results.

A 35,000-Kw. Induction Frequency Converter

Description, Operating Characteristics and Test Data

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Review of the Subject.—The application of the induction frequency converter is not new, but its use for connecting together very large systems had not been undertaken until the machines described in this paper were put in operation.

This paper describes the rating and mechanical construction of the two units, with special attention to the induction machine.

The testing of the machines was done after installation, and the results of these tests are given.

The theory of the operation of the induction unit is developed, being simplified by neglecting losses and resistance, thus avoiding complicated equations, but securing results agreeing quite closely with the more accurate calculations.

The behavior of the set under short circuits is discussed.

THE rapid extension of the idea of interconnection of power systems has resulted in a greatly increased use of frequency converters as a tie between lines of different frequencies, and has led to a corresponding increase in size of the individual units until to-day there are installations capable of transferring 35,000 kw. from one system to another.

The type of frequency converter most frequently used up to the present time consists of two direct-connected synchronous machines, and acts simply as a power tie between the systems. The power transfer is directly dependent on the governing characteristics of the prime movers of the generating units, up to the load at which one of the converter units pulls out of synchronism. The voltage regulation of one system is entirely unaffected by that of the other system as long as the power transfer is not changed. A voltage disturbance on either system, which is not severe enough to drop one of the converter units out of step, has practically no effect on the other unit or its system. In case one unit loses synchronism, the load transfer will decrease to a low value and its current will fluctuate widely. The other unit will operate at the reduced load as a synchronous machine. The voltage on the undisturbed system will remain at almost exactly its normal value.

This characteristic of the synchronous frequency converter is desirable in many installations, but there may be operating conditions where it would be desirable to have the voltage of both systems affected similarly by a disturbance on either. In case of sectionalized systems of the same frequency, a tie through reactors, transformers, tie-lines, or combinations of these methods tends to equalize the voltage on the various sections.

With systems of different frequency the voltage equalization may be obtained by the use of an induction frequency converter set, consisting of an induction machine direct-connected to a synchronous machine. This type of set has been used in a few cases, and a number of high-frequency generators direct-connected to induction motors instead of synchronous machines, has been built in small sizes. The theory of these sets is not new, and has been given in several books and other publications. However, the construction of machines of sufficient size to operate successfully between very large metropolitan systems is a new application and this paper is being written to give the general theory, application and test data of such a set.

An induction frequency converter was selected for the New York systems from the following considerations.

1. It is reversible and may be used to supply power either from 25 to 60 cycles or from 60 to 25 cycles. It is therefore practically the equivalent of a spare unit for both systems.

2. When operating between the systems, it will tend to supply reactive kilovolt amperes to either of them whenever the voltage of that system drops below normal. It will therefore tend to hold up the voltage of the disturbed system, although at the expense of transmitting some disturbance through to the other. Its use however, is analogous to the interconnecting of stations of the same frequency, which in general is considered good practise.

3. In the operation of synchronous converters, it is desirable to parallel at the source of a-c. supply, if the d-c. ends are to be connected to a common bus. Where one generating station supplies all the power, the feed for the converters is taken from a common bus or from several sections of bus connected together through suitable reactors. When more than one station of common frequency supplies power, the several stations are connected through suitable tie-lines. Where stations of different frequencies supply power, all stations of a common frequency are paralleled through suitable tie-lines, and the two systems of different frequency are tied through the induction synchronous frequency changer set. Improved parallel operation of converters has been obtained by this scheme of operation as applied to individual systems, and it follows that similar results can be obtained through a frequency converter which forms a voltage as well as a power connection.

The induction machine of the New York set is of considerable interest on account of the mechanical and

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electrical problems encountered in its design and construction, and a complete account of these features is given. The synchronous unit is one of the larger salient pole units of its class, but it does not have any very unusual features and, therefore, its design and characteristics are given only as they affect the operation of the set. A few illustrations are included to show the general construction details.

Rating

The frequency converter is designed for normal output of 35,000-kw. three-phase at unity power factor at either 25 or 60 cycles, and at 1.0 power-factor input from

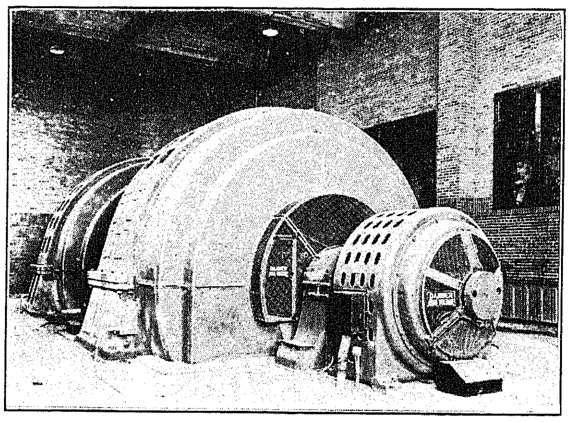


Fig. 1—Completed Installation of the 35,000-kw. Frequency Converter

the system of the other frequency. This input is slightly under 37,000-kv-a. and the 60-cycle induction unit has a nameplate rating of this kv-a. input as a motor. The normal rating of the set is for power transfer from 60 to 25 cycle, that is, normally the induction machine is a motor and the synchronous machine a generator. The 60-cycle voltage rating is 13,800 volts, and the 25-cycle 11,400 volts.

The induction machine has 14 poles. When operating at 300 rev. per min. the rotor revolves at a frequency corresponding to 35 cycles. If the rotor is then excited from the 25-cycle system with the phase rotation corresponding to the rotation of the rotor, the resultant frequency generated in the stator is 60 cycles.

The voltage generated in the rotor corresponding to the rated voltage of the stator is 2730 volts, but due to the fact that the excitation must be supplied through the rotor, it is necessary to raise the voltage applied to the rotor to overcome the reactance drop, due to the magnetizing current. The actual impressed voltage on the rotor to produce rated open-circuit voltage on the stator is approximately 2800 volts.

The rotor is connected to the 25-cycle bus and to the stator of the synchronous unit through a 17,100-kv-a. 3-phase 25-cycle air-blast transformer with taps in the primary to give either 2950 volts or 2850 volts on the secondary side. These taps permit variation of voltage ratio to control the power-factor of the induction unit for different operating conditions.

The mechanical output of the induction rotor through the shaft to the synchronous unit is approximately 35/60 of 36,000 kw. or 21,000 kw., and the electrical output to the transformer is approximately 25/60 of 36,000 kw. or 15,000 kw. The difference between the 37,000 kw. input and the 36,000 kw. output of the induction unit represented approximately the losses in this machine. The output of the transformer is approximately 14,600 kw., there being about 400 kw. required for transformer loss and input to the blower motor. The output of the synchronous machine is approximately 20,400 kw., with between 500 and 600 kw. loss in this unit. The magnetizing current of the induction unit must be supplied through the rotor to maintain unity power-factor on the stator, and the synchronous machine is rated 25,000 kv-a. 0.85 power-factor to furnish reactive kv-a. for this magnetizing current.

Starting of the set is accomplished by the use of a 1600 h. p. 3-phase 25-cycle 11,400-volts induction-starting motor with wound secondary for speed control by external resistance. The synchronous speed of this motor is 375 rev. per min., so that exact synchronous speed of the set can be obtained. The motor is of sufficient capacity to start the set and bring it into synchronism with both units excited for norm alvoltage.

A 115-kw. 250-volt exciter is direct-connected to the 25-cycle end of the set to furnish excitation for this unit.

Two 225-h. p. 3-phase 25-cycle induction motors are provided for direct connection to a blower which supplies ventilating air to the induction unit and air blast trans-

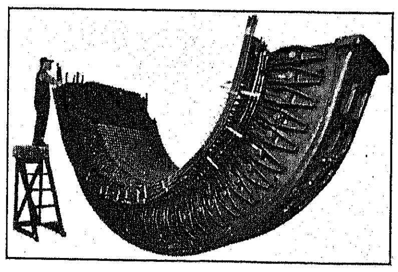


Fig. 2—Upper Half of Stator of 25,000-Kv-a. 25-Cycle, 3-Phase, 11,400-Volt A-C. Generator

former. The motors may be tied solidly into the low-tension side of the transformers, and the fan is therefore sure to start up as soon as the voltage is brought up on the induction unit and transformer. Only one motor is connected to the power supply at a time, the other being a spare unit.

Mechanical Design of Synchronous Unit

The stator of this machine is of standard construction for this class of apparatus and an extended description is not necessary. Fig. 2 is an illustration of the upper half of this stator and shows the double-layer type

winding, binding of the ends of the coils as well as other general details of its construction.

The rotor (Fig. 3) has laminated poles secured to the spider by a number of dovetails. The spider is made up of steel plates riveted together to give a very substantial structure of a material that is practically free from defects. The amortisseur winding consists of brass bars silver soldered to copper end segments, and the segments in turn are bolted securely to solid steel end rings. These rings were cut from solid plates and therefore have no joints. Shrouded fans attached to the rotor spider at each end just below the bottom of the pole add materially to the fan effect of the rotor for supplying air to ventilate the machine.

Mechanical Design of the Induction Unit

Stator. The stator is very similar in mechanical design to the synchronous machine stator, and to many other machines of its class.

It was built in two sections to facilitate shipment.

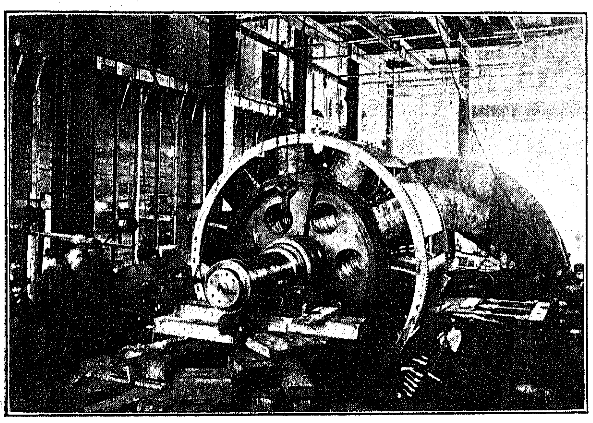


Fig. 3—Rotor of 25,000-Kv-a. 25-Cycle, 3-Phase 11,400, Volt A-C. Generator

The core is made up of laminations of high grade silicon steel. They are held in the frame by dovetail keys and cast steel end clamping flanges. These flanges are secured by bolts extending entirely through the stator frame. The stator slots are open, the formed and insulated coils being held in place by wooden wedges. The assembled coils are securely laced to three binding bands at each end of the stator. The stator coils, clamping flanges and lacing of the coils are very similar to these features of the synchronous unit as shown in Fig. 2.

The use of high grade steel in the core and liberal design with respect to copper results in low operating temperature and good efficiency.

Rotor. The rotor of this unit is the largest of its kind that has been built and it presented a number of very difficult problems in design.

It had to be designed to stand in emergency a run away speed of 514 rev. per min. as it is an induction machine and in case of short circuit on the 25-cycle system it would immediately start to accelerate toward this speed. However, on account of the size of these units, it was deemed desirable as an additional precaution to provide speed-limiting devices, which will trip the 60-cycle line switches at about 15 per cent over speed.

The rotor spider (Fig. 4) consists of two wheels

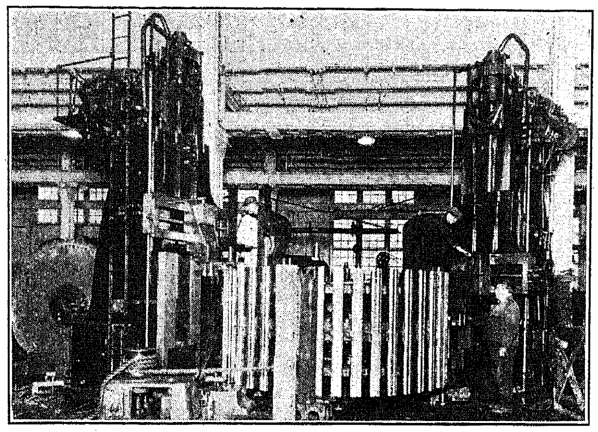


Fig. 4—Machining Rotor Spider of 37,000-Kv-a. 60/25-Cycle 3-Phase, 13,800/2700-Volt Induction Frequency Converter

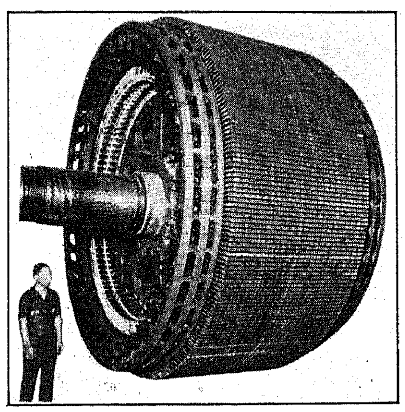


Fig. 5—Rotor of 37,000-Kv-a. Induction Frequency Converter Ready for Winding

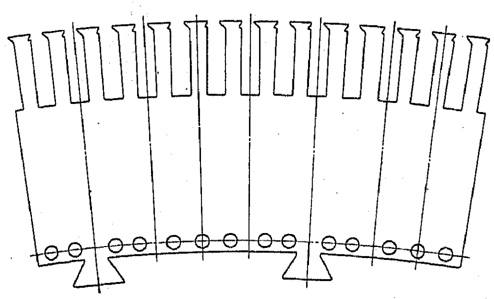


Fig. 6—Lamination for Rotor Core of 37,000-Kv-a. Induction Frequency Converter

bolted in the middle of the rotor. Each wheel has a set of arms at both ends, making a total of four sets of arms for this rotor.

The laminations (Fig. 6) are attached to the spider by dove tails and are also held together by steel pins driven through the completed assembly of the laminations. These pins are located as near as possible to the bottom

of the punchings, to avoid losses and heating. The laminations were made by a blanking die and the slots indexed later. They are indexed to give four different kinds of punchings, and in stacking they are arranged to overlap at the joints, giving three laminations crossing the joint for each break between laminations. The whole rotor is then tied together by the steel pins, so that the strength with the holding effect of the dovetails, pins and friction of the tightly clamped punchings approaches that of a complete circle of laminations. The core is clamped together by steel ring-type end flanges drawn up against the ends of the rotor spider by bolts extending entirely through the spider. The end flanges are of cast steel and each is formed of three rings tied together by cross webs. The two outer rings of each flange are sectionalized to prevent losses and local heating due to magnetic flux leakage. The core has radial air ducts opposite the stator ducts for ventilation. Fig. 5 is an illustration of the rotor ready for winding and shows the general details of its assembly. The sectionalizing of the rotor flanges is very well illustrated by this figure.

The stator slots are open, and it was considered advisable to use partially closed slots for the rotor, to avoid excessive tooth losses. Fig. 7 is a cross section

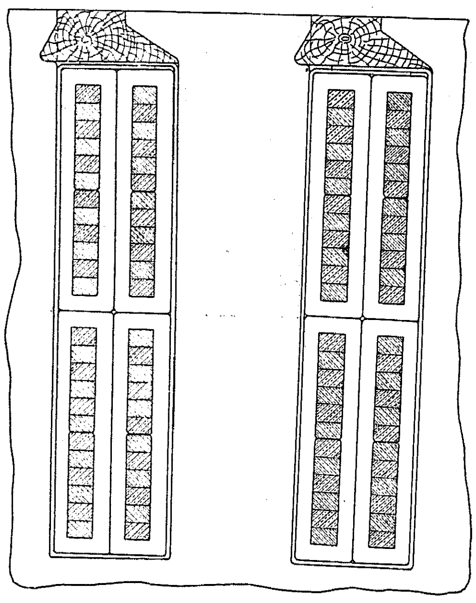


Fig. 7—Cross Section of Rotor Slot of Induction Frequency Converter

of the rotor slot, showing the overhanging tooth and the general arrangement of the coils. These coils are of the double-layer type similar to those in the stators of both units, but had to be made in pairs, each coil about equal in width to one half of the slot width. One coil of a pair was put into the slot and pushed over under the overhanging tip, after which its mate was assembled as it would be with an open slot.

The method of winding and also the fact that collector rings and brushes must be used to connect the winding to the system made it impractical to use full bus potential on the rotor. A stepdown transformer was consequently used, and incidentally gave a convenient and practical way of adjusting voltage for various conditions.

The rotor conductors are made up of several cotton-insulated strands, the individual turns being taped with mica tape. The external insulation consists of several tapings of mica tape with a protective cotton taping filled with insulating varnish. This insulation was

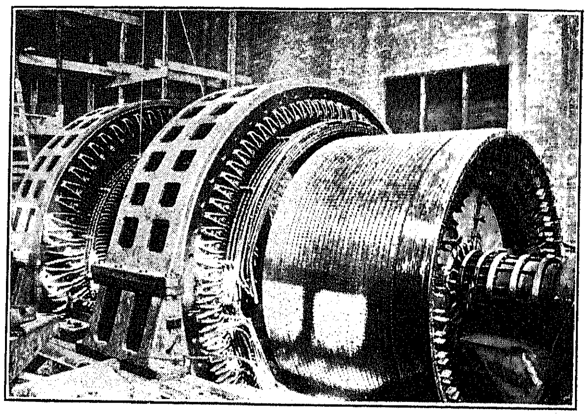


Fig. 8—Assembly of 35,000-Kw. Frequency Converter Set with Stator of Induction Unit Moved over to Clear Rotor

selected on account of its mechanical strength under compression, which is necessary to resist the centrifugal forces to which the coils are subjected.

The ends of the coils required very strong binding bands to prevent distortion due to the rotational forces. The total weight of binding wire is about 2400 pounds. Due to the proximity of conductors with heavy currents, special precautions were taken to keep down the losses and heating in these bands. The layers are insulated from each other by mica separators, and the various layers are sectionalized into small insulated sections decreasing in width from the ends farthest from the core to the inside ends next to the core. The crossovers between sections were spaced with references to polar spacing to neutralize the effects of the leakage flux. Fig. 8, which is a view of the set during installation, shows the details of the completed rotor.

Ventilation

The synchronous unit is an enclosed self-ventilated unit taking air in from under the base, and discharging this air through holes in the stator frame above the base. The air enters through air chutes at each end of the machine, and passes into the rotor fans through circular openings around the shaft. Most of the air is then caught by the poles and forced out through the radial air ducts of the stator core into the space in the stator frame back of the core. The remainder of the air passes through the end windings of the stator and then through the holes in the heads of the stator frame

under the air shields. The total ventilating air is then discharged into the room through the holes in the stator frame.

The arrangement of the induction unit is similar to that of the synchronous machine with regard to entrance of the air, except the air is supplied under pressure from the external blower. A double-shrouded-type fan on the rotor forces the air into the space back of the rotor core. The spaces between the rotor arms at both ends are covered by vertical plates to form an enclosed chamber inside the rotor spider. The plates on the collector end can be seen on Fig. 8, and the rotor fans are also visible in this figure just outside these plates. From the inside of the rotor the air passes through the radial rotor ducts to the air gap, and then through the stator ducts to the space in the stator frame back of the core. The part of the air that cools the end windings of the rotor and stator follows a path similar to that of the synchronous unit. The holes in the stator frame are closed to a plane somewhat above the horizontal center line, and a ventilating hood attached to the top of the stator frame. This hood has baffles to reduce the windage noise of the machine.

There is considerable resistance to air flow in the stator and rotor ducts, and it was not considered that the machine itself would furnish a sufficient supply of cooling air, hence an external blower was provided to secure additional air pressure. This blower was designed to deliver 122,000 cu. ft. of air per minute in the pit below the machine at a pressure of 7 in. of water. The actual quantity of air delivered per minute is somewhat over 100,000 cu. ft. for the induction unit and about 22,000 cu. ft. for the transformer.

Assembly

The set has four bearings each piped for water cooling. The two end bearings are supported by the base, and the two middle bearings are carried on bridges extending across between the two sides of the base, and supported by cast iron pillars extending down to foundation plates grouted into the foundation. The base, bridges, and the foundations are very strongly constructed and the set operates practically without vibration.

The bearings are also equipped with a high pressure oil system which lifts the shafts and greatly reduces the starting torque. The pressure required to lift the shaft initially is from 1000 to 1200 lb. per sq. in. and decreases to about 500 lb. per sq. in. after the shaft lifts and allows the oil to escape around it. As the rotor speeds up, the pressure gradually decreases to a little over 300 lb. per sq. in. at normal speed.

Movement of either stator frame for inspecting, cleaning or repairing the stator or rotor can be accomplished without removing the rotors. The bearing pedestal, bridge, and supporting pillars adjacent to either unit and the air shields of that unit are removed, and the stator lifted by special bolts in the feet. Rollers are then inserted in grooves in the feet, the frame dropped down on the rollers, and rolled along the base until the rotor and stator winding are clear from each other. The stator can then be lifted, rolls removed and the stator lowered to rest on the base. Fig. 8 shows the induction machine stator moved over clear of the rotor. Access to the stator is obtained from the bridge remaining between the two units.

Shipment

The transportation of the various parts to their destination is of interest as some of them were very close to the limit of the facilities available. All parts except the two rotors were shipped by rail, as they were sectionalized enough to permit transportation by this means. The diameter and length of rotors made it impossible to ship them completely assembled by rail, and an investigation of the method of assembly of these parts indicated that considerable time and expense could be saved by building them complete at the factory. They could be handled easily on the New York State Barge Canal and Hudson River, but the problem of loading them on the barge was more difficult and was only solved by securing the services of two railroad wrecking cranes.

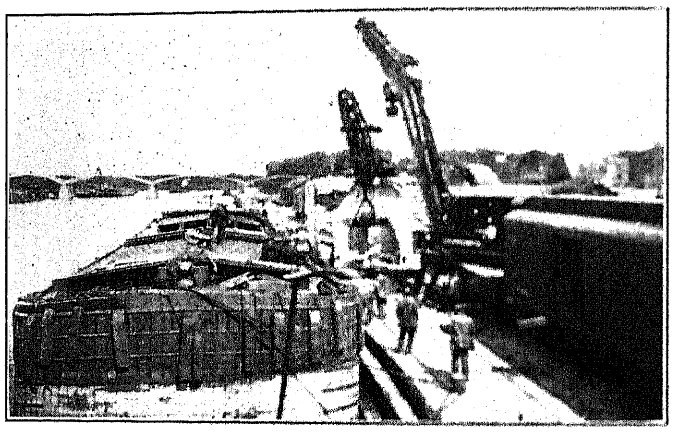


Fig. 9—Railroad Wrecking Cranes Loading Rotor of 25,000-Kv-a. Generator on Canal Barge

Fig. 9 is a view of the synchronous machine rotor being lifted out over the barge, which carried it to the Hell Gate Station.

The rotors were unloaded at destination by one of the derrick lighters available at New York City.

Parallel Operation and Phase Displacement Under Load

The relative phase relations on the 25 and 60-cycle ends of this set are definitely fixed for any particular load condition, the same as a standard synchronous frequency converter, and it is therefore necessary to synchronize on the proper pole of the 25-cycle unit to operate in parallel with other frequency converters. The poles are shifted by reversing the field of the

synchronous machine until the proper phase position is secured.

Assume a second set operating unloaded between the two systems, holding the same relative phase position between them, and the synchronous unit of the induction frequency converter operating from the 25-cycle end. A synchroscope between the 60-cycle end of the set and the 60-cycle system will then show the phase displacement of the 60-cycle voltage of the induction set. If the synchroscope indicates "in phase" position, reversing the field of the 25-cycle unit will cause the unit to drop back one pole or 180 deg. and the synchroscope will move back 1.4 × 180 deg. = 252 deg. or the equivalent of a forward displacement of 108 deg., because one pole of the 10-pole unit covers 1.4 times the mechanical angle that is covered by one hole of the 14-hole unit.

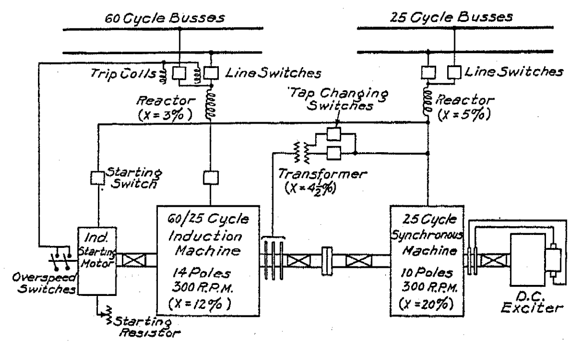


FIG. 10—DIAGRAM OF CONNECTIONS OF SET, TRANSFORMER AND REACTORS

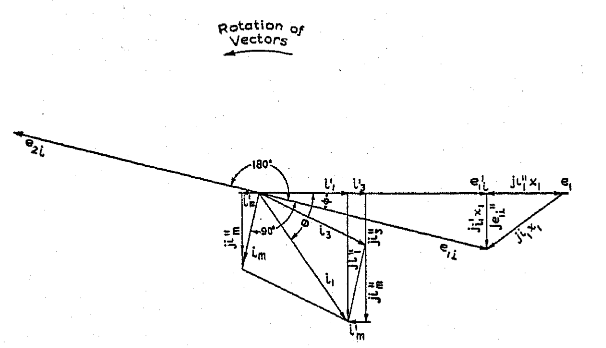


FIG. 11—VECTOR DIAGRAM OF INDUCTION FREQUENCY CONVERTER. PART 1

A tabulation of phase position for the various poles of the 10-pole unit is as follows.

Pole	Phase Angle
1	0 deg.
2	108 deg.
3	216 deg.
4	324 deg.
5	72 deg.
6	180 deg.
7	288 deg.
8	36 deg.
9	144 deg.
10	252 deg.

It is evident from this tabulation that there is just one pole on which the set can be synchronized for parallel operation with other sets, and it is necessary to either synchronize on this pole, or shift poles by reversing the field until it can be reached. The use of two synchroscopes for synchronizing will enable the operator to secure the proper phase relations without much difficulty.

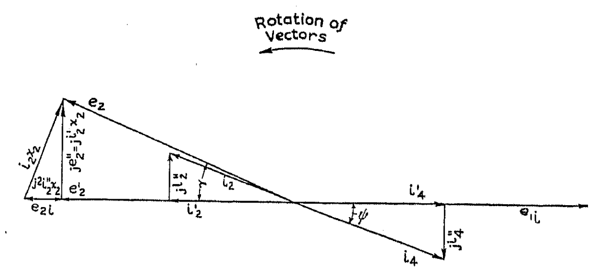


FIG. 12—VECTOR DIAGRAM OF INDUCTION FREQUENCY CONVERTER. PART 2

TESTING

The machines were not completely assembled at the factory and the tests had to be made after the final installation. The testing was done in cooperation with the operating department of the Hell Gate Station of the United Electric Light and Power Company. The excellent facilities available for testing, especially for loading the set, and the large capacity of the power supply made it possible to obtain practically all the

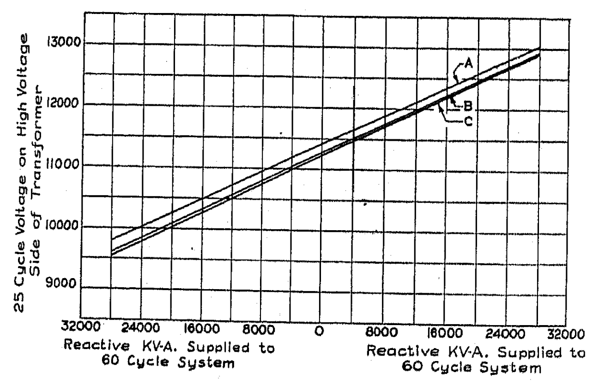


FIG. 13—CALCULATED OPERATING CHARACTERISTICS OF 35,000-KW. INDUCTION FREQUENCY CONVERTER SET. CURVES OF REACTIVE KV-A. ON 60-CYCLE STATOR OF INDUCTION UNIT, FOR VARIOUS VALUES OF 25-CYCLE VOLTAGE ON HIGH VOLTAGE SIDE OF TRANSFORMER CONNECTED TO ROTOR RINGS. 13,800 VOLTS HELD ON 60-CYCLE BUS

Curve *A*. 37,000-Kw. Input to 60-Cycle Stator
B. 18,500-Kw. Input to 60-Cycle Stator
C. No-Load Output from Set.

data desired for the set. The starting motor operated from the large 25-cycle system and was not subjected to variations in frequency. Also the secondary resistance could be varied for speed control on the no-load tests. The converter could be tied to the system at one end and to a special bus supplied by one or more large turbine generators on the other end for heat runs and regulation tests. This method of operation made it

TABLE I

Calculation of 25-cycle voltage on high voltage side of transformer.
Full load—37,000 kw. input to frequency converter.
13,800 volts held on 60-cycle bus.
See Fig. 11 for vector diagram.

Power Factor	Sin θ	i_1'	i_1''	$i_1'' x_1$	$j i_1' x_1$	$e_{1i}' = e_1 + i_1'' x_1$	$j e_{1i}'' = -j i_1' x_1$	e_{1i}	Cos ϕ	Sin ϕ	$i_m' = \frac{e_{1i}''}{x_m}$	$j i_m'' = \frac{-j e_{1i}'}{x_m}$	$i_3' = i_1' - i_m'$	$j i_3'' = j (i_1'' - i_m'')$	i_3
0.80 lag	0.60	1.0	−0.75	−0.072	j0.096	0.928	−j0.096	0.931	0.996	0.089	−0.035	−0.340	1.035	−0.41	1.113
0.90 lag	0.44		−0.488	−0.047		0.953		0.958	0.994	0.109		−0.349		−0.139	1.044
0.95 lag	0.312		−0.328	−0.032		0.968		0.973	0.995	0.100		−0.355		0.027	1.038
1.00	0		0	0		1.0		1.004	0.996	0.089		−0.366		0.366	1.098
0.95 lead	0.312		0.328	0.032		1.032		1.036	0.997	0.078		−0.376		0.704	1.250
0.90 lead	0.44		0.488	0.047		1.047		1.049	0.998	0.063		−0.383		0.871	1.352
0.80 lead	0.60		0.75	0.072		1.072		1.074	0.998	0.064		−0.393		1.143	1.540

TABLE II

Continuation of calculations in Table I. See Fig. 12 for vector diagram.

Power Factor	i_4	$i_4' = \frac{e_1 i_1'}{e_{1i}}$	Cos ψ	Sin ψ	$i_4'' = i_4 \sin \psi$	$e_{2i} = -e_{1i}$	$i_2' = -i_4'$	$i_2'' = -i_4''$	$i_2'' x_2$	$e_2' = e_{2i} + i_2'' x_2$	$j e_2'' = -j i_2' x_2$	$e_2 = e_2' + j e_2''$
0.80 lag	1.113	1.073	0.964	−0.266	−0.296	−0.931	−1.073	0.296	0.029	−0.902	0.106	0.908
0.90 lag	1.044	1.043	1.000	0	0	−0.958	−1.043	0	0	−0.958	0.103	0.963
0.95 lag	1.038	1.030	0.992	0.126	0.131	−0.973	−1.030	−0.131	−0.013	−0.986	0.102	0.991
1.00	1.098	0.995	0.906	0.423	0.465	−1.004	−0.995	−0.465	−0.046	−1.050	0.099	1.055
0.95 lead	1.250	0.966	0.773	0.635	0.794	−1.036	−0.966	−0.794	−0.079	−1.115	0.096	1.116
0.90 lead	1.352	0.954	0.705	0.709	0.953	−1.049	−0.954	−0.953	−0.094	−1.143	0.094	1.148
0.80 lead	1.540	0.932	0.605	0.796	1.225	−1.074	−0.932	−1.225	−0.121	−1.198	0.092	1.198

possible to hold any desired load from no load to the full capacity of the set with very little fluctuation.

Friction and Windage

The set ran from the starting motor without excitation on either unit, and a number of readings was taken

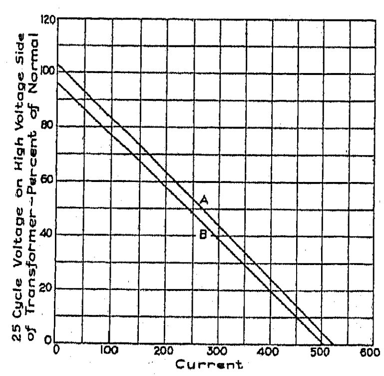


Fig. 14—Curves of Stator and Rotor Current of Induction Frequency Converter. 13,800 Volts Held on 60-Cycle Bus. No-Load Transfer through Set. Voltage on High Side of 25-Cycle Transformer in Rotor Circuit Varied from Normal to Zero

of input to this motor for various speeds as determined by secondary resistance. Fig. 15 is the test curve of friction and windage obtained from these readings.

Saturation and Synchronous Impedance of Synchronous Machine

The saturation data were taken at speeds varying somewhat from normal, but the curve in Fig. 16 is plotted with the voltage values corrected to 300 rev. per min.

The synchronous impedance as given on the same figure was taken under slightly varying speed conditions, but no correction is necessary in this case.

Impedance of the Induction Machine

A special testing generator in the Hell Gate Station was connected through a 2 to 1 step down transformer to the primary of the 17,100-kv-a. transformer. The secondary of this transformer is permanently connected to the rotor of the induction unit and no special test connections were necessary, beyond the connection to the primary of the transformer. The stator of the

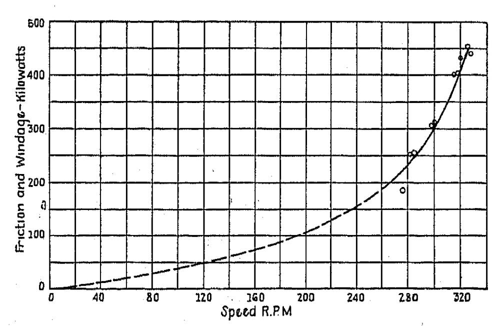


Fig. 15—Test Friction and Windage of Set

main induction machine was short-circuited, and the voltages applied to the rotor for various values of rotor, and stator currents were read. Since this gave the total impedance drop it was necessary to estimate the division between stator and rotor reactance from the calculated values. The total reactance, of course, is a test value and the error in the individual values would be quite small.

Open Circuit Core Losses

The core loss of the synchronous unit was obtained by running at practically normal speed with the field excited for approximately normal voltage. The kilowatt input to the induction starting motor was measured and the output to the set calculated by taking out the losses in this motor. The output then represented friction and windage of the set, exciter input, and core loss of the synchronous unit. The exciter output was measured, and its input calculated from its efficiency data, so that the core loss could be determined by subtracting the test value of friction and windage and the exciter input from the output of the induction motor.

In taking the core loss of the induction unit, it was necessary to excite it from the synchronous unit. One of the tap-changing switches in the high-tension circuit of the transformer (see Fig. 10) was closed, and input to the starting motor measured with the induction

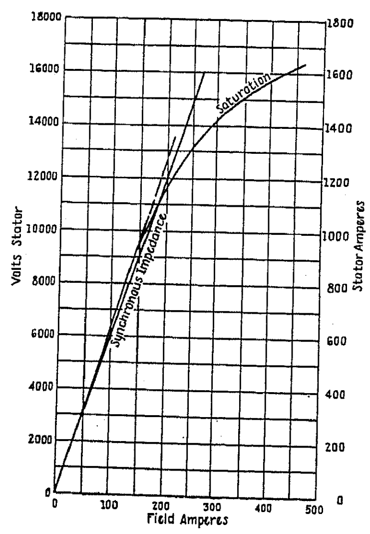


Fig. 16—Saturation and Synchronous Impedance Curves of 25,000-Kv-a. Synchronous Machine.

machine stator voltage brought up to normal. The starting motor output then represented friction and windage of set; exciter input; $I^2 r$ losses in synchronous machine stator, transformer and induction machine rotor for its magnetizing current; blower motor input; core loss of the synchronous unit, and core loss of the induction machine. Since all these losses except blower motor input and induction machine core loss had been previously determined, it was possible to obtain the induction machine core loss and blower motor input by subtracting the known losses from the total. The blower motor input could be closely estimated and thus the core loss quite accurately obtained.

It should be noted here that this test gives the entire running light loss of the set when operating with normal voltage on both units, and the total loss under load will therefore require only the addition of the $I^2 r$ losses, due to increase in the various currents, the additional load losses, and increase in exciter input. Errors in estimating individual losses from this test will therefore tend to compensate each other, and the final results of efficiencies will be very close to the actual values. Figs. 17 and 18 are the open circuit core losses of the two units. These curves are based on the test values at normal voltage and assume the loss varies as the 2.2 power of the voltage. This exponent is based on experience with other alternating current machines,

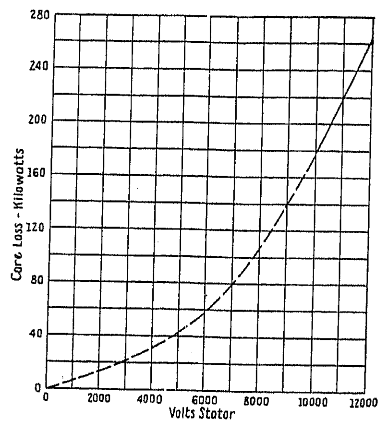


Fig. 17—Open Circuit Core Loss Curve of 25,000 Kv-a. Synchronous Machine. Curve from Test Values at Normal Voltage and Assumes Core Loss = K (Voltage)$^{2.2}$

and the curve will give quite accurate values for voltages in the vicinity of normal.

Magnetizing Current of Induction Machine

The machine was excited through the rotor, and the magnetizing current read for various values of stator voltage around the normal value. The results are plotted in Fig. 19.

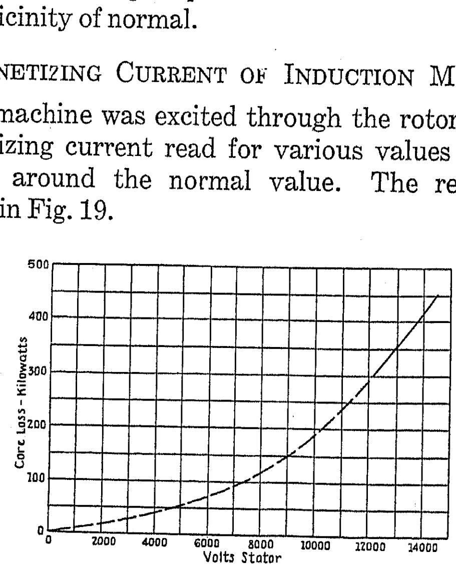
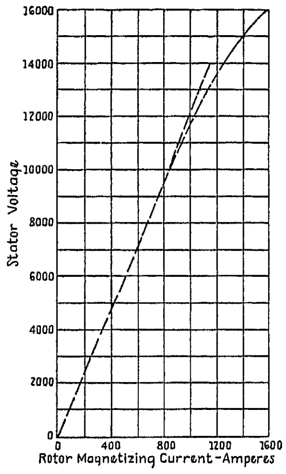


Fig. 18—Open Circuit Core Loss Curve of 37,000 Kv-a. Induction Unit. Curve from Test Values at Normal Voltage and Assume Core Loss = K (Voltage)$^{2.2}$

Short Circuit Core Losses

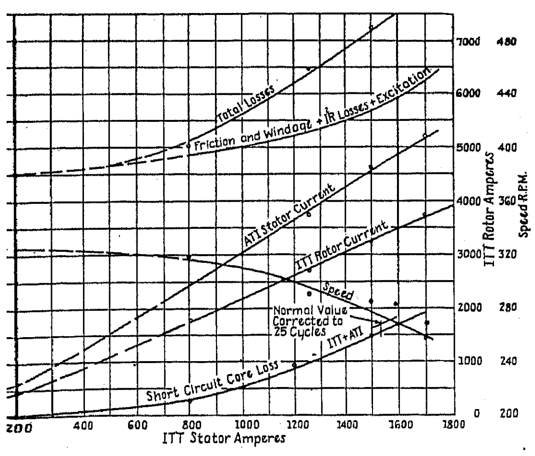
The stator of the induction machine was short-circuited, and the rotor tied in through the transformer to the stator of the synchronous machine. The blower motor was disconnected from the transformer. The input to the induction motor was read for various values of stator currents on the short-circuited machine. The output of the starting motor is given in Fig. 20 as "Total Losses." Friction and windage, $I^2 r$ losses

·ion losses were obtained from the tests pre- de on the set and from excitation readings, se tests. The total short-circuit core loss achines is the difference between the two es in Fig. 20 and is given in the lowest curve re. Test values of speed, synchronous ma- T I) stator current, and induction machine :or current are also plotted in Fig. 20. It ed, the current in the synchronous machine ot the rated value when the current in the machine stator current reaches its normal it is therefore necessary in the efficiency



›TOR MAGNETIZING CURRENT OF 37,000 KV-A. INDUCTION UNIT

Stator Open Circuit.



URVES FROM LOAD LOSS TESTS ON 37,000-KV-A. ON UNIT 25,000 KV-A. SYNCHRONOUS UNIT

s to estimate the proportion of loss to be th each machine. The law of the loss varia- :urrent can be obtained with the required om the shape of the curve of the short-cir- ·f both machines.

ELECTRICAL CHARACTERISTICS

trical operating characteristics of this type cy converter are developed quite fully in Steinmetz', "Theory and Calculations of Electrical Apparatus." Edition of 1917, Chapter XII.

The following theory is derived from the same general principles, but is simplified by neglecting resistances, and by making the calculations for the normal rotor frequency only.

The vector diagrams of the induction frequency converter, Figs. 11 and 12, are drawn for the machine operating as a motor, taking lagging current from the line, but the equations derived from them may be extended to the leading current condition simply by reversing the sign of reactive component.

NOTATION OF VECTOR DIAGRAMS

(See Fig. 11)

e_1 = line voltage on 60-cycle system.

$\dot{i}_1$ = line current on 60-cycle primary of induction frequency converter, that is in the stator circuit.

x_1 = sum of reactances of stator and series reactor between the stator and 60-cycle bus.

i_1' = energy component of $\dot{i}_1$.

i_1'' = reactive component of i_1, 90 deg. behind e_1. This current is negative for lagging-current input, and positive for leading-current input.

x_m = magnetizing reactance.

i_m = magnetizing current.

i_m' = component of magnetizing current in phase with e_1.

i_m'' = component of magnetizing current 90 deg. behind e_1.

$\dot{i}_3 = \dot{i}_1 - \dot{i}_m$.

i_3' = component of i_3 in phase with e_1.

i_3'' = component of i_3, 90 deg. behind e_1.

e_{1i} = component of e_1 which overcomes the internal induced voltage in the stator, and is equal to e_1 minus the reactance drop through x_1.

e_{2i} = internal induced voltage in rotor circuit.

It will be noted that the values of voltage are not laid out in proportion to the actual voltage, but on the basis of a percentage of the actual values, so that the vectors e_{1i} and e_{2i} are equal in length.

(See Fig. 12)

This diagram is a continuation of Fig. 11, but e_{1i} is now taken as the reference vector and i_3 is called i_4 to avoid confusion between the components of the current in the two diagrams.

e_{1i} and e_{2i} are same as in Fig. 11.

$\dot{i}_4$ = $\dot{i}_3$ from Fig. 11 and equal to it in scalar value.

i_4'' = component of i_4 in phase with e_{1i}.

i_4'' = component of i_4 90 deg. behind e_{1i}.

ψ = angle between i_4 and e_{1i}.

$\dot{i}_2$ = rotor current corresponding to $\dot{i}_4$ and 180 deg. in phase from $\dot{i}_4$.

i_2' = component of i_2 in phase with e_{2i}.

i_2'' = component of i_2 90 deg. behind e_{2i}.

x_2 = reactance of rotor and transformer in rotor circuit.

$i_2 x_2$ = reactance drop in x_2.
e_2 = voltage on high-voltage side of transformer, divided by transformer ratio to keep it in the same terms as e_{2i}.

Expressed as an equation in actual volts.

$$\frac{e_{2i}}{e_{1i}} = \frac{f_2}{f_1} \times \frac{n_2}{n_1}$$

where
f_1 = stator frequency.
f_2 = rotor frequency.
n_1 = stator turns in series per phase and circuit.
n_2 = rotor turns in series per phase and circuit.

The division of power output from the set may be derived from the usual equations of the induction motor. The power output from the collector rings, or secondary loss, if the power output of the collector rings is dissipated in an external resistance, is equal to the power input multiplied by the slip, and the mechanical output is equal to the power input multiplied by (1 − slip): That is

$$\text{Electrical output} = \frac{f_2}{f_1} \times \text{total output.}$$

$$\text{Mechanical output} = \frac{f_1 - f_2}{f_1} \times \text{total output.}$$

The electrical output of the converter is from the collector rings through the transformer, and the mechanical output through the shaft to the synchronous unit.

Let $e_1 = 1.0$ when its actual value is the rated voltage of the stator, 13,800 volts.

$e_2 = 1.0$ when its actual value is equal to

$$e_1 \times \frac{n_2}{n_1}\ \frac{f_2}{f_1} \text{ or 2730 volts.}$$

$i_1 = 1.0$ when it is the rated stator current.
i_3 and i_4 are expressed in same units as i_1.
$i_2 = 1.0$ when it is the rated stator current multiplied

$$\text{by the ratio } \frac{n_1}{n_2}$$

The calculations of the machine may be derived by assuming the magnetizing current to be supplied from either stator or rotor, and subtracting this current vectorially from the total current on that member. The ampere-turns, due to the resultant current in that member, will then be equal and opposite to the ampere-turns in the other member.

That is, when the currents are expressed on the basis of the above values as unity,

$$i_3 \text{ (which is also } i_4) = i_2$$

With the above equations as a basis, the equations from Fig. 11 are derived as follows:

$$\begin{aligned}
e_{1i} &= e_1 - j\, i_1\, x_1 \\
&= e_1 - j^2 i_1''\, x_1 - j\, i_1'\, x_1 \\
&= e_1 + i_1''\, x_1 - j\, i_1'\, x_1 \\
&= e_{1i}' + j\, e_{1i}''
\end{aligned}$$

$$\begin{aligned}
i_m &= \frac{e_{1i}}{j\ x_m} \\
&= \frac{e'_{1i}}{j\ x_m} + j\, \frac{e_{1i}''}{x_m} \\
&= -\frac{j\, e_{1i}'}{x_m} + \frac{e_{1i}''}{x_m} \\
&= i_m' + j\, i_m'' \text{ where } i_m' = \frac{e_{1i}''}{x_m} \text{ and } i_m'' = -\frac{e_{1i}'}{x_m}
\end{aligned}$$

$$\begin{aligned}
i_3 &= i_1 - i_m \\
&= i_1' + j\, i_1'' - i_m' - j\, i_m'' \\
&= i_1' - i_m' + j\,(i'' - i_m'') \\
&= i_3' + j\, i_3''
\end{aligned}$$

When the scalar value of e_{1i} and i_3 are determined, the values are transferred to Fig. 12, and the calculations carried out using e_{1i} as a new reference vector. The current i_4 in Fig. 12 is the same as i_3 in Fig. 11, but to avoid confusion between the components of this current in the two diagrams, it is changed to i_4 in the calculations made from Fig. 12.

$e_1\, i_1'$ = power input to the stator
$e_{1i}\, i_4'$ = power output of stator.

Since it assumed that there are no losses, these values must be equal.

$$\text{Then } i_4' = \frac{e_1\, i_1'}{e_{1i}}$$

Then from Fig. 12

$$\cos.\ \psi = \text{power factor of stator output} = \frac{i_4'}{i_4}$$

$$i_4'' = i_4 \sin \psi$$

The same flux cuts both rotor and stator conductors, and therefore the induced voltage is the same direction in both members, and when expressed in the units given above for e_1 and e_2 the induced voltages in both are equal in value.

But since e_{1i} is the voltage to overcome the induced voltage in the stator, it is 180 deg. out of phase with this induced voltage, and e_{2i} is therefore 180 deg. out of phase with e_{1i} and equal to it in value.

The following relations can now be derived from the preceding discussion.

$$\begin{aligned}
e_{2i} &= -\, e_{1i} \\
i_2 &= i_4 \\
i_2' &= -\, i_4' \\
i_2'' &= -\, i_4''
\end{aligned}$$

The rotor terminal voltage e_2 is determined by subtracting the reactance drop in the rotor from the induced voltage e_{2i}.

$$\begin{aligned} e_2 &= e_{2i} - j i_2 x_2 \\ &= e_{2i} - j^2 i_2'' x_2 - j i_2' x_2 \\ &= e_{2i} + i_2'' x_2 - j i_2' x_2 \\ &= e_2' + j e_2'' \end{aligned}$$

Table I is a set of calculations for e_{1i} and i_3 for full-load input to the converter with varying values of power factor, and is derived from the vector diagram Fig. 11.

Table II covers the calculation of i_2 and e_2, the secondary current and voltage on the high side of the transformer. The magnetizing current of the transformer is neglected in the calculations.

The constants used in these calculations were determined from tests and calculated data, and are as follows:

External reactance in 60-cycle lines—3 per cent based on 37,000 kv-a. at 13,800 volts.

Stator reactance 6.6 per cent.

Rotor reactance 5.4 per cent based on 15,400 kv-a. at 2730 volts.

Transformer reactance 4.5 per cent.

Equivalent reactance for magnetizing current—273 per cent.

Ratio stator to rotor current, based on effective turns = 0.475

Stator normal voltage = 13,800 volts
Stator normal current = 1545 amperes
Rotor normal voltage = 2730 volts
Rotor normal current = 3250 amperes.

In the calculations the values of x_1, x_2, etc. are expressed as the percentage values divided by 100.

Then total primary reactance

$$x_1 = 0.03 + 0.066 = 0.096$$

Total rotor reactance

$$x_2 = 0.054 + 0.045 = 0.099$$

$$x_m = 2.73$$

60-cycle bus voltage $e_1 = 1.0$ at 13,800 volts.

25-cycle voltage on high side of transformer e_2 1.055 for 11,400 volts, or 1.0 for 10,800 volts.

Ratio transformer = 10,800 to 2730
= 3.96 to 1.0

The curves in Fig. 13 are plotted on the basis of 13,800 volts held on the 60-cycle bus, and the transformer ratio held at the value to give 11,400 volts on the high side of the transformer at 1.0 power-factor input to induction unit with full load on the set.

Characteristics Under Short Circuit Conditions[1]

Fig. 14 shows a curve giving the relation of the stator current, and the current in the 25-cycle rotor and transformer circuit, to the voltage on the high side of the transformer for a variation in this voltage from normal to zero with transformer ratio to give 11,400 volts on high side and 2730 on low side. This curve was calculated by the same method as Fig. 13. The equivalent reactance is 19 per cent for calculations of kilovolt-ampere taken from the 60-cycle system, and 19.8 per cent for calculations of kilovolt-ampere delivered to the 25-cycle system. These percentages are based on the ratings of the respective circuits, 37,000 kv-a. for the stator and 15,400 kv-a. for the rotor.

In case of short-circuit on the 25-cycle system, the additional reactance between the transformer and the bus, which is 5 per cent based on 35,000 kv-a. or 2.2 per cent based on 15,400 kv-a., must be added. The synchronous reactance of this unit at rated load field current is 36 per cent based in this 15,400 kv-a. capacity of rotor circuit.

The reactance circuit for the 25-cycle side is thus a 35 per cent reactance and a 19.8 per cent reactance in parallel, and this combination in series with a 2.2 per cent reactance.

The combined reactance is 14.8 per cent, and the kilovolt-amperes delivered to the 25-cycle system is

$$\frac{15{,}400 \times 100}{14.8} = 104{,}000 \text{ kv-a.}$$

This value of kv-a. corresponds to the sustained short-circuit current, and is considerably less than the initial value, as the output of the synchronous unit would be limited only by the transient reactance which is 20 per cent based in 25,000 kv-a. or 12.3 per cent based on 15,400 kv-a.

The combined reactance is then 9.8 per cent and the input to the 25-cycle system on the initial current rush after short circuit would be equivalent to $\frac{15{,}400 \times 100}{9.8}$ = 157,000 kv-a.

The 104,000 kv-a. supplied to the 25-cycle system is divided inversely as the reactances in the synchronous machine and the rotor circuit of the induction unit.

Thus the kv-a. supplied by the induction unit is

$$\frac{35}{35 + 19.8} \times 104{,}000 = 66{,}300 \text{ kv-a.}$$

This kilovolt-ampere is equivalent to 430 per cent of normal current and the stator current for this rotor current is 450 per cent, and the voltage in the high side of the transformer and thus at the synchronous motor terminals is approximately 14 per cent.

The kv-a. from the 60-cycle line is $\frac{37{,}000 \times 450}{100}$ = 166,000 kv-a.

The author wishes to express his appreciation of the co-operation of Mr. H. Y. Hall, Superintendent of the Hell Gate Station of the United Electric Light and Power Company, and of Mr. H. W. Oetinger in the testing of the set, and to acknowledge the assistance of

1. See "Frequency Converter Ties Between Large Power Systems" by J. W. Dodge, *G. E. Review*, June, 1923.

Messrs. R. E. Doherty, R. F. Franklin, and D. S. Snell, in the preparation of the theoretical part of the paper.

Discussion

F. C. Hanker: There is some question as to the application of the machine described that I want to discuss, particularly on the basis of considerations that Mr. Shirley has outlined. He has given in his paper three considerations. If you will analyze them, you will find the first is applicable to either the synchronous unit or the type described. In the second, in the amount of inductive kv-a. returned to the system there appears to be a question as to whether the type described is as effective as the straight synchronous, because the induction machine must of necessity receive its magnetization from either the 25-cycle or the 60-cycle system, depending on the voltage condition. That means that you have to compensate for that excitation or magnetization from the line either through other apparatus connected to, say, the 25-cycle system, or an additional machine that would be supplied with the set. If you take existing conditions, it would not be very serious or very harmful to supply the magnetization from the 25-cycle systems but in other cases where the power factor of the 25-cycle load is much lower, then it is a handicap to the machine.

If you take the third consideration he has given, I fully agree with him as to the desirability of maintaining voltage in connecting the two systems, but it is questionable in my mind whether the reactance of the magnitude, that you obtain with the special machine, is going to be particularly effective and whether it is going to be of sufficient advantage to overcome the disadvantage of cost both in reduction in efficiency and in the complication of the machine.

If you take a similar machine which fortunately is available for comparison, the efficiency as given by Mr. Shirley shows losses of some 2000 kw. at full load. On the Brooklyn system they have a straight synchronous machine of practically the same rating, with about 500 kw. lower losses. In other words, the efficiency of the two sets inherently penalizes the special induction outfit. As the 500 kw. are largely constant losses, they are present all the time the machine is in operation. At 7000 hours a year they represent three and a half million kilowatt hours, which represents a considerable investment if capitalized, and would indicate the disadvantage to which this special outfit would be subjected.

From the other standpoint of the voltage, it is undoubtedly true that low reactance between stations of the same capacity is of considerable advantage, but it is also true that the tie must be of sufficient magnitude and low enough reactance in order to be effective in maintaining the voltage on the two systems. In the present instance that reactance is given as about twenty per cent, which represents a very appreciable drop, particularly when you consider the relative sizes of the two systems.

Here is a machine of some 35,000 kw. between systems of, 200,000 kw., or that order, of one frequency and at least 100,000 kw. of a different frequency, which means that the tie between those two systems is of comparatively high reactance. If you take the synchronous converters, only a small difference in voltage between the two supply circuits will cause reversal on one, which is the particular point that Mr. Shirley is endeavoring to overcome.

It would be interesting to ask if there has been any operating result that substantiates the advantages claimed for the induction type, and whether there has been any improvement observed in operation under abnormal conditions. It is only under those conditions that any difficulty may be expected from the reversal of synchronous commutating machines. The slight advantage if any is secured at a considerable expense, and the question of application should be very carefully considered before any of the special outfits are utilized.

It does involve complications and does cause both losses and increase in the magnetizing which every power company is endeavoring to keep down just as much as possible.

E. B. Shand (by letter): Mr. Shirley has described a type of frequency converter which has had a very limited application, therefore, it is quite justifiable to discuss it from a comparative standpoint with respect to the type of machine ordinarily used in similar applications, that is, the frequency converter set composed of two synchronous machines. This discussion intends to follow this above idea, using the data presented by Mr. Shirley to represent the induction or cascade type of machine.

Attention may be called to the three considerations enumerated by Mr. Shirley as determining the selection of the cascade type of machine for tying together the particular systems mentioned. The first of these, that of reversibility of operation, allowing the set to be considered a spare unit for either system, applies equally well to synchronous sets, and is rather a consideration for the installation of a frequency converter, regardless of type. The two remaining considerations deal with the ability of the cascade machine to transfer reactive kv-a. as well as power between the two interconnected systems. This is the main differentiation between the abilities of the two types of apparatus considered. In the synchronous set, the interconnection between the two electrical systems being through the common shaft is purely mechanical, so that power alone may be interchanged. In the cascade set, the interconnection is partly through the common shaft, and partly through the air-gap flux of the induction machine, this magnetic link allowing the interchange of the reactive kv-a. If my memory serves me rightly, this characteristic has given to the induction-type frequency converter the name "general transformer" for it serves approximately to connect systems of different voltage and frequency as the transformer does systems of the same frequency.

It may be noted that with the induction-type converter the synchronous machine plays no direct part in the interchange of magnetization, so that for the purpose of a simple analysis it may be assumed non-existent or disconnected. The frequency converter then becomes a wound-rotor induction motor with the stator windings connected to the 60-cycle system, and the rotor windings, through a suitable transformer, to the 25-cycle system. This combination is capable of interchanging magnetization, but is completely incapable of interchanging power. By analogy, therefore, this apparatus might be designated "the general synchronous condenser." The rotor will rotate at 25/60 of the synchronous speed at 60 cycles, and the air-gap flux interlinking both windings will generate voltages in the stator and rotor windings in the ratio of 60:25, on the basis of an equal number of turns in each winding. If the voltage ratio of the two systems varies from that determined from the design of the converter, the reactive kv-a. will flow until the reactive drop in the machine has equalled the voltage difference. The magnetizing current is supplied from the system with the relatively higher voltage. This is all similar to the flow of reactive kv-a. in a transformer, except that the frequency and the voltage per turn is not the same in the two windings. It thus happens that for every 60 kv-a. drawn from the 60-cycle system only 25 kv-a. is supplied to the 25-cycle system; and, conversely, 25 kv-a. drawn from the 25-cycle system becomes 60 kv-a. when taken from the stator terminals.

In the above mentioned arrangement of the "general synchronous condenser" there is no definite frequency ratio between the two circuits; this depends entirely on the frequencies as determined by the governors of the two systems thus interconnected. When the synchronous machine is connected to the same shaft, and electrically connected to the rotor of the induction machine, the frequency ratio becomes as definitely fixed as in the case of

the two synchronous machines mechanically connected together. The induction set, therefore, locks in synchronism the two systems which it interconnects. The relative phase displacement of the two systems will determine the load on the set, and if the phase displacement becomes too great, the set will drop out of step. The overload at which this will occur depends upon the design of the two machines, making up the set. The extreme overload capacity of such a set is of the same order as in the case of the synchronous type of set. With this in mind, it is believed that some of the statements in the second paragraph of Mr. Shirley's paper will be seen to give a deceptive idea of the relative limitations of the two types of sets with respect to pulling out.

Returning to the subject of the interchange of reactive kv-a. it will be seen that the ordinary transformer diagram may be used to determine the relative values of this interchange. Utilizing Mr. Shirley's data, the following impedances may be set down.

Stator reactance	6.6%
Rotor reactance	5.4%
Transformer reactance	4.5%
Magnetizing current	36.5%
External reactance (60-cyc. system)	.3%

It is assumed, in addition, that the synchronous generator is excited to furnish the magnetizing current, that is, that the set will operate at unity power factor at no-load. The results of these conditions will be as follows:—If the 25-cycle voltage drops so that the stator input is 37,000 kv-a. at 60 cycles, the output from the secondary alone will be 10,000 kv-a.; assuming, however, that the magnetizing kv-a. from the generator is added to this, the total 25-cycle output will be 15,000 kv-a., while the voltage drop in the set will be 17 per cent. Assuming now that the 60-cycle voltage drops until the stator output is 37,000 kv-a. the rotor input will be 21,000 kv-a., of which 5500 kv-a. will be supplied from the generator so that the net set input will be 15,500 kv-a. The corresponding voltage drop in the set will be 23 per cent. The above calculations check roughly with Fig. 13 of Mr. Shirley's paper.

It is considered that the logical conclusion from these figures is that the average interchange of reactive kv-a. should be

$\frac{37000 + 15400}{2}$ or 26,300 kv-a. with an average drop of

20 per cent. The set reactance based on its rating of 37,000 kv-a. would then be about 28 per cent from the standpoint of voltage regulation of the two systems. If the installed capacity of frequency changers of this type were considered to be from 20 per cent to 25 per cent of the capacity of smaller system, a figure which is not generally exceeded, it will be seen that the reactance of the tie between the two systems will be of a magnitude of between 110 per cent and 140 per cent, as compared with the smaller of the systems. With a tie of this value of reactance, it is evident that the effect of the induction frequency converter in equalizing the voltages of the two systems must be very limited, in fact, in considering a transformer tie of this nature, the regulating effect on voltage would ordinarily be disregarded.

The third consideration enumerated in the paper is closely connected with the above discussion; it regards the possibility of paralleling synchronous converters fed from the systems of both frequencies because of the tie formed by the induction frequency converter. The load distribution between two converters paralleled on the direct-current side depends upon the relation between their supply voltages. Under full-load conditions, one machine will reverse its power flow and the other carry double load when the supply voltage on one machine drops more than 10 per cent with respect to the other. The fact that the induction frequency changer set will allow double this voltage variation to carry full load reactive kv-a. signifies that the adoption of this type of machine will not be a deciding factor in determining whether or not converters may be paralleled under the conditions referred to above.

From the analysis as developed above, the writer's conclusion is that as a voltage tie between systems the capabilities of the induction type of frequency converter are negligible so far as any practical results are concerned, and that this characteristic of the induction converter should have no particular influence on the choice of sets.

O. E. Shirley: The discussions of this paper by Mr. Hanker and Mr. Shand both point out that the first point given for the selection of the induction frequency converter for the New York systems is applicable to the synchronous set as well. This, of course, is quite evident, and it was not intended to convey the idea that this characteristic was the exclusive property of the induction set, but simply to present, as completely as possible, the considerations that led to the choice of the induction frequency converter.

With regard to the question of magnetization of the induction machine it should be noted that under normal load conditions the magnetizing current for the induction machine is supplied by the synchronous unit and, therefore, the power factor on both sides, as it affects the systems, is unity. It is, therefore, evident that the additional machine suggested by Mr. Hanker is not necessary.

The comparison of the efficiency, presented by Mr. Hanker, shows a difference of about 1.4 per cent at full load. Part of this difference is accounted for by the assumption of a round number of 2000 kw. for the losses of the induction set. This value is somewhat high and, when used as a basis for calculating the difference of efficiency, it introduces an error of considerable magnitude. A comparison of efficiency of sets designed for the same characteristics as to insulation, break-out torque, and temperature rise shows that the difference is more nearly ¾ per cent than the value given above.

The break-out torque, which is mentioned in Mr. Shand's discussion, in the case of the induction frequency converter is largely dependent on the torque of the synchronous unit, since the torque of the induction machine with voltage maintained on the two systems is considerably above that of the usual synchronous machines. The maximum torque of this frequency converter with voltage maintained on both systems is over 200 per cent of normal, being very appreciably in excess of the 150 per cent torque of the usual design of unity-power-factor synchronous machines with a short-circuit ratio only slightly over unity. (Note that by short-circuit ratio is meant ratio of field amperes for no-load normal voltage to the field amperes for rated current on short circuit.)

The behavior of synchronous converters, operating in parallel on the d-c. buses with a-c. supply from systems of different frequency, is affected by a number of factors which have not been taken in account in either of the discussions, nor the original paper, but which must be considered in judging the operating characteristics of the converters in this way. These synchronous converters are usually located in substations, and the drop in the line and transformers will have a very decided practical effect in limiting the current taken by them during disturbances. The induction frequency converter being located at the central station and close to the bus will transmit a very considerable amount of reactive kv-a. balancing the voltages to a marked degree, and then the additional drop between the main buses and the synchronous converters should limit the current sufficiently to prevent injury to the converters under many conditions which would cause serious trouble if the induction frequency converter were not used.

The calculations by Mr. Shand from which he concludes that the effect of the induction set is negligible in causing a drop in voltage on one system, when a disturbance takes place on the other system, can best be answered by data taken from recording charts showing the results of a severe disturbance on

the 25-cycle system under actual operating conditions. The connected capacity on the 60-cycle system was 35,000 kw. at Hell-Gate, 14,000 kw. at Waterside, and 108,000 kw. at Sherman Creek. The voltage on the 25-cycle system dropped to 62 per cent of normal and on the 60-cycle to the following values:

Hell-Gate	71 per cent
Sherman Creek	81 per cent
Waterside	76 per cent
East 188 Street Sub-Station	76 per cent

These values of voltage drop do not bear out Mr. Shand's statement that "the regulating effect on voltage would ordinarily be disregarded."

A similar problem in parallel operation of synchronous converters from different stations of the same frequency had been encountered on the New York systems and this was worked out by tie lines between the stations. The size and characteristic of the induction frequency converter was based on experience with these tie lines, and since the tested values of reactances agreed very closely with the calculated values, there is no reason to believe that the operation with the frequency converter tie should not be as successful as with the ordinary tie lines.

The induction frequency converter has not been used to secure parallel operation of synchronous converters as outlined above, since the operating conditions of the two systems have not yet been such as to make this method of operation desirable. A second converter, however, is being built for parallel operation with the first and there is every reason to believe that the equalization of the voltages of the two systems will be successful whenever this method of operation is required, as the use of the two sets will take care of increased capacities which will be available and may be used later.

The Inertaire Transformer

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Review of the Subject.—*Oxidation is the cause of almost all of the troubles that develop in service in connection with transformer oil and it is thought by the authors of this paper that the obvious remedy for such troubles is to isolate the oil completely from contact with the oxygen of the air. A new method of providing this complete protection in the Inertaire transformer is described.*

In this transformer, a body of insert gas—nitrogen—is automatically created and maintained inside the case above the surface of the oil. This nitrogen is obtained from the outside atmosphere by a breathing process in which the breathed air passes into the transformer through chemicals which absorb the 21 parts of oxygen, leaving only the 79 parts of nitrogen to pass into the case.

A second purpose of the nitrogen gas is to eliminate the danger of fire or explosion above the oil surface for no fire or explosion can start or continue without oxygen.

A distinction is made in the paper between a "primary explosion," which term is used to designate a sudden abnormal pressure produced by gas expansion due to arcing in the oil, and a "secondary explosion" by which is meant an abnormal pressure produced by the combustion of an explosive mixture of gases. The hazard of the secondary explosion is eliminated by the absence of oxygen and effective protection against the primary explosion is given by the cushioning effect of the nitrogen body and a new form of diaphragm relief device.

A description of the Inertaire transformer equipment is given, also an account of a number of explosion tests which were made to demonstrate the effectiveness of the gas cushion in reducing the abnormal pressures due to arcing under the oil and of the new diaphragm device in relieving the pressures.

* * * * *

THE Inertaire transformer is an oil-immersed unit having a space above the oil surface filled with an inert gas composed almost entirely of nitrogen. The distinctively new feature of this type of transformer is that the body of nitrogen is automatically created and maintained above the oil surface through the controlled natural breathing of the transformer. The purpose of the new feature is to secure the decided benefits that are derived from contact with the oil of an inert and protective gas having no oxygen content. The very desirable results obtained are:

a. the prevention of oxidation of the oil,
b. the exclusion of moisture from the oil,
c. the elimination of risk of fire or explosion of inflammable gas above the oil surface,
d. the useful cushion of compressible gas above the oil surface, remarkably effective in case of a sudden increase of pressure due to an internal breakdown of the transformer.

A rise in temperature of the oil, due to load conditions in an oil-immersed transformer of the ordinary type, causes the oil to expand and exerts a pressure on the air above the oil level. If the transformer has an open vent, a portion of the air is expelled from the case. Conversely, a fall in the temperature of the oil causes it to contract and lessens the pressure above the oil level, thus drawing air into the case. This "breathing" action, which is unavoidable and which has heretofore been objectionable, is utilized through a combination of simple and automatic devices to create and maintain the inert gas body of the new Inertaire transformer. When conditions produce in-breathing, the air drawn in passes first through a quantity of deoxygenating and dehydrating chemicals. In the passage through these chemicals the oxygen content and whatever moisture is present are absorbed, and dry nitrogen is drawn into the gas space. A "breathing regulator" contributes to the economy and successful working of the equipment by its automatic control of the breathed air.

The new features of the inertaire transformer, which mark it physically as being distinctively different from the ordinary transformer having an air space above the oil level, are:

a. A cabinet mounted upon the side of the case, which contains a quantity of deoxygenating and dehydrating chemicals and a breathing regulator.
b. An improved form of relief device for abnormal pressures, placed in the cover of the transformer.

A detailed description of these parts and their functions will be found on page 4.

The Need of Inert Gas in a Transformer

Although the high-grade transformer oils of the present day are far superior to those of several years ago they have some undesirable characteristics which seem to be inherent in all oils of this class and these undesirable features apparently cannot be entirely eliminated even by the most careful refining. The principal undesirable characteristic is perhaps the tendency of oil to oxidize, with possible consequent sludge formation, when raised to moderately elevated temperatures in the presence of air. The darkened color of transformer oil after it has been in use for a time at ordinary temperatures is a manifestation of oxidation in a mild form. In a more pronounced form, a sludge gradually deposits upon the hottest parts of the core and windings of the transformer. This type of sludge seems to have little effect on lowering the dielectric strength of the oil but if it becomes pronounced it interferes with oil circulation and the dissipation of heat from the surfaces upon which it forms. Analysis of sludge of this type shows the presence of oxygen in abundance.[1]

Another form of sludge, called the "soap type,"

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is produced by oxidation when acids, liberated by slow decomposition of the oil, interact with metals inside the case[1]. This type of sludge has a great affinity for water and its presence has considerable effect upon lowering the dielectric strength of the oil.

Since oxidation is the great cause of sludging, the obvious remedy is to eliminate the possibility of oxidation. A rather common way of minimizing sludge formations is to fill the transformer case completely, providing an expansion tank to take care of the increase in volume when the oil heats up in service. This method is successful in a great measure. However, the volume of oil in the expansion tank, which diffuses through the whole structure, is exposed to the oxidizing effects of the atmosphere even though it is at a comparatively low temperature. The oil in the tank of an ordinary transformer may contain as much as 15 or

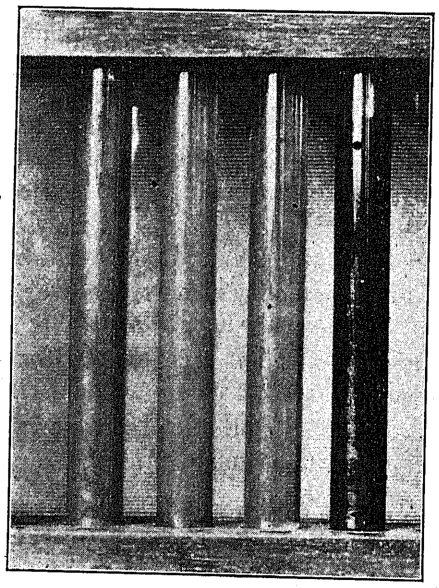

FIG. 1—COMPARISON OF FRESH GRADE "A" TRANSFORMER OIL WITH SAMPLES GIVEN ACCELERATED SLUDGE TESTS WITH AND WITHOUT THE PROTECTION OF INERT GASES

20 per cent of its volume of air carried in solution at ordinary operating temperatures. The oxygen in this dissolved air, being in such intimate contact with the oil, is very effective in producing oxidation.

A blanket of inert gas above the oil level is a complete remedy for oxidation troubles, for it effectively protects the oil against contact with oxygen. Furthermore, the air carried in solution in the oil gradually diffuses into the gas body and the inert gas takes its place. A striking illustration of the protective effects of inert gas in contact with oil is shown in Fig. 1. The test tube at the extreme left contains clean filtered grade "A" transformer oil sealed in the tube. The second tube contains oil of the same description having nitrogen sealed in with it. The third tube contains another sample of the same oil sealed in with carbon dioxide. The tube on the extreme right contains oil of the same grade but the tube was left open to the atmosphere. No heat tests were made on the first tube which was left in its original state for purposes of comparison, but the second, third and fourth tubes were subjected to an accelerated sludge test at a temperature of 200 deg. cent. for several days. The tubes in which the oil was protected by inert gas plainly show the entire absence of oxidation, while in contrast, the darkened color of the oil in the fourth tube clearly shows the effects of oxidation.

If an electric arc occurs under the oil level of a transformer, due to some internal fault, another undesirable characteristic of transformer oil develops. The effect of an arc is to cause breakdown of the oil and the sudden evolution of hydrogen, methane, carbon dioxide and gases belonging to the unsaturated series, with hydrogen forming by far the largest part of the liberated gases.[2] The evolution of these gases and the sudden increase of pressure due to their rapid expansion are for the purpose of this paper called a "primary explosion." The abnormal pressures due to a primary explosion are transmitted with almost no diminution by the practically incompressible oil to all parts of the case, if it is completely filled with oil, resulting often in strains in these parts which are excessive enough to disrupt the case. A gas or air cushion above the oil level helps greatly to reduce the internal pressures due to a primary explosion.

The vapors arising from transformer oil in normal operation, mixing with air above the oil level, are often thought of as explosive and therefore a menace to the transformer. However, it has been demonstrated that the danger of explosion with such mixtures is very remote.[3] Conditions are entirely different when the gases evolved from oil breakdown due to arcing rise above the oil level. When air is present above the oil level in sufficient amount, the mixture of air and hydrogen, which is the principal product of disrupted oil, is explosive within a wide range of proportions and a violent explosion follows if the gases reach the air space in an incandescent state or if their is arcing or corona to ignite the mixture. Such an occurrence for the purposes of this paper is called a "secondary explosion."

Another undesirable characteristic of transformer oil is its inflammability. Fortunately, troubles due to this characteristic are rare, but occasionally fires have been

1. Transformer Oil Sludge: by C. J. Rodman. *American Electrotechnical Society*, Sept. 19, 1921.

Oil Qualities Desirable for Transformers and Circuit Breakers: C. J. Rodman, *Electrical World*, June 24, 1922.

2. Oil Qualities Desirable for Transformers and Circuit Breakers: C. J. Rodman, *Electrical World*, June 24, 1922.

Causes and Prevention of Explosions in Power Transformers: O. H. Eschholz, *Electrical World*, Sept. 1, 1923.

3. Oil Qualities Desirable for Transformers and Circuit Breakers: C. J. Rodman, *Electrical World*, June 24, 1922.

Characteristics of Transformer Oils: O. H. Eschholz, *Electrical Journal*, February, 1919.

identified with primary and secondary explosions which have been excessive enough to disrupt the transformer case. When inert gas is used above the oil level of a transformer, secondary explosions and fires are impossible because there is no oxygen present to support them.

The advantages to be obtained from the use of inert gas in a transformer have been appreciated for some time and the idea of using it is not new. It has been tried in some form or other with little success, but it is thought that the principles presented in the Inertaire transformer are new and that the arrangement is the first commercially successful application.

Inert Gases

Of the various gases which might serve as protective means for transformers, there are two which are commercially available and which have been shown by laboratory tests to be entirely suitable for the purpose. They are nitrogen and carbon dioxide.

Nitrogen has commonly been called an inert gas because, except at high temperatures and under high pressures, it will not combine with other elements. It is harmless and entirely without action upon oil or any of the materials used in transformer construction. There are two general methods of producing it commercially, both using the cheapest and most abundant material—air. One method removes the oxygen by liquifying the air and then distilling off the nitrogen. This is by far the most common method of producing it commercially. The other general method is to remove the oxygen by allowing it to combine chemically with some material from which the residual gas is easily separated. This residual gas is chiefly nitrogen, but the gases found in the atmosphere which are truly inert—helium, argon, krypton and xenon—are present in relatively small amounts and are equally beneficial. No effort is made, with either of these processes, to separate these gases from the nitrogen of commerce unless there is a demand for them separately. The gas space of the Inertaire transformer contains these gases also, since they are not absorbed in passing through the deoxygenating chemicals.

The carbon dioxide dispensed in the liquid form in steel cylinders is made either from the gases given off in burning lime or by passing an excess of air through incandescent coke. The carbon dioxide resulting from either of these processes is then cooled and liquified by pressure.

The Inertaire Compound

A large amount of research work has been done in the Westinghouse Laboratories in connection with oxidizing agents ranging all the way from the extremely active but poisonous and inflammable yellow phosphorus to the simple, safe and effective mixture now used in the Inertaire transformer. Many of them failed to meet the exacting requirements of the problem involved in the Inertaire scheme on account of such objectionable features as: (a) instability at high temperatures, (b) too slow action at low temperatures, (c) the liberation of water or of hydrogen, or, (d) danger in handling or the fire hazard of supposedly spent material. Data referring to this work will appear later in technical papers.

The chemical compound adopted for the Inertaire transformer is made from an intimate mixture of finely divided copper and ammonium chloride mixed with enough kieselguhr to render the mass porous, and a small amount of concentrated calcium chloride solution. Before it is used, the mixture has a color about like that of cocoa powder, but as it absorbs oxygen it turns a beautiful robin's egg blue. The compound is used in a

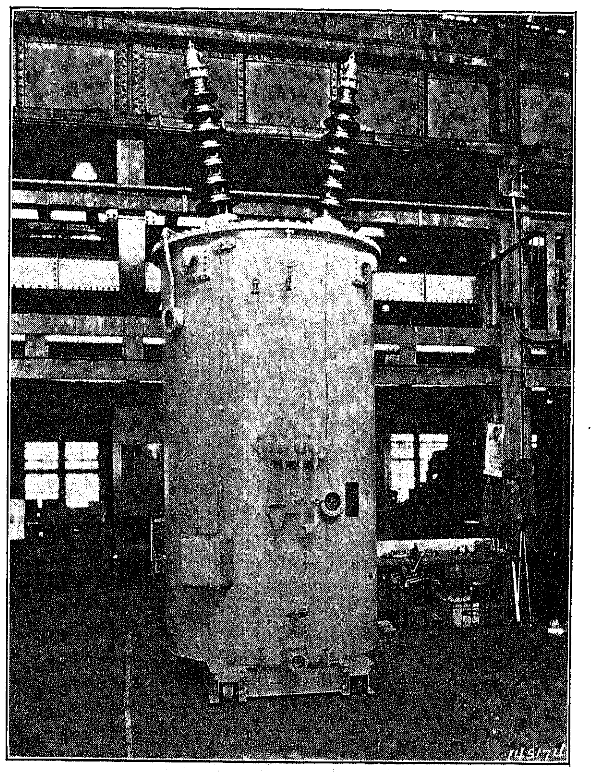

Fig. 2—The Inertaire Transformer

glass container and as the difference of color between the unused and spent portions is very pronounced, it is always easy to see how much unused material there is and hence to form a good estimate of the time when renewal will be necessary.

The reaction by which the compound removes oxygen from the air liberates a very small quantity of ammonia and, like nearly all reactions at moderate temperatures, requires a certain, although extremely small, amount of water. In order to prevent any water from this source or from humid atmosphere being carried over into the transformer, as well as to catch any traces of ammonia vapor which might be present, the nitrogen coming from the mixture is forced to pass through a generous charge of calcium chloride,

which is not only a very satisfactory dehydrator, but an extremely active and effective ammonia absorber as well.

An important and valuable feature of this deoxidizing mixture is its ability to function satisfactorily at all temperatures to which a transformer will ever be subjected. At −37 deg. cent. it continues to remove the oxygen completely, the only noticeable difference between this and at ordinary temperatures being a broadening of the line separating the unused and spent portions. It has also been operated successfully at 85 deg. cent. at which temperature it behaves just as it does at room temperatures.

The Inertaire Transformer in Detail

The equipment referred to in the opening paragraphs of this paper, as it appears on the Inertaire transformer, is shown in Fig. 2. The several features are described in detail in the paragraphs that follow.

The Chemical Container

The deoxygenating and dehydrating chemicals are contained in a glass jar which fits in the cabinet shown in the illustration. This jar contains about 50 lb. of the deoxygenating compound and a relatively small amount of dehydrating material. The cover is sealed in place like the cover of a battery jar with a compound which makes it gas tight. The chemicals, which of course, would begin to function immediately if exposed to the air, are in this way sealed at the manufacturer's works and they need not be handled from that time until they have become exhausted and the jar is discarded. Charging the Inertaire equipment with chemicals

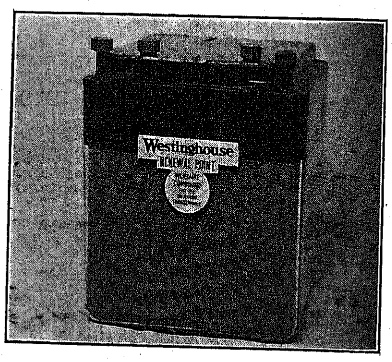

Fig. 3—Inertaire Compound in its Glass Container

requires simply placing the glass jar in the cabinet and making the four pipe connections which are necessary to complete the breathing circuit.

Air is drawn into this container at *a*, Fig. 4, by the suction which in-breathing action on the pipe *d* creates. It is drawn into the bottom of the glass jar and from there it passes upward through the deoxygenating compound. In this passage the oxygen is absorbed by the chemicals and nitrogen passes out at *d* and into the dehydrating chemical at *b*. Whatever moisture and traces of ammonia it contains are absorbed in its progress through this chemical. The dry nitrogen leaves the container at *c* on its way through the breathing regulator and into the gas space of the transformer.

There is a direct relation between the life of this charge of deoxygenating compound and the load cycle of the transformer, for changes of load by their temperature effects produce changes in oil volume and the expansion and contraction of the oil produce breathing. If the load is fairly steady, the breathing is moderate

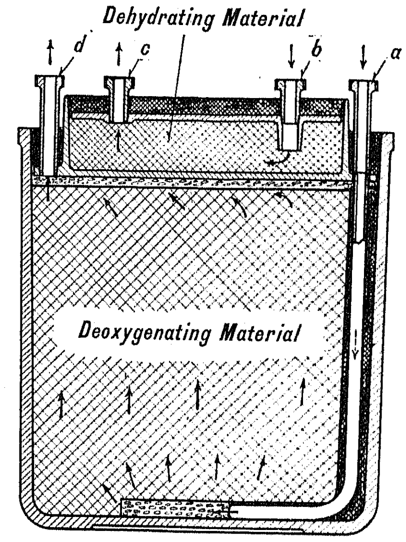

Fig. 4—Showing the Path of the Air through the Chemicals

and a charge of Inertaire compound should last as long as a year. If the load is widely fluctuating the charge may have to be replaced at the end of six months.

The decided change in the color of the deoxygenating compound as it functions is a useful characteristic. The line of demarcation between the used and the unused chemicals slowly advances from the bottom to the top of the jar as the deoxidizing action progresses. It can be seen clearly through the glass walls of the container and it forms an index of the condition of the chemicals and a guide for replacement.

A stock of Inertaire compound can be kept conveniently available for renewal purposes, since the sealed construction of the chemical containers makes it suitable for storing for indefinite periods.

The Breathing Regulator

The breathing regulator is the mechanism that controls and limits the natural breathing action of the transformer. It permits sufficient breathing to create and to maintain the inert gas body. In addition, it helps to conserve the gas body when it is once established. Lastly, by limiting the breathing, it reduces the burden upon the chemical compounds.

The breathing regulator functions as a two-way valve having characteristics which differ with the

direction of the pressure. It permits in-breathing of nitrogen into the transformer case with very little hindrance but when the conditions favor out-breathing, it interposes a barrier which prevents and delays actual out-breathing of nitrogen until the pressure within the transformer case reaches a certain predetermined point. At this point the regulator opens and allows the escape of sufficient nitrogen into the atmosphere to maintain the internal pressure below the predetermined point. In-breathing occurs only when the pressure within the transformer case becomes less than atmospheric. After a rise of oil temperature and an accumulation of pressure, due to the functioning of the breathing regulator, the oil temperature may decrease considerably before this accumulated pressure is reduced to the

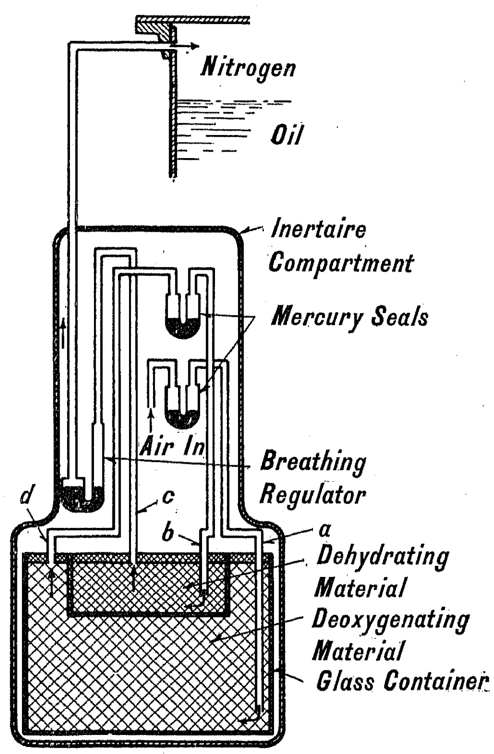

FIG. 5—DIAGRAMMATIC SKETCH SHOWING THE ARRANGEMENT OF THE PARTS OF THE INERTAIRE EQUIPMENT AND THE PATH OF THE BREATHED GAS

atmospheric point. During this period of cooling, which without the regulator would be an in-breathing period, no actual breathing action takes place. The regulator, therefore, not only conserves the gas body when oil temperatures are rising, but also delays and reduces in-breathing when temperatures are falling. Both actions reduce the demands made upon the chemicals and result in economy in the production of the nitrogen.

It is possible, though very unusual, that the load carried by a bank of ordinary transformers might be so steady and so free from fluctuations that the oil level would remain practically stationary and the breathing be nil. The condition of practically no breathing can be maintained with the inertaire equipment, even when there are fluctuations of load so long as they are within the range of the breathing regulator. There are, no doubt, many installations where Inertaire transformers can carry their ordinary cycles of load within the range of the breathing regulator and with no actual breathing and no burden upon the chemicals.

The breathing regulator is simply a mercury manometer, or U-tube, having refinements of structure

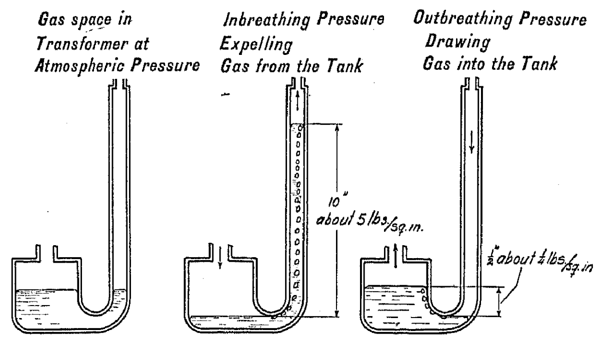

FIG. 6—SKETCHES SHOWING THE U-TUBE ACTION OF THE BREATHING REGULATOR

which cause it to carry out its functions without the possibility of blowing the mercury out of its proper channels when it operates. It is normally a closed valve which cuts off communication between the transformer case and the atmosphere. It becomes an open valve when the pressure in either leg is sufficient to lower the level of the mercury below the bend of the U-tube. When this point is reached communication is established between the two legs of the tube, and gas

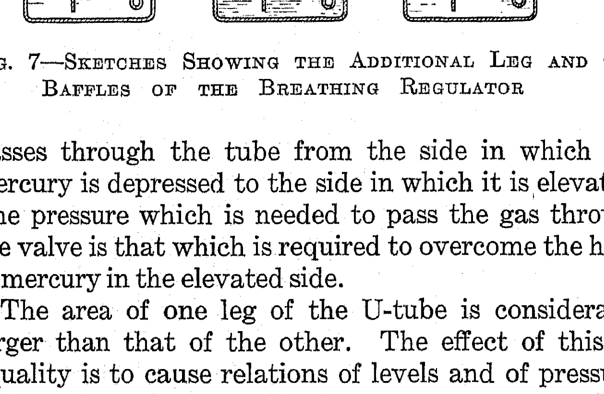

FIG. 7—SKETCHES SHOWING THE ADDITIONAL LEG AND THE BAFFLES OF THE BREATHING REGULATOR

passes through the tube from the side in which the mercury is depressed to the side in which it is elevated. The pressure which is needed to pass the gas through the valve is that which is required to overcome the head of mercury in the elevated side.

The area of one leg of the U-tube is considerably larger than that of the other. The effect of this inequality is to cause relations of levels and of pressures which differ with the direction of the pressure. It is

evident from Fig. 6 that greater pressure is needed to pass gas from the leg of large area to the leg of small area than in the reverse direction.

The large leg is connected to the gas space above the oil level of the transformer. The small leg is connected to the jar containing the chemicals. To breathe outwardly, the pressure in the transformer case must be enough to raise the mercury level in the small leg to a height of about 10 in., *i. e.*, 5 lb. per sq. in. In-breathing, on the other hand, can take place against a head of ½ in. of mercury, or about ¼ lb. per sq. in.

Fig. 7 shows schematically how the actual construction of the regulator combines the principle of the simple U-tube with a duplicate small leg and a set of baffles. When the excess is in the large leg of the regulator, the mercury is depressed in that leg and elevated in both of the small legs *a* and *b*. When the pressure reaches 5 lb. per sq. in. in the transformer

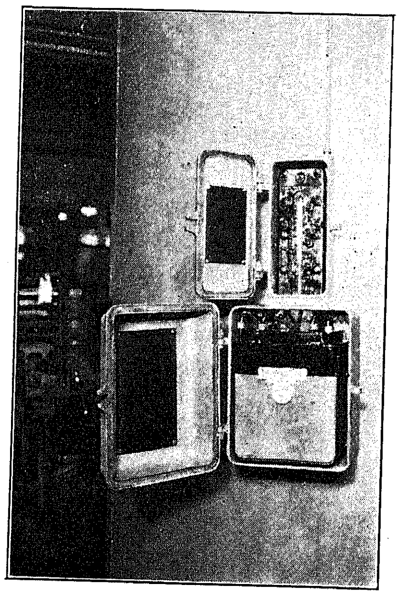

Fig. 8—The Cabinet Containing the Inertaire Compound and the Breathing Regulator

case and in the large leg, a passageway is opened into the leg *a*, but the mercury still seals off the connection into leg *b*. The nitrogen consequently finds its way up through the mercury in leg *a* and out to the atmosphere through the chemical container.

The purpose of the extra leg and the baffles is to prevent beyond question the carrying over of mercury with the gas when movement of the gas in either direction takes place. When the gas passes upward through the leg *a*, whatever mercury might be thrown over the top of the tube by a particularly high breathing rate is deflected back into the tube *b* by the baffle shown at *c*. When the action is in the reverse direction, the movement of nitrogen is down the leg *a* and the baffle at *d* serves to return to the well of the large leg any mercury that might tend to pass over with the nitrogen when the in-breathing rate is particularly high. This type of breathing regulator has been tested with breathing rates in both directions, ranging from small values to rates which are twenty times as high as the maximum that normally occur in power transformer service. No mercury has been carried over with the gas in these tests and the construction has been demonstrated to be an effective guard against such an occurrence.

In the actual construction of the breathing regulator, shown in Fig. 8, the mercury in leg *a* is visible through a window at the front of the regulator. A scale marked in pounds per square inch is provided in the cover adjacent to this window. The level of the mercury in connection with this scale indicates directly the condition of pressure within the transformer case. The small mercury seals, which are shown diagrammatically in Fig. 5, are placed in the middle portion of the breathing regulator. At times when there is no breathing, one of these seals isolates the deoxygenating chemicals from the atmosphere and one serves as a barrier between these chemicals and the dehydrating chemical. The levels of the mercury in these seals are visible through the small windows shown in Fig. 8.

The curves in Figs. 9 to 16 show the breathing action of an 8333-kv-a. transformer under various conditions. They illustrate the performance of the breathing regulator in limiting the breathing action, thus conserving the nitrogen body and the deoxygenating and dehydrating chemicals.

TABLE I—DATA FROM FIGS. 9 TO 16, SHOWING REDUCTION OF BREATHING RATES DUE TO THE BREATHING REGULATOR

Load Conditions	Air Breathed, in Cu. Ft. per Hr.		
	Without Regulator	With Regulator	Reduction of Breathing
Full load, Figures 9 and 13,.......	16.9	10.0	41 Per cent
" " " 10 and 14,.......	11.3	3.53	69 Per cent
¾ " " 11 and 15,.......	4.2	0	100 Per cent
" " " 12 and 16,.......	4.2	0	100 Per cent

Initial Displacement of Air from the Gas Space

Since it is desirable to start a transformer in service with inert gas in the gas space rather than to wait for the natural breathing gradually to dilute and displace the air originally present, some means should be provided for either removing the oxygen quickly from the air already in the transformer, or replacing it at once with an inert gas. While the former method is entirely feasible, the method of displacing the air immediately with inert gas from a cylinder is so simple and so convenient that it is recognized as the preferred practise.

The Inertaire equipment includes a fitting at which a gas cylinder may be connected. It is provided with a special device which restricts the flow of gas, with the high pressure of the cylinder behind it, to a reduced

rate which effectively and safely displaces the air from the gas space in a short time. When the cylinder is exhausted, it may be returned to the nearest of the numerous points of manufacture of nitrogen and a fresh supply may easily be obtained, if at any time it becomes necessary to open the transformer case for inspection.

the abnormal pressure following an arc is often great enough to disrupt the tank at its weakest point if no effective relief means are provided.

The accepted means of protecting a transformer case against abnormal internal pressure is to provide a thin diaphragm of some suitable material which is ruptured by the pressure should it become dangerous.

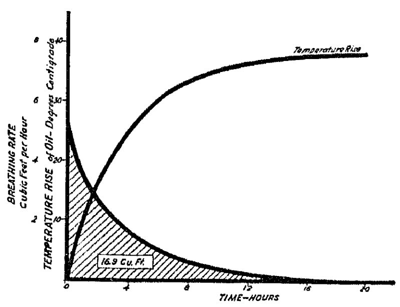

Fig. 9—Full Load, Starting from Ambient Temperature without Breathing Regulator

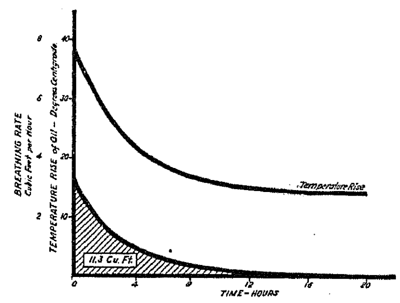

Fig. 10—Full Load Cut Off and Water Continued. No Breathing Regulator

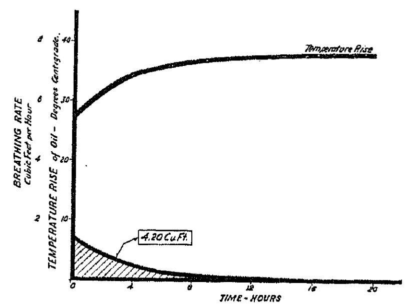

Fig. 11—Full Load, Starting from Three-Fourths Load Temperatures, without Breathing Regulator

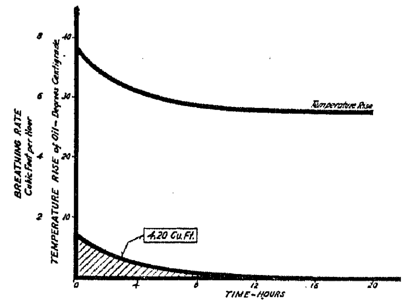

Fig. 12—From Full Load to Three-Fourths Load, Water Continued. No Breathing Regulator

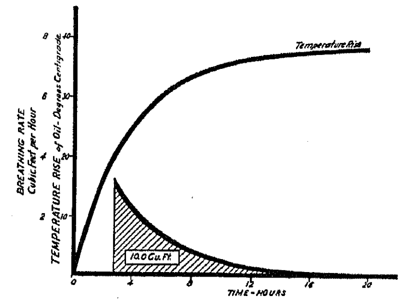

Fig. 13—Full Load, Starting at Ambient Temperature, with a Breathing Regulator

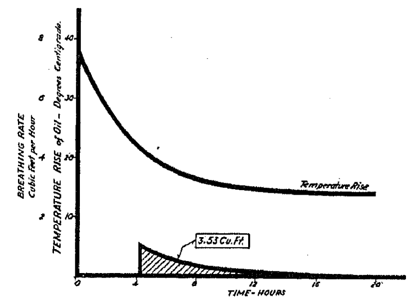

Fig. 14—Full Load Cut Off and Water Continued, with a Breathing Regulator

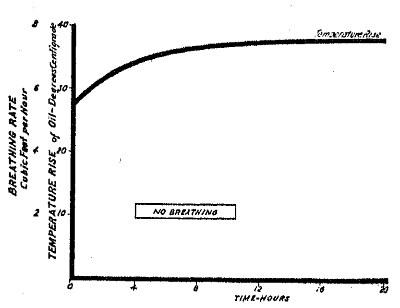

Fig. 15—Full Load, Starting from Three-Fourths Load Temperatures, with a Breathing Regulator

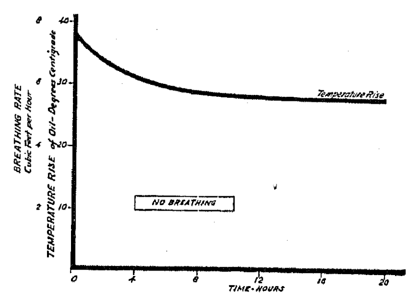

Fig. 16—From Full Load to Three-Fourths Load, Water Continued, with a Breathing Regulator

Curves Showing the Temperature Rise of Oil and the Breathing Rates of an 8333-Kv-a. Transformer. The Shaded Areas Show the Air Breathed and Deoxidized by the Inertaire Equipment

The Relief Device for Abnormal Pressure

The inert gas body above the oil level of an Inertaire transformer makes explosions in that space impossible. It is always possible, however, that some fault under the oil level may result in a primary explosion. While the wave-front of pressure created in this way is not so steep as that of a secondary explosion of hydrogen and air above the oil level and the results are not so violent,

The relief device of the Inertaire transformer forms part of the manhole opening in the cover. It consists of a circular diaphragm of thin sheet micarta which is clamped to a support placed directly beneath the manhole cover. The manhole cover itself rests upon the gasketed surface of the diaphragm support and it is free to move upwardly except for its weight and the retarding action of a number of heavy springs. The diaphragm is clamped in position with a ring and a

gasket, forming under normal conditions a seal which is gas and oil tight. The completely assembled mechanism is bolted down against a gasket on a machined flange which forms part of the transformer cover. This construction has the advantage of effectively exposing a large diaphragm area directly to the seat of internal disturbances and it requires no room in addition to that taken up by the usual manhole opening.

If an abnormal pressure develops inside the transformer case, the diaphragm is ruptured. The impact of the pressure against the manhole cover raises it almost instantly and forms an annular opening around

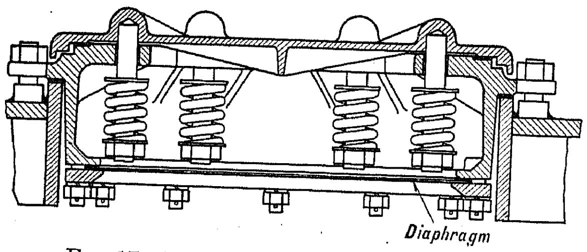

Fig. 17—The Inertaire Relief Device

its periphery through which the pressure is relieved. The springs absorb the energy imparted to the rising cover and quickly bring it back to its original position, in which it closes the annular opening. This prevents the discharge of oil or the entrance of water or diffusion of air into the case.

When the manhole opening is to be used for entrance into the transformer case, the outer row of bolts is removed and the complete assembly is withdrawn, leaving a full-size opening in the cover. It is unnecessary at such a time to disturb the diaphragm

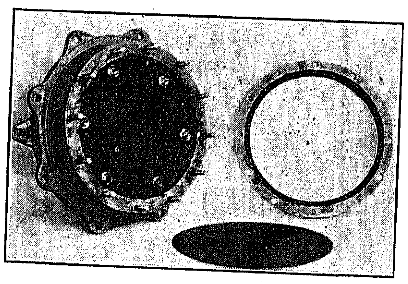

Fig. 18—Parts of the Inertaire Relief Device

itself. Fig. 17 shows a line drawing of a section of the complete mechanism and Fig. 18 shows the parts.

The selection of a suitable material for the diaphragm is a matter of some importance. In order to be reliable it must have a uniform rupturing characteristic under the conditions presented in relieving abnormal pressures. It must, at the same time, be a material of sufficiently substantial nature to be handled easily without the danger of accidental breakage. Of the three principal materials available for the purpose—sheet micarta, glass and thin metals—sheet micarta has been proven to be the most satisfactory. It is a laminated fibrous material which is bon[d] bakelite under heat and pressure. It is una[ff] oil, gas or water. It has strength, tough[n] flexibility and yet when subjected to shock in of a thin sheet securely clamped around its edge an almost brittle characteristic not unlike glass causes it to be shattered with a very fair [d] uniformity in its performance. The mica[r] phragms used in the Inertaire relief device are in thickness and 13 in. in diameter. These dia[p] rupture at approximately 10 lb. per sq. in.

An alarm mechanism is provided as part Inertaire relief device which gives a visual sem indication at the transformer and a distant alarm when the diaphragm is ruptured. When the m cover rises, a rod which is hooked in a lug on th is released. A spring causes it to project a sem over the edge of the cover and to close a set o contacts. The semaphore enables an attend

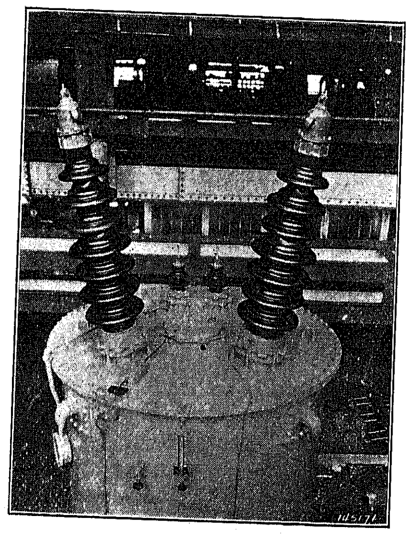

Fig. 19—Showing the Manhole Relief and Alarm D

locate the particular transformer in which troubl occurred. The alarm device may be seen on the of the transformer shown in Fig. 19.

Tests of an Inertaire Transformer with Abnormal Internal Pressures Suddenly Applied

A number of tests was made in a preliminary way on a small scale to illustrate what happens in a when arcing occurs under the oil level. A round b iron tank, 16 in. in diameter and 28 in. high, desi for a circuit breaker, was used for these tests and were created between electrodes placed under th level and fed by a 50-kv-a. transformer.

Fig. 20 shows a time-pressure curve obtained wh gas space above the oil level, with a depth of 4½ was filled with nitrogen. The rise in the pres curve is very gradual and the maximum pressur limited to a very small value. It will be observed f

the curve that there is no secondary explosion because the nitrogen above the oil level makes such an explosion impossible.

Fig. 21 shows what happened with all the conditions the same except that the space above the oil level was filled with air. At first, the rise of pressure was gradual, as before, but when the mixture of disruptured oil

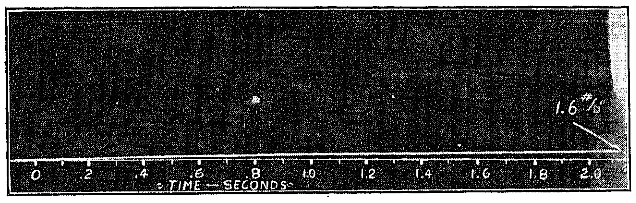

FIG. 20—"PRESSUREGRAM" OF A SMALL-SCALE PRIMARY EXPLOSION, WITH A CUSHION OF NITROGEN

gases and air in the air space came within the explosive limits, a secondary explosion occurred and it will be noticed that the pressure jumped very suddenly to a comparatively high value.

Fig. 22 shows the pressure resulting under the same conditions, except that the tank was filled completely,

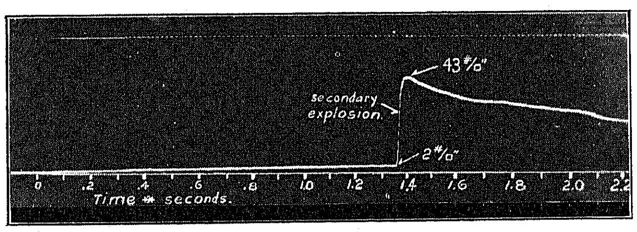

FIG. 21—"PRESSUREGRAM" OF SMALL-SCALE PRIMARY AND SECONDARY EXPLOSIONS, WITH A CUSHION OF AIR

with no gas or air space. There is, of course, no secondary explosion but the pressures due to the primary explosion are seen to be very much greater than those due to primary explosions of Figs. 20 and 21. These curves illustrate the protective effects of a compressible cushion of gas or air in reducing the

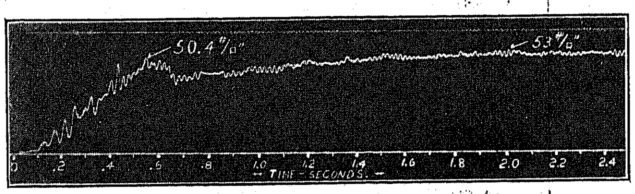

FIG. 22—"PRESSUREGRAM" OF A SMALL-SCALE PRIMARY EXPLOSION, WITH TANK COMPLETELY FILLED WITH OIL

internal pressures due to arcing, and the value of the inert gas as the cushion.

Following these small-scale tests, a complete set of explosion tests was planned and carried out on a full-size transformer tank to demonstrate first, whether the standard case construction for a power transformer would successfully withstand internal pressures equivalent to those created by arcs under the oil level; second, to determine how the Inertaire diaphragm relief device would function when called upon to relieve internal pressures of an explosive nature; and third, to get actual data regarding the effects on internal pressures of gas cushions of different volumes.

As a convenient method of obtaining definitely correlated results in large scale tests, a plan of using gunpowder bombs for producing explosive effects was worked out and adopted. These bombs were designed and made up in standard units, 1 ¼ in. in diameter and about 3 in. long, containing 40 g. of Dupont FFFG black powder. Considerable care was taken in weighing the powder and in loading the tin containers in order to get uniformity. An electric squib was embedded in the powder for firing purposes. Having a standardized unit of this kind made it possible, by using a number of them in multiple, to get a range of explosions of definitely related intensities.

Considerable thought was given to suitable means for measuring the instantaneous pressures during the tests. Through the courtesy of the engineers of the Bureau of Mines in Pittsburgh, the engineers of the

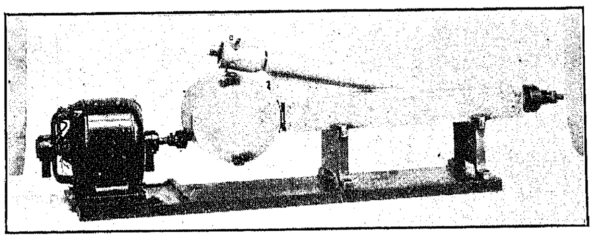

FIG. 23—PRESSURE INDICATOR, BUREAU OF MINES TYPE

Westinghouse Company were fortunate enough to obtain instruments which had been developed and used successfully for the purpose of recording the instantaneous pressures occurring in mine explosions.

This pressure indicator, shown in Fig. 23, has a specially hardened circular steel disk clamped around its periphery in a frame and having one face exposed to the pressure to be measured. Deflections of this disk are transmitted by a small pin to a pivoted mirror, which reflects a beam of light from a fixed point source onto a strip of sensitized film on a revolving drum. Instantaneous pressures exerted on the steel disk are faithfully reproduced by the line traced on the film. A time scale is also recorded on the film. With this time scale and a calibration of pressures, the resulting pressuregrams may be thoroughly analyzed and studied.

The transformer case used for the explosion tests was a round boiler iron tank, 72 in. in diameter and 134 in. high, belonging to a 6667-kv-a. 66000-volt transformer. A flat boiler-iron cover of standard design was bolted down on a ring flange with a cork gasket between. The usual bushings were removed, since they played no part in the tests to be made and their openings were covered with blind flanges. The Inertaire diaphragm

relief device was mounted in the cover and used in all tests except those in which the tank was completely filled. A transformer was placed inside the tank in order to produce the same interference or reflections of pressure waves from the surfaces of the transformer that might be present in ordinary service. One of the pressure indicators was used to measure pressures in the gas space, while three others of the same type were placed to record the pressures at different heights under the oil level. The gas given off by the exploding bombs consisted almost entirely of carbon dioxide

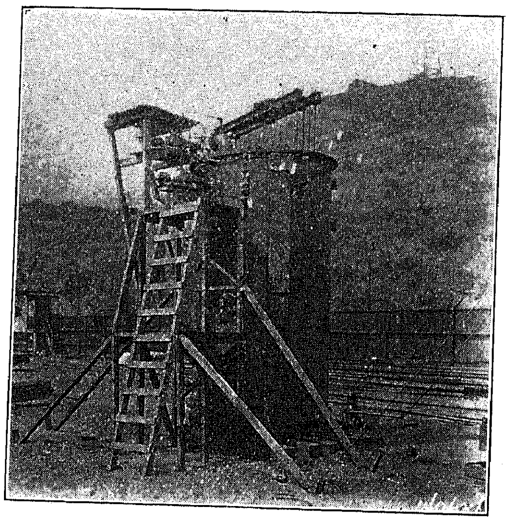

Fig. 24—Set-Up of Transformer for Explosion Tests

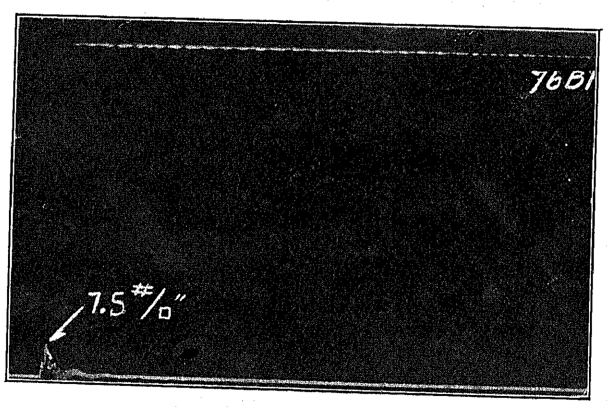

Fig. 25—"Pressuregram" of a Large-Scale Explosion with 120 Grams of Powder. Depth of Gas Space, 12 in.

and nitrogen, and since in this case there was no danger of a secondary explosion above the oil level, no effort was made during the tests to maintain an inert gas in the gas space.

The pressure indicators and the firing of the bombs were controlled electrically from a distance with all of the operations effected automatically in their proper sequence. The throwing of a single switch first started the d-c. motors operating the film drums and after allowing them sufficient time to attain constant speed, lighted the timing lamps and exploded the bombs. Fig. 24 shows the arrangement of the transformer case, the pressure indicators and the wires leading to the control point.

Approximately 90 explosion tests were made with the

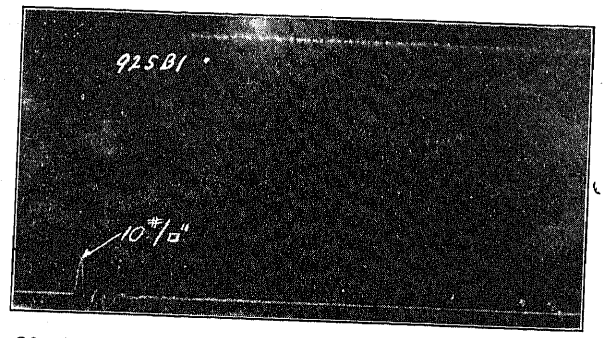

Fig. 26—"Pressuregram" of a Large-Scale Explosion with 120 Grams of Powder. Depth of Gas Space, 10 In.

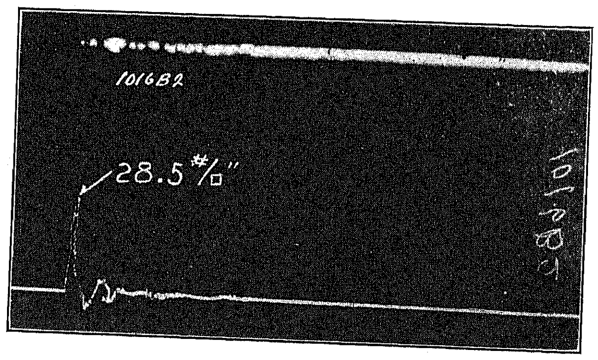

Fig. 27—"Pressuregram" of a Large-Scale Explosion with 120 Grams of Powder. Depth of Gas Space, 4 In.

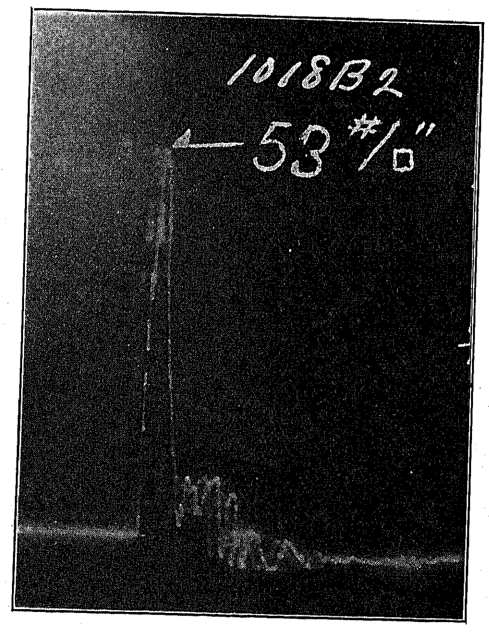

Fig. 28—"Pressuregram" of a Large-Scale Explosion with 120 Grams of Powder, with Tank Completely Filled with Oil

gunpowder bombs. During these tests gas spaces were used ranging from a depth of 12 in. below the cover to zero, where the tank was completely filled with oil. In the tests with the tank completely filled with

oil, an expansion tank was used for the overflow, and the usual relief diaphragm was placed in a pipe elbow above the oil level in the expansion tank. In all tests the bombs were placed uniformly 24 in. below the cover, near the center of the tank.

A detailed report of the results of these explosion tests must be reserved for other papers on the subject, since it would be too lengthy to include in the present paper, but typical results are shown in the pressuregrams in Figs. 25 to 28.

Fig. 29 shows a curve of maximum peak pressures measured in the gas space for different depths of gas space measured in inches. The curve shows in a striking way the important effect of the gas cushion in reducing internal pressures. It is interesting to note that the curve is beginning to flatten out when gas spaces corresponding roughly to the present air space practise are reached.

A rough idea of the equivalent arc-energy of these gunpowder explosions may be obtained from cal-

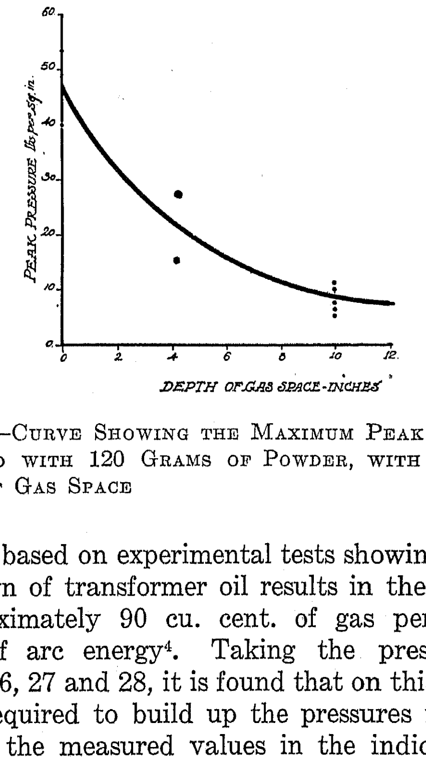

Fig. 29—Curve Showing the Maximum Peak Pressures Developed with 120 Grams of Powder, with Different Depths of Gas Space

culations based on experimental tests showing that the breakdown of transformer oil results in the evolution of approximately 90 cu. cent. of gas per kilowatt second of arc energy[4]. Taking the pressuregrams in Figs. 26, 27 and 28, it is found that on this basis the energy required to build up the pressures in the gas space to the measured values in the indicated time interval appear as given in Table II.

TABLE II
CALCULATED EQUIVALENT ARC-ENERGIES FOR THE EXPLOSIONS OF FIGS 25 TO 27.

Figure	Depth of Gas Space	Peak Pressure Lb./Sq. Inch	Time Interval to Peak-Seconds	Kilowatts Seconds	Kilowatts
26	12 in.	7.5	0.016	4550	285,000
27	10 in.	10.	0.018	5050	280,000
28	4 in.	28.5	0.025	5800	232,000

4. Arc Action on Some Liquid Insulating Compounds, by C. J. Rodman, *American Electrochemical Society*, Sept. 25, 1922.

These particular explosions were each made with three 40-g. bombs representing the same explosion energy and the values for arc energy check each other within reasonable limits of accuracy. They indicate that it is reasonable to think that the explosions made with 120 g. of gunpowder are not of a different order from those which can be expected from a fault in a transformer under modern conditions.

Explosion tests were made with as many as twelve 40-g. bombs and from the foregoing tabulation it is evident that they represent arc energy in extremely large amounts. In one of the early tests made with 200 g. of powder, one of the vertical welded seams of the tank was opened up for a distance of several inches; it was found that in making the tank these vertical seams had been welded only on the outside, instead of both inside and outside. After the seams were welded also on the inside the case successfully withstood all of the later explosions of greater intensity. In all of the explosions made with gas cushions, the Inertaire relief device functioned perfectly and relieved the internal pressures without damage to the case, with the exception of the incident just noted. With the exception of those explosions made with gas spaces of such small volume that the oil was on a level with, or above, the diaphragm, which is not a condition to be met with in practise, the quick operation of the relief device in returning to its closed position prevented the throwing of oil, which is remarkable, considering the severity of the explosions.

The principal conclusions to be drawn from these tests are:

1. Secondary explosions with their steep wavefronts are eliminated by the gas space of the Inertaire transformer.
2. The gas cushion is very effective in reducing the internal pressures due to a primary explosion under the oil level.
3. Round boiler iron tanks of standard construction, when protected by a gas cushion and an effective relief device, are sufficiently strong to withstand the shock of extremely abnormal internal pressures of short duration.
4. The Inertaire relief device is remarkably effective in protecting the transformer case against the shocks of extremely abnormal internal pressures and are quick enough in operation to prevent the throwing of oil at such times.

Resume of the Advantages of the Inertaire Transformer

The blanket of inert gas gives the maximum of protection to the oil against deterioration from oxidation or moisture. Sludging troubles are minimized and no filtering or cleaning of the oil is necessary. It is expected that the oil will even improve with service, due to the gradual elimination of the dissolved air.

The possibilities of fires and explosions above the oil level are eliminated. It should represent a better fire risk than other types of transformers.

The destructive effects of internal pressure due to arcing under the oil level are minimized by the gas cushion and the relief device.

It is simple and automatic in operation and the cost of maintenance is thought to be not greater, and perhaps less, than the usual cost of filtering and cleaning the oil.

The authors wish to acknowledge the work done in connection with the Inertaire Transformer by L. H. Hill, A. J. Maslin and L. E. Rossel, of the Westinghouse Company's Engineering Department, and to C. J. Rodman, A. H. Maude, W. C. Wilharm, C. A. Styer and J. G. Ford of the Westinghouse Research Laboratory.

The chemical part of the research work on deoxidizing materials has been under the immediate direction of C. J. Rodman. It is very largely due to his vision, persistence and faith, that the problem has been solved and the authors wish to express here their appreciation of his work.

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Discussion

A. H. Maude: Transformer oil, in common with most liquids dissolves the gases of the atmosphere to some slight extent. Though the quantity of oxygen which can be held in solution by transformer oil is very slight when expressed in weight units and insufficient to produce any appreciable quantity of oxidation products, it would nevertheless, be sufficient to destroy the effectiveness of the inert atmosphere of the Inertaire transformer should any large part of it diffuse from the oil into this atmosphere. It is well known that any gas thus held in solution will readily diffuse out into an atmosphere of some other gas until a certain equilibrium depending on the partial pressures and solubilities of the gases present is reached.

The above facts being indisputable it became necessary in the course of the development of the Inertaire transformer to go pretty thoroughly into this matter and determine some solubility constants not heretofore published and work out methods to analyze oil for these gases which will be published in a future paper. However, the solubility of the gases of the atmosphere in "Wemco A" oil can now be stated as follows:

	Gas temp. Deg. Cent.	Coeff. of absorption cc. of gas per cc. of solvent.	Solubility g. of gas per g. of solvent
Nitrogen.......	25	.0925	.000127
Oxygen........	25	.1705	.000266

From these figures by the application of the law of partial pressures it follows that this oil when in equilibrium with air will contain per cc. the following in solution:

Nitrogen...............	.0734 cc.
Oxygen................	.0353 cc.
Total..................	.1087 cc.

Now, if this oil has stood in contact with air so long as to reach an equilibrium with it and is then used in an Inertaire transformer it will, as soon as the air above the oil has been swept out by the inert gas, commence to liberate the oxygen it holds in solution.

Another factor of importance in this discussion is the chemical combination which is known to go on between oxygen and transformer oil and possibly goes on with structural materials of the transformer as well.

To find out what influence these factors, *i. e.*, physical solution and chemical combination, will have on the Inertaire transformer atmosphere it seems desirable to answer, at any rate approximately, the following questions.

1. How much oxygen will, in this way, be introduced into the inert atmosphere?
2. Will this oxygen be chemically reabsorbed and, if so, how quickly?
3. How quickly will oil exposed to air dissolve oxygen?
4. How much oxygen does transformer oil customarily carry in solution?

From the experimental data available it may be said that, if air-saturated oil be charged into an Inertaire transformer in which a gas space of the usual proportions is used, between 7 and 8 per cent of oxygen may be expected in the atmosphere one or two days after all air has been swept out, this quantity of oxygen having diffused out of the oil in which it had been held in physical solution. The oxygen will after this be slowly reabsorbed by chemical combination with the oil, but would take many weeks to be eliminated.

These difficulties can be overcome by using oil free from dissolved oxygen. The oil could be freed from this either by treatment under vacuum, by bubbling nitrogen through it or else the atmosphere can be cleared of oxygen on 2 or 3 successive days as has been heretofore done in the installation of the inertaire transformer. Otherwise, the nitrogen used initially to displace the air in the transformer can be bubbled through the oil thus displacing oxygen in solution as well as the oxygen in the atmosphere.

With regard to the rate at which oil will take up a gas by solution, this seems to be considerable and leaves no doubt that oil left exposed to the atmosphere or handled with pumps drawing in air around glands or pistons will contain some considerable quantity of oxygen in solution.

It was experimentally determined that 10 sq. cm. of tranquil oil surface with a large depth of gas-free oil beneath it will absorb 1 cc. of gas in 7 minutes.

The following results were obtained by analyzing various samples of "Wemco A" oil for dissolved gases:

	Per cent by Volume of Gas. N_2	O_2	CO_2	Total
Oil from sealed can..........	6.1	.2	.0	6.3
Oil from pump from storage tank (No. 1)............	6.0	2.5	.2	8.7
Oil from pump from storage tank (No. 2)............	9.1	3.3	.2	12.6
Oil from transformer with air atmosphere............	7.0	2.2	.2	9.4

These figures point to the conclusion that oil except when it comes from a sealed can must be expected to contain oxygen in solution and precautions must be taken in some one of the above ways of preventing this oxygen from contaminating and reducing the effectiveness of the inertaire gas cushion as a preventative of explosion and as a preserver of the oil.

L. H. Hill: It was my privilege to supervise the installation of the first Inertaire transformers, seven 8333 kv-a., 66000 volt units at the Grand Tower Station of the Middle West Power Company.

One of the interesting things noted in connection with this installation was that the oxygen content in the gas space before any nitrogen was blown or drawn in was only about 17 per cent, instead of 20.7 per cent, which is the oxygen content of the atmosphere. These transformers had been set up and the gas space closed some two or three months before these analyses were made and no breathing is believed to have taken place in that time. The reduction in oxygen content is largely due to slow oxidation of the oil. Oxygen is also somewhat more soluble in oil than nitrogen, which would also make for a reduction in oxygen content in the gas space.

This question of absorbed gases in the oil appears in another connection. The oxygen was initially removed from the gas space of the Middle West Transformers by blowing in nitrogen from a cylinder. The oxygen was reduced to less than 1 per cent by this method. After the transformer had stood over night it was found that the oxygen had come up to 3 or 4 per cent. After blowing down again to less than 1 per cent, the oxygen came up to about 2 per cent the next day and remained around this figure.

Tests previously made have shown that practically perfect mixing of the gases is obtained during the blowing-in process, so it is logical to assume that the increase is due to the oxygen coming out of the oil.

During the first week that the transformers were in operation the load curve showed about one-third to one-half full load for eight hours during the day, with no load during the remainder of the time. The breathing regulators prevented breathing entirely during this period, which shows the value of using a breathing regulator with the Inertaire transformer.

D. R. Dalzell: I think we will all agree that there are two distinct types of explosions that have occurred in transformers, namely, the rapid generation of gas due to decomposition of the oil by internal arcing, or, as it is termed by Messrs. Dann and Kellogg, a primary explosion; and also an explosion of secondary nature, due to the ignition of a gaseous mixture above the oil level in the tank. In the first case, the rate of increase of pressure is a function of the energy liberated at the fault, and occasionally may be so great as to produce effects much the same as those following a secondary explosion.

In the case of the ordinary "open" type of transformers not having their tanks completely filled with oil, a few secondary explosions have been experienced. These usually resulted in the cracking or breaking of the cover and sometimes the tank, and in a few cases burning oil was thrown about, with serious damage to the station. The usual cause of these explosions was arcing or static discharge on the exposed portion of the leads above the oil level, igniting a mixture of air and gases due to internal arcing in the oil.

The primary explosions, having a less sudden increase in pressure, usually blew off the manhole cover, and were not generally attended by burning oil above the surface. These were caused an internal failure in the windings.

It was realized that although explosions had occurred on only a very small percentage of transformers, yet their seriousness warranted all possible study to determine means of preventing them and minimizing their consequences.

Of the study made on this subject, beginning as early as 1913, it will be of interest to relate briefly those tests illustrating that a relief diaphragm would not protect the tank from secondary explosions.

Tests were made to determine whether an explosive mixture of gas and air could be produced above the oil level by arcing under oil, and also whether a diaphragm covering the manhole opening would break and prevent damage to the tank in case of a secondary explosion. A large transformer tank was nearly filled with oil, and arcs passed between electrodes under the oil, thereby generating gas which mixed with the air. Horn fiber diaphragms of thickness ranging from 5 to 10 mils. were placed over the manhole opening. A spark-gap was located above the oil to explode the gas.

Preliminary tests were made to determine the most explosive mixtures of gas and air, and when this value, about 40 per cent gas, was obtained, the gas exploded, resulting in the breaking of cover bolts, breaking and lifting of the cover, and scattering of burning oil. The diaphragm was destroyed, but failed to protect the cover.

Similar tests were made using an oil seal between the cover and tank. In this case, the tank was nearly filled with water, then oil added up to the normal oil level. Gas was introduced to the air space and the mixture ignited. Again the cover was blown into the air, proving that the seal was of no value.

One of the earliest steps (about 1910) taken to prevent the ignition of gas above the oil, was to place a metal shield around the lead, extending from the cover to a point below oil level, and grounded to the cover. This ground shield of course prevented arcing between leads or any static discharges, and has since been generally adopted as a standard construction in terminal bushings.

The next scheme tried was that of filling the space in the tank above the oil with a gas chemically inert, using a gasometer with which to maintain slight constant outward pressure and a tube of gas from which the gasometer was replenished when necessary. Patent No. 1,326,049 covering this combination, was issued to Mr. F. C. Green, a General Electric Field Engineer, (now deceased). The first test, using carbon dioxide gas, was on some 750-kv-a. transformers at the Remington Arms Plant at Ilion, N. Y., in March, 1916. In May of the same year, a 5000-kv-a. transformer of this type was installed at the Narragansett Electric Co., Providence, R. I. Considerable difficulty was experienced in reducing the outward leakage of gas to a reasonable amount, since the sealed cover construction, as we now know it, had not yet been completely developed. While this scheme no doubt would have been effective, yet it was at best an inconvenience to the operator to maintain.

In the meantime, the conservator was being developed by the General Electric Co., the first installation being made at Laurinsburg, N. C., later in 1916. This construction, with its completely oil-filled main tank, absolutely prevented any further secondary explosions. A pressure relief, consisting of a thin metal diaphragm mounted above the oil level in the end of a large pipe on the cover, was provided to take care of any increase in pressure due to generation of gas from internal failures.

In addition to this function, the conservator also prevented moisture from entering the main transformer tank and prevented air from coming in contact with hot oil, together with the re-

sulting possibility of sludging. Therefore, since we believed that the conservator provided all the advantages of inert gas, without requiring any attention from the operator, and that the possibility of damage to the tank from primary explosions was so remote, no further experiments of this nature were made.

Our subsequent experience with this type of transformer has been that to date, with about 1600 units in service having a total rating of over 8,000,000 kv-a., so far as we know there have occurred explosions in only 15 units, all of these being of the primary type of explosion previously mentioned. In only 5 has any damage resulted to the tank or cover, the diaphragm in all other cases relieving the pressure. Where tank damage resulted, it was in the earlier types using a metal diaphragm, which was found bulged, but did not break quickly enough to prevent the manhole cover from being blown off or the cover from cracking. Of course, as the diaphragm bulged, it automatically increased its strength against bursting.

These cases emphasized the necessity of using a diaphragm having a definite breaking point, independent of the suddenness of the applied pressure. With this in view, thin glass was adopted several years ago, and in no case has this failed to function properly.

The service record of the conservator units just outlined indicates that the standard construction of conservator with pressure relief will dissipate any except extraordinary pressures without damage to a reasonably strong tank.

In the application of an inert gas as a further precaution a against harmful pressures, we have favored the maintainence of the gas pressure by an auxiliary supply tank rather than by an automatic breather, primarily because of the uncertainty of the latter type and secondarily because of its demand on operating attention. It should not be overlooked that if a slight leak develops in any of the joints above the oil level when an automatic breather is used, there is no very convenient means of detecting the presence of oxygen and moisture which may be drawn in through such leaks, whereas with a slight pressure maintained at all times by an auxiliary tank, we have an easy and positive method of discovering leaks in the same manner as oil leaks are discovered in a conservator transformer.

Any inert gas may be applied to the conservator transformer by extending the pipe from the auxiliary oil tank down into the main tank far enough to form a pocket for the gas which may be introduced from commercial containers. Any outward leakage of gas would be indicated by a gradual dropping of the oil in the oil gage on the conservator. Expansion and contraction of the oil takes place in the usual way in the conservator, and the gas-cushion remains undisturbed. Such a pocket may also be formed by inverted chambers located at any convenient place in the tank directly under the cover, and when so located, there is retained the advantage, inherent in the conservator construction, of having any leak made visible by oil. This construction also obviates the necessity for periodic changing of the deoxidizing chemicals in the breather of the "Inertaire" construction.

C. F. Scott: The authors of the paper have set out the desirable results to be obtained in the handling of transformer oil and then have shown how, by a very simple means, these results are accomplished. The simplicity of the results is sometimes an indication of the difficult path and the many problems which have occurred in the solution of the problem. The result in this case seems to be a very happy one as the effective prevention of oxidation is secured by means which can be added in a simple way to present transformers provided their cases are tight.

The operation of electrical apparatus meets various limitations. Temperature is one of the most serious and in general the reason why temperature is serious is because it causes deterioration of materials. I remember the story of a discussion between Mr. Westinghouse and a theoretical engineer who was quibbling about rise of temperature and insisting upon some very low figures. Mr. Westinghouse suddenly asked him why not operate the machine red hot if it did not damage the insulation? The present paper does show that when oxygen is not present the oil maintains its integrity at a very much higher temperature, hence the temperature limitations usually applied to transformers can be very considerably raised in so far as the oil itself is concerned. If other materials are found which can be used as solid insulating material in transformers which are durable at higher temperatures than at present, then the way is open for operating them at present temperatures with higher factors of safety. Under emergency conditions of extreme overload, Inertaire transformers should be found serviceable and safe.

This transformer, therefore, appears to me to meet satisfactorily some of the present difficulties and limitations in transformer operation by preserving the oil, and it also indicates a way of making safe the operation of the transformer at higher loads and higher temperatures.

W. M. Dann: The discussion on this paper seems to bring out the fact that it is agreed that an inert gas is a very desirable thing to use in a transformer. A point that ought not to be overlooked is that while the freedom from the dangers of both primary and secondary explosions which the Inertaire transformer gives is a decided advantage from the insurance point of view, it is by no means the prime consideration in advocating its use. The elimination of the injurious effects of oxygen on the oil is a feature which, apart from the insurance against explosions, makes the use of the Inertaire transformer economically well worth while.

I think that Mr. Dalzell's feeling that the automatic breathes of the Inertaire transformer involves uncertainty is wrong and is probably due to his not having actual experience with its operation in service. It is very simple and is very much freer from uncertainly than the auxiliary supply tank with its reducing valve which must be depended upon to protect automatically the transformer from the high pressure of the auxiliary tank.

A New Type of Single-phase Motor

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Review of the Subject.—This paper deals with the development of a constant speed a-c. motor for application to single-phase current. A description is given of the derivation of this new type of motor from the plain repulsion motor. The novel feature lies in the armature which consists of a combination of three elements, namely: a commutated winding, a squirrel cage of high reactance, and a commutating device consisting of shielding metal strips.

It is shown that this motor in starting and acceleration possesses the characteristics of a series motor and when up to speed has the characteristics of a shunt motor. The changing over from the series to the shunt type is accomplished without the use of any automatic device, this change taking place due to the inherent qualities of the motor.

An explanation is given of the high power-factor possessed by this motor and also an elementary theory of commutation, which has proved to be as good as that in a d-c. motor with commutating poles.

WHEN looking back on the development of the electric motor, it appears that the traction motor with its severe service has had a great influence on both the mechanical and electrical design of the stationary motor for general purposes. Both for d-c. and a-c. applications, the traction motor has served as a model and particularly has its influence been strongly felt in the evolution of the a-c. motor. Great contributions to the art of the a-c. motor are found in the records of the A. I. E. E., which records describe not only discoveries of new means of improving results, but also establishing the theories governing the operation. Chief among these contributions are the papers by Lamme on the Series Motor; Steinmetz, on the Repulsion Motor; Alexanderson, on the Series Repulsion Motor; and Milch on the Repulsion Induction Motor. All of the motors mentioned, except the Milch motor, have the characteristics of the series type, the Milch motor approaching shunt-motor characteristics.

The new motor which will be described in this paper has the characteristics of a shunt motor over the running range, and the series type characteristics during starting and acceleration. The change over from the characteristics of the series motor to that of the shunt motor is accomplished in this new motor without the use of any external automatic device. Since the motor is a combination of a repulsion motor and a squirrel cage induction motor, for want of a better name, it will be referred to as the squirrel cage repulsion motor.

Through the work of Steinmetz and Alexanderson above referred to, the characteristics of the plain repulsion motor are well understood. In Fig. 1 is shown diagrammatically, Professor Elihu Thomson's well-known repulsion motor. This motor has in many respects the same characteristics as that of a series type d-c. motor. It differs, however, from the operation of such a motor in some respects and in examination of such differences, one in particular is rather striking; namely, the difference in the speed and torque curve. The series type d-c. motor will attain very high speeds if the load is removed; whereas, the repulsion motor will not run away, but will stop at a certain limited speed. The reason for this limitation in speed is due to the influence of the currents in the coils short-circuited by the brushes, and it may be stated that the larger these currents are, that is, the poorer the commutation, the lower will be the running free speed.

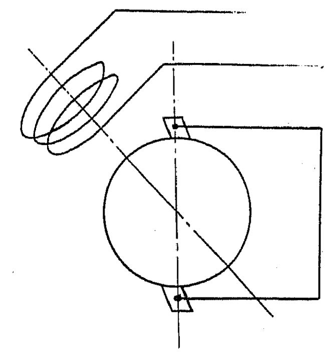

FIG. 1

With this observation in mind, the writer conceived the idea that if it were psosible by artificial means to exaggerate the influence of these short-circuited currents then it should be possible to so limit the running free speed that the result would lead to a nearly constant speed motor. In order to carry out this thought, a motor was made a number of years ago in accordance with Fig. 2. The field carries a single winding corresponding to the field winding in Fig. 1. The armature carries a commutated winding with its short-circuited

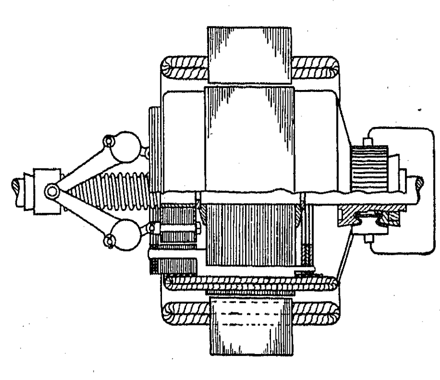

FIG. 2

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

brushes in accordance with Fig. 1, but in addition, there are placed in the same slot as the commutated winding, the bars of a squirrel cage. These bars are extended, as shown in Fig. 2, at one end projecting through a laminated ring rigidly attached to the shaft. Inside this laminated ring moves a circular core which is acted upon by a centrifugal mechanism in such a fashion that when the motor is at standstill, the moving core is entirely inside of the laminated ring, and when the motor speeds up, the inside core is moved out, until, near synchronism, the core is entirely moved outside the laminated ring. This mechanism, therefore, provides the squirrel cage with an automatically adjustable reactance which is a maximum at standstill and a minimum when running near synchronism.

When the motor is at standstill, the squirrel cage carries a comparatively small current due to the high value of the adjustable reactance, and the current is, therefore, flowing mainly in the commutated winding. Therefore, at standstill, this motor possesses about the same torque and starting current as in the case of the plain repulsion motor, Fig. 1. It is well understood that at synchronism the plain repulsion motor sets up a circular revolving field similar to that existing in the polyphase motor. Hence, at synchronism, the squirrel cage revolves at the same speed as this revolving field and is, therefore, at this speed, not producing any torque. Above synchronism, currents are generated in the squirrel cage, the condition being very similar to that of an induction generator. The torque in the squirrel cage is then a generator torque, thus opposing the motor torque in the commutated winding. Tests show that this motor will only exceed synchronism by 1 to 2 per cent. As soon as the speed drops below synchronism, the squirrel cage becomes active, producing a motor torque, and at these speeds, both the squirrel cage and the commutated winding pull together. Tests show that the maximum output, which occurred at about 10 per cent slip, was 250 per cent of the normal output; that is, very similar to a well designed polyphase induction motor. The power-factor was very excellent; *i. e.* close to unity, which is far better than that of either a plain repulsion motor or a squirrel cage induction motor, for reasons which will be explained in the following. The commutation of this motor was excellent, as good as that of a well designed d-c. motor with commutating poles. A brief theory of the commutation will be discussed later.

Although this motor performed in accordance with expectations, it was felt that it was capable of further development toward the ends of eliminating the moving parts. The thought naturally presented itself as to whether or not it would be possible to make the adjustable reactance an integral part of the motor proper.

The next step in the development was an armature structure shown in Fig. 3. The squirrel cage S is placed underneath the commutated winding R and the slots carrying these two windings are interconnected through the narrow slot L.

No external cores are used, the result being that the squirrel cage is given a permanent high reactance. Since this is the type of motor which was ultimately adopted for production, no further account will be taken of the motor, shown in Fig. 2, which has merely been mentioned as being one link in this development.

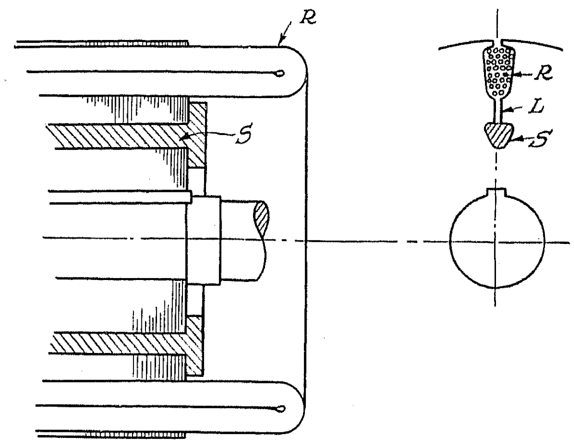

FIG. 3

In order to explain the working of the squirrel cage repulsion motor, shown in Fig. 3, the conditions existing in the plain repulsion motor will first be briefly considered. Following Steinmetz' paper of 1904, the field winding is resolved in two parts at right angle in space shown in Fig. 4. That portion of the field winding which magnetizes in direct opposition to the armature is called the compensating or transformer winding T. That portion of the field winding which magnetizes

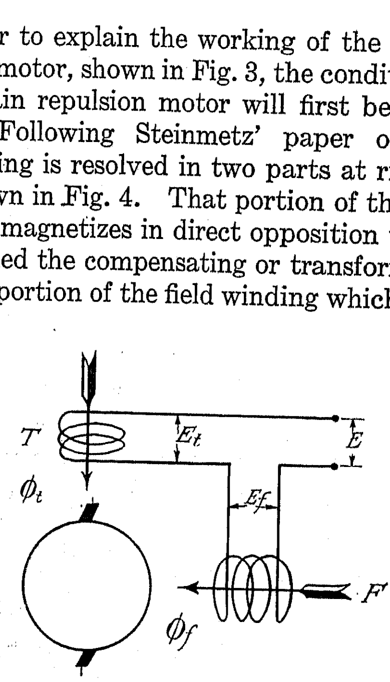

FIG. 4

at right angle to the armature is called the main field winding F. The transformer and the armature windings constitute a transformer, inter-linked by the mutual flux ϕ_t. The field winding F has no corresponding secondary winding on the armature except the coils, which are short circuited through the brushes. Since the influence of these short-circuited currents is small, below synchronism at least, these currents will be

neglected and then the field flux ϕ_f acts as pure self-induction.

When running at speed, there are two e. m. fs. induced in the armature, one E_{at} by alternation of the transformer flux ϕ_t and a second E_{af} set up due to the rotation of the conductors through the field flux ϕ_f. Since in a reactive coil the flux and the current are in phase and since the e. m. f. of rotation must be in phase with the field flux, the vectors I, ϕ_f and E_{af} are in time phase as shown in Fig. 5. Since the armature is short-circuited (neglecting the impedance drop) the e. m. f. of alternation must be equal and opposite to the e. m. f. due to the speed as shown in Fig. 5. Since the mutual transformer flux is 90 deg. ahead in time of the secondary e. m. f., the vector of the flux ϕ_t is placed accordingly in the diagram from which it now may be seen that the fluxes ϕ_f and ϕ_t are in quadrature in time.

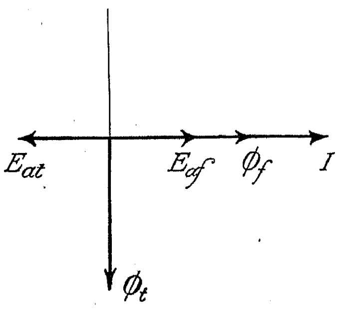

FIG. 5

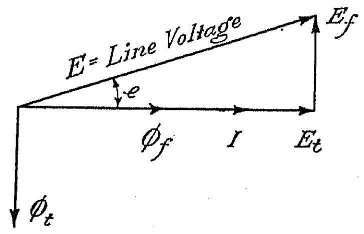

FIG. 6

Numerically, $E_{at} = C \times f \times \phi_t$, $E_{af} = C \times f_0 \times \phi_f$ where C = constant, f = primary frequency, f_0 = frequency of rotation. Hence, $C \times f \times \phi_t = C \times f_0 \times \phi_f$. At synchronism $f = f_0$. Hence, $\phi_t = \phi_f$, thus confirming the statement already made that at synchronism a circular revolving field exists.

A repulsion motor may, therefore, be looked upon as formed of two elements; namely a transformer loaded on non-inductive load in series with a reactive coil. Since, in a transformer loaded on a non-inductive load, the impressed voltage and the primary current are nearly in phase, in Fig. 6, is shown as a first approximation, the line current I in phase with the voltage E_t which is impressed on the transformer winding. If E_f is the voltage impressed on the field winding, this voltage leads the current by 90 deg. Cos φ is then the power-factor and thus Fig. 6 represents in an elementary way, the diagram of the plain repulsion motor.

It is obvious that by making E_f small as compared with E_t the power-factor will be good, which means running a weak field as compared with the armature reaction. In a repulsion motor of the size here considered, the power-factor at full load would range between 80 and 85 per cent. Test on the squirrel cage repulsion motor reveals, however, the fact that the power-factor at full load is close to unity. Thus, by adding a highly reactive squirrel cage, the power-factor has been considerably raised, a fact which at first appears paradoxical. The explanation is as follows:

In Fig. 6, we assumed that the current I in the field was in phase with the field flux ϕ_f. This is, however, not quite true even in the plain repulsion motor, since, due to the losses in the iron, there always exists an angle of hysteretic advance, and in addition, the coils short-circuited by the brushes act like the secondary of a transformer loaded on mixed reactance and resistance. In such a transformer, the primary current leads the mutual flux by a certain angle. In Fig. 7, the current I leads the field flux ϕ_f by a certain angle α. Completing the diagram, it is easily seen that the power-factor has improved since the whole voltage triangle has been turned in the diagram so as to decrease the

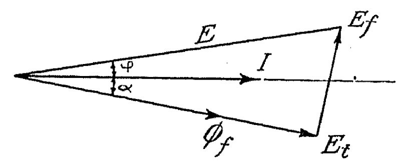

FIG. 7

angle φ. In the plain repulsion motor, the angle α is comparatively small, but nevertheless, there exists in such a motor, below synchronism, a certain amount of power-factor compensation.

It is now obvious that if a separate squirrel cage is used, a similar effect to that due to the currents short-circuited by the brushes is taking place, although, in a much more pronounced way. At full load, the squirrel cage actually delivers about the same amount of energy as the commutated winding. The primary corresponding to the squirrel cage consists of two phases; namely, the transformer winding and the field winding, each contributing about one-half of the output. Thus, at full load, each of these phases delivers about one-quarter of the output and hence, the angle α has become considerable at full load and the improvement of the power-factor is, therefore, appreciable. In Fig. 8 is shown a break test of a motor rated 3 h. p. and it may be seen that the power-factor at full load is close to unity.

It has thus been proven that by adding a highly reactive squirrel cage to the commutated winding in a repulsion motor, the power-factor, below synchronism, is improved,—a result which may at first appear contradictory, and the explanation as shown above

may best be summarized by the following quotations from Steinmetz:

"Exciting the field by a lagging current in the field, a lagging e. m. f. of rotation is produced which is equivalent to a leading current. As it is easy to produce a lagging current by self-induction, the commutator thus affords an easy means of producing the equivalent of a leading current. Therefore, the a-c. commutator is one

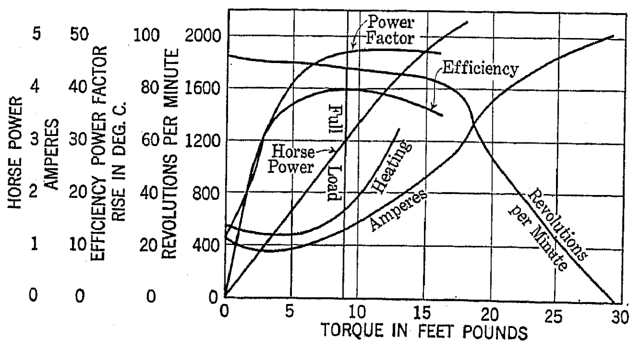

Fig. 8

of the important methods of compensating for lagging currents."

Similar conditions exist in the plain series motor in which power-factor compensation can be obtained by turning the time phase of the field flux by shunting a resistance across the exciting winding, which proposition, I believe, was first suggested by McAllister. In this case the efficiency suffers slightly due to the loss

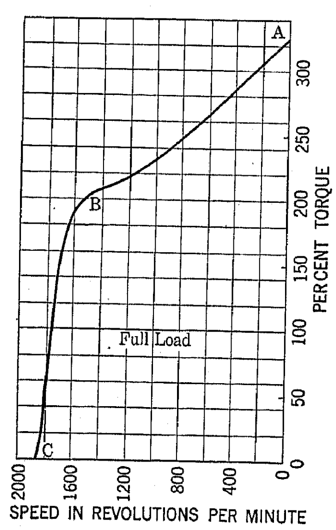

Fig. 9

in the resistance, and in addition, such an arrangement introduces a certain mechanical complication. In the squirrel cage repulsion motor, the addition of the squirrel cage cuts down the losses in the secondary, thus improving the efficiency. Thus, the squirrel cage raises both the efficiency and the power-factor, which conditions may be confirmed from the break test shown in Fig. 8.

Fig. 9 gives the typical relation between the speed and the torque for this type of motor. From this figure it may be seen that the torque curve may be divided into two branches; namely, the branch $A\ B$ corresponding to starting and acceleration and the branch $B\ C$ corresponding to the normal running conditions of the motor. During the period $A\ B$ the motor has, obviously, the characteristics of a series motor. During the period $B\ C$ it possesses in a marked degree the characteristics of a constant-speed shunt motor.

As the speed decreases and the motor performs along the curve $B\ A$, the current increases, due to the fact that the field is excited by the line current. The field flux increases with the decreasing speed. At standstill, corresponding to point A, we have the following conditions:

The squirrel cage has inherently, due to its construction, a high inductance. It is then penetrated by a current having primary frequency. Its reactance, therefore, is quite high. The commutated winding on

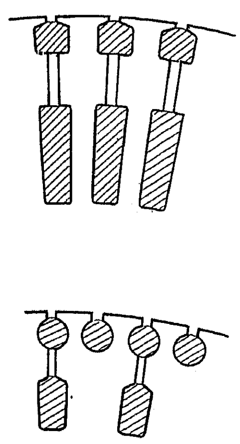

Fig. 10

the other hand has inherently a low reactance and the current, therefore, will flow mainly in this winding. We may also look upon this from a somewhat different viewpoint by stating that the magnetic flux seeks the path located between the squirrel cage and the commutated winding and thus the flux, which acts as self-induction for the squirrel cage winding, is torque-producing with respect to the commutated winding.

From the shape of the curve $A\ B\ C$, it will be seen that the torque during retardation increases without a break from B to A, thus securing a high "pull-up" torque, guaranteeing that the motor will pull up to speed the load that it starts. Over the branch $C\ B$, it possesses, as stated, the characteristics of a shunt motor to a marked degree, the full-load speed regulation being from 3 to 4 per cent. The branch $C\ B$, therefore, corresponds to running conditions generally obtained in a polyphase motor with low secondary resistance.

Since the mechanical construction possesses a certain similarity to the Boucherot double squirrel cage induction motor, it becomes of interest to examine whether

such a similarity exists from an operating standpoint. The double squirrel cage induction motor, may be constructed in a number of various ways as shown in Fig. 10. The main principle consists in providing one squirrel cage of high resistance and low reactance and a second squirrel cage of comparatively high reactance and low resistance. This principle aims at improving the starting conditions of the squirrel cage induction motor. In starting, when the frequency in the secondary corresponds to the primary frequency, a large portion of the current flows through the high-resistance winding, thus securing a high torque efficiency. When running at speed, the frequency in the secondary is the slip frequency; that is, only a small fraction of the primary frequency, and then the secondary current seeks the path of lowest resistance, consequently, taking the path of the low-resistance winding. Thus, near synchronism, this motor has a good speed regulation and a high efficiency. The power-factor of this motor suffers, however, for the following reason. In its reaction upon the primary, the secondary currents are of primary frequency, since they are always located in space in such a manner as to oppose the primary currents and thus the primary and secondary may be considered connected in series. It is, therefore, from the standpoint of power-factor, equally as bad to introduce reactance in the secondary as in the primary. Hence, the power factor, as well as the maximum output, suffers in the Boucherot type of motor. From this point of view, the squirrel cage repulsion motor, as has already been discussed, has opposite characteristics to the Boucherot motor, since both the power-factor and the maximum output improve by the use of the two windings.

Another distinction is the fact that the excitation in the squirrel cage repulsion motor has, in many respects, the character of that of a series motor, *i. e.*, the field flux increases somewhat with the line current. In the Boucherot motor, conditions are exactly opposite since it has the characteristics of a plain induction motor, the exciting flux (the mutual flux) decreasing with the increasing load. The two types of motors are, therefore, from an electrical standpoint, widely different.

One of the great difficulties with a-c. commutator motors has always been the question of commutation. Different means have been adopted for improving the commutation, such as the use of high-resistance commutator leads in the plain series motor in accordance with Lamme, or the use of fractional-pitch winding in the repulsion motor as suggested by Alexanderson, as well as providing for the proper commutating field also suggested by Alexanderson in his series repulsion motor. In the motor we are considering, another means for obtaining good commutation has been adopted.

In the plain repulsion motor, when it is running near synchronism, the field revolves with about the same speed as the armature conductors and, therefore, it is reasonable to assume that at such speeds the commutation is fair. The transformer flux, which is set up by the mutual action of the transformer winding and the armature, may be considered a commutating flux since it has not only the right time phase, but also the right strength to compensate for the electromotive force induced in the short-circuited coil by the field flux. On the other hand, there is another electromotive force induced in the short-circuited coils which causes sparking,—namely, the electromotive force of self-induction. In the squirrel cage induction motor, the squirrel cage is inter-linked with the leakage flux which causes self-induction in the short-circuited coils as shown in Fig. 11. The leakage flux, therefore, serves as a mutual flux of transformation between the short-circuited coil and the squirrel cage, thus transferring the energy, which would otherwise appear as a spark, to the squirrel cage. Part of this leakage flux over the length L is, however, not inter-linked with the squirrel cage and this leakage flux would, therefore, cause a slight amount of sparking. In order to eliminate this condition, a thin sheet of metal M is introduced in this slot. This metal strip is of

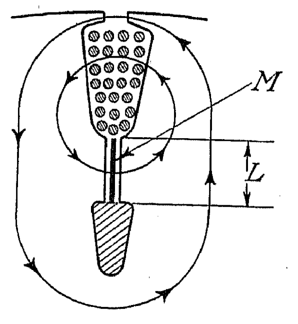

Fig. 11

comparatively high resistance. It should now be observed that the leakage flux over the length L must penetrate this metal strip and the corresponding energy is, therefore, dissipated in heat. In other words, instead of appearing as a spark, this energy appears as heat in the metal strips. Due to the fact that the frequency of commutation is very high, it is obvious that the resistance of the metal strips can be made quite high so as not to interfere with the flux distribution in starting, which conditions have already been discussed.

Thus, the armature of this motor consists in reality of three elements, each of which has an important function. These elements are the commutated winding R, the squirrel cage S and the commutating strips M. The commutated winding secures for this motor a high starting torque and it also contributes to the output when running at speed. Due to the fact that it is connected to a commutator, it secures power-factor compensation. The squirrel cage winding procures the characteristic of a constant speed type of motor over the running range and also secures perfect commutation in combination with the commutating strips.

Discussion

V. Karapetoff: An impression may be gathered from the paper that the power factor of the motor is improved because the squirrel-cage winding is highly inductive. It seems to me that the squirrel-cage winding necessarily has to be highly inductive so as not to interfere with the series characteristics of the motor at the start. If it were not for an additional mechanical complication (Fig. 2), it would be better to make this winding less inductive at and near synchronism. A transformer diagram at constant current will readily show that the common flux is lagging behind the primary current at a non-inductive load more than at an inductive load. Therefore, the less inductive the squirrel-cage winding, the more the flux ϕ_f is lagged behind I (Fig. 4). But the flux in T must be in time quadrature with ϕ_f in order that the total e.m.f. between the brushes be equal to zero. Therefore, lagging ϕ_f also causes ϕ_t and E_t to lag more behind I; in other words, it causes the vector of the current to come nearer that of the total voltage E. Thus, the power factor is improved even though the squirrel-cage winding has a high inductance, and the power factor would be improved more, if it were advisable to reduce the inductance of this winding in normal operation.

E. Bretch: About twenty to twenty-five years ago when the single-phase motor business was developing, considerable trouble was experienced with off-frequency circuits. In those days the single-phase generators were comparatively small and were designed for lighting service. When motors were connected to them, difficulty was experienced in keeping up the voltage and the generator was often speeded up to build up the voltage.

Single-phase motors with a centrifugally operated device to change from repulsion to induction were in difficulty on these off-frequency circuits, as they would short circuit the rotor at a fixed speed which on high frequency would be too low for the motor to come up to speed, causing severe fluctuations on the line as well as annoyance to the motor user. In considering the problem of constructing a motor that would be responsive to slip rather than speed in making the change from repulsion to induction, the writer conceived the idea of using a rotor with both the repulsion winding and the short-circuited winding permanently in circuit, the short-circuited or squirrel-cage being "loosely coupled" so as to be inactive until near synchronous speed.

During 1904 and 1905 several experimental motors were built and tested, and a patent (No. 848719) was issued to the writer April 21, 1907. This patent covered broadly the idea of a commutated winding at the surface of the rotor, a squirrel-cage or short-circuited winding beneath the commutated winding and by-pass or leakage path whereby the magnetic flux would pass between the squirrel-cage and commutated windings at stand still, or low speeds, when the alternations of the flux were high; but at, or near synchronous speed the flux of slip frequency would penetrate the squirrel-cage and give the motor the constant speed characteristic. The squirrel-cage from its inductive relation to the commutated winding also eliminated the sparking at the commutator.

During the next few years several hundred of these motors were placed in service, and were successful electrically and mechanically. However, due to the prejudice, at that time, against single-phase motors with brushes in service continuously and to the fact that some people insisted "wrongly" upon making the no-load losses a criterion for judging the efficiency of the motor, this line of motors was later equipped with a short-circuiting device and the squirrel-cage omitted. A description of these motors with the double-wound rotor was published in the *Electrical World*, July 1907.

Various combinations and proportions are possible between the three elements of the combination, the commutated winding, the squirrel-cage winding and the magnetic by-pass. In the motors as originally constructed, the control of the flux in the by-pass was accomplished by proportioning the cross section of the iron in the magnetic bridge between the two windings, and depending on the saturation of the iron.

Various other methods have been proposed, such as short-circuited loops encircling, or partially encircling the by-pass, making the by-pass of solid magnetic material and utilizing the choking effect of the eddy currents in the solid material, making the squirrel-cage conductors themselves of magnetic material, air gaps in the by-pass, as well as the one outlined by Mr. Bergman. The methods of using solid magnetic bars as a by-pass, and of leaving a thin bridge of iron in the rotor punchings between the two windings, have been in use for many years.

As to commutation, Mr. Bergman's statements are well founded. Only a few weeks ago, one of these motors with the double-wound rotor that had seen service for fifteen years, was returned for repairs. The rotor was perfect, and the commutator not as much worn as would be expected on a direct-current motor of similiar age.

As pointed out in the specifications of the above mentioned patent, the fact was then noted that the power factor was higher than that of the straight induction, but the reason for it was not understood. The problem had been to produce a sparkless motor that would change from repulsion to induction in response to slip rather than speed, with no particular thought of increasing the power factor.

I have a test sheet, dated Jan. 17, 1907 of a ¼ H. P. motor of this type, showing practically the same characteristics as Mr. Bergman's tests:

V. A. Fynn: A thing which strikes one in reading Mr. Bergman's paper is his disregard of that which has been accomplished by others. Others have worked on the very type of motor which Mr. Bergman now claims as new, have built such machines and have long ago published their results.

The motor of Mr. Bergman's Fig. 2 has a squirrel-cage and a commuted winding on the rotor but both of these windings are in the same rotor slots. Series conduction and induction motors so built were proposed many years ago by Lundell, they have no magnetic shunt or bridge between the two rotor windings and differ widely from the machine shown in Mr. Bergman's Fig. 3. The laminations shown at one end of the squirrel-cage are not the equivalent of magnetic shunts.

The motor of this Fig. 3 was patented in this country on September 2, 1907. See U. S. P. 848,719. The patentee, Mr. Edward Bretch, of St. Louis, has built and sold thousands of these machines and the fact that his patent is about to expire does not rob him of his priority. The motor shown by Mr. Bergman in his Fig. 3 does not differ from the Bretch motor in principle. There is nothing new about this machine. It differs from that of Lundell in that it provides a magnetic bridge between the two rotor windings.

In discussing the theory of the so-called repulsion motor, Mr. Bergman omits all consideration of the all important self-compensating feature of such machines. It is useless to go into this matter here since this point is fully explained, for instance, in my "Classification of Alternating Current Motors," PROCEEDINGS A. I. E. E., May 1915, page 972. Nor does Mr. Bergman's theory take into account the fact that in the motor of his Fig. 1 the rotor ampere-turns are in excess of the cophasal stator ampere-turns.

The power factor of the Bretch motor is high at some loads because below synchronism both the squirrel-cage and the commuted windings contribute to the motor field, a condition which results in a retardation of the phase of the resultant motor field as against that of the motor field which the squirrel-cage alone would produce. Just how such a retardation affects the power factor of a single-phase motor was shown by me, for instance, in "A New Single-Phase Commutator Motor," see *Proceedings* I. E. E., March 8, 1906.

It would be interesting to know just how the "field winding" of a motor, see page 3, column 2, line 13 from last of Mr. Bergman's paper, can contribute "about one half of the output." Field windings do not transfer energy. The fact is that the field winding of which Mr. Bergamn speaks, and which is shown in his Fig. 4 at *F*, does not exist. His motor is excited from the rotor and not from the stator as he supposes.

I cannot see how the flux "which acts as self-induction" spoken of on page 4, column 2, line 16 from last, which must be a leakage flux merely linking with the two rotor windings, can possibly produce a torque available on the shaft of the motor.

As to the short circuit *M* in the path of any flux threading the magnetic bridge between the commuted and the squirrel-cage windings, this is by no means novel. I have patented such a shield on June 6, 1911, see U. S. P. 994,381 and similar means on November 28, 1911, see U. S. P. 1,010,135.

I may further point out that I have done considerable developing work in connection with single-phase motors having a commuted and a squirrel-cage winding on the rotor with a magnetic bridge therebetween, producing different forms of series induction motors (Mr. Bergman would call them "repulsion motors") with an added "sunk" squirrel-cage and have fully discussed the theory of these machines. I refer to my several and old patents on this subject and to my paper "Single-Phase Squirrel-Cage Motor" which describes a machine showing considerable advance over the Bretch motor. Among other features my motor shows leading power factor at no load and practically unity power factor at most loads. When operated without auxiliary or external phase-compensating means it exhibits a power factor very near unity over a considerable range of loads as evidenced by Figs. 15, 16, 22, 25 of the paper last named, see PROCEEDINGS A. I. E. E., October, 1915.

Recently in discussing my paper entitled "A New Self-Excited Synchronous Induction Motor," Mr. Bergman found fault with the presence of a commutator and notwithstanding the fact that the machine in question is compensated at all loads and that its commutator carries nothing but exciting current and unidirectional exciting current at that. Now Mr. Bergman advocates a motor which although it has a commutator is not compensated at light loads, is not fully compensated at any load and is of such design that the commutator carries load current of full line frequency. If the criticism he directed against my machine is justified, is it not obvious that the same objection applies to a prohibitive extent to the motor he now describes.

P. M. Lincoln: The motor characteristics given in the paper, both the calculated and the observed, are very satisfactory. They show a very good shape of speed-torque curve, and I would like to ask about the commutation. Are they satisfactory in commutation, both at start and under running conditions, and how is that commutation obtained?

They are so satisfactory in comparison with the standard squirrel-cage polyphase induction motors, that undoubtedly if they can be made at a cost comparative to the cost of the standard squirrel-cage motor, they would receive a large application in commercial practice.

I would like to ask how about that cost. How much above the standard squirrel-cage motor will it be?

Also, how large a motor can be made in this manner?

G. H. Garcelon: Mr. Bergman's description of his conception of this motor is very interesting to me, as it varies somewhat from my own experience a few years ago in carrying out the commercial development of a line of motors similar in principle and operating characteristics.

Designers have long realized the more or less uncertain operation and expensive construction of mechanically operated devices used to combine the desirable characteristics of two or more types of motor, and have striven for a purely electrical means of securing such combination.

For starting and accelerating, the series and repulsion windings were most suitable, with the repulsion type lending itself more readily to commerical use, because of the following characteristics:—

1. Armature voltage is independent of line voltage, permitting reconnection of stator for different voltages without affecting the rotor winding.
2. Armature voltage may be selected for best mechanical and electrical design.
3. Simplicity of brush rigging.

For running, the squirrel-cage induction winding was most desirable because of its close speed regulation and mechanical simplicity.

Considering also the well known generating characteristics of the induction motor above synchronous speed, these two types of winding seemed to be ideal for combination except for the fact that their starting characteristics were normally antagonistic; as the squirrel cage would draw large currents at standstill and set up a field in opposition to the normal repulsion-motor field, greatly reducing, if not entirely annulling, the desirable starting characteristics of the repulsion winding.

The problem then seemed to be that of minimizing the effect of the squirrel cage at starting, and using it as much as possible after the rotor attained speed.

It was appreciated that the high-frequency rotor flux at starting could be damped to a great extent by using a high-reactance squirrel cage, and that the circular rotating field produced by the repulsion winding at synchronous speed would induce a low frequency in the squirrel cage which would minimize its reactance and cause it to develop appreciable running torque.

Experiments with various means of accomplishing these results proved the correctness of the theory, although several presented structural difficulties.

In order to secure power factor compensation to the marked degree shown in the paper, abnormally large magnetic parts are required so that saturation may be avoided.

By alterations in rotor design this type of motor lends itself to a variety of speed-torque characteristics and still maintains the other general characteristics mentioned by Mr. Bergman.

A line of motors of this general type, having a somewhat drooping speed-torque curve, has been on the market for three or four years and has been giving excellent service on such applications as pumps and compressors as well as in general service.

H. C. Specht (by letter): The motor with such a performance as described will necessarily be somewhat larger in dimensions than the ordinary repulsion-starting induction-running type. It also seems to me that the mechanical construction of the rotor laminations is such that very expensive equipment will be required and the punching cost will be high.

During the development of a similar motor various rotors were used, and it was found that those having a high reactance and low resistance in the squirrel cage showed a break in the speed curve at lower torques than those with relatively higher resistance and lower reactance. It was also found that the high-reactance rotors showed higher current and lower power factor at light loads than did the rotors of relatively higher resistance. However, the low-resistance rotors showed better speed regulation with better efficiency and power factor under overload conditions. This only goes to show the variations of some characteristics at the expense of others, and indicates the possibility of considerable choice in performance.

The improvement in power factor due to the use of the squirrel-cage winding with low resistance and high reactance becomes effective only after the load has caused the speed to drop below synchronism, whereas above synchronism the reverse effect is true. Therefore, it is not desirable to keep the speed of the motor above synchronism over a very large portion of the operating range. There is really very little object in comparing the power factor with that of a straight repulsion motor as this type is not

suitable for general application. If we compare the power factor of this motor with an induction motor having a short-circuited armature winding, the increase in favor of this type of motor is very marked due primarily to the lack of cross-field magnetizing current being reflected in the primary.

Mr. Bergman points out that the thin metal strip in the slit between the two windings improves the commutation under running conditions, and this is no doubt true, but, inasmuch as it is a very easy matter to secure excellent commutation near synchronous speed and the metal strip decreases the starting torque to some extent, as well as slightly increases the losses, it would seem to be an undesirable addition.

William Cramp (Communicated after adjournment): With reference to the article by Mr. S. R. Bergman in the JOURNAL of the A. I. E. E., July, 1924, it may interest your readers to refer to the *Journal* of the London I. E. E., Vol. 57, page 287, wherein I think they will find the prototype of Mr. Bergman's new machine.

Several motors working on this principle were designed and sold about 1904, but by his ingenious addition of metal plates in the slot openings to improve commutation, Mr. Bergman has undoubtedly improved the machine, especially when it is required for large horse power. The original motors gave no trouble at all as far as commutation was concerned, but they were built chiefly for driving fans, where the starting conditions are easy and the power comparatively small. I should like to add that my experience with these motors showed that the fall in speed from nothing to full load, was a good deal more than that of the ordinary induction motor. It is difficult to tell from Fig. 8 of the JOURNAL what the regulation on Mr. Bergman's machine is like, and it would be interesting if he could give further information on this point.

S. R. Bergman: In regard to Professor Karapetoff's statement, I pointed out in my paper that the reason for the good power factor in this motor was the action of the squirrel cage which is rather curious due to the fact that this squirrel cage possesses a high reactance. Professor Karapetoff is correct in his statement that the motor characteristics would be even better during running conditions if the squirrel cage possessed less reactance, a fact which I believe is illustrated in my paper by the description of the first motor having an adjustable reactance of the squirrel cage, this reactance being great in starting and low in running. Thus, I believe that Professor Karapetoff's conclusions are in full accord with the theory given in my paper.

Mr. Bretch has contributed a very interesting discussion showing that he, at a very early period, had some quite interesting ideas. Of course in the light of our present knowledge it may be readily understood why the motor built in accordance with the Bretch patent was abandoned.

The motor which is described in my paper is mechanically and electrically something quite different from the Bretch motor. It is, in my judgment, not possible to satisfactorily control the flux in the by-pass by proportioning the cross section of the iron between the two windings so as to depend upon the saturation of the iron for results.

Mr. Fynn discusses the novelty of this motor at some length, referring particularly to the Bretch patent. He forgets however, the very important fact that the Bretch motor has been discontinued in production. The motor proposed by Lundell I am not familiar with, but it seems from Mr. Fynn's description to be substantially different in principle.

Mr. Fynn seems to severely criticise the theory given in my paper. I wish to suggest that Mr. Fynn read the paper giving the details of the theory, which was presented at this Convention by Mr. West, entitled: "The Theory of the Squirrel Cage Repulsion Motor." Mr. West's theory is based on the general equations for the alternating current motor, single or polyphase, as first published by Steinmetz. Mr West's theory is carried out in complex quantities and agrees with the few graphical illustrations I have given in my paper. Results of careful tests at all speeds for both rotations, check our theory surprisingly closely. It seems therefore, plausible that such a theory is correct and it has well served our purpose of predetermining a complete line of motors of this type, which has recently been put on the market by the General Electric Company.

Mr. Fynn further states that the two windings into which we have resolved the field winding, do not exist and furthermore that field windings do not transfer energy. I wish to point out that we have built motors with two field windings and by inserting wattmeters in these windings it is an easy matter to check our theory.

Mr. Fynn also states that he cannot see how the flux, which acts as self induction, can possibly produce torque. The flux which penetrates the space between the two windings of the armature may be considered leakage flux with respect to the squirrel cage winding and mutual flux with respect to the repulsion motor winding. This is in accordance with the general accepted views on leakage and mutual flux. Hence, the flux which acts as self induction with respect to one winding, acts as torque producing with respect to the other winding.

Mr. Fynn also seemed to criticise the name of "Repulsion Motor" which I have given to the commutated winding. The brushes on this winding are short circuited and therefore, conform to that type of machine which was first brought out by Professor Thomson and called by him "Repulsion Motor." Professor Thomson's contribution in inventing the repulsion motor is one of the most notable contributions to the electrical art and in recognition of this contribution, I think we should maintain the name "Repulsion Motor."

Finally, Mr. Fynn has again brought up for discussion, his paper called "A New Self Excited Synchronous Induction Motor." This paper, which was presented at the Spring Convention in Birmingham, Ala., describes a polyphase induction motor possessing a commutator as well as slip rings having two armature windings in addition to two field windings. I merely have pointed out that in the case of polyphase motors which do not need to depend upon a commutator for operation, it certainly is a complication to add a commutator as well as slip rings, as well as four separate windings. A polyphase motor starts very satisfactorily with a squirrel cage and all these complications have been added to a polyphase motor in order to improve the power factor. As is well known, a single phase motor is not self starting, hence there is a good reason in order to start such a motor, to add a compensator. I wish to emphasize that in my motor the armature possesses only a single wire winding and the field also only a single winding. Furthermore, my motor does not possess any slip rings. It is, therefore, obvious that this new single phase motor is much simpler than Fynn's polyphase motor.

Turning now to Mr. Lincoln's inquiry as to the commutation in this motor, the theory of commutation is given at some length in my paper. The commutation in this motor is perfect during running conditions and as stated, is as good as that of a direct current motor with commutating poles. The reason for the good commutation is the fact that the motor runs near synchronism and the leakage flux is damped out by the squirrel cage and the metal wedges. The effect of the squirrel cage and these metal wedges is therefore, similar to the action of commutating poles in a direct current machine, since both of these devices take care of the self induction of the coils undergoing commutation.

As to Mr. Lincoln's inquiry about the cost of this type of motor, I may state that the material is efficiently utilized and that due to the simple mechanical arrangement of the windings, the process of manufacture is attractive.

As to how large a motor we can build in this manner is a question which is rather difficult to answer. There is nothing in the

theory of this motor which would indicate that size would be a limiting feature, which I believe will answer Mr. Lincoln's question.

Mr. Garcelon states that a few years ago he brought out a line of motors similar in principle and operating characteristics to this new motor. I believe Mr. Garcelon is referring to a motor built on the principle of using a squirrel cage in the same slots as the repulsion motor, this squirrel cage consisting of high resistance magnetic bars. This motor will give quite different operating characteristics from the motor I described. I go so far as to state that, in my opinion, the underlying principle is dissimilar in these two motors and the operating characteristics certainly justifies this statement.

There is one statement made by Mr. Specht which I wish to briefly touch upon. Mr. Specht states that the metal strips decrease the starting torque and the core loss and furthermore, as it is an easy matter to secure perfect commutation near synchronism it would seem undesirable to add these wedges. Of course, before standardizing such wedges we built numerous motors of identically the same design with and without such wedges. The efficiency and the starting torque showed no difference, but the commutation was markedly different inasmuch as the omission of these wedges made the commutation unsatisfactory. I agree with Mr. Specht that these wedges introduce small additional losses but as they improve the commutation, the commutation losses are eliminated and thereby counter-balance the losses in the wedges. Therefore, this is the explanation why the efficiency is not affected by these wedges.

Theory and Calculation of the Squirrel Cage Repulsion Motor

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Review of the Subject.—*A brief history of the development of the squirrel cage repulsion motor and a physical explanation of the principles of its operation have been given in the preceding paper "A New Type of Single Phase Motor" by Mr. S. R. Bergman. It is the purpose of the present paper to give the results of an analytical study of the operation of the motor. The derivation of the method of calculation is outlined and some of the more interesting results are given.*

THE only known method that has thus far been worked out for the calculation of the performance characteristics of the squirrel cage repulsion motor is a method which has been derived from an analytical solution of the fundamental voltage equations in a manner similar to the method given by Steinmetz[1] in his general equations for a-c. motors. The method that has been developed differs somewhat from that given by Steinmetz in the form and arrangement of the results. As a result of this difference in the treatment, not only was a workable method worked out for the calculation of the performance characteristics of the squirrel cage repulsion motor, but it was also found that accurate and useful methods for calculation of performance characteristics of different types of a-c. motors, such as, for instance, the double squirrel cage induction motor, could be worked out along similar lines. The derivation of the method of calculation will therefore be explained in the hope that similarly derived methods may be found useful in the study of other types of machines.

The squirrel cage repulsion motor, as described by Mr. Bergman, consists of a repulsion motor with a squirrel cage placed well below the commutated rotor winding. The brushes are set at a larger angle, usually about 30 electrical degrees, from the neutral axis than in the case of the plain repulsion motor. The amount of copper in the commutated winding and the size of the commutator and brushes of a squirrel cage repulsion motor are less than in a repulsion motor, since the squirrel cage carries part of the rotor current. The construction is otherwise essentially the same as that of a plain repulsion motor. The relation of the squirrel cage to the commutated winding is indicated in Fig. 1.

In the following, it will be assumed that all m. m. fs. and fluxes are sinusoidally distributed in space. In order to further simplify the solution, the angle of hysteretic lag between flux and m. m. f., and the effect of the local currents in the coils short-circuited by brushes are neglected. In calculation of losses the core loss can be calculated separately and considered either as extra input, or as a frictional drag, or as partly extra input and partly frictional drag. The motor operates at speeds near synchronism throughout its normal operating range where the currents induced in the coils short-circuited by the brushes are negligible. The effect of these currents is therefore neglected in the general equations. These assumptions effect a considerable simplification in the solution of the equations, which in its present form is at best too complicated for general routine design work.

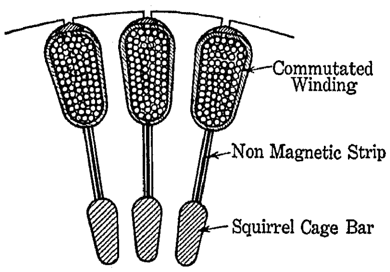


FIG. 1

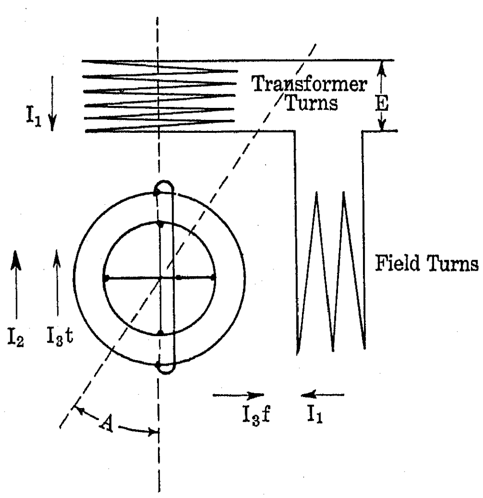


FIG. 2

Following the usual treatment of the plain repulsion motor, the stator winding of the squirrel cage repulsion motor is resolved into two component windings whose axes are at right angles to each other; the transformer winding whose axis, called the transformer axis, coincides with that of the commutated winding as deter-

1. Theory and Calculation of Electrical Apparatus, Chapter 19.

Presented at the Annual Convention of the A. I. E. E., Edgewater Beach, Chicago, Ill., June 23-27, 1924.

mined by the brush position; and the field winding whose axis is at right angles to that of the commutated winding. In accordance with the assumption of sinusoidally distributed m. m. fs., the number of effective turns in the transformer and field windings is equal to the effective turns of the stator winding as a whole, multiplied by the cosine and sine, respectively, of the angle formed by the magnetic axis of the commutated winding, as determined by the brush position, with the magnetic axis of the stator winding.

The squirrel cage can be considered as equivalent to a commutated winding, the corresponding brushes of which are short-circuited in two rectangular axes. For purposes of analysis, the motor can therefore be considered as consisting of four simple circuits; the stator circuit, the commutated winding, and the two equivalent circuits of the squirrel cage. The motor can be represented diagrammatically as in Fig. 2. By applying Kirchoff's Law to these four circuits, we obtain four simultaneous equations, from the solution of which the performance characteristics of the motor can be determined.

Symbols

E = voltage applied.
r_1 = resistance of stator winding.
r_2 = " " commutated winding.
r_3 = " " squirrel cage winding.
X_m = mutual inductive reactance corresponding to flux mutual to all three windings.
x_1 = leakage reactance of stator winding.
x_2 = leakage reactance of commutated winding (which is mutual to the squirrel cage).
x_3 = leakage reactance of squirrel cage (corresponding to the permeance of the leakage path between the commutated winding and the squirrel cage).
A = angle of brush setting from neutral.
I_1 = current in stator winding.
I_2 = current in commutated winding.
I_{3t} = current in squirrel cage in the transformer axis.
I_{3f} = current in squirrel cage in the field axis.
f = frequency of applied voltage.
N = effective turns in each of the windings.
S = speed as a fraction of synchronism.
s = slip.
ω = $2\pi f$.
σ = $S\,\omega$ = "angular speed."

The resistances and reactances should all be expressed in terms of the number of turns in the stator winding. The symbols for voltage and current all represent time vector quantities. The positive senses of the currents are indicated by the arrows in Fig. 2.

The fluxes linking the four circuits of the motor can be resolved into mutual and leakage components which can be expressed in terms of the various reactances and currents as follows:

The transformer flux ϕ_{mt} which is the component of the mutual flux of stator and rotor in the transformer axis,

$$\phi_{mt} = \frac{X_m}{2\pi f N}(I_1 \cos A - I_2 - I_{3t}) \qquad (1)$$

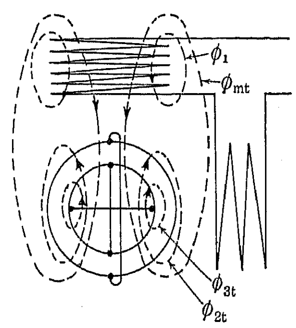
The field flux ϕ_{mf} which is the component of the mutual flux of stator and rotor in the field axis,

$$\phi_{mf} = \frac{X_m}{2\pi f N}(I_1 \sin A - I_{3f}) \qquad (2)$$

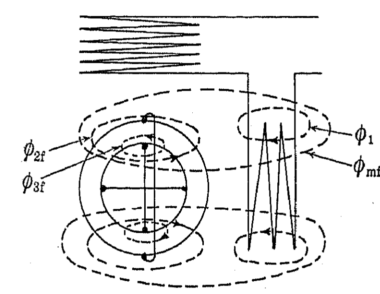
The leakage flux of the primary,

$$\phi_1 = \frac{x_1 I_1}{2\pi f N} \qquad (3)$$

The leakage flux linking the commutated winding in the transformer axis,



(a)



(b)

Fig. 3

$$\phi_{2t} = \frac{x_2(I_2 + I_{3t})}{2\pi f N} \qquad (4)$$

The leakage flux cut by the conductors of the commutated winding in the field axis,

$$\phi_{2f} = \frac{x_2 I_{3f}}{2\pi f N} \qquad (5)$$

The leakage fluxes in the transformer and field axes respectively, linking the squirrel cage conductors only,

$$\phi_{3t} = \frac{x_3 I_{3t}}{2\pi f N} \qquad (6)$$

$$\phi_{3f} = \frac{x_3 I_{3f}}{2\pi f N} \qquad (7)$$

The relations of these flux components to the circuits and to each other are indicated in Figs. 3a and 3b.

Voltage applied to the stator terminals must overcome the resistance drop $r_1 I_1$ and the voltages induced by alternation of ϕ_1, ϕ_{mt} and ϕ_{mf}. That is,

$$E = r_1 I_1 + j\,2\,\pi f\,(N\,\phi_1 + N\,\phi_{mt}\cos A + N\,\phi_{mf}\sin A)$$

Substituting for the fluxes their values given in equations (1), (2) and (3), we have,

$$E = r_1 I_1 + j\,x_1 I_1 + j\,X_m \cos A\,(I_1 \cos A - I_2 - I_{3t}) + j\,X_m \sin A\,(I_1 \sin A - I_{3f}) \qquad (8)$$

In the commutated winding, voltage is induced by transformer action of ϕ_{2t} and ϕ_{mt}, and by rotation through ϕ_{2f} and ϕ_{mf}. The vector sum of these voltages must equal the resistance drop $r_2 I_2$. This gives the equation,

$$O = r_2 I_2 - j\,2\,\pi f N\,(\phi_{mt} + \phi_{2t}) + S\,2\,\pi f N\,(\phi_{mf} - \phi_{2f})$$

or

$$O = r_2 I_2 - j\,X_m (I_1 \cos A - I_2 - I_{3t}) + j\,x_2\,(I_2 - I_{3t}) + S\,X_m\,(I_1 \sin A - I_{3f}) - S\,x_2\,I_{3f} \qquad (9)$$

Similarly, for the squirrel cage in the transformer axis,

$$O = r_3 I_{3t} - j\,X_m\,(I_1 \cos A - I_2 - I_{3t}) + j\,x_2\,(I_2 + I_{3t}) + j\,x_3 I_{3t} + S\,[X_m\,(I_1 \sin A - I_{3f}) - (x_2 + x_3)\,I_{3f}] \qquad (10)$$

and in the field axis,

$$O = r_3 I_{3f} - j\,X_m\,(I_1 \sin A - I_{3f}) + j\,(x_2 + x_3)\,I_{3f} + S\,[-\,X_m\,(I_1 \cos A - I_2 - I_{3t}) + x_2\,(I_2 + I_{3t}) + x_3 I_{3t}] \qquad (11)$$

The above four equations, (8), (9), (10) and (11) containing the four unknowns I_1, I_2, I_{3t} and I_{3f} can be solved giving equations from which the performance characteristic curves of the motor can be calculated.

It will be noted that these equations differ in form from the general equations of Steinmetz[2] for a-c. motors having any number of windings in both stator and rotor, in that his equations do not contain terms corresponding to $S\,x_2 I_{3f}$ in equation (9), $S\,(x_2 + x_3)\,I_{3f}$ in equation (10), and $S\,[x_2\,(I_2 + I_{3t}) + x_3 I_{3t}]$ in (11). In his treatment of polyphase motors, he took care of the phenomena involved by considering the leakage reactance of the rotor windings as being proportional to the slip. This gives exactly the same results, and is an exactly equivalent method when applied to polyphase motors. However, in cases where the mutual flux of stator and rotor is resolved into two components at right angles to each other in space, it seems preferable to treat the leakage flux in the same way and not to consider it as a flux revolving at synchronous speed. This applies particularly to the case of commutator motors where the current in the individual conductors is of line frequency. In single-phase motors where the leakage reactances of the rotor windings cannot in general be considered proportional to the slip, the omission of the terms representing voltages induced by rotation of rotor conductors through leakage fluxes results, in the usual case, in slight and negligible inaccuracies, but in the case of the squirrel cage repulsion motor would result in inaccuracies so great as to completely destroy the value of the results.

It is of interest to note that the reasoning on which the above equations are based is exactly similar to that followed by Arnold[3] in formulating the fundamental equations from which he developed the equivalent circuit for the single-phase induction motor according to the cross field theory.

Rearranging terms in equations (8) to (11) and solving for I_1, I_2, I_{3t} and I_{3f}, we get equations of the following forms:

$$I_1 = E\,\frac{F_1 + (1 - S^2)\,F_2 + j\,[F_3 + (1 - S^2)\,F_4]}{U + j\,W} \qquad (12)$$

$$I_2 = E\,X_m \cos A \left(\frac{G_1 + S\,G_2 + (1 - S^2)\,G_3 + S\,(1 - S^2)\,G_4 + j\,[G_5 + S\,G_6 + (1 - S^2)\,G_7]}{U + j\,W}\right) \qquad (13)$$

$$I_{3t} = E\,X_m \cos A \left(\frac{H_1 + (1 - S^2)\,H_2 + S\,(1 - S^2)\,H_3 + j\,H_4}{U + j\,W}\right) \qquad (14)$$

$$I_{3f} = E\,X_m \cos A \left(\frac{S\,J_1 + (1 - S^2)\,J_2 + j\,[J_3 + (1 - S^2)\,J_4]}{U + j\,W}\right) \qquad (15)$$

where $U + j\,W = F_5 + S\,F_6 + (1 - S^2)\,F_7 + j\,[F_8 + S\,F_9 + (1 - S^2)\,F_{10} + S\,(1 - S^2)\,F_{11}]$ (16)

and F_n, G_n, H_n and J_n are functions of the motor design constants, r_1, r_2, r_3, X_m, x_1, x_2, x_3, and A, and are independent of the speed. The complete expressions for these functions will be given in the appendix.

The torque produced by the motor at any speed can be considered as made up of three components produced respectively by interaction of the currents I_2, I_{3t} and I_{3f} with the fluxes $(\phi_{mf} - \phi_{2f})$, $(\phi_{mf} - \phi_{3f})$, and $(\phi_{mt} - \phi_{3t})$ through which the corresponding conductors rotate. The torque of the motor at any speed is, in synchronous watts:

$$T = 2\,\pi f N\,\{\,[\,(I_2\,(\phi_{mf} - \phi_{2f})\,] + [I_{3t}\,(\phi_{mf} - \phi_{3f})\,] + [I_{3f}\,x\,(\phi_{mt} - \phi_{3t})\,]\,\} \qquad (17)$$

where the expressions $[I_2\,(\phi_{mf} - \phi_{2f})\,]$ etc., represent the products of the in-phase components of current and flux. Substituting for the fluxes and currents

2. Loc. cit. 1.

3. Wechselstrom technik, V. I. Chapter 8.

their values according to equations (5), (6), (7), (13), (14) and (15) we get the following equation.

$$T = \frac{E^2 X_m^2 [M_1 + S\, M_2 + (1 - S^2)\, M_3 + S\,(1 - S^2)\, M_4 + (1 - S^2)^2 M_5 + S\,(1 - S^2)^2 M_6]}{U^2 + W^2} \tag{18}$$

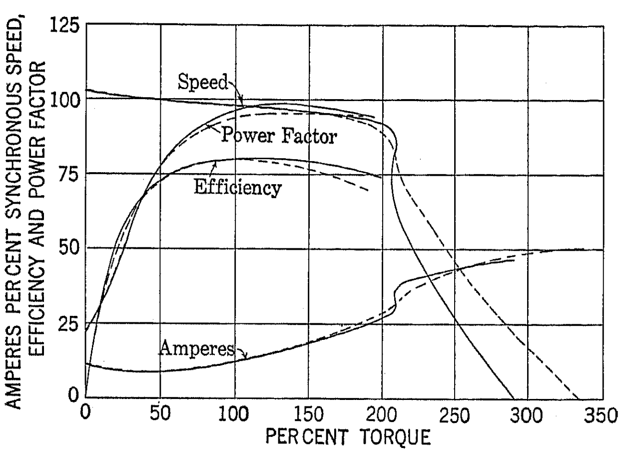


FIG. 4—PERFORMANCE CURVES OF S. C. R. MOTOR
Motor-rated 3-h. p. 1800-rev. per min.
220-Volt 60-Cycle
Calculated values——————
Tested values - - - - - - -

where M_1, M_2, etc., are functions of the motor design constants. The complete expressions for these functions will be given in the appendix.

By the use of suitable calculation forms, the torque developed and the current taken by the motor at any speed can be readily calculated from equations (12)

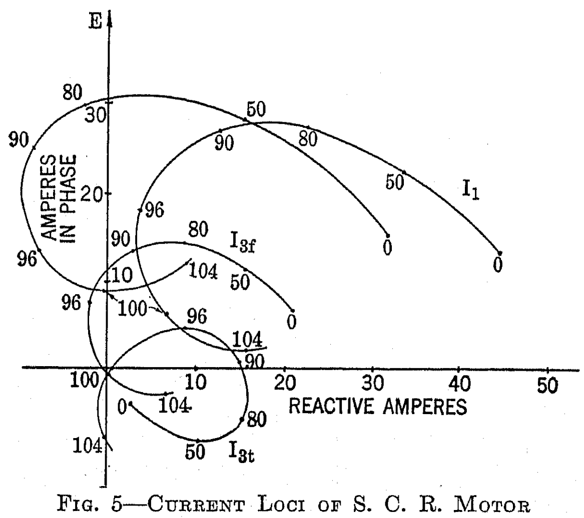


FIG. 5—CURRENT LOCI OF S. C. R. MOTOR

and (18). Since the value of current obtained by solution of (12) is given in terms of its power and reactive components, the power factor and input can then be determined. From the torque and speed, the output is determined; then after allowing for friction and core losses the efficiency is determined. If it is desired, the currents I_2, I_{3t} and I_{3f}, and the corresponding components of the torque can be calculated in a similar manner.

In Fig. 4 are shown the performance characteristics of a 3-h. p. 1800-rev. per min. 60-cycle squirrel cage repulsion motor as calculated from equations (12) and (18). Tested values are shown for comparison in the broken curves. It will be observed that the motor has a starting torque of approximately three times full load torque, and for speeds from standstill up to about 90 per cent of synchronism has a speed torque characteristic similar to that of a plain repulsion motor. For speeds in the neighborhood of synchronism the speed torque characteristics are those of an induction motor, from no load to double load. The motor therefore combines the characteristics of a repulsion motor and of an induction motor in operation, as well as in mechanical construction. The power factor at full load and overload is seen to be from ten to fifteen per

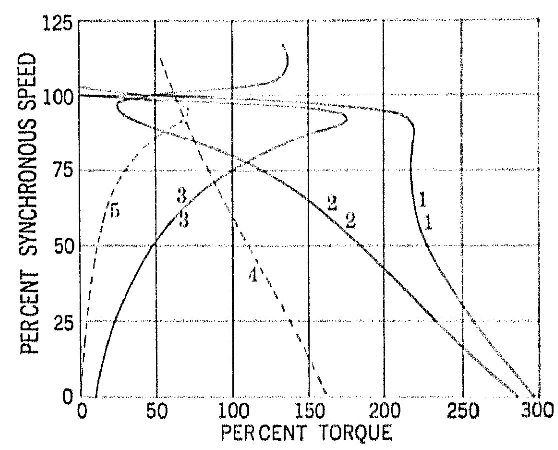


FIG. 6—DIVISION OF TORQUE BETWEEN ROTOR WINDINGS OF S. C. R. MOTOR (CALCULATED VALUES)

1. Total Torque Developed by Motor.
2. Torque Developed by Commutated Winding Currents.
3. Torque Developed by Squirrel Cage Currents.
4. Torque that Would be Developed if Squirrel Cage were Removed.
5. Torque that Would be Developed if Commutated Winding were Removed.

cent higher than would be obtained from either a plain repulsion motor or a plain single-phase induction motor of the same rating.

In Fig. 5 are shown the loci of the current vectors of the same motor, calculated from equations (12) to (15). The speeds corresponding to the different values of current are indicated in terms of synchronous speed on the curves. It will be observed that the stator current has its minimum value at synchronous speed, and that the variation in power input with change in speed in the close neighborhood of synchronism is accounted for principally by variation in power factor and not by variation in the magnitude of the current.

At full load, the current in the commutated winding is equal to approximately 85 per cent of the current in the stator winding.

The current I_{3t} in the transformer axis of the squirrel cage is very small for starting conditions, the squirrel

cage in this axis being effectively shielded by the commutated winding. In the field axis, the squirrel cage current I_{3f} has its maximum value at standstill. For normal operating speeds, these currents I_{3f} and I_{3t} are approximately equal in magnitude and 90 deg. out of phase in time, I_{3f} leading I_{3t}.

The division of the torque between the commutated and squirrel cage windings is shown in the calculated curves of Fig. 6. The speed torque curves of the motor when running with only the squirrel cage in the rotor, and with only the commutated winding, are shown in the dotted curves. It will be noted that the torque developed in the squirrel cage is roughly twice that which would be developed if the commutated winding were removed. In a general way, this is explained by the fact that for speeds in the neighborhood of synchronism, the motor has a nearly uniform revolving field like that of a polyphase induction motor. The explanation for this revolving field is the same as for the plain repulsion motor. That is, since the brushes bearing on the commutator are short-circuited, it follows that the voltages induced in the commutated winding by alternation of the transformer flux and by rotation through the field flux must be approximately equal. The transformer and field fluxes must therefore be 90 deg. out of phase in time as well as being at right angles in space. The resultant flux is therefore a revolving flux produced largely by the m. m. f. of the stator and commutated winding currents, independently of the squirrel cage currents. The squirrel cage therefore operates much the same as if it were in a polyphase motor, and it may be said to develop polyphase motor torque, the maximum torque thus developed being, roughly, twice the maximum torque that would be developed if the commutated winding were not used. For this reason the squirrel cage is much more effective at speeds in the neighborhood of synchronism than at standstill, and although the squirrel cage reactance is high enough so that the standstill conditions are comparable to those in a plain repulsion motor, the torque developed at speeds near synchronism is sufficiently large to give excellent speed regulation and to hold the no load speed to a value very slightly above synchronism.

The curve of torque developed by the commutated winding currents is of the same general shape as that of a plain repulsion motor, excepting in the neighborhood of synchronous speed where it has a definite maximum and minimum. The peculiar shape of this speed torque curve in the region of synchronous speed is of little practical significance or interest, except that it calls attention to the fact that the action of the commutated winding is greatly affected by the action of the squirrel cage current and vice versa, and that it is associated with the question of phase relations of the field flux, which, in turn, determine the power factor. This variation in torque is not due to any material change in the magnitude of the commutated current I_2 or the flux, $(\phi_{mf} - \phi_{2f})$, with which it reacts to produce torque, but is caused almost entirely by variation of the phase difference between the current and the flux.

When the motor is running at exact synchronous speed, the current in the squirrel cage can, for all practical purposes be considered equal to zero. The motor is therefore operating with the same current, power factor, efficiency, and torque as though the squirrel cage was removed, or, in other words, as a plain repulsion motor. If the load on the motor is increased so that the speed drops slightly below synchronism, currents are induced in the squirrel cage and induction motor torque is developed. The effect of these currents, as pointed out by Mr. Bergman, is to cause the field flux to lag the line current in time phase, resulting in power-factor compensation. That this phase difference is considerable for loads in the neighborhood of full load is shown by the fact that the torque produced by the commutated winding drops to a very low value.

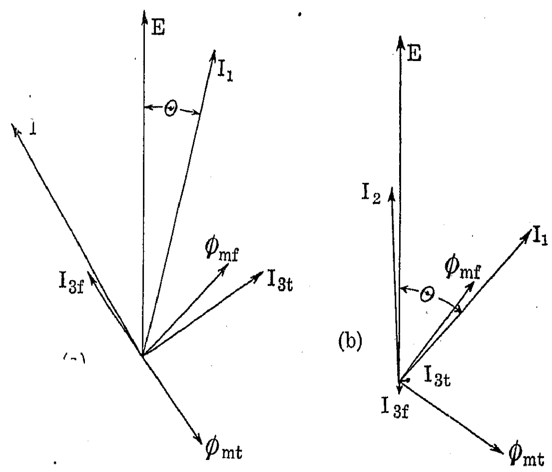


FIG. 7—VECTOR DIAGRAMS OF S. C. R. MOTOR
a. Full Load.
b. Synchronous Speed (Approximately Half Load.)

At exact synchronous speed, the squirrel cage repulsion motor may be said to operate exactly the same as a plain repulsion motor, since the current in the squirrel cage is then practically zero. A plain repulsion motor operating with the same brush position will be found to develop at synchronous speed a torque equal to approximately 50 per cent of the rated full-load torque of a squirrel cage repulsion motor of the same design constants. It is evident then that the squirrel cage repulsion motor develops approximately 50 per cent of full-load torque at exact synchronous speed, and for all loads greater than half load, the speed is below synchronism, so that power-factor compensation is obtained for all loads throughout the most important range.

Vector diagrams showing the phase relations at synchronism and at full load are shown in Fig. 7.

In a properly designed squirrel cage repulsion motor

the ratios of the resistance and leakage reactance of the squirrel cage to the resistance and leakage reactance of the commutated winding must be held within fairly close limits. In order to obtain the best starting torque per ampere of starting current, the squirrel cage reactance should be as high as possible. On the other hand, the higher the impedance of the squirrel cage, the higher will be the running free speed at which

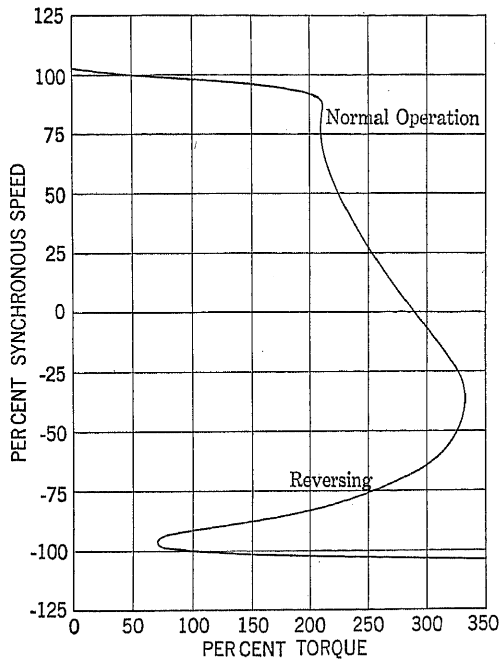


Fig. 8—Speed-Torque Curve of S. C. R. Motor for Positive and Negative Speeds

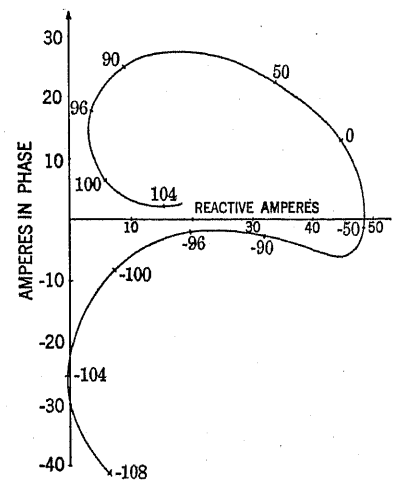


Fig. 9—Locus of Line Current—S. C. R. Motor

the generator torque, developed by the squirrel cage, will neutralize the motor torque, developed by the commutated winding. The currents, and therefore the losses, will be found to be correspondingly higher, so that we may say that the higher the squirrel cage reactance, the higher the no-load losses. There is a narrow range of values of squirrel cage reactance for which the starting conditions are comparable to those of a plain repulsion motor, and the no-load losses will be well within reasonable limits.

Since the full-load torque of the motor is furnished largely by the squirrel cage, it is evident that the speed regulation must be roughly proportional to the slip. The resistance of the squirrel cage should therefore be reasonably low in order to give good speed regulation.

A squirrel cage repulsion motor for reversing service may be obtained by providing a stator with distinct transformer and field windings, one of which can be reversed; or if desired, reversal may be accomplished by shifting the brushes. The reversing characteristics of the motor are shown in Fig. 8, which is the speed torque curve for both positive and negative speeds, and in Fig. 9, which is the locus of the stator current vector.

The current locus shows that although the motor does not return power to the line when driven at a speed above its no-load speed, it does operate as a generator with good characteristics, if driven at approximately synchronous speed against its normal direction of rotation as a motor. Its efficiency as a generator under these conditions is practically the same as its efficiency as a motor in normal operation, and as shown by the current locus, the power factor may be very high. The commutating conditions in this region also compare with those in normal motor operation, the explanation being the same as in the case of motor operation, that is, the voltages in the commutated winding induced by transformer action of and rotation through the transformer and field fluxes are approximately equal, and therefore the transformer and field fluxes must be approximately equal in magnitude and 90 deg. out of phase in time, giving as a resultant a rotating field, revolving with the rotor and therefore giving practically the same commutating conditions as in normal motor operation. A squirrel cage repulsion motor can therefore be reversed while running at full speed with no trouble from commutation. As shown by the current locus, the current taken by the motor is only very slightly greater at certain speeds during reversal than at starting.

It is well known that the current in an individual conductor of the rotor of a plain single-phase induction motor consists of two components of different frequencies, one component being of slip frequency and the other of approximately twice line frequency. The amplitude of the double frequency component is greater than that of the slip frequency component. In the squirrel cage repulsion motor, the current in an individual squirrel cage conductor also consists of two components, but in this case the double frequency component is small in comparison to the slip frequency component. This is to be expected, since the mutual flux of stator and rotor is, roughly speaking, a revolving field produced largely by the m. m. f. of I_1 and I_2 independently of the squirrel cage currents. The conditions affecting the squirrel cage can be compared

with those affecting the rotor of a polyphase induction motor with slightly unbalanced voltage applied to the stator; that is, the mutual flux of stator and rotor consists of a uniform revolving flux with a small pulsating flux superimposed in one axis. The ratio of the amplitude of the slip frequency component of current to the double frequency component, which is, roughly, the same as the ratio of the revolving component of flux to the pulsating component, can be determined for any particular speed from the calculated values of I_{3t} and I_{3f}. This nature of the squirrel cage current is of interest, in view of the fact that metallic strips or wedges are placed in the narrow slits connecting the squirrel cage slots with those of the commutated winding. These metallic strips serve the purpose of improving commutation by furnishing to the coils undergoing commutation local secondary circuits which absorb the energy that might otherwise appear in the form of very slight sparks at the brushes. The question of commutation and the action of the squirrel cage and these metallic strips in improving commutation has been rather fully discussed in the preceding paper by Mr. Bergman.

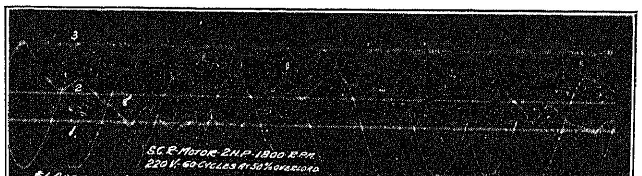


FIG. 10

Alternation of the leakage flux which threads these thin metal strips induces eddy currents in the strips, which has the effect of reducing the effective leakage reactance of the squirrel cage. The metal strips are therefore designed with sufficiently high resistance so that the eddy currents induced in them by the full-line frequency alternation of leakage flux at standstill will be so small as to be practically negligible. Otherwise, the effective leakage reactance of the squirrel cage would be appreciably reduced and the starting torque per ampere of starting current would be decreased.

When the motor is running at normal speed, the leakage flux threading the strips varies in the same manner as the squirrel cage current, *i. e.*, with a large slip frequency component and a small double frequency component. The effect of the slip frequency component in producing eddy currents in the strips will be entirely negligible on account of the very low frequency. The double frequency component can have only a very slight and practically negligible effect, on account of the small amplitude. The total effect of the metal strips on the operation of the motor is therefore quite negligible except for the improvement in commutation, and as is evidenced by the close agreement between the tested and calculated values shown in Fig. 4, it is perfectly legitimate to ignore the presence of the strips in calculating the performance characteristics of the motor, provided the strips are so designed as to properly take care of the standstill conditions.

In Fig. 10 are oscillograms showing the nature of the currents in the squirrel cage bars, the eddy currents in the metal strips, and the current in an individual coil of the commutated winding. In order to get an oscillogram showing the equivalent of the eddy currents in the wedges, an exploring coil was wound in the rotor with one side in the bottom of a commutated winding slot, and the other in the upper part of the corresponding squirrel cage slot. The wave shape of the voltage induced in this coil shows that the eddy currents in the metal strips consist largely of irregular high-frequency pulsations, which represent energy that is transferred during commutation from the commutated winding to the metal strips.

ACKNOWLEDGEMENT

The writer wishes to acknowledge his indebtedness to S. R. Bergman and to P. L. Alger for helpful suggestions in connection with the development of the theory and the method of calculation.

Appendix

Let $X' = X_m + x_2$

$X'' = X_m + x_2 + x_3$

Using these abbreviations, the complete expressions for the functions in equations (**12**) to (**18**) are:

$F_1 = r_2 r_3^2 - 2 r_3 x_3 X'$

$F_2 = - r_2 X''^2 - r_3 X'^2$

$F_3 = 2 r_2 r_3 X'' + r_3^2 X'$

$F_4 = - x_3 X' X''$

$F_5 = r_1 F_1 - x_1 F_3 - r_2 r_3 X_m (X_m + 2 x_2 + 2 x_3) - r_3^2 x_2 X_m - r_3^2 X_m^2 \sin^2 A$

$F_6 = - 2 r_3 x_3 X_m^2 \sin A \cos A$

$F_7 = r_1 F_2 - x_1 F_4 + x_2 x_3 X_m X'' + x_3^2 X_m^2 \sin^2 A$

$F_8 = r_1 F_3 + x_1 F_1 - 2 r_3 x_2 x_3 X_m - 2 r_3 x_3 X_m^2 \sin^2 A + r_2 r_3^2 X_m$

$F_9 = r_3^2 X_m^2 \sin A \cos A$

$F_{10} = r_1 F_4 + x_1 F_2 - r_2 (x_2 + x_3) X_m X'' - r_3 x_2 X_m X'$

$F_{11} = - x^2_3 X_m^2 \sin A \cos A$

$G_1 = - 2 r_3 x_3$

$G_2 = - r_3^2 \tan A$

$G_3 = - r_3 X'$

$G_4 = x_3 X'' \tan A$

$G_5 = r_3^2$

$G_6 = - 2 r_3 x_3 \tan A$

$G_7 = - x_3 X''$

$H_1 = - r_2 r_3 \tan A$

$H_2 = - r_2 X''$

$H_3 = - x_3 X' \tan A$

$H_4 = r_2 r_3$

$J_1 = H_4$

$J_2 = - [r_3 X' + r_2 X''] \tan A$

$J_3 = r_2 r_3 \tan A$

$J_4 = H_3$

$M_1 = [2 r_2 r_3^3 X' + 4 r_3^2 x_3^2 X' + r_3^4 X'] \sin A \cos A$

$$M_2 = r_2\, r_3^2\, [-\, r_2\, r_3 + 2\, x_3\, X' \cos^2 A - r_3^2 \sin^2 A - 2\, x_3\, X'' \sin^2 A - 2\, x_3^2 \sin^2 A]$$
$$M_3 = r_3\, x_3\, X' \sin A \cos A\, [r_3\, X' - 2\, r_3\, x_3 + 2\, r_2\, X'']$$
$$M_4 = r_3\, [2\, r_2\, r_3\, x_3\, X'' \sin^2 A + r_2\, r_3\, X'^2\, (1 + \sin^2 A) + r_2^2\, X''^2 + 2\, x_3^2\, X'^2 \sin^2 A + r_3^2\, X'^2 \sin^2 A]$$
$$M_5 = x_3^3\, X'\, X'' \sin A \cos A$$
$$M_6 = -\, x_3^2 \sin^2 A\, [r_2\, X''^2 + r_3\, X'^2]$$

The first step in calculating the performance characteristics of a squirrel cage repulsion motor from its design constants is to calculate the constants F_1 to F_{11} and M_1 to M_6 given above. Using the constants thus obtained, the calculation of current, power-factor input, torque, etc., can be carried out for all desired speeds simultaneously. Taking advantage of the fact that the constants to be calculated contain many terms and combinations of terms in common, and by using suitable calculation forms, the calculations can be completed in much less time than would be expected, judging from the lengths of the equations.

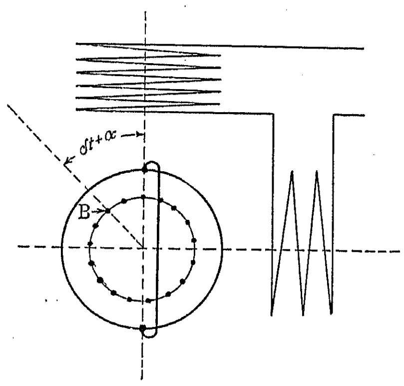


FIG. 11

An analytical expression for the instantaneous value of the current in an individual conductor of the squirrel cage would undoubtedly be so complicated as to be entirely out of the question. If it is desired to determine the wave shape of this current for any particular speed, it can be readily determined from the calculated values of I_{3t} and I_{3f}. Referring to Fig. 11, the instantaneous value of the current in the bar B is given by the equation

$$i_B = \sqrt{2}\, [\, |I_{3t}| \sin(\omega\, t + \theta_1) \sin(\sigma\, t + \alpha) + |I_{3f}| \sin(\omega\, t + \theta_2) \cos(\sigma\, t + \alpha)\,]$$

where $|I_{3t}|$ and $|I_{3f}|$ represent absolute values, θ_1 and θ_2 are the phase angles of I_{3t} and I_{3f} referred to the line voltage and α is the angular position of the bar when $t = o$. Transforming this equation into one involving functions of the sum and difference of two angles, we get the equation,

$$i_B = I_B' \sin[(\omega - \sigma)t + \beta_1] + I_B'' \sin[(\omega + \sigma)t + \beta_2]$$

or $i_B = I_B' \sin(s\,\omega\, t + \beta_1) + I_B'' \sin[(2 - s)\, \omega\, t + \beta_2]$

where I_B', the amplitude of the slip frequency component is given by the equation,

$$I_B' = \sqrt{\tfrac{1}{2}\, [|I_{3t}|^2 + |I_{3f}|^2 + 2\, |I_{3t}|\, |I_{3f}| \sin(\theta_2 - \theta_1)]}$$

and I_B'' the amplitude of the component whose frequency is $(2 - s)\, f$, is given by the equation,

$$I_B'' = \sqrt{\tfrac{1}{2}\, [I_{3t}^2| + |I_{3f}^2| - 2\, |I_{3t}|\, |I_{3f}| \sin(\theta_2 - \theta_1)]}$$

and β_1 and β_2 are constants.

Discussion

H. C. Specht (by letter): The calculation method shown by Mr. West will, without doubt, interest all motor designers. The way of demonstrating the different fluxes gives a very clear picture which can be easily understood. On the other hand, such a method of calculation without the use of diagrams and saturation curves seems to me rather complicated for routine design work, in spite of the fact that the effect of short-circuit current under the brushes and iron losses have been neglected. It is also assumed that all m. m. f. and fluxes are sinusoidal, which does not often correspond to the type of winding usually employed in a commercial motor. Another assumption is that of constant reluctance, whereas this will vary inversely with the saturation, which will have a very marked effect on the characteristics of the motor at overloads and when starting.

Possibly the neglect of these items is responsible for a discrepancy in the calculated curves shown in Fig. 6. Curve No. 4, as drawn, indicates that the starting torque of a repulsion motor will be increased by the addition of a squirrel-cage winding. This cannot possibly be the case, and Curve No. 4 should cross Curve No. 1 somewhere below synchronous speed, and show a starting torque considerably larger than that shown by Curve No. 1 representing the combined windings.

Using diagrams, in connection with the saturation curve for magnetizing voltage and flux, simplifies the calculation method, particularly when the motor is first calculated as the straight repulsion type and thereafter corrections are made for the effect of the squirrel-cage winding, short circuit current under the brushes, iron loss, etc. This method also gives a clear picture of the effect of the different items and any errors in calculation may be more easily noticed.

P. L. Alger (by letter): The element in design of the squirrel-cage repulsion motor that is all-important is the choice of the constants of the squirrel cage, and this is the point in which the new motor described by Mr. Bergman differs most from all previous motors of the same class.

Theory shows that in so far as starting is concerned, the squirrel cage should have the highest possible impedance, together with the lowest possible ratio of resistance to reactance. The ratio of resistance to reactance must be low in order to hold the axis of the squirrel-cage current as far away from the axis of the cross-field flux as possible, since this will give the least possible reduction of cross-field flux for a given squirrel-cage current. During normal operation, the squirrel cage should have the lowest possible impedance and a fairly definite, but rather low, ratio of resistance to reactance. This is true because maximum power factor at normal load is secured with a particular value of squirrel-cage reactance and because suitable values of pull-in torque and of losses are obtained with a particular value of resistance. Finally, in so far as commutation is concerned, the squirrel cage should have the lowest possible impedance, and a high ratio of resistance to reactance, since in this way the energy of commutation is most readily and completely dissipated without sparking.

Consideration of methods of securing these diverse values of the squirrel-cage impedance shows that it is very important to have no saturation in the reactance flux paths for the squirrel cage, since any such saturation would reduce the ratio of starting reactance to running reactance, which ratio should be made as large as possible. Also, the squirrel cage should have a low

d-c. resistance, with eddy currents of such a magnitude as to be inappreciable up to double line frequency, but considerable at commutation frequency. As the eddy currents give the effect of a reduction of reactance as well as an increase of the effective resistance, they are altogether objectionable at starting and altogether desirable for commutation.

In the new motor, these requirements have been met in a way which seems theoretically the best possible. The squirrel-cage leakage flux path has an air gap in it calculated to be of such a magnitude as to avoid all saturation at starting, and this air gap is filled with a metal wedge of such a resistivity as to give eddy currents that are inappreciable at operating frequencies, but are considerable at commutation frequency.

K. L. Hansen (by letter): Although Mr. West has made use of certain simplifying assumptions with reference to the angle of hysteretic advance and the effect of eddy currents in the shielding commutating strips, it must be conceded that the agreement between the tested and calculated values is sufficiently close to justify these assumptions, especially when the complexity of the calculations is taken into consideration. It is rather unfortunate, however, that neither design constants nor dimensions are given in the paper, thus making it impossible to compare the results with those obtained by a method differing somewhat from that of the author.

It is clearly set forth in the paper that the ratio between the constants of the squirrel cage and the constants of the commutated winding must be held between close limits, otherwise the no-load losses become excessive or the starting torque per ampere will be too low. The fact that at no load the motor develops two torque components neutralizing each other instead of just sufficient torque to overcome the friction losses, as is usually the case, tends to make the no-load losses high. The no-load losses of the particular machine discussed in the paper appear to be approximately 25 per cent of the full-load output. That may not be excessive in a 3 h.p. motor, but the same percentage losses in a larger machine would be excessive, and it would be of interest to learn what results have been obtained on motors of say 10- or 15-h.p. capacity.

The analysis of the squirrel-cage repulsion motor may be approached from a different viewpoint from that which Mr. West has chosen as his starting point. About thirty years ago Professor Ferraris suggested that single-phase induction-motor operation could be explained by considering the single-phase alternating field as made up of two components revolving in opposite directions. It seems that this method has not been looked upon with favor as a starting point for quantitative analysis, but I have found it exceedingly useful in the solution of just such problems as that discussed by Mr. West.

Analysis of the squirrel-cage repulsion motor from this viewpoint also leads to four equations with four unknowns, so the numerical calculations may not be much simplified, if any, but the forming of the equations from physical considerations is made more simple and direct by this method. It is not necessary to resolve the primary winding into two components at right angles to each other, nor to consider the transformer and rotational voltages separately. Equally, there is no need of considering the squirrel-cage as the equivalent to a commutating winding with the brushes short-circuited in two rectangular axes. From the viewpoint of two oppositely rotating fields the squirrel cage is considered as a polyphase circuit with two current components flowing in it of frequencies proportional to s and $(2-s)$, where s = slip. These current components are physical realities and are plainly shown on the oscillogram in Mr. West's paper. The reactances which these current components meet are likewise proportional to s and $(2-s)$.

I have not derived the formulas of the squirrel-cage repulsion motor from the viewpoint of two rotating fields, mainly because the constants are not given in the paper so that results can not be compared anyway. Another reason is that I have applied this method in considerable detail to a number of similar problems in a paper entitled "The Rotating Magnetic Field Theory of Alternating-Current Motors," which will be published in the JOURNAL at some future time.

I am indebted to Mr. Specht for calling attention to a point which should perhaps have been explained more fully in the paper. It is natural to believe that Fig. 6 of my paper is wrong as has been claimed by Mr. Specht, because at first thought it would seem that the addition of a squirrel-cage to a repulsion motor must necessarily reduce the starting torque. Paradoxical as it may seem, however, this is not true, and it is a fact that if the squirrel cage is removed from a properly designed squirrel-cage repulsion motor, the starting torque will be materially reduced unless the brush position is changed.

The explanation is that although the ampere-turns of the squirrel cage partially neutralize the field ampere-turns of the stator winding, and thus reduce the field flux per ampere of line current, the starting impedance is much reduced so that the starting current is greater. The starting torque is roughly proportional to the product of the starting current times the field flux, and therefore may or may not be decreased by the

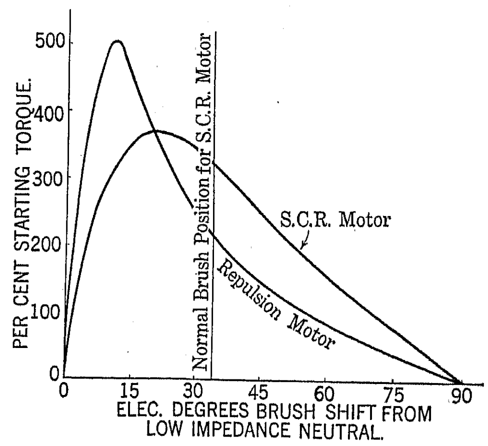


FIG. 1—CURVE OF COMPARISON BETWEEN STARTING TORQUES OF A REPULSION MOTOR AND A SQUIRREL-CAGE MOTOR FOR DIFFERENT BRUSH POSITIONS

addition of a squirrel cage, depending on the relative design constants of the motor and squirrel cage, and the angle of brush setting.

The curves of Fig. 1 give in a general way a comparison between the starting torques of a repulsion motor and of a squirrel-cage repulsion motor for different brush positions. It will be noted that for brush positions close to the low-impedance neutral, the plain repulsion motor has the higher starting torque, but as the brushes are shifted farther from neutral a point is reached beyond which the squirrel-cage repulsion motor has the higher starting torque.

It is of course true that the maximum torque obtainable is less for a squirrel-cage repulsion motor than for the corresponding plain repulsion motor. It should also be noted that the starting torque of the squirrel-cage repulsion motor is less sensitive to slight changes in brush position than is the plain repulsion motor.

The omission of certain minor factors in the derivation of the equations is, in my opinion, fully justified. If all factors that affect the motor operation were taken into consideration, assuming that it were possible to do so, the results would differ from those obtained by the method given in the paper by percentages in comparably smaller than the percentage difference between the starting torque values of curves 1 and 4 of Fig. 6 which Mr. Specht mentioned. In calculating starting torque, it is sometimes necessary to make certain allowances, such as those for saturation and the effects of the currents in the short-

circuited coils. The calculation of starting torque is so very much easier and shorter than the general calculation that the necessary allowances can easily be made.

I feel safe in saying that it is impossible to make a reliable quantitative predetermination of the performance of a squirrel-cage repulsion motor by first calculating the performance of a plain repulsion motor and then making allowances for the effect of a squirrel-cage unless experimental or other accurate data for motors of somewhat similar design characteristics are available. For example, I do not believe that the design of a 2-pole, 25-cycle motor could be most advantageously worked out on that basis if experimental or other reliable data were available for 4-pole, 60-cycle motors only. The conclusion which Mr. Specht arrived at concerning the curves of Fig. 6 might be said to be characteristic of the results that might be expected from the use of that method, for as I tried to show by the curves of Fig. 6, the action of each rotor winding is altogether different from that which it would have if the other winding were removed.

The method of calculation which I have outlined in the paper is too complicated for general routine design work, but, in my opinion, the design of a motor of which many thousands will be built, is of sufficient importance to justify the carrying out of a number of complete calculations so as to determine the most economical use of materials, to get a better idea of the best proportions of the design constants, and to learn how the operating characteristics of a motor will be affected by a change in any one of the design constants.

I am in complete agreement with Mr. Alger except in regard to the way in which squirrel-cage resistance affects the starting torque. At standstill, the current in the squirrel cage is almost all in the field axis, since the commutated winding effectively shields the squirrel cage in the transformer axis. The effect of the squirrel-cage resistance is to cause the field flux to lag the line current in time phase, thus making it less effective in producing torque.

In reply to Mr. Hansen's question about the no-load losses, it may be said that the no-load losses need never be objectionable. The larger the motor, the lower the percentage no-load losses may be made. For a 10-h. p. motor, the no-load losses are approximately 11 per cent of the full-load input.

I am glad that Mr. Hansen has succeeded in working out the theory of single-phase commutator motors by the revolving-field method, for it seems that it is necessary to use the revolving-field method in order to allow properly for the effects of eddy currents in the squirrel-cage bars.

The revolving-field method and the cross-field method both have their advantages, and in my opinion neither method should be used to the exclusion of the other. The revolving-field theory allows for the variation of inductance and resistance with frequency variations in the rotor conductors and would give the slip-frequency and double-frequency components of the rotor current directly. On the other hand, the cross-flux theory is, in my opinion, easier to work with, especially for commutator motors, and in the case of the squirrel-cage repulsion motor gives a clearer picture of the effect of the squirrel-cage currents on the power factor than would be obtained from the revolving-field point of view. The "current in the transformer axis of the squirrel cage" and the "current in the field axis of the squirrel cage," which are given directly by the cross-field method, have definite and very useful physical meanings.

I regret that Mr. Hansen has not completely worked out the equations for the squirrel-cage repulsion motor following the revolving-field method. I believe that the equations which he would obtain by that method, starting with the same assumptions, would be identical in form with mine. At least it should be possible to transform the equations obtained by one method into those obtained by the other. Possibly his final equations would appear in a form that would give an easier method of calculation than mine. A direct comparison of equations would be more convincing and probably easier than a comparison of calculated data.

Technical Committee Annual Reports 1923-1924

TRANSMISSION AND DISTRIBUTION COMMITTEE

To the Board of Directors:

The increasing demand for electrical energy has made necessary the interconnection of present systems and the development of additional hydroelectric power projects. These new developments are in many cases at points remote from load centers, and the transmission problems involve distances, amounts of energy, reliability, and voltages for their economic transmission in excess of previous operating practise.

The insulator problem continues to be one of the chief sources of difficulty in overhead transmission, and although considerable progress has been made, both in the design and manufacture of suspension type insulators, there still remains room for additional investigation on the electrical and mechanical effects influencing leakage resistance, deterioration, etc. Some flashovers of insulator strings are still unexplainable and research work is being done to try to determine the cause and cure of these flashovers. The cause of the flashover possibly has a bearing on many of the so-called lightning disturbances.

The successful automatic operation of oil switches with relay protection at voltages as high as 220 kv. is an accomplished fact and this prevents the damage to insulators on account of flashovers. It may be said that the transformers and switches operate satisfactorily at these high voltages. The improvement of the insulation to prevent flashovers and many of the so-called lightning disturbances seems necessary, as the flashovers do not seem to be caused by high-frequency or transient voltage, since the rise in voltage (sometimes double voltage) due to switching, does not of itself cause flashovers. The mechanical stress, due to continued loading, vibration of aerial spans, temperature, humidity, and atmospheric conditions, in particular, require further study.

The result of experience this year on the 220-kv. transmission lines in California is very encouraging. In the northern and southern California systems 220-kv. transmission seems now an absolute necessity, as a result of the large power loads to be transmitted. The leading charging kilovolt-amperes supplemented by synchronous condensers is proving a valuable asset in voltage maintenance.

Transmission research work is being carried on in California and at other places, to determine all the factors entering into a system of transmission that will give the greatest practical reliability of service and the greatest stability and efficiency. Those responsible for financing and cooperating in carrying out this work are to be commended.

In the eastern section of the United States, various companies have, or are contemplating, changing their distribution systems from 3-wire 3-phase 2300-volt, or ¼ phase, 2300-volt, to 3-phase, 4-wire, 2300-4000-volt. The special problems involved in such change-overs include, in general, arrangement of wires, identification of neutral wire and phase wires, transformer connection, re-winding motors, etc.

The increased demands for electrical energy in urban districts are likewise tending towards the use of higher distribution voltages in order that larger amounts of energy may be delivered more economically, and the congestion of overhead routes avoided. This increase of distribution voltages makes necessary the development more of economical and efficient types of lightning arresters, fuses, switches, and other protective apparatus.

The demand for rural service continues to be felt throughout the country, and types of construction and material economically suited for this service are being given careful attention. The low-load factor, in general, experienced in rural service has made advisable some form of transforming equipment which will reduce the core loss during light or no-load periods, and progress is now being made in the development of equipment for this class of service. Also research work is being done to see how much power can be used by the rural areas in different sections of the country.

Continued progress is being made in the work of standardizing materials used for aerial construction the economical principles of which are apparent to both the producing and consuming branches of the industry. In order that the benefits of such standardization may be realized, the consumers should, in so far as possible, use such standardized equipment, and the manufacturer should continue to cooperate in the production and improvement of these various types of material. Where manufacture has been stabilized, such standardization can be accomplished through the American Engineering Standards Committee.

CABLE RESEARCH

The most important development during the past year has been the evidence that the cable specifications of the N. E. L. A. and the present Standards of the A. I. E. E. do not insure a satisfactory cable for the higher operating voltages.

This subject is now receiving attention from the manufacturers as well as from the users of high voltage cable and as a result of these studies, it is hoped that it will be possible: *First*, to make the necessary changes in manufacturing processes and materials so as to secure a satisfactory cable for operation at the

higher voltages, and *Second*, to devise a method of testing high voltage cable which will determine its operating characteristics in advance of its installation.

The Cable Research Subcommittee is cooperating with similar committees of other organizations in the development of the necessary tests and specifications for this purpose.

During the past year a 500,000-cm. single-conductor underground cable installation has been made from the Sherman Creek Station of the United Electric Light & Power Company in New York City northward a distance of about eleven miles into Westchester County, which is intended for operation at 44 kv. 3-phase. An eight-mile line of similar size has been installed by the Cleveland Electric Illuminating Company for operation at 66 kv. 3-phase. These are the first high-voltage single-conductor underground transmission lines to be installed in this country and the results of their operation will be watched with considerable interest.

The committee is cooperating with the technical colleges in connection with their researches into the properties of impregnated paper insulation, as follows:

Massachusetts Institute of Technology.

The effect of heat on impregnated paper insulation.

Harvard Engineering School.

Dielectric loss and ionization.

Cornell University.

Dielectric loss, ionization and the mechanism of cable failures of impregnated paper insulation.

Similar investigations are now under consideration by Washington University (St. Louis, Missouri).

FRANK G. BAUM, *Chairman.*

LIGHTING AND ILLUMINATION COMMITTEE

To the Board of Directors:

Pursuant of the recent practise, the report of this committee is submitted in two principal sections; the first being an outline of the committee's plan and work, and the second a brief resumé of the advance in lighting for the year.

COMMITTEE ACTIVITIES

The Committee held an investigation meeting at the A. I. E. E. Headquarters, October 11, 1923, in which the plans for the year were generally formulated. Mr. Skiff was reappointed in charge of Illumination Items, and Dr. Shackleford in charge of solicitation of convention papers.

Various plans were discussed, and it was understood that the Chairman would continue the discussion among the membership of the committee by correspondence.

Illumination Items. The Committee has kept the Editor of the JOURNAL continuously supplied with articles for inclusion in this section. Some of these are original contributions, others are reviews of papers or reports published elsewhere. Concerning the latter, it often happens that information of value to practising engineers is accompanied with much mathematics or involved discussion of theory. The committee has endeavored to extract the essential conclusions and data in convenient form for the use of the membership. Inquiries concerning various topics indicate that the articles are read with considerable interest.

In connection with this work the committee supplies the Editor with short notes, for use as fillers.

Convention Papers. The analysis of the situation by members of the committee, leads to the conclusion that it is not desirable at the present time to submit many papers on lighting at general conventions. Relatively little material of note, in the strict field of the committee, has come out in the past year.

It seems to be the general opinion, and that of the committee, that pure lighting practise papers are more suitable for the Illuminating Engineering Society. Most of the material of this sort is better suited for Illumination Items than for the convention programs.

Guided by these considerations, and the congested condition of convention programs, the committee has found it expedient to arrange for only one paper—namely an authorative treatment of the general subject of street lighting, which is being submitted for the June convention.

Lighting Publicity. It has been suggested by C. F. Scott, that the material published in Illumination Items was of sufficient value to warrant steps to encourage wider use. Several plans for encouraging the copying of Illumination Items have been investigated. The following procedure was finally adopted as the most expedient. The committee will submit to the Editor of the JOURNAL, with each piece of copy, a list of periodicals likely to be most interested. The Editor thereupon will secure a few extra proofs, and forward to designated publications, with memo of release date. It is expected, of course, that in using such material credit will be given to the JOURNAL, as the source. This program is just being started, so that it is not yet practicable to judge how effective it will prove.

Cooperation with Sections. It has been the desire of the committee to cooperate with sections, to encourage and assist them in securing high-grade discussions of lighting questions.

No practicable way has yet been found for initiating such an activity on a large scale. However, a plan has been formulated for starting in to cooperate with college sections, under the direction of collegiate members of the committee. Owing to lateness of this development it was not found expedient to undertake this during the current year, and it is commended to the attention of next year's committee.

Designation and Scope of Committee. A communication has been received suggesting a change of name for the committee, and a definition of its scope.

This proposal is under discussion within the committee, and it is hoped to make suggestions in this connection before final action is taken.

PROGRESS IN LIGHTING AND ILLUMINATION

Based upon the radical improvements in illuminants, there have for several years been many improvements in lighting equipments, which have not been taken full advantage of in practise. Therefore, at the beginning of the Institute year, the development of lighting equipments was far in advance of their application. This was especially true of economic features, in that practise was far behind in taking advantage of the value available.

As a result of a growing realization of this condition, there has been a rapid accelleration in the application of improved lighting, in both new and revised installations; so that the year stands out as one of raising standards, successful commercial campaigns, etc., more particularly than as one of engineering development.

While this is regarded in the lighting industry as a commercial achievement, it is also interesting because it represents a growing tendency to apply good engineering principles to lighting practise.

As mentioned in last year's report, the sale of incandescent lamps provides the best numerical measure of the advance in extent and intensity of artificial lighting.

The sale in the United States of so-called large incandescent lamps for 1922 as reported last year was 203,000,000, a gain of about 22 per cent over 1921. The sale for 1923 is reported to be approximately 245,000,000, or a further gain of over 20 per cent.

The lamps are those used upon power circuits and as such are reported annually by the Lamp Committee of the National Electric Light Association to indicate the growth of that phase of electric lighting in which the central station industry is interested.

In addition, there has been growing up a very large use of lighting supplied by batteries and other sources, usually separate from central station lines, as for example, automobile lighting, flashlights, surgical lamps, etc.

Lamps for these services have been considered as a separate class, designated as miniature lamps. Previous information has been less complete and accurate with regard to these lamps. Last year's report carried an estimate that 85,000,000 miniature lamps were sold in this country during 1922. More complete estimates now show that nearly 130,000,000 were sold in 1922. The latest compilation shows the sale for 1923 to have been approximately 175,000,000, nearly all being tungsten lamps. The remarkable extent of the lighting represented by miniature lamps has not been generally appreciated.

This is no doubt due to the fact that growth has been of such recent date. The earliest records now available are for the year 1908 when a little less than 2,000,000 miniature lamps, mostly carbon, were sold. The widespread use of electric-lighted automobiles and of flashlights are the largest factors in this growth.

Illuminants. There have been no outstanding developments during the past year, though many minor improvements have been or are being made.

In the manufacture of incandescent lamps, the improved machinery referred to in last year's report has been further improved and more widely applied. Besides increasing the flexibility of adapting manufacture to meet the demand, these developments have shown an influence in reducing cost. It is reported that tungsten lamp prices are now at least 30 per cent less than in 1920, in spite of increased costs of labor and materials.

Last year's report also referred to the fact that it had been found practicable to make tipless lamps. This has now progressed to a point where practically all the more common types and sizes of tungsten lamps are being made without tips.

There have been a number of changes in filament form. For example, the 200-watt 110-volt lamp is now made with a ring-shaped rather than a saw-tooth or loop filament.

Bowl-frosted lamps of the vacuum type have been widely used in open reflectors and shades for home lighting. When this finish was adopted an all-frosted lamp was considered extravagant, due to the loss of light through internal blackening of the bulb. Recent tests with various types of equipment in common use indicate that the all-frosted lamp is nearly as efficient. Where the bowl-frosted lamp was used in connection with open reflectors and some styles of frosted globes, the filament was exposed to view. The all-frosted lamps are therefore being recommended in place of the bowl-frosted lamps in order to insure diffusion and better appearance.

In the focus-type miniature lamps used for motor vehicle headlighting, an effort is being made to reduce the tolerance of dimensional variations, with the view of forming the basis of avoiding the necessity of refocusing upon replacement of lamps. The improvement should render practicable a fixed focus lantern. Such equipment if properly constructed, would do much to improve headlighting practise, which is today suffering from the failure of the motorist to focus.

Arc lamps have practically disappeared from all of the ordinary classes of lighting, with the exception of street lighting. In the ornamental form, a considerable number of arc lamps with magnetic electrodes are being used, especially in "white way" districts. It is reported that electrodes of increased efficiency have been developed during the year.

Arc lamps are, of course, still used extensively, for projection and photographic work.

Mercury arc lamps are used in photographic and industrial lighting and the quartz tube lamps for sterilization, fading and other chemical applications. No radical improvements have been reported.

The fact that the efficiency of present day illuminants falls far short of the theoretical ideal, is a continual incentive to investigators to search for improved methods of light production.

Much attention is being given to arcs and dis-

charges in gases. While some interesting experiments have been tried, no results giving immediate promise of practicability have been reported.

Lamp Equipments. There are a multitude of light-modifying equipments for use in connection with incandescent lamps. New types and improvements are continually being developed. Many of the improvements have to do with artistic appearance or mechanical convenience. No developments have been reported which incorporate radically new engineering principles.

The most distinctive development of the year is in connection with street lighting luminaires, following up the idea suggested by the recent highway lighting units.

Various characteristics of asymmetrical distribution have been incorporated in equipment for residence streets and thoroughfares, for the purpose of concentrating more light in useful directions, and reducing the light where useless or undesirable. The problems involved are rather complex, requiring consideration of degree of redirection, as well as relative values of light in different directions. There is still considerable difference of opinion as to what is most desirable in various classes of streets, with and without foliage, and considerably more practical experience is necessary before a general agreement among engineers can be expected. It seems likely, however, that some degree of asymmetry of light distribution will be found desirable for certain classes of streets, while on the other hand, symmetrical distribution will predominate in others, for example, "white ways" and business streets.

A number of new types of traffic signals has been developed both in those used for the control of traffic and those which simply warn drivers of busy intersections. The former group are practically all electric lighted. Owing to the recent improvements in electric equipment, a growing proportion of the warning signals are electric lighted, either from central station circuits or self-contained batteries.

Lighting glassware by its very nature is subject to certain variations as to light transmission and diffusing qualities. Such variations affect both the efficiency and appearance of luminaires. These variations can be minimized by care in manufacture and in some cases by grading. This, of course, means additional cost, so that unless there is a criterion for evaluating superior glassware, there will be a tendency for the cheaper products to predominate. An association of glass manufacturers has been making a study of the problem with the view of providing more suitable methods of specifying the desirable characteristics.

Practise. Standards of lighting are advancing in practically all fields; notably homes, streets, stores, show windows, signs and schools. The improvements involve both higher levels and better diffusion.

Home lighting, involving as it does the elements of art and tradition, has been found a difficult problem for the illuminating engineer. Too often the application of modern incandescent lamps and higher levels has been rendered ineffective because the equipment has not been suited for the more brilliant and powerful light sources.

About a year ago, the shallow diffusing globe unit was found to provide a considerable improvement in kitchen lighting. This application found a remarkable acceptance, and under aggressive commercial campaigns, this practise has spread so that over 300,000 of such equipments are now reported to be in use. The indications are that there will be even greater activity in this field in the near future. Moreover, by educating the home-maker to the possibilities of good illumination, it seems likely to facilitate the improvement of illumination in other portions of homes.

From the humane viewpoint, it is important that hospital operating rooms be as well lighted at night as in the daytime. During the year a number of installations have been made which supply not only a high level of diffuse illumination but also light of a color approximating closely to daylight. This is reported to have advantages in identifying particular classes of tissue, and so facilitating accurate work on operations.

Certain athletic sports, such as tennis, have for years been carried on under artificial light. Artificial lighting has also been tried for baseball and football, but little has been heard of it after the first try-out. During the past year unusually good results have been reported in several games of each, the method of illumination being floodlighting.

Considerable experimentation has been carried on by the Government Air Mail Service, toward the improvement of airplane landing lights, field lighting and beacons for mail routes. Considerable advance is reported and new beacons are now being set up.

For a number of years considerable interest has been shown regarding the question as to whether or not there is something related to light as music is to sound.

Most of the attempts along this line have been the projection in sequence of various light colors, in varying intensity, without definite form. Some experimenters have used the effect to supplement musical productions.

One development which has attracted considerable attention during the past year has employed colors in more or less indefinite form, without musical accompaniment. Another development along similar lines, but for which no musical analogy has been claimed, automatically projects interesting kaleidoscopic patterns on a screen.

Various surveys and tests of lighting practise have been made and reported. A very important and extensive test of industrial lighting is now projected.

Codes and Standards. The state of Massachusetts has adopted an industrial lighting code similar to the American Engineering Standard. This is the ninth

state to adopt such a code. The Pennsylvania Industrial Lighting Code has been revised.

Last year reference was made to the organization of a sectional committee of the American Engineering Standards Committee to consider the project of a School Lighting Code, under the sponsorship of the Illuminating Engineering Society and the American Institute of Architects, (the A. I. E. E. being represented). This project has been carried on by the sectional committee and is understood to be ready for the final action of the American Engineering Standards Committee.

Another lighting standard referred to last year is the project relating to colored signal lights for traffic and other purposes. This question, which has many ramifications, is understood to be still under consideration.

American Engineering Standard on Illuminating Engineering Nomenclature and Photometric Standards, under the sponsorship of the Illuminating Engineering Society is receiving preliminary study for revision and further coordination with the corresponding European Standards.

Educational. For a number of years leading lamp manufacturers have conducted courses in illuminating engineering and lighting practise for representatives of the lighting industry. The past year has witnessed a considerable increase of interest in this direction.

Arrangements have been made by the National Electric Light Association for a course to be conducted under the auspices of the Illuminating Engineering Society for the purpose of training illuminating engineering specialists for central stations. This is to be held in the summer and fall of this year and is to include visits to a number of the larger cities.

Another educational undertaking about to be launched by the National Electric Light Association is a nation-wide movement to teach better lighting of homes.

American success in improving the practise in lighting has attracted much attention abroad and many visitors from various foreign countries have come to study our methods. Recently, a group of eight engineers from European lamp manufacturers inspected lighting in the large cities as far west as Chicago, and studied the American practises. Lighting demonstrations similar to those established in this country during the last two or three years have been installed in London and Paris.

In the United States the demonstration method of teaching good lighting seems to be firmly established. Various types of portable demonstrations are used in connection with lighting lectures.

A notable permanent exhibit, covering the principal applications of incandescent lamps, is attracting considerable attention.

A cooperative street lighting demonstration, covering about one-half mile of street has been installed in Cleveland to show the effect of various heights, spacings, locations, types and sizes of street lighting units. This has proved very helpful to municipalities studying methods of improving their street lighting.

At the Illuminating Engineering Society Convention at Lake George, New York, in September, 1923, one of the interesting presentations, was the solution of a hypothetical street lighting problem, by eleven different engineers representing a number of viewpoints. A similar method is being applied to an office-lighting problem with three contributors. The scheme seems to have possibilities of coordinating ideas on lighting practise.

The 1924 meeting of the International Commission on Illumination referred to in last year's report, is to be held at Geneva, Switzerland, Supplementing the usual reports, it is planned to have papers from various countries.

The United States National Committee will present a group of American contributions in this connection.

Research. That scientific research is still giving considerable attention to light, is evidenced by a few of the outstanding discoveries reported.

A new primary standard of light has been produced and described by Herbert E. Ives. While candlepower standards are carefully preserved in the form of incandescent lamps at several national laboratories, it is highly desirable that an accurate means of primary check be available lest there be a tendency over a considerable period of years for the standards to drift. The primary standards of the past have lacked accuracy of reproduction at the hands of different experts and under different local conditions. The new standard seems likely to insure greater accuracy in this connection.

An improved quality of transparent optical quartz has been reported as produced in a laboratory at Lynn, Massachusetts, by E. R. Berry.

The gap in the radiation spectra, between the infrared and the electric waves has finally been investigated by E. F. Nichols and J. D. Tear.

It was the final victory of Dr. Nichols' useful life, and he passed quietly away while presenting his results in an address before the National Academy of Sciences.

Conclusion. The remarkable accomplishments of electric lighting, are all the more notable when it is remembered that this art is so new and that many of the original pioneers are still alive and in active service. It, however, has reached the age at which losses may be expected to be numerous.

It does not seem suitable to close this report without mention of those outstanding Americans who have concluded their valuable contributions in this field, namely, Louis Bell, Charles P. Steinmetz and Ernest L. Nichols.

G. H. Stickney, *Chairman.*

MARINE COMMITTEE

To the Board of Directors:

The committee was organized, subcommittees were appointed and the first meeting was held November 8th. Three subsequent meetings were held. A good portion of the time of the committee's members has been devoted to the work of the Sectional Committee of the American Engineering Standards Committee for Electrical Installation on Shipboard.

The shipbuilding industry, while slightly improved from a year ago, is far from normal. The activities of your committee can only be reflected in proportion to that industry and greater improvement is hoped for during the coming year.

Owing to the success attending past installations of electric motors on shipboard, it is now almost universally recognized that electricity will be the future power for ships' auxiliaries outside of machinery spaces for all vessels, for all auxiliaries in Diesel engine propelled vessels and a portion of engine room auxiliaries in steam vessels.

Perhaps the most important and urgent work of the committee is the licensing of engineers, electrical, for vessels. The ship owners and operators must continue to bear the expense of educating the personnel, as no steps have been taken by the licensing authorities to improve the present conditions and, furthermore, the licensing authorities have candidly stated they can see no necessity for a change, although your committee has labored earnestly with them and presented what was thought to be conclusive evidence warranting action in that direction.

Your committee realized the importance of some action to insure proper maintenance of electrical apparatus on shipboard. We believe the matter has been presented to the shipowners and operators in such a manner that they are also convinced that some changes are desirable and your committee proposes to continue its activities, until some relief is obtained. It is to be regretted that developments did not warrant the presentation of the paper on the subject at the November 1923, meeting of the Naval Architects and Marine Engineers, as anticipated.

The Power Apparatus Committee is preparing, and no doubt will complete next year, some definite detailed recommendations for the kind of current, type of apparatus and control for various auxiliaries for several types of vessels.

The depression in shipping and shipbuilding has not given the Propulsion Committee the opportunity expected to collect and compile data from electrically propelled ships. There have been a few Diesel electric ships completed during the past year and some data should be available in the near future.

The Publicity Subcommittee have issued several interesting articles relative to marine electrical installations.

The other subcommittees on Standard Appliances, Historical, Radio, Wires and Cables, Interior Communication Apparatus and Editing have taken care of current work with nothing special to report.

The committee as a unit has worked harmoniously. The coming year will tax the committee with matters pertaining to the licensing of engineers and considerable energy will be require to have the Marine Rules adopted as an American Engineering Standard.

The committee is to be congratulated for its consistant efforts and achievements.

G. A. PIERCE, *Chairman*.

INDUSTRIAL AND DOMESTIC POWER COMMITTEE

To the Board of Directors:

During the past Institute year this committee had charge of an Institute Session at the Midwinter Convention in Philadelphia where two papers on the "Electric Elevator" were presented followed by a very interesting discussion of this important topic.

The committee assisted the New York Section in arranging a meeting on "Electric Drive for Ventilating Fans" which was held in New York, November 14th, 1923.

At the present time increasing interest is being taken in the application and control of motors for industrial purposes. Commercial motors are limited to relatively few types, but the control apparatus is capable of wide variations and in many cases control can be designed which will permit the use of a standard motor for most applications. The control system has, therefore, been receiving an increasing amount of attention, as it is realized that the success of many applications of electric drive depends upon selecting suitable control equipment.

Surface indications would lead one to believe that the type and variety of control is being greatly increased; the contrary, however, seems to be the case. As every art develops, engineers at first solve each problem in a new way as standard apparatus is limited and it is necessary to broaden their experience; as these accumulating data are analyzed it becomes possible to group the various problems and develop apparatus which will take care of the more frequent applications. It is also possible to subdivide the problem into units which can be standardized, so that where special systems are necessary they can be assembled from standard units.

Control engineering is working in the second stage where standardization is becoming of increasing importance. To a very great extent, the unit parts of

control equipment are being standardized and a number of combinations is being eliminated as it is found that during the earlier developments of the art systems were listed with only minor differences. By consolidating similar systems of control, a number of combinations now listed as standard can be eliminated. This standardization is a direct benefit to the art, as it reduces the new combinations and new apparatus developed on customers' orders; it enables the customer to obtain prompt service; and it enables the apparatus to be produced in a more economical manner, thus reducing the first cost of the installation and making electric drive more popular.

During the past year there has been some interesting applications and some new apparatus placed on the market.

The following is a partial list of new applications:

Silk Spinner Drive—In Textile Mill

The first commercial installation of a three-motor automatic drive for combination spinning, doubling, and twisting machine drive was placed in operation during this year. Each one of the three decks is driven by a one-horse power vertical motor, all three of which are mounted on the same end stand of the frame and in the proper location to drive the spindle belt.

An automatic magnetic trip throws the feed rolls out of action when the power is shut off or when it fails, and a fly-wheel on the motor, which drives the upper deck, continues the twisting on this deck for a short period, after the lower deck stops, sufficient to take up the slack in the yarn and thus prevent kinking.

The feed rolls are driven by a gear from the upper deck motor, thus insuring a positive twist.

The method of drive practically eliminates all kinks and broken ends due to failure of power and shutting down of the machine and also insures a positive and uniform twist.

Individual Motor Drive for Cotton Card

The first commercial installation of an individual motor drive on cotton cards was accomplished this year. The installation consists of 63 of the latest type of cotton cards installed in a new cotton yarn mill, each driven with a 1¼ h. p. gear-connected motor, mounted as an integral part of the machine. Control is accomplished with a special combination of snap switches in one cabinet to give normal forward operation and reverse direction of the machine for grinding the card clothing. Stripping is taken care of by rope and sheave drive, direct from the motor shaft.

The method of drive insures freedom from specks, dirt or oily lint, usually present with the line shaft and belt method of drive.

Electric Elevators

During the past year, there have been a great many installations of passenger elevators in office buildings, equipped with automatic leveling devices. The leveling being accomplished principally through the medium of an auxiliary motor which drives the hoist rope sheave through a set of spur and worn gears.

There has also been developed, a system for automatically leveling the elevator car at floor landings without the use of auxiliary motor, use being made of the main motor operating at a slow speed when leveling and controlled automatically when running at this slow speed. When the operator desires to stop at a floor, he moves his car operating switch to the neutral point when within a given distance of the floor at which he wishes to stop and the controlling mechanism then functions automatically to slow down the main motor and bring the elevator car to practically a level with the floor sill.

Another system of elevator control has been developed and has just been put in operation which has been termed, the Signal Control. This control operates somewhat on the order of the automatic push button elevator in that a passenger desiring to go up or down from any given floor in the building, presses the push button for the desired direction and the first approaching car will then automatically stop at that floor without any action on the part of the car operator, providing the elevator car is not loaded to capacity. When the passenger has entered the car, the operator closes the door and presses a push button in the car corresponding to the ground floor or the top floor and the elevator automatically starts off and runs to the terminal landing unless stopped by another passenger who has pressed a button on another floor. The operator in the car, when asked to stop at a particular floor, presses the button in the car corresponding to that floor and the car then automatically stops at that floor. This signal control system is also designed to automatically open and close the hatchway doors and the car gate.

These automatic leveling and automatic stopping elevators, can be operated on either direct or alternating current, as the motive power is usually supplied from a small motor-generator set which can be driven by a d-c. or an a-c. motor. The generator end of the set always being direct current.

The cars of these elevators are now in successful operation at 600, 700 and 800 ft. per min.

There has also been developed and put in operation recently, a new type of worm gear traction elevator, consisting of a *V* groove traction sheave driven by two worms operating on one gear wheel. The two worms being mounted vertically and each directly connected to the armature of a vertical-type d-c. motor. The acceleration and retardation of this type of winding machine when used with field weakening motors having a speed range of one to two or one to three, is remarkably smooth and rapid and is practically free from back lash.

Oil Well Drilling

One of the most interesting developments in the petroleum industry for many years is the Hild System of oil well drilling. This system is especially adapted for rotary drilling and makes use of practically standard "draw works" but introduces several features which have been previously lacking. The electrical equipment consists of two wound-rotor induction motors with suitable reversing controllers and regulating resistance, together with a gear unit which combines a speed reduction and a differential unit.

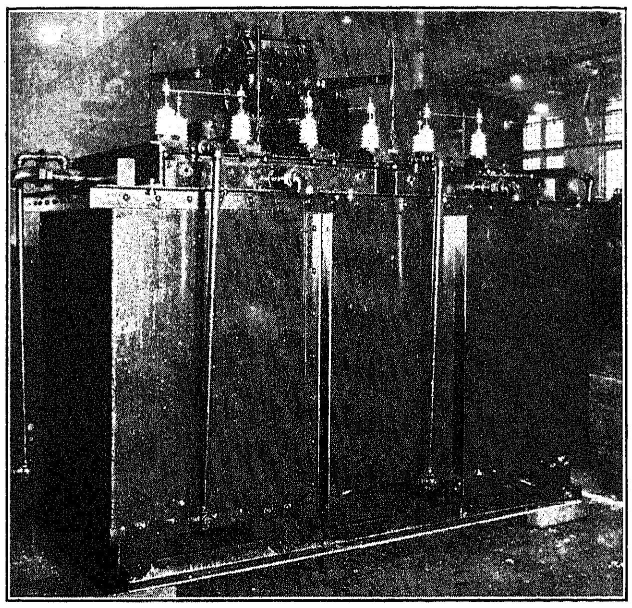


Fig. 1

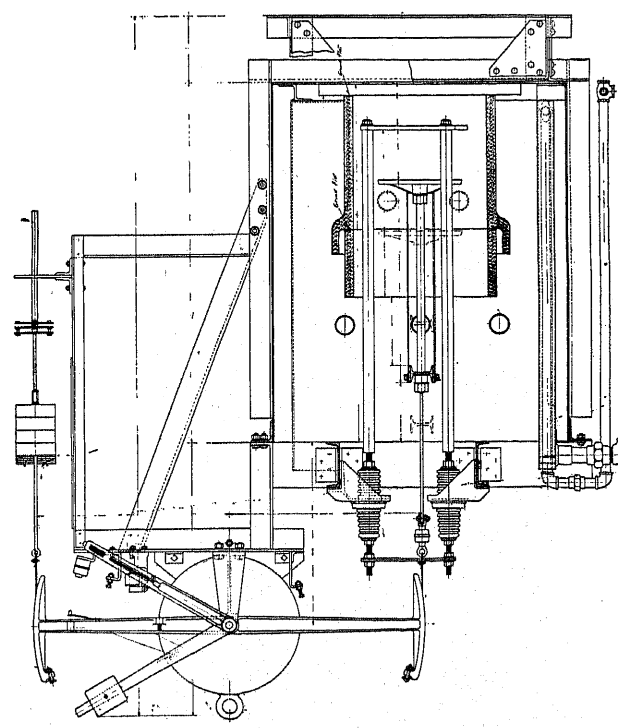


Fig. 2—Lewis Type Slip Regulator

This combination of motors, control and gears provides automatic feeding of the bit so that in sand or soft rock a high rate is obtained, while in hard or sticky formation the feed is slow, thus insuring a straight hold and reducing or eliminating the chances of twisting off the drill pipe. The forcing of the bit with no knowledge of the pressure on the bottom has probably been the cause of most of the trouble with rotary drilling; the Hild System eliminates this guess work.

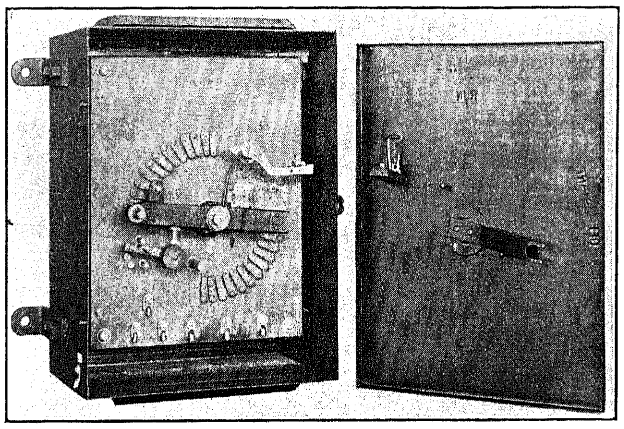


Fig. 3—Safety Starting Rheostat for Slip Ring Induction Motor

Some of the new electrical equipment placed in service during the past year is as follows:

Slip Regulator. Figs. 1 and 2 show a 1500-h. p. slip regulator for a wound-secondary induction motor. This regulator is of the liquid type and differs from the previous regulators in the construction and arrangement of the cells constituting the resistor element for each of the three phases. These cells are located within the electrolyte tank and have the terminals of the stationary electro brought out at the top of the tank.

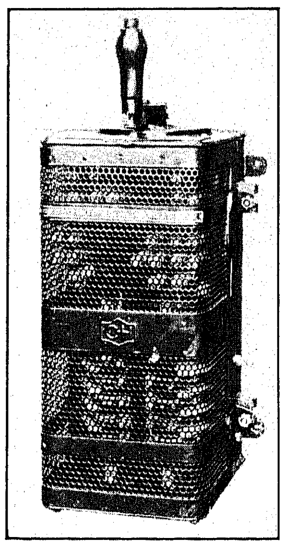


Fig. 4—Reversible Compound Drum Controller

This arrangement avoids leakage of the electrolyte, either through the connections at the bottom of the cells or by reason of failure of the cell walls.

Starter for Wound Secondary Induction Motors. This starter introduces resistance in the secondary of the motor during the starting period, its general features are illustrated in Fig. 3.

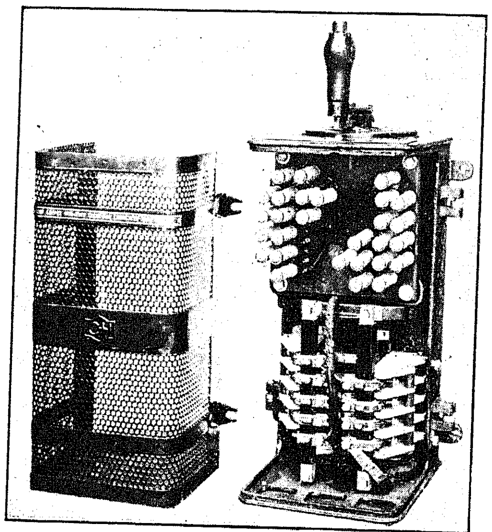


FIG. 5—REVERSIBLE COMPOUND DRUM CONTROLLER

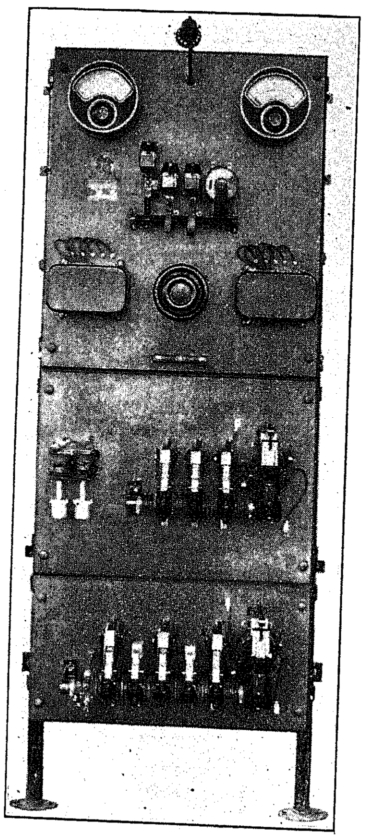


FIG. 6—AUTOMATIC STARTER FOR 3-PHASE SYNCHRONOUS MOTOR 150-AMPERES 550-VOLTS

Machine Tool Control. The distinctive feature of this controller is the mounting of the field controlling resistors within the case and adjacent to the drum fingers for commutating it. Both the resistor units and the controller fingers connect to this resistor and are mounted on a single slate base; this eliminates lead wires. The resistor units can be removed by loosening one nut at the end of the stud on which they are mounted.

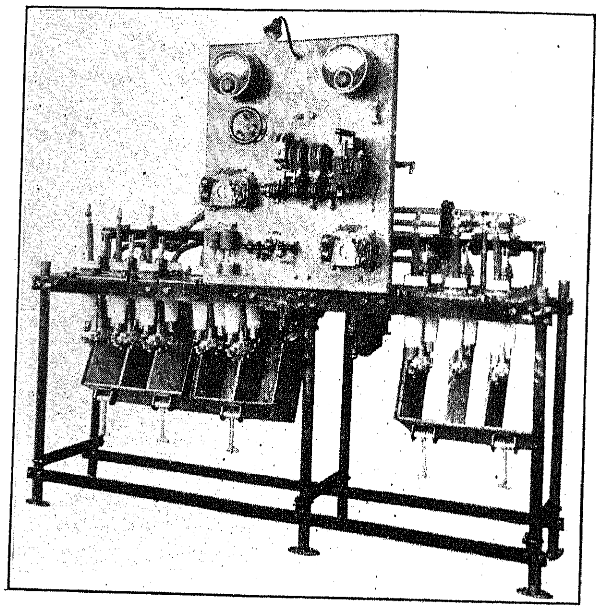


FIG. 7—AUTOMATIC SYNCHRONOUS MOTOR STARTER

Synchronous Motor Starters. These starters somewhat resemble the auto-transformer starters used for squirrel-cage induction motors. A starter of the 550-volt class is shown in Fig. 6, and for 2200-volt service in Fig. 7. The special features of these controllers are two motor-driven time relays, one for controlling the contactors which connects the motor to the auto-transformer to give reduced starting voltage, afterwards changing the motor connections to full voltage and the

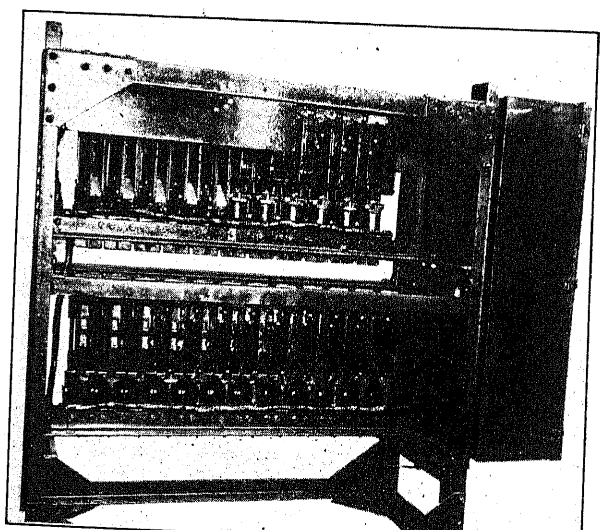


FIG. 8

other relay closes the field contactor after the motor has reached full speed.

Logging Donkey. The control illustrated in Figs. 8 and 9 uses pneumatic contactors for handling the main motor circuits. These particular controllers operated a 300/200 h. p., 2-speed yarding motor and two 250-h. p. loading motors. The illustration shows only one

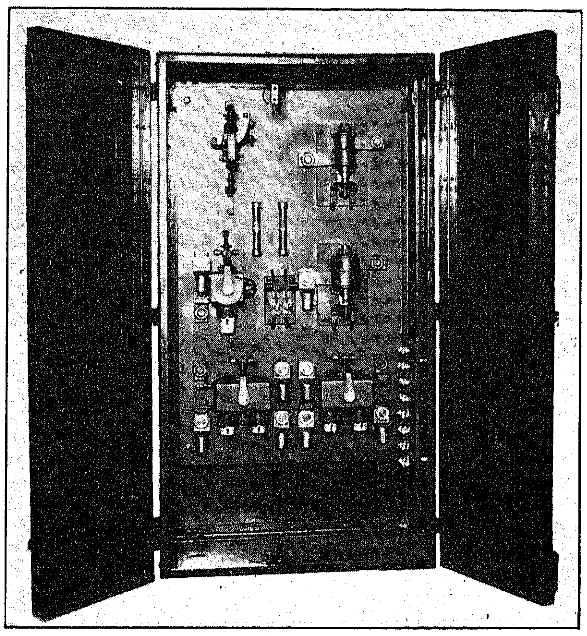


Fig. 9

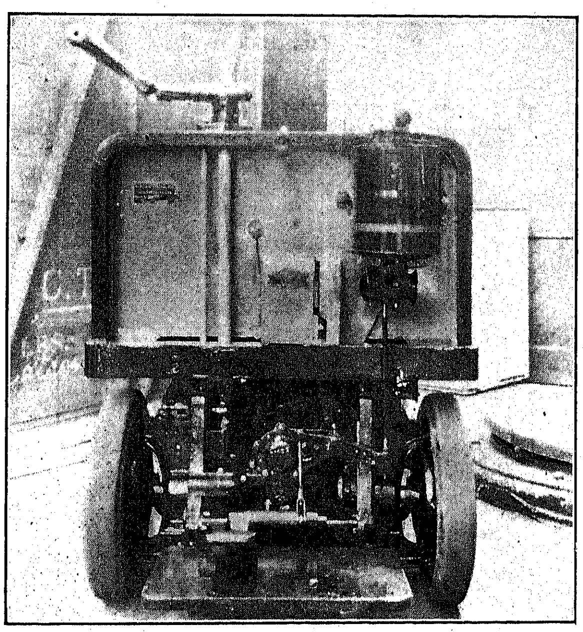


Fig. 10—Controller Mounted on Industrial Truck

controller. Air-operated contactors of this type are held open by a heavy spring and closed by an air cylinder. Their operation is still free from shock or jar and it is not necessary for them to be mounted in any definite position.

Industrial Truck. The control equipment shown in Fig. 10 is very interesting, not only from the apparatus standpoint but because it shows an application that is rapidly increasing in favor, namely, the storage battery. The control proper is of the drum-type, having mounted within the case a line switch interlocked with the brake pedal in such a way that the line switch is open if the operator applies the brake when the controller is in the running position. It is

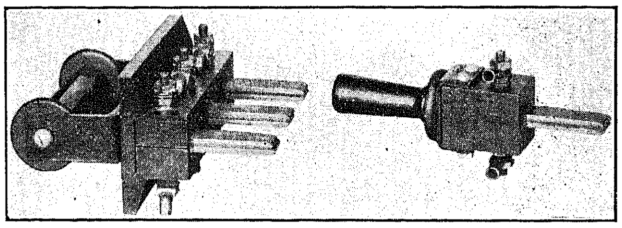


Fig. 11—Plugs for Use with Safety Type Jack Disconnecting Switch

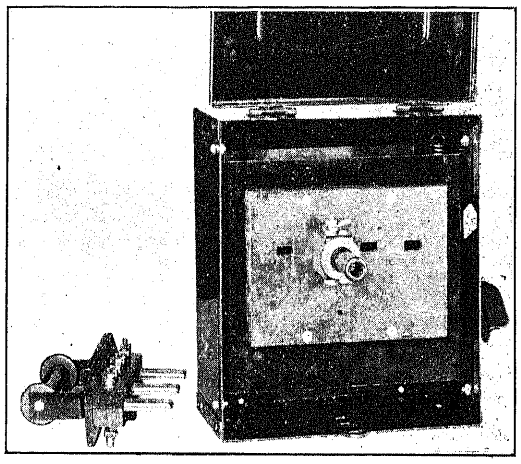


Fig. 12—Safety Type Jack Disconnecting Switch

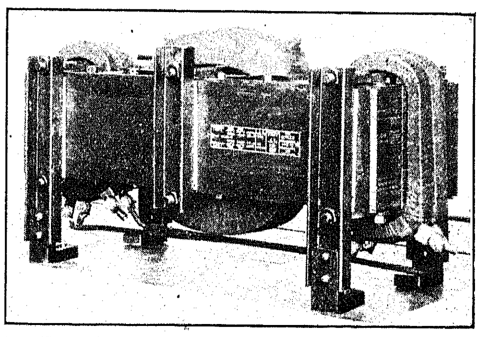


Fig. 13—Ten Kilowatt Magnetic Dimmer Element

then necessary to return the controller to the starting position before the line switch can again be closed.

Jack-Type Disconnected Switch. This device is illustrated by Figs. 11 and 12. The jack switch can be used for disconnecting control equipment and can also be used with ammeter and wattmeter jacks to connect these instruments in circuit without opening the line. Arrangements are made which permit this switch to be locked in the closed position, to prevent interference by unauthorized persons.

Magnetic Dimmers. A three-wire dimmer of the

magnetic type is shown in Fig. 13. This device is similar to a transformer, the central coil being connected to a source of d-c. power; the outer coils are connected in circuit with the lamps to be controlled. The power delivered to these lamps is varied by changing

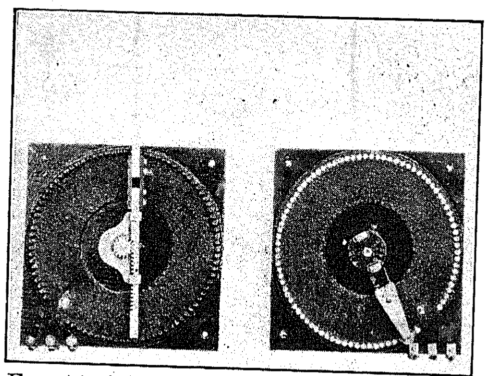


Fig. 14—Magnetic Dimmer Control Plate

the current in the d-c. coil, which changes the reluctance in the magnetic circuit of the a-c. coils. Fig. 14 shows the control plate for this magnetic dimmer.

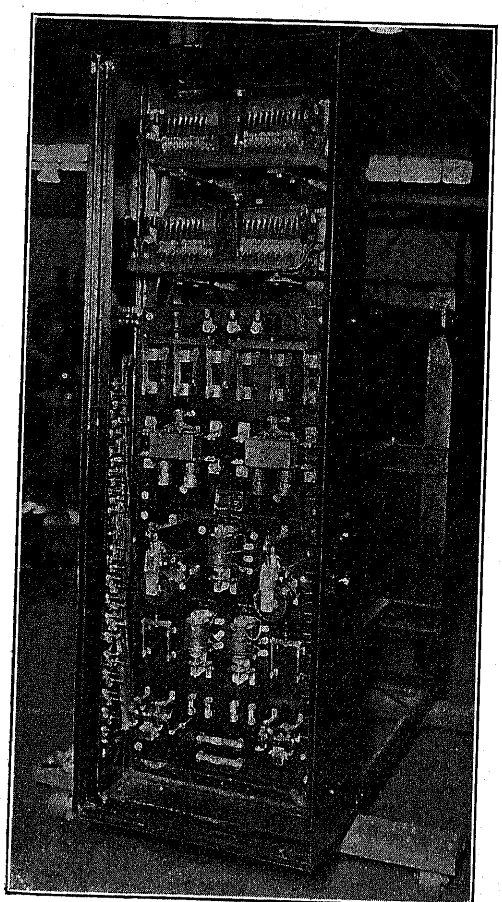


Fig. 15

A-C. Shovel. This is another application of the pneumatic contactor for industrial uses. This type of contactor was originally developed for railway service and is well adapted for industrial applications subject to continual vibration and shock. The design is made very compact and relatively light in weight. The electric shovel is similar to a large freight car filled with machinery; it is self-propelled and is often moved over rough and uneven surfaces. In many cases it is not practicable to operate the shovel with the floor level on account of the nature of the ground. The particular control illustrated is part of the equipment which

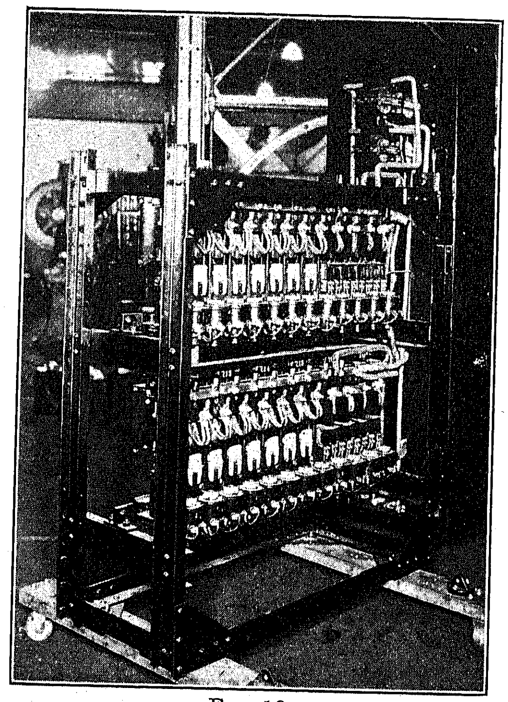


Fig. 16

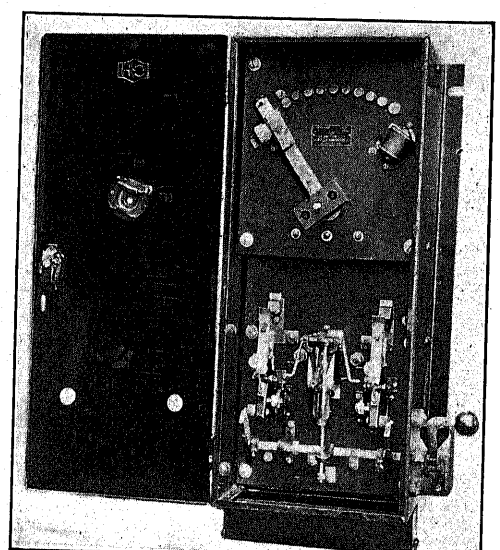


Fig. 17—Enclosed Manual Motor Starter with Circuit Breaker

consisted of two 150 h. p. hoist motors, one 80-h. p. thrust motor and one 80-h. p. swing motor.

D-C. Selenoid Brakes. These brakes are of the standard shoe type, the braking pressure being applied by a single spring and released by a solenoid magnet. They can be adjusted by hand and are so arranged that

the proper position of the adjustment is indicated by a pointer on the brake arm. The brake coils are weatherproof and have type B insulation which will withstand high temperatures. The travel of the magnet core is short, which increases the life of the working parts.

Controller for Building Equipment. The controller illustrated in Fig. 17 consists of a line switch and a starting rheostat, both operated from the outside of the case. The rheostat arm is held in the "off" position by a spring, it is operated in the usual manner after the line switch is closed and is held in the running position by a magnet. The arm automatically returns to the "off" position when the line switch is open. The line switch is so arranged that only one pole is closed at a time. If a ground or short circuit existed during the closing period, one pole would be free to trip open and relieve the circuit at the time the second pole was closed.

Definite Time Accelerating Relay. This relay is designed to give a maximum interval of approximately five seconds and is for use on a-c. circuits. The principle of this relay is an armature energized by an alternating magnetic flux. This armature is drawn across the face of the magnet under spring pressure. Each time the magnetism passes through zero, the armature moves a small distance. The repeated moving and gripping of the armature by the magnetic flux results in a ratchet movement and introduces the desired time element.

H. D. JAMES, *Chairman.*

POWER STATIONS COMMITTEE

To the Board of Directors:

SCOPE OF THE COMMITTEE

The Committee has not departed in the year past from the four main activities outlined in last year's report as:

First. Routine analysis and recommendation of papers submitted by the Meetings and Papers Committee.

Second. Securing of papers on subjects related to the Committee's jurisdiction for publication in the A. I. E. E. JOURNAL.

Third. Special research investigation with reports and papers resulting therefrom.

Fourth. Resumé of the year's progress on power station work.

In connection with the foregoing and the subject matter, as well as the treatment of the report, which follows your attention is directed to the extract from the "Tentative Report (touching this committee) of the Committee to Review Technical Activities of the Institute" as follows:

COMMITTEE OF POWER GENERATION

Field: Cognizance of all matters in which the dominant factors are the requirements, selection, relation, installation and operation of machinery and devices necessary to the generation and supply of electrical energy, including the economic questions thereto.

Note: The expectable functioning of this committee should be initiatory and determinative in the matters within its province. Subject to the approval of the Board it would have power in all matters arising therein, except such formulation of standards as is the function of the Standards Committee, including those of contact with other bodies and of arrangement for joint action where such is indicated.

Mention is made of this point because of certain criticism of two sorts which has come to the attention of the committee:

1st. That in previous years it has rather perfunctorily duplicated some of the reports of the organizations avowedly chiefly interested in operation, and

2nd. That the character of the reports has not been consistent with the dignity of the Institute primarily as a scientific body.

It is difficult to see how compliance with the procedure applied by the latter criticism can be made consistent with the field proposed for the realignment of the committee activity, if indeed there are sufficient matters of purely technical and scientific interest arising yearly within the scope of this committee to justify its continued existence without in any way treating of practical operating conditions.

As to the first criticism it may be sufficient to say that there is a feeling among several members, at least, of the committee that such practical operating reference as have heretofore appeared would be of value to a certain portion of the Institute not connected with associations of operating companies in various kinds of power station activity.

It would seem desirable that your body take under advisement from year to year the questions whether the active functioning of a committee like this could to advantage be, at least temporarily, discontinued; whether the committee is functioning along the most desirable lines; whether the committee is functioning in these lines to best advantage both intensively and extensively?

ACCOMPLISHMENT

During the year there has again been routine consideration of papers submitted by the Meeting and Papers Committee.

As to initiating the publication of papers, some progress has been made and is still in the making.

The idea of interesting manufacturers and colleges equipped with suitable laboratories in special research investigation along lines kindred to the activity of this committee has been considerably advanced. Several such laboratories are in a position to make at least initial analysis of the requisites of the problem as soon as specific cases can be laid before them. This calls for initial action from the members of the Institute through this committee. It must not, however, be expected that such research, except in special cases, can be carried on without providing the necessary funds for it. But it is much to know that facilities in both equipment and personnel actually exist for carrying on many problems, for which many might suppose a

considerable period of preparation and large expense would be necessary.

Résumé of the Year's Progess in Power Station Work

It is scarcely possible to attempt to review power station progress without trenching upon ground covered by the 1923 reports of the Electric Apparatus and Prime Movers Committee of the N. E. L. A. and the Power Generation Committee of the A. E. R. A. to which particular reference is made for a wealth of information and detail. Certain points, however, deserve special mention and are given with somewhat definite treatment.

In the preparation of this portion of the report each member of the committee was made responsible for certain topics; failure of presentation of any subject that might appear important in a report of this character is to be attributed to the inability of the individual to furnish the appropriate data. Several members of the committee have expended no little time and energy in the preparation of their contributions.

General

The most impressive feature of the year's power station work was the unusually large building activity of both steam and hydraulic plants, amounting to approximately 5,000,000 kv-a. of generating capacity and the continued tendency toward larger stations and larger units.

Hardly in any other year than during 1923 have so many schemes been introduced to increase the steam power plant efficiencies. The insecure and the varying grades and qualities of coal have given fuel oil and pulverized coal a very strong impetus. With fuel oil there happened to be a heavy over supply, therefore bringing a low price, and with pulverized coal the recognized advantage of being able to use various grades of coal within the same furnace without great trouble, its flexibility and ability to maintain high efficiencies were the major inducements for so many changes and new installations. It appears that although for some time pulverized coal had been used, no uniform practise has developed. The mode of firing, especially, appears to lack satisfactory solution. On the other hand a considerable number of improved stokers has been installed, and may be still further developed in the future under the pressure of the competition of pulverized coal. The sizes of boilers and furnaces have rapidly increased to considerable dimensions and it has been proposed to build a single boiler to supply a 30,000 kw. turbine-generator unit. Such a boiler would have a heating surface larger than the biggest boiler now used, by about one third. Recent boiler designs show a tendency to increase the heating surface exposed to radiant heat and some contemplate lining the entire furnace with heating surfaces. Should this be successful, the difficulties with refractories would be overcome, and the operation of boilers at high rating with preheated air would gain increased impetus. Investigations of superheated steam temperatures in connection with varying boiler loads have led to various superheater developments. Hand in hand with high ratings the furnace volumes have so rapidly increased in size and cost that future growth may be limited by economical considerations. High ratings brought higher flue gas temperatures and therefore the economizer was brought to the front again. For the same reason the demand for better feed water developed more efficient water treatment plants, evaporators and deaerators.

The striving for high efficiencies has brought higher steam pressures and with it various new cycles and larger capacities of generator units which have increased to 62,500 kv-a. These larger units are mostly of cross or tandem compound arrangement, especially where higher steam pressures are contemplated in connection with a reheating cycle.

With the innovation of bleeding main turbines for feed water heating, the house turbine direct connected to the main unit has been developed. As for steam pressures, installations are being made for 400 and 500, 1200 and 3200 pounds, the last one by the English Electric Company, at Rugby, England. The maximum temperatures considered for the above pressures are between 700 and 800 deg. fahr. With these high steam pressures and temperatures the reheating or Ferranti cycle, the regenerative cycle and the combined reheating and regenerative cycle are given most attention.

Toward the end of the year much was heard of the mercury turbine which has been under development by the General Electric Company since 1914.

As far as valves and fittings are concerned, it has been reported that tentative standards for pressures up to 900 lb. per sq. in. and at a temperature of 750 deg. fahr. have been agreed upon. Besides this rapid departure from established steam practise, the economic success of which only time can prove, the leading manufacturers have carried on research and improvements on established equipment. Two out of the three largest builders of equipment are giving turbines resonance tests before leaving the shops, and much thought is given to generator cooling, of which the closed system of ventilation appears to be gaining in favor. The operating results of modern steam turbines have improved markedly.

The development in hydraulic plant equipment has followed the general trend of steam plants, both in improvement of design and construction, and in size of units. The tendency toward large sized vertical units continued, and more attention was given to the question of suitability of design for the hydraulic conditions and load requirements, and careful working out of the details of design of auxiliary equipment required for the proper control and protection of the main units. The operation of the various control mechanisms electrically from the switchboard has come into more general use, and the details of switchboard control mech-

anisms as well as refinements in the setting, and the adjustments of such mechanisms have been worked out successfully.

The tendency in large hydroelectric stations, consisting of two or more vertical units seems to be in favor of a single floor station, with the main floor at approximately the elevation of the top side of the turbine casing, or about the center line of the main turbine steady bearing. With this type of station layout the generator is generally supported on reinforced concrete piers and the switchboard and remote control mechanism are located in a gallery off the main generator room, at about the elevation of the generators. This arrangement gives the operator vision of the generators, turbine equipment, governors, and all auxiliary apparatus.

Outstanding, as to size, are the 70,000 h.p. hydraulic turbines for the Niagara Falls Power Company, operating under a head of 213½ feet, at a speed of 107 rev. per min., and connected to a 65,000-kv-a., three-phase, 25-cycle, 12,000-volt generator. These units are also equipped with air brakes for bringing them to a stop quickly when shutting down.

The hydraulic turbine of the Oak Grove plant, of the Portland Railway Light & Power Company, at Portland, Oregon, has the highest head reaction wheel built to date. This is a 35,000 h.p. vertical turbine designed for operation under an average net effective head of 875 feet, the static head being 960 feet. The unit will operate at a speed of 514 rev. per min. and is complete with governor operated, automatic water economizing pressure relief valve; 72-in. water and electric motor-operated remote control butterfly valve; Moody type spreading draft tube and special oil pressure governor. The penstock providing power water for this unit is comparatively short but on account of its combination with a flow line several miles in length, the regulation problem involved has been solved by the installation of a Johnson differential surge tank hewn from the solid rock. A Johnson plunger type valve is provided at the upper end of the pipeline just below the surge tank, equipped with remote control and with emergency control features to provide for its mechanically closing down in the event of rupture to the pipeline or for any other reasons which would increase the velocity of the flow within the line above normal.

A notable improvement in hydraulic turbine design during the year, has been the use of the rubber seal rings or clearance rings in a number of large turbine installations. Former constructions have required metal rings at the clearance spaces between the rotating runners and the stationary portions of the casing. Usually these rings have been made in pairs, of which the rotating part is steel and the stationary, bronze.

The governor problem has been given special attention, and much improvement in speed control and regulations has resulted. Present-day governors are made with various accessories and auxiliaries, the use of which affords practically absolute safety and protection to the generating equipment under all phases and conditions of operation. Owing to the large size of the present-day electrical systems and extremely large inherent flywheel effect available for speed regulation, it is now very seldom that it becomes necessary to require that any additional flywheel effect be provided in the generator, or the hydraulic prime mover, to satisfy speed regulation requirements, as the inherent flywheel effect of the machines added to that of the system into which they will be connected is generally more than ample. With this condition there is also a strong and proper tendency on the part of the operators to simplify their pipeline and surge problems by increasing the length of time required for the governor stroke. With modern governors the stroke time may be regulated to meet conditions exactly, and the time for the opening stroke may be made different from that of the closing stroke.

Considerable progress was made during 1923 in various design features and in the control mechanism of butterfly valves. The remote electrical control of such valves when placed at the upper end of a pipeline or at the outlet of a surge tank has been quite generally adopted. A system of distant control has been developed which insures the correct operation of the valve at times of emergency without danger of the valve being operated inadvertently by accident to the control circuit through lightning or mechanical injury from falling trees, etc. Such valves can, of course, be readily operated by means of the controller at the valve and by means of suitable relay mechanism can also be operated from the powerhouse. Most installations are arranged to allow closing of the valve from the powerhouse but not opening it from that point, thus providing for the necessity of closing the valve in case of a serious accident at the power house, or to the pipe line, but not allowing the automatic opening of the valve from the power house with the pipe line empty, as this might cause serious damage or even loss of life. Indicating lamps are provided to show at the power house switchboard the open or closed position of the butterfly valve, and the operating motor of the valve is equipped with limit switches to prevent overtravel of the valve at either end of the stroke.

Attention has been given to increasing the life of generators as well as improving their stability. Use of relays for protection, particularly of the thermal type, has been quite a factor.

More confidence in extremely high-voltage transmission has developed from the results of the 220,000-volt systems put in operation during the year by the Southern California Edison and the Pacific Gas & Electric Companies. Transformers and switching gear for such service contributed much to the success and indicate that distinct progress has been made in the design and construction of such equipment.

For the higher voltages, switching apparatus and transformers are generally installed outdoors on ac-

count of excessive housing costs, and there is a tendency for the lower voltages to isolate the phases and isolate the entire switch house from the main power station building.

Mention should be made of supervisory control of generating stations from a central point which is generally in the load dispatcher's office, where is located the system diagram board on which switches are indicated by small colored lamps which at all times give complete information concerning operating status of the entire system. The development of the carrier-current system of telephony, which effects communication by means of high-frequency currents superimposed on the power conductors of the transmission lines, has increased the scope of usefulness of supervising control, particularly of hydroelectric stations located in remote districts.

Switches and Circuit Breakers

During the last few years the rapid increase in capacity of a number of large systems has reached the point where it has become a difficult matter to secure satisfactory oil circuit breakers of sufficient interrupting capacity to protect the system even when current-limiting reactors, to limit the short-circuit current, are used in every way practicable. The manufacturers of oil circuit breakers are making herculean efforts to keep pace with the requirements of the industry but to date no breaker with an interrupting capacity greater than 1,500,000 kv-a. has been installed.

The manufacturers have been handicapped in their design on account of their inability to test their breakers to destruction due to lack of generator capacity. To date very few large central stations have been willing to allow high capacity tests to be made on their system. It is believed that oil circuit breakers of very much greater interrupting capacity than have been manufactured, can be successfully made providing the need is great enough to justify the cost.

In order to limit the number of short circuits in power stations and minimize the damage done in case trouble occurs on the bus or switching equipment, a segregated phase scheme, either horizontal or vertical, is used in a number of the latest and largest stations built during the last two years.

Gang-operated disconnecting switches interlocked with the oil circuit breakers are to a great extent replacing single pole switches operated by a switch hook. In one of the newest large central stations practically no disconnecting switches are used for disconnecting the oil circuit breakers from the bus or line. The disconnection of the breakers is accomplished by lowering them below their normal position, thus breaking the connections and isolating the breakers.

The use of truck type panels for controlling station auxiliaries is increasing. A number of the latest large stations have installed or are contemplating the installation of equipment of this type.

Waterwheel Type Generators 1923 Development

The outstanding developments regarding capacity of units are:

a. 65,000-kv-a., 107-rev. per min., 25-cycle, 12,000-volt generators for the Niagara Falls Power Company.

b. 55,000-kv-a., 187½-rev. per min., 25-cycle, 12,000-volt generators for the Queenston Development of the Hydro Electric Power Commission of Ontario.

c. 28,000-kv-a., 428-rev. per min., 50-cycle generators, suitable also for 60 cycle operation at 514 rev. per min. with a guaranteed overspeed of 85 per cent for the Southern California Edison Company.

Some features worthy of mention in the case of one design for the 65,000-kv-a. units, are the elimination of the lower guide bearing on the generator, and the incorporation of a 650-kv-a. 2200-volt service generator within the main unit, the rotor of which is immediately above the main rotor and the stator suspended from the main upper bracket. The main rotor spider is made up of two wheels each consisting of seven cast steel sectors bolted to cast steel hubs. The main upper bridge is also of cast steel with ten radial arms bolted to a central hub.

The 55,000-kv-a. generators are very similar to the 45,000-kv-a. units built a year or two ago. In the design of one of the manufacturers an extra wide space is provided in the stator iron to facilitate removal of the armature coils. The first of these units went into operation in slightly less than one year from date of placing the contract.

There has been considerable change in the general design of smaller units; generators have been developed with the thrust bearing located immediately below the rotor spider with the top bracket eliminated entirely. With this type of construction reducing the turbine pit diameter to an absolute minimum there should be a considerable reduction in the cost of the unit. This type of design should also require considerably less head-room for erection, with a consequent saving in power house superstructure cost. In fact, it might be possible to dispose of the power house superstructure entirely by providing a weather-proof housing over each unit, which could be readily removed if necessary, and all operations carried on by chambers provided in the substructure.

Automatic generating stations are receiving more attention, as evidenced in the increasing number of installations such as some developments of the Hydro Electric Power Commission of Ontario, where in each case two generating stations each containing three 2000 kv-a. generators will be operated from a third station located four and six miles from the automatic station respectively.

A contract for generators placed during the year and worthy of mention is that placed by the Quebec Development Company for eight 30,000 kv-a., 13,000-volt, 112-rev. per min. units. This is notable not alone on

account of the size of the units but on account of the large number included in the contract.

Fly-Wheel Effect of A.-C. Waterwheel Generators

In connection with the design of waterwheel generators the question as to the most suitable flywheel effect of generator usually comes up for discussion, as very frequently the flywheel effect which is inherent in a normal design of generator is not considered sufficient from the point of view of waterwheel speed regulation and the advisability of adding extra weight to the generator rotor must be taken into consideration. It was thought therefore, that a brief study of this feature might well be considered in the present report of your Power Stations Committee.

The basic formula generally used for computations of speed regulation of water wheels is as follows:

$$d = \frac{800{,}000 \times H.\,P. \times T}{W\,R^2 \times (R.\,P.\,M.)^2} \tag{1}$$

Where T is the governor time in seconds, *i. e.* the time taken to operate the gates, h.p. is the change in horse power load which causes the governor to act; $W\,R^2 =$ flywheel effect of the rotating elements including the water wheel runner and shaft (expressed in lb. $-$ feet^2); rev. per min. the initial speed expressed in revolutions per minute. The resulting speed regulation d as given by this formula is expressed as a decimal; ($100 \times d =$ per cent speed change). This formula is approximate but sufficiently accurate for average conditions.

Formula (**1**) is more frequently used in another form as follows:

$$d = \frac{800{,}000 \times T}{K} \tag{2}$$

$$\text{Where } K = \frac{W\,R^2 \times (R.\,P.\,M.)^2}{H.\,P.} \tag{3}$$

K is known as the regulating constant and h. p. is the full load rating of the water wheel.

The constant K may range from 2,000,000 with small machines, particularly direct current generators, to 10,000,000 or more with large capacity, medium, or high-speed units. Flywheels or extra weight in the rotor are sometimes added, but K in commercial plants rarely, if ever, exceeds 12,000,000. There is some difference of opinion as to the desirable minimum value of K for governor operation; some makers have gone as low as 3,000,000, but in general 4,000,000 may be taken as a reasonable lower limit. A value of 3,000,000 is suitable only for open flume conditions where there is no large moving column of water to handle as in a closed penstock.

If the governor time on the average is taken as 2 seconds and $K = 4{,}000{,}000$, Formula (**2**) gives d a value of 0.4 or 40 per cent for a full horse power load change. This figure is of course, rarely reached in practice while the unit is operating on load, but serves as an indication of the extreme that might be reached if the unit were suddenly disconnected from the line. It is also a rough indication of the regulating characteristics of the unit. It should be remembered that the majority of waterwheel units are not isolated, but are tied in on a system with a number of other generators. The system is, therefore, usually sufficient to make the load changes on any one unit small in comparison to its rating, and the total connected flywheel effect of the system is such that the load changes have relatively little effect from the standpoint of speed change. Modern systems are becoming so large that the governor as such is practically inoperative under usual conditions and serves mainly for convenience in starting and stopping the unit, for adjusting the load from the switchboard, or as an emergency shut-down device in case of emergency. With isolated units the governor is, of course, necessary for regulation in most cases.

Plant operators and operating departments have, however, handled units to such a great extent with governors, that as a rule they are of the opinion that governors are inherently required for the control of hydroelectric units. This general attitude has resulted in the requirement of a regulating constant of 4,000,000 or more in most modern plants, in spite of the fact that the generator frequently works out with such flywheel effect as to give normally a smaller regulating constant. Naturally weight added to the generator rotor increases the cost and with vertical machines increases the weight on the thrust bearing, thus in many cases increasing the size and cost of the latter. Increased $W\,R^2$ over and above that inherent in the normal design will also result in most cases in a slight reduction in generator efficiency owing to the increased windage and friction loss.

Some hydraulic engineers as well as those engaged in the operation of hydroelectric plants are of the opinion that the usual hydroelectric unit arranged for an extended interconnected power system of considerable magnitude, particularly such systems as have a portion of their generating capacity in steam, do not require a governor, at least not one as usually built, and that it is not necessary to load the generator with extra weight for purposes of regulation. The reason for this viewpoint is found in the fact that a hydroelectric unit is intended primarily for the purpose of delivering the maximum possible number of kilowatt-hours per year to the system from the flow in the river and at the available head. A governor, as such, defeats this purpose to a greater or less extent, as any movement of the gates away from the point of best efficiency, decreases the net kilowatt-hour output; and a governor can regulate only by moving the gates. This loss of output is overcome in plants where the operation is most closely watched by running the governor on what is known as the load limit device. When the governor is so operated, it is ineffective from a speed regulation standpoint

unless the load is reduced so as to cause the governor to close the gates either partially or wholly. In large systems, such as are becoming prevalent, the regulation of an individual unit is of little consequence and, on this account, excess WR^2 in generator is becoming of less and less value. This factor together with the feature of maximum kilowatt-hour production will doubtless influence the design of units for operation on extended networks, in the direction of a gate opening device rather than a governor of usual type, this device being combined with an emergency shut down mechanism which would become operative in case the unit should lose its load or the speed tend to increase unduly. Such an arrangement would be much simpler than the regular type of governor. With control of this kind the gate mechanism would be controlled by hand from the switchboard and by float from the head water level. A number of units are at present operated successfully in this way.

Assuming that no extra weight is added to the generator rotor to increase the flywheel effect, the question arises as to what WR^2 may be reasonably expected in a generator of normal design and of given kilovolt-ampere output at given speed. Can a formula be developed from which the WR^2 can be determined? From the nature of the problem it is plain that no general formula can be developed that will give the WR^2 because a machine of given kilovolt-ampere output at given speed may be designed with different diameters of rotor. One design may use a rotor of relatively large diameter, high peripheral speed, and short axial pole length. The same kilovolt-ampere rating might on the other hand be made with a rotor of relatively small diameter, lower peripheral speed and longer axial length of pole. Both machines might be entirely satisfactory as generators, but their flywheel effects would be quite different. Again, the mechanical design of the rotor may have a decided influence on the WR^2, so that on the whole there can be a wide variation for machines of the same rating, even if the generators are equally good in other respects.

It has been felt by the Committee that some information as to the normal WR^2 of waterwheel generators as actually built would be of value as it would at least give an approximate idea as to what might be expected in machines of various ratings. With this in view, information was collected on quite a large number of units covering a wide range in speed and output and designed by three leading manufacturers. These flywheel effects have been plotted, using the ratio $\frac{\text{kv-a.}}{\text{R.P.M.}}$ as abscissas, and WR^2 expressed in lb.-ft.^{2} as ordinates. The kilovolt-ampere rating in each case was taken as the maximum rating of the generator and the WR^2 is that inherent in the design of the machine; *i. e.* it includes no extra added weight in the rotor. The curve shows an approximate average but this is added as a matter of interest only, and Fig. 1 is chiefly of value as indicating values of flywheel effect as they worked out in waterwheel machines as built and in operation. The general tendency seems to be to eliminate added WR^2 especially for machines used on large systems so that generators built in the future will doubtless correspond more nearly with those indicated in Fig. 1, as having the lower values of flywheel effect rather than the higher except in cases where a high WR^2 may be the result of mechanical features necessary to take care of overspeed, etc.

Temperature Standards

The question of temperature standards has been for some time in the hands of a subcommittee of the Institute Committee on Standards and the American Engineering Standards Committee for official action, to the progress reports of which as well as to the Committee on Electrical Machinery reference should be made for details of the present status of the recommendations.

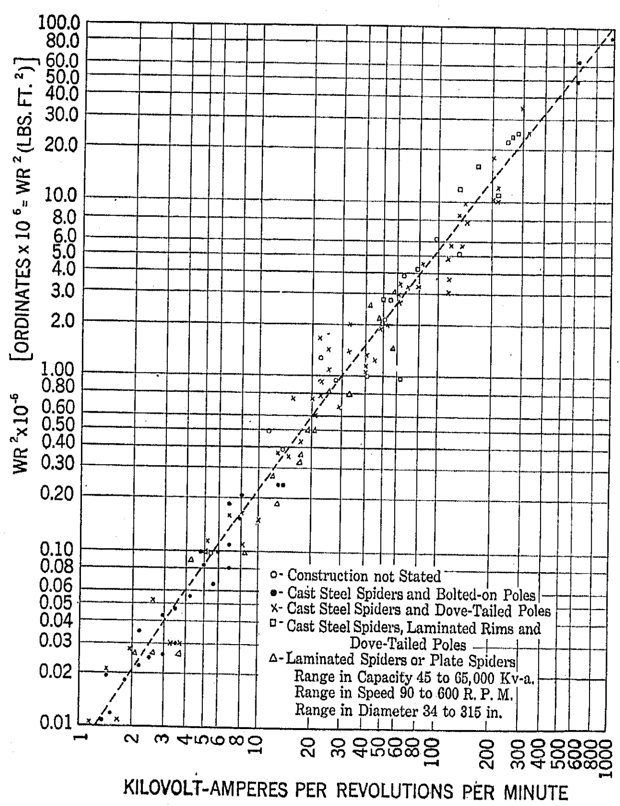


Fig. 1

Electric Drive and Control of Auxiliaries

Practically all the auxiliaries at the new station of the Edison Electric Illuminating Co. of Boston now under construction on the Weymouth Fore River will be driven by a-c., 60-cycle motors. The use of alternating current for this service, was decided upon after a careful comparison with direct current.

Motors and control that are essential to continuous

plant operation are designed to be automatically started, upon restoration of power, after an interruption.

On the larger constant-speed motors, a double squirrel-cage winding is used which reduces the starting current to approximately one-half that of the usual squirrel-cage design and allows the motor to be thrown directly across the line.

Maximum reliability and efficiency of the station service energy supply are obtained by the use of a 2500-kv-a., 2300-volt auxiliary generator direct connected to the main generator shaft and supplying power to a definite group of auxiliaries. Relay service is provided from buses which connect through transformers to the main 14,000-volt buses. An automatic face plate voltage regulator is used with each auxiliary generator.

It is intention of this report to describe the motor and control equipment of the station auxiliaries giving the reasons which led to its selection. No attempt will be made to compare electric with steam, or steam electric drives, as it seems to be generally recognized that station auxiliaries should be as completely motorized as possible.

Description of System

Fig. 2 shows a one-line wiring diagram of the auxiliary power system. There is a 2500-kv-a., 2300-volt auxiliary generator H on the same shaft as the main generator. This auxiliary generator is connected to a 2300-volt group bus J to which are connected all the auxiliaries required for the main unit, and in addition, a portion of the auxiliaries in the boiler house. This group bus is relayed from two 2500-volt buses K and L which are supplied with power from two 2000-kv-a. transformers M and N connected to the main 14,000-volt buses and also from a spare 1250-kv-a., 2300-volt turbo-generator U. These 2300-volt buses are designated "Common Station Service Buses" and supply power normally to all auxiliaries, such as cranes, elevators, and coal handling equipment, the operation of which may be temporarily interrupted without affecting power production. The boiler house auxiliaries and also the important auxiliaries for the main units, have selector breakers which allow them to be supplied with power from either their own generator group bus or from one other generator group bus. The only auxiliaries in this station that are steam driven are a high-pressure-boiler feed pump, a low-pressure-boiler feed pump and a steam-driven exciter. Normally, the steam-driven units will not be running. All other auxiliaries are driven by alternating current motors. The valves, however, are operated by 220-volt, direct-current motors.

The design is laid out to afford an independent system of two common station service and four generator group buses for every four main turbo-generators.

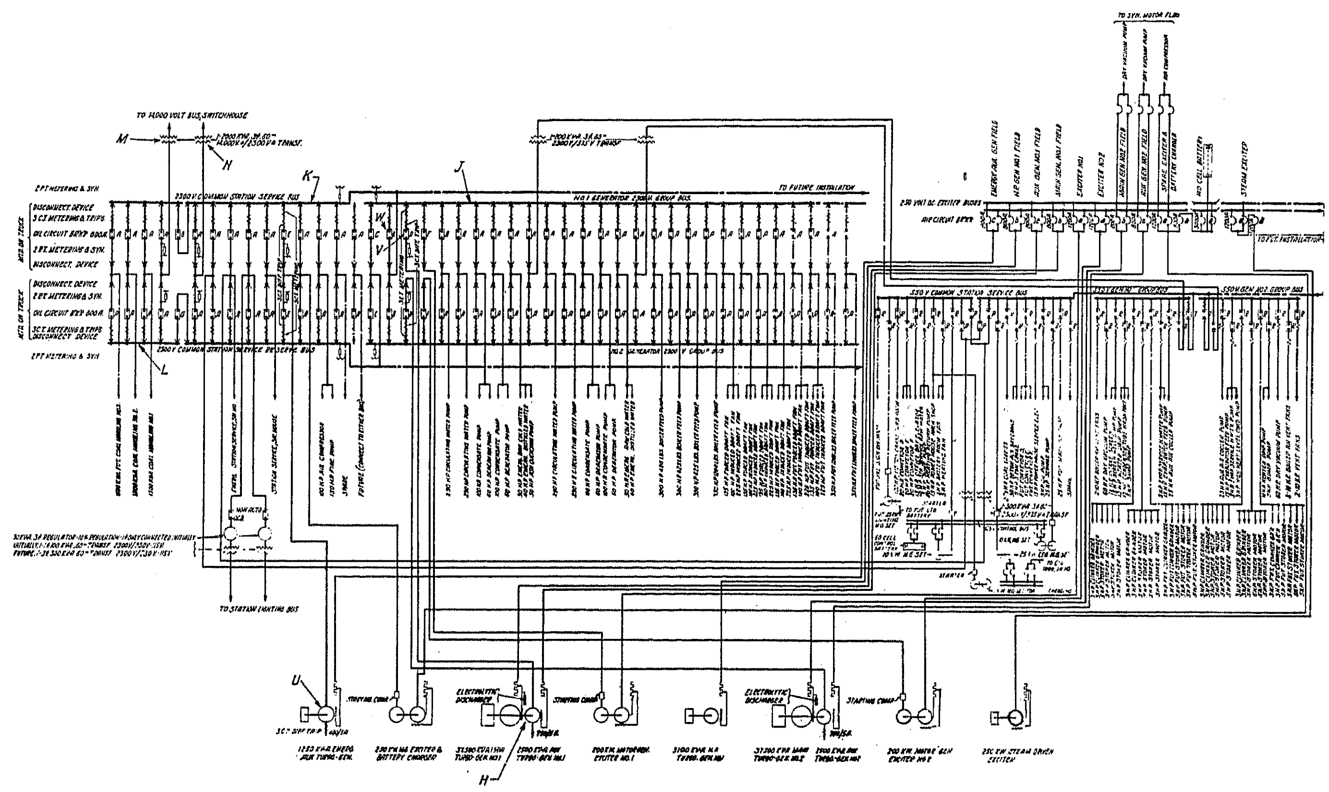


Fig. 2—One Line Diagram—Electrical Equipment Weymouth Power Station

H 2300-Volt Auxiliary Generator 2500 Kv-a.
J 2300-Volt Group Bus
K 2300-Volt Station Service Bus
L 2300-Volt " " "
M 2000-Kv-a. Transformer
N 2000-Kv-a. Transformer
U 1250-Kv-a. Emergency Generator
V Oil Circuit Breaker for Auxiliary Generator No. 1
W Oil Circuit Breaker for Group Bus No. 1 to Station Service Bus

ALTERNATING VS. DIRECT CURRENT

One of the first considerations in determining the type of electric drive, was the choice between alternating-current and direct-current motors. Determination of the relative economy of the two types of equipment depended largely upon the duty cycle assumed for the various auxiliaries and also upon the load factor of the plant. Comparative figures were made for both, using various schemes for obtaining the power supply.

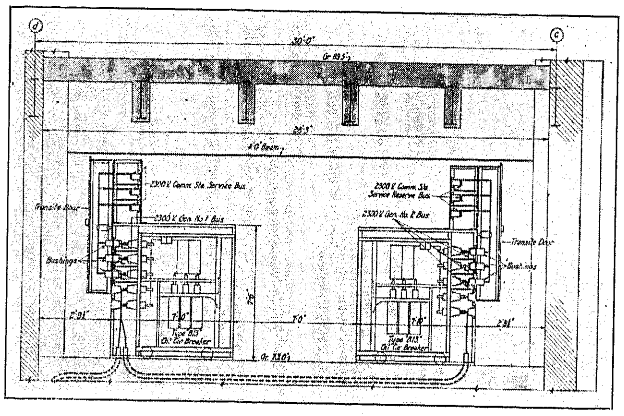


FIG. 3—CROSS SECTION-STATION SERVICE ROOM WEYMOUTH POWER STATION

and the layout was simpler. Smaller cables and conduits could be used and thus possibly wall and floor thicknesses could be reduced. Alternating-current motors were, therefore, adopted.

POWER SUPPLY

The auxiliary generator on the main generator shaft not only offers a power source of maximum efficiency but also allows unit operation through the group bus, thus securing a very high degree of reliability. It is estimated that the normal auxiliary power required by each 30,000-kw. unit will be about 800 kw., with about 450 kw. additional for the boiler house auxiliaries necessary to operate one turbine generator, making a total of approximately 1250 kw. for one main unit. One-half of the boiler room auxiliaries for three high-pressure generators add 300 kw., making a total of about 1550 kw. under normal conditions. Under emergency conditions, when it may be necessary to force some boilers up to their maximum rating, it is estimated that the load on one auxiliary generator will be approximately 2100 kw., which it will carry for a period sufficiently long to start other boilers.

The auxiliary generator is capable, not only of supplying power for its own auxiliaries and its portion of

TABLE I

COMPARISON OF AUXILIARY DRIVE FOR THE WEYMOUTH POWER STATION OF THE EDISON ELECTRIC ILLUMINATING CO. OF BOSTON, WEYMOUTH, MASS.

Assumes 6—30,000 Kw. Units, Maximum Load 150,000 Kw. Does not Include Common Station Service

Equipment	Rated kw. or kv-a.	No.	Hours per Year in Service	Eff. %		Scheme 1—D-c. Motors except Common Station Service. Auxiliary Generators. Rotary Converters. House Turbine.				Scheme 2—All A-c. Motors. Auxiliary Generators. Transformers House Turbine.			
						Annual Costs				Annual Costs			
				D. C.	A. C.	First Cost	Fixed Charges & Maintenance	Power @ ½c.	Total	First Cost	Fixed Charges & Maintenance	Power @ ½c.	Total
Auxiliary Generators	2000	6	5000	97	97	$90,000	11,600	6,960	$18,560	$90,000	11,600	6,600	$18,200
House Turbo-Generator	2000	1	7000			41,500	6,250	3,900	10,150	..	..	..	..
House Turbo-Generator	1000	1	..			..	..	..	..	22,000	3,300	..	3,300
Transformers, 14,000-v., 3-phase.	2000	2	8700			..	..	..	..	15,000	1,500	700	2,200
Transformers, 14,00-v., 3-phase	1500	2	8700			11,000	1,100	..	1,100	..	..	..	..
Syn. Converters	2000	6	5000	98		123,000	18,500	3,480	21,980	..	..	..	..
Syn. Converters—Spare	2000	1	7000	..	..	20,500	3,100	..	3,100	..	..	..	..
Wiring				96	99.3	136,000	16,300	8,100	24,400	70,000	8,400	1,500	9,900
Switching for Units						150,000	18,000	..	18,000	250,000	30,000	..	30,000
Motors				90.0	85.4	475,800	71,340	192,210	263,550	548,000	77,770	202,870	280,640
Totals				80.8*	81.8*	$1,047,800	146,190	214,650	$360,840	$995,000	132,570	211,670	$344,240

*This figure is not the product of the efficiencies in the table on account of the common station service.

The most economical direct-current scheme seemed to be one having an auxiliary alternator on the main generator shaft feeding direct to a 550-volt direct-current synchronous converter. The most economical alternating current scheme appeared to be the one adopted. A comparison of these schemes is shown in Table I.

This analysis indicated that for this station, alternating current was at least as economical as direct current

the boiler house load, but also under normal conditions of carrying the auxiliaries required to start another unit. Such operation is provided for through selector oil circuit breakers as previously described. The transformer capacity between the 14,000-volt bus and 2300-volt bus has been made sufficiently large to supply the common station service load and in addition, to relay one of the 2500-kv-a. auxiliary generators. In case, however, of a complete power station shutdown it

would be possible to start the plant by means of the steam-driven exciter, the spare 1000-kw. steam-driven generator and the steam-driven boiler feed pump.

Excitation System

For each main turbo-generator there will be a 200-kw., 250-volt motor-generator set for excitation, the motor being connected to the 2300-volt generator group bus. In order to provide excitation in starting up the station independently of any source of power supply, a 250-kw. steam-driven exciter will be installed. The turbine of this exciter will supply low-pressure steam for heating. To insure a source of excitation which will be instantly available in an emergency, a 140-cell type *G*, exide storage battery and end cell switch is to be installed together with a 250-kw. motor-generator set for charging. This motor-generator set can also be used as a spare exciter. The battery has a discharge rate of 2106 amperes for thirty minutes. All exciters are connected through selector air circuit breakers to duplicate exciter buses. The fields of all synchronous generators are connected through selector breakers to these buses which are also provided with sectionalizing switches.

Power Distribution

Truck type switchboards are used to control the 2300 volt circuits. These trucks are equipped with oil circuit breakers of the *B*-13 type, redesigned, however, for finger type contacts and also for clearances of at least 5 in. between phases and 3 in. to ground. The rupturing capacity is ample to take care of the service.

Analysis indicated that the use of reactors with smaller circuit breakers in each station feeder would cost more than high capacity breakers without reactors and the latter arrangement was adopted. The reactors also have the disadvantage of introducing a very large voltage drop when motors are being started.

Operation

The switching arrangement adopted for the auxiliaries allows considerable latitude in the method of operation. At the present time, the following is contemplated:

In starting up the plant, the 2300-volt auxiliary bus will be energized either from the main 14,000-volt bus connected to the system or from the steam-driven auxiliary house turbo-generator. Assume that No. 1 main generator is to be put into service. When No. 1 main generator has been synchronized to the 14,000-volt bus, the No. 1 auxiliary generator oil circuit breaker *V* will be closed and the breaker *W* connecting the generator auxiliary group bus to the common station service bus will be open manually. In shutting down a unit, the auxiliaries that are required to remain in service can be transferred to another generator group bus. This is accomplished by means of a single control switch which automatically opens one selector breaker and closes the other after the first one has opened. If it is found undesirable, however, to interrupt power to the motors momentarily under normal operation, the generator group bus can be connected to the common station service and the circuit breaker between the auxiliary generator and its bus can then be opened. No difficulty is expected due to any phase displacement through the station transformers, as the auxiliary generator is so wound and constructed with respect to the main unit, that at no load, both generators are in synchronism. On account of the high reactance in the circuit, it is not expected that there will be any disturbance if it should become necessary to synchronize the auxiliary generator with the main generator through the transformers.

Reliability of Power Supply

Interruption of auxiliary power supply might be caused by any of the following conditions:

1. Failure of the auxiliary generator or loss of its field.
2. Failure of the main generator.
3. Overspeeding of the turbo-generator.
4. Failure of an oil circuit breaker on the auxiliary bus to function, or a short circuit on the generator group bus.

In case the auxiliary generator should fail, the differential relays would trip out the generator breaker and close a selector oil circuit breaker to the station service bus, thus keeping power supplied to the auxiliaries. In case the field of the auxiliary generator should fail a low voltage relay would trip out the oil circuit breaker of the generator and automatically close the breaker to the station service bus. Failure of the main generator would cause its differential relays to operate, which in turn would operate the differential relay of the auxiliary generator and close the selector breaker to the station service bus. In case of a severe disturbance on the system which would be sufficient to overspeed the generator and shut off the steam, it is estimated that the inertia of the rotor would be sufficiently great to keep the auxiliary generator running at sufficient frequency to supply power to the auxiliaries, while the main oil circuit breaker is tripped and the throttle again opened. It is expected, however, that the auxiliary load remaining on the unit will prevent overspeed sufficient to trip the throttle even though the main load be entirely lost. Failure of an auxiliary oil circuit breaker or a short circuit on the group bus, would affect only that particular group until such time as an operator can transfer to another bus.

Regulation

Each auxiliary generator is equipped with a face plate regulator, which automatically operates the field rheostat and maintains constant voltage. Relays are provided to ring an alarm in case of excess or low voltage due to sticking contacts.

Motors and Control

General Description. In general, all motors over 25 h. p., are rated 2300-volt, 60-cycle, 40 deg. All motors

of 25 h. p. or less are rated 550 volts. Motors that are fed through trolleys are rated at 550 volts. Motors for operating valves are rated at 220 volts direct current.

The motor and control equipment of all auxiliaries upon which continuity of station output is dependent is so designed that on restoration of auxiliary power after an interruption, the auxiliaries will automatically be restored to service.

This is accomplished on squirrel-cage motors of 25 h. p. or larger by means of a double squirrel-cage winding which permits the motor to be thrown directly across the line requiring a starting current of a little over four times the rating of the motor. On slip ring motors, this is accomplished by means of an automatic contactor, the opening of which inserts a block of resistance in the rotor circuit which resistance is automatically short circuited after the current has declined to a definite value. On brush shifting motors a pilot motor restores the brushes to starting position.

Forced Draft Fan. Each forced draft fan is driven by a B. T. S. 135-h. p., 2200-volt, 1200-rev. per min. commutator type, brush shifting motor. The motor is thrown on the line by means of contactors which are controlled from a push button station mounted on a control board located in front of the boiler and from a similar station located at the motor. The brushes are shifted mechanically by the Bailey combustion control system. Variations in the steam pressure or the furnace draft, cause the Bailey mechanism to increase or decrease the speed of the motor. Upon loss of power the main line contactors open and cut the motor off of the line. When power is restored, the brushes are shifted back to the starting position by means of the Bailey mechanism; the main line contactor closes and the motor comes to the speed required by the combustion control system. In order not to reduce the volume of air while transferring the motor from one bus to another a time-delay prevents opening the main line contactors or returning the brushes to the starting position during this operation.

Induced Draft Fans. Each induced draft fan is driven by a 100-h. p., 2200-volt, 600-rev. per min. slip ring motor, and a 225-h. p., 2200-volt, 900-rev. per min. slip ring motor. Each motor is connected to the fan by means of a flexible coupling. The 100-h. p. motor is thrown on the line by means of a contactor which is controlled from a push button station mounted on the control panel located at the boiler and from a push button station located at the motor. A drum controller with the necessary resistors in the secondary circuit, provides for speed control of this motor. This drum controller is operated mechanically by the Bailey combustion control system. When the load exceeds the capacity of the 100 h. p. motor, the line contactor in the 225-h. p. motor circuit is energized through contacts on the 100-h. p. motor drum controller and the 100-h. p. motor is disconnected from the line. The 100-h. p. motor is designed to withstand speeds up to 900 rev. per min. without injury. The drum controller for the 225-h. p. motor is also operated by the Bailey mechanism. Upon loss of power, the contactors open and the motor shuts down. Upon restoration of power, the drum controller is returned to the starting position by the Bailey mechanism, the 100-h. p. motor is energized and the fan comes up to speed in the normal way. As in the forced draft control, a time delay is provided in the control to allow for transferring the motor from one bus to the other without returning the controller to the starting position.

A description of the motor and control equipment for other auxiliaries is given in Table II.

Switchboards

Benchboards for a number of large installations have been made completely of sheet steel including panels which heretofore have been of slate. A new type of switchboard lamp has been developed, arranged for mounting on the front of the board. A resistance in series with it replaces the usual small fuse. For some installations in metropolitan districts where space is at a premium, control apparatus of unusually small dimensions has been used satisfactorily. In a number of instances asbestos board barriers have been installed between wiring of adjacent bench board panels. Such barriers serve to localize serious control trouble, and are a convenient mounting place for terminal boards.

Special care is being given to arrangement of switchboard equipment and wiring to eliminate congestion and provide a maximum of operating convenience and accessibility.

Circular benchboards have been used in the control rooms of several large stations. This type of construction usually involves extra expense, and must be justified by the benefit to be gained in any particular installation.

Terminal Rooms

In metropolitan stations from which large numbers of feeders are served, the multiplicity of control required has in some instances necessitated the use of terminal rooms directly beneath control switchboard rooms.

In the Hudson Avenue station of the Brooklyn Edison Company, now under construction, such a terminal room is provided. In this particular installation, conduits are of uniform size and come up through the floor in groups to steel terminal boxes located in a circle directly under the benchboard on the floor above. There is convenient access to all control and instrument wiring. Terminal rooms of this sort increase the over-all cost of the station where their use necessitates increase in cubic feet of building. If they can be fitted in without such increase in building cost, the over-all station cost should not be materially more because of their use. In stations where terminal rooms would have increased cost to an extent not justified by the benefits gained, the following method of construction has been used with satisfactory results. The floor underneath the bench-

TABLE II

MOTORS AND CONTROL—WEYMOUTH POWER STATION

THE EDISON ELECTRIC ILLUMINATING CO. OF BOSTON

For	Manu-facturer	No.	Motor				Control
			H. P.	Volts	R. P. M.	Type	
Circulating Water Pumps	G. E. Co.	4	290	2200	360	S. R.	Hand operated non-automatic oil circuit breaker. Hand operated drum controller and resistors for 25% speed reduction. Contactor and resistance for automatic restarting and transferring from one source of power to another.
Condensate Pumps	"	4	100	2200	1200	F. T.	Hand operated non-automatic oil circuit breaker to throw the motor directly on the line.
Deaerator Hot Well Pumps	"	4	60	2200	1200	F. T.	Hand operated non-automatic oil circuit breaker to throw the motor directly on the line.
Dry Vacuum Pumps	"	2	60	550	120	Syn.	Automatic contactor panel for throwing motor directly on the line. Permits transferring from one source of power to another but does not provide for automatic restarting.
425 lb. Boiler Feed Pumps	"	3	300	2200	1800	S. R.	Hand operated non-automatic oil circuit breaker. Contactor panel in secondary circuit controlled by master switch and drum controller operated by Ruggles-Klingemann regulator to give 20% speed reduction in five steps with automatic regulation at each step. Provides for automatic restarting and transferring from one source of power to another.
1200 lb. Boiler Feed Pump	"	1	350	2200	3600	S. R.	Hand operated non-automatic oil circuit breaker. Contactor panel in secondary circuit controlled by master switch and drum controller operated by Ruggles-Klingemann regulator to give 40% speed reduction in four steps with automatic regulation at each step. Provides for automatic restarting and transferring from one source of power to another.
Forced Draft Fans	"	4	135	2200	1200	B. T. S.	See text.
Induced " "	{ "	4	100	2200	600	S. R.	" "
	{ "	4	225	2200	900	S. R.	
Emergency Raw Water Pumps	"	2	50	2200	1800	F. T.	Solenoid operated, non-automatic oil circuit breaker with float switch or push button control.
" Dist. " "	"	2	40	2200	1800	F. T.	Solenoid operated, non-automatic oil circuit breaker with float switch or push button control.
Ash Quenching Pump	"	1	50	2200	1800	F. T.	Hand operated non-automatic oil circuit breaker to throw motor directly on the line.
200 kw. Exciters	"	2	300	2200	1200	F. T.	Hand operated non-automatic compensator with external switches.
250 " "	"	1	370	2200	1200	F. T.	Hand operated non-automatic compensator with external switches.
Fire Pump	"	1	150	2200	1800	F. T.	Hand operated non-automatic compensator with external switches.
Air Compressor	"	1	106	2200	257	Syn.	Hand operated compensator with external switches with undervoltage release and field protective contactor.
Stokers	"	16	5	550	1650	B. T. A.	Hand operated non-automatic oil circuit breaker to throw motor directly on the line. Pilot motor for shifting brushes for speed variation controlled by push button or Bailey combustion control system. Provides for automatic restarting and transferring from one source of power to another.
Clinker Grinders	West.	8	3	550	600	S. C.	Hand operated non-automatic oil circuit breaker to throw motor directly on the line.
Station Service Pump	G. E. Co.	1	10	550	1800	S. C.	Hand operated non-automatic oil circuit breaker to throw motor directly on the line.
Heater Drip Pumps	"	2	10	550	1200	S. C.	Hand operated non-automatic oil circuit breaker to throw motor directly on the line.
Evaporator Feed Pumps	"	2	15	550		F. T.-P. C.	Hand operated non-automatic oil circuit breaker to throw motor directly on the line.
Condenser Tube Wash Pump	"	1	15	550	900	F. T.	Hand operated non-automatic oil circuit breaker to throw motor directly on the line.
Auxiliary Cooler Pumps	"	2	25	550	1800	F. T.	Hand operated non-automatic oil circuit breaker to throw motor directly on the line.
10 kw. Control M. G. Sets	"	4	20	550	1200	F. T.	Hand operated non-automatic oil circuit breaker to throw motor directly on the line.
25 " Lighting " "	"	3	40	550	1200	F. T.	Hand operated non-automatic oil circuit breaker to throw motor directly on the line.
High Heat Level Cond. Pump	"	1	7½	550	1800	S. C.	Hand operated non-automatic oil circuit breaker to throw motor directly on the line.
Evaporator Service Pumps	"	2	3	550	1800	S. C.	Magnetic switch with float switch or push button control.
Sump Pumps	Louis Allis	1	5	550	1800	S. C.	Float switch.
Bilge Pumps	" "	2	5	550	1800	S. C.	" "

TABLE II—*Continued*

For	Manu-facturer	No.	Motor H. P.	Volts	R. P. M.	Type	Control
Intake screen	G. E. Co.	1	7½	550	1200	S. C.	Non-automatic compensator.
110 ton crane	"			550			
20 " "	"			550			
5 " "	"			550			
Office Bay Pass. Elevator	Otis	1	15	550		S. R.	
Power House Freight Elevator	"	1	13	550		S. R.	
Switch House Pass. "	"	1	20	550		S. R.	
" " Freight "	"	1	13	550		S. R.	
Coal Larries	G. E. Co.	2	7½	550		S. R.	
Coal Breaker	West.	1	150	550	450	S. R.	Hand operated oil circuit breaker with under-voltage release. Drum controller with starting resistance.
Conveyor "A"	"	1	100	550	900	S. R.	Hand operated oil circuit breaker with under-voltage release. Drum controller with starting resistance.
" "B"	"	1	25	550	900	S. R.	Hand operated oil circuit breaker with under-voltage release. Drum controller with starting resistance.
" "C"	"	1	125	550	900	S. R.	Hand operated oil circuit breaker with under-voltage release. Drum controller with starting resistance.
" "D"	"	1	20	550	900	S. R.	Hand operated oil circuit breaker with under-voltage release. Drum controller with starting resistance.
" "E"	"	1	20	550	900	S. R.	Hand operated oil circuit breaker with under-voltage release. Drum controller with starting resistance.
" "F"	"	1	20	550	900	S. R.	Hand operated oil circuit breaker with under-voltage release. Drum controller with starting resistance.
Coal Unloading Towers	"	2		550			Hand operated oil circuit breaker with under-voltage release. Drum controller with starting resistance.
Coal Stock & Reclaiming Bridge	"	1		550			Hand operated oil circuit breaker with under-voltage release. Drum controller with starting resistance.
Misc. Vent. Fans, Heating Pumps, Oil Pumps				550		S. C.	Hand operated non-automatic oil circuit breakers to throw motor directly on the line.

S. R. —Slip Ring
S. C. —Squirrel Cage
F. T. —Double Squirrel Cage
F. T.-P. C.—Double Squirrel Cage, Pole Changing
Syn. —Synchronous
B. T. S. —Brush Shifting varying Speed
B. T. A. —Brush Shifting adjustable Speed

board itself has been dropped approximately three ft. All conduits terminate in a steel trench box below the benchboard. Ample room is provided for all connections and terminal boards, and workmen can stand erect in the pit below each panel. Thus, repairs and changes in connections can be made almost as conveniently as in a terminal room. The question of whether or not a terminal room should be installed must be settled for each particular power station on the basis of the cost involved and the benefit to be gained.

Bus Structures

Practically all of the larger metropolitan companies have adopted, in their new stations, the isolated phase arrangement of switching equipment. This form of construction has been almost universally used where large numbers of feeders are supplied directly from the power station buses. With the continued increase in capacity of generating stations, it has become necessary to reduce the hazard of bus short circuit in every way possible.

The isolated phase scheme of construction has eliminated in some instances the conventional bus structure. The need of a totally enclosed compartment for the bus becomes of secondary importance. In many cases busses have been run entirely in the open. There are several companies that prefer to enclose the bus in the usual manner, and use through bushings for making connections to the bus.

Concrete or brick bus structures for auxiliary circuits are in many instances being eliminated by the use of truck type circuit breakers.

It should not be assumed that three-phase bus construction is obsolete for large stations or that it cannot be safely used where special attention is given to insurance against short circuit.

Where three phase bus construction is used, it has been found important to eliminate the passage of gases from one compartment to another in the case of breakdown to ground. All inspection openings in bus structures are enclosed by means of hinged or removable doors, and where conductors pass from one compartment to another through bushings, closure of the air passage is effected by various methods. In this way, with station neutrals grounded through resistance and all structures equipped with adequate grounding buses, disturbances on one phase, thus limited in severity, have little opportunity to involve the other phases and cause a bus short circuit.

The design of isolated phase equipment is now well through the development stage, and with the operating records of such equipment in service, which will be available within the next few years, a definite value can be placed upon the protection against short circuits which results from this type of construction.

The question whether isolated phase or three-phase bus construction should be used, must be solved in each particular case on the basis of the conditions to be met.

Turbo-Generator Units

The future of the electrical industry is now so dependent on steam turbo-generator development that advancement and improvement in design and construction of this type of prime mover is of extreme importance in every detail affecting the reliability of operation, the efficiency and the initial cost of the unit.

Improvements made and the most notable advances incorporated in the designs of several large power stations now under construction involve the use of higher steam pressures and temperatures, interstage reheating of steam, and stage steam bleeding.

The movement in the direction of higher steam pressures was initiated in 1917 at the North Tees power station in England where a gage pressure of 475 lb. per sq. in. and total steam temperature of 725 deg. fahr. was adopted. Interstage reheating was contemplated but was abandoned because of a few necessary modifications in turbine design and detail of piping which could not be undertaken during the war period. These improvements, however, have since been made. Although no consistent operating results are as yet available at this plant, the possibility and practicability of satisfactory operation at pressures of 500 lb. and 700 deg. fahr. have been demonstrated. A number of minor difficulties has been overcome and a great deal of valuable data and experience added to our knowledge of the use of higher steam pressures, all of which do honor to the pioneers who had courage enough to blaze a new trail in steam power-plant practise.

On the other hand, very little has been added to our knowledge of the properties of steam at high pressures, but the innovations adopted indicate clearly that an appreciable gain in economy should be expected in returning heat to the boilers, through stage bleeding, instead of carrying it out to the condensers.

Steam turbine designers do not anticipate any particular difficulty in producing machines adapted for pressures of 1200 or 1500 lb. per sq. inch. For these high pressures two or more cylinder machines will be used, the high pressure cylinder being small and operating with as good a factor of safety as the lower pressure cylinders.

Regarding steam temperatures, the limitations of strength of metals now available do not permit the use of temperatures in excess of 750 deg. fahr. Research for metals capable of safely withstanding higher temperatures for turbines, as well as for valves and fittings, is being conducted.

The rapid growth of turbo-generators to the sizes now being installed indicates the trend of development of the industry in general. Single cylinder units of 30,000 kw. have now become well standardized, and in answer to the demand for still larger sizes, 50,000-kw. turbo-generators of the single-cylinder type have been developed, and seem to be the next step indicated in size of single-cylinder units.

Considerable progress has been made in the last four years in turbine design and construction with a view to securing increased efficiency and reliability of operation, and, at the same time, to improving the steam cycle conditions. More stringent steam and vacuum conditions, in order to increase the heat-drop between the throttle valve and the exhaust outlet, have brought about many changes in turbine design and construction as well as in the selection and arrangement of plant equipment and layout.

With the increasing demand for larger sized units, and with the tendencies toward higher steam pressures and temperatures, there are indications that new designs will be adopted involving the subdivision of the turbine into two or more cylinders and shafts.

Single barrel units of 35,000 kw. and tandem compound units of 60,000 kw., both types for operation at 550 lb. and 750 deg. fahr., are now being installed; 2600-kw. and 4000-kw. turbines, designed for a steam pressure of 1000 and 1200 lb., are under construction. These machines are an entirely new departure and involve special boiler equipment and piping arrangement. The turbines themselves, however, are simple in design and present no obstacles to the ultimate success of the system adopted.

Tandem compound units with single shaft, both single- and double-flow types, in sizes ranging from 30,000 to 60,000-kw. have been successfully developed for speeds higher than for units of the single-cylinder type. Designs of large units with two shafts and three cylinders have been completed which reduce the number of alternators to only two.

Some increase in efficiency may be expected from these new designs or turbines, and the multi-cylinder arrangement will greatly simplify the application of steam reheating between high- and lower-pressure cylinders. The reheating cycle seems to be, of necessity, a feature of increasing importance, depending on the steam pressure adopted.

Quite a number of important problems remain to be completely and satisfactorily solved as further advance is made in the development of steam turbines, especially now that more severe operating conditions are imposed. Among these problems are; suitable material for blading and means to safeguard its reliability. The lack of an entirely satisfactory material that is sufficiently mechanically strong, that maintains its strength at high temperatures, that is reasonably cheap, easily machined, resistant to corrosion and erosion, and, most important of all, thoroughly reliable, is one of the limiting

factors in the designs for large outputs and high speeds. Chemical corrosion, due to air dissolved in the feed water, is now almost eliminated by the use of deaerators, evaporators and close circulation systems; but erosion is becoming more serious, since the present limit of safe initial steam temperature has been practically reached, and therefore, progress toward high initial pressures must result in larger moisture percentages in the last stages.

Next in importance are the questions of blade fastening to the wheels or barrels, the lacing or shrouding, the balancing of rotors, the keeping of length of blades, blade speeds and stresses within reasonably safe limits, while reducing and leaving losses to a minimum. To reduce the leaving losses, several methods have been recently developed, such as the "duplicate exhaust" and the "multi-exhaust" with directing steam vanes.

The bleeding of steam from one or more of the turbines stages for feed water heating has been advocated by turbine manufacturers for some time, but it is only recently that power station engineers have considered the subject with the interest which it deserves.

Stage bleeding is applied for the purpose of improving the efficiency of the heat cycle as a whole, but since this improvement is due in part to the effect of bleeding on the performance of the turbine unit, the subject is properly mentioned at this point. The diversion of a considerable fraction of the input steam at intermediate stages reduces the leaving losses and the effects of choking in the lower stages; a smaller quantity of steam is left to be condensed; the vacuum is improved, and less heat is passed on to the condenser.

To obtain the full advantage of stage bleeding, normal operation of auxiliaries must be almost entirely electrical.

The first commercial equipment which includes a mercury boiler and turbine exhausting into a condenser boiler which in turn generates steam for an existing steam turbine station, was put in service in the fall of 1923. The results which have been obtained confirm experimental tests and there is every reason to believe that this development will be of great value in reconstructing existing stations.

Generators

Improvements in design and construction of the generator have kept pace with those of the steam turbine. These simultaneous advances have made possible the supply of the demand for larger, more reliable and economical units. The additions made in 1923 to the generating capacity of the steam-electric plants in the United States totalled over 2,000,000 kv-a.

The progress made in recent designs is the result of efforts directed towards reducing the mechanical and electrical weakness of older machines, improving the ventilation, thus permitting the use of longer rotors and the production of generators of larger capacity. As to reliability, marked progress has been made in the insulation, both as to class of material and its application. There are more effective methods of holding the rotor coils, and bracing and clamping the stator end connections, thus increasing the reliability of the windings under stresses of short circuits or system disturbances.

As to size of generators, 62,500-kv-a. steam driven units are now being produced. The largest units built, however, are water-wheel generators having a rating of 65,000-kv-a., 12,000-volt, 25-cycle, 107 rev. per min.

The closed system of generator cooling, which has been installed in a number of large stations during the year, has many points of advantage. The air, having once been cleaned by the washer, filter or other device, remains clean indefinitely, reducing the deposit-forming material circulated through the windings practically to zero. By the closed system also, relatively little oxygen is available to support combustion, and the fire hazard, which has been quite serious with the open system, is greatly reduced.

In line with the general efforts to improve the economy of generating stations, some of the newer installations are equipped to cool the air discharged from the generators, using condensate as the cooling medium, thus retaining the heat lost in the generators.

With the trend in new generating stations towards stage bleeding for feed water heating, the use of electric driven auxiliaries is indicated for normal operation as to obtain the greatest advantage. To supply the essential auxiliaries, consideration has been given to the use of an auxiliary generator, connected to and driven directly from the shaft of the main unit. This method combines high reliability and economy. Several such installations with a-c. auxiliary generators and one installation with d-c. auxiliary generator are definitely planned.

Power Station Ventilation

With the growth in the size of power stations, the problems of ventilation, heating and lighting have increased and are receiving special consideration by both the architect and designing engineer.

Ventilation in the large modern power station presents entirely new and different problems involving the provision of features which were considered of trifling importance in the smaller power stations of the past.

Today the problem of ventilation involves the satisfaction of three main requirements:

1. Supply and movement of air for respiration.
2. Supply and movement of air for cooling apparatus.
3. Supply of air for combustion purposes.

These requirements must be satisfied at all times and under wide fluctuation of climate conditions. The conditions vary according to the geographic location and operation of the power station; *i.e.*, whether the station is continuously operated or subject to occasional shutdowns.

Ventilation and daylight illumination are closely allied, as well as problems of heating, where such requirements are necessary.

The first requirement; *i.e.*, providing a supply and movement of air for respiration, fortunately, offers no serious difficulty in the power station where large spaces are generally available and which, because of the relatively few men to be accommodated, do not require an active movement of air. In the modern power station however, dependence can no longer be placed on securing an adequate supply of fresh air for respiration through leakage around windows and doors. Other and more substantial means of ventilation must be provided. These, as far as possible, should depend on natural effects rather than artificial circulation of air, except where it is absolutely necessary to introduce purifying or conditioning devices.

The second requirement,—the provision of air movement for the cooling of apparatus—is a large item in power station design. This problem involves the absorption and dissipation of heat units generated and radiated by the station equipment. Future improvements, however, will doubtless serve to recover these losses and return them to the system.

At the present time, it is necessary to provide a sufficient volume of cool air for the removal of these heat units, not only from the electrical apparatus but also from steam and hot water piping and other apparatus.

One of the problems which has been practically solved is the prevention of condensation in the turbine room, and satisfactorily solved, too, under the very much more difficult conditions found in dye houses and paper mills, where tons of water are evaporated each day by the processing machinery used in these plants. Turbine room conditions, fortunately, are not so serious and by proper insulation of the roof structure and ventilation, it is possible to maintain the room atmosphere below its dew point, thereby preventing the condensation of warm, moisture-laden air which is found in the atmosphere of the turbine room.

The last requirement involves the supply of a large volume of air for combustion. It has been found in cases where no provision was made to meet this need that during the winter months the air supply for combustion was entirely insufficient. This condition would not exist in summer when the air could be admitted through open doors and windows. With the increased size of boilers, and with the tendency toward air preheating, this problem becomes one of great importance and requires special study for both the forced and induced draft. Ideal conditions will be met when the warm air is taken from the atmosphere of the boiler room and returned to the furnace, thus accomplishing the dual function of ventilation and heat-loss recovery.

Lightning Arresters

The Protective Devices Committee of the Institute has an efficient subcommittee actively at work on lightning arresters, and its report will thoroughly cover the ground in its technical aspects. A brief survey of the field, however, from the viewpoint of some of the manufacturers may be appropriate here in considerations of power station design. The manufacturers are of the opinion that they are furnishing apparatus which is quite effective, if properly installed and used; that their studies now under way will result in improvement, but they all point out the fact that a great deal depends upon intelligent cooperation on the part of the users.

Arcing grounds on non-grounded circuits, over-dynamic voltage, such as is caused by overspeeding of waterwheel generators, and faulty grounding of lightning arresters, are some of the causes of failures over which the manufacturers have no control. Doubtless, these sources of trouble will largely disappear when the users become aware of their seriousness and their undesirable effect upon an important piece of protective apparatus.

Two of the large manufacturing companies have testing equipment using the so-called lightning generator of Steinmetz. These devices afford excellent opportunities for studying the action of arresters under artificial lightning disturbances of an impulsive character.

Briefly, the lightning generator has revolutionized the testing of lightning arresters and their performance in actual service can be closely predicted. This fact should be emphasized and operating companies should more fully realize what fundamental characteristics a lightning arrester should possess and that these characteristics can now be determined in the laboratory. It is not essential to wait for data on hundreds of lightning arresters over a period of years before the true value of any particular arrester can be determined.

The lightning generator, however, shows chiefly the effect of the impedence of arresters to high current impulse of very short duration. They do not show the effect on arresters of all of the disturbance that may occur on transmission lines or distribution circuits. That disturbances other than impulses of extremely short duration do exist, is hardly a matter of doubt; they should be carefully studied. An arrester may be satisfactory from the standpoint of laboratory tests in discharging impulses and yet may be vulnerable to disturbances having an appreciable time element. At present this can be determined only by several years of service operation.

The real present need of the manufacturer for future progress with arresters is, as he says, not so much in actual apparatus development as in getting more specific and definite data on conditions to be met in service. He needs to know much more definitely not only what the surge voltages are, in magnitude, duration and frequency of oscillation and of occurrence, but also to what values these surge voltages may be reduced before reaching the apparatus to be protected. It is also necessary, since economics plays an important part

in the choice and use of arresters, to evaluate less than full protection in terms of average yearly injury to apparatus.

With this sort of information available, he believes he can add to the assurance that the characteristics of present designs actually meet conditions of service as fully as is now believed, and moreover that he can apply arresters with greater precision and with fuller assurance of obtaining the desired results.

Transformers

All the manufacturing companies disclaim any new radical changes in the design of transformers during the past year. A few things, however, have been done for the sake of protecting the transformer in case of breakdown. Changes have been made in construction methods and some companies still disagree in regard to polarity of distribution transformers.

The problem of protection of transformers from explosions has been attacked in various ways: a manufacturer placed on the market last year an "Inertaire" transformer which is claimed to be practically immune to explosions or internal fire. This is accomplished by equipping the transformer with an air tight cover and connecting the free space between the top of the oil and cover with an inertaire respirator which deoxidizes the air which passes through it. This respirator is filled with a substance which is said to be able to deoxidize the air as rapidly as it enters, allowing only nitrogen, an inert gas, to pass into the transformer.

The idea of using an inert gas in transformers to prevent explosions is not new; another manufacturer used CO_2 gas in transformers constructed several years ago, but later did away with the practice, developing instead the oil conservator.

So far as is known, there has been no trouble developed from the use of the "Inertaire" attachment. The following criticism might be offered, however. Since the deoxidation of the air is accomplished by chemical absorption of the oxygen by the compound in the respirator it follows that sooner or later the compound will depend upon the rate at which the transformer "breathes." The rate at which a transformer will "breathe" will depend upon varying load conditions, change of surrounding temperature, and seasonal changes. Two transformers alike in every respect, but working under different conditions, even on the same system, would have different rates at which the deoxidizing compound would be dissipated. This would make necessary frequent inspection of the compound in the respirators and irregular renewals. If a transformer were allowed to go a few days beyond the point of complete dissipation of the compound it would fill itself with ordinary air, which would mix with the gases from the oil and form an explosive mixture.

This device, however, offers a protection to the oil since it minimizes surface oxidation of the oil.

Winding of Disk Coils for Core-Type Transformers. During the past year one of the principal manufacturers developed a new method of winding flat disk coils for core-type transformers. This method makes it possible to wind a complete stack of either primary or secondary disk coils with one or more continuous conductors. The first coil in a stack is wound from the outside to the inside and so on until the complete stack is wound. There are two distinct advantages gained by this method. One is the elimination of all connections between coils and thereby eliminating the possibility of trouble arising from poorly soldered joints. The second benefit comes from lessening the handling of the coils, which reduces the possibility of injury to the insulation and insures better electrical conditions. Another advantage is that the new method speeds up production, which is to the advantage of the user.

Polarity of Transformers. Since 1922, when the N. E. L. A. transformer standards providing for standard polarity for single-phase transformers were adopted and put into use by the two largest manufacturers, some confusion in overhead distribution systems has been caused by diverse practise on the part of some of the smaller manufacturers. It is highly desirable that there should be uniformity of manufacturing practise in this respect.

Temperature Indicators

Several manufacturers were requested to supply information as to the progress made in the past year in the field of "Temperature Indicators." That no radical changes have occurred was admitted by most of them. In general, the progress consists in the addition of refinements or improvements by slight changes of construction to standard equipment.

One manufacturer has improved the optical system in his radiation pyrometer so that better accuracy can be obtained when used on small objects. Another has placed in production a Precision Portable Potentiometer for use with thermocouples, one scale division of which, is equivalent to approximately 1 deg. fahr. when used with an iron constantan thermocouple. Some minor improvements also have been added to instruments actuated by change of resistance of the bulb unit and also those actuated by expansion of gas contained in the bulb unit. Another manufacturer, in development of an instrument for indicating or recording temperatures at a distance, has reached the point where the instrument has been shown to be thoroughly practicable but he is not ready as yet to disclose the principle of operation or the type of construction.

Some tendency is evidenced to apply to commercial thermoelectric pyrometers and resistance thermometers, principles perhaps never so used before, although known and used in an experimental way twenty-five years or more ago.

Electric Drive in Fuel Preparation

Much interest was exhibited by the various public service and other companies, including some of the

TABLE III

No. Plant Reporting	A	B	C	D	E
1	Crushers bucket elevators, belt conveyors	6 motors, total 210 h. p., G. E. and Wagner 3-phase 25-cycle	1923 tons 187,002	Per ton 2.9 kw-hr.	Received by rail, 2 locomotive cranes, 2 locomotives, and 2 gondolas, used in handling storage.
2	100% electric drive	21 motors, total 1175 h. p. 3-phase 60-cycle	362,800	1.38 kw-hr.	Received by barges, clam shell bucket to crushers, conveying belts to bunkers, by rail for storage, clam shell bucket and crane.
3	100% electric drive	8 motors, total 175 h. p. a-c.	146,000	.65 kw-hr.	Received by rail, dumped to hopper, pan conveyor to crusher, inc. belt conveyor to top boiler room. Horizontal belt conveyor to bunkers, bridge crane for storage.
4	100% electric drive	6 motors, total 160 h. p., d-c.	146,000	Not given	Received by rail, dumped to pit, bridge crane with crushers. Transfer cars to coal bunkers.
5	100% electric drive	8 motors, total 145 h. p.	233,875	2.8 kw. (1) 5.7 kw. (2)	Rail-bridge crane to crushers from storage cars to crushers by winches, crushers to belt conveyors and bucket conveyors.
6	100% electric drive	11 motors, total 1339 G. E. Allis - Chalmers West. 3-phase 25-cycle	407,412	1.059 kw-hr.	Coaling, tower from scows at dock to crushers, 4 ton cars to bunkers, aeri cableway for storage.
7 Pulvzd. anthracite	100% electric drive	30 motors, total 1055 h. p. 3-Phase 60-cycle	63,352 Anthracite slush	See E	Recovery from storage pile to boiler bins, inc. drying, pulverizing, conveying 25.93 kv-hr. per ton.
8 2 plants combined	100% electric drive	22 motors, total 710 h. p. 2- and 3-phase 60-cycle	142,663	2.5 kw-hr.	Barge to coal tower, to coal car, to crusher by locomotive, to coal conveyors, to bunker bins, Gantry crane for storage handling.
9	100% electric drive	19 motors, total 677 h. p. a-c. 60-cycle 440-volt	479,750	No record	By rail to storage by locomotive crane, by cars to car dumper to shaking feeder, to conveyor belt to breakers, to crushers, to bucket conveyor, to boiler bunkers.
10 Pulverized bituminous. Not in operation	100% electric drive	13 motors, total 419 h. p. 3-phase 60-cycle 440-volt	Max. capacity 105,000 12 tons per hr.	Not operating under construction	Rail, track scale, to track hopper, skip hoist to crusher, to dryers, to pulverizers, to cyclones, to screw conveyor to boiler bins.
11 2 plants combined	1-50%, 1-100% electric drive	31 motors, total 1258 h. p. a-c. & d-c. motor	780,000	1.0 kw-hr.	Barges to coal tower to receiving hopper, to crushers, to weighing hoppers, to belt conveyors to bunker bins in boiler room.
12	100% electric drive	51 motors, total 2750 h. p. a-c. squirrel-cage and slip ring	300,000	3.0 kw-hr.	Steamer movable towers, to crushers, to cable cars, to station bunkers, or to storage lot. From latter reclaimed by skip hoist. From bunkers to weigh lorries to furnace hoppers.
13 2 plants combined	100% electric drive	No data	(1) 135,000 (2) 60,000	.75 kw-hr.	Cars to track hopper, to crushers, bucket conveyor belts, distributed to bunker bins.
14	100% electric drive	18 motors, total 837 h. p. a-c. 230 & 2200-volt	482,335	.43 kw-hr.	Cars to pits, conveyor to crusher, to carriers elevated above boilers, cross conveyor belts, distributed to bunker bins.
15	100% electric drive	9 motors, total 590 h. p. a-c. 220 & 2200-volt	108,797	.6175 kw-hr.	Ditto
16 Pulverized bituminous under construction	100% electric drive	32 motors, total 1558 h. p. a-c. 230 & 2200-volt	Pulvd. coal est. 78,126 7 months 1924	See E	Do.-Crushed coal, to coal preparation house, gravity to dryers, to pulverizer mills, then by Fuller-Kinyon Pumping System to boiler pulverized coal bins. All power estimated 22½ kw-hr. per ton.
17	100% electric drive	13 motors, total 1115 h. p. a-c. 2- & 3-phase, 60-cycle	276,032	.755 kw-hr.	Barge to crusher in tower, conveyor to top of building, cross conveyor to boiler bunkers, coal storage by locomotive cranes, capacity 3000 tons per day.
18	100% electric drive	9 motors, total 1060 h. p. 2-phase a-c. 60-cycle	174,408	.954 kw-hr.	Barge to crusher in tower, cable road to boiler bunkers. Coal storage by locomotive crane. Capacity 1000 tons per day.

TABLE III—*Continued*

No. Plant Reporting	A	B	C	D	E
19	90% electric drive	16 motors, total 900 h. p. 2-phase a-c. 60-cycle	249,456	1.00 kw-hr.	Ditto but coal is crushed twice.
20	50% electric drive 50% steam	6 motors 145 h. p. steam storage locomotive yard and 2 locomotive cranes	166,250	Not quoted	Rail-cars handled by steam locomotive, dump to hopper-hoist by movable tower to roof, to crushers, to boiler room. Storage by 2 locomotive cranes.
21 2 stations reporting	100% electric drive	6 motors 205 h. p. d-c. (no unloading inc.)	438,000 365,000	0.25 kw-hr. 0.31 kw-hr.	Barge-coal towers to crushers, to cable road belt conveyor, to boiler bunkers.
22	100% electric drive	2—700 h. p. m-g. sets towers, 24 motors, 1748 h. p. d-c. & a-c. 3-phase 60-cycle	Est. 226,000 1924	1.08 kw-hr.	Barge, 2 coal towers to crushers, sampled, to cars automatically operated, to boiler bunkers. Storage 50,000 tons—Staten Island, for emergency.
23	Hoisting by steam. Balance 50% electric drive	10 motors 140 h. p. d-c. 230 & 125-volt	336,000	No hoisting .161 kw-hr.	Barge, steam hoists to crushers, to electric driven coal cars, to boiler bunker bins. Storage mentioned No. 22 for this plant also.
24	Hoisting by steam. Balance 50% electric drive.	11 motors 161 h. p. d-c. 115 & 220-volt	197,000	No hoisting .161 kw-hr.	Ditto. Inc. transfer some coal by belt conveyor to boiler bunkers.
25	40% electric drive	3 motors 27 h. p. a-c. 60-cycle 3-phase	7,350	0.90 kw-hr.	Rail to crusher hopper, conveyor to bunker bins, locomotive crane for storage.
26	100% electric drive	3 motors 45 h. p. a-c. 3-phase 60-cycle	40,411	Est. 4.5 kw-hr.	Rail to derrick to crusher, to belt conveyors to bunker bins.
27	100% electric drive except for storage-locomotive crane	4 motors 150 h. p. a-c. 3-phase 60-cycle	50,400	.8 kw-hr.	Rail, skip hoist to crusher above bunker bins, belt conveyor to latter. Locomotive crane for storage.
28	100% electric drive	10 motors 480 h. p. a-c. 3-phase 60-cycle 2200 & 440-volt	240,000	.52 kw-hr.	Direct from mine, weigh baskets, to pan feeders, to crushers, to inclined belt conveyor, to bunkers. Drag conveyors for storage also locomotive crane. 71,000 tons storage.
29 Pulverized coal	100% electric drive coal from bunker bins	4 motors 300 h. p. a-c. 3-phase 220-volt	Capacity 2 mills, 10 tons hr. 36,000 tons	See E Pulverized coal	Coal from bunker direct to pulverized coal mills, to dryer, to cyclone, to burner feeders, 27 kw-hr. per ton, coal contains some bone and silica.
30	95% electric drive pulverized coal. Dryer fans by steam	22 motors 477 h. p. a-c. 440-volt 3-phase, induction	50,000 Steam heating operating 8 months per year	See E Pulverized coal	Electric cars to raw coal bunker, drying, conveying, pulverizing, conveying to boiler bunker bins, 22.3 kw-hr. Emergency storage several miles distant. Locomotive crane, clam shell bucket.
31	100% electric drive	14 motors 578 h. p. a-c. & d-c. 440 & 230-volt	540,000	.55 kw-hr.	Rail to bins, to crushers, bucket elevators, belt conveyors, to bunker bins. Gantry crane for coal storage.
32	100% electric drive	5 motors 105 h. p. a-c. 3-phase 220-volt	91,250	1.4 kw-hr.	Rail to hopper, to crusher, belt conveyor, to bucket elevator, to top of bunker scraper conveyor to bunkers. Storage-crane-clam shell bucket.
33	Unloading steam driven towers. Balance 100% electric	1495 h. p. 37 motors d-c. & a-c. 550-volt	389,623	1.734	Barge, belt conveyor to crushers, conveying belt to bunkers in boiler room. Storage from barge conveyor belts to field.
34	100% electric drive	11 motors 183 h. p. a-c. 60-cycle 3-phase 550-volt	276,000	No data	Rail, bridge crane, belt conveyor, bucket elevator to belt conveyor to bunker bins, slack coal, do not use crusher. Storage by steam, or electric cranes.
35	100% electric drive	5 motors 107 h. p. a-c. 3-phase 60-cycle 220-volt	75,000	0.6 kw-hr.	Rail to crusher, belt conveyor to bucket elevator, to scraper, conveyor to boiler bins. Storage by horizontal boom crane.

largest and most efficient plants in the country, in submitting to the Committee the accompanying tabulated data, the various plants being listed for obvious reasons by number only.

The plants reported in the tabulation had a total coal consumption for the year 1923 of 6,767,528 tons.

The average power per ton of coal handled is 0.98 kw-hr.

The total horse power installed averages 0.0019 h. p. per ton.

The following key is to be used with the foregoing data table:

A. Extent or use of electric drive in coal preparation
B. Number and size and characteristics of motors
C. Annual tonnage handled
D. Estimated kilowatt-hours per ton
E. System used in coal handling and storage equipment.

Automatic Equipment

The general trend in generating station design for the year 1923 has been to lean strongly toward automatic operation of apparatus where practical, particularly so where a personal hazard might otherwise be involved. The tendency has been toward:

A general increase in the size of automatic or remote-controlled hydroelectric generating stations; the elimination of the human element, by making automatic the sequence of operations in connection with high-tension switching, where a failure to follow the proper sequence might result in a serious hazard or damage to equipment;

the electrification of station auxiliaries with automatic restarting or remote control;

the providing of the system operator with automatic line and system load indication, for better supervision and control.

A considerable saving has been shown in the past by automatic operation of small hydroelectric generating stations, but the tendency now is toward making wholly automatic or remote-controlled stations of even larger capacity. Illustrations of this are Sprite Creek Station of the Adirondack Power Company, where a 7500 kv-a. generating station is automatically controlled, and an installation of three 1750 kv-a. units of the Wisconsin Valley Electric Company. There are some 60 installations of this kind with nearly 100 generating units.

While larger steam generating stations are not being made wholly automatic, every year records the addition of more automatic equipment. In the broader sense, where automatic operation includes interlocking to prevent a wrong sequence of operations, the advance is more marked, as illustrated in the Weymouth Station of the Edison Electric Illuminating Company of Boston, the Commonwealth Edison Company's Calumet Station at Chicago, and the Cahokia Station at St. Louis, where the disconnectors on the line-switches are interlocked with the oil-switch, to prevent wrong sequent operation. In some cases the disconnectors are automatically operated by the oil switch mechanism.

Station auxiliary apparatus is being electrified and the operation effected with remote automatic control. One feature of recent design is the automatic restarting of the essential auxiliary motors when power is restored after a shut-down.

Some of the larger systems are giving serious consideration to the question of providing the system operator with automatic indications of the system line and load conditions by means of automatic totalizing meters to indicate the load on the various generating stations, together with the total system load.

A further advance in automatic operation is the automatic indication at the system dispatcher's office of the open or closed position of the line-switches, and some thought has been given to the question of giving the system dispatcher control over some line-switching, independently of the station operator, so that the system dispatcher is not only a general supervisor of operations but has independent control of the general power system as well.

The manufacturers seem to be putting forth every effort to make their systems of remote and automatic control more reliable, so that the operating companies will have more confidence in this type of equipment and will undoubtedly find further use for it.

Nicholas Stahl, *Chairman.*

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High-Tension Disconnecting Switches, H. J. Crabbs, *Elec. Journal*, August 1923.

Explosion Pots Reduce Switch Troubles, *Electrical World*, July 28, 1923.

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ARRESTERS AND REACTORS

Lightning Arrester Grounds, H. M. Towne, *Electrical World*, January 19, 1924.

Pellet Type of Oxide-Film Lightning Arrester, N. A. Lougee, A. I. E. E. JOURNAL, October 1923.

Operating Experience with Current-Limiting Reactors, N. L. Pollard, A. I. E. E. JOURNAL, September 1923.

Short-Circuit Forces on Reactor Supports, R. E. Doherty and F. H. Kierstead, A. I. E. E. JOURNAL, August 1923.

Forcing Most Economical Load Division by Reactors, A. H. Sweetnam, *Electrical World*, June 23, 1923.

Developments in Auto-Valve Lightning Arresters, A. L. Atherton, A. I. E. E. JOURNAL, May 1923.

AUTOMATIC CONTROL

Control and Checking System for Automatic Stations, R. J. Wensley, *Electrical World*, November 24, 1923.

Supervisory Systems for Remote Control, C. E. Stewart and J. C. Field, *Electrical World*, September 29, 1923.

Automatic Plants Aid Western Water-Power Development, E. R. Stauffacher and Gustaf Clinwald, *Electrical World*, June 2, 1923.

Development of Automatic Control, H. C. Hoyt, *G. E. Review*, June 1923.

Full Automatic Hydroelectric Station, E. B. Collins, *Electrical World*, May 19, 1923.

COMMUNICATION AND SIGNALS

Communication over 140,000-Volt Line, C. A. Boddie, *Electrical World*, December 22, 1923.

Signal Systems for Power Stations, C. D. Gray and M. M. Samuels, *Electrical World*, August 4, 1923.

Baltimore-Holtwood Carrier Current, *Electrical World*, May 12, 1923.

MISCELLANEOUS

Power-Station Lighting, Raymond A. Hopkins, *Electrical World*, November 3, 1923.

Simplified Method of Analyzing Short-Circuit Problems, R. E. Doherty, A. I. E. E. JOURNAL, October 1923.

Ventilation of Central-Station Buildings and Equipment, Report of Subcommittee on Ventilation of N. E. L. A. Electrical Apparatus Committee, October 1923.

Operating Performance of a Peterson Earth Coil, J. M. Oliver and W. E. Eberhardt, A. I. E. E. JOURNAL, September 1923.

Generator Voltage Regulators, J. H. Ashbaugh, *Power*, September 25, 1923.

Electric Machinery of Large Capacity, K. Sachs, *Electrotechnik and Maschinenbau*, September 2, 1923.

The Paradox of Hydro-Steam, G. H. Moore, *Engineering News-Record*, August 30, 1923.

Characteristics and Performance of Conversion Apparatus for Edison Systems, T. F. Barton and C. M. Fulk, *G. E. Review*, June 1923.

ELECTROCHEMISTRY AND ELECTROMETALLURGY COMMITTEE

To the Board of Directors:

This committee has during the past year continued the activity which was begun the previous year. As in 1923, it also in 1924 initiated a technical session held at the Spring Convention, devoted to electrochemical and electrometallurgical subjects. At this session, very excellent papers were presented which tended to indicate advances in the art and the status of the art in the respective fields covered. These papers were as follows:

(1) Maximum Demand Regulator for Electric Furnaces, by E. T. Moore.

(2) Manufacture of Phosphoric Acid in Electric Furnaces, by Theodore Swann and F. V. Andrea.

(3) Effect of Impurities on Battery Electrolyte, by G. W. Vinal and F. W. Altrup.

No regular committee meetings were held as all necessary business could be quite successfully conducted through correspondence. Four or five of the seven members of the committee have shown an active interest in the work. These were all unanimous in the belief that it would be a disastrous mistake to change the function and field of activity of this committee as proposed by the tentative report of the committee to review technical activities of the Institute, dated January 22, 1924. This committee also feels that the

tentative report referred to proposes an improperly narrow and restricted basis for the whole activities of the Institute and much prefers to support the higher conception of the Institute and its technical committees, as outlined in the general remarks on Technical Committee Activities which were approved and passed at a meeting of the Meetings and Papers Committee on May 2, 1924.

It is proposed to continue the activity of this committee during the next year by obtaining papers relating to the engineering requirements and operation of electrical equipment in electrochemical and electrometallurgical processes and to initiate a technical session at which such papers may be presented and discussed.

This committee especially provides the technical liason between the A. I. E. E. and the American Institute of Mining & Metallurgical Engineers and the American Electrochemical Society, so far as electrochemical and electrometallurgical applications are concerned. It is therefore the province of this committee to report that the two engineering societies mentioned have given considerable attention during the past year, and have, in fact, conducted several technical sessions at which a number of excellent engineering papers have been presented on subjects in which many electrical engineers, and accordingly the Institute, should be interested. As a matter of record, and for the sake of reference, a list of these papers is being included in this report. At the Midwinter Convention of the American Institute of Mining and Metallurgical Engineers held in New York on February 18th to 21st, 1924, the following papers of mutual interest to electrical and metallurgical engineers were presented:

(1) Direct Electrolysis of Black Copper Anodes of High Nickel-Lead Content, by M. H. Merriss.

(2) Electric Welding of Large Storage Tanks, by Harold C. Price.

(3) Greenwalt Electrolytic Copper Extraction Process, by Wm. E. Greenawalt.

(4) Electrolytic Zinc from Complex Ores, by U. C. Tainton.

At the Spring Convention of the American Electrochemical Society, Philadelphia, Pa., April 24th to 26th, the following papers were presented:

(1) Review of Progress in Electrolytic Refining of Metals, by S. Skowronski.

(2) Electrodeposition of Copper by the Union Miniere du Haut Katanga, by H. Y. Eagle.

(3) The Electrolytic Tank House, Chile Exploration Co., Chuquicamata, Chile, by C. W. Eichrodt.

(4) Electrolytic Silver Refining at Pachuca, by G. H. Clevenger.

(5) Electrolytic Tin, by Charles L. Mantell.

(6) Electrolytic Refining of Tin, by J. R. Stack.

(7) Progress in Electrolytic Iron, by Donald Belcher.

(8) The Electrolytic Production of Beryllium, by B. S. Hopkins and A. W. Meyer.

(9) Electrodeposition of Tellurium, by F. C. Mathers and H. L. Turner.

(10) An Attempt to Electroplate Tungsten on Iron, by C. A. Mann and H. L. Halvorsen.

There was also a session at which a series of seven papers on applications of organic electrochemistry was presented. The subjects were largely treated in a manner of value to the electrochemist, and therefore while they might be of interest to the electrical engineer if differently treated, a list of these papers is not being recorded.

Of all the electrochemical and electrometallurgical processes in which progress has been made during the past year or two, the one perhaps of greatest interest and possible future value to electrical engineers is that of electrolytic iron. It is worthy of record that the electrolytic production of high grade iron is receiving very great attention in a number of quarters in America, as well as in Europe, and that in America several small but commercial plants are already in operation, the latest of which has been started only within the past few months. Details will probably be available for publication later in the year.

J. L. McK. Yardley, *Chairman.*

IRON AND STEEL INDUSTRY COMMITTEE

To the Board of Directors:

At a meeting of this Committee in September, 1923, at which each member was present, plans for the ensuing year were discussed. It was agreed that the Annual Report should assume the form of either:

A technical paper prepared by members of the committee, dealing with some vital problem of the steel industry, for presentation at joint meetings of the Association of Iron and Steel Electrical Engineers and the American Institute of Electrical Engineers; or,

A Topical Report indicating briefly the progressive development of electrical applications in the steel industry.

The first suggestion had the enthusiastic and unanimous approval of the Committee. The subject selected for the paper, "Economic Balance of Thermal, Electrical, and Mechanical Energy" presents the greatest problem which the engineers of the steel industry have to face. A very comprehensive outline of this paper was prepared under such general subdivisions as:

A Sources of Heat and Power.
- a. Water Power.
- b. Natural Fuels.
- c. Prepared Fuels.

B Direct Application of Heat.
- a. Power Plant.
- b. Coke Ovens.
- c. Blast Furnaces.
- d. Open Hearth.
- e. Soaking Pits.
- f. Reheating Furnaces.

C Direct Application of Power.

D Heat Balance in Typical Steel Plant.

a. Energy requirements for steel plant of 100,000 net tons per month of finished and semi-finished steel.
b. Number of 600-ton blast furnaces.
c. Net tons of ore, coke, and limestone.
d. Cubic feet of gas evolved.
e. Cubic feet of gas required for stoves.
f. Cubic feet of gas available for other purposes.
g. Capacity of by-product plant to supply coke required by blast furnace.
h. Cubic feet of available coke oven gas.
i. Number of 100-ton open hearth furnaces required.
j. Quantity of gas, oil, or pulverized coal required.
k. Reclaimed energy from open hearth waste heat boilers.
l. Equivalent capacity in 50-ton electric furnaces.
m. Kw. hour and B. t. u. consumption of electric furnaces.
n. Number and capacity of soaking pits.
o. Pulverized coal, oil or gas required.
p. Tons of steel through Blooming Mill.
Kw. hours and B. t. u.
q. Tons of steel through 24 in. Billet and Bar Mill.
Kw. hours and B. t. u.
r. Tons of steel through two 18 in. Billet and Bar Mills
Kw. hours and B. t. u.
s. Tons of steel through 14 in. Merchant Mill.
Kw. hours and B. t. u.
t. Tons of steel through 10 in. Rod Mill.
Kw. hours and B. t. u.
u. Estimate of energy for all auxiliaries.
v. Summary expressed in terms of total B. t. u. in coal used for coke or as fuel for direct heating including B. t. u. value for fuel oil, also B. t. u. consumed or reclaimed in each application whether in form of heat or mechanical power.
w. Discussion of purchased power versus local generation.
x. Interconnection with Public Utilities.

Considerable work was done in the accumulation of data for this paper, and an effort made to arrange for joint meetings with the Local Sections of the A. I. E. E. in the more important steel centers. These efforts met with little success and the excellent work of certain members of the committee was rendered useless by the inability of all to devote the necessary time to this difficult but very important subject.

The committee, therefore, reluctantly gave up the preparation of this paper, and can only express the hope that under more favorable circumstances the work may be carried to a successful conclusion.

In place of this proposed technical paper the committee submits the following topical report of developments of the past year.

I. Generating Units

The steam turbine still holds first place as a prime mover in the steel plant power station, although excellent results are being obtained with the slow-speed gas engine using blast furnace gas under local conditions of high-load factor. A combination of gas engines and steam turbines often makes the ideal steel plant generating unit. In some instances the best economy is effected by the purchase of power during periods of low-load factor while carrying the base load of the steel plant at high-load factor on gas engines.

Much development work is being done with gratifying results in an effort to perfect the Diesel oil engine for small plants where these requirements do not exceed 5000 kw.

II. Distribution

It is almost impossible to appreciate fully the part which electricity plays in the steel plant of today. Cumbersome and inefficient steam and hydraulic lines are already largely replaced by a distributing network of electrical lines transmitting power to all parts of the plant in any desired quantity from local generating stations or from sources many miles distant. Instead of the many small and relatively inefficient prime movers formerly scattered about the plant, electricity has made possible the high economy of the modern central station, operating independently or in conjunction with other power systems.

If electric power is purchased from a source outside the steel plant, it is obtained at some high voltage such as; 22,000, 33,000, 66,000, etc., and is then stepped down through transformers to the working voltages of 6600 or 2200. The larger steel plants usually generate their own power at 6600 volts, as energy is easily transmitted about the plant with very little loss at this voltage. The 6600-volt power, either 60 or 25-cycle is commonly used for main-roll motors and stepped down through transformers to 230 volts a-c. or through motor-generators or synchronous converters to 230 volts d-c. for use on the smaller auxiliary motors.

Continuity of service through the distributing system, one of the most important, if not *the most important*, factors in a steel plant, seems to be best secured electrically through the use of the so-called loop or ring system, either overhead, underground, or a combination of both. This method of distribution lends itself to segregation of faults by the use of oil circuit breakers for the high voltages; of carbon circuit breakers or magnetic switches for the lower voltages. By controlling the breakers or switches with relays and interlocks, it is possible to isolate automatically any part of the system on which faults occur and without any material disturbance of the load on other parts of the system.

III. Yard Electrification

The economies found in the electrified yards of steam railway lines are found in even a greater degree in the steel yard system, due to the very intermittent nature of the work performed by a switching or transferring locomotive or car.

In a steam-operated yard there are individual self-contained units, generating motive power, handled more or less carelessly, and as a rule, with very low economies. The electrically-operated yard also has many individual self-contained units which, however, take their power from a central station fully equipped to produce power in large quantities at low cost. These electric locomotives with full automatic control are usually in the

hands of a single skilled operator and show a very fair operating economy.

The success or failure of an electrified yard depends to a large degree upon the method of getting power to the locomotive. Results obtained to date indicate that yard electrification is best secured through the use of the third-rail method, preferably the under-running type, of supplying power to the electric locomotive or car rather than the overhead trolley. The under-running third rail can easily be stepped over without personal danger. In a highly congested yard the use of a third rail demands a careful layout of switches, cross-overs, etc., but will adequately meet most requirements. In some cases a combination storage battery and third rail may be desirable.

One of our largest steel plants, after a year's experience with a 40-ton storage battery locomotive in general yard service reports most favorably. This locomotive is in operation during two 8-hour shifts and is charged during the third shift with occasionally one or two short-time high-current boosts during the 16 hours of continuous operation.

Among the many advantages of electrified yards over steam-operated yards are: one-man operation; no loss of time for getting coal and water or dumping ashes; cost of power about one-third that of steam; maintenance of electric locomotives on the order of one-ninth that of steam; two electric locomotives coupled together and operated by one crew; depreciation of electric locomotive materially less than that of a steam locomotive; greater flexibility in operation.

IV. Electric Furnaces

Thirty-seven new electric furnaces were installed during the year of 1923 in the United States and Canada, bringing the total number installed from 456 on January 1, 1923 to 493 on January 1, 1924. Of this number 442 are in the American industry and 51 in Canada. The new installations are mostly of small capacity, of 5 tons or less. Notable among the larger furnaces is one of 60-ton capacity. Development during the past year has been largely on the arc furnaces. Of the arc furnaces the three-phase, three-electrode type seems to be operating the most satisfactorily.

A number of repulsion-induction brass-melting furnaces was placed in commercial operation during the past year. This type of furnace utilizes the force of electro-magnetic repulsion existing between transformer high tension and low tension windings to cause a circulation of molten metal in the melting pot. A notable accomplishment of this type of furnace was the successful melting of pure copper on a commercial basis.

The largest horizontal single-ring type induction furnace in the United States has a maximum capacity of six tons and in normal service will melt three tons every three hours. It requires 800 kw. at 2200 volts, single-phase, 8½ cycles, supplied from a specially designed synchronous motor-generator. The motor of this set is rated at 1400 kv-a., 3-phase, and is operated at 80 per cent leading power-factor in order to compensate for low power-factor conditions in the electrical systems of the mill. In this way a low power rate is secured and the disadvantages of the inherent low power-factor of induction furnaces are avoided.

V. Electric Heating

Electric heating finds an important application in low-temperature heat treatment of small steel parts. Such installations, however, are more numerous in the steel-products industries than in the steel-making industry.

During the past year, a device has been developed for electrically pre-heating sheet-mill rolls preparatory to rolling hot steel sheets. Methods previously in use involved a series of gas flames, or hot-water jets played on the rolls during the warming-up period. By the use of electric heat, the rolls are more uniformly heated, the warming-up period is shortened, roll breakage is decreased, and a better steel sheet results.

Electric space heaters are now being installed in the frames of rotating machines in order to keep the windings warm and prevent sweating during long periods of idleness. This results in a materially longer life for the insulation, and lessens the danger of breakdown.

During the year 1923 electric ovens were, for the first time, successfully used for the bright annealing of copper wire. The oven is mounted on wheels and is moved back and forth over two hydraulic platforms, one of which can be loaded while the other is in the heating chamber. A water seal keeps the air from this chamber. The wire is uniformly annealed at all points in the oven and the product is bright and free from scale.

The connected load is 300 kw., 550 volts, 3-phase, and 7000 lb. of copper wire can be charged at one time and fully annealed in 75 min., the energy required averaging about 1 kw-hr. for each 25 lb. of wire.

VI. Automatic Control

During the past year there has been a steady movement toward simplified control and improved safety devices but no radical departures from past practise.

The ever-present demand for increased production has resulted in higher speeds for mill cranes and has consequently necessitated particular attention in design to secure safe and accurate control of the loaded hook, as well as high speeds for lowering the hook when empty. This is particularly the case with hot-metal cranes where dynamic braking is being extensively and satisfactorily used. Magnetic holding brakes are, of course, used in addition, one on the motor shaft and one on the intermediate shaft. The higher speeds coming into general use have also brought about the development of a safe limit switch, positive in action yet compact and admitting of mounting in various positions. There is a decided and increasing tendency

toward the use of enclosing features as a safety measure.

As mentioned elsewhere, the desirable characteristics of the synchronous motor have, in some few instances, overbalanced its objectionable features for steel-mill drives and a new type of automatic pushbutton starter has been developed for these motors which affords better protection and greater convenience than the older type of manual starter.

The use of automatic substations is spreading rapidly in the steel mills and during the last year several papers on this subject were presented before various societies. There is no doubt but that the automatic substation is an important item in the electrical net work of the mill. It is an excellent example of what can be accomplished safely and satisfactorily with automatic control.

Automatic hoists are being installed in the mills for handling coal, ashes and similar material. Balanced and unbalanced types are used calling for full reverse and dynamic lowering controllers. High-speed operation is provided with adequate slow-down features through automatic controllers. The use of two-speed a-c. motors has been an important addition to this field.

Two blast-furnace skip hoists have been installed recently using standard mill-type motors with mill-type shoe brakes of large size. One has a single motor equipment and is provided with full automatic push-button-operated control. The other uses two motors with a full automatic, series parallel controller to get high-speed operation with adequate slow-down and accurate stop. A large 36 in. shoe-type magnetic brake is used on each motor shaft.

The shoe type of magnetically-operated brake is fast replacing all others. A considerable number of brakes of a size larger than ever before attempted has been made with wheels 36 in. in diameter and 12 ½ in. face which have been very successful on blast-furnace skip hoists, large cranes, and car dumpers.

The application of magnetic type of friction clutch for use with synchronous motors and other power drives has been extended during the last 12 months.

Too little attention has been given in the purchase of control equipment to the duty cycle which it must meet. A more careful analysis of the time-torque-speed values is of utmost importance if the maximum of satisfaction is to be obtained under operating conditions.

VII. Main-Roll Drives

The reliability and high efficiency of what has come to be known as the steel-mill motor has given it a status such that only in rare cases is the steam engine ever mentioned as a competitor for main roll drives.

During the past year a very considerable part of the equipment sold for driving main rolls, reversing or non-reversing, has been for the purpose of replacing existing steam engines. Twelve units with an aggregate normal continuous rating of 55,500 h. p., including four d-c. reversing motors, supplied from suitable flywheel motor generators, have replaced engines, and one new 40-in. Blooming Mill has been equipped with a 6500-h. p., continuous rated, 50 rev. per min. d-c. reversing motor and flywheel set.

There has been the usual percentage of constant speed induction motors applied to continuous mills, sheet mills, bar mills and 3-high plate mills. Most of these installations are of interest only as an indication of the continuously increasing use of electric power. One installation comprising an 8000-h. p. 240 rev. per min., 13200-volt constant-speed induction motor, driving an interrupted continuous blooming mill, is notable as being the first steel mill motor for operation above 6600 volts.

The use of individual motors for the finishing stands of certain types of continuous mills, with the most exacting speed requirements, is increasing. Thus far the condition has best been met with d-c. motors in spite of their somewhat higher first cost and lower efficiency as compared with a-c. motors. The use of rolls in tandem, operating at high surface speeds and with but a short distance between stands, as in the case of continuous hot-strip mills, has made necessary a design of motor with close inherent regulation over a wide range of speeds. The success of such designs has been fully demonstrated in recent strip-mill installations.

In the normal expansion of an engine-driven plant, it is often desirable to disconnect a part of the rolls and drive them from one or more suitable motors. Due to the poor speed regulation of the engine, it is necessary to use adjustable speed motors provided with a special synchronizing control which will enable the motor automatically to follow closely all variations in engine speed. Several types of such control have been placed in successful operation.

Rolling mill engineers have long desired to apply synchronous motors to main rolls on account of the beneficial effect on the plant power factor. The deficiency of the usual synchronous motor in starting torque, has, however, almost entirely prevented such applications. The development of the revolving frame synchronous motor has overcome this objection and two such motors, each rated 580 kv-a. 70 per cent power-factor (500 h. p.) 360 rev. per min. have been installed for driving copper rolling mills.

An entirely new application and important development are the building of two BTA brush-shifting motors for driving rolling mills. One has a 3 to 1 speed range with 600 h. p. at maximum speed and the other 2 to 1 speed range with 500 h. p. at maximum speed. The operation of these machines will be watched with considerable interest, as they are expected to meet a long-felt need for an adjustable speed a-c. drive where the power required is too small to make Scherbius or Kraemer apparatus feasible on account of their high initial cost in small installations.

The largest Kraemer equipment in this country has been built during the year and will be used to drive a 14-in. Merchant Mill. This equipment is rated 4500/4500 h. p. 500/300 rev. per min. 2200 volts.

An interesting example of the attempt to secure the maximum flexibility in mill design is found in a 20-in. strip mill recently placed in operation. The roughing train is driven by a constant-speed induction motor, the intermediate train by a single adjustable-speed d-c. motor and the three finishing stands are driven by three independent adjustable-speed d-c. motors. The use of synchronous converters for supplying direct current for the adjustable-speed motors represents a departure from the usual steel-mill practise of using motor-generators for this purpose.

The following summary includes only main roll motors 300 h. p. and above, built by the three principal electrical manufacturers in this country.

		June 1923	June 1924
Continuous h.p.	60 cycle..........	452,840	478,390
" "	25 "	475,825	490,225
" "	direct current....	299,670	324,860
	Total.......	1,228,335	1,293,475

This shows an increase of 65,140 h. p. in the aggregate of main roll motors supplied by these three companies during the past year.

VIII. Arc Welding

Arc-welding processes and equipment are well standardized. The metallic arc has almost completely supplanted the carbon arc as it has been found that reliable welds can be made by a less skillful operator and the deposition of metal is just as rapid. Arc welding is being extensively used about the steel plant in the repair of worn pinions, wabblers, rolls, driving spindles, etc.

There appears to be a decided tendency toward the use of greater welding currents, and single-operator machines of high capacity are being produced to meet these requirements. The application of automatic arc-welding processes is expanding as the increased production and lower costs obtained become known.

One of the most important achievements has been a marked improvement in the construction of the electrode which makes it easy to combine the desired welding flux within the body of the electrode, thus securing all of the advantages which could be obtained with an externally coated electrode without its disadvantages. The flux is enclosed in an annular space between the core and a sheath, the amount being controlled by the thickness of the annular space.

It is customary to incorporate with the flux of coated electrodes, materials which merely serve as a binder. These materials add to the slag produced making it difficult to avoid inclusions in the deposited metal. The enclosed flux cannot flake off before it actually enters the arc.

The sheath, which receives the current and conducts it to the arc, always presents a clean surface to the holder or current-feeding device. This is particularly advantageous with the automatic welding machine where current is led in through the feed rolls or nozzle, as the electrode wire travels through in a continuous length.

* * * * *

There is a marked interest in the generation of electric power for sale as a means of realizing the benefit from the surplus gas from merchant blast furnaces. The single furnace of the Trumbull-Cliffs Furnace Company at Warren, Ohio has developed approximately 5000 kw. continuously over and above the power required for the plant itself and sold it to the local public service company. This sale of power materially affects the profit per ton of pig iron, placing the merchant furnace on a par with the furnace located at the steel plant, which in the past has been in better position to utilize the excess gas.

F. B. Crosby, *Chairman.*

PROTECTIVE DEVICES COMMITTEE

To the Board of Directors:

In addition to the five subcommittees which have been working during the past few years on protective devices, a new subcommittee on Automatic Stations was authorized and appointed last year and has been doing very active work.

Thus the committee's activities have been carried out along the lines of the six subcommittees, namely:

1. Lightning Arresters
2. Oil Circuit Breakers, Switches and Fuses
3. Relays
4. Automatic Stations
5. Current-Limiting Reactors
6. Grounding of Systems

The dependency on automatic devices on which the central station companies are constantly insisting is becoming stronger each year and the emergency functions are being taken away from the operator and placed with the properly selected equipment which is found more dependable than the human operator. The automatic features are found to be on the job twenty-four hours a day, 365 days in the year and can be relied upon to function in a moment's time and thereby maintain service at a higher standard with less operating cost than a completely manually-operated equipment.

During the past year the committee work has been very well supported and excellent results have been obtained, as indicated in the following subcommittee reports.

Subcommittee on Lightning Arresters

F. L. Hunt, *Chairman*

The work this year of the Lightning Arrester subcommittee has been chiefly that of preparing and agreeing on a statement which may be used as a basis of lightning arrester performance. This has been com-

pleted and was published in the June issue of the A. I. E. E. JOURNAL under the title "Basis of Comparison in Lightning Arrester Performance."

This statement is believed to be a start toward standardization in testing and classification of lightning arresters and it is the intention of the subcommittee to follow it up by the formulation of definitions of the features brought out in this statement.

The several different angles of lightning arrester design, performance and testing have been brought up in the papers fostered by the subcommittee and presented at the Spring Convention in Birmingham under the authorship of A. L. Atherton, E. R. Stauffacher, W. F. Young and C. E. Bennett.

During recent years there has been a steady increase in the emphasis which is placed on continuity of service by central stations and as the systems have grown in size and complexity, the results of failures become more serious. This has increased the emphasis and need of more effective protection against lightning and surges which has resulted in improved types of arresters being manufactured.

There is apparently some distance to travel before we find an ideal arrester, although the new developments in the Autovalve and Pellet type of arresters have gone a long way in bringing out an effective and simple arrester.

The necessity of good grounding conditions in connection with lightning arresters has been brought out forcibly this year, and attention is called to the fact that even an ideal arrester would be of little value on a system where good grounding conditions were not made effective.

Lightning arresters are very seldom installed today on completely underground systems and there is some question regarding their desirability on high-potential transmission lines of 200,000 volts and above.

Subcommittee on Oil Circuit Breakers, Switches and Fuses

E. C. STONE, *Chairman*

The subcommittee on Circuit Breakers, Switches and Fuses in cooperation with the representatives of the National Electric Light Association and the Electric Power Club have agreed on a definition of the interrupting duty of an oil circuit breaker. This definition as recommended to the A. I. E. E. Standards Committee is—

Interrupting Rating of Oil Circuit Breakers

The interrupting rating of an oil circuit breaker is a rating based upon the highest r. m. s. current at normal voltage that the breaker can interrupt under the operating duty specified.

The value of the current shall be taken during the first half cycle of arc between contacts during the opening stroke.

Operating Duty of Oil Circuit Breakers

(a) The operating duty of an oil circuit breaker shall consist of a specified number of unit operating cycles at stated intervals.

(b) Each unit operating cycle shall consist of a closing of the circuit breaker followed immediately by its opening without purposely delayed action.

Interrupting Performance of Oil Circuit Breakers

(a) An oil circuit breaker shall perform at or within its interrupting rating without emitting flame.

(b) At the end of any performance at or within its interrupting rating, the circuit breaker shall be in the following condition:

(1) *Mechanical*

The breaker shall be substantially in the same mechanical condition as at the beginning.

(2) *Electrical*

The breaker shall be capable of carrying rated voltage, and its main current-carrying parts shall be substantially in the same condition as at the beginning.

After performance at or near its interrupting rating, the interrupting ability of the breaker may be materially reduced and it is not to be inferred that it may be reclosing after such performance without inspecting, and, if necessary, making repairs.

Standard Operating Duty of Oil Circuit Breakers

The standard operating duty of an oil circuit breaker shall consist of two unit operating cycles at a two-minute interval.

The manufacturers are rating all new breakers in accordance with this definition and are also in position to give the rating of their old oil circuit breakers on this basis.

Although there have been no radically new developments in oil circuit breakers during the year and the largest breaker is still limited to 1,500,000-kv-a. rupturing capacity, the manufacturers are considerably advanced in the art and if larger breakers are required we feel reasonably sure that they can be supplied. There has been a marked improvement in the development of oil circuit breakers in the truck or quick removable type, allowing very much better facilities for inspection and repairs. This type of breaker is becoming very popular in the central station industry and will be of considerably greater use in the future.

A number of operating companies has started tests on oil circuit breakers to determine if the ratings guaranteed by the manufacturers are met. The operating industry expects to carry this very much further during the next year and a very definite set of specifications for testing oil circuit breakers so that the different sets of tests may be brought into a comparable basis, has been prepared. While it is expected that experience will possibly result in some modification in detail of this specification, it is believed that it covers the essential points and it is recommended that in order to secure truly comparative results, power companies conduct their oil circuit breaker tests in

accordance therewith. Copies of these specifications can be obtained at the A. I. E. E. headquarters.

There has been a distinct improvement in the satisfactory use of high-voltage current interrupting fuses and a number of tests has been conducted showing that it is possible to rely upon fuses in certain places where the short-circuit current does not reach excessively large values.

At a technical session of the Spring Convention at Birmingham, devoted completely to oil circuit breakers, papers were presented by H. J. Scholz, Alabama Power Company, A. J. D. Hilliard, General Electric Company and J. V. Jenks, West Penn Power Company.

Subcommittee on Relays

A. H. SWEETNAM, *Chairman*

The Relay Handbook is being published jointly with the National Electric Light Association, as mentioned in last year's report. It is practically completed and publication may be expected during this fall. We feel sure that this book fills a need throughout the industry and its publication will be welcomed by all. On account of the large number already subscribed for, there will be 10,000 volumes printed which reduces the price to N. E. L. A. or A. I. E. E. members to $4.00 per single copy with quantity discounts for 100 copies to one address at $3.00 per copy. The price to non-members is 50 per cent above these figures.

Development of greater accuracy of the transformer and the development of lower ampere windings on relay have made it possible to use bushing current transformer for relay protection, thereby saving considerable space and extra investment on new installations.

Greater use is being made of the indicating pointer and targets on the relays. Some companies are using the indicator pointer as an ammeter on the front of the control board. This not only gives them an indication of the load on the feeder but assures them that the relay circuit is intact. The targets have proven very valuable in analyzing troubles showing that a switch tripped out due to relay operation on certain phases.

Of the new type of relays that came out in production last year, the impedance relay is probably the most promising. This relay works on the principle of combining voltage and current with time so as to make the relay operate first which is next to the trouble. Two elements, one of potential and one of current, are so designed and arranged that the time of closing is inversely in proportion to the current and directly proportionate to the voltage or, in other words, depends on the distance between the relay and the fault.

A new relay designed to protect against open-phase, reverse-phase and unbalanced-phase in rotating machinery has been produced and shows promise of good application. This relay combines reactors and resistors in a network with an induction relay so that no current flows through the relay as long as the polyphase service is balanced and the phase rotation is correct. Upon the occurrence of the abnormal condition above mentioned, the relay received an appreciable amount of current and operates.

Greater care has been taken in the testing and maintenance of relays, and to the equipment available for this work has been added the portable phase meter which will indicate in electrical degrees the vector relationship between the current and voltage in any circuit. This addition to the testing equipment will make it possible to give greater assurance in the connections of directional relays.

The adoption of a-c. networks has made it desirable to install in certain places relays in manholes, and although the work in this connection is not very far advanced, there has been considerable emphasis placed on the development of relay cases for installation in moist or water-filled manholes and we can look forward to the development of real equipment for this work during the next year.

Subcommittee on Automatic Stations

W. H. MILLAN, *Chairman*

The newly organized Subcommittee on Automatic Stations has put in a year of very active work and has brought before the industry the dependability of protective devices for automatic stations in the six papers to be presented at one session of the Annual Convention in Chicago, by C. W. Place, H. L. Wallau, C. A. Butcher, Herman Bany, F. D. Wyatt and R. J. Wensley. The work of the next year has been very definitely laid out and the numerous subdivisions of this subject will be studied in considerable detail for a more comprehensive report next year. It is definitely planned to investigate the possiblity of applying automatic devices to perform all of the functions ordinarily included in the operators' duties even as far as such matters as the leakage of oil from transformer or switch tanks.

Subcommittee on Current-Limiting Reactors

N. L. POLLARD, *Chairman*

From the results of this year's study it appears that the reactors have filled the requirements more completely during the past year and reactor failures have been few. The manufacturers of this piece of equipment have studied the special requirements of the tremendous forces and potential strains and have produced a reactor which apparently meets the requirements in every respect.

In order that the special requirements of reactors may be brought before the industry, the committee has prepared a specification for current-limiting reactors which was published in the June issue of the A. I. E. E. JOURNAL. The disputed question of the desirability of using resistors with reactors is apparently no nearer solution than it has been for the past few years.

One of the large manufacturers recommends the use and two do not. There has been no operating experience which shows clearly that either the resistances are a hazard or that they are a necessity. The present status of the manufacture has been brought out in the three papers to be presented at the Annual Convention by H. O. Stephens and F. H. Kiersteadt, W. M. Dann, and S. I. Oesterreicher.

Subcommittee on Grounding of Systems

E. R. STAUFFACHER, *Chairman*

Last year's committee covered the Eastern operating views on grounding of systems and this year's committee has studied the same problem from the angle on the Pacific Coast.

There is still considerable discussion as to the amount of resistance which should be inserted in the neutral of a system which varies from several hundred ohms down to a solid ground. The very high-voltage systems have in general adopted this solidly grounded neutral and considerable apparatus is specified on that basis. Some of the more moderate transmission systems having 22,000 volts have adopted the solidly grounded neutral last year and there has been considerable extension of the solidly grounded 3-phase, 4-wire distribution system.

HARRY R. WOODROW, *Chairman.*

ELECTRICAL MACHINERY COMMITTEE

To the Board of Directors:

DURING the administrative year 1923-24 the Electrical Machinery Committee has conducted the following activities:

1. *Papers*—Under the direction of the Meetings and Papers Committee, thirty (30) papers have been secured by the Committee for Institute meetings, including the Annual Meeting of this year. These may be grouped as follows:

a.	Factors which influence design of electrical machinery	9 papers
b.	Generator design and construction	9 papers
c.	Motor design and construction	5 papers
d.	Frequency-Converter design and construction	1 paper
e.	Transformer design and construction	5 papers
f.	Static-Condenser design and construction	1 paper
	Total	30 papers

Opinions have been expressed to the effect that the presentation of so large a number of papers is not accompanied by sufficient advantages to justify the heavy expenses associated with their publication. It has to be admitted that any one paper is rarely even read by or could be useful to more than from 20 to 200 people and the publication of 20,000 copies in order that 200 copies or less shall reach the people who will profit by them seems at first thought very wasteful. But any other known method is liable to fail to place copies in the hands of the particular individuals who will make to the community the best return from the knowledge or inspiration received from them. The expenses are greatly reduced by the Institute's present practise of publishing in the JOURNAL merely an abstract of not over four papers (and in some instances merely a statement of the title) accompanied by the statement that a copy of the complete paper will be sent to any member applying for it.

The presentation of these papers and their discussion affords the opportunity for the specialists in the design, construction and operation of electrical machinery to interchange experiences and thereby advance in knowledge and ability.

Several of the papers, as examples of which may be mentioned:

Eddy Current Losses in Armature Conductors
Tooth Pulsations in Rotating Machinery
Harmonics Due to Slot Openings
Shaft Currents in Electric Machines

are of the nature of researches leading to information of which general use is made by all designers until such time as still better and more complete results are made available. In other words, while the discussions bring out valuable criticisms with respect to the assumptions made, the methods of investigation employed and the conclusions drawn, there is general satisfaction at the increase in the store of available knowledge represented by the papers.

A second group of papers (really a subdivision of the first group) describes researches into such characteristics of electrical machinery as require to be analyzed before Standards can be established. We may give the following examples of this group:

Effects of Expansion and Contraction on Insulation of Long Armature Coils.

Effects of Time and Frequency on Insulation Tests of Transformers.

Effect of Altitude on Temperature Rise of Electrical Apparatus.

Temperature Rise of Stationary Electrical Apparatus as Influenced by Radiation, Convection and Altitude.

As examples of a third group of papers may be mentioned:

The Inertaire Transformer.

Recent Advances in the Manufacture and Testing of Static Condensers in Power Sizes.

A New Type of Single-Phase Motor.

A New Synchronous Induction Motor.

In papers of the class of which these are examples, radically novel features of design and operation and novel methods of construction are described. The propositions put forward by the authors of papers in this class generally embody radical alternatives to usual types of machines and in the ensuing discussions the propositions are subjected to careful analysis by specialists in the respective fields.

As a fourth kind may be cited those papers consisting in clear and timely descriptions of especially important installations of machinery. We may take as examples:

65,000-kv-a. Generators at Niagara Falls.

The 35,000-kv-a. Frequency Converter for Hell-Gate Station.

While these descriptions are primarily valuable for record and for reference, the discussions often include valuable opinions with respect to characteristics and features of the designs described and of alternatives preferred or at any rate described by those contributing to the discussion.

Committee Deliberations on Matters Referred to It

Various matters are referred to the Committee for opinions or action. Recently the Board's Committee to Review the Technical Activities of the Institute referred its Tentative Report to the members of all technical committees. At a meeting of the Electrical Machinery Committee, the Chairman of the Board's Committee, Mr. Berresford, accepted the Electrical Machinery Committee's invitation to attend and assist the Committee in its consideration of the matter. The parts of the report specifically dealing with the Electrical Machinery Committee are reproduced in the following extracts:

EXTRACTS FROM TENTATIVE REPORT OF COMMITTEE TO REVIEW TECHNICAL ACTIVITIES OF INSTITUTE

1. Committee on Electrical Machinery

Field: Cognizance of all matters in which the dominant factors are the design and construction of devices and machinery for the generation, transformation and translation of electrical energy and the requirements determining design and construction, except where such requirements are specifically otherwise assigned.

Note: The expectable functioning of this committee should be initiatory and determinative in the matters within its province. Subject to the approval of the Board it should have power in all matters arising therein, except such formulation of standards as is the function of the Standards Committee, including those of contact with other bodies and of arrangement for joint action where such is indicated.

Standardization:

In your Committee's opinion, a degree of standardization work by technical committees is not only permissible, but desirable. Composed, as it is, of individuals active and experienced in a given field and associated for the purpose of contact with the operations and possibilities of that field, the need for and the possibilities of standardization therein should first present themselves to the Technical Committee.

There are two steps in the making of a standard—

a. The perception of the possibilities, the evaluation of the desirability and the determination of the degree to which standardization may be practicable, including preliminary formulation.

b. The actual and final formulation of the standard.

Your Committee believes that "a" will best be accomplished by the Technical Committee and "b" under the auspices of the Standards Committee, the working committees of which, under these conditions, would instinctively be composed in large parts of the men responsible for "a".

The Electrical Machinery Committee expressed its agreement with these sections of the Tentative Report of Mr. Berresford's Committee.

Committee Consideration of Standardization Matters in Their Initial Stages

In the tentative report of the Board's Committee it is stated that prior to the "actual and final formulation of standards" relating to electrical machinery, there is occasion for the Electrical Machinery Committee to make valuable contributions in "perceiving possibilities," "evaluating the desirability" and "determining the degree to which standardization may be practicable" and also in the "preliminary formulation."

At two meetings of the Electrical Machinery Committee held respectively on the 21st of April and the 5th of June, careful consideration has been given to some matters of considerable importance relating to the basic principles underlying the standardization of electrical machinery. These have included propositions drawn up to more clearly distinguish between "rating" and "service conditions." The whole subject is of such immediate interest as to justify including a review in this Report.

Review. Some ten years ago a general plan of Standardization which had particular reference to Electrical Machinery was adopted by the A. I. E. E. This plan was reasonably in agreement with the I. E. C. proposition and has since been very widely used in America and abroad. With respect to temperature recommendations certain limitations were established. These limitations varied with the class of insulation employed, the type of enclosure of the machine, the nature of the cooling medium and the method specified for the measurement of the temperature. But in a general sense or at any rate as an example, it may be stated that for open-type machines built with Class A insulation and cooled by air, the limiting temperature rise as determined by thermometers applied to prescribed accessible parts was established as 50 degrees. This limiting rise was permitted for places where, and at times when the temperature of the surrounding air did not exceed 40 degrees, with the further limitation that the recommendation only applied to machinery operating in places whose altitude was not more than 1000 meters (3300 feet) above sea level. While this general plan was adopted after much thorough discussion in this country and abroad in which many experienced people participated, the original impetus in America came chiefly from Dr. C. P. Steinmetz, Mr. B. G. Lamme, and Prof. A. E. Kennelly, who at that time applied a great deal of study to the subject of *Standards for Electrical Machinery*. It was not the intention either of those in Europe who contributed to this plan, nor of Steinmetz, Lamme, Kennelly and their associates to advocate these values as anything more than *limits*. They fully realized that in the ordinary course of events a very few machines might occasionally come to be subjected to these limiting conditions simultaneously. They recognized that such simultaneous occurrence of these limiting conditions frequently or for long periods would not be desirable but there appeared no sufficiently simple way of specifically discountenancing such operation at these limits without reducing the limits for *all* machines. This would have resulted in the use of

uneconomically large machines for almost all cases in order to avoid rather severe conditions in an almost negligible number of instances. Furthermore in this practically negligible number of instances it was known that any disadvantageous consequence would be simply a reduction in the life expectancy of the machine and would not consist in its failure. Naturally approved construction in all respects and the use of approved materials are assumed in these statements.

This general standardization proposition has gradually come into very wide use not only in America but abroad. In recent years, however, due partly, it is believed, to gradually losing sight of or mistaking the underlying ideas, a great deal of study has been directed to the development of proposals for utilizing machines frequently and for long periods under conditions which may occasion sustained temperature rises equalling and even exceeding the *fifty degree limit.*

Thus, if a consumer's motor was located where the surrounding air never or rarely exceeded 30 degrees, it seemed to him, (and the suggestion was sometimes made to him), that if there were no other limitation, such as stalling load or commutation or mechanical strength, he could advisedly place on the motor any load not occasioning more than 60 degrees rise.

Also if he had knowledge that a certain motor with a 50-degree rating actually on test showed a margin of, say, 7 degrees, the rise at rated load being only 43 degrees instead of the limiting value of 50 degrees, then even if the surrounding air had a temperature of 40 degrees he considered that he could place on the motor a greater load up to the point where he had used up this 7 degrees margin; and if the surrounding air had a temperature of only 30 degrees, he considered that he had a margin of 17 degrees extra rise which he could properly utilize. It should be needless to state that no such interpretations or consequences were contemplated when the system of limits was established. Had it been supposed that such interpretations were liable to become general, it would have been necessary to establish correspondingly lower limits. Indeed it was expressly stated in the Standards that the fact of a surrounding air temperature lower than the limiting temperature of 40 degrees was not to be taken as justification for loads occasioning a temperature rise in excess of the 50 degree limiting rise.

Also with respect to altitude, the original purpose in broadly endorsing the use of machines rated on these principles at any place whose altitude does not exceed 1000 meters (3300 ft.) for carrying the same loads as at sea level, was dictated by the desirability of having a simple basis of rating and it was believed that the few degrees greater temperature rise sustained by a machine carrying a given load at the upper limiting altitude as compared with the sea level rise, would at the most only slightly decrease the life expectancy, especially since it was realized that the coincidence of the occurrence of the conditions of the limiting altitude of 1000 meters and the limiting cooling air temperature of 40 degrees would be extraordinarily rare and it was believed that it would not justify the introduction of correction complications.

But the more general knowledge of the influence of altitude on temperature rise is leading people to observe that their machine is described as adequate to carry its rated load at the limiting altitude of 1000 meters. They add this to their knowledge that at 1000 meters altitude the machine has, at its rated load, a temperature rise greater by, say, 5 degrees, than at sea level. They note that on the occasion of its test at sea level it was only required to come within the limit of 50 degrees rise. They consequently consider that the machine must be regarded by its makers to be adequate to carry loads occasioning 55 degrees rise if the temperature of the surrounding air is not over 40 degrees. They may even go further and conclude that if the temperature of the surrounding air is only 30 degrees there is also available a 10 degree higher rise for that reason, giving them license to impose any load which will not occasion over 65 degrees rise. If, furthermore, instead of having, on test, the limiting rise of 50 degrees the rise was only, say, 43 degrees, they can equally reasonably add this further 7 degrees and conclude that the correspondingly increased load is in accordance with the proposition.

Meanwhile, although not as yet recognized in any way in the Institute Standards, manufacturers have established the practise of entering into undertakings applying to the operation of motors with *off normal circuit conditions.* The following is an example:

> Motors shall operate successfully at rated load and frequency, with voltage not more than 10 per cent above or below the name plate rating, but not necessarily in accordance with the standards established for operation at normal rating.

The user presumably reasons that he wants his motor to operate *successfully.* It is of less importance to him if it is "*not necessarily* in accordance with the standards established for operation at normal rating." Let us take the case of a consumer whose circuit voltage is 220 and who has available a 240 volt motor. If he operates it at its rated load on the 220 volt circuit, (on the security of the above quoted clause), the current will be increased about 9 per cent and the copper loss about 18 per cent. The core loss will be somewhat decreased, so the thermometric temperature rise may be only increased by (let us say) some 5 degrees. But if we add this 5 degrees to the 5 degrees relating to altitude which we have already discussed, then, with a cooling air temperature of only 30 degrees the user will be concluding that he will not be impairing the life expectancy by having his motor carry such loads as will occasion a temperature rise of 70 degrees.

But the average user cannot reasonably be expected to know that the 50 degrees limiting rise by surface thermometers is usually associated with a further rise

of some 15 degrees in the insulation in contact with the copper and that this further 15 degrees becomes

$$\frac{70}{50} \times 15 = 21 \text{ degrees}$$

additional when the surface temperature rise is 70 degrees. This 6 degrees further rise together with the 5 degrees further rise for altitude and the 5 degrees further rise assumed to correspond to the *successful* use on a circuit with a voltage 9 per cent lower than that on the name-plate aggregate 16 degrees addition to the "hottest-spot" temperature of $40 + 50 + 15 = 105$ degrees which is taken as the *Limiting Hottest Spot Temperature* for Class A Insulation. The hottest-spot temperature corresponding to this *successful* operation thus becomes $105 + 16 = 121$ deg.

The well-meant and entirely reasonable and practical standardizing proposition built up by Steinmetz, Lamme, Kennelly and their associates and based on suitable *limits* is thus seen to have become a web of inconsistencies and absurdities when interpreted and amended in ways which could never have been reasonably anticipated.

During the few months immediately preceding his death, Dr. Steinmetz was again directing his close attention to this subject and was suggesting a revision of the general proposition to cope with the new circumstances. It was the privilege of some of us to be working with him in this matter.

One of the tentative propositions to which this has led consists in having two entirely distinct statements relating respectively to Service Conditions and to Rating. The statement relating to Service Conditions aims to present a carefully considered recommendation as to when and where a machine may carry its rated load, (or more than its rated load), and when and where the load should be restricted to values less than the rating.

The plan proceeds from definitions of (1) *usual service conditions*, and (2) *unusual service conditions*. For the former the machinery should be adequate to carry its rated load, but for the latter it *may* sometimes be desirable to restrict the load to values less than the rating. These are *application recommendations*, and are distinct from the *rating*, which is a simple value corresponding to defined test conditions.

If this plan should be employed in a standardizing proposition, it is probable that the text would have to be somewhat different for different kinds of machines. The following text entitled SERVICE CONDITIONS has been suggested as suitable for use in *Standards for Transformers* and is being considered as an alternative to a more usual test.

SERVICE CONDITIONS

Usual Service Conditions: Usual service conditions are those in which none of the limits listed below are exceeded, nor two or more of these limits simultaneously closely approached.

Limits:

1. The temperature of the cooling medium should not exceed 40 deg. cent. for air or 25 deg. cent. for water.
2. The altitude should not exceed 1000 m. (3300 ft.)
3. The voltage should not be more than a specified limiting per cent above or below that set forth on the rating plate.
4. The frequency should not be more than a specified limiting per cent above or below that set forth on the rating plate.
5. The power factor should not differ more than a specified limiting per cent from that guaranteed.
6. The maximum load should not exceed the rated load, in combination with a 24 hour load factor approaching 100 per cent.

Unusual Service Conditions: All other service conditions should be designated as "unusual service conditions." Such would be

(a) If one of the above listed limits is exceeded.

(b) If two or more of those limits are simultaneously closely approached.

APPLICATION RECOMMENDATIONS

Under *Usual Service Conditions*, the apparatus should be expected to carry its rated output.

Where either of the two previously stated classes of *Unusual Service Conditions* occurs, it *may* sometimes be desirable to restrict the output to values less than the rating.

For example, a machine may be required for operation where:

a. The cooling medium frequently and for considerable periods approaches the limiting value and

b. The altitude approaches 1000 meters and

c. The machine is required to operate continuously at rated load for periods approaching 24 hours per day and every day and

d. The voltage differs from the rating plate voltage frequently and for considerable periods by amounts approaching the specified limiting per cent deviation and

e. The frequency differs from the rated value often and for considerable periods by amounts approaching the specified limiting percentage deviation.

The need for special consideration in these matters is apparent when it is pointed out that if the prevailing value of (a), the temperature of the cooling medium is sufficiently low, then other limits may be approached without concern; or as another instance, if (c) the load is very fluctuating, the average load being sufficiently low, then other limits may be reached without concern, etc.

In cases where transformers are operated under technical supervision it will be permissible to make

use of special loading instructions furnished by the manufacturer.

For example, the instructions may be in the form of maximum available capacity for different ambient temperatures or for different water temperatures and different rates of water flow for water-cooled types, without the maximum temperature exceeding approved values for continuous operation.

Or, where the nature of the load is fluctuating, these instructions may permit loads in excess of those approved for continuous operation but of such a magnitude that the deterioration of the insulating materials is no greater than that resulting from rated load in average ambient temperatures.

H. M. HOBART, *Chairman.*

EDUCATIONAL COMMITTEE

To the Board of Directors:

The outstanding event in the field of engineering education during the past year has been the inauguration by the Society for the Promotion of Engineering Education of a comprehensive project of investigation and development with provision for the active cooperation of the faculties of the engineering colleges, the professional societies, the employing industries, federal and state bureaus of education and other agencies.

It will be recalled that in 1907 the Institute joined with its sister professional societies and the Society for the Promotion of Engineering Education in undertaking a detailed study of engineering education. Finding the task to be beyond their financial resources, this group of societies proposed to the Carnegie Foundation for the Advancement of Teaching that it take over the problem. After due examination of the project, the Foundation generously acceded to the request and finally selected Dr. Charles R. Mann to make an extended investigation and report.

The report appeared just as the nation was developing its maximum war effort. The resulting disturbances in the work of the colleges assisted educational progress in details but retarded fundamental changes. As normal conditions returned it was apparent that the broader problems remained as before, plus the added complications arising from the war. It seemed necessary to supplement Dr. Mann's report by gathering additional evidence in order to deal intelligently with both the old and new problems confronting the colleges.

Through the generosity of the Carnegie Corporation, the Society for the Promotion of Engineering Education has been enabled to undertake (1) a project for the cooperative gathering of evidence from the past experience of the engineering colleges; (2) a survey of the occupational demands confronting the schools, to be carried out in cooperation with the organized industries and professional societies; (3) a study of the ways and means by which the engineering schools may be effectively related to the organized life of the engineering profession; (4) a study of recent advances in the principles and practises of education and psychology which are adaptable to the work of engineering schools; and (5) a study of corresponding problems and activities in Europe.

An important part in this project is being taken by the national engineering societies. At its 1923 convention the Society for the Promotion of Engineering Education voted to invite the four founder societies to appoint advisory councillors on education who should confer with the Board of Investigation and Coordination having the oversight of the general project. All four societies accepted the suggestion. The Institute appointed Messrs. Gano Dunn and F. B. Jewett to serve as councillors and the other societies named men of equal distinction and ability. The Board of Investigation and Coordination held a conference with the councillors, to which the secretaries of the four societies were also invited, on May 15, 1924.

The conference group recognized that its function is purely advisory and formulated agenda of studies to be pursued rather than action to be accomplished. The present circumstances seem particularly favorable to an inquiry into the professional status of engineering and the implications which arise from it with regard to engineering education. In recent years there has been a strong trend toward standardization in professional education for medicine, dentistry and law. In each case the respective groups of schools and related national professional associations have set up joint agencies of educational standardization. This movement has been closely related to the comprehensive investigations conducted by the Carnegie Foundation for the Advancement of Teaching and has been productive of important gains in the standards of educational preparation for these professions. On the other hand, there have been some results at least temporarily less beneficial.

With a comprehensive study of engineering education now in progress, it is fitting that these movements in other professions should be studied under the joint auspices of the schools and the professional bodies with an entirely open mind. It seems evident that engineering occupies a professional status dissimilar in important respects to the older, highly individualized professions of divinity, medicine and law, and that the formal relations between education and professional life which have been built up in these professional fields, do not necessarily constitute precedents of compelling logic. It is equally true that the national engineering societies might lend a far greater body of support to the engineering colleges without any trespass on their independence or autonomy.

As a step toward the clearer understanding of these problems, the conference group has agreed to sponsor and direct studies of the following subjects, to be carried out by the staff of the Director of Investigations of the

Society for the Promotion of Engineering Education:

1. The criteria of professional status, with a view to defining more accurately the status of the professional engineer.

2. The present status of relations between other professional bodies, particularly those concerned with medicine, dentistry, law and architecture, and the corresponding groups of professional schools, including the background of the present schemes of standardization and rating of schools by professional bodies, the evidences of detrimental as well as beneficial results from such standardization, and the present state of relations between engineering schools and engineering societies abroad, particularly in Great Britain and Germany.

3. Minimum standards which may properly be established for the recognition of any institution as an engineering school or any course of study as an engineering course.

4. Standards of educational attainment, in other than technological fields, which should underlie the professional training of engineering students. This would include languages, history, literature, economics and psychological and sociological sciences.

5. Standards of educational attainment in the common group of mathematical and physical sciences and of technological studies which should underlie the professional training of engineering students.

6. Sanctions concerning the normal length and the degree of specialization of engineering curricula to which the societies represented may be willing to give support.

7. Sanctions concerning the desirable qualifications of teachers who deal with professional engineering subjects, their professional and economic status and the appropriate scale of compensation for engineering teachers, to which the societies represented may be willing to give support.

8. The determination of aptitudes as a basis for admission to engineering colleges.

9. The extent to which the relations of the professional engineering societies to affiliated student groups may advantageously be unified or coordinated.

10. The contributions which the profession-at-large and the business and industrial organizations closely allied to it may make to the fund of vocational information relating to engineering and the means which may be employed to bring such information to the attention of parents, teachers and students at the time of selection of a college or university and of a course of general or professional college study.

11. The recognition to be given to graduation from an engineering college in the requirements for admission to professional engineering societies.

12. A survey of the occupational demand for engineering graduates in the more distinctly professional fields, as a complement to the surveys of demand in industrial fields now being undertaken.

W. E. WICKENDEN, *Chairman*,

ELECTROPHYSICS COMMITTEE

To the Board of Directors:

ADVANCES IN ELECTROPHYSICS 1923-1924

No attempt will be made to point out outstanding advances for the past year. Progress, in general, is continuous and not in marked steps. For this reason, it has been the policy of this committee to keep the membership informed by means of lectures by noted physicists and research workers rather than by an annual catalog of steps in progress. Such lectures have taken place during the year on general and specialized subjects. It has also been our policy to arrange popular lectures or reviews on the latest status of electrophysics. These lectures have probably always been better attended than those on any other subject. In passing, it may not be out of place to mention that wonderful advance has been made in the past few years in the knowledge of the interior of the atom and the radiations or energy changes that occur from its very heart. It is suggested that the committee next year arrange a popular lecture covering the latest knowledge of the atom.

LECTURES AND PAPERS

During the past year the Electrophysics Committee has had technical sessions at the Pacific Coast, the Midwinter, the Annual and the Regional Conventions. All of these sessions have been very well attended and the discussions have been at least as extensive as those at any other technical sessions.

The papers have been on a large variety of subjects such as insulation, transmission, ionization, magnetics, heat convection, transients and oscillations, vacuum tubes and detectors, radio and mathematics.

The committee expects an especially interesting session at the coming Pacific Coast Convention.

It is hoped that our Institute will continue to encourage this work.

F. W. PEEK, JR., *Chairman*.

TELEGRAPHY AND TELEPHONY COMMITTEE

To the Board of Directors:

In this report we are giving a brief summary of the advances which have been made or which have come into prominence in the communication art during the past year. The papers which have been presented under the auspices of this committee have, in general, been a record of such advances, and they are mentioned under the appropriate headings in the report.

TELEPHONE EQUIPMENT

Transformers perform so many and such valuable functions in telephone circuits that the paper on "Telephone Transformers," read at the February Convention, covers a field of the very highest importance. Some interesting special requirements in tele-

phone transformers have been introduced by the comparatively high impedance of many vacuum tube circuits.

One phase of the very important subject of machine switching was presented to the Institute during the year in a paper on "The Economic Development of a Step-by-Step Automatic Telephone Equipment."

Telephone Transmission

A "telephone transmission unit," differing from the commonly used "mile of standard cable," has been adopted for general use by the companies of the Bell System. A paper entitled "The Transmission Unit and Telephone Transmission Reference Systems," to be presented at the June Convention of the Institute describes the unit and the reasons for adopting it.

As telephone circuits, particularly long distance circuits, are necessarily elaborate and complicated, it is very important that convenient and rapid methods be available for checking their transmission efficiency at frequent intervals. A February Convention paper on "Measuring Methods for Maintaining the Transmission Efficiency of Telephone Circuits" discusses this important problem.

In all engineering of communication circuits and apparatus it is important to know accurately the characteristics of the currents and voltages which are being carried by the circuits, whether these be the signaling currents themselves, or interfering currents. A very convenient tool for analyzing currents into their component frequencies was described in a February Convention paper on "An Electrical Frequency Analyzer."

Two papers giving interesting and important discussions of long distance telephone transmission were given at the Pacific Coast Convention under the titles "Telephone Transmission over Long Distances" and "Applications of Long Distance Telephony on the Pacific Coast."

Outside Plant Practises

The Western Union Telegraph Company are employing, after an investigation of about three years, a method of treating existing pole line timber for the purpose of prolonging its life. They consider this method so promising that the apparatus for its application has been well standardized, and the practise of treating poles in this manner extensively adopted during the past year.

In this treatment the earth is first excavated from around the pole to a depth of about 18 in., decayed matter thoroughly cleaned off, and where practicable the pole allowed to air-dry for a few days. The section to be treated is then heavily charred by means of a portable kerosene burner, and while the pole is still hot, creosote that has been previously heated to a temperature between 180 deg. to 212 deg. fahr., applied by means of a high-pressure spray until the temperature of the pole has been lowered to the temperature of the oil, and all vaporization ceased.

The heat of the charring flame thoroughly sterilizes the timber and destroys any existing decay fungi, bacteria or borers; dries the wood and drives out all moisture; opens up checks and cracks to at least as great an extent as will probably occur in ultimate seasoning; heats the wood to a temperature higher than that of the oil so its subsequent cooling during the spraying process will actually contract the air in the cells and draw in the oil; and changes the thin, hard, outer layer of wood into a porous sponge of charcoal which acts as a reservoir to store up a considerable quantity of free oil that will afterwards gradually soak into the underlying layers of wood.

Telegraphy

The development of long telephone cables of 19 and 16 gage, capable of operating for distances of 1000 mi. and upward, has also led to the development of telegraph systems capable of operating through these cables. Two telegraph systems especially designed for the purpose are in commercial operation over such cables. It is expected that papers describing these systems will be presented to the Institute during the coming year.

In telegraphy there are certain definite relationships between the possible speed of operation over a circuit, the frequency range available for the circuit, the alphabet used, and other factors. Some of the fundamentals of these relations were brought out in a paper read at the February Convention, entitled "Certain Factors Affecting Telegraph Speed."

Submarine Telegraphy

The Western Union Telegraph Company expects to complete the laying during 1924 of a radically new type of ocean cable developed by the Western Electric Company. The cable to be laid will extend from New York to the Azores, there to connect to points on continental Europe. It is being manufactured by the Telegraph Construction and Maintenance Co. of London, England. The manufacture of this cable involves the use of a new material and new processes. In order to have a practical confirmation of the results which laboratory tests and theory indicated should be obtained, the engineers of the two companies made the unprecedented experiment of laying a temporary trial length of 120 miles of this cable in a loop from the island of Bermuda, and carried out comprehensive tests over it.

The essential difference between this cable and cables previously laid lies in the provision of a thin layer of "permalloy" placed in the cable between the copper conductor and the gutta percha insulation. "Permalloy" is a recent development in magnetic materials made by the Western Electric Company, consisting of a nickel-iron alloy with about 80 per cent nickel. At small field strengths its permeability is many times greater than that of any material hitherto known.

The layer of permalloy is obtained by putting the alloy in tape form and wrapping it around the con-

ductor. This increases the self-inductance of the conductor per mile—that is, the cable is "loaded." This reduces the attenuation and distortion of telegraphic signals and permits transmission at higher speeds over the cable. The new cable is no heavier in weight or larger in outside diameter than any of the cables recently laid by the Western Union Company, yet its message capacity, as shown by the recent tests, is expected to be fully three times as high as cable of the old type of similar weight and length.

With cable of this efficiency it may become practicable and economical to make considerable increases in the lengths of sections between terminals, thereby cutting down the number of stations where meassges are repeated with obvious operating advantages.

Radio

The great public interest in radio telephone broadcasting has continued to grow during the year, and such broadcasting is already established as a very important facility in our modern civilization. Much remains to be done, however, in improving the quality of the transmission received, and in overcoming interference. Two papers of interest, particularly in connection with these matters, were presented at the February Convention, one on "High Quality Transmission and Reproduction of Speech and Music," and the other on "The Function and Design of Horns for Loud Speakers."

The rapid development of radio broadcasting left the part of the National Electric Code covering the installation of radio sets and antennas in an unsatisfactory form. This has now been taken care of in the new code adopted at a convention of the National Fire Prevention Association in Atlantic City, and going into effect from December, 1923. Since the total of radio installations is now in the millions, the importance of this work is evidently very great.

In last year's report there were recorded tests of radio telephone transmission from America to England carried out in cooperation by the American Telephone and Telegraph Company and the Radio Corporation of America. Following this work, the British Post Office appointed a committee to consider transoceanic telephony, and this committee has now recommended that the British Post Office install at its new Rubgy radio station a 200-kw. transmitter of a type similar to that which was used by the American engineers in their transmission to Europe. If the committee's recommendations are followed, it will result in a station in England which can talk back to America, thus permitting two-way tests between the two countries.

Tests of transmission from America to Europe have been carried out at practically weekly intervals throughout the year, and this data, together with the data which will be made possible by two-way tests, will determine the practicability of a United States to London transoceanic telephone service. Under favorable weather conditions in winter, it is the expectation that telephone subscribers in the United States and in London can be connected so they can talk together as over ordinary telephone circuits. The difficulty of giving a continuous service, however, can be appreciated from the fact that in spite of the long wave lengths used, the amount of power necessary to give satisfactory speech in London at one time may be as much as 10,000 times as great as that required a few hours earlier.

An outstanding development in radio has been the use of short wave lengths of 100 meters and less for transmitting material for rebroadcasting and for other purposes. Under some conditions these shorter wave lengths appear to travel with less loss and less interference than do the wave lengths ordinarily used in radio broadcasting. Under some conditions, however, the short waves appear to involve rapid fluctuations which are not yet completely analyzed and understood. There is not yet sufficient data available to determine just how great the practical value of these short waves may become.

A paper on "Radio Telephone Signaling—Low Frequency System" read at the February Convention describes important developments which have been made in radio calling systems to permit any one of a considerable number of radio stations to be "rung up" without interference with each other, and with a high degree of reliability.

Many forms of selective circuits are in use, or have been proposed for reducing the effects on radio transmission of static interference. A theoretical discussion of the possibilities and limitations of such circuits will be given at the June Convention in a paper entitled "Selective Circuits and Static Interference."

Telephone, Telegrpah and Radio Terminology

The Bureau of Standards has been carrying on active work on the question of telephone, telegraph and radio terminology, which they believe will lead to much more satisfactory definitions of the terms used in the communication art. One feature of this work is that key terms are first defined, such as "telephone," "telephone station," "telephone line," "central office," etc., and then related and associated terms are defined on the basis of their relationship to the key turns. Illustrative examples of definitions made up along these lines have been prepared, and it is expected that a paper on this whole subject will be available for submission to the Institute during the coming year.

Military Signaling

The potential value of radio signals as an aid to the navigation of aircraft at night and in fogs has long been recognized, but the application has been delayed largely because special receiving equipment of considerable bulk and weight seemed to be necessary in the air where both space and lifting power are limited. The Signal Corps of the Army has sought a solution of the problem which might keep all the special equipment on the ground and has during the past year built an

experimental station which has given most satisfactory results. The system in brief, consists of a radio transmitter at a ground station which is coupled to two crossed loop antennas alternately in such a way that double dashes sent by an automatic transmitter are received with equal strength in one vertical plane only through the station. This plane may be given any direction by adjustment of the special coupling device, or it may be made to rotate continuously at the rate of one complete revolution per minute. The former is desirable for aircraft flying over fixed routes. In this case the pilot simply steers a course which keeps the two received signals of equal intensity. The rotating equi-signal plane is better adopted for military operations of aircraft from their base. In this case, a special timing signal is automatically sent out from a non-directional antenna at the instant the equi-signal plane extends due north, and the double dashes follow that timing signal at 10-sec. intervals. A pilot hearing the timing signal, simply counts the signal pairs heard until he hears a pair of equal intensity. If that equality comes in the seventh pair following the timing signal it is evidence that the line joining his position and the transmitting station makes an angle of 7 × 10 deg. = 70 deg. with the North-South line through that station. The direction of rotation of the equi-signal plane is determined by the direction of rotation of the coupling device and is necessarily made known to pilots in advance. No special equipment of any kind is used in the air, the ordinary radio receiving equipment serving for both communication and position finding.

During the past year, the Army has put into service two radio stations, using water-cooled vacuum tubes of 5 and 10-kw. rating, and delivering 10-kw. to the antenna. The transmitters are remotely controlled from receiving stations where both automatic transmission and recorded reception are employed at speeds up to 75 words per minute. These two stations, as well as several others of lower output, are daily handling practically the whole of the War Department telegraph traffic, as well as traffic for several other government departments.

The Signal Corps of the Army reports that it is also actively developing an a-c. system for submarine cable telegraphy, and a uniform code for all forms of transmission, *i. e.* wire, cable and radio. Certain of these developments will be tried out shortly on the Alaskan cable system, and it is hoped that a paper on these results will be made available for the A. I. E. E.

Carrier Telephone Operation over Power Lines

A number of different systems has been developed for permitting the superposition of a telephone channel on an operating power line by employing carrier current methods. Four papers have been presented to the Institute on this subject during the year, three at the California Convention, entitled "Recent Developments in Carrier-Current Communication," "Carrier-Current Telephony on the High Voltage Transmission Lines of the Great Western Power Company" and "Some Experiences with a 202-Mile Carrier-Current Telephone," and one at the Birmingham Convention entitled "Carrier Telephony on Power Lines."

Supervisory Signaling Systems for Power Circuits

Several supervisory signaling systems for the control and indication of remote power equipment in power distribution systems have appeared during the year. With these systems an operator at a central point may know continuously the condition of operating units at one or more distant stations which may be unattended and may control these units as he may desire.

Audiometers

Instruments for measuring the acuity and quality of hearing have been developed. In these instruments a thermionic tube oscillator is arranged to produce pure tones which may be varied in pitch and intensity throughout the range of auditory sensation. The determination of the amount of tone required for hearing as the pitch is varied over the range, gives data of great importance in studying the condition of the hearing of those who are partially deaf.

Transmission of Pictures over Telephone Lines

A demonstration was recently given to the press of the capabilities of a new system for electrically transmitting pictures over long distance telephone lines developed by the engineers of the Bell Telephone System. The sending apparatus was located in Cleveland, Ohio, and the receiving apparatus in New York City, a long distance telephone line connecting these points.

In this system a photoelectric cell, responding to variations in intensity of a beam of light shining in turn through successive elements of the picture to be transmitted, is employed to modulate an alternating current. This modulated alternating current transmits the picture over the telephone line. At the receiving end this alternating current controls a beam of light by means of a device called a "light valve" so that successive elements of a sensitized film are exposed in accordance with the lights and shades in the original picture. The picture to be transmitted and the sensitized film at the receiving end are mounted upon cylinders, these being rotated in synchronism by means of a low-frequency alternating current which passes over the same telephone circuit which transmits the picture current.

The system was shown to be adaptable to the rendering of commercial service. It is possible to transmit not only photographs suitable for reproduction in a newspaper, but also line drawings, printing, and handwriting.

It is stated that the extent to which the system will be used in conjunction with long distance telephone lines will depend upon the demand which arises for this type of service. The system is also applicable to radio

transmission of pictures when atmospheric conditions are such that steadiness of transmission and freedom from interference can be assured.

The Committee is especially indebted to its following members for their activities in obtaining material for this report: E. H. Colpitts, H. W. Drake, R. D. Parker, F. A. Raymond, Edgar Russel, F. A. Wolff.

O. B. BLACKWELL, *Chairman.*

RESEARCH COMMITTEE

To the Board of Directors:

PART I

Activity in the field of electrical research has been very pronounced and has covered a very wide range during the past year. A number of results of outstanding importance may be recorded, although it does not appear possible to single out any one of them as being preeminent over the others.

In the field of molecular physics and the constitution of matter notable progress has been made, resulting in greater certainty as to the nature of the structure of the atom, particularly with reference to the relations of electron orbits, the quantum theory or radiation, and the functions of the electron structure as determining chemical affinities and reactions. Also from the physical laboratories, we have important new data on gaseous conduction at low pressures, and on the production and behavior of electronic emission in high vacuua. The application of these new data has led to great improvements in the construction of vacuum tubes, particularly in the directions of higher power and greater efficiency.

Special mention should be made of the intensive studies which have been devoted to the field of magnetism. Noteworthy papers have been printed on the relation of the magnetic and mechanical properties of steel. Increased knowledge is available as to alloys having high permeability at low flux densities. An especially noteworthy contribution has been made on the importance of carbon, even in the smallest quantities, on the magnetic properties of the iron-silicon alloys and the exact nature of its influence.

There has been unusual activity in the study of the properties of high-voltage insulation. These studies have taken the form of a continuation of tests under the conditions of practise, as for example, in the study of high-voltage cables, and also of laboratory investigation aiming at a better understanding of the underlying processes. Studies in this field, either completed or under way, cover the following variety of topics: The insulating properties of glass as affected by moisture, the pyro-electric theory of breakdown with special reference to paper insulation, internal ionization and its influence on the life of insulation, dielectric absorption in built-up insulation, the importance and exclusion of moisture in built-up insulation, the transmission and dissipation of heat in insulation and reviews of theories of insulation.

In the high-voltage field a number of observations on 220-kv. lines has resulted in further knowledge of the conditions to be met in this important new field. Unusually high values in voltage in both laboratories and on power lines have been the means for further study with particular reference to the question of line insulation. These studies are resulting in a steady extension of our knowledge and in the improvement in the methods for handling higher values of voltage for transmission.

Numerous other problems have been attacked, among which may be mentioned the study of net works of conductors for transmission and distribution, the performance of storage batteries with special reference to impurities in electrolytes, the standardization and accurate measurement of radio frequencies, and the properties of electrolytic rectifiers.

PART II

The committee has had before it as its principal object a thorough review and digest of the literature under the eight headings of the Problem of Insulation, as published in the JOURNAL of the Institute for June, 1923, and which was a joint report of the Committee on Research of the A. I. E. E., and the Committee on Electrical Insulation of the Division of Engineering, National Research Council. The committee has addressed itself first to the securing of volunteers for carrying out this examination of the literature, and to this end has published appeals at various times in several journals, and has addressed letters to a large number of authorities and workers in the field of insulation, always requesting cooperation and assistance.

While the number of volunteers for the work of the examination of the literature has been disappointingly small, a beginning has nevertheless been made in connection with the principle sections of our report. A chairman of a subcommittee has been appointed in connection with each of the sections, and the assignments are as given in the following list:

I Reviews and Compendia of Work Already Done Subdivided among the following sections:

II Nature of Dielectric Absorption
J. B. Whitehead, Chairman, C. A. Adams, Michel G. Malti, H. H. Race

III Phase Difference in Dielectrics
Delafield DuBois, Chairman, LeRoy Clark

IV Electric Strength
W. A. Del Mar, Chairman, F. W. Peek, Jr., G. B. Shanklin

V Dielectric Constant
T. S. Taylor, Chairman

VI Restivity
H. L. Curtis, Chairman

VII Flashover Voltage
F. W. Peek, Jr., Chairman, R. H. Marvin

VIII Theories
V. Karapetoff, Chairman
B. Whitehead

A standard form for recording the results of the examination of a single paper has been prepared and forwarded to all workers. The use of this form should not only facilitate collection of data and the preparation of individual reports, but it is hoped that it will also assist greatly in the collection and coordination of all the investigations on particular topics.

Reports from the several subchairmen indicate that substantial progress has been made, particularly under headings, II, IV, VI and VII. So far as the actual examination of the literature is concerned, the particular value of this work, will be found in the summaries and conclusions under the special headings, which the committee hopes will include statements as to the present problem under these headings with suggestions for the most profitable line of experimental attack.

The committee held a meeting at the recent Philadelphia Convention of the A. I. E. E. at which nine members were present, and at which the various topics in connection with the program were discussed. Of particular interest was the report of Professor Karapetoff of Cornell University of a grant of money from the Heckscher Foundation for experimental work in the field of insulation, and that this committee had rendered him material assistance in the securing of this grant. The committee endorsed the general outline of the work proposed by Professor Karapetoff.

While it cannot be said that the work of the committee is progressing rapidly, it must be remembered that the character of the work now before it is of such a nature as to require time and sustained effort. Surveys in the field of insulation can only be made by experienced and competent men, and under our present plan we are relying entirely on the voluntary efforts of a comparatively small number. All of them are busy men, and with them the work of the committee must necessarily take a subordinate place. The committee is endeavoring to point out that the examination of this literature is a research work of first importance, and that in each case it will constitute a valuable publication. Nevertheless the present method of procedure undoubtedly means that the progress made will be very slow. Apparently it can only be greatly accelerated by the securing of the full time or perhaps the half time of two or three competent men. Obviously, for this purpose financial support is necessary. The work which the committee has outlined cannot properly be done by graduate students or inexperienced men. The committee, therefore, recommends that the Institute consider the advisability of the raising of a small fund for the purpose mentioned, or possibly in some other way secure the more extended services of men competent in this advanced field.

J. B. WHITEHEAD, *Chairman.*

INSTRUMENTS AND MEASUREMENTS COMMITTEE

To the Board of Directors:

The Instruments and Measurements Committee submits the following report, giving briefly a summary of its activities during the past year:

At the Mid-Winter Convention held at Philadelphia, February 4th to 8th, one session was assigned to the committee. The following papers were presented:

1. *Method of Testing Current Transformers*, by F. B. Silsbee. This paper described eleven methods which might be used and gives the advantages and disadvantages of each, together with results which might be expected.

2. *Recent Developments in Kilovolt-Ampere Metering*, by B. H. Smith and A. R. Rutter.

This paper described two suggested types of commercial kilovolt-ampere meters.

3. *Automatic Transmission of Power Readings*, by B. H. Smith and R. T. Pierce.

This paper described seven methods by which meter and instrument readings may be transmitted to distant points and results which might be expected from each.

4. *Quadrant Electrometer for Measurement of Dielectric Loss*, by D. M. Simons and W. S. Brown.

This paper described a new zero method of using an electrometer for testing cables.

This group of papers was well received and an interesting discussion followed.

Standardization of Electrical Measuring Instruments, by H. B. Brooks.

At the annual convention in June, 1923, as mentioned in our previous report, the first draft of specifications for standardization of instruments was presented. The committee has done nothing since that time, preferring to wait for the results and comments covering this first section before going ahead with additional sections. It is expected, however, that in the near future this committee will again actively take up this work with a view of ultimately developing a complete set of specifications on instruments.

Developments. The principal development during the year is probably the announcement of a commercial type of kilovolt-ampere-hour meter. The Westinghouse Company announces that it will have a commercial type meter available during the Summer of 1924. The Committee also understands that the General Electric Company and the Sangamo Company will have models available in the near future.

The tendency of the art toward the use of small indicating instruments as mentioned in last year's report seems to be continuing and we would expect larger demands for the smaller instruments in the future.

A number of operating companies have installed systems of remote metering and totalizing of power readings. The benefits derived by having at one

central point the readings of all power stations is greatly appreciated by system operators and executives. It is anticipated that considerable progress along these lines will be seen in the immediate future.

Bottom-connected meters in polyphase and direct current types of meters are appearing on the market and finding considerable favor. The General Electric Company has announced a new bottom connected d-c. meter and the Sangamo Company a bottom-connected polyphase meter.

The report of I. B. Smith, summarizing the activities abroad in the instrument field is attached. This gives in a very complete and convenient form the summary of the work being carried on in foreign fields and is presented for the convenience of our members and for the sake of its historical value.

G. A. SAWIN, *Chairman.*

Measuring Instruments and Methods 1923

ABSTRACTS OF FOREIGN ACTIVITIES IN THIS FIELD

Ampere-Hour Meter

Ampere-Hour Meters for Alternating Current: Designed to give a linear relation between current and torque over about 90 per cent of the working range of the meter. Accomplished, by adjusting the length of the iron path, constricting it at various places, by adjusting the difference of phase between the fields and, by taking into account the stray field. Meter begins to register at 2½ per cent of full load.
J. Busch, (*Elekt. Zeits.*, Sept. 7, 1922).

Breakdown Voltage

A "Dielectrimeter" for testing the breakdown voltage of insulating oils. It is described as portable and inexpensive.
Empsom Electrical Engineering Co., Ltd., *Journal of the Institution of Electrical Engineers*, June, 1923.

CO_2 Meter

The Econometer CO_2 Tester: This apparatus is suitable for taking snap tests of flue gases. It consists of a cylinder and graduated piston, the gas is drawn into the cylinder and then passed into the absorbent, and the piston is pushed down until the final pressure is the same as the initial. The percentage of CO_2 is read from the graduated rod.
Engineering, April 27, 1923.

Ranarex CO_2 Recorder: An attempt is made to get over the defects of many of the existing CO_2 recorders, *viz.*, the delicacy of their construction, amount of attention required, and length of time for test. The essential parts of the present device are two small fans, run by a motor, and each run at the same speed. In the chamber adjacent to each fan is a disk with vanes so mounted that it tends to rotate owing to the friction of the air or gas set in motion by the vanes of the driven fan. Its tendency to rotate is proportional to the density of the gas in the chamber, and this fact is taken advantage of to determine the percentage of CO_2. The instrument is adapted for testing motor engine exhaust gases, and it is claimed that it has an accuracy of 1 per cent.
Engineering, April 6, 1923.

Electrical CO_2 Recorder: The general principle of the method, which was developed by G. A. Shakespear, of Birmingham University, is that two identical spirals of platinum wire are enclosed in two separate cells, in a metal block, and that each of the spirals is connected to one arm of a Wheatstone bridge, the other two arms being of manganin. Current flowing through the bridge heats the spirals, and causes them to lose heat to the walls of the cells. If the two cells contain gases of different thermal conductivities, the spirals will cool at different rates, and the difference in temperature will cause a deflection of the galvanometer, the extent of which will depend on the difference in the conductivity of the two gases. The construction is such that equal changes in the temperatures of the two gases affect both sides of the bridge equally, so that if one of the cells contains a pure gas and the other the same gas mixed with some other constituent, the deflection will be a measure of the amount of the second gas present, and the galvanometer can be calibrated to show directly the percentage composition of the mixture. In practise one cell is filled with air saturated with moisture, and the other is exposed to the gas under test. The difference in conductivity in the gases in the two cells therefore depends solely on the percentage of carbon dioxide present.
Electrician, Nov. 10, 1922.

Current and Voltage Transformer

Transformers for Measuring Apparatus: The pressure and current transformers for voltmeters, ammeters, relays, etc., are generally a weak point of a plant, as their degree of security against excess pressures and currents is inferior to that of the generators and power transformers, owing to lack of space. The author points out some remedies for this state of things, which can be summarized as follows: (1) Limitation of the number of such apparatus, (2) Increase of their power, (3) Insertion of ohmic resistances in parallel with current transformers and in series with pressure transformers, (4) Loading the secondary of the pressure transformers, (5) Grouping of several transformers in series, (6) Establishment of different series of transformers in relation to the importance of the plant, and (7) Reducing as much as possible the number of the terminals.
G. Campos, *Elettrotecnica*, May 15, 1923.

Compounded Current Transformers: In an ordinary current transformer, the ratio and the angle of displacement vary with the load on account of the varying permeability. In this article the values of the errors involved are fully discussed mathematically. The compounding effect is obtained by providing an exciting winding, having no mutual inductive action on the main windings in order to bring the iron to a condition corresponding to the maximum permeability for the materials, so that the permeability remains practically constant at all loads.

Two methods employed by the Compagnie pour la Fabrication des Compteurs are described. In the first, there are two closed magnetic circuits side by side. The primary and secondary coils are wound over the two adjacent limbs together, while the exciting winding, consists of two equal coils, one on each of the outer limbs, connected so that the flux due to the exciting winding opposes the main flux in one limb and assists in the other. In this arrangement, the exciting coils must be wound by hand, and in order to keep the number of turns low, it is advisable to use a small auxiliary transformer. To obviate this, the method shown in the Figure is used, in which the exciting winding is wound directly on a separate portion of the iron circuit.
Iliovici, *Elect. Soc. Franc. Bull.*, Feb., 1923.

Current Transformers: A new tube type current transformer is advertised for use with instruments and meters.
Chamberlain & Hookham Co., Ltd., *Electrician*, Oct. 26, 1923.

Current Transformer: Designed to give a large number of ranges with precision accuracy in one piece of apparatus. The instrument has five self-contained primary ranges, and is arranged so that by "looping through" the primary winding, ten additional ranges, from 150 to 1800 amperes, may be obtained. This instrument should be exceptionally valuable as a standard for use with current transformer testing sets. It apparently is somewhat in advance of any American instruments in which three self-contained ranges is the maximum number that has been combined with "loop-through" construction.
Elliott Brothers, London, *Electrician*, Jan. 18, 1924.

Calorimeter

A recording and Integrating Gas Calorimeter: The article is a brief description, with illustrations, of C. V. Boys' calorimeter, described by him at the annual meeting of the Institution of Gas Engineers on June 22. The calorimeter is of the water-flow type, the same water being circulated continuously through the apparatus. The fundamental features of the instrument are: water and gas are doled out positively at the correct respective rates, and the correction for gas volume as affected by temperature, pressure, and humidity, are likewise effected by a positive operation. A number of extremely ingenious features are embodied in the design.

J. S. G. Thomas, *Nature*, Aug. 19, 1922.

Capacity and Phase Angle Measurement

Measurement of Small Capacities by the Resonance-Suspension Method: The possibility by means of cathode tubes of producing oscillations of the greatest stability has enabled processes to be devised for the measurement of very small capacities and changes of capacity. Reference is made to the publications of Pungs and Preuner, Herweg, and Hammer on such processes. Each of the latter, as well as the one now described, depends on the principle that a particular oscillation frequency in an electrical system alters with capacity changes, so that the latter may be determined by means of the former. Section 1 deals with procedure; Section 2 discusses mathematically the measurement of capacities by an increase of normal capacity, and derives a formula thereto; Section 3 gives details for the measurement of small self-inductions by the resonance-suspension method.

W. Glitsch, *Phys. Zeits.*, Dec. 15, 1922.

Condenser Leakage

Measurements of Leakage Conductance: A bridge method for testing imperfect condensers which will be useful in technical practise rather than in the high precision work of the laboratory. After describing briefly the usual method of comparing the capacitance and leakage conductance of a condenser with a standard condenser, the author suggests as preferable the use of the Maxwell bridge. In this the imperfect condenser in one arm is compared with an adjustable inductance and resistance in the opposite arm. The two remaining arms contain fixed non-inductive resistances. The standard condenser with its accompanying troubles is therefore eliminated. The resistance of the variable inductance is compensated by a resistance connected in parallel with the test condenser; balance is obtained by adjustment of the inductance and the variable resistance in series with it. Expressions are deduced for the capacitance, leakage conductance, and loss-angle of the condenser, full account being taken of residual errors. Earth capacity troubles are removed by use of the Wagner earthing device. A discussion of the conditions for sensitive working is given, together with a complete diagram of connections of the network. The paper concludes with a description of several bridge arrangements for the direct measurement of the working capacitance of a cable, and also shows how to adopt the Maxwell bridge to enable the working leakage conductance to be determined.

U. Meyer, *Elekt. Zeits.*, Aug. 16, 1923.

Capacity

Capacity of Fine Wire Coils: For purposes of measuring the resistances of coils possessing great self-induction, the usual processes involving bridge arrangements are inapplicable, and an oscillation method has to be adopted. This not only demands special precautions, but is dependent on the availability of the Thomson formula $t = 2\pi\sqrt{LG}$. The method now described gets over both types of complication, and very full apparatus details with illustrative diagrams are included. The principles of the arrangements employed are as follows: (1) A primary coil is coupled so loose with that to be measured that the oscillation frequency of the latter is not appreciably influenced. (2) By change of frequency of the current, the tension in the primary coil is kept constant and the maximum tension in the other coil evaluated. (3) Different supplementary capacities are employed in conjunction with the unknown. (4) The tension measurement at the primary and secondary coils is found by means of a high ohm valve-voltmeter arrangement using a highly sensitive mirror galvanometer. The frequency measurement is made acoustically. Tables of data are given.

E. Marx and A. Karolus, *Phys. Zeits.*, Feb. 1, 1923.

A Simple Compensation Method for Measuring the Capacity and Leakage of Telephone Cables: Capacity and leakage measurements on telephone cables are generally made by Wien's bridge method with Wagner's auxiliary connection, three measurements being required to make possible the calculation of the working value. The method here described gives the working values by a single measurement. It is a compensation method and offers advantages compared with the bridge methods previously proposed for the same purpose, particularly in point of simplicity and accuracy in the hands of relatively unskilled observers. It is shown analytically and by numerical examples that the present compensation method is not appreciably affected by asymmetry in the supply circuit, hence there is no need to take precautions such as are required by bridge methods, to avoid asymmetry. By a method of screening described in the admittance of the leads can be eliminated from the observed result.

J. Kuhle, *Elekt. Zeits.*, Sept. 28, 1922.

Condenser Loss: A small condenser without losses can be measured with considerable accuracy by the method of oscillations. Another equally accurate method applicable only to perfect condensers has been given by Falckenberg, and it is the object of the author to give a modification of the method to enable imperfect condensers to be measured. In principle two resonance circuits are equally coupled to a source of oscillations, the natural frequency and damping of one circuit being capable of adjustment so that the resonance curves of the two circuits for variations of the source frequency can be made to coincide. The condenser to be measured is then introduced into the adjustable circuit and the natural frequency and damping restored to the previous values by changes in the circuit adjustments. From these changes the capacitance and loss resistance of the condenser can be deduced. The paper concludes with a brief discussion of the means of increasing the sensitivity of the method and claims an accuracy of the order of 10^{-5}.

S. Benner, *Zeits, f. Physik.*, 100-102, 1923.

Cable Capacity and Leakage: In a two-core cable with earthed lead sheath, if c_1, c_2 and c_{12} are respectively the earth capacities of the two cores and the Intercapacity between them, the working capacity between the cores is given by $c = c_{12} + (c_1 c_2)/(c_1 + c_2)$, a similar relation holding between the corresponding leakage conductances. The author points out that c cannot be measured in a simple Wein bridge without considerable error, unless the sheath can be maintained at earth potential. Wagner has ensured this by the application of his well-known earth connection to the detector branch-points; three measurements and a calculation then serve to determine c. To reduce the work several ways have been suggested to enable c to be found by a single measurement. By a consideration of the pressure drops in the arms of a Wein bridge, the author devises a new test connection, consisting of a double bridge arrangement, which enables the sheath to be held at zero potential, whether the earth capacities of the two cores be equal or not. Tabulated results of measurements made by various methods are given for purposes of comparison.

E. Wellmann, *Elekt. Zeits.*, May 17, 1923.

Cable Capacity and Leakage: A simple compensation method is described for measuring the capacity and phase angle of condensers and cables. A comprehensive description requires

diagrams. The method has been tested out and is stated to give accurate results.
W. Geyger, *Archiv. f. Elektrot.*, June 30, 1923.

Condensers

New Electric Condenser: The new condenser consists of plates or ribbons of a special quality of celluloid, called cellon, upon both sides of which silver armatures are deposited by a chemical process. The cellon employed can stand 40,000 volts per mm. and has a specific inductive capacity of 4. The dielectric losses are very low, of the order of 1 per cent at 50 cycles. The losses at the edges are reduced to a very low value by surrounding the armature edges by an extremely thin, high resistant, border of silver. In the case of a battery for power-factor improvement, the edge losses are reduced to 0.1 or 0.2 per cent of the apparent power.

The condenser takes the form of a porcelain cylinder closed by metallic caps, so that a number of cylinders can be connected in series by simply superimposing them. Applications of the new condenser are considered. The claim is made that when it is used as a "passing-through" insulator for high pressures, it allows a very efficient and reliable apparatus to be obtained. It is also remarked that, owing to the perfect adherence of the armatures to the dielectric, very silent working is obtained by the use of this condenser in radiotelephony.
E. Pfiffner, *Eletrot.*, Oct. 15, 1922.

Dielectric Loss

Losses in Insulating Materials: A Wattmeter method of measuring dielectric loss at high voltage in which errors due to brush discharge, phase displacement in the wattmeter, etc., can be easily corrected.

The current coils of the wattmeter are earthed and carry the test current. The voltage coils are connected to the low tension side of the transformer in series with a high resistance and an inductance also one winding of a variometer. A specially designed adjustable air condenser of 4×10^2 microfarads capacity is included in the circuit. The phase error of the transformer is compensated when the wattmeter reads zero by adjusting the inductance.

The variometer compensates for the change of mutual inductance in the wattmeter. The special iron core wattmeter has exceptionally high sensitivity.

The paper concludes with an interesting series of characteristic curves for micarta and other insulating tubes showing the losses as related to temperature, time and voltage.
S. Rump, *Brown Boveri Rev.*, Aug., 1923.

Electrometer

String Electrometer: A platinum string of 2 μ thickness is used under variable tension. The electrodes are movable and with different tensions on the string, the range of the instrument is 0.001—1000 volts. There is a nearly linear relation between the voltage and displacement of the string.
C. W. Lutz, *Physik. Zeits.*, April 15, 1923.

Fluid Velocity

Stream Velocity: The relatively low velocities of the cooling media in electrical machines and apparatus may be measured conveniently by a method based on variations in the coefficient of cooling devices exposed to the stream in question. The heat W lost from any body to a stream flowing with velocity v increases rapidly with v, when the latter is small. The value of $d\,W/d\,v$ decreases as v increases, because the particles of the cooling medium have less time in which to pick up heat by conduction. It is most important to know the exact velocity of cooling medium when this velocity is low, and its value is best determined by a method subject to the same laws as the cooling of the machine. The actual temperature of the cooling medium being T_o, the time taken for a thermometer bulb to cool from T_1 to T_2 (both higher than T_o) is a measure of the stream velocity, and the latter can be read directly from an empirical calibration curve. In air at 5 m./sec. a certain thermometer bulb cooled from 80 deg. excess temperature ($T_1 - T_o$) to 30 deg. excess temperature in 25 sec. and this, or a lower velocity could be determined with a high degree of accuracy. In liquids, the rate of cooling is more rapid, but good results can be obtained within the excess temperature range 15 to 5 deg. by enclosing the thermometer in a copper shell with a rubber sheath, to increase its heat capacity and retard the cooling. An electrical method uses a spherical coil of insulated copper wire which is maintained at a constant temperature, say 116 deg. cent., by varying the current through it until balance is obtained in a bridge circuit, the other three resistances of which are constant. The watts required to maintain the coil at constant temperature can be plotted against the velocity of flow of the cooling medium. Typical calibration curves for both the mercury therometer and the electrical methods of measurement are given in the original.
F. Müllner, *Elek. u. Maschinenbau*, April 8, 1923.

Velocity of Flow in Pipes: An electrical effect which can be utilized is the sudden change which occurs in the electrical properties of a fluid when a charge of salt is inserted. A method of measuring consists of noting the time required for such a charge to pass through a measured length of pipe, and determining the time of its arrival at the discharge end or some other point by the jump of the needle of a sensitive voltmeter connected to two electrodes placed in the discharge. Thin sheets of zinc and copper about 6 in. long, about ¼ in. apart, insulated from each other and from the pipe end, were used for poles, and connected to the terminals of a sensitive voltmeter. During ordinary discharge the needle indicated a difference of potential between the poles of about 0.2 volt, but when the salt solution arrived it jumped to about 0.5 volts, dropping again to 0.2 volt as the salt passed. The accuracy of this method was found by comparison to check very closely with the potassium permanganate measurements which were carried on simultaneously.
I. E. Houk, *Engineering*, May 25, 1923.

Fluid Velocity Meter

Velocity of Water: This paper describes experiments made to test the usefulness of a method of measuring water currents at low velocities based on the fact that the resistance to the passage of an electric current between two electrodes immersed in water depends on the velocity of motion of the water. The experiments were made with tubular electrodes and with rods of brass, copper and iron arranged 2 to 10 ft. apart in pipes from 4 in. to 10 in. in diameter. Tests on the various pairs of electrodes showed that the resistance, when measured by an alternating current was independent of the motion of the water in the pipe. It therefore appeared that the changes observed when a direct current is used must be done to the removal or liberation of the products of electrolysis by the moving water. With a voltage below 1.5 volts the instability of the conditions was such that no reproducible results could be obtained. A battery of 4 or 5 cells giving an e. m. f. of 3 or 3.75 volts across the electrodes was therefore used, but even then it was exceedingly difficult to obtain a repetition of any measurement. It is suggested that the phenomena must be of an electrolytic nature depending on the effects of over-voltage and transfer-resistance at the electrode surface. Curves are given showing the results obtained by Cleverdon and by the author.
M. A. Hogan, *Engineering*, Jan. 19, 1923.

Galvanometers

Instability of Differential Galvanometer: In the course of certain experiments in which a differential galvanometer was employed, the author remarked that when the current in the coils exceeded a certain value the equilibrium of the needle became unstable. As a matter of fact if the currents in the two coils were established when the needle was slightly deflected towards the right, the needle was further deflected towards the right; and if it was slightly deflected towards the left, it went

further towards the left. The author studies this phenomenon analytically and compares it to that shown by a balance which, when the load exceeds a certain value, becomes unstable, because its center of gravity is higher than the center of oscillation. In any case, the phenomenon is rendered possible by the coils not being perfectly coaxial.
A. Sellerio, *Elettrotecnica*, Dec. 5, 1922.

Galvanometer for Zero Reading: The telephone is generally more convenient than contrivances giving visible readings; but with thermionic valves to rectify the alternating currents the sensibility and the precision may be made much greater than with the telephone, while the rapidity of measurement is at least equal. This is due to the amplifying effect of the thermionic valve. But a thermionic valve has a high resistance, and requires a high-resistance galvanometer. Much smaller resistances may be used if the thermionic valve be replaced by a crystal-detector, as used in radio-telegraphy; the best is a crystal of argentiferous galena, against which a copper point is pressed by an adjustable spring.
L. Mazza, *Accad. Lincei*, Atti, Dec. 5—Dec. 17, 1922.

Ballistic Galvanometer: Conditions for sensibility of ballistic galvanometers in closed circuits with damping.
A. J. Staring, *Arch. des. Sciences*, Sept.-Oct., 1922.

Einthoven String Galvanometer: The discussion of the theory of the string galvanometer has been in general of too mathematical a character and has not considered many of the cases commonly occurring in practise. The author in this paper gives a short and simple presentation of the theory, taking account of some practical cases which are specially important.
A. Pochettino, *Accad. Sci. Torino*, Atti, 1921-1922.

A New Galvanometer: This galvanometer has resulted from the demand for a very sensitive instrument occasioned by a previous discovery of a new type of thermo-element specially suitable for ultra-red radiation. The principle involved is the pendulum-like oscillation of a thin metallic frame in a magnetic field. The metal is almost non-magnetic and the magnetic field is produced by weak permanent magnets capable of such arrangements as are convenient for producing the required sensitiveness. The paper gives a very complete description of the instrument with five illustrative figures. All galvanometer requirements are claimed to be fulfilled by this instrument.
R. Mechau, *Physik. Zeits.*, June 1, 1923.

Improved Mirror Galvanometer: For bringing about a refinement of the mirror reading, a Fresnel double mirror is used instead of the ordinary galvanometer mirror and the interference fringes read by means of a micrometer. The utility of the instrument is thereby improved enormously. The paper contains a detailed description of the modified instrument and its working.
W. Mobius, *Phys. Zeits.*, May 15, 1923.

Ballistic Galvanometer Sensitivity: In connection with certain magnetic researches the author has been led to investigate the means available for increasing to the optimum the sensitivity of a moving coil ballistic galvanometer working on closed circuit. Diesselhorst has studied the performance of the instrument, assuming all its dimensions to be given, and has worked out the field strength which must be provided by its magnet to give the greatest sensitiveness. The author in the present article investigates, by a slightly simplified analytical solution of the differential equation, the independent effects of the various quantities (dimensions of moving coil, control torque, field strength, constants of the closed circuit, etc.) entering into the equations upon the optimum sensitivity. Details of design procedure to provide optimum sensitivity under stated experimental conditions are given. The paper concludes with the description of an instrument in which the moving coil system can be easily replaced by one suitable for the work in hand. Since the optimum sensitivity depends on the nature of the closed circuit, some means of adjusting the sensitiveness of the galvanometer to suit a circuit of given resistance must be provided. In the author's instrument an electro-magnet is used for this purpose. In common with the observations of other workers it was found that, with the strong fields provided by the electromagnet (1200 gauss), the paramagnetic impurities in the moving coil produced a marked increase in control and a loss of sensitiveness; this effect was overcome by the addition of small pieces of diamagnetic bismuth to the moving system.
A. J. Staring, *Arch. des. Sciences*, Mar.-April, 1923.

Moving Coil Galvanometer: This is a description of a moving-coil instrument with pointer made by the Siegfr. Guggenheimer A.-G. The field is variable by means of a magnetic shunt. Aluminum wire is used for the coil, of which the weight is 0.48 grams. The instrument is for use as a thermoelectric pyrometer with thermocouples.
Gorgas., *Zeits. Elektrochem.*, Sept. 1, 1922.

Loop Galvanometer: It is called a "loop galvanometer" because it has a single loop of wire which hangs in the magnetic field and acts as the moving part. It is air damped and nearly dead beat. The motion of the loop is observed through a microscope, which makes a very compact and portable outfit. The sensitivity is between 3×10^{-7} and 7.5×10^{-9} amp. according to the magnification used in the microscope. It has a resistance of 6-10 ohms.
Robert Mechan, *Physik. Zeits.*, 24-1923.

New Galvanometer: A very interesting instrument, using the thermal-conductivity method devised by Dr. Shakespeare. This instrument is called an "Electrical Micrometer," and is designed to magnify very small deflections of indicating instruments. The magnification is produced by causing the pointer of the instrument which gives but a small deflection to stretch one heated spiral and at the same time to compress another heated spiral in the opposite arm of the Wheatstone bridge. Apparently the elongation and contraction of the heated spirals causes a difference in the thermal emission, and thereby changes the temperature of the spirals. This unbalances the Wheatstone bridge to an extent enabling the detector in the bridge circuit to magnify the initial motion by a factor of approximately 1000.
Cambridge & Paul Company, *Electrician*, Jan. 11, 1924.

Gap Voltmeter

Sphere-Gap Voltmeter: A simple form of sphere-gap voltmeter is described equipped with screw adjustment and vernier. The screw adjustment can be thrown out of action so that the platform carrying one of the spheres can be moved back quickly, leaving the vernier to be read at leisure. The spheres are 3 inches in diameter and a table of voltages for various spark-gaps is given for this particular size of sphere.
E. A. Owen, *Rontgen Soc. J.*, Oct., 1922.

Instrument Design

The Mechanical Design of Scientific Instruments: The scientific instruments here considered are mechanical. The author deals successively with the questions of definite position, turning pair, frictional aberration, the screw pair, backlash friction and variance, accuracy and sensitivity.
A. F. C. Pollard, *Roy. Soc. Arts J.*, Sept. 29, Oct. 6, 13, 1922.
Also *Engineering*, June 9, 16, 23 and 30, 1922.

Magnets

Magnet Steel: This article gives further particulars about the new type of steel, which has been found by the Reichsanstalt to give better results when used for magnets. Weiss and Preuss discovered that an improvement was made by adding cobalt to alloys of iron and manganese: the author shows that an improvement results from a further addition of chromium. The article contains the results of tests on various alloys, hardened at different temperatures.
E. Gumlich, *Zeits. f. Physik.*, 241-252, 1923.

Friction of Pivotal Movements: A description of researches

made with very small pivots (0.9 mm. diameter) working between jewels, such as used in watch-making. Taking the case where the spindle is horizontal, it was found, after studying the influence of speed of rotation, that the results presented the same general character as those of Stribeck (working on spindles several cm. in diameter), who interprets the hydrodynamic theory of Sommerfeld. The coefficient of friction diminishes rapidly at first as the speed increases, passes a known minimum, then immediately increases indefinitely. The viscosity of the lubricant plays a most important part. The results are represented empirically by the equation:

$$f = f_o\ e^{-\lambda\omega} + A\ \omega$$

where f is the coefficient of friction, ω the angular velocity, and f_o, λ, and A constants depending on the bodies in contact.

A. Jaquerod, L. Defossez and H. Mugeli. *Arch des. Sciences*, July-Aug., 1923.

Totalizing Meters

Wattmeter: It is often necessary to sum up in a central position the energy or the currents supplied to, or received from, different parts of a main. This problem presents itself, for instance, if the energy is furnished to an electric railway at different points from the same power station. The author has solved this problem by suppressing the spring of, say, the different wattmeters installed at the feeding-points and mechanically connecting their moving coils with the moving coils of suitable ammeters. For each value of the energy indicated by each wattmeter a given current will be required in the ammeter coil to hold the apparatus in equilibrium. An arm connected to each apparatus can establish a current through a small electric motor, in one sense or the other, so as to increase or diminish the value of a variable resistance and, consequently, the current through the ammeter, and to bring again the apparatus into equilibrium. The currents from the different wattmeters are sent in parallel to the central position through the general ammeter, which can indicate total watts, or through a meter, etc.

Two wires, or a single wire and earth return, are required for connections; these wires can be of small section and even of iron. The same system can be used for the summation of intensity, etc.

G. Campos, *Elettrotecnica*, Oct. 5, 1922.

Megger

Constant Pressure Meg: The "Meg" is a small size megger, and apparently the addition of the constant pressure feature is something new.

Evershed & Vignoles Ltd., *J. Inst. of Elec. Eng.*, Jan., 1924.

Ohmmeter: An insulation testing ohmmeter, similar to the megger type of instrument and somewhat reduced in size.

Record Electrical Co., *Electrician*, Jan. 18, 1924.

Magnetic Measurements

Magnetomotive Force Meter: This instrument, devised by Rogowski and Steinhaus, 1912, consists of a thin, flat, flexible strip of wood fiber or celluloid wound, with the utmost attainable uniformity, with fine insulated wire. The author shows how it may be employed in determining with great accuracy the number of windings in a coil, and if this be known, the number of short-circuited windings, if any. He also shows how its use may sometimes simplify phase-angle measurements.

V. Engelhardt, *Archiv. f. Elektrot.*, Aug. 20, 1922.

Magnetic Flux: It is known that if an electric current flows through a fluid, which is at the same time in a magnetic field perpendicular to the current, the liquid is subjected to a force at right angles both to the current and to the field. The author has imagined a permeameter and a fluxmeter based upon this phenomenon. Both apparatus are essentially formed of a German silver cylindrical helix comprised between two coaxial cylinders of insulating material, the helicoidal space between the turns being filled with mercury. Two metallic disks form the basis of the cylinders, which are disposed horizontally. Two vertical glass tubes filled with colored alcohol form the natural continuation of the mercury helix. The current flows parallel to the axis of the cylinders, alternatively through the German silver and the mercury. The magnet, the flux of which should be measured, is introduced inside the cylinders. The force acting on the mercury produces a difference in the levels of the two alcohol columns. The sensitiveness obtained was 20 Maxwells per mm. of level difference of the alcohol columns, when the apparatus carried a current of 10 amps. per sq. cm.

Hysteresis Measurement

Magnetic Hysteresis: Many investigations have been carried out on rotary magnetic hysteresis, yet the recent results show great discrepancies. This inconsistency is thought to be due chiefly to the systematic errors of various methods of measurement. Taking the irregularity of the electromagnetic field brought into existence by the unsymmetrical forms of test pieces and Joule's effect as the chief disturbing factors, the author describes a comparatively accurate method of measuring the torque of a solid spherical test piece in a rotary magnetic field and the correction of Joule's effect.

T. Miyazaki, *Inst. El. Eng.*, Japan, Nov., 1922.

A Simple Method for Determining Hysteresis Loss; with Notes on Relations between Magnetic Constants: The author has investigated and improved upon a formula originated by Anderson and Lance for the calculation of hysteresis loss from the maximum induction and the corresponding coercive force. As given by Anderson and Lance, the formula is: $W_h = a\,B_m\,H_c$; where W_h = hysteresis loss in ergs/cm.^{3}; B_m = maximum induction; H_c = coercive force; and $a = 0.2133 + 0.01082 \times 10^{-3}\,B_m$. In this form, the expression for W_h is not applicable to high values of induction because when saturation is approached W_h and H_c become constant, but B_m and a continue to increase. Also, due to differences between materials, the hysteresis loop broadens to a greater or less extent in the neighborhood of the knee and the value of W_h may vary widely though H_c and B_m be the same.

The author's tests on dynamo core plates show that a does not vary linearly with B_m, but rather according to the equation: $a = 0.225 + 0.000889 \times 10^{-3}\,B_m + 0.000861 \times 10^{-6}.\ B_m^2$. Values for the coefficient a up to $B_m = 18{,}000$ are given in the original. The improved formula is sufficiently accurate for purposes of approximate estimates. No single formula can be applicable to both hard and soft material.

The following relations, determined experimentally, are used in the Reichsanstalt. For normal magnetization curves of uniform material, the remanence of which lies between 5000 and 12,000: μ max. $= k\,B_r/H_c$ very nearly; where μ max. = maximum permeability; B_r = remanence corresponding to a high magnetization; H_c = corresponding coercive force; and $k = 0.476 + 0.00568\,H_c$. For the smaller values of H_c, it is permissible to take $K = 0.5$. The field strength corresponding to maximum permeability is approximately 1.3 H_c. From these expressions it follows that the induction $B\ \mu_{max}$. corresponding to maximum permeability is given by: $B\ \mu_{max} = 1.3\ (0.476 + 0.0057\ H_c)\ B_r$; and can be determined without plotting the actual permeability curve. For soft materials it may be taken that $B\ \mu_{max} = 0.62\ B_r$ but for cast iron and hard steel, the complete formula must be used. Comparisons between calculated and observed values for $B\ \mu_{max}$ show that the formula gives results accurate within $\pm$ 5 per cent (often considerably closer).

E. Gumlich, *Elekt. Zeits.*, Jan. 25, 1923.

Meter Testing

Portable Testing Equipment: A full description is given of the construction and use of a portable testing equipment used by the Lech Electricity Works, Augsburg, for calibrating high-tension meters. The equipment includes a standard three-phase meter for unbalanced loads, a precision, dynamometer-type, three-phase wattmeter, a voltmeter, ammeter and rotary field indicator, and

sets of precision current and pressure transformers. By alternately short-circuiting the coils of the three-phase watt-meter, two readings are obtained from which the power factor of the load can be calculated, provided that the load is nearly balanced. The connections of the test circuits are shown in the original and data are given from a typical test.
D. Freyer, *Elekt. Zeits.*, Feb. 8, 1923.

Oscillograph

Braun Tube: This paper gives details of a graphical method of obtaining Cartesian curves from the figures obtained with a Braun tube whose field plates are shunted across an inductance and resistance in series in an alternating-current circuit, the magnetic field coils being in series with the inductance and resistance. A numerical example is given for the case of a 500-cycle alternator.
G. Joos and E. Mauz, *Jahrb. d. drahtl. Tele.*, April, 1922.

Braun Tube: A description is given of a high-voltage cathode-ray oscillograph which is employed in the study of a-c. wave-forms of all frequencies up to 10^7 cycles per sec. The oscillograph is essentially a development of the Braun tube, but is adapted for photographic recordings as well as for visual observation. Methods of generating a true axis on the records are described. In the case of very low frequencies a mechanical method of translating the film is sufficient, but at higher frequencies the inertia factor becomes too serious, and it becomes necessary to resort to electrostatic and electromagnetic means of deflecting the pencil of cathode rays rapidly across the film. Excellent reproductions of records are made at all frequencies up to 220,000,000 per second. The oscillograph has obvious applications to high-frequency research as in wireless telegraphy, where the ordinary forms of oscillograph are quite useless.
A. Dufour, *Onde Elec.*, Nov., Dec., 1922 and Jan., 1923.

The Cathode Ray Oscillograph: The paper deals with a new form of cathode-ray oscillograph adapted for commercial production and laboratory use. The instrument described is of the low-voltage type, in which a hot cathode is employed as a source of the electron current. This low-voltage type of oscillograph is much more sensitive than the high-voltage cold cathode type designed by M. Dufour. Various methods are described for focusing the cathode-ray stream, and a proposal has been made for an oscillograph with external (*i. e.*, outside the vacuum) photgraphic film. Experiments have been made to determine the most suitable photographic film or plate. Ordinary gelatine coated roll films or plates are unsuitable, owing to the marked absorption of the cathode-rays by the gelatine. The best results have been obtained with Schumann plates containing a proportion of calcium tungstate. This material phosphoresence with a light rich in ultra-violet, and consequently the secondary luminous effect on the Schumann plate is very great.

Mechanical, electrostatic and electromagnetic methods are described for generating a time-axis on the records. For certain purposes this time-axis is sinusoidal, whilst for others it is linear. Numerous records of high-frequency A. C. wave forms and of impulsive electrical phenomena have been obtained, and a few of these are reproduced in the Paper. Brief reference is made in conclusion to the applications of the oscillograph to research and electrical engineering problems, where other well-known forms of oscillographs (Duddell, Einthoven, etc., types) cannot be employed on account of the inertia of the moving element.
A. B. Wood, *Physical Soc. of London*, Feb. 15, 1923.

Oscilloscope

Oscilloscope: This instrument, called an "Ondoscope" by the author, is an oscilloscope so mounted that it can be rotated about an axis parallel to the tube and thus used for obtaining the "current-time" curve of a high-tension discharge apparatus; and, if the applied voltage exceeds the ionizing voltage of the oscilloscope tube, is suitable for any alternating or fluctuating current. It can be used on circuits from 200 volts up to 250,000 volts or more, and will give the current wave-form when the current has a maximum value between 2 milliamperes and 50 or more.
F. L. Hopwood, *Rontgen Soc. J.*, Jan., 1923.

Peak Voltmeter

Peak Voltmeter: The method described consists essentially of introducing a fraction of the high potential to be measured into the anode circuit of a triod valve and varying the grid potential till the anode current vanishes. A known anode potential serves to calibrate the arrangement. The fraction of the high potential is obtained from a static potentiometer consisting of four equal and parallel plates arranged with their centres in line. The high potential is applied to the two outer plates and the fractional potential obtained from the two inner ones.
A. Hund, *Jahrb. d. drahtl. Tele.*, May, 1923.

Potentiometer

A-C. Potentiometer: This paper describes an a-c. potentiometer which measures two rectangular components of the voltage under test. The instrument contains two elements; one, called the "in-phase" potentiometer, is supplied with alternating current through a sensitive reflecting dynamometer, the readings of which can be standardized by means of direct-current and a standard cell. The second, or "quadrature" potentiometer is supplied with current in quadrature with the preceding; this element contains the primary of a mutual inductance, the secondary of which can be connected to the potential tappings of the first potentiometer through a vibration galvanometer. The mutual inductance is of such a value that, with the "in phase" potentiometer tappings at a chosen reading, balance can be secured by adjustment of the rheostat in the circuit of the "quadrature" potentiometer; the latter is thus indirectly checked against the dynamometer. With both elements standardized in this way a given voltage can be measured by connecting it to the instrument through a vibration galvanometer, the potential tappings of the two elements being joined in series therewith. By operation of the two elements and of phase-reversing switches, if necessary, balance can be secured between the unknown voltage and the sum of its two quadrature components tapped off the instrument.
D. C. Gall, *Electrician*, April 6, 1923.

Thermocouple Potentiometer: A thermocouple potentiometer expecially designed for measuring the electromotive force of thermocouples.
Siemens Brothers & Elliott Ltd., *J. Inst. Elec. Eng.*, Aug., 1923.

Precision Potentiometer: Five dials with a total resistance of 11,000 ohms are employed. Using a current of 0.1 milliampere, the range is 1.1 volts which can be read to 10^{-5} volt, with a current of 1 milliampere the range is 11 volts which can be read to 10^{-4} volt. Using a voltage divider or multiplier, the measuring range can be raised to 110 or 1100 volts when using a current of 0.1 milliampere in the potentiometer. With the corresponding shunt resistance the range can be decreased to 0.11 or 0.11 volts. Three of the dials have double brushes which make contact so that the two resistance coils between every second block of the dial are in parallel with the entire resistance in the next lower dial. In this way each position on the lower dial represents 1/10 the voltage of that obtained on the higher dial.
Zeit. f. Instrument, 43, 288, 1923.

Power Factor

Electrometer Method: The paper proposes null methods of using a quadrant electrometer to enable measurements of power factor and effective resistance in a-c. circuits to be made. Such measurements have hitherto been almost exclusively made by deflectional use of the electrometer; it is then necessary to calibrate the instrument and to take account of the influence of electric control on the needle. If the voltages on needle and quadrants are arranged so that the deflection is reduced to zero, the instrument need not be calibrated, and the electric control has no effect. Such a null use of the electrometer possesses marked advantages due to simplification of theory, to the fact that steadi-

ness of voltage is not so essential as in deflectional use, and to the usual general merits of null methods.

Two methods of connection of the instrument to attain the desired object are described. The theory is worked out and practical tests are recorded. It is also shown how the methods can be adapted for use at high voltages. The theory of the electrometer is worked out with a view to investigations of the effect of electric control and to emphasize the advantages of the null use of the instrument. In the discussion, various speakers brought out practical points arising in the use of the methods. E. H. Rayner drew attention to important details in the design of electrometers. A. Rosen pointed out the need for great constancy of frequency in tests of condenser power factor.

D. Owen, *Phys. Soc. Proc.*, April, 1923.

Phase Meter: The purpose of this paper is to describe the action of several instruments made by Hartmann & Braun for power factor and allied measurements. The well-known phase meter for use in a single-phase circuit is first described, in which a uniform rotating field is produced by stationary coils displaced by 90 deg. in space and carrying currents in quadrature. Freely pivoted in this field is a coil supplied with current from a current transformer connected in one of the lines, this moving coil setting itself in such a direction that its angular displacement from an initial line is equal to the phase-angle of the circuit. In the form described in the paper the rotary field is produced by an ironclad stator producing two component fields displaced 120 deg. in space and 60 deg. in time, the windings being excited from the mains, one having a resistance and the other a choker in series. The moving coil is fed from the secondary of a current transformer. The influence of the distortion of the rotary field, arising from changes in the angle of phase displacement between the component fields and from the effects of the iron magnetic circuit, upon the scale of such instruments is discussed. By provision of a suitable elliptic field a scale about 65 deg. long covering a range of power factor from 0.4 — 1 — 0.4 can be produced, a recording phase meter embodying this feature is described.

A phase meter for polyphase networks is described in which current is led into the moving coil from a special transformer, the secondary of which is pivoted coaxially with and rotates together with the moving coil, thus eliminating any need for leading in ligaments to the moving system; a scale covering 360 deg. is thus secured, since the coil can make a complete revolution, and thereby indicate a change of phase rotation or a phase reversal. The application of the principles embodied in these instruments to power-factor meters for unbalanced loads, is given. Instruments for the measurement of idle power ($E\,I \sin \phi$), wattless current ($I \sin \phi$), and for the indication of phase equality in the paralleling of alternators are described.

F. Voller, *Elekt. Zeits.*, April 5, 1923.

Pyrometer

Optical Pyrometer: Employing an incandescent lamp the filament of which is placed in front of an opening in the furnace, is described, with the methods of obtaining a balance in the portions of the field of view illuminated by the lamp and by the furnace. Experiments to determine the effect of the distance between the pyrometer and the furnace are described, and it is shown that this distance can be increased to 2 or even 3 meters without loss of accuracy. The effect of dust and smoke in the air between the furnace and the pyrometer is considered, and experiments described on the effects produced by water-vapor and CO_2 in this space. There is very little physiological effect as between different observers.

U. Retzow, *Zeits. Vereines deutsch. Ing.*, Feb. 24, 1923.

Resistance Measurements

Contact Plugs: When using the Thomson bridge to measure small resistances in machines, cables, or other cumbersome equipment, the source of current and the bridge with comparison resistance and galvanometer are mounted on the test bench, and four leads are run to the unknown resistance, two for the main circuit and two for the bridge. If circumstances do not permit these leads to be connected in a permanent manner, the connections may be broken; or made in incorrect sequence, so that the galvanometer is subjected to a dangerously heavy current. This risk is eliminated by the special keys described. A copper pin 2 mm. in diameter is surrounded by, but insulated from, a brass tube of 4 mm. internal diameter and 1 mm. thickness. A spiral spring presses the tube forwards so that it makes contact first when the plug is applied to the terminal of the unknown resistance. As the plug is pressed down, the brass tube recedes and thus allows the terminal to come into contact with the end of the internal pin. In one of the plugs, the internal pin is itself spring-supported and provided with an insulated head which, when the plug is pressed to its full extent, closes a key in the galvanometer circuit. The connections are such that when the first plug is applied the main circuit (connected to the pin) cannot be closed until the bridge lead (connected to the outer tube) has been connected. The second plug makes the bridge circuit, main circuit, and galvanometer connections in the order stated. Disconnections are automatically made in the reverse sequence. The voltage drop at the main circuit connections does not enter into the measurement when these keys are used. The resistance of the plugs and their bridge leads is generally negligible. It has been found that, however carelessly the plugs are used and whatever the form of the surface to which they are applied, the connections are made and broken in the correct sequence.

H. Schering, *Elekt. Zeits.*, Jan. 4, 1923.

Non-Inductive Resistance Coils: The authors consider the theory and design of a double-wound non-reactive resistance coil, and give an example of a 100,000-ohm coil, having a phase angle less than 5 deg. at a frequency of 5000.

H. Nukiyama and Y. Shoji, *Inst. El. Eng.* Japan, J., Aug., 1922.

Manganin: The resistivity and temperature coefficient of copper-manganese alloys, and the effects of other elements, such as nickel, iron, aluminum, and silicon, are investigated. The authors find that a straight copper-manganese alloy gives the best manganin, and that there is no need to add other elements to improve the resistivity and temperature coefficient. The alloy containing 13 per cent manganese has a specific resistance of 45 michrons per centimeter cube, the temperature coefficient at 22.5 deg. cent. being 0.00000348.

S. Kimura and K. Sakamaki, *Elekt. Lab.*, Tokyo, Sept., 1922.

Effective Resistance: The Wien bridge method of measuring the effective resistance of an inductance coil requires a knowledge of the losses in the condenser. The author describes a method which allows the losses of the inductance coil and condenser to be separated by making use of two different forms of bridge. The inductance and capacity are first joined in series in one arm of a bridge, the other three arms of which are non-reactive resistances, one of the adjacent arms being adjustable. Either the capacity or frequency must be finely adjustable. A balance having been obtained on this bridge, another bridge is formed in which the inductance coil and condenser, unchanged in values, become the opposite arms, the other two arms being non-reactive resistances, of which one must be adjustable. The results are obtained in the form $R_L = 1/2\,(R' + R'')$ and $R_c = \frac{1}{2}\,(R' - R'')$, in which R_L is the effective resistance of the inductance, R_c is the effective resistance of the condenser, R' is the apparent resistance of the inductance in the first bridge, and R'' that in the second bridge.

H. V. Higgitt, *Electrician*, Feb. 2, 1923.

Recording Instruments

"C. G. S." Relay Graphers: This paper describes recording instruments working on the relay principle which embody some original features. The instruments are made by the officine Mechanische Italiane. They employ a relay motor, controlled by the movement of the measuring system, to operate the record-

ing gear and at the same time to restore the moving system to its equilibrium position. In the "C. G. S." recording wattmeter the moving system consists of two coils, one mounted on each of two arms projecting from a pivoted spindle; each moving coil lies within a fixed current coil. The two current coils and two potential coils are connected to a three-phase circuit according to the well-known two-wattmeter method of measuring power. Electrodynamic action between the fixed and the moving coils deflects the latter through a small angle and causes an arm carried by the spindle to make contact with one or other of two pins, thereby making the circuit of a small relay motor. The spindle carries at its lower end a flat spiral spring, the relay motor being geared thereto in such a way that a torque is put on the spindle to oppose the deflecting torque and thereby to restore the moving system to its mid-position. At the same time the motor traverses a recording pen over the chart mounted on a clock-driven drum.

This principle is also applied to a recording phase-meter, this instrument being a recording wattmeter with the restoring spring removed. The moving system is now energized from the rotor of a phase-shifter geared to the relay motor, the stator of the phase-shifter being fed from the three-phase supply. The fixed coils are connected one in each line. The apparatus is arranged so that the relay motor turns the rotor of the phase-shifter until the voltage impressed on the moving coils is in quadrature with the current in the fixed coils; there is then no torque on the moving system, which will stand in its mid-position. If the power-factor varies, one or other of the relay contacts will be closed and the motor will operate the rotor of the phase-shifter so as to restore the moving system to zero; at the same time a recording pen is traversed over a chart calibrated in power-factor.

By combination of a phase meter with a recording voltammeter a record of power-factor and reactive voltamperes can be secured on an unbalanced three-phase system. To meter the kv-a. and the maximum demand a phase-meter can be used in combination with a watt-hour meter and a maximum demand indicator. The instruments have a high torque, are free from friction error, and have very great sensitiveness.

Electrician, Nov. 3, 1922.

Remote Indicator: This is an instrument designed in accordance with the Fawssett patent. This instrument is designed to indicate or record at a central station, the voltage, current, power or power-factor at a remote station, which may be many miles away.

Cambridge & Paul Instrument Co., Ltd., *Electrician*, Dec. 28, 1923.

Machine Timing: A special type of recording instrument which comprises a recording ammeter together with an operation recorder. The object of the arrangement is to provide a record of the timing of one or more operations of a machine or group of machines simultaneously with a record of current or power consumption.

Evershed and Vignoles, *Electrician*, Jan. 18, 1924.

Ionization Recorder: The photographic registering drum is mounted on the same shaft as the switch, which regulates the voltage between the incandescent cathode and the grid, accelerates the electrons, and produces the velocity which ionizes or excites the molecules of the gas under investigation. Light from the mirror of the galvanometer, which measures the ionization current, or other current produced in the tube, falls on the photographic film, the deflections being along the length of the drum, or at right angles to the displacements due to the voltage changes. A device is employed in which a small additional lamp is lighted at regular intervals, and prints a series of lines on the film, parallel to the axis of rotation, corresponding to equal increments of voltage. This enables the curves taken to be measured with accuracy. The galvanometer lamp can be automatically switched on and off in a regular manner during a run; so that the line recorded is dotted in a distinguishing manner, and a number of curves may be taken on the same film, and easily identified. It is possible to shift the apparatus parallel to the axis, so that the different curves taken are separated from one another. The shaft can be driven at constant speed, which can be varied within large limits, by means of a special motor, and special switching arrangements were made so that measurements could be carried out in the darkened room, which could only be lighted by means of a red lamp, which did not affect the photographic film. The following investigations have been carried out with this apparatus; ionization and excitation of light in helium and in hydrogen (not as yet completed); ionization of the halogen compounds of hydrogen, and of complicated compounds such as acetic acid; these last have as yet given no definite values, but show the enormous influence of small quantities of water-vapor; ionization of water-vapor (incomplete); exit work of the electrons from different incandescent wires.

P. Knipping, *Zeits. f. Instrumenten.*, Aug., 1923.

Stress and Strain

Electric Telemeter: The authors describe a form of instrument used for measuring rapidly varying pressures, stresses, and strains, and recording these at a distance. The principle is that of the variation with pressure of the resistance of a stack of carbon plates. A method has been devised for mounting these in such a way that the resistance is stable and free from hysteresis. By using two stacks in a Wheatstone bridge arrangement it has been found possible to give the instrument a linear characteristic. The carbon plates are mounted in a steel frame cut from a solid piece of metal, and they are mounted under a certain definite pressure to ensure stability of resistance. In the actual instrument described annular rings are used instead of plates, about 50 being generally employed. The change of resistance found is of the order of 46 per cent for a change of length of the stack of 0.00217 in. The two stacks mounted in the Wheatstone bridge are so arranged that the resistance of one is decreased and the other increased by the force or displacement in measurement. In use, the bridge is first balanced. Then as change takes place in the resistance of the carbon stacks the balance is upset and the bridging instrument deflected. As long as the current taken by the bridging instrument is small in comparison with the total current in the bridge circuit, these deflections will be proportional to the total change of resistance of the stacks, and the latter being proportional to the force or displacement under test, the ultimate result is a uniform scale reading of the bridging instrument. This enables calibration directly in thousandths of an inch in the case of strain gages, or total load or pounds per square inch in the case of stress or pressure gages. The leads from the carbon stacks to the other portion of the bridge circuit may be of almost any length, some of No. 16 copper wire, 100 ft. long, having been used successfully. This enables simultaneous readings or records to be obtained at a number of widely separated points by a single operator, the fixed resistances and bridging instruments being combined in one suitable panel. The instrument has found a number of different applications. In addition to measuring deformations it may be used as a strain indicator for rigid members, and the necessary modifications for this purpose are described in the paper. An example is given of the results obtained by two separate instruments for measuring the longitudinal compression and the transverse tension of a member at various loads.

O. S. Peters & R. S. Johnston, *Engineering*, 116 pp. 253-254, Aug. 24, 1923.

Inductance

The Thomson Bridge with Alternating Current: This article deals especially with the use of the Thomson bridge for measuring the small inductances of non-inductive shunts. The method can be also used for determining the transformation ratio and the phase-angle between the primary and secondary currents in current transformers. The theory of the method and the results obtained for some shunts are given.

E. Biffi, *Elettrotecnica*, Aug. 5, 1923.

Steam Flow Meter

The Metering of Steam: An account is given of the meters developed by the author and his firm, George Kent, Ltd. These depend upon the pressure difference set up when steam flows through an orifice in a pipe. The theory underlying the instruments is given, and the paper is accompanied by numerous diagrams.
J. L. Hodgson, *Inst. Naval Architects Trans.*, 195-202, 1922.

Synchronizers

Synchronizing Voltmeters: Some recent improvements in the devices for synchronizing alternators are described in this article. After recalling the study of Styff and some devices of Siemens, the author briefly describes a device imagined by him and used in Italy during the war, when, owing to coal shortage, it became necessary to connect power stations in parallel so as to relieve one another. This device is characterized by its simplicity and low price. It consists essentially of a two-coil voltmeter, each coil being fed by a phase of the three-phase system, the connections being taken to opposite sides of the paralleling switch. The coil connected to the leading phase is in series with a choke producing a lag of 60 deg., while the other coil is in phase with a suitable ohmic resistance. The paralleling switch can be closed when the voltmeter passes through zero.
B. Usigli, *Elettrotecnica*, Sept. 5, 1922.

Tachometer

Measurement of Speed of Rotation by a Stroboscopic Method: A beam of light passes through a hole in a disk attached to a string and illuminates a white disk with a radial black line, on the end of the rotating shaft. When the string vibrates the first disk moves to and fro in its own plane, and the second disk is only illuminated when the hole passes its position of rest. The tension of the string, and its vibration frequency, can be regulated by turning a graduated drum, until the black mark on the second disk appears to be at rest, owing to persistance of vision; and the graduations can be arranged so as to give the speed of rotation.
A. Guillet, *Comptes Rendus*, May 22, 1923.

Temperature

Relative Effects of Radiation and Convection: The rates of cooling of two large alcohol thermometers, one of which has its bulb silvered and polished, are observed under the conditions to be tested. Combining these with the rates of cooling of the unsilvered thermometer when enclosed in a high vacuum and in still air surrounded by an enclosure at the given temperature, the relative effects of radiation and convection are determined.
L. H. Nichols, Roy. Soc. Canada, *Trans.*, 333, 1922.

Thermometer

Bolometer: The conditions for realizing industrial electric thermometers of the bolometer type are theoretically studied, and two types of such apparatus as constructed by the firm of Allocchio, Bacchini & Co., are described and illustrated. The first type requires adjustment of the arms of the Wheatstone Bridge: the second type is of the direct-reading type. A special three-wire connection is required between the variable resistance and the system of the bridge resistances, battery and ammeter in order that the resistance of the connecting wires may be neglected. The constancy of the e. m. f. of the battery can be verified from time to time by inserting a given resistance in place of the variable one. This is made of nickel wire and, like the other resistance of the bridge, is of the order of 100 ohms. The scale of the direct-reading ammeter is practically uniform and is marked in degrees. These thermometers are employed for determining at a distance the temperature of transformer oil, dynamo bearings, etc.
C. Bacchini, *Elettrotecnica*, Oct. 5, 1922.

Temperature Bridge: The bridge is a temperature difference instrument designed for use with resistance thermometers. It apparently employs a shunted-decade arrangement, similar to that used in the Mueller bridge.
Dr. F. E. Smith, *Electrician*, Jan. 11, 1924.

Recording Pyrometer: A multiple-point temperature recorder, operating on the millivoltmeter principle, and using several colored inks for identifying the various records.
Siemens Brothers & Co., *Electrician*, Jan. 18, 1924.

Ammeters and Voltmeters

Voltmeter: A direct-reading Thermionic Voltmeter. The feature of this voltmeter is that it is arranged for measuring very low values, such as one or two volts without absorbing any appreciable power from the circuit of which the voltage is being measured. The instrument is apparently in commercial form, and appears to be of the type manufactured by Cambridge and Paul. The instrument utilizes a uni-pivot direct-current galvanometer, in conjunction with a vacuum tube rectifier.
E. B. Moullin, *J. Inst. Elec. Eng.*, Feb., 1923.

Avometer: A portable electrical measuring instrument designed to measure current from .001 to 12 amperes, voltage from 0.1 to 600 volts (or higher if required) and resistance from zero to 5000 ohms. It is claimed that this instrument is direct reading, no calculations being necessary, but no explanation is given as to just how direct readings of resistance are obtained.
Automatic Coil Winder and Electrical Equipment Co.,
J. Inst. Elec. Eng., Jan. 19, 1924.

Switchboard Meters: A series of instruments for mounting on industrial switchboards. The instruments are of the moving iron type, and the feature of the instrument seems to be that while front connections are used, these connections are completely enclosed in a separate chamber.
Electrical Apparatus Co., Ltd., *J. Inst. Elec. Eng.*, Jan., 1924.

"Cirscale" Meters: A series of circular-scale indicating instruments under the name "Cirscale." These are apparently direct-current instruments having a scale almost 300 deg. in length.
Record Electrical Co., Ltd., *J. Inst. Elec. Eng.*, Feb. 19, 1924.

"Oval" Meters: Permanent magnet moving coil instruments for switchboard mounting. This apparently has been designed to fit in with a similar line of "Oval" instruments which has been in use for many years on a-c. circuits. The instrument probably finds its greatest demand for switchboard mounting where space is limited.
General Electric Co., Ltd., *Electrician*, Feb. 1, 1924.

Ammeters and Voltmeters: Measuring instruments with their accompanying resistances arranged in a side compartment so as to be self-cooling, the air being drawn in downwards around the periphery and rejected upwards through the central part.
Nalder Bros. & Thompson, *Electrician*, Jan. 18, 1924.

Small Current Meter: A device is described for use in connection with the compensation method of measuring currents of 10^{-9} ampere or less. It consists essentially of a current divider, which divides a variable current in a ratio very large compared with unity. The small current is used for compensating the unknown current; the large current is read in a relatively insensitive instrument. The device consists of a thermionic cathode and an anode pierced by a small hole. The large current is that received by the anode; the small current is the fraction passed through the small hole in the anode. The currents are varied by changing the temperature of the thermionic cathode.
General Electric Co., Ltd., *J. Scientific Instruments*, Nov., 1923.

Volt-Ampere Meter: A knowledge of the consumed volt-ampere hours is required in devising an accurate method of charging for the energy supplied to an a-c. circuit. The paper gives a simple two-wattmeter method of measuring the volt-amperes consumed in a three-wire, three-phase circuit with balanced load. The current coils of the two instruments are connected in lines 1 and 2 respectively. The voltage circuit of the first wattmeter is joined across lines 2 and 3, and has a total

resistance of r_1 ohms. The voltage circuit of the second instrument is connected between lines 1 and 2; its total resistance can be varied by means of a sliding contact on the voltage coil series resistance to a value $r_1 - x$. The total power indicated by the two instruments is

$$W = 3\,E\,I\left\{\frac{\sin\phi}{\sqrt{3}} + \frac{1}{\sqrt{3}}\,\frac{r_1}{(r_1 - x)}\cos(30 - \phi)\right\}$$

By adjustment of x the bracketed factor can be made equal to unity for a given value of ϕ, and the sum of the readings of the two wattmeters will be the total volt-amperes. The variation of x with $\cos\phi$ and $\sin\phi$ is traced. It is shown that x is zero when $\phi = 60$ deg., and that its variation with $\sin\phi$ can be taken to be nearly linear reckoned from that point. Some form of ϕ or $\sin\phi$ indicator can be utilized to actuate the sliding contact and to give automatically the correct value of x for various power factors. Assuming the linear variation, the instruments give substantially correct readings of volt-amperes for values of $\cos\phi$ exceeding 0.2, the maximum error being 2.4 per cent.
C. Breitfeld, *Archiv. f. Elektrot.*, May 28, 1923.

Corona Voltmeter: Corona occurs at a lower voltage when the wire is negative than when it is positive. If F be the voltage gradient at the surface of a wire of radius a, then $F = A\,\theta\,(1 + B/\sqrt{\theta\,a})$, where A and B are constants and θ the density of the gas in which the discharge takes place. If the wire be surrounded by a coaxial cylinder of radius b, then the voltage gradient at a point distance r from the common axis is $E/(r \log_e b/a)$, so that corona will occur when

$$E = a \log_e b/a\left[A\,\theta\left(1 + \frac{B}{\sqrt{\theta\,a}}\right)\right] \quad .$$

By suitably choosing a and b, the discharge can be limited to corona, sparking being prevented. It is found that rods or tubes are preferable to wires, as the diameter may be considerable. Two methods can be employed for measuring corona voltage. The first, which is rather expensive, is to mount cylinder and rod in an air-tight case. The rod is connected to one terminal of a high-voltage generator, and the cylinder to the other, and the pressure (and so the density of the density of the air) is adjusted till corona just starts. The second method is to mount cylinder and rod in air, rods of varying radii being employed for the different voltages, the current being adjusted through the primary of the high-voltage generator, so that corona always just forms. The voltmeter connections are shown diagrammatically.
J. R. Clarke, *Rontgen Soc. J.*, Jan., 1923.

COMMITTEE ON MINES

To the Board of Directors:

The Committee on Mines has very little to report this year. There has been but one session held, at which papers pertaining to mine electrification were presented —namely, at the Birmingham Meeting. At this meeting a partial mining session was held, and three papers were presented on mining subjects. One of these was by L. C. Ilsley, Electrical Engineer of the Bureau of Mines, and was entitled, "Electrical Safety in Coal Mines." C. E. H. Von Sothen of the Industrial Engineering Department of the General Electric Company presented a paper on, "Automatic Sub-Stations for Mines;" and F. R. Grant and the writer presented a paper on "Tests on Mine Hoist Control," describing the operation of a large Ward Leonard hoisting equipment. Mr. Grant is also of the Industrial Engineering Department.

There has been no meeting of the committee this year, as the members are so widely scattered throughout the country and there have been no matters of sufficient importance to warrant calling them together.

No real constructive work has been undertaken by the committee, as there does not seem to be any field for such work at this time. The most urgent requirement in the mining field today, as far as electrification is concerned, seems to be the want and need of a satisfactory code for the safe installation of electrical apparatus underground, and this has been prepared by the American Mining Congress and the Bureau of Mines. This code is at present on the way to the Engineering Standards Committee for final approval. The majority of the members of your committee have been in touch with this work, and this field is very satisfactorily covered.

As to suggestions for future procedure, I feel the only way we can stimulate general interest in the work, of this committee, and possibly within the committee itself, will be by the holding of a joint session with some one of the important mining societies, and I would strongly recommend that such a joint session be considered. I believe that if this was put through it would be possible to secure quite a number of interesting papers on engineering subjects pertaining to electrification of mines; but electrical engineers intimately connected with mining work, I am afraid, do not feel that the A. I. E. E. is sufficiently interested in their work to warrant their presenting papers.

F. L. Stone, *Chairman.*

TRACTION AND TRANSPORTATION COMMITTEE

To the Board of Directors:

Just before the close of the last administrative year, a special committee was appointed to consider and report to the Board of Directors:

1. As to the desirability of continuing the Traction and Transportation Committee.

2. If the committee is to be continued, as to the character of the work which should be expected of it.

3. As to a specific program recommended for work of the committee for the ensuing year.

The committee reported on October 6 and recommended, in brief, substantially as follows:

1. That the Institute should have a technical committee to cover, broadly, the field of transportation.

2. That the most important function of such a technical committee would consist in guiding the activities of the Institute so that authoritative papers could be secured, bearing on the most important topics. Various subjects were suggested under this heading.

3. For the ensuing year, the committee recommended that the plan which the Meetings and Papers Committee had initiated for having a Railway Session at the Midwinter Convention in Philadelphia be carried

out; that this meeting be devoted exclusively to the production of papers by railway executives and operating men, if suitable papers could be secured.

The report was adopted by the Board and the present committee was appointed early in November with instructions to follow out the plans suggested. The recommendations of the committee concerning the Midwinter Convention were carried out. Two Transportation Sessions were held on February 5, one in the afternoon, the other in the evening. The latter was one of the largest ever staged by the Institute; it was held in the Metropolitan Opera House in Philadelphia and the proceedings were broadcast all over the country by radio. Five addresses by prominent railway and other executives were of a high grade and much appreciated. They were of a general character, of interest to everyone and as such they should be preserved.

The afternoon session was notable from the fact that the railway problem was discussed largely by railway operating men and brought out a good many interesting side-lights. The committee undertook to portray a vision of the future of rail transportation in this country and to gain a better understanding of the problems to be solved. Both were realized to a certain extent and it is the intention of the committee to continue the plan of getting the railway operating men to cooperate with us in this way.

The principal afternoon speaker was Mr. L. G. Coleman, Assistant General Manager of the Boston & Maine Railway, who gave an excellent paper in which he painted a graphic picture of the steam locomotive—its virtues and limitations. A number of other operating men joined in the discussion and contributed largely to the success of the meeting.

The past year has been distinctly one of progress in the field of the electric railway. No particularly spectacular events have been noted but the steady development going on in all lines promises to handle the transportation problem most satisfactorily when, finally presented.

Several new types of locomotives have been developed in the past year and improvements have been made in line construction, current collection, distribution systems, etc. Indications are that several large electrification projects will be started as soon as the financial situation will permit the railroads to undertake such improvements.

It is recommended for next year that:

1. Suitable papers be presented covering the points brought out in the Philadelphia meeting.
2. A suitable paper covering the subject of power supply for electric railways.
3. Papers covering the latest developments in electric locomotives.
4. Papers covering the development of the oil-electric or steam-electric locomotive.

N. W. Storer,
Chairman.

The Hysteresis Character of Corona Formation

BY HARRIS J. RYAN and HENRY H. HENLINE
Fellow, A. I. E. E. Associate, A. I. E. E.
Both of Leland Stanford, Jr. University

Review of the Subject.—To begin with the hysteresis character of 60-cycle corona formation was indicated by two factors: (1) the value of the crest voltage was of dominating importance in the corona loss-voltage relation and (2) the value of the power lost in corona from a high voltage transmission line approximated that given by the product of the voltage in excess of the critical voltage and the line charging current.

Diagrams of the cyclic instantaneous voltage-charge relation in corona formation were obtained and revealed the facts that in a given case the charge lags behind the corresponding voltage by an approximately **constant amount** *and that the energy dissipated per cycle is proportional to the product of the crest voltage and the difference between the crests of the voltage and the critical voltage. The corresponding equation for the full corona loss-voltage relation is*

$$P = 4 f C (E^2 - E E_0).$$

60-cycle brushes, coronas and streamers are due to the activities of ions carrying charges largely of the same sign as that of the conductors from which they emanate. The factors that control brush formation are considered. Local corona is due to an incomplete unstable brush pattern. Full corona is correspondingly due to a complete stable brush pattern. In relation hereto critical voltage, crest voltage and irregularity factor are defined. The difference in routing of linear 60-cycle and curvilinear radio frequency discharges is found to be due to the activity of ions carrying like charges in the former and a mixture of unlike charges in the latter.

* * * * *

YEARS ago in studying the nature of corona losses occurring from a small high-voltage laboratory line the cathode ray tube was used to observe the cyclic relation of corresponding instantaneous values of line voltage and charge.[1] It was found that the sides of the resulting diagrams were practically parallel, causing them to resemble hysteresis cards. The significance of this was not recognized at the time. More recently two items were encountered that have called for a critical study to be made of these same diagrams: (1) the value of the power lost through corona from a transmission line closely approximates that given by the product of the charging current and the voltage in excess of the critical voltage; (2) the dominating importance of the crest voltage in relation to corresponding corona loss. These items clearly indicated *the hysteresis character of corona formation.*

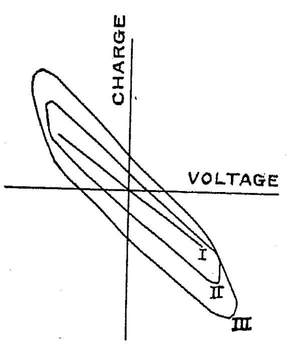

FIG. 1

Fig. 1 is a copy of Fig. 6 which appeared in the paper referred to above. The diagrams in this figure were obtained while using a pair of wires 0.085 inch in diameter spaced 12.5 inches apart. Card I was taken at 44,000, card II at 53,400 and card III at 64,000 root-mean-square approximate sine wave volts.

1. Numbers refer to the Bibliography.

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., Oct. 13-17, 1924.

The heavy line diagram in Fig. 2 represents the general form of card obtained in such cases. When the crest voltage is below the value E_0, the loss is zero, and the card is a straight line lying on the line X Y. At a crest voltage very slightly below E_0, the card is the line g h, and the maximum charge is Q_a. The charge varies directly with the voltage, and all the energy stored in the charge Q_a at the critical voltage, E_0, is

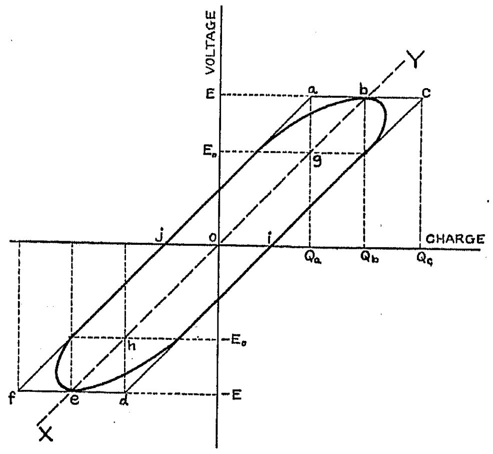

FIG. 2

returned to the source when the voltage is reduced to zero. As the voltage is increased beyond the critical value, E_0, to the value E and again reduced to E_0 the charge increases because of corona formation from the value Q_a to that of Q_b. The net result of this action is to increase the charge permanently through the succeeding half cycle by the amount $Q_b - Q_a$. Assuming that the cycle has now been formed up, the same thing occurs when $-E_0$ increases to $-E$, the isolated charge $Q_b - Q_a$ having a value of $o\,i$ is first discharged without return of energy and then built up to the same negative value, $o\,j$. Then as the cycle proceeds while $-E$ changes again to $+E$ the charge Q remains at all times reduced correspondingly by the same amount $Q_b - Q_a$. Thus a loss of energy per cycle is caused

through the corona formation determined substantially by the area of the card $a\,c\,d\,f\,a$.

From these boundaries the value of the power lost in corona may, therefore, be written as the product of the frequency and the $E - Q$ area determined by the corona cycle. Since the areas

$a\,c\,i\,j\,a = a\,c\,Q_c\,Q_a\,a$

$$P = 2\,f\,E\,(Q_c - Q_a) \text{ and} \tag{1}$$

because

$$Q_a = E_0\,C;\; Q_b = E\,C;\; Q_c - Q_a = 2\,(Q_b - Q_a)$$

it follows that:

$$P = 4\,f\,C\,(E^2 - E\,E_0) \tag{2}$$

wherein

P is the value of the corona loss in watts per conductor.

f is the value of the frequency.

E_0 is the crest value of the critical voltage to neutral.

E is the crest value of the line voltage to neutral.

C is the capacitance in farads of one conductor to neutral.

The reason for the close agreement of the values of corona losses and their corresponding products of line charging currents and line voltages in excess of critical voltage may now be made apparent, *i. e.*

$$P' = i_c\,(e - e_0) \text{ or}$$

$$P' = 2\,\pi\,f\,C\,e\,(e - e_0) \text{ or} \tag{3}$$

$$P' = \pi\,f\,C\,(E^2 - E\,E_0) \tag{4}$$

The value of P is only $4/\pi = 1.272$ greater than that of P' which accounts for the close proximity of *measured corona loss* P_0 and the corresponding value for the product of charging current and voltage in excess of critical voltage as in equation (**3**).

The form of these *voltage-charge* diagrams produced by corona is specific and may be clearly understood by a study of the ionization existing around a conductor subjected to high voltage. If the voltage does not at any time during the cycle exceed the value E_0 in Fig. 2, there is no corona loss and the cathode ray diagram is a straight line. If the voltage increases beyond E_0, the diagram opens out with sides virtually parallel, indicating that some change has occurred in the atmosphere adjacent to the conductor. The air, has, of course, been broken down or completely ionized about the conductor radially outward as far as the critical voltage gradient has been exceeded. Ions carrying charges of unlike sign to that of the conductor move in quickly to its surface and are discharged. Those carrying charges of like sign are repelled and move quickly to the boundary stated above, whereat the electric intensity has fallen to the critical ionizing voltage gradient. Thereafter their velocities are far lower, so much so that their positions remain in effect much the same until a reversal of the action occurs by a corresponding reversal of the voltage crest from E to $- E$.

Fig. 3 illustrates the state of things in the electric field about a conductor due to corona formation when the voltage has just passed through the crest value E, and is near the value of E_0, diminishing. The electric field attached to the conductor was set up by the critical value of voltage, E_0. The rest of the field due to the increase of E above E_0 was terminated upon the ions that were lodged just beyond the envelope of ionized atmosphere as specified. It follows that the electric intensity adjacent to a high-voltage conductor in the air cannot exceed critical value.

The value of the field thus detached from the conductor is $Q_c - Q_b$ in Fig. 2. Having been carried into place by ions it cannot be removed by the ordinary action of a displacement current through the charged dielectric to the electrode conductor. Such ion-attached field will persist until discharged by ions of unlike sign repelled from the conductor when the succeeding negative crest value of the voltage occurs. The energy in the ion-attached field cannot, therefore,

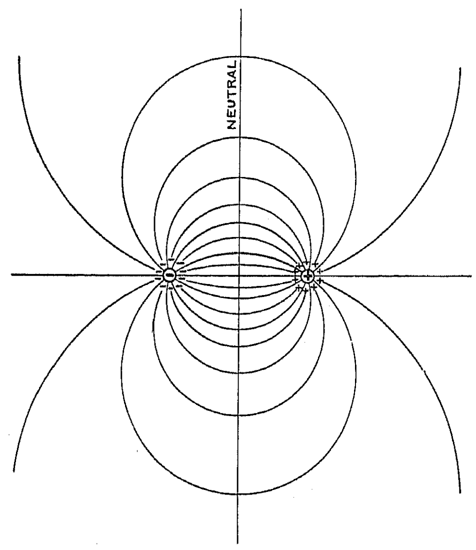

FIG. 3

be returned to source through the conductor. Such field can only be reduced to zero by degrading the energy stored in it to heat.

Fig. 4 illustrates the same corresponding state of things when the value of the voltage is near $- E_0$, increasing. The field attached to the ions now terminates on the conductor, and no longer on the neutral surface. Thus, it becomes understood that whereas the charge set up by the voltage, E_0, from conductor to neutral is Q_a without corona formation; with corona formation and E diminishing from E_0 it must have a value *larger* than Q_a by the amount of the field attached to the ions, *i. e.*,

$$Q_a = (E - E_0)\,C = Q_c - Q_b;$$

Likewise, with corona formation and E increasing toward E_0 it must have a value *smaller* than Q_a by this same amount of the field attached to the ions.

Thus it is that the chief factor in 60-cycle corona formation is the production of a hysteresis relation of volt-

age to charge. The charge lags behind the voltage a definite amount throughout the cycle. For any given value of the critical voltage, E_0, the energy thus lost in corona hysteresis per cycle is dependent only upon the crest value of the voltage, E. In the voltage-corona loss relation, therefore, the value of the crest voltage is of dominating importance. The root-mean-square value of the voltage, as such, has no relation to the magnitude of the energy lost in corona per cycle.

Critical and crest voltages to neutral, capacitance

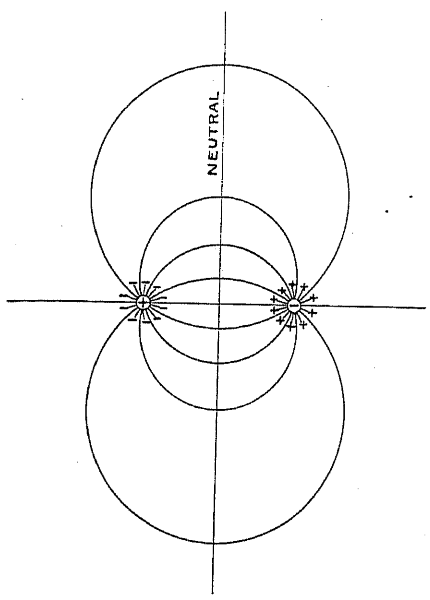

FIG. 4

to neutral and frequency have thus been found to be the primary controlling factors in corona formation. The value of the critical voltage, E_0, has long since been known to be dependent upon a number of secondary factors. Of these the *irregularity factor* is at once the most important and difficult to manage.[2]

Irregularity factor is the ratio of the average to the maximum intensity of the electric field adjacent to the high-voltage conductor. The value of this ratio is dependent upon other related factors such as the mechanical, physical and chemical irregularities of the conductor surface, the magnitude of the supply of free ions in the air about the high-voltage conductor and the manner and extent to which such free ions are associated with moisture, dust particles or with other finely divided matter.[3,4]

Critical voltage has a crest value that is just sufficient to produce break-down or ionization in the air adjacent to a high-voltage conductor. When the corresponding electric field is attached to the conductor with uniform density, the value of the critical voltage is maximum. When, from some cause, the density of the electric field is irregular, the corresponding value of the critical voltage is given by the product of the values of the maximum critical voltage and the irregularity factor.

These third-order factors may be absent to such an extent as to raise the irregularity factor to unity. In that event full corona will be found to occur near to and above the critical voltage. No isolated brushes will have been formed. Or these third-order factors may be present to such an extent that the resulting irregularity factor may have been forced to a very low value, 0.5, more or less.

At the corresponding critical voltage a few stray brushes are formed. As the voltage is raised, the number of brushes is increased. Through their mutual shielding effect the irregularity factor is also raised. This action continues with increase in voltage until a point is reached whereat no space is left in which to accommodate more brushes.[4] Thereafter the brush pattern is fixed and so are the irregularity factor and critical voltage constituting the condition of full corona formation.

From these considerations, it was evident further progress in the studies required that the voltage-charge relation should be understood for brushes occurring singly or in multiple, as well as for full corona formation.

To study the voltage-charge relation for single brush discharges the set-up diagrammed in Fig. 5 was used. Large metal screens were set in vertical planes about two feet apart and grounded. The cathode ray tube was placed between the screens. The 36-inch sphere was placed about three feet from the screen on one side, and the point which was the source of a brush discharge was placed about 30 inches from the other screen. It was connected to the wattmeter as shown. Its height above the concrete floor was about four feet. The potential reducing plate used on each side was insulated

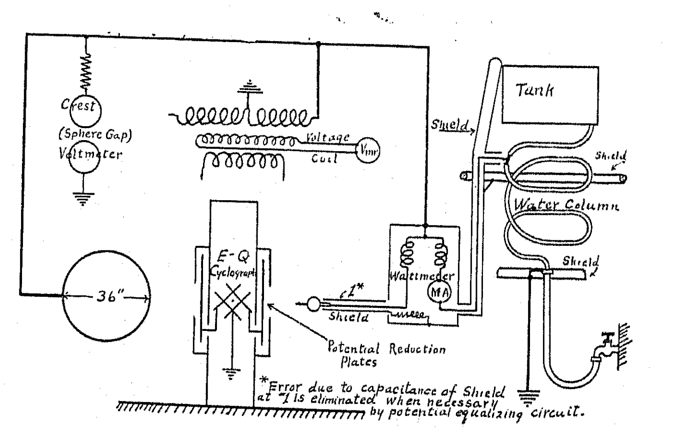

FIG. 5

from the large screen by a layer of beeswax. One deflector plate of each pair in the cathode ray tube was grounded, and the others were connected to the potential plates. Grounded shield plates partly covered the potential reducing plates for adjusting the amount of the deflection of the cathode ray beam along each axis so as to obtain a card of suitable dimensions.

The 36-inch sphere was used to produce the voltage deflection. Since there was no corona on it, the electric field set up was proportional to and in phase with the

voltage, and the potential reduction plate gave a certain percentage of the voltage between the sphere and ground. The point produced a good supply of ions on the other side even at relatively low voltages. Hence, the potential reduction plate on that side produced a deflection proportional only to the electric field or charge and not to the applied voltage. The appearance of the apparatus is shown in Figs. 6 and 7.

The point was connected to the source through the wattmeter[4] shown in the diagram whereby the powers consumed in the brush formed from the point at various

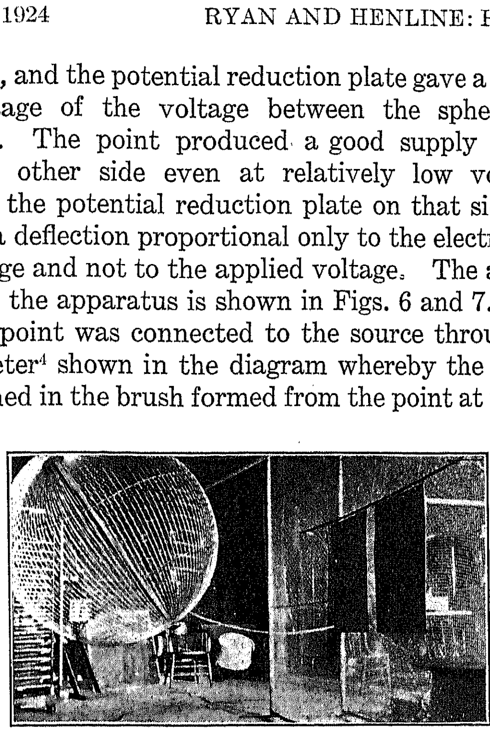

Fig. 6

voltages could be observed. The diagrams obtained at 20, 30, 50 and 80 kilovolts to ground are designated as A, B, C and D, respectively, in Fig. 8. The following table gives a summary of the results obtained.

Kilovolts to Ground	Watts by Wattmeter	Areas of Diagrams sq. in.	Ratios Watts/Areas
20	0.0	0.00	..
30	0.8	0.06	13.3
50	3.1	0.26	11.9
80	11.3	1.10	10.3

Because the deflections of the cathode ray were not exactly proportional to the actuating potentials, it

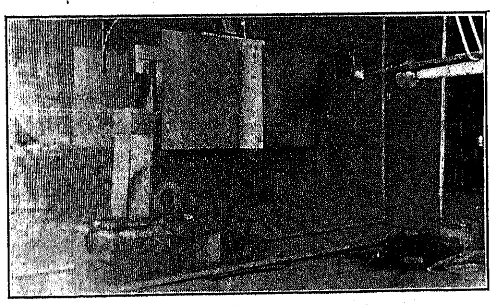

Fig. 7

follows that it was not proper to expect equality in the ratios of the areas and their corresponding powers observed by wattmeter. The fact, however, that the ratios did not differ greatly shows that the diagrams are good indications of the nature of the mechanism of the corona loss occurring in brush discharges.

Looking at the corona-brush diagrams critically one learns that their hysteresis character remains complete without mutilation until the value of the applied voltage is more than double the critical voltage at which the brush was started. Finally, as the voltage was raised the length of the brush became too great for all of the ions to reach their boundaries promptly. The space within the brush remained occupied to some extent with ions of both signs. Hereby gas conduction developed as it is ordinarily understood. A compound diagram resulted due to the combination of two, one because of hysteresis and the other resistance. The form of the one is obliquely rectangular and of the other eliptical.

For the study of corona brushes in multiple, brush patterns on cylindrical conductors were produced artificially by means of short tacks or water drops placed at regular intervals.[4]

The heads of the tacks were cemented to the surface of the conductors. The high-voltage wattmeter was used to determine the corona loss-voltage relations. The conductor length was 5 feet; diameter 0.532 inch; and the surface was studded with tacks ¼ inch long. The corresponding areas of conductor surface per tack, critical voltages, applied voltages, losses and values

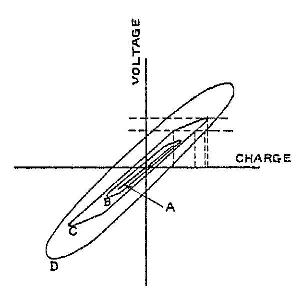

Fig. 8

of x in $P = k\,(e - e_0)^x$ at e are given in the following table:

Square Inches per Tack	e_0 Critical Voltage Kv.	e Applied Voltage Kv.	Loss Watts	x in $P = k$ $(e - e_0)^x$ at e
0.0625	40	90	50.5	1.56
0.25	40	90	50.5	1.56
1.00	40	90	50.5	1.56
1.64	43	93	44.5	1.64

From these results it is clear that the full corona loss-voltage relation remains unchanged over a wide variation of the brush pattern. It is also clear why Peek and his coworkers who first studied this relation found it to be unchanged by the degree of free ionization present in the air about the high-voltage corona-forming conductors.

As the value of the critical voltage rises from any or all of several causes specified above, local corona by brush formation diminishes and practically disappears when its maximum value has been attained. The corresponding loss-voltage curves obtained by measurement straighten out with rising critical voltage.

Thus, the difference between Peek's law of corona for a given frequency

$$P = k\,(e - e_0)^2 \text{ and}$$

equation (2)

$$P = 4\,f\,C\,(E^2 - E\,E_0) \text{ is}$$

of little importance. The essential difference is in the values of e_0 and $0.707\,E_0$. Each must be obtained by substituting the same observed values of P and applied voltages, e, and $0.707\,E$ in their respective equations. The cause of the difference will be apparent when the two methods of attack have been recalled and compared. E_0 can be observed by the voltage-charge indicator and by calculation, while e_0 can be obtained only by calculation from the full corona loss-voltage relation. In the case of Curve 3, Fig. 9, these calculated values were found to be for e_0, 116 kv. and for $0.707\,E_0$, 133 kv.

For comparison of the actual values occurring in the corona loss-voltage relation and the corresponding values calculated by equation (2) and Peek's equation No. 34 (High Voltage Engineering, 1920 Ed.) the authors are indebted to Messrs. J. C. Clark and Frank F. Evenson for the July and August, 1923 data by which the curves numbered 1, 2, 3 and 7 were located in rectangular coordinates in Fig. 9. The coronas were formed on cables made up of 49-strand copper, rope-laid. The conductor diameters, single-phase center to center horizontal separations and meterological conditions are specified in the figure. By means of an improved high-voltage wattmeter Curve 3 was relocated April 3 and again April 5, 1924 and the corresponding results were chartered also in Fig. 9. The curves thus located, but not drawn in, have been numbered 4 and 6. They were found to be in close agreement with Curve 3. Values corresponding to those in Curve 3 were calculated from Peek's law of corona and equation (2) and are given in the following table.

CORONA LOSS
Kw. per Mile per Conductor

e Kv. to Neutral	Peek's Equation	$P = 4fC$ $(E^2 - E\,E_0)$ Curve 5	Curve 3	Curve 4	Curve 6
120	0.24	*	0.80	0.52	0.80
125	1.21	*	1.11	0.87	1.10
130	2.92	*	1.57	1.50	1.75
135	5.38	*	2.56	3.50	3.40
140	8.58	5.7†	6.9	7.50	6.20
145	12.5	10.8†	13.0	..	..
150	17.2	16.6†	18.9	..	..
155	22.7	22.8†	24.6	..	..

*Local Corona Section.
†Full Corona Section.

As the voltage is lowered, if the full corona brush pattern breaks up and local corona is formed, full corona critical voltage cannot be determined by voltmeter. It can only be determined from the voltage-charge diagram. Without such diagram it must be obtained by calculation from the observed full corona loss-voltage relation. This relation within fair limits is never complex. The corresponding equation can be derived with no difficulty. It may then be used to calculate the value of the critical voltage. In curve 3, Fig. 9, the full corona part of the loss-voltage curve was found to be virtually a straight line. The value of the corresponding critical voltage was found, therefore, by extending such line to the point where it intersected the

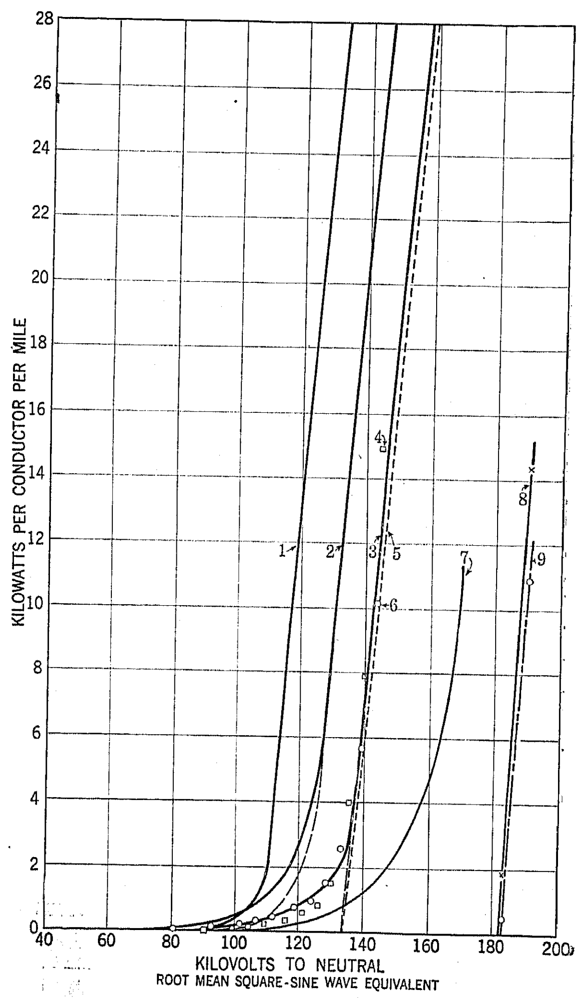

FIG. 9—CORONA LOSS

1. Rope Lay Copper 0.614 in. Dia.—Clark & Evenson.
2. " " " 0.80 " " " " "
3. " " " 0.91 " " " " "
   Aug. 2, 1923 — Clear; Windy.
   Barometer 30.01 in.
   Temperature 68 deg. fahr.
   Relative Humidity 52 per cent.
4. Rope Lay Copper 0.91 in. Dia., Carroll, Peterson & Stray.
   Apr. 5, 1924 — Clear; Light Breeze.
   Barometer 30.17 in.
   Temperature 68 deg. fahr.
   Relative Humidity 20 per cent.
5. Rope Lay Copper 0.91 in. Dia.
   Loss Computed from Equation
   $P = 4\,f\,C\,(E^2 - E\,E_0)$
   $e_0 = 134$ Kv. from Curve 3.
6. Rope Lay Copper 0.91 in. Dia., Carroll, Peterson & Stray.
   Apr. 3, 1924 — Cloudy.
   Barometer 30.05 in.
   Temperature 54 deg. fahr.
   Relative Humidity 51 per cent.
7. Rope Lay Copper 1.24 in. Dia., Clark & Evenson.
8. Concentric Strand Aluminum 0.952 in. Dia., Bright.
9. Ditto, Black.

Curves 8 and 9 plotted from results obtained on 6 ft. 19½ in. lengths in laboratory and corrected to basis of 17 ft. spacing.

Spacing: Curve 2 17 ft.
Curves 1, 3, 4, 5, 6, 7 18 ft.

voltage axis, viz. 134 kv. The capacitance to neutral per mile of conductor was 0.0145 μf. With these values by means of equation (2) Curve 5 was located in the same diagram, Fig. 9, and is in close agreement with curve 3.

The corona formed about large cables is virtually always pervaded by a brush pattern. Such pattern may be so complete as to occupy in corona formation substantially the whole of the electric field and, therefore, of the capacitance attached to the conductor; or the pattern may be less complete, occupying correspondingly less of the capacitance. When the pattern is nearly complete, as under the conditions for which the data locating curves 3, 4 and 6, Fig. 9, were obtained, the corona loss-voltage relation given by the equation

$$P = 4 f C (E^2 - E E_0) \qquad (2)$$

should be substantially correct. When the brush pattern is stable but less complete, not all of the electric field and, therefore, not all of the capacitance will be occupied in corona formation and the value of C in equation (2) will be correspondingly reduced. With this understanding of the values of capacitance, C, and the corresponding critical crest voltage, E_0, equation (2) should give the correct corona loss-voltage relation for every variety of 60-cycle corona formation with stable brush patterns. When the brush patterns are unstable the values of the capacitance, C, and the critical crest voltage, E_0, become unknown functions of the applied crest voltage, E, rendering the equation unavailable. This phase of the hysteresis character of corona formation is here presented more especially for the further understanding of the nature of corona than, as yet, to offer practical means for the economic study of conductor sizes in relation to corona losses. The economic forms of conductors are those which develop local corona only. In the present state of the science a large amount of experimental study remains to be done before the economic sizes can be predetermined with precision.

That the magnitude of the supply of free ions is a powerful factor in local corona formation may be understood through the results obtained in the following experiment: A bright, clean, 10-foot section of concentric strand aluminum-steel core cable, having a diameter of 0.952 inch, was mounted in the laboratory horizontally, 3 ft. 6.5 in. above the concrete floor neutral. It was connected through the wattmeter to the high-voltage source and properly shielded so that the only power values indicated by the wattmeter were those due to corona formation on the actual cable specimen. The corona loss-voltage values thus found were corrected to correspond to a distance to neutral of 8.5 ft. or 17 ft. between centers in a single-phase circuit. They were then used to locate Curve No. 8 in Fig. 9. The various active high-voltage leads and terminals present in the indoor laboratory drained the space of free ions to such an extent that the formation of an appreciable brush pattern was prevented, resulting in the absence of local corona formation. Full corona formed at an increment of voltage above the maximum critical voltage. Without correcting for the shielding effect due to the proximity of the high-voltage connecting leads the calculated value of the irregularity factor exceeded unity.

After aluminum cables of the sort just specified have been in service with corona formation, they will, in some circumstances, become coated with a definite layer of finely divided carbon, dust particles and cementing material causing the coating to adhere firmly. Through the understanding of corona formation herein presented it appeared reasonable to expect that the corona loss from the roughened carbon-coated specimen should be less at a given voltage than from the bright, smooth and clean specimen of what would otherwise be the same cable. The film of oxide covering the bright cable would thus inevitably interfere a little with the ready access of the incoming ions to the raw surface of the conductor. It would interfere to a degree with the uniformity of distribution of the ions moving toward and away from the conductor. On the other hand, the incoming ions for the carbon-coated cable would strike everywhere a conducting surface that should facilitate the development of full corona with a more nearly complete absence of brush formation with a corresponding increase of irregularity factor and of critical voltage. The test was, therefore, made for the black cable and the corresponding results were used to locate Curve 9 in Fig. 9. From these curves it was found that at 190 kv. the losses per conductor-mile for the bright and black cables were respectively 13.3 and 10.2 kilowatts. These bright and black cable specimens were very kindly furnished for the purpose by Mr. H. A. Barre, executive engineer, Southern California Edison Co.

For 10 years it has been known that 60-cycle discharges are routed substantially over the shortest paths between opposing high-voltage electrodes and that in contradistinction thereto radio frequency discharges occur over the routes of the tubes of force established by the impressed voltage.[5,6] The hysteresis character of 60-cycle brushes leads to an understanding of the cause of the difference in the routing of these two forms of discharge. At 60 cycles there is ample time for ions carrying charges unlike those attached to the high-voltage conductor to move up to it and be discharged; and for the ions of like charges to be repelled from the conductor and to reach their boundaries just beyond the ionized zone of air covering the conductor. Such surviving ions are drawn in a direct route to the opposing electrode by the tension of the electric fields attached from them to it. In the radio frequency discharges the state of things is quite different. The time between voltage crests is too short for the segregation of the ions to any considerable extent. The result is that the brushes which become discharges are occupied with ions of both signs. Their fields cancel

and they are propelled only in the direction of the original fields, due to the source voltage applied between the electrodes.

Conclusions

1. Sixty-cycle corona in all ordinary circumstances develops the character of a gas dielectric hysteresis.

2. The values of *crest voltages* are controlling in respect to losses by corona formation.

3. *Critical voltage* can have a definite value only when the brush discharge pattern in corona formation is stable. Its value should be understood to be given by the product of the maximum critical voltage and the irregularity factor.

4. *Irregularity factor* should be understood to be the ratio of the average to the maximum electric intensity of the field adjacent to the high-voltage conductor.

5. The rational corona loss-voltage relation is given correctly by equation (2) only within those limits of full corona formation wherein the capacitance value and brush pattern remain fixed.

6. Local corona loss varying from 0 to 8 kw. per mile depends upon too many variable factors to permit of calculation without the use of knowledge to be obtained only through a large amount of further study by measurements.

7. The strong ion-formed fields that are responsible for the corona hysteresis are the cause that routes a 60-cycle discharge by the shortest path between electrodes. Without such fields, as in radio frequency discharges, the discharge routes must be along the tubes of force of the original electric fields.

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Discussion

For discussion of this paper see page 1162.

The High-Voltage Wattmeter

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Review of the Subject.—A complete laboratory set-up for measuring losses at low power factors from high-voltage circuits is assembled around a moving water column resistance multiplier which permits the direct use of standard low-voltage instruments. A shielding system is used to eliminate capacitance effects in the loading and voltage circuits. The paper contains structural details of the wattmeter aggregation and the loss loading equipment, and the operating procedure for conducting atmospheric and insulator loss measurements.

* * * * *

Introduction

The standard dynamometer-type low-voltage wattmeters may have their voltage ranges ordinarily extended to 1000 volts or thereabouts, by means of resistance multipliers. Higher voltage ranges are feasible only when the requisite multiplying resistances are constructed so as to be free from inductance and capacitance effects. Though expensive, this practise has been occasionally employed for voltages as high as 50,000.

The study of some of the problems encountered in 220,000-volt power transmission projects has required the use of voltage and power meters on extra high voltage circuits, accurate at low power factors. To provide satisfactory instruments for this class of research work, Professor Harris J. Ryan proposed to the authors to abandon the traditional use of "low loss" current control resistances in the voltage circuits and to substitute therefor moving water column resistances. The water columns are inexpensive, quickly erected, free of appreciable inductance and may conveniently be shielded from capacitance effects. When such resistances are designed to limit the currents in the voltage circuits of the instruments to 50 milliamperes at the highest voltages to be used in the studies, ordinary low-voltage standard makes of dynamometer-type voltage and power meters may be used. However, the experimental resources in these undertakings must include the requisite large amounts of power which must thus, necessarily, be consumed by such use of these measuring instruments on high voltage circuits. For example, the voltage circuit of a single instrument as herein presented will require 100 milliamperes, 50 for the instrument proper and 50 for the necessary shield to be considered later. The power thus consumed in a 350,000-volt circuit for the voltage circuit of a single instrument will amount to 35 kilowatts. In some of the application trials of the wattmeter reported upon herein, the low power-factor loading currents were less than 50 milliamperes. To increase the sensitiveness of the wattmeter, the loading current was passed through the moving coil and the voltage circuit current which was doubled (consuming 70 kilowatts) was passed through the field coil. Even so, such cost of power is small when compared with the total of the remaining costs involved in making studies of this character.

Plan

The wattmeter circuits are shielded in such a manner that all appreciable electric fields terminate upon the enveloping shields rather than upon the instruments and the connections. Fig. 1 is a schematic diagram of the shielded wattmeter. The power supply from the high-

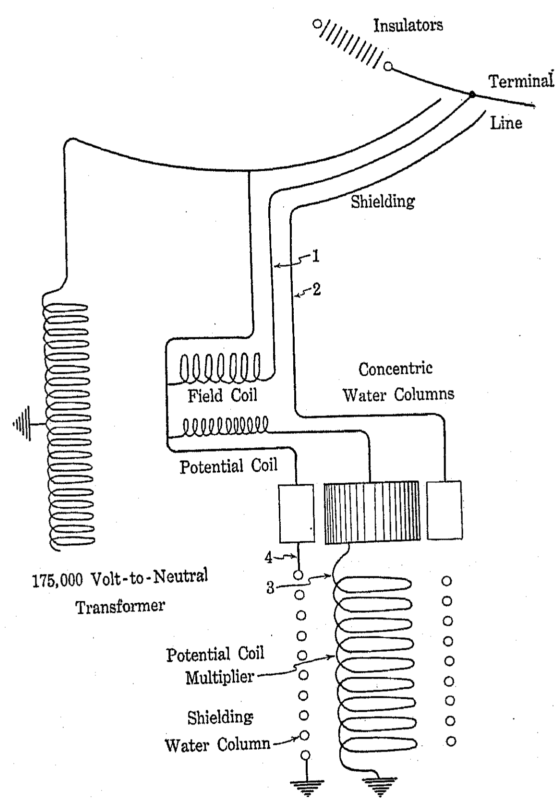

Fig. 1—Schematic Diagram

voltage source is divided into four circuits, (1) the power circuit which supplies power through the instruments to the line, (2) the main shielding circuit which surrounds the power circuit and supplies all losses and electric fields which would otherwise occur therefrom, (3) the potential circuit which contains the water resistance multiplier and (4) the multiplier shielding circuit.

Fig. 2 is a connection diagram of the assembly of

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., October 13-17, 1924.

instruments and auxiliaries which constitutes the high-voltage wattmeter. The source of high voltage is a single-phase testing transformer which has the mid-point of the secondary grounded. A voltmeter, V_m, connected across the voltage winding of the transformer permits an accurate determination of the secondary r. m. s. voltage. A terminal which facilitates the connection of the wattmeter to different experimental lines permits a direct connection of the power circuit to the line and insulates the end of the shielding circuit

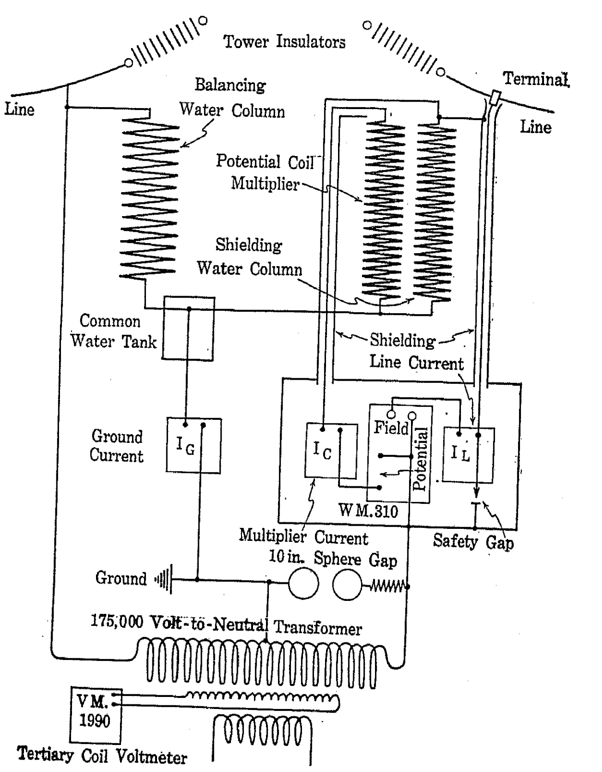

FIG. 2—CONNECTION DIAGRAM

from the line for a voltage that is equal to the resistance drop through the instruments.

The indicating instruments which must be used at high potential consist of a wattmeter, W_m, and two milliammeters located in the shielding cage which is insulated from the ground. One milliammeter, I_L, is connected in series with the field coil of the wattmeter and measures the current flowing through the power circuit to the line. The other milliammeter, I_C, is connected between the potential coil of the wattmeter and its multiplier and measures the current in the potential circuit. Because the resistance of the water column in the wattmeter voltage circuit changes with temperature, its value must be determined for each set of observations from the readings of instruments I_C and V_m. The multiplier is shielded by a concentric water column, the lower end of which is connected to the low-potential terminal of the multiplier. A balancing water column is necessary to load the other side of the transformer and avoid errors due to phase displacement. The low potential terminal of this balancing water column and the low potential terminal of the multiplier are joined and grounded through a milliammeter, I_G. A zero ground current indicates that the loads to neutral on the transformer are in balance. A sphere-gap voltmeter provides a means for determining the crest values of the voltages to neutral.

VOLTAGE CIRCUIT RESISTANCE MULTIPLIER

The multiplier is shown in the center of the illustration, Fig. 3. It is connected in series with the potential circuit of the wattmeter and consists of a water column contained in 71.5 ft. of ½-in. rubber hose. A cylindrical galvanized sheet iron tank, 20 in. deep and 19 in. in diameter, serves as a reservoir of sufficient capacity to supply the multiplier column with water continuously for half an hour. The water column which serves as a shield for the multiplier column is contained in 78.5 ft. of ½-in. hose. The reservoir which supplies water to this column is an annular galvanized sheet iron tank,

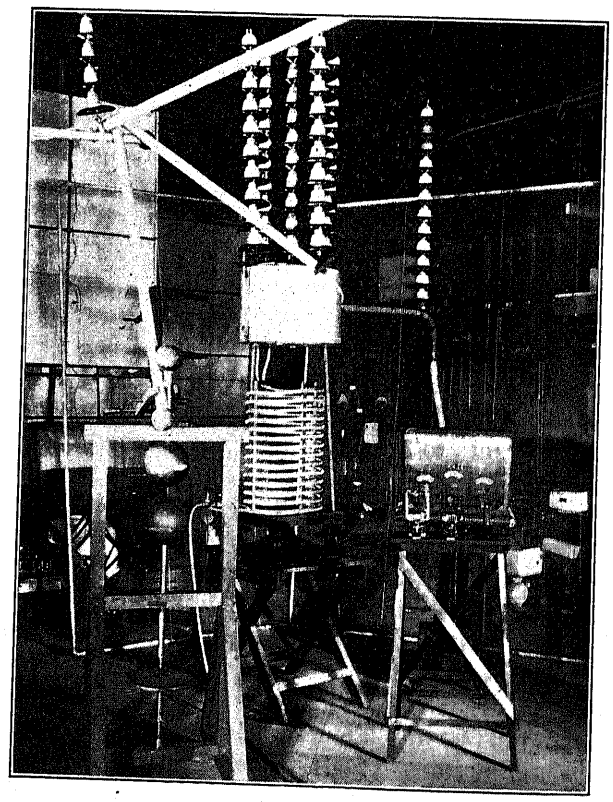

FIG. 3

19 in. deep, 21 in. inside diameter and 29.5 in. outside diameter. Four wood seperators maintain the tanks in their relative concentric positions. A glass gage on the side of the shielding tank indicates the level of the water supply.

For effectiveness in shielding and economy of space, the hose water columns are formed into two concentric coils with the shielding coil on the outside. The coils are 36 in. high and are mounted vertically on four 1-inch by 1-inch upright bakelite rods 39 in. long which

are placed at quadrant points on a 20-in. circular center line between the coils. Each coil forms a 14-turn helix with a 2½-in. pitch. The helices are fastened to the bakelite strips by small cotton cords which pass through holes in the bakelite. The maximum radial potential difference between corresponding points on the helices is that due to the drop of potential through the instruments. Since this drop is small, no special precaution is necessary in insulating the helices from each other. The axial potential difference between adjacent turns on the helices is 12,500 volts when the line voltage is 175,000 volts to ground. The voltage gradient is 6250 volts per inch, which gives a reasonable factor of safety, assuming the flash-over voltage of air to be 10,000 volts per inch.

Dowel pins are used in the construction of the multiplier frame in fastening wood spreaders to the upper ends of the bakelite rods as metallic pins would be possible sources of corona formation. The tops of the hose columns are connected to taps in the bottoms of their respective reservoirs by means of standard hose couplings. The lower ends of the water columns are joined so that they discharge through a common globe valve. The rods supporting the water columns are fastened to a wood base which rests upon four pin-type insulators. The annular reservoir is supported by four ten-unit insulator strings, while the inner reservoir is supported by a single ten-unit insulator string. Turnbuckles permit the proper distribution of the load between the insulator strings.

The resistance which provides the balancing load on the transformer is a water column contained in 75 ft. of ¾-in. rubber hose. This water column has approximately the same resistance as that of the multiplier and its shield in parallel. The column itself is connected to a 30-gallon galvanized iron tank which is suspended from a ten-unit insulator string. A glass gage on the side of the tank indicates the level of the water. The flow of water is controlled by a valve at the lower end of the column. A flexible 2-in. conductor connects the top of the column electrically to the transformer bus.

The water discharged from the columns flows into a common discharge tank insulated from the ground. Wire connections provide paths of low resistance from the low potential terminals of the water columns to the discharge tank.

The dimensions of the water columns are those which proved satisfactory for the tap water available at Stanford University. This water has an approximate resistivity of 1400 ohms per centimeter cube at 15 deg. cent.

Indicating Instruments

The instrument shield consists of a platform and a cage mounted on a 30-in. porcelain pedestal which provides sufficient insulation for a working voltage of 175,000 volts to ground. The platform is a 22-in. by 30-in. board clamped to the upper end of the pedestal. Wooden cylinders are grooved to fit flush with the top of the board so that the edges of the platform have a two-in. radius of curvature. The platform is covered with copper fly screen.

The cage consists of a wood frame 33 in. high, 30 in. long and 22 in. deep, covered with copper fly screen. The frame is doweled and glued together and the screen is fastened with brass tacks. All corners of the cage have a two in. radius of curvature to limit corona formation. The leads from the instruments pass through a brass plate screwed to the back of the cage. Brass hinges hold the cage to the platform and allow the cage to be raised in order that connections and adjustments may be made to the instruments.

A standard dynamometer-type low-voltage wattmeter, when used as the instrument in the high-voltage wattmeter, functions as a calibrated electro-power-dynamometer. A variable multiplying factor must be introduced to determine watts from the readings of the wattmeter aggregation. The power dynamometer used in the high-voltage wattmeter when employed for corona loss studies was designed to have a full scale deflection at 20 per cent power factor. The maximum voltage ranges were 75 and 150 volts. The maximum current ranges were (1), with the fields in series, 0.5 ampere, and (2), with the fields in parallel, 1. ampere. The scale had fifteen large divisions subdivided into tenths. This instrument was not sufficiently sensitive for accurate results when the power loss to be measured was less than 50 watts at 350,000 volts. In the present state of the art a more sensitive instrument may be had from the manufacturers for use when small power losses are under investigation.

The current, I_C, in the wattmeter potential circuit, and the line current, I_L, were measured by milliammeters which have full scale deflections of 75 milliamperes. The ground current, I_G, was measured by a milliammeter which had a full scale deflection of 15 milliamperes. The voltmeter, V_m, for measuring the voltage coil voltage had a maximum range of 450 volts. The instruments in the line circuit were protected from surges by a safety gap mounted within the instrument cage as shown in Fig. 2. A wire spring connected to the line conductor and a copper plate connected to the shield when separated by a sheet of tissue paper formed such gap. The breaking down of this tissue paper would short circuit the line current ammeter and the current coils of the wattmeter. Three telescopes, mounted on a suitable stand several feet in front of the instruments as shown in Fig. 3, were used in reading instruments I_C, I_L, and W_m. When the telescopes were focused on the meters within the cage, the screen did not interfere with the observer's vision, provided sufficient distance was allowed between the instruments and the screen.

The source of variable high voltages used for the corona loss integrity trials was a 150-kv-a. transformer with a maximum voltage rating of 350,000 volts. A voltage testing coil which gives a faithful 1/1000 reproduction of the secondary voltage is strategically located

in the windings of the transformer. The testing coil voltage in volts is equal to the secondary voltage in kilovolts. A 2300:230-volt multi-tap transformer serves as a primary supply for the high-voltage transformer. A hand-operated induction regulator provides uniform variation throughout the 23-volt interval from one tap to another of the multi-tap transformer.

The sphere-gap crest voltmeter as shown in the foreground of Fig. 3 was an essential unit in the assembly of the high-voltage wattmeter for the measurement of atmospheric losses. The 10-in. spheres were mounted according to the specification given by Farnsworth and Fortescue[1]. The calibration curve used for this sphere-gap was based on data given for 250 millimeter spheres with one sphere grounded in Table 204 of the A. I. E. E. Standards for 1922. A 300,000-ohm protective resistance was connected in series with the sphere-gap to limit the current at spark-over.

Structural Details of Atmospheric Loss Loading Equipment

Fig. 4 is an illustration of the general assembly of the

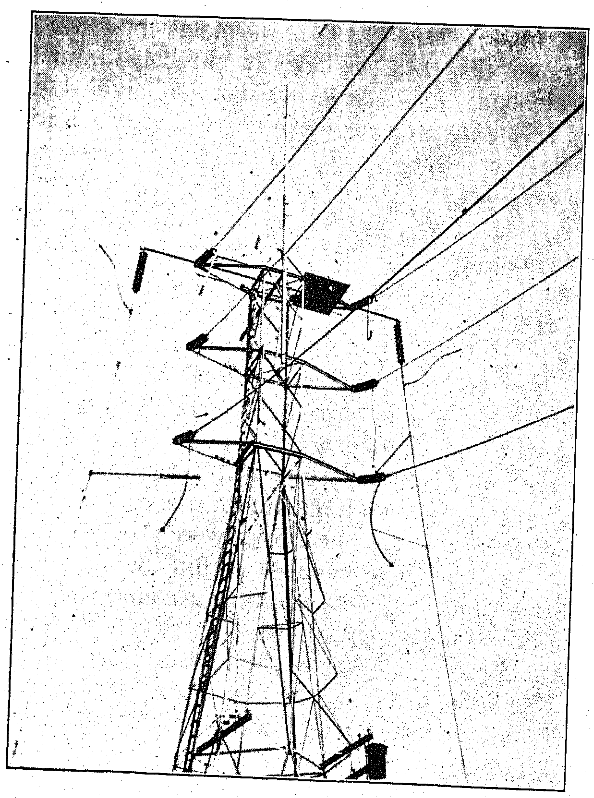

Fig. 4

switching equipment on the tower at the source end of the experimental line. Each length of cable is dead-ended to the tower cross-arms by a ten-unit string of 10-in. cap and pin-type suspension insulators. The vertical distributing busses which supply power to the transmission cables are insulated from the top of the source tower and from the ground by ten-unit insulator strings. These busses are equipped with projecting arms to facilitate the interchange of connections between the busses and the cables. Each bus is pivoted so that it may be rotated in either direction until the desired connection is made from the projecting arm to the corresponding conductor of the single-phase line under test.

1. Trans. A. I. E. E., Vol. XXXII, 1916, p. 733.

Fig. 5 illustrates the high-voltage terminal projecting from the vertical distributing bus. The line conductor is connected to this terminal by means of a 6-ft. hook-

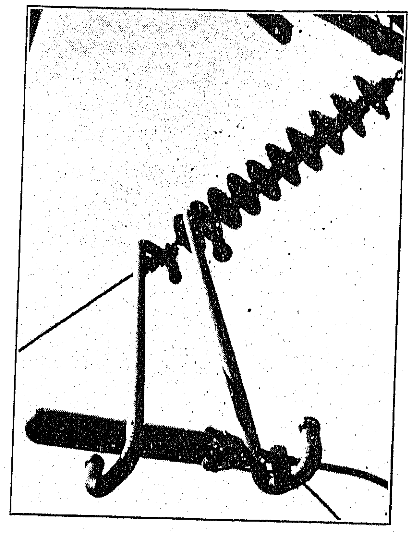

Fig. 5

link of 2½-in. galvanized sheet iron tubing. The main part of the terminal consists of a cylinder of galvanized sheet iron, length 30-in., diameter 5-in., one end terminating in a hemisphere of thin copper. The cylinder is mounted and insulated (1) from the hollow shaft of the terminal by two corrugated bakelite disks and (2) from the rain protecting funnel on the shaft by a small air gap. The conductor of the power circuit is a No. 14 insulated copper wire shielded by a flexible 5/8-in. armor. This cable is wound spirally about the vertical distributing bus and the end of the conductor is soldered to the galvanized cylinder of the terminal. The flexible armor is insulated from the cylinder and is fastened to the shaft of the terminal. To separate the insulator loss from the line loss, the loss over the insulators at the terminal end of the line may be eliminated by connecting the cap of the insulator next to the line to the shielding circuit as shown in Fig. 5. In this case the shielding circuit supplies the insulator losses. An extension of the shielding circuit to all of the insulators for the direct elimination of their losses from the power measuring circuit may be made when desired.

Operating Procedure for Atmospheric and Insulator Loss Measurements

The sphere-gap is set for the voltage at which observations are to be taken. The valves of the water columns are opened. Voltage is then applied and

gradually increased until the sphere-gap flashes over. The readings of the four meters, W_m, I_C, I_L and V_m just in advance of the flash-over are noted and recorded. The barometric pressure and temperatures by wet and dry bulb thermometers are recorded at the beginning and end of each test.

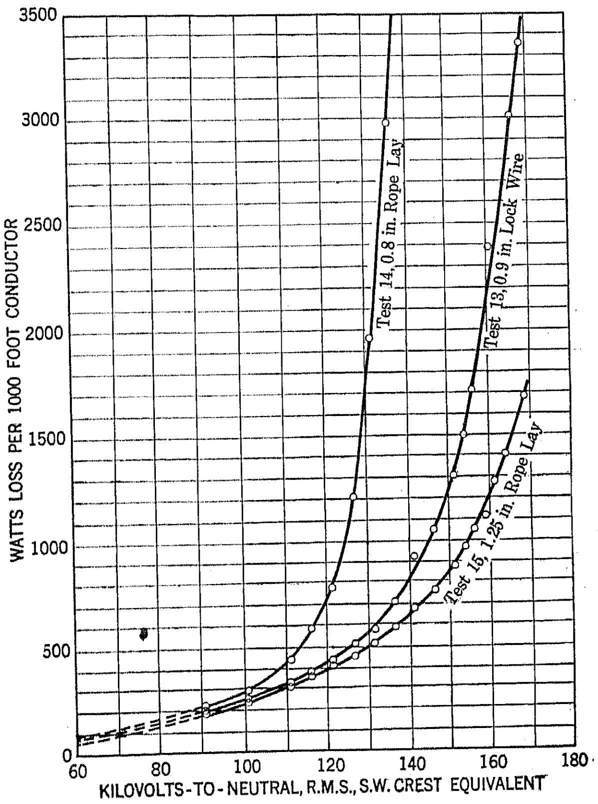

FIG. 6—TRANSMISSION LINE LOSSES MEASURED BY HIGH-VOLTAGE WATTMETER, STANFORD UNIVERSITY, CAL.

Fair weather, temperature 60 deg. fahr., humidity 58 per cent, barometer 30 in., frequency 60 cycles.

Test	13	14	15
Conductors.....	Smooth Lock Wire	375,000 Cm. Rope Lay	900,000 Cm. Rope Lay
Diameter.......	0.9 in.	0.8 in.	1.25 in.
Spacing........	17 ft.	17 ft.	18 ft.
Length.........	234 ft.	241 ft.	241 ft.

A typical test consisted of about 30 sets of readings with observations at 5-kilovolt intervals from 40 kilovolts to ground to 175 kilovolts to ground. Such a test under ordinary conditions required about 20 minutes and the services of at least five attendants.

Actual trials soon established the fact that the wattmeter could handle voltages up to the maximum operating limit of the transformer long enough to accomplish satisfactory observations. At 170,000 volts to neutral the total power loss for 234 ft. of 0.9-in. diameter single conductor amounted to about 900 watts in fair weather. This corresponded to a 1.5-watt deflection on the 15-watt scale of the low-voltage range indicating wattmeter. The minimum operating range of the high-voltage wattmeter aggregation was determined by the sensitivity of the indicating wattmeter. At 50,000 volts to neutral the total atmospheric power loss from the 0.9-in. conductor was less than 20 watts and no readable deflection of the indicating wattmeter, as used, could be observed.

During April, 1923, this wattmeter aggregation was used to determine the single-phase atmospheric power losses from three pairs of large conductors in lengths of 240 ft. approximately, each, as follows: lock-wire, smooth cable, 0.91-in. diameter and rope-laid cables 0.8- and 1.25-inch diameters. For illustration, the results of three of these tests are chartered by means of rectangular coordinates in Fig. 6.

The possibility of error, due to neglecting the loss over the insulators, was investigated by making two tests,—(1) with the insulators normally connected to the line, and (2) with the insulator loss supplied by the shielding circuit as shown in Fig. 5. The insulator loss was so small that the indicating wattmeter was not sensitive enough to give quantitative results in clear weather, although a slight loss was perceptible.

CONCLUSION

The high-voltage direct-measuring wattmeter as reported herein was in a pioneer state. The application trials showed that it is not only practicable but advisable to use a wattmeter directly on high-voltage circuits. The next advance in the development of the wattmeter was the evaluation and elimination of the error powers due to capacitance effects in the loading and voltage circuits. Such study was undertaken by Messrs. Carroll, Peterson and Stray in the Stanford high-voltage laboratory. Their results have been embodied in a paper entitled Power Measurements at High Voltages and Low Power Factors presented concurrently to the American Institute of Electrical Engineers.

Discussion

For discussion of this paper see page 1162.

Power Measurements at High Voltages
and Low Power Factors

BY JOSEPH S. CARROLL
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THOMAS F. PETERSON
Student Member

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Student Member

Graduate Students, Leland Stanford Jr. University

Review of the Subject.—*A description is given of the apparatus for the measurement of power as low as a fraction of a watt at power factors approaching zero and voltages as high as 175 kv. to neutral. A standard make of portable wattmeter was used having maximum ranges of 1.5 watts, 37.5, 75 and 150 volts and 20 per cent power factor. To this wattmeter was adapted a three-megohm water column resistance multiplier.*

In the original form of the wattmeter the origins of certain errors, due to capacitances chiefly, were studied and methods for their elimination were subsequently determined. Integrity tests were developed by which the values of error powers were obtained and used as guides in making subsequent adjustments for the reduction of such errors to zero. These integrity tests were checked by using the wattmeter for the measurement of known amounts of power.

Reconnaissance studies were made of the voltage-corona power loss relations for rope laid copper, 0.91-in., lock wire smooth copper, 0.91-in. and concentric strand aluminum, 1.006-in. diameter transmission line conductors in rain and fair weather at differing degrees of humidity, temperatures, and barometric pressures, from "initial" to "full" corona formation. This class of studies was extended to corona loss values as offered by a wide variation of the "surface roughness" of conductors subjected to high voltages. The losses for a single brush were measured and the shielding effect of groups of brushes was studied. Losses to single strings of insulators under different conditions were determined.

THE sensitive, low-power factor, high-voltage wattmeter has been awaiting solution through twenty years or more. In the meantime, because of the lack of this wattmeter, satisfactory dimensional data over a wide range of important high-voltage actions have been difficult if not impossible to obtain. The promise for success of the high-voltage wattmeter using a water column resistance multiplier, leads to the conviction that it could be improved considerably, and that a series of high-voltage reconnaissance tests could be undertaken that would define lines along which further helpful studies could be made of the losses produced in brush discharges, local corona and insulators on high-voltage power transmission lines under conditions varying as to equipment forms, climate and altitude. The primary purpose of the present undertaking was, therefore, to make a series of reconnaissance tests of the above sort after improving the accuracy of the high-voltage wattmeter to the requisite degree.

The following study was made of the origin, nature and elimination of the errors occurring in the high-voltage wattmeter reported upon concurrently by Clark and Miller,[1] which will be referred to hereafter as the 1923 wattmeter.

At the outset of the work, when engaged upon laboratory transmission line corona loss measurements, certain peculiarities were observed in the indications of the wattmeter when the coil connections were changed. There were many possibilities as to the cause of the trouble. A systematic study of them was, therefore, made.

The usual low-voltage wattmeter errors such as self- and mutual-inductance of coils, electrostatic attraction, wave form, frequency, etc., were soon found to be negligible in the present case. The most serious of these, self-induction of the potential coil, produced, at the most, only 1 per cent error. The important source of error was found to be the capacitance in multiple with instrument coils. The schematic diagram of connections, Fig. 1, shows that the leads and their shields in the water column and line constitute condensers which shunt the wattmeter coils. These act to produce error by causing currents in the coils to differ from those in the water column and line, both in magnitude and

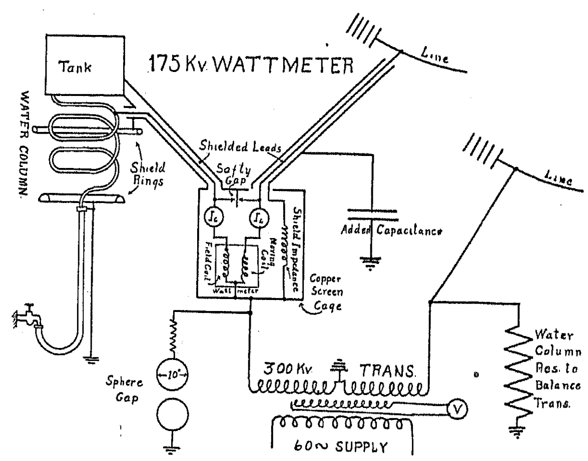


FIG. 1

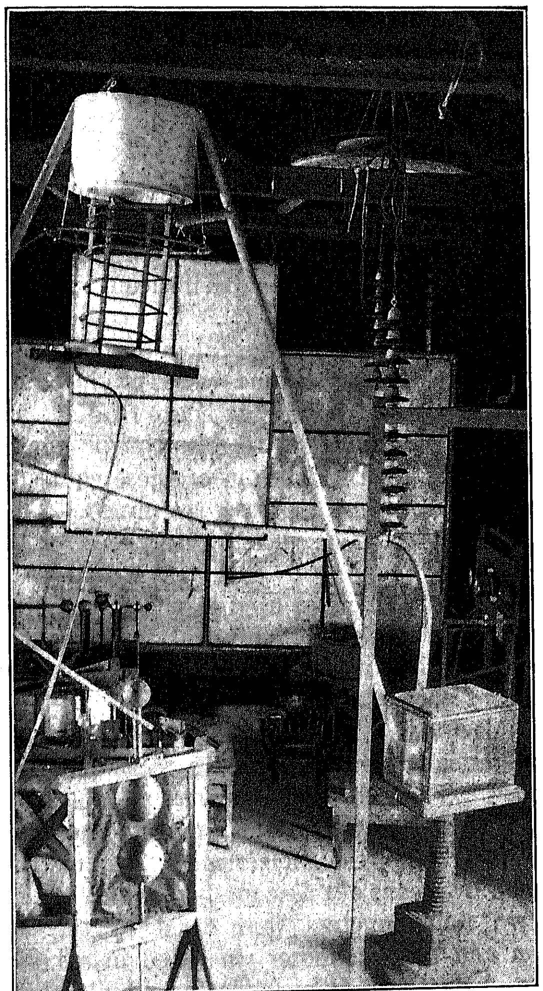
phase position. Of these, the shift in phase is the only one of importance and is constant for fixed connections. The error reading, due to capacitance across the coil in series with the line, varies as I_L (current to line) and I_c (current to water column). That due to the other capacitance varies as $I_L\,I_c$. Since I_L and I_c are both very nearly proportional to voltage, E, the net error power varies as E^2.

The error due to the capacitance between the coil lead to the line and its shield was eliminated by equal-

1. Philip C. Clark and Charles E. Miller, The High Voltage Wattmeter, TRANS. A. I. E. E., Vol. XLIII. p. 1125, 1924.

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., Oct. 13-17, 1924.

izing their potentials. This was done by connecting the shield to the source through a suitable impedance mounted within the instrument shield cage as indicated in Fig. 1. The resistance-inductance ratio of this impedance was made the same as that of the coil and corresponding milliammeter connected to the line. The values of this impedance and the capacitance of lead-to-the-line shield to neutral were then adjusted so as to consume the same voltage as that consumed in the coil and milliammeter by the line charging current.

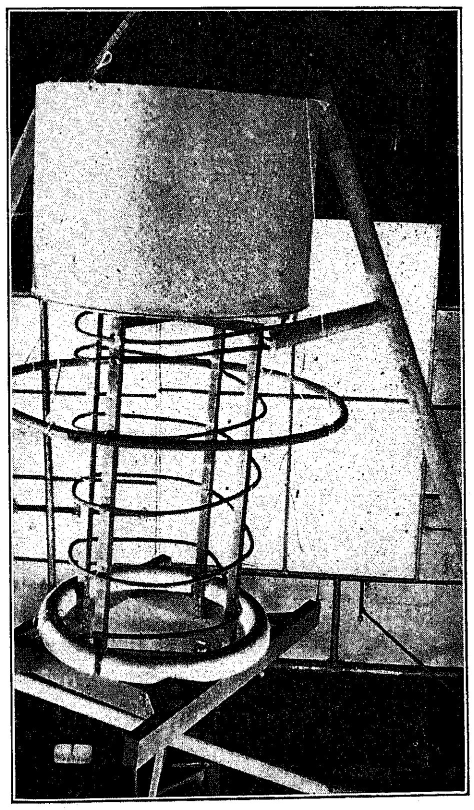


General View of High Voltage Wattmeter Showing Instruments in the Copper Screen Cage, the Water Column Multiplier and the Sphere Gap

The difference of potential then applied to the error-making capacitance was thus brought to zero and the error eliminated.

The companion error due to the capacitance between the voltage coil and milliammeter lead to the water column multiplier and its shield was reduced to a negligible value by a sufficient structural reduction of such capacitance. This was accomplished by making the lengths of the lead and shield a minimum and by using a relatively large diameter (2.5 in.) of shield and correspondingly small diameter of lead (No. 14 B. & S. gage) mounted axially within the shield. Thus arranged, the errors due to the capacitances shunting the wattmeter coils and milliammeters were eliminated.

Further studies of the 1923 wattmeter revealed another difficulty. It came up in the effort to repeat measurements of fixed corona losses. It was found that the values could not be repeated with sufficient agreement. The cause of the difficulty was finally traced to the highly undesirable manner in which the outer shielding water column functioned. The water multiplier and water shielding columns did not, in warming up, form exactly the same temperature patterns. The conductivity of ordinary potable water such as was used, changes rapidly with temperature. Varying potential differences were, therefore, caused between the multiplier and shielding water columns. Corresponding charging currents were, as a consequence, set up through the comparatively large capacitance that existed between the two columns giving rise to corresponding error-powers that would develop



Shielded Water Column Resistance Multiplier for Wattmeter

and vary as the temperature patterns changed. It was found impracticable to maintain equality of such temperature patterns. The shielding water column used in the 1923 wattmeter was, therefore, entirely abandoned.

Another form of water column multiplier was designed and constructed so as to reduce the errors due to capacitance substantially to zero. A quarter-inch air hose was used which allowed shortening the net length to 20 feet or one fourth the length of the hose in

the former column. It was wound on four bakelite strips and formed a helix eighteen inches in diameter and about three feet long. To insure a sufficient flow of water through such a small hose to keep it from overheating, the pressure of the tap water supply was used to force the water through it up into the tank above. In mounting it, the water column was placed as far above the ground as practical, which was about 12 ft. The capacitance of the tank was eliminated by connecting the wattmeter just one turn of hose below the tank, the latter being connected directly to the high-voltage supply. It is true that the water in this section of five feet of hose shunts the wattmeter coil; however, the resistance of this part of the water column was about half a megohm, whereas the total impedance of the wattmeter coil and milliammeter was only 1160 ohms,

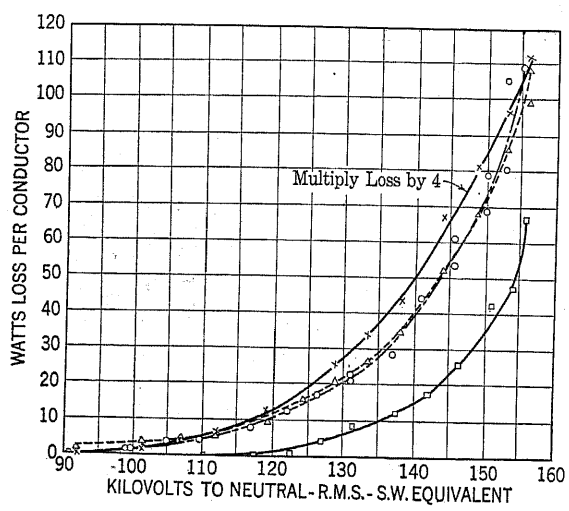


FIG. 2—CORONA LOSS. CONCENTRIC STRAND CABLE
Diameter 1.008 in., length 231.5 ft.

Curve	×	△	⊡	⊙
Date	3	3	5	22
Barometer	76.327	76.327	76.620	75.743
Temp. °F	54	54	64	62.5
Rel. Humidity	..	68%	20%	51%
Vapor Product	..	0.1935	0.0239	0.1445
Time	11:20 a. m.	4:00 p. m.	10:10 a. m.	3:44 p. m.
Weather	Lt. Rain	Fair	Fair	Fair

so that the error introduced by the connection was negligible. With this arrangement the capacitance of the water column was very small. The tank itself served as a good shield, and only a little additional shielding was found to be necessary. This was accomplished by placing a ring, made out of one-inch metal tubing, around the water column. The ring was suspended from the tank to which it was connected and stood out concentrically from the water column about eight inches. The distribution of the shielding field was further improved by placing another ring at the bottom of the water column, connecting it to the ground wire at the end of the small hose.

When the true power was known to be small, it was found that the shields could be so placed that the wattmeter could be made to give a negative reading. The meaning of this was that the error power through adjustment of the shields could be reversed in sign. From this it followed that if the values of the error powers were known they would furnish a guide to find the positions of the shields whereat the magnitude of the error power would be reduced to zero.

A method was, therefore, developed to segregate the true and error power constituting the corresponding aggregate indicated by the wattmeter as follows: The real power due to a few watts (say 30 to 50) of line corona loss and the error due to the water column multiplier capacitance were held rigidly constant through fixed frequency and crest voltage. The latter was controlled by a sphere-gap By varying the flow of the water in the column, its temperature and thereby its resistance could be changed. In this way I_c (in phase with the voltage) could be varied by any fractional amount. Under these conditions the part of the wattmeter reading due to actual power increased by the same proportionate amount as I_c, while the residual part of the reading due to capacitance error remained constant (independent of I_c). Remembering these facts, with two sets of readings taken at different values of I_c, two simple equations were produced for the equalities of the wattmeter aggregate readings and the sums of the corresponding true and error power portions thereof. By the solution of these equations the value of the error reading was determined. Then, by trial, shielding could be varied until the error became zero.

The Weston wattmeter used in these measurements was a sensitive specially designed instrument. The full scale reading of 1.5 watts would be obtained at 20 per cent power factor with maximum rated current through each coil. The currents to the line and water column resistance multiplier were measured by milliammeters. These three meters were all inclosed in the copper screen cage which was connected to the high-voltage terminal of the transformer. The instruments were read by means of telescopes about six feet distant. The two milliammeters were used in the vertical position and the wattmeter was used in a horizontal position, being read from a platform built above the screen cage.

As a test of the ability of the wattmeter to measure actual power, a special test was devised. A 6000-ohm resistance consisting of two Thomson wattmeter multipliers was inclosed in a cylindrical metal shield with hemispherical ends, one end of the resistance being soldered to the shield and the other end brought out through an insulating bushing. A regular set of readings was taken at some definite sphere-gap setting then this resistance of 6000 ohms was inserted in the circuit at the point where the wattmeter lead connects on to the line and another set of readings taken at the same sphere-gap setting as before. Knowing the line current and this added resistance, the power absorbed by it could be computed and compared with the increase in

power as shown by the wattmeter. These results checked very well.

As a check on the reliability of the method of separating real and error power by changing the resistance of the water column by diminishing the flow of water, the following test was made: A set of readings was taken at a definite line voltage with the water flowing freely through the hose; the voltage was there held constant and the resistance of the water column decreased by restricting the flow of water. When the current through the water column had reached a predetermined value the meters were again read. The power was computed from the differences of the two wattmeter readings and the two currents through the water column. The resistance was then inserted in the line circuit and the procedure repeated at the same voltage as before. The difference in the power computed from these two tests should be the $I^2 R$ loss in the resistance. The following are the results of these tests at two different voltages:

Line Kv. V_H	I_L	I_C	W	Actual Power*	Mean Value
		No resistance in line			
240	31.9	38.6	0.062		
240	31.3	70.0	0.081	14.7	
240	31.8	38.6	0.058		
240	31.2	70.0	0.078	15.5	15.1 (a)
170	23.1	26.1	0.010		
170	22.8	60.0	0.010	0	
170	23.1	26.1	0.011		
170	22.8	60.0	0.012	.5	.2 (b)
		5989 ohms res. in line			
240	31.6	38.4	0.065		
240	31.0	70.0	0.097	24.7	
240	31.5	38.5	0.065		
240	30.9	70.0	0.093	21.6	23.1 (c)
170	23.0	26.1	0.012		
170	22.7	60.0	0.022	5.1	
170	23.1	26.1	0.013		
170	22.7	60.0	0.022	3.5	4.3 (d)

*Computed from the increase in wattmeter reading produced by the increase in I_C.

As can be seen from the data, the measured power increased when the resistance was inserted. The amount of power absorbed in the resistance is given by the difference of (a) and (c) for the one voltage and from (b) and (d) for the other voltage. These values are 8.0 watts and 4.1 watts and the loss computed from the resistance and the line current is correspondingly 6.0 watts and 3.1 watts. It was indeed gratifying to know that such small values of power could be measured with fair accuracy at such high voltages and low power factors. The accuracy here was not quite 100 per cent, however, considering the magnitude of the wattmeter reading, and also that the greater part of it was error power, the results are as close as would be expected. It was this test which showed that true power could be separated from error power by the method of changing the water column resistance.

The test was made before the water column had been sufficiently shielded to give correct power from a single set of readings. The shielding was then adjusted until the computed power was the same, irrespective of the resistance of the water column. The following are data of tests in which the power due to corona loss

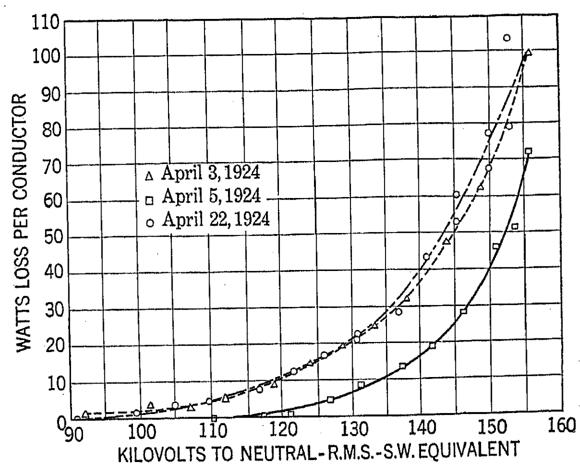


FIG. 3—CORONA LOSS. CONCENTRIC STRAND CABLE

Diameter 1.008 in., length 231.5 ft. Corrected to vapor product of 0.15 and air density of 1.000.

on the line was measured with different currents through the water column:

V_H	I_L	I_C	W	Power: By Watt-meter	Power: Computed from $W_2 - W_1$ and $I_{C2} - I_{C1}$
		April 3, Aluminum Con. Cable in Light Rain (I_L held constant)			
190	..	43.5	0.018	8.0	
190	..	70.0	0.028	7.7	7.3 (Water up)
210	..	44.3	0.050	24.0	
210	..	70.0	0.077	23.4	22.4 (Water up)
		April 4, Lock Cable. (Dry)			
210	27.6	49.5	0.0005	0.21	
210	27.6	70.0	0.0005	0.15	0 (Water down)
210	27.6	44.0	0.004	1.9	
210	27.6	70.0	0.006	1.8	1.6 (Water up)
		April 8, Lock Cable. (Dry)			
255	34.1	61.4	0.090	38.0	
244	34.2	70.0	0.103	38.1	39.6 (Water up)

The results were very satisfactory, showing not only that the water column capacitance error power had virtually been eliminated but also that the power loss below visual corona, if any, was extremely small.

THE CORONA LOSS—CREST VOLTAGE RELATION AT VOLTAGES UPWARD FROM CRITICAL VALUES

The value of the crest voltage in all corona and related actions is controlling.[2] The measured values

2. Harris J. Ryan and Henry H. Henline, The Hysteresis Character of Corona Formation. See Page 1118 of this Volume.

of power are believed to be as accurate as necessary as long as the sphere-gap must be used to determine crest voltage. It is only accurate to two per cent, but a two-per-cent change in crest voltage may cause a far greater change in corona loss because of the character of the corona loss-power relation.

The method of test was to set the sphere-gap and raise the voltage until the sphere-gap sparked-over. At that instant the voltmeter, wattmeter, and both milliammeters were read. This procedure was repeated at different sphere-gap settings until a sufficient number of values had been obtained. Power was

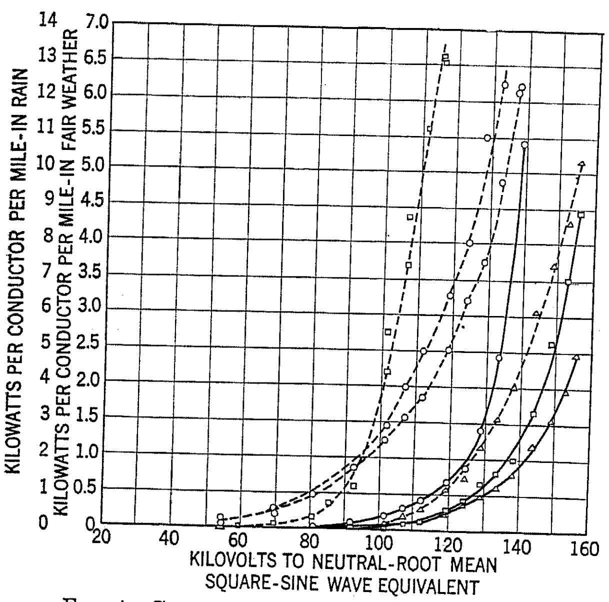


FIG. 4—CORONA LOSS PLUS INSULATOR LOSS

Curve.......	⊙	⊡	⚠	⊙	⊡	⚠
Cable.......	rope lay	lock wire	conc. str.	rope lay	lock wire	conc. str.
Material.....	copper	copper	aluminum	copper	copper	aluminum
Diameter....	0.91 in.	0.91 in.	1.008 in.	0.91 in.	0.91 in.	1.008 in.
Circular mils.	500,000	700,000	806,600	500,000	700,000	806,600
Date........	3	3	3	1	1	3
Barometer...	30.05 in.	30.05 in.	30.05 in.	29.98 in.	29.98 in.	30.05 in.
Temp. °F....	54	54.5	54	53.5	53.5	54
Rel.humidity	51%	66%	68%	..	..	..
Time........	4:45 p.m.	4:20 p.m.	4:00 p.m.	2:20 p.m.	3:00 p.m.	11:20 a.m.
Weather.....	Clear	Clear	Clear	Lt. rain	Mod rain	Lt. rain
Spacing.....	18′	17′	17′	18′	17′	17′

computed from the meter readings by the following formula:

$$P = \frac{W_m \dfrac{V_m \times 1000}{2}}{4931 \times 0.001\, I_c}$$

Where W_m is wattmeter reading, in watts, V_m is voltmeter reading in volts, I_c is current to the water column in milliamperes, 4931 is the wattmeter constant. P is the power in watts, and the other constants take care of voltage to neutral, and the proper units. I_L does not enter into the computation of power but was measured for other purposes.

The cables used in the tests were rope-lay, concentric strand and lock wire. The rope-lay and concentric strand cables are familiar to everyone. The lock wire cable is similar to that used on aerial tramways and has a smooth exterior.

The curves of corona loss in Fig. 4, show that the loss is less than that found by most previous investigators. Corona loss starts at approximately the visual corona voltage and the loss below this point, if any, is negligible. The rope-lay cable had a much rougher surface than the other cables and therefore might be expected to have a larger loss. At 130 kv. to neutral, the loss in fair weather was from 1.4 to 1.6 kw. per conductor per mile. At 120 kv. the loss was from 0.5 to 0.7 kw. per conductor per mile. In rain, the loss at 130 kv. is 9.5 kw. per conductor per mile. The loss at 120 kv. is 6.0 kw. per conductor per mile or twelve times the fair weather loss. The loss in rain starts at below 50 kv. to neutral while the loss in fair weather starts at from 70 to 90 kv. to neutral.

In fair weather the loss on the lock wire cable is very low. At 140 kv. to neutral the loss is from 0.35 to 0.9 kw. per conductor per mile while at 120 kv. the loss is only from 0.02 to 0.25 kw. per conductor per mile. In rain the loss is much greater. At 110 kv. to neutral the loss is 9.5 kw. per conductor per mile or nearly 200 times the fair weather loss for the same voltage.

The loss on the aluminum concentric strand cable is much less than would be accounted for by the fact that it is somewhat larger in diameter than the others. In dry weather at 140 kv. to neutral the loss is from 0.9 to 1.25 kw. per conductor per mile. At 120 kv. to neutral the loss is from 0.15 to 0.30 kw. per conductor per mile in fair weather. In rain the loss is about 4.5 kw. per conductor per mile at 140 kv. and about 1.2 kw. per conductor per mile at 120 kv. to neutral. This means that the wet weather loss is about four times the fair weather loss through a specified range of voltage.

From an inspection of the curves shown in Figs. 2 and 3, it is seen that the amount of moisture in the air has a great influence on corona losses of this order. Mershon[3] found that corona loss was a function of the vapor product. He defined vapor product as the product of the relative humidity in per cent and the vapor pressure in inches of mercury. Approximate equations of Mershon's corona loss vapor product curves were obtained and by them our results were reduced to the same vapor product. As can be seen by the comparison of Fig. 2 and Fig. 3, the curves of the concentric strand cable were somewhat closer together after correcting for vapor product. Although corona loss may vary as a function of the vapor product, the results of these tests do not follow Mershon's curves. Mershon found that the vapor product affected the "critical point" or point where the law of the curve changes. The critical point on his curves corresponds to the voltage

3. Mershon. High Voltage Test. TRANS. A. I. E. E., Vol. XXVII, p. 845, 1908.

where corona loss starts on our curves or approximately visual corona voltage.

The moisture in the atmosphere seems to affect the starting point of corona and the part corona loss, but it has little effect on the full corona loss. From the results of present tests it would seem that not only was the starting point of the curve shifted but the shape of the curve was changed by the amount of moisture in the air. When these values of loss are plotted on semi-logarithmic paper (Fig. 5) they seem to indicate the existence of three laws since three intersecting straight lines are formed. The slopes of the upper and lower straight lines are practically independent of the

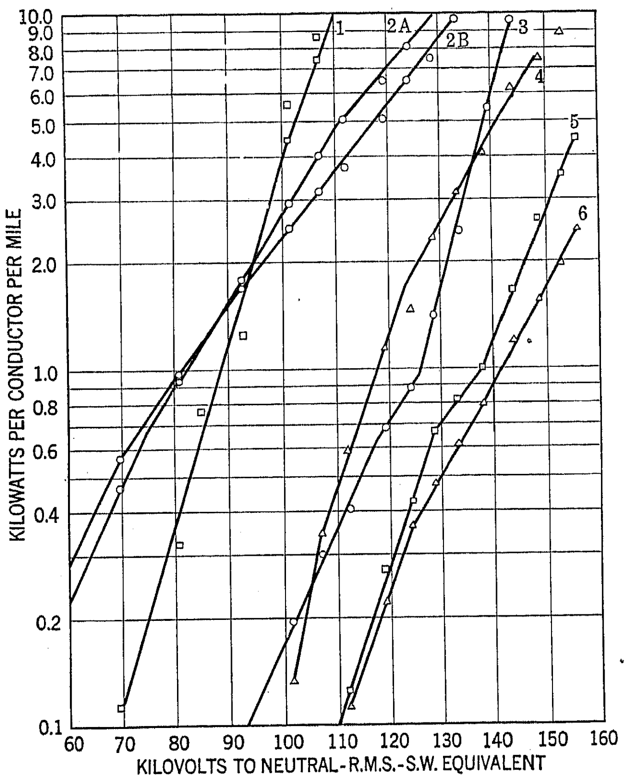


Fig. 5—Corona Loss Plus Insulator Loss

1. Lock Wire—Rain
2. A. Rope Lay—Rain
3. B. Rope Lay—Fair
4. Concentric Strand—Rain
5. Lock Wire—Fair
6. Concentric Strand—Fair

humidity, but the slope of the middle line seems to depend on the amount of moisture in the air. It has been suggested that these three lines refer to brushes from clamps, part corona and full corona.

Insulator Losses

Insulator losses on a ten-unit string were measured in the laboratory building at 150 kv. to neutral. For cap and pin insulators with skirts the loss was about 7 watts dry and dusty, 5 watts dry and clean, 4¼ watts wet on top, 4¾ watts when sprayed from top; 9 watts when wet and dirty and 450 watts when wet all over. At 150 kv. the loss on a ten-unit string of core-and-tine insulators was found to be about 10.25 watts dry and clean; 10 watts when sprayed from top and 150 watts when wet all over.

Through corona loss tests on the laboratory yard tower line by difference between corona loss with one supporting insulator string excluded and again included in the wattmeter circuit, the loss on a 9-unit string

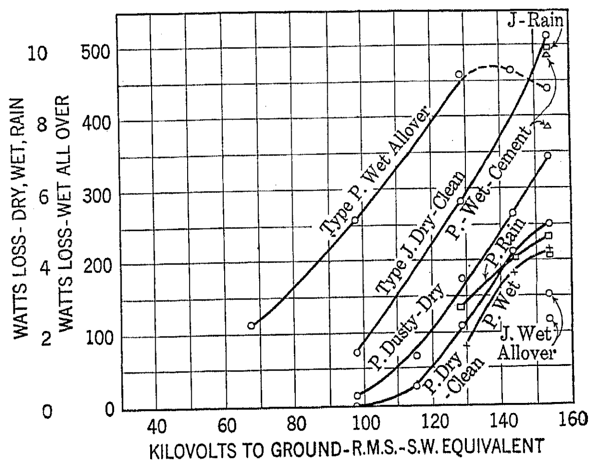


Fig. 6—Insulator Losses on 10-Unit String

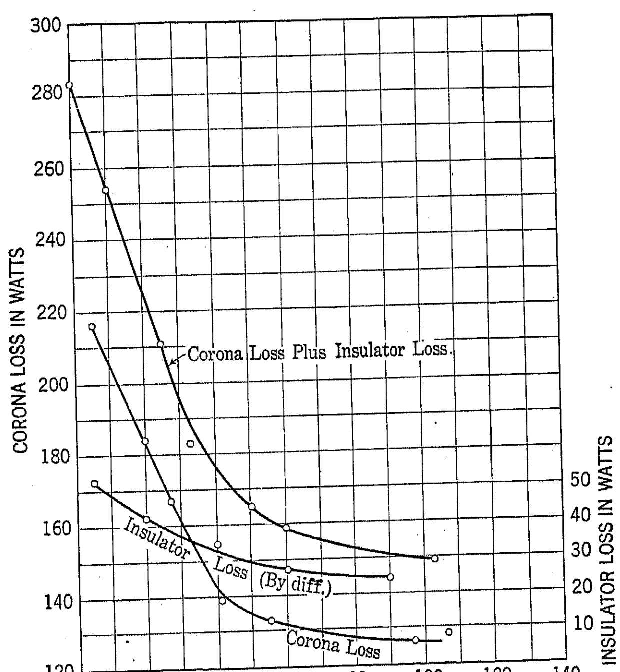


Fig. 7—Corona and Insulator Losses

Lock Wire Cable. Temp. 57 deg. to 62 deg. fahr. Barometer 29.94 in. Relative Humidity 77 to 67 per cent. Time 9:10 to 11 a. m., April 12, 1924. High Fog—to clear 151.3—150.0 kv.

of cap and pin-type insulators with skirts was found to be about 50 watts at 150 kv. when wet with dew. By the same method the loss on the same string of insulators was found to be 20 watts at 135 kv. in the rain.

From these results it is seen that insulator losses

in dry weather are negligible. In light rains the loss is still low but in heavy rains accompanied by wind the insulator losses may become very high. From the few observations made it seems that the loss when dew is on the insulators is greater than during light rains.

Brush Losses

Measurements were made of the power loss-voltage relation for a variety of corona brushes, singly or in multiples. When the number of brushes was small, 1 to 6, a wire was run out from the high-voltage source terminal through the wattmeter, a distance, generally, of about six feet. With its concentric shield, 2.5 inches in diameter, it was supported horizontally on insulators about four feet up from the concrete floor. The tubular shield was terminated with a three-inch sphere.

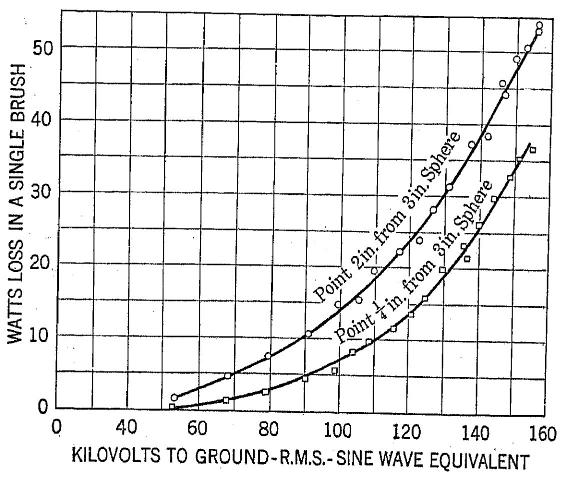


Fig. 8—Brush Losses

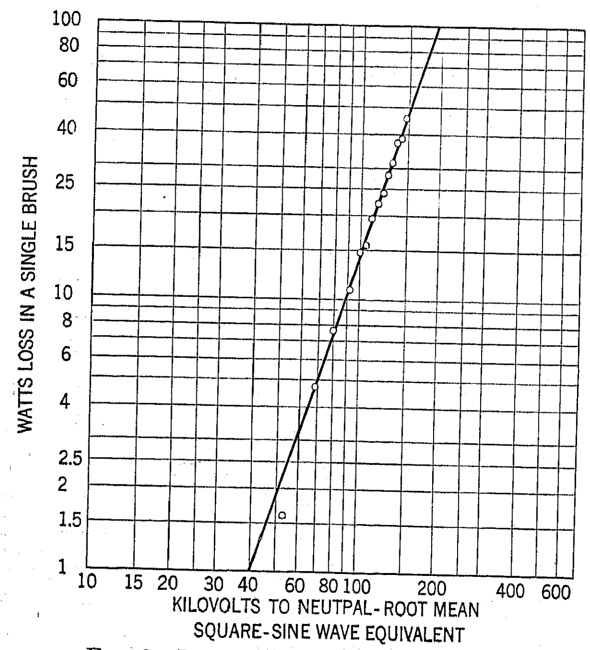


Fig. 9—Power Loss in a Single Brush

Barometer 29.89 in. Dry Bulb Temperature 51 deg. fahr. Wet Bulb Temperature 52 deg. fahr.

The lead wire was extended a convenient distance beyond the spherical end of the shield as specified later. From it from one to six points were mounted from which to set up brushes.

When the brushes to be formed were many (from a few to 500) the requisite points from which to form them were provided by cementing the heads of small (*one-quarter-inch*) tacks to the surfaces of metal tubes having diameters and lengths to be specified later. Both ends of the tubes were suitably shielded. Thus arranged, the wattmeter measured only the losses set up from the points as desired.

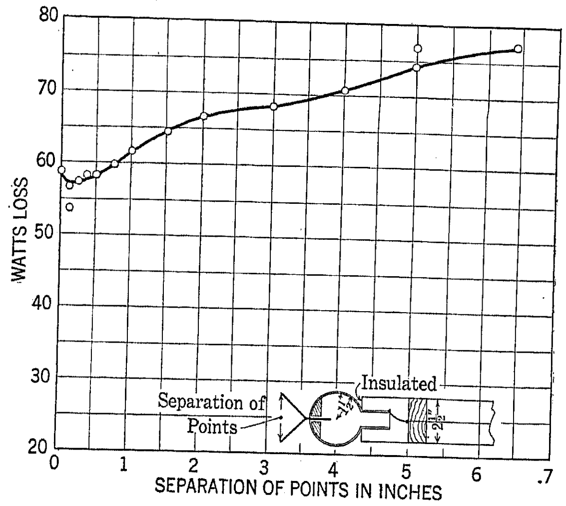


Fig. 10—Brush Loss

2 Points in a "V" Temp. 70 deg. fahr. Barometer 29.96 in. 154.0 kv. Relative Humidity 20 per cent. Points 3 in. from 3 in. Sphere.

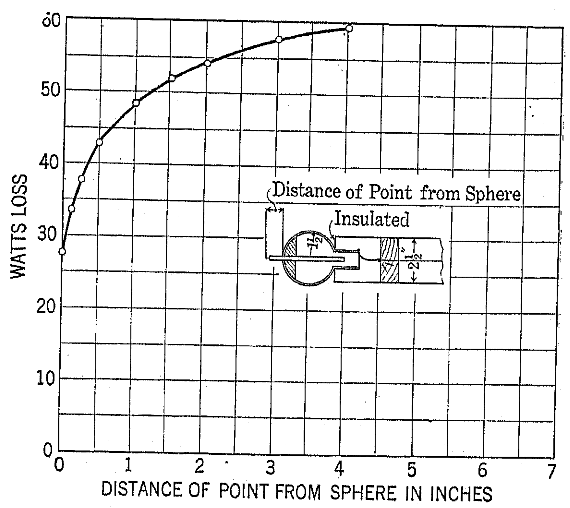


Fig. 11—Effect of Point Length on Brush Loss

Temp. 70 deg. fahr. Barometer 29.94 in. 154.3 kv. Relative Humidity 33 per cent.

The loss from a brush from the end of a wire or rod is much larger than the loss from a single brush on a line conductor cable. The brushes on a cable crowd each other. It is not surprising, therefore, that the loss per brush is less when there are more brushes.

The curve of a single brush located on logarithmic paper by losses and corresponding voltages (Fig. 9)

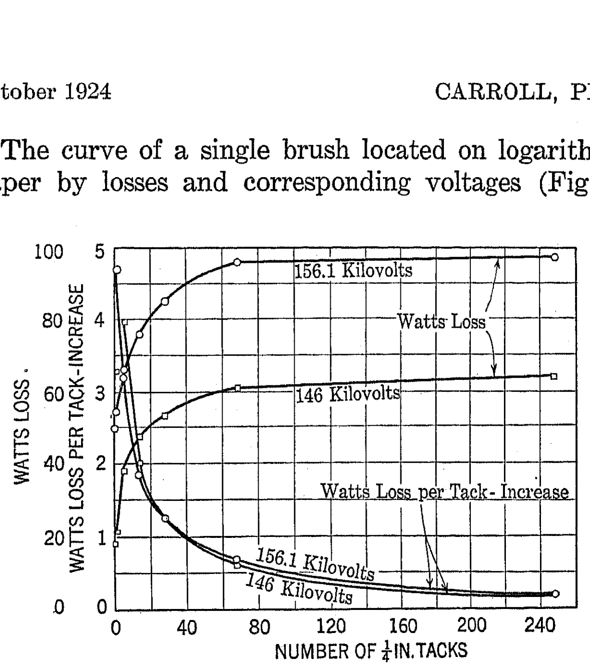


FIG. 12—EFFECT OF TACKS ON CORONA LOSS FROM CONDUCTORS
Length 2 ft. 3 in. Aluminum Tube Diameter 17/32 in.

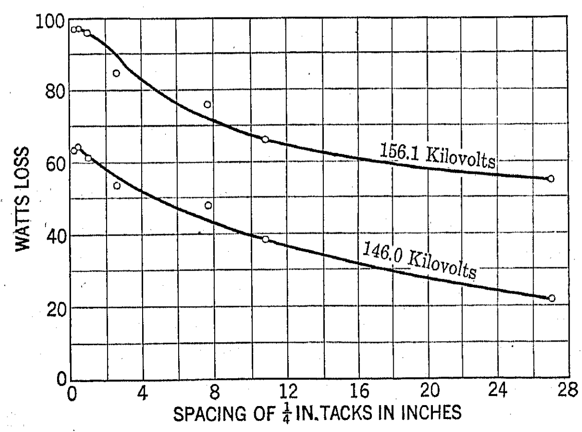


FIG. 13—EFFECT OF SHIELDING ON BRUSH LOSS FROM CONDUCTORS
Length 2 ft. 3 in. Aluminum Tube Diameter 17/32 in.

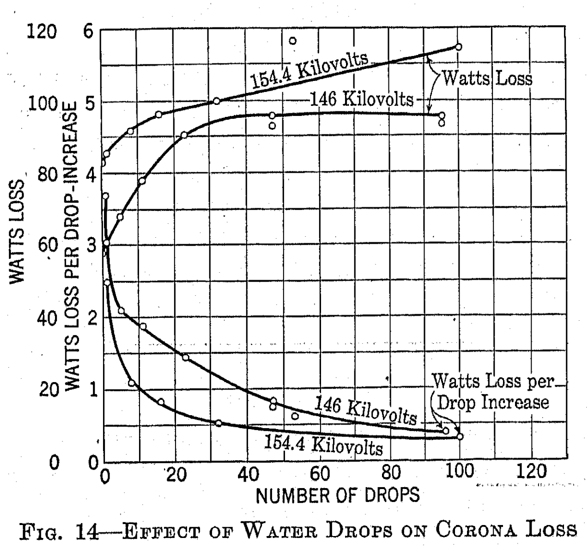


FIG. 14—EFFECT OF WATER DROPS ON CORONA LOSS FROM CONDUCTORS
Length 6 ft. Brass Tube Diameter ½ in.

indicates that the brush loss in the particular circumstances varies over a wide range with the cube of the voltage. For other brush losses the curve seems to be similar but the variation of loss is not as the cube but as some other power of the voltage.

The curve of Fig. 10 shows clearly the shielding effect of one point upon another. The curve of Fig. 11 likewise shows the shielding effect of the sphere from which the point projects.

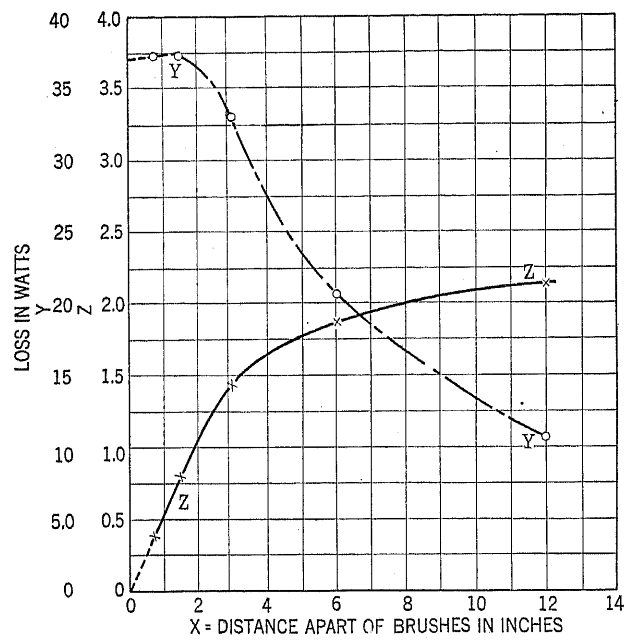


FIG. 15—CORONA LOSS TESTS

Effect of Mutual Shielding. Loss due to Brushes on ½ in. Polished Brass Tube 6 ft. long at 138 kv. R. M. S. to Neutral. Brushes Formed on Water Drops. Dry Tube Loss—57.5 W. $Z-Z$ Loss per Brush. $Y-Y$ Loss due to all Brushes on 6 ft. Length. $Y = \left(\frac{72}{x} - 1\right) Z$

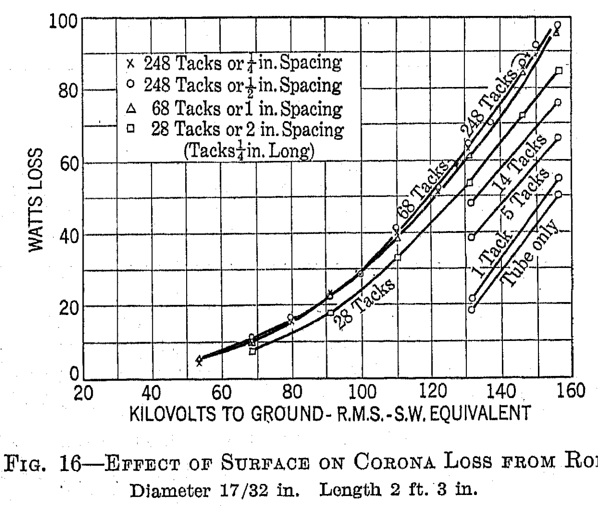


FIG. 16—EFFECT OF SURFACE ON CORONA LOSS FROM ROD
Diameter 17/32 in. Length 2 ft. 3 in.

Figs. 12 to 15 show the effect of surface irregularity and of water drops on corona loss as well as the shielding effect of the brushes from water drops and tacks. Figs. 16 and 17 show the effect of surface roughness on corona loss. Tacks were equally spaced on a rod and the corresponding curves were obtained as shown.

Loss from Conductors as Affected by the Surface

The curves of loss from a brass tube are especially interesting because they are virtually straight lines. The loss with a polished tube was considerably lower than it was when coated with soot from a candle flame. The loss from the tube when its surface carried its natural tarnish was still higher than when soot covered.

It is interesting to note that the loss from a section of the cable from the Big Creek Line covered with a natural deposit of carbon was three fourths that from a section of new cable under the same conditions.

From the curves of brush and corona losses as affected by surface conditions it would seem that the loss increases as the number of projections on the surfaces up to certain points and then remain practically constant or they may decrease slightly as the number of projections is greatly increased. This may be understood from the brush loss curves. The brushes stretch out quite a distance into the air around them and therefore are affected by what is taking place near them. When they are too close together the loss per brush will be decreased, but since there must be more brushes to accomplish this the total loss will be more. There is a final limit where the loss per brush decreases faster than the number of brushes increases and therefore the total loss becomes less. This explains the distribution of brushes on a conductor. Where opportunity for the formation of brushes is favorable they will form only so close together that their field will not decrease the total loss. When they are too close together their number is decreased and the total loss is again increased to the maximum. In other words, brush formation stabilizes toward the production of maximum loss.

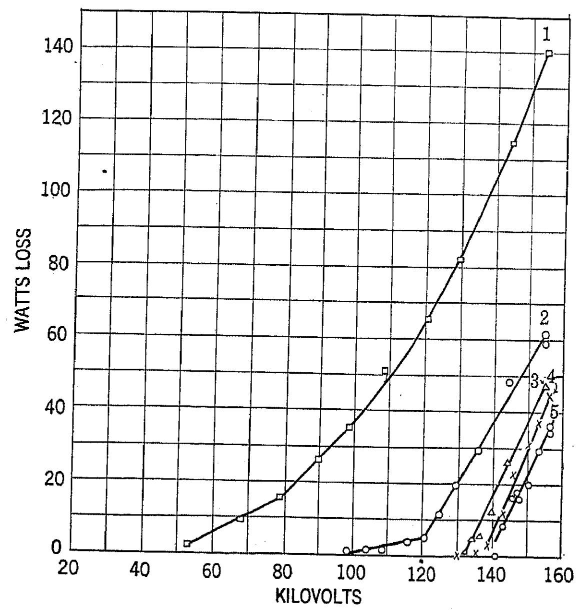


Fig. 17—Effect of Surface on Corona Loss

Curve	1	2	3	4	5
Conductor	Al. tube	Al. tube	Brass tube	Brass tube	Brass tube
Surface	Wet string	Dry string	Natural	Soot coated	Polished
Length	5 ft.	5 ft.	5 ft.	5 ft.	5 ft.
Diameter	17/32in. 5/8in.	17/32in. 5/8in.	5/8 in.	5/8 in.	5/8 in.
To ground	3 ft. 7 in.	3 ft. 7 in.	3 ft. 6 in.	3 ft. 9 in.	3 ft. 9 in.
Barometer	29.99 in.	29.99 in.	29.99 in.	30.08 in.	30.08 in.
Temp. °F	69 deg.	69 deg.	69 deg.	64 deg.	64 deg.
Rel. humidity	39%	39%	39%	45%	45%

Conclusions

1. It has been found practicable to separate true and error power.
2. The power loss from transmission conductors below visual corona, if there is such a loss, is extremely small.
3. Humidity is a "part" or "local" corona loss factor.
4. Losses on dry insulators are small; when completely wet may be relatively high.
5. Rain increases corona losses considerably, the amount of such increase depending upon the type of conductor.
6. For a given length of conductor with points on it, the loss per brush decreases with the number of brushes and the total loss increases with the number of brushes up to a certain value and then remains constant or nearly so.

Acknowledgments

Though the names of the authors will usually be associated with the results of these studies, when credit is to be conferred, several others must be considered. Prof. Ryan, always inspiring, has been a very dependable source of ideas for the solution of problems. Prof. Henline has greatly assisted, not only in the operation of equipment but in many other ways such as pointing out errors in methods of measurement. It is with great pleasure that we acknowledge this and express our deep appreciation to them as well as to others who have been involved in the work.

Discussion

For discussion of this paper see page 1162.

Fair Weather Corona Losses at 60 Cycles
on Large Overhead Power Cables

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Review of the Subject.—*After defining briefly the problem of the selection of a proper high-voltage transmission conductor, the authors discuss the difficulties that caused the failure of their earlier attempts to measure corona losses at the Stanford University laboratory.*

The equipment used for the present investigation, as well as the methods of test and the procedure followed in the reduction of test data, are described. The corona loss is taken as a function of the crest value of the voltage in accord with the theory of Professor Ryan. Insulator losses in fair weather are found negligible.

Errors existed in the authors' measurements of the losses, causing them to be high, but a correction is made by subtracting the "error power" from the gross power curves.

The cable samples are briefly described.

Complete tables of tabulated data are presented in Appendix A; and some idea of the consistency and reliability of the results may be obtained from these tables, and from the various curves.

INTRODUCTION

ATTENDANT upon the design of a large-power high-voltage overhead transmission comes the problem of the choice of a conductor having not only all the obviously essential mechanical properties, such as strength, flexibility, durability in abnormal as well as normal operation, but it must possess a desirable electrical character in addition to the requisite conductivity: namely, such a diameter and configuration of section that corona loss will not exceed an economic limit at the selected operating voltage and under the imposed conditions of weather and altitude for the proposed line. The urgent need of more data on the corona loss from medium and large-size cables in order to meet this problem has resulted in an attempt to obtain such data experimentally at the high-voltage laboratory of Stanford University.

EARLIER STANFORD EFFORTS—METHODS AND EQUIPMENT

Prior to the summer of 1923, efforts to measure corona

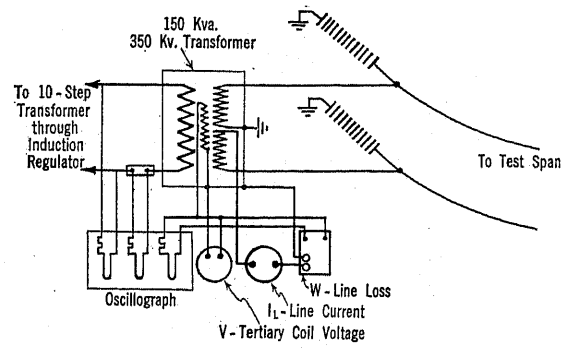


FIG. 1

loss with a wattmeter connected as shown in Fig. 1 were unsuccessful, probably due mainly to the fact that the amount of power going into corona loss on the connections from the high-voltage transformer to the test-cables was very large in comparison with the loss on the actual test-cables. It is also quite possible that variations in the wave form of the testing voltage were responsible for many of the great discrepancies encountered in corona loss curves made at different times, but with apparently identical conditions as to tem-

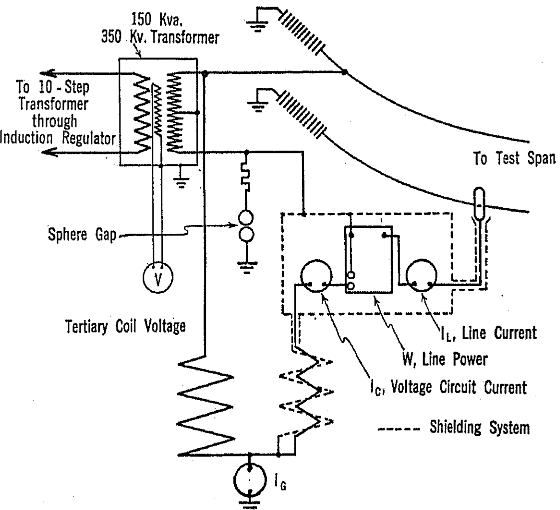


FIG. 2

perature, barometric pressure, and humidity. It is not possible to say that the earlier measurements could have been made successfully had the foregoing obstacles been removed, since there may have been still other sources of trouble that were quite masked by these prominent ones. However, it seems highly probable that the method could have been studied and refined until small losses were capable of accurate measurement.

EQUIPMENT—SUMMER OF 1923

The Source Transformer. The source of high voltage in the Stanford laboratory is a General Electric testing transformer bearing the following name-plate data: "No. 1198244, Type K, Form EV, Capacity 150 kv-a., volts 300,000-200/400/800." The transformer con-

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., October 13-17, 1924.

tains a Hendricks tertiary "voltmeter coil" giving a faithful reproduction in volts of the secondary voltage in kilovolts. Continuous adjustment over the range of 0-350 kv. between terminals, or 0-175 kv. to ground is secured by employing a 3.6-kv-a. hand-operated induction regulator arranged by a suitable switching arrangement, including the full use of both its bucking and boosting actions through reversal of its primary winding, to cover the range between adjacent taps of a 50-kv-a. 10-step secondary 2200-220-volt transformer receiving the 60-cycle power from the University substation.

The Sphere-Gap. The sphere-gap used was constructed according to dimensions given by Farnsworth and Fortescue (TRANS. A. I. E. E., Vol. XXXII, 1913, page 733). 10-inch spheres were used and the calibration is that given in the A. I. E. E. Standards, 1922 edition, Table 204, for 250 mm. spheres. In operation, the bottom sphere was grounded and the distances from the gap to extraneous objects and to external parts of the circuit were maintained many times greater than those specified as minima in the Institute Standards.

Test-Cables. The facilities of the Stanford Laboratory permitted mounting only about 235 ft. of test-conductor in a single span between steel towers. Spans were supported at each end by a string of from 10 to 12 suspension insulator units. To form a balanced single-phase circuit, another piece of identical conductor is mounted on the other side of the towers. Voltage from the grounded-middle high-voltage transformer winding is applied to this test-circuit, but, since the measurements were made from line to ground, the losses per mile for the various conductors reported herein are based upon a test length of single conductor.

High-Voltage Low Power-Factor Wattmeter. During the winter of 1922-'23, Professor Harris J. Ryan developed at Stanford a wattmeter for direct observation of small amounts of power at high voltage and low power-factor. This instrument, in the earlier form in which the authors used it, is fully described in the paper at this Convention entitled "The High-Voltage Wattmeter" by P. C. Clark and C. E. Miller. The reader is advised to consult that paper for details of construction. The following very brief description of the instrument is given here for the reader's convenience. The feature of outstanding importance in the arrangement used by the present authors was that the wattmeter was operated at line voltage above ground. A direct connection from it to the line carried the line exciting current, while a complete system of electrostatic shielding was provided so that corona losses occurring in busses and connections were not included in the power measurement. Thus, in the application here described, the wattmeter was designed to read only the power consumed in corona losses on the test-cables or in insulator losses.

Fig. 2 is a diagram of connections, as this instrument[1] was used in the summer of 1923 in the measurement of the corona losses on the various specimens of line conductors that are reported in this paper. The large rectangle represents a shielding cage of copper screen completely surrounding those instruments which were operated at high potential. This cage was supported on an insulating pedestal and the instruments were read from a safe distance by means of grounded telescopes. The multiplier used for the watt-meter consisted of a 71-ft. length of ½-inch rubber garden hose coiled into a helix and carrying ordinary tap water supplied from a cylindrical tank hung by suspension insulators just above the helix. The shielding for the multiplier hose consisted of a second helix of similar hose concentric with, and surrounding, the multiplier helix and carrying tap water supplied from an annular tank surrounding, and forming a shield for, the first-mentioned tank. Approximately 25 kw. of power were consumed in the watt-meter multiplier and its shielding resistance when maximum voltage of 175 kv. to ground was employed. This amount of power supplied from only one-half of the high-voltage winding of the transformer would be sufficient to cause serious unbalancing of the two voltages applied to the single-phase test line from either cable to ground. Hence, during all measurements, balance was preserved by loading the other side of the transformer by a water-column connected from bus to ground, and carrying tap water from a supply contained in an insulated tank at the top. Valves were employed at the grounded ends of all water-columns to permit convenient adjustment of water flow. All water-columns discharged into a common watertank sufficiently well insulated from ground to permit the use of a milliammeter connected between this tank and ground to read the combined ground current, I_G. Obviously, when this current was reduced to zero by adjusting the rates of water flow on the two sides of the system, the transformer secondary voltages to neutral were balanced.

TEST METHODS—SUMMER OF 1923

General Procedure. The procedure to determine the power loss at any desired voltage was as follows: The sphere-gap was set according to the Institute curve for the one-sphere-grounded case. The water was started flowing at the proper rate in all three water-columns. By the use of the switches and the induction regulator in the primary circuit of the testing transformer, the voltage was increased continously until the sphere-gap sparked over. At the signal thus given, simultaneous readings were taken of the voltmeter, V; milliammeter, I_C; and wattmeter, W. If a sufficient

1. The instrument in its present refined form is described in the paper presented at this Convention by Carroll, Peterson, and Stray entitled "Power Measurements at High Voltage and Low Power Factor."

number of observers were available, the line exciting current, I_L, was also read. Observations were frequently made of the temperature of the metal of the tower in the shade and in the sun at points which were the approximate average height above ground of the conductor under test, and the average of these two readings was taken as the conductor temperature. Records were also kept of the readings of wet-, and dry-bulb thermometers (stationary-air type), barometer, time, and weather.

A Further Detail of Procedure. It was found expedient to begin taking readings for the corona curve of a conductor at its highest voltage point in order to have cooler, hence higher, resistance water in the water-columns. This reduced the likelihood of the wattmeter's voltage circuit current exceeding a safe value before the sphere-gap sparked over. Subsequent points at lower and lower voltages were less difficult to take since the water columns heated less rapidly on lower voltage. At very low voltages, it was generally found helpful either to reduce the rate of water flow, or to raise the voltage more slowly to allow time for the water-columns to heat and thus to increase the reading of the wattmeter, the increased reading being in proportion to the increased voltage circuit current.

Reduction of Test Data

Power. By dividing into the r. m. s. secondary voltage to ground, as given by the voltmeter coil of the transformer, the milliammeter reading, I_C, expressed

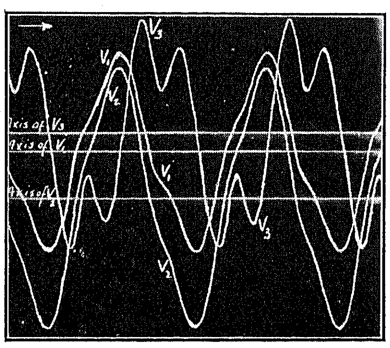


FIG. 3—OSCILLOGRAM TAKEN WHEN TRANSFORMER WAS SUPPLYING APPROXIMATELY 90 MILLIAMPERES TO BUSSES AND TEST-SPAN OF ½-INCH CABLE AT 285 KV.

V_1 transformer primary voltage.
V_2 transformer voltmeter coil voltage.
V_3 current in grounded middle of high-voltage secondary, approximately 90 milliamperes.

in r. m. s. amperes, there was obtained the total resistance in ohms of the wattmeter voltage circuit, and hence the value of the multiplier to be applied to the wattmeter readings due to the increased resistance in its voltage circuit.

Voltage Values and Wave Form. Correction for air density of the sphere-gap voltage reading was made in accordance with Section 2370, A. I. E. E. Standards, 1922 Edition.

At the higher voltages, when fairly large exciting currents were supplied to the test cables and buses, the induction regulator produced considerable distortion of the voltage applied to the primary winding of the testing transformer and, since exactly the same wave form was developed in the tertiary voltage coil (see Fig. 3) it seems reasonable to assume that the secondary (high) voltage is of practically identical distorted shape.

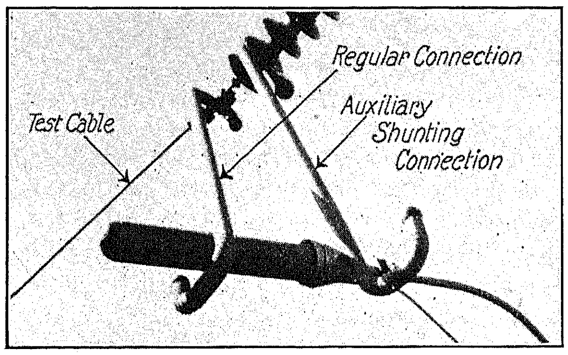


FIG. 4—METHOD OF SHUNTING OUT INSULATOR LOSSES

The sphere-gap was used for measurement of high voltage and the corona loss curves in this paper are plotted using for abscissas the r. m. s. values corresponding to crest values as given by the sphere-gap. This is in accord with the gas hysteresis theory of corona as established in the paper by Professors Ryan and Henline presented at this Convention. In that paper, it is shown that, with voltage waves within reasonable limits of distortion, corona loss is a function of the maximum, not the r. m. s. voltage.

Insulator Losses. By the use of an auxiliary connection from the shielding system to the cap of the insulator next to the cable under test, the power taken by one insulator string was shunted around the wattmeter, causing it to be carried by the shielding system. This auxiliary connection is shown in Fig. 4. In fair weather, no appreciable difference could be detected between a curve of corona loss taken with this connection and one taken without it. Therefore, it is concluded that the fair weather loss in the two insulator strings used to support each length of cable is negligible. A direct measurement of this loss on two strings in parallel mounted inside the laboratory confirmed this conclusion.

Errors in First Results and Methods of Correction

Using the experimental methods described above, the authors spent several weeks in the summer of 1923 in a strenuous effort to obtain as much test data as possible on fair weather corona losses for various cables. It was not suspected that considerable errors existed in the

power readings as given by the wattmeter; in fact, the anxiety to secure many readings while the laboratory was available operated against giving much critical analysis to the integrity of the results. The curves being secured were of a shape that seemed not unreasonable, although the apparent presence of appreciable power loss at voltages below the visual corona voltage was somewhat surprising and led to speculations as to the nature of a so-called atmospheric conduction loss, since it had already been established that insulator leakage losses were altogether negligible.

In the fall of 1923, J. C. Clark conceived the idea that the distributed self-capacitance of the helically-coiled water-hose multiplier might be the cause of some error in the watt-meter readings. Rough measurements of this capacitance were attempted and the result indicated that a very considerable subtractive correction for the leading component of the wattmeter voltage circuit current should be applied to the wattmeter readings. In the winter of 1923-'24 and the following spring, the foregoing and other errors of the original high-voltage wattmeter arrangement were thoroughly studied by Messrs. Carroll, Peterson and Stray. Their paper describes the work that was done to eliminate or to minimize various sources of error. As pointed out in the Carroll-Peterson-Stray paper, the aggregate error in the original arrangement is proportional to the square of the voltage to neutral. That the lower portions of all the curves observed in the spring and summer of 1923 obeyed a parabolic law, $W = k E^2$, was early noticed by Mr. Evenson, and for some months this fact had been held to indicate that there existed at such lower voltages a true conduction loss, $W = G E^2$, where G was thought to be a combination of insulator surface conductance and a possible "atmospheric conductance."

The curves shown in Figs. 7 to 11 inclusive are derived from the original ones observed in the summer of 1923 by the following process. First, the lower portion of the curve is fitted by a parabola, which, as shown by Messrs. Carroll, Peterson and Stray, is entirely composed of "error power;" *i. e.*, the aggregate effect of all the sources of error. This parabola is extended by the use of its equation over the entire range of voltages; finally, the complete parabola is subtracted from the original curve.

Cable Samples

An illustration of the cables tested is given in Fig. 5, and the principal physical data are given in the table of Fig. 7. The values given in the table for the pitch of the spiraling are only roughly approximate since they were the result of a very few direct measurements, and not the accurate values that would be obtained by counting the number of turns in many feet of cable.

Concentric Lay Lock-Wire Cable, Fig. 6. So far as the authors are aware, Sample F, the concentric lay lock-wire cable is new as a proposed electrical transmission conductor. In this cable, the individual strands in the outer layer may be roughly described as having a distorted dumb-bell section. These strands fit together in such a way that they hold each other in place. The periphery of the cable section is fairly smooth, closely approaching a circle. Sample F was made up on special order using dies that existed for the manufacture of

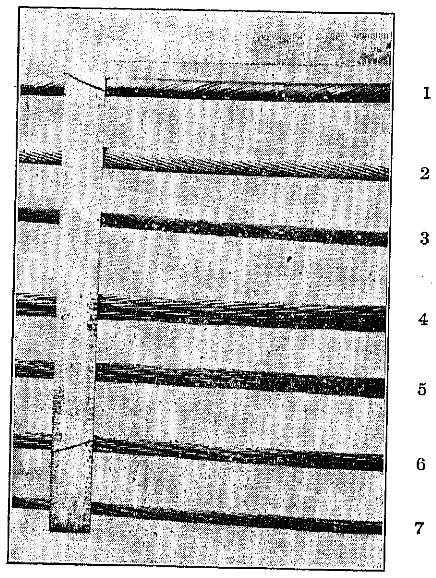


Fig. 5—Cable Samples
1. 700,000 Cm. Lock-Wire
2. 806,000 Cm. Concentric Lay
3. Steel Elevator
4. 900,000 Cm. Rope Lay
5. 500,000 Cm. Rope Lay
6. 375,000 Cm. Rope Lay
7. 225,000 Cm. Rope Lay

steel tramway cable. It is thought that, by the use of more carefully designed dies for drawing the lock-wires, the cable might be built to have a smoother surface, and hence a better corona characteristic than that of Sample F.

Steel Elevator Cable. For brevity, this cable, Sample E, is designated as a 49-strand rope-lay cable in the table of Fig. 7. However, it is really composed of six outer 7-strand ropes, with a hemp core taking the place of a central steel 7-strand rope. So far as corona losses are concerned, it is, of course, the equivalent of a 19-

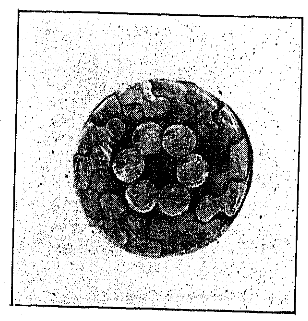


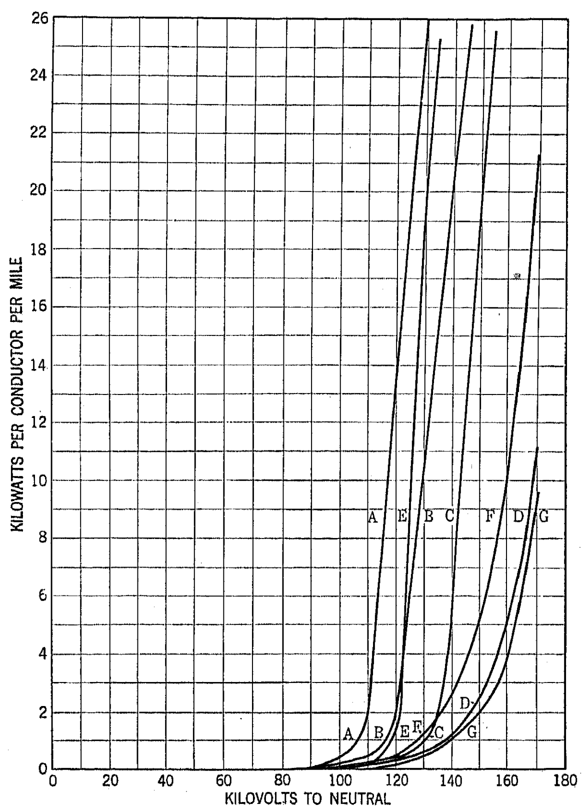
Fig. 6—Section of Lock-Wire Cable

strand cable. The strands within the 7-strand ropes and the 7-strand ropes themselves, have a fairly short pitch in spiraling. This results in a cable having a considerable smoother surface than that of any of the copper rope-lay cables, samples A, B, C, or D, which all have rather long pitch spiraling.

The Other Samples. The remaining samples tested are in more common use as electrical conductors, hence require no special comment. All samples had been weathered for at least one year.

Results

The curves of Fig. 7 are representative of the results



obtained. Fig. 8 shows the same curves with the scale for power magnified ten times and with the voltage scale doubled, in order to bring out the characteristics of the cables in a range of economic importance. Fig. 9

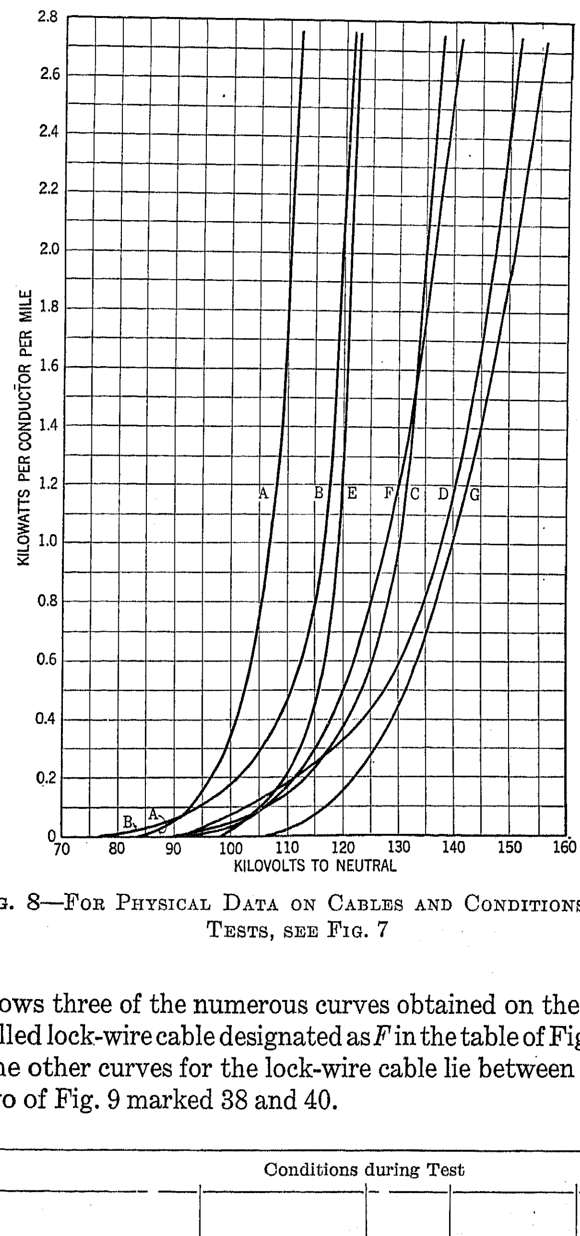


Fig. 8—For Physical Data on Cables and Conditions of Tests, see Fig. 7

shows three of the numerous curves obtained on the so-called lock-wire cable designated as *F* in the table of Fig. 7. The other curves for the lock-wire cable lie between the two of Fig. 9 marked 38 and 40.

Curve	Description of Conductor: Kind: Material	Kind: Lay	Kind: Strands	Sample	Pitch of Spiraling (Inches): Strands in 7-Strand Ropes	Pitch of Spiraling (Inches): Ropes in Cable	Conditions during Test: Dimensions: Extreme Diameter	Dimensions: Section	Dimensions: Test Length	Mean Temp of Conductor	Humidity: Wet Bulb	Humidity: Dry Bulb	Humidity: Per cent.	Mercury Barometer	Weather	Test No.
							(Inch)	Cm.	Feet	(degs. F)				(Inches)		
A	Copper	Rope	49	*A*	3½	13	0.614	225,000	235.5	84.1	62.0	78.0	39.4	30.02	clear, breezy	70
B	Copper	Rope	49	*B*	4¼	15½	0.80	375,000	233.0	80.6	64.0	73.8	58.2	30.00	clear, breezy	79
C	Copper	Rope	49	*C*	5	16	0.91	500,000	234.0	83.3	61.5	77.5	39.0	30.04	clear, breezy	76
D	Copper	Rope	49	*D*	6	27	1.24	900,000	232.0	76.5	62.0	73.8	51.0	30.04	clear, breezy	57
E	Steel	Rope	49	*E*	2	6½	0.741	?	234.3	79.7	63.8	75.8	51.5	30.04	clear, breezy	62]
F	Copper	Concentric "Lock-Wire"	?				0.91	700,000	233.8	74.3	61.0	69.0	63.4	30.01	clear, No breeze	42
G	Aluminum	Concentric	37				1.008	806,600	231.5	76.0	60.0	72.0	50.0	30.00	clear, breezy	96

Fig. 7—Kilowatts per Conductor per Mile

An effort was made to determine the losses over the insulator strings in an early morning hour when the dew was heavy, and Fig. 10 gives the curves obtained. The curves marked 92 and 93 were taken between 4 and 5 a. m., and they include, in addition to the loss on the cable, respectively the loss over one string, and two strings of insulators. Hence, by taking twice the differ-

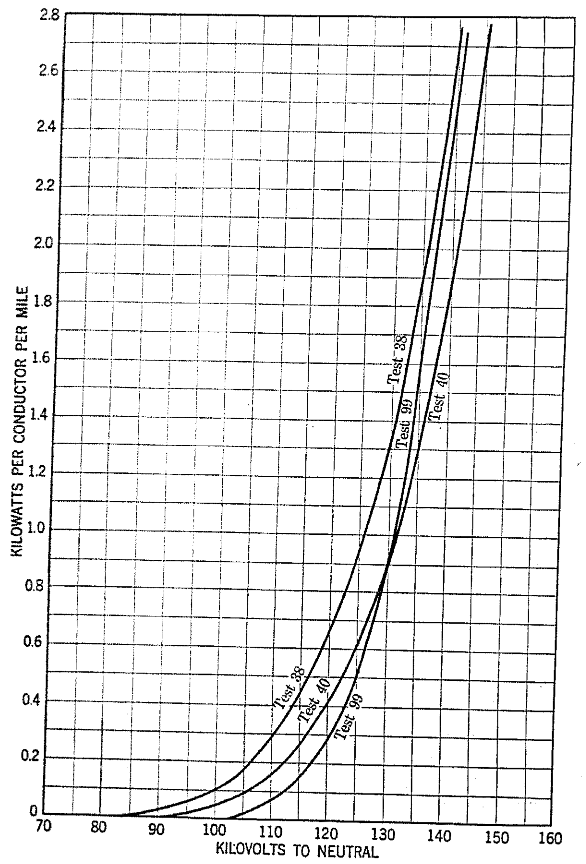


Fig. 9—Lock-Wire Cable
Runs Showing Probable Upper and Lower Limits of Loss
The Other Curves Obtained Lie between 38 and 40

Conditions of Test

Test No.	Mean Temp. of Conductor (Degs. F.)	Humidity: Wet Bulb	Humidity: Dry Bulb	Humidity: Per cent	Mercury Barometer (Inches)
38	74.3	60.0	70.0	55	30.05
40	77.9	58.0	65.0	65.5	30.01
99	67.0	57.8	68.3	52.0	30.01

ence between the two curves, the curve marked 2 (93-92) is obtained which may be assumed to represent the loss over two insulator strings. By subtracting this insulator loss from curve 93, the curve marked 93-2 (93-92) is obtained which may represent the loss occurring on the cable only. For comparison with this curve, the typical fair-weather non-dew run given by curve 42 is reproduced in Fig. 10. It is seen that, above about 135 kv., the curves are in rather good agreement, but that their lower portions are not in accord.

A series of measurements was made to determine the manner in which the loss for the concentric lay 1.008-inch aluminum cable (Cable G in Fig. 7) depends upon spacing. The results are shown in Fig. 11.

Numerous other runs were made besides those shown in the curves. Values taken from the corrected smooth curves are shown in the table of Appendix A in order that any one interested may plot the results of any of the fair-weather runs that were made.

For illustration of the nature of the original test data, the method of their reduction to values for plotting, and the correction for error power, see Appendix B and Fig. 12.

It will be noted from the curves and from the other data tabulated in Appendix A, that curves taken on the

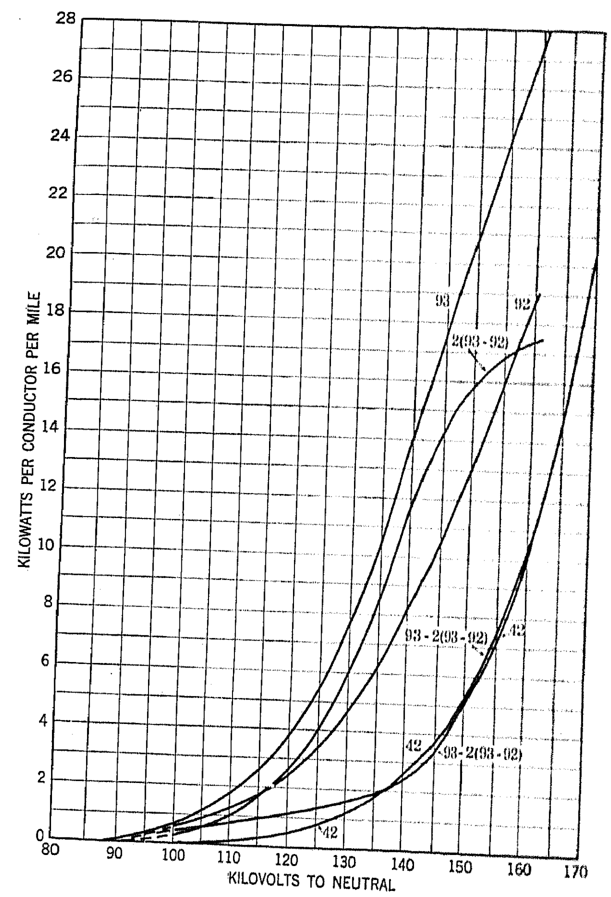


Fig. 10—Insulator Losses under Dew Condition
93 Lock-Wire Cable plus Two Insulator Strings.
92 Lock-Wire Cable plus One Insulator String
2 (93-92) Insulator Loss over Two Strings
93—2 (93-92) Lock-Wire Cable Loss Only.
42 Dry Lock-Wire Cable Loss (for comparison)

same cable vary from each other considerably. Independently of conditions of humidity, temperature, and barometric pressure which, of course, all cause some divergence among observations at any given voltage, there exist the difficulties of deriving the results by the methods available. Since the test-piece of cable was short, the readings on the scale of the wattmeter were

low. This made it necessary for a number of the lowest readings to estimate the position of the pointer to the nearest one-tenth (occasionally one-twentieth) of the lowest one of the 150 equal divisions of the 15-watt instrument that was used prior to test 94. For test 94 and subsequent tests, a 1.5-watt instrument was available and much improvement was made in the plotting of the lower portions of the curves. As will be seen by a study of Fig. 12, the process of subtracting rather large amounts of "error power" may be expected to yield curves for true corona loss showing considerable variations in their lower portions.

With the greatly improved facility now available in the present instrument which no longer includes appreciable "error power" in its readings, it should be possible to make measurements of the same order of accuracy as that of the readings of the calibrated instruments with which the measurements are made. The opportunity for such measurements on the cables discussed herein has not yet been available.

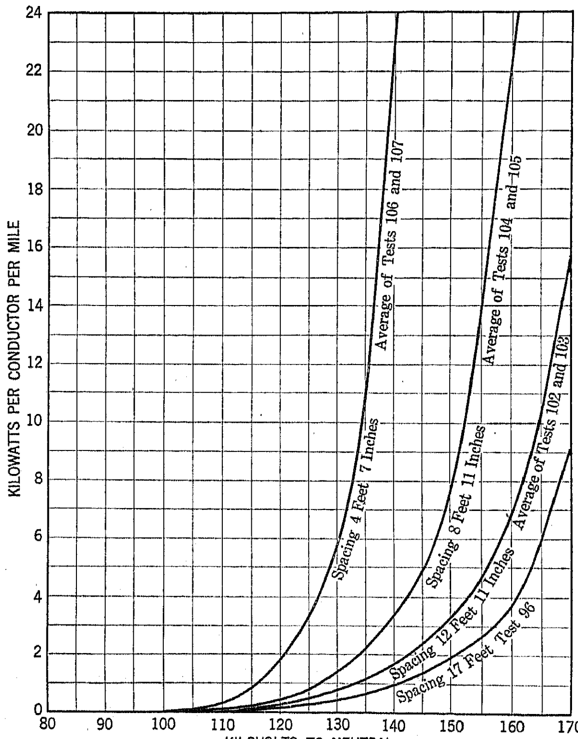


FIG. 11—ALUMINUM 37-WIRE CONCENTRIC STRAND LOSSES AT FOUR DIFFERENT SPACINGS

Conductor Diameter 1.008 inches.

ACKNOWLEDGMENTS

The authors are greatly indebted to the following who have aided this investigation in the ways stated:

The Pacific Gas and Electric Company for the donation of the 500,000-cir. mil rope-lay copper cable,

The Bureau of Power and Light, Department of Public Service, City of Los Angeles, for the donation of the 225,000-cir. mil, 375,000-cir. mil, and 900,000-cir. mil. ropelay copper cables; and the 700,000-cir. mil concentric lay lock-wire copper cable,

The Stanford University for the use of the High Voltage Laboratory, and for many other facilities; also for the power service during the tests,

Professor Harris J. Ryan for his cordial interest and encouragement manifested in innumerable ways. Without his high-voltage wattmeter, these measurements could not at this time be reported.

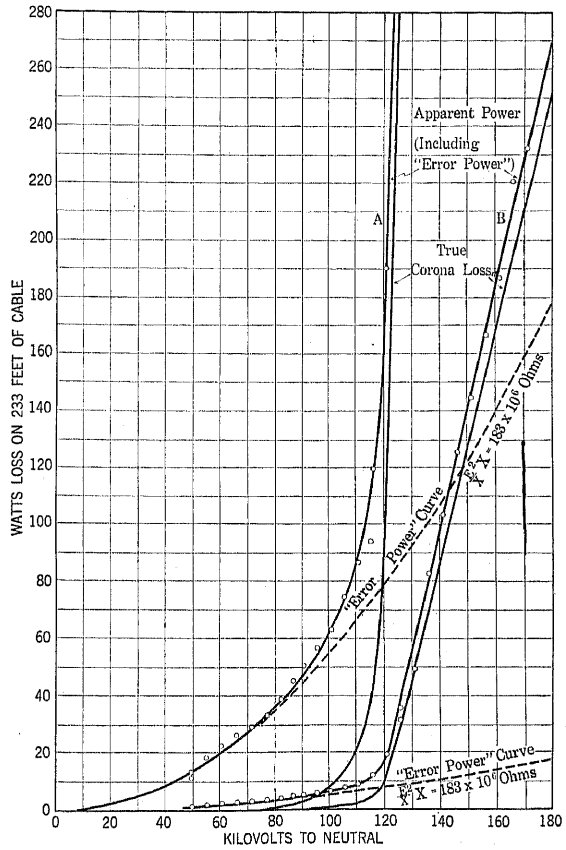


FIG. 12—375,000-CIRCULAR MIL ROPE-LAY CABLE 17-FOOT SPACING

Test Number	79
Date of Test	July 26, 1923
Time	11:31 A. M.
Temperature (Mean)	80.6 Deg. Fahr.
Humidity	58.2 Per cent
Barometer	30.00 Inches
Weather	Clear, Slight Breeze
Conductor	49-Strand Rope-Lay
Diameter	0.80 Inch
Area	375,000 Cir. Mils
Spacing	17 Feet
Length Tested	233 Feet
Conversion Factor to Mile	22.66
Test by	J. C. Clark and F. F. Evenson
Method:	Ryan's High Voltage Wattmeter

Note: Curves *A*, Read Directly. Curves *B*, Multiply Watts by 10.

Appendix A

CORONA LOSSES IN WATTS FOR TEST LENGTH GIVEN

	90 kv.	100 kv.	110 kv.	120 kv.	130 kv.	140 kv.	150 kv.	160 kv.	170 kv.	Cable Temp. deg. fahr.	Humidity			Baro-meter Inches Mercury	Time
											Wet Bulb deg. fahr.	Dry Bulb deg. fahr.	Per Cent		
225,000 C. M., Spacing 18 ft. Rope-Lay Test Length 235.5 ft. Factor for one mile, 22.42															
Test No. 70	2.3	14.8	78	590	1120	1640	2150	2670	..	84	62	78	39.4	30.02	3:05 p.m.
" " 71	0.0	5.0	115	615	1110	1605	2100	2600	..	84	61.5	78	37.8	30.03	4:00 p.m.
" " 73	7.0	18.0	90	575	1080	1580	2070	2570	..	80	59.8	74.5	40.5	30.04	5:35 p.m.
375,000 C. M., Spacing 17 ft. Rope-Lay Test Length 233 ft. Factor for one mile, 22.66															
Test No. 36	8.2	19.5	47.5	120	400	910	1425	1945	2460	73.2	60	71.5	50.4	29.93	4:35 p.m.
" " 37	7.0	18.0	48	119	525	1000	1490	1970	2445	72.5	59.5	70	53	30.05	2:20 p.m.
" " 79	2.5	7.5	20	90	465	875	1290	1695	2105	80.6	64	73.8	58.2	30.00	11:30 a.m.
" " 80	0.0	3.3	16	75	400	860	1325	1790	2250	88.5	65.8	79.5	49.2	29.97	2:00 p.m.
" " 81	6.2	12.1	24	69	415	850	1285	1720	2155	88.7	65.3	81	42.3	29.97	3:07 p.m.
500,000 C. M., Spacing 18 ft. Rope-Lay Test Length 234 ft. Factor for one mile, 22.57															
Test No. 76	0.0	2.0	6.4	16.5	44.5	240	835	1435	2035	83.3	61.5	77.5	39	30.03	4:13 p.m.
" " 100	3.0	8.0	17.0	32.5	70	300	830	1360	..	68	57.3	67.5	52	30.01	4:42 p.m.
" " 101	0.0	1.6	6.8	25	84	290	785	1340	1890	65	56.8	65.5	58	30.01	5:33 p.m.
900,000 C. M., Spacing 18 ft. Rope-Lay Test Length 232 ft. Factor for one mile, 22.75															
Test No. 39	0.5	3.2	9.0	23.5	45.5	81	146	280	710	74.3	60	70	55	30.05	4:45 p.m.
" " 56	0.0	0.0	3.0	10.0	23.4	49.5	102.5	210	490	75.6	59.5	72	47.2	30.08	4:40 p.m.
" " 57	0.0	3.0	7.6	14.5	26.0	52	111	224	520	76.5	62	73.8	51	30.04	1:54 p.m.
Steel Elevator, Spacing 18 ft. Rope-Lay Test Length 234.33 ft. Factor for one mile, 22.53															
Test No. 62	0.0	1.2	9.5	60	810	1375	1895	2410	..	79.7	63.8	75.8	51.5	30.05	1:55 p.m.
" " 65	0.2	2.1	8.0	37.5	725	1295	1820	2340	..	84.2	62	77.5	40.5	30.00	4:40 p.m.
" " 67	0.3	2.8	9.5	45	550	1140	1650	2150	2660	64.9	59.3	70	54	30.02	7:53 p.m.
Aluminum Concentric-Lay Test Length 231.5 ft. Factor for one mile, 22.8															
Test No. 94, Spacing 17 ft.	0.0	0.0	1.3	11.0	31.0	57.4	96.5	173	..	73.4	61	69	63.4	30.00	11:05 a.m.
" " 95, " 17 "	0.0	1.0	3.9	10.4	24.4	49.8	90.5	170	..	79.7	62	72.8	52.8	30.00	2:50 p.m.
" " 96, " 17 "	0.0	0.0	1.0	6.8	19.6	44.5	85.0	162	..	76.0	60	72	50	30.00	5:05 p.m.
" " 97, " 17 "	0.0	0.0	0.0	3.6	13.9	34.5	67.5	137	..	71.6	58.3	65.8	64	30.01	9:08 a.m.
" " 102, Spacing 12 ft. 11¼ in.	0.0	0.5	3.1	9.8	30.4	73.5	146	308	730	67.1	60	68.5	60.6	30.10	11:00 a.m.
Test No. 103, Spacing 12 ft. 11¼ in.	0.0	0.0	2.4	13.0	40.0	77.0	145	295	640	66	60	69	58	30.09	2:36 p.m.
Test No. 104, Spacing 8 ft. 10½ in.	0.0	1.0	6.6	23.8	67.5	152.5	368	1045	..	68	60	69	58	30.09	3:40 p.m.
Test No. 105, Spacing 8 ft. 10½ in.	0.0	1.2	4.2	12.2	55.0	146.0	309	930	..	68.5	59.3	68.3	58	30.09	4:40 p.m.
Test No. 106, Spacing 4 ft. 7 in.	1.1	5.3	20.6	86.0	255	1010	2210	..	..	67	58.5	67	60	30.09	5:35 p.m.
Test No. 107, Spacing 4 ft. 7 in.	0.0	0.0	8.6	72.0	255	1040	2210	..	..	64	58.5	67	60	30.09	6:18 p.m.
Lock-Wire, Spacing 17 ft. Concentric Lay Test Length 233.84 ft. Factor for one mile, 22.58															
Test No. 38	1.2	4.4	12.5	29.3	60.3	123	246	465	928	74.3	60	70	55	30.05	3:55 p.m.
" " 40	0.0	2.0	7.0	18.5	40.3	84	170	355	715	77.9	58	65	65.5	30.01	8:32 a.m.
" " 42	0.0	1.5	7.4	22.0	54.0	116	232	450	910	74.3	61	69	63.4	30.01	10:26 a.m.
" " 45	0.6	3.3	10.5	28.5	60.0	128	254	500	1000	71.6	63	71.5	64.4	29.98	1:40 p.m.
" " 47	0.6	3.2	10.0	24.8	53.8	113	225	520	1000	80.2	64	74	57.4	29.97	3:52 p.m.
" " 49	0.5	2.8	9.7	24.5	53.5	107.5	217	455	900	79.0	62.8	72.8	56.7	29.97	5:17 p.m.
" " 51	1.4	5.0	14.5	38.5	82.5	151.5	261	458	840	59.9	57	64.5	63	30.00	8:29 p.m.
" " 52	0.5	3.5	10.0	23.3	50.0	107.5	198	395	885	59.0	57	64.5	63	30.00	9:48 p.m.
" " 84	0.0	1.3	6.8	23.6	59.0	121.0	233	473	988	87.8	71.2	82.3	57.9	29.92	6:45 p.m.
" " 85	0.0	0.3	2.8	17.0	47.5	97.3	196	402	890	77.5	68.5	79.5	57.0	29.92	7:30 p.m.
" " 86	0.0	1.2	5.6	18.5	47.0	100	203	465	980	71.2	64.8	75.3	56	29.96	8:45 p.m.
" " 87	0.0	0.0	2.8	13.7	43.0	95.2	183	352	710	67.6	62.5	72.5	56.5	29.97	9:40 p.m.
" " 88	0.0	0.5	4.5	19.4	52.0	131	256	453	730	64.4	62.3	71.0	60.7	29.97	10:35 p.m.
" " 89	0.0	0.5	6.2	26.0	69.0	156	322	583	922	62.6	61.8	69.8	64.0	29.97	11:36 p.m.
" " 90	0.0	6.0	20.2	53.0	116.0	219	376	625	960	61.5	61.5	68.0	69.1	29.97	12:33 a.m.
" " 91	0.0	0.0	11.5	39.8	100.0	208	394	635	930	59.0	61	68	67	29.97	1:52 a.m.
" " 92	9.5	26.0	54.5	110.0	207	364	580	835	1115	57.6	59.5	64.8	73.6	29.97	4:18 a.m.
" " 93	4.5	31.0	80.5	166	337	625	932	1225	1515	55.4	58	63	74.2	29.97	4:50 a.m.
" " 98	0.5	2.0	5.3	14.6	36.2	90	187	.375	870	67.0	57.5	68	52	30.01	11:35 a.m.
" " 99	0.0	0.0	2.8	12.8	40.0	112	231	435	890	67.0	57.8	68.3	52	30.01	2:40 p.m.

Appendix B

TEST NO. 79—CORONA STUDY AT STANFORD UNIVERSITY 375,000-cm. Rope-Lay 0.80-inch Diameter Copper Cable, Length 233 ft., Spacing 17 ft., Frequency 60-Cycles.

July 26, 1923—11:16 a. m. to 11:46 a. m. Clear Weather, Slight Breeze. Barometer, 30.00 in. Atmospheric Density Factor, 1.0091, Sphere-Gap Factor, 1.0084.

Temperatures: Wet Bulb 64 deg. fahr: Dry Bulb 72.5 deg. Fahr.
64 " " " " 75 " "

On Tower: Shade 78.8 deg. fahr: Sun 81.5 deg. Fahr.
79.7 " " " 80.6 " "

NOTES

The word "corrected" in the column headings means simply that the original readings have been corrected by the use of calibration curves for the various instruments.

To obtain the factor by which the corrected wattmeter reading (column 5) is multiplied to give the true watt loss (column 7): Divide the r. m. s. voltage to neutral by tertiary coil voltage (column 4) expressed in volts by the multiplier current through water column (column 6) expressed in amperes. This gives the total resistance of the wattmeter voltage circuit. The ratio of this resistance to that of the normal 150-volt wattmeter voltage circuit, 4432 ohms, gives the desired factor.

The following is an illustration using the figures for the first observation in the above table:

$$\frac{174800}{0.07469} \times \frac{1}{4432} \times 4.937 = 2320.$$

IMPORTANT: The above calculated losses in watts are for the test length of this cable, 233 feet. The losses in Column 7 of the table include the "error power," and to obtain the net true power, there must be subtracted from the curve for the given *test length*, the error power curve:

$$W = \frac{E^2}{183 \times 10^6}$$

where E is in volts,
W is in watts.

To obtain the losses in watts for one mile of cable, multiply the net corona loss curve by 22.66.

Setting of Sphere Gap cm. S	Sphere Gap R. M. S. Sine Wave kv. to Neutral E_{SG}'	Corrected kv. to Neutral using Sphere Gap Weather Factor E_{SG}	Corrected R. M. S. kv. to Neutral by Tertiary Coil E_T	Corrected Watt-Meter Reading W_m	Multiplier Current through Water Column Milli-Amperes I_C	Watts Loss by Calculation W
10.0	170	171.2	174.8	4.397	74.69	2320
9.6	165	166.2	171.9	4.301	75.55	2205
9.2	160	161.2	165.8	3.458	69.41	1867
8.83	155	156.2	162.2	3.100	68.22	1665
8.5	150	151.1	158.7	2.961	73.20	1447
8.13	145	146.1	153.6	2.653	73.41	1252
7.8	140	141.1	148.8	2.332	75.66	1033
7.45	135	136.1	143.9	1.774	69.90	825
7.1	130	131.0	132.9	1.09	66.34	493
6.75	125	126.0	128.93	0.744	69.21	312.5
6.75	125	126.0	129.62	0.865	71.27	354.4
6.4	120	121.0	123.44	0.481	70.73	190
6.1	115	116.0	118.32	0.297	66.34	119.7
5.8	110	110.9	111.89	0.226	65.75	86.9
5.5	105	105.9	107.83	0.226	73.62	74.6
5.2	100	100.8	102.61	0.205	75.33	63.0
4.9	95	95.7	97.79	0.194	75.55	56.6
4.6	90	90.6	91.98	0.178	73.08	50.5
4.34	86	86.6	87.48	0.162	70.73	45.2
4.1	81.5	82.1	82.78	0.156	75.02	38.85
3.8	76.5	77.1	77.11	0.130	68.32	33.2
3.5	71.0	71.5	71.45	0.114	63.13	29.1
3.2	65.5	66.0	64.61	0.103	57.65	26.1
[illegible].9	60.0	60.5	58.95	0.086	51.58	22.4
2.6	54.5	54.9	53.30	0.070	46.06	18.3
2.33	49.0	49.4	47.80	0.043	40.57	11.42
2.33	49.0	49.4	47.95	0.049	41.12	12.9

Observations by J. C. Clark, F. F. Evenson, I. R. Harcourt.

Discussion

For discussion of this paper see page 1162.

Corona Loss Tests on the 202-Mile 60-Cycle 220-Kv. Pit-Vaca Transmission Line of the Pacific Gas and Electric Company

BY ROY WILKINS
Associate, A. I. E. E.
Pacific Gas & Electric Co., San Francisco, Cal.

Review of the Subject.—*A description is given of the physical and electrical characteristics of the Pit-Vaca 220-kv. transmission likely to affect corona loss. The line contains three distinct configurations and two sizes of conductor. Corona loss measurements were made from Pit Power House on the three configurations and from Vaca Substation on one of the configurations. Three methods of measuring corona losses were used at Pit and four at Vaca.*

The measured corona losses were found to follow exponential laws in three distinct phases: below visual corona, visual corona and visual corona with losses sufficiently high to produce voltage distortion. At no point did the losses follow a quadratic law. A review of the reports of the results of corona loss tests on other lines indicates that the corona losses followed exponential laws.

At the time the Pit No. 1 plant was put in operation, late in 1922, the transmission line to Vaca-Dixon Substation went into service at 110 kv. first, because some parts of the line hardware and insulators were not available in sufficient quantities to equip the line for 220 kv., and second, because there was no need for the higher voltage to transmit the power then available.

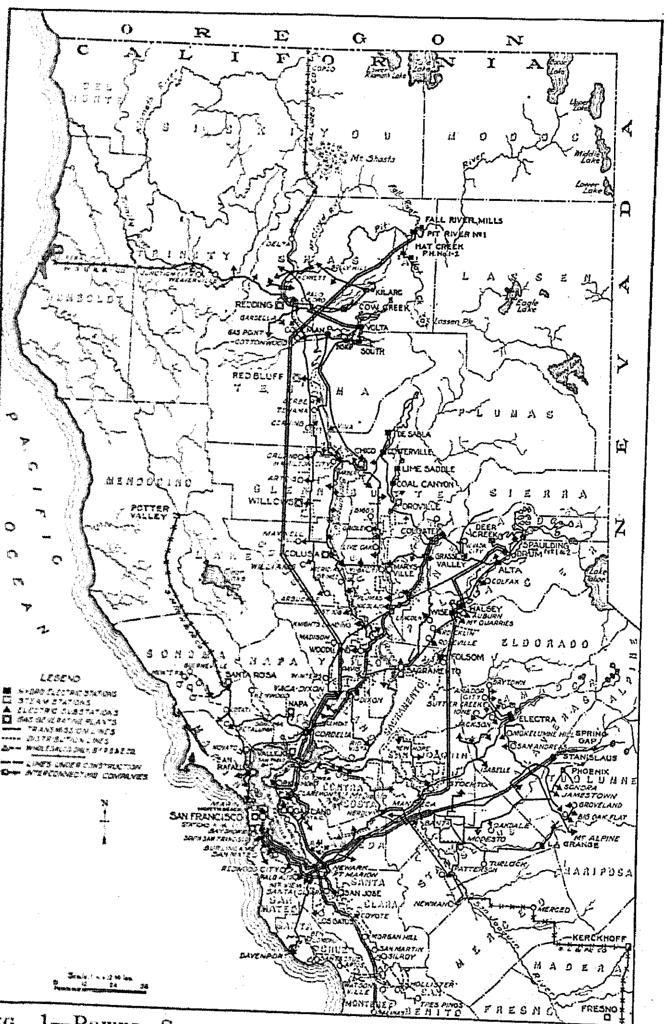


FIG. 1—POWER STATIONS. PACIFIC GAS & ELECTRIC CO., SAN FRANCISCO, CAL.

Between that time and November 1st, 1923, the original line was insulated for 220 kv. and the second line was built and insulated for 220 kv., but both were operated at 110 kv. Before the original line was put into operation at 220 kv. during the first part of November, 1923, corona loss tests were made quite carefully on such sections of the line as could conveniently be handled at the power house.

These lines will both be operated at 220 kv. as soon as the power from Pit No. 3 is available, sometime in 1925. In the meantime they will be operated as they were at the time of the test, one at 220 kv. and the other at 110 kv.

An idea of the geographical location may be gained from the map of the Pacific Gas and Electric Company's transmission system and an idea of the elevations from the following table:

TABLE I

Location	Miles	Elevation
Pit	0	2800 Ft.
Maximum elevation	15	5200 "
Round Mountain	27.5	2157 "
Elevation	40	1000 "
Cottonwood	60	433 "
Williams	143	125 "
Vaca	202	83 "

The line insulation is shown in a chart included, and the tower data on the following table:

For the first 27½ mi. from the power house, these lines are of steel core aluminum and each circuit will ultimately be on its own tower line with a horizontal spacing of 19 ft. between wires and 75 ft. between center wires of the two circuits.

At the present time the 110-kv. line has its towers complete as called for in the ultimate plans.

The 220-kv. line was at the time of test and is now operating on "*H*" type wood poles with standard single-

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., October 13-17, 1924.

circuit towers at angles and places where it was thought that the poles were inadequate.

These poles have a conductor spacing of 16 ft. and are spaced much closer together than the towers on the 110-kv. circuit, giving the required 30 ft. minimum clearance

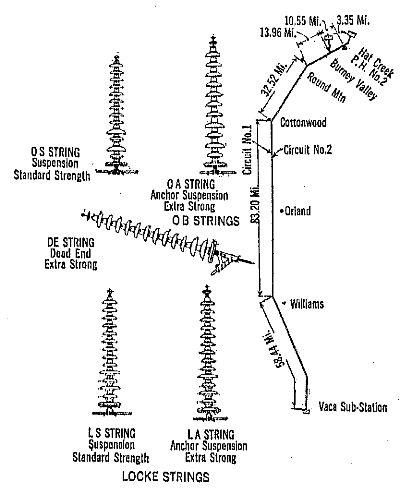


Fig. 2

Section	From	To	Conductor	
			Circ Mills	Material
1	Pit River PH No. 1	Hat Creek PH No. 2	518,000	SC. A1.
2	Hat Creek PH No. 2	Round Mtn.	518,000	SC. A1.
3	Round Mtn.	Cottonwood	500,000	Cu.
4	Cottonwood	Williams	500,000	Cu.
5	Williams	Vaca	500,000	Cu.
6	Pit River PH No. 3	Burney Valley	518,000	SC. A1.

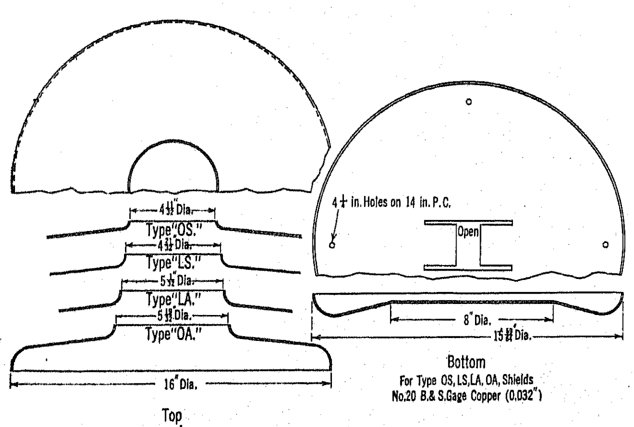


Fig. 3

to ground, but having an average clearance of less than would be indicated by the tower data.

There are 202 pole structures and 98 towers from Pit to Round Mountain and 320 pole structures and 39 towers from Round Mountain to Cottonwood. These give, as nearly as can be determined from the profiles, an average of 17 ft. from Pit to Round Mountain and 16.3 ft. from Round Mountain to Cottonwood.

The aluminum conductor is made up of 42 strands 0.112 in. in diameter around a 19 strand steel core, each strand of which is 0.111 in. in diameter.

The weight per mile is 5900 lb., of which the aluminum is 2560 lb.

The next 32½ mi. is of 49 strand rope lay copper, 500,000 cm. in area with the configuration and spacing as noted above.

The copper is of 7 by 7 strand M. H. D. rope lay, made up of 7 strands 0.101 inch in diameter with a left

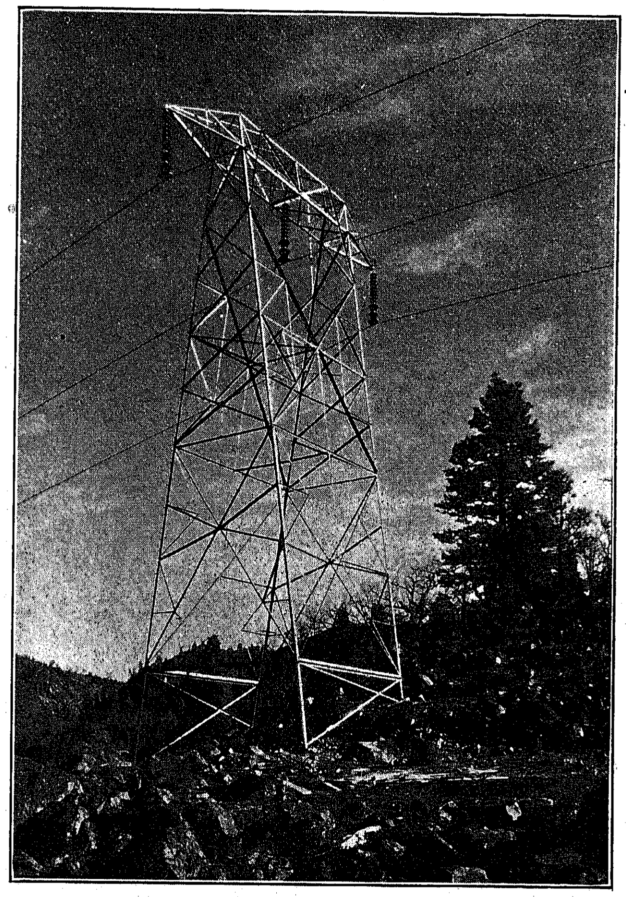


Fig. 4—Tower in Horizontal Configuration, Aluminum Section

hand lay, and a pitch of 5 inches; of these, 7 are in turn cabled into a rope with a 15 in. pitch right hand lay. A circle enclosing the cable is 0.91 inches in diameter.

From this point 142 mi. to Vaca-Dixon Substation, both lines are on the same twin circuit tower with 15 ft. vertical spacing and 24 ft. between circuits, using the copper conductor described above. The lines are completely transposed with respect to the ground and communication circuits, and each circuit with the other. Complete barrels have an average length of 15.5 mi. as shown in the diagrams.

The transformers at the power house are 16,667-kv-a. single phase, 127,000 to 11,000, connected Y-Delta

TABLE II
DATA ON STANDARD TOWERS OF PACIFIC GAS AND ELECTRIC COMPANY 220-KV. TOWER LINES

Item	Single Circuit Towers		Double Circuit Towers	
	Type SA	Type SC	Type M	Type O-98
Height				
Overall	55½ ft.	55½ ft.	97 ft.	98 ft.
To Lower Crossarm	51½ ft.	51½ ft.	62 ft.	63 ft.
Width				
At base	20 ft. by 20 ft.	20 ft. by 20 ft.	20 ft.	20 ft.
At crossarm	4 ft. by 20 ft.	4 ft. by 20 ft.	6 ft.	6 ft.
Conductor Separation				
Between phases	19 ft.	19 ft.	15 ft.	15 ft.
Between circuits	..	..	24 ft.	24 ft.
Conductor				
Max. size (copper)	500,000 cm.	500,000 cm.	500,000 cm.	500,000 cm.
Loading	8 lb. wind, ½ in. ice	8 lb. wind, ½ in. ice	8 lb. wind, no ice	8 lb. wind, no ice
No. broken	3	3	3	6
Span				
Normal	500	500	800	800
With angles	..	1000	..	800
Max. on tangents	750	1500	1000	1800
Max. contributing wt.	1500	2100	1050	1800
Max. Angle	None	22½ deg.	None	22½ deg.
Type of Cond. Support	Suspension	D. E. or Susp.	Susp.	D. E. or Susp.
Weight of Tower	5100 lb.	7270 lb.	8440 lb.	12,950 lb.
Foundations				
Concrete (cu. yd.)	6.28	11.8	7.8	15.2
Excavation (cu. yd.)	18.1	60.4	25.5	84.5

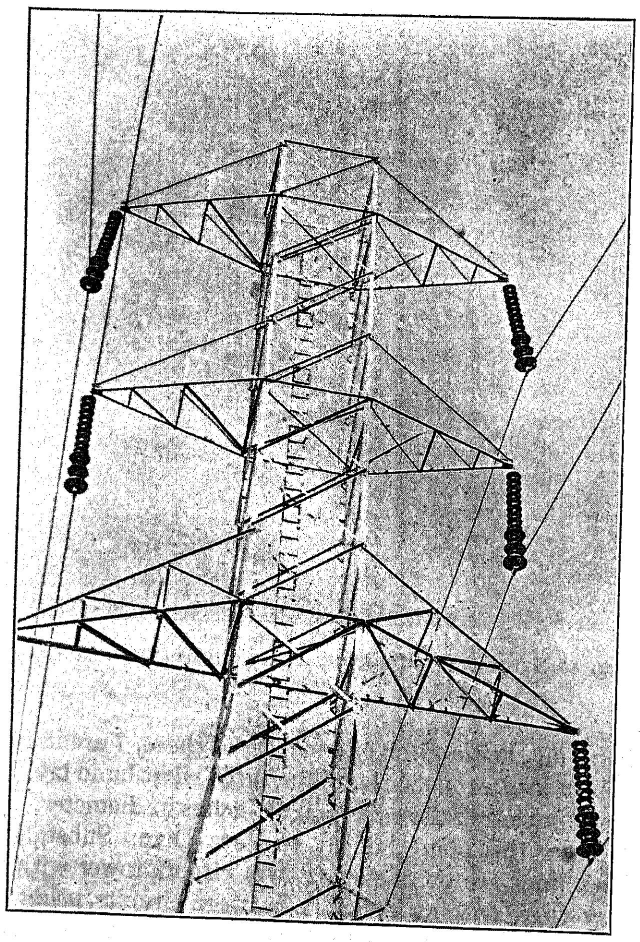


Fig. 5—Tower in Vertical Configuration, Copper Section

with the ground terminal of the high tension permanently grounded, giving 220,000 normal operating voltage on the line. Those at the substation are 16,667-kv-a., 115,500 to 63,600 auto-transformers with an 11,000-volt delta winding which is connected to 20,000 kv-a. synchronous condensers.

The lines are equipped with a reverse power and

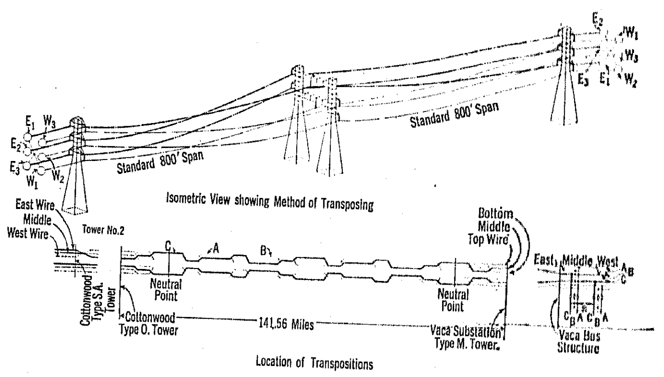


Fig. 6

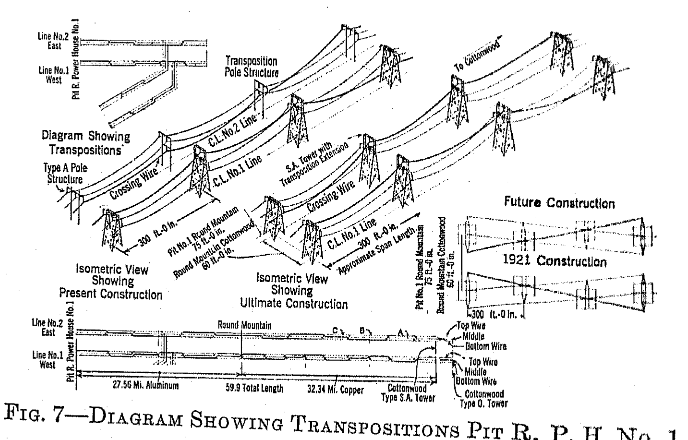


Fig. 7—Diagram Showing Transpositions Pit R. P. H. No. 1 to Cottonwood

residual relay combination allowing the clearing of a line in trouble on much less than normal current for grounds or phase unbalances, and having directional

features so that it functions equally well with either one or both lines in operation.

The transformers are protected by balanced relays and the generators by over-voltage and balanced relays.

Switching is done on trouble on the 220 kv. in the same way that the lower voltages are handled.

When switched out, the lines are energized by charging with a generator at reduced frequency, starting a condenser at Vaca, building up the speed and paralleling at the substation.

When the tests were contemplated, it was decided to use at least two methods of measurement with calibrated instruments and instrument transformers on the low-tension side of the transformers for one set of readings and the current in the ground side of the high-tension winding of the main transformers, together with a calibrated water hose resistance, for the other set.

This hose resistance is made up of two ½ in. garden

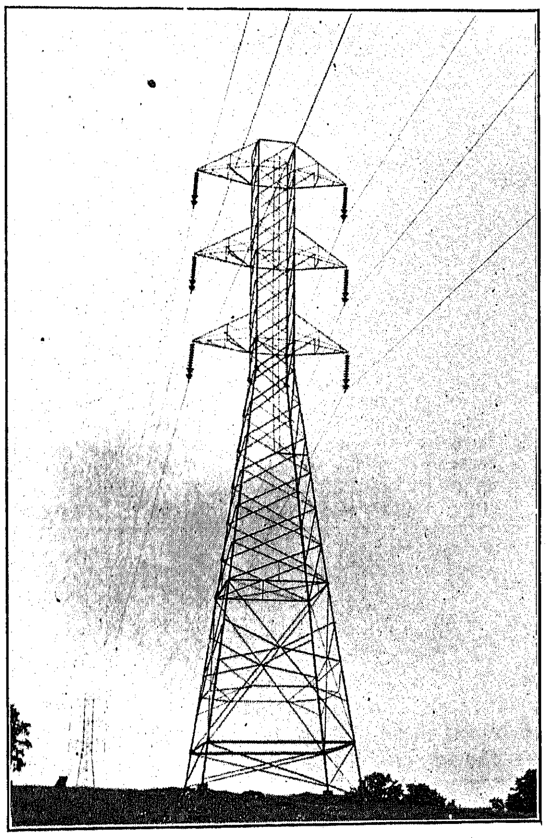


FIG. 8—LINE IN VERTICAL CONFIGURATION WITH TOP OF TRANSPOSITION IN BACKGROUND

hose approximately 75 ft. long in parallel, connected to the 220-kv. line at the top and grounded at the bottom. Water is forced up one hose and flows down the other, and the voltage used is shunted off a few inches of the ground ends.

In addition, the turbine gate opening was calibrated in generator kilowatts output, giving a fairly accurate check on total losses by a totally different method. At each reading, oscillograms were taken and some of the representative curves are included in this report.

Transformer iron losses were accurately measured on the transformers in place, and copper losses were calculated and checked against factory measured values.

The $I^2 R$ due to charging current in the line was calculated by the method proposed by Jacobsen appearing in the A. I. E. E. PROCEEDINGS. This was checked against the method used by Lewis in the A. I. E. E. of June 1921 and was finally determined by graphically integrating for five-mile intervals.

Using the low-tension readings as a standard, the

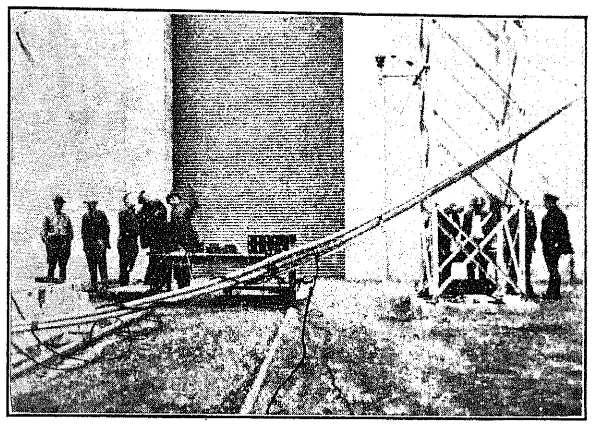


FIG. 9—HOSE AT PIT NO. 1

several loss curves were plotted, using values corrected for instrument and instrument transformers and having the transformer losses subtracted.

These values having the $I^2 R$ of the line subtracted were next plotted and finally the values per wire per mile were plotted. As originally taken, the values for the 15-ft. vertical spacing were obtained by subtraction of two tests and correcting for voltage rise, inasmuch as these values were lower than those on the 19-ft. horizontal spacing. A test was made at Vaca-Dixon Substation charging back 59 mi. to Williams. The agreement between these two curves, 29 and 31 is very good and as all the values at Pit are checked against the

TABLE III
LINE CONSTANTS FOR CORONA TESTS
PIT TO ROUND MOUNTAIN 27.5 MILES

Res. per Mi.	Reac. per Mi.	Cap. Susp. per Mi.	Diam. Al. Cond.	Spacing	Aver. Height at Tower	Min. Clearance
0.1815	0.793	5.372	1.08 in.	19 ft. Hor.	50 ft.	30
Round Mountain to Cottonwood 32.5 Miles						
0.117	0.812	5.224	0.91 in.	19 ft. Hor.	50 ft.	30
Cottonwood to Williams 83 Miles						
0.117	0.784	5.423	.91 in.	15ft. Vert.	55 ft.	30
Williams to Vaca 59 Miles						
0.117	0.784	5.423	0.91 in.	15ft. Vert.	55 ft.	30

TABLE IV

CORONA TESTS PIT NO. 1 POWER HOUSE—NOVEMBER 13, 1923
PIT TO ROUND MOUNTAIN 27.5 MILES OF ALUMINUM 17 FT. HORIZONTAL SPACING

Volts Low Tension Read.	Volts Hose Read.	K. V. between Conductors by Hose	Amperes High Tension	Transformer Copper Loss kw	Transformer Iron Loss kw.	Total Transformer Loss	Kw. Low Tension corrected for Inst.	Net kw. to Conductors	I^2R due to High Tension Amperes	Net Corona	Temp. Deg. Cent.	Barometer Inches	Film No.	Time
														P. M.
48	37	9.73	9	..	35	35	..	..	..	..	9.5	26.91	8 a	1:25
57.3	44	116.3	10.5	2	50	52	..	..	..	..	9.5	26.91	8 b	1:36
71.5	54.5	143.5	13	3	76	79	120	41	2.8	38	9.5	26.91	8 c	2:48
80.75	62.0	163.0	14.5	4	92	96	144	48	3.3	44	9.5	26.91	9 a	3:03
93.5	72.5	190.5	17.3	6	126	132	240	108	5.0	103	9.3	26.90	9 b	3:15
101.0	78.75	206.5	18.4	7	146	153	288	135	5.7	129	9.3	26.90	9 c	3:25
111.3	87.5	229.5	20.3	8	181	189	480	291	7.0	284	9.0	26.89	10 a	3:37
119.6	94.7	249.0	21.9	10	220	230	624	394	8.0	386	8.9	26.89	10 b	3:54
128.5	102.8	271.0	24.1	12	280	292	1247	955	9.7	947	8.5	26.89	10 c	4:07

Transformers 127,000 Y grounded to 11,000 volt delta.
Low Tension Potential Ratio 100/1
Weather Clear.

TABLE V

CORONA TESTS PIT NO. 1 POWER HOUSE—NOVEMBER 14, 1923—PIT TO COTTONWOOD
PIT TO ROUND MOUNTAIN 27.5 MILES OF ALUMINUM 17 FT. HORIZONTAL SPACING
ROUND MOUNTAIN TO COTTONWOOD 32.5 MILES OF COPPER 16 FT. HORIZONTAL SPACING

Volts Low Tension Read.	Volts Hose Read.	K. V. between Conductors by Hose	Amperes High Tension	Transformer Copper Loss kw.	Transformer Iron Loss kw.	Total Transformer Loss	Kw. Low Tension corrected for Inst.	Net kw. to Conductors	I^2R due to High Tension Amperes	Net Corona	Temp. Deg. Cent.	Barometer Inches	Film No.	Time
														A. M.
	37	97.3	19	7	34	41	96	55	12	43	7 deg. cent.	27.05	11 a	10:25
	43	112.5	21.8	8	48	56	129	73	16.3	57	6 deg. cent.	27.05	11 b	10:43
	52.5	138.6	27	14	69	83	192	109	24.5	84	7.6 deg. cent.	27.05	11c and 12a	10:55
	54.0	142.0	28	16	73	89	206	117	26.9	90	5 deg. cent.	27.05	..	11:12
	62.3	164.0	32	20	93	113	288	175	35.0	140	5 deg. cent.	27.05	12 b	11:18
	72.0	189.0	36.5	24	120	144	480	335	45.4	290	5 deg. cent.	27.05	12 c	11:25
	79.3	208.0	39.8	29	141	170	734	564	53.6	510	5 deg. cent.	27.05	13 a	11:37
	89.4	236.0	45.0	36	182	218	1,440	1,222	69.4	1,153	5 deg. cent.	27.05	13 b	11:45
	96.4	254.5	49.0	43	220	263	2,688	2,425	81.0	2,344	5.3 deg. cent.	27.05	14 a	12:00
	106.0	280.0	54.8	52	290	342	4,990	4,648	101.2	4,547	5.4 deg. cent.	27.05	14 b	12:07

Weather fair—light clouds

TABLE VI

CORONA TESTS PIT NO. 1 POWER HOUSE—NOVEMBER 25, 1923—PIT TO WILLIAMS
PIT TO ROUND MOUNTAIN 27.5 MILES OF ALUMINUM 17 FT. HORIZONTAL SPACING
ROUND MOUNTAIN TO COTTONWOOD 32.5 MILES OF COPPER 16 FT. HORIZONTAL SPACING
COTTONWOOD TO WILLIAMS 83 MILES OF COPPER 15 FT. VERTICAL SPACING

Volts Low Tension Read.	Volts Hose Read.	Kv. between Conductors by Hose	Amperes High Tension	Transformer Copper Loss kw.	Transformer Iron Loss kw.	Total Transformer Loss	Kw. Low Tension corrected for Inst.	Net kw. to Conductors	$I^2 R$ due to High Tension Amperes	Net Corona	Temp. Deg. Cent.	Barometer Inches	Film No.	Time
69	55.6	146.4	68	74	72	146	570	425	300	125	9 deg. cent.	26.98	25 c	
74.5	59.9	157.5	75	100	81	181	692	511	359	152	9 deg. cent.	26.98	26 a	
81	66.4	174.6	81.5	120	94	214	928	714	424	290	9 deg. cent.	26.98	26 b	
85.1	71.3	187.6	86.5	132	103	235	1,244	1,109	478	531	9.5 deg. cent.	27.00	26 c	
92.5	77.2	203.0	93	154	121	275	1,920	1,645	550	1,095	9.5 deg. cent.	27.00	27 a	
100.0	84.8	223.0	102.5	186	143	329	3,697	3,368	673	2,695	10 deg. cent.	27.00	27 b	
104.5	89.0	234.2	108.0	206	156	362	5,600	5,238	746	4,492	10 deg. cent.	27.00	27 c and 28 a	
111.0	95.8	252.3	122.0	265	162	427	9,840	9,413	950	8,463	11 deg. cent.	27.00	28 b	
111.5	98.9	260.0	130.0	300	198	498	13,670	13,172	1,080	12,092	11 deg. cent.	27.00	28 c	
120	102.6	270.3	143.0	370	224	594	18,860	18,266	1,300	16,966	11 deg. cent.	27.00	29 a and 29 b	
126	106.0	280.0	154.0	440	258	698	24,480	23,782	1,515	22,267	11 deg. cent.	27.00	29 c.	

TABLE VII

CORONA TESTS—PIT-VACA LINE—JANUARY 13, 1924

VACA-DIXON SUBSTATION TO WILLIAMS—59 MILES OF COPPER OF 15 FT. VERTICAL SPACING

Volts Low Tension	Volts Hose	Kv. Bet. Cond. by Hose	High Tension Amps. Multiply by 2	Transf. Losses	Kw. Low Tension Cor.for Inst.	Net Kw. to Conductors	I^2R due to High Tension Amps.	Net Corona	Net Kw. from Williams in Line End of Transf. and Hose	High Tension Kw. from Williams in Ground End of Transf. and Hose
83	81.1	169.1	14.0	83	150	67	1.4	66	245.7	−3,560
89	87.5	181.1	15.3	103	183	80	1.6	78	291.6	−4,180
95.7	94.3	193.9	16.6	134	260	126	1.9	124	352.8	−4,720
102.7	102.3	207.8	18.0	176	440	264	2.2	262	429.3	−4,980
107.2	106.8	215.2	18.7	203	600	397	2.4	395	505.8	−4,940
112.2	112.2	224.6	19.6	244	1,200	956	2.7	953	675.0	−4,400
128.6	129.7	252.5	23.5	420	4,380	3,960	3.8	3,956	1,902	+8,400
				With No. 1 Line Dead						
89	76	159.3	13.4	70	140	70	1.3	69		
94	92.5	190.5	16.6	127	200	120	1.9	118		
111.2	114	227.0	19.9	258	1,100	842	2.1	840		
115.0	116.4	232.8	20.4	277	1,400	1,123	2.9	1,120		
118.5	120.6	238.2	21.3	310	2,020	1,710	3.1	1,707		

waterwheel output, it is felt that reliable results are obtained.

Test Results:

There are included four sets of data:—

1—A test from Pit to Round Mountain from which Curves 21 and 27 are plotted.

2—A test from Pit to Cottonwood from which Curves 22, 25 and 28 are plotted.

3—A test from Pit to Williams from which Curves 23, 26 and 29, are plotted.

4—A test from Vaca-Dixon to Williams from which curves 24 and 30 are plotted.

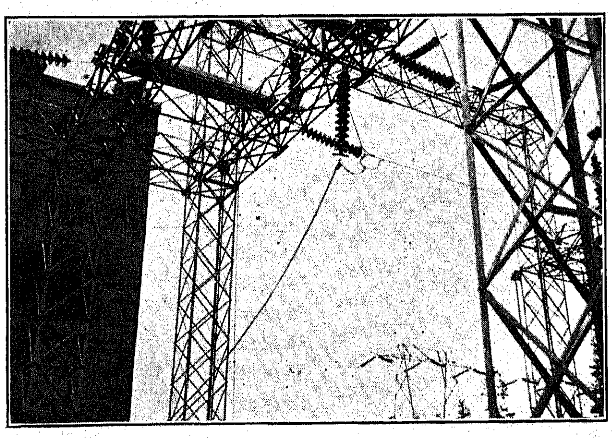


FIG. 10—HOSE AT PIT NO. 1

There is included a curve of the hose calibration, the water-wheel calibration, the transformer losses, one comparing the high-tension readings, and finally, curves plotted on semi-log paper showing the general law of the loss measured. There is also given measured single and three-phase impedance.

Calibration Low-Tension Readings. The instruments were calibrated at the approximate power factor used before the test and after completion rechecked at the test power factor and load readings. The instrument

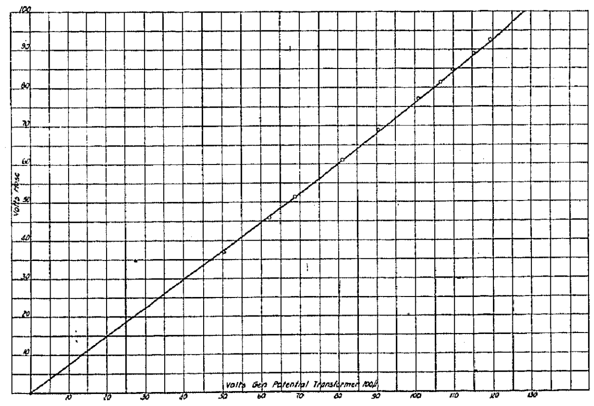


FIG. 11—HOSE CALIBRATION OF 2½ IN. HOSE IN PARALLEL

50 ft. 3 in. Long Resistance 5.5 Meg. Ohms

7 ft. between V. M. Terminals

V. M. Res. 3623 Ohms

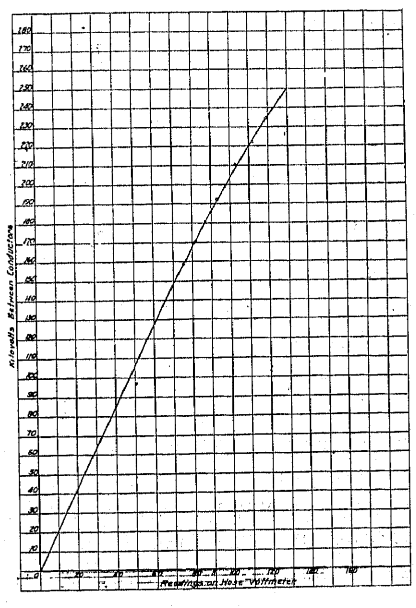


FIG. 12—HOSE CALIBRATION

transformers were calibrated for ratio and phase angle and the values for correction taken from curves. These corrections individually were never more than 1.2 per cent when applied to the low-tension readings, and when combined, were less than the individual corrections as they were in error in opposite directions.

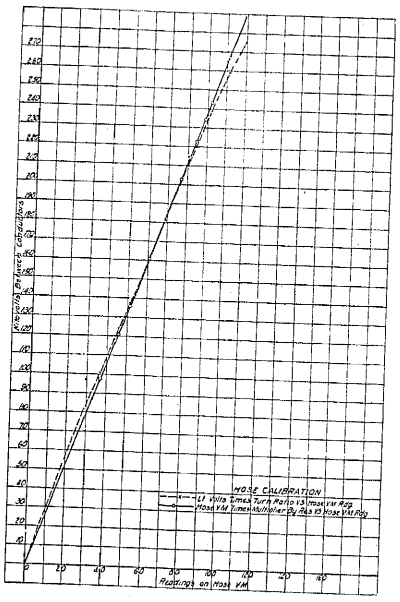


FIG. 13

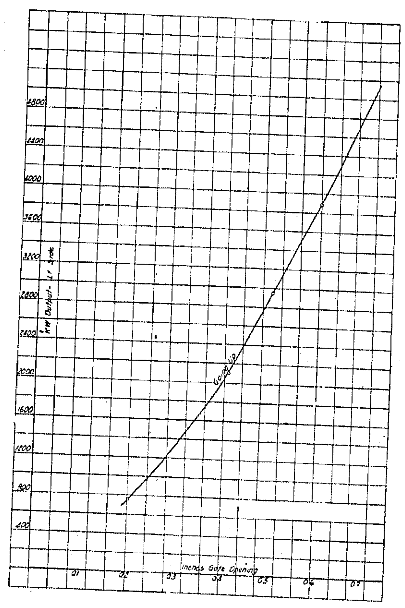


FIG. 14—CALIBRATION OF NO. 2 WATER-WHEEL KW. READ ON 11 KV. AT 1 POWER FACTOR

These corrections were applied, although it is believed they are very close to the accuracy with which it was possible to read the meter. Inasmuch as the losses were considerable, at times over 20,000 kilowatts, the readings were for the most part well up on the meter scale.

Calibration High Tension. The voltmeter on the

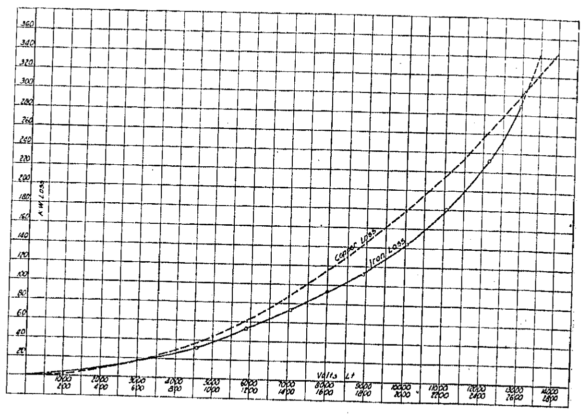


FIG. 15—THREE-PHASE TRANSFORMER LOSSES NO. 2 BANK PIT NO. 1 3-16667 KV-A. 220 KV. Y-GROUNDED TO 11 KV.

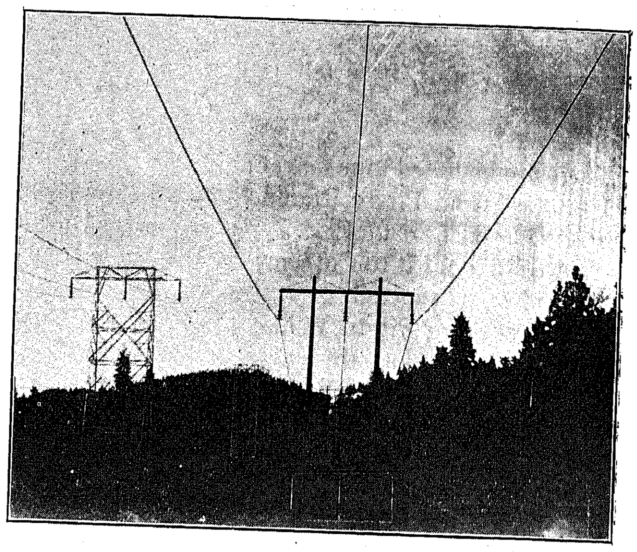


FIG. 16—ALUMINUM LINE AT ROUND MOUNTAIN LOOKING NORTH TOWARD PIT FROM THE FIRST TOWER ON THE ALUMINUM SECTION. ONE HOUR EXPOSURE, PANCROMATIC FILM

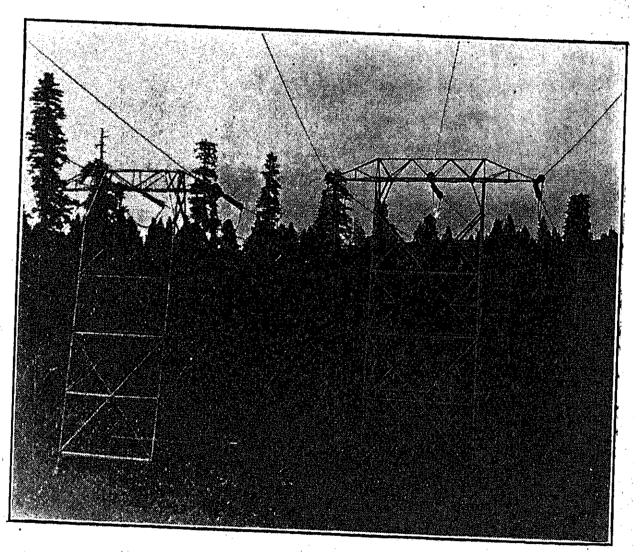


FIG. 17—VIEW OF COPPER LINE AT ROUND MOUNTAIN LOOKING TOWARD PIT FROM THE FIRST TOWER IN THE COPPER SECTION. ONE HOUR EXPOSURE, PANCROMATIC FILM

high tension was calibrated, first, by using the *lowtension* voltage at the turn ratio with the transformer alone

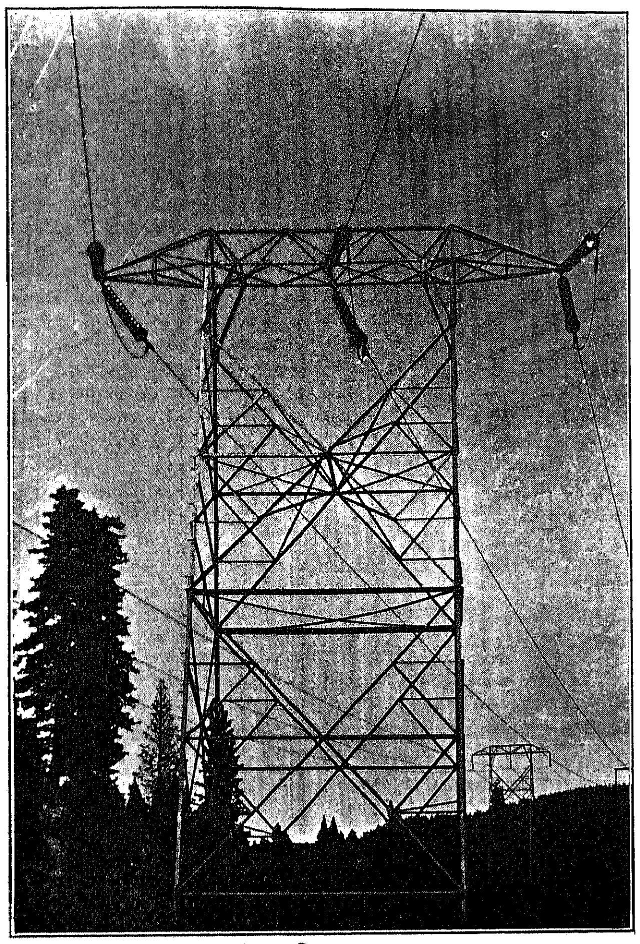


FIG. 18—VIEW OF ALUMINUM LINE, FIRST TOWER NORTH OF ROUND MOUNTAIN, DEAD END. ONE HOUR EXPOSURE, PANCROMATIC FILM

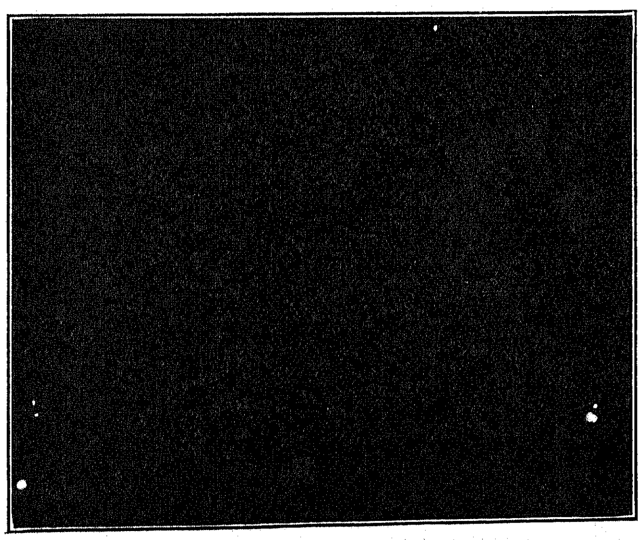


FIG. 19—TOWER AND INSULATORS ON ALUMINUM SECTION. QUARTZ LENS. TEN-MINUTE EXPOSURE, PANCROMATIC FILM

excited from the low tension, and second, by measuring the hose resistance by two independent methods and using this resistance in the usual multiplier formula, correcting by several approximations for the shunted part of the hose. Both of these calibration curves are plotted.

The high-tension wattmeters were calibrated for approximate power factors but on test showed very consistent curious reversals which were studied somewhat further and will be discussed later.

Water-Wheel Calibration. The water-wheel was calibrated in kilowatts generator output against gate

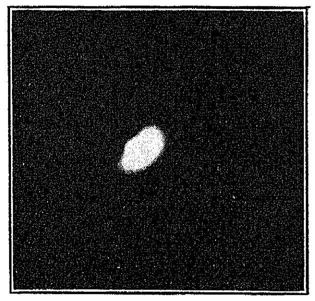


FIG. 20—CORONA SPOT ON ALUMINUM CONDUCTOR. QUARTZ LENS, SIX-MINUTE EXPOSURE, PANCROMATIC FILM

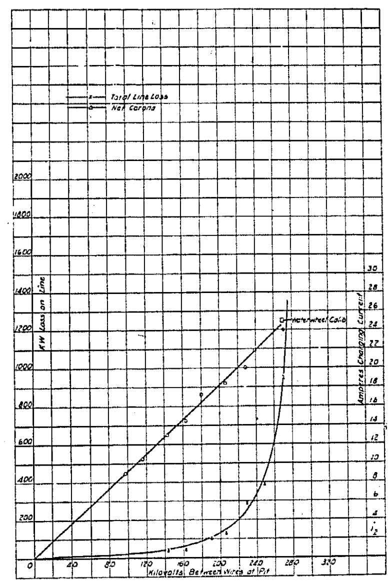


FIG. 21—CORONA LOSS AND CHARGING CURRENT FROM PIT NO. 1 TO ROUND MT. CORRECTED FOR TRANSFORMER LOSS—27.5 MILES OF ALUMINUM HORIZONTAL 17—FOOT SPACING

opening, using special small current transformers at unity power factor.

While somewhat crude, these readings are manifestly unaffected by power factor, phase angle, etc., and give considerable assurance when they check the more precise measurements.

Barometer and Temperature. Barometer and temperature readings were taken and are included in the data. No attempt has been made to reduce these values to sea level and 25 deg. cent. for the reason that

the only correction formulas available contain several unknown factors not directly measurable, and inasmuch as the results depart from the general law of which these factors are a part, it was thought best to give the data as secured.

Considerable effort was exerted in trying to find out if there were any corona or other emanations or disturbing influences present around the insulators that could be photographed, all giving negative results. The views given with their titles are self-explanatory and are representative.

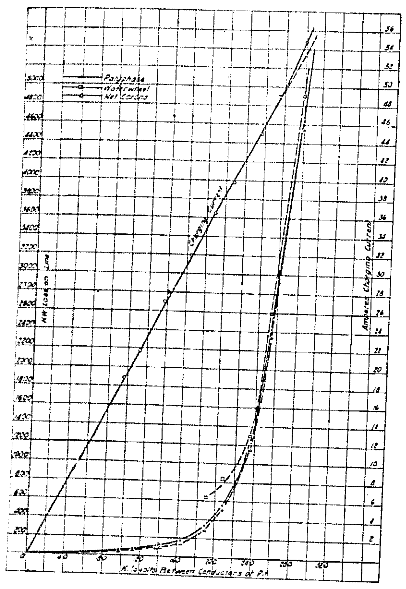


FIG. 22—CORONA LOSS AND CHARGING CURRENT FROM PIT NO. 1 TO COTTONWOOD

Corrected for Meters and Transformer Losses 27.5 Miles of Aluminum 17 ft. Horiz. Spacing: 32.5 Miles of Copper: 16 ft. Horiz. Spacing

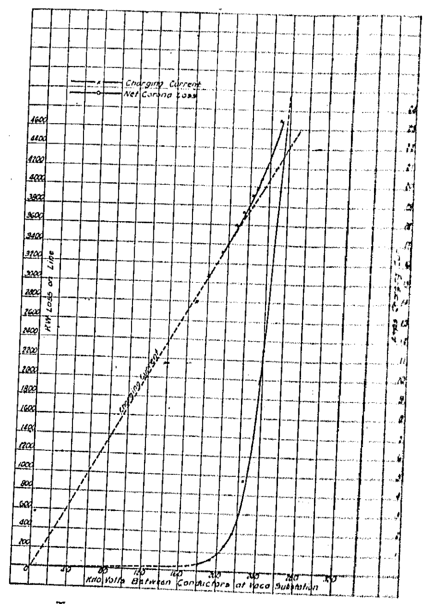


FIG. 24—CORONA LOSS AND CHARGING CURRENT FROM VACA—DIXON TO WILLIAMS

Corrected for Meters and Transformer Losses 5.9 Miles Copper 15 ft. Vertical Spacing

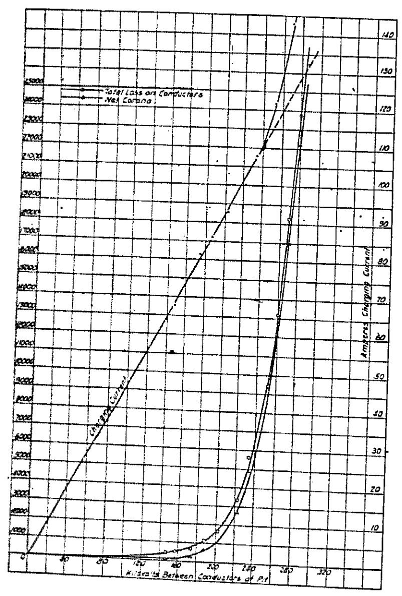


FIG. 23—CORONA LOSS AND CHARGING CURRENT FROM PIT NO. 1 TO WILLIAMS

Corrected for Meters and Transformer Losses.
27.5 Mi. of Aluminum 17 ft. Horiz. Spacing
32.5 Mi. of Copper 16 ft. Horiz. Spacing
83 Mi. of Copper 15 ft. Vert. Spacing

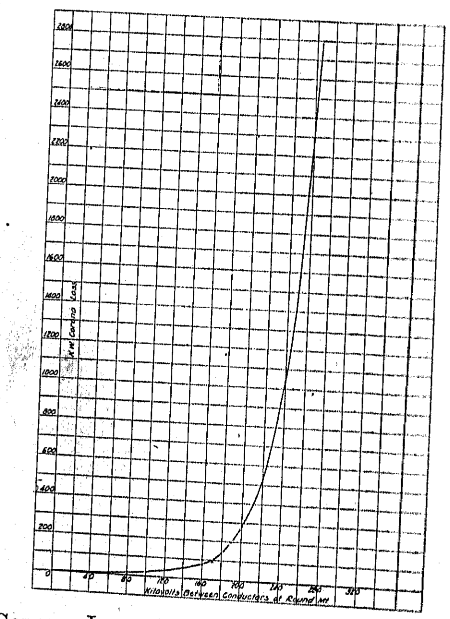


FIG. 25—CORONA LOSS—NET. ROUND MT. TO COTTONWOOD

From Difference of Two Tests. 32.5 Mi. of Copper 16 ft. Horiz. Spacing. Corrected from Voltage Rise from Pit to Round Mt.

Observed Corona. Under normal running conditions photographs of the line with a very high grade commercial lens and a quartz lens loaned by Mr. Peek, were made and are included.

It should be borne in mind that most of the exposures are for considerable time and therefore show more corona than would be visually observed under the same conditions; and that a film of the character used is much more sensitive than the eye to corona discharge.

It is noticeable, as before noted, (Lewis, TRANS.

A. I. E. E., p. 1079, 1921) that corona is the most profuse on the middle conductor and in proximity to ground; also it can be noted that the conductor next to the parallel circuit on the horizontal spacing has more corona than the outside one. Inasmuch as the conductors are transposed on the circuits under test, the individual conductor losses check, as demonstrated repeatedly during test. It is difficult, however, to include any such effect in a general formula, taking account only of the mean geometric spacing of conductors.

No storm losses are so far available but observations of storm conditions were made on the valley section of vertical spaced copper conductor and are included.

Only a representative set of data is given for each test. For instance, four complete curves were taken from Pit to Round Mountain during two consecutive days, in order to check the high-tension wattmeters, giving points that fall exactly on the same curve, as nearly as can be plotted.

The several instruments were read by more than one person to avoid any personal errors and every pre-

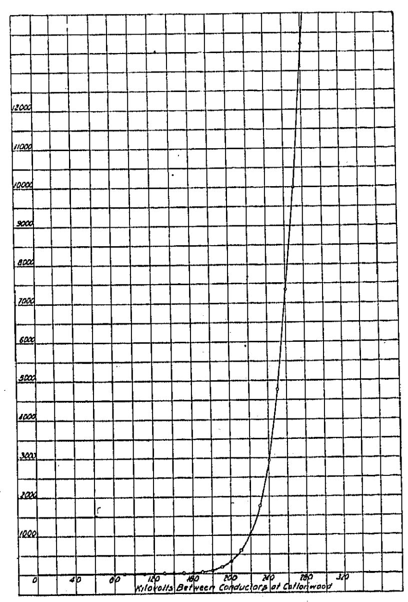


FIG. 26—CORONA LOSS—NET. COTTONWOOD TO WILLIAMS

From Difference of Two Tests. 83 Miles of Copper, 15 ft. Vertical Spacing. Corrected for Voltage Rise from Pit to Cottonwood.

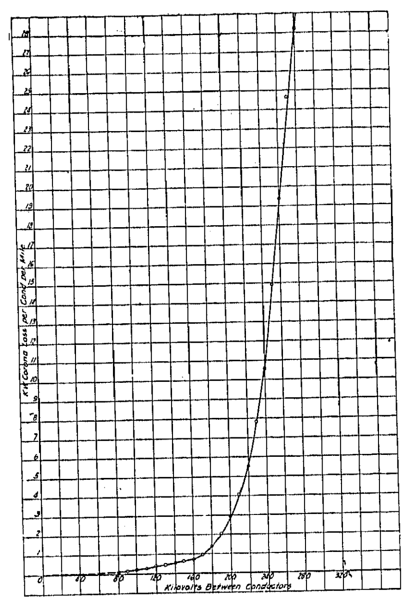


FIG. 28—CORONA LOSS ON ROPE STRAND

500,000 Cm. Copper. 17 ft. Hor. Spacing at 11 Deg. Cent. and 27 in. Bar. Wires to Ground 50 ft. at Tower. Wires to Ground 30 ft. Min. in Span

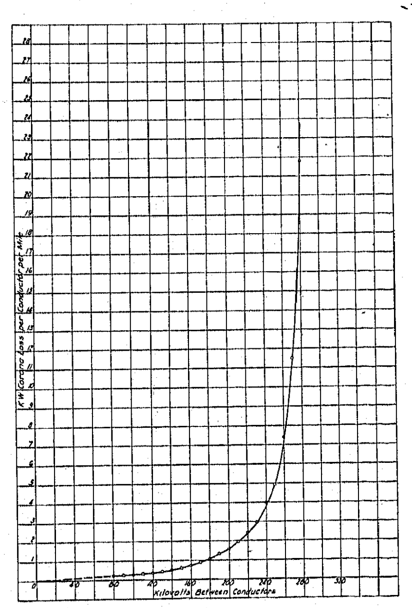


FIG. 27—CORONA LOSS ON STEEL CORE

52684 Cu. Cm. Aluminum 1.06 in. Diameter. 17 ft. Hor. Spacing at 10 Deg. Cent. and 27 in. Bar Wires to Ground 50 ft. at Towers. Wires to Ground 30 Min. in Span

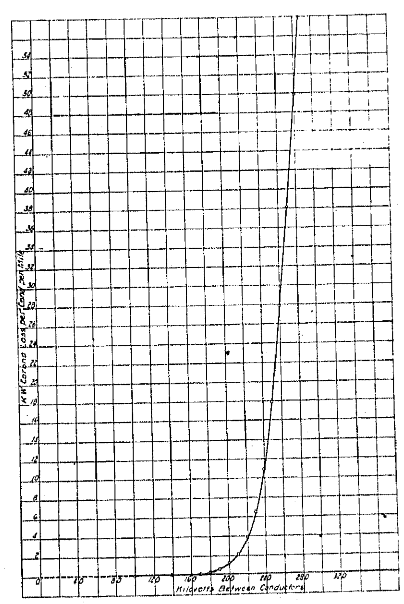


FIG. 29—CORONA LOSS ON ROPE STRAND

500,000 Cm. Copper. 15 ft. Vert. Spacing at 11 Deg. Cent. and 28 In. bar. Bottom Wire to Ground 55 ft. at Tower. Bottom Wire to Ground 30 ft. Min. in Span. Derived Curve

caution used to see that no faulty connections or equipment were used.

In all there were used:

1—Calibrated turbine gate opening.

2—Low-tension k-w., amperes and volts.

3—High-tension voltage from hose.

4—High-tension kw. from the ground end of the high-tension transformer winding, and voltage from the ground end of the hose.

5—High-tension kw. from the line end of the transformer high-tension winding and the high-tension end of the hose.

6—High-tension kw. using the ground end of the transformer high-tension winding and potential from the spare transformer and an 11-kv. potential transformer.

7—Oscillograms at each reading.

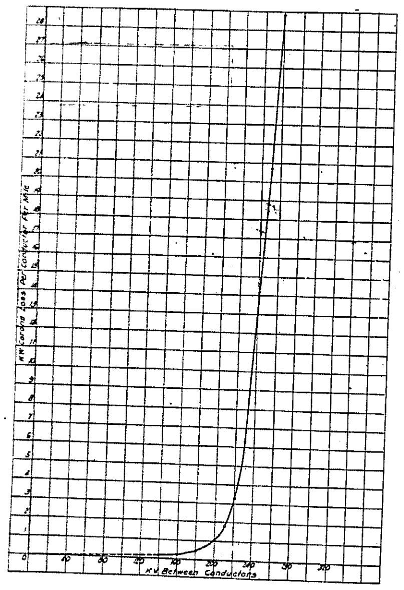


FIG. 30—CORONA LOSS ON ROPE STRAND 500,000 CM. COPPER

A study of each of these leads to the following conclusions:

1—The turbine gate opening calibrated in kw. generator output at unity power factor is, of course, approximate, due to the difficulty of reading accurately, but being affected by neither power factor nor phase angle it gives an added assurance when it checks the more accurate methods.

2—The low-tension kw. were used as a standard, and every precaution used to secure accurate data.

The curves given are based on these readings and the several other methods considered as checks against them.

3—High-tension voltage was measured directly on a voltmeter and the hose described above.

This hose was left in service several days to determine whether there would be any deterioration or change due to difference in water resistance.

For the same conditions no change could be detected, though for conditions which caused considerable leading current to flow through the transformers, the voltage rise above the turn ratio was greater than the usual calculations indicate. (See Lewis, A. I. E. E., 1921).

4—In the original scheme it was decided to use the current in the ground side of the transformer high

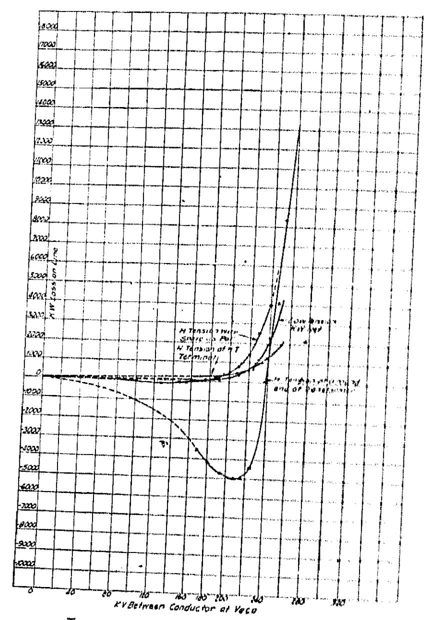


FIG. 31—CORONA LOSS CURVES FROM VACA-DIXON TO WILLIAMS Cor. for Meters and Transformer Losses. 59 Miles Copper. 15 ft. Vertical Spacing

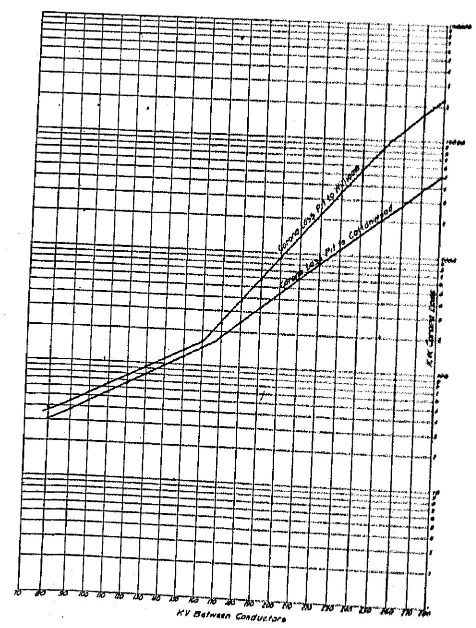


FIG. 32

tension and potential from the ground end of the hose, giving the loss on one conductor.

In practise, it was found that the wattmeter reversed part way up the curve and gave apparent losses much higher than the known generator load.

It was finally decided that this was due to phase angle, but no practical field method of calibrating it could be found.

An attempt was made to shield the hose by enclosing it in a larger hose, running water up the inside and down between the two sizes of hose and using the inner hose on the wattmeter with no practical change in results.

5—The wattmeter was shielded and its current coil put directly in the line using the top part of the hose for potential.

This gave high results at low voltages and low results at high voltages.

6—During the time that the tests were carried on at Pit, an attempt was made to measure the loss in one conductor from Vaca to Williams using the ground of the transformer high tension and the spare transformer, together with an 11-kv. potential transformer to supply voltage.

This gave similar results to the original hose combination, being negative at low voltage and high at high voltages, the negative values being much less as shown in curve 31.

Some two weeks later this was checked in addition to low-tension readings and for the same physical conditions even though read by a different person on another meter. The curve, so far as could be read, was exactly the same and showed an appreciable negative loop at low voltages.

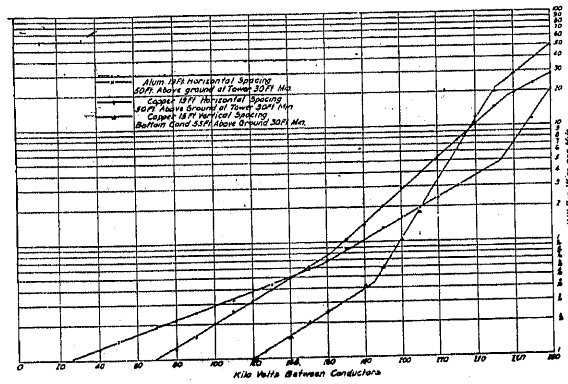


Fig. 33—Corona Loss in Kw. per Wire per Mile

The low-tension readings checked very closely the losses determined at Pit for the same configuration.

These several combinations of high-tension readings are plotted in curve together with the low tension for the same configuration.

By applying a correction for phase angle, these check the low-tension readings and while the phase angle could be calculated approximately, no direct measurement could be taken. They are therefore included only as a matter of interest and an indication of the difficulties of direct high-tension measurements.

The method of correcting for instruments, transformers and $I^2 R$ losses has been described and mention has been made of the voltage correction.

From Pit to Round Mountain the voltage rise was of very little importance being in the order of one kv.

With more lines added, as from Pit to Williams, the rise is appreciable and since it was desired to determine the loss from Round Mountain to Cottonwood and Cottonwood to Williams, the voltage rise was calculated, using the values of line constants determined by the test in the convergent series formulas.

It was discovered that the losses as taken followed an exponential law and plotted a straight line on semi-logarithmic paper. This allowed us to correct the losses in small enough sections that the voltage rise over that section was inappreciable.

This voltage rise correction was not thought to be essential but was carried out for the loss from Round Mountain to Cottonwood and from Cottonwood to Williams.

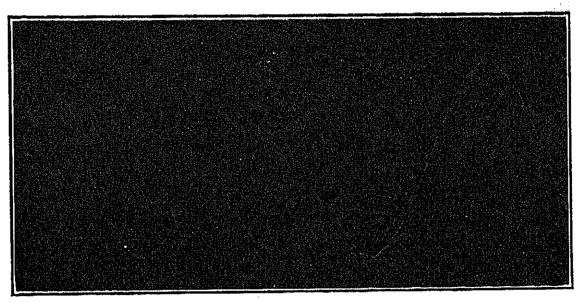


Fig. 34—Pit No. 1. Wave Form No. 2 Generator. Normal Speed, Normal Volts Disconnected

This last was checked by an entirely separate test from Vaca to Williams taken at Vaca and plotted for voltage at Vaca.

This checks almost exactly the corrected curve from Cottonwood to Williams, indicating that the method used was correct.

The final results are given in loss per mile per wire on a line in which the rise in voltage was very small and so can be used to build up the losses on a similar line of any length, provided the voltage along it is known and short sections used; it also allows us to determine what the losses under load conditions would be on a line with a lower voltage at the receiving than at the sending end.

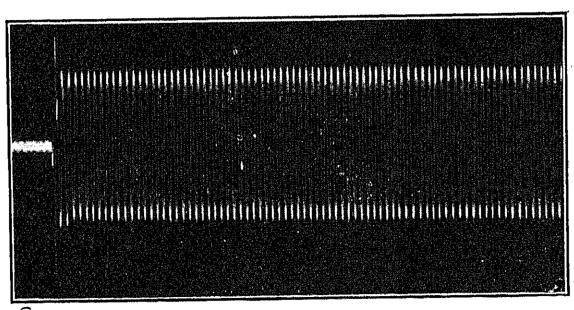


Fig. 35—Corona Loss Tests Pit-Vaca 220 Kv. Line

Switching on Line at Vaca. Record at Pit with Line open except for Hose to Ground. Steady state 133,700 Volts to Ground at Pit.

From the final results it will be noticed that the portion of the line having a comparatively low horizontal spacing has a higher loss for low voltages (even with the

conductors farther apart), than the higher vertical spacing.

On the higher voltages the closer spacing has the higher loss. This indicates that the distance to ground has considerable effect on the loss, and may at times overshadow the distance between conductors.

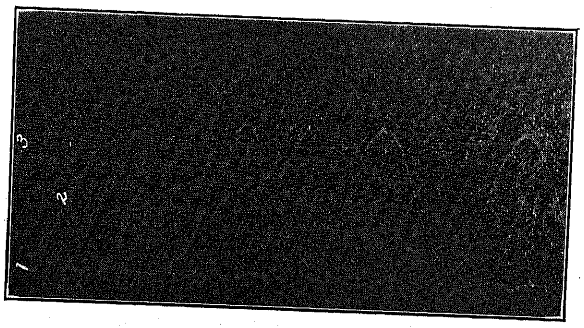


FIG. 36—CORONA LOSS TESTS. PIT-VACA 220-KV. LINE. CORONA LOSS PIT TO WILLIAMS
1—Ground Current Less than 10 Amperes
2—Line Amperes 108 eff.
3—Line Voltage to Ground 135,400 eff.

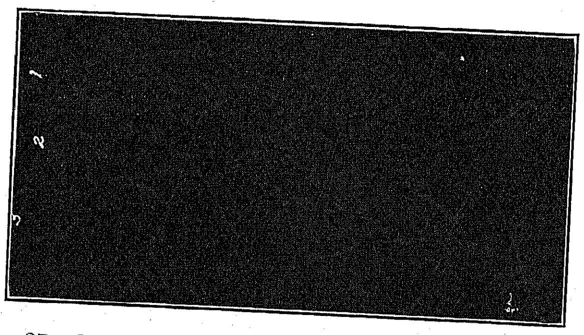


FIG. 37—CORONA LOSS TESTS. PIT-VACA 200 KV. LINE. CORONA LOSS PIT TO WILLIAMS
1—Ground Current 10 Amperes eff.
2—Line Current 121 Amperes eff.
3—Line Voltage to Ground 144,600 eff.

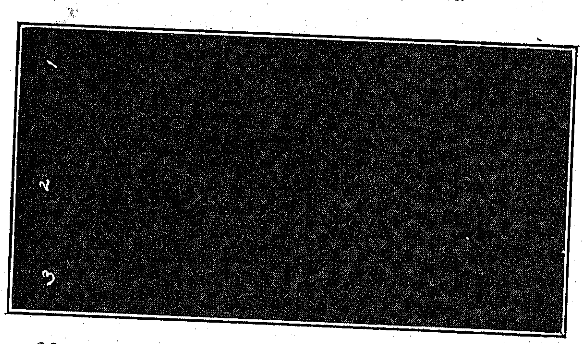


FIG. 38—CORONA LOSS TESTS. PIT-VACA 220-KV. LINE CORONA LOSS PIT TO WILLIAMS
1—Ground Current 45 Amperes eff.
2—Line Current 143 Amperes eff.
3—Line Voltage to Ground 156,200 eff.

As the voltage is increased, a point is finally reached where the slope of the curve changes so that the increase in loss per kv. increase in voltage is less than on the lower part of the curve.

At this point the current wave begins to be distorted and a 180-cycle ground current flows, which at the higher voltages, reaches values of one-third of the total charging current.

This is clearly shown in Figs. 36, 37 and 38 wherein the oscillograph remained unchanged, only the line voltage being raised. This has been discussed in previous papers, though the fact that the actual corona loss does not increase as fast with increase in voltage as at the lower voltages does not seem to have been noted. (Peek, Lewis, p. 1155, p. 1079, TRANS. A. I. E. E., 1921).

For the aluminum wire this point could not be reached because of the larger conductor and comparatively short line.

There is, however, a break in the aluminum loss line at about 155 kv. between conductors. This is below the visual corona and it has been suggested that it is connected in some way with the insulating film on the conductor and the ionization around it.

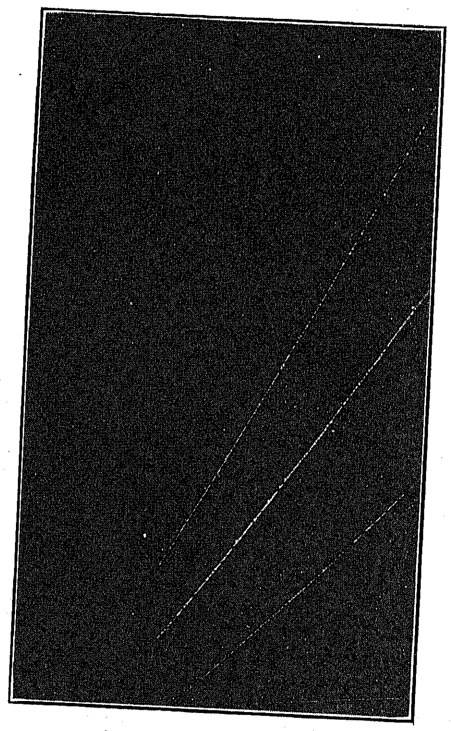


FIG. 39—CORONA AT NIGHT IN RAIN. PIT-VACA 220-KV. LINE AT 3 MILES FROM VACA

The curves secured seem to divide the corona loss into three phases as regards line voltage:

1—Losses below the point where the corona is visual, called by Professor Ryan dark brushes and discharging free ions.

2—The visual corona in which the losses are considerable and the increase in kw. loss per kv. rise in voltage is greater than in Case 1.

There, visual brushes are formed and ionization by collision is the important loss medium.

3—At a point sufficiently high in voltage a considerable third harmonic flows, deforming the current wave (see oscillograms Figs. 36, 37 and 38, and decreasing

the slope of the curve, *i. e.*, having less loss increase in kw. per kv. increase than 2.

The loss has the nature of a gas electrical hysteresis with half arc characteristics and at this point the charging current deviates from a straight line indicating that the dimensions of the conductor have in effect increased (see Curve Fig. 23) as suggested by Peek (A. I. E. E., July 1921).

There has not so far been sufficient data accumulated to make any attempt to formulate a law or fit to the

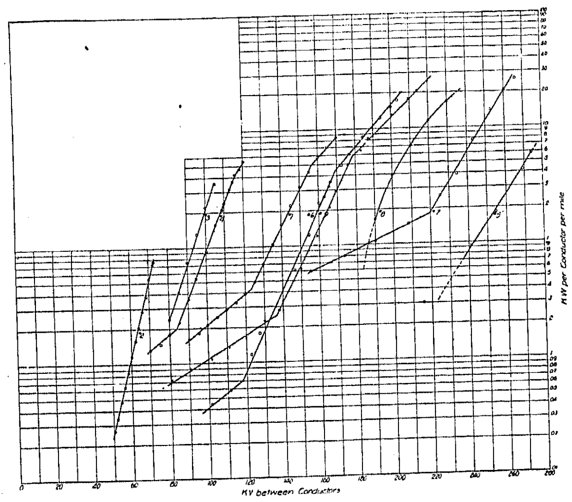


FIG. 40

No. 1. Lewis Curve No. 18, page 1109, A. I. E. E. Data divided by 3 × 101.5 to get Kw. per Conductor per Mile

No. 2 Jakobsen Curve "C," page 103, A. I. E. E. 1918 Data divided by 3 ×68.4 to get Kw. per Conductor per Mile

No. 3. Faccioli, page 341, A. I. E. E. 1911. Data divided by 3 × 153.5 to get Kw. per Conductor per Mile

No. 4. Faccioli Curve "C," page 342, A. I. E. E. 1911, Data divided by 3 × 63.5 to get Kw. per Conductor per Mile

No. 5. Wood, page 723, A. I. E. E. 1922. Kv. multiplied by 1.73 to get Kv. between conductors

No. 6. Peek, High Voltage Engineering, page 248, Test 146 Reduced to Miles

No. 7. Clark & Ryan. Pit-Vaca Conductor 18 Spacing, Bar 30.01 In. Temp. 67.5 Deg. Fahr.

No. 8. Peek. Pit-Vaca Conductor Dec. 1921, 16 ft. 10 in. Spacing 25 Deg. Cent. 76 Cm. Bar. $M_0 = 68$

No. 9. Peek. High Voltage Engineering, page 122. Reduced to Miles

existing laws in order to determine what the loss would be for conditions other than those under which the data were taken.

So far no actual mesaurements have been taken during storm conditions. Observations have been made, however, and a kodak picture taken during a comparatively heavy rain storm at night.

After dark and during the time that rain is actually falling the conductors are faintly luminous and can be traced between towers.

On inspection at close range the luminous area is made up of corona tufts or brushes, some two inches long, starting in a bright core and ending in a blue brush at the points where a rain drop comes near to the conductor.

Their position would give an average spacing of from 12 to 17 in. during heaviest rainfall, and at times when few rain drops fell, of probably two or three feet.

There is no regularity in either their position or direction and they seem to flit back and forth along the conductor remaining only a fraction of a second in one place.

It was noticed that no rain drops collected on the 220-kv. conductors as they did on the 110 kv. of the same size conductor in the same tower.

In the higher elevations the same effect is noticed with the snow, it being harder, of course, to see just what happens to the brushes.

Roughly, from station meters, the corona loss seems to be approximately doubled, dropping back to normal again as soon as the storm ceases.

A great many engineers have stated that it is advisable to run below the visual corona point. This statement is open to question and the final solution is economic rather than technical.

The $I^2 R$ loss on the line under discussion at full load is 8500 kw. approximately. As the voltage is lowered to decrease corona the $I^2 R$ increases and conversely when the voltage is raised to decrease $I^2 R$ corona in-

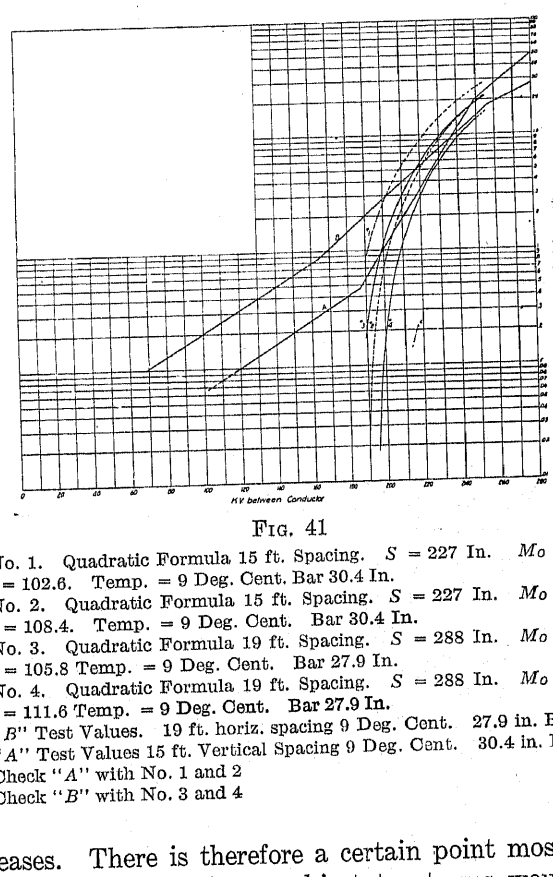


FIG. 41

No. 1. Quadratic Formula 15 ft. Spacing. $S = 227$ In. $M_0 = 0.68$. $C_0 = 102.6$. Temp. = 9 Deg. Cent. Bar 30.4 In.

No. 2. Quadratic Formula 15 ft. Spacing. $S = 227$ In. $M_0 = 0.72$. $C_0 = 108.4$. Temp. = 9 Deg. Cent. Bar 30.4 In.

No. 3. Quadratic Formula 19 ft. Spacing. $S = 288$ In. $M_0 = 0.68$. $C_0 = 105.8$ Temp. = 9 Deg. Cent. Bar 27.9 In.

No. 4. Quadratic Formula 19 ft. Spacing. $S = 288$ In. $M_0 = 0.72$. $C_0 = 111.6$ Temp. = 9 Deg. Cent. Bar 27.9 In.

"*B*" Test Values. 19 ft. horiz. spacing 9 Deg. Cent. 27.9 in. Bar.

"*A*" Test Values 15 ft. Vertical Spacing 9 Deg. Cent. 30.4 in. Bar

Check "*A*" with No. 1 and 2

Check "*B*" with No. 3 and 4

creases. There is therefore a certain point most efficient which in territory subject to storms would be lower than that for the same conductor in a more advantageous climate.

At present there seems to be no way of determining what the corona loss on a proposed line will be at the voltage present practise gives it to operate, and this is the particular region in which power company engineers are interested.

The losses at voltages below visual corona are con-

siderable and if known might in certain cases vary the size of conductor used 100 per cent (see Jakobsen, TRANS. A. I. E. E., p. 91, 1918).

On the line tested, the losses are greater for a line close to the ground even though it has a greater spacing. This is not accounted for in present formulas.

In lines where several constructions of line are alternates this will affect the design and therefore the cost, and is important.

The curves as finally plotted, together with such tests as have been published from time to time by the Institute, give a series of straight lines on semi-logarithmic paper throughout their entire range corresponding to the several physical changes noted.

These curves are plotted from the original data as given in the several references, the changes necessary to reduce them to kilowatt loss per conductor per mile being also noted.

In tests such as those of Lewis the condition nearest approximating the 220-kv. test configuration and connections was plotted.

These curves were plotted to compare with the form of curve observed on the test and all of the actual transmission line tests obtainable seem to show this exponential form.

From the original test made in 1921 on a 300-ft. section of the copper conductor, the irregularity factor was given as 0.68. After reviewing the tests as made on the line, this has been changed to 0.72. Both of these values are used in the quadriatic formula under test conditions of temperature and barometer and plotted together with the values of the test in Curve 20.

The observed loss would indicate for the test conditions approximately 22,000,000 kw-hr. per year, the value of 0.68 and the quadratic law 43,000,000 kw-hr. per year and the value of 0.72 approximately 24,000,000 kw-hr. per year.

These represent losses in the order of $100,000 per year and bear out the statement that it is desirable to know the loss under operating conditions. A more thorough understanding of the physical causes and mechanics of corona loss is gradually being reached. This is hopeful and if, from lines existing and now being built, it is possible to secure the data for determining these losses, it will be more and more worthwhile as longer lines and higher voltages come to be used.

Discussion

PAPERS ON CORONA

(RYAN & HENLINE, CLARK & MILLER, CARROLL, PETERSON & STRAY, CLARK & EVANSON, WILKINS)

PASADENA, CAL., OCTOBER 13, 1924

F. W. Peek, Jr.: It is my opinion that apparent discrepancies in the corona papers presented are not due to error in the different investigations, but rather to the difficulty of comparing different conditions. To illustrate what I mean by one example: An aluminum cable strung at Stanford in a grassy field and another strung at Pit after being dragged over the sharp lava rocks should be expected to give different losses near the critical voltage. It would be difficult to predict the result of the mutilation by the lava beds without direct measurements.

I will point out here briefly a few of the outstanding facts concerning my early work which may apply to the present measurements:

Tests were made on indoor lines and on outdoor lines in actual towers under all kinds of weather conditions and at temperatures varying from below zero to 90 deg. fahr. Probably one hundred thousand different readings were taken. Measurements were always duplicated on both the high-and low-voltage sides of the transformer by wattmeters especially calibrated for low power factor. The transformer loss was small. These readings checked. I have great faith in readings made on the low-voltage side because the probably error can be determined readily. It

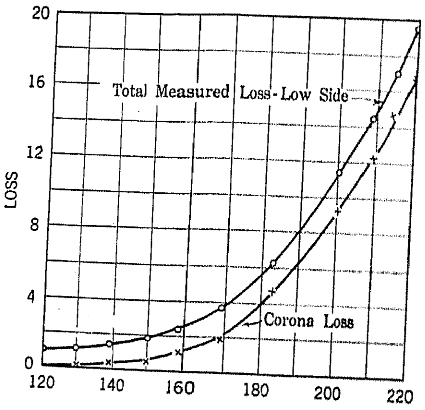


FIG. 1

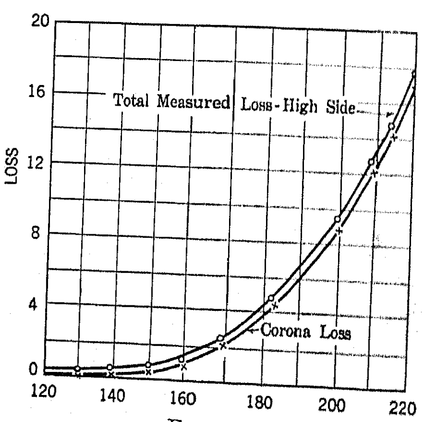


FIG. 2

FIGS. 1-2—COMPARISON OF HIGH AND LOW SIDE CORONA LOSS MEASUREMENTS

was small. Wave shape and the maximum of the wave was determined by an oscillograph. Voltage was not measured by gaps but determined by calibration of the voltmeter coil with step-down transformers. A small error in voltage is equivalent to a larger error in power. Figs. 1 and 2 herewith give an example of the total loss measured on the high and low side and the net corona loss. It is not difficult to determine the possible error, which is small. In Fig. 3 the losses obtained on high and low sides are plotted on a single curve. There is good agreement.

A study of these data showed that the quadratic law applied very closely above the critical voltage. The criterion for the quadratic is that the curve between the voltage and the square root of the power loss is a straight line. The test for the quadratic law is thus quite simple. Near the critical voltage there was an excess loss depending upon irregularities, dirt, moisture, etc., on the conductor surface. This caused a loss in excess of the

quadratic law which followed the probability law. I found no loss due to direct leakage through the air. What excess loss there was below the true critical voltage was found to be due to

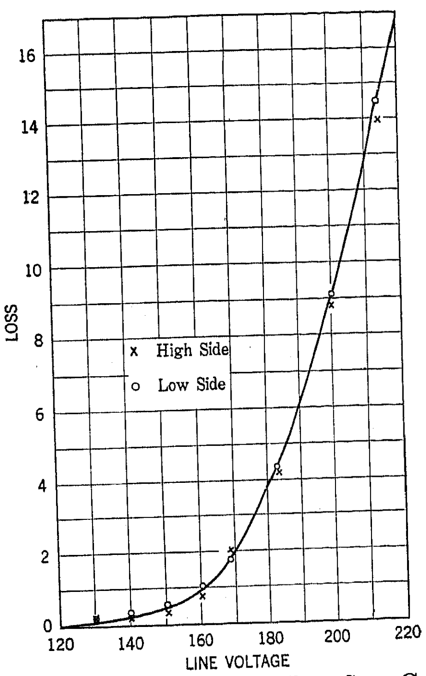


Fig. 3—Comparison of High and Low Side Corona Loss Measurements (See Table I, "Law of Corona," A. I. E. E. 1911)

3/0 Seven-strand cable; 10 ft. spacing; Total length of cond. 3600 ft.; Bar 0.75 cm.; Temp. 10 deg. cent.

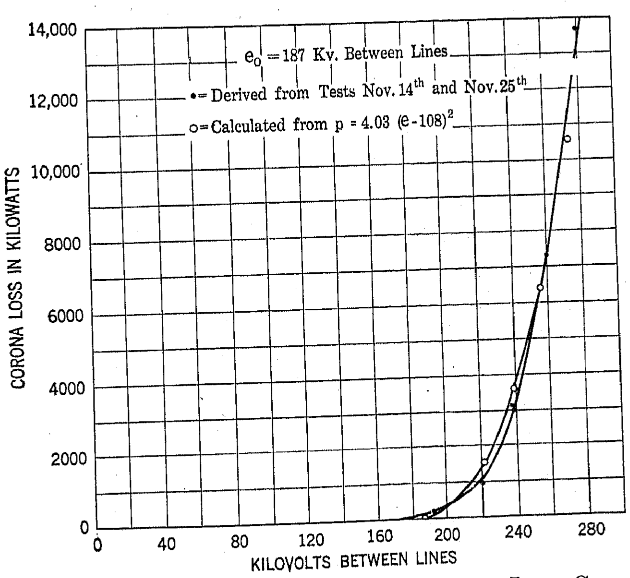


Fig. 4—Calculated and Measured Corona Loss Cottonwood and Williams Section of 220,000-Volt Pit-Vaca Line

Length = 83 mi.; 500,000 cm., 7 by 7-strand copper cable; Rope lay O. D., 0.910 in.. M_o = 0.72; Spacing = 15 ft. vertical.

	Nov. 14	Nov. 25	
Bar. Pressure	27.05 in.	26.99 in.	At Pit No. 1
Temperature	5.63 deg. cent.	10.0 deg. cent.	

δ = 1.03

actual brushes at local points. The loss at this part of the curve is very sensitive to dirt and moisture on the conductor surface. This is fully discussed in my first paper.

Professors Ryan and Henline presented today a very interesting paper on the theory of the mechanism of corona. I am pleased that the corona values which they have measured and calculated check so well the values calculated by my quadratic

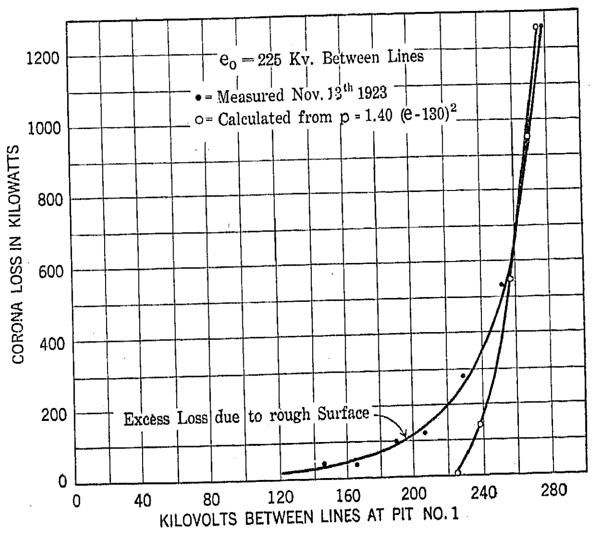


Fig. 5—Calculated and Measured Corona Loss, Pit to Round Mountain Section of 220,000-Volt Pit-Vaca Line

Length = 27½ mi.; 642 cm. 61-strand, steel-core aluminum cable; Concentric Lay O. D. = 1.08 in.; Cable new and scratched, M_o = 0.78; Spacing = 19 ft. horizontal

Bar. pressure = 26.90 in. } At
Temperature = 9.2 deg. cent } Pit No. 1
δ = 0.95

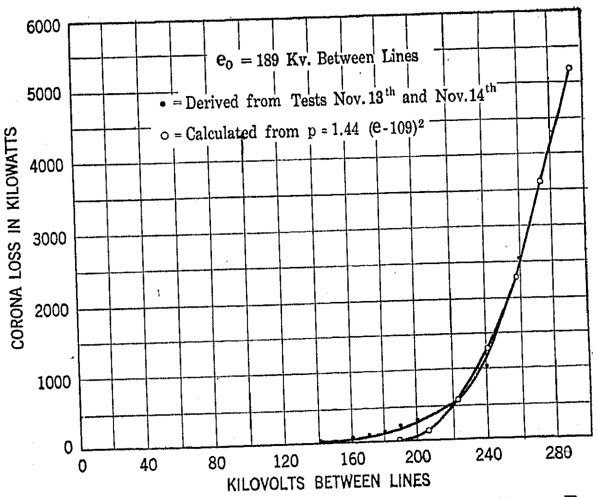


Fig. 6—Calculated and Measured Corona Loss, Round Mountain to Cottonwood Section, 220,000-Volt Pit-Vaca Line

Length = 32½ mi.; 500,000 cm., 7 by 7 strand copper cable; Rope lay O. D. = 0.910 in.; M_o = 0. 72; Spacing = 19 ft.

	Nov. 13	Nov. 14	
Bar. Pressure	26.90 in.	27.05 in.	At Pit No. 1
Temperature	9.2 deg. cent.	5.63 deg. cent.	

Conditions for this section
Assumed = 28.0 in. and 9.2 deg cent.
δ = 100

law as shown in their table. There is no doubt that part of the corona loss is a per-cycle loss, but in addition there is a component loss which does not depend upon the frequency. There is also considerable corona loss on direct current, The loss on a wire

apparently varies for the two halves of the a-c. wave. For this reason, I have never thought of this loss as a hysteresis loss.

I have always thought of the quadratic law as rational. Energy is stored in the dielectric until the critical point is reached on the voltage wave. At this point the brushes start to short circuit part of the dielectric. Part of the energy is not returned. This energy should be as the square of the excess voltage above the critical voltage.

The measurements of Professors Clark and Evenson are interesting. I would like to call attention to the fact, however, that the correction which they find it necessary to make in order to get the net corona loss is much larger than the transformer-loss correction for measurements made on the low side. Such

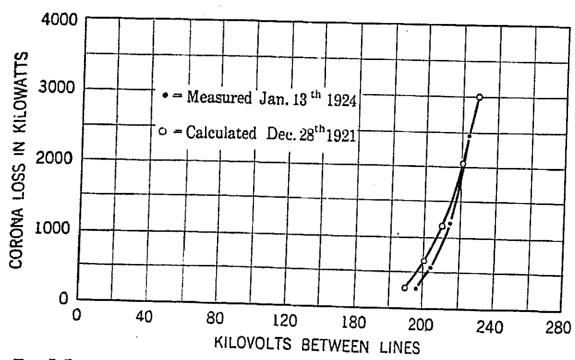


FIG. 7—MEASURED AND PREDICTED LOSSES ON WILLIAMS TO VACA SECTION OF 220,000-VOLT PIT-VACA LINE

Length = 59 mi.
500,000-cir. mil, 7 by 7 strand copper cable
Rope lay O. D. = 0.910 in.
M_o = 0.72
Spacing = 15 ft. vertical
Corrected to standard conditions of temperature and barometric pressure
t = 25 deg. cent.
p = 29.9 in.
δ = 1.00

a correction is uncertain, whether it is made on paper or in the instrument.

The high-voltage wattmeter when fully developed should be a very useful tool.

Mr. Wilkins' measurements are very important from both the theoretical and practical standpoint. Investigations of this character are of great value to the industry. Two types of conductors are used: Concentric-lay aluminum cable in the lava country of the Pit and rope-lay conductor in the valley. Since the rope-lay conductor is irregular in a uniform way, the surface should not be much affected during stringing. The uniform aluminum conductor should be quite badly mutilated by dragging in the lava and should, for this reason, be expected to give large excess loss near the critical voltage. It is naturally difficult to estimate the effect of such treatment for any particular case. Mr. Wilkins' measurements show this excess loss for the aluminum conductor. The measured and calculated losses check very closely for the rope-lay cable. This is illustrated for the two different types of conductors in Figs. 4 and 5 herewith giving calculated loss and measured points taken from Mr. Wilkins' paper. Fig. 7 shows the measured and predicted loss for the line. Mr. Wilkins finds that the measured losses do not always follow a quadratic. This should be the case on a long line for the following reasons: There would be at least several critical voltages along the line due to the different conductors, difference in elevation and temperature. The wattmeter reads the sum of the losses for all of the sections at different critical voltages. If the losses in each section follow the quadratic law, the sum of these quadratic curves, which is what the wattmeter reads, could not be expressed by a single quadratic but would be a curve similar to the ones found by Mr. Wilkins. This is shown in Fig. 8. Each section should be calculated separately and added to obtain the total. A small error in voltage would also make a large error in power. I note that there is a considerable difference in voltage measurement on the low side and by the hose on the high side. This would also contribute towards the difference noted.

Mr. Wilkins states that no corrections were made for temperature. This would also cause difference between his measurement and calculations. I would like to ask if loss measurements were also made during the warm summer months. The quadratic law shows that the loss would be much higher at the higher temperature. If Mr. Wilkins has such measurements they would be a check on the quadratic law.

It is my opinion that it will generally not be economical to operate a line above the fair-weather e_o voltage. The loss during fair weather at this voltage is generally not of great importance since the controlling factor is the rain loss. With

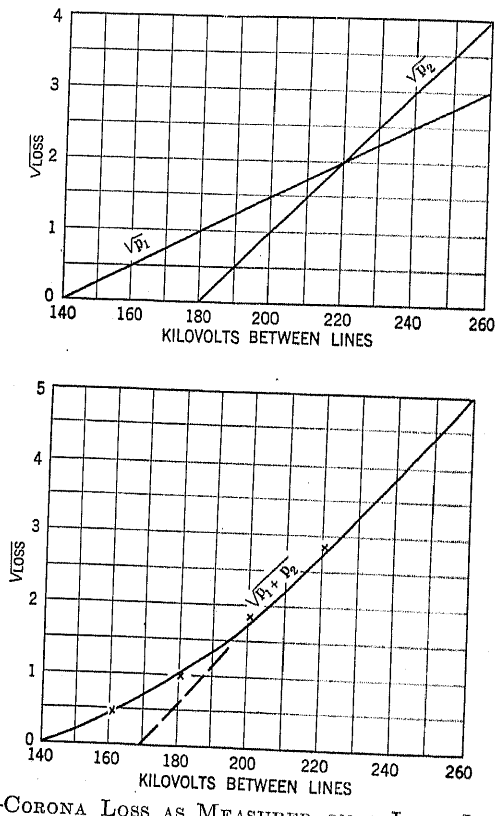


FIG. 8—CORONA LOSS AS MEASURED ON A LONG LINE WITH DIFFERENT CRITICAL VOLTAGES

Upper Curve—Two quadratic corona loss curves with different critical voltages

Lower Curve—If two corona loss curves with different critical voltages are added, the sum cannot be represented by a quadratic, although each component curve is, in itself, a quadratic.

rain only 10 per cent of the time the greater part of the annual loss would occur during storms.

The large amount of power transmitted on 220-kw. lines will generally require conductors so large that the critical voltage is above the operating voltage. If this does not occur, the diameter of the conductors can be easily increased by a central core to meet this condition. In other words, why have any appreciable loss when it is so readily prevented? The exact calculation of the annual loss is thus not always as important from the practical standpoint as from the theoretical standpoint.

In conclusion, the loss measurements on which the quadratic

law was based were made in duplicate on high and low sides of the transformer and were always in good agreement. Voltage was measured by step-down transformer and voltmeter coil. The wave shape was determined by oscillograph.

This law holds over a very large range of voltage as shown by the curves in my discussion of Professor Harding's paper.

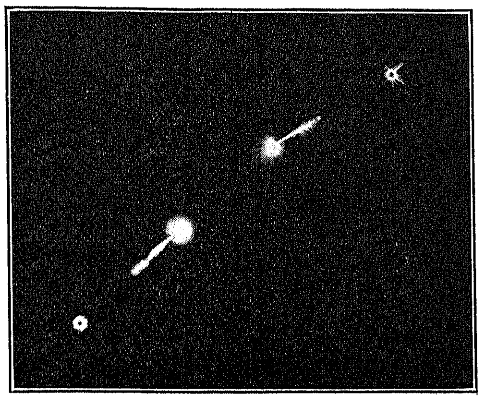


Fig. 9

Measurements made in 1924 check those made in 1910 although made by different men with different apparatus.

The reasons for apparent and actual deviation from the law indicated by some of the curves in the papers presented today were for the most part observed and discussed in my 1911 paper.

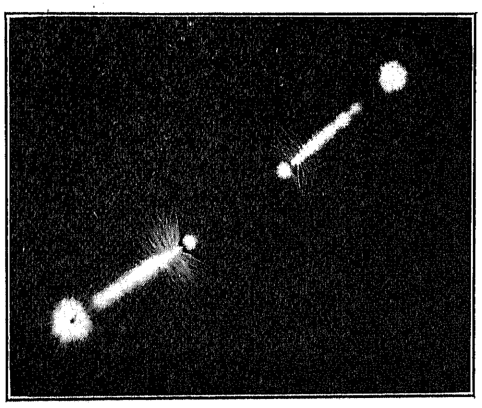


Fig. 10

Predicted loss is in good agreement with the actual loss on the 220-kv. lines.

It will generally be desirable to operate below the e_0 voltage. If the conductors are not large enough the diameter can be increased by a core.

R. W. Sorensen: The Steinmetz-Hayden paper, entitled High-Voltage Insulation, presented at the 1923 Pacific Coast Convention held in Del Monte, showed some very interesting phenomena, the reasons for which we would all like to know. Thinking that some Lichtenberg figures might help to work out the reasons, I have some photographs which may also help explain the hysteresis character of corona loss as found by Professor Ryan and his colleagues.

Fig. 9 is made with about 10,000 volts d-c. applied to two needle points spaced about 1 3/8 in. apart. The point at the right is connected to the positive terminal and the one at the left to the negative terminal. In these figures is seen the characteristic difference between the positive and negative records, both as to appearance and distance from the electrode, marked by the emanating electrons.

Fig. 10 shows two similar points with alternating current impressed upon them.

Fig. 11 shows a cylinder and a needle point, the nearest approach one can make to having a plate represent the discharge between a point and a sphere. In this case, the voltage applied was alternating current. The voltage on all slides shown is below visible corona voltage.

Fig. 12 shows a double phenomena, the cylinders being charged

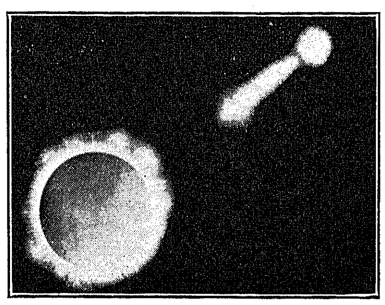


Fig. 11

positively and negatively used in conjunction with negatively and positively charged needle points, respectively. This photograph was made by Mr. H. E. Mendenhall, a graduate student at California Institute of Technology.

An examination of these photographs shows the greater range of electron travel from the positive electrode as compared to the range from the negative one.

An inspection of these figures leads one to the logical conclusion

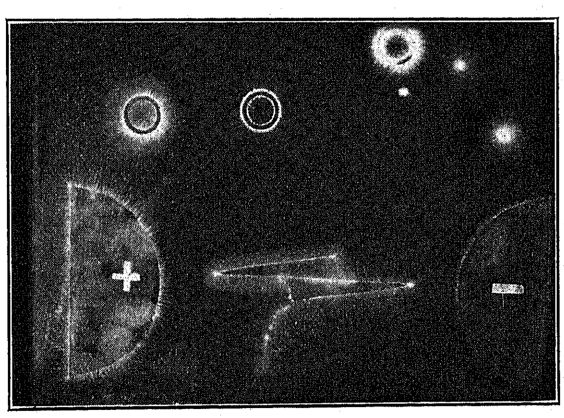


Fig. 12

that the reason for the results of the tests made by Steinmetz and Hayden is that when a point is connected to the positive terminal, the electrons travel a much greater distance for a given applied potential gradient than is the case when the point is negative and the sphere positive. Extending this reasoning to explain the results of tests which are now under discussion, is it not possible to assume with a fair degree of certainty that this also explains the reason for hysteresis effect of a-c. corona. During the half cycle a conductor is positively charged its electrons have a larger field of travel than during the half cycle it is negatively charged, thus making it impossible to account for a complete balance or neutralization of charges.

W. A. Hillebrand: While the electro-motive force increases up to a certain point, your charge is directly proportional until

your critical voltage is reached. As you descend the curve and reach zero electro-motive force there is a residue charge which should be neutralized. A somewhat similar relationship occurs during the other half wave. That means that within certain limits the energy loss per cycle is constant regardless of frequency. To me that has been extremely significant. The time effect as commented on by Professor Ryan does not enter over the lower range of frequency; as frequency increases the time effect does enter. We have another relation of interest: That you have a loss by electrical conduction which is a nice parabola. Is it possible that at a certain point we begin to develop the self-propagating streamers? That is, the Peek high-frequency effects? If that is so, it is to my mind important that we understand still more about the fundamental periods at which a transmission line will oscillate (I am not referring to the frequency of the line as a whole) in order to determine whether or not the fundamental frequency is at a point where you will develop the possibility of self-propagating streamers.

A wattmeter that will measure one watt, or a fraction of a watt at 110,000 volts, is, I think, as everyone who deals with measurements will agree, a remarkable achievement. I hope that it will be found possible, through the medium of this instrument, to develop further information with regard to the current-carrying capacity of liquid films that will lead to understanding the question of surface leakage.

For example, there is a transmission line in our neighborhood that has gone out simply due to surface loss over the insulators. It is a 60-kv. circuit. When the line is dead, and suddenly energized, the loss is too great to start it going.

J. B. Whitehead: In the first of these papers we have a further contribution to the problem of the measurement of small losses at low power factor in high-voltage circuits. It is a problem that has been attacked for years; and which has caused great trouble because of the difficulty of maintaining the current in the voltage circuit of a wattmeter, of whatever type, exactly in phase with the applied voltage.

Professor Ryan's work is notable in three particulars, I think: First of all he is putting the entire wattmeter system in the high side of the line. Ordinarily, in work of this kind, we find that instrument at the central neutral point. He does not eliminate however the problem of screening the voltage coil and resistance.

The next point is the method used for screening the voltage circuit and making certain that the current in that circuit is not only proportional to the voltage, but that it shall be in phase with the voltage. Care must be taken of both of these things. Professor Ryan has done it by adjusting the relation of the series resistance, or voltage multiplier, as he calls it, to the electrostatic field. I like, particularly, the ingenious method that he has used for maintaining a uniform electrostatic potential gradient over the resistance. He has placed it between two external shielding rings and has studied the field between those rings making sure that the water coil, as it spirals down, takes a constantly descending electrostatic potential as well.

The use of water is the third point. I think it is not very common to use water as a resistance. The difficulties are due to low conductivity and to heating. Professor Ryan has attempted to avoid them by letting his water flow continuously and I see no reason why that is not a perfectly sound method for meeting the difficulty provided, only, that the temperature is properly regulated.

It is not, perhaps, as well known in this country as it should be that the screening of such resistances, so as to make sure the electrostatic error is avoided, and a method for studying the degree of such error, and controlling it, has been published. It was published in Germany first by Professor W. B. Kouwenhoven of Johns Hopkins University, and subsequently by Washington University, of St. Louis. It is a very complete examination of this question of electrostatic error in connection with resistances and well worthy of examination by anyone who is contemplating making measurements at high values of voltage.

Professor Ryan and his associates in the second paper, are investigating the question, why the measurements of different observers on corona loss, particularly, in the initial stages, are not always concordant and are subject to doubt. The first reason, is the question of the accuracy of the measurements to which we have just referred. The second reason is due to the fact which Mr. Peek has mentioned, and which, is also clearly in Professor Ryan's mind, that it is impossible to obtain perfectly uniform conditions on a conductor in the open and, particularly, in a long line of such conductors, with alternators in the character of the conductors themselves, the spacing, etc. Most work that I have done has been within the laboratory where we could control conditions. There is never any doubt in such experiments as to what is going to happen after you have made a number of those experiments and the results may usually be given as definite laws. In the open your conductors are not smooth; you may have cable of various make-ups; these cables are scratched and dirty; you no longer have the question of what is the law of corona, but you have the question as to how dirty your wire is and how many needle points it has on it, or how many drops of water it has on it. It seems to me that that is the principal reason why so many of these studies that are brought before us lack the punch of authority.

Now, Mr. Peek, in his first work, met this situation squarely by recognizing the fact and pointing out we must increase the voltage above the initial loss value before we get to the region subject to definite law. We must reach a voltage where the irregular point discharges unite in increasing the diameter of the conductor and making it equivalent to a smooth conductor and for that reason Peek found it necessary to introduce two critical voltages. One was the voltage at which the conductor would start into corona had it been perfectly smooth. The visual critical voltage was something lower.

I am not sure that we are going to get anywhere by trying to study this intermediate range. It seems to me that it is a question of entirely indefinite nature, and the corona voltage may vary from point to point and from line to line. I like very much better the plan adopted by Mr. Peek of simply inserting an engineering constant or factor, having different values for different conditions, into the law for smooth conductors. That seems to me a sound engineering method.

Professor Ryan's second paper deals with this intermediate region of irregular corona formation. He is trying to find some law, as I read his paper, which will enable us to predict what is going to happen within this initial stage. The pictures that he shows and also those by Mr. Peek, make me question whether there is not perhaps something else to the matter. The location of these individual brushes, the fact that they stick as you raise the voltage, and the fact that the pictures show no approach to a uniform appearance as the voltage is raised, makes me ask the question, whether it is permissible to say that above a certain voltage corona becomes a uniform cylinder and that further increase with voltage will be in accordance with a definite law. Frankly, I don't know how to feel about it. Professor Ryan has opened the question anew, and shows that we must study further before we can say that we know all about the law of corona.

I now pass to Professor Ryan's paper in which he is suggesting that the phenomena of corona has a hysteresis character. There have been numerous explanations of the process of breaking down of the air. They ask us to think about the motion of electrons between molecules and to picture the process of ionization by collision. But I don't think that present physical theory is easily followed by engineers. It is, therefore, perhaps just as well if we can confine ourselves to some picture of the loss process due to corona which makes a less extended demand upon our attention. So, Professor Ryan has made this very interesting

suggestion that the loss due to corona may take on the character of hysteresis. He has been led to it by the shape of the power curves taken with his electrostatic wattmeter and taken originally, I believe, with the cathode-ray tube.

Magnetic hysteresis is distinctly a molecular phenomenon, so far as we know it. The nearest we can come to picturing the mechanism of it is that it is some kind of a friction between or within the molecules. Magnetic hysteresis, for years, was the only type of hysteresis that we knew anything about. We next heard of dielectric hysteresis, the loss in dielectrics due to an alternating electric field. Here too, such attempts at explanations as we have had, picture these losses as due to some type of successive orientation on the part of the molecule, or the atom, causing frictional loss.

Now, for these reasons, I am raising the question as to whether the use of the term "hysteresis," which implies molecular friction, is as helpful in explaining corona as the idea of straight electric conduction; that is, conduction through the air by means of the motion of charged molecules or aggregates of molecules known as gaseous ions? You can study the motion of these ions; you can feel them, under certain circumstances; if you put a hole in a grounded cylinder in the middle of which there is a corona-forming conductor, you can feel the motion of ions that come out of that hole under the impetus of the electric field. There are numerous other evidences of conductivity of the air. Experiments with continuous voltage will show you conductivity directly; you can put obstacles in the path and stop the flow of the ions and I don't see exactly how, with Mr. Peek, we are going to think of the hysteresis character of corona, or think of corona as a hysteresis effect and at the same time take care of these evidences of conductivity.

Now, the suggestion of the hysteresis nature of corona by Professor Ryan is based on the shape of the curves taken with the electrostatic wattmeter in which he finds a maximum value of the induction, a shape similar to the magnetic hysteresis loop, and a loss in proportion to the area of that loop, all of which are most suggestive. But it seems to me that the shape of these curves can be completely explained in terms of the conductivity of the air. Taking his Fig. 5, the discharging point is directly opposite one of the plates of the wattmeter and as soon as the point begins to discharge, the plate takes up some charge; that charge is going to stay on that plate because it is insulated, until it is neutralized by an opposite charge in the next half cycle. While the charge is on the plate it will naturally modify the distribution of the electrostatic field that was there before the point began to discharge, consequently, he has a shifting of the sides of his curves, but these remain straight. Why? Because they have a definite charge upon them from the preceding half cycle and he has simply changed the potential of the plate by a uniform amount and the central line of his diagram is carried out to the side of the loop, but always parallel to itself. As he pushes the voltage higher and higher the sides of the cycle will become more and more curved because it takes a longer and longer time to neutralize the charge due to corona on the preceding half cycle.

R. J. C. Wood: I was a little disappointed at the stand that Professor Whitehead took, or, perhaps, I misunderstood him. What I gathered was that we should not try to formulate any laws about this region of corona, which is between the starting point and the point where it follows Mr. Peek's law. On the other hand, Professor Whitehead said we should get all the data we could in that region so that we could establish factors of safety. Perhaps, I misunderstood him entirely and we all mean the same thing. We all want this data because it pertains precisely to that region on the voltage curve, wherein we most probably will have to work for economic reasons.

There is another point of view on this question, namely, that in order to employ anything like 220,000 volts there must be a very large block of power to transmit and a very great distance to transmit it in order to make it an economic success. Having that very large block of power and long distance the economic conductor will be so large that corona losses will be practically eliminated.

There are some points in the paper that I would like to mention. The great trouble with all this corona data has been in bringing the results of different observers into conformity. There are one or two things which occur to me that do not seem to have been specifically mentioned, which may account for these differences. One of them is the deposit which forms on the conductor after it has been in use at a high voltage. Our Big Creek line, of aluminum conductors, was first operated at 150,000 volts and down in the lower altitudes where there was considerable smoke and dust in the air the conductor became coated with what might be called a black enamel. It was hard and so closely adherent to the conductor that it required some force to scrape it off with a knife blade. Up in the higher altitudes, above 3000 or 4000 ft. where the air was pure, that conductor was not so black, and, in fact at the highest altitude of 5000 ft. it was almost as bright after several years as at the time when it was first installed. When we went to 220,000 volts the conductor very quickly blackened, even in the higher altitudes. Now, it was noticed when we were running our experimental line, 27 mi. long, previous to the energizing of the total line up to 220,000 volts, that when we first energized that line some new connections we had put in of new bright cable were very luminous and emitted a considerable noise. We could hear the crackling of corona quite a distance, but after the voltage had been on for two or three weeks these conductors blackened like the rest of the line and the noise decreased very much. That brings up the thought that in some of these semi-laboratory tests, by which I mean tests where there is a length of line in the open energized from a laboratory, possibly the conductor may not have reached the ultimate surface condition. For example somebody wants to test a piece of new kind of conductor with a naturally bright surface when it is received, and it is hung up on the experimental line and tested, possibly before it has gone into the steady condition. A line will afterwards be built of that same kind of conductor and after it has been in service for a little while the black deposit may have formed and then, naturally, the tests on the line as a whole will not conform with the laboratory tests made on the new conductor.

I have a suggestion, which may or may not be of any value as to the measurement of these losses. The great difficulty lies in the low power factor and it occurred to me that, perhaps, a way out of the difficulty would be to raise the power factor, and that can be done by putting a synchronous condenser on the end of the line. In case of a long transmission line there would be two advantages, a better power factor at the sending end where the meters were and the ability to control the rise of voltage over the line and have practically uniform voltage over its whole length. The only objection I see would be in evaluating the losses in the condenser, but I believe one could study these and arrive at what they were quite closely.

J. S. Carroll: The water-column resistance used as the multiplier for the high-voltage wattmeter is of vital importance. The studies made at Stanford last gave us a little insight into the solution of some of the problems of electrostatic shielding. To begin with, the original water column was built of 75 ft. of ½-in. garden hose. Last year it was shortened to 20 ft. using ¼-in. air hose. Just before coming to the convention some brief measurements were made with a water column 5 ft. long and 0.095 in. in diameter. A stout glass tube was used in this case. On account of this smaller diameter the water pressure had to be raised to about 130 lb. per sq. in. in order to keep the temperature within reasonable limits. The water column was mounted in the vertical position between two horizontal metal plates of sufficient size to produce a fairly uniform field at the center where the water column was located. The bottom plate and the lower end

of the water column were connected to ground, the upper end of the column was connected to the instruments and the upper plate was connected directly to the high-voltage terminal of the transformer.

Unfortunately, in this case, water has a very large negative resistance-temperature coefficient, so that the voltage gradient will not be exactly uniform down the column on account of the heating effect of the current. However, it is possible to adjust the field between the two plates until it very nearly matches the gradient down the water column. Even if the two fields are not exactly the same throughout their entire lengths the total error produced will be very small on account of the small capacitance of this short water column. In regard to the test as to when the fields are matched, a similar method will be used that was used last year in which the resistance of the water column was decreased by cutting down the flow of water and allowing it to warm up. There is a little objection to this method but it was not as serious last year as I had thought, but now we are getting down into corona and related matters, to where we are having to split hairs over those values near the x-axis. To test for correct shielding of this short water column, instead of changing the conductivity of the water by reducing the flow which would change the shielding slightly, we will change the conductivity by the introduction of a salt solution and the temperature of the water will be kept constant by controlling the flow. No doubt there will be a few problems to solve before this method will be free from objection, one of which will be the proper mixing of the solution beforehand to prevent an unsteady flow of current. However, with the crude apparatus now set up this method seemed to work very satisfactorily. With the voltage held constant the current was gradually raised from 30 milliamperes to 60 milliamperes. With this arrangement we have an ideal multiplier the multiplying factor of which can be controlled easily and made any value between rather wide limits. Not only can we evaluate error due to the capacitance of the water column and tell when it is eliminated but we can also keep the current through the voltage coil of the wattmeter at a relative large value for the low values of power which will increase the deflection of the wattmeter four to six times.

This year we hope to be able to keep the wattmeter on the line continuously. Tanks which have hitherto been used and require emptying or filling at intervals will not be used. A nozzle will be used to spray the water to the ground from the upper end of the water column. It was found by test that a 6 ft. spray effected a complete break in current at 175 kv.

J. P. Jollyman (by letter): Mr Wilkins' paper is a noteworthy contribution to the knowledge of corona loss. The Pit transmission system offers an unusual opportunity to secure accurate data on the corona loss of three distinct configurations and two conductor materials.

The comparison of the results of the several methods of measurement employed is most interesting and clearly demonstrates the difficulties of high voltage measurements. The determination of generator output from the output of the prime mover is to be especially commended as a method of check. This confirmation of the accuracy of the readings taken on the low-voltage side of the step-up transformers gives great confidence in the results.

A study of the results of the tests as shown in Fig. 33 reveals some interesting information. The corona loss is found to be a function of the total applied voltage with a law of the form:

$P = Ke^v$

where P = KW corona loss
K = a constant
V = total voltage in kv.
e = naperian base.

For a given configuration K has a certain value up to a certain value of v which may be regarded as a "critical voltage." Above this value of v, K has a different value such that P increases more rapidly for a given increase in v. At a still higher value of v or possibly of P, K again changes and the increase of P for a given increase of v is less than in the preceding stage. This second change in the law marks the beginning of a voltage-wave distortion. This point was not reached on the aluminum conductor due to limitation of voltage which could be applied. Not enough data are available to determine with certainty whether the second point of change is determined by the total loss per mile or by total load on the generator. The indications are that the total loss per mile for a given configuration is the important factor.

Mr. Wilkins calls attention to a curious change in the corona loss of the aluminum conductor below the "critical voltage" not found with copper conductors.

The discovery that the measured corona loss on the Pit transmission followed exponential laws with exact precision led to a review of the data on the best available tests reported by Lewis, Faccioli and others. This study is shown in Fig. 40. Every curve but one follows exponential laws and this curve records the results of tests on a short sample. The tests made on the long lines where carried over a sufficient range of voltage exhibit the three stages of loss observed on the Pit tests.

The most important conclusions that can be reached from Mr. Wilkins' paper are:

1. Corona loss follows exponential laws both below and above the "critical voltage."
2. Factors not heretofore considered important, such as capacity to earth and configuration, have a marked effect on corona loss.
3. Aluminum has certain characteristics with respect to corona not possessed by copper.
4. Losses below the "critical voltage" are large enough to be of commercial importance.
5. Direct high-voltage measurements by methods heretofore employed are likely to contain errors due to phase angles and change in ratio, the correction of which are extremely difficult, if not impossible.
6. A new investigation of the subject of corona loss is required to establish the laws of corona loss within the commercial range. Such an investigation should be made on outdoor three-phase circuits of considerable length, so arranged that the configuration may be varied to conform with the several arrangements used in practise and transposed to give a complete barrel.

The commercial range of corona loss may be regarded as including corona losses from small values up to values about equal to the $I^2 R$ losses considered reasonable for a certain conductor. Under present-day costs, an increase in conductor size may be justified when corona loss exceeds one-quarter of the $I^2 R$ loss.

C. A. Jordan (by letter): It is merely the desire in this discussion to call attention to a fact which may be significant, but which appears to have escaped attention of Messrs. Ryan and Henline.

In Figs. 1 and 8 of this paper, it will be noticed that the longitudinal axes of the corona loops obtained with increasing voltage rotate about the origin with increasing angular displacement from the straight-line no-loss card as the applied voltage becomes higher. This denotes a gradual change in the voltage-charge relationship, and hence a change in the capacitance which determines that relationship.

It has been commonly, and apparently reasonably, supposed that ionization of the surrounding atmosphere increases the effective capacitance of a conductor. Unfortunately conclusive evidence on this point is not forthcoming from the paper under discussion. In Fig. 8, the axes of the corona-loss cards lead the straight-line no-loss card, indicating a decreasing capacitance with increasing loss. In Fig. 1, the axes of the corona-loss cards lag behind the zero-loss card, indicating an increasing capaci-

tance. May it be supposed that the two sets of cards, made at times separated by an interval of years, were not obtained under the same conditions of instrument polarity, or that one set of cards has been incorrectly sketched for reproduction? If it be accepted that the effective capacitance of a conductor does increase with corona loss, some interesting conclusions are possible.

Referring now to Fig. 2, attention is invited to the loop axis, X-Y. The reciprocal of the slope on this line is the effective capacitance. The axis X-Y as drawn implies a constant capacitance, not only for all applied voltages, but for all values of the same applied voltage. If, however, the capacitance is increased by ionization, the axis cannot be a straight line. In this case (considering the first quadrant of the figure and starting with zero applied voltage) as the voltage is increased the conductor charge will vary in linear relation, determined by the reciprocal of a constant capacitance, until the critical disruptive voltage is reached. At this point, capacitance will begin to increase and the slope of the axis to decrease by increasingly greater amounts until the crest value of applied voltage is reached. The same thing will occur in the third quadrant in the opposite sense, and the axis will then represent an elongated letter S, being concave downward in the first quadrant and concave upward in the third quadrant.

Having disintered the skeleton, may we attempt to construct the body? Such a skeleton immediately suggests the classical hysteresis loop teminating in sharp points. If this be the case and were it possible to determine for all physical, electrical and meteorological conditions the factors which determine the shape and size of the loop, it is probable that a relatively simple, exact law for corona would result. Undoubtedly Equation 2 of the paper is an approximation of such a law, but the equation seems subject to error through the use of a constant conductor capacitance.

It remains to reconcile the actual corona diagrams given in the paper and the familiar hysteresis loop surmised above. Two thoughts immediately come to mind: (1) Is the cathode-ray cyclograph capable of faithfully indicating or reproducing the instantaneous relations between voltage and charge? and (2) Do the diagrams indicate corona loss alone, or a combination of true corona and gaseous-conduction losses?

Picture a number of sharp-pointed hysteresis loops of increasing sizes, and possibly slightly different shapes, drawn about a common origin with sides generally parallel. Then consider the pointed ends rounded off as might be done by a lagging instrument. Are not the resulting diagrams similar in shape and inclination or rotation to those given in the paper, especially when it is remembered that the slope of the experimental loops is determined by the *average* capacitance?

Again, were test voltages carried to such high value that, in addition to corona, gaseous conduction occurred, the resulting cyclograph diagrams would be deformed from the pointed loop to an elliptical form, as stated so well on Page 4 of the paper.

M. H. Gerry, Jr. (by letter): There is an important difference between conditions affecting the transmission of power and its distribution, although many of the problems are common to both. In many discussions of late there has been a tendency to confuse the two.

High-tension distribution is an economic necessity for the future, but it does not require voltages beyond the range of present practise. It involves delivery of power over an area and in practise functions through a network. Its problems are those of regulation, continuity of service and commercial efficiency.

Actual transmission of power is a distance problem and involves the transfer of energy in quantity from its source to a far removed center of distribution. Its further extension requires the employment of higher voltage.

Consider now, the transmission of power as the problem of transferring a large block of power a distance of say, a thousand miles. Can this be done commercially, and if so, how? At the present time power is being transmitted about three hundred miles, which appears to be near the distance limitations for sixty cycle three-phase current at the voltages now permissible.

A great forward step in the art is required if there is to be a material extension of the distance of power transmission, and only in this way will it be possible to make available in the centers of population the vast undeveloped water power resources of the nation.

The papers presented at this meeting go to the heart of the problem. Engineers must employ higher voltages for transmission but in what form or how applied we do not know. One important step is a clearer understanding of the nature of corona losses and another is their quantitative determination. Such losses must be suppressed or reduced to a small amount before progress is possible in this direction.

The study by Professors Ryan and Henline of the hysteresis character of corona is not only a contribution to knowledge and a clear experimental demonstration of certain facts, but it points the way to still more important fields of investigation and should give the engineering profession courage to believe that the limitations of voltage as applied to the transmission of power has not yet been reached.

Harris J. Ryan: Replying to Mr. Peek: In our studies of corona losses occurring from power transmission lines so far, we have found only "per cycle" losses. Succeeding *half-cycle* losses are not independent but parts of the total loss per corresponding cycle. We are partly but not altogether in agreement that the corona loss is due to the collapse of the air as a dielectric adjacent to the conductors. In our understanding the loss occurs in reversing the charges on the free ions that surround the conductors in approaching and receding from voltage crests that are higher than corresponding critical voltage. The charge between opposing groups of ions is automatically limited to equality with the increase of charge between the conductors due to increase of applied voltage from critical to crest values. The *ion-attached charges* must be reversed in passing each voltage crest. The energies stored in such charges are not reactive as is the case for *conductor-attached charges*, and must all be degraded to heat, *i. e.*, molecular friction. This is the understanding that led to the derivation of equation (2) in our paper. In his discussion of Professor Harding's corona paper Mr. Carroll has shown that this equation and the corona loss-voltage relations found therein are in excellent agreement.

Corona, when existing on actual power lines is known to consist in few or many individual brushes, as the case may be. We have called attention to the hysteresis character of corona formation for a better understanding of brush formation and the loss of power therein. Our endeavor has been to accomplish a better understanding of the corona loss-voltage relations for power transmission lines when the losses are below 7.5 kw. per conductor-mile, or thereabouts. That is the range in which our equation (2) does not apply because therein the brush pattern is not fixed. We regard equation (2) as a criterion for fixing the corona-loss limits, below which, brush patterns are unstable.

In continuous-voltage corona formation the ions of like signs that are driven forth from a charged conductor meet and combine with corresponding ions carrying unlike charges driven forth from the oppositely charged return conductor. This is conduction by convection. Manifestly, the frequency can be lowered and the distance between the conductors lessened in alternating-voltage corona formation until conduction by convection also occurs. Physicists have determined in conduction by convection that, for an air-density factor of unity, negative and positive ions travel 1.83 and 1.35 centimeters per second in an electric field due to one volt per centimeter. The restricted conditions for which alternating-voltage corona will produce loss by convection may, therefore, be calculated.

Professor Sorenson's photographic studies are yielding helpful results. Years ago in an effort to observe the E-Q relation in

the glass dielectric of a plate condenser fitted with tin-foil electrodes, the corona at the edges of the tin foil on the opposite faces of the glass plate yielded well defined hysteresis loops. The study of similar phenomena photographically must surely add to our understanding of the differing behavior of the free positive and negative ions in determining the hysteresis character of corona.

Mr. Hillebrand helpfully calls attention to the importance of knowing the carrying capacities of liquid films that will lead to an understanding of the problem of surface leakage over insulators. The high-voltage wattmeter is sufficiently sensitive and accurate for the promulgation of such studies.

Dr. Whitehead has contributed a number of helpful criticisms and suggestions with most of which we are in hearty accord. Formerly we shared his views and those of Peek himself that "We must increase the voltage above the initial loss value before we get to the region subject to definite law. We must reach a voltage whereat the irregular point discharges unite in increasing the diameter of the conductor and making it equivalent to a smooth conductor." Our understanding is that "the region subject to definite law" is due to the formation of a stabilized brush pattern and not that "the irregular point discharges unite in increasing

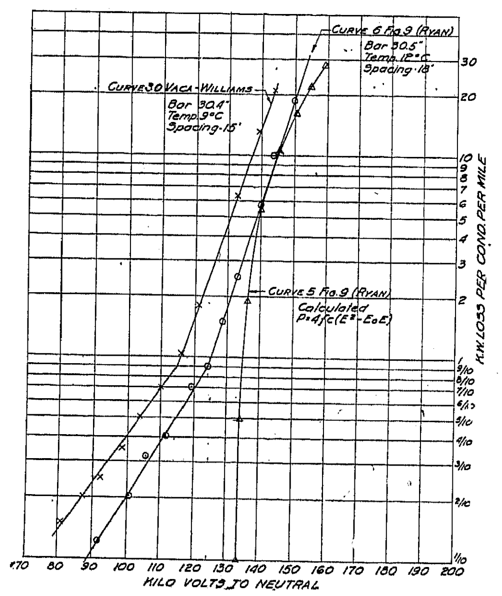


FIG. 13

the diameter of the conductor and making it equivalent to a smooth conductor." Mr. Jollyman points out that "under present-day costs an increase in conductor size may be justified when conductor loss exceeds one quarter of the $I^2 R$ loss. Such loss occurs entirely within the *unstable-brush-pattern range.*" Ardent study of the corona loss-voltage relation occurring in unstable brush patterns must, therefore, continue. As Mr. Wood states, these data are wanted because "they pertain precisely to that region on the voltage curve wherein we most probably will have to work for economic reasons." The unstable-brush-pattern losses are related to the total line voltages. We are convinced that law and order exist among them, though the factors are many and too little is known about them as yet. When the corona losses have increased sufficiently to sustain stable brush patterns in clear weather their values vary much as do squares of the crest voltages in excess of the corresponding critical crest voltage. We cannot, however, conceive of the effective use of engineering constants or factors by which to derive unstable brush-pattern losses from the corresponding losses due to stable brush patterns formed at higher voltages.

Magnetic hysteresis is due to the presence of definite quota of molecular magnets left stranded in their polar orientation by atomic friction that cause alternating flux to lag with respect ot the corresponding alternating m. m. f. These residual molecular magnets are reversed as each flux crest is passed with a cor-

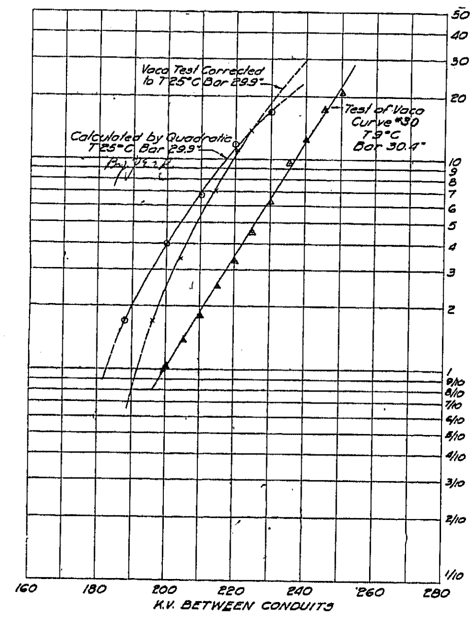


FIG. 14

responding loss of energy. In a strikingly similar manner when 60-cycle electric fields of ionizing strength are set up in air, residuals of ions are left about the conductors that cause the electric field also to lag by definite amounts with respect to their cor-

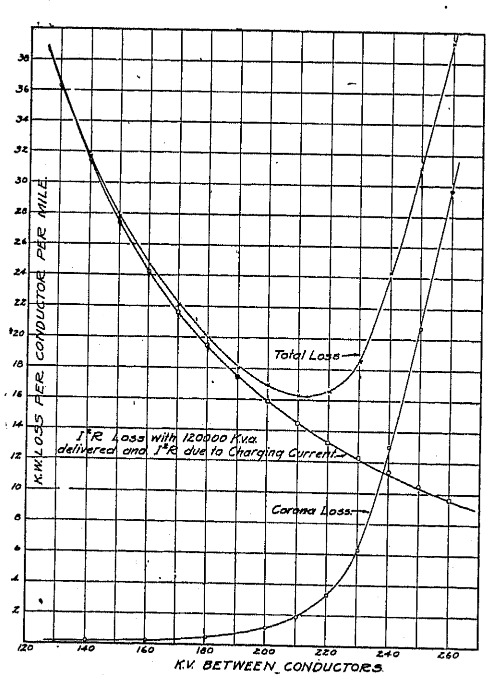


FIG. 15

responding e. m. f. s. Such stranded ions have their charges reversed as each voltage crest is passed. The energy stored in placing these charges about the conductors must be degraded to heat through molecular impact (friction) when by recombination and replacement their charges have been reversed.

Mr. Jordan notes the gradual rotation of the longitudinal axes

of the corona loops with increasing voltage and concludes that such effect is due to a corresponding change in capacitance. This use of the term capacitance is apt to be misleading. Capacitance of a conductor is the quantity or charge it accepts per volt. The quantities recorded in the cyclograms in excess of the corresponding conductor charges were employed in setting up the space charges on the free ions left by the corona between voltage crests. Conductor capacity is not altered in value by the presence of such space charges. In the course of our work many of the highly reasonable conjectures of the character brought forward by Mr. Jordan were also encountered. It would, however, have made our paper too long to have included a record of them and what we did in their regard. Equation (2) was the result of a large amount of study beyond that of the make-up of the voltage-charge diagrams.

R. Wilkins: Fig. 13 herewith shows Prof. Ryan's Curve No. 6, Fig. 9 plotted on semi-logarithmic paper, together with a curve from the data of the test on the Vaca-Williams section of the Pit line at approximately the same temperature and barometer and a spacing of 15 ft. All of this section of the line had an elevation of less than 1000 ft.

There is also his calculated Curve No. 5, of Fig. 9, and I would point out that at 127,000 volts to ground we are running on a part of the curve which does not check.

By reference to Dr. Steinmetz, it seems necessary for comparison, to plot the curve in such a manner that the test data will form a straight line or a series of straight lines. Fig. 14 herewith shows this same test curve No. 30 with the temperature and barometer as indicated under constant weather conditions and taken at an altitude of less than 1000 ft. plotted on semi-logarithmic paper giving a straight line. There is also plotted a calculated quadratic curve for these conditions and also the curve shown by Mr. Peek of a correction of this test to a temperature of 25 deg. cent. and a barometer of 29.9 inches. By using a correction developed for a quadratic curve on the test data an entirely different character of curve is developed. It would seem more accurate to calculate the curve for test conditions.

Fig. 15 bears out the statement made in the paper that it is not economical to run below the corona point at least on this particular line. There is a decided minimum to the loss curve for a given kw. transmitted.

In closing, I will say that for values of corona loss on 300-ft. sections of the rope-lay cable taken in two of this country's best high-tension laboratories before the line was erected the variation from the test values taken on the Pit line in place at the operating voltage was somewhat over 500 per cent and the errors were in different directions. Any such variations in losses having a value of $100,000 per year is worth further study.

It seems worth while to make a plea for tests under practical operating conditions because as higher voltages and longer lines are used, it means dollars to the ultimate consumer.

The Corona as Lightning Arrester

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Review of the Subject.—Occasional suggestion has been made in recent years that the properties of the high-voltage corona might be utilized as a protection against lightning and other similar types of disturbance on transmission lines. The idea in these suggestions is that since the ionization attending corona renders the air conducting, the excess voltage following the lightning discharge will be relieved or lowered by leakage between lines or to ground. It has been stated that lines operating in the highest range of transmission voltages are more immune from lightning disturbances than those of lower voltage, and that this is due to the fact that they operate nearer to corona-forming voltage, and that abnormal rises of voltage are thus prevented by the relatively low value at which corona forms. This paper aims to present additional evidence that the suggestion mentioned is not only sound but has important possibilities. Experiments are described indicating a relatively simple and inexpensive method of equipping transmission lines to take advantage of the protective properties of the corona, without incurring its disadvantages.

Conductivity Due to Corona

The conductivity of the air, resulting from corona, constitutes a leakage conductance of the line. This general constant of the electric circuit is usually assumed to be zero in a transmission line, although its presence and importance in telephone circuits is well known and understood. Shunted or leakage conductance increases the attenuation or rate of decay of electric waves or impulses, and if present when a disturbance due to lightning passes over a line, must of necessity tend to retard or smooth it out.

We may estimate the value of the conductance due to corona from Peek's[1] expression for the power loss. The loss increases with the excess of voltage above the critical value and also with the frequency, hence the conductance will also have a wide range of value. As examples, two No. 00 wires, discharging corona at 10 kv. above the critical value, might under ordinary conditions have a corona loss of 2.8 kw. per kilometer, and this is equivalent to a shunted conductance of 0.77×10^{-6} mhos per kilometer. Also, suppose this line to be placed 30 ft. above the earth, that it operates at 50 kv., and that it is arranged, by methods described below, for a critical corona-forming voltage of 55 kv. If this line should suffer a lightning disturbance causing a rise of voltage to a value of 60 kv. above ground, the resulting corona power loss would be about 13.8 kw., equivalent to a conductance of 3.8×10^{-6} mhos per kilometer. The foregoing values of conductance pertain to a frequency of 60 cycles. For higher frequencies and for higher voltages the values of conductance will also be higher in accordance with Peek's formula.

Assuming, as now generally accepted, that a chief danger from lightning disturbances lies in the steepness of wave front associated with high-frequency oscillations or sudden pulses, the question before us is as to the influence of leakage conductance, in the amounts suggested above, on the rate of decay of high-frequency transients on power transmission lines.

The Attenuation Due to Corona

The question of the change in shape of the front of a pulse or wave is one of the attenuation or decay of its component frequencies. In considering the value of corona as a protective device, it is necessary to interpret its functions as closely as possible, in terms of the usual constants of the line, and their occurrence in the general equations of wave propagation. The presence of leakage conductance in certain types of telephone circuits has been known a long time and its influence on speech distortion is well understood. In the analysis of transmission line phenomena, however, leakage conductance has always been considered negligible, and the values of the normal circuit constants to remain the same at all frequencies. It is only recently that Steinmetz[2] has studied the variation of the attenuation constant with the frequency, and proposed an explanation of the known rapid decay of steep wave fronts on transmission lines. He concludes that owing to the skin effect and to electromagnetic and electrostatic radiation, the attenuation constants pertaining to the higher ranges of frequency rise to enormous values, and that these frequencies constituting the steep slopes of wave pulses, are rapidly damped out, thus accounting for the relatively limited distances over which the steep fronts of such pulses persist. The values for the attenuation constant as computed range from 70 at commercial frequencies, through, 287 at 10,000 cycles, to 7×10^6 at 5×10^6 cycles; the enormous values at high frequencies are due principally to the loss of energy radiated into space.

In these estimates of the values of the attenuation constant, the leakage conductance is assumed to be zero. If any is present it must, therefore, still further increase the attenuation, and accordingly tend to slow up all frequencies still more rapidly. Now it happens that the values of leakage conductance due to corona lead to values of attenuation constant much higher in the lower range of frequency than those computed by Steinmetz. The latter increases with the frequency more rapidly than the former in the upper range, but the corona attenuation is the higher over the entire range up to 1.5×10^6 cycles where the two values are about equal; beyond this the Steinmetz value is the

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higher. Thus the effect of corona is to greatly increase the attenuation constant over the entire range of the more important frequencies entering into the usual high frequency line transient. It is at once evident, if these estimates and assumptions are correct, that the corona must have the effect of slowing up high frequency and steep wave front transients even more rapidly than now observed and computed.

To arrive at an estimate as to the influence of corona, we may conveniently follow Steinmetz, using both his examples and his method for determining the progressive alteration of several types of transients but using new values of attenuation constant to include the influence of corona. He considers the alteration by attenuation of three types of disturbance likely to arise from lightning or other similar cause; the quarter-wave oscillation, the periodic rectangular wave, and the flattening of a steep wave front. In each case departure is taken from the summational expression representing the Fourier series and involving the succession of terms of increasing frequency making up the wave or pulse. The influence of the attenuation constant, u_n, appears always as $\epsilon^{-u_n t}$, u_n however taking continually increasing values with increasing frequency. The values of u_n are computed as influenced by unequal current distribution (skin effect), electromagnetic, and electrostatic radiation, but the effect of each is reduced to the equivalent series resistance r, or shunted conductance g, per unit length, for ready adaptation to the well-known general circuit equations, as treated by Steinmetz, through the value

$$u_n = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right)$$

To include the influence of corona we may add another term to the above expression for u_n, by increasing g by the amount due to corona leakage conductance at the frequency n. Using the proper values of conductance and capacity, we find for the case of two No. 00 wires 6 ft. apart and equipped so as to give a normal corona discharge at 40 kv., and undergoing a 60 kv. lightning pulse, a value of u_n due to corona alone of 4800 at 10,000 cycles, with values increasing approximately proportionally with the frequency. And for the same two wires 30 ft. above ground and equipped for corona discharge at 55 kv. between lines, when subject to a pulse of 60 kv. above ground a value of u_n due to corona alone of 24,000 at 10,000 cycles; the value of u_n at this frequency if no corona is present, is 288. Thus under conditions quite likely to arise in practise, it is possible to look for a greatly increased attenuation constant over that of the normal line, by arranging the line so that a normal corona discharge curve may begin a few kilovolts above normal line voltage. This may be accomplished in various ways, one of which, suitable for existing lines, is described below.

Now it happens that the value of u_n due to corona, is many times greater than the value as computed on the assumption of no leakage conductance, throughout the entire lower range of frequency, and in fact is exceeded by the latter only at frequencies above 1.5×10^5 cycles. Thus for the line described above the attenuation constant, no corona present, is 70 at 10^3 cycles, 287 at 10^4 cycles, 3710 at 10^5 cycles, and 291,500 at 10^6 cycles. The corresponding values due to corona are 495, 4800, 48,300 and 483,000. The effect then of ordinary corona discharge is to greatly increase the attenuation constant throughout the lower range of frequency and to add substantially to its value in the upper ranges. There is good evidence, therefore, that corona may be expected to slow up and smooth out irregular pulses even more rapidly than the rates computed by Steinmetz for corona-free lines.

Steinmetz's assumption that the attenuation constant is rapidly increased at higher frequencies through the phenomena of the electro-magnetic and electrostatic radiation has been seriously questioned.[7] However, the influence of these factors on the values of the total attenuation constant including corona formation is very small. The values of u_n, as deduced by Steinmetz, are far surpassed by the value on a corona-forming line for all frequencies below 1×10^6 cycles. At 1.5×10^6 cycles, the Steinmetz value, and that due to the influence of corona alone, are about equal, while for higher frequencies the Steinmetz values become greater than those for corona. If, as claimed, therefore, the Steinmetz assumption as to radiated energy is erroneous, the effect on the conclusions below is to be found only at frequencies above 1.5×10^6 cycles per second. As the range of frequencies above 1.5×10^6 cycles plays but a small part in the total ranges of the several types of wave considered below, the conclusions reached would not be as seriously affected.

Question may arise as to the time interval required for the setting in of corona, and its power loss, at high frequencies. There is little available experimental evidence here, but such as there is appears to be in the direction of assisting the attenuation influence of corona. In the lower range of frequency a number of observers[3] have noted a slight decrease in the critical voltage with increasing frequency. Ryan's experiments indicated a marked lowering at 40,000 cycles, and those of Peek[4] that the law of corona power loss holds up to 100,000 cycles.

The Damping Influence of Corona on Lightning Disturbances

In the following, the attenuation, caused by corona, of three types of lightning disturbance on a line consisting of two No. 00 B and S wires, 30 ft. above ground, and 100 km. long, is studied and compared with behavior of the corona-free line. The examples are those of Steinmetz and his ingenious methods of approximation to the true values have been followed, with the exception, however, that the attenuation constants at the various frequencies have been increased so as to include the effect of corona.

Quarter-Wave Oscillation. A particular case of a steep wave front is the rectangular wave resulting from the connection of voltage to a transmission line open at the far end, or the sudden short-circuit of such a line. In these cases there is the possibility of the natural frequency or quarter-wave length oscillation, with superposed harmonics, of the line against ground. It is found that except for points very near the end of the line where the disturbance arises, the maximum voltage gradient, or steepness of the wave front, at time t, and distance $1 = 3 \times 10^{10}\, t$ is, approximately:

$$G = \frac{E}{l_o} \sum_{0}^{\infty} {}^{n}\, \epsilon^{-u_n t}$$

in which E is the line voltage, l_o the length of the line and u_n the attenuation constant for the frequency $(2\,n + 1)\, f_o$, f_o being the natural frequency of the line.

Table I gives the values of maximum gradient of wave front, in volts per meter, length of wave front in meters, and equivalent frequency, for a 100 km. No. 00 B and S two-wire line 30 ft. above ground with no corona present, and also the values of the same quantities when the line is discharging under corona at a voltage about 10 per cent above the critical value. The rapid fall of the voltage gradient on the wave front, and the consequent lengthening of the wave

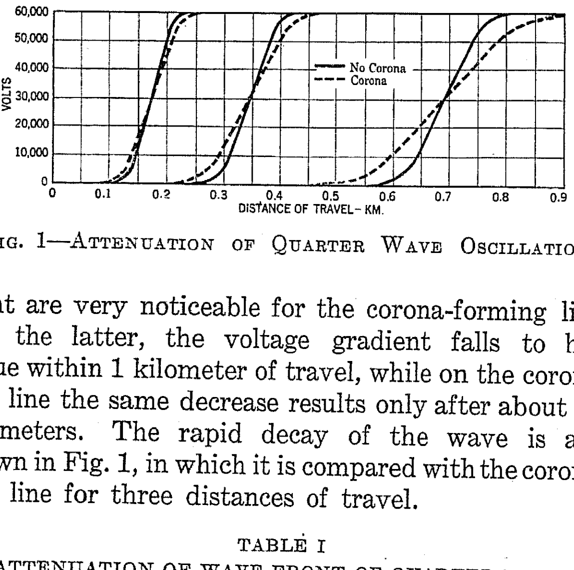

Fig. 1—Attenuation of Quarter Wave Oscillation

front are very noticeable for the corona-forming line. On the latter, the voltage gradient falls to half value within 1 kilometer of travel, while on the corona-free line the same decrease results only after about 2.5 kilometers. The rapid decay of the wave is also shown in Fig. 1, in which it is compared with the corona-free line for three distances of travel.

TABLE I
ATTENUATION OF WAVE FRONT OF QUARTER WAVE OSCILLATION ON CORONA FORMING LINE

Time t micro-seconds	Distance of Travel km.	Voltage Gradient Without Corona	Voltage Gradient With Corona	Wavefront Meters Without Corona	Wavefront Meters With Corona	Equiv. Kilocycles Without Corona	Equiv. Kilocycles With Corona
0.575	0.1725	792	584	119	161	630	465
1.15	0.345	575	369	164	256	456	293
2.3	0.69	414	217	227	435	330	173
11.5	3.45	184	53	510	1780	147	42
230.0	69.00	36	3	2500	..	30	..

Rectangular Wave. In the case of the pure rectangular wave, made up of all odd harmonics of the fundamental frequency of 60,000 cycles, and represented by

$$e = \frac{4\,E}{\pi} \sum_{0}^{\infty} {}^{n}\, \frac{\epsilon^{-u_n t} \operatorname{Sin}(2\,n + 1)\,\varphi}{(2\,n + 1)}$$

the high values of u_n, due to corona at the lower frequencies, cause a rapid vanishing of terms with increasing values of n and t. This causes the wave to lose its rectangular shape rapidly. As shown in Fig. 2, in less than 10 microsecond, or 3 km. of travel, on a line on which corona is present, the rectangular wave has assumed practically a sine shape, and its amplitude is rapidly decreasing. If the line is not discharging under corona, only the highest frequencies are rapidly damped and the wave is still flat-topped after 40 microseconds or 12 kilo-

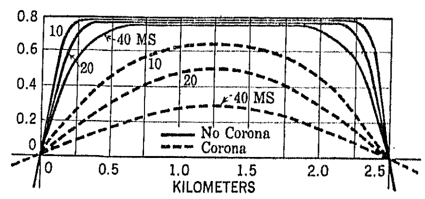

Fig. 2—Attenuation of 60,000-Cycle Rectangular Wave

meters travel, and its amplitude is still higher than that at 10 microsec. (3 km.) on the corona forming line.

Flattening of Steep Wave Fronts. The general expression for the maximum voltage gradient of any steep wave front or pulse of any length is shown to be:

$$G = \frac{2\,E}{3 \times 10^{10}} \int_{-\infty}^{+\infty} \epsilon^{-ut}\, f\, d\,(\log f)$$

in which t is the time since the origin of the pulse, and u, the attenuation constant, a function of f, the frequency. The relative values of this integral for one No. 00 B. and S. wire 30 ft. above ground and for values

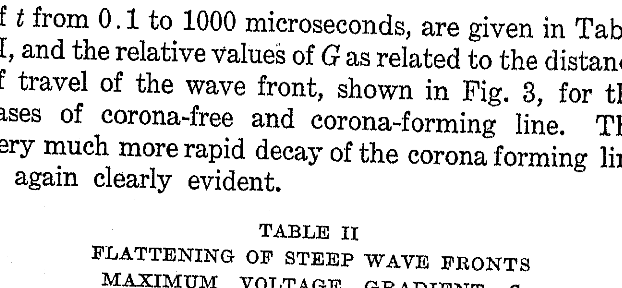

Fig. 3—Flattening of Steep Wave Front by Corona

of t from 0.1 to 1000 microseconds, are given in Table II, and the relative values of G as related to the distance of travel of the wave front, shown in Fig. 3, for the cases of corona-free and corona-forming line. The very much more rapid decay of the corona forming line is again clearly evident.

TABLE II
FLATTENING OF STEEP WAVE FRONTS
MAXIMUM VOLTAGE GRADIENT G

Time, seconds	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}
Corona-free line	20.2	6.84	1.86	0.635	0.119
Corona-forming line	16.8	3.71	.672	.07	0

In each of the foregoing examples a most striking improvement as regards the elements of danger of a lightning disturbance is shown for the corona-forming line. While the figures must be taken as only the roughest of approximations, it is nevertheless believed that they are of the correct order of magnitude. Particularly important is the indication that only relatively short lengths of corona-forming line are necessary, for this minimizes the elements of complication and danger, and reduces costs. In view, therefore, of the apparent advantages to be gained, it remains to examine the problem of a corona-forming line in the light of some of the conditions of normal line construction and operation.

Disadvantages and Difficulties

Looked at from the standpoint of application to existing lines and normal operating conditions, there are several questions which arise and which may constitute serious disadvantages to the use of corona as a protective device. Perhaps the most important is the variation in the value of corona-forming voltage with atmospheric conditions. Corona voltage is raised by an elevation of pressure, and lowered by an elevation of temperature; the influence of temperature is the more important. An annual range of temperature of 60 deg. cent. in any one location is not unusual, and this would mean an elevation of 25 per cent of the cold weather, over the hot-weather, critical voltage. When, however, we note that very short lengths of corona-forming line are indicated as necessary for protection, there would appear to be no difficulty in setting the corona-forming voltage at the value insuring protection under all conditions, and allowing the line to discharge continuously during the periods of high temperature. On an average line, an elevation of voltage of 12½ per cent above the corona-forming value would result in a continuous loss due to corona, of the order of about 1 kw. per mile, so that if the line were set for the average conditions as regards density of the atmosphere, a good measure of protection would result in cold weather, and excellent protection in warm weather, at the expense only of the relatively low value of loss mentioned.

Several writers have attempted to estimate the extent of the protection available through corona, by comparing the probable amount of energy stored in a line at the time of lightning discharge, with the normal rate of power loss from corona. A cloud overhead induces charges on a part of the line. When the cloud discharges, charges are left on the lines immediately raising their potentials. If the rise of voltage exceeds the flashover voltage of the insulators, it is discharged at once; if not, the wave front of voltage advances down the line oscillating back and forth and gradually leaking to ground. If the flash-over voltage of the insulators be taken as the maximum value reached by the lightning voltage, and a length of line of the order of 20 miles be taken, it may be shown that the total amount of energy liberated will be of the order of 2000 or 3000 joules. If this energy is estimated to traverse the line at the velocity of light, the instantaneous values of power are seen to reach very high values, values in fact which are not appreciably lowered by the known rates of dissipation of energy by means of corona. For these reasons some doubt has been expressed as to the value of corona for the purposes in view. If the discussion in the preceding paragraphs of this paper is correct, it is obvious that the entire charge liberated by a lightning discharge cannot proceed down the line with the velocity of light, and it is not so much a question of the total amount of stored energy, as of the steepness of the wave front with which the charge, or voltage, proceeds down the line. Assuming then that the values of stored energy as estimated above are correct, the indications are that the initial steep and dangerous impulse would be quickly retarded, and the total amount of energy would finally dissipate itself by repeated oscillations back and forth along the line, with wave fronts of continually decreasing steepness.

There is also the question of the influence of frequency on the value of corona-forming voltage, as already mentioned. Experimental evidence was cited here, indicating that if frequency does have an influence, it is in the direction of lowering the corona voltage at higher frequencies.

It is quite obvious in connection with all of the above questions that experiment and trial only can answer them. The principal purpose of this paper is to point out the value of shunted conductance as a means of lightning protection, whether or not this conductance be obtained by corona formation, by the use of small line conductors, points on the usual size of conductor, or by any other form of distributed parallel-connected resistance. Furthermore, it is evident that owing to the short lengths of line, and the low values of loss involved, it would be a comparatively easy matter to make a test of the use of corona as a protective device.

Artificial Corona for Existing Lines

Examination of the relation between voltage, size of wire, and spacing for corona formation, shows at once that within the range of practise, only those lines operating at very high voltages and in high altitudes approach in any way nearly the critical corona-forming condition. Usually the corona-forming voltage is from two to three times the normal operating value. In order, therefore, to bring such lines to a corona-forming voltage differing by only a small amount from the operating value, it is necessary either to modify the spacing or to change the size of the conductor. For most values of transmission voltage and size of conductor, the spacing necessary for corona formation is so short as to constitute serious danger of the swinging together of the wires in the spans, or of flashover. The reduction of the size

of conductor is the more effective means of lowering the normal corona voltage on existing lines. For example, on a line of 1.6 cm. diameter, aluminum cables, with 9 ft. (274 cm.) spacing, reduction of the diameter of the cable one-half value, would result in a reduction of the corona-forming voltage of 33 per cent. A still further reduction in size of the conductor, therefore, would often be necessary in order to produce corona in accordance with the laws for round conductors. Such reductions in diameter would be possible in some instances, by the use of copper clad steel or other conductors of high tensile strength. It is probable, however, that this method of reducing the corona voltage would not be attractive to the average line superintendent. For these reasons it has appeared worthwhile to investigate the possibility of equipping existing lines of standard spacing, with some form of point or stud of such design that the line would give a corona discharge curve equal to, or better than, the normal corona power loss curve, at a voltage as nearly as desired to the normal operating value, and which, in addition, might be applied to the line with relatively little difficulty and expense.

No priority is claimed for this idea, except, perhaps, as regards this country. The power of point discharge for relieving high voltage has long been known. In particular, experiments with short barbed wire lines have been performed by R. Nagel[5] in Germany, who states that two power companies have announced intention to install experimental stretches of barbed wire as lightning protection. However, no notice of this work appears to have been taken in this country, and transmission engineers have not been impressed with the value of the suggestion of corona protection. These facts in large measure prompted the present work.

The experiments of Nagel are especially worth noting, as they give direct evidence of the value of corona or point discharge in reducing high voltage line pulses. He worked with two out-of-doors lines each 250 ft. long. One line consisted of smooth wires, and the other of commercial barbed wire, both of iron. The barbed wire line showed point discharge beginning at 30 kv. Each wire of each line was equipped with a reactance coil at the far end, and across these coils spark-gaps were placed. Spark-gaps were also placed across the two wires of each line. Electrostatic impulses of 40 kv. were sent at rapid intervals into each of the two lines. The voltage rises at the end of the line between wires was measured by the gaps between wires, and the steepness of the wave by the gaps across the reactances. The barbed wire line showed much lower voltages and flatter waves, the maximum gap lengths for discharge being from 30 per cent to 60 per cent of those for the smooth wires.

Barbed wire, while eminently suitable for such initial experiments, is probably not the best type of discharge conductor. Better results are likely with other and more regular shapes of discharge point. Moreover, the suggestion to replace a section of transmission line with a length of barbed wire is so unattractive that it would probably be refused at once by most transmission engineers. The following experiments then were undertaken with the aim of devising a simple form of stud or clip which may be readily applied to existing lines, which may be so designed as to discharge at any desired value of voltage, and give a discharge power curve as steep or steeper than the normal corona loss curve.

The Experiments

The method of studying the relative corona conductivities of different forms of discharge points is indicated in Fig. 4. A is the conductor to be studied, B a surrounding metal cylinder 30.5 cm. in diameter, and 178 cm. long. This cylinder is perforated with a large number of 1.6-cm. diameter holes, over the whole surface of a central region 76-cm. axial length. Immediately outside this central perforated section, and

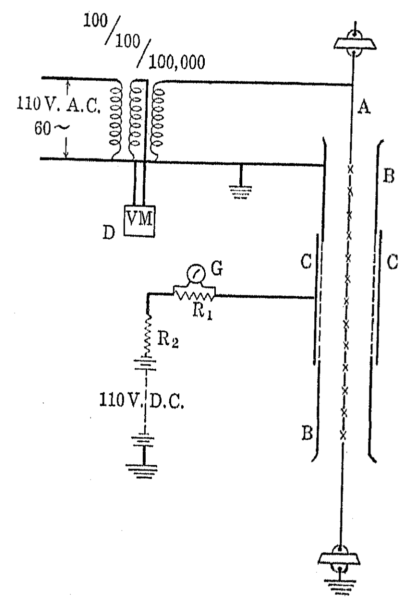

Fig. 4—Diagram of Connections, Conductivity Due to Corona

separated from the outer wall of the cylinder B, is an outer surrounding cylinder of sheet metal, C. This outer cylinder C, well insulated from B, is connected to ground through the resistances R_1 and R_2 and a source of continuous voltage, usually 110 volts. A sensitive galvanometer G is connected across the terminals of the resistance R_1. Cylinder B is connected to ground, and the galvanometer, therefore, indicates the conductivity of the air layer between cylinders B and C. This conductivity sets in as soon as corona appears on the central conductor A, due to the elevation of voltage by the transformer T, the value of voltage being read by the voltmeter D connected to a tertiary coil of the transformer. This is a very simple and convenient method of studying the relative conductivity of different types of corona-forming conductor. It has been described in detail in the author's papers on the corona voltmeter.

As typical of the types of discharge curves to be observed with this equipment, see Fig. 5. The two curves C are given by a nickel-plated steel rod with carefully smoothed surfaces, as used in the corona voltmeter[6]. These curves show clearly the sharply marked critical corona voltage, the absence of conductivity below the critical voltage, and the steep ascent of the discharge curve. Curves A and B were taken on a 1.58-cm.

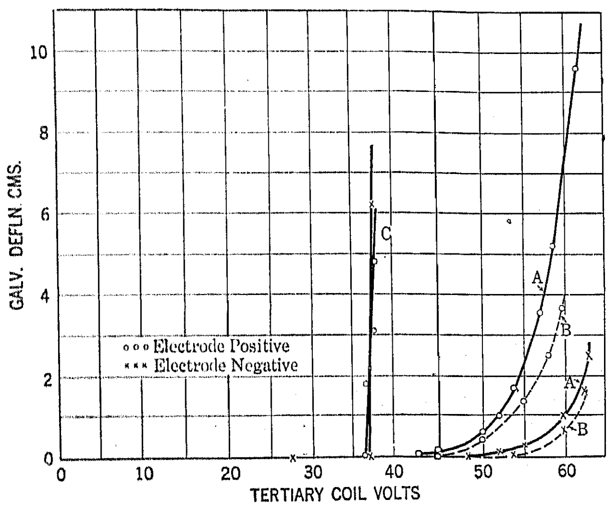

FIG. 5—CORONA DISCHARGE

A. Al. Cable, 19-Strand 5/8 In. Diam. New
B. " " " " " Weathered
C. Nickel-Plated Steel Rod, 0.188 In. Diam.

diameter, 19-strand aluminum cable; A in the fresh state as taken from the reel, and B after operation for a number of years on the transmission lines of the Pennsylvania Water and Power Company. Two curves are given in each case showing the galvanometer deflections with the electrode C positive and negative. The curves with positive electrode begin earlier and rise more rapidly. This is due to the greater velocity of the negative ions and indicates that for experiments of the present type, the electrode C should be charged with positive potential as being most sensitive to the beginnings of corona. The slightly lower critical voltage of the new cable is probably explained by the fact that its outer layer of strands was not tightly in place. The pitch of the spiral of the weathered cable was nearly twice that of the new, and all the strands of the former were snugly in place.

The Pennsylvania Water and Power Company's lines operate at 70 kv. three phase, 25 cycles, and the samples of cable were kindly furnished by the officials of that Company. This cable was, therefore, taken as a convenient example for a study of possible methods of altering its normal corona discharge curve. The discharge curve of Fig. 5 indicates that this line is operating at a voltage somewhat less than one-half the value of that at which corona would form under normal conditions. The experiments were, therefore, directed toward the development of a suitable point or stud on this cable which would give a discharge curve starting in the neighborhood of 22 volts (tertiary coil) and which should rise from that value as sharply as possible.

The properties of commercial forms of barbed wire were first studied. Six different types were investigated, consisting of twisted pairs of various forms of round and square wires, with points at intervals of from 7½ cm. to 15 cm. The points in all cases were about 1.25 cm. radial length, with jagged points resulting from ordinary cutting processes. The curves of three of these wires are shown in Fig. 6 and are typical of them all. The curves indicate, especially that of sample 3, that barbed wire may be constructed so as to give a fairly steep curve of discharge. All of the types of barbed wire investigated, however, have the disadvantage that their discharge curves all begin at about the same value of voltage and that they are, therefore, not readily susceptible to selection for a specific value of voltage. Moreover, the value of the voltage of initial discharge is

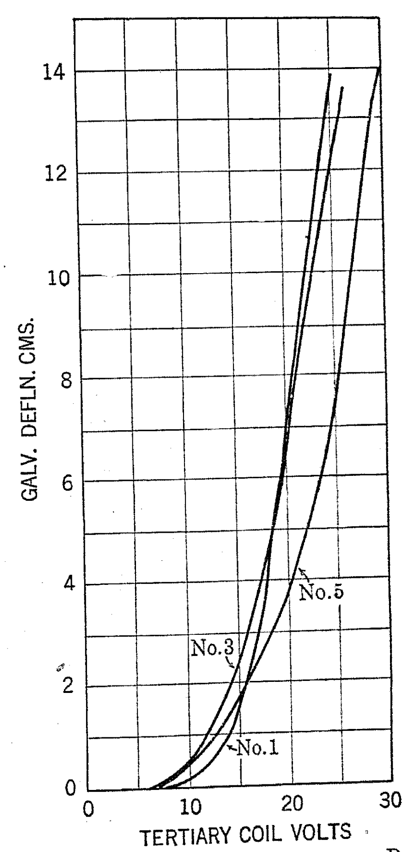

FIG. 6—DISCHARGE CURVES. COMMERCIAL BARBED WIRE

No. 1. Four 5/8-In. Points 3-In. Spacing
No. 3. Four 5/8-In. " 6 3/8-In. "
No. 5. Two 5/16-In. " 5 1/2-In. "

very low, being, for the case of the line in question, about 25 per cent of the value of the normal operating voltage.

The influence of the number and the spacing of the points on ordinary barbed wire was also studied. A number of special samples was constructed, consisting of two 10 B. and S. copper wires twisted together and with points ½ inch long placed in pairs at different distances of separation. The points were those result-

ing from the cutting of the wire with pliers. The curves of Fig. 7 show some of the results. Curve No. 0 shows the discharge of the twisted pair without any points; No. 1 with points spaced at 25 cm.; No. 2, at 12½ cm., etc. It will be noted that the effect of increasing density of points, *i. e.*, the number of points per unit length, is to increase the steepness of the discharge curve up to a certain point. In the present case the spacing of 3½ cm. shows the maximum and steepest discharge curve. A closer spacing as shown by curve 5 evidently has the

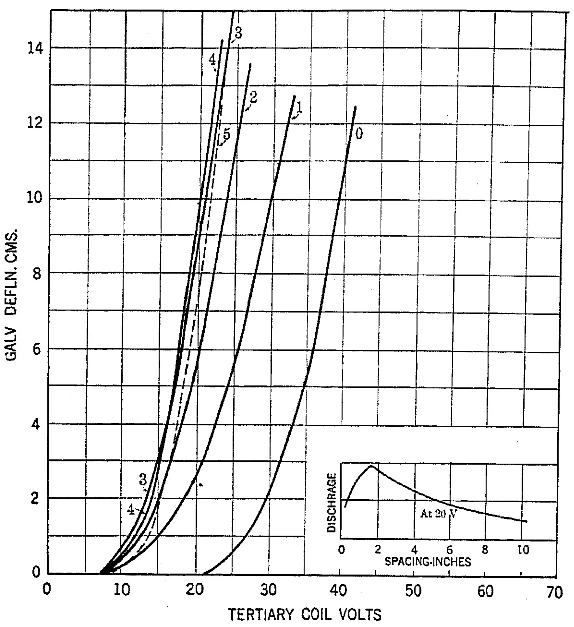

Fig. 7—Influence of Number of Points on Discharge Curve, Uniform Spacing

0	No Points	
1	Spacing 25	Cm.
2	" 12.5	"
3	" 6.25	"
4	" 3.1	"
5	" 1.75	"

effect of reducing the electric intensity at the ends of the points, due to their mutual electrostatic influence. The small inset curve shows the variation of the discharge with the density of points, for a particular value of voltage. The influence of point spacing is also shown in the curves of Fig. 6 in which curve No. 3 with a spacing of points twice that of curve No. 1, nevertheless shows a steeper gradient. The curves of Fig. 7 also show the disadvantage of barbed wire mentioned above, that the discharge begins at a very low value of voltage, and is not susceptible to a variation of critical value.

A large number of different types of discharge point was tried with the 19-strand aluminum cable. In one form narrow bands of sheet iron or aluminum were clamped around the cable with points cut in the metal and bent outward. Points of this character having angles of from 15 deg. to 30 deg. give discharge curves similar to those of barbed wire; that is, they begin from low values of voltage. For a point cut square on the end, the result is only slightly better than that for sharp points. Conical bosses stamped into sheet metal result in higher value of critical voltage, but show discharge curves with a very slow rise.

Some of the results of these experiments are shown in Fig. 8. Curve No. 1 is that pertaining to two 30 deg. sheet-steel points 1.25 cm. long, projecting radially at opposite ends of a diameter. Curve No. 2 is that of two 0.32-cm. diameter brass studs 1.25 cm. long, also at opposite ends of a diameter. Curve No. 3 is that resulting from wrapping around the cable in spiral form a No. 8 B. and S. copper wire at the same pitch as that of the strands of the cable, but in opposite direction. Curve 4 is that of the bare aluminum cable. Curve No. 1 for sheet steel points is not suitable for reasons already mentioned. Curve No. 3 is the steepest curve observed throughout the experiments and, therefore, appears to have marked advantages. However, it should be noted that this curve has a critical value of voltage about the same as that pertaining to the bare cable. It is possible that a lower value might be obtained by the selection of a different size of wire, or perhaps a ribbon of special type of cross-section. Since, however, the application

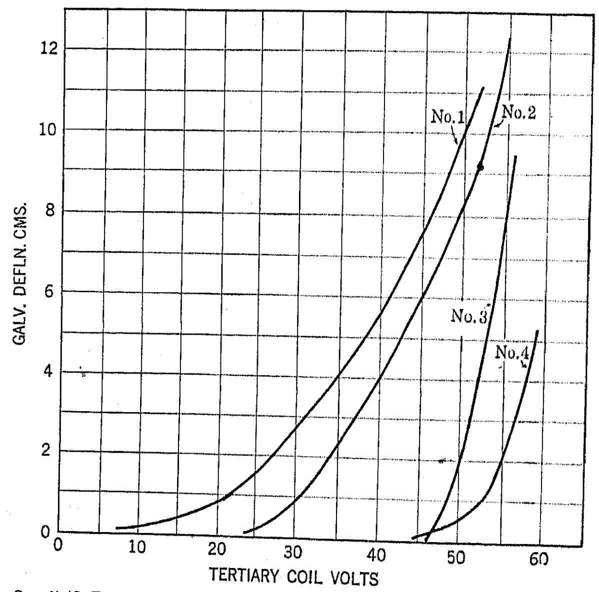

Fig. 8—5/8-In. 19-Strand Cable. Relative Discharge Curves

No. 1. Two Sheet Steel Pts. (30 Deg.) 1.25 Cm. Long
No. 2. Two Brass Studs, 0.32-Cm. Diam., 1.25 Cm. Long
No. 3. No. 8 Wire Spiralled on Cable
No. 4. Bare Cable

of such a spiral or ribbon to existing lines would be troublesome, if not thoroughly impracticable, experiments were not carried further in this direction.

The studs used for Fig. 2 were the only simple and convenient method encountered in which the point of initial discharge could be readily controlled. These studs were turned from brass rod and were of uniform cross-section throughout their length and their ends were rounded to hemispherical shape. The length of stud, diameter, and the radius of curvature of the end,

determine the initial value of discharge voltage; the spacing or linear density of the points along the cable determines the steepness of ascent of the discharge curve. A number of studs of this type were tried and the dimensions given are those selected for a curve starting at about 20 volts (tertiary). The curves of Fig. 9 show the influence of the number of points in increasing the steepness of the discharge curve, and that a 15-cm. spacing gives a discharge curve somewhat steeper than that of the bare cable. A closer spacing might give an even steeper curve, but the gain would be small as compared to the number added. For these reasons it is concluded that the corona-forming voltage of any existing line may be adjusted to any value below that pertaining to the bare cable, by mounting on it in pairs radially projecting studs at a spacing between pairs of about 15 cm. The studs should be of uniform circular cross section with rounded ends. The dimen-

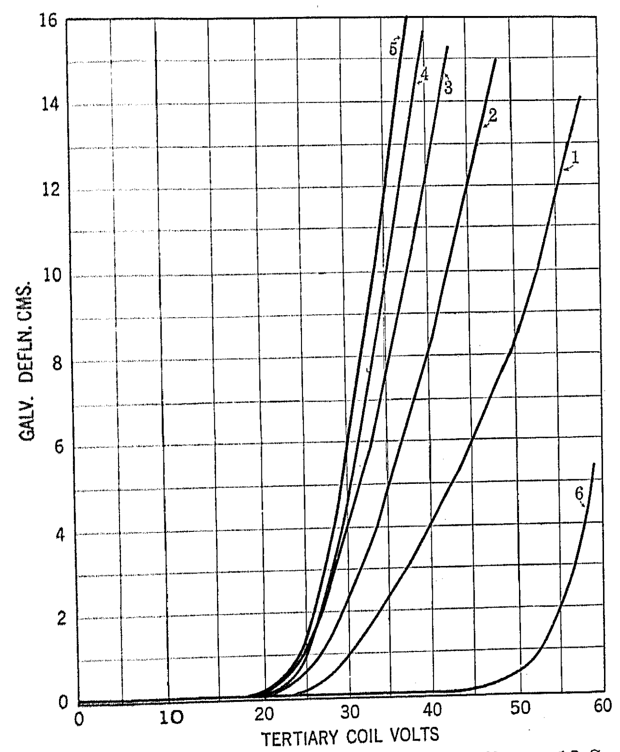

Fig. 9—0.5 In. by 0.125 In. Studs on 0.625 In. Diam. 19-Strand Cable

No. 1. Single Pair
No. 2. 2 Pairs 10-In. Spacing
No. 3. 3 " 5-In. "
No. 4. 4 " 6-In. "
No. 5. 8 " 6-In. "
No. 6. Bare Cable

sions of the studs will determine the voltage of initial discharge, and they can be determined most readily for each new case, by the simple experimental method described above.

A simple and effective method for mounting the points on the cable, is shown in Fig. 10. The brass studs have thin flat heads, somewhat in the manner of rivets. The studs are placed, points outward, through holes in the ends of a thin but stiff spring clip, which is then slipped over the cable. The clip embraces more than half the circumference of the cable, and the two studs are thus a little nearer together, on the open side of the clip, than one half the circumference. By this means the heads of the studs grip the strands of the cable, resulting in a firm attachment of spring clip and studs.

Experiments to test the value of the suggestions

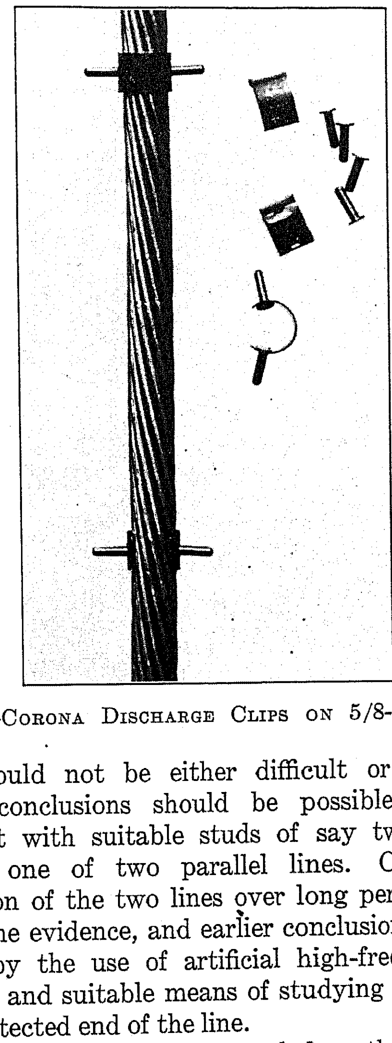

Fig. 10—Corona Discharge Clips on 5/8-In. Cable

above should not be either difficult or expensive. Definite conclusions should be possible from the equipment with suitable studs of say two terminal miles of one of two parallel lines. Comparative observation of the two lines over long periods should afford some evidence, and earlier conclusions would be possible by the use of artificial high-frequency disturbances and suitable means of studying their effects at the protected end of the line.

The author gratefully acknowledges the assistance of Mr. N. Inouye throughout the experiments.

Conclusions

1. The conductivity resulting from corona on transmission lines is sufficiently high to cause large values of attenuation constant, and the value of corona as a protection against high-frequency and steep wave fronts resulting from lightning and other causes is pointed out.

2. It is indicated that only short lengths of corona-forming line, say one or two miles, are necessary for a large measure of such protection.

3. To take advantage of this method of protection, the corona-forming voltage should be slightly above the operating value. On existing lines this may be

accomplished by reducing the spacing and size of conductor, resulting in uniform corona, or, when this is objectionable, by equipping the conductors with suitable discharging points.

4. The relative discharge values of various types of point are studied experimentally. A simple form of spring clip and stud is proposed, which gives a discharge equivalent to that from corona, and which may be readily mounted on a transmission line conductor.

5. The method has some disadvantages, and its predicted value depends on several assumptions requiring proof. In view of its simplicity and the short lengths of line involved, experiments on the scale of practise should be not only practicable, but attractive, in view of the high degree of protection promised.

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Discussion

F. W. Peek, Jr.: In a paper that I presented at Swampscott last year I described some tests in which the variation of the steepness of the wave front and the voltage of a wave were measured as it traveled along a line on which there was considerable corona loss.

It is not necessary to repeat this description here. Briefly, the tests show that the voltage and steepness of the wave rapidly decrease as the wave travels over the line. In this work the effects of choke coils, capacity, open-ended lines, series resistance, etc., were also investigated.

The method of making the tests was simple. Some years ago I found that the sphere-gap measured the correct voltage of a transient while the needle indicated a voltage that depended upon the steepness of the wave. The ratio between the needle and sphere voltage was called the impulse ratio. The character of the wave is determined by making measurements by sphere and needle at different parts of the line.

There is no doubt that corona and other losses reduce the voltage of the wave and the steepness of the wave front as it travels over the line.

John C. Damon: Lightning arresters for high voltages have been remarkably expensive and if the transmission line could be the lightning arrester, too, it apparently would save all of the arrester expense. The trouble is, however, that the transmission line has to be designed to carry the load, to prevent corona from giving too much loss, to give reasonable regulation and to string at such a tension that the towers will not be unduly expensive. These are not independent variables. It has recently become possible to get cable with the outside diameter independent of the cross section of the conductor which will doubtless permit the future design of lines to act as lightning arresters in addition to their other functions.

V. Bush (by letter): The influence of corona on traveling waves is an exceedingly interesting topic, and I greatly appreciate the courage of Dr. Whitehead in making a mathematical attack on the problem. In such a complex phenomenon it is, of course, necessary to make some limiting assumptions in order to make it possible to formulate the problem at all; and it is hardly just to be too critical of mathematical short cuts when employed in connection with such a difficult matter. Yet I feel that he will agree that a discussion of the nature of his assumptions, and of the logic employed, will be of assistance to those who are intimately interested in this subject.

Dr. Whitehead represents the effect of corona as a constant ohmic leakance. Of course its effect is really cyclic in nature and complicated in many ways. As is so ably considered by Prof. Ryan and Prof. Henline, there is a hysteresis effect, so that corona leakance cannot be expressed even as a discontinuous function of voltage alone; but the function, if it can be formulated, must contain the time also. It has also been considered by some that there is also a cyclic change in capacity due to to corona, but in view of later work, and especially some at the Massachusetts Institute of Technology shortly to be published by Mr. M. F. Gardner, it is certain that this capacity effect is small, if it exists at all. In view of the complexity of the actual phenomena it is good judgment to make the first, admittedly only roughly approximate, formulation of the problem by the assumption used by Dr. Whitehead of a constant corona leakance.

But having made this assumption I cannot understand why the author attempts a new mathematical treatment. The progress of waves on wires, under the assumption of four constant line parameters, was first solved by Heaviside (Electromagnetic Theory, Vol. I, Ch. IV; Vol. II, p. 312, Electrical Papers, Vol. 2). It has received much attention by Poincare, Fleming, and later by Carson and Manneback. The mathematical solution has also been experimentally checked at M. I. T. for the exact case considered.[2]

Moreover, this solution of Heaviside's shows clearly that, when the four circuit parameters,—resistance, inductance, capacitance, and leakance,—are assumed constant, a perpendicular wave front of current or voltage remains strictly perpendicular as it propagates at the velocity of light. The wave front is attenuated in magnitude, and there may be distortion of the tail of the wave, but there is no change in perpendicularity at the wave front itself.

Corona may indeed cause the wave front to become non-perpendicular, for corona is not properly represented by a constant leakance; but any mathematical treatment which makes the assumption of four fixed constants should lead to a sustained perpendicular front, or else should substantiate its direct clash with the classic treatment.

The mathematical method of the paper was earlier used by Steinmetz. It represents a traveling wave by a Fourier series, gives physical meaning to the separate terms of this series, and applies to each a modifying factor, due in this case to the effect of corona. This method is bound to lead to error. Having formulated the differential equations of a system on the basis of certain constants, and made a solution, we cannot then introduce the effect of new factors into the separate mathematical terms of this solution without endangering the correctness of our conclusions. We should instead reformulate the differential equations for the conditions obtaining with the new factors introduced as a part of the original premises. Stated in another way, we cannot solve for the Fourier series expressing a traveling wave on a line, and then modify separate terms of our solution in accordance with the effect which leakance would produce on terms of that nature if existing independently, without destroying the dependability of our results.

The danger involved will perhaps be clearer if we carry the method to extremes. The Fourier expression for a rectangular wave of current traveling down a line is a function of distance

x and time t, such that it sums up to zero everywhere outside the wave, and to a constant value of current everywhere inside the wave. The separate terms of this series are sinusoids. Suppose we ascribe to each a separate physical existence, and assume that each produces independently its own resistance loss. Since these terms are continuous along the line we are led to the absurd result that we will compute a loss in the wire at points to which the wave has not yet penetrated. The result is exactly the same if we treat a wave of voltage, and the leakance loss.

The paper states that it is a generally accepted belief that a chief danger from lightning disturbances lies in the steepness of wave front. I agree, but not that this is *the* chief danger. The chief dangers are voltage amplitude, and duration of application of voltage due to traveling waves. The steepness of wave front is sometimes of interest, but not often. We usually are concerned with the amount of voltage our terminal apparatus is called upon to sustain, and for how long. We are rarely concerned with how fast it is applied. Steepness of wave front has been overemphasized. We apply the steepest possible wave front, without worry, every time we connect a transformer bank directly to live bus bars. Why be concerned if it is nearly as steep as this when a line intervenes? We do care, though, about amplitude. Corona has its effect on amplitude also of course.

J. B. Whitehead: I regret that I have failed to include reference to the experimental results of Mr. Peek on the attenuation of the voltage pulses on a corona-forming line. Under his reminder I now recall them well. They are strong indications in support of the assumption that corona has value as a lightning arrester.

Professor Bush does me too much honor in suggesting that I have made a mathematical attack on this serious problem. I have not done this. The complexity of the phenomenon, emphasized in the paper and reiterated by Professor Bush, was a sufficient deterrent. The purpose of my paper has been, as clearly indicated, an approximate answer only. While I am well aware of the criticism which has been directed to the use of the Fourier analysis in problems of this character, I have been glad to find at hand in the Steinmetz paper referred to a series of computations which lent themselves so readily to the addition, to other attenuation factors, of that due to corona. Professor Bush appears to have overlooked the fact that the Steinmetz analysis does not assume fixed values of circuit constants. The fact that the analysis takes account of the changes of both resistance and conductance with frequency was one of the reasons why it appealed to me for my present purposes. The method is especially helpful if the conductance is assumed to vary with the frequency as has been done in making the corrections indicated in the first paragraph above.

As for the use of Heaviside's methods, Professor Bush's partiality for them seems to have arisen since the appearance of his very valuable paper on Transmission Line Transients in December 1923, for in that paper he emphasizes clearly the difficulty of using the Heaviside treatment for cases in which the transmission engineer is interested, and also the laborious computations involved in the attainment of numerical results. I suppose he will hardly recommend Heaviside's analysis or any of its extensions for cases in which the resistance and conductance are unknown functions of the frequency. Perhaps the most important thing is that we now have good experimental evidence that the presence of corona on lines not only tends to diminish the gradient of a pulse but also to decrease its maximum value. If, as seems to be true, Steinmetz's method of analysis is open to serious criticism it is obvious that the application of some more correct method to this problem would be a service of great value. I hope that Professor Bush will be led to undertake this, using those methods which seem to him more suitable and convenient.

Corona Losses Between Wires
at Extra High Voltages—II

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Review of the Subject.—Results of corona loss tests upon three sizes of cables at voltages varying from 100 to 620 kv. and spacings from 18 to 38 feet are reported from the Engineering Experiment Station of Purdue University. These tests results, reduced to the standard 1000 feet of transmission line, are compared with corresponding values calculated from Peek's formula for similar conditions of operation. A description of the tower line and method of measurement of the losses in the high-voltage circuit is included.

An empirical equation has been developed which approximates quite closely the relation between corona loss and voltage for different spacings. The variations of the empirical coefficient of this equation are indicated for 2/0 and 4/0 cables.

Three methods of attacking the problem of modified transmission line design for the elimination of excessive corona losses between wires at extra high voltages have been outlined for further research and study.

I. Calculation of Capacity and Corresponding Radius of Equivalent Coronal Conductor and Its Relation to Voltage between Wires.

II. Determination of Corona Losses between One Wire and Ground.

III. Photographic Reproduction of Equivalent Electrostatic Field Surrounding Model Conductors with Proportional Spacings between Wires and Ground.

The paper should be considered a progress report to be enlarged and analyzed further at a later time.

CORONA losses upon transmission lines may reach values worthy of the serious consideration of the designing and operating electrical engineer at or above potentials of 100 kilovolts between wires, depending upon the size and spacing of the wires, the weather conditions and the elevation of the line above sea level.

The critical voltage at which corona occurs upon a given transmission line and the magnitude of the power loss due to corona for definite line conditions may be calculated by means of the formulas developed by F. W. Peek, Jr., based upon certain empirical constants which were determined by him in early laboratory and experimental transmission line tests.[1] These tests included voltages up to a maximum of 250 kilovolts.

Tests carried on at Purdue University upon an experimental line 1380 feet in length were reported to the Institute in 1912.[2] These results were compared with those obtained upon other lines already in operation, as well as those calculated from Peek's formula. These tests were made with instruments connected in the high-voltage circuit and therefore were subject to no error depending upon the efficiency or power factor at which the step-up transformers were operating. The following significant conclusions may be quoted from the latter paper:

"Corona loss may be readily and accurately determined with instruments connected directly into the high-tension circuit.

"Corona loss curves are parabolas, the constants of the equations being different above and below the visual critical voltage.

"Test values checked results calculated from Peek's formula for points above the visual critical voltage with a fair degree of accuracy, especially at the wider spacings between wires.

"Variations from Peek's formula were in the direction of greater losses for a given voltage than those given by the formula. This was also found to be true of the tests which have been made upon operating lines, when the latter were reduced to a common standard for comparison."

This paper sets forth the results of tests carried on at Purdue University during the past two years with a single-phase line designed for 600 kilovolts. The corona losses were measured upon three standard sizes of cables at various spacings by means of a wattmeter in the high-voltage circuit operating at voltages as high as 620 kilovolts. These losses have been compared with values calculated by means of Peek's formula for similar cable size, spacing and operating conditions and representative curves have been plotted for both test and calculated values. The applicability of this formula and the possible error to be expected in its use therefore became apparent for calculations of corona losses at voltages in excess of those within which the original empirical constants were determined. In all cases, the test and calculated results have been reduced to the standard length of line of 1000 feet. Since there is now a practical demand for transmission lines to be operated at voltages in excess of those considered a maximum at the time the former tests were completed, it becomes a matter of particular concern to determine and record these corona loss measurements and their relation to calculated values.

The line upon which these tests were made was 1710 feet in length. It consisted of three equal spans of cables supported upon semi-flexible steel towers 65 feet in height. The towers at either end of the line were of similar design longitudinally guyed to withstand the

1. See Paper by F. W. Peek, Jr., on "Law of Corona" TRANS. A. I. E. E., Vol. XXX, Part III, p. 1889, 1911.

2. "Corona Losses between Wires at High Voltages I" by C. Francis Harding, TRANS. A. I. E. E., Vol. XXXI, Part I, p. 1035, 1912.

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., October 13-17, 1924.

dead-end stresses. The cables were supported by means of standard suspension insulators of fifteen units each, hung from movable carriages which were designed with rollers to operate transversely upon a steel cross arm 40 feet in length supported upon each tower. The spacing between the two cables in the same horizontal plane could thus be readily changed by increments of from two to four feet between the limits of 18 feet and 38 feet. Three sizes of stranded aluminum cables with steel cores rated as 2/0, 4/0 B & S and 500,000 cm., have been tested to date.

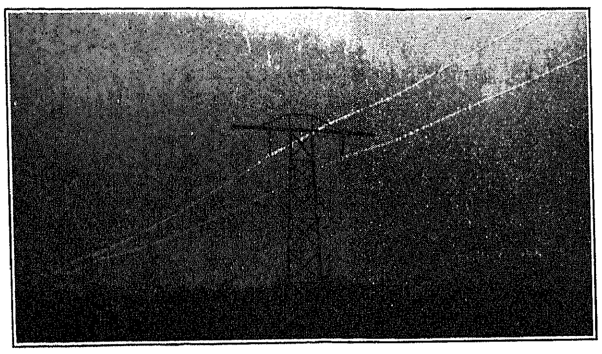

Fig. 1—Corona at Night 600 Kv.

The feeders connecting the test line with the transformers of the high-voltage laboratory were of particularly large cross-section and were spaced relatively far apart in order to reduce the tare losses which were subtracted from the gross power measurement to determine the net corona losses upon the test line.

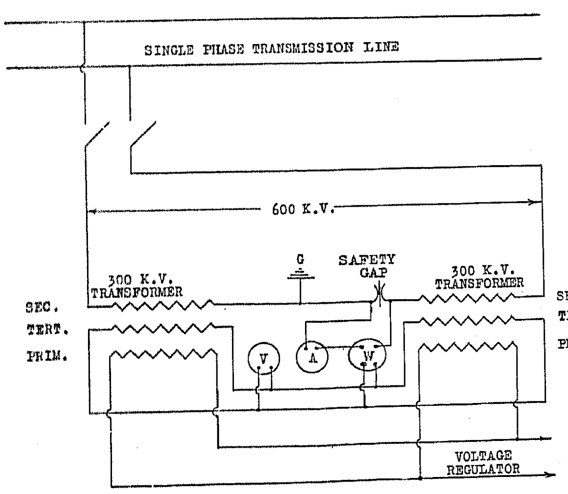

Fig. 2—Diagram of Transformer Connections

Two identical transformers, purchased from the Westinghouse Electric and Mfg. Co., rated as 135 kv-a., 300 kv., 60-cycle oil-insulated units with 1900-volt double-coil primaries were used for the power supply. One secondary terminal of each transformer was connected to one side of the single-phase circuit through a condenser bushing, while the other end of each secondary winding was connected to ground through the indicating instruments. Safety spark-gaps were shunted around the instruments for protection in case of an open circuit. The secondaries of the two transformers were connected in series for line voltages in excess of 300 kv., while the two coils of each primary winding might be connected either in series or in parallel for the varying ratios of transformation desired.

The variation of voltage was accomplished very satisfactorily without wave-form distortion, by means of an auxiliary regulating transformer with tapped primary, whose connections were varied by means of a motor-driven drum controller. The entire ranges of voltage, up to maximum values of 150, 300, or 600 kv., could be readily traversed throughout 480 equal increments at a definite predetermined rate, or any desired voltage could be held constant, or varied slightly at will, by means of the auxiliary hand control.

The gross power output of the two 300-kv. transformers with their secondary windings connected in series (Fig. 2) was measured very accurately by means of a specially calibrated indicating wattmeter whose current coil was connected in series with the high-voltage transformer winding. The voltmeter and the potential coil of the wattmeter were supplied with a voltage proportional to the line voltage through the agency of a tertiary coil in each transformer. These tertiary coil voltages were very carefully calibrated by means of the A. I. E. E. standard 50-cm. sphere-gap measurements under the two different conditions of the line being connected and disconnected from the step-up transformers.

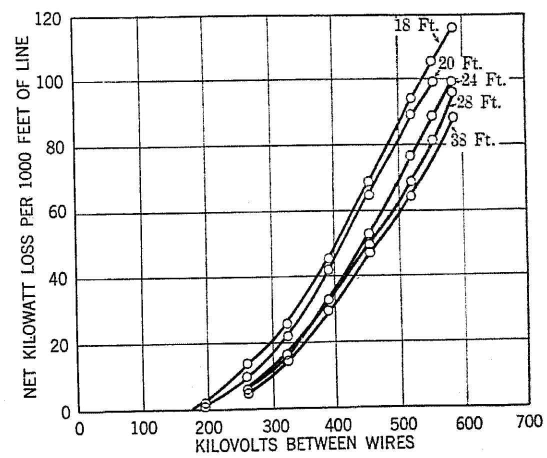

Fig. 3—Net Corona Loss per 1000 Ft. of Line 2/0 Cable

An additional check was made to insure the accuracy of ratio of line to tertiary coil voltage applied to voltmeter and wattmeter measurements under test conditions with large corona loads upon the line. A standard A. I. E. E., 50-cm. sphere-gap with neither sphere grounded was connected across the line, and readings taken of tertiary coil voltmeter at the moment of spark-over of the sphere-gap at various standard settings. These calibrations were made over the entire

range of voltage involved in the tests. As the experimental line, although open-circuited at the further end, is not long enough to cause abnormal voltages to be induced therein, it is believed that the losses due to corona, at the voltages herein specified, have, therefore,

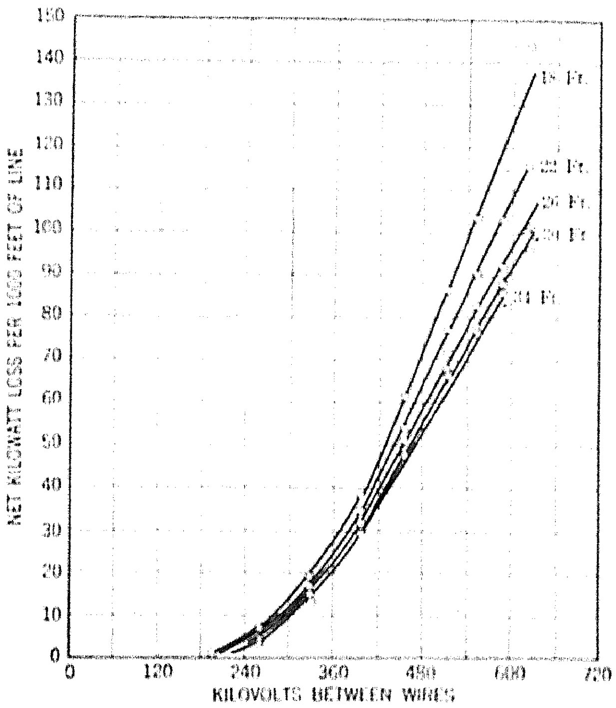

Fig. 4—Net Corona Loss per 1000 Ft. of Line: 4/0 Cable

been accurately determined. Furthermore, it is probable that the leakage losses over insulators which necessarily form a part of the net losses on the line proper after the feeder corona and insulator tare have been

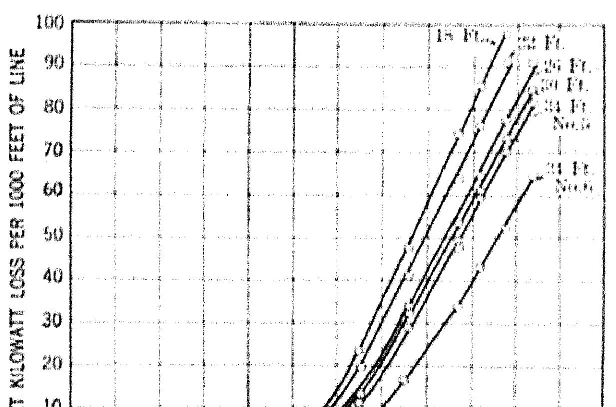

Fig. 5—Net Corona Loss per 1000 Ft. of Line: 500,000-Cm. Cable

subtracted, are entirely negligible. This may be assumed to be true because only four suspension insulators per cable were involved and for the further reason that great care was exercised to make sure that all units were in good condition throughout the series of tests. In previous tests reported to the Institute[3] in which precautions were taken to subtract the line insulator leakage losses, it was found that such were negligible with the insulators in first-class condition.

Results of Tests

In the Appendix, Tables I to XIX inclusive, will be found the results of the tests upon the three sizes of wires under consideration for five or more spacings. The weather conditions, including temperature and barometric pressure at the time of the test, are also recorded. The latter data provide means for calculating the values given in the last column of the tables from Peek's formula, for which a typical solution will be found for one test condition in the Appendix as an illustration of the method followed.

The curves of Figs. 3, 4 and 5 also represent the net corona losses, reduced to a standard length of 1000 feet of line for the three sizes of cables tested at spacings

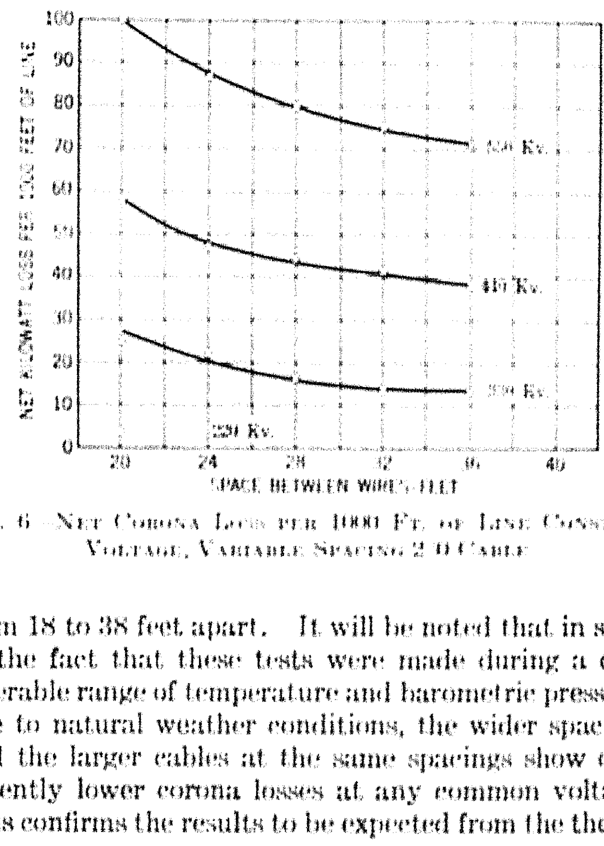

Fig. 6—Net Corona Loss per 1000 Ft. of Line: Constant Voltage, Variable Spacing 2/0 Cable

from 18 to 38 feet apart. It will be noted that in spite of the fact that these tests were made during a considerable range of temperature and barometric pressure, due to natural weather conditions, the wider spacings and the larger cables at the same spacings show consistently lower corona losses at any common voltage. This confirms the results to be expected from the theory of corona.

The relation between corona losses and cable spacing are shown in Figs. 6, 7 and 8 at three definite voltages, determined graphically, from curves of Figs. 3, 4 and 5. These points, with two minor exceptions, fall upon smooth curves, apparently following a definite law which indicates, as would be expected, a marked and regular decrease in loss with increased spacing between cables.

3. "Corona Losses between Wires at High Voltages I" by C. Francis Harding, Trans. A. I. E. E., Vol. XXXI, Part 1, p. 1036, 1912.

These curves further indicate the zones of voltage and spacing for the three sizes of cables within which the losses are not excessive from the practical standpoint. For example, if it soon becomes desirable to operate a line at 330 kilovolts, the losses are found from Fig. 8 to be moderate for the 500,000 cm. aluminum stranded cable. However, with 2/0 and 4/0 cables these losses

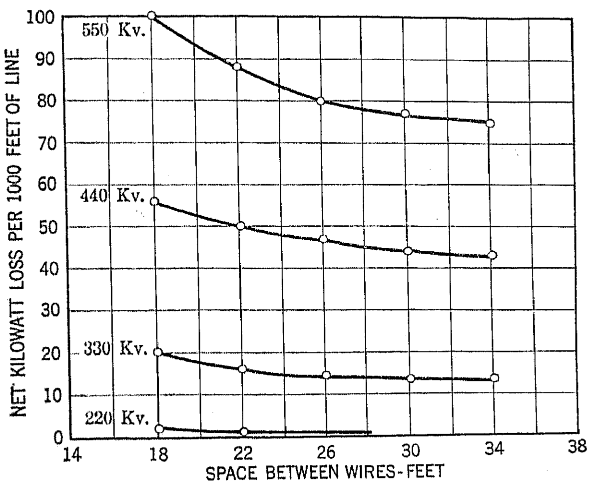

FIG. 7—NET CORONA LOSS PER 1000 FT. OF LINE CONSTANT VOLTAGE, VARIABLE SPACING 4/0 CABLE

are objectionably large, particularly at the narrow spacings. At 220 kilovolts between cables, a potential at which some lines are already operating, the corona losses are negligible, especially for the largest size of cable.

It should be noted that all results in this paper are plotted for single-phase circuits. The corresponding

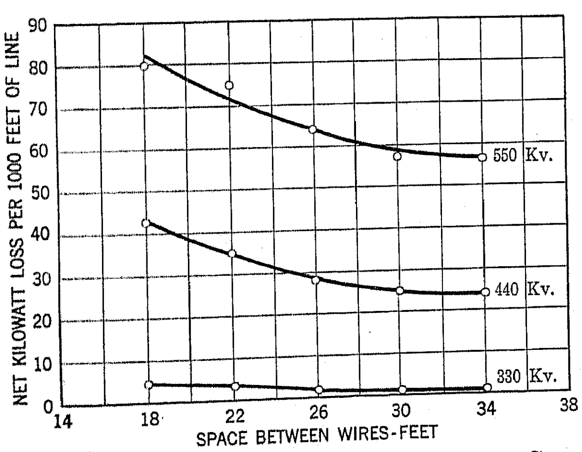

FIG. 8—NET CORONA LOSS PER 1000 FT. OF LINE CONSTANT VOLTAGE, VARIABLE SPACING 500,000-CM. CABLE

losses per 1000 feet of the normal three-phase line can be obtained from these curves by referring to a value of kilovolts between wires $\frac{2}{\sqrt{3}}$ times as great as the single-phase line voltage and by multiplying the resultant net loss in kilowatts taken from the curves by $\frac{3}{2}$ to include the loss upon all three wires.

For example, to determine the net three-phase corona loss per 1000 feet of line consisting of three 500,000-cm. cables spaced 26 feet apart at a voltage of 220 kv. between cables, the corresponding single-phase curve of Fig. 5 should be consulted for a voltage between wires of

$$\frac{220 \times 2}{\sqrt{3}} = 254 \text{ kv.}$$ The corresponding power loss is found to be 0.33 kw. This is the loss per 1000 feet of two-wire line having the same voltage to neutral as a 220-kv. three-phase line. The three-wire, three-phase line will, therefore, have a loss of $\frac{3}{2} \times 0.33 = 0.495$ kw. per 1000 feet, or 2.62 kw. per mile. A similar

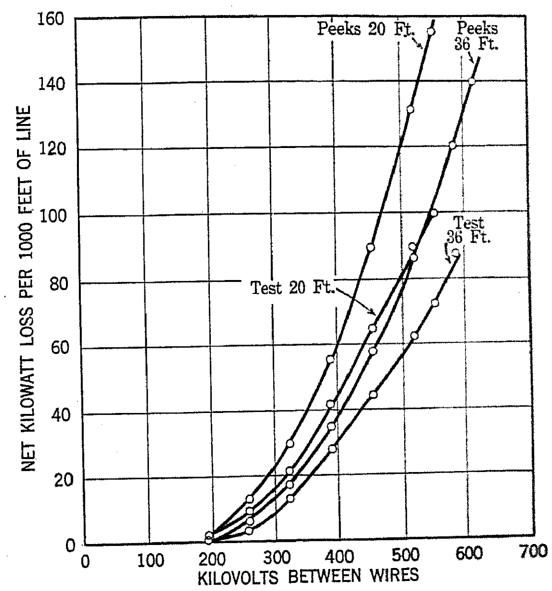

FIG. 9—NET CORONA LOSS PER 1000 FT. OF LINE COMPARISON WITH PEEK'S FORMULA 2/0 CABLE

calculation for a possible 330 kv. three-phase line having the same size of cable and the same spacing, indicates a probable loss of $\frac{13.5 \times 3}{2} = 20.25$ kw. per 1000 feet, or 107.0 kw. per mile. This would obviously be excessive and would, therefore, involve a change of design for a line to operate efficiently at such a voltage.

Comparison with Calculated Values. A complete analysis of the comparative results of tests and corresponding losses calculated for the same weather conditions, based upon Peek's formula, was not possible in the time available since the completion of the tests.

It is evident, however, from values in the tables and Figs. 9, 10 and 11, in which such comparable results are plotted, that the calculated values are consistently higher than test values *throughout the higher ranges of*

voltage. This departure of actual losses determined by the tests from the calculated losses seems to be particularly noticeable at the narrow spacings and with the smaller sizes of cable. For example, it is apparent from

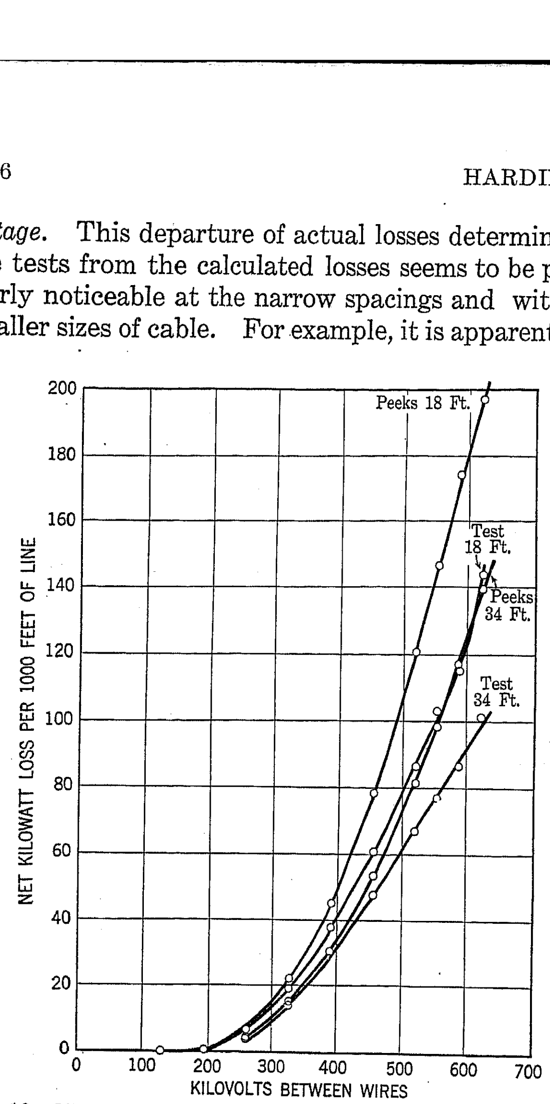

FIG. 10—NET CORONA LOSS PER 1000 FT. OF LINE COMPARISON WITH PEEK'S FORMULA 4/0 CABLE

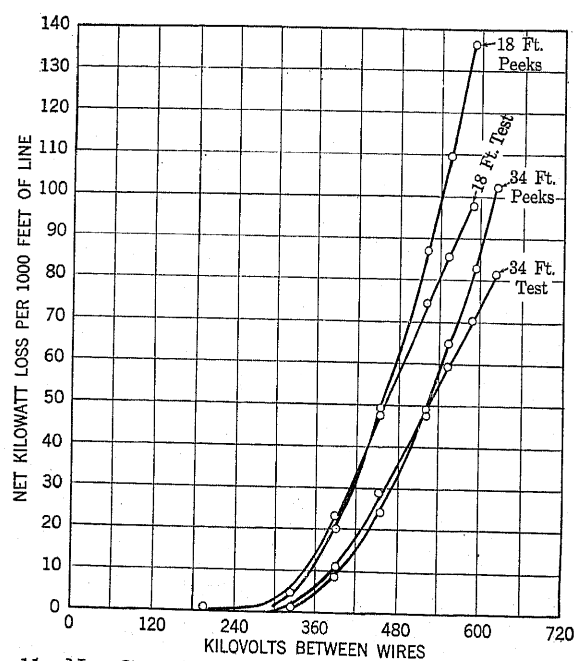

FIG. 11—NET CORONA LOSS PER 1000 FT. OF LINE COMPARISON WITH PEEK'S FORMULA 500,000-CM. CABLE

Fig. 10 that for 4/0 cable the calculated and test values are practically coincident for 18-foot spacing up to voltages not exceeding 300 kv., while the 34-foot spacing for the same cable indicated a coincidence up to 400 kv. For the 500,000-cm. cable, whose net test losses due to corona are superimposed upon the theoretical curves in Fig. 11, the calculated and test results are practically coincident up to 450 kv. for the 18-foot spacing and 520 kv. for the 34-foot spacing, the formula giving higher values of losses above and lower values below these voltages. Fig. 9 indicates a marked departure at the higher voltages of test from predicted values in the case of 2/0 cable for both narrow and wide spacings.

Test results are consistent with those reported at lower voltages in the first paper, since the theoretical calculation indicates losses less than the test values throughout the lower range of line voltages, while the former calculated values have a steeper slope upward with increased voltages than those found in the tests. In fact, the net losses throughout the higher range of line voltages, during the latter tests, seemed to follow a straight line or a modified quadratic law, rather than the parabola of the formula, thereby showing lower values of loss than those calculated.

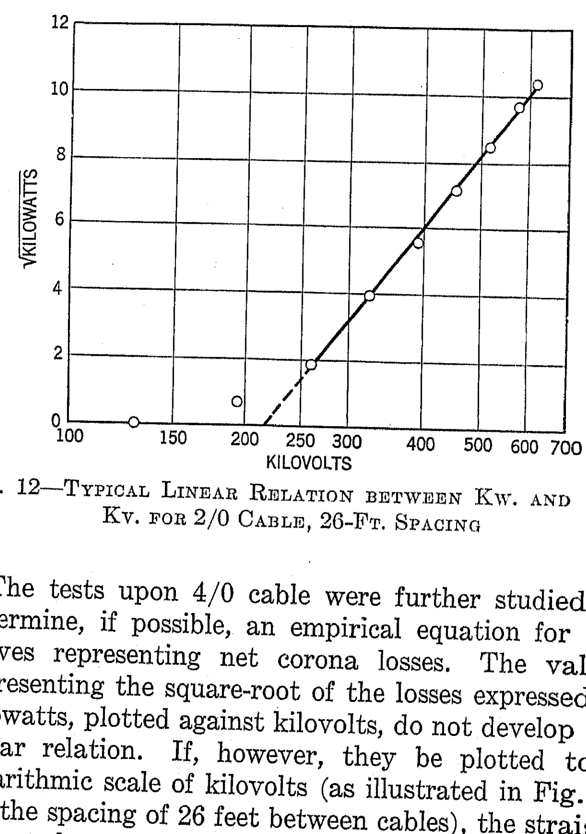

FIG. 12—TYPICAL LINEAR RELATION BETWEEN KW. AND LOG KV. FOR 2/0 CABLE, 26-FT. SPACING

The tests upon 4/0 cable were further studied to determine, if possible, an empirical equation for the curves representing net corona losses. The values representing the square-root of the losses expressed in kilowatts, plotted against kilovolts, do not develop the linear relation. If, however, they be plotted to a logarithmic scale of kilovolts (as illustrated in Fig. 12 for the spacing of 26 feet between cables), the straight line results. This indicates an equation of the form:

P (Corona loss in kw.) $= C \log_{10}^2 \text{kv.}/\text{kv.}_1$ where C is a function of the spacing (s), as indicated for both 2/0 and 4/0 cable in Fig. 13.

The voltage kv.$_1$ determined by the intercept for zero corona loss, as indicated in Fig. 12, is found to be 215 kv. for 4/0 cable or $P = C \log_{10}^2 \dfrac{\text{kv.}}{215}$.

It is rather significant that the values of C, which are

obviously proportional to the corona loss, approach a minimum value as the spacing between cables is increased. An apparent critical or unstable condition seems to exist for these two cable sizes, for spacings between 30 and 38 feet under these test conditions. It will be of interest to determine by subsequent tests whether this is due to the distortional effect upon the electrostatic field of the conductors at wide spacings

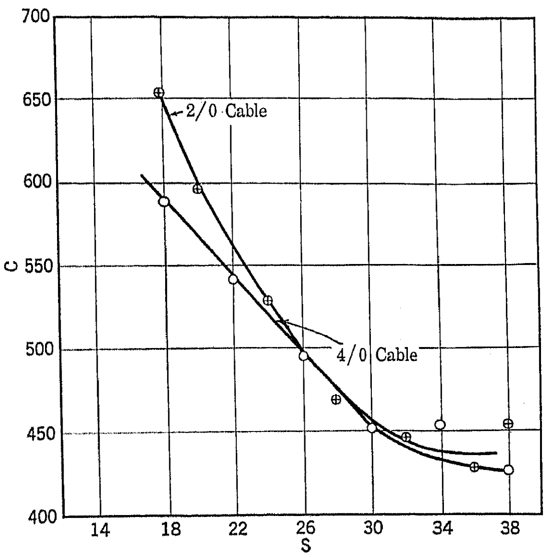

FIG. 13—RELATION BETWEEN EMPIRICAL COEFFICIENT (C) AND SPACING FOR 2/0 AND 4/0 CABLE

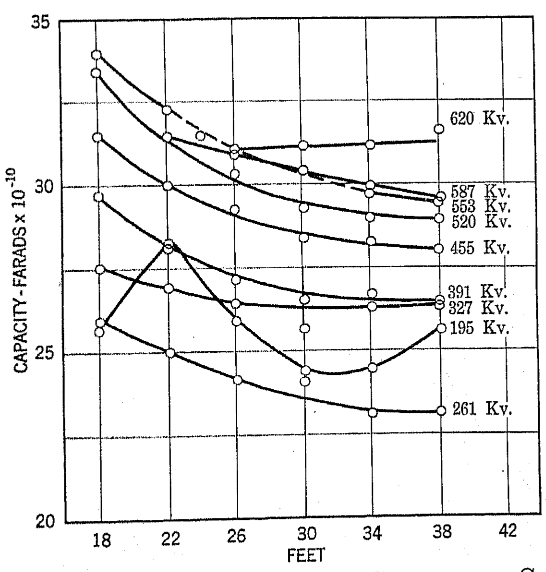

FIG. 14—RELATION OF EQUIVALENT CAPACITY OF CORONAL CONDUCTOR AND SPACING AT VARIOUS CONSTANT POTENTIALS FOR 4/0 CABLE

resulting from the proximity to the ground, or whether the "equivalent coronal conductor," if it may be called such, created by the ionization of the air surrounding the line wire, under excessive corona, becomes of such diameter or form as to produce a distinct localized minimum, or possibly a constant minimum loss independent of further increase of spacing.

Such possibilities immediately suggested three further studies, which have been undertaken in part as follows:

I. Calculation of Capacity and Corresponding Radius of Equivalent Coronal Conductor and Its Relation to Voltage between Wires.

II. Determination of Corona Losses between One Wire and Ground.

III. Photographic Reproduction of Equivalent Electrostatic Field Surrounding Model Conductors with Proportional Spacings between Wires and Ground.

The tentative results of these three investigations are briefly summarized as follows:

I. Capacity and Corresponding Radius of Equivalent Coronal Conductor.

The capacity of an equivalent conducting body of

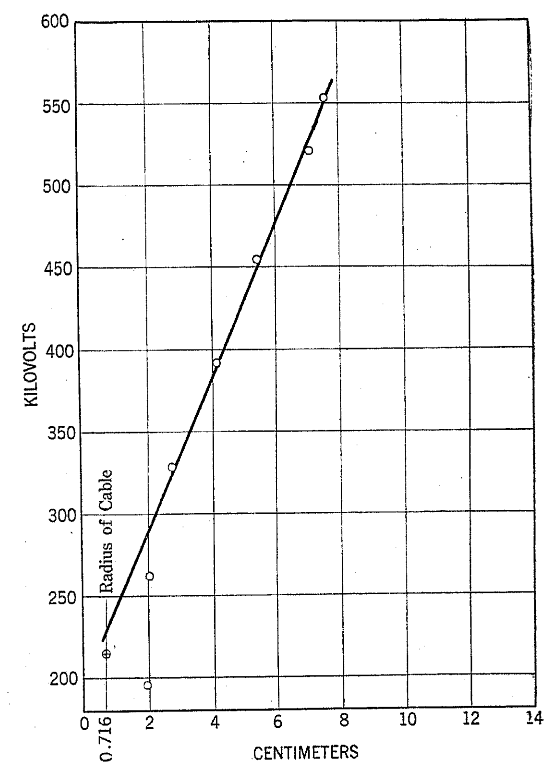

FIG. 15—RELATION OF RADIUS OF EQUIVALENT CORONAL CONDUCTOR TO VOLTAGE BETWEEN WIRES AT 18-FT. SPACING

corona or apparent corona, which has been assumed to be a cylinder, concentrically located with respect to the line wire, may be readily calculated from the test data available in the tables of the Appendix. The net corona power loss for each voltage and the corresponding current and frequency being known, the power factor was determined and recorded in column 6 of each table. The reactive or capacity component of the total current becomes known, therefore, and the equivalent capacity and radius of the enlarged hypothetical conducting coronal cylinder may be determined for any test frequency. A sample calculation of such capacity and radius is included in the Appendix as an illustration.

The relation between such calculated capacities and the spacing between two 4/0 line wires for certain

indicated constant voltages ranging from 195 kv. to 620 kv. is plotted in Fig. 14. It is particularly interesting to observe that for such wires at 620 kv., the capacity is practically constant for spacings extending

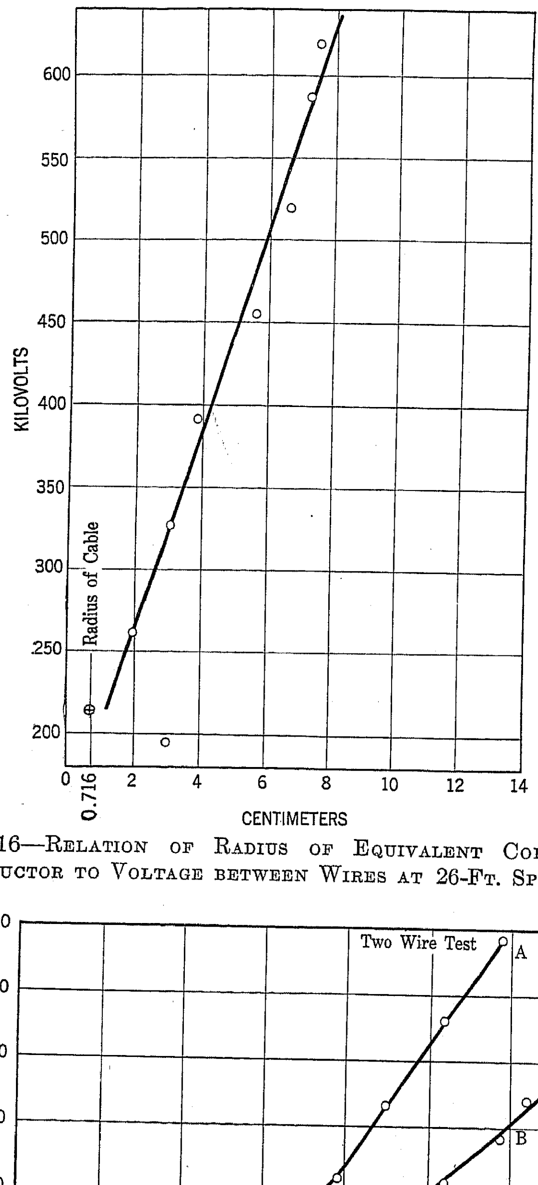

FIG. 16—RELATION OF RADIUS OF EQUIVALENT CORONAL CONDUCTOR TO VOLTAGE BETWEEN WIRES AT 26-FT. SPACING

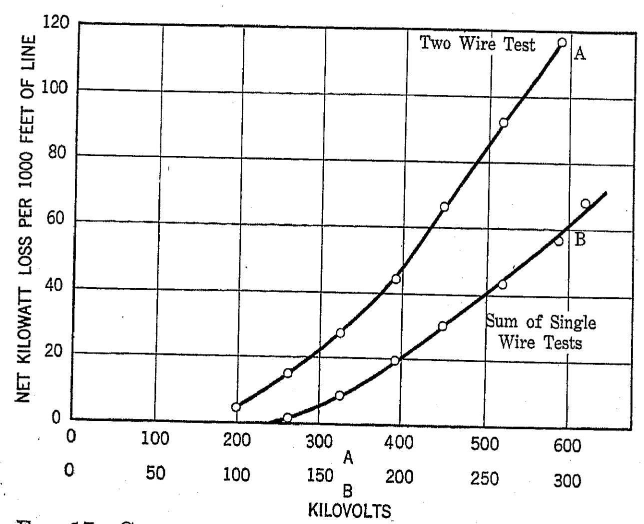

FIG. 17—COMPARISON OF CORONA LOSSES OF ONE 4/0 WIRE TO GROUND WITH LOSSES BETWEEN TWO 4/0 WIRES, 18-FT. SPACING

from 26 to 38 feet and probably beyond if the tests be extended, while at lower voltages the trend is toward smaller capacities as the spacings are increased and as the voltages for a given spacing are decreased. This condition is such as would be expected from the theory of the electrostatic circuit involved. The erratic variation of the equivalent capacity indicated by the curve for 195 kv. is also of interest, since this voltage is below the critical corona-forming voltage for such a cable. The irregularity of the curve is probably another confirmation of the well-known unstable condition surrounding such a wire when operating at a voltage approximating the critical corona potential.

If this investigation be extended to the determination of the radius of the equivalent coronal conductor, as previously indicated, the relations between such radii, expressed in centimeters, and the potentials impressed between two 4/0 wires of such a transmission line, spaced 18 and 26 feet apart, are available for further study in Figs. 15 and 16 respectively. As a basis of reference, the actual radius of the cable, which is 0.716 cm. in this case, has been plotted at a point corres-

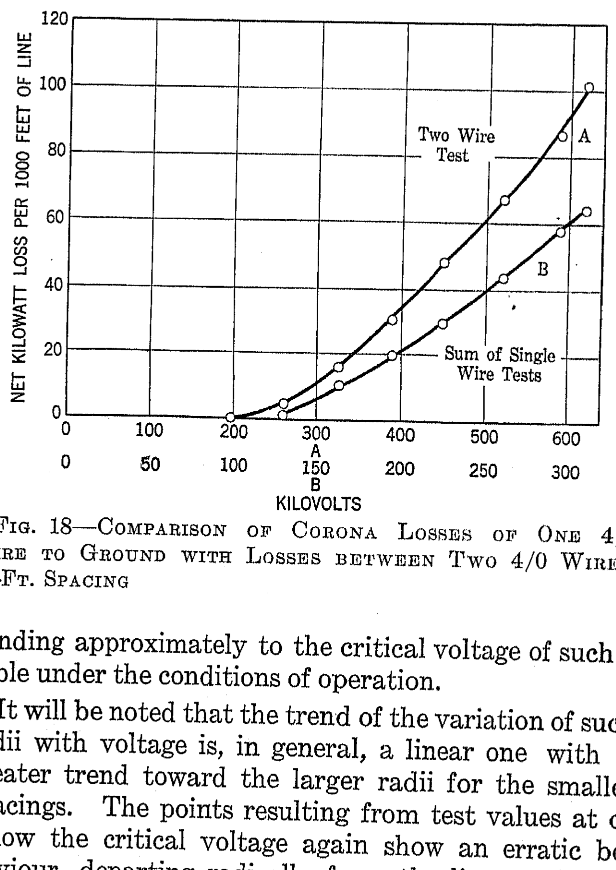

FIG. 18—COMPARISON OF CORONA LOSSES OF ONE 4/0 WIRE TO GROUND WITH LOSSES BETWEEN TWO 4/0 WIRES, 34-FT. SPACING

ponding approximately to the critical voltage of such a cable under the conditions of operation.

It will be noted that the trend of the variation of such radii with voltage is, in general, a linear one with a greater trend toward the larger radii for the smaller spacings. The points resulting from test values at or below the critical voltage again show an erratic behaviour, departing radically from the linear relation. In some of the other charts analyzed, but not reproduced in this paper, a very marked but probably purely incidental symmetry of fluctuation of such alternate points upon either side of the linear trend graph was particularly noted.

Although the practical bearing of this latter investigation may be far from obvious, it is probable that it may have a part to play in the determination of conductor diameters and spacings to be used for the higher voltages for power transmission in the future.

Corona Losses between Wires and Ground

Tests were conducted with the various sizes and spacings of wires upon the experimental transmission line with voltages increasing to a maximum of 300 kv. between one wire and ground. The other wire remained in place, at a definite spacing, but was disconnected from the high-voltage source and ungrounded.

An indication of the trend of such test data is evidenced by Tables XX and XXI in the Appendix and

Studies of Electrostatic Fields Looking at Ends of Parallel Wires at Various Spacings. All 65 Mm. Above Ground (Fig. 19-26)

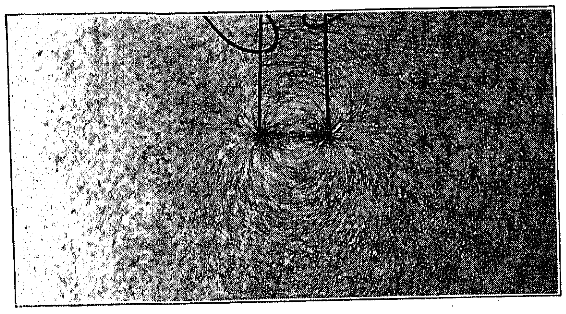

Fig. 19—Spacing, 18 Mm.

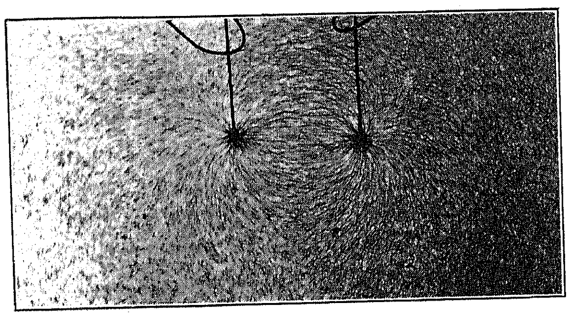

Fig. 20—Spacing, 38 Mm.

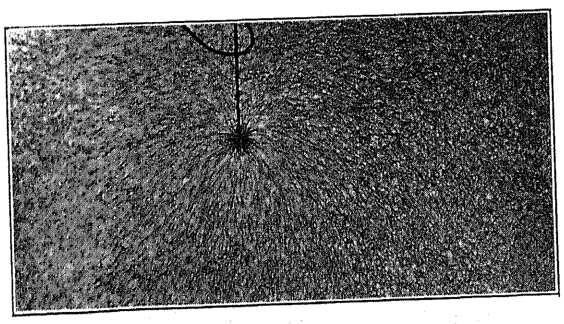

Fig. 21—One Wire and Ground

the composite curves of Figs. 17 and 18. As the spacing between wires is increased, especially if they are supported from relatively low towers, the effect of the neutral plane of the ground itself evidently becomes of importance, especially in determining proper corona eliminating designs for multi-circuit, super-voltage lines. It is to be expected for such spacings, as confirmed by these tests, that the arithmetical sum of the power losses due to corona between each wire individually and ground is very much less than the loss between wires at the same spacing. As the spacing is increased, however, this difference becomes less marked. The limiting factors in this relation at the higher voltages are yet to be determined, as they involve the relative costs of extra high towers as compared with wide rights of way for such long distance lines to be constructed in the future.

Reproduction of Electrostatic Field between Wires and Ground

The attempt which was made to apply the method of equivalent coronal conductor capacities and their radii, previously described herein, to the effect of the ground in cases involving wide spacings of wires, re-

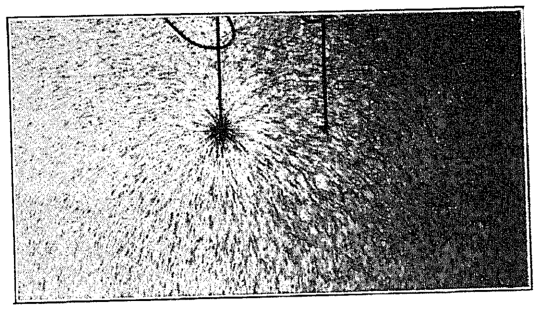

Fig. 22—Spacing, 32 Mm., Right Wire Disconnected

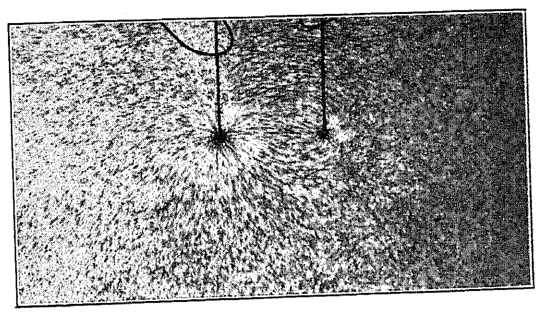

Fig. 23—Spacing, 32 Mm., Right Wire Grounded

sulted in erratic values, probably due to necessarily badly distorted resultant electrostatic fields.

The physical conception of this condition will, no doubt, be made clearer by the use of a method of photographing an equivalent model electrostatic field which has been recently originated by R. H. George, Research Assistant of the Engineering Experiment Station of Purdue University. Although this method was developed for another purpose, which will be published at an early date[4], its adaptation to this problem will be evident from inspection of Figs. 19 to 26 inclusive.

The scale of spacing in the model is proportional to the transmission line under consideration, one milli-

4. Forthcoming Bulletin, Engineering Experiment Station, Purdue University, Lafayette, Ind. "Improved Method of Visualizing and Studying The Electrostatic Field" by R. H. George, Research Assistant.

meter of the model corresponding to one foot on the actual line. In order that the effects of the same relative changes in spacing of conductors and the application of corresponding potentials between one and two wires and ground might be duplicated as closely as possible, the voltage for Figs. 19 to 26 was held constant. The captions of the illustrations will, no doubt, adequately explain the effects of ground potentials upon field distortion for wide conductor spacing.

Tests Under Abnormal Conditions. Two tests which

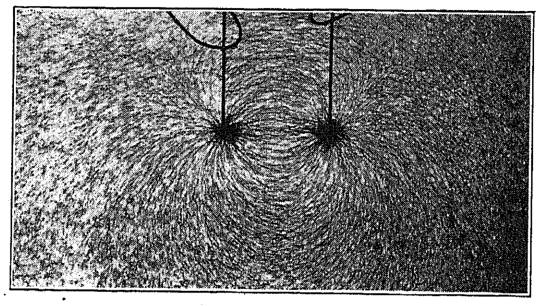

Fig. 24—Spacing 32 Mm., Potential between Wires

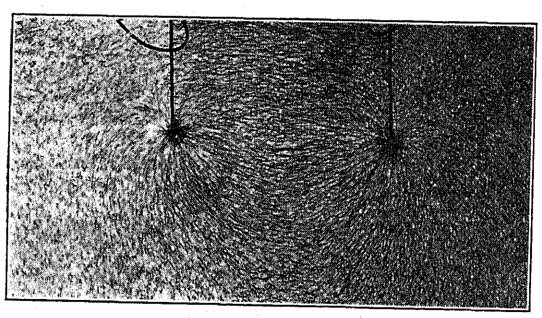

Fig. 25—Spacing, 65 Mm., Potential between Wires

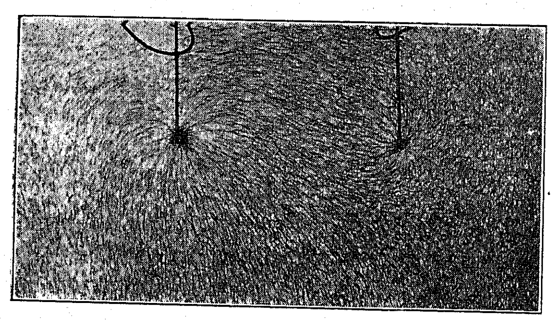

Fig. 26—Spacing, 65 Mm., Right Wire Grounded

were made under rather unusual weather conditions, seem to merit especial comment. Reference to Fig. 5 will reveal two tests upon 500,000-cm. cable at 34-foot spacing. Curve 5 in this graph represents a test during a rain storm with a temperature of + 3 deg. cent. and a barometer reading of 73.4 cm. The losses were relatively high in spite of the low temperature which would tend to produce lower losses than those of the average temperature of the other tests. Curve 6 was run upon a clear cold winter day with sleet upon the cables. The temperature was − 7.5 deg. cent. and the barometer 75.2 cm., as a result of which with the possible added effect of the enlarged conductor due to sleet formation, the loss was exceptionally low.

The tests are being continued upon other sizes of cables with varying conditions of operation, in order that a more detailed analysis may be made in a subsequent paper of actual results throughout this range of voltage.

Acknowledgment

The author desires to express his appreciation of the cooperation extended to the Engineering Experiment Station and School of Electrical Engineering of Purdue University by the following manufacturing companies, staff members and students, who provided materials and time for the design and construction of the line and for the tests reported in this paper:

The towers were furnished by the Bates Expanded Steel Truss Co., and the cables by the Aluminum Company of America. Insulators were used upon the line from the General Electric, Locke, Lapp, Westinghouse, Thomas, Ohio Brass and Jeffry Dewitt Companies.

Dr. W. E. Edington, Assistant Professor of Mathematics, Purdue University, who has devoted a portion of his time to the work of the Engineering Experiment Station, has assisted materially in the preparation of this paper. R. H. George and K. A. Oplinger, Research Assistants in the Engineering Experiment Station, operated the transformers, supervised the tests and checked all calculations. The following students of electrical engineering carried out much of the detailed construction upon the transmission line, took the readings under immediate supervision, and made calculations of the tests under the direction of the author:—

Design and Construction of Line: R. J. Rhinehart, A. Lorber, J. H. Brecheisen and H. O. Mathews.

Tests on 4/0 Cable: F. M. Holaday, P. W. Harrison, J. R. Parnin, W. F. Spaulding and W. J. Guenther.

Tests on 500,000 cm. Cable: F. C. Jones, M. S. Watson, K. O. Thorp, E. J. Archbold, R. C. Goodwin, K. T. Kwo and W. A. Sevedge.

Tests on 2/0 Cable: B. D. Holley, I. H. Hollis, W. J. Rannells, R. S. Merchant, D. Rasmussen and R. J. Morrison.

Appendix

TABLE I

Test No. VII. Spacing 18 ft. Size of wire 2/0 B & S.
Temp. +15.5 deg. cent. Bar. 741 mm. Date April 7, 1923
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.15	..	0	0
195.5	3.3	0	3.3	0.22	0.08	1.9	2.7
261.0	22.8	0	22.8	0.34	0.26	13.3	13.8
326.0	49.0	5.2	43.8	0.47	0.32	25.6	33.5
391.0	88.0	9.8	78.2	0.59	0.38	45.7	61.8
456.0	133.5	16.3	117.2	0.75	0.39	68.6	98.8
521.0	187.5	25.8	161.7	0.88	0.41	94.6	144.2
554.0	211.8	31.9	179.9	0.95	0.41	105.2	170.6
586.0	231.0	33.3	197.7	0.98	0.40	115.3	199.0

TABLE II

Test No. VI. Spacing 20 ft. Size of wire 2/0 B & S.
Temp. +14.5 deg. cent. Bar. 748 mm. Date April 6, 1923
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.15	..	0	0
195.5	1.63	0	1.63	1.21	0.04	0.95	2.14
261.0	16.95	0	16.95	0.33	0.21	9.92	12.1
326.0	42.4	5.2	37.2	0.45	0.29	21.7	30.0
391.0	81.5	9.8	71.7	0.59	0.36	41.9	55.6
456.0	127.0	16.3	110.7	0.73	0.38	64.7	89.6
521.0	179.5	25.8	153.7	0.87	0.40	89.7	131.5
554.0	202.0	31.9	170.1	0.92	0.40	99.5	155.1

TABLE III

Test No. V. Spacing 24 ft. Size of wire 2/0 B & S.
Temp. −2.2 deg. cent. Bar. 756 mm. Date March 31, 1923
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.16	..	0	0
195.5	0	0	0	0.24	..	0	0.8
261.0	9.8	0	9.8	0.30	0.12	5.7	7.6
326.0	32.6	5.2	27.4	0.42	0.20	16.0	21.2
391.0	66.0	9.8	56.2	0.55	0.26	32.8	41.4
456.0	108.4	16.3	92.1	0.68	0.30	53.9	68.8
521.0	156.0	25.8	130.2	0.83	0.30	76.2	102.0
554.0	183.0	31.9	151.1	0.90	0.31	88.5	123.0
586.0	203.0	33.3	169.7	0.95	0.31	99.1	143.5
619.0	228.0	..	..	1.00	..	..	166.5

TABLE IV

Test No. IV. Spacing 28 ft. Size of wire 2/0 B & S.
Temp. +1.1 deg. cent. Bar. 749 mm. Date March 30, 1923
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.15	..	0	0
195.5	0	0	0	0.22	..	0	.84
261.0	9.78	0	9.78	0.30	0.13	5.7	7.4
326.0	31.0	5.2	25.8	0.41	0.23	15.1	20.4
391.0	65.2	9.8	55.4	0.53	0.31	32.4	40.0
456.0	101.0	16.3	84.7	0.66	0.34	49.5	65.8
521.0	143.5	25.8	117.7	0.79	0.35	68.6	98.1
554.0	171.5	31.9	139.6	0.88	0.35	81.6	117.0
586.0	197.0	33.3	164.7	0.99	0.36	96.3	136.5
619.0	220.0	..	..	1.00	0.36	..	..

TABLE V

Test No. III. Spacing 32 ft. Size of wire 2/0 B & S.
Temp. +0.5 deg. cent. Bar. 753 mm. Date March 28, 1923
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.13	..	0	0
195.5	0	0	0	0.22	..	0	0.5
261.0	6.5	0	6.5	0.29	0.09	3.8	6.1
326.0	27.7	5.2	22.5	0.40	0.22	13.2	17.7
391.0	58.4	9.8	48.6	0.53	0.28	28.4	34.9
456.0	94.5	16.3	78.2	0.65	0.32	45.7	58.5
521.0	137.0	25.8	111.2	0.78	0.34	65.0	87.7
554.0	162.2	31.9	130.3	0.84	0.35	76.1	105.0
586.0	182.5	33.3	149.2	0.89	0.35	87.4	123.0
619.0	200.0	..	..	0.93	0.35	..	143.5

TABLE VI

Test No. II. Spacing 36 ft. Size of wire 2/0 B & S.
Temp. +10 deg. cent. Bar. 752 mm. Date March 23, 1923
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.15	..	0	0
195.5	0	0	0	0.22	..	0	0.6
261.0	6.5	0	6.5	0.29	0.09	3.8	6.3
326.0	27.7	5.2	22.5	0.39	0.22	13.1	17.6
391.0	57.1	9.8	47.3	0.51	0.29	27.7	34.8
456.0	91.6	16.3	75.3	0.64	0.32	44.1	57.5
521.0	132.0	25.8	106.2	0.76	0.34	62.2	86.0
554.0	155.0	31.9	123.1	0.82	0.34	72.0	102.8
586.0	182.2	33.3	148.9	0.87	0.35	87.1	120.5
619.0	205.0	..	..	0.94	0.35	..	140.0

TABLE VII

Test Ia. Spacing 18 ft. Size of wire 4/0 B & S.
Temp. −3.5 deg. cent. Bar. 745 mm. Date January 19, 1924
Weather Snow Storm. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.14	..	0	0
195.5	8.0	0	8.0	0.25	0.17	4.67	0.04
261.5	26.2	0.16	26.04	0.37	0.27	15.24	5.56
327.5	52.1	4.56	47.54	0.50	0.32	27.81	32.75
391.5	85.0	9.23	75.77	0.63	0.35	44.3	42.6
455.0	129.0	15.98	113.02	0.77	0.37	66.2	73.5
520.0	175.0	24.30	150.7	0.92	0.37	92.0	111.0
553.0	208.0	29.55	178.45	0.99	0.38	104.4	134.0
587.0	234.0	35.0	199.0	..	..	116.4	

TABLE VIII

Test Ib. Spacing 18 ft. Size of wire 4/0 B & S.
Temp. 8.8 deg. cent. Bar. 754 mm. Date February 15, 1924
Weather Cloudy. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.13	..	0	0
195.5	1.63	0	1.63	0.22	0.04	0.95	0.17
261.5	11.13	0.16	10.97	0.31	0.14	6.4	6.74
327.5	37.2	4.57	32.64	0.43	0.26	19.1	22.6
391.5	73.8	9.23	64.57	0.57	0.34	37.8	45.0
455.0	120.2	15.98	104.23	0.72	0.37	60.9	78.5
520.0	172.3	24.3	148.0	0.88	0.39	86.5	121.0
553.0	206.0	29.55	176.45	0.96	0.39	103.2	147.0
587.0	232.0	35.0	197.0	1.15	0.34	115.2	174.0
620.0	288.0	40.75	247.25	..	..	144.5	197.0

TABLE IX

Test II. Spacing 22 ft. Size of wire 4/0 B & S.
Temp. 0.6 deg. cent. Bar. 755 mm. Date February 25, 1924
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.13	..	0	0
195.5	1.36	0	1.36	0.24	0.03	0.79	0
261.5	7.17	0.16	7.01	0.30	0.09	4.19	4.32
327.5	32.95	4.57	28.38	0.41	0.24	16.6	17.0
391.5	68.25	9.23	59.02	0.55	0.32	34.5	36.2
455.0	109.0	15.98	93.02	0.69	0.35	54.4	63.7
520.0	156.0	24.3	131.7	0.83	0.36	76.3	99.0
553.0	183.2	29.55	153.7	0.91	0.36	89.7	120.5
587.0	211.0	35.0	176.0	0.95	0.38	103.0	143.5
620.0	238.0	40.75	197.3	..	..	115.4	165.0

TABLE X

Test III. Spacing 26 ft. Size of wire 4/0 B & S.
Temp. 4.0 deg. cent. Bar. 749.5 mm. Date February 26, 1924
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.12	..	0	..
195.5	0.82	0	0.82	0.22	0.02	0.48	0
261.5	6.25	0.16	6.09	0.29	0.08	3.56	3.72
327.5	30.2	4.56	25.6	0.40	0.23	15.0	15.5
391.5	63.3	9.23	54.07	0.53	0.31	31.6	33.9
455.0	102.8	15.97	86.82	0.67	0.34	50.7	58.2
520.0	146.8	24.3	122.5	0.80	0.36	71.7	90.5
587.0	194.0	35.0	159.0	0.92	0.36	93.0	130.0
620.0	224.0	40.7	183.3	0.97	0.35	107.1	154.0

TABLE XI

Test IV. Spacing 30 ft. Size of wire 4/0 B & S.
Temp. 6.0 deg. cent. Bar. 746.5 mm. Date February 27, 1924
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.12	..	0	0
195.5	0.65	0	0.65	0.21	0.02	0.38	0
261.5	5.59	0.16	5.43	0.29	0.07	3.12	5.9
327.5	28.2	4.56	23.64	0.40	0.22	13.12	14.0
391.5	60.3	9.23	51.07	0.52	0.30	29.83	30.75
455.0	97.9	15.97	81.93	0.66	0.33	47.9	54.5
520.0	140.4	24.3	116.1	0.78	0.35	68.0	84.0
587.0	186.2	35.0	151.2	0.91	0.35	88.5	121.0
620.0	214.0	40.75	173.3	0.98	0.35	101.25	142.0

TABLE XII

Test V. Spacing 34 ft. Size of wire 4/0 B & S.
Temp. 10.05 deg. cent. Bar. 741 mm. Date February 29, 1924
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.12	..	0	0
195.5	0.98	0	0.98	0.21	0.02	0.57	0
261.5	7.5	0.16	7.34	0.29	0.10	4.28	3.49
327.5	31.44	4.56	26.88	0.40	0.24	15.72	14.0
391.5	61.0	9.23	51.77	0.52	0.30	30.25	30.8
455.0	97.4	15.98	81.42	0.65	0.33	47.6	53.4
520.0	139.0	24.3	114.70	0.77	0.35	67.0	81.4
553.0	160.5	29.55	130.95	0.83	0.35	76.5	98.0
587.0	183.4	35.0	148.40	0.89	0.35	86.6	117.2
620.0	214.0	40.75	173.25	0.98	0.35	101.3	139.5

TABLE XIII

Test VI. Spacing 38 ft. Size of wire 4/0 B & S.
Temp. 5.0 deg. cent. Bar. 740 mm. Date March 1, 1924
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.13	..	0	0
195.5	0.43	0	0.43	0.22	0.01	0.25	0
261.5	4.94	0.16	4.78	0.28	0.07	2.79	2.73
327.5	26.9	4.56	22.34	0.39	0.21	13.05	11.95
391.5	56.2	9.23	46.97	0.52	0.28	27.42	27.09
455.0	91.6	15.98	75.62	0.65	0.31	44.1	48.20
520.0	130.0	24.3	105.70	0.76	0.33	61.17	74.90
553.0	150.5	29.55	120.95	0.83	0.33	70.7	89.10
587.0	172.1	35.00	137.10	0.88	0.33	80.25	104.00
620.0	196.0	40.75	155.25	0.94	0.34	90.09	126.50

TABLE XIV

Test No. I. Spacing 18 ft. Size of wire 500,000 cm.
Temp. +6.2 deg. cent. Bar. 750 mm. Date November 14, 1923
Weather Cloudy. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.15	..	..	..
195.5	0.72	0.40	0.32	0.25	0.02	0.187	..
261.0	1.8	1.74	0.06	0.32	0.03	0.035	..
326.0	14.2	6.5	7.70	0.40	0.11	4.5	4.18
391.0	50.2	11.6	38.6	0.53	0.24	22.6	19.9
457.0	99.0	18.4	80.6	0.63	0.38	47.1	48.0
520.0	154.0	26.5	127.5	0.82	0.36	74.6	86.0
553.0	179.0	31.4	147.6	0.88	0.37	86.4	110.7
587.0	203.5	36.7	166.8	0.92	0.37	97.5	138.5

TABLE XV

Test No. II. Spacing 22 ft. Size of wire 500,000 cm.
Temp. +16 deg. cent. Bar. 744 mm. Date November 21, 1923
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.16	..	..	..
195.5	0.76	0.16	0.60	0.25	0.02	0.35	..
261.0	1.63	0.74	0.89	0.32	0.02	0.52	..
326.0	10.75	5.38	5.37	0.40	0.18	3.14	3.78
391.0	43.3	10.38	32.92	0.51	0.22	19.25	18.4
457.0	87.7	17.26	70.44	0.65	0.30	41.2	44.3
520.0	133.8	25.8	108.0	0.77	0.34	63.2	70.8
553.0	166.0	31.2	134.8	0.86	0.35	78.8	102.5
587.0	192.3	36.4	155.4	0.92	0.36	91.0	129.0

TABLE XVI

Test No. III. Spacing 26 ft. Size of wire 500,000 cm.
Temp. +16 deg. cent. Bar. 751 mm. Date November 28, 1923
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.14	..	..	..
195.5	0.49	0.17	0.32	0.24	0.01	0.187	..
261.0	1.30	0.74	0.56	0.31	0.02	0.328	..
326.0	6.65	5.38	1.27	0.37	0.06	0.744	2.3
391.0	31.7	10.38	21.32	0.48	0.17	12.35	14.0
457.0	73.4	17.26	56.14	0.61	0.26	32.3	36.2
520.0	117.4	25.8	91.6	0.73	0.31	53.6	66.7
553.0	142.0	31.2	110.8	0.79	0.32	64.7	86.5
587.0	169.3	36.4	132.9	0.86	0.33	77.6	117.0
620.0	197.0	42.4	154.6	0.94	0.34	90.5	135.0

TABLE XVII

Test No. IV. Spacing 30 ft. Size of wire 500,000 cm.
Temp. +16 deg. cent. Bar. 751 mm. Date December 1, 1923
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.14	..	0	..
195.5	0.38	0.16	0.22	0.23	0.01	0.13	..
261.0	1.30	0.74	0.56	0.30	0.02	0.33	..
326.0	7.43	5.38	2.05	0.38	0.06	1.20	1.6
391.0	29.7	10.38	19.32	0.47	0.16	11.3	11.6
457.0	69.7	17.26	52.44	0.59	0.28	30.6	31.4
520.0	110.8	25.8	85.0	0.71	0.30	49.7	58.9
553.0	134.5	31.2	103.3	0.74	0.33	60.5	77.2
587.0	160.0	36.4	123.6	0.84	0.33	72.3	98.5
620.0	187.2	42.4	144.8	0.91	0.33	84.5	120.5

TABLE XVIII

Test No. V. Spacing 34 ft. Size of wire 500,000 cm.
Temp. +3 deg. cent. Bar 734.5 mm. Date December 15, 1923
Weather Rain Storm. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.14	..	0	..
195.5	0.81	0.40	0.41	0.24	0.02	0.24	..
261.0	2.28	1.74	0.54	0.30	0.03	0.32	..
326.0	8.70	6.50	2.20	0.38	0.07	1.29	0.7
391.0	31.9	11.60	20.3	0.47	0.18	11.9	8.2
457.0	69.1	18.40	50.7	0.58	0.26	29.6	24.8
520.0	108.0	26.5	81.5	0.70	0.30	47.6	48.6
553.0	132.5	31.4	101.1	0.76	0.31	59.2	64.3
587.0	157.5	36.7	120.8	0.84	0.32	70.5	82.9
620.0	182.0	42.5	139.5	0.89	0.25	81.5	103.3

TABLE XIX

Test No. VI. Spacing 34 ft. Size of wire 500,000 cm.
Temp. −7.5 deg. cent. Bar. 752 mm. Date January 3, 1924
Weather Clear. Loss between two wires.

Kv.	Kw. Gross	Kw. Tare	Kw. Net	Sec. Current	Power Factor	Kw. 1000 ft.	Kw. 1000 ft. Peek's Formula
130.5	0	0	0	0.14	..	0	..
195.5	0.33	0.16	0.17	0.26	0.01	0.10	..
261.0	1.04	0.74	0.30	0.31	0.01	0.18	..
326.0	2.93	5.38	..	0.38	0.02	..	0.05
391.0	14.85	10.38	4.47	0.46	0.06	2.61	4.9
457.0	45.0	17.26	27.74	0.57	0.17	16.2	18.1
520.0	83.2	25.8	57.4	0.67	0.23	33.6	38.3
553.0	103.8	31.2	72.6	0.72	0.26	42.5	52.0
587.0	125.4	36.4	89.0	0.78	0.28	52.0	68.2
620.0	153.0	42.4	110.6	0.85	0.29	64.6	86.0

TABLE XX

Comparison of Corona Losses of One 4/0 Wire to Ground with Losses between Two 4/0 Wires, 18 foot Spacing
Net Totals per 1000 Feet

Kv.	East Wire Net kw.	West Wire Net kw.	Sum Net kw. E. & W. Wires	Net kw. Loss Two Wires 18′ Double Voltage between Wires
66.2	0	0	0	0
99.1	0	0	0	0.95
130.4	0.594	0.706	1.3	6.4
163	4.44	4.38	8.82	19.1
195.5	9.82	9.61	19.43	37.8
225	15.45	14.81	30.26	60.9
261	21.95	21.12	43.07	86.5
295	28.64	27.95	56.59	115.2
311	34.61	33.34	67.95	

TABLE XXI

Comparison of Corona Losses of One 4/0 Wire to Ground with Losses between Two 4/0 Wires, 34 foot Spacing
Net Totals per 1000 Feet

Kv.	East Wire Net kw.	West Wire Net kw.	Sum Net kw. E. & W. Wires	Net kw. Two Wires 34′ Double Voltage between Wires
66.2	0	0	0	0
99.1	0	0	0	0.57
130.4	0.701	0.645	1.346	4.28
163	5.39	5.33	10.72	15.72
195.5	10.38	9.36	19.74	30.25
225	14.97	14.45	29.42	47.6
261	21.79	21.61	43.4	67.0
295	29.17	28.28	57.45	86.6
311	33.18	30.37	63.55	101.3

Typical Calculation of Capacity and Radius of "Equivalent Coronal Conductor"

4/0 Cable, 26 ft. Spacing, 391.5 Kv.
(Refer to Table X, Test III and Fig. 16)

$$C = \frac{I_c}{2\,\pi\,f\,E} \tag{1}$$

$$C = \frac{19.4 \times 10^{-9}}{\log_{10} \dfrac{S-R}{R}} \text{ (farads per mile, parallel conductors)} \tag{2}$$

$$I_c = I \sin\theta \text{ where } \cos\theta = \text{power factor} \tag{3}$$

From test data of Table X, $\cos\theta = 0.31$, whence $\sin\theta = 0.951$. Substituting values in (1) for gross capacity

$$C_{gross} = \frac{0.53 \times 0.951}{2\,\pi \times 60 \times 391{,}500} = 34.15 \times 10^{-10} \text{ farads}$$

C_{tare} (Feeders and Transformers)
C_{net} =

$$= \frac{6.97 \times 10^{-10} \text{ farads}}{27.18 \times 10^{-10} \text{ farads}}$$

This may be equated to (C) of Formula (2) thus:

$$27.18 \times 10^{-10} = \frac{19.4 \times 10^{-9}}{\log_{10} \dfrac{792.48 - R}{R}} \times \frac{1710}{5280}$$

whence $R = 3.83$ cm. (See Fig. 16).

Typical Calculation of Corona Loss by Peek's Formula

2/0 Cable, 20 ft. Spacing, 554 Kv.
(Refer to Table II, Test VI)

$$\text{Density Factor } (D) = \frac{3.92 \times B}{273 + T} \tag{1}$$

$$\text{Critical Voltage } (l_0) = g_0\, M_0\, r\, D\, 2.303 \log_{10} s/r \tag{2}$$

$$\text{Corona Loss } (P) = \frac{344\,f}{D} \sqrt{\frac{r}{s}}\,(e - e_0)^2\, 10^{-5}\, \frac{1.61 \times 2000}{5280} \tag{3}$$

Substituting values in (1)

$$D = \frac{3.92 \times 74.8}{273 + 14.5} = 1.02$$

$$e_0 = 21.1 \times 0.87 \times 0.566 \times 1.02 \times 2.303 \log_{10} \frac{610}{0.566} = 74 \text{ kv.} \tag{2}$$

$$P = \frac{344 \times 60}{1.02} \sqrt{\frac{0.566}{610}} \left(\frac{554}{2} - 74\right)^2 \times 10^{-5}\, \frac{1.61 \times 2000}{5280} = 155 \text{ kw.} \tag{3}$$

Discussion

F. W. Peek, Jr.: Prof. Harding has given us some curves up to 500,000 volts on conductors good for operation at 220 kv. or less. His loss measurements under some conditions show a close agreement with the quadratic law over a reasonable range of voltage for the conductors. The difference on several of the conductors is, I believe, due to the fact that sphere-gaps were used for voltage measurement. Regarding the divergence at the upper part of the curve where the arc-over voltage is approached, we have sometimes noticed a somewhat similar

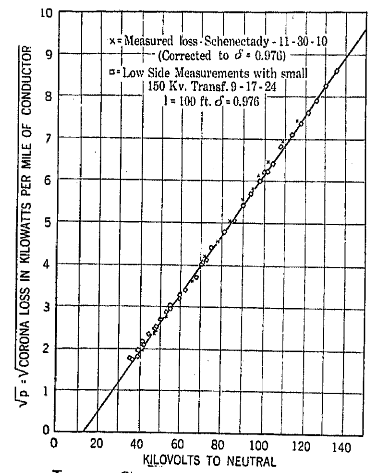

Fig. 1—Corona Loss Curve 0.066 Diameter Galvanized Iron Wire at 162 In. Spacing—60 Cycles

tendency on measurements that we have made up to one million volts. This has occurred on large conductors at spacing relatively small compared to diameter. It seems to be due to the distortion in the dielectric field that takes place because of the great amount of corona. The ratio of spacing to effective radius becomes small. The flexible corona conductor is distorted and no longer a cylinder. At the voltage when the divergence starts the corona begins to separate into huge cart

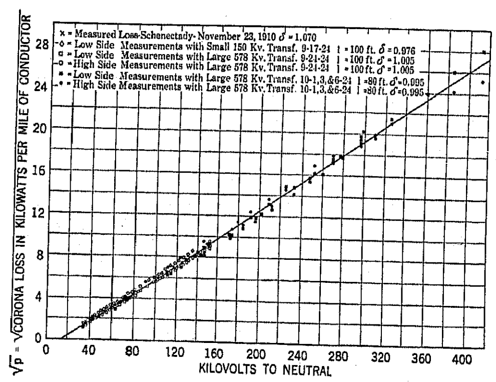

Fig. 2—Corona Loss Curve up to 800-Kv. Between Lines for 0.066 In. Diameter Galvanized Iron Wire at 162 In. Spacing 60 Cycles

wheels. These have a shielding effect preventing or lowering the loss between wheels. This does not mean that the quadratic law would not apply for calculation of the loss for million-volt conductors. Larger conductors and greater spacing would be used and the law would hold in the usual way. The tendency to diverge thus seems to apply only to extreme conditions or when spark-over is approached and distortion results.

We have never found the corona loss to vary, as Prof. Harding has, from the quadratic. Fig. 1 herewith is plotted from loss measurements made at Schenectady on a long outdoor line in 1910 and from measurements made in the Laboratory in Pittsfield on a short line in 1924. These measurements were made 14 years apart by different men with different apparatus, yet the check is exact. The curve is plotted between volts and the square root of the power because if a straight line results, it proves a quadratic. The range in voltage here is ten times the starting voltage. Measurements made on the same wire in 1924 up to over 1,000,000 volts or 60 times the critical voltage show that the quadratic law holds over this range. This seems to be a remarkably good confirmation of the quadratic law since the 1924 tests were made on a short line and check measurements were made on both high and low sides. The transformer-loss correction at 1,000,000 volts was rather large compared with the corona loss because of the short length of line. A wire with a small radius compared with the spacing was used to give a condition without distortion. Many curves similar to the above, all confirming the quadratic law, were made in 1910 up to about 250 kv.

Fig. 3 shows a similar curve for a cable like the one used by Prof. Harding. The tendency to diverge at about 500 kv. can be noted. The quadratic law is followed up to this point where distortion starts due to the large diameter of the corona. This distortion can be readily observed by visual measurements as noted above.

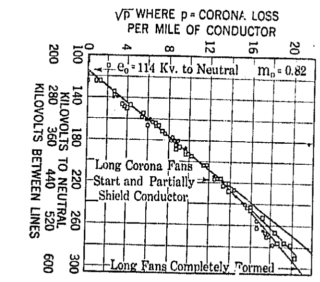

Fig. 3—Corona Loss Curve for 61 Strand, 600,000 Cir. Mil Cable at 15 ft. 9½ In. Spacing 60 Cycles, Single-Phase

$r = 0.445$ in.

$\delta - 0.964$

$p = 0.0166\,(e - 114)^2$ kw. per mile of wire.

I do not believe that Prof. Harding can be serious in his statement that 330 kv. could be used on a 500,000 cir. mil conductor. While Prof. Harding's tests are very interesting, the data cannot be applied to lines above 220 kv. because the tests were made on small conductors.

In comparing the loss between a single wire and ground with that of one of two wires, the spacing of the single wire to ground should be half the spacing of the two wires and the voltage of the single wire to ground should be half the voltage between the two wires. This would give equivalent stress.

L. P. Ferris: I should like to ask Mr. Harding whether he has taken any account of the change in wave form, which takes place with corona, in calculating the "radius of the equivalent coronal conductor?" I judge not, from the discussion on the sixth and seventh pages of his paper and the formula given in the appendix. Apparently the capacity is assumed to be directly proportional to the charging current per volt, which, of course, it would be if the wave form remained constant. It is well known that corona introduces harmonics as shown in the papers by Prof. Ryan and Mr. Wilkins. The question then naturally arises, how much of the increase in charging current can be accounted for by the increased admittance at the harmonic frequencies of a capacity fixed by the dimensions of the wires themselves and their spacings? There is apparently no doubt

that corona is accompanied by a cyclic variation of leakage or capacity or both, but it would be very desirable to have a definite picture of the mechanism of corona and its resulting distortion of wave form. We should then be able to agree on whether or not the capacity is truly increased, and if so, to what extent.

J. S. Carroll: The extraordinary increase in capacitance that Prof. Harding found has been somewhat difficult for me to understand. In some of our tests at Stanford the charging currents were about 10 per cent more than those calculated from the dimensions of the line and the line voltage as given by the voltage coil on the transformer. This bothered me for some-time until I got hold of the oscillogram of the voltage wave on the line. The oscillogram was taken from the Hendrick's coil

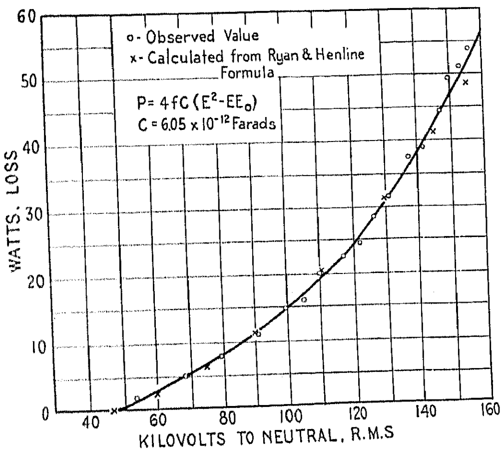

FIG. 4—SINGLE BRUSH LOSS FROM A POINT EXTENDING 3 IN. FORM A 3 IN. SPHERE

of the transformer. After the wave had been analyzed into its harmonics and these used in computing the line charging currents they were found to be within about 2 per cent of the observed values. The charging currents in the results of Prof. Harding seem to run rather high and the question of wave form, of course, has been previously brought up. These results of Prof. Harding are rather interesting from the standpoint of theory. I think there isn't much doubt that he is well above the economic range of operation, but it is interesting to know the power losses in this region.

I made some computations, using the corona-hysteresis formula that has been mentioned here, and this formula, by the way, fits the condition of the single brush as shown herewith in Fig. 4.

In these cases, until we known more about the conditions, we have to work backward, so to speak, with the formula. It would be a difficult problem to compute the exact capacitances of the arrangement we have there, so I took two points on the observed curve to determine the two unknowns, which would be the critical voltage and capacitance. These values were substituted in the formula and the power loss was calculated for various voltages with the results as shown in the curve of Fig. 4. It is interesting to note that the capacitance of this brush remained constant over a wide range of voltage.

Now, in Prof. Harding's tests. we have a line which is unquestionably being fairly well in corona, but there is some question as to whether it is in full corona. In Mr. Peek's pictures lines about this size are shown at a potential of a million volts and there are still some portions of the line that can be seen, apparently shielded by the brushes. In the use of the hysteresis formula with Prof. Harding's results we used the calculated capacitance as obtained from the dimensions of the line but the calculated values of power loss did not exactly agree with the observed values. However, when this calculated capacitance was multiplied by a constant the calculated and observed results were very close. These were only three curves out of the several of Prof. Harding's. With others tried, some fit as closely as these, others not quite so well. (See Fig. 5).

Now, this capacitance term we must think of in a little different way from the ordinary when applied to this formula. As used here it *means that capacitance which is attached to corona and not the capacitance of the conductor computed from its dimensions.* If the conductor is in full corona in every sense of the term then the field attached to the corona will be the same as that which would be attached to the conductor if no corona were present. In case of the brush formation on the conductor, if those brushes do not cover the whole field then only part of the capacitance of the line as computed from the dimensions should come in use on this formula. Of course, these curves run right into the x-axis at the critical voltage. That is because Prof. Harding's results begin well above the initial brush formation. Of course, the extreme lower part of these curves is not correct as shown by results in which lower-range instruments were used. At the early brush formation, we have a few brushes here and there, in

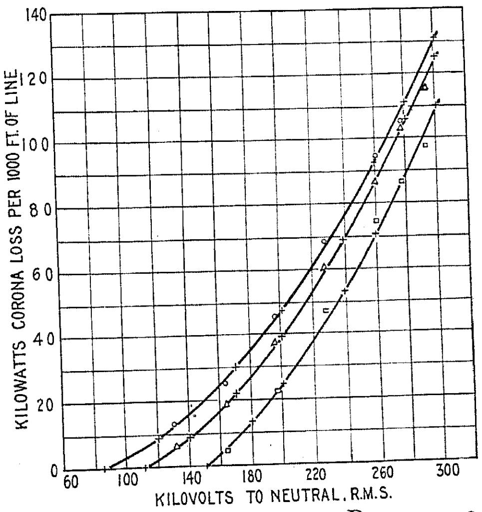

FIG. 5—CURVES OF BRUSH PATTERN IN RELATION TO C AND E_o IN EQUATION

$P = 4fc\,(E^2 - E_o\,E)$, Ryan and Henline

Using data given by Prof. C. F. Harding in his Pasadena Paper.

Curve	⊙	△	⊡	X
Conductor	2/0	4/0	500,000 cir mils.	Calculated from Henline Formula Using $K\,C$ instead of C
Diameter	0.447 in.	0.564 in.	0.904 in.	
Spacing	18 ft.	18 ft.	18 ft.	
Capacity M. F. per 1000 ft.	0.00246	0.00255	0.00275	
K	0.85	0.90	0.93	

some cases Mr. Wilkins says they are something like 100 ft. apart. It is the capacitance attached to these brushes that we want to consider, and not the full capacitance of the line, when this formula is used. The lower voltages used by Prof. Harding were the higher voltages used in most of the tests made at Stanford. What we are interested in is to find information so that we can predetermine the losses in the economic range. I have done some calculating along this line but the more calculating I do the more I see the need of experimental data and the tests actually made on the lines are extremely valuable in these respects.

Just one other word in regard to this capacitance. In the laboratory tests inside where we fixed up the rods with points

on them, and water drops just where we wanted them—in other words, where the brush formation was fixed, the corona-hysteresis formula fits the results exactly, just as it did in the case of the single brush, but when we come up to the order of a polished rod where the brush formation is not fixed, then we have trouble. I might say that in one case I tried a mathematical experiment similar to what Mr. Peek did, splitting the curve up into two component parts, you might say, with two different values of capacitance and two different critical voltages. I assumed we had a certain set of brushes at a certain critical voltage and that would have, of course, a certain capacitance; then, later on there was another set of brushes starting in with a higher critical voltage and an added capacitance. It came out very well.

C. Francis Harding: In closing the discussion of this paper, to which contributions of interest have been made by Messrs. F. W. Peek, Jr., L. P. Ferris and J. S. Carroll, it seems necessary again to emphasize the fact that the corona losses presented in the paper are actual, accurate results of tests under standard conditions of line construction and not extrapolations from empirical laboratory data. It is recognized by the author, as stated in his paper, that changes in both the design and construction of transmission lines for voltages above 220 kv. will be found necessary in order to avoid excessive corona losses, but the most practical way to determine the obstacles to be avoided in such designs is to determine the actual losses and other unknown phenomena resulting from higher voltages impressed upon present standard line construction.

The "difference on several conductors . . . . due to the fact that sphere-gaps were used for voltage measurement" suggested by Mr. Peek for the variation between values presented in the paper from theoretical calculated results has not been explained sufficiently by reference to the use of the sphere-gap, since the latter is recognized as the standard method of voltage measurement.

That the divergence between test and calculated corona losses, at the extra high voltages, may be due to distortion of the dielectric field, as suggested by Mr. Peek, is quite probable. This cause was mentioned on page 937 of the paper when, in reference to Figs. 14 and 15, it was found that the capacitance and diameter of the "equivalent coronal conductor" were not only unexpectedly large at very high voltages, but were erratic in behaviour at critical voltages and spacings. It is significant that these results have been confirmed by Mr. Peek in Fig. 3 within the range of 440 and 600 kv. between lines, although the size and spacing of conductors have not been given. The results submitted by Mr. Peek in Fig. 1 of his discussion represent values at much lower voltages, with no indication of the sizes or spacings of wires tested. The losses of Fig. 2, which are not referred to directly in his discussion and for which no cable sizes or spacings are listed, apparently indicate a considerable departure from the straight line determining the quadratic law if the curve be drawn through the points indicated as "High Side Measurements." The "Low-side Measurements" of net losses are necessarily made less accurately, although the methods of calculation and the conditions of test adopted are not indicated in his discussion. The conclusion seems to be warranted therefore, that between 440 and 600 kv. the divergence of such net losses from the calculated results may be considerable, if the conditions of line construction are such as to permit the formation of considerable corona.

With regard to the use of 500,000 cir. mil. conductor at 330 kv., no statement in the paper will be found to the effect that "330 kv. could be used on a 500,000 cir. mil. conductor" as incorrectly quoted by Mr. Peek. Losses between *two such cables* at spacings of from 30 to 34 ft. are shown in Fig. 8 of the paper to be "moderate" as compared with smaller cables and higher voltages, but the determination from test values of a loss of 107.0 kw. per mile of three-phase, three-conductor circuit of 500,000 cir. mil. cables at twenty-six foot spacing is stated on page 1185 to "*obviously be excessive and would, therefore, involve a change of design for a line to operate efficiently at such a voltage.*" Although it is probable that larger conductors will be used for voltages above 220 kv. when such lines are constructed, the possibility of increasing spacings between wires and the reduction of clearances to ground in some sparsely settled districts may be found to be a more economical means of reducing corona losses to an efficient minimum than by providing all of the adjustment by means of increased cable sizes.

No high-voltage transmission lines at present maintain the clearances between cables and ground equal to one half the spacing between the cables. It is recognized, as pointed out by Mr. Peek, that such an arrangement would provide equivalent electrical stresses. The test results of Figs. 17 and 18 were inserted in the paper in order to illustrate the saving in corona losses which may be found possible in the future by increasing spacings between cables by installing each cable or two cables of the same potential of a multiple circuit line upon individual relatively low towers, installed transversely upon the right-of-way at relatively wide spacings as compared with the height of such cables above ground.

In reply to Mr. Ferris concerning wave-form, it may be said that the calculations of capacity and the resultant "radius of the equivalent corona conductor" were made upon the basis of the sine wave having a maximum voltage established by the sphere-gap calibration. Although there is no doubt that harmonics are introduced by the corona formation upon such a line, particularly under partial and variable corona conditions, yet previous results reported to the Institute[1] by the author upon another experimental transmission line were determined from the oscillograms of both voltage and current, which seemed to indicate only very slight wave distortion at the higher power factors resulting from the very high voltages and consequent large corona losses. However, some of the seemingly erratic capacity variations at critical voltages noted in Fig. 14 may possibly be accounted for later upon the theory of distorted wave form. It is hoped that further studies may be made upon the effects of wave form upon this line.

The conclusion expressed by Mr. Carroll confirming the "need of experimental data and the tests actually made on the lines" is very pertinent, as it is believed that only the results of actual trials of extra high voltages upon lines duplicating future operating conditions with proposed new methods and designs for corona loss reduction will prove conclusively what the most economical, efficient, safe and entirely satisfactory line construction is to be for 330 kv. and above.

1. See footnote appended at bottom of page 1182.

Lightning

Does Lightning Oscillate? As a First Approximation, What is the Voltage, Current, Resistance, Energy, Power, Damping, Potential Gradient, and Rate-of-Change of Current?

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Review of the Subject.—*By the use of some new experimental data of the resistance of conducting vapors the conclusion is reached that the current in a streak of lightning oscillates. Starting with experimental value of 4500 volts per centimeter, as the average potential gradient for the electrostatic field between the thunder-cloud and earth, some astonishingly high values for the factors involved in lightning resulted. For more than a decade the average current in a lightning stroke of 10,000 amperes has been accepted. The calculations in this paper show a value as great as one-and-a-half-million amperes. Some of the other factors correspondingly large are as follows: The energy stored in the electrostatic field is 700 kilowatt-hours. The maximum power expended in the discharge is 860-billion kilowatts. The frequency for the particular stroke calculated, one mile long, is lower than former estimates,—about 50,000 cycles per second.*

Calculations were made of a bolt of lightning which struck a wooden pole protected by a No. 6 wire. The results agree with the independent calculations of the factors given above of a lightning stroke.

In a later paper the subject will be pursued further, and the equation will be recorded.

CONTENTS

THE object of this paper is to present the principal data which indicate strongly that the usual strokes of streak-lightning are oscillatory.

Significance of Oscillations

A pertinent question would be; What if lightning does oscillate? What significance has it? A few answeres will be given.

1. To the electrical engineer the protection of electrical apparatus and continuity of service are of prime importance. If lightning oscillates with a definite frequency a particular type of protective device must be used to protect against resonance. If the natural frequency of a coil corresponds to the lightning frequency the total energy of a traveling wave of electricity produced by lightning will concentrate by resonance in the coil. This local concentration of the electric charge endangers the insulation between turns of the coil.

The oscillatory type of lightning surge has an effect different from the simple high-voltage impulses which require only a lightning arrester to relieve the immediate potential stress. Furthermore, the oscillating surge differs from the abnormal currents which demand circuit breaker protection. A surge with a frequency may not possess either dangerously high voltages, which will jump through the path of the lightning arrester to ground, nor dangerous values of current, yet through resonance it is a menace to insulation.

2. Still more important is the protection of persons from electric shock, especially in a house struck by lightning. The presence of oscillations adds to the difficulty of protection.

3. Oscillations enter also into the protection of chimneys, powder magazines, oil tanks, and the like, the destruction of which causes at least a heavy financial loss.

4. To establish, in general, an understanding of the fundamentals of lightning theory and action,—in fact, to understand why lightning should strike from cloud to earth, it is necessary to know if the conditions are such as to permit oscillations. If experimental data and mathematical calculations combined with scientific speculations should prove that conditions of oscillation must exist before lightning can strike from the usual thunder-cloud to earth, then conversely oscillations are established because lightning does strike from the usual thunder-cloud to earth, as many can testify.

Where Lightning is Not Oscillatory

Oscillations are not generally accepted. The information comes to me from E. F. W. Alexanderson that

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radio-engineers have had experiences with noises coming from "static" (an inclusive term) which lead them to conclude from their observations that the effects of lightning are consistent either with unidirectional current or otherwise, if oscillatory, with oscillations of no fixed frequency. Several eminent men of science who have taken an interest in lightning phenomena have assumed and measured direct currents induced by thunder-clouds. Furthermore, their measurements show changes of induced potential remarkably slow. De Blois[1] made measurements of induced current ten years ago, using an antenna and an ordinary bifilar oscillograph. C. T. R. Wilson[2] in making calculations of lightning current from his experimental measurements assumes a single unidirectional discharge. H. Norinder[3] made measurements of induced voltages due to thunder-clouds and again the changes of voltage were comparatively slow, even less than in a 60-cycle circuit and the current measured was unidirectional. Long before any of the foregoing observations were made Elihu Thomson, from experiments and observations, had concluded that lightning was non-oscillatory. If I understand correctly the part of the thunder-cloud considered by these eminent investigators, I am in agreement,—there are no oscillations. Later an endeavor will be made to show that the measurements made by De Blois, Wilson, and Norinder were potential changes due to the outlying parts of the thunder-cloud and not to the main streak of lightning. The large brilliant streak is the only part of the discharge with an electric resistance low enough to allow oscillations. The situation will be clarified by the statement that no proof is offered herein to cast doubt on the measurements made by these investigators. So far, none of their measurements was made near enough the main streak of lightning or otherwise with suitable instruments to record the sudden changes and high frequency oscillations of the streak itself.

Furthermore, be it understood that calculations of lightning current and oscillations, herein given, are not yet definite matters in the realm of engineering. Step by step during the past score of years a closer approach has been made toward an accurate understanding. There is yet much work to be done but the time seems opportune to record several factors which approach an engineering nature.

Scope of This Paper

Herein the results of new experimental data are presented. Also a new method of approaching the solution of the problem is given. It is not possible to present briefly the results and at the same time give mathematical justification. The merest statements of the results become voluminous without including the proofs. Therefore, it is planned to present the subject matter in readable form and, later, after the completion of the theory of formation of lightning storms, the conditions will be more propitious to show the formulas involved, to discuss their applicabilities and limitations, and state definitely the assumptions made. At present, then, comes the readable story.

However, in the lack of detailed calculations in this paper, an explanation is due those who "shake a right smart mathematical pencil." The particular group of well-established equations chosen for calculating numerical values includes those developed for "distributed" inductance and capacitance. Numerical values are mostly rounded off. Approximations are quite consistent with the knowledge of lightning and also with this first step in the presentation of the subject.

Standardized Electrostatic Field of Force

Initially there is needed a standard thunder-cloud. But pictures of thunder-clouds must first be developed before it is possible to choose one for a standard. The

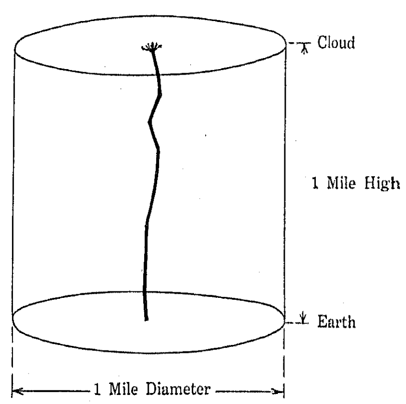

Fig. 1—Choice of Size of an Electrostatic Field of Force Due to a Thunder Cloud as a Basis of Calculation and Reference,—Designated in this Paper as Standard Field.

term "standard" in this case does not signify that every thunder-cloud has to be equal to the standard, any more than that any distance on the earth has to be equal to the standard yard, deposited in the archives at Washington. Like the standard yard, the standard thunder-cloud is a matter of reference. The standard thunder-cloud is later to be built out of the static field of force.

In the English system the standard field of force of a thunder-cloud is to be cylindrical in form one mile in diameter and about one mile high, Fig. 1. The lower surface of the field is the surface of the earth where one electric charge may be definitely located. The upper surface of the static field is that section of the thunder-cloud which may be considered the opposite plate of a condenser. But the cloud charge is not located on a definite surface like the earth. The total charge is made up of tiny charges on small drops of water distributed throughout the body of the cloud. Thus the visible thunder-cloud which produces the standard

1. Some Investigations on Lightning Protection for Buildings. Trans. A. I. E. E., Vol. XXXIII, Pt. I, p. 519, 1914.
2. *Phil. Trans. of the Royal Society*, 1920-21.
3. *Electrical World*, February 2, 1924.

static field must be placed, at present, somewhat indefinitely about a mile above the surface of the earth.

Why choose these dimensions? Clouds do not form much above an altitude of three miles and thunder-clouds are usually much lower. Perhaps, as a guess, a kilometer is closer to the usual height of thunder-clouds. The horizontal dimension of the part of a thunder-cloud which discharges in a single stroke is estimated between a quarter of a mile and—very indefinitely—three miles. There may be larger charged volumes than the standard static field, yet it may be conceded that for the present a mile in diameter is large enough to consider,—the values of the calculated factors are sufficiently terrific.

It is well to prepare the mind for a later metamorphosis of this static field. According to the theory to be built up, the energy of the static field may not enter directly into the streak of lightning. Indeed, the electrostatic energy of this initial field of force may be turned entirely into electromagnetic energy before it is finally used up in the vivid hot streak which connects the cloud and earth. In brief, this static field is not the dynamic lightning stroke. The static field is merely a starting point.

The Capacitance of the Standard Static Field of a Cloud. The capacitance is equal to that of a condenser built of ordinary window-glass 1/8 in. thick and about 2.5 ft. square (0.3 cm. thick and 80 cm. square). Numerically the capacitance is about eleven thousandths microfarad (11.18×10^{-9} farad).

The Maximum Voltage of the Thunder-Cloud to Earth. According to the line of reasoning which follows, a thunder-cloud a mile above the earth and about to start a lightning stroke cannot have more potential above the earth than 700-million volts. The voltage of the thunder-cloud relative to the earth may be less than 700-million volts and still discharge to earth,—how much less depends upon the type of thunder-cloud and the soundness of the theories herein presented, of oscillations and of low damping resistance in the path of the discharge. It is not possible to discuss intelligently the value of the minimum potential to discharge to earth until definite pictures of the process of formation of the path of discharge are given. At present the discussion will be confined to the probable maximum value of lightning potential.

Establishment of the Probable Voltage Gradient of Thunder-Clouds

Theoretical assumptions of the main factors involved in a lightning stroke are not necessary. The discharge through the clear atmosphere is apparently like the high-voltage discharges in the atmosphere of a laboratory. After the average voltage gradient is determined, volts per centimeter or volts per foot from earth to cloud, it is only necessary to determine the approximate height of the cloud to calculate its voltage. The extremes of the possible voltage gradient will first be discussed. It is certain that the average voltage gradient from cloud to earth is not equal to the corona value of 24,000 volts per centimeter at an altitude of one mile (161,000 cm.). There can be no doubt that this critical voltage, which causes a visible discharge of electricity through the air, must exist at the point in the cloud where the lightning discharge is initiated. A visible corona must appear somewhat like a headlight to the discharge as it strikes forward. But, so far, as already indicated, no such high-voltage gradient near the earth has ever been measured.

A. B. Hendricks, in testing his two-million-volt transformer outfit, obtains an average voltage-gradient between pointed electrodes of 5000 volts per centimeter (1/6 the corona value at sea-level). The distance between sharp points was 426 centimeters (14 feet). The applied voltage was 2,120,000 volts peak value. Dividing the voltage by the spark distance gives approximately 5000 volts per centimeter for the voltage-gradient.

The voltages of spark-over of a needle gap at values up to 160,000 volts are the same for direct current as for the peak value of 60-cycle alternating current. So far as these data go, therefore, the 5000 volts per centimeter at the surface of the earth, as the average gradient, is fairly acceptable.

At the lower surface of the storm-cloud, at a mile elevation, the conditions are different in several respects from those at the surface of the earth. Not only are there charged raindrops, humidity at the dew-point, but also the elevation itself will change the potential gradient. The test of spark-over voltage between sharp points in normal, dry air would lower the potential gradient from 5000 volts per centimeter average at the Pittsfield elevation to 4290 volts per centimeter average at one mile elevation. A higher humidity at the cloud might raise the gradient above 4290 volts per centimeter. The presence of charged raindrops free to move under the electrostatic force would have an unknown effect. Furthermore, if there is a local discharge at the surface of the cloud the conditions are no longer static. There may be a resultant oscillation or at least an impulse. The possible significance of a local discharge in the cloud is the production of ultra-violet light which ionizes the air in the neighborhood, making it more conducting. Also if there is a local oscillation, the spark voltage in the neighborhood is lowered as so well pictured by President Ryan in his May address in New York. These factors of humidity, charged raindrops, local discharges, ultra-violet light produce uncertainties in the average voltage gradient in the cloud, which would be disturbing if it were necessary to get an exact value. But it is not. Some of the factors of lightning are not known and it will be shown in this theory that one factor is incorrectly estimated by over 10,000 per cent. Therefore, it is quibbling to hestitate at a guess which apparently involves at most only a relatively few per cent error.

H. Norinder estimated a voltage gradient near the earth as great as 4000 volts per centimeter. It must be evident if the voltage gradient near the earth was 4000 volts per centimeter, and we picture the cloud ready to begin a brush discharge, that is to say about 24,000 volts per centimeter at the lower surface of the cloud, the *average* potential gradient between cloud and earth must have been greater than 4000 volts by some unknown value. Having now in a rough way found the probable values in the neighborhood of 4000 and 5000 volts per centimeter, an intermediate value of 4500 volts per centimeter will be arbitrarily chosen as a probable maximum for a tentative working value.

An average voltage gradient of 4500 volts per centimeter and a height of one mile (161,000 centimeters) give the maximum possible voltage of the standard static field, stated above, of 700-million volts.

The Maximum Voltage Induced by Lightning On a Transmission Line

Since the potential gradient near the surface of the earth must be less than the average potential gradient to the cloud, the maximum potential that can be *induced* on transmission wires must be less than the product of the maximum potential gradient of 4500 volts per centimeter and the height of the line expressed in centimeters. For transmission wires strung at 33 to 44 feet above the surface of the earth the maximum possible induced potential from lightning will be five- to six-million volts.

To avoid a possible misconception, distinction may be pointed out between induced charges and the instantaneous voltage of a bolt of lightning. The voltage just in front of the lightning bolt as it bores its way through the atmosphere to earth is approximately 30,000 volts for every centimeter. By this line of reasoning, a power line that is struck by lightning may possibly reach a voltage instantaneously of six times the maximum induced value, numerically 30-million to 36-million volts,—but not more.

Returning again to speculations on the induced values of potential on a line, there is a special significance in the possibility of reaching five- to six-million volts. Will transmission circuits ever reach a high enough insulation to prevent all accidental short-circuit arcs around insulators? It has been frequently observed by transmission engineers that each step in raising the power voltage of the line so improved the insulation of the line that fewer lightning discharges cause flashovers on the insulators. Speculations have been made regarding the possible number of flashovers on the new 220,000-volt line with one-and-a-half-million volts (impulse) arc-over of the insulators. What would take place if such lines had been constructed in lightning-infested districts? If the maximum induced voltage of six-million volts is established, then as frequently as it happens every transmission circuit subjected to this maximum will arc-over the insulators momentarily at least.

The energy stored in the electrostatic field is about 700 kilowatt-hours which, if it could be gathered for use, would, at 10 cents, bring \$70. In less practical units there are 3×10^9 joules.

Power of a Discharge

The power expended by the discharge of a standard electrostatic field, although it is only a part of the total energy of the thunder-cloud, is astounding,—860-billion kilowatts (860×10^{12} watts).

Contemplating this possible display of power indicates we must give up the habit of speaking of dynamic power in lightning arrester work as indicating generator power and not lightning power. There is more dynamic in lightning than there is in our so-called misnamed dynamic power.

Is it possible that such power exists? The use of a potential gradient of 4500 volts per centimeter substituted in established equations indicates it does. The value of 4500 volts per centimeter cannot be much in error because 4000 volts per centimeter has been measured.

R. A. Millikan uses heavy discharges in the laboratory to strip the outside electrons off the atoms of the first chemical group, leaving nothing but the helium nucleus. The proof of the condition of the atoms he finds in the spectrum. Lightning with its terrific power will do as much to the nitrogen atoms of the atmosphere. How much more will it do to the oxygen atoms? This speculation will be given a special significance later.

Lightning current of a standard electrostatic field may reach a million amperes r. m. s. or 1.45-million amperes peak value.

Lightning currents have been measured by the magnetization of pieces of basalt placed in the neighborhood of the bolt of lightning. In a few tests magnetic forces in the basalt were left which indicated currents as high as 20,000 amperes. Elsewhere, by assuming the duration of a lightning stroke to be 28 millionths of a second, Wilson was able to calculate as much as 140,000 amperes. With these figures in mind it was startling to calculate that the standard thunder-cloud would give a current of a million amperes. This high current is calculated on the assumption that the resistance in the path of the stroke is not sufficient to prevent oscillation.

What is the resistance of the path? How may we determine if the discharge is damped sufficiently to prevent oscillations? Measurements in our laboratory were made on the current of arcs ranging from 1000 amperes to 34,000 amperes. The resistivity of the vapor (.0084 ohm per centimeter cube) derived from these tests was used to calculate the size, that is to say the diameter, of a streak of lightning which would just give a resistance that would damp out oscillations.

The diameter of such a lightning discharge would be less than ½ inch (a centimeter in diameter). When a discharge reaches a diameter of a foot, as apparently some of them do, the resistance in the path of the discharge must be less than 1/1000 the critical damping resistance. Thereby the conclusion is reached that lightning is oscillatory. Properly taken photographs of lightning will give the data to make the foregoing conclusions more convincing.

The lowest frequency that the discharge streak of a standard electrostatic field may have is, in round numbers, 50,000 cycles per second for a discharge a mile long (more accurately, 43,600 cycles per second). There is a special significance to frequencies of this order of magnitude. The natural frequencies of many transformer coils are of this order of magnitude. Resonance between the impressed and natural frequencies means destruction of insulation.

There is no known reason why the standard lightning stroke cannot oscillate in segments and thereby exhibit a higher frequency.

It is well-known also that lightning discharges shorter than a mile take place which might give a higher fundamental frequency than 50,000 cycles per second. Discharges three miles long have been photographed.

Damping of the Oscillations. There are several factors of vital interest in the question, How is the lightning current damped? What sort of law does it follow from its peak current to its zero value?

In an early paragraph the statement was made that lightning currents of 20,000 amperes peak value had been measured by the magnetization of basalt. In a later paragraph the results of calculations gave a peak current in a standard lightning cloud of the order of magnitude of a million-and-a-half amperes. As matters stand the experimental measurement has a firmer basis for truth than the theoretical calculations. In other words, every theoretical calculation must square itself with experimental measurements. An analysis of this difference between 20,000 amperes (measured) and 1,450,000 amperes (calculated) will show that both values may be correct even in the same lightning stroke. In order to measure the magnetic force of a lightning stroke F. Pockels assumes necessarily that the discharge is non-oscillatory—that is to say, the discharge is a direct current which rises to a maximum value and immediately turns and decreases to zero. In order to bring these widely diverging values of 20,000 and 1,450,000 into harmony it is necessary only to prove that the million amperes is the first alternation of an oscillation which continues to oscillate, magnetizing and demagnetizing the basalt until the last oscillation is reached. Then it is necessary to muster the factors which show that the lightning stroke does not follow the usual gradual disappearance of a current that takes place in our familiar discharges of condensers through inductance and low resistance as measured in the laboratory. Fig. 2 shows this usual type of decrease of current of oscillation to a very low value. In the familiar laboratory circuit the resistance has a nearly constant value. To be sure, in the lightning stroke the resistance of the earth may, in some isolated cases, be high enough to predominate and cause quick damping of the oscillations, but where the earth conduction is fairly good, most of the damping occurs in the resistance of the lightning streak itself. It is well-known that the resistance of an arc increases rapidly as the current in the arc decreases. Therefore, the oscillations in a lightning discharge will decrease step

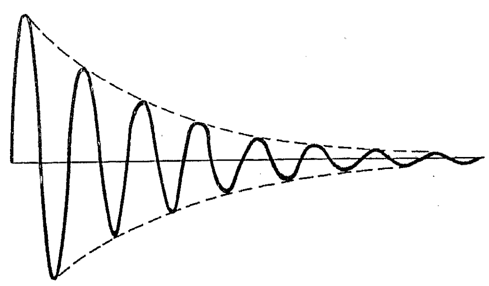

Fig. 2

by step in its peak value until the current arrives at some such value as 10,000 or 20,000 amperes, after which the resistance in the path of the stroke increases so rapidly that the oscillations are damped out and this last high value of current comes down to zero without overshooting in the other direction of current, as shown in Fig. 3.

If this theory is correct the measurements of current made by Pockels were of the final alternation of the lightning stroke. The point at issue at present simply

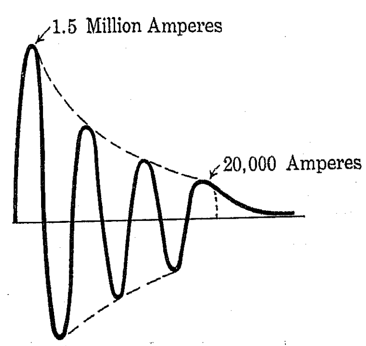

Fig. 3

is that the million ampere theory must square itself with incontrovertible experimental measurements. Has this not been made possible in the above explanation? If so, the truth of oscillations of lightning is a little more firmly established and we can pass on to the next step.

Time-Rate-of-Change of Lightning Current is of the Order of 400-Billion Amperes per Second

At the offset to clarify the position of the rate-of-change of current, it should be noted that we have

already dealt with an important factor, namely the rate-of-change of potential or potential gradient. But the potential gradient is a space change in the voltage, namely the voltage per centimeter of altitude in the atmosphere. There is another factor in the voltage change, namely the time-rate-of-change of voltage, especially in the unmade path of the advancing discharge. This factor of time-rate-of-change of voltage is pointed out at present for clarification purposes only. Later we may deal with the question of how long it takes a lightning charge to bore its way through the atmosphere from the cloud to the earth, or vice versa according to the authority. At present we are concerned with what takes place from the instant that the path is initially established in the hot streak to earth. The current must necessarily start from zero. It must rise according to some law, or combination of laws, to its peak value, possibly of the order of a million amperes. If the discharge is "standard" the current should rise to its peak value in about one two-hundred-thousandths of a second (.0000058 second). If, first, the tentative assumption is made that the current rises along a straight line from zero to one-million amperes, the rate-of-change of current will be two hundred and fifty billion (250×10^9) amperes per second (1,450,000 amperes divided by .0000058 second).

If, more correctly, it is assumed that the rise of current from zero to a peak value, instead of following a straight line, follows the more usual familiar sine curve, then the rate-of-change of current will be greater,—400-billion (400×10^9) amperes per second. This is the result for a cloud a mile high, a mile in diameter, and discharging at a peak current of 1.45-million amperes.

Experimental Confirmation of Rate-of-Change of Lightning Current

By an entirely different angle of approach, quite independently of the calculation on the standard lightning stroke, a reasonable check on the above results is obtained. In investigations of lightning on transmission lines my old associate, John A. Clay, Resident Manager and Engineer of the Western Colorado Power Co., at Durango, Colorado, has given me data of a direct stroke from which I can make tentative calculations of the rate-of-change of current. The details of this occurrence will be given in the A. I. E. E. Journal at some later date. At present I am privileged to use the data for calculations.

A wooden pole has an iron ring around its top and a No. 6 iron wire passes down the pole from the iron ring to the earth, ending in three widely spaced turns around the base. The pole was struck by lightning. In spite of the protection of the No. 6 wire in parallel with the wooden pole the potential induced along the pole was sufficient to cause a discharge along the wood in a parallel path to the protecting wire but on the opposite side of the pole. The discharge was severe enough to strip off the sap-wood for nearly the full length of the pole. Although the assumption may not be absolutely rigid, it seems reasonable to assume that the high voltage from the top to the bottom of the pole was due to a well-known law, namely the product of the inductance of the No. 6 wire on the pole and the rate-of-change of current. There are but three factors: voltage drop, inductance, and rate-of-change of current. The approximate inductance of the vertical straight wire can be obtained by simple mathematical calculations. We are seeking for a value of the rate-of-change of current, therefore we shall perforce have to assume how much voltage was required to ionize a path in the wood and establish a discharge which split and cast off the surface of the pole. In choosing this voltage a good deal of taste and discrimination may be used. However, the choice of voltage must be within a limited range set by the knowledge of spark potentials. Without attempting a thorough discussion, the first tentative assumptions will be two values: First, as the upper value, the corona voltage gradient not of 30,000 volts per centimeter but 24,000 (corrected for an altitude of 6000 feet); and second, as the lower limit, the average voltage gradient for a gap between points, 4000 volts per centimeter.

Assuming 24,000 volts per centimeter for a pole 33 feet (1000 cm.) high gives the absolute upper limit of rate-of-change of current as 960-billion (960×10^9) amperes per second. Incidentally the instantaneous voltage from top to bottom of the pole might have been 24-million volts for the maximum possible value, but it is far from probable that the maximum value was reached.

Of course, the splitting of the pole may take place gradually from end to end as might be done with an axe or other tool. In such a case it would seem that the electricity at the point of advancement has still a potential gradient of 24,000 volts per centimeter, but the average potential gradient from the top to the bottom of the pole may be something less. As a minimum it seems reasonable to assume the average potential gradient obtained by Hendricks for a fourteen-foot discharge between sharp points, namely 4000 volts per centimeter (corrected for altitude). Using the average voltage gradient at 4000 volts per centimeter, the rate-of-change of current in the direct stroke to the pole must have had at least the minimum value of 160-billion (160×10^9) amperes per second. Incidentally the absolute minimum voltage from top to bottom of the pole is now set at 4-million volts.

One of the joys of this speculative work is to attack an unknown factor from two independent standpoints and have the results agree. The gods occasionally supply such a gift; it is unseemly to look into the matter too critically. Yet perhaps some little devil of chance has falsely slipped the decimal point to make the two calculations agree. If later these results prove to be illusions it will only go to show that illusions in an engineer's experience are naturally short-lived.

Considerations of the Results

As matters stand, calculations show that the rate-of-change of current in the lightning stroke to the pole must have been between 160-billion and 960-billion amperes per second, according to which extreme of the potential gradient is chosen as able to split the pole. The previously calculated standard electrostatic field gave a rate-of-change intermediate, namely 400-billion amperes per second. This rate corresponds to an average gradient down the pole of 10,000 volts per centimeter. Thereby the total voltage from the pole-top to ground would have been 10-million volts (10,000 volts per cm. × 1000 cm. height). This remarkable agreement lends confidence to the earlier calculations of current of over a million amperes in the lightning stroke as reasonable and approximately correct.

The Colorado thunder-cloud cannot have forced on it the dimensions of a standard electrostatic field one mile in diameter and one mile high without engendering valid objections. The static field of that stroke was what it was and nobody knows. It may have been exactly equal to the adopted standard. The real point is that it could not have been greatly different in its equivalence.

No one saw the stroke on the pole. Even if witnessed, the dimensions and height of the cloud would remain unknown. Therefore it would be impossible to use these factors to calculate the current in the stroke and yet through the mathematical trick just employed it is learned that the current must have been of the order of magnitude of a million amperes.

The simplest illustration of this result is to show what the current could not have been. For example, let it be assumed that the lightning current in the stroke was 10,000 amperes, the value we were given and believed in, for the average lightning stroke, during more than a decade past. If this assumption is not true then its use in the equations will give unreasonable values. It does. The substitution of 10,000 amperes gives a frequency of the lightning stroke greater than six-million cycles per second,—a value quite impossible to accept. As a comparison, the frequency of 6⅓ million cycles corresponds to a quarter-wave length of line oscillations of 1200 centimeters,—a length equal to about the height of pole. This statement is only an analogy and does not prove anything. But the equation says very definitely that if the six-million cycles per second for the lightning stroke is not acceptable there is an alternative, namely to increase the diameter of the cloud. But to bring the frequency of oscillation of the lightning streak down to the value of the standard lightning stroke (43,600 cycles per second) and limit the current to 10,000 amperes would require an enormous cloud 146 miles in diameter—an absurd size for a thunder-cloud. Although the range of size of thunder-clouds is broad, nevertheless, there is a limit to the latitude allowed in the choice of the dimensions. As to altitude, clouds seldom form above three miles. Thunder-clouds are usually much lower. As to horizontal dimensions, the rain that falls during any brief time gives a rough estimate of the plan of the cloud. The usual thunder-cloud is comparatively small, a few miles across at most. Any result which requires a thunder-cloud of reasonable dimension is acceptable. To get reasonable results by the well-established equations demands a lightning current of the order of a million amperes.

Summary of the Factors

1. The maximum value that the *average voltage gradient* between cloud and earth may have is probably about 4500 volts per centimeter.
2. The maximum voltage of a thunder-cloud a mile high would therefore be about 700-million volts.
3. The current of a lightning stroke from a cloud a mile high has an r. m. s. value of about one-million amperes.
4. The lowest frequency of a lightning stroke a mile (161,000 cm. or 1.6 km.) long is about 50-thousand cycles per second. Strokes of greater or less length do not vary in frequency inversely as the length,—as in transmission circuits, but otherwise.
5. The energy in the electrostatic field is 700 kilowatt hours.
6. The power of the stroke is 860-billion kilowatts.
7. The rate-of-change of current in a lightning stroke is of the order of 400-billion (400×10^9) amperes per second.
8. The damping resistance decreases as the area of cross-section of the vivid streak increases. It is only the very thin discharges or discharges having high earth resistance that do not oscillate. If we can properly interpret our photographs of lightning strokes the diameter of the discharges in general is large enough to permit free oscillations. It will be left until later to produce data indicating that although lightning has an oscillation it may not have a fixed frequency even during the oscillations of a stroke.

Conclusions

The most pertinent questions are: What facts have been used and what assumptions have been made?

Some of the experimental facts used are:

1. The average potential gradient to produce a discharge in the normal atmosphere is about 4500 volts per centimeter. This gradient multiplied by the height of thunder-cloud gives the potential of the cloud.
2. The specific resistance (.008 ohm per centimeter cube) of arc vapors has been measured in the laboratory with currents up to 30,000 amperes.

Mathematical factors:

3. Very low specific resistance is proof positive that the resistance of the vivid streak is negligible initially and therefore that a simple equation may be

used (current = voltage of the cloud ÷ surge impedance. The surge impedance is $\sqrt{\frac{L}{C}}$).

The combination of these three factors,—average potential gradient, low specific resistance, and a simple well-established equation—gives a current in a lightning streak about a mile long of over a million amperes.

Assumptions involved:

1. That a streak of lightning is a straight conductor, the inductance of which may be calculated with close approximation by the established equations.
2. That the electrostatic field of the cloud is a capacitance that may be calculated with approximation by the formula for parallel plate condensers.
3. That the plate of the condenser as formed by the cloud has an equivalence at some depth in the cloud not known but a depth small as compared to the mile of height.

Since, so far, only a part of the thunder-cloud was considered, namely its lower surface, it may appear to the reader that the assumption was first made that the resistance was high in the path of the displacement current of the static field and subsequently that the resistance was low when the discharge gathered in the static field. Later it will be shown that as soon as the bolt strikes, the static field changes its location in the cloud from the lower surface of the cloud to the branches of the streaks in the cloud in which the electricity gathers.

As already indicated, it is not possible to bring this part of the speculative work to any great degree of completion in this paper, as it is dependent on the other parts to follow. Very considerable modifications will have to be made in the details of our standard thunder-cloud, as herein given, after an understanding of the formation of the lightning stroke is set forth. A complete understanding is not yet available in the records of human knowledge. It is only, however, by setting up pictures hypothetically of the various phenomena involved in the several kinds of lightning storms that rapid advance can be made. These theories are targets at which all observers can shoot. There is a rule to this game. Each step in the theory must square itself with the available experimental data and also with the preceding and following steps. At this point the matter must stand for the present.

Discussion

For discussion of this paper see page 1217.

Lightning and Other Transients on Transmission Lines

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Review of the Subject.—*A great many uncoordinated researches and observations have been made of lightning on transmission lines both in the field and in the laboratory. It is the purpose of this paper to coordinate this work with a view of determining the various types and order of magnitude of predatory voltages to which transmission lines are subject. No claim is made for exact results. If it were possible to obtain such results, they could apply only to specific cases. A knowledge of the general types of disturbances and their order of magnitude should be of great value in determining the best means of providing against them. Failures of apparatus can be provided against in a number of ways, for instance, by excessive insulation; by less insulation but a design to prevent local high stresses; by prevention of high lightning voltages by placing the line under ground; by greatly limiting the possible voltage by a ground wire; by limiting the time of application of the high voltage by arresters; or by combinations of the above. Obviously, the matter of design and economics is also of importance in considering the problem of protection.*

The technical part of the discussion will not be limited to cloud lightning, but will cover other disturbances.

Recent work of the author with his lightning generator and model transmission lines with and without ground wire will be included.

Voltage of a Lightning Flash

There has always been speculation as to the voltage of a lightning stroke, and various estimates have been made in more or less complicated or indirect ways. Recent laboratory tests have shown that up to about 2,000,000 volts maximum, 150,000 volts is required for every foot of spark.[1] If this rule held for great distances, the voltage of lightning could be calculated, since the length of the lightning flash as well as the height of cloud, etc., can be estimated. It had been generally concluded, however, that the voltage of the stroke was very much less than would be indicated by the length of the flash, or that when the flash-over started, it in some way continued to great distances at relatively low voltages.

In a lecture before the Franklin Institute the author described an experiment which seems to offer an almost direct means of measuring lightning voltages.[2] The means is quite simple. When a lightning flash occurs within a certain distance of a transmission line, a certain percentage of the voltage of the bolt is induced on the line. The voltage of the bolt cannot be measured, but its distance from the line and height of cloud can be estimated. The actual voltage induced on the line can be measured by gaps or estimated from insulator flashovers. The author has measured lightning voltages on transmission lines in Colorado as high as 500,000; insulator flashovers by lightning have occasionally indicated voltages as high as 1,500,000 or more.

A model was made to scale representing cloud and transmission line for a certain observed condition. By means of the lightning generator it was found that when a flash occurred from this model cloud 1 per cent of its voltage was induced on the model line. But we know by observation that the voltage induced on an actual line, under similar conditions, is sometimes of the order of 1,000,000. If this is 1 per cent of the voltage of an actual lightning flash, the voltage of the flash must be 100,000,000 volts. (See Fig. 1). This gives a voltage of 100,000 per every foot of spark, (330 kv/m.) which, considering the possible error, indicates that the needle gap spark curve may hold even at these extreme voltages. While the field produced by the charge is fairly uniform, it is probable that at the instant before spark-

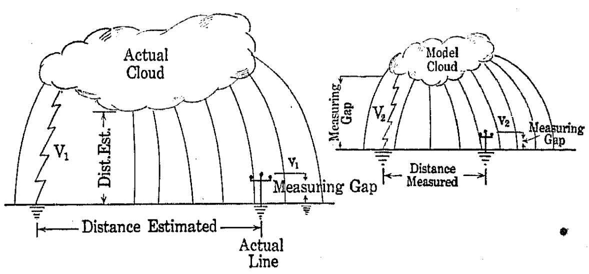


FIG. 1—MEASURING THE VOLTAGE OF LIGHTNING

Measured or Estimated Height of Cloud. Distance to and Length of Flash. Voltage Induced on Line. Lightning Voltage not Measured.

Lightning Voltage Determined from V_1 and Known Percentage Induced on Line from Model

Model Line to Scale from Actual Line. Lightning Voltage and Induced Voltage Measured. Therefore Per Cent of Lightning Voltage Induced on Line is Known.

$$\frac{v_1}{V_1} = \frac{v_2}{V_2} \qquad V_1 = \frac{v_1}{v_2} V_2$$

over a needle-like streamer forms and breakdown then corresponds to the needle gap distance. Needle gap spark-over requires less than 20 per cent of the 30 kv./cm. required for a uniform field. The sparking distance should usually correspond to a continuous voltage because there is generally no large transient until after the spark starts.

It thus appears by approximately direct measurement that the order of voltage of a severe lightning stroke to ground may be about 100,000,000. The lightning voltage during a storm will, of course, vary over a very

1. F. W. Peek, Jr., Tests at 1,000,000 Volts, *Electrical World* December 31, 1921.

2. F. W. Peek, Jr., High-Voltage Phenomena, *Journal* Franklin Institute, Jan. 1924.

Presented at the Pacific Coast Convention, Pasadena, Cal., October 13-17, 1924.

wide range, sometimes much higher but generally lower than the value above. The author has observed that during a severe thunderstorm there may be many induced strokes at very low voltages, a less number at moderate voltages and so on to very few at the extreme voltages.

It will be noted that the above conditions require a gradient of 100 kv./ft. (330 kv./m.) in the most dense part of the dielectric field where the flash occurs, and a

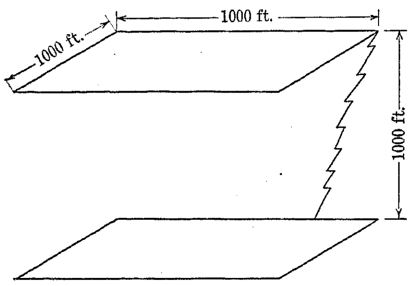


Fig. 2—"Cloud" Assumed for Calculation of Waves Shown in Fig. 3

gradient of less than half of this value a short distance from the flash. (See Fig. 4.)

Norinder has recently measured voltage gradients during a lightning storm which confirm the values given above.[3] To quote from his article in the *Electrical World*—"field intensities of the order of 100 kv. to 150 kv. per meter (30 to 45 kv./ft.) are common during thunderstorms at about the height of an ordinary transmission line. Pressures of 200 kv. per meter (60 kv./ft.) have also been recorded. A close study of the records shows that in the regions near the lightning path field intensities of the order of 300 kv. to 400 kv. per meter (90 to 120 per ft.) may exist." De Blois observed corona brushes of considerable dimensions. This also is an indication of gradients of the above order.

Nature of a Lightning Discharge

The experimental evidence seems to show that for the most part lightning discharges are impulses of very steep wave front, although some discharges are of impulses of slanting wave front and some are oscillatory. Oscillographic measurements made by De Blois on discharges from antennas showed that 60 per cent of the discharges were of steep wave front.[4] Similar studies recently made by Norinder indicate impulses of slanting wave front. However, it is not possible to tell from oscillographic records just how steep such waves are because of the relative slowness of the apparatus in responding. Other observations, however, indicate the order of the steepness of the wave front. When an impulse voltage is measured by a needle-gap and a sphere-gap, the sphere indicates approximately the true value of the voltage while the needle indicates a much lower value.[5] The ratio of these voltages indicates the steepness of the wave front. From observations on transmission lines the author has found impulses corresponding to single half cycle of 200-kv. sine waves and also impulses of very much greater steepness with an impulse ratio of two.

It is of interest to make certain assumptions as to size of cloud and length of discharge and calculate the wave to see if the above observations seem reasonable. Although such a check is quite rough, it should give good indications as to whether steep wave front impulses are likely to occur.

Assume that a cloud 1000 ft. (305 m.) square and 1000 ft. (305 m.) high, uniformly charged, discharges to earth. (See Fig. 2). The capacity of such a cloud is approximately 27×10^{-10} farads and the inductance of the path is 0.000488 henrys. If the resistance of the discharge path is 1000 ohms, the discharge is an impulse and non-oscillatory. The time is conveniently measured in microseconds. See Fig. 3, which is quite in agreement with observations. Fig. 3 also shows that the wave is practically an impulse with a resistance of 500 ohms for the discharge path, and that it is a damped oscillation for 100 ohms resistance. It is certain that

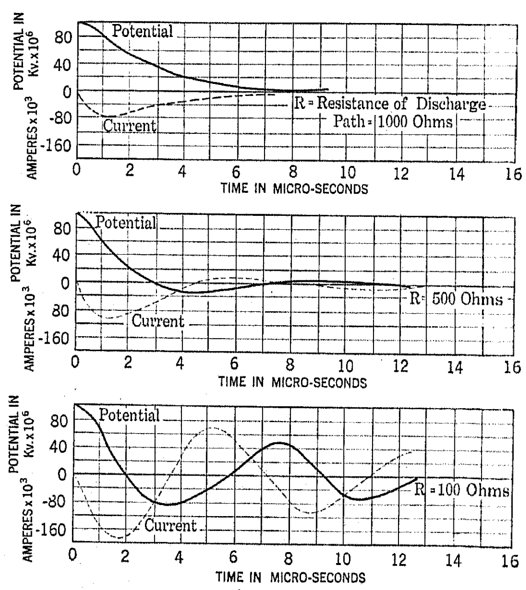


Fig. 3—Transients for "Cloud" Discharge to Earth (See Fig. 2)

$C = 27 \times 10^{-3}$ Microfarads
$L = 0.488 \times 10^{-3}$ Henrys

the cloud condenser is not as simple as the example taken and the probabilities are that there are local flashes in the cloud before the final flash occurs.

At 100,000,000 volts the maximum current would be 78,000 amperes. The stored energy would be 13,500 kw/sec.

3. H. Norinder, Electric Thunderstorm Field Researches, *Electrical World*, Feb. 2, 1924.

4. L. A. DeBlois, Some Investigations on Lightning Protection of Buildings, Trans. A. I. E. E., Vol. XXXIII, Pt. I, p. 519, 1914.

5. F. W. Peek, Jr., The Effect of Transient Voltages on Dielectrics, A. I. E. E. XXXIV, p. 1857, 1915; Lightning, *G. E. Review*, July 1916.

LIGHTNING ON TRANSMISSION LINES

Most lightning disturbances on transmission lines occur by electrostatic induction and not by direct stroke. A charged cloud causes an electrostatic field to earth. Part of the field will terminate on a transmission line within its area. The line is said to have a "bound charge." If the voltage between earth and cloud becomes high enough, a lightning flash will occur. Although this flash may be a mile away from the line, the charge on the line is released and the insulated line increases from earth potential to some value above with polarity opposite to that of the cloud. The effect is of a voltage suddenly applied between line and ground. The field that extended between line and cloud now extends between line and ground. The voltage wave travels over the line at the velocity of light. If the line insulators are strong enough or have a high enough impulse ratio, the impulse may travel to the powerhouse to break down apparatus or to be harmlessly discharged to ground over the arrester if it has low resistance and low impulse ratio. As this lightning wave travels over the line, it becomes gradually dissipated by losses. The voltage that the line assumes at the instant of discharge is that of the equipotential surface at the point in which the line is located. This is a certain percentage of the potential of the cloud above earth, or, in fact, a certain percentage of the voltage of the lightning bolt. In studies in Colorado, as already stated, the author has actually measured induced lightning voltages on transmission lines as high as 500,000 volts. Insulator flashovers have occurred that indicate induced voltages as high as 1,500,000 volts, although the greater percentage of voltages induced on transmission lines are very much lower than this. These figures as already shown afford a means of estimating the voltage of a lightning flash.

Lightning Voltage vs. Height of Line. Fig. 4 shows the lines of force from a charged cloud. Transmission lines of equal height are shown, one right under the cloud and others some distance away. It will be noted that for the line under the cloud the gradient is 100 kv/ft. (330 kv/m.) but that it is much less a short distance away. The gradient is high immediately under the cloud. It will also be noticed that the voltage for any line will vary approximately as the height of the line because the field over the relatively short distances is approximately uniform. No attempt is made, because of the scale, to show the flux terminating upon the line. When a discharge takes place from cloud to ground, the line takes the potential of the equipotential surface in which it is located. The induced lightning voltage on a transmission line thus varies approximately as the height of the line. It is also approximately equal to the height of the line times the voltage gradient. Note that the voltage decreases very rapidly with increasing distance from the cloud.

Maximum Possible Induced Lightning Voltage on a Transmission Line. It was shown above that the maximum possible voltage gradient during a lightning storm was about 100 kv./ft. (330 kv./m.). This gradient would determine the voltage induced on the line if it were directly under a cloud charged to a voltage sufficient to discharge it to earth or to the line. Generally this condition will exist only in cases of direct stroke and gradients for very severe storms will be of the order of 50 kv./ft. (165 kv./m.) while for the majority of storms, gradients will be much lower than this. Low gradients will also exist for cloud to cloud discharges, which are generally in the majority.

It is of interest to tabulate height of tower vs. theoretical maximum voltage at values for very severe storms. The voltages in Table I were found by multiplying the height of tower by the gradient caused by the cloud. In the second column, the maximum voltage was found by using a gradient of 100 kv/ft. As previously stated, this is the gradient necessary to cause the lightning flash, and can only apply to the line when it is directly under the storm center. The voltages in the third column were found by using a gradient of 50 kv/ft. Such a gradient occurs when the line is about 1000 ft. from the storm center. It is a severe condition and does not usually occur. In column 4, a gradient of 100 kv/ft. was used, while a gradient of 50 kv/ft. was used in columns 5 and 6. All these conditions are severe and unusual. The voltages are for the instant that the flash occurs, and before possible insulator arc-overs can take place. These figures are, of course, not exact, but probably give the order of the voltage that might occur on a badly exposed line in a very severe storm directly over the line. Data obtained in Colorado for 24 miles of a badly exposed line on a high mountain ridge showed only one or two direct strokes in a season of fifty severe storms. Of the many lightning impulses induced on the line very few exceeded 50 kv. during a storm.

Referring to Table I, note that the ground wire practically cuts the voltage values in half. The ground wire will be very completely considered in a later section. The lightning spark-over voltages for insulators and bushings are given in the last column.[6] It will be noted that a 220-kv. line is generally free from lightning voltages high enough to cause insulator flashovers.

Propagation of Lightning on Transmission Lines. A laboratory study was made of the propagation of lightning on transmission lines.[7] These tests show that the voltage decreases in value and becomes less steep as the lightning impulse travels along a line. On striking an end, it increases in voltage and steepness of wave front as it is reflected. When an impulse strikes an inductance or choke coil, part is reflected but the voltage on the far side may increase to several times the in-

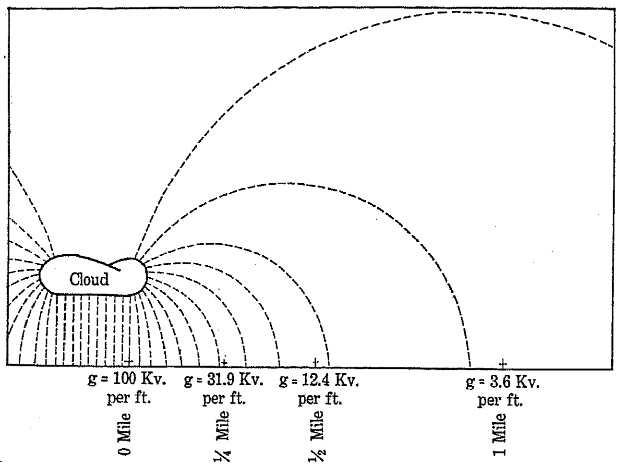
6. F. W. Peek, Jr. The Effect of Transient Voltages, A. I. E. E., Vol. XXXIV, p. 1857, The Insulation of High Voltage Transmission Lines, *G. E. Review*, Feb. 1922. High Voltage Power Transmission, A. S. C. E. Vol. LXXXVI, p. 725.

7. F. W. Peek, Jr., The Effect of Transient Voltages—III, TRANS. A. I. E. E., p. 940, June 1923.

TABLE I
LIGHTNING VOLTAGES ON TRANSMISSION LINES

Height of Tower Ft.	No Ground Wire		Ground Wire			Lightning Flash-over Voltage of Line		
	Theoretical Highest	Probably Usual Highest	Direct Stroke to Ground Wire (One Wire)	Usual Highest (One Ground Wire)	Usual Highest (Three Ground Wire)	200 Kv. Line	154 Kv. Line	110 Kv. Line
75	7500	3750	3750	1875	900	..	..	..
50	5000	2500	2500	1250	625	1800	..	..
40	4000	2000	2000	1000	500	1800	1400	900
30	3000	1500	1500	750	375	1800	1400	900
20	2000	1000	1000	500	250	..	..	..

For a grounded line, the voltage above ground just before the discharge would be zero. For an isolated system the line would not assume the above voltage before the discharge as there would be leakage over the insulators and arresters and much "static." There would also be a reduction of the static voltage by the section of the line not under the clouds.



Distance from point directly beneath cloud	g.	Voltage induced on a line twenty ft. high
0	100 kv. ft.	2000 kv.
0	3.19 "	638 "
½ "	12.4 "	248 "
1 "	3.6 "	72 "
2 "	0.9 "	18 "

Cloud 1,000 ft. above earth and at a potential of 100,000,000 volts.

FIG. 4—ELECTRIC FIELD AND POTENTIALS IN SPACE CAUSED BY CHARGED CLOUD

coming wave. This is not the case if the choke coil is shunted by resistance. A large choke coil without resistance may be a source of danger.

The corona loss in a line helps to lower the voltage of the wave. This was noted by measuring the voltage as the wave traveled along the line.

High Frequency and Oscillations Due to Secondary Discharge, Switching and Arcing Grounds. When an insulator is arced-over or a break occurs to ground, an oscillation takes place. In 1907 and 1908 the author conducted some tests in the mountains of Colorado for E. E. F. Creighton and J. A. Clay.[8] A 24-mile idle line was available. The lightning was permitted to discharge to ground through a large gap in series with a small auxiliary gap in a dark box. A rapidly revolving photographic film on a steel disk recorded the discharge. Many of these discharges were photographed. In all cases, following the first impulse, a very highly damped oscillation took place at the natural period of the line or about 1900 cycles. Fig. 5 gives an example of one of the many records taken. A few of the records are given in Table II.

TABLE II
DURATION AND FREQUENCY OF A LIGHTNING DISCHARGE FROM A TRANSMISSION LINE
("High Line" 24 mi. Long)

Film No.	Half Oscillations	Duration Seconds	Frequency	Gap Setting Voltage to Ground Kv.	Fuse Blown
1a	12	0.0031	1930	20	
1b	6	0.0018	1660	20	
2a	8.5	0.0022	1930	20	
2b	8.5	0.0025	1700	20	Multiple
	6.5	0.0017	1880		Stroke
2c	4.5	0.0013	1730	20	
2d	5	0.0016	1560	20	Multiple
	4.5	0.0014	1600		Stroke
2e	5	0.0017	1470	20	
2f	6	0.0017	1800	20	
2g	6.5	0.0019	1700	20	
3a	10	0.0031	1600	20	1 strand 0.0036
3b	9	0.0029	1550	20	Multiple
	6	0 0026	1500		stroke
4a	1	0.0002	2000	35	1 strand 0.0036
4b	1	0.0002	2000	35	
4c	1	0.0002	2000	35	
4d	1	0.0002	2000	35	
4e	1	0.0002	2000	35	
4f	1	0.0002	2000	35	
5a	8	0.0020	2000	20	
5b	7	0.0019	1840	20	
5c	11	0.0026	2100	20	1 strand 0.005 broken
5d	7.5	0.0020	1850	20	

The duration values given in Table II are a measure of the duration of the secondary oscillations and not of the lightning stroke. The "duration" measurements by De Blois are of the same order 0.006 seconds, while measurements by Norinder are of the order of 0.05 seconds. This instrument did not record the first impulse but the oscillation following the arc-over. Other observations indicate that in general there is the very steep impulse, followed, when breakdown occurs, by a highly damped oscillatory discharge of the line at its natural period. This oscillation is of comparatively low frequency.

8. E. E. F. Creighton, Measurements of Lightning, TRANS. A. I. E. E., Vol. XXVII, p. 669, 1908.

When there is an arcing ground, especially on a non-grounded delta system, oscillations occur. As far as can be observed, these oscillations are damped and have the effect of lightning impulses.[9] They rarely reach double voltage on a transmission line but may reach higher values when confined by inductance coils and may build up high local voltages in transformers. Although large numbers of tests have been made by producing arcs on power systems, undamped oscillations have not been observed. Fortunately, the severe oscillations produced by arcs on non-grounded systems are disappearing with the adoption of the grounded neutral system. With the grounded neutral systems the oscillations of arcing grounds are no longer a problem.

Disturbances due to switching are of the same nature as impulses or damped oscillations.

The reason that oscillations are highly damped seems apparent. Corona loss increases as the square of the excess voltage above the starting voltage and directly

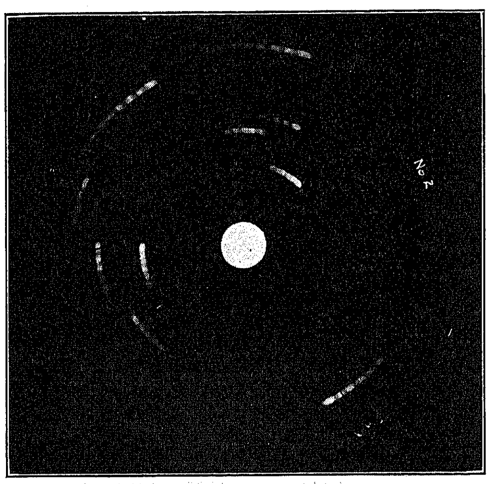


FIG. 5—LIGHTNING DISCHARGE FROM TRANSMISSION LINE PHOTOGRAPHED ON REVOLVING FILM

as the frequency. A simple calculation shows that it would require thousands of kilowatts at high frequency to supply the high-frequency loss on a comparatively short line at voltages considerably below the high-frequency flashover voltage of a line insulator. Furthermore, with arcs to ground or between lines the oscillations could not be isolated to short sections of a line. This also applies to damped oscillations of high train frequency.

High-frequency low-voltage undamped oscillations can exist and do exist locally on transmission lines. Such oscillations have been observed at towers where there were incipient arcs due to loose connections on the insulators or to faulty units in a string. The energy of such oscillations is necessarily low, since it is caused by the discharge of the very small capacity of the insulator. Probably the principal harm that these oscillations can cause is disturbances in carrier current systems.

A full size 220-kv. tower with insulators with and without grading rings was set up in the laboratory to try to obtain high-frequency voltages by arcs. The short section of line was excited at 60 cycles. To simulate a line, the high-voltage condensers from the impulse generator were inserted between the short line and ground. The dampening of such an arrangement should be much less than on a transmission line of similar capacity. In considering available energy it must be remembered that breakdown is caused by voltage. A large condenser with low losses in a short line can maintain the voltage more readily than the capacity of a long leaky line. Arcs made on the condensers failed to cause the insulators to arc-over. Loose contacts were made on insulator strings and varied to cause incipient arcs as well as arcs of considerable length. There was no measurable increase in voltage. A frequency meter showed a frequency of about four million cycles and very low voltage for local insulator arcs. Corona discharges five feet in diameter at a million volts failed to cause any measurable increase in voltage. Arc-over at normal voltage was readily caused by dirt.

TABLE III
ACTUAL DIMENSIONS OF MODEL LINES

Dimension of Cloud: 5 ft. by 7½ ft. (152 cm. by 229 cm.) Horizontal plate with rounded edges

Height of Cloud above Ground: 43½ in. (112 cm.) unless otherwise stated

Height of Line	Size of Conductor	Spacing
	Diameter	
12 in. (30.5 cm.)	0.040 in. (0.102 cm.)	3 in. (7.6 cm.)
6 in. (15.2 cm.)	0.020 in. (0.051 cm.)	1½ in. (3.8 cm.)
3 in. (7.6 cm.)	0.010 in. (0.025 cm.)	¾ in. (1.9 cm.)
1½ in. (3.9 cm.)	0.005 in. (0.012 cm.)	⅜ in. (0.9 cm.)

Ground Wire: Same size as line wire and, unless otherwise stated, conductor spacing from nearest conductor.

Lightning Generator Constants: Resistance 5000 ohms. Cap = 1.31×10^{-3} mf. $L = 2.88 \times 10^{-2}$ mh.

It is, of course, possible to apply persistent oscillations or damped oscillations of high train frequency to a short section of transmission line by means of a powerful oscillator. The effects of such voltages are quite characteristic and unlike anything that has been observed on a practical line. (See page 708). Insulators are not punctured and shattered, but cracked by excessive heating. At some sharp point on the line large corona losses cause great hot streamers. The ionization persists from cycle to cycle or train to train. If this point is removed or covered with porcelain, a similar arc starts at some other place where the gradient is high. These hot streamers or "electric needles" cause arc-over at about half of 60-cycle arc-over voltage.[10] To produce the thermal effect and to reduce the insulator

9. G. Faccioli, Electric Line Oscillations, TRANS. A. I. E. E., Vol. XXX, p. 337, 1911. W. W. Lewis, Switching Operations, *G. E. Review*, Oct. 1913. F. E. Terman, Measurement of Transients, TRANS. A. I. E. E., Vol. XLII, p. 462, 1923.

10. F. W. Peek, Jr., Dielectric Phenomena in High Voltage Engineering, McGraw-Hill.

flashover voltage, the oscillations must persist for a relatively long time. Although high-voltage persistent oscillations or oscillations of high train frequency do not occur on transmission lines, they are of interest in an academic way and make a spectacular demonstration.

The arc-over voltage is always higher for the types of oscillatory voltages that occur on transmission lines than it is for 60-cycle voltages.

Oscillation may cause high local internal voltages to build up in transformers.[11]

The Ground Wire

Quite an extensive laboratory study has been made on the protective value of the ground wire. This investigation was made on a model in which size of conductor, conductor spacing, height of line, etc., were all reduced to scale.[12] The results show quite conclusively that under favorable conditions the ground wire offers a very high degree of protection for both induced and direct strokes. Under such conditions and practical conductor arrangements it is possible to reduce induced lightning voltages to one-third of the value of those induced on unprotected lines. Under these conditions the ground wire also offers almost complete protection against direct strokes. When it is possible to obtain the value of protection mentioned above, the higher voltage lines will be practically immune from insulator flashovers due to lightning.

Some very interesting theoretical studies of the ground wire have been made.[13] Methods of making the calculations are also found in text books.[14] In general, the experimental work checks the theoretical work. There have been many conflicting reports from operating companies as to the protective value of the ground wire. Probably approximately half of the reports are in favor and half against the ground wire. The reason for this seems to be apparent from this investigation. The ground wire gives the high value of protection mentioned if the ground is good and the reactance of the ground connection is low. In a dry country, with poor grounds, its protective value against induced strokes would be low. On the other hand, in a damp country its protective value should be high. Its protective value would be low if grounds were made infrequently, since there would then be a considerable length of wire or reactance between the ground wire and the ground connection.

Complete data are given in the tables for the various factors affecting the ground wire. These will be discussed in detail.

Induced Strokes. The arrangement for studying induced strokes on transmission lines is shown in Fig. 6. The plate represents the position of the cloud which causes a steady electrostatic field to ground in the vicinity of the transmission line. The complete cloud includes the condenser of the lightning generator. This field is in reality a 60-cycle a-c. field but at the instant

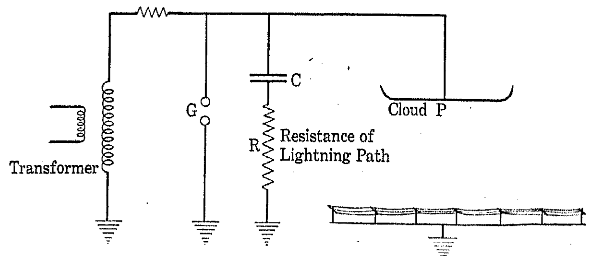


Fig. 6—Connections Used in Studying Induced Voltages on Transmission Lines

of discharge of the clouds to earth is in effect a steady field. Due to this field, all points in the intervening space have a definite potential. The space in the vicinity of the line has a certain percentage of the lightning potential above earth. When the condensers discharge, the charge on the transmission line is released and the line assumes the potential of the equi-potential surface in which it is located. See Fig. 4. The voltage of the cloud P and of the transmission line can be accurately measured; the voltage of the cloud to ground by spheres and the voltage of the lines to ground by small needle-gaps mounted to give minimum capacity. The ratio of the conductor diameter to spacing, etc., was selected to correspond to practical conditions. Tests were made with the transmission line grounded through high resistance to represent the grounded neutral system and also without grounds. The results

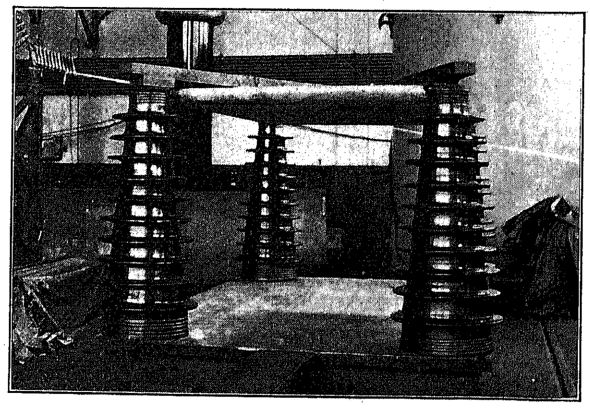


Fig. 7—Study of Voltage Induced upon Transmission Lines. Illustration Shows Overhead Cloud and a Section (to Scale) of one of the Lines Studied During the Investigation

11. J. Murray Weed, Prevention of Transients in Windings, A. I. E. E., Sept. 1915, Feb. 1922. L. F. Blume and A. Boyajian, Abnormal Voltages in Transformers, Trans. A. I. E. E., Feb. 1919.

12. F. W. Peek, Jr., High Voltage Phenomena, Franklin Institute, Jan. 1924.

13. W. Peterson, The Protective Value of the Ground Wire, E. T. Z., Jan. 1914. E. E. F. Creighton, Theory of Ground Wires, Trans. A. I. E. E. 1916, p. 948.

14. E. J. Berg, Electrical Engineering, Advanced Course, McGraw-Hill.

were practically the same. The model cloud and line are shown in Figs. 7 and 8.

The actual height of cloud is not important. It can be readily shown mathematically that since the field is practically uniform between the line and earth the induced voltages depend only on the voltage gradient in this part of the field. A given voltage gradient may be caused by a high cloud and high-cloud voltage or a low cloud with low-cloud voltage. Any condition may thus be simulated by a fixed cloud distance and varying voltage.

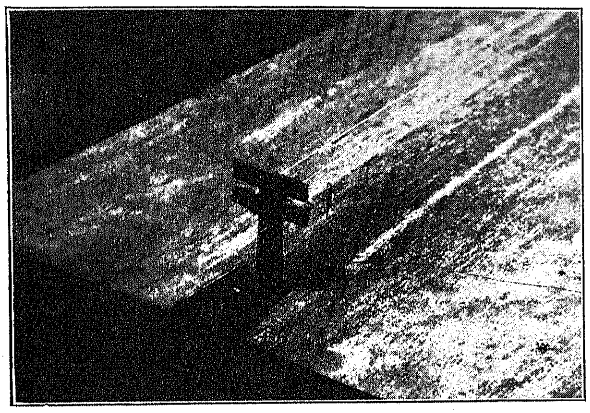


FIG. 8—SHORT SECTION OF A THREE-PHASE LINE WITH OVERHEAD GROUND WIRE UPON WHICH A STUDY WAS MADE OF INDUCED VOLTAGES OF LIGHTNING. ILLUSTRATION SHOWS ONE OF THE "TOWERS" AND ONE OF THE SPARK-GAPS USED TO MEASURE VOLTAGE AT THIS POINT

TABLE IV
PROTECTIVE VALUE OF THE GROUND WIRE
Induced Voltages on Transmission Lines With and Without Ground Wire
Line by Scale 30 ft. (9.1M) Conductors Spaced 7.5 ft. (2.3 m.)
Size 1.2 in. (3 cm.) Lightning Voltages 372 Kv.

Arrangement	Scale Distance to Cloud above Line	Induced Voltage	Per Cent of Lightning Voltage Induced on Line	Protective Ratio of Ground Wire Voltage with Ground Wire / Voltage without Ground Wire
No ground wire o o o	216ft.(66m.)	33	9	
Ground wire above center o line o o o	216ft.(66m.)	14.7	4	0.43
Three ground wires above o o o line o o o	216ft.(66m.)	8.7	2.3	0.26
No ground wire o o o	864ft.(264m.)	8.0	2.1	..
Ground wire above center o line o o o	864ft.(264m.)	3.4	0.9	0.43
No ground wire o o o	Irregular Field	6.3	1.70	
Ground wire o o o o	"	3.3	0.9	0.52

TABLE V
EFFECT OF GROUND WIRE ARRANGEMENT

Arrangement	Height of Line: Actual of Model	Height of Line: To Scale	Voltage induced on Line: Actual of Model	Voltage induced on Line: To Scale	Protective Ratio
No ground wire O←S→O←S→O	6in. (15.3cm.)	30 ft. (9.2 m.)	29.8	1788	..
One ground wire... dist; "S" above center wire. . o o o	" "	" "	14.3	858	0.48
Two ground wires.. . . o o o	" "	" "	10.2	612	0.34
Three ground wires. . . . o o o	" "	" "	7.3	438	0.24
One line wire...... 4 ground wires . . o . .	" " " "	" " " "	1.7	102	0.06
One line wire 3 ground wires..... . . o .	" "	" "	4.7	282	0.16
One ground wire S/2 above center wire...........	" "	" "	12.9	774	0.43
One ground wire 2S above center wire...........	" "	" "	17.0	1020	0.57
Four ground wires.. . s . s o s o s o s . s .	" "	" "	2.7 2.7 2.7		0.09 0.09 0.09
Three ground wires. . A B C . o o o .	" "	" "	A7.2 B4.2 C7.2		0.24 0.14 0.24
Three ground wires. . o o o . . .	" "	" "	7.2 5.2 7.2		0.24 0.18 0.24

$s = 1\frac{1}{2}$ in.

By reducing the line spacing, size of conductor and height to scale, the capacity and inductance *per unit length* remain practically the same as for the full size line. The only factor that does not correspond is the resistance. It is relatively higher. However, it is not believed that this materially affects the results. It might also be well to point out that with equal voltage gradients the induced voltage will be much lower in the model than on an actual line. This follows because the voltage is approximately equal to the height of line times the gradient.

The length of the artificial cloud has no particular meaning except that it is long enough to produce a practically uniform field.

It will be noted from Table IV that the protective

value of the ground wire is quite high and the same for all cloud heights as theory would indicate. With one ground wire the voltage is practically cut in half. This also conforms with theory. The last column in Table IV is called the protective ratio. A ratio of 0.43 means that the induced voltage on the line with a ground wire is 0.43 of that on a line without a ground wire.

Ground Wire Arrangement. Table V shows the effect of various ground wire arrangements in limiting the lightning voltage. This table gives sufficient data to estimate the value of any practical arrangement.

TABLE VI
COMPARISON OF INDUCED LIGHTNING VOLTAGES ON GROUNDED AND NON-GROUNDED NEUTRAL SYSTEMS

Line Arrangement	Height of Line		Voltage Induced on Line		Protective Ratio of Ground Wire
	Actual of Model	To Scale	Actual of Model	To Scale	
No ground wire.	6in. (15.3cm.)	30 ft. (9.2 m.)	29.8	1788	..
Ground wire above center line..........	" "	" "	14.4	864	0.48
No ground wire power lines grounded through resistance ("neutral grounded")...	" "	" "	33.1	1986	..
Ground wire "neutral grounded"...	" "	" "	14.7	882	0.45

Under favorable conditions one ground wire reduces the voltage to one-half; two to one-third and three to one-fourth. The effectiveness of the ground wire increases as the distance between it and the line wire is decreased. The protective ratios given above are for a spacing equal to the conductor spacing which closely approximates the practical condition.

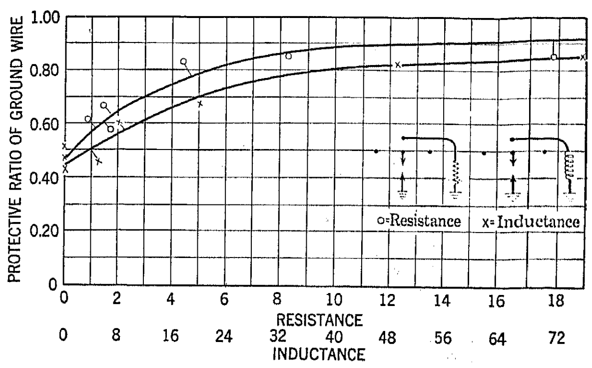


FIG. 9—EFFECT OF RESISTANCE AND INDUCTANCE UPON THE PROTECTION OFFERED BY AN OVERHEAD GROUND WIRE

Comparative Value of Ground wire on Grounded and Non-Grounded Neutral Systems. Table VI shows that the ground wire is equally effective on grounded and non-grounded neutral systems. It is interesting that the induced voltage is generally slightly higher on grounded neutral systems.

Variation of Induced Voltages and Effectiveness of Ground Wires for Lines of Different Heights. Table VII shows that by doubling the height of the line the

TABLE VII
COMPARISON OF INDUCED VOLTAGES FOR CONSTANT CLOUD AND DIFFERENT HEIGHTS OF LINE

Line Arrangement	Height of Line		Voltage Induced on Line Kv.		Protective Ratio of Ground Wire	Average Gradient near Line	
	Actual of Model	To Scale	Actual of Model	To Scale		Kv. ft.	Kv. M.
No ground wire..................	6 in. (15.3 cm.)	30 ft. (9.2 m.)	23.3	1398	..	46.6	152
Ground wire above center of line....	" "	" "	11.0	660	0.47	..	..
No ground wire..................	12 in. (30.5 cm.)	60 ft. (18.3 m.)	48.7	2922	..	48.7	159
Ground wire above center line......	" "	" "	24.0	1440	0.47	..	..

TABLE VIII
EFFECT OF CLOUD VOLTAGE

Arrangement	Height of Line		Voltage Induced or Line		Protected Ratio of Ground Wire	Lgt. Voltage Per Cent
	Actual of Model	To Scale	Actual of Model	To Scale		
No ground wire..................... Ground wire........................	6 in. (15.3 cm.) "	30 ft. (9.2 m.) "	33.0 14.7	1980 882	.. 0.45	100 "
No ground wire..................... Ground wire........................	" "	" "	16.2 7.3	972 438	.. 0.45	50 "
No ground wire..................... Ground wire........................	" "	" "	8.2 ..	492 ..	.. ..	25 "
No ground wire irregular field......... Ground wire irregular field...........	" "	" "	6.3 3.3	378 198	.. 0.52	.. ..

lightning voltage *induced on the line is approximately doubled.* The protective ratio of the ground wire is not changed with change in the height of line. Table VIII shows that the protective ratio is independent of cloud voltage.

Effect of Type of Discharge on The Value of the Ground Wire. So far, the results discussed have been for voltages induced by cloud to cloud or cloud to ground discharge with a steady field between cloud and line at the instant previous to discharge. A quite different type of discharge might occur. There might be a cloud at near ground potential directly above the line, while above this cloud there might be another at a very high potential. See Fig. 10. A discharge between these clouds could in turn cause an impulsive discharge between cloud and ground. This would induce a voltage on the line. A model of this arrangement was made with the results as given in Table IX. The protective value of the ground wire is the same as for the other types of discharge.

Relative Values of the Ground Wire Where the Soil is Conducting and Non-Conducting. The Effect of Inductance or Resistance in Series. Tests were made to determine the relative value of the ground wire when used in countries with dry and damp soil. A long box with a metal bottom was filled with 6 in. of sand. The 6 in. line was placed upon the top of the sand. Referring to Table X, the first two tests were made with a metal plate on the surface of the sand as a conducting ground. The next three tests were made with the sand ground and with the "water level" below the surface at a distance equal to the height of the line. By comparing tests 1 and 3, it is seen *that under equal conditions the induced voltage is higher in dry countries.* This follows because the flux extends from cloud to water level.

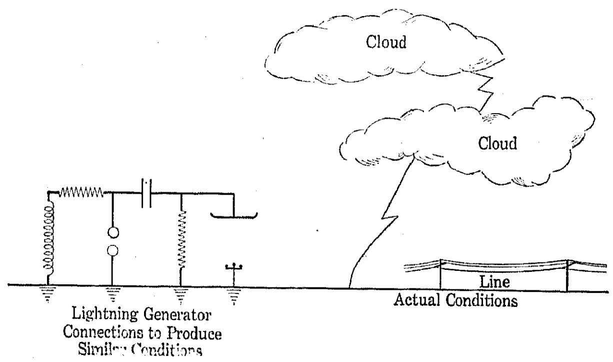


FIG. 10—IMPULSIVE DISCHARGE (DISCHARGE TAKES PLACE FROM UPPER TO LOWER CLOUD THEN FROM LOWER CLOUD TO GROUND)

The effect is that of increasing the height of line. Test 4 shows that the ground wire is effective providing connections are made to water level. Test 5 shows that it is not effective with a poor ground connection.

The ground wire reduces the voltage by reducing the initial charge or the flux terminating on the line and by

TABLE IX
COMPARISON OF INDUCED LIGHTNING VOLTAGES FOR CLOUD TO GROUND DISCHARGE AND IMPULSIVE DISCHARGE FROM CLOUD TO CLOUD

Line Arrangement	Height of Line		Voltage Induced on Line		Protective Ratio of Ground Wire	Average Gradient before Discharge	
	Actual of Model	To Scale	Actual of Model	To Scale		Kv./ft.	Kv./m.
No ground wire..................	6 in. (15.3 cm.)	30 ft. (9.2 m.)	23.0	1380	..	46	150
Ground wire above center line......	"	"	11.1	666	0.48	..	..
*No ground wire..................	"	"	17.9	1074	..	0	0
*Ground wire above center line.....	"	"	8.2	492	0.46	..	..

*Impulsive discharge to "dead" cloud above line and to ground. See Fig. 10.

TABLE X
COMPARISON OF INDUCED LIGHTNING VOLTAGES WITH CONDUCTING AND NON-CONDUCTING SOILS

Line Arrangement	Height		Nature of Soil	Voltage Induced on Line		Protective Ratio of Ground Wire	Average Gradient near Line	
	Actual of Model	To Scale		Actual of Model	To Scale		Kv./ft.	Kv./m.
No ground wire..................	6in. (15.3cm.)	30 ft. (9.2 m.)	Conducting	33.0	1980	..	66	216
Ground wire above center line......	"	"	"	14.7	882	0.45	..	..
No ground wire..................	"	"	Top dry sand 6 in. to water level	37.0	2220	..	74	..
Ground wire above center line. Grounded at water level..........	"	"	Top dry sand 6 in. to water level	19.8	1188	0.53	..	..
Ground wire above center of line. Poor ground made by driving nail in sand.........................	"	"	"	34.6	2078	0.98	..	..

TABLE XI
EFFECT OF RESISTANCE IN "GROUND"

Arrangement	Height of Line		Voltage Induced on Line		Resistance or Reactance	Protection Ratio
	Actual of Model	To Scale	Actual of Model	To Scale	Ohms or Milli-henrys	
No ground wire....................	6 in. (15.3 cm.)	30 ft. (9.2 m.)	33.1	1990	∞	..
Ground wire above center—Resistance in series with ground connection....	"	"	15.7	942	0	0.47
	"	"	18.4	1100	855	0.55
	"	"	20.0	1200	1430	0.60
	"	"	17.5	1050	1700	0.53
	"	"	24.9	1500	4400	0.75
EFFECT OF REACTANCE IN GROUND WIRE						
No ground wire....................	6 in. (15.3 cm.)	30 ft. (9.2 m.)	33.1	1990	∞	..
Ground wire above center—Induction coil in series with ground connection.	"	"	13.8	828	0.08	0.42
	"	"	16.6	995	0.11	0.50
	"	"	18.4	1100	0.13	0.55
	"	"	14.8	888	0.46	0.45
	"	"	25.0	1500	0.98	0.75
Long wire line to ground............	"	"	13.8	828	10′ *	0.42
	"	"	13.8	828	50′	0.42
	"	"	17.4	1045	100′	0.52

*Length in feet.

increasing the capacity to ground. This capacity has in series with it, however, the inductance and resistance of the ground connections. If the lightning discharge took place without time, the initial instantaneous voltage would be the same with or without the ground wire. The tests so far have shown that there is sufficient delay so that the inductance does not produce an appreciable effect with short wire connections. Inductance was added to the ground wire connection with the result shown in Fig. 9. Tests were also made with long ground connections and with series reactance. They show that the protective value of the ground wire could be greatly reduced in practise by resistance in the ground connection or by considerable distance between grounds causing high inductance. Thus, in a long span the induced voltage would be highest in the center of the span and minimum at the tower.

The actual values of resistance and inductance used, as well as length of ground connections, are given in Table XI. These values do not apply directly to a practical line, since the voltage and the energy on the actual line would be much higher. It is difficult to correct these to equivalent conditions for an actual line. It is safe to say that the values should be much smaller than those indicated to make an effective ground wire. They emphasize the importance of short low resistance low reactance ground connections. Fortunately, the modern steel tower affords a low inductance path to ground.

Electromagnetic Induction. The voltages measured in the line wires protected by ground wires are made up of the electrostatic induction and electromagnetic induction due to the induced current in the ground wire. The electromagnetic induction was found to be negligible. High impulse currents from the lightning generator were sent through the ground wire. The voltage induced on the line was 400 volts per ft. per 1000 amperes.

Direct Strokes. The ground wire, if favorably installed, is undoubtedly of great value in case of direct strokes. In the experimental work on direct strokes, the voltage was increased so that the discharge would take place almost directly over the ground wire. It will be seen by referring to Table XII that the ground wire offers good protection from direct strokes. There seems to be greater chance for an unprotected line to be struck in a dry country than in a wet country. In these tests the connection used was that in Fig. 6.

TABLE XII
PROTECTIVE VALUE OF GROUND WIRE FOR DIRECT STROKES

Arrangement	Number of Strokes Applied	Number Striking Line	Number Striking Ground Wire	Number Striking Ground
Connections Fig. 6				
No ground wire. Conducting soil. No needle on cloud..............	105	14	..	82
Ground wire. Conducting soil. No needle on cloud.................	102	0	35	67
No ground wire 6 in. sand. No needle on cloud.....	102	59	0	
Ground wire 6 in. sand. No needle on cloud.....	102	1	76	
No ground arcs 3 in. sand. Needle on cloud........	104	91	..	
Ground wire 3 in. sand. Needle on cloud........	102	0	96	
Connections Fig. 10				
No ground wire. Needle on cloud conducting soils.................	100	100	..	
Ground wire. Needle on cloud. Conducting soils.................	100	2	98	

Tests were also made with the impulsive discharge of Fig. 10. It is interesting that this type of discharge was early recognized by Lodge. It is more difficult to predict where this type of discharge will strike than for the steady discharge. In these tests about 2 per cent of the hits struck under the ground wire to the line. The data will be found in Table XII. It appears that the ground wire will give almost complete protection against direct strokes on the line.

These tests indicate why direct strokes are so rare in practise. With everything deliberately arranged for the spark to take place to the line projecting above a "plain", it very frequently took place elsewhere. In practise the chances of a direct stroke would be very small, first, because the cloud arrangement would not be deliberate and, second, because the chances would be lessened by trees, hills, etc. The same would apply to high induced strokes because they necessitate a very high-voltage cloud near the line. Here, too, hills and valleys would be helpful.

Insulating to Withstand Lightning Voltages
Lightning Breakdown Voltage of Apparatus

Transient voltages are more likely to cause concentration of stress in apparatus containing inductance and capacity than normal frequency voltages. In designing inductive apparatus to withstand lightning voltages, the problem is greater than merely putting enough insulation between line and ground to withstand the highest probable lightning voltage as in the case of the insulator string. This has been recognized in the use of heavy end turn insulation in transformers. However, it is not always possible or economical to insulate local turns or coils to withstand the full voltage of a lightning impulse. It is obviously of great importance to design apparatus so that transients do not concentrate on a few turns but divide evenly over the total insulation. This has been done by shields.

Fig. 11—Lightning Striking the Overhead Ground Wire of a Transmission Line During Study of Direct Strokes and Induced Lightning Voltages on a Three-Phase Line during a Lightning Storm

Shields have a preventative action like the ground wire. Two good examples will be mentioned here. The ring insulator shield is used at high voltages to distribute the normal frequency stress more uniformly. By its use, the operating stress is reduced to about 25 per cent of the stress on a non-shielded string. The maximum lightning stress is also reduced to 25 per cent of that for a non-shielded string.

There is great danger of concentration of stress in transformers. At the first instant that lightning strikes a tranformer, the coils act as if they are open circuited; only the capacities function and the transformer is in effect a string of capacities or insulators with capacities to ground to cause unbalance. By placing a proper shield on the line end the effect of the capacities to line is to neutralize the capacity to ground and there is no voltage concentration. The transformer shield differs from the insulator shield in that the insulator shield controls the stress at both normal and transient operation, whereas the transformer shield operates only for transients.

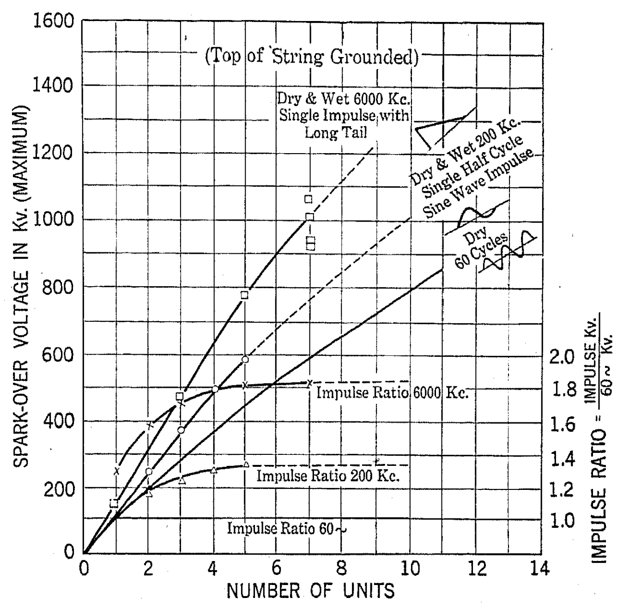


Fig. 12—Comparison of 60-Cycle and Lightning Flash-over Voltage on Suspension Insulators

The general principle of shielding is to place the coil between shields so that the electrostatic field established by the transient causes each turn to take the same relative stress as it does at normal frequency. An oscillation is thus not possible. The shield is of equal value in preventing high voltages from building up locally on the apparatus by high-frequency oscillations on the line. If even voltage distribution is obtained the problem is simplified.

For lightning strokes, switching surges and other high-voltage transients that occur on transmission lines, the breakdown voltage is much higher than for 60 cycles. The range of these surge voltages is given for line insulators in Fig. 12. The highest values represent steep wave front lightning voltages. For such voltages the wet and dry arc-over values are the same.

The steeper the wave front or the higher the frequency the higher is the breakdown voltage.

Persistent oscillations cause breakdown at decreasing voltages with increasing frequency, if the ionization persists from cycle to cycle. Hot electric needles are formed which reduce the spark-over voltage. However, as already stated, because of excessive losses, such oscillations cannot reach high values on transmission lines. This also applies to damped oscillations of high train frequency. Where the train frequency is over about 1000 cycles the breakdown voltage begins to decrease below the 60-cycle value with increasing train frequency. On transmission lines a damped train of oscillations may occur for each half cycle. The train frequency is then 120 per second. The breakdown and spark-over voltages are higher than for the 60-cycle. The breakdown voltage for the oscillation of the "Tesla Coil" excited at 60 cycles is also higher than the 60-cycle breakdown voltage.

Oscillations on a line can only follow after an arc occurs. Thus the original break must always occur in

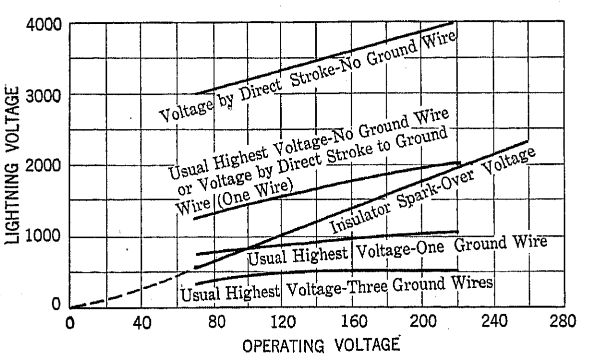


Fig. 13—Comparison of Insulator Arc-Over Voltage with Maximum Lightning Voltage for Severe Storm Directly Over Line

some other way as by lightning or by dirt or faulty construction. It is important to prevent the initial failure. Failures by dirt have been too frequently attributed to high frequency.

The lightning breakdown voltage of liquid and solid insulations is often five or more times the 60-cycle value when the stress is equally distributed.

The lightning flashover voltage of insulators is plotted with operating voltage in Fig. 13. On this same figure are plotted curves of the probable highest lightning voltage for lines with and without ground wires. A direct stroke on a line without a ground wire causes voltages much higher than the insulator arc-over voltage. The second curve down represents the usual highest voltage on a line without a ground wire. It happens that this same curve corresponds to the voltage by a direct stroke on a line with one ground wire. This curve crosses the insulator spark-over voltage curves for an operating voltage of 220 kv. This indicates that a 220-kv. line without a ground wire is not likely to have insulator trouble from lightning except in case of direct stroke. It further indicates that a 220-kv. line with a favorably installed ground wire is not likely to have trouble from lightning under any circumstance. The next curve gives the usual highest voltage for a line with one ground wire. Under usual conditions very little trouble should be expected for lines insulated for over 100 kv. operation. The lowest curve shows that three ground wires reduce the probability of lightning trouble to still lower operating voltages. The probability of trouble with direct strokes would be reduced in proportion.

The values of voltages taken in the above curves are for the most severe storms directly over exposed lines. Such conditions might not occur during a year or several years. It is a well established fact that during any storm there are likely to be many low-voltage induced impulses, a less number of moderate voltage impulses and frequently none at high voltage.

Lightning Arresters

The methods for guarding against lightning so far discussed have been preventative, such as the ground wire; better distribution of stress by the shield and by extra insulation. The remaining method is the arrester.

The object of the arrester is to permit transient or other excess voltages above a given value to discharge to earth and to suppress the dynamic arc and prevent oscillations. Since the transient currents are likely to be very high an efficient arrester must have low resistance. An arrester must also have low time lag, otherwise the transient voltage may rise to high values before the discharge occurs. A good arrester is of unquestionable value at the low and moderate operating voltages. The question of the extent of its use to take care of the unusual conditions at the very high voltages is an economic one.

Because arresters have gaps they cannot prevent low-voltage oscillations from building up high voltages in transformers. However, there is very little to be feared from such trouble in modern apparatus with shields. Where special cases require it, such oscillations are readily absorbed by placing a resistance in series with a condenser without a gap across the line.

Conclusions

Voltage and Energy of a Lightning Flash. The voltage of a severe lightning flash is probably of the order of 100,000,000. The current may be 78,000 amperes and the stored energy 13,500 kw./sec. The discharge is usually non-oscillatory and often takes place in a few microseconds.

Voltage Disturbances on Transmission Lines. Most lightning disturbances on transmission lines are steep wave front impulses that occur by electrostatic induction. There may also be impulses of slanting wave front and damped oscillations. The lightning arc-over of insulators is always higher than for 60 cycles and is not greatly affected by moisture.

The usual induced voltage is probably below 1000 kv. The voltage increases almost directly with the height of the line. The maximum possible voltage can be estimated by multiplying the maximum gradient of 100 kv./ft. by the height of the line in feet.

A lightning wave travels over the line and is dissipated to a considerable extent by corona loss. When it strikes an inductance or the end of the line it increases in value and is reflected. If the inductance is not shunted by resistance, high values of voltage may be built up. When breakdown occurs there is a damped oscillation at the comparatively low frequency of the natural period of the line or some section of the line.

Switching surges and other high-voltage disturbances that occur on transmission lines are damped oscillations. Such disturbances always require a higher voltage to cause insulator flashover than 60 cycles.

Persistent or continuous undamped oscillations at high voltage cannot exist on transmission lines because of the enormous losses. This also applies to damped oscillations of high train frequency.

A high-frequency oscillation requires an arc. It is, therefore, a secondary disturbance following a breakdown caused by lightning, dirt or some defect. Oscillations are not serious on grounded neutral systems.

The Ground Wire. A single ground wire when properly installed under favorable conditions reduces the induced lightning voltage to 48 per cent of that without a ground wire; for two ground wires the reduction is to 34 per cent while for three ground wires it is to 24 per cent.

The reduction is the same for isolated and grounded neutral systems.

The ground wire is apparently not effective in a dry country unless grounds can be made in conducting soil. Induced lightning voltages are higher in a dry country than in a wet country.

The effectiveness of a ground wire decreases as the resistance or inductance of the ground connection increases.

The ground wire is also a good protection against direct strokes.

Insulating to Withstand Lightning Voltages—Lightning Breakdown of Apparatus. Insulating inductive apparatus to withstand lightning voltages is a greater problem than simply adding insulation. Shields to prevent localization of stress are important and function in a manner somewhat similar to the ground wire.

A plot of the lightning strength of insulators and the probable highest lightning voltage for different operating voltages is given in Fig. 13. The indications are that a 220-kv. line with a ground wire should be almost free from lightning troubles. It cannot be said that there is any definite voltage where immunity begins, since the unusual may always happen. The danger becomes less and less with increasing operating voltage.

The extent to which protection should be used is a combined engineering and economic problem. The technical problem, which has been the subject of this discussion, is to determine the strength of apparatus, and the voltages to which it is likely to be subjected. With these factors known, the economic problem is to balance the cost of insurance against the value of better service and reduced liability of trouble.

Discussion

LIGHTNING AND OTHER TRANSIENTS ON TRANSMISSION LINES (Peek) LIGHTNING (Creighton)

Pasadena, Cal., October 14, 1924

F. W. Peek, Jr.: In some respects Mr. Creighton's conclusions are not as far from my own as might appear at first glance. The effect on transmission lines is determined wholly by the gradient. The fact that he has very high energy values is due to the high voltages or high cloud heights that he has assumed.

Mr. Creighton finds lightning oscillatory because of the very low value of resistance that he takes for the spark or bolt. In this I cannot agree with him. The specific resistance that he has taken was for a short very high-current dynamic arc. Such resistance is low because it is determined by the metallic vapor from the electrodes—in other words, a huge arc-light.

The conduction of a lightning spark is determined by the ionization of the gases of the air and not by metallic vapor. The specific resistance in gas conduction is very much higher than in metallic-vapor conduction—in fact they are not of the same order. Even in sparking-over an insulator string at 60 cycles, the initial break is gas conduction and it takes an appreciable time before the metallic vapors come into the arc. This initial spark can generally be seen in insulator flash-over photographs as a bright line independent of the rest of the arc. If specific resistance corresponding to gas conduction is taken it will be found that the discharge will be either non-oscillatory or very highly damped.

J. B. Whitehead: Since the time of Franklin there has been no question as to the electrical nature of lightning, but not until recently have we been able even to approach a reasonable certainty as to the particular forms of discharge and the values of electrical quantities occurring in lightning strokes. Even now our knowledge is very approximate, and the conclusions we have drawn are likely to be upset at any time. Messrs. Peek and Creighton compute the values of voltage, current and energy of a lightning discharge, from the known values of spark discharge in the air, and the assumption that a thunder-cloud and the earth constitute a parallel-plate condenser. The experimental evidence on which the conclusions are based, are the laws of spark discharge as we know them from direct tests, observations of potential gradients at the surface of the earth during lightning storms, and now, in Mr. Peek's case, experiments with laboratory models of transmission lines and cloud, and in Mr. Creighton's case, deductions from values of resistance of high-voltage electric arcs. The conclusions in each case are in approximate agreement as to the probable gradient at the surface of the earth at the time of the discharge. Mr. Creighton's estimated value of the voltage of the cloud is seven times that of Mr. Peek's, which leads to a corresponding difference in the values of current and energy. The two papers differ particularly in their estimate of the form of the discharge; Peek holding that usually the discharge consists of a single pulse, with occasional heavily damped oscillatory discharges. Creighton, on the other hand, concludes that the lowest frequency of a lightning stroke is about 50,000 cycles per second. Peek's conclusion is in conformity with

that of a number of others who have worked in this field, and Creighton is making a radical departure from a considerable mass of expert opinion, when he assumes that the discharge is always oscillatory. He bases his conclusions entirely on the low value of the resistance of a high-power arc, assuming that the discharge path of the charged condenser formed by cloud and earth is such an arc. I do not think that he has made out a good case for this. There appears to be two serious objections to his view. The first is the assumption that all of the resistance of the discharge lies in the spark or arc itself. In the passage of the spark, electric charge is drawn from wide areas of both earth and cloud. As is well known, the earth may have considerable values of resistance. Our knowledge of the effective resistance to the passage of charge through the mass of a cloud is not very definite, but I believe that in this case it would be found that this resistance is considerably higher than that of the surface of the earth. For example, the heavily ionized air between two corona-forming lines unquestionably has a very high resistance. The second objection is the assumption that the specific conductivity of the lightning streak is the same as that of the vapor of a sustained high-current arc. In the latter case the electrodes between which the arc is formed are vaporized, and it is this vapor which constitutes the conducting medium of the arc. No such source of conducting ions is present in the lightning streak.

It is probable that lightning discharges take a wide variety of forms. It would appear, however, from these two papers, that the evidence that we now have is quite good as to the unidirectional pulse as the commonest type of discharge, that such oscillations as do occur are very heavily damped, and that there is little or no evidence of a prevailing type of oscillatory discharge of relatively high frequency.

H. Michener: The portion of Mr. Peek's paper which deals with the ground wire is of particular interest to me as the value of the ground wire has been discussed at great length by various members of the Edison organization. Our present 220,000-volt lines have always been equipped with the ground wire with the exception of about 60 mi. of one line. The operation of this 60-mi. section, without ground wire, has been the same as that of the 60 mi. of line parallel to this and which has a ground wire. These two 60-mi. parallel sections are located in the San Joaquin Valley.

In planning for a new line from Los Angeles to Big Creek it was necessary for us to decide for or against the ground wire. This line will be located in the mountains and the foothills nearly all of the way and 75 per cent of the footings will be in rock. We have interpreted the results of the experiments that Mr. Peek has reported in his paper to mean that a ground wire on a line in a location such as this will give only 2 per cent more protection for the line than it would have without any ground wire. On this basis we have decided to build the line without a ground wire, but to design the towers so they will carry a ground wire safely in case we should ever want to install one.

H. T. Plumb: There is not enough caution thrown out similar to that of Mr. Peek about seeing that the ground wires and lightning arresters are properly grounded. In this Western country where there is not much soil, and where that soil oftentimes is thoroughly dried out, it is difficult to get ground connections and I want to suggest something which I have been advocating and using for several years. That is, making a coupling with the earth, not through conductance, but through capacitance; connecting to something that is extensive and that is a good conductor, and let it make the connection to ground by capacitance. I believe that will very materially add to the protective value of many lightning arresters which are not now properly grounded.

J. Slepian: The very interesting quantitative results which Mr. Creighton obtains, although admittedly somewhat speculative in nature, have a very great practical value for those engaged in combating the effects of these atmospheric disturbances upon power systems. It is very gratifying to find a general agreement as to the magnitudes of the voltages which may be induced on transmission lines, and to find that there is a logical relation between these magnitudes and the dielectric properties of air.

Mr. Creighton has not limited himself to the study of induced surges on systems, but has estimated the quantities in the lightning stroke itself, voltage, current, power, frequency, etc. These quantities depend on the capacity, inductance and resistance of the discharge path. The first two factors are geometrical in nature and may be calculated with considerable confidence. The resistance of the broken-down air making up the lightning stroke is a much less certain quantity. Mr. Creighton, by taking the results on the conductivity of heavy-current arcs in the laboratory makes it seem very plausible that resistance in the lightning path is small compared to the reactive ohms in the lightning circuit, and so neglects it in calculating current and power. The current and power so obtained are appalling. The practical conclusion to draw is that against the direct stroke to a power system we are helpless. Our puny lightning arresters are futile, discharging their hundreds or even thousands of amperes from the stroke involving millions of amperes.

We are better situated, however, with respect to the induced surges. Here the currents involved are well within the capacity of modern arresters. And since, fortunately, direct strokes are rare, while induced surges are frequent, we may feel confident that these arresters are really rendering an effective service.

By assuming that the resistance in the lightning stroke is small, Mr. Creighton arrives at another conclusion which at first sight would seem to be of very great importance for induced strokes. This is the oscillatory character of the lightning discharge. A remote stroke then, might induce on a system a surge of such low voltage that it would not discharge through the arresters, and yet which on account of its oscillatory nature, might develop a dangerous resonance in the vitals of a transformer. I wish to show that this apparent danger is actually non-existent.

I need merely to point out, that developing a resonance is a a cumulative phenomenon and therefore takes place only for wave trains which are sustained or only slightly damped. The first cycle of the train starts the resonant system oscillating; the second cycle finds itself in phase with this oscillation and increases it further; the third cycle likewise finds itself in phase and feeds still more energy into the resonant system. Thus the resonance builds up, and the energy stored in it will amount to many times the amount supplied by the first cycle. Things are otherwise if the wave train is strongly damped. The second cycle in the train, because of its smaller intensity, adds only a little to the energy supplied by the first cycle; the third cycle still less, and so on. Thus the total energy supplied to the resonant system by the damped train is little more than that supplied by the first cycle, and a cumulative resonance does not exist.

Now, I shall show that a lightning stroke is always of this strongly damped character. Where is the damping resistance? In the radiation! The column of highly conducting air, stretching up one mile high is the most effective antenna one could find for radiating the 50,000-cycle energy from Mr. Creighton's cloud. It is, in fact, as all lightning strokes must be, granting low ohmic resistance in their paths, an open oscillator oscillating at its own natural period. Such oscillations are always strongly damped by their own radiation. Textbooks on radio telegraphy show that in such natural oscillations, the second cycle is only about 30 per cent of the amplitude of the first; the third cycle will then be less than 10 per cent of the first, so that practically, we may say that the train is limited to two cycles. Hence a cumulative resonance is impossible.

It must not be thought that the radiation resistance which causes this great damping will seriously affect the calculations of Mr. Creighton. Very considerable resistance may be added to an oscillating circuit without affecting the maximum current very greatly. For example, the maximum current in a critically damped circuit is equal 1/2.718 times the maximum current in that same circuit with resistance reduced to zero.

John B. Taylor (by letter): "Does lightning oscillate?" The question is not a new one in our discussions. Sixteen years ago (see TRANS. A. I. E. E., Vol. XXVII, p. 684, 783 and 795, 1908) Mr. Creighton was assuming oscillations in lightning flashes with frequencies of 1,000,000 cycles, and I was characterizing such a figure as unreasonable for a mile-long stroke because the velocity of propagation in space is wholly insufficient to move a charge back and forth over a mile in a millionth of a second. This objection related only to the estimated figure for frequency, irrespective of low or high resistance in the discharge circuit which determines whether the discharge oscillates or not.

Mr. Creighton, in his present paper, defends the idea that the typical cloud discharge is oscillatory and now calculates or estimates 50,000 cycles per second for the one-mile-long discharge between earth and a cloud one mile in diameter. The capacity of the cloud to earth is given as 1/100 micro-farad which, in round figures, requires an inductance of 1 millihenry to give the oscillation frequency of 50,000 cycles. This value of inductance seems much too low for a straight-away conductor a mile in length well separated from any return circuit. A value of inductance two or three times as great appears more reasonable.

However, this point does not appear so important as the omission of anything in the picture to indicate what becomes of the moving charge or current at the two ends of the mile-long flash. His argument for the presence of oscillations is that the flash itself has a definite diameter greater than ½ in. and approaching 1 ft., and that the resistance of a flash of such size is low enough to permit the discharge to persist in oscillations. But the charge and discharge must move in the cloud from the main flash to the edges. This will add perhaps a mile to the length of the discharge path, increasing the inductance, and properly further affecting the estimates of frequency. It is hard to picture a thunder cloud with moisture particles or drops in such contact as to provide an extended conducting path of low resistance. A suitable picture of the cloud itself seems quite as essential to the theory as a picture of the main flash.

Mr. Peek straddles the question by saying that *some* lightning discharges are oscillatory. He does not tell just what class of experimental evidence is taken to justify his conclusion that oscillations at times occur. For his cloud of assumed dimensions (1000 ft. square, 1000 ft. elevation, a smaller and lower cloud than Mr. Creighton's) he figures that there will be oscillations with frequency of about 140,000 cycles if the resistance of the discharge path is below 1000 ohms.

Messrs. Creighton and Peek both make reference to experimental work on the other side of the Atlantic by Mr. H. Norinder who reported in *Electrical World*, Feb. 2, 1924. Mr. Norinder made experimental observations of induction from natural lightning flashes, and, with a cathode-ray oscillograph, secured evidence that none of a number of lightning flashes studied were oscillatory. As the cathode-ray is not subject to the mass and natural period limitations of the vibrator in the more familiar oscillograph, it serves well for studying electrical oscillations at frequencies of a million or more cycles. Mr. Peek must have overlooked this fact when reading the Norinder article for he states immediately following the mention of Norinder's work:

"....it is not possible to tell from oscillographic records just how steep such waves are because of the relative slowness of the apparatus in responding."

Mr. Creighton disposes of Norinder's conclusions by making a distinction between oscillations in the cloud and in the flash, and promises to discuss this at a later date.

From these two papers and from the Norinder article, it appears that Mr. Creighton holds a brief for there being oscillations in a lightning discharge, basing his case on assumptions as to dimensions and physical characteristics.

Mr. Peek says there may or may not be oscillations but invites the question: "What is the evidence?"

Mr. Norinder gives experimental evidence that oscillations are absent in lightning flashes.

E. R. Stauffacher: Mr. Michener has told you that we were somewhat reluctant to give up the idea of using an overhead grounded wire on the new Big Creek line, but it was finally decided that the possibility of the ground wire breaking and coming in contact with the conductors outweighed the protection which the ground wire would afford. I should like to ask Mr. Peek one question regarding the value of overhead grounded wire as applied to the location of this wire with reference to the conductors and the structure of the tower.

The Big Creek transmission towers are constructed approximately as shown in Fig. 1 herewith with overhead grounded wire fastened at point A. Can we assume that the values given by

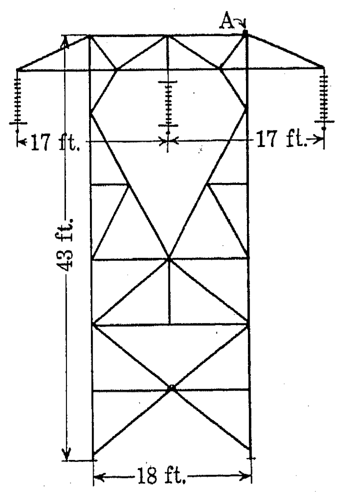


FIG. 1—STANDARD BIG CREEK TOWER SHOWING LOCATION OF OVERHEAD GROUNDED WIRE WITH RELATION TO CONDUCTORS

Mr. Peek for the protective value of overhead grounded wire apply to transmission lines having the conductors in horizontal configuration, separated a distance of from 17 ft. to 22 ft. to the same extent as if the conductors were placed in vertical configuration for example, and spaced closely together? If this distance between conductors were 11 ft. instead of 22 ft., can we assume that these values given, that is, the protective values of the ground wire, will be the same in both cases?

It has been the practise of the Southern California Edison Company during the past year to use an extensive system of grounding at the major substations, and we are now beginning that practise at our minor substations. We feel that one ground wire to a ground well did not give us sufficient protection and we are now adopting the policy of connecting everything together, thus approaching a condition of the station building and equipment sitting on a copper mat.

As regards lightning arresters, the ground leads are brought to pipes driven immediately along the row of arresters. From these pipes connections are made at several points to the network of underground conductors. Mr. Plumb's remarks regarding the use of some metal object, such as the framework of a building, or a fence around a substation, leads me to ask Mr. Peek this question: Suppose the grounded wire on an overhead transmission line

is connected to steel towers that are set in rock, which is an insulator, do we or do we not have a good ground for the dissipation of external discharges and thereby gain advantage of the protective value of an overhead grounded wire?

W. A. Hillebrand: Referring to Mr. Peek's paper, and the discussion of lightning phenomena, and also to Mr. Norinder's referred to therein, if you have a bound charge, after the lightning discharge breaks, that bound charge, presumably, will proceed to travel in opposite directions. If, for instance, the discharge should be six miles long the frequency would be in the order of 30,000 cycles per second. Now, this charge is passed from a highly charged region, that is, where the dielectric is charged, into a region where the dielectric is not charged, so correspondingly it will be absorbed in the dielectric and the energy will be dissipated. As shown by Dr. Whitehead, these damping losses are absorbed in the dielectric region, as the charge passes, and it is reasonable to assume that energy will be returned and oscillate at this frequency through the circuit.

What bearing has the ground wire on this? To a man with radio experience a ground wire is very similar to a counterpoise used in high-power radio transmitting stations for the purpose of reducing the ground resistance, increasing the current, the energy of oscillation, and the voltage is developed directly proportional thereto. Now, if there is anything in the possibility of developing an oscillation, as a result of a traveling wave, then anything that tends to reduce the rate of damping will increase the vigor of that oscillation and the voltage developed in direct proportion thereto.

One of the most complete ground wire systems that I have ever seen had three ground wires on top of a steel tower and three conductors directly below it. This vertical distance between was roughly six feet; the voltage of the circuit is 33,000 volts; the length of the system is about 100 miles, there were about 18 miles of steel towers carrying this combination. Now, as a result of their troubles, due to lightning, on that 33,000 kv. circuit, they would not consider an insulator with less than a normal 66-kv. rating that measures, roughly, 13 x 12 inches, having a 60-cycle flash over of about 200,000 volts. Now, is it possible that you have two compensating effects that counteract one another? The undoubted shielding effect of the ground wire system, also this other effect reducing the ground resistance, increase the oscillation that may result from a closely coupled counterpoise, and of the vigor of that coupling, the closeness of it, there is ample evidence from a number of sources.

Now, it is possible that the nature of the ground itself has something to do with the intensity of lightning disturbances and of their frequency. According to the fundamental electrostatic law the tube of force terminates in the cloud at one end and the charge in the ground at the other. This charge within the earth is drawn up as an amperage current in the earth. There is presumably only a certain amount of energy for the accumulation of the charge. If the ground resistance is high, is it possible that that in itself is a limiting factor? The ground resistance is low in a well watered valley, or on the surface of a subterranean water system and there you will get characteristic thunderstorms of greater violence than elsewhere.

If general corona is allowed to develop the energy is probably immediately or quickly dissipated. On the other hand, both experience and theory tell us that if that energy is allowed to break loose at a single point, at a clamp, or some other point, you will get a streamer that may run to a great length. It is disputed whether or not this thing does exist. There are many evidences that point unquestionably to the fact that we have something of that kind.

The experiment that Mr. Peek shows so beautifully of artificial lightning splitting blocks of wood, has been performed by natural lightning for many years. Over a year ago, in Northern California, I saw a pine tree, four or five feet in diameter, an old tree the center of which was partially rotted out, struck by lightning and it broke off about half way up the trunk and the flash continued down, cork-screwing down the tree. I don't know how many kilowatt-hours, but probably not more than half a dozen, would be required to shatter that tree. The interesting thing which Mr. Peek did in his experiment, was to dissipate the energy where he wanted to. If he wanted to burn up No. 6 iron wire weather-proofed, he subjected the antenna lead to a stroke of lightning and the wire evaporated leaving the weather-proof covering intact. When he wanted to split a stick of wood, his conductor was large enough and the energy was dissipated in the wood. That was what happened in that tree; this current of an unknown number of amperes split the tree to pieces, came down to the bark and where it entered the ground there was no sign of any disturbance whatsoever. I looked around the roots of the tree to see if there was any evidence, but apparently the conductivity at that portion of the circuit was too high.

R. P. Jackson (by letter): In the exceedingly interesting discussion of the possible limits of lightning discharges from cloud to ground, it does not appear that any consideration was given to the fact that the upper plate of the conductor, that is the cloud, is not a metal plate of comparatively low resistance, but is a great group of small segregated, conducting particles. The assumption that the earth resistance is negligibly low is probably quite correct. The resistance of the main path of the arc might readily be within the range indicated. In the cloud itself, however, the arc or current path must branch into a multitude of filaments ultimately reaching each individual droplet. What the resistance of this umbrella or mushroom-shaped portion of the spark path is, is difficult to surmise. It is probable that even if the discharge would otherwise be oscillatory, there is a powerful damping action due to the inability of successive alternations to spread out to the whole cloud and a rapidly increasing resistance is thereby interposed. The effect therefore may be quite different from that of an ordinary discharge through a reasonably fixed resistance and although the initial wave front may be steep it would appear as though the damping action of the cloud resistance would be very great.

F. W. Peek, Jr: Mr. Michener and Mr. Stauffacher have both asked regarding the ground wire. As I understand it, in considering a new line, the question came up as to whether a ground wire should be used or not. It was decided that a ground wire should not be used, not because the ground wire when properly and favorably installed, would not give good protection, but because, under the conditions of this line it could not be well grounded, and, therefore, would not materially effect the lightning induced on the line. I think that was a correct conclusion because if the ground wire cannot be well grounded it gives very little protection. The factor of protection, to a great extent, depends upon the resistance.

In the tests that I made, the ground wire was, in most cases, approximately at a distance from the conductor equal to the spacing between adjacent conductors. My data is applicable to Mr. Stauffacher's line. There is one interesting point that I would like to make clear. When the conductors are arranged in a horizontal plane the voltage induced on each conductor will be about the same so there will be practically no lightning voltage between them. The lightning voltage will be between conductors and ground. When the conductors are in a vertical plane, there will be considerable voltage between conductors, as well as between conductors and ground.

One interesting fact about the ground wire is that one ground wire will protect any number of other conductors. If one conductor is protected and other conductors are placed, they will receive protection without reducing the protection on the first conductor.

Answering Mr. Hillebrand, a number of years ago in Colorado we made experiments on a 25-mi. idle line. In these tests the voltage between line and ground was measured by gaps while

the nature of the discharge was determined by means of a rapidly revolving photographic film. The gaps indicated that the first discharge was a steep wave front followed by the oscillation of the line at its natural period, which, in this case, was about 1800 cycles, or at very low frequency. With the longer lines, the frequency would, of course, be much lower than this. The effect is thus that of a steep-wave-front impulse which may be dangerous, followed by a more or less harmless low-frequency oscillation.

The ground wire tends to increase the damping, just as a short-circuited turn in a transformer would. The actual resistance of the ground itself is very low, because of the great cross-section. It is, in fact, much lower than the resistance of the ground wire. The ground resistance is high where it is part of the ground-wire circuit. This follows because of the small contact area where the ground connection is made.

E. E. F. Creighton: In formulating a closure on the several interesting points brought out in the discussions of my paper it seems pertinent to state my attitude of mind. As a developer of electrical protective devices I am endeavoring to learn the severest conditions to which the protective devices will be subjected. If there are oscillations a certain type of protection should be provided. If finally it should be proved by measurements that lightning is never oscillatory it will simplify the design of electrical protective apparatus. Meanwhile continuity of electrical service is too important to permit us to assume no oscillations when possibly they exist. The cathode-ray oscillograph, properly applied, will tell the tale and put an end to our speculations.

My speculative paper is founded primarily on some new experimental evidence of low resistance of sparks. By the term sparks is meant the accepted designation of conduction by gases and is to be distinguished from conduction by metallic vapors. I think the metallic vapor was driven from the conducting stream in which currents as high as 30,000 amperes were recorded by the oscillograph. Mr. Peek differs with me in my assumption that the lightning streak has the same low value of specific resistance as I measured in these tests. I cannot be absolutely positive that metallic vapor was not present in the three-foot length of arc. Unfortunately these tests could not be included in my brief paper. Metallic vapor travels about 2000 ft. per second. The driving forces in the arc and spark were great enough, I think, to give greater velocities to the metallic vapor which should have driven it bodily out of the conducting stream.

Be that as it may, the latest researches by physicists, Dr. Millikan for example, have shown that the outer layer of electrons on such atoms as nitrogen and oxygen may be torn loose under extremely heavy current densities. These free electrons in great numbers might supply high conductivity in the spark stream. Getting electrons free from gases, like nitrogen and oxygen, requires more agitation of the atoms and bombardment than from metallic vapors. Under low-current densities nitrogen and oxygen in a spark have admittedly lower conductivities than metallic vapors. It seems to me, however, there is great risk of being wrong to assume that under the intense forces of a lightning discharge the ionization of nitrogen and oxygen is not appreciably greater than when measured at lower current densities. The electrons exist in great numbers in the gas atoms. It is a matter of experience that the electrons under bombardment can be separated from the atoms and thereby become conductors of electricity. The degree of conductivity is the question debated.

In Dr. Whitehead's discussion I wish to correct a misunderstanding. He says, "Creighton is making a radical departure from a considerable mass of expert opinion when he assumes that the discharge is always oscillatory." I do not assume the discharge is always oscillatory. I agreed with these experts (second page of my paper) that they are right. I think many lightning discharges are non-oscillatory and the cloud part (the blue brush and weak spark portion) of every discharge must necessarily be non-oscillatory. The resistance of the path, or in other words wide-spread dissipation of the energy, is too great to permit oscillations,—according to the accepted mathematical theory. I discussed this matter of location of resistance at the time De Blois published his paper. In brief, the situation may be stated as follows: If we mentally replace the highly conducting streak, say a mile high, by a metallic conductor the top of the conductor will act as one plate of a condenser and the earth as the other. The electromagnetic energy will be stored up around the conductor. Since the oscillating current (assuming it exists will be greatest near the earth, the electromagnetic energy will be greatest there. According to this understanding the lightning bolt should fade away, beginning at the top and rapidly extending downward. The main streak of the lightning stroke is indeed a vertical antenna and it radiates a part of its energy as has been pointed out by Dr. Slepian.

Radiation of "wireless waves" by the lightning is admittedly a factor too important to neglect. Aside from the heavy drain on the energy in the lightning bolt by electromagnetic waves there is also the further drain of energy in the form of heat and light, both at relatively high frequencies. To offset these losses and thereby to keep up the possible oscillations there are sources of supply of energy which may be drawn on after the first cycle of natural oscillation. One source of energy is in the thundercloud itself. We know definitely by measurement that the energy in some thunderclouds is not given up immediately and thereby exhausted during the first stroke. Correspondingly, therefore, we may infer that the energy is not all given up by the cloud to the bolt during the first part of the stroke. This speculation is appropos to a continuation of the oscillation.

Furthermore, there is another source of energy which may be designated as "burning the atmosphere." It requires a great expenditure of energy to disrupt the atmosphere during the initial formation of the lightning bolt. Why should this energy not be returned in part, on recombination of the elemental parts,—just as for example in the combining of carbon and oxygen.

The million cycles per second as a natural frequency of lightning, that Mr. Taylor mentions in his discussion, was derived from some measurements made in Colorado about eighteen years ago in conjunction with Clay and Peek. The frequency was obtained not from lightning, but from resultant oscillation in the power transmission circuit. It is admitted that these oscillations may have been local in the circuit and not due to the lightning bolt.

In closing, I wish to point out again the characteristic attitude herein that we should look to the worst effects of lightning, with the understanding that protection successful against the worst will make all the lesser effects negligible.

Transmission at 220 Kv.
on the Southern California Edison System
A SYMPOSIUM

Review of the Subject.—*The object of this composite paper is the presentation of a fairly complete description of this 220-kv. system and its operation, together with an account of some work which is being done in preparation for a third line to Big Creek.*

The first section is descriptive of the system and the flashover troubles, giving a detailed account of the bird theory for the cause of flashovers and of the evidence substantiating that theory, also of the measures being taken to prevent the birds from causing flashovers.

The balanced relay protection for the lines and the relay installations to control a flashover or other accidental ground in case the balanced relays are not allowed to function, together with some information obtained from a study of flashovers are contained in the second section.

Section three is devoted to a study which is being made to determine the mechanical and electrical characteristics which will give a most economical third line from Big Creek to Los Angeles. Both aluminum and copper conductors of various large sizes and working at various tensions are considered, and tower locations on ten miles of profile were made.

Section four reports that vibration, particularly in the longer spans, has apparently caused some failures of ground wires and possibly of conductors. Frequencies of 13 to 30 cycles per second and amplitudes up to one inch have been recorded. The vibrations are believed to be due to air currents, but no means of preventing them has been discovered.

In order to minimize the effects of vibration, it is proposed to reduce the weight of the dead-end clamps so that the shocks will be transmitted to the tower connections instead of being absorbed at the outer end of the clamps as at present. A new light weight dead-end clamp has been designed and is being tested under service conditions.

In section five it is concluded that for a high voltage line of large capacity the high cost per mile and heavy tonnage to be transported during construction require careful location work to strike the most economical balance between a straight line and one most easily accessible for construction and maintenance. Right-of-way cost must also be given consideration, together with many other factors which make up the total cost.

The complexity of the problem and the large amount of money involved with consequent opportunity for saving, warrant unusual methods of reconnaissance and survey, and purchasing right of way.

Section 1—Description of System and Operating Experiences

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THE single line diagram (Fig. 1) shows the 220,000-volt system of the Southern California Edison Company. The lines from Big Creek No. 1 through Big Creek No. 3 to Eagle Rock are 243 miles long. The taps to Laguna Bell start about three miles from Eagle Rock and are 26 miles long. From 1913 to 1923 the lines from Big Creek to Eagle Rock operated at 150,000 volts. Since May 1923 the system as shown has been operating at 220,000 volts.

Fig. 2 shows the anchor tower with dead-end insulators.

Fig. 3 shows the standard tower with suspension insulators and with a 10-foot extension under it. On account of the increase in the legal minimum clearance from conductor to ground, it was necessary, when reconstructing for 220,000 volts, to install five, ten or fifteen-foot extensions of this type under 2150 towers out of a total of 3340. The greater number of these was raised with the lines energized. Fig. 4 shows one being raised.

The conductors on the lines from Big Creek to Eagle Rock are of 605,000 cm. aluminum with 78,500-cm. steel core. The insulators have 12 units in a suspension string and thirteen units in each of the two parallel dead-end strings which are three units and two units more respectively, than when operating at 150,000 volts.

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., October 13-17, 1924.

Figs. 5 and 6 show the two types of towers for the branch lines to Laguna Bell. These are new lines designed for 220,000 volts. The conductors are 666,600-cm. aluminum with 85,400-cm. steel core. The insulators have 13 units in a suspension string and 15 units in each of the two parallel dead end strings which are the numbers that would have been put on the old lines if the tower dimensions had permitted.

The shield rings are made of cast aluminum, with the exception of those at the top of the suspension strings. Copper or iron shield rings would be less damaged by an arc and would constitute less fire hazard. When flashovers occur, the aluminum of the shield ring melts and actually burns while falling to the ground. This sets fire to any inflammable material with which it comes in contact. Several bad grass fires were started in this way and it was necessary to clear the grass away from the towers. Flashovers had caused fires before the aluminum shield rings were installed but to a less extent in proportion to the number of flashovers. As a laboratory check on the belief that the aluminum actually burns, *i. e.*, combines with the oxygen of the air with sufficient rapidity to cause heat and fire, arcs were established between small wires of various kinds of metal. The copper and the iron wires melted, the molten metal became a dull red before reaching the floor and did not set fire to the shavings which had been sprinkled on the floor. When the arc was established between the aluminum wires there was a shower of sparks which remained white hot and burning for an appreciable length of time after reaching the floor and which immediately set fire to the shavings.

Practically the only operating troubles ever experi-

enced on the Big Creek lines have been insulator flashovers. During the ten years of operation at 150 kv. they occurred at the rate of about 16 per year. After the line was energized at 220 kv., the number of flashovers increased materially and were for the various months from May, 1923 to June, 1924 inclusive, as follows: 6, 5, 12, 14, 9, 10, 8, 1, 4, 3, 3, 0, 0, 1. When August, 1923 passed with its 14 flashovers, it was necessary to abandon the policy of trying one preventative at a time in search for the one which would stop the flashovers and, instead, to try at once all the possible preventative measures that could be conceived. This work was started early in September and, together with the more systematically planned work of the immediately preceding months, began to bear fruit in decreased numbers of flashovers.

Two or three years ago one of the men in charge of the lines saw a bird, just as it was leaving the tower, drop a stream of stringy excrement which extended from the tower member above the insulator to a point as low or lower than the conductor. The observer was apprehensive of a flashover until the stream had fallen clear of the tower without coming in contact or within arcing distance of the conductor. The bird was reported to have been an eagle.

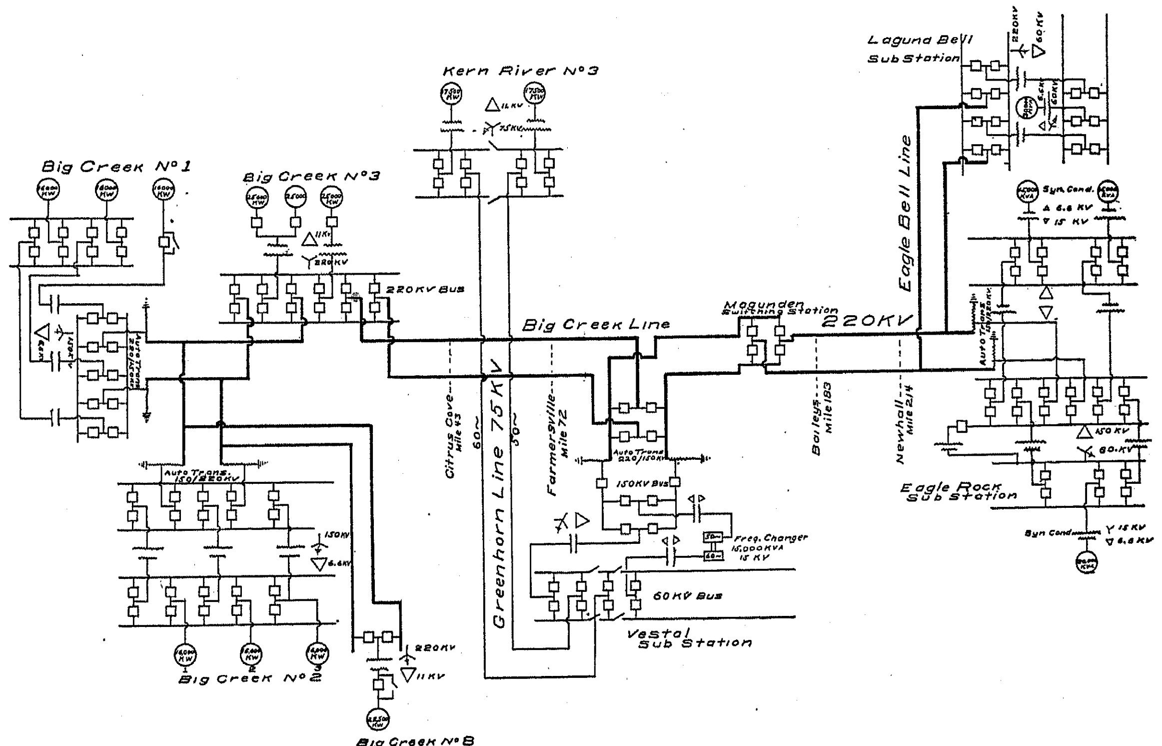

FIG. 1—SINGLE LINE DIAGRAM OF THE 220,000-VOLT SYSTEM OF THE SOUTHERN CALIFORNIA EDISON CO.

A large part of the work done in the effort to prevent flashovers was predicated on the belief that they were caused by the semi-liquid excrement of large birds, hawks and eagles, at the time it fell over the string of insulators or through the air close to them, thus short-circuiting a considerable portion of the air along the insulator string.

The bird theory of flashovers was given consideration several years ago, soon after the lines began to operate at 150 kv., but no evidence to support it could be obtained. The idea in mind at that time was that the accumulation of bird dirt on the insulators would be the only way that the birds could be responsible for flashovers. Some of the strings which fastened over were found to be clean so the bird theory was discarded as a cause of any considerable number of the flashovers. The flashovers occurred so infrequently in those days that the number of observations from which to draw conclusions was small. It took five years after operation began to accumulate as many flashovers as occurred during the first year of 220-kv. operation.

At nearly all the towers where flashovers have occurred during the last two years, direct evidence that a bird was the cause has been found. To facilitate the description of this evidence assume a particular case. The direction of the line is north and south. Three or four feet east of the point of support of the insulators a streak of bird excrement and a spot burned by the arc were found to coincide on the crossarm member. Burned spots were found extending from this point along the crossarm member to a point eight or ten feet on the west side of the insulator.

On the east side of the shield ring a spot of bird excrement and a burn from the arc were found to coincide and burns from the arc extended around the ring to the west side.

The conclusions drawn are that the bird was sitting on the tower some distance to the east side of the point of support of the insulator string and the wind was blowing toward the west with sufficient force to cause

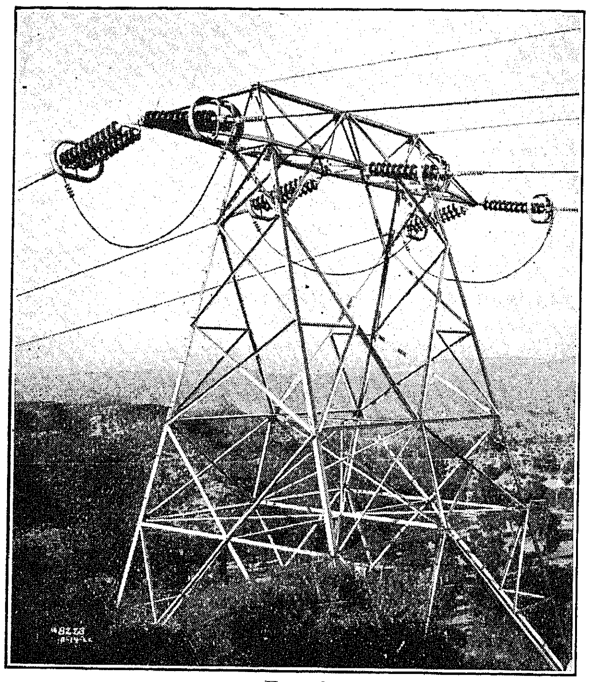

Fig. 2

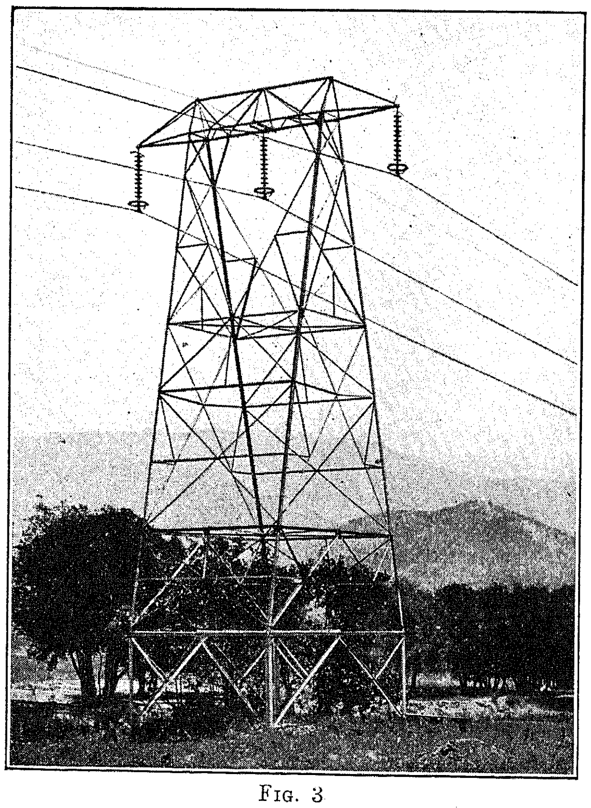

Fig. 3

the stream of excrement to strike the shield ring and start the arc. The wind then blew the arc across the insulator string, the top traveling along the tower and the bottom around the shield ring.

The above is a special case and is subject to many modifications according to the position of the bird and the direction and strength of the wind. It is not necessary that the stream of excrement actually strike the energized parts. In the case of a dead-end tower the arc is started between the jumper loop and the crossarm members.

The greater number of flashovers has occurred on the sections of the lines where there have been the most

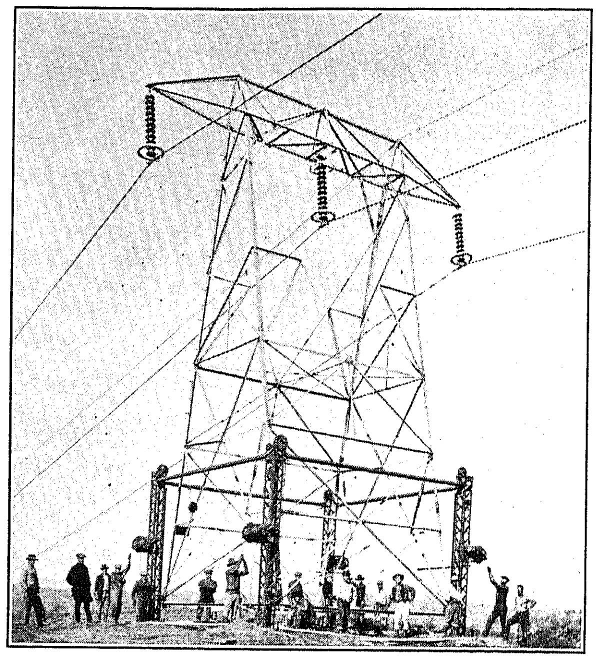

Fig. 4

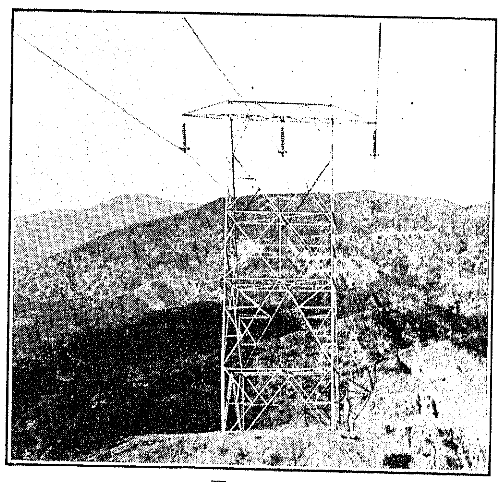

Fig. 5

birds as indicated by the greater amount of bird dirt on the insulators when cleaned a few months ago. Also the insulators on the center conductors have had more than seventy-five per cent of the flashovers during the last four years. These insulators were much more dirty from birds than those on the outside conductors.

F. W. Peek, Jr., in his high voltage laboratory, set up an 11-unit string of insulators, mounted a $\frac{3}{8}$ in. tube,

with cork in the bottom, on the tower near the top of the insulator string. Then by putting one or two ounces of a starch-salt solution in the tube and pulling the cork out by a string, flashover occurred at normal voltage when the stream flowed down over the insulators and also when it fell through the air near the insulators but touched neither the insulators nor the energized parts at the bottom of the insulators.

The first attempt to prevent the birds from causing flashovers consisted of placing guards made of steel strips in an inverted V on the favorite roosting places of the birds over the center string of insulators. Also wires were stretched about four inches above the horizontal members of the crossarm for a distance of four or five feet on each side of the center. Above each outside string of insulators a single spike was placed so that it interferred with a large bird perching there (See Fig. 7). These guards were put on the greater part of the line about two months after 200-kv. operation began but the flashovers instead of decreasing in number, increased very appreciably. The indications were that the birds, having been forced out of their accustomed places, were perching on the top shield ring of the center insulator string and thus causing more trouble than before. The upper shield rings were then removed. At the same time one more unit was added to the suspension strings making twelve units per string, and the insulators were all cleaned and meggered, and the defective units replaced. This work was done during September, October and November with a very little running into December and January.

The flashovers continued at the rate enumerated above. This was due partly to the persistence of the

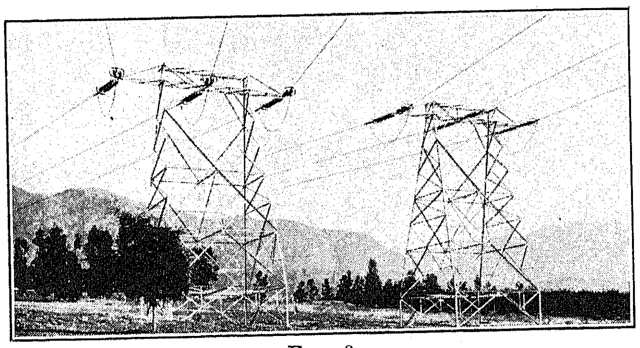

Fig. 6

birds in crowding into their old perching places and partly to the fact that no bird guards were placed on the dead-end towers. The proportion of flashovers occurring on dead-end towers increased after bird guards were placed on the suspension towers.

Because the flashovers were continuing, though at a diminished rate, 60 miles of line were equipped with galvanized iron pans in the crossarm above the center strings of insulators. They were about 4 by 8 ft. with the center above the insulators. They were installed in Feb. 1924, and no flashovers have occurred in this section since then. However, only four flashovers have occurred on the whole line during that period.

It is believed that guards which will not allow any large bird to perch above the conductors will eliminate nearly all flashovers which cannot be traced to other mechanical interferences or to external lightning.

There is a possibility that flashovers may be caused by an accumulation of any kind of dirt on the insulators in combination with dew or fog. Laboratory

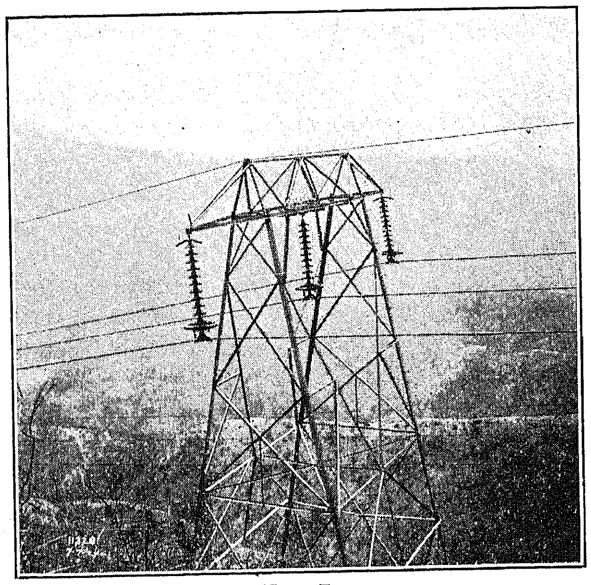

Fig. 7

tests have shown that this condition can occur. Mr. Peek has been able, by a combination of dirt and dew, to flashover a 9-unit string at about 20 per cent its dry flashover voltage. This condition is difficult to obtain but the fact that it can be indicates that the insulators should be kept clean, especially in regions where soluble salts, such as alkali and sea salt are deposited on the insulators.

Section 2—Automatic Protection—Balanced Relays and Flashover Control

BY. E. R. STAUFFACHER

Associate, A. I. E. E.

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Throughout the history of the Big Creek transmission line there have been flashovers which would occur occasionally at no particular interval and at no particular season of the year or time of day. This was the case when operating at both 150 kv. and 220 kv. These flashovers would occur in the proximity of an insulator string, would result in grounding one of the wires and would cause an interruption to the service. The operators became expert in handling these flashovers with the original installation of two units in each Big Creek No. 1 and Big Creek No. 2 and the synchronous condensers in Eagle Rock and the interruption resulting from a flashover was very short. When

the system became more complicated by the addition of the frequency changer and the Kern River No. 3 connection at Vestal, with more generating stations at Big Creek, the manual control of flashovers did not give such good results. The procedure under manual operation was as follows:

The ground current resulting from a flashover was indicated in the power houses and the substations by means of ground ammeters and bell alarms. The operators at the power houses then immediately began to reduce the exciter fields manually until the transmission voltage was lowered sufficiently for the flashover arc to break and thereby free the line from this ground or short circuit. After this the generator field was restored to normal, thus building up the transmission voltage again. At the receiving stations the practise was to open the field on the synchronous machines upon the occurrence of a flashover, thus allowing these machines to operate as induction motors until the flashover was cleared by lowering the voltage at the generating end. When the voltage was up to normal, the frequency changer and the synchronous condenser fields were again closed. As a result of the successful use of balanced relays on some of the important trunk lines of the 60-kv. transmission network, similar relays were adopted for the 220-kv. system, and at the same time it was decided to install equipment which automatically would accomplish the function of lowering the voltage of the generators and opening the fields of the synchronous apparatus at the substations in a manner quite similar to the method heretofore followed manually.

The single line diagram, Fig. 1, shows the 220-kv. Big Creek circuits are operated in parallel through busses at each of the power houses and substations and at Magunden. When a flashover occurs in any one of the four sections the proper functioning of the balanced relays will cause switches to be opened automatically to isolate the section of transmission line upon which the flashover occurs, with no disturbance to the rest of the system and with no interruption to service. The only indications that a flashover has occurred are the tripping out of the switches and a drop in voltage for a period of two to five seconds.

At each sectionalizing point the balanced relays installed are connected to bushing-type current transformers in the oil circuit breaker terminals. Upon the occurrence of unbalance in the current flowing in the two lines, these balanced relays close their contacts, trip out the proper switches and thus disconnect the section of line in which the trouble has occurred. At some of the stations the relay system is more complicated than at others. At Vestal, for example, it was necessary to balance the sum of the currents flowing in two oil circuit breakers and one auto-transformer connected to one line against the same for the other line. This is accomplished by means of paralleling the current transformers in the apparatus concerned. Due to the possibility of a dangerous rise in voltage should some automatic operation separate the generating end from the receiving end of the transmission line, the tripping connections of the oil circuit breakers were so arranged that after the circuit breakers of one line are operated, those of the second line would immediately become non-automatic. With such connections, the removal of one line from service for any purpose renders the remaining line non-automatic and leaves the service exposed to the risk of a comparatively long interruption with only manual protection. To provide against such a contingency automatic flashover control equipment operating on the generator voltage was installed.

A master relay connected in series with the second-

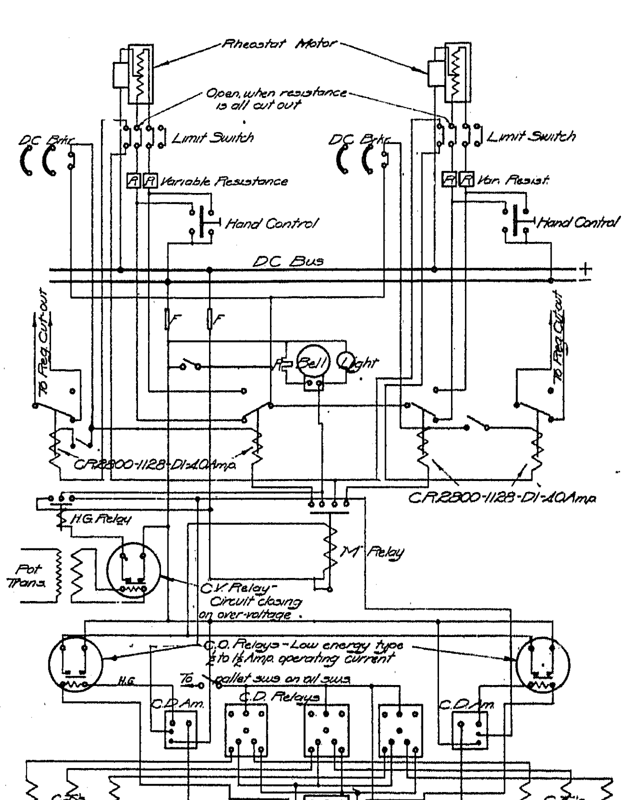

FIG. 8

aries of the current transformers in the grounded neutral of the power transformers is so arranged that when the ground current passes through this relay, due to a flashover, the relay contacts will close and operate a number of auxiliary relays. These auxiliary relays will cause the motor-driven rheostats to cut in resistance in the field of the exciters at the various generating plants until the voltage is lowered sufficiently to break the arc, thus eliminating the flashover from the line. The master relay is then returned to its original position, which, by means of the auxiliary relays, will cause the motor-driven rheostats to go back to their original position, thus bringing the voltage back to normal. Simultaneously with the lowering of the voltage at the various generating plants, the ground currents at the

terminal substations and at Vestal operate relays which automatically lock open the master circuits of the field regulators of the condensers and frequency changers, thus introducing the maximum resistance in the exciter

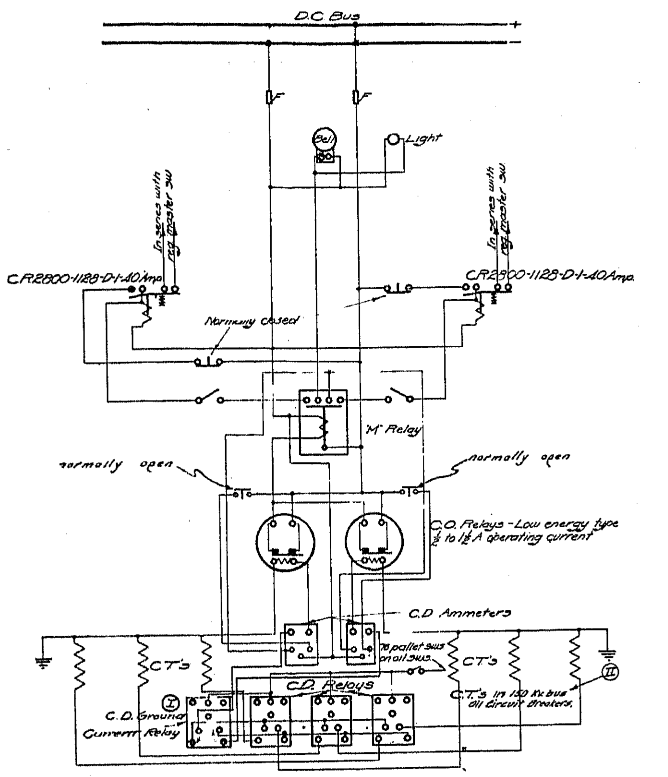

Fig. 9

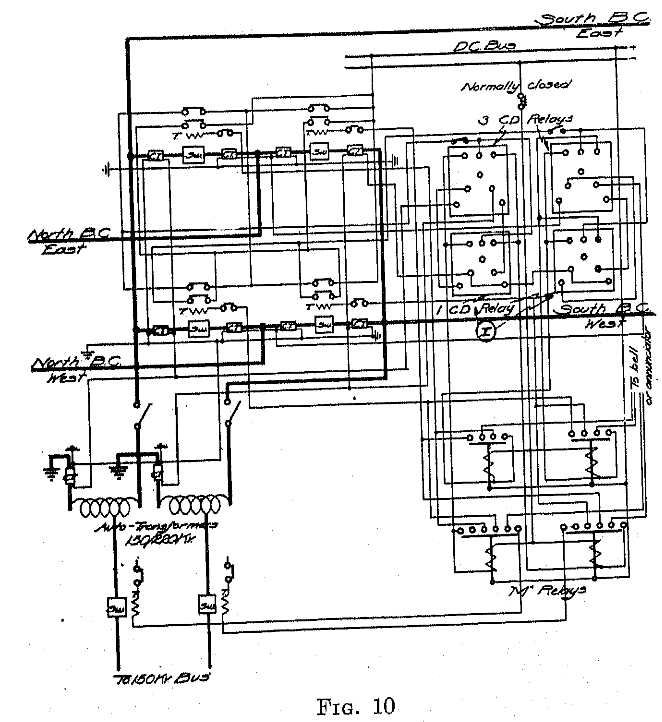

Fig. 10

field. When the arc is broken and the condensers and frequency changers are returned almost to synchronous speed, the master relay is released by a push button, thus allowing the regulator to bring the excitation of the condensers and the frequency changer back to normal.

The time required to handle the flashovers, when two lines are in parallel and the balanced relays operative, is from two to five seconds. When line conditions are such that the balanced relays cannot operate, about 30 seconds is required to handle the flashover, either by the manual method or by means of the automatic suppressor. Of this time 10 seconds is required to bring the voltage down to the value where the arc will break and approximately 20 seconds is consumed

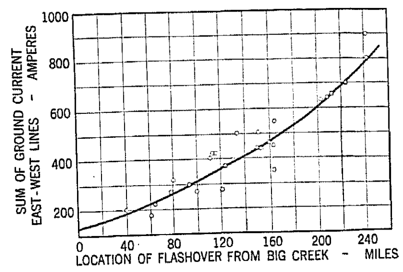

Fig. 11—Ground Current—Eagle Rock 220-Kv. Flashovers East-West Lines. (East-Line Short Circuits)

in restoring the voltage to normal. However, a rheostat in series with each of the motor circuits is so connected that it will permit varying the above time interval in lowering and raising the voltage as experience dictates. The use of these rheostats makes it possible to calibrate all equipment at generating plants, so the voltage can be lowered simultaneously throughout the Big Creek generating system regardless of the characteristics of the individual generators. Figs. 8, 9 and 10 give detailed wiring diagrams and explanation of the connections for the balanced relays and the

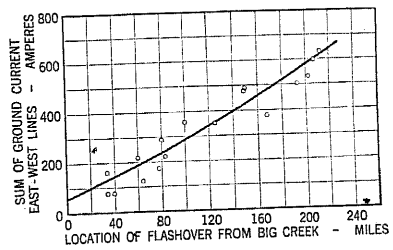

Fig. 12—Ground Current—Eagle Rock 220-Kv. Flashovers East-West Lines. (West-Line Short Circuits)

automatic flashover control for Big Creek No. 1, for Eagle Rock, and for the balanced relays only at Vestal. The connections at the other stations are similar with only slight differences where local conditions demand. Knife switches are included in the connections so that any part of the equipment can be rendered non-automatic at will.

Careful studies of the flashovers have been made with reference particularly to location, time, duration weather at point of flashover, voltage, ground current, and kv-a. connected at each of the stations, damage

done by the flashover, and relay operation. Cases in which the trouble was cleared by the relays caused less damage than others.

A study of the ground currents at the time of a flashover shows that these currents vary roughly with the distance between the flashover and the major generating plants or major substations. Curves indicating this are given in Figs. 11 and 12. In the curves the locations are given in terms of miles from Big Creek and it will be noted that as the distance from Big Creek increases, the amount of ground current flowing from Eagle Rock also increases. This is undoubtedly due to the decrease in ground circuit resistance, as the distance between Eagle Rock and the point of flashover is decreased.

Since the time of placing the balanced relays in operation (August, 1923), the performance has been very satisfactory. A comparatively short time has elapsed since the entire 220-kv. line has been placed under automatic protection and there has been no opportunity to study the clearing of flashover in the section south of Magunden. There have been a number of operations on the two sections between Big Creek No. 3 and Vestal and from Vestal to Magunden. At the time of the greater number of these relay operations there was no interruption to service. The relays were first set for a current unbalance of 350 amperes. On the first two cases of trouble following the installation of the relays, those on the end of the section in trouble toward Big Creek operated properly but those on the end toward Eagle Rock did not operate, because there was not sufficient energy supplied from the Eagle Rock end to cause as great unbalance as 350 amperes. After the second occurrence, the relay setting was lowered so that only 300 amperes unbalance was necessary to cause the relay contacts to close. Since that time the relay operations have been satisfactory in all cases, clearing the trouble from two to five seconds, and giving since installation 13 perfect operations out of 15 opportunities.

Section 3A—Economic Studies of Transmission Line Design with Particular Reference to the Mechanical Features

BY C. B. CARLSON

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The concrete example of the studies for a new 220-kv. line to the Big Creek water power plants will be used in presenting methods for arriving at the economic conditions of design for this article. The method used was as follows:

Ten miles of actual profiles of an existing transmission line which traverses practically the same route as that to be followed by the new line were taken and so chosen as to represent three miles of level profile; three miles of rolling profile and four miles of mountainous profile. On these profiles were plotted the actual tower locations for the following various materials and conditions.

Tension—between 12,000 lb. and 24,000 lb. on steel reinforced aluminum cable having aluminum of 666,600, 1,000,000, 1,250,000 and 1,500,000-circular mil sizes. Copper cables of equal conductivity to the size of aluminum mentioned above were tried with several working stress values. There were naturally several heights, strengths and spacings of towers and also numbers of extensions required to meet the conditions of the various sizes of wires and unit stress variations.

The spacing of the conductors was fixed at 22 ft., because of the requirement to have six feet of clearance to ground under maximum conditions of swing.

A sufficient number of points was plotted for each condition to permit of the drawing of a curve. The span and the tension which would give the most economical cost were then taken from these curves and referred to the calculations for electrical conductivity economy, the different phases of which are discussed in another part of this paper. The electrical and mechanical studies are interdependent and were conducted in close cooperation.

In the case of the aluminum cable, it was found for the amount of current to be carried about a million circular mils of aluminum were necessary. The economical tension was about 12,000 lb. and by insisting that a factor of safety in the steel core be such that the maximum working load be 10 per cent less than the elastic limit of the steel core, a cable with the following characteristics was determined on: 1,033,500-cm. steel reinforced cable with either 7 or 19 strands of steel of equal cross sectional area and 54 strands of aluminum.

The same set of economical studies was made with reference to the use of copper conductor. The different tension and tower locations were plotted on the same profiles as were used for the aluminum cable. One of the electrical requirements in the use of the copper was that the outside diameter of the cable be not less than 1.1 inches, so investigation was made on a recently manufactured cable which had two layers of copper stranded on a flexible tube. By plotting total values of cost for completed line for the various tensions, tower locations and tower heights, curves were drawn between the points and the minimum condition was determined, and the point where a rise in cost started was also determined which gave a little range in conditions to permit them to be fitted to the varying profile of an actual line.

A certain limitation was imposed in the use of tension values which was fixed as a maximum, the use of two strings of high strength insulators having a predetermined maximum tension value. This tension, used in its maximum proved economical for both the aluminum and for the copper. To date, however, there has been no definite decision as to which of the materials for cable will be used, hence no definite conclusions can be

stated, but for the copper cable the following sizes were determined:

One of 650,000 circular mils stranded on flexible copper tube having an outside diameter of 1.1 inches. Also a stranded copper cable with 800,000 cir. mils having an outside diameter of 1.03 inches. Of the two sizes the 650,000-cir. mil cable used at a maximum tension value of 12,000 lb. was found to be the more economical. The tower design was based on using a steel of high elastic limit, which material has been found to represent a saving in the cost of the line.

Specially designed attachment details were developed and tested. The principal point which was especially considered was a form of attachment to the cable which would minimize the effect of vibration. Tests are now being conducted on the attachment to determine this effect.

It will be noted that a tremendous amount of calculating is necessary to arrive at the minimum values, but it seems that changes in tension, heights of towers, and spacing of towers impose so many variables that no system better than the curve plotting method will satisfy.

Section 3B—Economic Studies of Transmission Line Design with Particular Reference to the Electrical Features

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Initial Conditions

Length of line was to be 275 miles. This was determined by the fact that the present Big Creek lines from Big Creek to Eagle Rock Substation are 243 miles in length and the lines from Eagle Rock to Laguna Bell Substation are 26 miles or a total length of 269 miles. Assuming that the future lines would terminate at Laguna Bell or in the same neighborhood, the round figure of 275 miles was used.

The spacing of conductors was to be 22 ft. using a flat constructure and was determined by the studies on tower design as outlined elsewhere in this paper.

The voltage was to be 220,000 volts at the generating end and 200,000 volts at the receiving end; this was determined by the voltage of the present Big Creek lines. The power factor of the load is 80 per cent lagging.

Line Characteristics

With the above conditions fixed, the first step was to determine the auxiliary line constants for different size cables, varying from 600,000 to 1,600,000 circular mils, the lower limit being just below the size of the original Big Creek line. Due to corona, it would not be desirable to use a cable with as small a diameter as the present line.

The constants of the line alone were calculated by the usual hyperbolic method, then the constants of the line in combination with the sending and receiving transformers were found by the method outlined by R. D. Evans and H. K. Sels in the *Electric Journal* of August, 1921. The capacity of the transformers was sufficient to deliver 150,000 kw. at the receiving end.

The final results are shown in Table I.

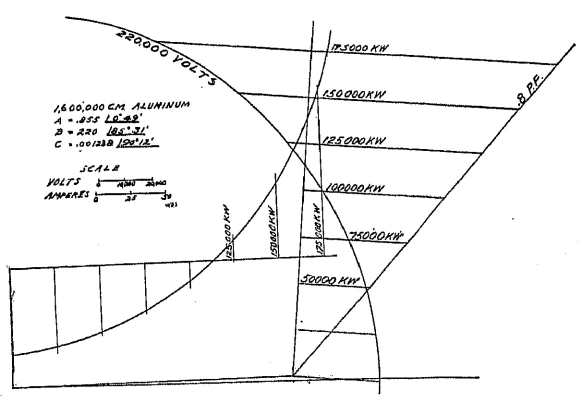

Fig. 13

The 650,000-cir. mil copper is of special design wound over a hollow copper tube which gives it a larger diameter than the ordinary stranded copper cable of the same cir. mil cross section.

With the constants determined, the graphical method as outlined in C. H. Holladay's paper in the A. I. E. E. Journal for November, 1922, was used to determine

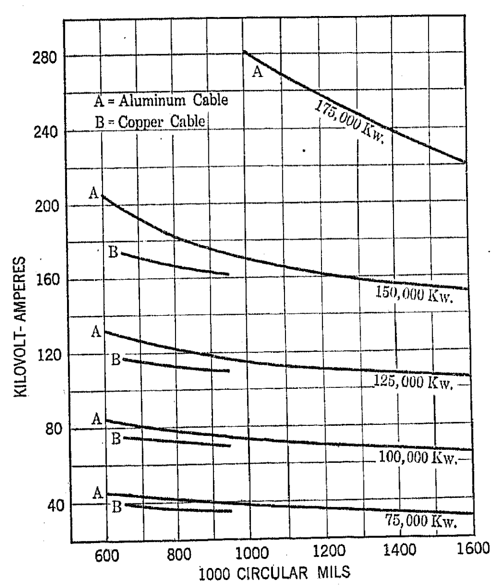

Fig. 14

the condenser kv-a. necessary and the line and transformer losses under different conditions of load. Fig. 13 gives the diagram for the 1,600,000-cir. mil aluminum cable and Fig. 14 gives the condenser kv-a. necessary for different sizes of aluminum and copper cables under various loads. In order to determine the most economical conductor, it is necessary to find the one

TABLE I
AUXILIARY CONSTANTS

Circular Mils....	600,000	650,000	800,000	950,000	1,000,000	1,600,000
Material....	Aluminum	Copper	Copper	Copper	Aluminum	Aluminum
Diameter..	0.96 in.	1.1176 in.	1.031 in.	1.123 in.	1.24 in.	1.544 in.
A.........	0.858 / 1deg.18min.	0.857 / 1deg. 7min.	0.859 / 0deg.46min.	0.857 / 0deg.49min.	0.857 / 1deg. 8min.	0.855 / 0deg.49min.
B.........	236 /80deg.14min.	229 /83deg.37min.	232 /85deg. 5min.	229 /85deg.22min.	227 /83deg.32min.	220 /85deg.31min.
C.........	0.001136/90deg.29min.	0.001163/90deg.18min.	0.00115 /90deg.47min.	0.001177/90deg.18min.	0.001184/90deg.18min.	0.001238/90deg.12min.

for which the sum of the variable costs are at a minimum under any stated condition.

These variable costs were determined and put in the form of annual cost per kilowatt year delivered. The different items making up the variable costs consisted of the annual depreciation and interest on the condensers, cable and towers, together with the annual cost

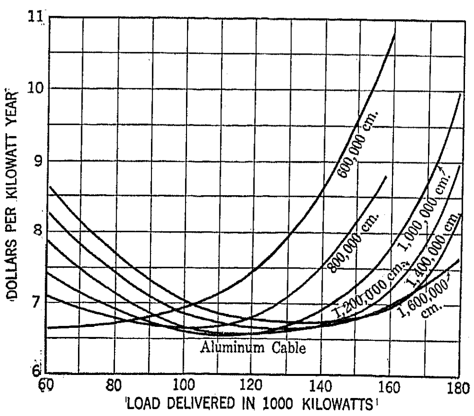

FIG. 15

of energy wasted in the transmission circuit which includes the transformers and condensers, as well as the line itself.

The cost of energy lost was determined by the cost of reproducing it by steam at the same load center, taking into consideration the annual load factor. The

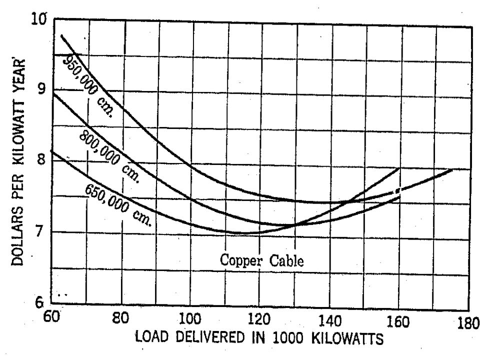

FIG. 16

variation in cost of towers, due to the use of different size conductors was determined by the economical study of tower design given in another part of this paper.

Fig. 15 gives the comparative variable costs per kilowatt year for different loads and different sizes of aluminum conductor. Fig. 16 gives the same thing for copper conductors.

An analysis of the curves shows that 150,000 kw. is the maximum economical load, since the annual costs start to increase either before or at that point even on as large a cable as 1,600,000 cir. mils. There are several sizes of conductors where the cost curve is comparatively flat between 100,000 kw. and 150,000 kw. delivered.

The curves also show that there is no clear-cut decision available as to the correct size of cable. Before the final decision is made consideration must be given to several factors. For instance, the prices upon which the curves are based are those of the early spring of 1924 and are subject to revision, the total initial cost may be a deciding factor and the load conditions under which the line will operate also affects the answer.

Section 4—Vibration of Conductors and Overhead Ground Wires

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Southern California Edison Co., Los Angeles, Cal.

Among the problems connected with the 220-kv. lines, that of the vibration of conductors and ground wires has claimed its share of attention. On account of the larger diameter of the conductors this phenomenon is clearly visible, and exaggerated reports are frequently received that the conductors are jumping up and down 6 in. or more. There have also been complaints of the rattling of towers, even on still nights. In one instance a conductor failed at the outer end of a dead-end clamp and the appearance of the broken strands indicated that vibration might have caused the break. A second conductor was discovered with 28 aluminum strands out of a total of 54 broken at the end of a dead-end clamp. Numerous cases of broken ground wires have been found, the strands breaking at the point of attachment to the towers. In a few cases ground wires have fallen but a larger number of broken strands has been discovered by inspection and repaired before complete failure of the cable. These troubles led to an investigation of the vibration problem as it appears in the spans of the Big Creek and Eagle Bell lines. While this investigation has not yet led to definite conclusions as to the cause of vibration and method of preventing it, the information obtained is presented here as a part of the operating record.

The motion of the conductors is always in an approximately vertical plane and no horizontal displacement of the conductor has been observed. It is comparatively easy to secure records of the amplitude, frequency and duration of the vibrations. This is done by means of the recorders shown in Fig. 17.

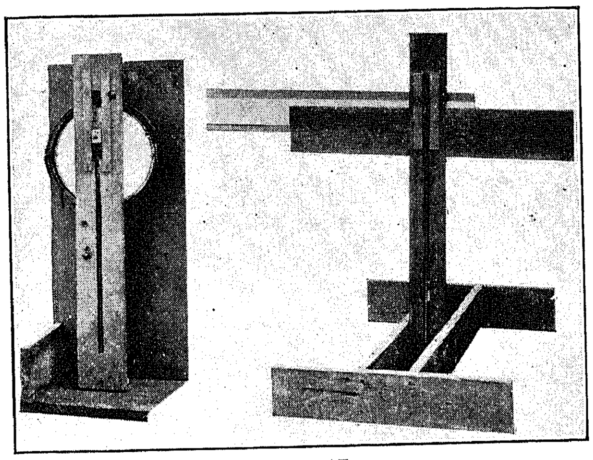

FIG. 17

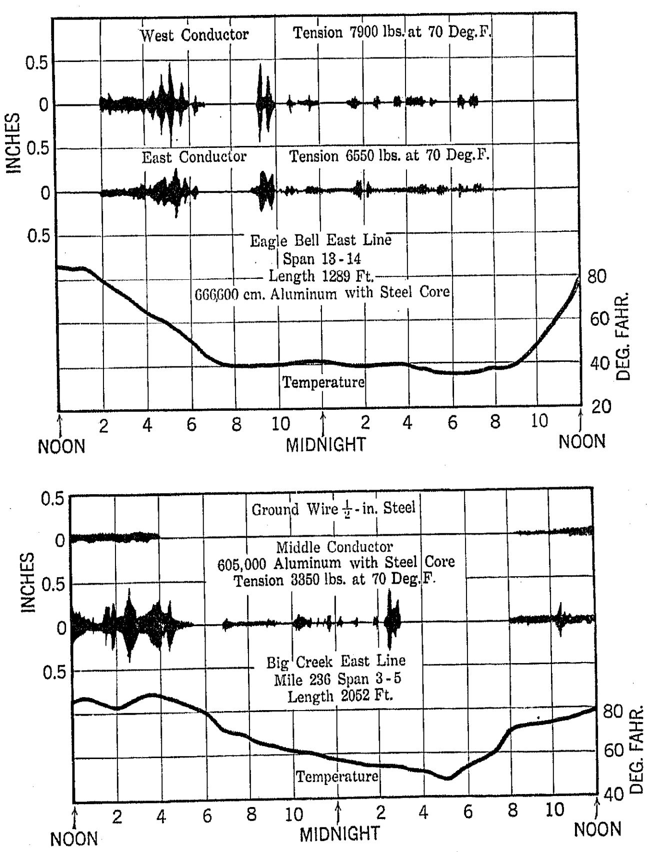

FIG. 18

The motion is transmitted to the recorder by means of a paraffined string thrown over the conductor and attached to the sliding block, which carries the pencil. A spiral spring below the sliding block keeps the string taut. For the frequency records the paper is moved horizontally by hand, the motion being timed with a stop watch. The 24-hr. duration records are obtained by applying a similar device to the circular record chart of a Bristol mechanical recorder. Simultaneous records of vibration and temperature were taken to determine a possible relation between these conditions. These records fail to show any such relation and appear to establish the fact that vibration is generally independent of temperature. A great many spans apparently never vibrate while some are particularly subject to vibration; the longer spans being particularly susceptible. Vibrations are intermittent and the records show that they start and stop at irregular intervals throughout the day. Fig. 18 shows typical records of amplitude and duration. These graphs were transcribed from the original records

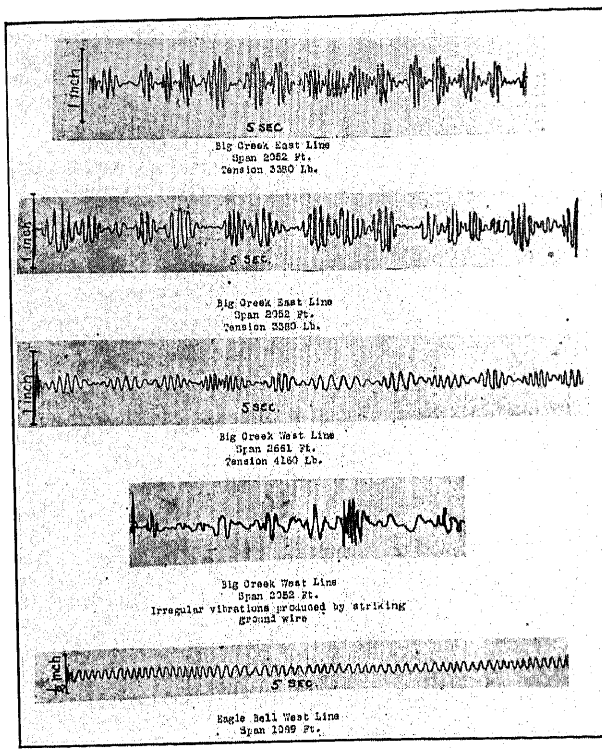

FIG. 19

to eliminate the effect of the stretch of the string which makes an irregular line on the circular chart and obscures the meaning of the record.

Spans subject to vibration were observed under various conditions, such as the sudden changes of temperature, lines dead, lines being energized, lines being dropped, etc., and it appears that vibration is not effected by any of these changes. Apparently this is a purely mechanical phenomenon and might have been encountered on lines of any voltage just as well as on the 220-kv. system.

In span 13-14 of the Eagle Bell line vibration is especially prevalent and it was at this point that one conductor failed. When temporary repairs were made, this conductor was left with considerably less tension than the others. Simultaneous records were taken

of the tight and loose conductors in this span and both were found to vibrate during approximately the same periods throughout the day, although the amplitude and frequency were less in the loose conductor. Apparently conditions other than tension and temperature produce the vibrations. There appears to be no difference between center and outside conductors on the tower as far as vibration is concerned. Several simultaneous records show practically the same periods of vibration for each position.

Fig. 19 shows typical records of frequency, character and amplitude of vibration. Some of the records show simple vibrations of a single frequency, while in others various lower frequency components are present. Amplitudes as great as 1 in. were observed, while frequencies varied from 13 to 30 cycles per second in the spans tested. Standing waves 12 to 16 ft. in length were found in most cases where vibration was present. The normal sustained vibrations are regular in frequency and character and this quality appears to be essential to maintaining the vibration of the conductor. Attempts to set-up vibrations in the ground wire by striking it with the hand resulted in irregular vibrations which soon died out.

Vibration is believed to be due to air currents. Strong winds do not usually produce vibration but appear to have the effect of damping out the oscillations by setting up swaying motions of the wires. A slight air current appears to be sufficient to start and sustain the vibrations.

Means of preventing vibrations are still under consideration. It seems probable that an irregularity in the weight of the conductor might tend to damp out the vibrations and such a plan has been considered. Weighting of the conductor at various points might accomplish the same result. However, the places most affected by vibration are at dead-end clamps and other points where the waves are reflected, and there is objection to increasing the number of such points by clamping weights to the conductor. The most violent cases of vibration have been noted in spans which have been in service for 10 years or more and the fact that no failures have occurred in such spans indicates that vibration alone is not much to be feared. The suspicious failure of steel reinforced aluminum occurred in the Eagle Bell line where the mechanical stress is higher than in the older line, and it seems probable that high stress combined with vibration is liable to cause trouble. However, the evidence is not conclusive as to how much the vibration would contribute to the failure.

The most promising method of preventing the damage due to vibration at points of reflection appears to be to provide a joint where movement can take place without bending or shock to the cable, and new clamps have been designed with this in view. Fig. 20 shows a new cast aluminum dead-end clamp which is being tested. The mass of the clamp has been reduced to a minimum so that it will move about its supporting pin, vibrating as a part of the conductor instead of reflecting the waves from its outer end by reason of its inertia. Simple mechanical devices are provided for holding the aluminum strands and steel cores. The aluminum is held by the standard compression joint which has proven by long service to be electrically and mechanically good. The steel core is held separately by long taper conical wedges which have been tested to the ultimate strength of the core. Several other improvements are embodied in the design. Cast aluminum terminals are attached to the jumpers by means of compression joints and a plain bolted connection is made between the jumper terminal and dead-end clamp. This arrangement eliminates the troublesome process

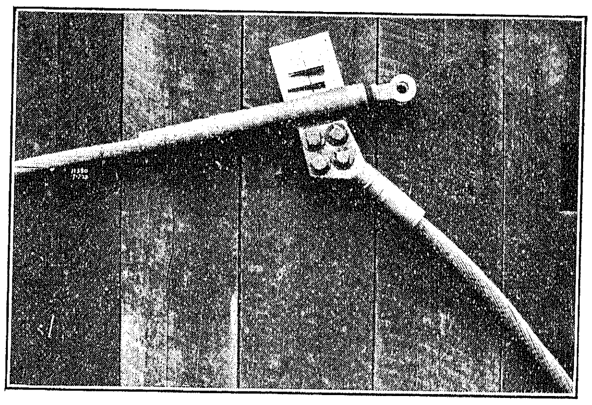

Fig. 20

of bringing out the aluminum strands separately at the side of the clamp for attaching the jumper, and the more or less uncertain bolted clamps for splicing the jumper. Ample current capacity is provided in the cast aluminum body of the clamp and current-carrying connections are all made with the standard compression joint or by means of flat bolted surfaces. This clamp has been installed on an experimental span and will be tested to determine its performance under vibration conditions duplicating those on the line.

Section 5—Location and Right of Way

BY V. D. ELLIOTT
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Transmission line location is an art which has been somewhat neglected as compared to other branches of the electric utility industry, or, as compared to railroad or highway location. This has come about in a natural way because transmission lines in the beginning were comparatively cheap affairs and even with the advent of steel tower construction and higher voltages the cost per mile was far below that of a railroad or well constructed highway. Often much time and engineering skill were expended on power houses, dams and pipe lines and even on transmission line design, but the location and construction of the

line were put off until the last moment and then rushed through in a haphazard fashion with resultant losses.

The cost of 220-kv. circuits on separate parallel lines of towers, such as the recently constructed Eagle Bell line, is in the same class with that of a railroad or highway, hence the business of selecting and securing a right of way is worthy of considerable thought and careful balancing of all factors involved. Many points which are of minor consideration on lower voltage lines become of real importance on the 220-kv. line.

In choosing a location for the Eagle Bell line and for a new line to Big Creek, known as the Vincent Line, which is still in the process of being located, the same general methods that are applicable to any transmission line location work have been used but with modifications or refinements which will be described.

In rough or mountainous country the controls are the natural topographic features and the main points to be borne in mind are safe and suitable locations for towers and a general alinement such that material may be delivered by motor truck via existing roads, or such that new roads can be built without too great an expense. With high tonnage of material and men to be delivered, this becomes a very important item and must be given more consideration than with smaller lines.

In flat or agricultural lands the controls are all artificial or man-made and influence the right-of-way cost.

The final choice of location must usually rest on reconnaissance work which proceeds the actual line survey. This reconnaissance is therefore a very important part of the job and holds great possibilities for saving or spending money unnecessarily on construction or right-of-way purchase.

The problem naturally divides itself into the two classifications previously indicated, *viz.*, the case of rough or mountainous country of little or no commercial value and the case of relatively flat, tillable land highly developed with crops, highways, railroads and farm or residential buildings where land values are high and natural topographic features of little or no importance. There are, of course, gradations between these two extremes in which may be included land of low commercial value possessing no controlling topographic features. In this case no problem exists and a straight line is laid out.

A portion of the Eagle Bell line is located on a right-of-way 250 ft. wide, in a highly developed territory near or within incorporated city limits where land values range from $2,000 to $5,000 per acre. Aeroplane reconnaissance was found to be a very valuable help and aerial maps of this territory were particularly useful in projecting the line among improvements to the best advantage and buying parcels of land or negotiating for a strip without the inevitable advertising resulting from a survey. It should be understood that useful as these maps and aeroplane reconnaissance were, they did not reduce the amount of final survey work necessary nor entirely eliminate ground reconnaissance. The actual saving, due to the speed and accuracy with which the purchases proceeded, cannot, of course, be determined but all indications are that it was many times the cost of the maps.

A location through land of this character giving minimum total cost resulted in a very crooked line.

Aeroplane reconnaissance over mountains or hilly lands gives one a comprehensive general view and impression which cannot be secured in any other way. At a height of from four thousand to six thousand feet above the ground details are visible in good weather but the topographic features assume their proper proportions and relations to one another so that important ones stand out and produce the proper impression on the mind of the observer. When observing from a high point on the ground, part of the terrain is hidden from view and a certain distorted impression is gained due to the different distances from the eye of the various features in the range of vision.

Despite this defect in ground observation it is a necessary supplement to the aeroplane view in choosing a final location. In addition to this preliminary or trial lines are often justified. These may ordinarily be run by stadia and only the major controls or those of especial bearing on the question taken. While a profile of this kind does not accurately represent the country, it is a great aid and will usually contain enough information to enable one to make an intelligent choice between two or more alinements. When it is considered that the cost of one tower is equal to the cost of several miles of such preliminary line the wisdom of this refinement is evident.

Experience with aerial maps of mountainous territory show them to be difficult to produce with satisfactory accuracy and to be of little benefit. Dangerous and adverse flying conditions and scale distortions, due to uneven ground elevations, multiply enormously the cost and mechanical difficulty of producing accurate maps and the information conveyed concerning relative elevations and other controlling topographic features is so meager that their cost cannot be justified.

Having chosen a final alinement, a survey must be made which will show to a predetermined accuracy the horizontal and vertical distances between points on the line and also the relation of the line to property corners and legal subdivisions of land. This last is, of course, purely for right-of-way purposes. This general statement regarding survey is true of a transmission line of any voltage but with the 220-kv. line, the cost and importance of it justifies methods of survey which will give a higher degree of accuracy and are less likely to contain errors than those frequently used.

Two methods have been tried thus far. In the first case, conditions were ideal for choosing an alinement,

setting signals on probable tower locations and tying these points into a triangulation system from base lines conveniently located. Stadia topography was taken in the vicinity of the probable tower points and at points of doubtful clearance in between. No center line profile was run but one was plotted from the topography. In the second case, a center line profile was taken by means of slope chaining in the roughest country and horizontal chaining with a line of levels where the country was not so rough. While the triangulation method with local topography worked out fairly well where it was used, the second method appears to be better and of more general application.

The Eagle Bell right-of-way is intended to accommodate three tower lines of horizontal construction, the towers being 78 feet apart, center to center. The right-of-way is 250 feet wide and with a conductor spread of 22 ft. 3 in. leaves only 24 ft. 9 in. from the outside wire to the edge of the right-of-way. With a calculated conductor swing of 18 ft. 6 in. in the normal span, a clearance of 6 ft. 3 in. remains to the edge of the right-of-way. This is not all that could be desired but on the high priced land this was considered the maximum permissible width. On public lands in the National Forest and private lands of lower value a 300-foot right-of-way was secured. On the new Vincent line to Big Creek a 200-foot right-of-way is being secured to accommodate two tower lines.

Projecting and securing a right-of-way as wide as this becomes a real problem, not only because of the acreage involved but also on account of the difficulty of finding wide enough spaces between obstructions.

On the Eagle Bell line the right-of-way was purchased in fee. Due to its proximity to cities and towns and rapidly developing territory this was considered a wise precaution in order to make the position of the line more secure. With a right-of-way of this width it is practically necessary to purchase in their entirety small land parcels which are hopelessly cut up by such a strip. The left-over pieces can, however, be grouped together or rearranged and sold to advantage when they are under one ownership. This was done on the Laguna Bell line and, due to the rising real estate market, showed a profit in many cases.

On the Vincent Line, easement is being secured except for a few cases where fee is being purchased for the same reasons as on the Eagle Bell line. The protective clauses in the easements are more effective than usual and for the class of territory traversed easements are believed to be adequate.

The best job of right-of-way location and survey work may be set at naught by careless or injudicious location of towers and special effort should be made to insure that this part of the work be accurately and carefully done. The old reliable centerline profile and celluloid template of wire curve are found to operate on 220 kv. as well as on 60 kv., but due to the wide spread of the conductors, it is necessary to take account of the cross slope of the ground at critical places to a greater extent than with the lower voltage. This is especially true of single-circuit horizontal construction in which the spread is greatest.

The experience in tower locating on the Eagle Bell line showed that in rough country the exact location of towers, as well as their elevation and type of extension, if any, must be determined on the ground. Reasonably good topography with a one-foot contour interval was found inadequate for properly locating towers, mainly because of soil or other conditions which could not well be shown on a map.

Discussion

PAPERS ON SOUTHERN CALIFORNIA EDISON SYSTEM

(Michener, Stauffacher, Shaw, Carlson, Gaylord, Elliott)

Pasadena, Cal., October 14, 1924

H. A. Barre: As you know, our initial problem was something very different from building a 220,000-volt line. We had the existing Big Creek Line. We didn't have time to build a new line; we had to get in and make that one work at a higher voltage to carry the additional amount of power that we had to transmit. That was quite a serious problem.

The failures that have occurred on the line have been very instructive. One incident will be sufficient to show the things we ran into. In making the shield rings we worked over a number of different possibilities and finally hit on the idea of using aluminum alloy. We were told by the Aluminum Company that aluminum was a metal which would melt and drop away under heat and hit the ground in a comparatively cool condition and eliminate one of the causes of trouble we had which was setting fire to the whole countryside during our dry summers. Of course, you know what happened The concentrations of power were so enormous that the aluminum rings actually ignited and burning pieces fell like a star-shell and set the ground afire. This condition applies to the shield and not the aluminum conductor.

Now, in operating the line we clear away the grass around the towers and in that way eliminate the trouble.

In regard to the matter of relays, I will say that, of course, we know that relays are not 100 per cent perfect; however, they are better than nothing, very considerably better.

R. Wilkins: I would like to talk on Section 2, the "Protective System." In the past most of the time and energy has been spent on the economic construction of transmission lines and very little time spent on the possibilities of being able to operate those lines economically after they are constructed.

Relays have made such networks as are now operating on the Pacific Coast possible and economically feasible and in the final analysis it is switching and relaying that will determine how many and what kind of lines there shall be to deliver large blocks of power.

In the company by which I am employed it has by trial been proved that it is not feasible to break up that system into sections in times of trouble and the accepted method now is to separate completely the smallest practicable section of that network in such a manner as to give the least possible disturbance.

We use a relay system in which each line is complete in itself and which allows selective directional tripping on any number of lines. In this work relays relying on direction of trouble are superior to those using time for selectivity, and relays using current only for direction are superior to those using both current and voltage on account of power-factor troubles.

With the power available in a large network it is essential that a line in trouble be cleared at a low current value and very

quickly in order to prevent serious damage. Emergency repairs on a 500,000-cir. mil line is a real job and takes considerable time.

On the high-tension network of the Pacific Gas and Electric Company we now clear selectively on grounds and unbalances of about 25 per cent and in all cases less than ½ of the normal line current. During the year 1923 out of 1569 high-tension relay operations there were 1490, or 95 per cent correct.

If the relays don't clear a line attached to a system which can deliver approximately half a million k-va., it takes but just a fraction of a second to do considerable damage. The lower current for which you can get the relays set and still make them selective, the better relay system you will have.

J. Mini, Jr: I am glad to see that Mr. Elliott does not hesitate to speak of transmission-line location as an art. In the final success of the working of a transmission-line system, and without a careful consideration of this problem much money can be squandered. Regarding the use of airplane surveys, I will say that we have tried that to a small extent. It gives, as stated in the paper, a comprehensive picture as a whole and saves running a number of preliminary locations, but in the end the final detailed location work is not in any way minimized. Another important point is, I think, that it does avoid preliminary advertising that goes with a survey party and which immediately causes property values to sky-rocket over night.

A wide right-of-way is desirable especially through timber country to better avoid accident and forest fires. It is possible, sometimes, to have a narrow right-of-way by arranging to cut all of the tall trees outside of the right-of-way which later might fall across the line. In a rough hillside country it seems important that a proper fill be made for the particular conductor on the uphill side of the right-of-way. While we have, in the past, made office locations, with final checks in the field, we are changing our practice and doing more of the locating directly in the field without the necessity of taking notes to the office and having them returned again later to the field.

The paper by Mr. Gaylord, on the vibration of conductors, is an interesting and important one. We had some trouble from this cause on our long Carquinez span. The details of this trouble and how it was remedied, were discussed by Mr. L. J. Corbett last year at the Del Monte Convention and I will say nothing further here.

It has been our practise to string our conducters with moderate tensions, meaning more sag and lighter towers. This matter was presented in a paper last year by our Mr. Dreyer at the Del Monte Convention.

The paper by Messrs. Carlson and Shaw about "Economic Studies of Transmission Line Design with Particular Reference to the Mechanical Features," is a modified form of Lord Kelvin's law abreast of the times, and includes many factors not thought of in the day of Lord Kelvin. Since some of the intermediate steps are not shown I am not sure whether the corona loss was figured in but I presume so. It is fortunate that the economic zone extends over such a wide value of kilowatts delivered because it must be remembered we cannot afford to lower them to the economic limit for when one of the birds that Mr. Michener speaks of comes along we are going to be deprived of that circuit for awhile, at least, and the remaining circuit must have a margin of capacity left to take care of the loss temporarily.

With reference to Mr. Michener's paper, I will say that some flashovers on the Pacific Gas and Electric Company's system have taken place, on both the 110-kv. and 220-kv. systems. The trouble on the 220-kv. line consisted of thirteen flashovers while still operating as a 110-kv. system. There were a number of flashovers on the 110-kv. system and only a few on the 220-kv. system.

I will classify flashovers as of four kinds: (1) That due to being struck directly by lightning; (2) dirty insulators, with light drizzling rain; (3) dirty insulators with dew formation. The two last classes of flashovers occur during the night. We have never had very much experience with dirty insulators where the trouble has occurred in heavy rains, but in light rains, or heavy mists, there has been considerable trouble. The dew-formation trouble is of somewhat a different character. The insulator is dirty, as in the former case, but the trouble usually does not happen until the early morning hours, practically at sunrise, or on cold nights, but the insulator has been lowered in temperature during the night period, and in the early morning hours, practically after the time the sun rises, dew will rise off of the surface of the earth as high as the insulators in the towers. When this highly humidified air reaches the cold surface of the insulator, beads of water are condensed on the surface of the insulator and often form to such a large extent that the different beads are almost connected, making a heavy sheet of water, and, under this condition it is not hard to see how flashovers may occur.

Then, there is the fourth class of insulator flashovers which we call the mysterious type. It occurs on insulators that are in the very best of condition regarding their surface resistance. It occurs in the middle of the day, in the bright sun when the temperature is high and of the insulators that have been examined the surface was found in perfect condition. As I have said, we call those the "mysterious type." We have, for some years, had various devices on the lines trying to see if there was some transient condition of rising voltage which might be responsible, but so far we have not been successful in finding any evidence to that effect. Nevertheless, we still believe that such a possibility is there.

Now, as regards the "bird theory:" I am in perfect accord and willing to blame the bird for a portion of these flashovers of which I call the "mysterious type." We have had absolute evidence that this is so in a few cases. I refer, particularly, to types of strings of insulators that were formed, or hung on the tower in the form of an inverted "V" with the apex at the cross-arm and insulators spread at the base where they were clamped at the wire. I know of one case, in particular, where the flashover was seen by a rancher and the trouble was found by the patrolman after he had notified him. In that case the evidence was so clear that there was no doubt about it. The bird excrement had run down the insulators, which formed a series of steps, being hung at about a 45-degree angle, and the arc absolutely followed the path of this excrement. There have been a great many strings of flashovers found perfectly clean and no evidence of any bird soil on them. We have suspected for sometime that the theory advanced by Mr. Michener was possible. As far back as 1915 we attempted some experiments, which are described in the paper as being credited to Mr. Peek, but we were not as successful, however, as Mr. Peek was, in our experiments. We poured some white-lead paint down alongside the insulator string, but due to the fact that the testing transformer was of small capacity we were not able to establish an arc following the path of the paint. However, I think if the testing transformer had been of a large capacity, as no doubt Mr. Peek's was, we probably would have established a similar experiment. I think the way to make this experiment, however, is actually to take a real transmission line with lots of power behind it, get it out of service and then pour down some liquid of that type. I think the experiment will show, without doubt, that it is entirely feasible to cause flashovers and that they do happen by this very method.

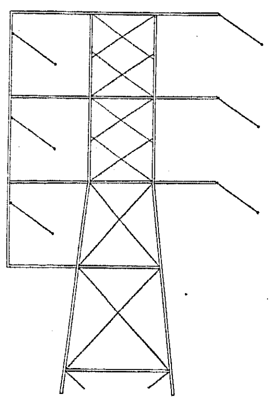
I have some photographs which I took out of a string of insulators which had been on the tower but a few months. The bottom of the string has an insulator shield of spun copper 16 in. in diameter. That string of insulators has in it units 10 in. in diameter and several units 13 in. in diameter. That is an overhang on each side of 1½ inches. The photographs plainly show some of this excrement had caught on the 1½-in. ledge and yet the string of insulators was perfectly clean.

H. C. Sutton: The papers seem to indicate that vibrations are more serious in the longer spans. We had a case with one of

our companies where vibrations were observed in 100-ft. spans on a 11,000-volt copper wire where the vibrations were quite excessive in a fairly light wind. I have also heard of a case where vibrations on a similar line were great enough to lift the insulator off the pin. This particular type of pin was of the lead-tip type where the lead had been disintegrated due to the vibration.

We have experienced trouble recently in Connecticut on 66,000-volt lines with normal spans of 800 ft. The lines were built of 2/0 and 4/0 copper. During a severe wind storm with the wind at an exceedingly high velocity, there were four locations in the line, which is 30 mi. long, where the wind apparently deflected from the side of the hill and raised the conductor a distance of at least 8 ft. and wrapped the conductor around the conductor above it. Now this trouble may have been due entirely to the excessive wind and not to vibration, however, vibration may have contributed to the trouble.

It has also been observed in some places that there are certain spans that are more subject to vibration than others and these spans are not the longest spans in these particular lines. Apparently there is some harmonic condition set up where the vibration reaches a considerable magnitude, the vibration being sufficient to bring the conductors together. This harmonic



Cut Showing Recent Method of Avoiding Dead-End Construction on Angles

vibration has been corrected by changing the sag in the conductor.

Now, in the particular instances where we had trouble with the 66,000-volt lines in Connecticut, we decided to change the sag in the wire. We decided that if a harmonic condition was in any way responsible for the trouble, at least we could correct this condition by changing the sag in the wire. We would like to have sent observers to check up these spans and to determine definitely if a harmonic condition contributed to the movement of the wire, but we felt that we could not afford to run the risk of having a reoccurrence of this trouble.

In looking at some of the high-tension lines in California, I note that there seems to be a considerable number of dead-end points in the line, that is, where the insulators are in dead-end position. Some of these dead-end points are at locations where no angles exist. If vibration is serious, of course the dead-end points should be avoided as far as possible. There are other reasons for avoiding dead-end construction, in that dead-end construction causes other trouble. With the higher voltages the wire loop at dead-end points is excessively long. These loops sometimes swing into the tower and this in turn makes necessary the holding of these loops down by some complicated mechanical means.

In the State of Connecticut where we built a number of 66,000-volt lines, we have avoided the use of dead-end construction as far as possible. We cannot construct our lines on perfectly straight right-of-way. It therefore becomes necessary to install angles in the line. At these locations we have used the so-called pull-down type of construction, installing the pull down on the side of the tower where the line would swing into the tower. On the other side of the tower we allow the conductor to swing away, putting in a single string of insulators. There are other methods of avoiding dead-end construction on angles. A recent method is shown in the illustration. This will allow the use of single strings of insulators on each side of the tower and will avoid some of the objectionable features of the pull-down type of construction.

D. I. Cone: The problem of the location of a high-voltage line, as so interestingly discussed by Mr. Elliott, has a deep interest for the communication engineers. Actual experience on the Pacific Coast has shown that with these very high-voltage lines the situations of proximity between power and communication circuits are limited to occasional crossings and situations in which there is parallelism at separations of the order of 1000 ft. Under these circumstances the fields of influence, with which we are concerned, are chiefly the magnetic fields, due to current returning through the earth and to balanced load currents. The latter can be cared for by transposing. Thus, the chief problem becomes that of guarding against the effects of earth-return currents arising from star-connected transformers with grounded neutrals, the corona effect that has been discussed here in previous papers, or other causes.

We have two situations to guard against: transient disturbances, at time of flashovers or other accidents, and the steady-state or normal, induction. The data given in the symposium by Messrs. Michener and Stauffacher, on the occurrence of flashovers and the amounts of ground current, are a most valuable contribution to the knowledge of the communication engineers, enabling them better to gage the frequency of occurrence and severity of these disturbances. Likewise, for the steady-state conditions, the data presented in such papers as that of Mr. Wilkins are of great value.

J. A. Johnson: I would like to add a small contribution on the subject of vibration and also on the subject of insulator flashovers.

Some years ago the Canadian Niagara Power Company built a transmission line from their Canadian Plant to Buffalo, which crosses the Niagara River between Fort Erie and Buffalo, with two spans, one of about 1900 ft. and one of 2200 ft. These conductors, originally, were all aluminum and this vibration trouble was experienced on that crossing so severely that the conductors were crystallized at the point of attachment to the towers and in a few cases broke and fell into the river. The solution of the trouble adopted at that time was to interpose a chain between the conductor and the point of attachment to the tower thereby changing the constants of the vibrating system and introducing an ineriat effect from the weight of the chain so that the vibration was considerably mitigated. However, this solution as applied there was not completely effective. The conductors continued to break and were later changed to copper-clad steel. However, it would seem to me that a solution of this character, with possibly a graded weighting of the conductor as it approaches the point of support, might constitute a solution of this particular trouble.

Now, in the matter of insulator flashovers due to deposits on the insulators: One of the most recent 60-kv. lines which we have built passes in fairly close proximity to the chemical plants at Niagara Falls where one of the products is hydrochloric acid. This acid is stored in a tank outside of the plant proper and this tank gives off a considerable amount of fumes. This particular line is located immediately on the bank of the Niagara River and in the Spring and Fall is subjected to a considerable amount of fog. During these fogs, in the early morning usually, we have had cases where the insulators, four units of Jeffrey-Dewitt, have flashedover. We made an investigation of this trouble, taking samples of the material which we found on the surface of the insulators, which is, when moist, a slimy substance that is

practically impossible to wipe off the insulator. We found it contained a mixture of zinc and calcium chlorides. The zinc chloride came from the galvanizing of the hardware of the insulators, which was completely removed from the iron. The calcium came from a nearby carbide plant which belches into the air a large quantity of lime dust. The solution we found was to wash the insulators and we have been doing that now for nearly a year, both on these particular transmission towers and also at our substation, which is in this same neighborhood. We do this while the line is alive. The substation is a mass of insulators of both suspension and post type. We use a fire hose with about a ¾ in. stream and play it over this mass of insulators, allowing the water to fall on the insulators like a heavy rain. On the transmission towers we have placed pipes up the towers with nozzles arranged to play a stream on the insulators and we periodically visit them with a pump mounted on a truck and drop a suction hose into the river and play the water on to the insulators. We find this effective in preventing this trouble.

It will ill become an Eastern engineer, to whom mother nature has handed her water on a silver platter, you might say, to attempt to suggest to Western men how you might get the water to the insulators. However, I have no doubt that your hydraulic engineers, who have shown themselves so efficient in taking water from the place where it is and is not wanted, to the place where there is none and it is desired, will find some way to do it if the electrical engineers decide they want the water there.

L. M. Klauber: In washing insulators we have been compelled to reduce the amount of water used because of inaccessibility of the lines and the scarcity of water along the routes. We have recently used an air-compressor outfit, applying a combined air-water spray with a nozzle of the type used in acetylene welding and by this means have succeeded in cleaning insulators using only a few gallons of water per mile of line. We find that with a nozzle of this type an increase in the quantity of water above a certain amount lessens rather than increases the cleansing effect. The air actually does the cleaning and the water serves only to moisten slightly the deposit so that the air can blow it off. This may be of interest to those who operate in arid or mountainous territory.

The work of cleaning insulators by using compressed air is not only better, from the standpoint of water haul, but is likewise a labor saver. We have found that one crew of men, with one compressor, can clean approximately four miles of line per day, whereas, with the old method of washing the insulators by hand, which frequently necessitated actually removing the insulators from the line in order to clean them properly, a crew of men could hardly cover two miles per day. The insulators washed in the new way are washed with greater thoroughness than by hand washing because the air-water jet thorougly cleans the grooves underneath the insulator. We washed about eighteen miles of line about six weeks ago; prior to the washing we were having flashovers at the rate of from three to six per night, depending on the condition of the atmosphere. Since washing them we have not had a single flashover.

Interconnection of Power Systems
in the Southeastern States

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Review of the Subject.—*Superpower systems have become one of the prominent economic and political factors of the day. Because in the Southeastern States interconnection of systems and interchange of energy have been practised longer and to a greater degree than almost anywhere else. The author reviews briefly the history of interconnection in this section and the underlying principles that brought this about.*

The advantages of interconnection in reducing capital investment, operating expenses and outages, as compared with isolated operation are shown.

The necessity of thorough engineering studies to get the best use of the interconnected systems is brought out.

The importance of having the proper kind of interchange contract is stressed and the workings of some interchange agreements are illustrated.

The load dispatching and other operating problems are discussed, bringing out the necessity of careful voltage and frequency control, accurate adjustment of protective relays and the proper handling of wattless current.

The value of close cooperation through engineering and operating committees in studying the most economical handling of present plants and the proper sequence of new development are stressed.

* * * * *

SUPERPOWER, or giant power, as Governor Pinchot prefers to call it, has been very much before the public for the last two years. General Tripp with his clear vision and masterly exposition of this great subject has done much to clarify and make understandable to the general public the problem involved and the effect of its solution. The appeal to the imagination of a completely electrified country has resulted in great public interest in the matter, so that it is but natural for the politicians seeking an issue and the advocates of government ownership to give their attention to this new problem as being fraught with more possibilities than the more settled railroad, telephone and telegraph systems. While there is a tremendous engineering problem involved, there are other and equally important phases that must be carefully considered before superpower and complete interconnection will be an accomplished fact.

To those of us actively in public utility work, the widening out of the field has been so gradual that unless we mentally turn back ten or fifteen years we cannot properly appreciate what tremendous advances have been made, nor yet how much greater fields there are ahead. Our problem is always to serve the public, but who is the public? Thirty years ago it meant the dozen blocks that could be reached at 110 volts or 220 volts d-c. from the old Edison bipolar. Twenty years ago it meant a radius of 25 miles around a plant with the beginning of real central station development and transmission. Ten years ago systems had widened to take in many hundred square miles of territory with straightaway transmission of power over distances up to 250 miles at 132,000 volts. Today great systems in adjoining states are being interconnected so that we have one great system covering many states, and serving thousands of square miles of territory and millions of people.

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., October 13-17, 1924.

Our great problem for the next ten years is to increase the capacity of these interconnections, to develop the distant waterpowers, the mine mouth, or other strategically located (from an economic standpoint) high-capacity steam plants and connect them with the great load centers by means of superpower networks at 154,000 volts or higher.

Up to this time the growth and development of our utilities has been rather haphazard. We have been in an unstable condition, due to a number of causes, and the load has usually been ready in advance of the power plant and system. The reversal of the government's attitude of 10 or 15 years ago, culminating in the passage of the federal water power act, and sane public service commissions have been two great factors in stabilizing our business and in giving the public confidence in our securities. As a result public utility securities are now enjoying an unprecedented popularity. This makes the financing of needed extensions and new developments a much simpler problem and one that can be handled on a sound economic basis. If we are wise we will now plan, as some companies are doing, for ten years in the future. The use of central station power has become as universal as the use of the telephone and its growth can be predicted just as accurately. Planning ten years ahead will appall some of us because we find it hard to believe our own figures, and yet a look backward should convince us of their correctness.

In less than ten years nearly all of us will be faced with the problems of interconnection, power interchange and the operation of interconnected systems. While these problems differ in detail in different parts of the United States, there are certain elements in common. For this reason a review of what has been and is being done in the Southeastern States may be of interest to members of the Institute.

The territory embraces Mississippi, Alabama, Tennessee, Georgia, North and South Carolina. The

principal power companies in this territory are Southern Power Company, Carolina Power & Light Company, Georgia Railway & Power Company, Central Georgia Power Company, Columbus Electric & Power Company, Alabama Power Company, Birmingham Electric Company, Tennessee Electric Power Company and Memphis Light & Power Company. The accompanying map shows the territory and the principal transmission systems.

The drainage area is that of the southern extremity of the Appalachian Mountains. The flow characteristics of our rivers are very different from those of the far West or the North. We do not have the melting snows of high mountains to give us the maximum stream flow during the summer and fall seasons when our ice, cotton-gin and cottonseed-oil mill loads are heaviest. W. S. Lee, a distinguished member of our Institute, has well called them "the fugitive waters of our Southern streams." Variations of 1000 to 1 between maximum and minimum are common, and the wide variations in stream flow necessitate large steam reserves or storage reservoirs. The stream flow curves shown herewith are typical.

The topography of the country is such that the majority of power developments utilize heads of from 50 feet to 150 feet. The exceptions are the plants of the Georgia Railway & Power Company on the headwaters of the Tallulah River, where a maximum head of 608 feet is developed in one plant and the plants on the upper Ocoee River of the Tennessee Electric Power Company where heads of 250 feet are developed.

There are no coal deposits in Georgia or the Carolinas or Mississippi. Freight rates from the Alabama, Tennessee, Kentucky or West Virginia fields to Georgia or the Carolinas are high. Therefore, the great steam reserve plants of the future for this district will be located in Alabama or Tennessee, with the probability of the connection of the North Carolina systems with mine mouth steam plants in West Virginia. The Southern Power Company in its Bridgewater plant has developed a water storage equivalent to 110,000,000 kilowatt hours of energy. The Georgia Railway & Power Company, in its Burton and Mathis Dams has developed approximately 90,000,000 kw-hr. of storage. Alabama Power Company, in its Cherokee Bluffs project, now building, will have an available storage of 60,000,000,000 cu. ft. of water, which is equivalent to 300,000,000 kw-hr.

Hydroelectric development in the South on a large scale was started by James Duke and W. S. Lee of the Southern Power Company about 20 years ago. Mr. Duke started the cotton mill industry to manufacture cotton goods from the raw cotton near the point of production, using power from the hydroelectric plants he built to run these mills. As a result, the marvellous growth of North Carolina has not been exceeded by any state in the Union. The system serving this territory has one of the greatest transmission networks in the country and its peak load is in excess of the 275,000 kw. It was followed by the Yadkin River Power Company in North Carolina, the Georgia Railway & Power Company on the Tallulah, The Tennessee Electric Power Company on the Ocoee, the Columbus Electric & Power Company on the Chattahoochee, and the Central Georgia Power Company on the Ocmulgee River. Development in Alabama did not start until 1912 when construction of the Lock 12 Plant on the Coosa River was commenced. The load was developed very rapidly by Alabama Power Co. and today it ranks second to the Southern Power Company in output.

The first interconnection between large systems in the South was made in 1912 near Atlanta between the Georgia Railway & Power Company and the Central Georgia Power Company. In 1914 the Georgia Railway & Power Company connected with the Columbus Electric & Power Company at Newman, Georgia, and about this same time with the Tennessee Electric Power Company near Rome, Georgia, and with the Southern Power Company at their Tallulah Plant. In 1921 Georgia Railway & Power Company and Alabama Power Company joined their systems at the state line by a 110,000-volt line from Gadsden, Alabama, to Lindale, Georgia. This year a 110,000-volt line was completed between North Auburn, Alabama, on the Alabama Power Company's system and West Point, Georgia, on the Columbus Electric & Power Company's system. The Georgia Railway and Power Company is now completing a new 110,000-volt line between Lindale, Georgia, on its system, and the Ridgedale substation at Chattanooga on the Tennessee Electric Power Company's system. There is under consideration a line at 110,000 volts between Huntsville, Alabama, on Alabama Power Company's system and Hales Bar, Tennessee, on Tennessee Electric Power Company's system. We have a complete interconnected network embracing ten companies and extending over five states. An important feature is that the interconnections have what until recently would have been considered high interchange capacity, that is, of the order of 20,000 kilowatts to 25,000 kilowatts.

In the Birmingham District, the Tennessee Coal Iron & Railroad Company has ore and coal mines and steel mills with a demand in excess of 40,000 kilowatts of electrical power. The advantage of an interconnection with a large system was realized and an interchange contract was made in 1914 with the Alabama Power Company. At night and over week-ends the Tennessee Coal Iron & Railroad Company is able to return a large amount of power at low cost as the by-product gas used in firing the boilers would otherwise be wasted. The interconnection has also proven of great advantage to the steel company because it has permitted rapid expansion of its finishing plants without the delay and cost of construction of additional power facilities.

The fall line marking where the rivers of the Central States and of Tennessee and Alabama break through to the costal plain swings down from Keokuk on the Mississippi to Muscle Shoals, Alabama, on the Tennessee, to Lock 18 a few miles above Montgomery on the Coosa, to Tallassee on the Tallapoosa River and to Columbus on the Chattahoochee River. There is no water power of any magnitude south of these points to the Gulf of Mexico. If, therefore, Mississippi and Louisiana are to have the benefit of widely distributed hydroelectric power, it must come from Alabama. It is logical to anticipate, therefore, that within the next five or ten years power from Muscle Shoals and other Alabama hydroelectric plants will be transmitted to Memphis, Tennessee, across Mississippi and even to New Orleans, Louisiana, a distance of approximately 300 miles. When this is done, seven states with a population in excess of 16,000,000 people will be served from what, due to interconnection, will be virtually one great system.

Prior to 1920 the lack of any federal water power laws hindered hydro development on so-called navigable streams and forced the building by the Alabama Power Company, in 1916, of the Gorgas Steam Plant about 35 miles from Birmingham, Alabama, in order to take care of the rapidly growing load which exceeded the low water capacity of Lock 12. The Gorgas plant is exceptionally located from the economic standpoint, situated as it is on the shore of Lake Bankhead, formed by the government navigation dam at Lock 17 on the Warrior River, and in the center of a large coal field with good steam coal mined within 1000 feet of the plant. Here it might be noted that the location of a large mine mouth steam plant is not the simple matter some of the popular writers on superpower systems seem to think, for the tremendous amount of condensing water required for a plant of 100,000 kw. or 200,000 kw. capacity is seldom found near a coal mine.

The entrance of the Unites States into the world war and the War Department's decision to build a great nitrate plant at Muscle Shoals led to a 30,000-kilowatt extension of the Gorgas Steam Plant, the construction at Muscle Shoals of a 60,000-kilowatt steam plant in 1919 and the start of construction on the Muscle Shoals Dam No. 2 or Wilson Dam, all of which have been more or less in the public eye for the past two years, due to the controversy as to whether the Wilson Dam and the nitrate plants should be sold for a pittance to Henry Ford, or leased to the Southern Power companies and the energy not used in fertilizer manufacture distributed throughout the entire South.

From the foregoing, it will be evident that much pioneer work had been done prior to 1917. The leaders in this were the Southern Power Company and the Georgia Railway and Power Company. The world war emphasized the necessity of coordination of resources. In 1918 a survey was made by the War Industries Board, under the direction of Frederick Darlington, of the available power in these states and of how the power resources should be developed and coordinated to give the greatest service. This report recommended large capacity tie lines between companies and showed clearly that the most economical use could be made of the power to be generated at Muscle Shoals by operating it in connection with the plants of the existing companies.

Following the close of the war, there continued to be a serious power shortage, particularly during dry seasons. This led to the construction of a line between Gadsden, Alabama, and Lindale, Georgia. This was very advantageous, because it permitted the Alabama Power Company's mine mouth steam plant to operate at 100 per cent load factor, 24 hours a day, the surplus power above that required in Alabama going into the Georgia

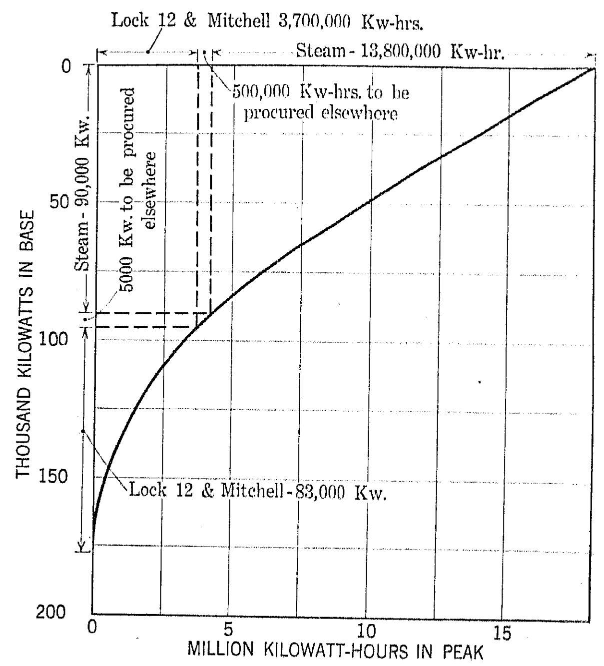

FIG. 1—EXAMPLE OF USE OF LOAD DISTRIBUTION CURVES FOR LOW FLOW CONDITIONS

Company's system where it could be absorbed even at night, as this company had ample storage capacity and so could completely shut down its hydroelectric plants after the peak load hour in the evening.

Eastern Georgia and the Carolinas are one hour ahead of Tennessee and Alabama and this causes a difference in the time of the peak on the different sytems. The Georgia and Carolina companies' industrial load being very largely cotton mills, their annual load factors are in the neighborhood of 33 per cent, whereas the Alabama and Tennessee companies with mining and a more diversified industrial load have annual load factors of about 57 per cent. The dominating part played by the diversified industrial load in Alabama is well shown by the fact that the peak on the Alabama Power Company system from Monday through Friday always occurs between 9.30 A. M. and 2.30 P. M. Not even the

heavy lighting and street railway load of the Birmingham Electric Company in the months of November and December were sufficient to change this.

An interesting operating feature has been the working together for maximum economy of the Georgia Railway & Power Company and the Alabama Power Company systems during the rainy seasons. The Lock 12 and Mitchell Dam plants of the Alabama Power Company are essentially run-of-river plants with only day-to-day storage, whereas the Georgia Company normally

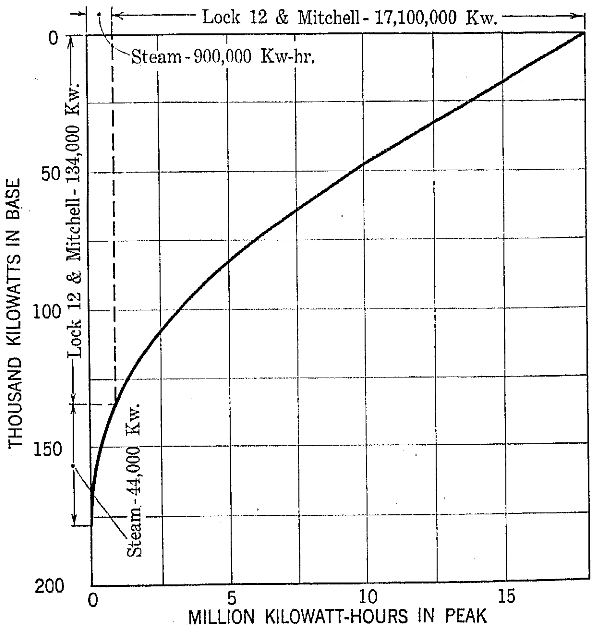

Fig. 2—Example of Use of Load Distribution Curves for High Flow Conditions

starts the rainy season with storage reservoirs, nearly, if not entirely depleted. The Alabama Power Company day loads even during periods of maximum river flow exceed its hydro generating capacity, necessitating considerable steam generation. The Georgia Company's installed hydro generating capacity is in excess of its day peak. Under these circumstances steam generation has been reduced to a minimum by the Alabama Power Company carrying the Georgia Railway & Power Company night load and getting back the equivalent amount of energy during the day time. This, of course, can only be done during the period of filling the storage reservoirs.

There is a considerable variation in rainfall through the district. It reaches a maximum of somewhat over 60 in. in the northwestern corner of Georgia. The normal rainfall for the Coosa River drainage is 55 in. That of the Tennessee, Catawba and Chattahoochee Rivers is about the same. Although the regular rainy season is practically coincident for the entire district, being from December through March, there is a great variation in summer rains and showers, making interconnection valuable in getting full benefit of summer floods at run-of-river plants located on different water sheds.

The value of interconnected power systems was clearly brought out by E. A. Yates in the hearings on Muscle Shoals at Washington in May, 1924.

As illustrated in Fig. 3 in an average year, there is a capacity at Dam No. 2 or Wilson Dam, and Dam No. 3 of 850,000 h. p. for four months of the year. During the three low months of the average year, there is 250,000 h. p. available, while in years of low flow there is a capacity of only 100,000 h. p. in the low water period. Assuming an installation of 500,000 h. p., this being available about 50 per cent of the time, there is required a steam standby of 400,000 h. p., if the system is operated independently and not interconnected with other great power systems. Government regulations require that the flow of the river below 10,000 second feet be passed continuously. This flow is necessary to generate 100,000 h. p., so that it could not be used in the peak of the load curve to increase the horse-power capacity supplied by Dams No. 2 and No. 3 and reduce the 400,000 h. p. of steam needed.

Now if Muscle Shoals is used in connection with the Tallapoosa River development, the steam standby required in a low year will be only about 160,000 h. p., while in an average year no steam will be used. Similarly, the storage and reserve steam plants of the other

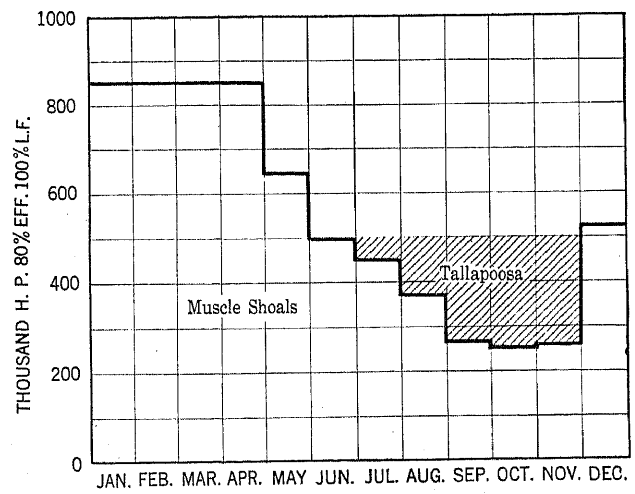

Fig. 3—Tennessee River, Dams No. 2 and No. 3. Power Available in Year of Average Flow. Muscle Shoals Operated Independently Year of Average Flow

Installation	
Dam No. 2	600,000 h. p.
Dam No. 3	250,000 h. p.
Total	850,000 h. p.

companies in the southeast would valorize additional blocks of secondary power at Muscle Shoals which would otherwise be practically worthless.

The estimated growth in load of the Southeastern States, including the seven utilities mentioned and Memphis, New Orleans, and Mobile, will require the following kilowatt hours generation up to 1935:

1923 about	3,250,000,000	kw-hr. (Used)
1926 will require	4,325,000,000	" "
1930 " "	5,860,000,000	" "
1935 " "	7,600,000,000	" "

This is an increase in 1926 of 1,075,000,000 kw-hr. over 1923 and on a basis of 40 per cent load factor requires an additional installation of 306,000 kw. This is practically the number of kilowatt hours that can be generated at Muscle Shoals on a 50 per cent load factor with 300,000 kw. installed.

If a single isolated industrial company attempted to use 1,075,000,000 kw-hr. from Wilson Dam at Muscle Shoals, even on an 80 per cent load factor in 1926, it would require 125,000 h. p. of steam plant capacity and would have to make 200,000,000 kw-hr. of steam to supplement the available hydro power. The Power Companies on the other hand would get this low water kilowatt capacity from their present hydro plants and would not be required to run steam. For example, the Alabama Power Company's Lock 12 and Mitchell Plants on the load of last year, carried about 60,000 kw. during the peak load of the day in low-flow period although the 24-hr. average flow was only about 20,000 kilowatts.

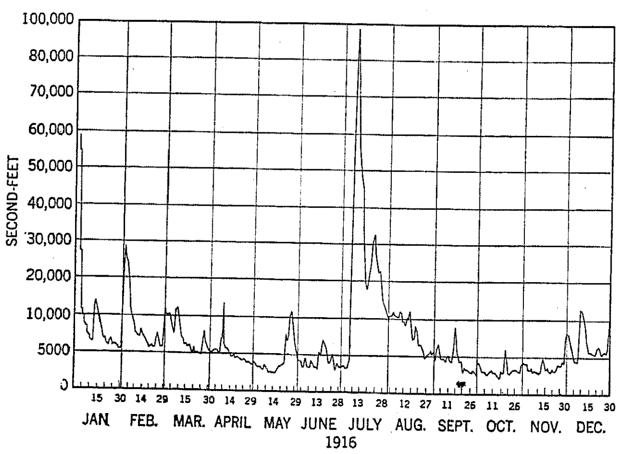

FIG. 4—HYDROGRAPH OF CHATTAHOOCHEE RIVER AT COLUMBUS, GA. FOR 1916

In 1922 the power shortage in Georgia and the Carolinas was even more acute than during the previous two years. There had not been sufficient winter rains to completely fill the Georgia Company's storage reservoirs and there was an unusually dry summer in the Carolinas. All cotton mills served by the Southern Power Company and the Carolina Power & Light Company were put on part time operation and those on the latter company's system were facing almost complete suspension. The situation was brought to the attention of the Secretary of War with the request that the Power Companies be permitted to temporarily lease the 60,000-kw. Muscle Shoals Steam Plant, which was then idle. Following an investigation by army officers proving that the case had not been overstated, the Secretary of War leased the plant and the Alabama Power Company has since operated it for the companies interested. In 1922, 44,137,000 kw-hr. were generated at Muscle Shoals steam plant and 44,646,400 kw-hr. went over the tie line to keep the mills of Georgia and the Carolinas going. In 1923, 46,910,000 kw-hr. were generated at Muscle Shoals and 48,460,000 kw-hr. supplied to Georgia and the Carolinas. Incidentally, during this period over $400,000 has been paid the United States for rental of the Muscle Shoals steam plant.

Interchange between the Southeastern power companies proved so successful that 18 months ago executives of the various power companies decided that still closer cooperation would be beneficial. An operating subcommittee was formed with one representative from each company's operating department. This committee meets once a month, exchanges information in regard to load and rainfall conditions, energy generated on each system by hydro and steam and discusses such matters as voltage regulation, load dispatching and system protection. The work of this committee has proven of very definite benefit. Its successful work has caused the executives to go a step further and an engineering subcommittee has now been formed, consisting of the chief engineer or executive engineer of each company and studies are being made of the possibilities of a coordinated development program, with the object of avoiding construction of two or more large plants by different companies at the same time, if one development plus interchange will handle the load. In this manner one company might save the fixed charges on a very large investment for two or three years while the other company would load its new development from the date of completion with not greatly increased operating costs. Under such circumstances it ought to be possible to work out a mutually advantageous interchange.

While this engineering committee has just organized, there is every reason to expect good results. An assistant engineer or a computer from each company is assigned to the work and these men are putting in full time in the same office assembling data on stream flows, studying rainfall and run-off records, plotting present load curves for both wet and dry seasons and projecting the growth of each company's load for the next ten years and determining the proper sequence of developments to carry the loads. While the proximity of the various systems and the necessarily restricted location of large steam reserve plants may make this a more immediate problem in the Southeast, I believe the principles involved are applicable to advantage elsewhere.

These statements show the value of having an engineering committee working on a coordinated study of all available resources and their most economical development. After the engineers have completed their estimates of what the tie lines will cost and the operating men have worked out their scheme of operation, an interchange contract must be made before the lines can be built and power interchange actually take place. Upon the type of agreement

between the interconnected companies is largely dependent the success of the project and its most economic use. Quite naturally, the first tie lines were built to enable a company having a surplus of energy to sell it to another company which was short of power. Following this came agreements covering emergency service, and later, agreements for straight interchange. All these different conditions must be covered by contract, and rates established. Furthermore, rates must be varied, dependent on whether surplus or dump power, steam power, with a clause varying the rate as the price of coal varies from the base price, or hydro storage power is being supplied. It is readily seen, therefore, that the drawing of a contract is not simple. Fortunately, after the contract is drawn with all its legal phraseology, carefully protecting every right of both parties, and arranging at great length for the arbitration of disputed points, it is usually forgotten, except the rate clause, and operation under it is decidedly simple. O. N. Hollis, of the Detroit Edison

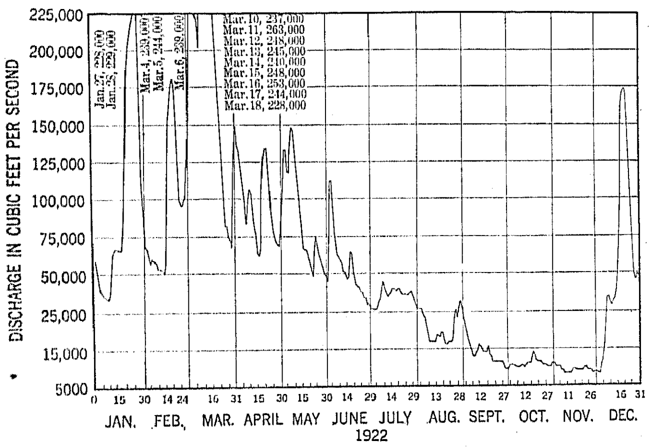

Fig. 5—Daily Hydrograph. Tennessee River at Florence, Ala., Alabama Power Co.

Company has just delivered a most excellent paper before the Association of Edison Illuminating Companies, on the subject of interconnection rates and contracts which brings out in detail many of these points.

In the southeast all these forms of contract are in use. There are the firm contracts for definite portions of the capacity of the Muscle Shoals Steam Plant. If the capacity is used there is a definite flat rate per kilowatt hour, based on the price of coal. The method of billing for the energy under these contracts is interesting, showing how simple a complicated problem can sometimes be made. For example, power destined for the Carolina Power & Light Company is delivered by the Alabama Power Company to the Georgia Railway & Power Company, at the state line and is in turn delivered by the latter to the Southern Power Company at Tallulah Falls and by the Southern Power Company to Carolina at Wateree. Only one bill is rendered, and that by the Alabama Power Company against the Carolina Power & Light Company for the full amount of power delivered to the Georgia Railway & Power Company at the state line. To compensate for line losses, the Georgia Railway & Power Company and the Southern Power Company each deducts a percentage of the power delivered to them for transmission and passes on the remainder to the Carolina Power & Light Company.

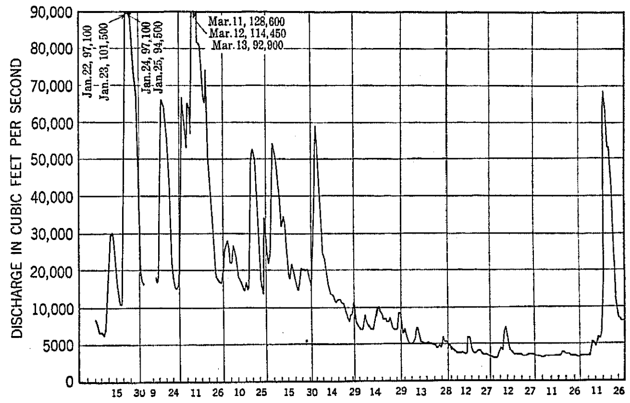

Fig. 6—Daily Hydrograph, Coosa Lock 12 Development, Alabama Power Co.

Interchange contracts under which one company supplies power at night at one rate and gets it back in the day time at a higher rate are common. Nearly all interchange contracts are optional on both parties. This is, if A has power to sell and B desired to buy, power moves; otherwise not, and vice versa.

With the contracts signed and the tie lines built, it

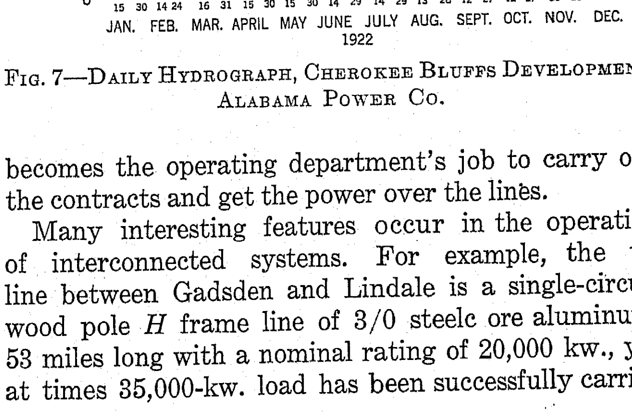

Fig. 7—Daily Hydrograph, Cherokee Bluffs Development, Alabama Power Co.

becomes the operating department's job to carry out the contracts and get the power over the lines.

Many interesting features occur in the operation of interconnected systems. For example, the tie line between Gadsden and Lindale is a single-circuit wood pole H frame line of 3/0 steelc ore aluminum, 53 miles long with a nominal rating of 20,000 kw., yet at times 35,000-kw. load has been successfully carried

over the line. With a load of 20,000 kw. at 95 per cent power factor, the voltage at Gadsden on the 110,000-volt bus is approximately 110,000 volts and the voltage at Lindale drops to approximately 102,000. The voltage at Atlanta, Georgia, is approximately 110,000 and at Tallulah Falls approximately 115,000 volts. The voltage maintained at the generating plants on the Alabama Power Company's system is approximately 115,000 volts under the above conditions. Of course, in transmitting power in

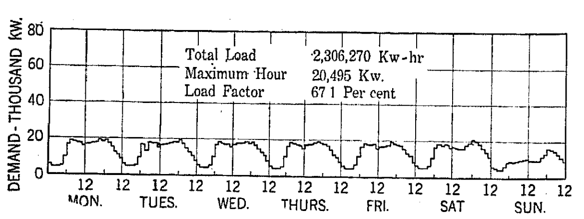

FIG. 8—AVERAGE WEEKLY LOAD CURVE, FEB. 12-18; MAY 7-13; AUG. 6-12; NOV. 19-25, 1923

blocks of 15,000 to 25,000 kw. over these tie lines, power-factor correction is absolutely essential. It is desirable to operate these lines at as near unity power factor as possible. Synchronous condensers are being installed on the various systems for power factor correction. During the year 1923 the Alabama Power Company installed on its system approximately 20,500 kv-a. in synchronous condenser capacity and during 1924 an additional 27,500 kv-a. will be installed. These condensers, of course, are located near the load centers to secure the maximum possible benefit of power factor correction. The Alabama Power Company is now installing a 25,000 kv-a. turbo generator at its Gorgas steam plant and provisions are being made for disconnecting the generator from turbine and operating the generator as a synchronous condenser. This is especially desirable since in the wet season the steam plants are shut down and as the hydro plants are located in the opposite section of the system, operation of the generator as a condenser materially improves conditions throughout the system. During the dry season hydro generators are floated for power-factor correction. By shutting the wicket gates and breaking the vacuum in the draft tube on the machine it is possible to operate generators for power factor correction with only a small loss of water. This same practise is, of course, followed by other companies on the interconnected network and the system supplying energy is relieved, in so far as possible, of excess reactive component by systems receiving the energy, these systems using their spare generator capacity for power-factor correction. Although without doubt the proper place for the major part of power factor correction is at the customers' load, a certain amount of correction is necessary in large load centers where large capacity condensers can be installed and definite control of voltage conditions can be secured.

High-voltage metering equipment is necessary for the interconnecting lines and it has been the practise to install this equipment in substations to which the lines connect. For instance, on the line between Gadsden and Lindale, meters are installed at the Gadsden plant and at Lindale, and energy delivered or received is based on the net amount supplied or received at the State line, though no meters are installed there the losses being prorated. Comparatively little trouble has been experienced in the high-voltage metering equipment. Only one or two cases of current transformer and potential transformer failure have occurred during the past three years. It is customary to carry one reserve potential and one reserve current transformer for each metering equipment.

In general, 110,000-volt oil circuit breakers are used for synchronizing purpose. Inherently, these breakers are slower than the low-voltage breakers in generating stations; however, with a little practise the operators are able to synchronize systems with no adverse effect to either of the systems. Of course, it is necessary to carefully check circuit breaker adjustments so that at all times they will close as quickly as possible.

To successfully operate the transmission systems in parallel and at the same time secure the proper load adjustments between the various generating stations and between the systems themselves, it is essential that the frequency be rigidly maintained at 60 cycles. At first some difficulty was experienced in maintaining a fixed frequency and on numerous occasions in the early days of the operation frequency variations were such as to overload the tie lines to such an extent that the overload relays tripped out the switches, separating the systems. Some difference was always found in the frequency indicators in spite of careful calibration but this difficulty has been overcome by the installation

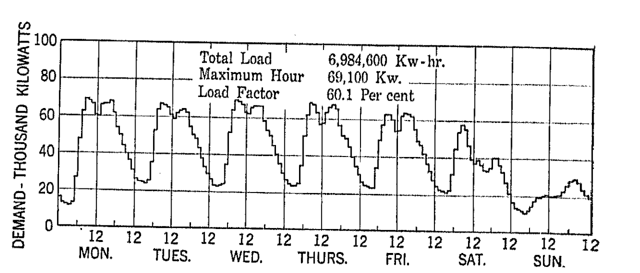

FIG. 9—AVERAGE WEEKLY LOAD CURVE, FEB. 12-18; MAY 7-13; AUG. 6-12; NOV. 19-25, 1923

of Warren Master Clocks at either the principal regulating stations or at the Load Dispatcher's office. The Master Clock makes it possible to maintain an average frequency within very close limits.

It has been found that the regulation of frequency must be controlled by the base load stations and especially the base load station of the system supplying the bulk of the power, all other stations making the necessary governor adjustments to fall in step. The governors at all stations taking the load fluctuations

should be equally sensitive so that all systems can properly carry their share of the fluctuations.

On the system of the Alabama Power Company the base load is carried by the hydro plants and load fluctuations by steam plants in the wet season and vice versa in dry seasons. During the wet season, therefore, the frequency is controlled at Lock 12, and in the dry season it is controlled at the Gorgas Steam Plant. Master Clocks are installed at both stations for the purpose. There is no appreciable difference in manner in which the hydro and steam stations handle load fluctuations.

Operation is handled between load dispatchers on the various systems. It has been found advisable to establish a central load dispatching point at Atlanta and all messages concerning the dispatching are usually relayed through Atlanta to the interested systems. Where a company desires to purchase energy the Operating Superintendent or Engineer customarily advises the Central Load Dispatcher of the amount of power desired on interchange and this is relayed to the dispatchers of the proper systems. In case of emergency, the Load Dispatchers have authority to act and it has frequently been possible to save very important loads by quickly bringing in additional generating capacity. In this way interruptions due to line trouble are reduced in number and duration, because in most cases important load centers can be served over two entirely different routes. For instance, The Alabama Power Company has very important loads at Gadsden and Huntsville, Alabama. This load is served by the Gadsden Steam Plant and by two lines from the hydro plants as well as the interconnecting line to the Georgia Railway & Power Company's system at Lindale. Lightning storms may occur when the steam plant is not in service, causing outages to the

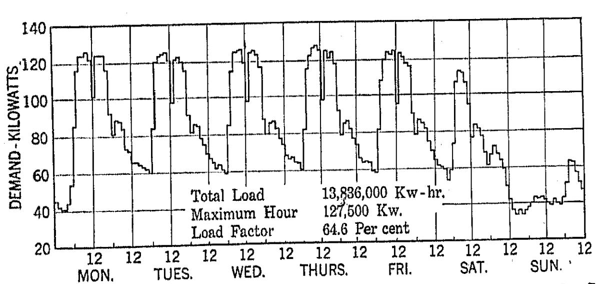

FIG. 10—AVERAGE WEEKLY LOAD CURVE, FEB. 12-18; MAY 7-13; AUG. 6-12; NOV. 19-25, 1923

two 110,000-volt lines connecting the hydro plants. In such cases the tie line is used to supply these load centers until the steam plant can be brought in or the lines repaired. The supply to the Georgia Railway & Power Company's system is from hydro plants located approximately 100 miles northeast of Atlanta and trouble on this company's lines may cause serious interruptions to customers around Atlanta, and the tie line is used to serve this load while the repairs are being made.

Effective relay protection is essential for the successful operation of the interconnected systems and each company has made many improvements in protective layouts during recent years. It is desirable to immediately relieve the system of short circuits by tripping out the faulty line or equipment as quickly as possible. Differential protection for transformers and generators and, in so far as possible, balanced protection for transmission lines have been applied. A diagram of the protective scheme in use on the Alabama Power Company's system is shown herewith. It will be noted that

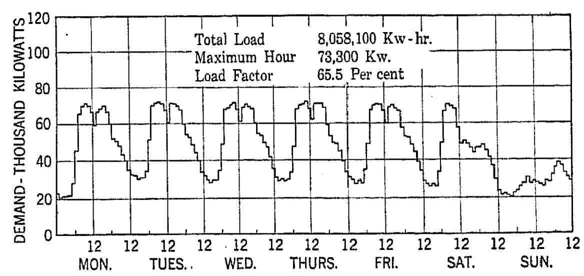

FIG. 11—AVERAGE WEEKLY LOAD CURVE, Feb. 12-18; MAY 7-13; AUG. 6-12; NOV. 19-25, 1923

on this system lines are run in pairs between important load centers and generating stations, and it is therefore possible to apply balanced relays which allow the quick disconnection of the faulty line, allowing the remainder of the circuit to continue carrying the load. The operation of balance relays and differential relays has as a general rule proven successful. Continuous studies are being made with a view of providing the best protection which can be afforded. Short circuit studies are essential in the application and setting of relays as well as of oil circuit breakers. A short-circuit calculating table has been in use on the Alabama Power Company's system for several years for this purpose, and at this time the company is constructing a new board, with permanent resistors for existing transmission lines, generators, and transformers and variable resistors for extensions. The system is reproduced in miniature so that a short circuit may be placed at any point and read by pushing an ammeter button. The total short circuit, as well as the component current in various lines can be determined.

The problem of voltage control is very important since it affects the distribution of reactive current between the various systems, the system maintaining the highest voltage taking more of the reactive component. If the system supplying energy also supplies the reactive component associated with it, then the voltage drop over the tie line will vary according to the load on the line; *i. e.*, from zero at no load to a maximum at full load. Where the range of load is great, too wide a range of voltage is required for satisfactory service. It is possible, however, to maintain constant voltages at both ends of the tie line over the entire range by varying the magnitude of the reactive component transmitted. This method has the disadvantage of increased losses

due to the flow of large reactive currents at light loads over the tie line. The best method perhaps is to strike a compromise and allow the voltage drop over the tie line to vary within permissible limits, allowing reactive currents to flow to maintain these limits. With a heavy load on the tie line this may mean that the receiving system must supply all of the reactive current of the load received and in addition, some reactive current to the line so that the load can be transmitted at a leading power factor.

Typical hydrographs are attached for the Tennessee, Coosa, Tallapoosa and Chattahoochee Rivers. Fortunately for those making hydraulic studies, stream flow and rainfall data are available on these streams over many years. On the Tennessee, for example, rainfall and stream flow data are available over a period of 40 years for the Florence gaging station.

All studies have been made on a weekly basis, both as to available water and to power consumption. Yearly flow curves are plotted, using the 52 weekly flow points instead of 365 daily flow points. While studies have been made of the wettest and driest years, a careful analysis indicates that studies of plant capacities and generation based on what we have called a median year give best results. A median year flow is a theoretical year built up by using for each of the 52 calendar weekly flow points that week which has the same number of years showing this same calendar week with a greater flow as there are years showing it with a lesser flow in the total number of years for which the data are available. There is no such thing as an average year and in our efforts to get a safe typical year and not have to make a study of all load conditions for all the years for which data are available, we have arrived at the median year as here defined which seems to be safe and to materially lessen computations. Typical weekly load curves are shown herewith.

There is necessarily a great deal of tiresome computation in hydraulic studies covering periods of years. To lessen it, Mr. C. James, Assistant Chief Engineer of the Alabama Power Company, has developed a very interesting curve showing distribution of load between different plants, which has proven exceedingly convenient and accurate. While many other forms of curves are used, the one we use lends itself readily to a determination of the position of a given plant in the load curve.

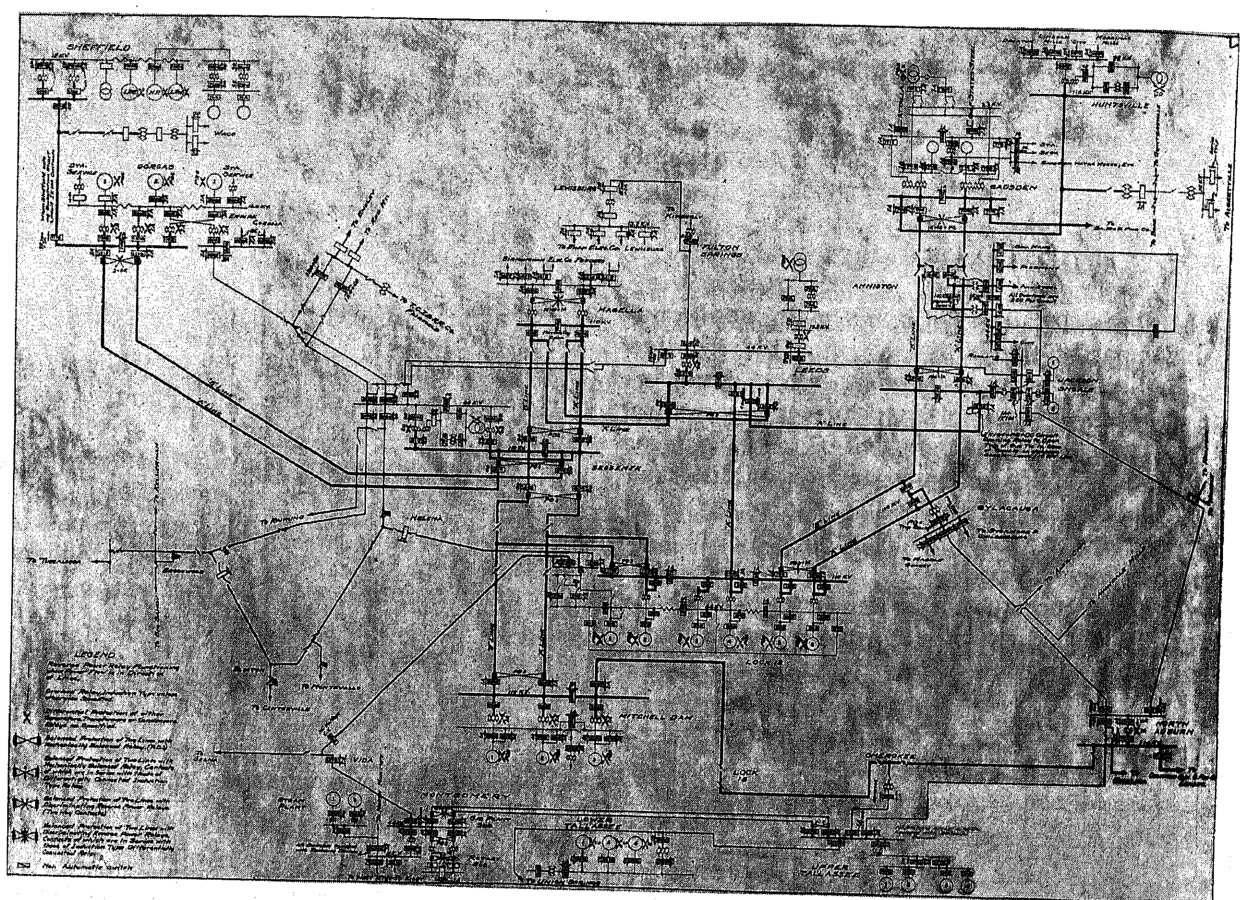

Fig. 12—Single Line System Diagram Showing Principal Relay Protection, Alabama Power Company

The usual weekly load curve as plotted by the Alabama Power Company shows the kilowatt hours generated for each hour of the week consecutively, from Monday to the following Sunday, inclusive. In the power studies it is often important to know the kilowatt hours which a certain plant can generate, knowing

its kilowatt capacity and position in the load curve and the steam flow. This is an area on the load curve as plotted and considerable time would be necessary to determine it in each of the many cases when it is needed. The distribution curves above referred to are in the terms of kilowatts and kilowatt hours, so that for any known kilowatt demand and position on the load curve, the kilowatt hours can be read on the curve or vice versa.

Fig. 1 is an illustration of the use of the curve for low-flow conditions. Under these conditions, in order to use as much capacity of the hydro plants as possible, they are used in the peak of the load curve and steam in the base. Using the entire flow of the river and placing the hydro plants in the peak, we wish to know how much capacity will be obtained. Knowing the installation of steam available, we also wish to know how many kilowatt hours this installation can furnish if used in the base of the load curve, in order never to use a greater amount in our study. Assume a weekly load of 18,000,000 kw-hr. and a peak hour of 178,000 kw. During the low-flow week, the Lock 12 and Mitchell Dam plants of Alabama Power Company will furnish 3,700,000 kw-hr. Our curve is in terms of kilowatt hours in the peak and kilowatts in the base, this being the most convenient form to draw and read the curve. Entering the curve with 3,700,000 kw-hr. in the peak, we read 95,000 kw. in the base. A base load of 95,000 kw. must therefore be furnished in the base load steam or other hydro plants. The kilowatts used in the peak then, by Lock 12 and Mitchell Plants will be 178,000 minus 95,000 or 83,000. Now start on the problem from the opposite angle. Assume we have 90,000 kw. of steam available. We wish to know how many kilowatt hours these steam plants can furnish when used in the base of the load curve. Entering the same curve as used above with 90,000 kw. in the base, we read 4,200,000 kw-hr. in the peak. Subtracting 4,200,000 kw-hr. from the total load of 18,000,000, we have 13,800,000 kw-hr. available with the limits of the load curve from 90,000 kw. of steam. If our load were such as to be able to use 90,000 kw. continuously for the entire 168 hr. in the week, we could, of course, obtain 168 times 90,000 or 15,120,000 kw-hr.

Fig. 2 is an illustration of the use of the curve under high-flow conditions. Under these conditions the hydro plants are used in the base of the load curve and steam in the peak, unless there is enough hydro capacity available to meet the demand without steam. Assume the same weekly load and peak as used under low-flow conditions, 18,000,000 kw-hr. and 178,000 kw. In this case, we wish to know how many kilowatt hours can be obtained from the hydro plants when used to the best advantage in the load curve assuming the flow is sufficient to furnish this amount. We also wish to know how much steam is needed in the peak. The installation at Lock 12 and Mitchell Dam are 80,000 and 54,000 kw. or a total of 134,000 kw. Reading the curve for 134,000 kw. in the base, we find 900,000 kw-hr. in the peak. Subtracting 900,000 kw-hr. from the load of 18,000,000, we have 17,100,000 kw-hr. which can be furnished by Lock 12 and Mitchell Dam, as long as the flow is high enough. If there are no other hydro plants available, the remaining 900,000 kw-hr. and the balance of the demand on the system, or 44,000 kw. must be steam.

In conclusion, it may be said that the various power companies are fully aware of the benefits of interconnection and are intelligently planning to take the maximum advantage of it in the future. Executives, engineers and operating men are working in the closest cooperation. To meet the needs of the rapidly growing industrial load of the southeastern states will tax the ability of the public utilities in this district. Tie lines much larger than the present must be constructed, and their design is a serious engineering problem, when proper consideration is given to protection of service, and to voltage and power factor control. It may well be that a high-tension network of the type proposed by Percy Thomas in his paper before the Institute last April is the ultimate solution. That the problem is worthy of the best technical study is evident when the present capacity of the interconnected systems is considered:

Installed Hydroelectric Generating Capacity......................	782,000 kw.
Installed Steam Generating Capacity	353,000 kw.
Hydro Storage Capacity..........	225,000,000 kw-hr.
System Output in 1923...........	3,250,000,000 kw-hr.
System Peak....................	740,000 kw.

and the further fact borne in mind that, according to the best data available, these figures will be doubled in 1930.

Discussion

P. H. Thomas: Mr. Mitchell's paper is particularly timely since it indicates not only the most advanced point to which widespread interconnection of large systems has progressed in actual practise, but points out the probable solution of some of the problems now being faced in the further extension of interchange.

I have in mind such matters as voltage control at individual stations on a tie line, when the direction of energy flow changes, the proper distribution of true energy between plants, with changes of load and other conditions throughout the 24 hours and the placing and control of wattless. These features would involve no difficulties if the systems could be laid out new, with operation interconnected in view, but may cause very serious trouble if large systems are merely interconnected without proper study of their characteristics.

The scheme of Mr. James utilizing a curve connecting the "peak kw-hr." and the "base kw." for assigning the position of the generating units is most ingenious and useful. I have used a similar curve built up by integrating the kw-hr. shown by the load curve below any given kw. generating capacity. I have extended it to distinguish between minimum stream flow hydro

power and pondage power and to make assignments to individual stations. It is a most useful analysis.

There are two principal points to which I will call attention.

First—In any large interconnection, even where there is no dominating water power plant or single steam station of supreme importance, there will be a considerable amount of installation in tie lines, condensers and power stations in which the interest of the whole will not be the the same as that of some of the individuals. For example, a tie line might be of great advantage to an outlying system but of small value to the system in whose territory it lies, or the construction of a line by a different route or at a higher voltage might be of great future importance to the group as a whole but of no immediate value to the company operating in the territory to be crossed.

The conclusion to be drawn is that there will apparently be a great advantage in creating a common entity or organization, controlled by and working for the interest of the whole, which could properly undertake work that it would not be fair to ask any one company to undertake. The existence of such a unity would greatly facilitate interchange contracts for power. Indeed, it is difficult to see how the full benefit of interconnection can ever be established otherwise.

Second: My second point is to emphasize the necessity for looking ahead and planning a complete skeleton system for the future covering the full zone in conformity with which new installations can be made. Without this there will inevitably be much waste. For example a tie line laid out for an immediate interchange may be directly overbuilt by a higher voltage line a few years later.

If it turns out that single-circuit lines as proposed in my paper presented at the Birmingham convention are of advantage, the building of a two-circuit line may later require the addition of another line to reach some parallel point, which might have been reached by separating the original two-circuit line and running the circuits by different routes.

Again: The divided conductor has a very great theoretical advantage in a widespread interconnected system, both in increasing the capacity to balance lagging current and in reducing inductance to facilitate tie-line interchange of current. If this is to prove the best construction, the sooner this is determined the better.

The full importance of lagging current and wattless component is not generally realized. As long as the lagging could be cared for by running the generator at low power factor and lines were short, wattless current cost little. But with very long lines, wattless current must be largely eliminated at the receiving end and this must often be done by installing new machinery to carry wattless current kv-a.—a very expensive process. Therefore, the gain in charging current by dividing the line conductor may be more important in the future than it has been in the past.

It is not unlikely that very large sums of money may be saved in the next 15 years by proper planning ahead for the interconnection and this is particularly true in the zone treated by Mr. Mitchell.

Large Steam Turbine Generators

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Review of the Subject.—*The authors discuss the manufacture of large steam turbine-driven generators touching on what is considered the best practises of the present day. A description of a 62,500-kv-a. 60-cycle generator is included together with test data. The fact is brought out that even on the largest generators yet built, moderate temperatures may be expected. The losses and ventilation problems involved in this type of apparatus are discussed, and predictions made as to the probable sizes at given speeds which may be expected in the future.*

* * * * *

THE power companies that generate and distribute power in the largest amounts have continually asked for larger and larger steam turbine units, with the result that 30,000-kw. units have become quite numerous. The design of the turbine and of the generator at every step to larger sizes involves distinct problems that in nearly all cases are worked out by two distinct groups of engineers. These two groups are in continual competition as to which can produce the larger machine at any given speed,—steam turbine or electric generator.

It is gratifying to the designer, the manufacturer and the user of the largest steam turbine generators that have been built, to find that they prove in service as good in practically every respect as the best of the older and much smaller units. The extremely large generators at the present time are possible, by reason of improvements in materials in both their electrical and mechanical properties, and improvements in details of construction that have in view the protection of the insulation against small movements that are produced by mechanical stresses and by changing temperatures. Progress is dependent, to a large extent, upon research work, improvements in construction and the development of better materials for any given purpose; for example, retaining bands of greater strength to hold the field coils in place at heads of the rotor, or steel laminations for stator cores of better characteristics in the matter of permeability or hysteresis and eddy current losses. Improvements in the mechanical strength of materials in the rotor permit of higher peripheral velocities. At the present time good practise is limited to speeds of about 25,000 feet per minute. Any discovery or development that would permit of an increase of 10 per cent would at once allow the possible size of unit to be increased approximately 33 per cent, since the possible largest size varies approximately as the cube of the peripheral speed.

Installations of several of the largest steam turbine-driven generators built in America have been made during the past year. They are 6-pole 62,500-kv-a. 0.8 power factor, 1200-rev. per min. self-ventilated units. The first of these put into operation is in the Hudson Avenue Power House of the Brooklyn Edison Company—see Fig. 1. Some idea of the size of these units may be obtained from the following weights: rotor 205,000 lb.; stator 205,000 lb.; shields, etc., 25,000 lb.; total, not including base, pedestals or bearings, 435,000 lb. There are some 41,250 lb. of copper in the machine. The stator has 37.6 miles of insulated wire, and the rotor 4.5 miles.

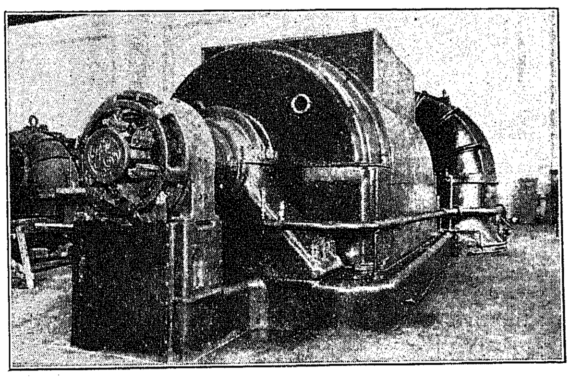

FIG. 1—62,500-KV-A., 60-CYCLE, 1200 REV., 80 PER CENT POWER FACTOR, 13,800-VOLT GENERATOR IN HUDSON AVENUE POWER HOUSE OF BROOKLYN EDISON COMPANY

ROTOR

The rotors of the largest generators consist of steel forgings, many of them of a single forging, which serves as a through shaft with journals for the bearings, and machined to receive coupling at one end for connection to steam turbine, and at the other end for connection of direct connected exciter, if one is supplied; the main body of the forging serving as the field part of the magnetic circuit and machined to receive the field windings. The rotor of the 62,500-kv-a. 1200-rev. per min. generator consists of three forgings, the middle one a hollow cylinder.

One of the most interesting processes of machining in the shop is the milling of the slots in these large rotors. Fig. 2 shows two cutters at work at diametrically opposite points on a 4-pole rotor. A great amount of material must be removed by these cutters, especially in the case of a rotor for a 6-pole machine of as great as 62,500 kv-a. capacity. The cutters employed for this work have been developed to their present degree of perfection by little inventions from time to time during

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., October 13-17, 1924.

the past few years, such as methods of securing cutter teeth in place and removing them quickly. At the same time great improvements have been made in the treatment of the steel used in the teeth. The result of all this improvement is that the quantity of material now removed in a given time is practically twice that of a few years ago; at the same time the cutters themselves

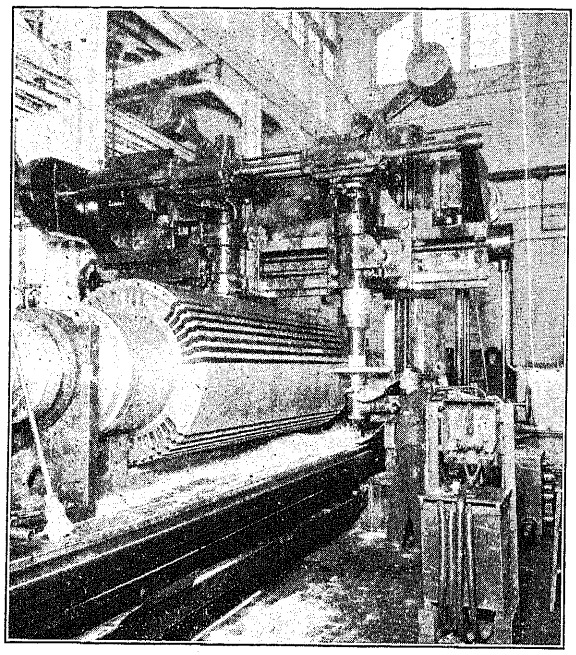

Fig. 2—Milling Two Coil Slots at Once, 7 In. Deep by 1.375 In. Wide, in 4-Pole Rotor for 35,300 Kv-a. Generator. The Cutter Concealed from View by the Rotor Body is Milling the Slot Diametrically Opposite

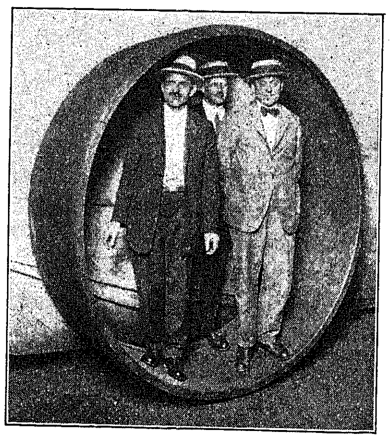

Fig. 3—Rotor Retaining Ring for 62,500-Kv-a., 1200-Rev. per Min. Generator, Ready for Expanding by Heat and Shrinking On

require much less attention, the cutter teeth remaining in fit condition a much longer time before sharpening is necessary.

The forming up of the field coils for such extremely large machines has been made possible by the development of special machines for winding the coils. The winding machine not only bends to a small radius the heavy copper strip on edge, but changes the pitch every turn, since the winding must be assembled turn by turn in radial slots.

The method of securing the windings against centrifugal force by metallic wedges driven into the dovetail grooves in the sides of the slot throughout the body as well as the retaining rings used at the heads of the rotor, remains practically the same as for many years but continual improvements have been made in small details. At the present time steel wedges are employed throughout the slot portion. These are of magnetic steel in the slots immediately adjacent to the pole center and of non-magnetic to prevent magnetic leakage in the outer slots of every pole. The retaining bands regularly used in these machines are of nickel steel, or a similar steel alloy, and are, consequently, of

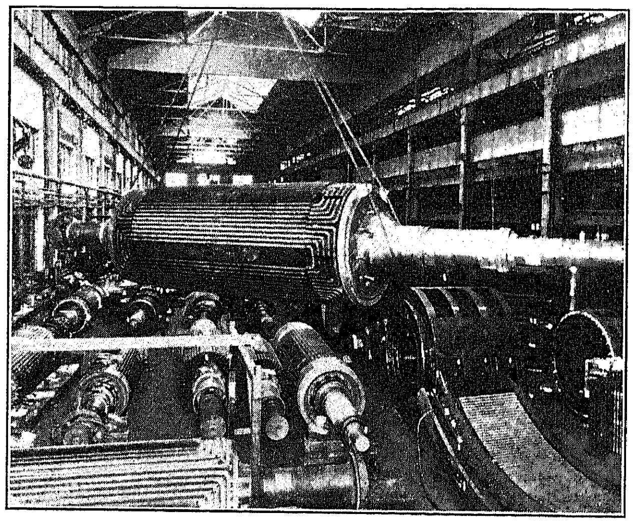

Fig. 4—Rotor of 62,500-Kv-a. 13,800-Volt, 60-Cycle, 1200 Rev. per Min. Steam Turbine Generator

35 ft. long, 205,000 pounds' weight, completely wound and supported from crane for transit through building to receive binding bands in shop where both stators and rotors are wound.

magnetic material. It would be desirable to have these retaining bands of non-magnetic material, if workable material were available that had sufficient strength. Fig. 3 shows the retaining ring or binding band for one end of one of the 62,500-kv-a. rotors. A good idea of the size of this ring can be formed from the fact that three men of average size are standing side by side inside the ring.

Some of the most important problems that have had to be worked out in connection with these machines is the securing in place of the insulation on the ends of the several field coils, as also the securing in place of the completely insulated coil so that the immense stresses of centrifugal force, as well as expansions and contractions of changing temperature will not injure the insulation or disturb the balance of the rotor. As shown in Fig. 5, the present method employed is the encasing of the insulated coils in metallic saddles.

These are made of aluminum. The blocking between coils is of the same material.

Stator

Fig. 6 shows a view of the frame of one of these machines. It is of the well-known fabricated structure,

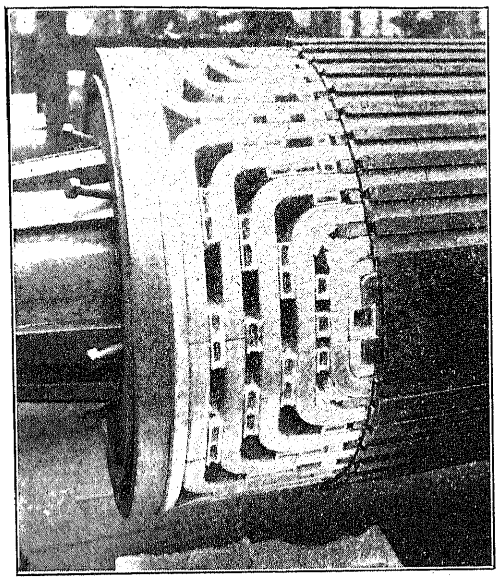

FIG. 5—VIEW OF END OF ROTOR OF A 35,300-KV-A. 1500 REV. PER MIN. STEAM TURBINE GENERATOR ROTOR

Ready for receiving retaining ring, showing field coils encased in aluminum saddles and mechanically supported by specially designed aluminum blocks.

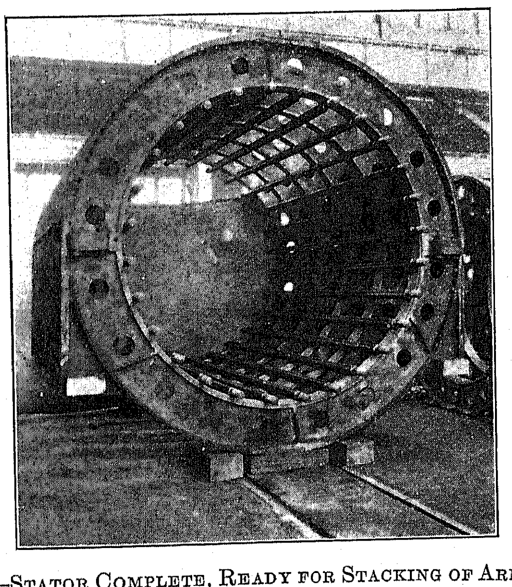

FIG. 6—STATOR COMPLETE, READY FOR STACKING OF ARMATURE CORE OF 62,500-KV-A., 1200-REV. PER MIN. GENERATOR

Showing multiplicity of ring castings tied together by boiler plate riveted to the outer periphery and by steel bars bolted to inner periphery that carry dovetail slots for assembly of laminations and threads at ends for assembly of clamping flanges and nuts.

which has been successfully used for a number of years. As clearly shown, the frame consists of six annular cast *I*-sections, which are held at the outer periphery by heavy boiler plate riveted to the flanges of the sections. This results in an unusually light and strong structure. There are twenty-eight rectangular steel bars bolted to the inner periphery of the *I*-sections. These bars serve a triple purpose; first, they add stiffness to the frame; second, they are slotted so as to receive the dovetail keys and punchings; third, the ends are turned down and threaded so they act as clamping bolts for the flanges which restrain the core at each end of the machine.

As to the winding, there are 126 coils, each having three turns of 85.8 per cent pitch. Each turn consists of sixteen asbestos insulated strands having a section of 0.21 in. by 0.085 in. Connected with six circuits per phase, a current density of 1521 amperes per square inch is obtained. These six circuits per phase are separated into two groups of three circuits each. Inasmuch as the insulation of the winding and connections was guaranteed to withstand a high potential test of 40,000 volts for one minute, extra insulation was applied. The total thickness of insulation from copper to slot wall is 0.280 in. This high potential test at 60 cycles was successfully applied, using a 300-kw. transformer and a separate generator for controlling the voltage. The charging current measured in the grounded side of the 40,000-volt circuit was 5.6 amperes for each of the three phases. This corresponds to 224 kv-a.

The coils and turns are insulated by hand with mica tape throughout their entire length. High thermal conductivity is secured and compact solid insulation is obtained free from entrapped air by subjecting the insulated hot coils more than once for many hours to a vacuum treatment followed each time by injections of a hot sealing compound under pressure.

Nine insulated leads, instead of the usual six, are brought out from the armature connections through the terminal board. Of the three leads per phase, one goes to neutral and one to each of the two groups of circuits mentioned above. This arrangement permits the insertion of the current transformers used with the balanced relay system of protection for generators, also the insertion of a differential current transformer at the line end of each phase of the winding. Normally, current passes equally through the double primary coils of the differential current transformer from the two halves of the divided generator phase winding. Any unbalance of the currents in the two groups of circuits per phase produces currents in the secondary which can be made to trip out the generator at any desired setting in this secondary circuit.

Temperatures and Methods of Determining Same

Searching tests have been made in the shops as well as on generators in commercial service to determine:

1—maximum observed temperature rise on the bare copper of the stator windings:

2—maximum observed temperature rise as commercially measured (between upper and lower coils in the slot) by

(a) 20 in. long, ⅛ in. wide resistance temperature detectors;

(b) 10 in. long, ¼ in. wide resistance temperature detectors;

(c) thermo-couple;

3—differences in temperature at different locations longitudinally in the slot of the commercially embedded detector;

More of these tests have been made on the 35,300-kv-a. 85 per cent power factor (30,000 kw.) generators of various voltages than on any other size. The most recent of these tests was on 13,200-volt machines, when, fortunately, a pair of them came through the shops at the same time and it was possible to run them at full load, zero power factor. Information gleaned from the data obtained, indicates,—

1—that on machines from 12,000 to 14,000 volts, the temperature of the bare copper is approximately 15 deg. greater than the temperature determined by the 20 in. long detector, as located between top and bottom coils in commercial machines, when the latter is from 55 deg. to 60 deg. above temperature of ingoing air;

2—temperatures determined by resistance detector and thermo-couple on bare copper are approximately identical. Temperatures determined by the 20 in. long detector, as located in commercial machines, is approximately the same as determined by thermo-couple;

3—the temperatures at the half-way location in slot are approximately 2 deg. lower than at the quarter-way, which indicates a trifle better ventilation at the half-way.

The actual temperature rises for these 13,200-volt generators, with full-load current, were 48 deg. cent., as determined by 20 in. resistance temperature detector in stator slot, and 72 deg. cent. in rotor winding, as determined by resistance when operating with leading current and, consequently, greater excitation than required for 85 per cent power factor, full load output.

Tests on 62,500-kv-a. Generator

The results of the tests made under actual load on the 62,500-kv-a. generator of the Brooklyn Edison Company are here given in the form of curves:

Fig. 7 shows generator under test equipped with a tall stack for the purpose of determining the quantity of air flowing through the machine. The tests consisted in a series of runs following one upon the other until constant temperature was attained; 1st, at normal speed without excitation; 2nd—at normal speed, no-load, 13,800 volts (the rated voltage); 3rd, at ¼ load; 4th, at ½ load; 5th, at ¾ load and 6th, at full load. Owing to the conditions of the system into which the generator was feeding, the potential at generator terminals increased with load and attained 14,350 volts at the full 62,500 kv-a. The power factor was slightly less than 80 per cent.

The losses were determined from the quantity of air and the temperature rise of the air at the various conditions. The quantity of air was 112,500 cubic feet per minute.

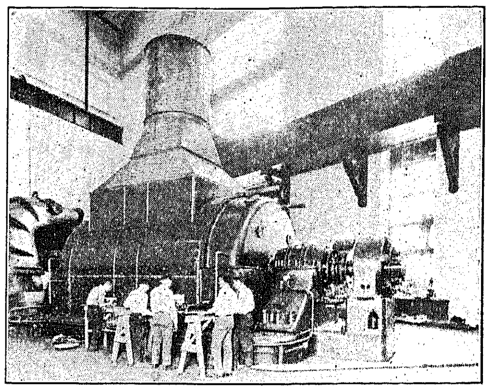

Fig. 7—62,500-Kv-a. 1200-Rev. Generator Brooklyn Edison Company as Equipped with Tall Stock for Measuring Quantity of Cooling Air during the Tests

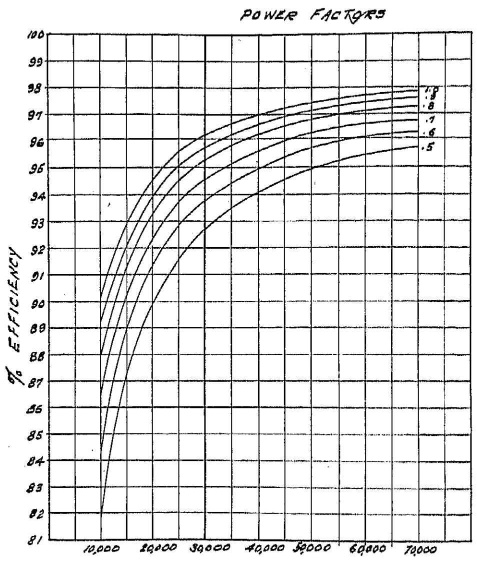

Fig. 8—Efficiency Curves Based upon the Actual Losses as Determined by Tests on 62,500-Kv-a. 13,800-Volt, 1200 Rev. Generator

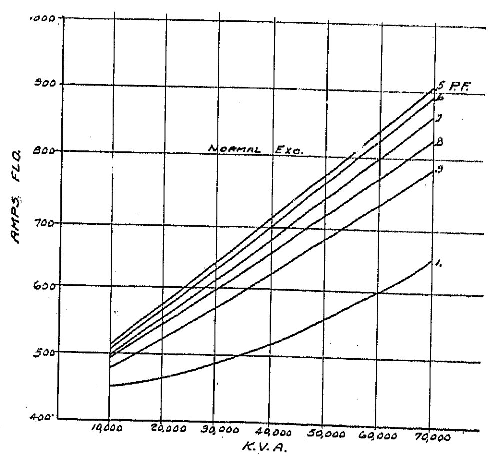

Fig. 9—Curves Showing Amperes, Excitation at 13,800 Volts for Various Loads and Power Factors of the 62,500-Kv-a. 1200-Rev. Generator

Fig. 8 shows efficiency curves as determined from the measured losses.

Fig. 9 shows excitation required for various power factors and loads at the normal volts—13,800.

Fig. 10 shows temperature rises attained under the different conditions of no excitation, normal excitation without load, and at partial loads and full load, with the voltage required by the system. It is to be noted that this generator has the remarkably low temperature of 50 deg. cent. rise under full load.

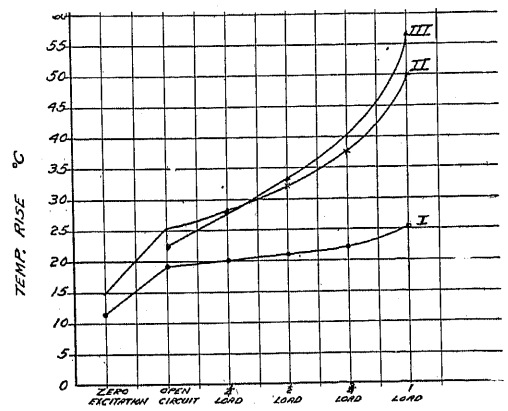

Fig. 10—Temperature Rises, 62,500-Kv-a. 1200-Rev. Generator

I. Temperature rise of air
II. Maximum temperature rise of armature winding by R. T. D.
III. Temperature rise of rotor winding by resistance

Ventilation

The ventilation of these large machines follows the well-known multi-path radial system used so successfully on generators of this type since 1913. Air is drawn in at each end of the machine through converging ducts formed by the end bells. Its pressure is elevated by multivane fans attached to each end of the rotor. These fans are so constructed that the air stream at each end of the machine is split into two parts, the inner section of fan feeding directly into the air-gap and thence radially out through the ½-in. air ducts with which the core is supplied. The air from the outer section of the fan is carried through sheet iron ducts to a center compartment in the machine, whence it flows radially inward to the air-gap, thence axially along the air-gap until it meets the first stream of air from the inner fan, whence it flows radially outward through the air-ducts. These two air streams are separated at the fan by a third, or inner shield, which also acts as a diffusing ring for the air which passes through the outer fan. See Fig. 11. This system of keeping the two air streams separate has shown better results than when the air is allowed to exhaust into a common chamber and seek its own distribution path through the generator. This is due in part to the diffusing effect of the inner ring and, second, to the characteristics of the fans themselves. On machines of this size it is sufficient to have only two multiple paths. In machines of greater length, or smaller diameter, it is necessary to resort to a larger number of multiple paths through the core. The decision as to the number of multiple paths that shall be used will depend on the velocities in the air-gap and through the air-ducts. A large number of multiple paths will result in a high air-duct velocity and low air-gap velocity, with poor cooling for the rotor and maximum cooling for the stator. Few multiple paths result in low air-duct velocity and maximum rotor cooling. The ideal distribution then would be one in which the air-gap was kept running full of cool air and the air-duct velocity in the neighborhood of 4000 to 6000 feet per minute. Beyond this point the pressure drop to force the air through is greater than the gain in cooling and, therefore, undesirable. In very long machines, where the limit of mechanical possibilities has been strained, the ability to take all of the air into the ends of the machine becomes extremely difficult. When such conditions are met it seems desirable to do away with fans on the rotor and resort to external blowers. The adoption of external blowers has never been viewed with much favor by the operating engineers, due to the fact that it means another auxiliary added to the already overburdened list, and, second, due to a feeling on the part of most operators that the external blower as now built is not as reliable as fans attached to the rotor. These considerations have in the past more than outweighed the advantage of the higher efficiency which was to be

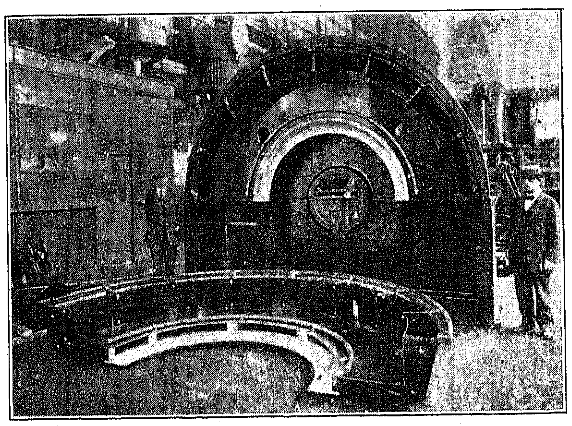

Fig. 11—One of the End Shields or Ventilation Housings for 62,500-Kv-a. 1200-Rev. per Min. Generator Showing in Foreground Upper Half Inner Shield Attached to Outer Shield, with Fire Protection Pipe at Inner Circumference

expected in the slow-speed blower. With the advent of the closed circuit system of ventilation in which the air is recirculated, the losses being removed by surface coolers, new possibilities are introduced which make the consideration of external blowers at this time quite pertinent.

Owing to the limited space, the high peripheral speeds, and the difficulty of recovering the velocity head of air leaving the fan, the design of fans attached to the rotors of large turbine generators becomes difficult

and results in a fan of rather low efficiency. Since the air passes through these fans before entering the generator, the losses of fans are absorbed by the air, which results in the air entering the generator at some 5 deg. or 6 deg. higher temperature than would be the case if the fan losses were not absorbed. When it is considered that the total rise of air through the generator is only 20 deg. to 25 deg., the large sacrifice which must be made to supply sufficient air for ventilation and the handicap which is placed on the generator in the matter of temperatures are at once apparent. If, on the other hand, an external blower is used on a recirculating system, and the blower is mounted ahead of the surface coolers, the temperature of the air entering the generator will be practically that of the air leaving the coolers, and the temperature of the generator, both stator and rotor, will be lower by the amount of heat which would have been absorbed in the air due to the fan losses. A second advantage of the external blower lies in the higher efficiency which it is possible to obtain. The question of reliability of these blowers is one which should not offer any serious obstacles when it is clearly known what the service requirements are.

General

The space for windings in the rotors of all steam turbine generators is limited by the high centrifugal forces. On the other hand, if the stator winding is correctly designed to be free from eddy currents, and slots of ample dimensions are provided, any desired low temperature of the windings may be secured when properly ventilated and insulated by materials of high thermal conductivity; hence, in so far as temperatures are concerned, the permissible electrical power output of a turbine generator need not be limited by the stator winding; provided the electrical stability is maintained, voltage regulation is commercially acceptable and the rating is maintained for which it is designed.

Although it may be said that no well-designed machine should be actually limited by any one element and in keeping with the principles of economy that the stator should be decreased until a balance is reached with the rotor, nevertheless, a certain freedom in design of the stator exists as to the control of losses and temperatures wherein no hard and fast boundary defines what is absolutely safe.

It will be impossible to include here any extended description of stator windings and the several methods which are now being used to partially or completely eliminate the circulating currents in the coils.

The stator conductors of either the bar or coil type consist of many thin rectangular strips of copper insulated by cotton or asbestos. Using a thin strand obtains small strand loss; the segregation of the strands permits the carrying out of the following methods of reducing the circulating current loss to a low value:

(a) Use of stranding continuously through two or more coils;

(b) Use of a total depth of copper in the slot, less than would be used if the d-c. resistance alone was considered;

(c) Use of a large number of circuits and turns per coil;

(d) Use of a two-turn coil turned over at one end only;

(e) Use of a twisted conductor;

(f) Use of coils having conductors segregated into groups and the groups carried separately through two or more coils by proper transpositions.

Losses

Fig. 8 shows the efficiencies of this generator at various power factors. Values upon which the efficiencies are based are made up of the well-known losses, such as friction and windage, core loss, $I^2 R$ of the armature and rotor, and also of certain losses which for a better term, and because they are usually associated with the loading of a machine, are called "load losses." It is not the purpose of this paper to go at any length into the source and magnitude of these last losses; the field is large enough to form the subject of a paper by itself. It is enough here to mention that losses do exist in all of what might be termed the inactive magnetic parts, as well as the active magnetic parts of the machine, and that these losses are of sufficient magnitude to have an effect on the efficiency of the machine. Under the heading of inactive magnetic parts might be classed the clamping flanges, the air shields, the dovetail ribs, frame, etc. Under the active magnetic parts might be classed the rotor core surface, retaining wedges, retaining rings, etc. The difficulties of actually determining magnitude and source of loss in these various parts, on, say, a 30,000-kw. machine, are so great, due to the difficulty in handling such enormous amounts of power, that the idea was conceived of building turbine-type generators in miniature and carrying out investigations on these little machines. Accordingly, two little machines were built which are in every essential detail an exact reproduction of the large machines, reduced in scale to one-third size, and, hence, in capacity to one-twenty-seventh. It is gratifying to record that the percentage losses shown by these little machines were in exact percentage agreement with the large machines. This does not include the $I^2 R$ of the copper, which is excessive. By substituting wood for the various inactive parts, such as shield, flanges, end fingers, etc., the magnitude of the losses in these various parts has been fairly accurately determined. The steel end rings, which retain the end windings of the rotor coils, have for sometime been under suspicion as being the source of considerable loss, although complete data are not available on these parts. Tests on the miniature generators revealed the fact that while there is magnetic field set up by the end turns of the armature winding, which rotates in synchronism with the end bells on the

rotor and, therefore, causes no loss, there is also a high-frequency harmonic flux which occasions a considerable loss in the end structure. These harmonic fluxes are probably associated with certain pitches of the armature winding and are worse with some pitches than with others. However, no exact data are available on this point. A very interesting feature in connection with these losses is that they bear a relation to the magnetic loading of the field. The armature end turns set up a magnetic circuit which interlinks the rotor end structure with the inactive magnetic, and a portion of the magnetic parts of the stator. Under conditions of sustained short-circuit, or of a leading power factor on the machine, under both of which conditions the magnetic loading of the rotor is small, the flux leakage paths are completely dominated by ampere-turns of the armature and windings, and the losses are large. Under conditions where the machine is loaded, or where the power factor is lagging, the end turns of the rotor establish a counter m. m. f. which greatly changes both the direction and magnitude of the end flux leakage and this results in much lower losses and heating.

The data that have been and are being taken on this subject lead us to hope that at no distant time the uncertainty regarding the losses at heads of machines will be done away with and that machines of even higher efficiency than at present will be built.

As intimated above, it may be stated that the temperature rise of the rotor winding is greater than

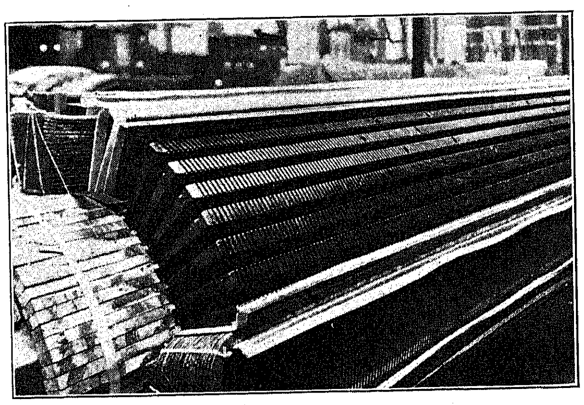

Fig. 12—Partially Wound Rotor of 31,250-Kv-a. Steam Turbine Generator Showing Circumferential Grooves in Solid Forged Rotor to Improve Ventilation and Reduce Pole Face Losses

it would be from the losses of the exciting current alone and the efficiency of the generator is reduced by the following additional losses of the rotor.

1. Friction of the rotor surface upon the cooling air.
2. Eddy currents at stator tooth frequency in the skin surface of the rotor steel. These are dependent upon the variable permeance of the slotted inner periphery of the stator core and the zigzag flux leakage to and from the rotor caused by the distribution of the m. m. f. in the stator slots.

It is now the general practise to groove the surface of the rotor body, see Fig. 12, and thereby secure lower temperature rises of the field winding and higher efficiency. Grooving of the surface with a coarse thread increases the cooling surface, reduces slightly the flow of air, and increases the turbulency of the cooling air in the air-gap. It also increases the resistance to flow of high-frequency eddy currents, thus improving efficiency as well as reducing temperature. In all cases when tested it has been found that grooving has been beneficial.

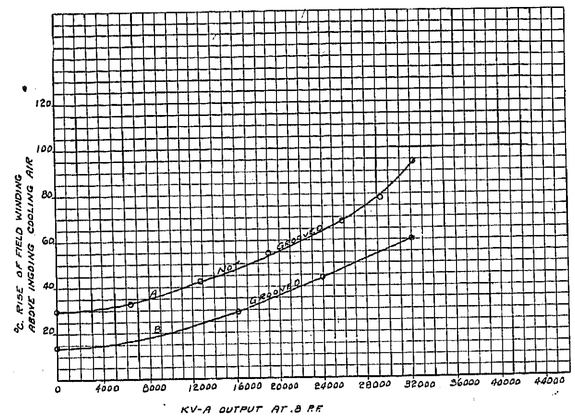

Fig. 13—Reduction in Temperature of Field Winding of 31,250-Kv-a. 13,200-Volt Generator Obtained by Grooving Rotor

An exceptional case of the improvement which has been registered by the grooving of the rotors is seen by observing in Fig. 13 the difference in temperature rises secured during tests upon two rotors identical in all respects except that one was grooved, the other not.

The tests determined the following:

1. The grooved rotor obtained a temperature rise of 58 deg. cent. at closely full-rated load as compared with a rise of 88 deg. cent. for the similar ungrooved rotor. A second grooved rotor tested in the same stator as the ungrooved rotor just mentioned obtained a temperature rise of 53 deg. cent. upon the field winding.
2. The core loss was reduced.
3. The rate of air flow before grooving was 69,200 cu. ft. per min. This was reduced to 54,000 cu. ft. per min. by grooving.
4. The stator was slightly reduced in temperature.
5. The efficiency was greatly improved.
6. The customer has had a third rotor grooved, tests upon which have not yet been made.

Fire Protection. It is particularly important, in connection with large totally enclosed generators, that they be equipped with protection against fire. Differential relay protection should be provided in order that excitation may be removed and the unit disconnected from the system in case of a break through the insulation. The differential relay protection is useful only in connection with dielectric failures. It sometimes happens that fire starts on the surface of the

insulation, without any electrical failure of the insulation. The exact causes of these fires are not always known but they are liable to occur under certain conditions, such as the accumulations of dirt, consisting of carbonized and other highly inflammable material. They are probably started by a spark resulting from some switching operation or sudden heavy short-circuit on the line. Such surface fires are hardly known, except in connection with large totally enclosed machines. It is possible in some cases to discover the presence of fire by means of smoke in the air discharge pipe, in which case it is usually best to immediately remove excitation, shut unit down and make examination. However, all large generators should be provided with equipment to put out fire, whether started by dielectric failure or on the surface of insulation by static spark or spark of any kind. It has become customary to equip these machines with pipes at both ends in proximity to the end windings, and with outlets so placed as to direct steam, water, or whatever agent is employed, in the most effective manner for putting out the fire. Steam and water are the agents that have been generally employed, although carbon dioxide gas has been installed in a few cases. The use of an inert gas, such as carbon dioxide, has a certain advantage over the use of steam or water, in that it causes no temporary or permanent injury to the insulation; furthermore, the operator who suspects a fire, will not wait so long to persuade himself that action must be taken if it is a matter of turning on the inert gas, as he would to turn on steam or water. Undoubtedly, the system of ventilation now coming rapidly into vogue, of recirculating the same air and carrying off the heat by water-coolers, has a decided advantage in the matter of fighting fire, since the oxygen in the air becomes rapidly exhausted and the products of combustion serve as a fire extinguisher. Tests that have been made indicate that a 25 per cent saturation of carbon dioxide is ample to put out a fire. This system of ventilation prevents the accumulation of dirt, etc., and thus does away with surface fires.

Operating Conditions. The handling of the rotor by operators is of extreme importance. Probably everyone will concur in the statement that the ideal condition to maintain in large generator rotors is a constant temperature of such magnitude as 90 deg. or 100 deg. This is not possible, but efforts should be made to maintain temperatures that vary within as small a range as possible.

The distributed field winding assembled in numerous slots and secured against centrifugal force by metallic wedges in the slots, and by metallic binding bands at the heads, is peculiarly susceptible to deterioration if temperatures within the winding change frequently and through a wide range. To appreciate this, it is only necessary to consider the nature of the structure. There are three different materials, one of them a more or less intricate compound, that are subjected to the changing temperatures, and each of the materials has a temperature coefficient of its own quite different from that of the others. The temperature coefficient of the copper is the highest and about one and one-half times that of the steel in which it is embedded, and approximately two times that of the insulation that stands intermediate between copper and steel. To intensify these differences, the copper itself is always subjected to a much wider range in temperature than either the insulation or the rotor body proper, and to a quicker response to changing load. Common practise points to temperature rises in the copper of about 85 deg. cent. above the cooling medium, as the proper range to allow between the no-load starting and the full-load constant running conditions. By simple arithmetical applications of the formula for change of volume, due to temperature coefficient, we find the length of the field conductor in a 10-foot long slot for a quick 85 deg. change, to become $\frac{1}{8}$ in. greater than the length of the envelope of insulation (except as the insulation stretches to accommodate itself to the pulling action of the coil) and 1/12 in. greater than the slot itself, in which the insulated coil is embedded. Undoubtedly, improved methods of forming up the insulation and of holding the winding against centrifugal force allow these movements to take place with very little deterioration, but it is greatly to be desired on the part of the operating people, that the variations in temperature be kept from exceeding 85 deg. cent.

A condition of operation to be avoided is that where the rotor is allowed, if the turbine is shut down, to reach a temperature approximating that of out-of-doors in our temperate climates during the cold season. Undoubtedly cases have existed in the near past, and probably at the present time, where the rotor temperature gets down to 0 deg. cent. after an overnight shut-down. If in such case the turbine is started up and loaded, and for reasons of personal comfort to operators the station air is used, the cooling air temperature will soon attain something like 25 deg. cent., and if for any reason an overload in kilowatts for short period is desired, or an unusual demand for power factor correction arises, requiring excitation amperes to be increased as much as 10 per cent over that stamped on the nameplate, the rotor temperatures may readily attain 135 deg. cent. In such case the unequal expansions of the various materials of the rotor that may occur within a few hours, undoubtedly exert mechanical strains that come with tremendous force on the insulation and must, when repeated often enough, result in disintegration. It is, therefore, strongly recommended to all operators of large steam turbine units, that they make a study of how to maintain the best conditions in the matter of temperature change in the rotor windings.

It is to be recommended:

1. that overloads in excitation be avoided;
2. that such arrangements of ventilation be adopted

as will permit as little cooling down of rotor when at rest as possible, and on no occasion permit the entrance direct of outdoor air;

3. that a practise be established of determining temperature of field winding at stated intervals by observing volts at collector rings and comparing it with that obtained at same load conditions when generator was first put into service;

4. that insulation resistance be measured at the same stated intervals;

5. if for any reason at any time the rotor at rest has become very cold, run it at low speed with excitation sufficient to bring temperature up to about 75 deg. cent. in about one-half hour before raising speed to normal.

Possible Future Units. The possible size of a generator at any speed is limited if the designer produces a "stiff shaft" machine; by this we mean a generator whose rotor at normal speed operates below the critical speed. Assuming adherence to this practise, and to the same type of ventilation at the several speeds, *viz.*, whether by means of self-contained fans or external blowers, the possible size at the several speeds varies inversely as the square of the speed; thus, for 60-cycle generators, 8000-kw. (10,000 kv-a.) at 3600 rev. per min., corresponds to 32,000 kw. (40,000 kv-a.) at 1800 rev. per min. and to 72,000 kw. (90,000 kv-a.) at 1200 rev. per min. The largest sizes at all these speeds should be of high voltage, say 10,000 to 14,000, to permit of the use of a sufficient number of slots and have designs that will not result in too large amperage per slot.

In the matter of size as related to ventilation adopted, it can be greatly increased by the use of external blowers, since the number of multiple paths can be increased and valuable space saved between the rotor proper and the bearings at the two ends, space otherwise occupied by the fans and the chambers below and beyond them for the ingoing air. At the present moment 62,500-kv-a., 14,000-volt 60-cycle generators are being built at 1800 rev. per min. and 60,000-kw. 11,000-volt 25-cycle generators at 1500 rev. per min.

Discussion

A. M. Rossman: The built-up rotor construction which Mr. Foster describes is interesting to us because of an experience which we have had within the last two or three months with a solid forged rotor.

A machine had been operating down in Texas for eight or ten years. The rotor winding had gone bad and the rotor was sent to the factory for rewinding. The factory examined it and found a crack in the rotor which extended two-thirds around the circumference, and rendered it unfit for going back into service. We then split it and found that the only good metal left consisted of a skin of metal about 1/8 in. or 1/4 in. thick on one side and, on the other side, a small wedge 2 in. or 3 in. in thickness extending from the center to the outer surface. How that machine continued to operate with the rotor in that condition is a mystery.

Going into the history of this rotor we discovered that it had been made in Germany and shipped to this country just before the war began. It had an exploring hole drilled from end to end and the factory records showed that a small crack from shrinkage had been detected. They did not, at that time, attach the significance to that small crack that they should have. Now, I understand, such a forging would be rejected. In the built-up rotor described by Mr. Foster such a crack could not easily develop.

Another point is this question of low temperatures. We are very glad, indeed, to see progress made toward cutting down the high temperatures. The earlier machines used to give considerable difficulty. The copper would expand when it was hot and shrink when it was cold, and before we knew it, the insulation had separated from the copper and was bowing out in the ventilation slots; the next thing we had a break-down and the machine would have to be rewound. Low temperature means low expansion and a lessening of mechanical troubles.

C. M. Gilt: The maximum size of turbo-generators has increased in a series of steps with periods of several years intervening in which the size has remained fairly constant. Recently another step has been taken, beginning with the 62,500 kv-a. units for Brooklyn and followed by machines of even larger capacity.

The amount of power developed by, and the money invested in, one of these large units is so great that the problem of adequately protecting it against damage, and of limiting the extent of the damage should a failure occur, is presented with increased force. The older practise of tying generators to the bus without any automatic protection against internal failure has been replaced by a scheme of differential protection of the windings. The method of balancing one end of each phase winding against the other is serviceable in case of a phase-to-phase short circuit, or phase-to ground when the unit is on a grounded system, but will not operate on a turn-to-turn short circuit within the same winding until the damage has spread far enough to develop into a ground or a phase-to-phase short-circuit.

As a further protection the groups of parallel windings within the same phase windings of the Brooklyn machines have been balanced against each other with cross-connected current transformers. This method of protection loses its sensitivity when the number of turns per phase is large for the circulating current sent around the closed loop by the unbalance of the voltage in the parallel paths is a small portion of the total current of each path.

We are considering a somewhat different method of balancing the windings of one of the machines on the system and from such studies as have been made it appears to have some advantages. The method consists in tying together the parallel windings at equipotential points at approximately the middle of the winding and inserting a current transformer and relay in the tie. Normally no current should flow across this tie and the current transformer can, therefore, be built of a size to give the best relay operation. In case of a turn-to-turn short circuit, a ground or a phase-to-phase short circuit occurring in the windings between the ends where they are joined together, a current will flow across the mid-point connection and cause relay operation. The advantage of this method over the scheme of balancing the windings with cross-connected current transformers lies in the fact that the relay operates directly on the cross flow of current and not on a small difference between two large currents. The disadvantage lies in the additional number of leads that must be brought out of the machine and the taps made at the mid-points. The additional leads brought out need not be full capacity, but built only to withstand possible short-circuit currents. While the scheme appears to be selective, there may be difficulty in picking out points on some machines as actually built so that the cross flow of current under all conditions of load and external short circuit will be enough smaller than that which would occur in case of a turn-to-turn short circuit to give posi-

tively selective relaying. This difficulty, however, applies with equal force to the other method of balancing.

We agree with the authors of the paper in favoring a closed system of ventilation for these large machines. Not only is it more effective than an air washer in keeping the machine clean, but we believe has also a decided advantage in case of a fire. The oxygen within such a system will not sustain combustion very long. In addition a gaseous extinguisher, such as carbon dioxide, can be used effectively within a confined space.

We are naturally keenly interested in the development of these large machines, their protection and operation. We believe that there is a real place for them on the larger systems and hope to see them perform creditably.

E. H. Freiburghouse: While it is possible, as Mr. Rossman points out, to inspect thoroughly the cylindrical body and stub ends of a rotor such as has been employed in this 62,500-kv-a. generator at Brooklyn, nevertheless, very rigid sensitive tests are made by which forging faults may be detected and located throughout the interior of the one-piece solid forged rotors.

It has been our practise for many years to use a solid forged steel rotating element having radial slots machined in it to receive the windings. The reliability of the solid forged rotor has been demonstrated by the successful operation of over 1000 units ranging in capacity from 2500 to 62,500 kv-a. and rotative speeds from 1200 to 3600 rev. per min.

No failures chargeable directly or indirectly to forging faults have occurred during operation either in the factory or in service. This is also true of the nickel-steel forged rings which support the end portions of the rotor winding.

Three inspections are made of the interior of the forging, the first being at the forge shop.

The number of rotor forgings rejected from all causes has been less than 2 per cent of the total number received to date.

The scheme of protection mentioned by Mr. Gilt for obtaining protection against internal faults, such as short-circuited turns, has been considered and seems to have desirable features, but to the writer's knowledge has never been applied.

Heating of Large Steel-Cored Aluminum Conductors

BY R. J. C. WOOD

Associate, A. I. E. E.

Southern California Edison Co., Los Angeles, Cal.

STUDIES of the most economical size of conductor for the Southern California Edison Company's third 220-kv. transmission line from Big Creek to near Los Angeles led to the choice of either a 1,033,500-cir. mil aluminum steel core or the equivalent 650,000-cir. mil copper cable.

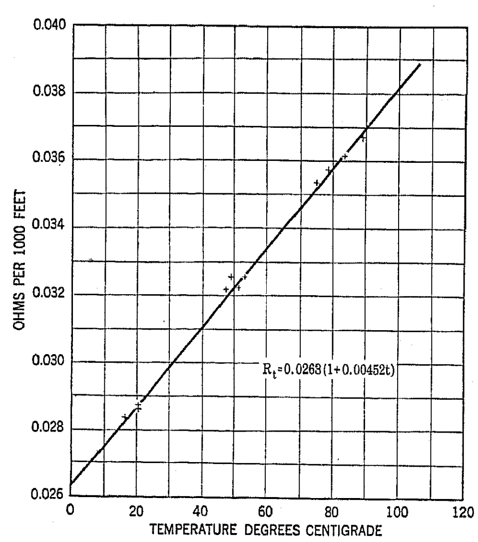

FIG. 1—TEMPERATURE COEFFICIENT OF RESISTANCE 605,000-CIR. MIL ALUMINUM STEEL CORE CABLE

The normal full-load current in this conductor will be about 450 amperes and when a fourth line is put into service, paralleled and cross-connected with the third circuit, it appears probable that sections of the line may be required to carry 900 amperes continuously over periods of time.

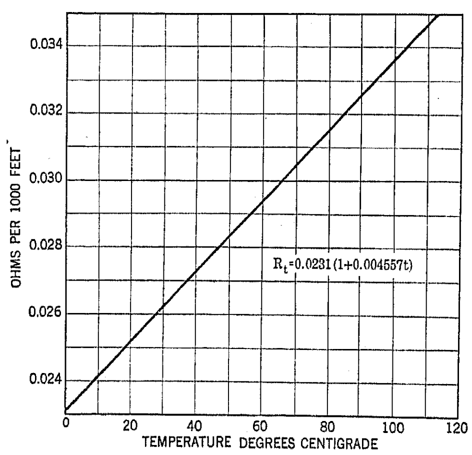

FIG. 2—TEMPERATURE COEFFICIENT OF RESISTANCE 666,000-CIR. MIL ALUMINUM STEEL CORE CABLE

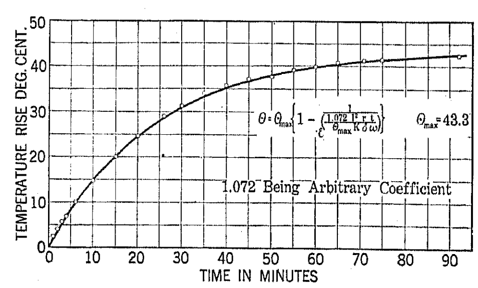

FIG. 3—HEATING CURVE OF 666,000-CIR. MIL CABLE CARRYING 602 AMPERES

The temperature rise of such cables carrying these large currents might prove to be a limitation which

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., October 13-17, 1924.

would have to be considered, not on account of fusing or burning, but because of the expansion and consequent increased sag in the transmission line which might reduce clearances unduly.

The rulings of the California State Railroad Commission require, at 130 deg. fahr. a minimum clearance to ground of 30 ft. over territory susceptible of cultivation and accessible to vehicles, and over highways and railroads. This provides an ample margin, but special cases occur, such as crossing other circuits either of the same or other companies' systems where insufficient clearances might be allowed, if the reduction in

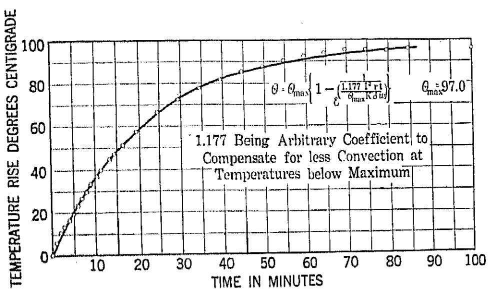

FIG. 4—HEATING CURVE OF 666,000-CIR. MIL CABLE CARRYING 900 AMPERES

them, due to conductor heating, were not taken into consideration.

A series of tests was therefore made to determine the temperature rise, both in still air and in wind, of three sizes of aluminum steel-cored cable; tests upon copper to be made at a future date.

The first tests were upon new clean cable of 666,000 cir. mils.

Afterwards, a sample of 605,000-cir. mil cable cut from the Big Creek transmission line was tested. This cable had become coated with a hard, smooth, black

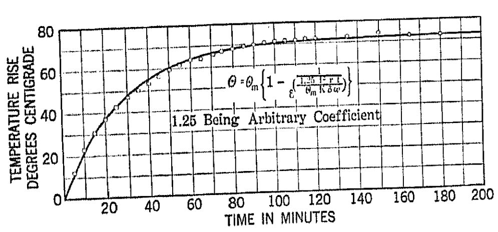

FIG. 5—HEATING CURVE OF 1,033,000-CIR. MIL CABLE CARRYING 1029 AMPERES

deposit which forms on the conductor at high voltages by electric precipitation from the air. This coating can only be removed with difficulty even by scraping; incidentally it reduces corona loss, probably by reason of the mechanical smoothness imparted to the conductor.

Finally, the heating of new clean 1,033,500-cir. mil cable was determined. It should be noted that these sizes as here given do not include the steel core.

The tests were made indoors in the corner of a large room remote from doors and windows. A length of about 20 ft. of cable was suspended so that about six feet was horizontal at a height of seven feet above the floor and six feet below the ceiling; the ends of the cable were brought around to the supply transformer furnishing 50-cycle low-voltage current. The loop of cable thus formed an oval about nine by six feet, its plane inclined to the vertical, to prevent heated air rising into the upper portion.

Attempts were first made to measure temperatures with thermometers, but these were abandoned in favor of a copper-constantin thermocouple having its hot junction placed in the second layer of stranding of the cable, counting from the outside. The cold junction was similarly embedded in a 24-in. length of cable suspended 12 in. below the horizontal portion of the cable under test. This arrangement gave consistent results, measured the temperature rise directly, and was sensitive to 1 deg. cent; stem effect or conduction of heat by the thermocouple wires was practically eliminated by wrapping them once round the circumference of the cable, insulated from it by a single layer of thin paper continued once around outside the thermocouple wires and bound in place.

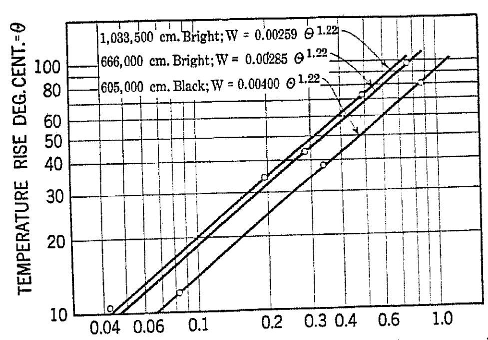

FIG. 6—HEAT DISSIPATED IN WATTS PER SQUARE INCH OF SURFACE

The final temperature rise resulting from different amounts of current was determined, as well as the rate of heating from the time the current was first turned on until a steady temperature was reached, this requiring from 1.5 to 2 hours. From these observed results the watts dissipated per square inch of surface were calculated, the surface being considered as that of a cylinder of diameter equal to the maximum outside diameter of the cable. The rate of heat emission per degree of temperature rise increases with temperature difference as shown in Fig. 6. As might be expected, the black coating referred to above increases the rate of heat emission.

The observed results differ materially from the data published by George E. Luke in the *Electric Journal*, Apr., 1923. Luke found that the temperature rise was practically proportional to the watts dissipated and that the color and nature of the surface had but little effect upon the specific rate of heat dissipation, whereas in our experiments the rate of heat dissipation was found to be about 66 per cent more at 100 deg. cent.

rise than at 10 deg. cent. rise and to increase 17 per cent even between 30 deg. cent. and 60 deg. cent. temperature rise. Blackened aluminum cable dissipated 40 per cent more energy than bright cable with equal temperature rise. It is not to be expected that Luke's curves for the temperature rise of aluminum cables would agree closely with observations made upon steel-cored cable, as the latter has a relatively larger surface.

TABLE I
CONSTANTS OF CABLES

Nominal Size Cir. Mils	Aluminum Cir. Mils	Steel Cir. Mils	Total Cir. Mils	Weight Al.	1000 ft. Steel	No. of Strands Al.	No. of Strands Steel	Dia. of Cable Inches
605,000	605,000	78,000	683,000	568	211	54	7	0.954
666,000	666,000	86,000	752,000	626	232	54	7	1.000
1,033,500	1,033,500	134,000	1,167,500	971	353	54	7	1.247

Wind was produced horizontally by from one to three 12-in. desk fans appropriately placed so as to distribute the air current as evenly as possible, the velocity of the wind was measured by an anemometer calibrated during the tests. This artificial wind differs from natural winds in being very much more constant and in lacking the gusts and eddies of nature, but since the additional emissivity due to wind is approximately a linear function of velocity over a considerable range, the cooling effect as determined should agree with that produced by natural winds of the same average velocities.

TABLE II
OBSERVED DATA, HEATING OF CABLES

Size cm. Nominal	Surface	Wind ft/min.	Current Amps.	Temp. Rise Deg. Cent.	Temp. of Cable Deg. Cent.	Ohms per 1000 ft.	Watts per Sq. In.
605,000	Black	0	314	12.0	32.8	0.03027	0.08295
"	"	0	600	38.0	59.0	0.03335	0.334
"	"	0	902	80.0	100.0	0.03823	0.8654
"	"	97	905	55.2	77.8	0.03558	0.8108
"	"	170	907	47.0	69.8	0.03462	0.7926
666,000	Bright	0	602	43.3	62.3	0.02965	0.2857
"	"	0	900	97.0	117.0	0.0354	0.7605
"	"	96	900	57.5	73.5	0.03083	0.6622
"	"	175	900	47.0	66.0	0.03004	0.6455
"	"	300	900	34.8	53.8	0.02877	0.6180
1,033,500	Bright	0	338	10.5	36.1	0.01751	0.04259
"	"	0	690	34.3	60.4	0.01915	0.1940
"	"	0	1029	72.6	98.7	0.02175	0.4903

When it comes to the application of the results to transmission line design, the effect of wind in cooling the conductor is of great importance, even an almost imperceptible breeze of 0.2 miles per hour has a very marked cooling effect. More information is needed upon the minimum air velocities that may be coincident with maximum air temperatures in different localities.

From the data, empirical equations were obtained for the watts lost per square inch of surface, both clean and black, and for the additional convection losses due to wind, and it is believed that these equations are reliable for engineering purposes within the range of sizes covered by the experiments, and may be used without probability of material error between the limits of 500,000 to 2,000,000-cm. cables.

Resistance measurements were made by drop of potential method at different temperatures as the cable cooled naturally after a heat run. The temperature coefficient of resistance calculated from this data is slightly higher than previously published; a probable explanation is that when heated the aluminum stranding, expanding more than the steel core, loosens somewhat, decreasing the contact pressure between strands and layers. In a suspended conductor under considerable tension this effect will be less or absent. In order to be on the safe side, the coefficient as found was used in the derived equations.

In Figs. 3, 4 and 5 are shown the observed rate of heating compared with an equation of rational form but containing an empirical coefficient.

It may be shown that when

I = current in amperes
r = resistance in ohms of unit length
a = temperature coefficient of resistance
U = coefficient of heat emission from surface
t = time
O = temperature rise above air
K = heat equivalent in watt seconds
σ = specific heat of cable
w = weight of cable per unit length

$$\text{Then } \theta = \frac{B}{A}\left\{1 - \frac{1}{e^{At}}\right\}$$

and the final temperature rise after a long time is

$$\theta_m = \frac{B}{A}$$

$$\text{So that } \theta = \theta_m \left\{1 - \frac{1}{e^{At}}\right\} \qquad (1)$$

$$\text{where } A = \frac{U - a\,I^2\,r}{K\,\sigma\,\omega}$$

$$B = \frac{I^2\,r}{K\,\sigma\,\omega}$$

always provided that U does not vary with θ. As a matter of fact U does vary with θ as is shown in Fig. 6.

It will be noted that the observations as plotted indicate quite consistently an exponential relation between temperature rise and watts dissipated and that within the limits of expected error the value of the exponent is the same for all three cables, including both bright and black surfaces.

The substitution of the equation for U in the differential equation of temperature rise leads to an unwieldy integral. The heating curve for bright cable may be closely approximated, however, by the equation

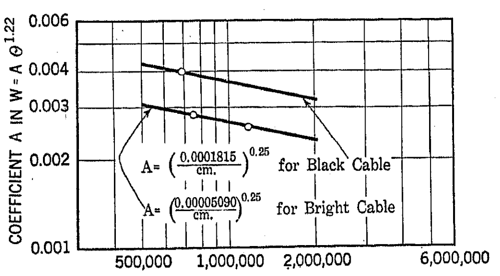

FIG. 7—COEFFICIENT OF HEAT DISSIPATED AS A FUNCTION OF THE SIZE OF THE CABLE

$$\theta = \theta_m \left\{ 1 - \frac{1}{e^{\left\{ \frac{I^2 r t}{[1 - (.0043\, I)^2]\, \theta_m k \sigma \omega} \right\}}} \right\} \qquad (2)$$

this is the equation of the solid lines in Figs. 3, 4 and 5, and is of the same form as equation (1) when

$$A = \frac{I^2 r}{[1 - (.00043\, I)^2]\, \theta_m K \sigma \omega}$$

The agreement with the observed rate of temperature rise is sufficiently good for practical purposes. The number of different sizes of cable used is, of course, much too few to establish a law connecting size with coefficient of heat emission, but the nature of the phenomena suggests an exponential equation which cannot be greatly in error within reasonable limits since the change in emission is small over a large range in cable size. In Luke's paper, above referred to, this matter is considered much more fully, and a curve of dissipation constant and cable diameter is given. This curve is an exponential one lending strong support to the assumption as to the form of the curve.

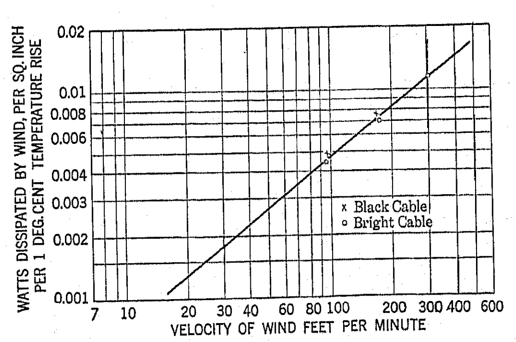

FIG. 8—HEAT DISSIPATED BY WIND IN WATTS PER SQUARE INCH PER DEGREE TEMPERATURE DIFFERENCE

Only one size of blackened cable was obtainable and it is assumed tentatively that the curves for black and bright cables are parallel, which is the same thing as assuming that the ratio of the dissipation constants of bright and black cables of the same size shall be constant and independent of their absolute size.

Fig. 7 shows how the heat emission varies with the size of the cable which is here given, including the area of the steel core. The equation of loss is

$$W_s = \left\{ \frac{0.00005090}{\text{cir. mils}} \right\}^{0.25} \theta^{1.22} \text{ for bright cable} \qquad (3)$$

$$W_s = \left\{ \frac{0.0001815}{\text{cir. mils}} \right\}^{0.25} \theta^{1.22} \text{ for black cable} \qquad (4)$$

where W_s = watts dissipated per square inch of surface. To obtain watts per square inch per 1 deg. cent. the exponent of θ will be 0.22.

The effect of wind is given in Fig. 8 and Table III.

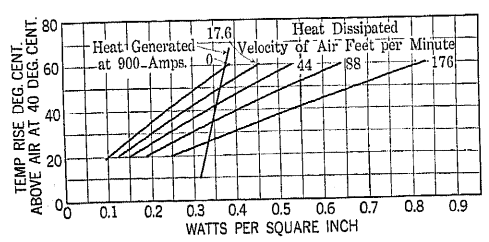

FIG. 9—HEAT DISSIPATED IN STILL AND MOVING AIR FROM BRIGHT 1,033,500-CIR. MIL CABLE, AND HEAT GENERATED AT 900 AMPERES.

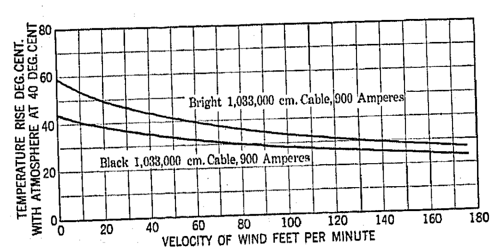

FIG. 10—FINAL TEMPERATURE RISE OF 1,033,500-CIR. MIL CABLE CARRYING 900 AMPERES

The additional heat loss due to wind over and above what would exist in still air is shown as a function of air velocity.

The loss is nearly the same for bright and black cable as would be expected. This loss is given by

$$W_w = 0.0001185\, V^{0.8} \theta. \qquad (5)$$

where W_w = watts dissipated per square inch

V = velocity of air feet per minute.

The total loss from the cable is therefore given by

$$W = \left\{ \frac{0.00005090}{\text{cir. mils}} \right\}^{0.25} \theta^{1.22} + 0.0001185\, V^{0.8} \theta.$$

for bright cable (6)

$$W = \left\{ \frac{0.0001815}{\text{cir. mils}} \right\}^{0.25} \theta^{1.22} + 0.0001185\, V^{0.8} \theta.$$

for black cable (7)

TABLE III
Watts Dissipated by Wind
666,000 cm. Bright Cable

Wind Ft/Min.	Temp. Rise Deg. Cent.	Watts per Sq. Inch per 1 Deg. Cent.		
		Total	Loss in Still Air	Loss due to Wind
96	57.5	0.01152	0.00700	0.00452
175	47.0	0.01373	0.00669	0.00704
300	34.8	0.01776	0.00626	0.01150
605,000 cm. Black Cable				
97	55.2	0.01468	0.00973	0.00495
170	47.0	0.01686	0.00943	0.00743

W being watts per square inch of surface.
The heat generated in the cable is

$$W = \frac{I^2 r_o [1 + a(\theta + t)]}{S} \tag{8}$$

where t = temperature of atmosphere
S = Surface area of a length of cable having resistance r_0 at O deg. cent.

The final temperature rise is easily obtained graphically at the intersection of the curves plotted from equations (6) and (8) or (7) and (8). The method of doing this is illustrated in Fig. 9, the curves giving heat dissipated as per equation (6) and the straight line giving heat generated plotted from equation (8). Intersections are in equilibrium giving the final temperature rise. A cross plot from Fig. 9 and other similar curves are shown in Fig. 10 which presents the final results in a more agreeable manner.

The probable increase of effective resistance of these steel-cored cables on alternating current has not been separately considered but is included in the empirical coefficients.

Discussion

A. M. Rossman: I was shown recently some results of a heating test of a cable passing through a clamp. The cable was 300,000-cir. mil copper cable and the clamp was a drop forging. In that case the current was raised to such a value as to give an appreciable heating of the conductor. The clamp showed a temperature several degrees higher than the conductor. This is contrary to the results which Mr. Wood obtained.

H. W. Dwight (by letter): The question of the effect of direct sunlight in raising the temperature of a large conductor and increasing the sag has a close reference to the problem investigated by Mr. Wood. If the measurements were made indoors under a skylight, the effect of wind would be eliminated. If they were made outdoors, the error due to reflection by the skylight would be removed. The results, though approximate, would give additional data for calculating the maximum sag.

The question of the best size of steel core is one which could be investigated with the apparatus used by Mr. Wood, if samples of cable were obtained having different sizes of steel core but the same cross section of aluminum or copper.

Among the advantages of a larger steel core are: first, reduction of corona loss; second, reduction in skin effect due to the aluminum or copper taking the shape of a thinner tube; third, reduction in resistance due to larger cooling surface and lower temperature, and fourth, reduction in resistance due to the current carried by the steel core, which, although a small percentage, is not negligible, but amounts to about 2 per cent. These advantages are all equivalent to increasing the conductivity of the aluminum or the copper. They should be balanced against the disadvantages due to the larger steel core, of greater cost, weight, ice load and wind load.

Such a comparison might result in the use of a core of low-grade steel for a large copper transmission cable. A core of low-priced steel has usually better conductivity than one of high-grade steel, for alternating current, though not always for direct current.

If accurate means are not available for measuring the watts loss with alternating current, each sample of cable could be calibrated by measuring its final temperature rise when direct current is flowing, for which case the watts loss can be accurately measured. The temperature rise by thermocouple can also be very accurately measured, as is shown by the test readings plotted on the curves in Mr. Wood's paper. Curves would be drawn for each sample showing the continuous temperature rise plotted against watts loss. Then, when alternating current is flowing, the watts loss would be deduced from the temperature rise, and the effect of the steel core on the conductivity of the cable for alternating current could be judged.

For example, if 400 amperes, alternating current, in a 600,000-cir. mil copper cable without a core produced a temperature rise which gave 107 watts in a certain length, from the calibration curve of that sample, and if the same current in a cable with 600,000-cir. mils of copper and a steel core produced a temperature rise which gave 100 watts on the corresponding curve, then the core had increased the conductivity 7 per cent, by reducing skin effect, by lowering the temperature and by adding the conductivity of the core itself. This total percentage increase in conductivity due to the core might be greater than the percentage increase in cost caused by the core, and there would be the added advantage, of probably more importance still, that the corona loss would be decreased.

In making such a test, the samples might be connected in series, so that there would be no doubt that they carried equal currents. Such a test, like many other measurements in physics, could well be made in a room in which a thermostat controls electric heaters of sufficient size to hold the room temperature exactly constant, which is a very easy matter to arrange except in hot weather.

The Development of a Suspension-Type Insulator

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Review of the Subject.—Increasing voltages employed in transmitting large amounts of power over long distances emphasize the need of a better line insulator for this purpose. This paper outlines the conditions under which a suspension-type insulator has been experimentally developed for such use. In the form described, it consists of metallic terminal members to suitably distribute the electric flux and an insulating, impregnated wood (or other material) mechanical strain member, concentric with the hollow electric field produced between the metallic terminal members. The unit described is designed for use on 110,000-volt power lines, two similar units in series for 220,000-volt lines and, when necessary, three units in series on 330,000-volt lines. The fundamental principles of the insulator are established and later continued experience in service may or may not modify detail.

INTRODUCTORY

ONE of the important problems involved in long-distance high-voltage power transmission at the present time is that of the line insulator. For the last 30 years, constantly increasing voltages have been employed for power transmission and each increase in line voltage has brought its problems of design of the several component parts, which together constitute such a system; transformers, bushings, line conductor for corona losses, regulation, control, etc.; insulation, insulators, etc.

Step by step the work of investigation, development, and resulting design of the several parts of high-voltage systems has kept pace with the commercial and economic demands for ever increasing line voltages. Some of the time, possibly, one or more of these factors has lagged behind the development demanded by the situation, but, in general, the engineering investigation and design of the various features have kept pace with and, in certain instances, have clearly pointed out a path of accomplishment in advance of economic needs.

Fig. 1 indicates what this advance in line voltage has been through the years. Voltage values for line insulators, bushings and power transformers, as distinguished from testing transformers, have followed a similar advance. Testing transformers and their bushings have ranged at from four to six times the voltage of the power transformer or line voltage for a given period.

We have been confronted, from time to time, in this progress by what appeared, at the time, to be insurmountable difficulties. First, the transformer could not be insulated, then, when it was insulated, the higher voltage could not be brought out through any available bushing. When this was accomplished, and the higher voltage was applied to the line, the known insulators failed and the line could not be insulated. After securing a line insulated from earth with improved insulators, the atmospheric and corona line losses were found to be such that the possible limit for line voltage was at one time set at 60,000 volts. Within three years of that time, it was shown that 60,000 volts was not the limit and within six years, the law of design for corona line losses was established. Recently we have had limits set for the amount of power which can be transmitted over long distances.

As frequently as some apparently impossible barrier has presented itself, it has been surmounted and this has gone on, time after time, through the efforts of many engineers and scientists, until the earlier and, at the time, really difficult barriers to surmount, now look as simple from our present point of view as our present

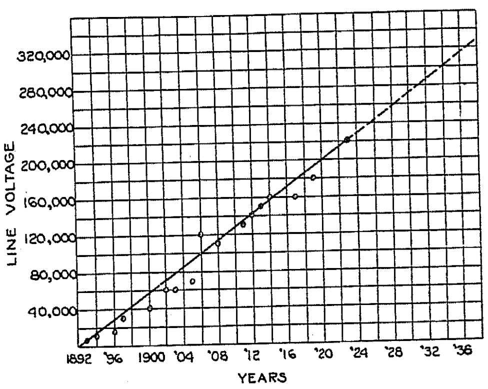

FIG. 1

"impossibilities" are likely to appear a quarter of a century from now.

The advance in power transmission line voltages, from the beginning of its advance over thirty years ago to the present time, has been consistent and uniform. It, at least, suggests the possibility of a continuance of a similar rate for some years to come. Whether we reach 300,000 or 350,000 volts for superpower transmission line potentials before or after 1935 or 1940 is perhaps not important. However, the case does appear to be clear that we are to advance in this direction, and that the times and the values for such line voltage increase are to be set, mainly, by economic rather than by purely engineering limitations.

These considerations, together with stimulation from

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work on other related factors of this problem, led the writer, early in 1920, to apply himself to the definite problem of the rational design of an insulator which might be better adapted to increasing line voltages than those then available. This paper will present the results thus far secured on that portion of the problem which relates particularly to the suspension type of high-voltage line insulator. Parallel work has been carried out on the application of these principles to the design of pin type insulators and to transformer and bushing design, but an account of this work will not be attempted within the limits of this paper.

In order to show the relationships of this problem, it may be well to outline most briefly the gradual development of the high-voltage line insulator. The first telegraph lines required insulators and, starting with the early split tube insulator of Ezra Cornell as an illustration, we have a duct with the insulating material (glass, pottery or porcelain) placed in compression. The development of the insulator, until most recently, has been along purely empirical lines upon this original basis of a capped duct or tube with the insulating material in compression. A fairly standardized glass, pottery or porcelain insulator for telegraph and telephone service yielded many years of satisfactory experience before there was need of insulators for the early light and power circuits. These early light and power circuits did not range above 700 volts, except certain series arc and incandescent systems which ranged up to 5000 or, in some cases, as high as 10,000 volts.

When 5000 volts or more were applied on the line, although below corona effects, difficulties were experienced and merely making a large capped duct or tube type telegraph insulator did not entirely answer the question. However, this type was still adhered to, and by making modifications of an empirical character, improved materials, petticoats, etc., improved puncture and leakage loss values, creepage surface, hygroscopic conditions, etc., etc., an insulator resulted which served its purpose for the higher voltages.

With the advent of corona forming voltages, there opened a new chapter of insulator development, in which much of the earlier experience was no longer applicable and a number of new theories and resulting designs appeared. These principles at first neglected the direct question of the surrounding electric field and were applied empirically to the preceding type of insulator,—still a capped duct with the line and tie wire at or near the top of pin and the insulating material still in compression. Corrugated surfaces, multiple parts, multiple petticoats, improved dielectric strength of materials, creepage current distribution, leakage losses, etc., etc., produced insulators which again served their purpose, although usually of faulty design and often concentrating electric flux unnecessarily so as to lower breakdown values they might otherwise have possessed.

A more careful study of this problem next led to a departure from the continued empirical development of the original tube or duct type and the rational application of theory to insulator design as based upon the study of dielectrics in air and the electric field of the insulator and its surroundings. The theory was stated by Maxwell many years ago, but its application to the improvement of insulator design was then new and is now firmly established in its application. The fact, then appreciated, that a conformity of the contour of the surface of the insulator to the electrostatic lines of force of the electric field gives a closer practical approach to the dielectric strength of surrounding air than can be secured by any other surface, was applied in the production of a new type of insulator. This insulator had introduced a new element into this class of design and, because the lines of mechanical stress were parallel to the electrostatic flow lines, an insulator was produced in which the material (porcelain) could be used otherwise than in simple compression and need be no longer of the simple tube or duct type. It was no longer necessary for the line and tie wire to surround or approximately surround the top of the pin. The resulting insulator introduced these principles together with surfaces of rain sheds conforming to equipotential surfaces of the surrounding electric field.

With further increase in line potentials above 100,000 volts, even this development of the pin-type insulator no longer met the demands and, even before this latter development, suspended strings of insulating (porcelain) units came into high-voltage line practise. This was another departure from the earlier empirical duct or tube type and appeared, at first, to take care of the difficulties imposed by steadily mounting line voltages. However, it was soon found that a string of such units, because of capacity relations, would not withstand a voltage equal to the sum of voltages which could be applied to the individual units. This effect, together with increasing costs and lengths of such strings of units for the higher voltages now in use or contemplated in the near future, leaves much to be desired and possibly presents another barrier.

Statement of the Problem

In former insulator designs, and until the application of the flow-line principle and the use of the string of suspension units, it has been the practise to design for higher and higher voltages by increasing the dimensions, improving the materials, multiplying the parts, etc., as based empirically upon previous experience with the tube or duct type insulator and its modifications. Even with the application of the flow-line principle to commercial designs, the surfaces of the porcelain were shaped to conform to the lines of force of a field resulting from previous practise as to location and shape of metallic terminals.

In all of these designs, the porcelain insulating surfaces, or some of them, were in the path of maximum potential gradient and along the path by which corona formation and final breakdown and arcing would occur.

In order to proceed effectively to the design of insulators for higher voltages, it appeared advisable to introduce certain elements and principles not heretofore employed and looking toward:—

(1) Placing the insulating surface under more favorable conditions for operation, especially as to corona and arcing.

(2) Simplifying the form of the insulating surface for convenience in manufacture.

(3) An insulator free from corona prior to breakdown.

(4) Reducing weight.

(5) Reducing over-all length.

(6) Reducing costs.

(7) Producing a higher voltage unit insulator.

(8) Elimination of porcelain, if possible.

These results may be attempted by the application, among others, of the following features:—

(9) The production of a hollow electric field[1], in which to place the insulating member.

(10) In combination with the hollow electric field, to so shape metallic electric flux distributing terminals as to produce along and near the axis of the field a field of force with lines so distributed as to conform to the surface of an insulating (mechanical strain) member which shall be most desirable from purely mechanical considerations of strength and manufacture.

(11) In combination with the above, to so proportion the metallic flux distributing terminal members as to eliminate corona formation anywhere on the insulator prior to a close approach to breakdown voltage, and well above operating voltage.

(12) An insulating mechanical strain member, to meet the purely mechanical conditions imposed by a higher voltage line insulator and, conveniently manufactured, may be an elongated cylinder or frustum of a cone or close approach to such forms.

(13) A mechanical strain member employed in the above combination should either be of some material of better mechanical properties than porcelain or of a porcelain having mechanical properties superior to those heretofore employed in insulator construction.

A proper combination of the above features should result in an insulator which will offer, among other advantages:—

(14) No possibility of corona or arcing along its single insulating surface; opening the possibility of the use of other material than porcelain.

(15) Maximum practicable dielectric strength along its insulating surface.

(16) Ample mechanical strength.

(17) A single unit suitable for use on 110,000-volt power lines, two such units in series for 220,000-volt lines and, when necessary, three units in series on 330,000-volt lines.

1. By a hollow electric field is meant an electric field surrounded symmetrically along its axis (plane or other surface) by a stronger field of higher average and maximum potential gradient.

Experimental Development

The preliminary experimental work of development

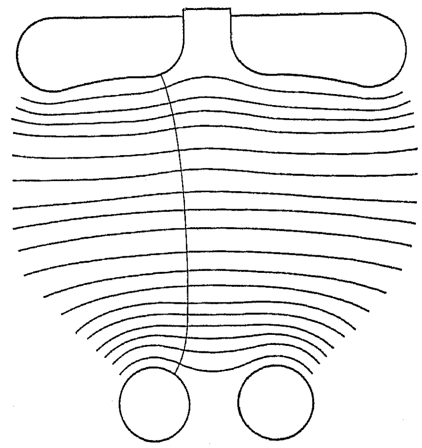


Fig. 2

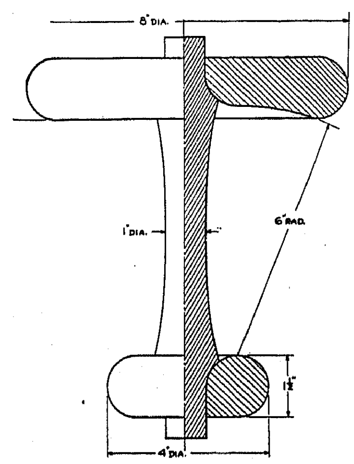


Fig. 3

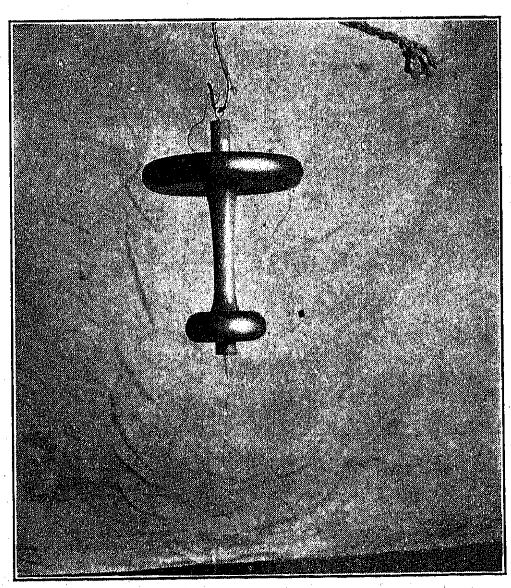


Fig. 4

of an insulator to meet the above conditions has now been completed and a few preliminary commercial

model suspension insulators have been built to place on high-voltage power circuits. This will give service experience with a view to detail modification, if necessary, before placing such insulators in general service in quantity.

The results obtained in this experimental development are of interest and, without attempting a full account of all the experimental work involved, may be outlined as follows:—

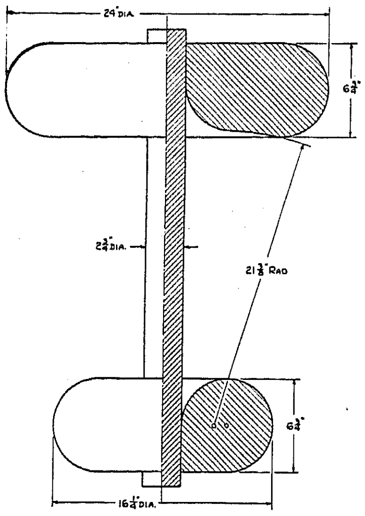


FIG. 5

An early attempt to determine, by theoretical analysis, a suitable form of insulating surface resulted in a form of spindle shown in Fig. 2 and dimensions as in Fig. 3.

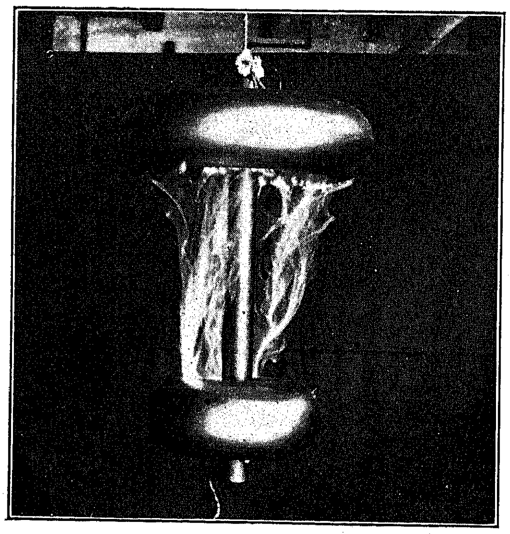


FIG. 6

This was constructed of wood with gilded terminal members giving appearance of Fig. 4 and yielded breakdown values of from 130 to 145 kv.

For convenience in manufacture, it appeared advisable to so shape the metallic flux distributing terminals that a cylindrical spindle, rather than the shaped spindle of Fig. 4, could be used, if possible, without too great loss of breakdown values. An insulator was, therefore, constructed as shown in Fig. 5 with gilded wooden terminal flux distribution members. Under test, this showed absolutely no corona up to nearly 400 kv. and at 400 kv. broke down without sign of previous distress. Fig. 6 shows appearance at breakdown with

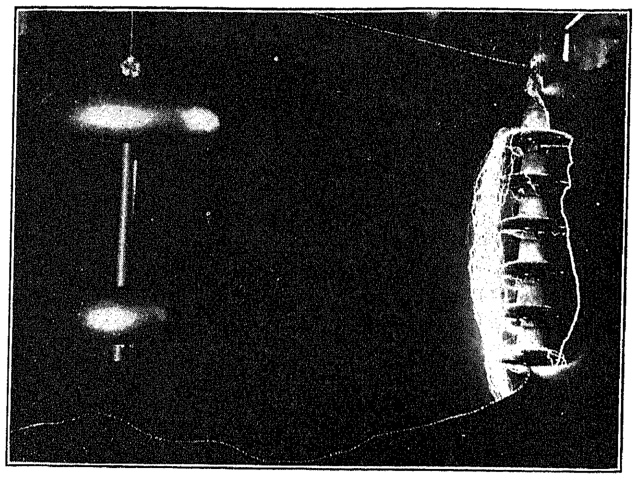


FIG. 7

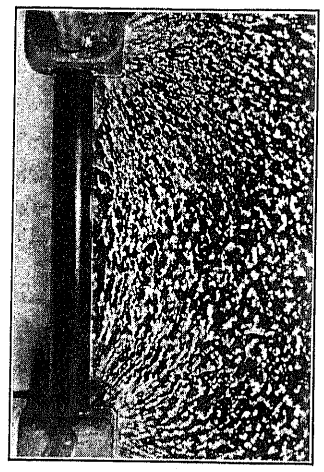


FIG. 8

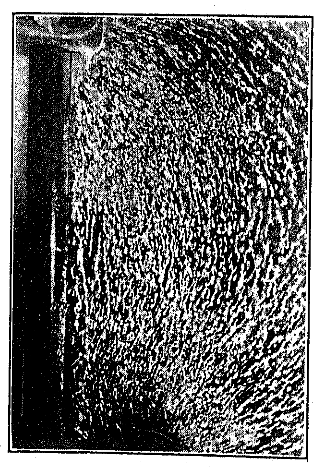


FIG. 9

no corona or arc along the insulating spindle. Hung in parallel with a string of six unit insulators of somewhat greater overall length, results were as shown in Fig. 7 at 360 kv. and no sign of corona on the single unit.

These results gave encouragement for continued work and considerable effort was expended in the exploration of the surrounding fields and on the study of field forms secured with shredded asbestos. Fig. 8 shows the field form around a cylindrical spindle with metallic rods at each end and at right angles to the axis of the spindle. This is a condition which, clearly, does not give an insulating surface conforming to the lines of force of the

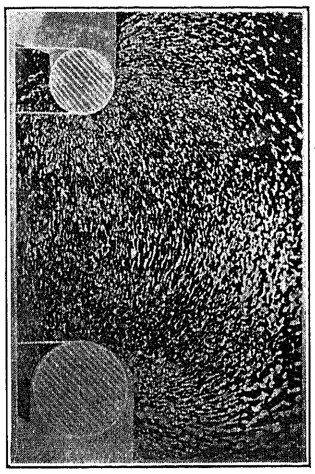


FIG. 10

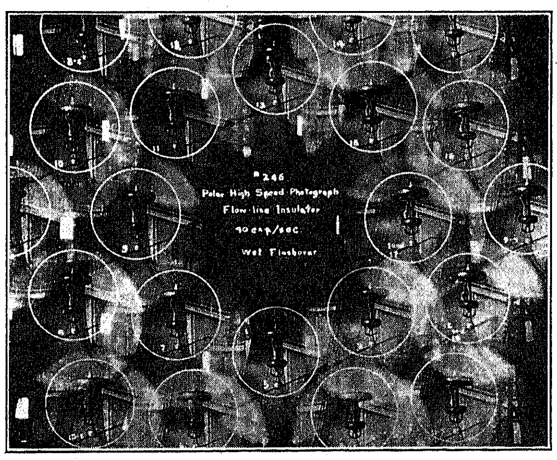


FIG. 11

field. Fig. 9 shows the effect of placing a metallic torus surrounding one end of the spindle. The field of force has flow lines more nearly conforming to the insulating surface. Fig. 10, with a torus surrounding each end of the spindle and concentric with its axis, gives a field with lines of force parallel with the surface of the insulating spindle throughout most of its length and also places the spindle in a hollow electric field where the strongest part of the field, with a maximum potential gradient, is a cylindrical belt immediately along the shortest path between the two toroidal surfaces and entirely surrounding the spindle.

Visual observation determined that with such arrangement undesirable corona formation did not occur up to about the breakdown voltage, but the eye could

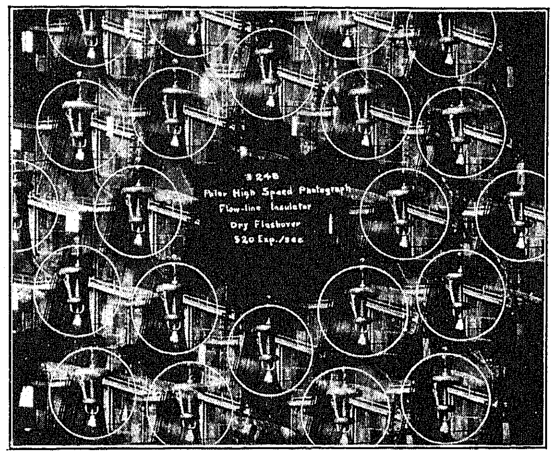


FIG. 12

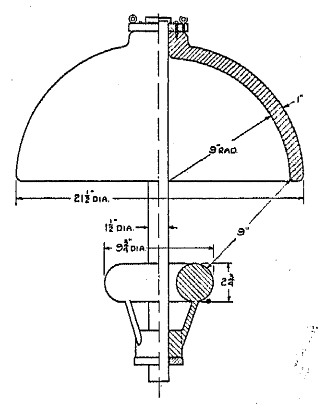


FIG. 13

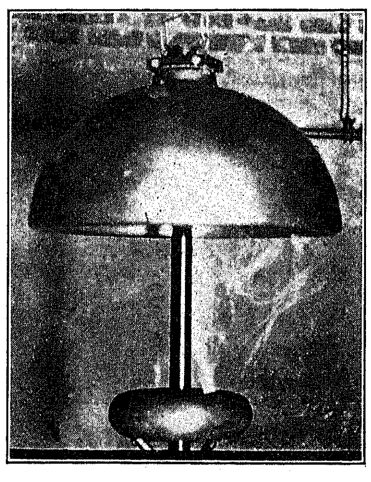
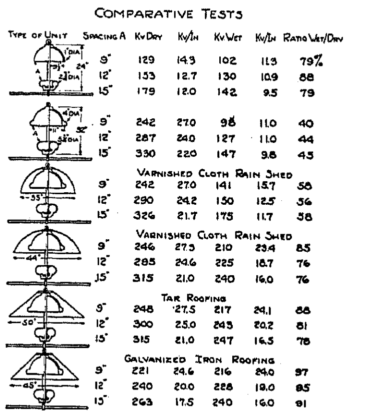


FIG. 14

not detect with certainty if static streamers or power arcs might not occasionally, and for an instant, traverse the surface of the insulating spindle. In order to determine this with certainty, polar high speed stereoscopic photo-

graphs were taken. Fig. 11 shows such a photograph at 48 exposures per second and Fig. 12 shows a similar photograph at 520 exposures per second. In no case was a streamer or arc formed along the insulating spindle nor could one be blown upon it by strong air currents. Up to this point no attempt had been made to secure results except under dry conditions, as it was believed



COMPARATIVE TESTS

Type of Unit	Spacing A	Kv Dry	Kv/In	Kv Wet	Kv/In	Ratio Wet/Dry
	9"	129	14.3	102	11.3	79%
	12"	153	12.7	130	10.9	88
	15"	179	12.0	142	9.5	79
	9"	242	27.0	98	11.0	40
	12"	287	24.0	127	11.0	44
	15"	330	22.0	147	9.8	45
Varnished Cloth Rain Shed						
	9"	242	27.0	141	15.7	58
	12"	290	24.2	150	12.5	56
	15"	326	21.7	175	11.7	58
Varnished Cloth Rain Shed						
	9"	246	27.3	210	23.4	85
	12"	285	24.6	225	18.7	76
	15"	315	21.0	240	16.0	76
Tar Roofing						
	9"	248	27.5	217	24.1	88
	12"	300	25.0	243	20.2	81
	15"	315	21.0	247	16.5	78
Galvanized Iron Roofing						
	9"	221	24.6	216	24.0	97
	12"	240	20.0	228	19.0	95
	15"	263	17.5	240	16.0	91

FIG. 15

that absence of complication of wet conditions would help to an understanding of the necessary fundamental relationships. The next step involved study of the modifying elements introduced by wet spray test and storm conditions.

A model corresponding to Fig. 13 was constructed and gave the appearance of Fig. 14 on breakdown. This

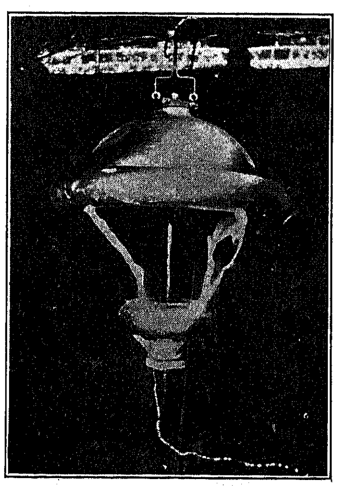


FIG. 16

did not yield as good results as hoped for, although the following results were obtained.

	Dry		Wet		
Spacing	Kv.	Kv./in.	Kv.	Kv./in.	Ratio
9 in.	201	22.3	102	11.3	51
12 in.	240	20.0	131	10.9	55

In order to determine the influence of the various factors involved, and to secure more favorable results, a long series of comparative tests was carried out on a variety of models and combinations. Of this series, a

FIG. 17

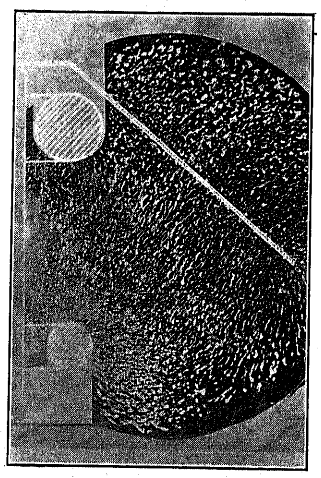


FIG. 18

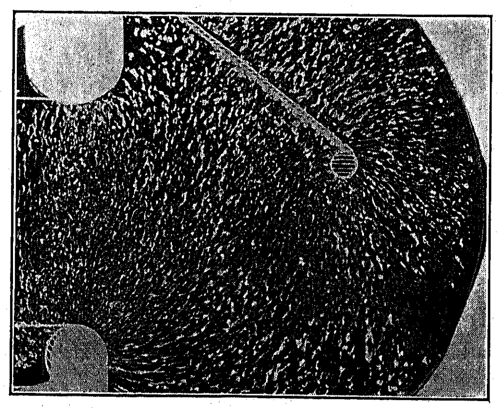


FIG. 19

few typical combinations and their results are given in Fig. 15. The appearance of two of these models, under arcing conditions, is given by Figs. 16 and 17.

These tests showed clearly the direction in which further development should proceed and a careful study

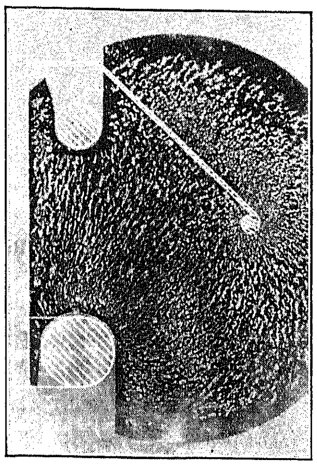


Fig. 20

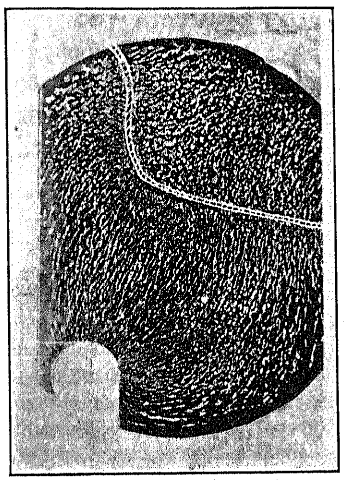


Fig. 21

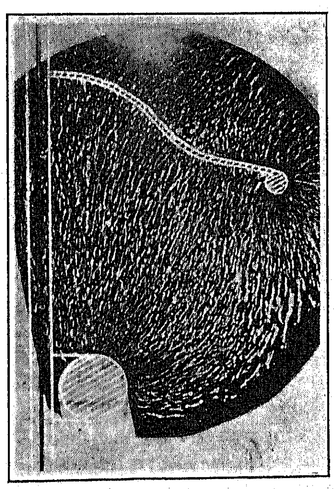


Fig. 22

of field forms was made on several models, in confirmation, and as shown in Figs. 18, 19, 20, 21, 22 and 23.

As a result of this, and work on other models, a final model of insulator was constructed in limited number with dimensions as shown in Fig. 24, for the collection of data under service conditions on high-voltage power lines with a variety of climatic conditions. The field

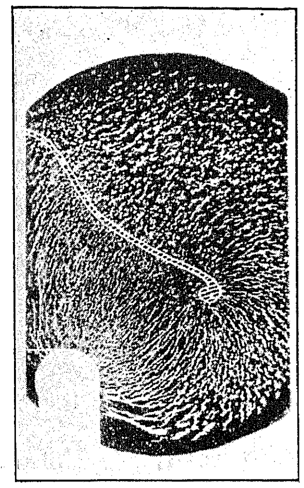


Fig. 23

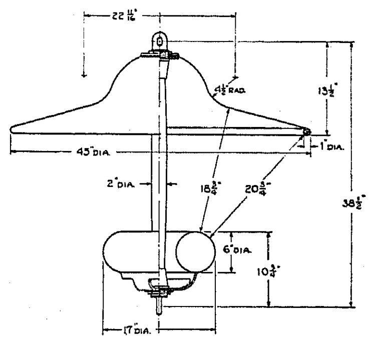


Fig. 24

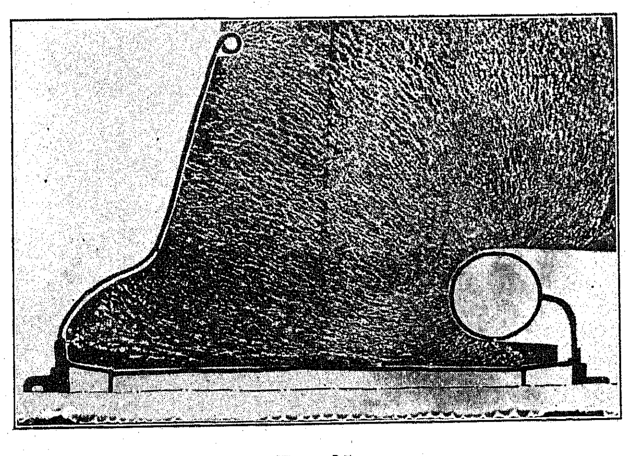


Fig. 25

form of this model is shown in Fig. 25. The appearance of this model, following severe snow conditions, is shown in Fig. 26 and under heavy arcing conditions at 280-kv. in Fig. 27.

With sufficient experience, under service conditions,

its rating can be established, or, if necessary, dimensions modified to meet any desired rating.

The test values on this insulator range at about 280 kv., or over, dry and 200 kv., or over, under standard wet spray test per unit. No corona is visible on any part of the insulator until within about five per cent of

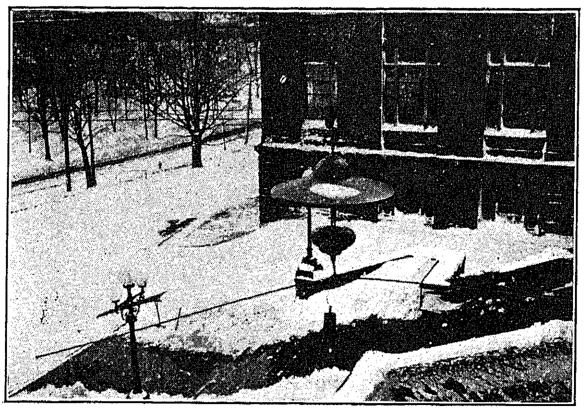


Fig. 26

breakdown voltage. Two units in series require about 500 kv. dry or 350 kv. wet for breakdown. These values are without capacity modification which could be readily secured to improve these values.

During the period in which the electrical features of

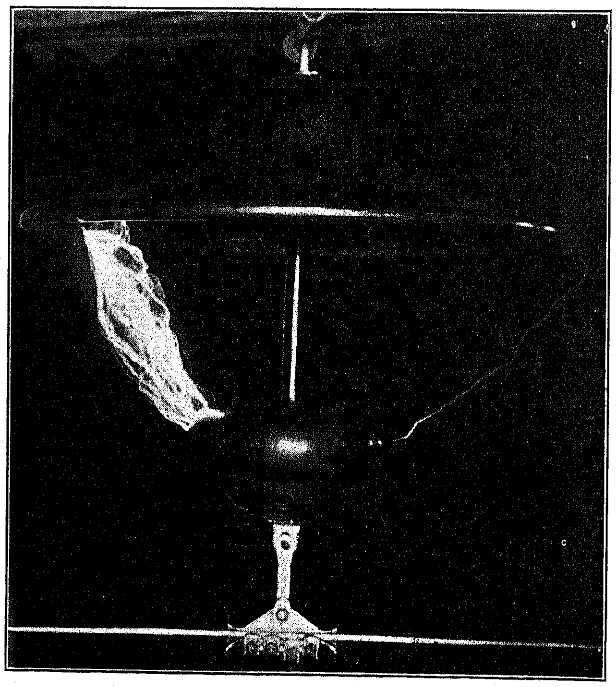


Fig. 27

this insulator have been under development, H. C. P. Weber, of the Westinghouse Research Laboratories, has been engaged in the development of impregnated and treated wood spindles for use in this insulator. His results are such that, with absence of corona and arcing conditions on the surface of the insulating member, there is definite promise of an insulating strain member with satisfactory dielectric properties and mechanical properties far superior to any porcelain yet available. The spindles used in demonstration tests accompanying this paper have been prepared under methods developed by Dr. Weber, but will not be further considered in a paper devoted to the electrical rather than the chemical features of this problem. Artificial aging and weathering tests of great severity accompanied by repeated electrical tests, extending over a number of months, have been applied to these spindles.

Special acknowledgment should be made for the thorough cooperation and great assistance which has been rendered in this problem by D. F. Miner and H. W. Tenney, both of the Westinghouse Electric and Manufacturing Company, through work which they have carried out in Pittsburgh.

Acknowledgment should also be made for the interested and valuable assistance which has been given on this problem by assistants who have worked with the writer in Worcester at one stage or another of the development of the problem. These are R. H. Bryant, H. W. Tenney, A. W. Hill, D. E. Howes, O. B. French, R. M. Field, S. T. Chen, C. L. Denault, R. L. Kimball, R. D. Paul and E. Topanelian, Jr.

Discussion

At Worcester

E. M. Hewlett: In trying various materials in experimenting on the first suspension-type insulator we found that any compound or any material that *can* carbonize, *will* carbonize. I am in hope that by distributing the strain in the way that Professor Smith has done, the carbonization will be reduced and the life of the rod lengthened, but it seems that in this electrical work, anything that *can* happen *will* happen. For instance, when an insulator is subjected to fog, you get a little dampness on the surface. Or when you get it out in the Middle West, in the Salt Lake section, where you get alkali dust, then you get a surface condition that will start a little static and start a little leakage. These conditions, with the available materials, are likely to result in deterioration.

The thing that we have always been looking for on all insulators, and I think in a good many other things, too, has been a suitable material.

Now, of course, if we can get a better porcelain, a porcelain twice as strong, or 50 per cent stronger, a little tougher, with a better dielectric strength, we can do quite a good deal, even with our present design. If we had a ceramic material, which we felt was safe under tension conditions and wouldn't crack and drop the line, why, then we could work something of this kind of a design. On the wireless antennas, they use a 50-in. long tube, 3½ in. in diameter, and they shield the ends with a ring very much in the same way as Professor Smith has done here. They do not use the rings for rain shields, but just as distribution rings.

On transmission voltages above 110,000 volts, it has been found that the lower disk on suspension-unit strings takes a larger percentage of the total electrical stress than the other units of the string. Therefore, on the higher voltages it has been found necessary to put grading rings on the strings in order to balance up the stresses. That process can be carried on so that the present types of suspension units can be used for any voltages that we know of at the present time.

We are in need of materials that will withstand arcing static and corona. The discovery of such materials will not only render possible numerous suggested designs, but will also open the field to many new designs of insulators for high-voltage transmission insulation.

C. F. Scott: In 1904—twenty years ago this summer—the International Electrical Congress was held in St. Louis. It had many sections. One of them was "Power Transmission." I happened to be its Chairman. One of the best groups of transmission people that had ever come together were there, presenting and discussing papers—something like half of them related to the line or the insulator.

It is interesting to read now the criticisms of the pin-type insulator, its construction, its theory, its size and its performance. V. G. Converse, who was the man who had made the underhung insulator that Mr. Skinner spoke of, had a long paper on insulators, a historical paper which described and illustrated all the different sizes and types. He refers to his underhung insulator or underscrewed insulator and goes on after saying that the pin insulator had been growing in size until it approximated a Chinese pagoda, to give another form of the same thing with an improved construction.

Dr. Perrine suggested the building of a little cottage over each insulator to protect it from the weather, a sort of a cubical, 4 or 5 ft. in size, open on two sides to let the line run through. Everybody was pointing out the insulator as the limitating element in transmission which had then reached 66,000 volts. We had the transformers but not the insulators.

M. H. Gerry, the man who started the first 50,000-volt plant for the Missouri River Power Company—spoke of his insulator experience, and made some very significant remarks. He described and illustrated some experiments showing the discharge over glass plates—simple static experiments—and he said, "The direction in which we will have to look in the future is a study of the electro-static conditions. They are coming into consequence at these higher voltages."

Three years later, in 1907, papers at the Niagara convention of this Institute by Buck and Hewlett presented a new type of transmission. The problem had been to hold the line in place, to hold it steady. It had been above the cross arm. They proposed putting it below. They proposed long spans hanging the wire from a succession of suspension insulators. They changed the law of the insulator from the third power to the first power. When you double the size of a pin insulator, you increase its weight as the cube; when you double the underhung type, you use two insulators increasing the weight by two instead of eight. There were two very significant things; one was the presentation of a new system, and the other was its reception. The insulator caused a new era in transmission, but the hearers didn't recognize it; the discussion was trivial.

The curve of increasing transmission voltages for the last 30 years or more runs up fairly uniformly to 1903 and then keeps on a pretty straight line at about 66,000 volts for five years. Then there is a sudden jump up to 110,000 all at once. Then it went on up. Why? The suspension insulator had come; it changed the whole trend of transmission.

Now we have again something new. When Buck and Hewlett discarded the upright insulator, because it was mechanically wrong, they got it mechanically right by shifting the position 180 degrees. What has Prof. Smith done? He has treated the problem in a broad, engineering way. Most people have tried to hang on another insulator or change the shape of the petticoats or change the metal clamp or improve the porcelain.

Professor Smith does not modify; he starts *de novo.* He proposes a dozen or more different requirements to be met and they seemed wonderfully exacting and almost impossible. But he had the courage to lay them out and then to meet them.

Now, what has it come down to? Why nothing at all but a metal umbrella with a wooden handle with an ornament at the bottom, but it apparently meets the mechanical and electrostatic requirements.

I wonder whether we are going to be as slow as the American Institute of Electrical Engineers was nearly twenty years ago in recognizing that maybe some new thing with big possibilities has come such as was presented by Buck and Hewlett.

V. Karapetoff: I should like to ask Professor Smith a question or two in regard to his design, if I may. Electrostatic systems may be divided into *glow* systems and *spark-over* systems. Take two large spheres, a short distance apart and raise the voltage to a point where there is ionization on the adjacent surfaces. The flying electrons will also ionize by collision the remainder of the space between the spheres. Therefore, there is no intermediate stage of corona formation and a breakdown takes place almost at the same voltage as the first corona appears. On the other hand, take two small spheres, placed quite far apart. As the voltage is raised, a potential gradient is reached at which ionization by collision begins at the surfaces of the spheres, where the voltage gradient is at a maximum. As the air is broken down, the diameter of the spheres is seemingly increased because the ionized portion may be considered as part of the metallic conductor. Therefore, a condition is reached at which a stable equilibrium is possible, because the voltage gradient beyond the ionized range is not sufficient to cause further ionization by collision. I call such a system a glow system. As the voltage is raised higher, the ionized layer increases in thickness and then streamers begin to form. Finally a complete breakdown takes place. This is, then, the difference between a spark-over system and a glow system.

If I understand correctly, the apparatus to be protected should rather have glow characteristics, while the protective apparatus should preeminently have spark-over characteristics. If I want a quick gap to protect some apparatus, naturally I would select a condition in which there is no intermediate corona stage; while if I want something protected then, I should judge, a glow arrangement is preferable. When the voltage rises beyond a certain limit, the apparatus becomes partly self-protecting by the formation of ionized corona regions, and when the voltage goes down, the apparatus field becomes normal again.

I understood Professor Smith to say that in his insulator a sparkover takes place without previous corona formation, and I should like to ask him if this is theoretically possible, with the shape of the guard ring that he has, or with the lower terminal and the umbrella above. His photographs show that the electrostatic flux spreads out from the lower ring reaching a maximum dielectric flux density there. So that, theoretically, at least, a stage is reached at which the voltage gradient at the ring exceeds that necessary for ionization by collision and an ionized layer is formed. The same applies to the edge of the umbrella. I do not say that with the construction used these layers are necessarily harmful; I only should like to know if they exist.

Another question which I should like to ask Professor Smith is this: He spoke of a hollow field. To me, a hollow field means a field which is more intense on the outside than on the inside. For example, if we have an iron pipe, longitudinally magnetized by a coil, we might say that it has a hollow field, in the sense that the flux density in the material of the pipe is greater than in the interior space. Now, the permittivity of wood is at least twice, if not more than twice, that of the air, so that with the same applied potential and the same average voltage gradient we should expect a higher flux density in the central wooden stick than in the surrounding air. Can such a field be called hollow? The field is very skillfully arranged and the lines of force are almost parallel to the outer surface of the wood, so that there is no corona formation on the surface. I understand this point. But is it not true, nevertheless, that the dielectric flux density is higher in the rod than it is in the air?

This may seem like splitting hairs, but seeing that we are now in the midst of a new and rational epoch in the development of

insulators, let us start our terminology right, before it is too late.

H. A. Stanley: All the experiments we have seen, and the talk we have heard, have had to do with the insulator in the vertical position. I assume that we are not going to get away from the use of insulators in the strain position. This particular insulator is not adapted for use horizontally. For instance, the lower bowl, I should judge, would hold water in the horizontal position, and I would like to inquire if the thought is to work out something different for use in the horizontal position?

The second point is the copper. I suppose that when the manufacturers get around to selling this device to us they will try to make it in some cheaper material. I would like to inquire if there is any reason, theoretically, why it couldn't be made of galvanized iron.

Mr. Bowlen: The point Mr. Stanley just brought out about the insulator being placed in a horizontal position, is, I think, of considerable interest, and I believe, in that connection, there is a strain insulator on the market at the present time which uses a wooden core as its principal tension member, covered with porcelain tubes filled with petrolatum or oil. That is used on a span something like 3000 ft. long and it is in successful operation; it has a means of re-filling the space between the wooden core and the porcelain. That might be the answer to putting it in the horizontal position.

Discussion at Pasadena

W. A. Hillebrand: Professor Smith, perhaps, has revived the first form of suspension insulator that was brought out. We have insulators today that operate and constitute a not unreasonable charge upon the system. If you take into account obsolescence, their charge is less than that of many other elements of the system. Today, it is probably less than the direct depreciation charge of wood poles and cross arms, and taking into account obsolescence, it is less than the charge upon the earlier and comparatively recent types of prime movers, generators and oil switches. It is a charge which the industry is able to bear without imposing an undue burden. I think it is the best practical insulator we have today because it works. The only question is that of making it a commercial success. It has got to be something better or cheaper, to offer equivalent service at a lower cost than that which we have today. The only question then is that of application. Unquestionably it will be tried out and in sufficient quantity to give a demonstration. The operation of this insulator depends upon the satisfactory maintenance under field conditions of an initially established gradient.

This matter of bird droppings causing flashovers, to my mind, is a question as to whether with two units your field will be strong enough to exclude foreign particles and moisture. There is a leakage problem that is, perhaps, the most serious of all. We have a new material, which is treated wood.

Now, there is one thing that is fundamental, and which applies to both the manufacturer and to the user of that type of insulator, that is, the failure of a single piece of dielectric. and that applies to any known dielectric, means the dropping of the line. That is, a failure from any cause whatever, straight mechanical failure, puncturing, burning due to leakage, arcing due to flashover—anything that causes a line interruption means a loss. Now, experience has shown that you can have as many as 30 per cent of the pieces of dielectric fail mechanically, or electrically before obtaining the first line interruption. That is solely a matter of the number of pieces used; that determines the reliability. With insulators of the types now in common use, the reliability is the function of power of the number of insulators in the string. That is one of the most commonly overlooked and one of the most fundamental applications in regard to insulators today. On the other hand, the probabilities of interruption with an insulator of this kind is in direct proportion to the number of pieces of dielectric in use. That is, that every additional piece of dielectric you add constitutes an additional hazard instead of an additional protection. It puts on the manufacturer an almost unbelievable requirement with regard to reliability. You may be able to obtain it, but on the basis of experience I would say it should be approached with the greatest of caution.

J. B. Whitehead: Professor Smith has adopted in his insulator two devices which we have known for sometime, namely, the arcing ring and impregnated wood as an insulating material. The particular aspect of his work to which I want to call attention is the study that he has made of the properties of the arcing ring, or screen, and the method that he has adopted in that study. We have had arcing rings that will take the arc and save the insulator for sometime, but Professor Smith has made a study of the shape of these rings in a systematic and scientific way and has arrived at a form of ring which gives a uniform potential gradient over the insulating member. He thus eliminates regions of high potential gradient and has, therefore, reduced to a minimum the probability of an initial brush, or corona, or spark-over. It is a scientific investigation and it should, therefore, be emphasized as an example which we should all attempt to follow in putting experiments of this kind under way.

I suppose that the methods are being followed in the study of the impregnation of the insulating member. Of course, the open question in connection with the insulator, is the life of the single insulating member. It would appear to me that it is going to be a question of the power of the surface of this member to withstand the action of the elements. We know that porcelain, originally apparently perfectly safe, eventually, under the action of the elements, temperature and so on, developed a cracked surface which eventually leads to failure. In the case of the impregnated wood member we would seem to have the possibility of avoiding in a large measure, that particular type of failure. If we can obtain an impregnated material, which is more or less plastic, and which, therefore, will yield to the expansions that are brought about by temperature change, it would appear that we might have a surface which would accommodate itself to existing conditions from day to day. It appears to me, however, that if we do not obtain such a type of impregnated material, there is a considerable element of danger in such an insulating element. For, if this impregnated wood disintegrates we will get a fibrous substance in the path of the potential gradient and this will lead to brush discharges.

C. E. Skinner: As I understand it, Professor Smith's investigation is not based primarily on the use of wood in a strain insulator; but on a study of the potential gradient and a design such as to permit the use of a strain member so placed that when an arc is formed it will not be along the surface of the insulation. This arrangement permits the use of wood, which in the past has proved most satisfactory in many other applications as an insulating material. It has the advantage of great mechanical strength and is readily obtainable. It has the disadvantage of being carbonizable and difficulty has been experienced from time to time in providing suitable impregnation.

Methods of impregnation have been studied and results have been obtained which seem to warrant the assumption that methods are now available which will justify the use of wood in this connection, provided it is not subject to arc-over, and Professor Smith's tests show that this need not be feared in this particular design.

Professor Smith's insulator is especially interesting as being the first radical departure from the type which has been generally accepted as standard for many years. Line insulation has for some time been considered the limiting factor in the use of high-voltage transmission. This insulator gives promise of removing this limitation.

I do not understand that Professor Smith is ready to advocate this insulator for commercial work until more field experience is obtained, and tests are under way which will give this field experience. The wood strain is merely incidental to this type of construction, and any other material which will satisfy the mechanical and insulation demands could be used to replace it.

H. F. Elliott (by letter): Electrostatic shields, or terminals, of the type proposed by Professor Smith, have been used with rod, tubular and other forms of insulators for radio antennas and associated services since 1914 or perhaps even earlier.

The high losses which accompany corona at radio frequencies, and the extremely destructive character of corona at the higher frequencies, forced the adoption of special types of insulators with suitable shields to control the electrostatic fields at this comparatively early date. The accompanying photographs, taken during 1916, 1917 and 1918, may be of some historical interest in this connection.

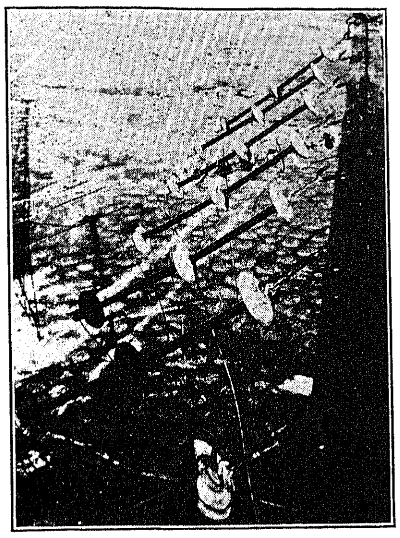


Fig. 1

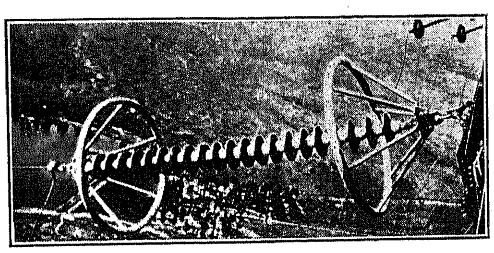


Fig. 2

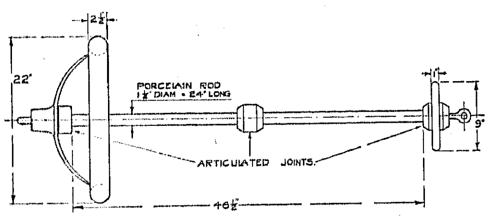


Fig. 3

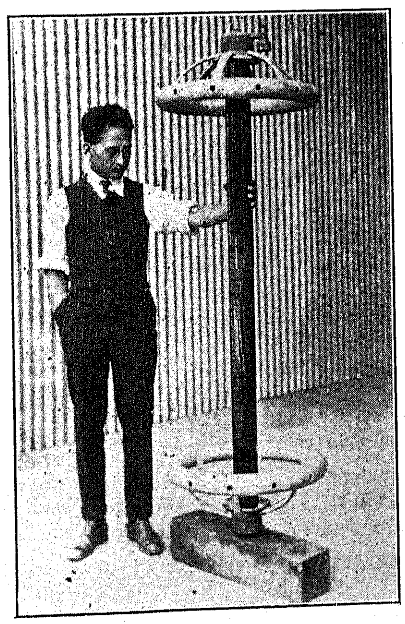


Fig. 4

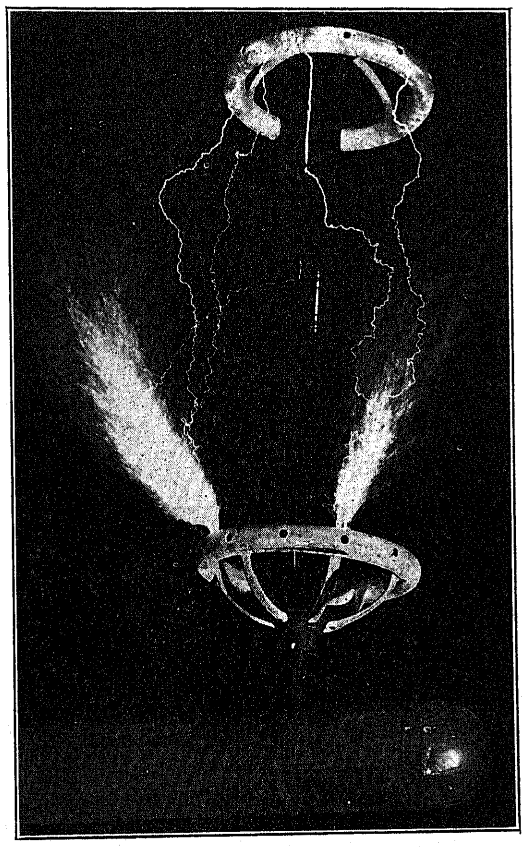


Fig. 5

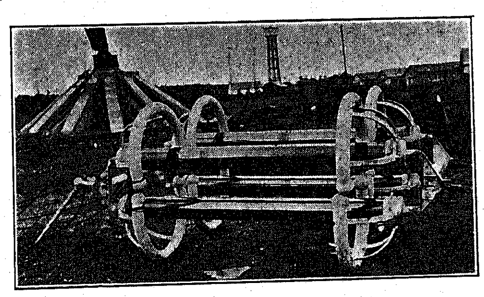


Fig. 6

Fig. 1 shows the type of porcelain-rod insulator and disk shields which were used for several high-power radio antennae during 1916 and 1917. Fig. 2 shows a specially shielded string of standard suspension units which was used with the same antennas. All of these insulators functioned satisfactorily and without visible corona at potentials of the order of 100,000 volts, r. m. s., and frequencies ranging from 15,000 to 50,000 cycles.

Shielded strings of standard suspension units in a variety of combinations were tested at the Stanford University high-voltage laboratory during 1917. The results of these tests will be found in the paper by Mr. Frank G. Baum entitled "Voltage Regulation and Insulation for Large-Power Long-Distance Transmission Systems," PROCEEDINGS A. I. E. E., Vol. XL, 1921, pages 1067 and 1068.

Fig. 3 is a sketch of a porcelain-rod antenna insulator designed during 1917 and tested at the Stanford University high-voltage laboratory during the same year. Using 60 cycles, and with the insulator dry, the first pin points of corona appeared at 140,000 volts; intermittent streamers occurred at 225,000 volts and large streamers but no flashover occurred at 350,000 volts, which was the limit of the testing equipment. With the insulator thoroughly wetted, there was some brushing at 100,000 volts, but the changing currents quickly dried the surface.

During 1918 porcelain-tube insulators of the type shown in Fig. 4 were developed. These have since come into general use and have proven highly satisfactory, both as to electrical efficiency and mechanical reliability. Fig. 5, shows such a unit under test at 183,000 volts, r. m. s., and 46,000 cycles. The shape of the surrounding electrostatic field is clearly shown by the position of the corona streamers, which are entirely clear of the insulator proper.

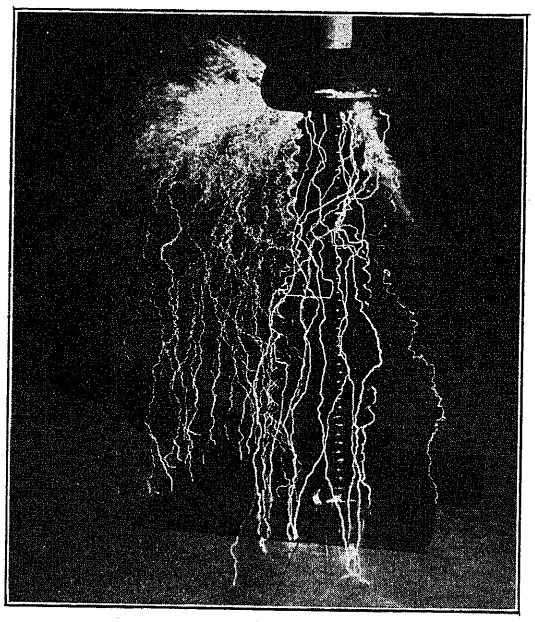


FIG. 7

Fig. 6 shows a special high-strength antenna insulator designed during 1918 for service involving potentials of 135,000 volts, r. m. s., to earth, frequencies of 12,000 to 30,000 cycles and a working load of 20,000 lb. *in tension*. Each of the four porcelain tubes comprising this unit was approximately 6 in. diameter by 6 ft. long and each tube had an ultimate tensile strength over 20,000 lb. Recently, similar units have been constructed whose ultimate tensile strength exceeds 35,000 lb.

Fig. 7 shows a pedestal-type insulator, with electrostatic shield, under test at 234,000 volts, r. m. s. to earth and 51,000 cycles. This unit was one of a number used for special radio-frequency switches which were operated, without the slightest indication of corona, at a potential of 135,000 volts. The shape of the electrostatic field as controlled by the shield is indicated in the photograph by the corona streamers.

The possibility of constructing the electrostatic shields of antenna insulators so as to shed rain was considered at an early date but was not immediately adopted because the insulators had to operate in a nearly horizontal position, as shown in Fig. 1. More recently, however, the practise of using cone-type electrostatic shields which also act as rain shields, has been adopted with great satisfaction for radio antennas employing insulators in a vertical position. An account of the development and testing of units of this type is given in a paper by Mr. W. W. Brown in the PROCEEDINGS of the Institute of Radio Engineers, October, 1923.

It will be noted that all of the insulators described in the foregoing are of porcelain. Many other materials have been tried, some with apparently excellent success in the laboratory, but none, except porcelain, has proven satisfactory in service for potentials above a few thousand volts.

H. B. Smith: In response to some of the questions that have been asked, I will make a few suggestions.

First, with respect to the suggestion regarding fog and dust which has been made—and I will combine with that the question Professor Karapetoff raised with respect to the hollow field.

Professor Karapetoff is correct in stating that there is greater density of the dielectric flux within the stick than immediately outside, but comparing the space along the length of stick, as compared with the short spacing between metal terminals, we have outside of the surface of the stick a density variation exactly conforming with his description for a hollow field and the stronger field on the outside tends to deflect particles of moisture, dust, etc., in the direction of that field and away from the stick.

Reference was made to the size of the insulator. It does look large, here in this room, but it doesn't look large out on a transmission tower for such a voltage as would be employed where such an insulator would be used. The parts are not heavy.

A question was raised as to the material. These, you understand, are the first insulators of this size, and for convenience, the hood is made of spun copper. That was just for the small number. It was not feasible to prepare dies for pressing, as would be done in quantity. As soon as that is done, the material cheapest for production, considering depreciation, will be used. There is nothing in the metal that is used that affects the operation of the insulator, theoretically.

If I may take just a moment in retrospection, referring to what Professor Scott has said, I would say that when his paper of 1898 was presented—depending upon wires of small diameter—it was felt that a limit for voltage might be reached of 55,000 or 60,000 volts. It stimulated us here in Worcester in 1900 to 1902 as soon as we had developed the transformer, (which is now in the room below and was used in the demonstrations this morning), to apply higher voltages to a transmission circuit that was placed along Boynton Street, and we used wires of larger diameter.

In a thesis presented by Cook, Davis and Wiard in 1901, the results are shown of the effect of increasing diameter and prove clearly that 60,000 volts was not prohibitive.

In those days, before Professor Ryan went to California, we were in close contact with each other—we were working together more or less on this question—and this fact was communicated to Professor Ryan and we had quite a little correspondence on the matter.

Soon after that, Mershon made other tests in Colorado and the results of those tests were sent to Professor Ryan. Mershon's results did not agree with our results in Worcester, so that both Professor Ryan and myself were somewhat skeptical as to the accuracy of either Mershon's work or the work that we had done here in Worcester.

Professor Ryan later sent to Worcester results of his experiments on the effect of pressure, and also going back to some old work on the same subject, made seventy-five years before—the work of Paschen, so that taking into account the difference in elevation of Colorado and Worcester, he found that our results agreed, substantially.

We then had confidence and Professor Ryan went on with his

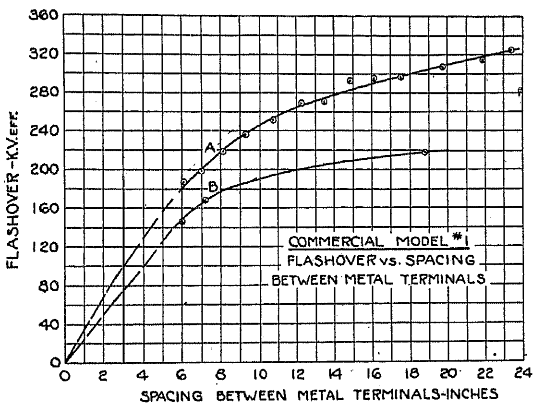
classical paper that he presented to the Institute in 1904, which gave us the foundation for the law of corona losses between conductors.

Reference was made to the large diameter of the hat of this insulator. You must remember that the hat is connected to earth. You can make one continuous hood for three or four insulators, if you choose, instead of splitting it up.

The question of deterioration of organic materials, under such conditions as these, is, of course, the important question, and it is a question that can only be answered finally by experience on the insulators in service. We have tried to put the insulating member under the most favorable conditions possible to prolong its life. We have attempted to remove visible corona. Whether ionization coincides with visible corona, may be a question. Visible corona does not appear until very close to the point of break-down, and we operate the insulator at half, or less than half that voltage.

With regard to the question of the horizontal strain insulator, there is no attempt to meet that condition with this present insulator. The same principles can be employed in such a type of insulator. I think there is a little misunderstanding as to the construction of this torus. It has a perfectly continuous surface.

The comment of the operating people who will have to deal with these insulators in service, is perfectly fair—that it should be tried out. That is the thing that we propose to do. We will then know just the limitations and just what may be necessary to



do to apply these same principles, which I believe to be right, to the actual conditions as imposed in service. We hope that we can meet them the first time they go into actual service but that isn't always done.

H. B. Smith: Possibly some of you may have thought that the insulator I am suggesting is radical because of its apparent departure from existing practise, but I cannot regard it in that way. It happens that in 1898 I succeeded in producing a transformer for 175,000 volts in a single unit. I think that at that time it was the highest voltage single unit transformer. Transformers had previously been used with high-tension circuits in series, that is, a number of units in series but not in cascade as we are doing now. In 1901 we built our single unit 500,000-volt transformer. As a result of having these transformers many people from all over the country were sending us insulators for testing. Therefore, the present insulator is the development of a quarter of a century and it is not a recent thing in my mind, except in its final form at the present time. It has been a gradual development of thought and experience through a great many years.

The pin type of insulator reached its limit and we all recognized the importance of the work that was done in producing the multiple-unit string in carrying us past a stumbling block at that time, and, in the present form of insulator the importance of the unit is recognized. The difference is mainly in producing a unit for a higher voltage per unit and an insulator which gives a uniform distribution of potential along its insulating surface, and terminal connections for that surface such that corona is eliminated until very nearly the flashover voltage. It is that early development of corona, preceding the final voltage limit,—a result of an excessive gradient, which overthrows the whole dielectric field and fixes a lower flashover limit.

The question of puncture, as we have had to deal with it in the porcelain insulator in the past, where at the head you have a relatively thin layer of dielectric stressed to the maximum, is eliminated in the present design; we have wholly different relationships.

If we plot flashover voltage against spacing between the metal surfaces we have a curve for dry flashover as shown in *A* of the accompanying illustration. With 18½ in. spacing between metal surfaces on the unit, we have about 300,000 volts flashover value. Now, you can reduce that to 8 or 9 in., and you then have the more pronounced characteristics of the sphere-gap, higher kilovolts per inch for flashover, and only a very small reduction in the total flashover voltage; with a very much lower spacing. With that smaller spacing, you will have capacity variation.

In reference to Mr. Wood's point of a standard unit, or of a variety of units, it may be a question for the operating engineer to answer whether he cares to use two standard units, with flashover of 300,000 volts each, or two together in series, as we saw yesterday, with a flashover of 520,000 volts, or whether he will use a shorter stick for one or both of those units and a more uniform distribution.

The wet flashover values run along in a flat curve as shown in *B* so that with the greatest spacing the ratio between wet and dry—for instance on the unit we saw yesterday,—is in the neighborhood of 82 per cent. As you go to lower spacing you have a higher ratio between the wet and dry flashovers. In fact, in many laboratory tests we have had unity ratio, when we had the right kind of water. With ordinary tap water and rain it will run up to 90 per cent or 92 per cent.

Now, I have not had as many rocks thrown at this, as many rifle shots at it, as I hoped might be experienced. Some of you saw the demonstration of a single unit in Worcester, many of you saw the demonstration of two units in series yesterday afternoon (at Pasadena). Perhaps, I ought to say, as it was difficult to announce the matter there, that on the two units that were hung up in the laboratory, were applied 520,000 volts before flashover and I presume you noticed that up nearly to the point of flashover you could not see (in a dark room) the location of the insulators. That showed that we really have succeeded in the elimination of corona to way above working voltage. That is one of the features that makes it possible to consider the introduction of such a material as impregnated wood. We should not consider that the experience of the past, under *other* conditions, is necessarily applicable in *this* case. We all know that the use of wood insulation has been disastrous in most cases, especially, where exposed to the weather. Nor are we limited to the use of wood in this case.

The insulating member, or stick, is placed in what I have called a hollow electric field. That is, the shape of the metal hat and the torus below it is such that we have entirely surrounded the wood stick by a field of higher potential and maximum gradient than that which the insulating surface sustains. We also have an insulating surface parallel to the lines of force along that surface which gives us a uniform distribution of potential along the surface. Now, the presence of the field of higher gradient outside of the insulating surface accomplishes two or three things. It, to a certain extent, deflects particles, moisture, dirt, even heavy particles, as pieces of straw, etc., so that they do not come in contact with the stick. I don't mean to say that it will wholly *prevent* moisture, or a heavy fog, settling on the stick, but it will minimize this effect and the stick will withstand the discontinuous moisture. You may have certain conditions in manufacturing, particularly in chemical manufacturing areas, or where exposed to fogs,—especially where, coming with other

deposits, where it may be necessary to clean the surface periodically, as is the case with other insulators but this insulator provides as good a surface for cleaning as I can imagine, if that proves to be necessary. Only experience in varied service will tell us what the needs will be in that respect.

You understand, I am sure, that this insulator has just passed through what I consider its developmental stage. We have now had, for some months, a group of these insulators, of which I will show you slides, on a tower in the Pittsburgh area where they are at line voltage and subjected to the Pittsburgh atmosphere near a large foundry where foundry gases are blown across them. Since I left Worcester I have received word that after several months in that service they have just passed through a period of 60 hours of continuous rain, fog and wind and there have been no failures as yet.

Now, as to the life of the sticks, we feel confident as to the thoroughness of the impregnation of the wood. The question of the weathering of the surface of the stick can only be told by service. Last winter we, for a number of months, had sticks soaking in tepid water throughout the day, freezing at night, and subjected to flashover tests twice a day. Now, I don't know how long a life period that would equal. Those are the facts of the case; you can form your own guess as to what length of time in normal service that would mean.

The plan that I hope to follow with this insulator is not to recommend its application on power lines at the present time with the view to superseding present methods of insulation. That would be very unwise. Following these preliminary service and life tests, I hope to place them in limited numbers, upon a number of service lines, under a variety of climatic and other conditions, so that the experience in such service will tell whether it is a better, a cheaper, or more desirable insulator for general power line application than those now in service. That will *then* be a question that anybody can answer for himself. Of course, I would not do this except that the evidence at the present time has impressed me of the very excellent probabilities before the insulator.

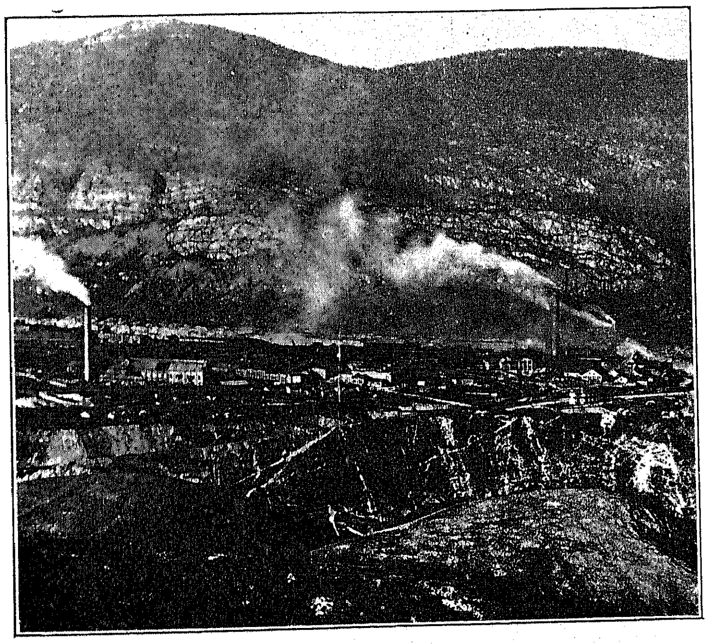
Electrical Equipment

Consolidated Mining and Smelting Company's Zinc Plant, Trail, B. C., Canada

BY R. N. LOCKYER
Member, A. I. E. E.
Asst. Supt., West Kootenay Power and Light Company, Bonnington Falls, B. C.

Introduction

THE great demand for supplies of zinc required in the manufacture of munitions, especially in the manufacture of brass shells which contain 66.6 per cent copper and 33.4 per cent zinc, led to an investigation in 1915 to ascertain the feasibility of producing metallic zinc in Canada. This resulted in the establishment of a zinc plant by the Consolidated Mining and Smelting Co. of Canada at Trail, B. C., at an initial cost of $2,500,000.



General View Trail Plant

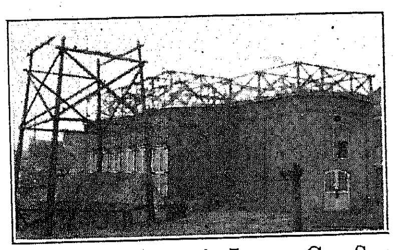
The production of zinc was increased from a daily output of 1000 pounds (453 kg.) to the present production approximately 80 tons (72,500 kg.) daily, having a purity of 99.9 per cent. The present annual capacity of the zinc plant is approximately 29,200 tons (29,500 metric tons). The starting up of the electrolytic plant at Trail and the almost simultaneous starting of one at Anaconda, Montana, and the placing of orders with both of these plants, broke the German strangle hold on high grade zinc, in fact other grades as well.

Until about 1915 all commercial zinc was produced by distillation, a process involving many difficulties. The resultant product was comparatively impure and there was also a very decided factor of loss due to certain peculiar tendencies of zinc vapor to solidify in undesirable forms. Today a great deal of the zinc which is produced in North America is deposited electrolytically; a process which results in a purer and more homogeneous product and therefore, one which is more desirable in the trades than that made in the old way.

It is not the intention of the writer to devote for any length of time on the zinc process as this article is intended to cover only the electrical equipment of the plant. For a complete description of this process the reader is referred to an article by L. W. Chapman in the *Chemical and Metallurgical Engineering Magazine*, August 11th, 1920, Vol. 23, No. 6, pages 227-237.

Power Supply

The power supply is derived from a distributing station located between the two generating rooms. This station being the property of the West Kootenay Power & Light Co., a subsidiary of the Consolidated Mining and Smelting Co. The Power Company is one of the pioneer concerns engaged in hydroelectric development in British Columbia which has played such an important part in the mining development of the Kootenay District, in fact without such power this development would have been impossible.



West Kootenay Power & Light Co. Substation

Power is transmitted to Trail, a distance of approximately 32 miles (51.2 km.) from the company's hydroelectric plant, over two transmission lines at a pressure of 60 kv.

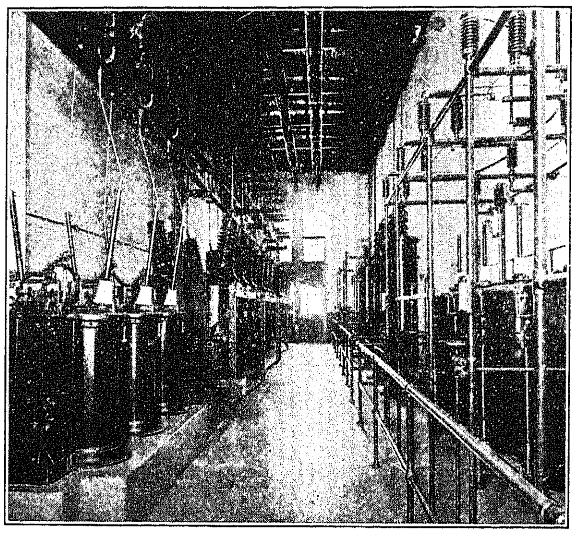
The substation is of solid brick construction, the incoming lines entering through the roof. The interior is subdivided into three sections, namely, transformers, high-tension switches and lightning arresters, and low-tension bus structure with the low-tension switches and control panels.

Electrical Equipment in Distributing Station

The transformers are of the Canadian Westinghouse Company's manufacture, each bank, of which there

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., October, 13-17, 1924.

are two, is located in a fire proof compartment and consists of three single-phase, 2500-kv-a. water-cooled, oil-insulated transformers with primary and secondary connected delta. It is to be noted that no spare transformers have been installed. In the event of a transformer breaking down, service would be restored by operating the damaged bank open delta while repairs were being made. Each transformer is mounted on a truck. The high-tension switches on both the incoming lines and transformers are of the Canadian



High-Tension Room, West Kootenay Power & Light Co. Substation

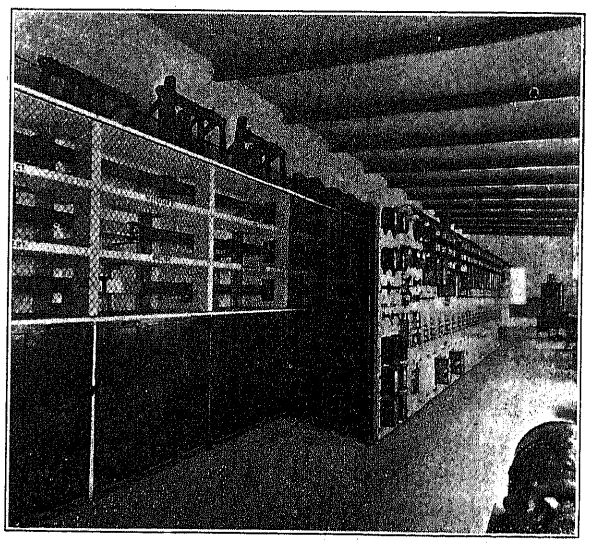
General Electric Company's manufacture; these being their form K-26-70 kv.-300 ampere. These switches are equipped with tank lifters, and are protected through inverse time limit series trips mounted directly overhead on the bus structure.

Two sets of aluminum lightning arresters protect the two incoming lines. Directly adjoining, the high-tension room is the low-tension bus structure which is of concrete. On this structure are mounted all the low-tension switches. Two form *H*-6 connect the secondaries of the transformers to the bus. Eighteen form *H*-3 switches are on the outgoing feeders, connecting the smelter substation and Lead Refinery substation. There are thirteen motor generator feeders and a circuit for lighting the city of Trail. All the switches are protected by time limit overload relays. A d-c. panel is mounted at one end of the control panels from which is operated a 5-kw. 125-volt motor generator set for charging a 60-cell storage battery. This battery energizes the operating bus for the remote control switches as well as supplying an auxiliary lighting source in case of power failure. This is accomplished by a quick throw over switch operated by a no-voltage release. The city lighting system is fed from a 3-phase automatic induction voltage regulator set for 10 per cent boost or buck. The street lights are controlled from a series mercury arc rectifier operating magnetite arc lamp.

Some mention should be made of the two 20-kv. lines passing through this station. Up to the time of building the Zinc Plant, the only lines coming into Trail were the 20-kv. lines feeding directly into the Consolidated Mining and Smelting Company's substation which has now been dismantled. These lines originally fed from No. 1 power house, but as the load conditions very seldom require both plants to be in operation at the same time, these lines are usually supplied from a 20-kv. bank of transformers in No. 2 power house.

In the event of conditions necessitating the operation of both plants, No. 1 plant can be paralleled with No. 2, or No. 2 can drop the load on the 20-kv. lines, leaving No. 1 plant to handle this. Generally No. 1 and 2 plants are operating in parallel through Trail, all phasing being done at the power house. These lines are controlled from Westinghouse Type "G. H." Oil circuit breakers and protected by reverse power relays.

The total maximum demand from this station to date has been 20,900 kw., in other words the equipment has safely withstood an overload of 22 per cent. R. A. Ross, M. E. I. C., Montreal, acted as consulting engineer for the power company.



Low-Tension Room, West Kootenay Power & Light Co. Substation

Generator Rooms

The generator rooms at Trail, although they do not embody any new features, are of interest due to the fact that it is another application of electric power to a new industry. An industry that has sprung up since the commencement of the late war. They are of hollow tile and steel construction. No. 1 Generator Room is 266 ft. (81.1 m.) in length and 30 ft. (9.14 m.) in width. No. 2 generator room is 218 ft. (66.4 m.) in length and 30 ft. (9.14 m.) in width. The floor and foundations of the motor generator sets are solid

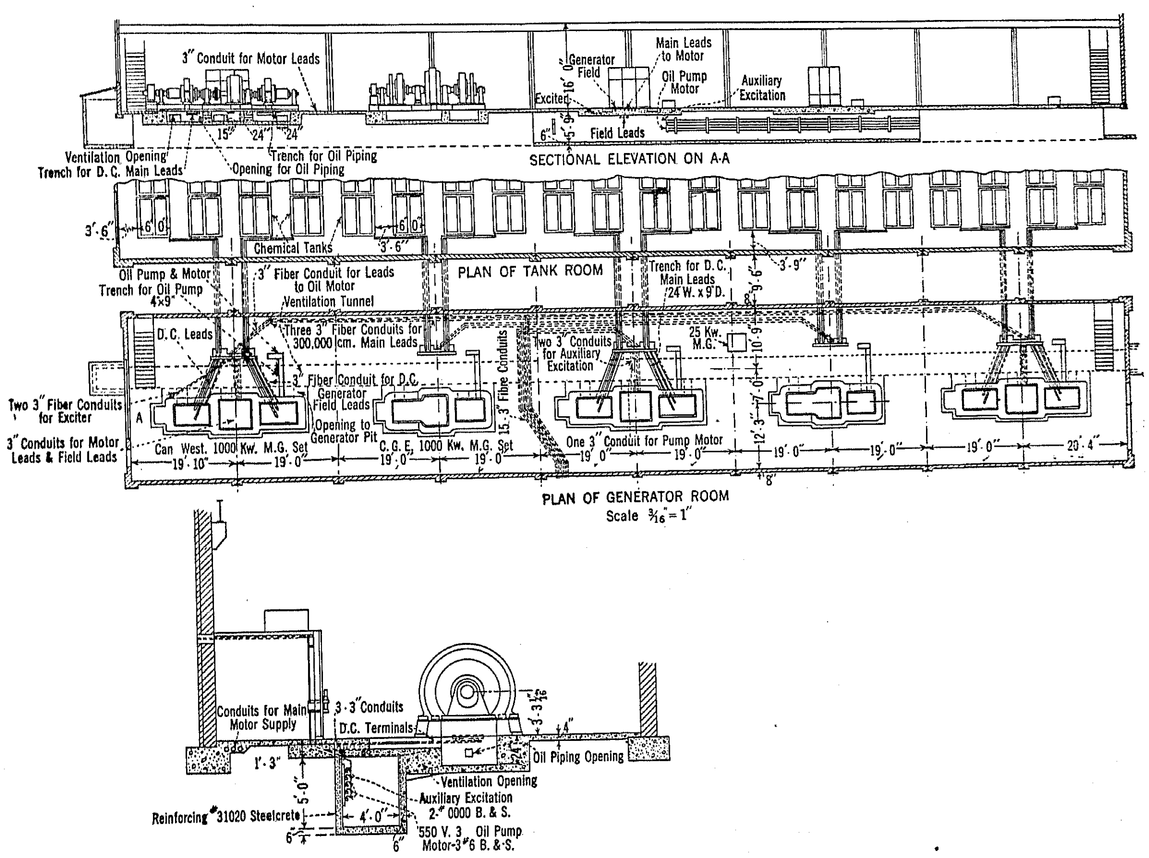
concrete. For the purpose of erecting and facilitating the making of repairs, these rooms are served with a 10-ton (9075 kg.) electric crane, having an electric travel and hoist, the traverse being manually operated. No. 1 Generator Room has seven double-ended sets while No. 2 has six, each consisting of 2500-kw. 125-volts, 4000-ampere (maximum) generators. The two d-c. generators are direct-connected to a 1150-kv-a. synchronous motor, the exciter for which is connected to one end of the shaft. All the machines are arranged longitudinally with the building as can be seen from the photograph.

ventilation. Approximately 30 tons (27,400 kg.) of copper was used in connection with this installation.

Motor Generator Sets

Let us now turn our attention to the motor generator sets. The over-all length of the Westinghouse bed plate which is one single casting is 28 feet (7.62 m.) and the total weight, including the oil pump, is approximately 71,000 lbs. (32,200 kg.) or 71 pounds (32.2 kg.) per kilowatt. Holding-down bolts are used, the bed plate being grouted in on top of the foundations.

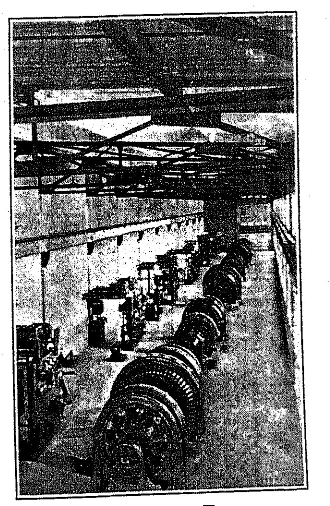
The generators are 8-pole shunt-wound interpole



The switch boards are located between the sets and the wall adjoining the tank room. This arrangement being decided upon so as to minimize the amount of copper bus used in connecting the tanks.

In the early development of the plant, considerable thought was given to the arrangement of a transfer bus in order that any set of tanks might be connected to any machine, thereby permitting more frequent inspection of the units. But when the yearly interest on a bus of the required size and the high cost of copper at that time was considered, it was decided the investment would not be justified.

The d-c. bus bars from the generators to the switchboards and from the switchboards to the tanks, are made up of four 5-16 inch (8 mm.) x 4 inch (10.1 cm.) copper bars separated by 5-16 in. (8 mm.) spacers for

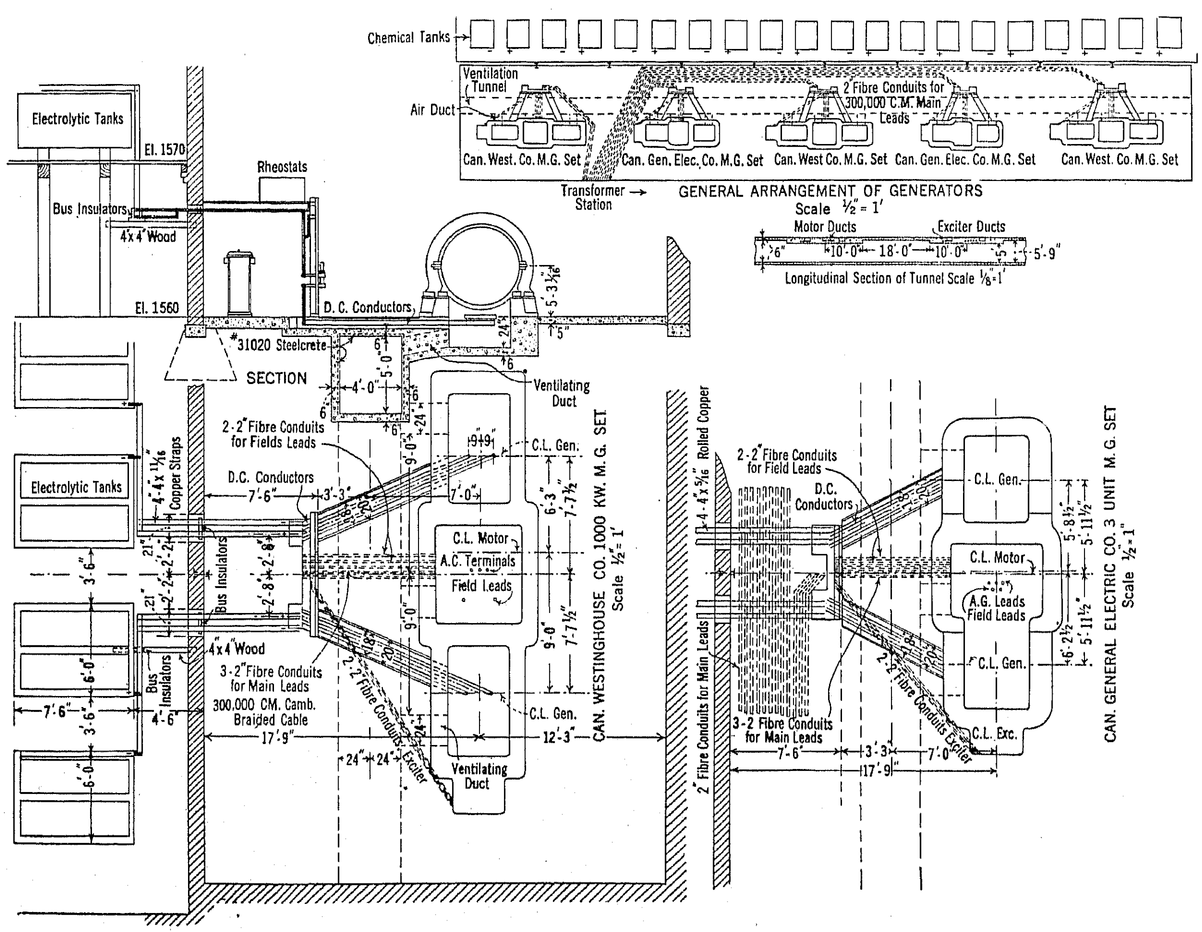


Generator Room

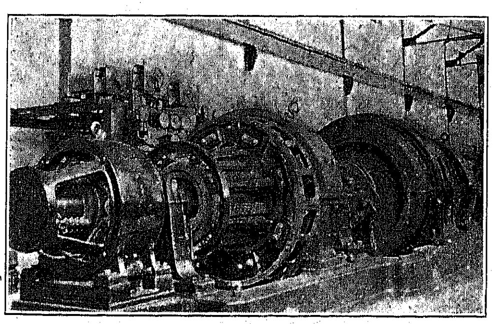
type of 500-kw. capacity and have a voltage regulation of from 50 to 125 volts. The speed is 600 rev. per min.

Two noteworthy features are the commutator and the interpole or commutating pole winding. The length of the commutator is 2 ft. 2.5 in. (67.2 cm.). The diameter is 1 ft. 10 in. (53.3 cm.) The peripheral speed is 3970 ft. (1210 m.) per minute.

With commutators of this length and speed, it has generally been found necessary to shrink a steel ring around the center of the commutator, but in the case



of three machines, this has been overcome by the use of three "V" rings which allows the whole face of the



CANADIAN WESTINGHOUSE MOTOR-GENERATOR SET

commutator to be utilized for the collection of current. 'll commutator bars, of which there are 128, are equipped with radiators to aid in expelling the heat generated.

"Le Carbone" soft graphite brushes are used, operating at a current density of 63.5 amperes per square inch (12.5 amperes per sq. cm.). A spring tension of 1.25 pounds per square inch (0.15 kg-m. per sq. cm.), *i. e.*, 1.75 to 2 pounds (0.79 to 0.90 7 kg.) per brush has proved to give the best satisfaction. This, of course, necessitates having the mica undercut to a depth of about 1/16 inch (1.5 mm.). The peripheral speed is sufficiently high to prevent dirt from sticking in the recesses. The commutators revolve in a leading direction, *i. e.*, they revolve from the heel of the brushes. This has given the best possible wear on the brushes and has minimized their tendency to lock in their holders.

The interpoles are arranged so that the coils receive only one half armature current. This being attained by dividing the total number of interpoles into two sets and connected series multiple. The field frame is of high grade iron and the poles so proportioned so as to minimize the armature reaction.

The shunt field winding is separately excited. The four pillars which support the bearings are insulated from the bedplate, thus preventing the circulation of stray currents through the bearings which would have a tendency to pit both the shaft and the bearings.

The generators were subjected to an insulation test

of 1500 volts a-c. for one minute in accordance with the rules of the A. I. E. E. The insulation will stand 90 deg. cent. without injury. The generators have an efficiency of approximately 91 per cent at full load.

As can be seen, the synchronous motors are located in the center of the set, being 1475 h. p. output, 1158 kv-a. input, 3-phase, 2200-volt, 60 cycles, 600 rev. per min. 314 amperes per terminal.

The stator or armature winding is a twelve-pole single-star connection. The stator consists of a cast iron frame. The inside of the frame being provided with ribs which have dovetails machined in them to receive the armature punchings which have dovetails to correspond. The outside of the frame is provided with large ventilating ducts to provide a good circulation of air. The armature core, which consists of built-up laminated sheet steel, is held in place by end plates, there being ventilating spaces provided at intervals across the armature.

The armature coils are machine wound and of the diamond type. These are formed and insulated before being placed in the slots, there being two in each slot held in place by fiber wedges. The ends of the coils project beyond the armature and are securely corded to a cold rolled-steel ring which is supported by cast iron arms bolted to the frame. This has been done to protect the coils against any undue magnetic strains.

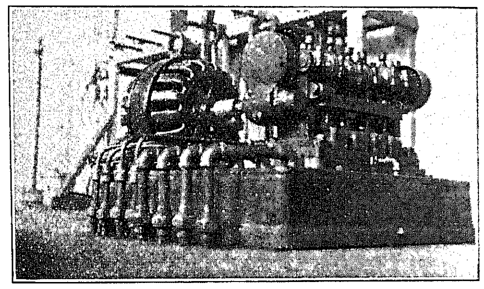
The starting winding is of the interconnected poleface or amortisseur winding.

The rotor consists of a cast steel spider with laminated steel poles and a copper strap winding for the field. The spider has machined dovetailed slots into which the dovetail projections of the poles are fitted and are held firmly in place by tapered keys. A small end plate is bolted on each side of the pole to prevent the keys from moving sideways. These plates are made wider at the top and adjusted in a radial direction so as not only to support the field coils on the bottom side, but also to make the coils fit the poles snugly and firmly, thus eliminating any trouble due to loose field coils, especially after the motor has been in use for a long time.

The bed-plate is so arranged that the armature can be slid clear of the rotating field by means of jack screws for this purpose which is of aid in making inspections and repairs.

The motor was subjected to the following insulation test as recommended by the A. I. E. E., field 1500 volts a-c. for one minute and armature 5000 volts a-c. for one minute. The operating efficiency of the motor at full load is approximately 94.5 per cent.

The exciter is a 125-volt, 200-ampere type "*S A*" 4 poles, 2 interpoles, compound-wound generator mounted on the extreme end of the shaft, in other words overhung. To minimize the starting current to approximately 400 kw. An oil-jack or high-pressure duplex pump forcing a film of oil under the bearings at a pressure of 1000 lb. (458 kg-m.) per sq. in. (6.4 sq. cm.) has been used. This eliminates the static friction to such an extent that the whole revolving member whose weight is approximately 9 tons (8225 kg.) can readily be turned by hand. This oil-jack is on the floor at the left hand side of the switchboard. The whole pump unit is self-contained, the base of same being the oil reservoir having an approximate capacity of 10 gal. (378 liters). The pump is driven with a 1-h. p., 110-volt, 3-phase type C. C. L. induction motor connected to the pump through a worm gear. The

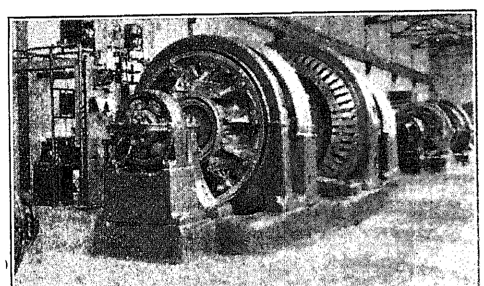


OIL-JACK

pump itself has four cylinders, each cylinder being equipped with high-pressure relief valve.

CANADIAN GENERAL ELECTRIC MOTOR GENERATOR SETS

The over-all length of the Canadian General Electric bed-plate which is built up in three sections, bolted together, is 21 feet, 9 inches (6.62 m.). The total weight of the set is approximately 81,000 pounds (36,700 kg.) or 81 pounds (3.6.7 kg.) per kilowatt. This bed-plate employs no holding down bolts.



CANADIAN GENERAL ELECTRIC MOTOR-GENERATOR SET

The generators are 10-pole shunt wound machines of the interpole type and of 500 kw. capacity. They have a voltage variation of from 0-125 and speed of 514 rev. per min.

The interpoles received full armature current, consequently are of a massive construction. The length of the brush bearing surface on the commutators is 1 foot, 7 inches (48.1 cm.), diameter 2 feet, 6 inch (76 cm.), giving a peripheral speed of 4025 feet (1228 m.) per minute. The commutator is built up of 140 bars and revolves in a leading direction.

The commutators are operated with a graphite

brush manufactured by the Morgan Crucible Company, having an operating current density of 47.6 amperes per square inch (12.2 amperes per sq. cm.). The spring tension being 1.5 pounds per square inch (0.15 kg. per sq. cm.). This, of course, necessitated having the mica undercut.

Note should be made of the strength of the yoke supporting the brush arms; this being a characteristic feature of the machine. The field frames and brush yokes are split horizontally which is of great assistance during erection or the making of repairs. This feature is also found on the Westinghouse sets. The bearings for both makes of machines are two part and are provided with oil ring lubrication, together with sight feed oil gages.

The temperature rise after operating at full load is in the armature 40 deg. cent., in the commutator 55 deg. cent. and in the fields 40 deg. cent. The generator efficiency at full load is 92 per cent.

The motor of this set is a Type *A T I* 1150 kv-a., 2200 volts, 60 cycles and runs 514 rev. per min. Two bearings are insulated from the bed-plate to prevent the circulation of shaft currents.

The stator winding is a 14-pole single-star connection. The bed-plate is so arranged that the armature may be slid clear of the revolving field, if necessary. The starting winding is of the interconnected pole-face type, the bars being of a special high-resistance monel metal alloy.

The operating efficiency of the motor is 95 per cent at full load. The exciter which is a 126-volt, 11 kw. six-pole machine is mounted on the end of the shaft.

Artificial Loading of Generators

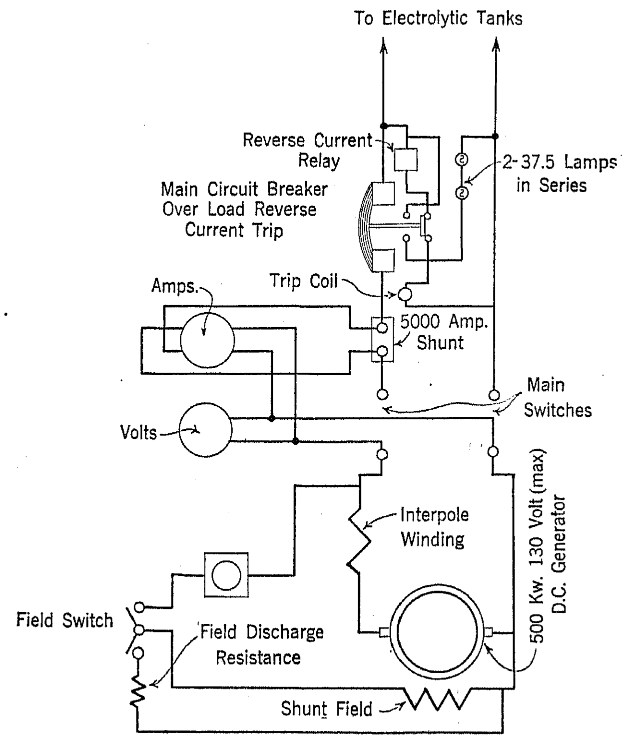
A noteworthy feature during construction was the ease with which the d-c. generators were artificially loaded before going into service, this being done principally for seasoning the commutators. It was only necessary to insert jumpers connecting the bus bars of each generator in multiple and apply the loading back test, commonly known as the "Hopkinson" test. The current taken by the motor being only that required to overcome the losses of the set.

The Auxiliary Exciter

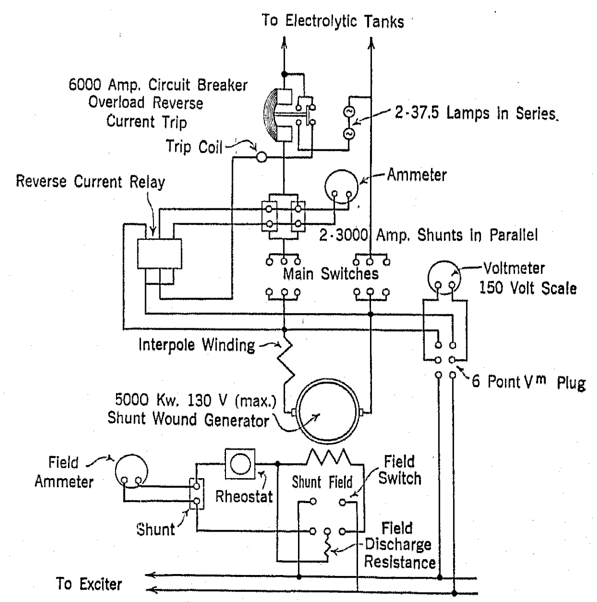
The auxiliary exciter consists of a motor generator set. The motor is of 40 h. p. and is 2200-volt type C. C. L. machine. The generator is rated at 25 kw. and 125 volts. This set not only serves as an auxiliary exciter but also as a source of supply for the cranes in the generator rooms and tank rooms as well as a source of power for a research laboratory. This set is provided with a suitable black marble switchboard on which is mounted the necessary instruments.

Ventilation

The frames of the generator sets and the neutral connections of all the instrument transformers, are securely grounded to a 250,000 cm. ground bus which uns the full length of the generator room. This bus is located in a tunnel under the floor space between the machines and the switchboard. It is 4 feet (1.22 m.) high and 4 feet (1.22 m.) wide, and serves for two purposes. One of these is that it contains the auxiliary exciter bus as well as a 3-phase, 110-volt circuit for the



Canadian General Electric Wiring Diagram



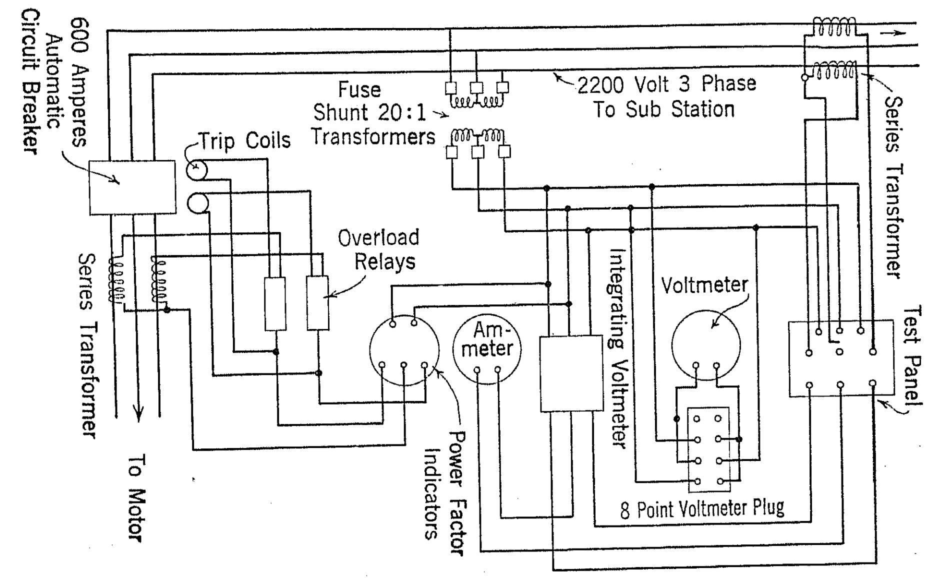
Canadian Westinhgouse Wiring Diagram

Westinghouse oil pumps, that it also contains the station lighting circuits.

For the purpose of ventilating the machines, air is drawn in through ducts connected to the tunnel; each motor and generator having its own duct, and the warm air is exhausted through cupolas on the roof.

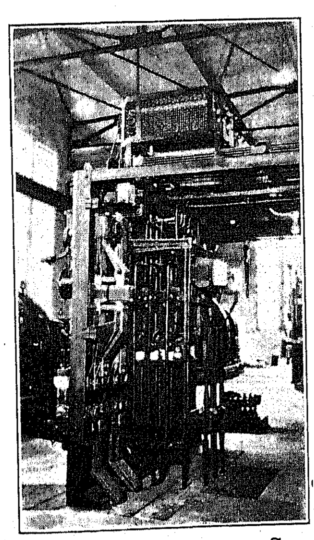
Switchboards

The switchboards are of the standard three-panel type on which is mounted the complete apparatus for the control of the sets. On the generator panel is mounted an ammeter which is connected in the tank circuit. These panels are equipped with a reverse current feature adjusted so that in case of power failure or the motor dropping off the line, the counter e. m. f. of the cells would not motor the set by acting as a primary battery. This counter e. m. f. is made use of as an auxiliary lighting source, each switchboard being equipped with lights, receiving current from the tanks

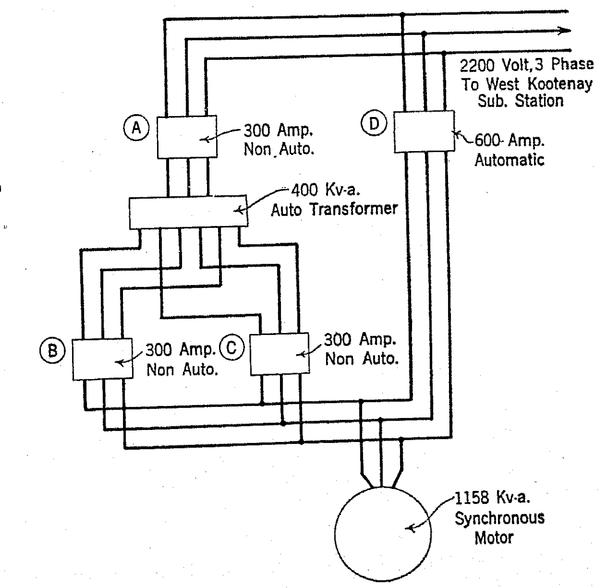


Instrument Wiring, Canadian Westinghouse Switchboard. Motor Panel

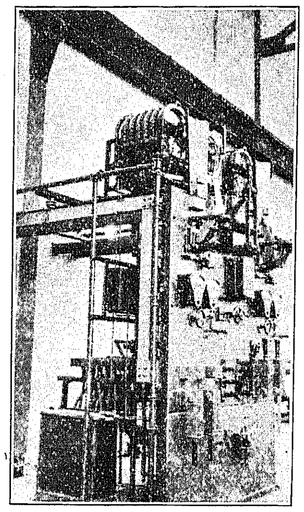
they supply. This is accomplished through a small automatic switch mechanically connected to the d-c. breakers which in turn are controlled from the reverse current relays. The a-c. or synchronous motor panel is marked by its simplicity; in fact, simplicity is one of the outstanding features of the whole plant. On the upper portion of the panels are mounted the indicating instruments. Test blocks have been installed, thus permitting the checking and testing of instruments under load. The motor is protected by inverse time limit alternating-current overload relays. The motor panels of the two manufacturers differ in some respects. With the Westinghouse panel the switches are mounted directly behind, one a Type B-non-automatic, 2500-volt, 300-ampere and the other a type B automatic oil circuit breaker of 600 amperes at 2500 volts. Two type *B* non-automatic, 300-ampere, 2500-volt oil circuit breakers are mounted on the wall directly at the rear of the switchboard over the auto-transformers and are mechanically connected with operating handles on the front of the board by bell



Rear Canadian Westinghouse Switchboard

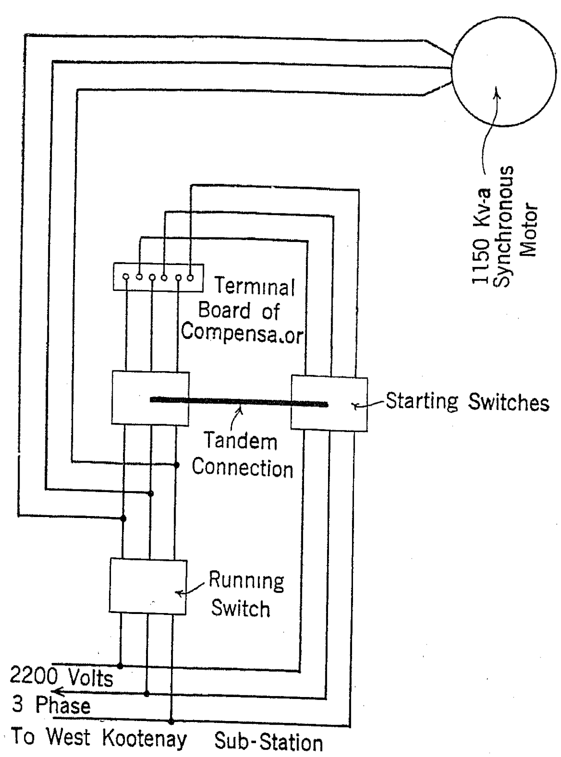


cranks and connecting rods. The 300-ampere non-automatic switch mounted directly at the rear of the panel is the magnetizing switch for the primary of the auto-transformer and is only in service during the period of starting. The next adjacent handle is the



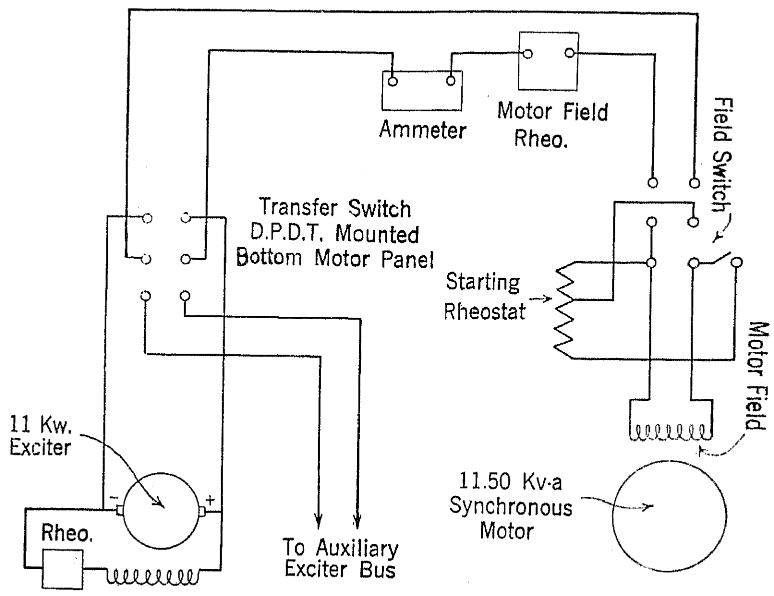
Canadian General Electric Switchboard

first or 50 per cent tap, *i. e.*, 1100 volts. The next handle is the 70 per cent tap or 1540 volts when 2200 volts is impressed on the primary. These switches are all mechanically interlocked to permit the proper sequence of operation. The auto-trans-



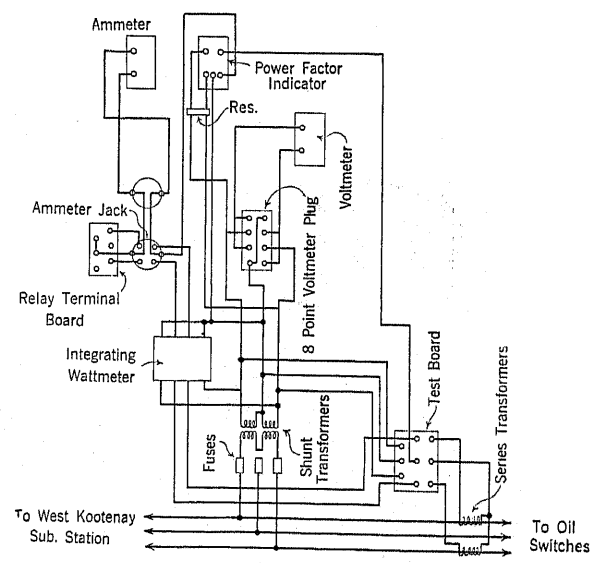
former, capacity 400 kw. is a self-contained, 3-phase oil cooled transformer with various taps having been brought out so that the following voltages may be obtained, 330, 485, 595, 770, 1100 and 1540.

The procedure of starting the sets is very simple. The motor being started as an induction motor, the rotor being equipped with an interconnected pole-face winding. After the machine has attained synchronous speed this winding acts as a damper to retard any



tendency to hunt. The revolving field is shunted through a 3.1-ohm resistance to decrease the induced voltage in the field at the start to 85 volts.

The Canadian General Electric switchboard is very similar to the Westinghouse with the exception of the



Instrument Wiring, Canadian General Electric Switchboard

motor panel. A unique feature of this board is the carbon break field switch mounted at the rear of the panel and operated from the front by means of spade handle and connecting rod. Only three oil switches are required; the magnetizing and starting switch are

mechanically connected in tandem, *i. e.*, they operate simultaneously. The method of starting is practically similar to that of the Westinghouse set. The motor being started as an induction motor during which time the rotating field is shunted through a resistance of 35 ohms. This reduces the induced voltage, which is due to the transformer action, to 850 volts at the moment of starting. On the motor reaching full speed, the revolving field is energized to full strength, *i. e.*, the field current required to maintain unity power factor at no load. When this is accomplished the motor is connected to the line. The motor attains synchronous speed in approximately sixty seconds. The switches are mechanically interlocked making the board as fool-proof as possible. The motor is protected through a double pole, overload inverse time limit relay operating a d-c. tripping coil.

Power Cables

All the power cables run from the Power Company's switches to the motor generators. They supply through 4-in. (10.1 cm.) "Orangeburgh" fiber ducts embedded in concrete under the floor. These cables are of 300,000 cm. and are insulated with varnished cambric which in turn is protected with a heavy braid impregnated with a moisture-repelling compound.

Load Characteristics

The load, which is wholly a synchronous motor load, is most desirable on account of its great flexibility as a power-factor corrector. This load has been a great help to the power house, cutting down the lagging kv-a. at the generators, which is mostly caused by the rest of the load on the power company's system being inductive. The average power factor at the power house which is 32 miles (51.2 km.) away is 0.98. Excellent voltage regulation is obtained at the power company's substation bus due to this synchronous motor capacity; the voltage never varying more than 2.5 per cent of the normal voltage.

The synchronous motors can be operated as synchronous condensers so long as the armature current does not exceed 15 per cent of full load current and the temperature rise does not exceed 40 deg. cent. If the temperature rise does not come up to 40 deg. cent. 25 per cent increase in armature current is possible.

For electrolytic purposes, advocates of the rotary converter are to be found; their main point of argument being their higher efficiency. They recommend the use of single units of about 5000 kw. capacity. However smaller units in d-c. generators have many advantages over the rotary converter; one of these being the ease with which the d-c. voltage can be regulated. The rotary converter requires expensive inductive regulators or synchronous boosters on the a-c. side also expensive starting equipment when compared with that of the synchronous motor.

With the large units, they are usually operated at about 500 volts on the d-c. side, which of course, means a large number of tanks in series. In the event of a breakdown, in either the tank system or the rotary, a much larger section of the plant is affected than would have been the case had smaller direct-current generators been used. Where a large corrective effect for low power-factor is desired, the rotary converter should not be used as a synchronous condenser, nor is it possible to obtain absolutely smooth operation from a rotary at the end of a long line having a high ohmic resistance. The rotary even with all its latest improvements is still a trouble maker at times and it is a question whether the saving in power brought about by its use will offset the cost of repairs and general up-keep when compared to a direct-current generator driven by a synchronous motor.

In one large plant it has been found necessary to install motor generators of sufficient capacity to keep the zinc from redissolving back into the solution when the rotary converters are down for repairs. To circulate a current to prevent the zinc dissolving, the motor generator would have to be approximately half the capacity of the rotary-converter. This greatly increases the electrical investment with accompanying decrease in earnings per kw. of generating equipment installed.

In the plant under discussion, this protection could be brought about by installing a transfer bus, when the unit shutdown could be transferred over to another generator, thereby not interfering with the operation of the plant to any great extent but as already mentioned it was not considered that the large investment in copper bus would be warranted.

Air Compressors

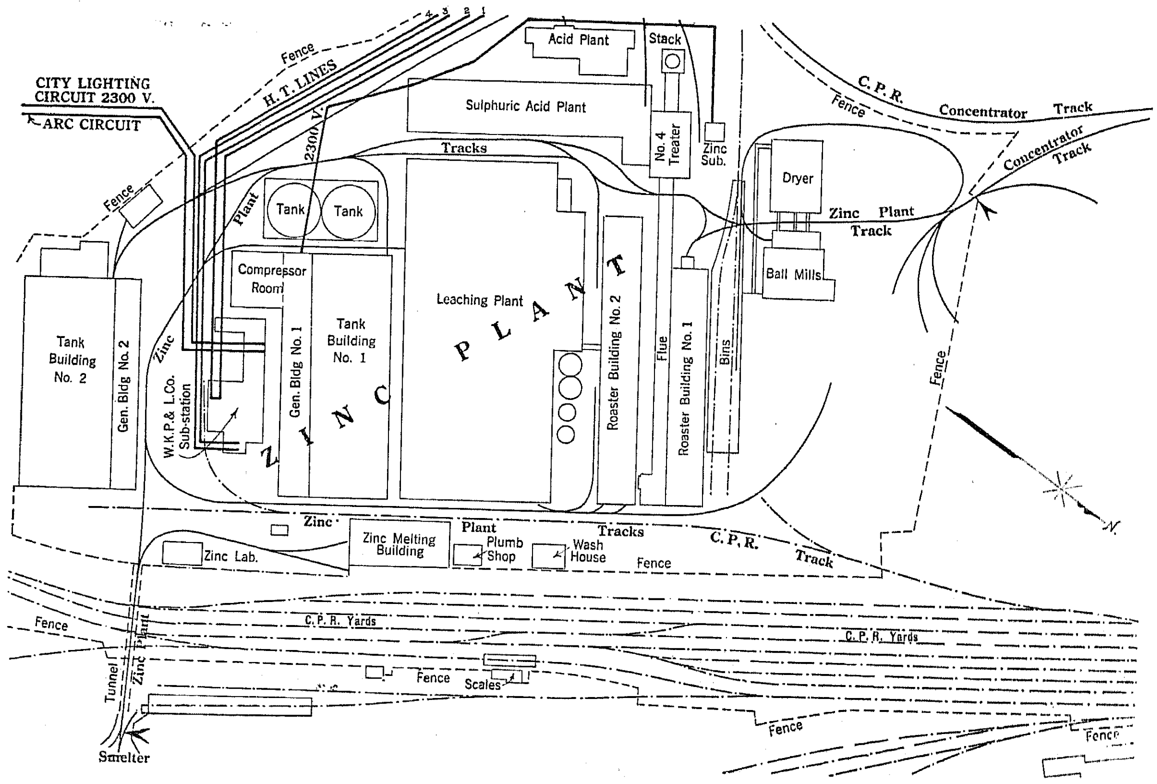
We will now turn our attention to the air compressors which are located at the far end of No. 1 generator room. This wing is of similar construction to that of the generator room. The compressors are operating at an elevation of 1558 ft. (475 m.) above sea level under an atmospheric pressure of 13.9 lb. per sq. in. (978 kg. per sq. cm.) barometer 28.3 in. (717.5 mm.) mercury. These compressors are a very vital part of the zinc plant, as air is used wholly for agitation, and the circulating of the different acid solutions. When the plant was first put into operation, the agitating of the solutions was done mechanically and the solutions were circulated by means of pumps. These pumps soon gave a great deal of trouble on account of the very severe operating conditions.

The first compressor installed was a Rand Ingersoll, with a capacity of 1650 cu. ft. (49.7 cm.) of free air per minute at a pressure of 35 pounds (15.9 kg.) speed 125 rev. per min. and belt-driven from a 200 h. p., 2200 volt, form "*L*" (internal resistance) Canadian General Electric induction motor. As the size of the plant increased, and the demand for air became greater, it became evident that the compressor plant was inade-

quate to cope with the increased demands and therefore it became necessary to install additional units. In the end, the capacity of the first compressor was increased by the addition of an additional cylinder which doubled its capacity.

This compressor has an automatic unloader working on both cylinders; in fact, when the other units are working, this unloader regulates the whole system. This compressor is now driven with a Canadian Westinghouse 300-h. p. type "C. C. L." 2200-volt, 490 rev. per min. induction motor. The motor is controlled from a panel on which are mounted, the starting switches, instruments, relays and low-voltage release.

The second compressor is of particular interest as this machine has been practically rebuilt, being taken from one of the company's mining properties where it had been operating as a steam unit for a number of years. It is a two-cylinder unit and by various valve arrangements can be operated; compounded, two cylinders in parallel or each cylinder separately. Usually it is operated compounded with a capacity of 3200 cu. ft. (90.5 cu. m.) of free air per minute, at a gage pressure of 80 lb. (36.3 kg.). With the two cylinders in parallel, the capacity is 4800 cu. ft. (136 cu. m.) at a pressure of 35 lb. (15.9 kg.) speed 67 rev. per min. This compressor is now electrically driven; this being made possible by installing a herring-bone gear on the crank shaft next to the fly wheel. The gear is driven from a pinion shaft connected to the motor through a flexible steel link coupling. The pinion shaft and pinion are in one piece having been machined from a solid steel forging. This unit is driven by a Canadian Westinghouse 600 h. p., 400 rev. per min., 60-cycle, 2200-volt, 144 amperes per terminal, squirrel-cage induction motor. The motor is controlled by a switchboard panel and a 1000 kw. auto-transformer with taps brought out by which the following voltages may be obtained: 650, 935, 1150, 1400, 1700 and 1900. That is when 2200 volts is applied to the primary. The motor is started on the 1400-volt tap taking 200 per cent full-load current and requiring from 50 to 70 seconds to attain full speed.



General Plan. Zinc Plant

No. 3 compressor has a capacity of 2000 cu. ft. (56.5 cu. m.) of free air per minute, speed 150 strokes per minute. This compressor only operates on the high-pressure system at an average pressure of 80 pounds per sq. in. (5.62 kg. per sq. cm.). Like No. 2 compressor, this machine came from one of the company's mining properties; the only change being found necessary was in the remodeling of the valve gear.

A 400-h. p. 2200-volt, wound-rotor Allis-Chalmers induction motor supplies the motive power. The motor is controlled from a suitable grey marble switchboard on which is mounted an oil circuit breaker and a six point Cutler Hammer drum controller, cutting out the different steps of resistance. On attaining full speed, the slip rings of the rotor are short-circuited. The motor has a full speed of 300 rev. per min. The starter being equipped with a "no-voltage" feature.

This compressor is rope-driven, the idlers and the tightning devices being fastened to the roof structure.

The three compressors are on one power feeder from the power company's bus. Each compressor has disconnecting switches connected ahead of the oil switches in order that the switches and starting equipment can be readily inspected.

Industrial Motors

In the rest of the plant there are practically only 550 volts induction motors varying from 5 to 200 h. p.

The 550-volt service is derived from a local substation which is within close proximity of the points of supply. The maximum capacity of this station is 1200 kv-a., there being 400 kv-a. water-cooled, single-phase transformers connected delta-delta. The primary voltage being 2200 and secondary 605. The primary has also a 1990 volt tap. Power is fed to this station over a 500,000 cm. line consisting of two circuits of 250,000 cm. This line is protected with a set of Canadian General Electric aluminum lightning arresters at each end. A three-panel switchboard controls the outgoing feeder circuits supplying, namely, a 1000-ampere circuit to the ball mills, a 600-ampere circuit to the roaster building and a 600-ampere circuit to the leaching plant. Practically all apparatus are individually driven, thereby the operation as a whole is not materially affected by the shutting down of any one machine.

The same reasons that hold for the preference of a-c. to d-c. motors in general industrial work make the a-c. motor more desirable for the zinc plant operation. There being two reasons for this: First, the relatively higher alternating voltage that can be used, *i. e.*, 500 or even 2200 volts in dry places, and second, the greater mechanical simplicity of the a-c. motors, particularly that of the squirrel-cage type which is very rugged and its initial cost, together with maintenance, being low. The operating conditions that exist in a cement mill are practically duplicated in the plant which is being described (see "Motors on Cement Industry" by R. B. Williamson, TRANS. A. I. E. E., Vol. XXXVLL, 1918, page 1241.)

On account of the heavy accumulation of dust in the motor windings which interfere with the ventilation, the machines have to be frequently blown out and cleaned off. It is the writer's opinion that in such places where the dust accumulations are heavy, at times plugging the air-gaps, a motor designed with a wide air-gap would be more suitable. Of course, a motor having this characteristic has a poor power factor, but this could be improved if the correction warranted the investment by connecting a synchronous or static condenser to the feeders supplying the induction motors.

This plant is especially well adapted to take care of power factor correction on account of the large capacity installed in synchronous motors.

Probably the most desirable motor for this class of work would be that of the enclosed type, which would permit a very close air-gap and consequently high power factor. Motors of this type are extensively used in the anthracite coal fields of Pennsylvania.

The argument generally advanced against the enclosed motor is that of the high initial cost, on account of the designer having to be more liberal with iron and copper. If, however, the enclosed motor were equipped with ball bearings, it seems only reasonable that the extra cost would be more than offset by the saving brought about in maintenance.

The roasters are arranged in two parallel lines, seven in one and six in the other. The roasters are of the wedge type; having seven hearths, each hearth having four sets of rabbles operated from the bottom by means of a bevel gear driven from a Westinghouse back geared motor. Coal is used as fuel and the roasters are fired by hand.

The gases issuing from the thirteen roasters feed into a general flue which in turn passes through the Cottrell plant (No. 4 treater). The elementary principles of the Cottrell system of dust precipitation are now generally known so that an extended theoretical discussion would be superfluous. The treater operates upon unipulsating direct current which is supplied from the rectifier room at a potential of 60 kv. The power consumption is 28.5 kw.

The electrical equipment consists of an Allis-Chalmers 38.5 h. p., 3-phase, 550-volt induction motor, a 28-kw. 220-volt, single-phase generator and a high-tension rectifier; all direct-connected so as to operate in synchronism, and an Allis-Chalmers single-phase, 220-volt to 150 kv. secondary connected to operate at 75 kv. Low-reactance transformer capacity 25 kv-a. The transformer is of special design and is constructed in such a manner as to be amply protected against all excessive strains placed upon the insulation due to the electrical surges arising at times from momentary short circuits, or arcing in the treater, as arise continuously from the use of the mechanical rectifier, which makes and breaks contact in the high-tension circuit.

The voltage in the treater is controlled from the switchboard through resistance control by means of which the field excitation in the generator can be varied over a wide range. This controls the generator potential which in turn varies the voltage impressed upon the treater. The voltage in the treater can also be regulated by shifting the position of the stationary contacts of the rectifier relative to the poles of the generator by which the proportion of the current wave can be controlled. This method of regulation, however, is only used for the making of general adjustments and is not resorted to in the course of ordinary operation.

The rectifiers are usually set once and for all in such a manner as to give contact to that part of the wave which will assure smoothest operation when the treater is operating under normal conditions. But when the conductivity of the gases varies radically, it becomes

desirable at times to change the position of the rectifier contacts.

The electrical equipment in connection with the concentrator consists of:

No. of Motors	H. P.	Type	Make	Speed	Motor Driving
1	15	K. S.	C. G. E.	1200	Bucket Elevators
1	25	K.	"	"	Filters
1	15	K.	"	"	Thickeners
1	5	K. S.	"	"	Ball Mill Classifier
1	15	K. S.	"	"	Conveyor to Ball Mill
1	7.5	C-C.L.-B.G.	C. W.	1120	Bins
1	5	K.	C. G. E.	1200	Oliver Filter
1	50	K.	"	"	Pump
1 x	200	L.	"	600	Flotation Machines
1 x	100	K. S.	"	720	Ball Mill

The above motors are 550-volt machines with the exception of those marked "X" which are 2200-volt.

The equipment also included two Ding magnetic separators, speed 440, operating on 110 volts d-c. The 550-volt service is obtained from two 30-kv-a., 2200-550 volts (pole type) Canadian General Electric transformers connected in open delta. At the time of writing, a mineral separator consisting of a battery of flotation machines was being installed in direct connection with the leaching plant. This installation will require approximately 200 h. p. in 550-volt induction motors.

The motor equipment in the different departments consists of:

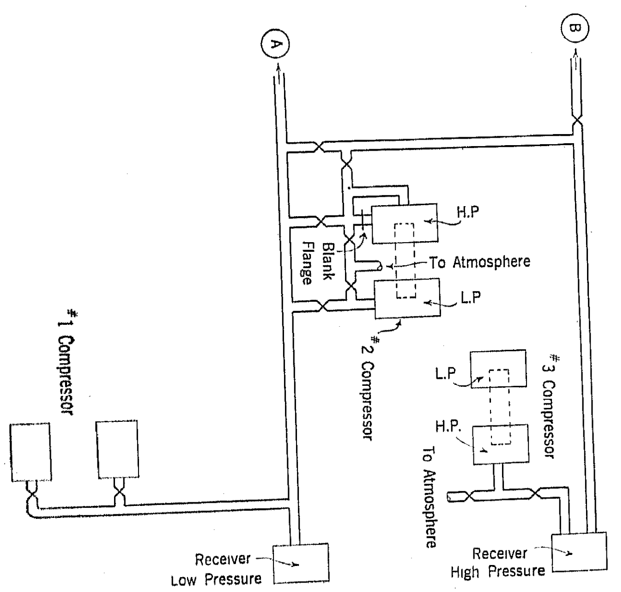
	No. of Motors	H. P.	Type	Make	Speed	Motor Driving
Ball Mills	3	175	K. 1	C. GE.	600	3 Tube Mills 5 ft. x 18 ft. (1.52 M. x 5.48 M)
	1	5	C.C.L.—B.G.	C. W.	1120	Belt Conveyor
Roaster Building	1	10	C.C.L.—B.G.	C. W.	1120	7 Hearth Wedge Roasters
	2	7—5	" "	"	"	" " " "
	10	5	" "	"	"	" " " "
	7	7—5	" "	"	"	Belt Conveyors
	1	7—5	" "	"	"	" "
	1	5	" "	"	690	" "
	1	5	" "	"	1120	" "
	1	10	K. 1.—B. G.	C. G. E.	1200	Spiral "
	2	7—5	C.C.L.—B.G.	C. W.	1120	Spiral "
	4	5	" "	"	"	" "
	1	5	I.L.T.—B.G.	"	"	" "
	7	7—5	C. C. L.	"	"	Fans "
	2	5	"	"	"	"
Dryers	2	10	C.C.L.—B.G	C. W.	1120	2 Dryers 5 ft. x 40 ft. (1.52 M. x 12.17 M.)
	1	7—5	K2 C	C. G. E.	1200	Belt Conveyor
Cottrell Plant	1	38—5	AllisChalmers		1750	Single Phase Generator 110—250 V. 113— 5 A.
Leaching Plant	2	30		Fairbanks Morse	900	2 Gear Driven Triplex Gould Pump
	2	20	K. S.	C. G. E.	1200	Storage Pumps
	1	15	C. C. L.	C. W.	1120	Pumping electrolyte from filters to tank room
	1	10	K.	C. G. E.	1200	Pump, Thickeners to Pachucas
	1	20		Allis Chalmers Bullock	830	" Pachucas to Thickeners
	1	20	K.	C. G. E.	900	" " " "
	1	30	C. C. L.	C. W.	650	2 Pumps, from thickeners to top of building
	1	5	K. S.	C. G. E.	1200	Pump, from filter to thickeners
	6	5	C. C. L.	C.W.—B.G.	1120	Dorr Thickeners
	1	5	C. C. L.	C.	690	Dorr Thickeners
	1	5	C. C. L.	C. W.	1120	Drag Classifier
	1	5	"	"	690	" "
	1	7—5	" —B.G.	"	1120	" "
	3	7—5	" "	"	"	Dorr Thickeners
	1	3	"	"	"	Filter
	1	5	K. T.	C. G. E.	1800	Pump
	2	5	C. C. L.	C. W.	1120	Spiral Conveyor
	1	5	" —B.G.	"	"	Belt Conveyor
	1	5	"	"	690	Bucket Elevator
	1	5	K. T.	C. G. E.	1200	" "
	1	10	C. C. L.	C. W.	1120	" "
	1	10	"	"	690	Machine Shop
Zinc Melting Room	1	7—5	C. C. L.	C. W.	1120	Drossing Machine
	2	5	"	"	1120	Pipe Threading Machines
Lead Burners	1	5	K. T.	C. G. E.	1800	Emery Wheel
Plumbing Shops	1	3	C. C. L.	C. W.	1120	Drill Press

C. W.—Canadian Westinghouse Company, Ltd.
C. G. E.—Canadian General Electric Company.
B. G.—Back Geared.

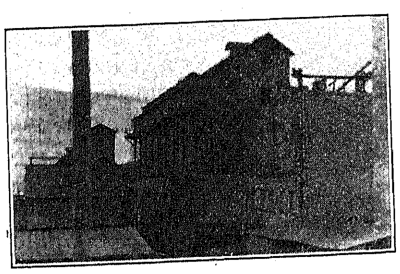
Summary of total equipment installed in the Zinc Plant:

Name of Departments	No. of Motors	Total H. P. Installed	Aver. H. P. per Motor Installed	Aver. Power Consumption	Aver. Load Factor
No. 1 Generator Room	11	10626—5	966—4	6200 kw.	78
No. 2 Generator	9	9046—5	1005—1	5000 kw.	74.25
Room, compressors, incl. Vacuum pump	5	1375	265	690 kw.	67.3
Ball Mill	4	530	132—1	243 kw.	61.5
Wedge Roasters, Dryers and Cottrell Plant	44	323—5	7.32	88.6 kw.	35.8
Leaching Plant, Zinc Melting Room	37	353	9.95	113 kw.	43
Lead Burners and Plumbing Shops Concentrator	10	4375	43.75	275 kw.	84.5
Total	120	22692	186	12609.6 kw.	74.25

The total horse power in motors installed per ton (916 kg.). Zinc is 215.8 h. p., this being figured from the theoretical daily capacity at 100 per cent efficiency, of the plant which would be 105.6 tons. (95,800 kg.). The theoretical capacity is computed from the total ampere-hour capacity for 24 hours of the generator equipment: this being 2,496,000 ampere-hours. Horse power in-



Air Compressor Layout



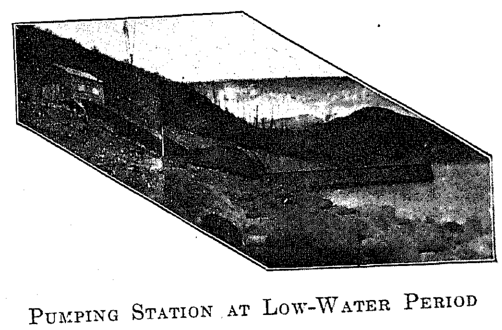
Cottrell Plant

stalled per ton figuring on the actual output of 85 tons (77,100 kg.) would be 266.9.

Water Supply

A very interesting development of vital importance to the continuous and successful operation of the reduction plant is the water supply system, this being of the most modern nature. After having experienced practically a total shut down one very severe winter due to a water shortage, plans were immediately made for the construction of a modern pumping station by which the plant as a whole would always be assured of an ample supply of water. In such a plant as is under discussion, there are very few departments that do not use water.

The pumping station is located about a mile (1.61 km.) up the Columbia River from the reduction plant; this point having been chosen on account of the water there being pure and free from sediment or taint which would make it unfit for domestic or other purposes. Also it is not affected by the drainage or seepage from the plant. The flat or bench on which the plant is situated is about 250 ft. (76 m.) higher than the average water surface in the river. The greatest obstacle to

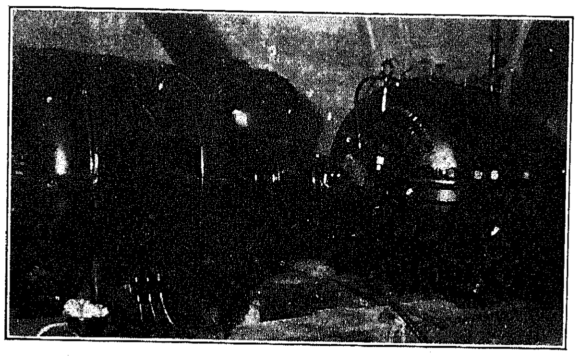


Pumping Station at Low-Water Period

be overcome was the great rise and fall of the water in the river due to the change of seasons and which at this point amounts to about 50 ft. (15.75 m.). The company's engineers overcame all these difficulties and have constructed a pumping station of large size and great strength at a point low enough to draw up the water when the river is at its lowest stage. With a sloping tunnel entrance running from a point up the river bank well above the highest known water mark, the whole has been covered and made absolutely water tight. This tunnel is 82 ft. (25 m.) in length. The pumps are housed in an arched concrete house of extremely heavy construction, being designed so that the total weight of the building and machinery overcomes the tendency of the structure to float at high water and at the same time to safely withstand the great pressure of water when the river is in flood. At

this time there is 30 (9.15 m.) or 40 feet (12.2 m.) of water over the roof of the pump house.

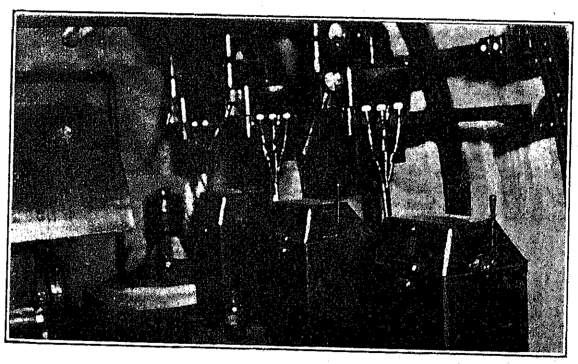
The floor of the pump house is 1350 feet (412 m.) above sea level. The building is 30 feet (9.15 m.) long and 20 feet (6.1 m.) wide with an arched roof 14 feet (4.27 m.) in height and 18 in. (35.8 cm.) in thickness. Through the bottom of the pump house three 10-inch (25.6 cm.) steel pipes lead downward to a covered suction forebay from which the supply is drawn. The



PUMPING UNIT

pumping equipment consists of three 4-stage turbo pumps manufactured by Goldie and McCulloch, Galt, Ontario. These pumps have a capacity of 1500 gal. (5675 liters) per minute against a maximum lead of 337 feet (102.6 m.) Each pump has a 10-inch (25.6 cm.) delivery connected to the 16 inch (40.7 cm.) main which runs up through the tunnel and is connected to the old system.

The pipe lines are built for two pumps, to be operating continuously, the third to act as an auxiliary. Under such conditions the plant has a capacity of over 4,000,000 gal. (15,130,000 liters) every 24 hours, or approxi-



STARTING EQUIPMENT FOR PUMP MOTORS

mately a sufficient supply for a city of 30,000 people.

The pumps are direct-connected to Canadian Westinghouse type *"H S"* induction motors operating at 1150 rev. per min. These motors are rated at 200 h. p. but will deliver 250 h. p. continuously with a rise in temperature of 55 deg. cent. They are started through auto-transformers which are mounted on the floor directly under an oil circuit breaker. The breakers are for motor protection and are equipped with overload tripping coils. An ammeter has been mounted on the lid of each switch. For taking care of all seepage and leakage a 3-inch (7.6 cm.) vertical sump pump is provided; having a capacity of 180 gal. (681 liters) per minute against a maximum lead of 50 ft. (15.25 m.). The pump is driven by a 5-h. p. 550-volt type C. C. L. Westinghouse induction motor and has a speed of 1750 rev. per min. The motor is controlled by an automatic float switch.

For priming purposes when the river is below the level of the pumps, two vacuum pumps are provided, being located on the landing at the entrance of the tunnel. These are shut down from the pump room by push buttons connected to solenoids in connection with the starting switches.

Power is derived from a bank of stepdown transformers, immediately outside the building consisting of three 125 kv-a. single-phase transformers connected delta. This bank is fed from a 2200-volt line running from the power company's substation, a distance of about a mile (1.61 km.). An outdoor type oxide film lightning arrester has been installed to further insure the station against a shut down.

Discussion

W. S. Peterson: I was employed at the plant referred to in this paper, which has rotary converters for its source of direct current. My experience at that plant, was that the rotary converters justify themselves in every way. The losses saved were roughly 5 per cent or 6 per cent of the total power used. The total power was 35,000 kw., which was maintained for 24 hours per day. You can understand that the power saved more than makes up for the increased maintenance of the rotary converter. The final result is greater economy.

I will now explain the use of the motor-generator set to hold the zinc from redissolving into the solution, when the rotary converters were off the electrolyzing tanks. The plant was originally laid out for five tank circuits, with six rotary converters, thus allowing one spare. Due to the necessity for greater production at one time, the plant was operated with six sets of tanks on the six converters, with no spare. The company had some motor-generator sets from an electrolytic copper refinery, which they were not using, and therefore installed them to use temporarily on a tank circuit while a rotary converter was down for repair work. This change probably worked out better than to have another rotary converter, due to the fact that the rotary converters in actual operation were down for repairs and maintenance not much over 1 per cent and certainly less than 1½ per cent of the total time. The main loss in a shutdown was not only the loss of production but the loss due to redissolving of the zinc. By having the motor generator carry one-third to one-half of the normal tank load, this latter loss could be avoided.

In the early years of the operation of this plant there was considerable trouble with the converters due largely to the lack of experience of the operators. For the past three years, I am told, the operation has been such as to give nearly 99 per cent availability per machine which seems remarkably good. Some may feel inclined to disbelieve this, but this record is taken over periods of nine or ten months continuous running at a time when there are no spare machines due to full production being required. A converter would be taken down once every week or ten days for cleaning and overhauling the brushes.

I would say from my experience with this kind of a plant, that the synchronous converter certainly deserves its place there due to its economical operation and is to be considered far superior to the motor generator for that work.

Electrometallurgical Applications

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Review of the Subject.—*This paper deals with a field of engineering in which electrical, chemical and metallurgical engineers are all interested, and have responsibilities. It contains a discussion of electrical power utilization developments, which are based upon fundamental principles of physical chemistry and electrochemistry. It outlines the particular function of the electrical engineer and indicates his close co-relationship with the chemist and metallurgist in the solution of such industrial problems. It deals only with metals which are ordinarily employed in construction or manufacture and in the production of which large quantities of electrical power are employed. Smelting and electrolytic methods are compared; data and illustrations on plants and equipment for the production of iron, zinc, copper, nickel, aluminum, etc., are included.*

ELECTRICAL engineers may perhaps be said to be inclined to confine themselves to that engineering function which consists of taking power generated by some natural resource and putting it in the form desired for some domestic or industrial use at some location. On the one hand, they are in contact with the mechanical, hydraulic and civil engineers and on the other, with the mechanical, chemical and metallurgical engineers. They are a sort of intermediary and as such, of course, perform an essential service. They are less likely to initiate than the one group and less likely to carry through to ultimate consummation than the other group, power utilization projects. From the one hand, they must take what is given and with the other, they do not equally join in reaping the final rewards.

It is an interesting thing to observe how regularly the electrical engineering student at college studies and absorbs all those laws that have been observed and formulated regarding electricity and then shortly thereafter, deliberately throws many of them away and thenceforth, speaks only in terms of the volt, ampere, ohm, henry, farad, gauss and gilbert, which, indeed, he pursues with relish in devious ways with all the mathematical weapons at his command. It is true that he joins with the mechanical engineer in taking interest in the laws for the conversion of electrical energy into mechanical form and vice versa. Daily, in industrial engineering applications, he converts kilowatts into horse power and horse power into kilowatts.

When considering energy, the electrical engineer seldom takes occasion to go back of the kilowatt hours. Having departed the alma mater, he never speaks more of the unit of energy, the Joule, nor of those electrical units of quantity, the Coulomb, and Faraday. He is not interested in those laws of Gibbs-Hemholtz and Faraday which give the relationship between heat of chemical combination and electricity and connect quantity of electricity with electrolytic production.

Those laws have been abandoned; but let us not think they have fallen upon stony ground or in barren places. Not at all! They are in the arms of the electrochemist and electrometallurgist who esteem them highly and regard them as their very own, the foundation stones of the arts and industries which they have fostered and developed.

It would seem that if he desired, the electrical engineer might have a more equal share with the chemist and metallurgist in those power utilization developments which are based upon the fundamental principles of physical chemistry and electrochemistry. What actually happens, however, frequently, is that the chemist or metallurgist is discovered to know far more about electrical engineering than the electrical engineer does about chemistry or metallurgy, so that he gets charge of the whole undertaking and the electrical engineer is in the position of subordinate.

Inasmuch as the electrical engineer should and must meet with the chemist and metallurgist on a common ground, in those industries based upon physical chemistry and electrochemistry, it is proposed to state and briefly discuss some of the more elemental things, in which electrical engineers as well as chemists and metallurgists are vitally interested, because they constitute starting points in the economic industrial utilization of power.

1. Every chemical combination or dissociation absorbs or gives out a certain definite amount of heat and takes place at either a definite temperature or within a certain temperature range.

2. Some chemical combinations in aqueous or molten solutions are capable of being dissociated or produced by the action of electric current flowing through them.

Upon the first statement is based the practise of smelting; and upon the first and second, the practise of electrolysis. In smelting, energy, whether in electrical form or otherwise, applied in producing the chemical combination or dissociation, must supply that quantity of heat and at that temperature. In electrolysis, the energy required or absorbed is not only determined as in smelting, by the heat of chemical formation, but it is also determined by a fact established by Faraday, that the number of grams of any metal which would take the place of one gram of hydrogen in a chemical combination will be deposited by a

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definite quantity of electricity called one Faraday (F), in amount equal to 96,540 Coulombs.

The electrical energy absorbed or produced in electrolysis, in volt Coulombs or watt seconds, for the number of grams of the metal in the chemical combination, which is equal to its atomic weight, is $e \times n \times 96{,}540$, where e is the electromotive force absorbed or produced in the electrochemical process and n is the valence or number of bonds uniting the metal with the other elements in the chemical compound. Thus: if electric current be passed through a pure solution of copper sulphate, $CuSO_4$, in water from an insoluble anode to a copper cathode, copper, Cu, is deposited at the cathode and oxygen, O, is liberated and sulphuric acid, H_2SO_4, is formed at the anode. $e = 1.22$ volts; $n = 2$; atomic weight of Cu = 63.57. Then, 63.57 grams of Cu are deposited by $1.22 \times 2 \times 96.540 = 235{,}500$ volt Coulombs, or watt seconds; or, converting to pounds per kw-hr., 1 kw-hr. theoretically deposits

$$\frac{3600 \times 1000 \times 2.2 \times 63.57}{235{,}500 \times 1000} = 2.135 \text{ lb. of Cu}$$

The Gibbs-Hemholtz equation definitely connects electrical energy absorbed or produced in electrolysis with heat of chemical formation and temperature of operation. According to it; $e\,n\,F_{(at\ temp.\ T)}$

$$= 4.187\,Q_{(at\ temp.\ T)} + n\,F\,T\,\frac{d\,e}{d\,T}\;; \quad \text{where } T$$

equals absolute temperature, Q equals heat of formation in gram calories and, since one gram calory = 4.187 Joules, $4.187 \times$ number of gram calorie = number of Joules, or watt seconds, corresponding. From this equation e the theoretical voltage absorbed or produced in electrolysis, may be obtained; thus $e = \dfrac{4.187\,Q}{n\,F} + T\dfrac{d\,e}{d\,T}$. The second term at the right is the temperature coefficient of the electromotive force multiplied by the absolute temperature. It may be considered as a term of correction or refinement and since it is small at ordinary temperatures, the approximate result obtained by using only the first term is sufficiently accurate. For example, with the aid of this equation, the decomposition voltage for copper sulphate, used above, may be obtained. As shown by the following equations, when an electric current is passed through a solution of copper sulphate, both $CuSO_4$ and H_2O are dissociated and H_2SO_4 is formed.

$CuSO_4 + H_2O$ + electrolysis

$= Cu_{(at\ cathode)} + H_2SO_4 + O_{(at\ anode)}$

Molecular weight:

$(63.6 + 32 + 4 \times 16) + (2 \times 1 + 16)$

$= 63.6 + (2 \times 1 + 32 + 4 \times 16) + 16$

Gram, kilogram or pound equivalent:

$159.6 + 18 = 63.6 + 98 + 16$

Heat of formation or dissociation—gram calories:

$197{,}500 + 69{,}000$ = heat of electrolysis $+ 210{,}200$.

Then heat of electrolysis $= 197{,}500 + 69{,}000 - 210{,}200$

$= 56{,}300$ gram calories; and $e = \dfrac{56{,}300 \times 4.187}{96{,}540 \times 2}$

$= 1.22$ volts.

So much for fundamental theory; but very frequently, the heat of formation of the material smelted or electrolyzed which, in any smelting or electrolytic operation, is the primary thing determining the amount of energy required is so masked by other factors that in calculating the final or overall efficiency or economy of a process, the initial starting point is either not determined accurately at all or is practically lost from sight. In the smelting process, heat is absorbed or is obtained from:

1. The various heats of combinations or dissociation that are operative in the rearrangements of chemical combinations which take place at or before reaching the smelting temperature.
2. Raising the valuable chemical combination and such other chemical combinations as are necessary to the requisite reactions, and the furnace to the smelting temperature.
3. Radiation, convection, conduction heat losses.
4. Electrical supply equipment.

In the electrolytic process, heat is absorbed or is obtained from:

1. The heat of formation or dissociation active in the main reaction.
2. The heats of formation or dissociation in the various secondary reactions that go on in commercial electrolytes at or near the anode or cathode.
3. Raising the electrolyte and electrodes to operating temperature.
4. Radiation, convection, conduction, heat losses.
5. Electrical loss in the form of or equivalent to leakage, thus reducing the ampere efficiency, that is, the number of grams of metal deposited per Faraday from the theoretical 100 per cent, which is as previously stated.
6. Electrical supply $C^2 R$ losses, which show up in voltage drop in the various component parts of the electrical circuit such as the electrolyte, the anode, cathode, contacts, bus leads or conductors and the electrical supply equipment itself and result in a total impressed voltage, E, being required, which is much larger than e. If no other external heat is provided, it is obvious that ampere efficiency (No. 5 above) $\times\, e/E$ is a measure of the inherent efficiency of the electrochemical process at a particular plant.

The electrical losses mentioned just above are all perhaps of a nature with which electrical engineers are generally acquainted, except those taking place in the electrolyte itself and these, accordingly, require special study to determine variations due to variations in temperature, acidity, current density and other factors.

For example, the copper sulphate solution of composition ordinarily employed in copper refineries has a negative temperature coefficient of approximately 0.5 per cent per degree fahrenheit within the range of temperature ordinarily employed; so that appreciable saving of power results from operating at the highest practicable electrolyte temperature, that is, around 150 deg. fahr. Other ohmic losses being small compared to the electrolyte loss, a power saving of 15 per cent results as compared to operation at 120 deg. fahr. This partly accounts for the economy of copper refineries which generate their own power from steam and operate in the neighborhood of 20 per cent non-condensing, so as to provide steam for electrolyte heating, and experience as a result a net saving somewhat more than 10 per cent. Mention should, perhaps, be made also at this point of the fact that in the electrolytic refinery the opposite reaction is going on at the anode to that taking place at the cathode, so that the resultant e is zero and E is the result of resistance drop purely. As a further example, the acidity of the zinc sulphate ($ZnSO_4$) solution at one of the largest electrolytic zinc reduction plants ranges between 5 and 9 per cent. With constant voltage applied, a change of one per cent in the acidity of the solution changes the resistance so as to produce about a 15 per cent change in current density. This is of importance because it is indicative of the fact that control of the electrolytic cell room conditions can readily be at least partially obtained through variations in acidity, thus relieving the electrical supply equipment and rendering it unnecessary that that equipment be provided inherently with an impracticable amount of voltage varying capability.

From a consideration of the various items of the total heat appearing in the smelting or the electrolytic process, it is apparent that the only really useful energy is that taking part in the final or main reaction or that energy anywhere else in the process that tends to reduce the amount required in the main reaction. The useful heat is a small part of the whole and all the various losses and secondary reactions must be taken into account in determining the overall economy. Where possible, the secondary reactions should be made to work in favor of, rather than against, the process and where external heat can be supplied more cheaply, as it has been shown can be done by steam in the copper refinery, than from electrical energy, this should be done. In smelting, the largest quantities of heat appear or are absorbed under headings 1, 2 and 3, above. In electrolysis, the quantities of heat involved under 1, 2 and 3 also constitute by far the major part. The radiation, etc., losses under 4, for electrolysis are so much less than under 3, for smelting, owing to the lower temperature of operation that this partly accounts for the comparative economy usually to be obtained.

To state the case briefly, the energy required for electric smelting is a function of the amount of furnace charge, whereas, the energy required for electrolysis is a function of the amount of metal deposited. This is the fundamental difference between the two processes and the one which determines the economic success of either in any application. It is not an appreciable disadvantage so far as efficiency is concerned in the electrolytic process, at least in aqueous solutions, to have large quantities of the materials in process. For example, in the case of a large electrolytic copper reduction works more than one-half month's production would be tied up in the electrolyte alone and for a daily production of 60 tons of copper there would be in the neighborhood of 30,000 tons of electrolyte. This large body of electrolyte, flowing in cycle, contains only about three per cent of copper when it enters the electrolytic cell room and still contains about two and one-half per cent of copper when it leaves it. For economic smelting, on the other hand, of the more common metals, it is essential that only small quantities of material in proportion to recoverable metal be in process. Low priced electric energy in conjunction with high priced fuel is a favorable condition for economic electric smelting, but high grade, concentrated materials in process are also essential.

In Norway and Sweden, particularly, there are large economic electric smelters producing pig iron from iron ore. A type of furnace, such as shown in Fig. 1 is employed, which is similar to the blast furnace. The gases are recirculated but there is no blast and external heat to produce the temperature necessary for the reaction is introduced through 6 electrodes from 3000-kw. transformers. The energy consumption is in the neighborhood of 2400 kw-hr. per long ton, or 1.09 kw-hr. per pound of pig iron.

It is of particular interest that the product equals 47.5 per cent of the furnace charge. The charge per ton of product is approximately, 3680 lb. of iron ore containing 61.5 per cent of iron, 132 lb. of lime stone, 815 lb. of charcoal and 17.5 lb. of electrode consumed, or a total of 4645 lb. The ore consists of 20.64 per cent Fe_2O_3 and 64.96 per cent Fe_3O_4, or 760 lb. and 2385 lb., respectively, per ton of pig iron produced. It is, of course, understood, that electric smelting, as fuel smelting, requires fluxes and reagents to produce chemical reactions at the smelting temperature. Hence, the amounts of limestone and charcoal stated above. In view of the high percentage of the final product in the initial furnace charge, owing to the high grade iron ore employed, an analysis of the way heat is absorbed or produced, in the main reactions at least, will be of interest as indicative of the efficiency and economic possibilities not only of this particular electric smelting operation but of other similar operations.

1. Reduction temperature 900 deg. cent.—probable equation:

$$Fe_3O_4 + 3C = 3\,Fe + 2\,CO + CO_2$$

Molecular weight—weight equivalent:

$$231.5 + 36 = 167.5 + 2\ (28) + 44$$

Heat of formation or dissociation—gram calories:

$$270{,}800 = 2\ (29{,}160) + 97{,}200 + \text{electric heat.}$$

$$\text{Then, electric heat} = \frac{115{,}280 \times 3.968 \times 1{,}000}{167.5 \times 3412} = 800$$

kw-hr. per long ton. As there are $\dfrac{2385 \times 167.5}{231.5}$ = 1725 lb. of Fe produced from Fe_3O_4, 627 kw-hr. will be required.

2. Reduction temperature 800 deg. cent.—probable equation:

$$2\,FeO + 2\,C = 2\,Fe + 2\,CO$$

Reduction temperature 700 deg. cent.—probable equation:

$$Fe_2O_3 + 2\,CO = 2\,FeO + CO + CO_2$$

Combining:

$$Fe_2O_3 + 2\,C = 2\,Fe + CO + CO_2$$
$$159.7 + 24 = 111.7 + 28 + 44$$
$$197{,}700 = 29{,}160 + 97{,}200 + \text{electric heat}$$

$$\text{Then, electric heat} = \frac{71{,}340 \times 3.968 \times 1000}{111.7 \times 3412} = 742$$

kw-hr. per long ton.

Then, the 535 lb. of Fe produced from Fe_2O_3 will require 181 kw-hr. Then, the total 2260 lb. of Fe (including 3 per cent loss) produced will require 808 kw-hr. theoretically.

3. The ore carrying the reduced Fe is raised to a much higher temperature 1400 deg. or 1500 deg. cent. in the crucible of the furnace by the addition of about 342 kw-hr. per ton of Fe where the Fe melts, uniting with 3 or 4 per cent of carbon and a fractional percentage of silicon to form the fusible product known as pig iron. There are several minor reactions in the reduction of the silicious gangue, in this case 14.4 per cent of the ore, and in the decomposition of the limestone, which fluxes the gangue and the ash of the reducing agent, if any, to form a fusible slag floating on top of the iron which is tapped and thrown away. These reactions do not materially affect the total electric heat required which, therefore, is approximately 1150 kw-hr. per long ton of white low silicon iron suitable for making steel.

Accordingly, the inherent efficiency of the process comparable to the energy efficiency, $e/E \times$ ampere efficiency, for electrolytic processes, is 1150/2400 = 48 per cent. Whereas, the furnace operating voltage is usually from 70 to 80 volts, equipment having a range of from 60 to 120 volts is required.

In Norway and Sweden also, there has been extensive economic electric smelting of zinc. Here the raw material employed has been high in the values produced. It has consisted largely of cheap, slagging lead zinc say 35 per cent zinc and 10 per cent lead ores, making beside metallic zinc, lead bullion and copper matte. In other words, it is really a separating process, each of the three products requiring further working to place it in marketable form. The power consumed is from 2 to 2½ kw-hr. per lb. of zinc pro-

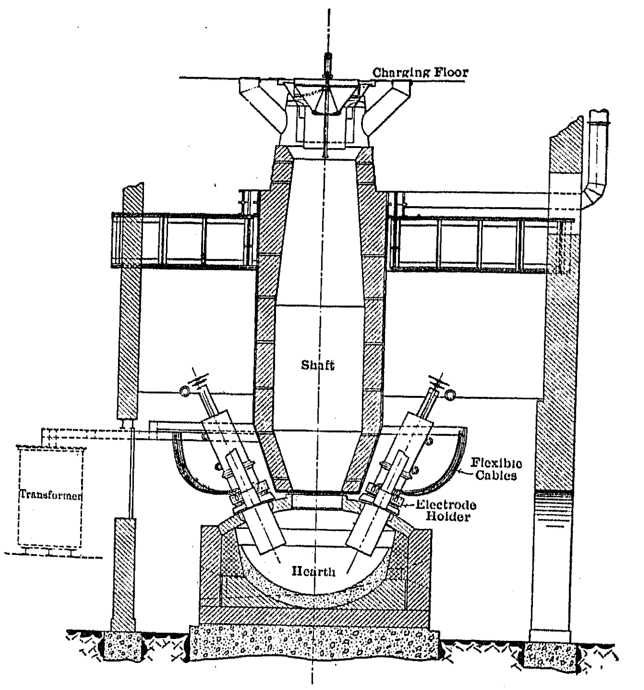

FIG. 1—SHAFT TYPE, ELECTRIC SMELTING FURNACE FOR PIG IRON

This type of furnace allows for circulation of gas so that about 25 per cent of the heat value of the gas formed is utilized within the furnace. Of the total kw-hr. supplied about 6½ to 7 per cent is lost in the cooling water and 20 to 25 per cent by radiation. There are reported to be about 30 electric smelting pig iron furnaces in the world, 21 of which are located in Sweden and Norway. The furnaces are mostly of 3000-kw. capacity, although several of 4500 kw. and 5000 kw. have been built.

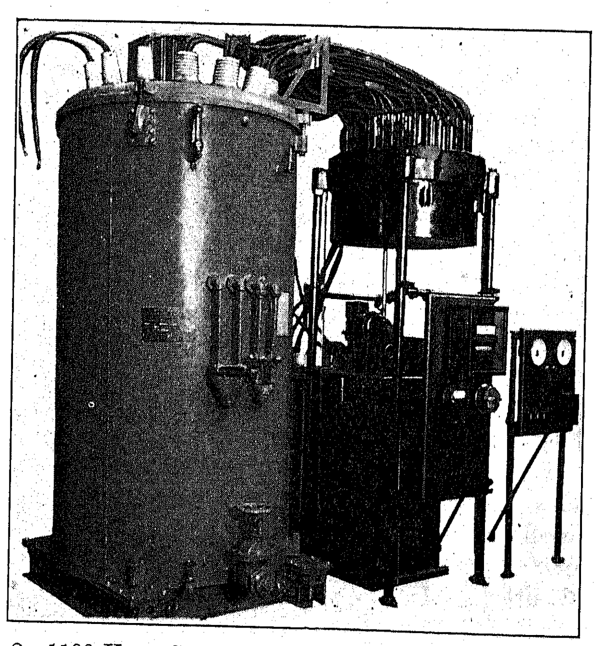

FIG. 2—1100-KV-A. STEP INDUCTION REGULATOR TRANSFORMER EQUIPMENT FOR ELECTRIC SMELTING FURNACE

This view shows control panel containing feeder type constant current automatic control.

duced. Electrical units of about 1000 kw. each have been largely employed. Equipment having a voltage range for the furnace of from 50 to 180 volts, or 90 to 180 volts, is required.

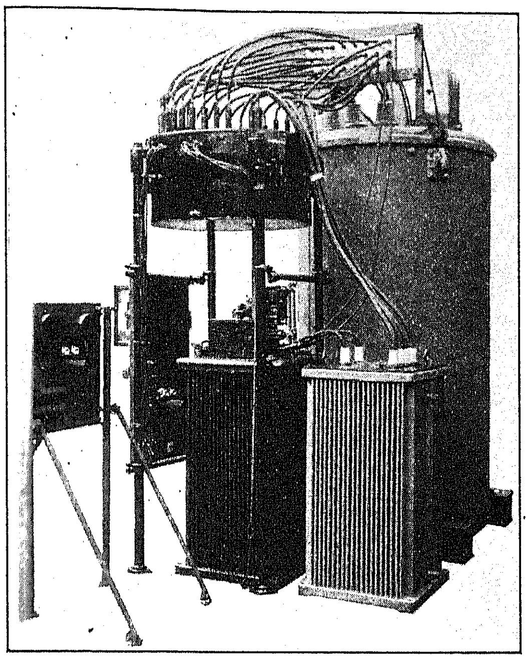

FIG. 3—VIEW OF 1100-KV-A. STEP INDUCTION REGULATOR TRANSFORMER EQUIPMENT FOR ELECTRIC SMELTING FURNACE WHICH SHOWS PARTICULARLY THE SMALL INDUCTION REGULATOR SERIES TRANSFORMER, TANK CONTAINING SELECTOR SWITCHES AND REAR OF CONTROL PANELS

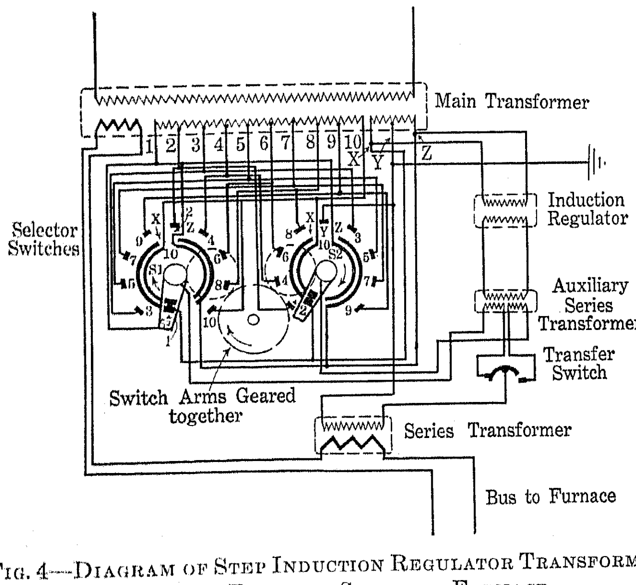

FIG. 4—DIAGRAM OF STEP INDUCTION REGULATOR TRANSFORMER UNIT FOR ELECTRIC SMELTING FURNACE

This diagram shows the scheme so clearly that taken together with descriptions with Figs. 5 and 6, further detail description is unnecessary. When the primary supply voltage is very high it is desirable to have a separate winding on the transformer of moderate voltage upon which the auxiliary control apparatus will operate. By this means, it will be obvious that the electrical apparatus such as selector switches, induction regulator, etc., will be of comparatively small capacity.

Figs. 2 and 3 show the type of step induction regulator transformer unit, which has been largely employed in the economic electric smelting of zinc. It is also the preferable type of unit for use in the economic electric smelting of pig iron. About 22 or more of these units have been installed, and a considerable number, 10 or so, of smaller units of 550 kv-a. capacity of a similar type. This type of control, as developed particularly for electric smelting, is shown in the diagrams, Figs. 4, 5 and 6, for the different electrical conditions usually met with in practise. The details of this are instructive, particularly to electrical engineers. Fig. 7 is a plan or layout of the first complete commercial electric steel smelter and rolling mill established in the Western Hemisphere. This is the plant of the Compana Electro Metallurgica Brazileria,

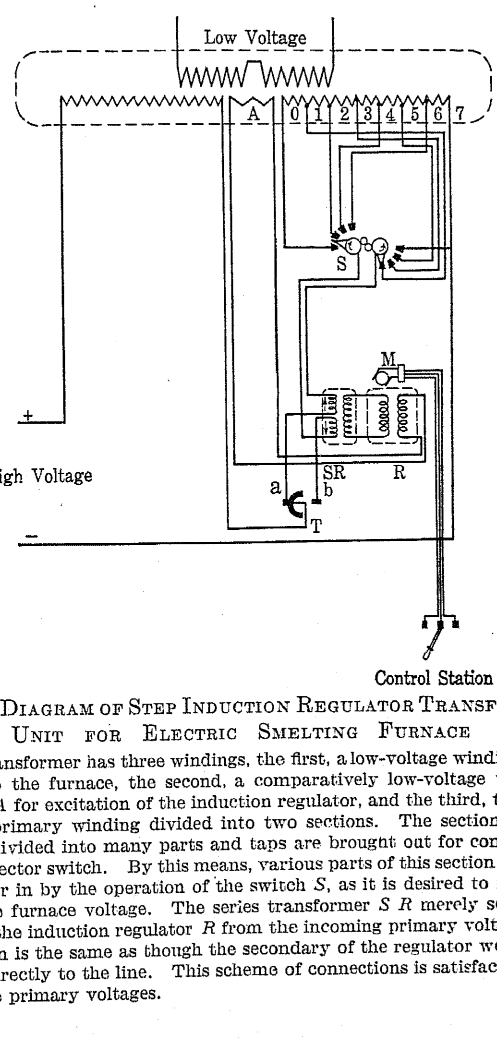

FIG. 5—DIAGRAM OF STEP INDUCTION REGULATOR TRANSFORMER UNIT FOR ELECTRIC SMELTING FURNACE

The transformer has three windings, the first, a low-voltage winding connected to the furnace, the second, a comparatively low-voltage winding marked *A* for excitation of the induction regulator, and the third, the high voltage primary winding divided into two sections. The section to the right is divided into many parts and taps are brought out for connection to the selector switch. By this means, various parts of this section may be cut out or in by the operation of the switch *S*, as it is desired to raise or lower the furnace voltage. The series transformer *S R* merely serves to insulate the induction regulator *R* from the incoming primary voltage and the action is the same as though the secondary of the regulator were connected directly to the line. This scheme of connections is satisfactory for moderate primary voltages.

with a capacity of 30 tons of steel per 24 hours. The equipment consists of:

Two shaft type furnaces (1 as standby), voltage range 60 to 120	3000 kv-a. each
Two 6-ton Bessemer converters (blowing engine for 1)	700 kv-a. "
One 6-ton Ludlum electric steel refining furnace	1500 kv-a. "
One 16-inch rolling mill	500 kv-a. "
One 10-inch rolling mill	500 kv-a. "
Miscellaneous 220-volt power for motors, cranes, shops and lighting	400 kv-a. "
Total Installation	6600 kv-a. "

TABLE I

TABLE OF ELECTROMETALLURGICAL DATA, REFERRING PARTICULARLY TO ELECTROLYTIC

Na	Mg	Al	Mn	Zn	Cd	Fe	Co	Ni	Sn	Pb	H	Sb	Bi	As	Cu
	Metal	Plant	Atomic Weight	Valence	Chemical Equivalent	Electro-Chemical Equiv.- Lb. Dep. p. 1000 Amp. Hrs.	Theoretical Electrolyte	Heat of Formation Calories	Theoretical Deposition Voltage	Theoretical Lb. Dep. p. Kw-hr.	Principal Components Actual Electrolyte	Anodes	Cathodes	Inches Center to Center Spacing	C° Electrolyte Temperature
	Hydrogen		1		1	0.0822	Water (H_2O)	68,400	1.48	0.0555	H_2O + 15% NaOH	Diaphragms Cast Iron Nickel Plated	used Cast Iron Nickel Plated		
	Oxygen		16	2	8	0.658	Water	68,400	1.48	0.445					
	Zinc Production Plants	Great Falls (1)	65.37	2	32.685	2.685	Zinc Sulphate $ZnSO_4$ H_2SO_4	248,000 210,200	2.32	1.155	$ZnSO_4$ H_2SO_4	Lead	Aluminum	2	35
	"	Trail (2)										Cast Lead	Rolled Aluminum	2	
	"	Risdon Tasmania (3)													
	"	Martinez Test Plant (4)										Perforated Lead	Sheet Aluminum		35
	Nickel		58.58	2	29.34	2.41	Nickel Sulphate $NiSO_4$	228,700	1.9	1.27	$NiSO_4$ $CuSO_4$	Diaphragm 65% Ni S Cu Fe	used at	anode	
	Copper Production Plants		63.57	2	31.785	2.61	Cupric Sulphate $CuSO_4$	197,500	1.22	2.135	$CuSO_4$ H_2SO_4	Lead	Copper		
	"	Chuquicamata					$CuSO_4$				$CuSO_4$ HNO_3 Fe Cl	Alloy of Cu Si Fe Pb Sn	Copper	2	40
	"	New Cornelia					$CuSO_4$	197,500	1.22	2.135	$CuSO_4$ $FeSO_4$	3½% Antimony Lead	Copper	2½	34
		Katanga Copper Panda Works Test Plant													
	Copper Refineries	Temporary Plant										Lead	Copper		40
												70% Copper 10% Pb 10% Ni 2% As 1% Sb	Copper		57
	"	(1)				2.61	$CuSO_4$					Poled Copper	Copper		55
	"	(2)				2.61	"				4% Cu 1% As 0 7% Ni 0.8% Fe	Poled Copper		2.4"	60
	"	(3)				2.61	"				4 2 Cu. As 0.18% Ni 1 8%			2½" for 2 batches 2" for third	57
	"	(4)				2.61	"				4.5% Cu	Blister Copper			50
	"	(5)					"					Poled Copper	Series System	(9/16")	59
	"	(6)					"					" "			65
	"	(7)					"					" "			60
	"	(8)					"					" "			55

TABLE I—*Continued*

PRODUCTION OR REFINING OF METALS ORDINARILY EMPLOYED IN CONSTRUCTION OR MANUFACTURE

Hg	Ag	Au	—Potential Series of Metals									
Acidity %	Current Density a.p.sq.ft	Practical Ampere Eff.	Practical Deposition Voltage	Practical Energy Eff.	Actual Lbs. dep. per Kw-hr-	Lbs. Annual Output	Yearly Kw-hr; Reqd. 8770 Hrs.	Kw. Electrolytic Capacity Reqd. (Excl. Stg. Sheets & Plating Out)	Installed Converting Equipment	% Elec. Eff.	Tank Room Circuits	Raw Material
		95	2.05	68.5	0.038 = 6.7 cu. ft.	1,800,000	47,400,000	5400	18 300 kw., 250 v. syn. m. g. sets		110 cells in series. 5000 to 6000 amps. per circuit.	
					0.304 = 3.34 cu. ft.	14,400,000						
6.75	22¼	83.5	3.55	54.5	0.63	110,000,000 +8% used in process	190,000,000	21,500	Voltage 500-580, 60 cycles, 6 5800 kw. syn. booster converters 10,000 amps. each & 2 600 kw. m. g. sets.	91.5	6 144 cell circuits	35% Zn concentrate roasted & leached 700 tons.
	24.2		3.6			55,000,000 (144,000,000	95,000,000 proposed)	11,000	13 60 cy. m. g. sets syn. motors and 26.500 kw., 125 v. gens.	85	26 32 tank circuits	
						75,000,000	125,000,000	15,000	7 3000 kw. syn. convs., 500-600 v. range by ind. reg., 50 cycles			
28	100	90	3 6	58	0.67				1 500 kw., 250 v. m. g. set operating 216 v. 1800 amps.		1 circ., 60 cells, 4 cathodes p. cell.	
		80	3.2	47.5	0.603	17,000,000	28 200,000	3,210	5 4000 amp., 260 v. syn. booster convs., 3 for nickel and 1 for copper—normal range, 230-290 v., 1 spare.		3 84 tank circ. for Ni.	High grade bessemerized matte (produced from 3¼% Ni & 1.7%, cu. 1500 tons, ore.) roasted & leached (residue melted obtaining 60 tons anodes—65% Ni.
		80	2.15	45.3	0.97	9,000,000	9,300,000	1,055			1 108 " " Cu.	
Cooling Towers not Reqd.	10.3	88.4	2.241	48.1	1.027	225,000,000	219,000,000	25,000	3 2500 kw. syn. m. g. & 4 2500 kw., asyn. m. g. sets, 2 900 kw. & 2 780 kw. syn. m. g. sets, 5 3480 kw., 12,000 amp. syn. booster convs. normal range voltage 202 290.	90.5	8 circuits, 894 tanks, total, incl. 64 & 46 stg. & plating down.	19,000 tons, 1.8% cu ore, 90% recovery.
1.7 to 2.1	7.05 6.2-7.56	71 73	2.03 2.08	41.7 42.8	0.915	41,000,000 +14,000,000	45,000,000 cement cu.	5,100	4 1700 kw. 170 v. syn. m. g. sets, 190 v. max., 2 15,000 amp. circuits.	86	2 76 tank circuits.	6250 tons, 1.631% cu. ore, 74% recovery.
6.4	9.13	83.3	1.94	52.3	1.12	2,640,000 (later 66,000,000 proposed)	2,360,000	300	1 50 cy. syn. motor, 2 75 v., 3000 amp. gens.			
preferred	12.8	84	0.55		3.96							
	18	90	0.35		Excl. Stg. sheets & plating out 6.7	750,000,000	112,000,000	12,800	8 954 kw. syn. booster converters, 94 126 v., 4 1215 kw., 200-250 v., 1 1800 kw., 200-250 v.		Waste heat boilers for heating electrolyte, largely used.	
15	24	89	0.43		5.40	180,000,000	33,400,000	3,800	5 1260 kw., 6000 amp., 210 v. max. 90% p. f. syn. m. g. sets, 7000 amps. range 140-180 v.		2-circuits, 480 tanks, each.	
15.5	19	87	0.310		7.375	155,000,000	21,000,000	2,400	1 2220 kw., 185 v. m. g. set, 1 1500 kw., 150 v. m. g. set, 2 300 kw. & 1 400 kw., 150 v.			
15	20	85	0.57		3.9	15,000,000	3,850,000	440	1 500 kw., 125 v. syn. m. g set.		176 tanks Waste heat boilers extensively used 1½ lbs. steam available per lb. cu. produced.	
	20	85	0.195		11.35	450,000,000	39,000,000	4,500	6,000 Kw. engine driven d. c. units or turbine syn. converter units. Approx. 20% of total run non - condensing for electrolyte heating.		550 tanks.	
	17	87	0.310		7.32	450,000,000	61,500,000	7,000	9,000			
	16	89	0.333		7.00	550,000,000	78,700,000	9,000	12,000		No. 1 1800, No. 2 1656 tanks.	
	18	88	0.35		6.56	250,000,000	38,200,000	4,350	6,000		1225 tanks.	

TABLE I—*Continued*

Na	Mg	Al	Mn	Zn	Cd	Fe	Co	Ni	Sn	Pb	H	Sb	Bi	As	Cu
	Metal	Plant	Atomic Weight	Valence	Chemical Equivlent	Electro-Chemical Equiv.-Lb. Dep. p. 1000 Amp. Hrs.	Theoretical Electrolyte	Heat of Formation Calories	Theoretical Deposition Voltage	Theoretical Lb. Dep. p. Kw-hr.	Principal Components Actual Electrolyte	Anodes	Cathodes	Inches Center to Center Spacing	C° Electrolyte Temperature
	Lead		206.9	2	103.45	8.5					10% H_2SiF_6 8% $PbSiF_6$	20 24" × 30" 98% Lead	21 Lead		32
	Tin		119.0	2	59.5	4.87					Stannous Tin Sulphuric Sulphonic Acid	90 to 99% Tin	Tin	2¼	35
	Iron	Production Experimental	55.84	2	27.92	2.29	$FeCl_2$ to $FeCl_3$	101,000 127,850	0 965	2 375	Diaphragm Reqd.	Insoluble (Graphite)	Polished Steel Mandrel	Rotating Stationary	70 70
		Refineries				2.29	150 $FeSO_4$ $+7H_2O$ 75$FeCl_2$ $+4H_2O$ 100 $(NH_4)_2SO_4$				5.5% Fe	Iron (Mild Steel)	Iron (Polished Sheet Steel)	4¼"	35 to 40
		(A. Boucher,	A. Bou	chayer &	C. P. Perin)		$FeCl_2$				14.4 to 18% Fe	Pig Iron Scrap Steel	Iron (Polished Sheet Steel)	rotating Stationary	70 to 75
	Aluminum		27.1	3	9.03	0.741	Al_2O_3	392,600	2.83	0.262	Al_2F_6-6NaF $+Al_2F_6+$ CaF +NaCl +15% Al_2O_3	Graphitized Silicon Free Carbon	Carbon		800
	Magnesium		24.32	2	12.16	1.00	Mg Cl_2 fused with NaCl & NH_4Cl MgO in fused flourides	151,200 143,400	3.27	0.306		Graphitized Carbon			
H	Hydrogen		1		1	0.0822	NaOH	112,500				Diaphragms Graphite	used		
NaOH	Caustic Soda						NaCl Solution in H_2O	97,690	2.32						
Cl	Chlorine		35.45	1	35.45	2.915		68,400		1.255					
	Sodium		33	1	23	1.89	Fused NaOH	102700	4.45	0.425		Nickel	Iron		320

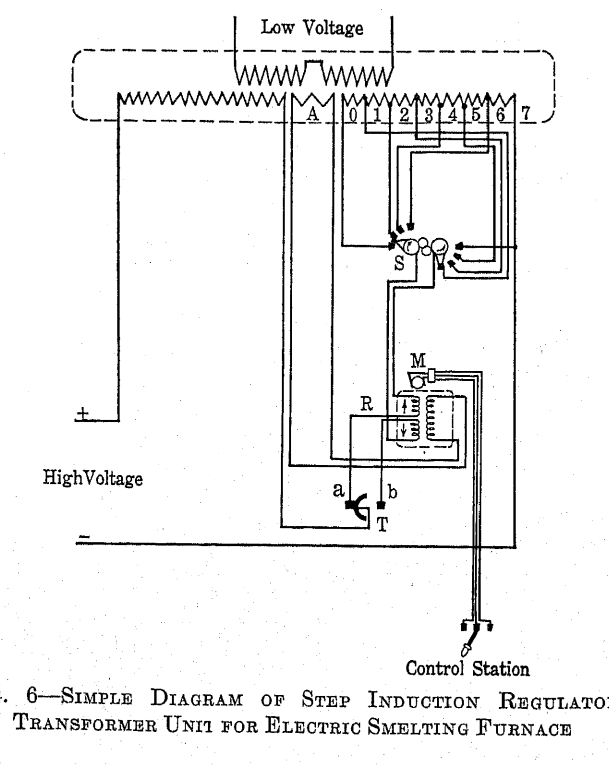

FIG. 6—SIMPLE DIAGRAM OF STEP INDUCTION REGULATOR TRANSFORMER UNIT FOR ELECTRIC SMELTING FURNACE

This is used when the primary voltage supply to the furnace unit is comparatively low so that it is unnecessary for the sake of economy to insulate either the induction regulator or the selector switches. The "buck" or "boost" of the induction regulator is equal to one-half the voltage existing between adjacent transformer taps. When it is desired to vary the furnace voltage, it is merely necessary to start the operating motor M of the regulator and if the regulator has been in neutral position, immediately a voltage is induced in the secondary of the regulator. In raising furnace voltage, this induced voltage would add to the supply voltage. With the regulator continuing to revolve, the induced voltage reaches its maximum which is one-half the voltage between adjacent transformer taps. At this point, the selector switch S being geared to the regulator, operates so as to connect the next transformer tap to the supply. At the same point, the transfer switch T also geared to the regulator, operates so that the induced voltage in the regulator subtracts from, rather than adds to, the supply voltage. This last operation is secured by having the secondary of the regulator made up of two windings, so that the reversal from boost to buck can be made through the transfer switch by reversing one of these windings. This feature allows the induction regulator to be rotated continuously in one direction for either raising or lowering the furnace voltage. It is obvious that this equipment may be controlled entirely automatically by a constant current regulator or a constant energy regulator.

Fig. 8 is a cross sectional view of the smelting furnace showing particularly details of the gas recirculating system and giving a good idea of the two parts of the furnace, the shaft in which the reduction of the ore takes place and the crucible in which the pig iron is formed at high temperature. Fig. 9 shows the layout of electrical connections between the regulator transformer and

TABLE I—*Continued*

Hg	Ag	Au	—Potential Series of Metals									
Acidity %	Current Density a.p.sq.ft.	Practical Ampere Eff.	Practical Deposition Voltage	Practical Energy Eff.	Actual Lbs. dep. per Kw-hr.	Lbs. Annual Output	Yearly Kw-hr. Reqd. 8770 Hrs.	Kw. Electrolytic Capacity Reqd. (Excl. Stg. Sheets & Plating Out)	Installed Converting Equipment	% Elec. Eff.	Tank Room Circuits	Raw Material
10	13	85	0.365		19.65	102,000,000 (216,000,000 proposed)	5,200,000	595	1 210 kw., 60 v., 3500 amp. gen. 1 396 kw. 110 v. 3600 amp. gen.		1 164 tank circ., 1 108 tank circ.	
	10	85	0.35		11.80	31,000,000	2,600,000	300	Steam engine driven gen. 125 v., 3600 to 4500 amps.		340 tanks.	
	100	90	6.0	14.5	0.345							Pyrrhotite ore, pyrite & pyrrhotite tailings.
	30	80	2.3	33.7	0.80							
NH_3 to keep Neutral or Alkaline	12 to 10	72.5	0.95		1.75	1,500,000						
	60 to 70	05	3.75		.58	1,500,000 (proposed 14,000,000)					2-30 tank circs. 1000 Amps. 100 to 125 volts each.	
					1.36							
	600	85	6.00	40	0.105				300,000 kw. syn. converters, syn. booster converters & w. w. driven d. c. generators.	92.5	About 42 cells on 250 v. & 84 on 500 v., 12,000 to 15,000 amps.	
					0.124							
		00	3.7	0.02 0.71 56.5	(or 3½ cu.ft.) .082						65 to 70 cells in series 5000 amp.	
				40	100							

smelting furnace electrodes. Provision is made by mounting the regulator transformer units on trucks traveling on tracks so that the electrical equipment can be shifted quickly from one smelting furnace to the other.

In Norway also, we have had the electric smelting of copper in large scale tests at an expenditure of 700 kw-hr. per long ton of charge containing 6 per cent or 132 lb. of metal and producing a 30 to 40 per cent copper matte. In other words, it has practically been merely a concentrating operation at an expenditure of 5.3 kw-hr. per pound of copper contained in the matte. It has been calculated that the concentrates from one of our western mines containing about 32 per cent Cu could be smelted by electricity, producing a matte of from 53 to 68 per cent Cu at an expenditure of approximately one kw-hr. per pound of Cu contained. This product would, of course, still require additional expensive treatment. By the electrolytic process, 700 kw-hr. will produce 700 lb., or one lb. per kw-hr. of high grade copper, requiring very little further treatment to be in merchantable form; and this is produced from a solution or electrolyte containing only a few per cent of Cu obtained by the action of dilute H_2SO_4 upon raw low grade oxidized ore containing only around one and one-half per cent Cu. The electrolyte may, of course, also be obtained by the action of H_2SO_4 upon concentrates of sulphid ores which have been oxidized by roasting.

Turning to the electrolytic processes, voltage, rather than temperature (at least usually) induces dissociation and, owing to the low temperature at which these processes (usually) may be conducted, efficiency, as has been stated, is not a function of the quantity of charge that is, the quantity of material in course of treatment, but is a function of the amount of metal deposited. In view of the growth and future prospects of electrolytic processes in the production of metals commonly used in construction and manufacture, electrical engineers must take more than a detached or theoretical interest in the subject. Table I shows considerable data with relation to the status of electrolysis as applied to the production of metals at the present time, particularly in the Western Hemisphere. Where not obtained from published sources, these data are only approximate, but a study of it will

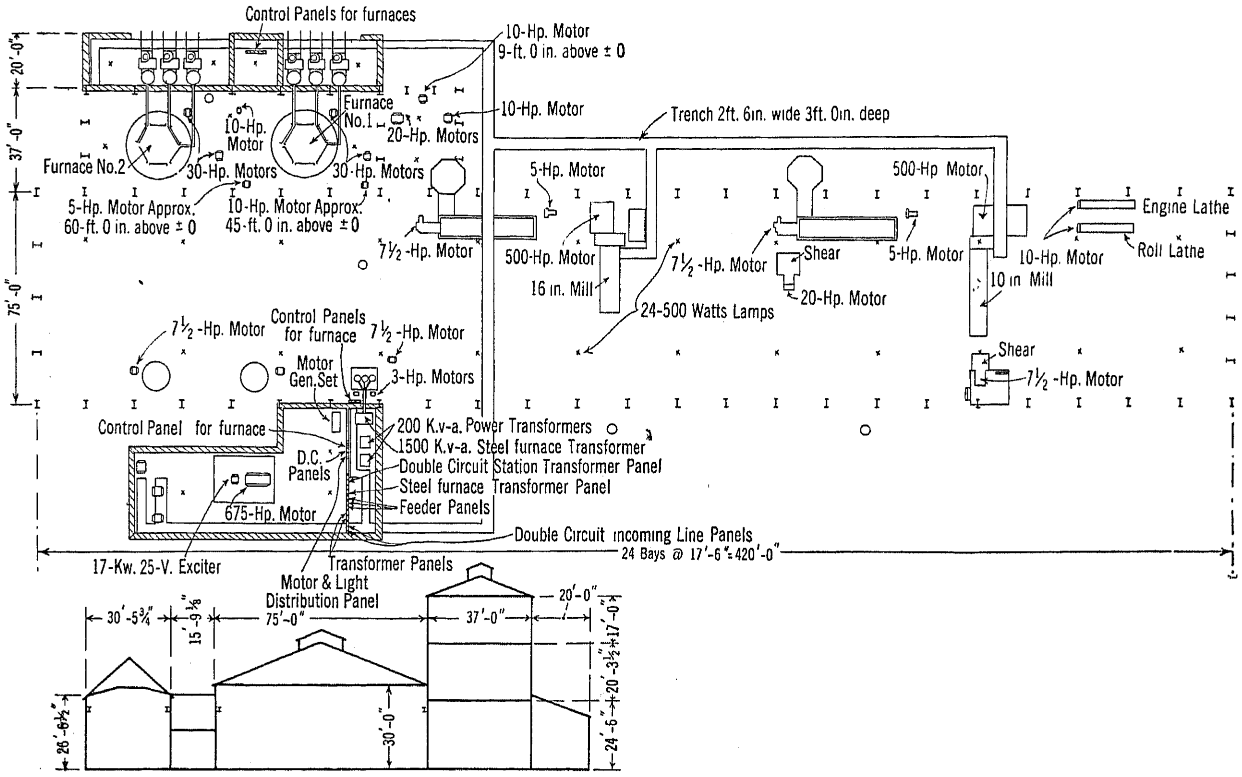

FIG. 7—ELECTRIC STEEL SMELTER AND ROLLING MILL—COMPANA ELECTRO METALLURGICA BRAZILERIA—30 TONS CAPACITY PER 24 HOURS. THE EQUIPMENT CONSISTS OF:

2—Shaft type furnaces (1 as standby), voltage range 60 to 120	3000 kv-a. each
2—6-Ton Bessemer converters (blowing engine for 1)	700 kv-a.
1—6-Ton Ludlum electric steel refining furnace	1500 kv-a.
1—16-in. rolling mill	500 kv-a.
1—10-in. rolling mill	500 kv-a.
Miscellaneous 220-volt power for motor, cranes, shops and lighting	400 kv-a.
Total installation	6600 kv-a.

indicate the correctness of many of the statements made.

The chart, Table I, shows how the problem in the metallurgical plant passes from the physical and chemical to the electrochemical and electrometallurgical and on to the purely electrical. Perhaps of all the columns, the electrical engineer is most interested in one near the center showing the overall efficiency of the electrochemical process and one to the right showing the efficiency of the purely electrical part of the installation. It is assumed that power is delivered at the electrochemical substation from the company's own power plant, or purchased from an external power plant in the form of alternating current; and the electrical efficiency given represents the efficiency of the electrical circuit from that point to the busses on top of the cells upon which the anodes and cathodes rest. The particularly interesting point is that in the relatively modern and efficient installation from 7.5 to 15 per cent of the power received at the electrolytic substation is lost before the current reaches the cells and that the cells themselves in a commercial economic plant are, in general, no more than 50 per cent efficient,

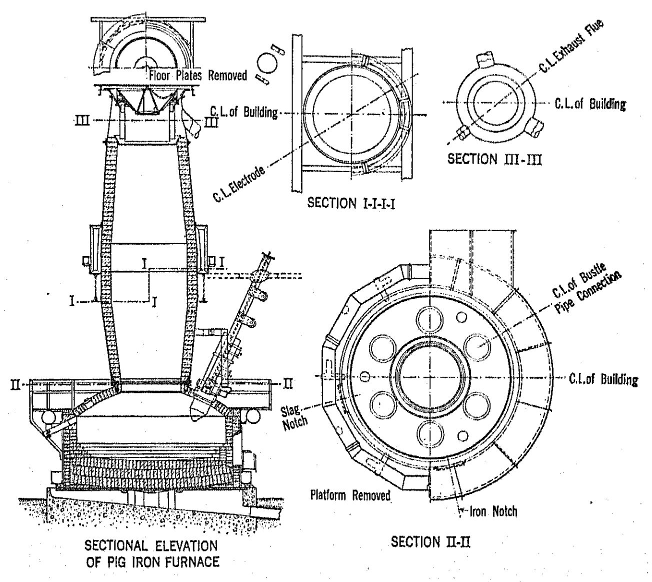

FIG. 8—CROSS SECTION OF 3000-KV-A. ELECTRIC SMELTING FURNACE FOR PIG IRON—COMPANA ELECTRO METALLURGICA BRAZILERIA

that is, they use upward of 100 per cent more power per pound of metal produced than would be calculated

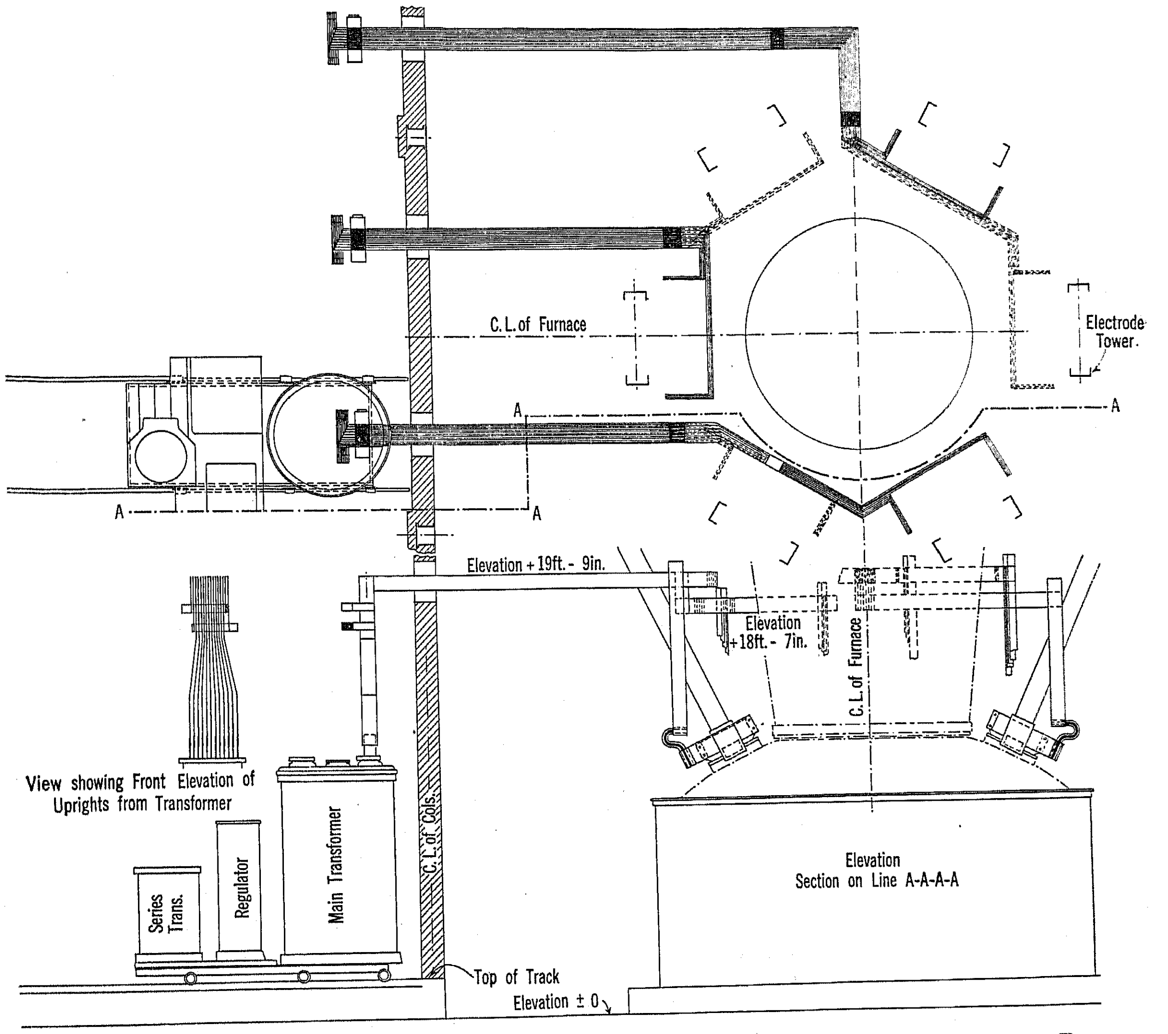

Fig. 9—Electrical Layout, 3000-Kv-a. Electric Smelting Furnace Showing Connections Between Transformers and Electrode Holders—Compana Electro Metallurgica Brazileria

by the theoretical Faraday and Gibbs-Hemholtz laws. This is well known to those chemists and metallurgists who are closely associated with this field of engineering, but it is being mentioned because it is not so familiar to electrical engineers. It may be said that the present day solution of the electrochemical problem gives an efficiency in the neighborhood of only 50 per cent of the theoretically possible, or rather impossible.

As the electrical problem was rather fully covered in my paper before the American Electrochemical Society, "The Substation Problem of the Electrochemical Plant," Volume 32 of *Transactions*, and in my discussion before the American Institute of Mining and Metallurgical Engineers, September, 1918, on "Electrolytic Zinc," further consideration at this time will be confined especially to features directly affecting efficiency.

Certain conclusions seem to have been borne out by accumulated experience. In general, it is not necessary or advisable to have great voltage range in the electrical supply apparatus. Great voltage range introduces costs and inefficiencies in the electrical equipment

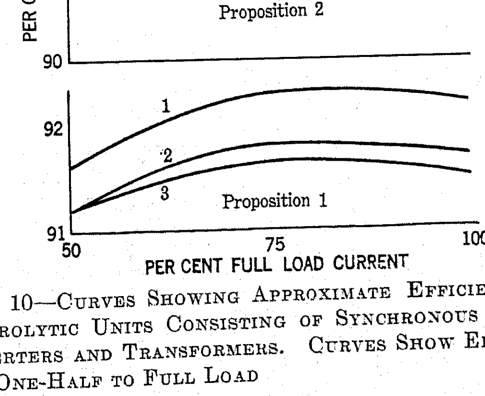

Fig. 10—Curves Showing Approximate Efficiencies of Electrolytic Units Consisting of Synchronous Booster Converters and Transformers. Curves Show Efficiency from One-Half to Full Load

TABLE II

ELECTROLYTIC SUBSTATION EQUIPMENT FOR ELECTROLYTIC COPPER REDUCTION WORKS

5000 Tons of Ore per Day Treated Producing 100,000 lb. of Copper with 1 kw-hr. per Lb.

25 Cycles Plan No.	Conv. No.	Kw. Each	Voltage Range	Ampere Each	Total Kw. Norm. Volt.	Transformers No.	Transformers Phase	Transformers Kv-a. Each	Transformers Kv-a. Total	Spare Conv. Capcy.	Spare Transf. Capcy.	Approx. Eff. Per Cent
1	3	2280	265-335	7,500	6840	4	1	1670	6680	2280	1670	92.2
1-A	3	2280	265-335	7,500	6840	3	3	2510	7530	2280	2510	92.4
2	4	1400	240-310	5,000	5600	4	1	1540	6150	1400	1540	91.9
2-A	4	1400	240-310	5,000	5600	4	3	1540	6150	1400	1540	92.3
3	8	650	110-150	5,000	5200	8	3	715	7520	650	715	88.3
4	6	950	94-126	8,600	5700	6	3	1050	6300	950	1050	88.3
5	2	4050	250-290	15,000	8100	4	1	1490	5950	4050	1490	91.7
60 Cycles	M-G. sets									M-G. set		
2-B	4	1500	200-300	6,000	6000	..	..	..	..	1500	..	88.5-13,200 Volts
6	4	1700	125-190	10,000	6800	..	..	..	..	1700	..	87.5- 2,300 Volts
Booster	Conv.									Conv.		
7	5	1076	122-165	7,500	5380	5	3	1200	6000	1076	1200	89.5

Continuous Capacity Kw. Max. Boost.	Cost-Convs. and Transf., or M-G. Sets, plus Switching Equipment	Daily Tonnage can Treat	$ Per Ton *	$ Per Kw.†
5000	$106,930 Approx.	5480	$19.50	$21.40
5000	104,700 "	5480	19.10	20.90
4650	98,510 "	5100	19.30	21.20
4650	99,800 "	5100	19.60	21.50
5250	189,400 "	5760	32.90	36.10
5450	206,950 "	5960	34.70	38.00
4350	128,100 "	4760	27.00	29.40
5400	135,000 "	5920	22.80	25.00
5700	150,000 "	6250	24.00	26.30
4950	130,000 "	5430	24.00	26.30

*Based on 5000 tons per 24 hours requiring approx. 4560 kw. d-c. capacity.

†Based on normal capacity at max. boost. voltage including cost for spare equipment.

The cost figures above are relative merely in an approximate way and should not be interpreted as indicative of present day prices.

which offset advantages gained in the cell room. The most economic all around installation is one in which the cell or tank room layout is designed with sufficient flexibility to demand comparatively small flexibility or voltage range in the electrical apparatus. Except that for all service of 125 volts d-c. and lower, the motor-generator set is the better application; Table I shows that the electrical efficiency is much greater for the higher voltage tank rooms (greater for 500 volts than 250 or 125 volts and greater for 250 than 125 volts) and when synchronous converters are employed rather than motor-generator sets. Synchronous booster converters with about a 12 per cent $\pm$ voltage range seem adequate, that is, say from 220 to 280 volts, or from 240 to 300 volts, or from 485 to 615 volts. In fact, in the aluminum and zinc industries which operate lines of cells at 500 volts and higher, the cells are so manipulated as to require very little voltage variation from the electrical equipment. A large part of the production of aluminum is accomplished by means of straight synchronous converters without boosters.

Fortunately, the synchronous booster converter is an economical machine to build and an efficient one to operate over the range which has been indicated. Sometime ago I made an analysis to determine the best electrical plant for an electrolytic copper reduction works, the data of which are typical and illustrative as well as anything could be, that the voltage should be as high as practicable and that unless external conditions, due to transmitted power, are too unfavorable, synchronous booster converters or straight synchronous converters should be used in preference to motor-generator sets. This analysis was made because it was known from data such as given in Table I, that some modern commercial plants have 100 per cent higher efficiency in the purely electrical part of the plant than do others. The curves in Figs. 10 and 11 show the efficiencies of a number of electrolytic units, that is, actual combinations of equipment, transformer and synchronous converters, which might be employed. While the service is a constant-current service at practically full load, it is noteworthy that high efficiency is obtainable over a considerable range of load and over the entire range of voltage. Indications were that for an electrolytic copper reduction works treating 5000 tons of ore, or more, per day, that the necessary electrical substation equipment would cost in the neighborhood of $22 per kw. of continuous supply to the tank room and $20 per ton of ore treated and that the cost of the electrical substation, building and equipment, would be from 15 per cent to 20 per cent

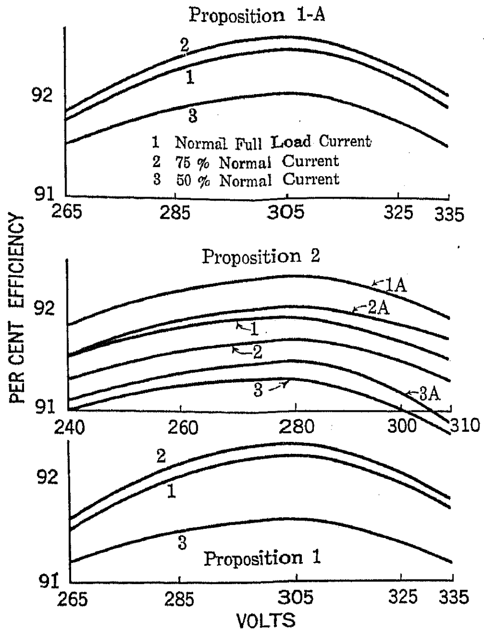

FIG. 11—CURVES SHOWING APPROXIMATE EFFICIENCIES OF ELECTROLYTIC UNITS CONSISTING OF SYNCHRONOUS BOOSTER CONVERTERS AND TRANSFORMERS. THESE CURVES SHOW EFFICIENCIES FROM MINIMUM VOLTAGE TO MAXIMUM VOLTAGE OF RANGE

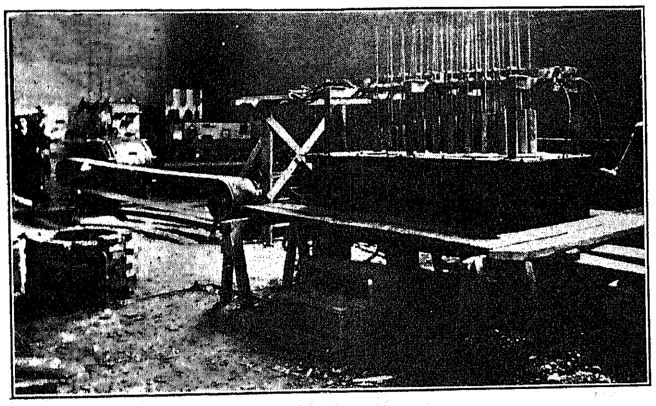

FIG. 12—EXPERIMENTAL PLANT FOR OBTAINING ELECTROMETALLURGICAL DATA ON ALUMINUM, SHOWING ELECTROLYTIC ALUMINUM CELL AND LOW-VOLTAGE D-C. GENERATOR, BELT DRIVEN BY INDUCTION MOTOR AND MEASURING INSTRUMENTS

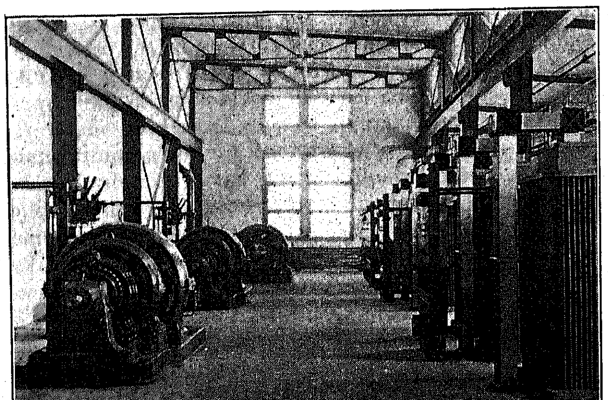

FIG. 13—SUBSTATION FOR ELECTROLYTIC NICKEL AND COPPER PRODUCTION

There are five synchronous booster converters, 4000-ampere capacity each, maximum volts 290. The booster gives a range of 30 volts ± and by a number of taps and switches on the secondary of the transformer any operating voltage down to 100 or less is obtainable. Wiring diagram which is typical of a station of this type is shown by Fig. 14.

of the cost of the tank house. (The ratio of cost of electrical substation to tank house for electrolytic zinc would be higher than this owing to the larger kw-hr. per pound taken by zinc and the greater current densities employed; refer to Table I.) A tabulation

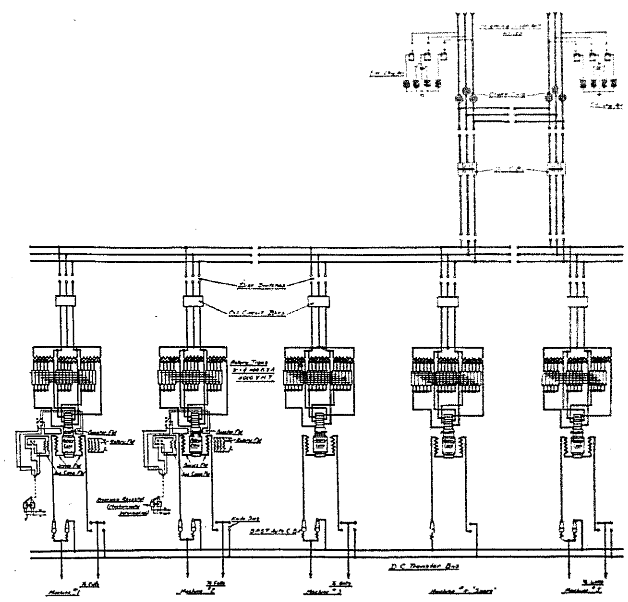

FIG. 14—ELECTRICAL CIRCUIT DIAGRAM FOR ELECTROLYTIC SUBSTATIONS FOR NICKEL AND COPPER HAVING 4 ACTIVE UNITS AND 1 SPARE

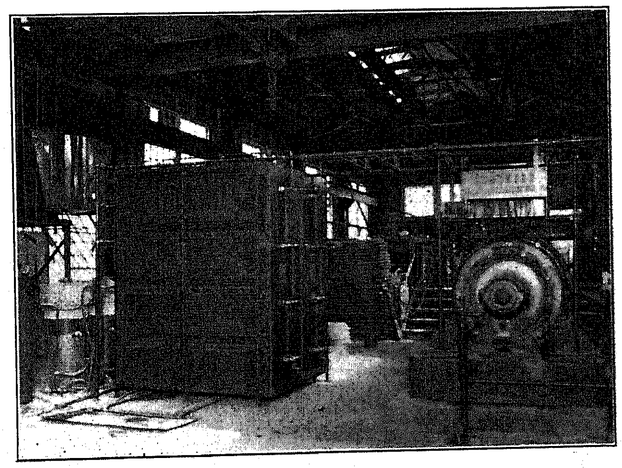

FIG. 15—ELECTROLYTIC IRON REFINERY, CAPACITY 15 TONS PER MONTH

To the rear, at the center, may be seen the electrolytic tanks and in front of them a pile of Armco iron anodes ready for use. To the right at the front may be seen a 100-kv-a. 5000-cycle alternator, which, together with the banks of condensers, shown to the left, form a power supply for high-frequency inductive melting furnaces, shown to the left of the condensers. At the extreme left is a hydrogen tank for the purpose of insuring a non-oxidizing atmosphere in the furnaces. The electroltyic iron is here melted together with other materials in the manufacture of high grade iron alloys. Power for the electrolytic cells is obtained from a 2000-ampere 20-volt motor-generator set which is not shown in the view.

of the equipment considered in the analysis above referred to, is shown in Table II. This shows first cost very much in favor of higher voltage and, considering the matter of efficiency of conversion once more, let us assume that the electrolytic copper plant

has an output of 60,000,000 lb. of copper per year at a cost of 10 cents per pound. Then, the difference in conversion of between 86 per cent and 91 per cent, that is, 5 per cent will mean something in the neighborhood of 0.8 per cent of the total production cost of the copper with power at one and one-half cents per kw-hr., or more than $45,000 per year, which is about one third of the cost of the electrical substation equipment.

It would be impossible to cover the relationship of the electrical engineer to the commercial development of economic electrometallurgical processes within the reasonable scope of an American Institute of Electrical Engineers' paper. It was merely desired to indicate his correlationship with the chemist and metallurgist in working out these problems. By a survey of the field, without being critical, it was hoped to be stimulating. Discussion has purposely been confined to the more common metals. Consideration of the electrolytic treatment of precious metals has been purposely omitted since the quantity of power involved is small. No reference has also been made to the electric smelting of metals such as chromium, tungsten, molybdenum, etc., which are used in the production of alloyed steel. Fig. 12 is a view of an experimental plant for the purpose of determining electrometallurgical data in the production of aluminum, which, perhaps, it should be mentioned can only be produced electrolytically at high temperature. Fig. 13 is a substation of synchronous booster converter transformer units employed in the electrolytic production of nickel and copper. Fig. 14 is a typical diagram of electrical circuits for such an installation. Fig. 15 is a small commercial electrolytic iron refinery.

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Discussion

C. J. Russell: Mr. Yardley's paper is timely and the present speaker desires to call attention to the importance of the subject of electrolytic and electrometallurgical processes to electrical engineers—particularly to those who are connected with public utilities. Entirely aside from the fields in which such processes are found, we are just on the threshold of tremendous developments along the lines of industrial heating by electricity. The question of the relation of the electrical engineer to all these applications is important because he must pass upon many points which may be decisive as to the commercial success of any undertaking. There have been many failures due to the applications of electricity for the accomplishment of some process which would never have been undertaken, if proper advice had been tendered or accepted. The application of electricity to meritorious processes has been seriously retarded by these failures. As against that, we find in France, Norway and Sweden a very large number of highly successful processes along the lines of electric-furnace operations which cover a field from electrometallurgical work to the manufacture of calcium cyanamid and nitrogen fixation.

This paper should result, particularly on the Coast, in a very active interest, on the part of electrical engineers, in the consideration of electrochemical and electrometallurgical applications along the lines of electrolytic and electric furnace processes.

Carl Hering (by letter): In referring to the very important matter of the energy which is set free or absorbed in nearly every chemical reaction, the author still repeatedly uses the

old term "heat." These energies have for many years been called "heats of combination" because in the older, simple, purely chemical reactions this energy always appears as heat or cold. But in many electrochemical processes this energy does not appear as heat, and it is confusing to the student to call it so. It appears as electrical energy, being set free as in batteries, or absorbed, as in electrolytic reduction processes. It is therefore not up-to-date in this electrical era still to refer to this energy as heat. It should be called the *energy* of combination, even though in books, the data are still always given in calories,—one of the units in which energy may be measured.

In stating the elementary principles near the beginning, the author should have added to his statement (1) the words "and pressure." Pressures are quite as important as temperatures in some chemical reactions, hence the useful autoclave. In his next paragraph, (2) it would have been more clarifying to have added that the action takes place only at the electrodes. None but secondary reactions can occur in any other parts of the electrolyte, and the energy of these secondary reactions may be entirely foreign to the electrical energy. Many years ago I suggested that the answer to the much-discussed question, as to whether a certain reaction is primary or secondary, might well be based on this distinction.

In the next paragraph of this paper, it might have been made clearer that Faraday's law does not refer to any energy. It is entirely independent of whether the energy involved is large or small. The voltage, with which Faraday's law is not in the least concerned, must be included to determine the electrical energy. Faraday's law deals only with ampere-hours or coulombs (ampere-seconds) and not with watt-hours. Like many others, the author omitted to say that this law applies directly only when there is but one atom to the molecule, and when the change of valence is unity. Serious mistakes may be made in some calculations by ignoring these limitations.

The latest and best value of the Faraday is 96,494, and not 96,540. A more convenient form is 26.804 ampere-hours, which divided by the atomic weight gives the quantity of electricity in ampere-hours per gram, or 12,158, which divided by the atomic weight gives the ampere-hours per pound, both for every unit change of valence. Faraday's[1] law, stated as usual in terms of valence (or worse yet, with that term omitted), fails to apply in some cases and is therefore not a universal law; using the term "change of valence" makes it a universal law.

In many of the kinds of electrochemical processes referred to by the author, the so-called "thermo-chemical" constants, or "heats of combination" (better called energies of combination) given in tables, are extremely important, as they may show that a proposed process will cost more in energy than the result is worth. I have repeatedly called attention to the fact that these constants as given in the tables, are in many cases the sum of two constants which ought to have been stated separately; they represent two radically different things, namely, the real chemical energy and the physical energy (temperature, pressure and change of state).

The reason for separating them is that the physical part, which may be quite large, might perhaps be reduced (by pressures and temperatures for instance) which the essentially chemical part, of course, cannot. The physical part of this energy may involve relatively large latent heats. Thus, when solid carbon is burned in gaseous oxygen, the end-product being a gas (CO or CO_2), the latent heats of liquefaction and vaporization of carbon have been absorbed or deducted from the chemical energy of the combination and only their difference appears. In the thermite process, the oxygen must be regarded as being solid to start with, and the end-product is solid, hence the high temperatures, there being no latent heat deductions. Or when solid magnesia (flash powder) is burned in air forming the solid oxide, the latent heats of liquefaction and vaporization of oxygen has been added to the chemical energy, hence the high temperature and bright light produced. If zinc can be reduced from its ore without vaporization (for instance by increased pressure or by electrolysis) its latent heat of vaporization, ordinarily lost, can be saved.

1. Electrochemical Equivalents. Hering and Getman, which contains further explanations and other useful figures.

Electricity in Mines

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Review of the Subject.—*In the following paper, the author has given a fairly complete description of the various uses to which electric power is put in and around the mines, together with some interesting figures in regard to the amount of electric power now used and that which is still to be applied.*

* * * * *

A COMPLETE treatise on the subject of this paper namely, "Electricity in Mines" would be exceedingly voluminous. Neither time nor space will permit me to enter into a detailed discussion of all the applications of electric power in the mining industry.

In describing various applications of electricity to mining apparatus, I will endeavor to be as brief as possible, and further, keep away from any great amount of technical discussion. My desire is rather to show you the various ways in which electricity is used around the mines and to give you some impression of the extensive field of application offered by the mining industry.

Electricity as a medium for the transformation of power lends itself so well to practically all power applications in and around the mines that, notwithstanding the gratifying advances that have been made, it is a little surprising to one who has studied the subject, that this form of energy is not more universally used.

In the case of a small operator in a territory where purchased power is not available, it is always a question whether or not it is best to install an electric generating station. Local conditions incident to such an installation must be taken into account and the problem analyzed carefully before a decision can be made.

Fortunately, however, power lines from large power companies have penetrated the majority of the important mining fields. Equitable rates have been established whereby the prospect or operator can afford to purchase power. Large distributers of power are, I believe, well satisfied that mining load is very desirable and are soliciting it vigorously. This is quite in contrast with the policy of a few years ago. At that time very few power companies were anxious to have mining loads on their lines. This is easily explained by the fact that to take on such loads it would have been necessary to increase the size of their transmission lines and generating stations, all of which involve large capital expenditures. This inability to serve caused many mine operators to install generating stations of their own. Many of these stations have been abandoned and the operators changed over to purchased power as soon as the Public Utilities were ready to furnish it at a reasonable rate.

Serious power interruptions have always been the constant dread of a mine operator, particularly if his mine be gaseous or collects water rapidly. Some mines give off explosive gases so rapidly that an hour's shutdown of the ventilation system changes the mine from a comparatively safe thing to an exceedingly dangerous thing, waiting only for some half-wit to light a cigarette to send himself and his fellow workers into the great beyond and incidentally totally wrecking the property.

Power interruptions today are fortunately rather rare and of short duration. It is only in exceptional cases that a standby source of power is required. When such an expediency is resorted to it usually takes the form of a gas engine and generator just large enough to run the vital parts of the machinery such as the hoist, the fan, (both at reduced speeds) and some lights. Needless to say, continuity of service is the prime factor in the supply of power to mines.

In order to give you some idea of the magnitude of the mining industry the following figures should be of interest.

There are employed in and about the mines of the United States in round numbers one million men. They receive an annual wage of approximately one billion five hundred million dollars. Of these one million miners, thirty-five per cent work above ground and sixty-five per cent under ground. The mines require a total of approximately 6,800,000 h. p. in motive power. Of this only 1,603,000 h. p. or twenty-three per cent is connected to Public Service lines and 1,206,000 h. p. in motors or 17.7 per cent is run from privately owned electric plants.

Even with this small percentage of purchased power, the amount paid annually to power producing companies is about thirty million of dollars.

So there is still a very large field for the sale of power to the mining industry.

Now as to the nature of the load, it is not steady and the load factor is quite variable, seldom exceeding fifty per cent. By load factor I mean the ratio of the maximum 15-min. integrated peak to the average load over 24 hr.

In Fig. 1 is shown a typical 24-hr. chart from a group of anthracite mines. This chart is fairly typical of a large majority of coal properties. It has a load factor over 24-hr. of approximately 42 per cent calculated on the basis above described.

The individual drives will now be taken up and

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discussed in as much detail as space and time will permit.

VENTILATION

The usual method of supplying air for the support of life and the rendering of the mine safe from dangerous gases is to install a fan at the top of an air shaft. The fan may be either of the exhaust type drawing air up the shaft or it may be of the blower type forcing air down the shaft, the former as an "up-cast shaft" and the latter as a "down-cast shaft." Frequently the doors of the fan house are so arranged that the fan can, by adjusting these doors, be operated either as an exhauster or a blower. Such equipments are known as reversible fans. Much discussion has taken place among mining engineers as to whether or not reversible fans should be permitted, the point being that in case of fire the air currents might and, in fact, have been reversed without proper knowledge of the inside conditions, thereby causing greater damage and loss of life

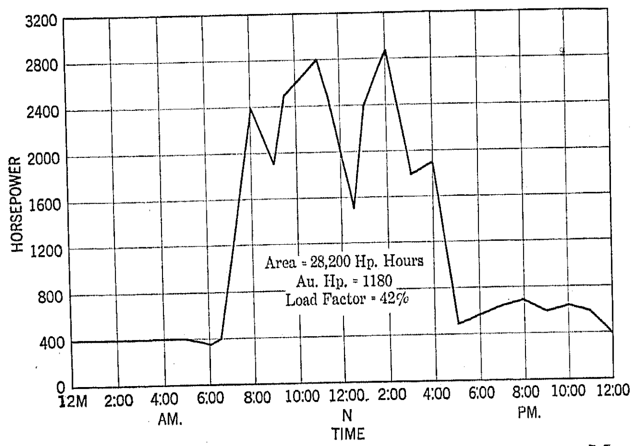

FIG. 1—24-HOUR LOAD CHART OF GROUP OF ANTHRACITE MINES

than would probably otherwise have resulted. This, however, is slightly aside from the electrical engineer's problem.

The load characteristics of the fan are similar to those of most centrifugal apparatus, the power required by a given mine fan being approximately proportional to the cube of the speed. The normal speed of large fans varies from 75 rev. per min. to 350 rev. per min. or even higher, depending on the size and design of the fan.

The starting load of large fans differs from most other loads in that the load on the driver increases very rapidly with the speed. Fans cannot to advantage be by-passed. It is impractical to close the intake or outlet because of the size and consequent pressure that would result. Furthermore, could either or both of these openings be closed, it would only reduce the load at full speed to about 75 per cent of full load. Many mines require variable speed fans as some operators feel that at times it might be necessary to increase the speed above normal and at other times advisable to drop it to some subnormal speed. I think, however, there are many more mines where fans are run at constant speed delivering sufficient air at all times to keep the mine free from dangerous gases than there are mines which require variable speed fans. Thus, it may be seen that the fan drive presents some interesting problems to the electrical engineer.

In the case of the constant speed fan, the problem is largely one of starting. The drive may be induction motor of the squirrel-cage type or slip-ring type geared, belted, or with chain drive. Due to the starting conditions, it would seem that the slip-ring motor has quite a decided advantage over the squirrel-cage motor. If the squirrel-cage motor be used, very high percentage taps must be resorted to in the starting compensator, otherwise the motor will be thrown across the line at a considerable percentage below full speed, thus putting an excessive stress on the apparatus and the system. With the slip-ring motor, the torque can be controlled nicely by the controller and increased gradually as required from zero to full-load speed.

Synchronous motors have been suggested for fan drive but the pull-in torque must be 100 per cent of full-load torque if the motor is just large enough to drive the fan. To meet this would be to impose design conditions, which, if met, would be very detrimental to other desirable characteristics in the motor. Synchronous motors have been installed, however, using a motor considerably larger than is actually necessary to drive the fan. This solves the pull-in problem, but on the other hand it gives the operator a motor of which only a part of the installed capacity is in use 99 per cent of the time. Synchronous motors have been successfully installed on fan drives using a clutch between the rotor and the load, bringing the motor to speed with the clutch disengaged and after the motor is in step bringing the load up on the clutch. Some installations as large as 750 h. p. have been so made.

To overcome the starting troubles on fans and other similar devices such as mills and crushers where there would be a decided advantage accruing from the use of a synchronous motor, one large manufacturing company has developed what it chooses to call a super-synchronous motor. This motor has, as far as the load is concerned, a starting torque equal to the break-down torque of the motor. This is accomplished by mounting both the stator and the rotor on bearings and placing a large brake around the outside of the stator. To start a fan with such a motor the brake is released and power is applied to the stator. This begins to revolve and comes to synchronous speed, no load. Field is then put on the rotor and the brake applied to the stator. When the braking effort is sufficient the stator begins to slow down, but simultaneously the rotor and load begin to accelerate, increasing in speed as the stator falls in speed. The sum of the two speeds is always equal to the synchronous speed of the motor. Finally, the stator is brought to rest and the rotor and load are at synchronous speed. I do not know of any

such installation having been made on a fan but some 50 of these machines have been installed on Ball and Tube mills in cement and metal mills with very excellent

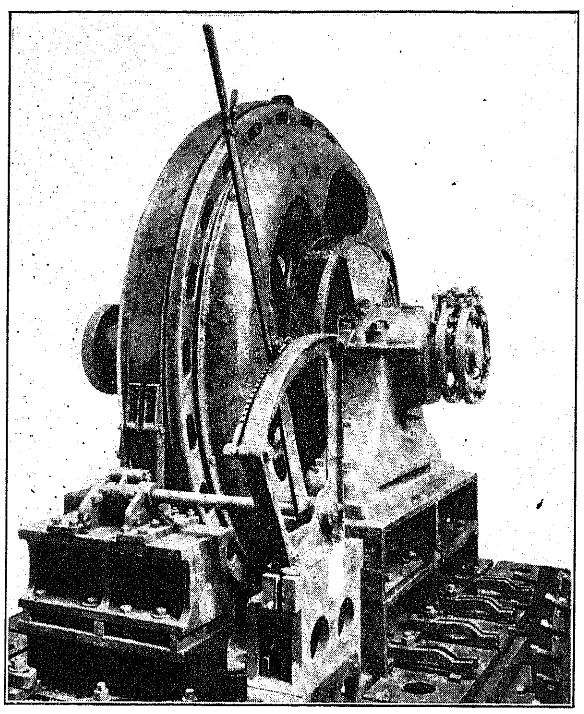

FIG. 2—SYNCHRONOUS MOTOR WITH REVOLVING STATOR SHOWING COLLECTOR RINGS AND BRAKE MECHANISM

450-H. P. 0.8-Power Factor, 187.5-Rev. per Min. 3-Phase 60-Cycle 2200-Volt

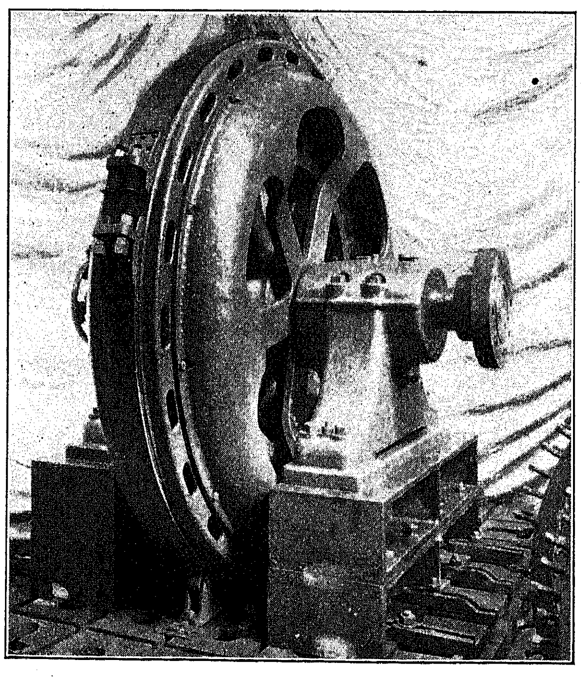

FIG. 3—SYNCHRONOUS MOTOR WITH REVOLVING STATOR SHOWING COUPLING END

450-H. P. 0.8-Power Factor 187.5-Rev. per Min. 3-Phase 60-Cycle 2200-Volt

results. Two views of one of these motors are shown in Figs. 2 and 3.

The synchronous motor also lends itself much better for direct connection to the fan than the induction motor on account of the poor power factor of the latter at slow speed that prevails with fan drives.

In view of the fact that high power factors are rewarded by many power companies, it would not be surprising to see many more synchronous motor drives for fans installed as the work of electrification proceeds.

Variable speed fans present an added problen. The starting conditions are similar to those of the constant speed fan but there is the additional problem of the variable speed control.

Where two or possibly three fixed speeds are sufficient and these speeds are some common fraction of full speed, such as one-half and three-quarters, a pole changing squirrel-cage induction motor has some field.

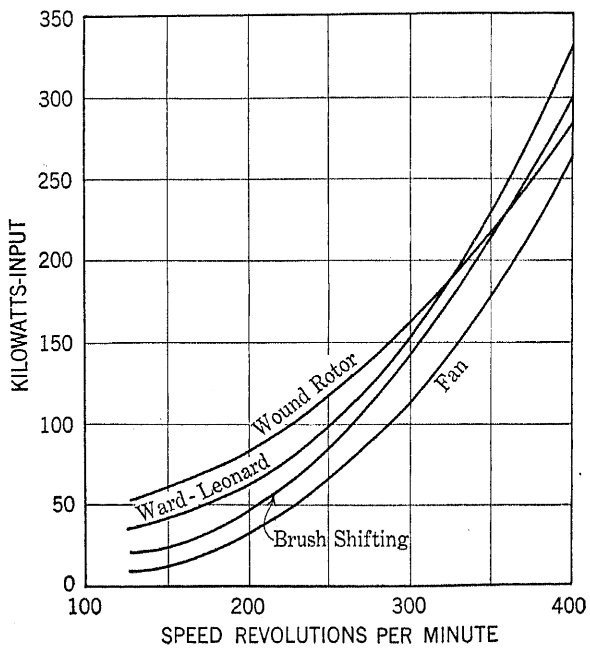

FIG. 4—SPEED KILOWATT INPUT CURVES OF MOTOR-DRIVEN FAN USING WOUND ROTOR INDUCTION MOTOR, BRUSH SHIFTING ALTERNATING-CURRENT MOTOR AND DIRECT-CURRENT MOTOR WITH WARD LEONARD CONTROL

On large drives, however, where variable speed is required intermediate speeds are usually necessary, which puts the application of the squirrel-cage motor out of the question.

For variable speed fan drives, we have left the slip-ring motor with controller and resistance, the brush-shifting a-c. motor, the Scherbius and Cramer systems of drive with various modifications and possibly to these might be added the Ward Leonard control system. (This latter has never as yet to the writer's knowledge been used for this work).

Fig. 4 shows the power required at the fan shaft for a given fan at various speeds from minimum to maximum. On the same sheet are shown the powers consumed to drive this fan at the various speeds using first the slip-ring induction motor, second the brush-shifting motor

and third the Ward Leonard system of control. Since the power factor of the slip-ring motor falls quite rapidly at light loads, curves are also shown in Fig. 5 giving the kilovolt-ampere input under the same conditions. Other sizes of fans and other speed variations may change the relative values of these curves. It is only necessary to capitalize the saving of one over the other to determine which is the cheapest form of drive.

Reliability is, in many installations (for reasons already referred to) the controlling factor in the choice. The slip-ring motor perhaps has a little the best of this argument, there being only one motor involved, and no commutators or other mechanism beyond the control and resistance. The other systems, however, are reliable and have been installed on a large number of fans.

The curves shown in Figs. 4 and 5 do not take into account the efficiency of the fan proper. This efficiency is just as important as the efficiency of the motor drive. For instance, the majority of fans will increase in

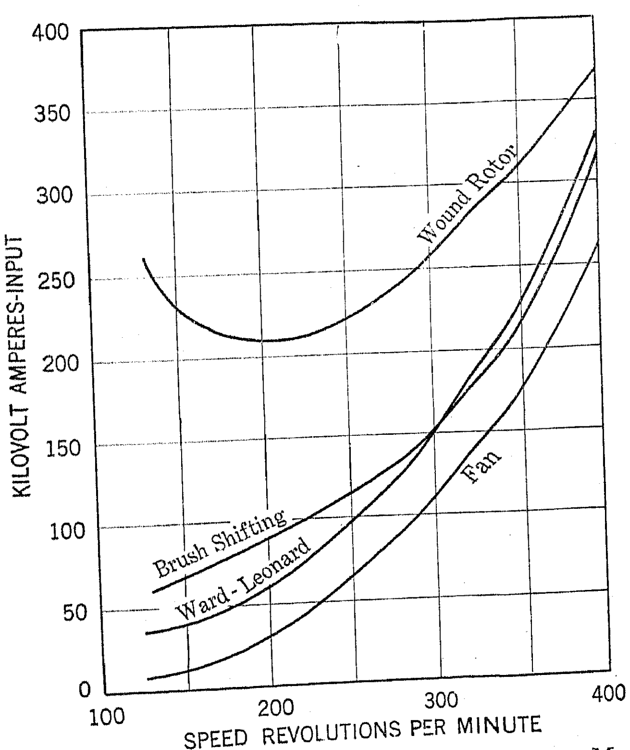

Fig. 5—Speed Kilovolt-ampere Curves of Motor-Driven Fan Using Wound Rotor Induction Motor, Brush Shifting Alternating-Current Motor and Direct-Current Motor with Ward Leonard Control

efficiency up to a certain speed and beyond this speed the efficiency falls off very rapidly. Thus, a most efficient motor drive might be selected, but on account of operating the fans at an inefficient point the overall efficiency of the installation might be low. The prospective purchaser is strongly urged to make a thorough analysis of his fan and drive before obligating himself beyond recall. A small percentage difference in efficiency is much more serious in the case of a fan than in almost any other motor application about the mine. The fan runs continuously and therefore the kilowatt-hours consumed pile up at an amazing rate so that a few per cent saved by improving the efficiency of this drive nets him a very large return over the year.

In this day of automatics, the fan drives have received their share of attention. They may now be built to start automatically upon the return of power after an interruption and to shut down and notify the officials in case of hot bearings or excessive temperatures in any part of the apparatus. They can be started and stopped by remote control.

Motor-driven fans, if properly designed and installed,

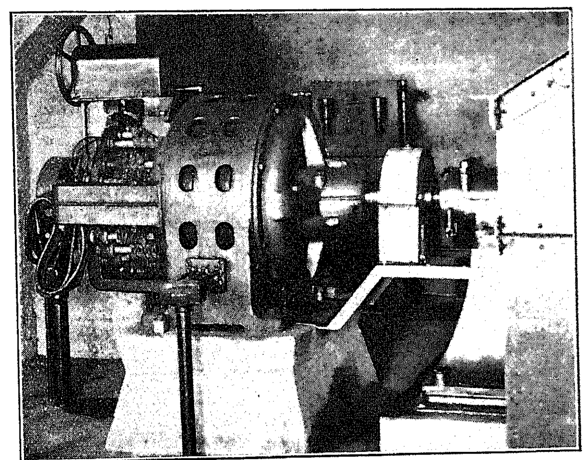

Fig. 6—Brush Shifting Variable Speed 150-H. P. Motor Direct-Connected to High Speed Fan

are thoroughly reliable and economical pieces of machinery.

The fan load is ideal from the power supply standpoint. It is usually steady for 24 hours a day and 7 days a week. From the operator's standpoint it helps materially to improve his load factor.

Fig. 6 shows a brush-shifting motor directly coupled to a high-speed fan. This does not happen to be a mine installation but is similar to the arrangement used for direct-connected fans.

Haulage

The installation of electric mine locomotives was the first achievement along the lines of economy in which electricity played the leading part. Long before power companies thought of supplying current to the mines, electric locomotives were busy underground hauling loaded cars from the working face and returning empties. Their power supply usually consisted of a small direct connected engine generator unit in sizes varying from one to three hundred kilowatts. The generators were generally very heavily over compounded to compensate for excessive line drop.

In this manner the early operators hoped to save expense in their copper line wire investments. Later experience, however, seems to indicate that it is better practise to maintain as high a voltage as practicable at

the substations and make a moderate investment in copper, thereby keeping the voltage up and the energy loss down.

The scheme of wiring and underground transmission varies considerably with local conditions. Sometimes cables are run down the main shaft, in other instances a borehole is put down as close as possible to the center of distribution and a feeder cable dropped through the borehole. Various parts of the mine wiring are usually sectionalized so that power may be cut off in any desired part of the mine. Since the early days the small engine generators have been shut down and modern substations installed.

The design of the locomotive has changed most materially as conditions surrounding its operation become more thoroughly understood by designing engineers.

Mine locomotives may be divided into two general classes namely; the main-line haulage locomotives and

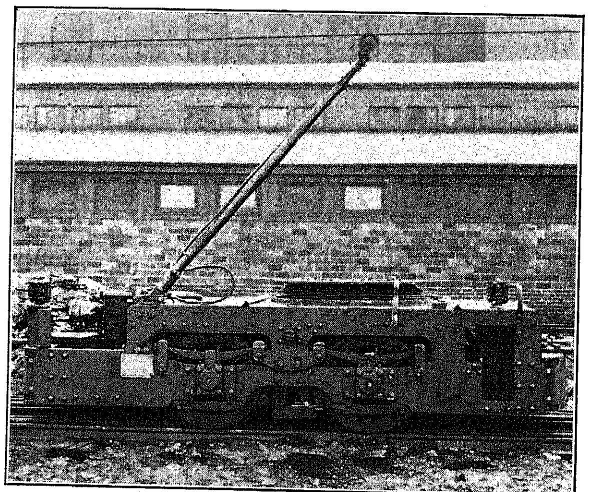

FIG. 7—FOUR TON GATHERING LOCOMOTIVE SHOWING EQUALIZING LEVERS AND CABLE REEL

the gathering locomotives, the former as their name indicates haul large strings of cars or "trips" from various central points to the foot of the shaft, while the gathering locomotives bring the loaded cars from the rooms to the central points where the trips are made up. The gathering locomotive differs from the main-line locomotive in that it is usually lighter, of shorter wheel base and carries with it a cable reel. The size of the gathering locomotives ordinarily used varies from three to eight tons. The cable reel is usually driven by a torque motor which when the reel is in action is left across the line permanently and continually exerts a torque against the reel. One end of the cable is electrically connected to the trolley or other source of power and when a locomotive enters the room the cable reel is unwound against the torque of the motor. When the locomotive comes out of the room, the torque developed by the motor is sufficient to reel up the cable as the locomotive returns. The action of this motor is very similar to that of an endless spring. Cable reels are built in two distinct forms, one a comparatively long drum of small diameter placed at one end of the locomotive and the other a large diameter drum with a very narrow face, placed on top of the locomotive with its axis vertical. The long-faced small-diameter reel referred to necessitates a mechanical guide to insure the proper coiling of the cable. The short-faced large-diameter reel requires no such device, but, on the other hand, usually adds a few inches to the overall height of the locomotive.

Cable reels of both types were originally mechanically driven from the locomotive axles but this was found decidedly unsatisfactory because in coming out of a room of any considerable pitch or grade the locomotive runner is very liable to lock the wheels which in the older style would stop the cable reel winding in and allow the locomotive to run over and destroy the cable. Gathering locomotives are usually designed to operate at a speed of from six to seven miles per hour at normal rated drawbar pull. Quite recently the question has come up as to whether it would not be better to drop this speed to approximately three or three and one-half miles per hour, it being found that very few gathering locomotives exceeded this speed and that the locomotive runners wasted a great deal of energy in the resistance, running the motors in parallel with a large quantity of resistance cut in series. A slow-speed gathering locomotive was designed and installed for one of the very large coal companies and it was found that a material saving resulted in kilowatt-hours per day with practically no reduction in tonnage. The question naturally suggests itself as to why (since the majority of locomotives have two motors and high speed is obtained by operating the motors in parallel) a series parallel controller could not be used, which would give practically half speed when operating the motors in series, and also give high speed with motors in parallel when desired. This scheme is entirely feasible and is open to but one criticism, namely, the locomotive runners will not operate their motors in series but prefer to run in parallel on resistance; this, notwithstanding all rules and instructions to the contrary.

The electrical equipment on the gathering locomotives generally consists of two motors totally enclosed and rated on a 75-deg. cent. rise full-load one hour. The design of the motor has been materially improved these past few years and today operates as far as commutation is concerned about as well as a standard industrial motor. It is equipped with interpoles and in the better designs with very liberally proportioned ball bearings. This latter feature alone has cut down the locomotive repair bill very materially. The old style bearings used to wear badly and since the inspection was not of the best, the locomotive would run until its armature struck the pole pieces which usually necessitated a complete rewinding. This complaint is practically unheard of with ball bearings.

Control

The majority of controllers is of the series parallel type, so wellknown that no description of them is necessary.

Within the past few years, however, one of the large manufacturing companies has built as a standard part of its line a controller of the series parallel type with an auxiliary cylinder, which, when moved into the proper position, gives the operator complete control of his trip by a dynamic brake action of the motors. This arrangement lightens the manual work of the mine locomotive runner and has proved very popular where installed.

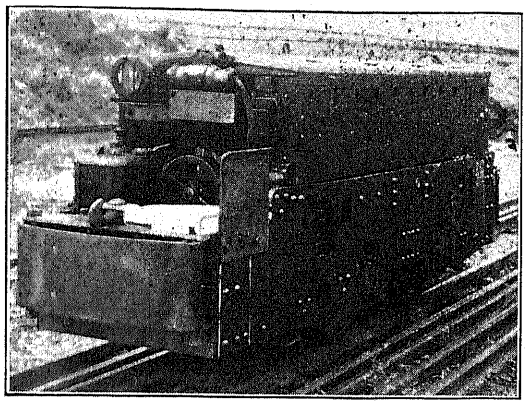

Fig. 8—Five-Ton Two-Motor Storage Battery Mine Locomotive Extensively Used in Metal Mines

Storage Battery Locomotives

The application of the storage battery locomotive is in the writer's opinion rather limited. At first thought, it would seem as though this type of locomotive would have a very wide application in mines. No trolley is necessary and the rails need not be bonded. The equipment could be made safe to enter gaseous places in the mine and altogether the outlook for this particular type seemed very bright. Several small manufacturers entered the field building locomotives equipped with storage batteries and turned their salesmen loose to secure business. A great many failures followed, due largely to improper application. Locomotives were put on long hauls that were designed only for short ones. The result was a very rapid battery deterioration and consequently very heavy renewal charges. From such information as I can gather the sales of storage battery mine locomotives have fallen off quite materially in the past year or year and a half and many instances are recorded where users have changed from the storage-battery type to the cable-reel type for gathering work. I do not mean to indicate by this that the storage battery locomotive has no field in the mine. In many cases it has a field but nothing like the general application as first anticipated. Properly designed and operated storage battery mine locomotives have a battery large enough to carry it through a day of normal operating conditions and after an all-night charge is ready for service the following morning. Batteries are usually mounted in a battery box which is easily removable from the locomotive and a new battery substituted when desired. The standard speeds of battery locomotives do not exceed four miles per hour at rated drawbar pull. The direct current for charging a storage battery is furnished usually from a motor generator set installed in the charging room and controller through a battery-charging panel.

Main-line locomotives are of the same general construction as the cable-reel locomotives. Their function as the name indicates is to operate along the main haulage ways carrying loaded cars to the foot of the shaft and empties to some central point. In sizes they run usually from 6 to 30 tons. The motors are rated on the same basis as the gathering locomotives and the control is of the series parallel type. One comparatively recent improvement has been developed for the haulage locomotives which is that in place of the ordinary series parallel controller, a master control and contactors have been substituted. The main-line currents are handled on contactors of the same general design as those used on street railway work. Locomotives so equipped have created a very favorable impression and this method of handling main-line currents on contactors will in all probability be adopted by all manufacturers of trolley-type locomotives.

Another important change of design in mine locomotives of all types is in the method of carrying the weight on the axles. Leaf springs with equalizing levers are arranged so that an equal weight is carried on each wheel irrespective of track conditions. This is obviously of great advantage when rough mine track conditions are considered. In the older types of locomotives when operating on uneven tracks, the weight on the wheels varied greatly, which in turn reacted upon the drawbar pull. The general arrangement of these levers and equalizing devices can be seen in the illustrations.

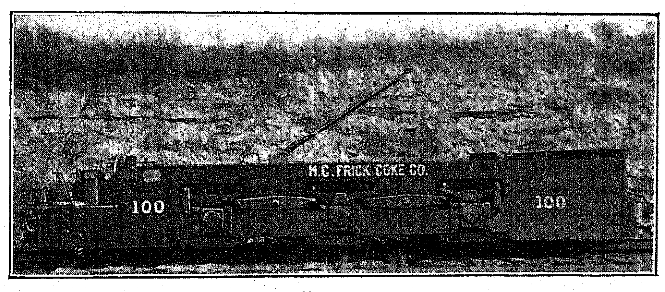

Fig. 9—Locomotive

The larger locomotives are equipped with air brakes and suitable compressor. The usual motor equipment on modern locomotives is of such capacity as to give approximately 10 h. p. per ton of weight on the drivers,

the rating being as before stated on a 75 deg. cent. rise at rated drawbar pull for one hour.

SUBSTATIONS

No practical mine locomotive has been built using a-c. motors. It, therefore, becomes necessary to equip the locomotives with d-c. motors.

Power is delivered to the mines in the form of high-voltage alternating current. This power must be transformed to a suitable voltage and then such amounts of it as are necessary must be converted to direct current for supplying the mine locomotives and in many coal mines the mining machines for undercutting the coal.

The step-down transformers are of the usual type transforming the transmission voltage to 2200 volts. The transformer end of the substation may be of the outdoor or indoor type and is designed along the same lines as similar step-down substations in other industrial work.

Conversion from alternating current to direct current is accomplished either through a motor-generator or a

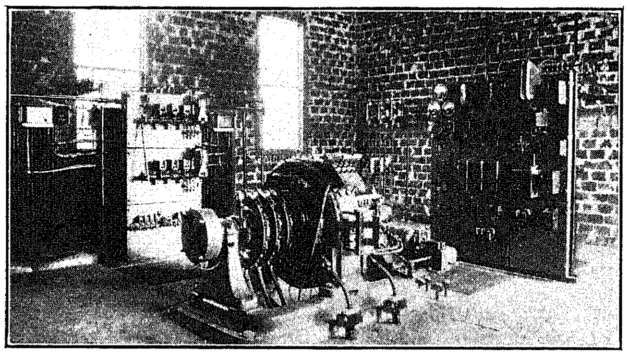

FIG. 10—AUTOMATIC ROTARY CONVERTER SUB-STATION

synchronous converter. If the synchronous converter is used, further transformation of a-c. voltage is necessary. If the motor generator is used, a motor can be operated directly from the 2200-volt bus.

The question as to the selection of a motor generator or synchronous converter seems to be quite badly misunderstood by many engineers. At first glance it would seem as though the synchronous converter on account of its higher efficiency should be used in all such places. It would seem to the writer that the determining factor in the choice lies largely in the nature of the power supply. If the operator is sure of a constant voltage supply and does not require any over compounding of the converter and is in no need of power-factor correction, then the indications point to a synchronous converter. If, however, the line voltage is variable, any percentage reduction in voltage on the a-c. side will produce a corresponding reduction on the d-c. side where a synchronous converter is used. This reduction in voltage means to the operator a reduction in speed of practically every piece of machinery operating on the d-c. system which reacts in practically the same percentage on his output. If he requires power-factor correction, standard mine motor-generators are designed to operate normally at full load with 80 per cent leading power-factor in the synchronous motor. Synchronous converters cannot be used for power-factor correction beyond operating them at unity power factor. The d-c. voltage of the motor-generator can be held constant irrespective of a-c. voltage fluctuations. The generators of the motor-generators can be readily compounded if it is so desired.

The load on the rotor or motor-generator is extremely variable and the set must be built to withstand and commutate satisfactorily 100 per cent over-load momentarily and should be capable of carrying 50 per cent over-load for two hours without any injurious temperatures resulting.

The control of these units and the units themselves must be capable of withstanding numerous short circuits. One side of the d-c. bus is always grounded. Consequently, the fall of a trolley wire in the mine, which is a very frequent occurrence due to faulty roof conditions, will cause a dead short circuit. The control must be capable of removing this short circuit from the generator as quickly as possible.

In large operations, particularly coal mines, it is very common to find underground substations working in parallel with stations on the surface. The a-c. supply for these underground stations may be taken down through a borehole or when the proper precautions are taken may be carried through the air-way or even through the haulage way to the substation. The voltage of the d-c. power supply for mines is either approximately 250 volts or 500 volts, the higher voltage installations becoming scarcer as time goes on.

AUTOMATIC SUBSTATIONS

Many of the modern substations are equipped with full automatic features exactly the same as many of the railway automatic substations with a possible exception that I know of no mine substation which shuts down on failure of load. This added feature would not be practical in a great majority of mines as the load is varying continually in the substation from a maximum to minimum at very short intervals. The automatic features, however, which are included are as listed below. These protective features apply to both synchronous converter and motor-generator substations containing one or more units and are arranged to protect against

A-c. over load
D-c. over load
D-c. reverse power
D-c. reverse polarity
A-c. under voltage
Loss of motor excitation
Loss of generator excitation
Single-phase starting
Imperfect starting
Over-heated bearings
Over-heated windings
Over-speed

Perhaps the most interesting feature of this automatic control is the d-c. circuit breaker which will open on an over-load or short circuit and will remain open until the short circuit has been removed, after which it will close automatically. In view of the numerous short circuits to which substation apparatus is subjected, some few operators have installed what is known as a high-speed circuit breaker. With the ordinary circuit breaker a considerable time elapses from the time the short circuit occurs until it is opened at the circuit breaker and frequently machines will arc-over under this condition; and even if they fail to arc-over considerable burning of the brushes and arcing of the commutator ensues. The high-speed circuit breaker will completely disconnect the machine from the line in less than 0.01 sec. This rapid action prevents the current from rising to any very high value beyond the setting of the circuit breaker and consequently it takes the shock of a dead short circuit from the machine.

Pumps

All mines require a drainage system of some sort. In many operations the problem of keeping the mine

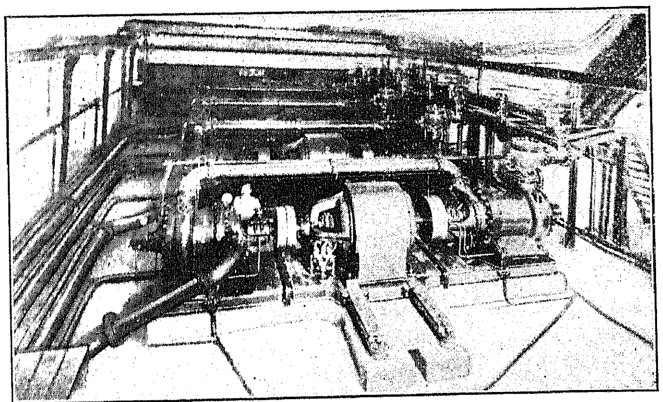

Fig. 11—Four 1750-H. P. 2000-Volt, 1500-Rev. per Min. Motors Driving Centrifugal Pumps, Randfontein Central Gold Mining Co., Ltd., So. Africa

free from water is one of the most difficult and most expensive. For instance, the average pumping condition throughout the anthracite field is 20 tons of water pumped to one ton of coal mined. Many of the mines, particularly in wet seasons, exceed this figure three or four times. Consequently, we find inside some of our large mines enormous pumping plants and sometimes the capacity is large enough to supply a fairly large city. The water pumped is usually acidulous to such an extent that in many instances pumps have to be made of very special and expensive acid-resisting metals.

Pumps are divided into two general classifications namely; the multi-cylinder plunger pump and the centrifugal pump. The former requires a moderate-speed motor usually geared, while the latter calls for a high-speed motor arranged for direct connection. In either case the problem of the electrical engineer is the same. The motor must have moisture-resisting insulation and must be very carefully constructed, particularly in the insulation of its coils. The atmosphere of an underground pump house is usually very humid and unless special precaution is taken the coils will breathe, taking in very moist air which deposits its moisture inside the coils tending to weaken the installation. The larger electrical manufacturing companies have designed a standard line of high-speed induction motors for the driving of centrifugal pumps.

There are no special features connected with the control of pumps as they are usually run at constant speed until the sump-level is lowered to a desired amount, after which they are shut down either automatically by means of a float switch or by hand.

Some of these pumping stations have been made fully automatic and arranged so that when the level of the sump rises to a predetermined value, a priming device comes into play which primes the pump, and after the priming is accomplished the main driving motor is thrown across the line and the pump started. The main motor cannot start until the pump is primed. Should the pump lose its suction for any reason, the equipment automatically shuts down. The most up-to-date installation of this sort, which it has been the writer's privilege to see, is in one of the mines of the Philadelphia-Reading Coal and Iron Company.

The selection of the proper centrifugal pump for a given job should receive very careful consideration. Pump manufacturers list the same pumps for various speeds and heads. The efficiency curve of a pump when plotted to speed in the majority of cases shows a very decided maximum and a rapid falling off either side of this high efficiency point. If a pump is selected to operate at a speed very different from the high efficiency point, the result, irrespective of the drive, will be a very inefficient installation.

In many wet mines we find what is known as gathering pumps which consist of small portable equipments which can be moved about the mine and take care of local flood conditions. These gathering pumps are usually of the centrifugal type and manually controlled, being driven by an induction motor or a d-c. motor taking power from the trolley supply.

The foregoing gives a very brief description of apparatus in general use about the mine. There are, of course, many small other applications for motor drive such as small booster fans, small portable air compressors, electric driven rock drills, etc., but neither time nor space will permit any description of these less important applications.

Hoisting

Of all the problems connected with the mine that of the proper selection of the hoist is perhaps the most interesting. Electric hoists have long since passed beyond the experimental stage and have innumerable times demonstrated their greater economy as compared

with any other form of hoisting. The electrical engineer is no longer asked to compare the economy of the electric hoist with that of the steam engine. One manufacturing company has, within the past 15 years sold 230,000 h. p. in electric hoist divided among 450 units. The only problems now to be solved are how large shall the drive for the hoist be, what is the

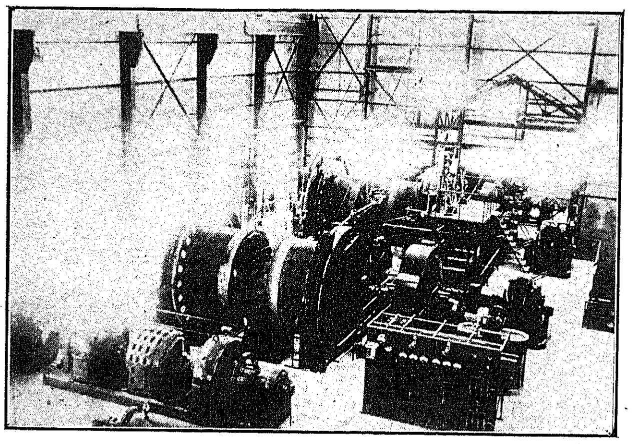

FIG. 12—AUTOMATIC MAIN HOISTS, INSPIRATION CONSOLIDATED COPPER CO., MIAMI, ARIZONA

best shape to make the hoist drum and finally what is the best system for the hoist in question.

The question of size of hoist can be readily determined from the duty cycle. This duty cycle can be calculated with great accuracy as shown by a paper presented before the spring meeting of the Institute at Birming-

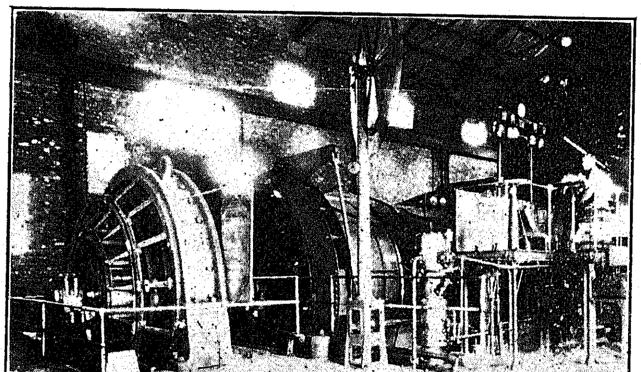

FIG. 13—ONE OF THE TWO DUPLICATE 5000-H. P. MINE HOIST EQUIPMENTS

Installed by the Randfontein Central Gold Mining Company in South Africa at their north and south shafts, each equipment consisting of two 2500-h. p. 160 rev. per min. d-c. motors, one of which is shown above. These two hoists are the largest electrically driven hoists in the world.

ham this year by the writer and F. R. Grant. This paper shows the original calculated duty cycle and the duty cycle that was actually obtained, not on a graphic meter, but on an oscillograph which read volts, amperes and speed. The areas and peaks checked within a negligible percentage.

The best shape of drum can only be determined after several duty cycles have been made using various shapes. In many instances, a plain cylindrical drum is necessary. This is particularly true if hoisting is to be accomplished from more than one level. The conical drum and the cylindro-conical drum have only a special field of application, but in this field the proper

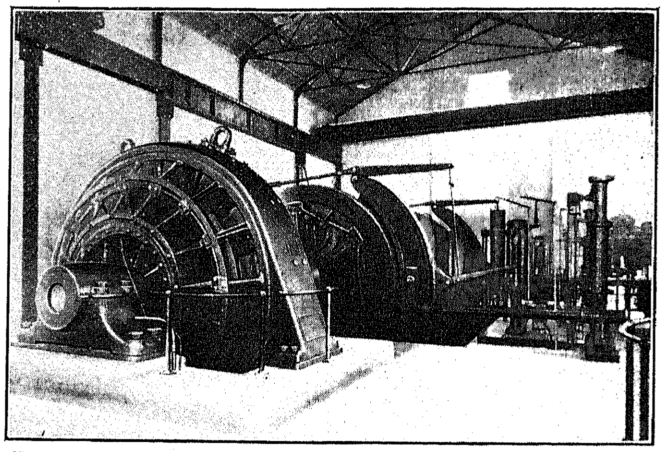

FIG. 14—TYPE MPC-16-1800-H. P. 80-525-VOLT HOIST MOTOR DRIVING FIRST MOTION DOUBLE CYLINDRICAL HOIST, ELU ORLU MINING COMPANY, BUTTE, MONTANA

selection of the shape is of quite vital importance. For instance, on short, fast cycles the rating of the main-drive motor can be reduced as much as 35 per cent by the use of a cylindro-conical drum as compared with a straight cylindrical drum.

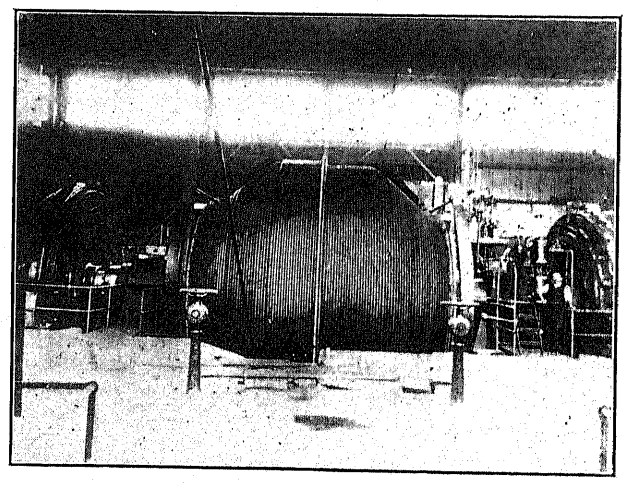

FIG. 15—4000-H. P. D-C. HOIST, SOUTH RAND SHAFT, CROWN MINES, LTD., SOUTH AFRICA, CYLINDRO-CONICAL DRUMS

In drums other than cylindrical, the up load is started on the small diameter while the down load starts with a large diameter. This difference in diameters reduces the starting torque very materially and in fast cycles the acceleration is frequently the controlling factor in the motor rating. This reduction of starting torque is therefore very important.

There are limits, of course, beyond which it is not well to go in proportioning the large and small diameters.

The small diameter is usually fixed by the diameter of the rope. Conservative hoist engineers feel that the ratio of rope to drum diameter should not be less than 1 to 60, otherwise the rope will deteriorate quite rapidly due to the bending stresses. Thus with the minimum diameter determined, the maximum diameter is limited by the weight and consequent $W R^2$ of the drum. It is very easy to defeat the entire object of the drum by running its $W R^2$ too high.

There seems to be no limit in the steepness of the spiral on which the rope climbs from the smaller to the larger diameter, drums having been constructed where the diameter increases as rapidly as two feet per turn.

The conical and cylindro-conical drums assist also in keeping down the peaks in very deep shafts where the weight of the rope is equal to, or as is frequently the case, far exceeds the weight of the material hoisted. An excellent illustration of this is found in one of the 4000-h. p. hoists installed at the Crown Mines, South Africa. This particular shaft has a vertical depth of over 3500 ft. and carried a weight of ore of 16,000 lb. The rope was two inches in diameter and had a total weight of more than 23,000 lb. For this reason a drum was selected 12 ft. small diameter, 20½ ft. large diameter, the drum being of the cylindro-conical type, the cone portion occupying about half of the face and the rope being wound in two layers on the large cylindrical portion. By the adoption of this drum the over balancing effect of the rope was nicely compensated for and the starting load is reduced very materially.

As to the proper system of use, we have the choice of a single induction motor usually geared to the hoist drum through one set of gearing, or we have the d-c. motor driven from a motor-generator set with Ward Leonard control, the motor-generator set consisting of a synchronous or induction motor set without flywheel. Finally, we have a d-c. motor driving the hoist from a motor-generator set with Ilgner control. By Ilgner control, I mean an induction motor-generator set with a flywheel large enough to remove objectionable peaks from the line. Very briefly, this control consists of a motor-generator set as above described and a slip regulator, which, as the load tends to rise above a predetermined value, inserts resistance in the rotor of the induction motor of the motor-generator set thereby holding the torque of the induction motor practically constant. The load on the d-c machine may call for a much greater torque than the induction motor will exert. The result is the set will slow down in speed, the flywheel giving up energy. As soon as the load is removed from the d-c. end, the induction motor being considerably below synchronous speed will draw sufficient load from the line to bring the set back to nearly synchronous speed and restoring to the flywheel the energy it gave up during the heavy demand. A properly designed Ilgner system will keep the input to the hoisting equipment practically at a constant value, if hoisting is carried on at a constant rate, or it will limit the demand to any predetermined value desired.

Thus you see, we have two distinct systems of hoist drive, one using a single induction motor of the slip-ring type with the speed controlled by a variable resistance in the rotor circuit. The second system involves a d-c. generator either direct connected or geared to the hoist drums, and a motor-generator set with or without a flywheel for driving the hoist motor. The second system naturally will cost three or four times that of the first. The question naturally arises why should the second system ever be used. The answer is as in all such installations total cost, namely, interest in fixed charges plus operating costs.

The control of the induction motor hoists is so simple that it needs little or no description. A large resistance is connected across the collector rings and is cut in and out at the will of the operator by means of contactors operated from the master controller. The control is, of course, equipped with current limits so that the acceleration peaks will not go beyond certain predetermined values, but these peaks come directly from the line and no type of resistance will reduce them. In other words, acceleration of a given mass is to be accomplished in a given time and it will require the expenditure of a certain number of horse power seconds. The acceleration of the hoist load by this method of control is exactly the same as the acceleration of any other load where a constant line potential is held. Approximately one-half of the energy taken from the line during acceleration will be dissipated in resistance.

With the d-c. system using Ward Leonard control there is practically no rheostatic loss, since the speed of the hoist is controlled by the generator voltage.

On short fast cycles this saving in acceleration losses amounts to so much that the difference in the kilowatt hours per trip when using induction motor as compared with the Ward Leonard is sufficient to warrant a large expenditure for the d-c. system.

Further, the d-c. system of control is very much more flexible and lends itself a little better to the application of safety and speed controlling devices. The d-c. hoist can be run at reduced speeds with no rheostatic losses. This is impossible with the a-c. hoist.

It is impossible to state with any degree of certainty which system is the more economical until a thorough analysis of the problem is made.

Automatic Hoisting

Due to the ease of control where the d-c. Ward Leonard system is used it has been found feasible to install completely automatic hoists. By this I mean, a hoist which can be started from a push button in the morning and run continuously for 24 hr. or longer, if necessary, without any manual attention whatever. A beautiful example of this is found in Inspiration, Arizona. These hoists have been in operation 9 years

without a single objectionable instant. A much larger hoist has been put in operation at the Inspiration property which embodies many of the automatic features found in the original installation. The hoist motor frame on this later installation is the largest single-hoist motor equipment in the world. It is rated 2150 h. p. at 53 rev. per min., the rating being given on 40-deg. cent. rise continuous operation.

Whether or not a fly-wheel is used for the equalization of the incoming power depends on two factors. First, if purchased power is used and instantaneous demands are penalized then a fly-wheel should be installed. Second, if the hoist is to be operated from a small plant, either privately owned or a public utility, the addition of a flywheel will frequently be of such value in reducing the peaks on the line that even though the plant be small no additional capacity will be necessary. This would not be the case if an induction-motor hoist or a motor-generator set without a fly-wheel were installed.

Undercutting Machines

These machines are used only in coal mines but there very extensively. The purpose of the machine is to cut a very thin channel across the bottom of the coal face. This channel is about six inches high and from six to ten feet deep. The mechanical parts of this machine are very ingenious. The cutting part usually consists of an endless chain carrying cutter teeth. The machine is started in one corner of the room and the cutter feeds itself under the coal until it has entered its entire length. Clutches are then engaged and the machine feeds itself across the face. It makes all motions under its own power. After the coal has been under-cut it requires very little shock to break it down.

The motor drive for these machines may be either direct current or alternating current. The motor is usually totally enclosed and rated on a one-hour continuous run basis. Service is extremely severe and only the very best of workmanship and material will survive this kind of operation.

Strip Mining

Numerous ore and coal deposits have been located so close to the surface that they can be attacked by removing the overburden. This system of mining is commonly known as strip mining. Very large electrically-operated shovels very similar to the steam shovel have been developed for this stripping work. The largest in use at the present time is what is known as the model 350, meaning that the entire weight of the equipment is approximately 350 tons. This shovel is capable of handling a ten-yard dipper and has a boom 90 ft. in length. The duty on the motors of an electric shovel is probably the most severe encountered anywhere. They are continually starting and stopping and being subjected to heavy over-loads. Notwithstanding this fact, however, the electric shovel is rapidly coming into its own.

The majority of electric shovels has a motor for each operation, namely, the hoist, the swing and the crowd. These motors are run from individual generators which form part of a four-unit motor-generator set. The control is a slight modification of the Ward Leonard, the difference being that each generator has a differential series field which limits the current output

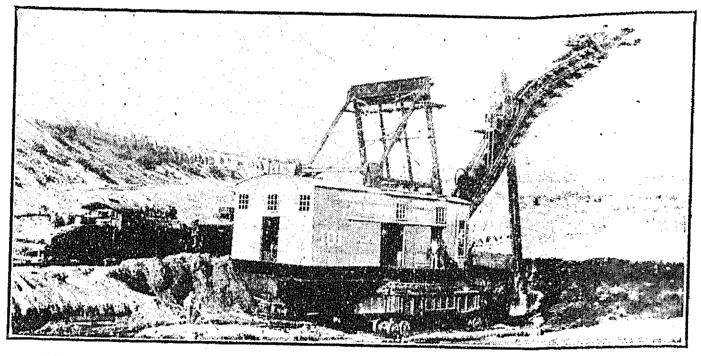

Fig. 16—Model 350 Marion Electric Shovel, the Largest of its Type Built

of the generator to a predetermined value. Thus it is impossible for the operator to abuse the electrical apparatus or the mechanical apparatus beyond a certain point.

Induction motors have been installed on electric shovels but from all information the writer has been able to secure the d-c. Ward Leonard control is much more dependable and the operating costs considerably lower.

Discussion

G. B. Rosenblatt: The paper apparently deals mostly with the application of certain electrical equipment to coal mining and makes little reference to metal-mining practise with which I am much more familiar. If it will be remembered that the remarks I have to offer are based on my experience in the metal-mining field and may fit in with coal-mining practise only in so far as they may apply to it, I would like to offer the following comments:

Emphasis is laid by Mr. Stone on the importance of maintaining ventilation underground. He then suggests for the drive of the fans to maintain this ventilation, a synchronous motor of the type designated as "supersynchronous," that is, the type whose stator revolves during starting. Admission is made that this type has never been applied to such work but the author advocates it. It is my opinion that the design of the so-called super-synchronous motor has not progressed to the point where it may be considered sufficiently reliable for such an important application. This type of motor has today been used almost entirely for the drive of ball or tube mills in cement plants. While continuity of operation is important in such service, it is not paramount. If a tube mill shuts down due to motor trouble, a certain production is lost—but if a mine fan is shut down due to motor trouble at critical moments, men die. In connection with investigations conducted by a prominent western mining organization which decided on synchronous motor drive for its tube mills, existing installations of the so-called supersynchronous motor were investigated. The results of

this investigation were that the synchronous motor with stator revolving for starting was not deemed altogether reliable. It may therefore be considered that this type of motor in its present stage of development is not suitable for mine-fan drive where human lives are at stake.

In discussing the applications of synchronous motors to mining work, it is to be regretted that no mention has been made of the really large field for such installations afforded by air compressor and pump drive. The introduction of synchronous motors into the mining industry is really a very important matter to all of the large power systems which supply such load as well as to the mining companies which generate their own power. All metal mines use a large amount of compressed air. Simple direct-connected synchronous motors are most readily applied to the drive of the requisite compressors. Of late, synchronous motors have also been very successfully applied to driving mine pumps and there are now some very large underground pump stations in western mines equipped entirely with synchronous-motor-driven pumps.

The author seems to feel that the field of usefulness for storage-battery locomotives underground is decidedly limited and possibly decreasing. My own experience does not bear this out. In metal mining particularly, we find the properly designed storage battery locomotive increasing in popularity and use. It is the poorly designed and improperly applied storage-battery locomotive that has caused the failure of certain installations of storage-battery haulage. Coming right down to fundamentals, the comparison is not fairly to be made between the trolley type or gathering-reel locomotive and a storage-battery locomotive, but should rather be made between the two types of haulage systems as a whole, including in such comparisons all items that go into the actual cost of hauling a unit of ore or coal over a unit of distance. The machinery designer—the factory engineer—may be much interested in different advantageous details of one type of locomotive as against another, but the purchaser and the user—the man that pays the bills—is primarily concerned with what it costs to haul his ore or coal per ton and per mile.

Mr. Stone states "induction motors have been installed on electric shovels but from all information the writer has been able to secure, the d-c. Ward-Leoanrd control is much more dependable and the operating costs considerably lower."

The relative advantages of a-c. versus d-c. for large shovel drive has been a moot question among interested engineers for the past several years and each system has had its staunch supporters. Some while ago, a prominent copper company which had for years been doing their mining with over 20 large steam shovels decided to electrify the shovel operations. The arguments submitted by the various supporters of the two systems of electric drive—a-c. versus d-c.—were carefully and painstakingly reviewed by the management of the mining company who finally came to the conclusion that all the arguments presented were based largely on opinions and deductions and not on known facts, so they set out to find the facts for themselves. They bought and installed two shovels—both identical mechanically—one equipped with Ward-Leonard d-c. equipment, the other with a-c. induction motors. The electrical installation of each type was supervised by the manufacturers advocating that particular type, so that both shovels had the advantage of the best electrical engineering talent. These shovels were then put into regular mining operation and their performance carefully watched and recorded by both the engineering and operating staffs of the mining company. After several months operations, the mining company decided that sufficient data had been collected to permit it to determine upon the type of shovel for its subsequent electrification. In order to assure itself that its judgment in making this important decision would be as correct as possible, the mining company secured the services of one of the best known American electrical engineers and a prominent member of this Institute to assist its own engineering department in selecting the most advantageous equipment. As a result they bought shovels equipped with a-c. induction motors—not the Ward-Leonard d-c. equipment.

The test records of this mining company covering the actual operation of both types of shovels under real mining conditions are most interesting. They indicate that the a-c. equipment: (1) Costs less to buy than the d-c.; (2) Is much simpler than the d-c.; (3) Should have materially less maintenance than the d-c.; (4) Consumes slightly—very slightly—more power than the d-c. per average ton of ore dug; (5) Can dig a bit faster than the d-c. equipment; (6) Imposes somewhat greater peaks and considerably poorer power factor on the power supply line.

These tests were made with big railway-type shovels mounted on caterpillars and the results, while conclusive, may not apply to all installations. However, the outcome of these large scale service tests certainly confutes Mr. Stone's statements regarding the general superiority of d-c. equipment for electric shovels.

C. H. Matthews (by letter): This paper seems to cover the application of electrical equipment to coal mines from the manufacturer's standpoint and I wish to make several comments that affect the practical use of the equipment described.

Mine ventilation, as Mr. Stone states, is of paramount importance and the most reliable and proven apparatus must be selected. The common design of synchronous motor with magnetic clutch is a very reliable piece of equipment and is suitable for driving fans of almost any capacity and speed.

Induction motors have proven efficient and reliable for driving mine fans but must be used with some form of speed reducer whereas the synchronous motor can be efficiently built for almost any speed for direct connection.

Where power-factor correction is necessary synchronous motor-generators and synchronous motors driving air compressors usually give the desired results, and the use of synchronous motors on mine fans and centrifugal pumps is desirable for lowering the power costs.

The use of storage-battery locomotives is a necessity in many metal mines if economical transportation is to be obtained and since the requirement of "permissible" equipment in gaseous mines the storage-battery locomotive seems to be getting a more permanent berth. There are many engineers who differ on the advisability of storage-battery applications, but there still remains a demand which must be satisfied.

The suggested speed of 3 to 3½ mi. per hour for gathering locomotives was probably based upon storage-battery locomotive speeds which speeds are maintained at a fairly constant rate over the working day. With trolley locomotives the speed depends upon the trolley voltage so that in the majority of mines a trolley locomotive designed for a speed of 3 to 3½ mi. per hour at rated voltage and draw-bar pull may not handle the same output as a storage-battery locomotive of the same weight and geared for the same speed. Since the voltage has a direct bearing upon the speed of trolley locomotives and as the trolley voltage is seldom maintained at normal value it would seem desirable to design for speeds of 4 to 5 mi. per hour at rated voltage and draw-bar pull. Some of the locomotive manufacturers used 500-volt motors on 250-volt power to obtain slow speeds for gathering. This was an easy way to secure data on slow-speed service but it reduced the rating of the motor equipment which from an operating point of view is not desirable. A study of motor curves on slow speeds of 3 to 3½ mi. per hour when using 500-volt motors on 250-volt power shows that the light-load speeds do not increase in the same manner as with motors designed for 250-volt slow-speed operation, so that a drop in trolley voltage causes a decrease in speed over the whole speed curve which is not compensated for to any great extent by light loads.

Graham Bright (by letter): I believe that a little more attention should be given to the question of the isolated power plant for coal mines. It is true that with central-station power available, in the large majority of cases there is little excuse for

the existence of the isolated power plant. There are, however, some localities where central-station power is still not available, and there are other cases where the central-station rates and form of contract are such that where the mining company has a fair supply of good water, and in some cases can utilize waste fuel of no commercial value, it would not only be feasible but would be economical to install an isolated plant.

The most important question, by far, in regard to the installation of an isolated power plant, is that of water supply. In most cases, the water available in the vicinity of a coal mine is very poor, and cannot be used in boilers without a treating plant. It is true that it is possible to treat water so that it will be very satisfactory for boiler service, but these treating plants require a certain amount of skill for their operation, and this skill is seldom available in the coal fields. In a great many cases, as soon as a man is educated to take care of the treating plant, he is likely to leave the locality, and the only one available for this particular situation is, figuratively, a man who knows nothing whatever of this kind of work, and no one has the time to teach him the details. The result is that the boiler plant gets in very bad shape and may be ruined in a short time.

Mr. Stone mentions the desirability of having a stand-by source of power where the mines are gaseous or collect water rapidly. A form of gasoline-engine-driven generator has been frequently applied, and Mr. Stone mentions that the hoist and fan. under such conditions, can be operated at reduced speeds. With the ordinary type of unit, operating at full frequency, this this can, of course, be accomplished with a fan only in case it is of the two-speed type, or of the variable-speed type. The hoist can be operated at reduced speed only by means of inserting resistance in the hoist motor, and this does not greatly reduce the amount of power required, since the extra power is absorbed by resistance. Where it is necessary to keep the fan and hoist in operation, the writer has proposed a scheme of supplying a gasoline-engine-driven generator, operating at reduced frequency. This permits the fan to operate at a lower speed, with very much less power, and permits the hoist to operate at reduced speed with an economical use of power. A plant of this type has been installed at one of the mines of the Y. & O. Coal Company, near Pittsburgh, and low-frequency emergency power can be made available within thirty seconds after loss of the central-station power.

Mr. Stone indicates that the load factor of the anthracite mine will be something like 42 per cent. I believe that in some of the mines where very heavy pumping takes place the load factor is even higher than 42 per cent. In the average bituminous mine, however, where the ventilation and pumping are both light, the load factor will average about 25 per cent, and in some cases may be as low as 15 per cent. This, of course, is one of the reasons why the cost of power at some of the smaller bituminous mines is rather high on a kilowatt-hour basis.

Mr. Stone illustrates a very ingenious scheme for allowing the use of a synchronous motor for operating a mine fan. The objection to the synchronous motor has always been that it has a very comparatively low pull-in, while for fan operation the motor should have a pull-in of 100 per cent. Unless a great deal of power-factor correction is required, the installation of a motor of sufficient capacity to produce the proper pull-in would not be economical. The use of a clutch has not been advocated much in the past, due to the added complication. I believe the time is coming when the synchronous motor will be used very commonly for the installation of mine fans, but I feel that the motor described by Mr. Stone is only a step in the ultimate direction. The objection to this type of motor is that extra collector rings are required, and in many cases these collector rings will carry a voltage of 2200. Another inherent objection to this type of motor is that for economical design, the peripheral speed of the rotating element of the motor should be fairly high. The stationary part of a motor is generally made of steel punchings, which furnish the required active iron surrounded by a shell of cast iron or cast steel. In case the stationary part is to rotate, as described by Mr. Stone, then the limit in speed would be determined by the outside diameter of the stationary part. Even when made of cast steel, this part would have to be carefully balanced and additional bearings supplied. This would mean that the peripheral speed of the rotor would be very much less than the economical speed, and for this reason, the motor would be more expensive than the standard type. As stated before, I believe that the motor described by Mr. Stone is but a step in the evolution of the synchronous motor, and the time is coming when we will have what is practically a standard motor with a magnetic or similar clutch, preferably inside of the motor, which can be operated automatically, and this motor will not cost a great deal more than the standard motor.

Mr. Stone's curves, showing the power taken by different types of motors, are very interesting, and indicate that the brush-shifting motor is well worth considering, where variable speeds can be utilized.

I am glad to know the stand that Mr. Stone has taken regarding the application of storage-battery locomotives to mines, as I know of a great many cases where misapplications have been made and the storage-battery locomotive has been given a "black eye" simply because of misapplication.

The application of contactor control for mine locomotives has a great many advantages. In the first place, it makes a much safer controller for the operator. A number of accidents have occurred where the operator has been seriously burned, due to the ordinary drum type of controller blowing out. With the contactor control, the operator has only the master controller close to him, and the danger of his being burned is very remote. Another advantage of the contactor control is that it forces the mine management to install sufficient copper to give a fair voltage regulation. This not only saves a considerable loss in power, but increases the output of the mine, and cuts down the maintenance of apparatus caused by burn-outs.

The application of contactor control also lends itself to dynamic braking, without seriously complicating the control itself. It also simplifies tandem operation in that only one power cable is required between locomotives.

The application of equalizer systems is becoming quite prevalent for both two- and three-motor locomotives, as a proper equalizing system will not only increase the power of the locomotive by keeping the load equally distributed on the wheels, but will make a much better tracking locomotive, thus preventing costly and annoying derailments. The usual method of applying an equalizer system to a two-axle locomotive is a rather difficult matter if a stable locomotive is desired. Unless the equalizer bars are restrained in some manner, the locomotive may become tilted to one end or the other and remain in that position. This is also true of the three-axle locomotive, where the axles are equalized all the way along each side. Where this type of equalizer is used, you will frequently notice that the locomotive is considerably closer to the track at one end than it is at the other. One way to eliminate this unstable condition is to equalize two axles on the sides and cross-equalize the third axle.

A feature illustrated in Fig. 9 of Mr. Stone's paper, but not mentioned by him, is the method of taking the end thrust on the end of the axle, rather than on the wheel hub. This is a distinct improvement on the older method, and is being almost universally adopted. It provides an easy method for taking up the end play and for lubricating the surfaces which take the end thrust. It also keeps the supply of lubricant for the main journals in a much cleaner condition, and for this reason the maintenance on the main journals and brasses should be much reduced.

I agree with Mr. Stone that the question of whether to use the synchronous converter or the motor-generator set for substations is largely determined by the nature of the power supplied.

With good voltage and frequency regulation there is no reason why the rotary converter should not give as good service as the motor-generator set, and it will have much better efficiency. In many of the bituminous mines, the substation will operate a considerable portion of the time at very light load. The losses with a synchronous converter are only about one-half of what they would be with a motor-generator set under these conditions. Where sufficient copper is installed, the constant voltage supplied by the synchronous converter will give very satisfactory operation. The over-compounding obtained by a motor-generator set in many cases indicates a considerable loss, and the high voltage near the substation during heavy loads is often injurious to compound and shunt-wound motors working in the vicinity. The automatic substation has come to stay, and with this equipment available there is no real excuse why good regulation should not be obtained in most of the working places in the average coal mine.

Mr. Stone's remarks in connection with the application of specially shaped drums to hoists are very much to the point, and where short, fast cycles are required, a very careful analysis must be made or the effect of the specially shaped drum will be entirely defeated.

In connection with electric shovels, the question of which is the best type of equipment to use has not as yet been finally settled. There seems to be a great deal of merit in the application of direct-current motors, using motor-generator sets and Ward-Leonard control. This type of equipment is rather complicated when it comes to the number of machines involved, but, of course, has an extremely simple and efficient control. The straight alternating-current motor, however, requires very much less equipment, and where the power system has sufficient capacity, this type of equipment has some advantages over the direct-current equipment. Just recently a large mining company in the West, which uses a great many shovels for a stripping operation, has conducted tests on both types of equipment, and have come to the conclusion that alternating-current is more efficient and less costly to install. From the results of the tests and investigations, they have decided to start equipping all of their steam shovels with the alternating-current system, using induction motors.

The direct-current shovel equipment in operation at this time using motor-generator sets and field control seems to be rather sluggish when compared to the alternating-current, and it is generally found upon comparison that the alternating-current equipment will dig and load more material in a given time than the direct-current of the same capacity.

F. L. Stone: Referring to Mr. Matthews' very interesting discussion, I agree with him entirely in his comments on storage-battery locomotives. There are places where their installation indicates good judgment on the part of the management. Like many other pieces of apparatus, it could almost be considered as a necessary evil under certain conditions. I tried to make this clear in the paper. It is the promiscuous and careless application of storage-battery locomotives to work that they are not suited to perform to which the writer objects.

Reference has been made to a prominent mining company in the West, where two practically duplicate shovels were installed, one being driven by a-c. motors and the other by d-c. motors, with variable voltage control. Before the tests on these equipments had reached a stage from which anything very definite could be deduced, the mining company purchased seven additional a-c. equipments. After the shovels had been in operation a year or more and some of the new a-c. shovels had been delivered and were put in operation, the mining company then felt that it had sufficient data to make an intelligent selection. It then purchased eight d-c. equipments. This, I think, together with the fact that many other companies have, on investigation, purchased d-c. drive, fully substantiates my statement.

In regard to Mr. Bright's discussion, will say that I am sorry I did not have time or space to go into great detail in connection with the problem of isolated plants versus purchased power.

Mr. Bright's comments on the super-synchronous motor show a very clear understanding of the subject. I might state that over fifty of these machines have been shipped and put in service. The collector-ring trouble which Mr. Bright refers to was taken care of by thoughtful and careful design. No trouble from this source has been brought to the writer's attention. The stator, as Mr. Bright points out, must be and is, very carefully balanced. The peripheral speeds of the stator are not at all excessive, and the factors of safety are very large. The ease of control while starting the load by means of the band brake is a very noticeable feature in this motor. The load can be brought up at a high rate of speed or at a low rate of speed, or the rates can be varied during the starting cycle at the will of the operator.

Practises in Telephone Transmission
Maintenance Work

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Review of the Subject.—*This paper describes the practical applications of transmission maintenance methods in a telephone system. The methods applicable to toll circuits of various types are first discussed, information being included in this connection on the maintenance of the amplifier circuits involved in telephone repeaters and carrier. Testing methods applicable to the local or exchange area plant are next described, the description including both manual and machine switching systems. The results accomplished in toll and local transmission maintenance work are considered from the standpoint of the kinds of troubles which can be eliminated and the effect which these troubles have on service.*

The methods described in the main body of the paper relate particularly to tests of volume efficiency. Certain other transmission maintenance testing methods directly associated with volume efficiency tests are briefly described in Appendix A of the paper.

* * * * *

IT is the purpose of this paper to present a general picture of the practical applications of methods of measuring transmission efficiency in the Bell System which have been developed by study and experience under plant operating conditions. The rapid growth of the telephone industry has made it necessary that these methods be such as to allow them to be applied on a large scale in a systematic and economical manner thereby providing for a quick periodic check of the efficiency of the various types of circuits as they are used in service.

Transmission maintenance can be broadly defined as that maintenance work which is directed primarily towards insuring that the talking efficiencies of the telephone circuits are those for which the circuits are designed. There are, of course, many elements which affect the talking efficiency and various d-c. and a-c. tests are available for checking the electrical characteristics of circuits and equipment to insure that these characteristics are being maintained in accordance with the proper standards. In the final analysis however, an overall test of the transmission efficiency of the circuit in the condition it is used in service will show at once whether it is giving the loss, or in the case of amplifier circuits, the gain which it should give. Transmission tests, therefore, offer a means whereby many of the electrical characteristics of circuits can be quickly and accurately checked.

In referring to transmission testing apparatus in this paper, four standard types described in papers previously presented before the Institute are involved. The first three types listed below were described by Best[1] and the fourth by Clark.[2] Reference in these papers was also made to the standard oscillators used in supplying the measuring currents for the sets.

1—*A Transmission Measuring Set.* This is an "ear balance" portable set suitable for loop transmission testing only and designed primarily for testing equipment and circuits in the smaller central offices.

3—*A Transmission Measuring Set.* This is a "meter balance" portable set suitable for both loop and straightaway transmission testing and designed primarily for testing circuits and equipment in the larger central offices.

4—*A Transmission Measuring Set.* This is a "meter balance" set suitable for both loop and straightaway transmission testing and designed for permanent installation at the larger toll offices primarily for testing toll circuits.

2—*A Gain Set.* This is a "meter balance" set designed for measuring amplifier gains.

Volume transmission efficiency is expressed in terms of Transmission Units[3] and measuring apparatus is calibrated to read these units directly. The principles involved in the use of these units in transmission testing work as well as the principles of the measuring apparatus employed were discussed in the papers referred to above. Certain other testing methods in addition to volume efficiency tests are also extensively used in transmission maintenance work and some of the more important of these are briefly discussed in Appendix A of this paper.

Since the routine procedures in testing toll circuits using the above apparatus differ considerably from those followed in the local or exchange area plant, the toll and local practises have been considered separately in the following discussions:

Transmission Tests on Toll Circuits

The importance of having available means for quickly checking the transmission efficiency of toll circuits and of economically maintaining the proper standard of transmission is evident when it is considered that in a plant such as that operated by the Bell System there are at the present time more than 20,000 toll circuits in service. The circuits making up this system are of various types and construction, depending on the service requirements and length, and also upon certain other factors determined by engineering and economical design considerations.

From the standpoint of maintaining transmission

1. See Bibliography.

Presented at the Pacific Coast Convention of the A. I. E. E., Pasadena, Cal., October 13-17, 1924.

efficiency between toll offices, the various types of toll circuits can be divided into three general classes: one, non-repeatered circuits, two, circuits equipped with telephone repeaters and three, circuits equipped for carrier operation. The latter two classes are alike in many respects as far as the maintenance methods are concerned and both require somewhat more attention than the circuits not equipped with amplifying apparatus. The length and number of repeaters involved are also important factors which must be taken account of in tandem repeater and carrier circuit maintenance. Very long tandem repeater circuits such, for example, as the long toll cable circuits described by Clark[2] require special maintenance procedures similar in many respects to those required in carrier maintenance.

The 4-A type of transmission measuring set generally used for testing toll circuits may be considered as a toll transmission test desk. Fig. 1 shows a picture of one of the latest models together with an oscillator for supplying the measuring current, installed at a toll office for use in routine testing. The set is provided with trunks to both the toll testboard and toll switchboard, and also with call circuits to toll operators' positions for use in ordering up circuits for test. The electrical measuring circuit is designed so that tests may be made on two toll circuits looped at the distant end, or straightaway on one toll circuit the distant terminal of which terminates in an office also equipped with a transmission measuring set of the same type.

To illustrate the application of this toll transmission

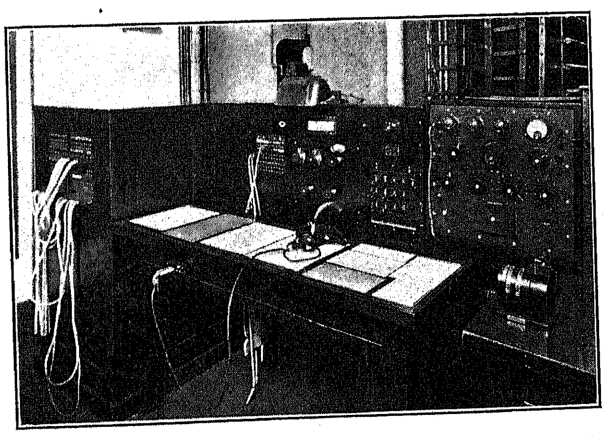

Fig. 1—Illustration of 4-A Transmission Measuring Set and 4-B Oscillator Installed in a Toll Test Room

test desk, Fig. 2 shows schematically an arrangement of four toll offices having circuits between them of the three general classes—non-repeatered, repeatered and carrier. Offices A and D are equipped with transmission measuring sets of the type shown in Fig. 1. A logical testing procedure for the arrangement in Fig. 2 is for offices A and D to test the non-repeatered circuits 1 to 4 and 10 to 13 by having them looped two at a time at the distant terminal offices B and C. By "triangulation measurements" on any three circuits in each group, the equivalent of each individual circuit can be readily computed.

For the circuits 5 to 9 extending between offices A and D equipped with telephone repeaters or carrier, straightaway measurements can be made in each direction with the two transmission measuring sets provided. Loop tests could, of course, also be made on the circuits from either office A or D, but this would require cutting the telephone repeaters out of one circuit or having available a non-repeatered or non-carrier circuit, since

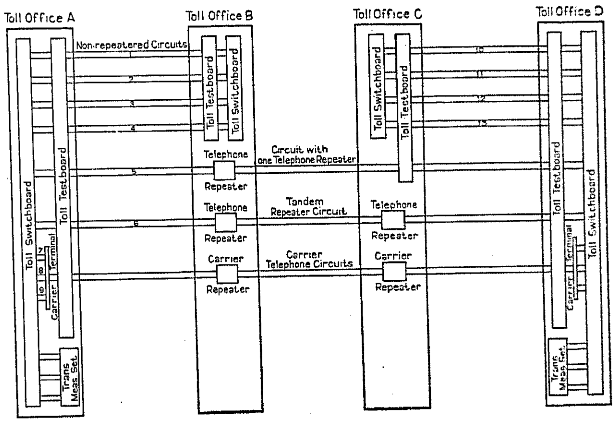

Fig. 2—Schematic Diagram of Typical Toll Circuit Layout to Illustrate General Method of Testing Non-Repeatered, Repeatered and Carrier Circuits

the gains of the repeaters in the two directions introduce variable factors in the overall equivalents which do not permit triangulation computations to be made. The overall tests on the carrier circuits do not differ in any way from the tests on repeatered or non-repeatered circuits, each carrier channel being tested as a separate circuit through the switchboards. The measuring current is modulated and demodulated in the same manner as voice currents under regular operating conditions and the measured equivalent, therefore, indicates the overall transmission efficiency.

The map of Fig. 3 shows the locations in the Bell System of transmission measuring sets of the general type described above. At a number of the larger toll centers, such as New York and Chicago, where the number of toll circuits to be tested require it, several transmission measuring sets are installed. There are now in operation between 40 and 50 of these sets, making it possible to test all of the longer and more important toll routes in the system. The shorter toll circuits radiating out from the large toll centers are also tested with these same sets. At the smaller offices where fixed transmission measuring sets are not warranted, the toll circuits which cannot be picked up by the larger offices are tested by portable transmission measuring sets of the 1-A or 3-A types in connection with other maintenance work.

One very essential requirement in carrying on a systematic testing program is to have records of the detailed makeup of the toll circuits which give both the

circuit layouts and the equipment associated with the circuits. Such a record is valuable, not only in giving the maintenance forces a picture of the circuits and equipment which they are testing, but it also furnishes a means for establishing the transmission standards to which they should work. When transmission tests indicate trouble, this record becomes of particular service in locating and clearing the cause.

Fig. 4 shows a sample of the type of toll circuit layout record card which has proven very satisfactory and is now generally used in the Bell System.

range of frequencies involved, and that conditions do not exist which will disturb the overall balance between the circuits and networks sufficiently to cause poor quality of transmission.

Considering telephone repeater maintenance, Fig. 5 shows a schematic diagram of a 22 type repeater and indicates the important tests which are made locally to insure that the apparatus is functioning in a satisfactory manner as a part of a toll circuit. The numbers applied to the different tests listed in the figure show approximately the points in the repeater circuit at

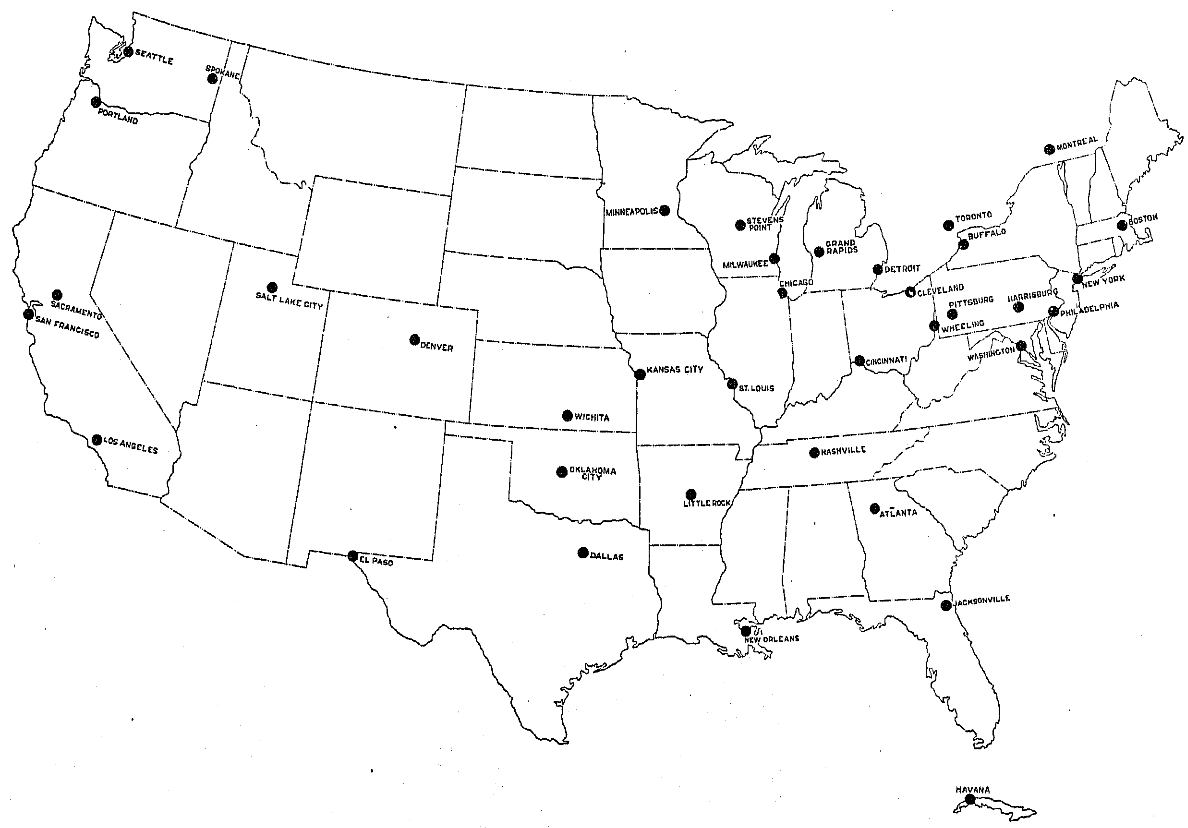

FIG. 3—MAP SHOWING LOCATIONS IN BELL SYSTEM OF PERMANENT TRANSMISSION MEASURING SETS

Telephone Repeater and Carrier Maintenance. Voice frequency telephone repeaters were discussed in a paper by Messrs. Gherardi and Jewett[4] and carrier systems in a paper by Messrs. Colpitts and Blackwell.[5] The various arrangements of amplifiers to provide for telephone repeater and for carrier operation as described in these papers make up integral parts of toll circuits and introduce elements in the circuits which have to be given particular local attention in maintaining the overall transmission efficiency. Since both telephone repeaters and carrier employ the same types of vacuum tubes with very similar arrangements for power supply, the maintenance requirements for the two are much the same. The chief items to be observed in both carrier and repeater maintenance are that the gains specified to give a desired overall transmission equivalent be kept as constant as possible, that these gains remain fairly uniform within the

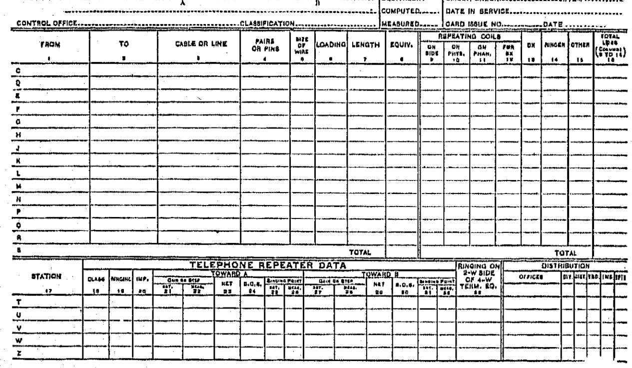

FIG. 4—SAMPLE OF A TOLL CIRCUIT LAYOUT RECORD CARD

which the tests are made, the purposes of the tests being evident from their names.

When carrier operation is applied to toll circuits, an

additional transmission system is introduced involving the use of currents of higher frequencies than those in the voice range. From a maintenance standpoint this means that certain additional testing methods must be employed which will insure the proper generation and transmission of the carrier currents and that the modulation and demodulation of the voice frequency currents is accomplished without distortion or excess loss in overall transmission.

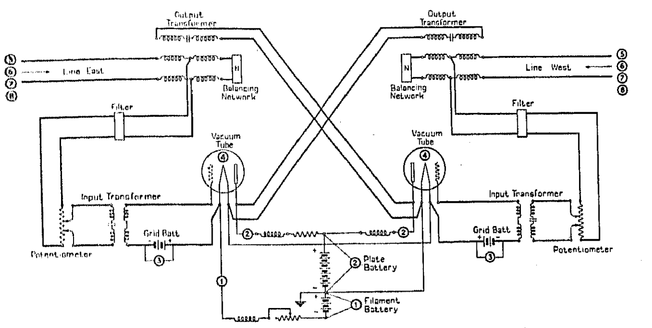

Fig. 5—Schematic Diagram of a 22-Type Telephone Repeater Showing Important Local Transmissiom Maintenance Tests

1. Filament Current and Voltage
2. Plate Current and Voltage.
3. Grid Voltage and Poling.
4. Vacuum Tube Activity Tests.
5. Gain Tests (Working Potentiometer Steps).
6. 21-Circuit Balance Tests.
7. Potentiometer Step-Gain Tests.
8. Gain Frequency Tests.

To give a general picture of the more important features involved in the transmission maintenance of carrier systems, Fig. 6 shows a schematic diagram of a carrier layout having one carrier repeater. The particular arrangement shown is for the type *B* system described by Messrs. Colpitts and Blackwell,[5] although the same general maintenance considerations apply to any of the present systems. It will be noted that three series of tests are required, one for the carrier repeaters, one for the carrier terminals and one for the system as a whole. The nature of these various tests and the approximate points in the carrier system where they are applied will be evident from the names and numbers used in the figure.

For both telephone repeaters and carrier systems, provision is made in the regular testing equipment so that the tests can be very quickly applied both as a routine proposition and also when required for trouble location.

Transmission Tests on Exchange Area Circuits

The transmission conditions in the exchange area plant are important not only from the standpoint of insuring good local service but also to insure good toll service, since the local plant forms the terminals of toll connections. The exchange or local plant offers a somewhat different transmission maintenance problem than the toll plant, particularly with respect to the routine testing procedures which must be followed to insure satisfactory transmission. This will be evident when it is considered that in each city and town a complete telephone system is in operation which involves the use of a large number of circuits of various types. There are also in use three general types of telephone switching equipments; manual, panel machine switching, and step-by-step machine switching, and in certain cities combinations of these equipments. It is estimated that at the present time in the Bell System there are in the neighborhood of two and one-half million exchange area circuits, exclusive of subscribers' lines, involving equipment other than contacts and wiring which may directly affect the transmission of speech.

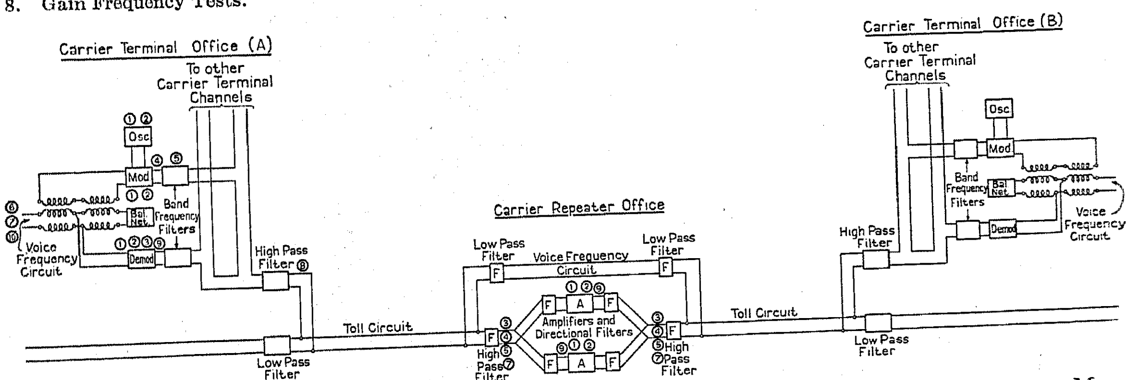

Fig. 6—Schematic Diagram of a Carrier Telephone System Showing Important Transmission Maintenance Tests for Carrier Repeaters, Carrier Terminals and Overall

CARRIER TERMINAL TESTS

1. Filament, Plate and Grid Battery Tests.
2. Vacuum Tube Activity Tests.
3. Channel Rectified Received Current Tests.
4. Modulator Output.
5. Modulator Band Filter Output.
6. Channel Loop Gain Tests.
7. 21 Circuit Balance Tests on Voice Frequency Circuits.

Overall Tests of Complete Carrier System

8. Tests of total Carrier Output Current into Toll Circuit.
9. Tests of Carrier Current at Repeater Outputs and finally Rectified Received Current at Distant Terminal.
10. Overall Transmission Tests.

CARRIER REPEATER TESTS

1. Filament, Plate and Grid Battery Tests.
2. Vacuum Tube Activity Tests.
3. Gain Tests.
4. Potentiometer Step-Gain Tests.
5. Gain-Frequency Tests.
6. Check of Frequency of Test Oscillator (not shown in figure).
7. High Frequency Singing Tests.
8. Output Current on Overall Test of System.

CARRIER TERMINAL TESTS Similar to those listed for Office A.

The general classes of exchange area circuits in both manual and machine switching offices, important from a transmission maintenance standpoint, are listed in Table I. The operating features of manual telephone systems are generally well known as are also the features of step-by-step machine switching systems, both having been in use for many years. The panel machine switching system which is a relatively recent development was described in a paper by Messrs. Craft, Morehouse and Charlesworth.[6]

TABLE I

CLASSIFICATION OF CIRCUITS IN THE EXCHANGE AREA PLANT IMPORTANT FROM A TRANSMISSION MAINTENANCE STANDPOINT

MANUAL OFFICES

Local Switchboards	P. B. X. Switchboards	Toll Switchboards	Toll Testboards
Cord circuits	Cord circuits	Cord circuits	Composite set circuits
Operators' circuits	Operators' circuits	Operators' circuits	Composite ringer circuits
Trunk circuits	Trunk circuits	Trunk circuits	Phantom & simplex circuits
Miscl. circuits	Miscl. circuits	Miscl. circuits	Miscl. circuits
	Subscribers' loops and sets		
	Operators' telephone sets		

MACHINE SWITCHING OFFICES

Panel	Step by Step
District selectors	Connectors
Incoming selectors	Toll selectors
Trunk circuits	Trunk circuits
Miscl. circuits	Miscl. circuits
Subscribers' loops and sets	
Operators' telephone sets for Special service positions	

General classes of exchange area circuits involving equipment other than contacts and wiring which affect telephone transmission.

While it may appear at first hand from the above discussion that transmission testing in the exchange plant is a complicated and expensive matter, this has not proven to be the case. It has been found by experience that the systematic use of transmission measuring sets, following the testing methods which have been developed provides a means for periodically checking transmission conditions with a relatively small amount of testing apparatus and with a small maintenance force. All of the transmission circuits exclusive of subscribers' lines in a 10,000-line central office, either manual or machine switching, can, for example, be completely tested by two men in a period of from two to four weeks, (five and one-half 8-hour days per week assumed) any trouble found being cleared as the testing work is done. The maintenance of the subscribers' lines is not included in this work since it is taken care of by other methods as outlined later.

In order to give a general picture of the application of transmission testing in the exchange telephone plant, a brief discussion of the methods employed in both manual and machine switching systems is given below. In either system the loop method of testing proves most satisfactory, that is, one measuring set is used and where both terminals of a circuit are available as in cord circuits, a loop test through the circuit is made. In testing trunk circuits two trunks are looped together at their distant terminals and a measurement made on the two combined.

Transmission Tests on Manual Exchange Area Circuits. In central office, P. B. X. and toll switchboards, the cord circuits and associated operators' circuits are tested by using a portable transmission measuring set, moving this along the boards as required to pick up the cords. Fig. 7 shows a 3-*A* transmission measuring set being operated at an *A* switchboard position. The cords are picked up and plugged directly into the set as shown and measurements made of the loss of both the cord and operator's circuits. Trunk circuit tests are made at the switchboards in the same manner as previously described for loop transmission tests on toll circuits, portable measuring sets such as shown in Fig. 7 generally being employed for this work.

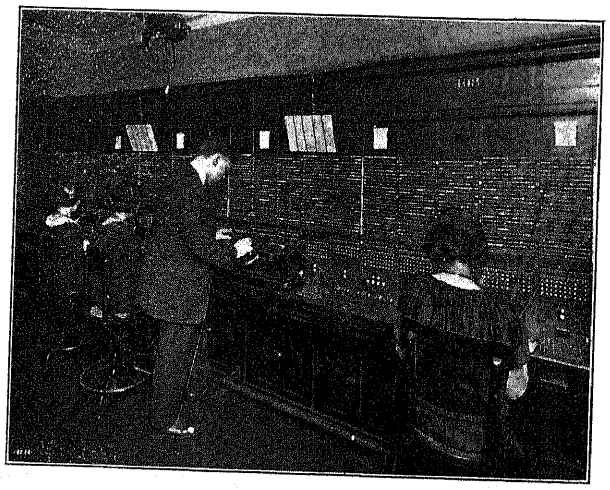

FIG. 7—ILLUSTRATION OF A 3-A TRANSMISSION MEASURING SET BEING OPERATED IN A MANUAL OFFICE

Operators' sets are inspected periodically and transmitter and receiver efficiency testing methods are under field trial which provide a means for testing these instruments in central offices. The miscellaneous transmission circuits in an office are tested at the points where they can be most conveniently picked up. The tests on toll test board circuits are made at this board and involve chiefly loop tests on the equipment associated with the toll circuits in the office and tests on the toll line circuits between the toll testboard and toll switchboard.

Transmission Tests on Machine Switching Circuits. The transmission circuits in panel machine switching systems are identical to those in manual systems, while these circuits in step-by-step systems are of a different design but essentially the same as far as transmission losses are concerned. Transmission tests on machine switching circuits are similar to those on manual cir-

cuits but involve special methods for picking up the circuits and holding them while the measurements are made. The standard types of transmission measuring sets are used in this work in conjunction with the regular testing equipment provided in the machine switching offices and the methods which have been developed offer a quick and convenient means for making the tests. In manual offices the circuits terminate in jacks or plugs at switchboards where they are readily accessible. In machine switching systems, provision is made for terminating the circuits in jacks at test desks or frames where they can be picked up by patching cords and tested as conveniently as the corresponding types of circuits in manual offices. Machine switching systems offer an important advantage in transmission testing work, particularly in trunk testing, in that the circuits to be tested can be looped automatically by the use of dials or selector test sets, thereby doing away with the necessity for having someone at the distant office complete the loops manually.

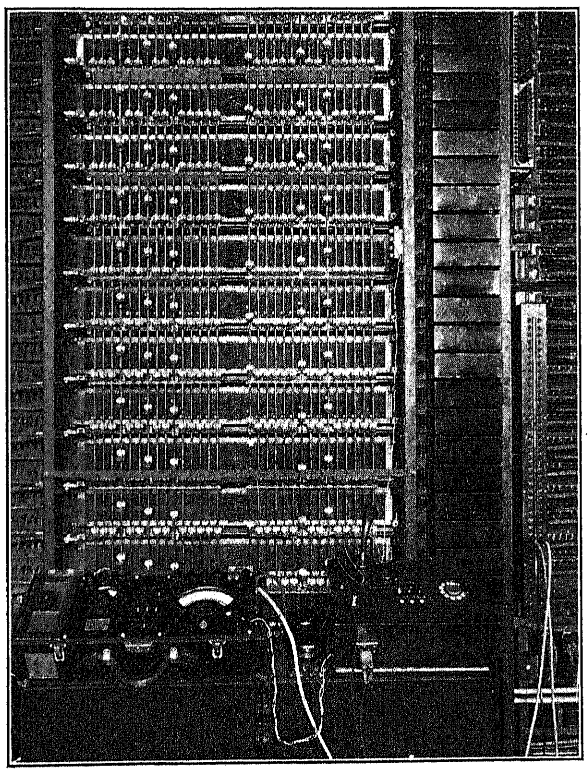

Fig. 8—Illustration of a 3-A Transmission Measuring Set, Set up in a Panel Machine Switching Office for Testing District Selectors

In panel machine switching offices the circuits involving transmission equipment corresponding to cord circuits are the "district" and "incoming" selectors. These are tested by setting up the transmission measuring set at the district or incoming frames and connecting the set to the test jacks associated with the circuits. Tests on trunks between manual and panel machine switching offices where both systems are in operation in the same exchange area are generally made from the manual office, the loops being dialed from the *A* switchboard, while trunks between two machine switching offices are tested from the outgoing end of the trunks.

Fig. 8 shows a 3-*A* transmission measuring set, set up in a machine switching office ready for making tests on district selectors. To illustrate the general method

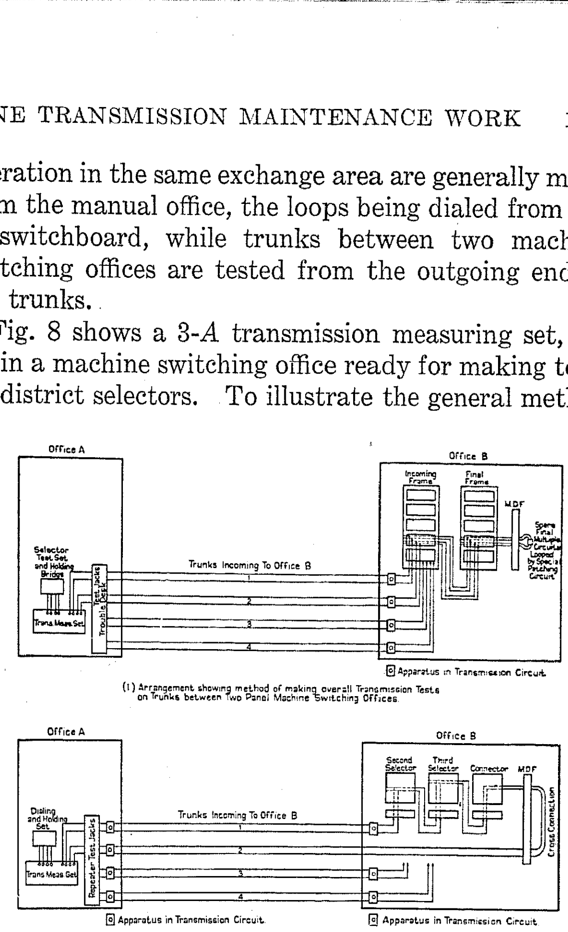

Fig. 9—Schematic Diagrams Showing Methods of Making Transmission Tests on (a) Trunks Between Panel Machine Switching Offices and (b) between Step by Step Machine Switching Offices

of testing panel machine switching circuits, the upper diagram of Fig. 9 shows the schematic arrangement for measuring trunks between two panel machine switching offices. The transmission measuring set is located at office A, and connection made to the outgoing end of the trunks to office B through the test jacks at the trouble desk. A standard selector test set used in local maintenance work and a high impedance holding coil are also connected to the trunks through the measuring set, these being used to establish the loop and hold this loop while the tests are made. At office B two spare multiple circuits are cross-connected at the main distributing frame. Any two trunks in the group can then be automatically looped together at office B by the use of the selector test set which functions to connect the trunks to the two spare multiple circuits previously cross-connected at office B.

In step-by-step machine switching offices the circuits involving transmission equipment corresponding to cord circuits are the connectors. Each connector is provided with a test jack through which connection can be made to a transmission measuring set and the loop completed over a test trunk by dialing. Local selectors do not contain any equipment other than contacts and wiring in the transmission circuits but these can be tested in the same manner as connectors if it is desired to check the wiping contacts and wiring. Toll selectors which involve equipment in the trans-

mission circuit can also be tested in the same manner as connectors. Trunks between manual and machine switching offices can be most conveniently tested from the manual office, the trunk loops being established directly by dialing.

To illustrate the general method of testing step-by-step machine switching circuits, the lower diagram of Fig. 9 shows the schematic circuit arrangement for testing trunks between two machine switching offices. The transmission measuring set is located at office A in a position so that it can be patched to the outgoing trunk repeater test jacks and an arrangement for dialing and holding is connected to the trunks through the measuring set. At office B the apparatus in one trunk is disconnected and this trunk used as a test trunk by cross-connecting it at the main distributing frame to a spare subscriber's multiple terminal. All trunks in the group can then be tested by dialing over them, from office A, the number of this spare terminal at office B which automatically loops them back over the test trunk.

Maintenance of Subscribers' Lines and Stations.

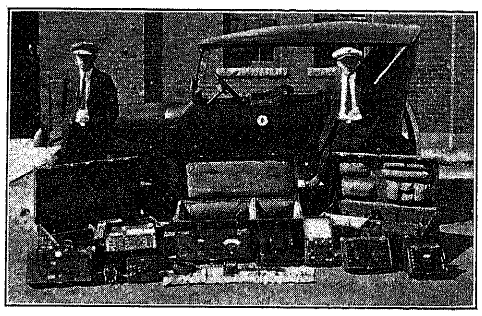

FIG. 10—ILLUSTRATION OF A TYPICAL TRANSMISSION TESTING TEAM LAYOUT

The circuits making up subscribers' lines from switchboard to instruments consist simply of pairs of conductors, almost always in cable, with the necessary protective devices. These can be checked by certain d-c. tests described in a recent paper.[7] Equipment is also provided in local test boards for use in making talking transmission tests between the station and the test board. Accurate machine methods for determining the efficiency of transmitters and receivers have been developed for testing new instruments and instruments returned from service.

General Scheme of Testing Exchange Area Circuits. The plan being followed in the Bell System for systematically checking the transmission conditions of exchange area circuits is to have all offices tested periodically by men equipped with portable transmission measuring sets who travel from office to office. It has been found by experience that after an office has once been tested and any transmission troubles eliminated, it is only necessary thereafter to make transmission tests at infrequent intervals, these subsequent tests serving primarily as a check on the local maintenance conditions. With a testing plan of this kind large areas can be covered by a small traveling force with a small amount of testing equipment. This results in a very economical transmission testing program while at the same time insuring that transmission conditions are maintained satisfactorily.

Fig. 10 shows a typical transmission testing team layout. The team is equipped with an automobile which proves an economical means of transportation between offices and exchange areas and provides a convenient method for carrying the testing equipment. During transportation this equipment is packed in padded trunks which insures against injury. In this particular case the equipment includes, in addition to transmission testing sets and oscillators, other apparatus such as a wheatstone bridge, crosstalk set and noise measuring set so that other maintenance work may be done in connection with transmission testing whenever this is desired.

RESULTS ACCOMPLISHED

The results accomplished in transmission maintenance work can best be appreciated by considering the kinds of troubles which adversely affect transmission and which can be detected and eliminated by routine testing methods. Consideration is first given to the general causes of troubles which are detrimental to both toll and local transmission, and later the features in this connection more particularly identified with telephone repeaters and carrier systems are discussed.

The different classes of circuits given in Table I are made up of various combinations of the following individual parts:

Repeating Coils	Plugs	
Retardation Coils	Jacks	
Relays	Keys	
Condensers	Heat Coils	
Resistances	Carbons	
Auto-Transformers	Wiring	Switchboard to M. D. F.
Induction Coils		Cross-connection
Loading Coils		Outside
Cords	Transmitters	
	Receivers	

The above parts are combined in various ways to make up the complete operating circuits such as cord circuits, operators' circuits, trunk circuits, etc. Each complete circuit causes a definite normal loss to telephone transmission which must be taken account of in designing the plant to meet various service requirements. If, however, any of the parts used are defective, if the wrong combinations of parts are used, or if the installation work is not correctly done, excess transmission losses will result which may very seriously affect the transmission when the particular circuits involved are employed in an overall connection.

Classification of Common Types of Troubles. An analysis of a large amount of transmission testing data has made it possible to develop a definite trouble classification which is particularly helpful in trans-

mission maintenance work and which permits the most efficient use of the results in eliminating transmission troubles. Experience has shown that the troubles found can be divided into two general classes, A—Troubles which can be detected either by simple d-c. or a-c. tests in connection with the regular day-by-day maintenance work or by transmission measuring sets, and B—Troubles which can be detected most readily by transmission measuring sets. The most important troubles in the above classes are as follows:

Class A	Class B
Opens	Electrical Defects
Grounds	Incorrect Wiring
Crosses	Wrong Type of Equipment
Cutouts	Missing Equipment
	High Resistance
	Low Insulation

If, in making transmission tests in a central office, a high percentage of Class A troubles is found the remedy is generally to instigate more rigid local maintenance routines paying particular attention to the type of circuits in which the troubles are located. The percentage of Class B troubles is not as a rule as high as the Class A troubles and experience has shown that when Class B troubles are once eliminated by transmission testing methods only infrequent subsequent tests are required to take care of any additional troubles of this class which may get into the plant.

In determining what constitutes an excess loss, the value of the transmission as well as the practical design and manufacturing considerations to meet operating limits are taken into account. An excess gain is also considered as a trouble on circuits equipped with amplifiers, since this may produce poor quality of transmission which is likely to be more detrimental to service than an excess loss. The value of transmission based on economical design considerations varies, depending on the first cost and annual charge of the particular types of circuits involved. A gain of one TU in the toll plant is generally worth more, for example, than one in the local plant, since it costs more to provide. In transmission maintenance work the cost of making transmission tests and clearing trouble is balanced against the value of the transmission gained for the purpose of establishing economical transmission limits to work to.

Specific Examples of Common Troubles Found and their Effect on Transmission. Certain kinds of troubles which are detected by transmission measuring sets do not cause excess losses which can be quantitatively measured. Such troubles are, however, readily detected by "ear balance" transmission measuring sets in that they cause noise or scratches and by the "meter balance" sets from fluctuations of the needle of the indicating meter. The most common trouble of this kind is due to cutouts or opens which may be caused by dirty connections, loose connections, improper key and relay adjustments, etc. While not causing a quantitative value of excess loss, this class of trouble is very detrimental to transmission and more serious in many instances than fixed excess losses. Indeterminate troubles of this nature are given an arbitrary excess loss value based on experience.

Considering troubles which give definite losses, the most common kinds are caused by electrical defects in equipment, incorrect wiring of equipment in circuits and wrong types of equipment. The other classes of troubles, such as crosses, high resistances, and low insulation, also generally give measurable excess losses but these are not as common in the plant, since troubles of this nature are more likely to affect the signaling and operation of the circuits and are, therefore, eliminated by the regular maintenance work. Missing equipment will in certain cases cause a gain in transmission but affects the circuits adversely in other ways.

Typical examples of common troubles, with the excess losses which they cause, are given in the following table:

Type of Circuit and Equipment	Cause of Trouble	Approximate Excess Transmission Loss
Repeating coils in cords, incoming trunk circuits, selectors, toll connectors	Electrical defects (Generally short circuited turns)	1.5 to 5.0 TU
	Incorrect wiring (Generally reversed windings)	2.0 to 13.0 TU
Supervisory relays in "A" cord circuits	Electrical defects (Open non-inductive winding)	About 2.5 TU
Bridged retardation coils or relays in toll cord circuits, composite sets, connectors and step-by-step repeaters	Electrical defects (Generally short circuited turns)	1.0 to 5.0 TU
Repeating coils on loaded toll switching trunks	Wrong type of equipment, incorrect wiring	1.0 to 4.0 TU
Induction coils in operators' telephone sets	Electrical defects. Incorrect wiring	1.0 to 13.0 TU

There are, of course, many other specific types of troubles detected by transmission tests which give definite quantitative losses but the above will serve to illustrate the value of this testing work in eliminating excess losses in a telephone plant.

Maintenance Features Peculiar to Telephone Repeaters and Carrier Systems. The same classification of troubles discussed above applies to repeaters and carrier systems. Amplifier equipment, however, employs certain features which are not common to the more simple telephone circuits and some of the troubles which may occur if the proper maintenance procedures are not followed will seriously affect service. It is for this reason that repeater and carrier installations are provided with special testing equipment which is always available for use either in routine maintenance or in locating and clearing any troubles which may occur in service. Automatic regulating devices are also provided wherever this is practicable in order to

reduce to a minimum the amount of manual regulation and maintenance.

The important elements in both repeaters and carriers which may directly affect transmission or cause service troubles in other ways are as follows:

Filament Batteries	Potentiometers
Plate Batteries	Filters
Grid Batteries	Transmission Equalizers
Vacuum Tubes	Signaling Equipment
Balancing Equipment	Patching Arrangements

The tests outlined in the main body of the paper aim to insure that the above essential parts of repeater and carrier circuits are functioning properly and that the equipment as a whole is giving the desired results in overall transmission efficiency.

Conclusion

The above discussions of testing methods and the results accomplished indicate how a comprehensive and economical transmission maintenance program can be applied to a telephone plant to check the volume efficiency of the circuits. Consideration is continually being given to new testing methods and their applications in order that further improvements in service may be effected and advantage taken of increased economies in testing.

Appendix A

Principles of Testing Methods Closely Associated with Transmission Efficiency Tests

Tests of volume efficiency often need to be supplemented by other methods of testing in transmission maintenance work. Transmission efficiency both as regards volume and quality may be seriously affected by noise or crosstalk, and tests for any conditions of this kind are therefore important in maintenance work. Furthermore when efficiency tests show excess losses or unsatisfactory circuit conditions other testing methods prove very valuable in locating the cause.

To illustrate this phase of transmission maintenance work the principles of some of the more important testing methods are briefly described below. Two of the tests described employ a method very similar to loop transmission testing while others employ the well known "null" method. A special method employing three winding transformers and amplifiers widely used to determine impedance balance conditions between lines and networks is also described. Several methods which involve simply current and voltage measurements have been mentioned in this paper but these are generally well known and therefore require no detailed description.

1. Measurements of Crosstalk

In the circuit shown in Fig. 11, if a-c. power is supplied to a circuit known as the "disturbing" circuit and unbalances exist between this circuit and a second known as the "disturbed" circuit, power will be transferred from one circuit to the other causing crosstalk in the second. A definite power transmission loss therefore takes place between the two circuits which can be measured by a loop transmission test similar to the efficiency tests described in the main body of the paper. An adjustable shunt called a "crosstalk meter" calibrated in either *TU* or in crosstalk units is substituted for the two circuits. With the same power supplied alternately to both the "disturbing" circuit and the meter and with the sending and receiving end impedance conditions as shown, the meter shunt is

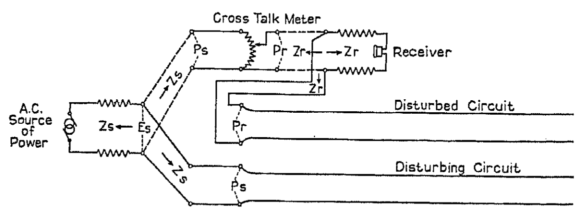

Fig. 11—Diagram Showing Principles of Crosstalk Measurements

adjusted until, in the opinion of the observer, the annoyance produced by the tone in the receiver is judged to be equal for the two conditions. The reading of the shunt if there was no distortion of the line crosstalk currents would then give the volume of crosstalk which could be expressed in *TU* as $10 \log_{10} P_r/P_s$ similar to loop transmission testing. However, this relation only holds approximately in practise since the line crosstalk measured is produced by various currents having different phase relations and a certain amount of distortion therefore occurs. The commercial form of crosstalk set now used is equipped to give the approximate impedance relations required and also

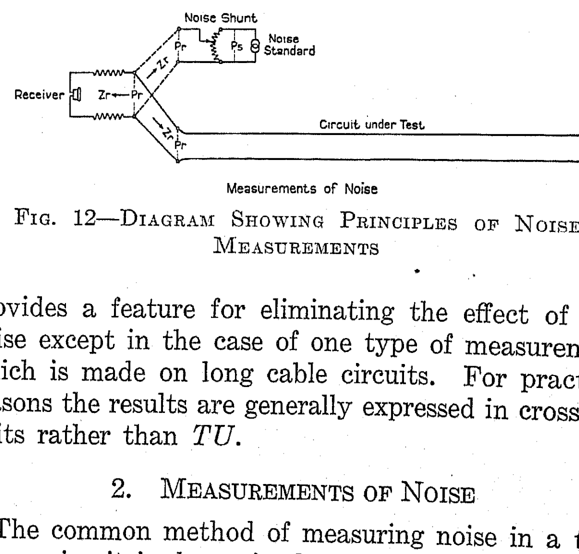

Fig. 12—Diagram Showing Principles of Noise Measurements

provides a feature for eliminating the effect of line noise except in the case of one type of measurement which is made on long cable circuits. For practical reasons the results are generally expressed in crosstalk units rather than *TU*.

2. Measurements of Noise

The common method of measuring noise in a telephone circuit is shown in the diagram of Fig. 12. In this test an artificial noise current produced by a generator of constant power P_s called a "noise standard" is substituted for the line noise current. If the

two noise currents were exactly alike as regards wave shape and the relative magnitude of the frequencies involved they would produce the same tone in the receiver and their volumes could be made equal by adjustment of the noise shunt. The power ratio, P_r/P_s, as indicated by the shunt, would then give a measure of the line noise in terms of the noise standard. This condition however, is not met with in practise due to differences in wave shape of the two noise currents. For this reason noise measurements are made by adjusting the noise shunt until the interfering effects of the noise on the line and from the shunt are judged to be the same for which condition the power supplied to the receiving network by the noise standard is not necessarily the same as that supplied by the line. The receiving end impedances however, are kept as nearly alike as practicable to prevent reflection losses.

3. Measurements of Line-Network Balance (21-Circuit Balance Test)

The testing arrangement of Fig. 13 shows the principle of the 21-circuit balance test referred to in the main body of the paper in connection with telephone repeater and carrier maintenance. In this test the gain of an amplifier calibrated in *TU* is used to compensate for the loss through a three winding transformer or output coil of a telephone repeater. If the impedances of the balancing network and line were exactly alike at all frequencies, *i. e.*, $Z_n = Z_L$, and no other unbalances existed in the circuit none of the power supplied by the amplifier to the input of the three-winding transformer would be transferred to the output, *i. e.*, the power ratio P_s/P_r would be infinity. However, this ideal condition cannot be produced in practise so that there is always a finite power loss between the input and output of the transformer which can be measured approximately by the gain of an amplifier calibrated in *TU*. An internal path for

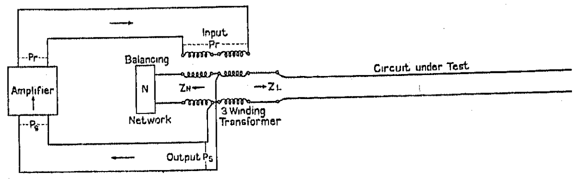

Fig. 13—Diagram Showing Principles of 21 Circuit Balance Tests

currents which may produce "singing" or a sustained tone is established if the gain of the amplifier P_r/P_s is greater than the loss P_s/P_r through the three-winding transformer. As unbalances between network and line become greater the loss through the three-winding transformer becomes less thereby requiring less gain in the amplifier to produce a "singing" condition. It should be noted in this connection that to produce the condition described above exactly, the current received around the "singing" path must be in phase with the starting current. In practise this condition obtains sufficiently accurately so that the gain of the amplifier required to produce "singing" gives an approximate measure of the impedance balance between line and network.

4. Measurements of Resistance, Reactance and Impedance

Diagram (a) of Fig. 14 shows the wheatstone bridge circuit for d-c. resistance measurements. It is unnecessary to describe the well known principles of this

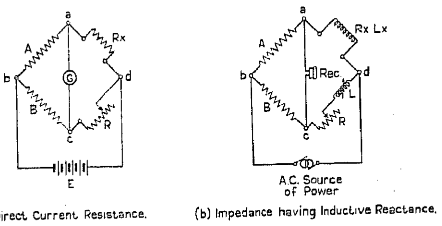

Fig. 14—Diagrams Showing Principles of Null Methods for Measuring Resistance, Reactance and Impedance

bridge but mention is made of it here in view of its importance and use in telephone maintenance work. It supplies an indispensible method of measurement for certain trouble locations, such as crosses and grounds and embodies the fundamental principles of all null tests.

Diagram (b) of Fig. 14 gives a bridge circuit for measuring impedance, the particular arrangement shown being for measurements of impedances having inductive reactance. The bridge measurements express impedance in terms of its resistance component and equivalent inductance or capacity. In measuring an impedance having inductive reactance at any frequency, *f*, for example, a balance gives $R = R_x$ and $L = L_x$. At the frequency *f*, the effective resistance is given directly by the value of *R* and the reactance by the relation, $2\,\pi\,f\,L$. The impedance is the vectorial sum of these two or $\sqrt{R^2 + (2\,\pi\,f\,L)^2}$. In maintenance work involving impedance measurements as will be noted in the next testing method described, the effective resistance component and the equivalent inductance are generally used directly without combining.

5. Measurements of Line Impedance and Location of Impedance Irregularities

Fig. 15 shows a telephone circuit connected to a bridge and terminated at its distant end in characteristic impedance. If the circuit has approximately uniform impedance throughout its length the resistance and equivalent inductance curves of this impedance within a range of frequencies will be fairly smooth as indicated by *A* and *C* of the figure. The curves are not perfectly smooth since it is not practicable to construct the line for perfect impedance uniformity. If at

some point in the circuit an irregularity is present such as an omitted loading coil, an inserted length of line of different construction, etc., which changes the impedance, this will produce a periodic change in the resistance and inductance curves A and C such as shown by Curve B. Curve C will be changed in the same way as Curve A but for simplification this is not shown on the diagram.

The change in impedance in the circuit reflects some of the current sent out back to the sending end where it adds to or subtracts from the sending current depending on the phase relations of the two currents at any particular frequency. Since impedance equals E/I its value changes as the value of I changes. This is made use of in line impedance measuring work to give

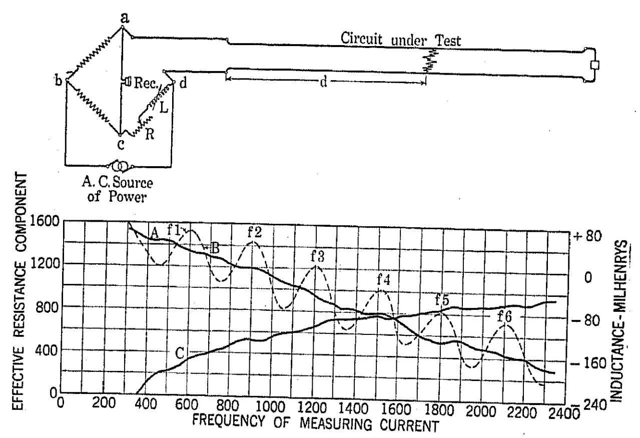

Fig. 15—Diagram and Impedance Curves Showing Principles of Line Impedance Measurements by Null Method and Location of Impedance Irregularities

a location of impedance irregularities which may exist somewhere in the line.

Referring to Fig. 15, let d equal the distance in miles to an impedance irregularity and f, one frequency at which the resistance component of the impedance is a maximum. The next maximum point will occur at a frequency f_2 such that as the frequency has been increased, one complete wave length is added in the distance traveled by the reflected current. Maximum points at f_3, f_4, etc., occur in the same way as the frequency is increased. Considering the two values f_1 and f_2 let

V = velocity of current in miles per second

W_1 = wave length at frequency f_1

W_2 = " " " " f_2

N = number of wave lengths in distance traveled by reflected current or 2 d.

At frequency f_1 then,

$$N = \frac{2\,d}{W_1}$$

and at f_2,
$$N + 1 = \frac{2\,d}{W_2}$$

also at f_1, $W_1 = V/f_1$

and at f_2, $W_2 = V/f_2$

Substituting above

$$N = \frac{2\,d\,f_1}{V} \text{ and}$$

$$N + 1 = \frac{2\,d\,f_2}{V}$$

Subtracting, $$1 = \frac{2\,d\,f_2}{V} - \frac{2\,d\,f_1}{V} \text{ or}$$

$$d = \frac{V}{2\,(f_2 - f_1)} \text{ which is the distance}$$

in miles from the sending end of the circuit to the point of impedance irregularity. The velocity of propagation V is not exactly constant within the entire frequency range but does not vary sufficiently to materially effect the accuracy of impedance trouble locations by this method.

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Discussion

For discussion of this paper see page 1343.

Telephone Circuit Unbalances
Determination of Magnitude and Location

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Review of the Subject.—*The maintenance of telephone circuits in a high state of efficiency with respect to balance is important since unbalances contribute to crosstalk between telephone circuits and to noise when such circuits are involved in inductive exposures. Methods and apparatus are described which afford operating telephone companies means for maintaining their circuits in the condition of minimum practicable unbalance.*

Different types of unbalance are discussed, also their effects under different conditions of energization of the unbalanced circuit and neighboring conductors.

Methods are described for determining:

(1) *The general condition of circuits with respect to balance by crosstalk measurements from their terminals,*

(2) *The approximate location of unbalances along a line by measurements over a range of frequencies with a bridge at one end of the line, and*

(3) *The final location of unbalances by field measurements with an unbalance detector which may be operated by a lineman and does not usually require interruption of telephone service, except momentarily.*

Toll circuit office unbalances are briefly discussed and a special bridge for detecting, locating and measuring the unbalances of composite sets is described.

In an appendix is given a mathematical treatment of the bridge method for locating unbalances and a discussion of the necessity of terminating the circuits involved in the tests in their characteristic line impedances.

Introduction

Noise induced in a telephone circuit may arise in two different ways: First, by reason of dissymmetry in its relation to the inducing circuit, unequal voltages are induced in its two sides; Second, by reason of dissymmetry of its two sides in their relation to ground or to other telephone conductors, the voltage induced between the conductors and ground or along the conductors causes a voltage between the two sides of the unbalanced circuit. In the usual case, the voltages induced to ground or along the conductors are large compared to their differences, so that unbalances of the telephone circuits themselves have an important bearing on the problem of preventing noise in such circuits.

Dissymmetry with respect to the inducing circuit involves, of course, only that part of the disturbed circuit within the inductive exposure. This type of unbalance involves properties of the inducing circuit as well as of the telephone circuit and is therefore dealt with no further in this paper. Unbalances of the telephone circuits themselves may involve those parts of the circuit within the inductive exposure as well as the unexposed parts of the circuit and the terminal apparatus.

A telephone circuit would be balanced if its two sides had equal self and mutual impedances and equal admittances to ground and to other conductors in each elementary section of its length. In other words, the sides of a balanced circuit would be symmetrical in their relations to ground and to other nearby circuits. Such a circuit would have no difference in voltage between its two sides when equal voltages are induced between them and ground at any point of the circuit or when equal voltages are induced along them.

It should be noted that this definition does not require longitudinal uniformity as a condition of balance. It merely requires that corresponding constants of the two sides of the circuit shall be equal at the same point and that this condition shall be satisfied for every point along the circuit. In practise it is neither necessary nor possible that this definition be rigorously satisfied, but a close approach to this ideal state of balance is obtained by transposing or twisting the conductors as a pair so that unbalances in neighboring sections of any circuit tend to neutralize, and by careful attention to the design and maintenance of the line and all connected apparatus.

Types of Unbalances

Unbalances of telephone circuits are of two general types. The first may be called self unbalances involving only the conductors of the circuit and ground, and the second, mutual unbalances involving also the other conductors. Thus, a self unbalance would exist even though all conductors except those of the circuit in question were removed to a great distance. Since in the usual case, both sides of circuits are constructed of wires of the same size and conductivity and at the same distance above the earth, there are no inherent self unbalances. Such unbalances are largely accidental and arise from such causes as defective joints, defective loading coils, broken insulators, tree leaks, or from dissymmetry in apparatus inserting series impedances in the circuit, or connecting admittances between the sides of the circuit and ground. Unbalances of this type do not depend upon the condition of surrounding circuits, although the noise effects caused by these unbalances may so depend.

The resultant unbalanced current or voltage in a circuit depends on both the self and mutual unbalances and upon the mode of energization and terminal impedances of both the circuit in question and neighbor-

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ing conductors. In practise, it is not possible to measure the effects of the two types of unbalance separately.

It will be of interest to discuss briefly the relative importance of the two types of unbalance in their effects on noise and crosstalk with a view to examining the adequacy of various methods which may be used for measuring and locating unbalances.

Fig. 1 indicates a circuit which is perfectly balanced. This circuit consists of two similar conductors parallel to each other and to the plane surface of the earth, and removed to a great distance from all other conductors. Such a circuit can have only self unbalances, which in the practical case might consist of slight differences in conductor diameter, in conductivity and in sag, variations in the contour of the earth, resistance in joints and differences in series impedances or shunt admittances introduced by connected apparatus.

Fig. 2 shows the four parallel untransposed conductors of a phantom group. In this case the phantom would be unbalanced to each of its side circuits. If equal voltages were induced to ground or along each of the four conductors (similar apparatus being assumed connected to the two side circuits), a voltage would be caused in each side circuit, while no voltage would be caused in the phantom.

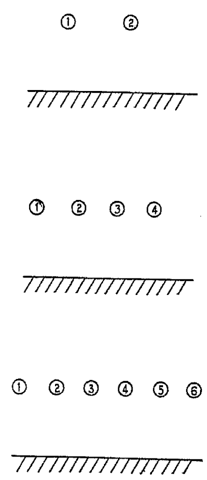
Fig. 3 indicates three circuits, each consisting of two similar untransposed parallel conductors. In this case, assuming perfect construction, each circuit would have no self unbalances. However, no one of the circuits would be symmetrical in its relation to each of the other conductors. If equal voltages were induced to ground or along the two sides of circuit 3-4, and both 1-2 and 5-6 were left either free or connected to earth, no voltage would be caused between the sides of 3-4. The same would be true if equal voltages were induced along all six of the conductors or from them to ground. Voltages would be caused, however, in both circuits 1-2 and 5-6 under this condition. A voltage would be set up in circuit 3-4 if equal voltages were induced from its sides to ground or along its sides, while different conditions were imposed on circuits 1-2 and 5-6 as, for example, if 1-2 were isolated and 5-6 connected to ground.

In the practical case, involving multi-wire lines, none of the circuits on the line are inherently balanced in respect to their mutual relations to all other conductors. A close approximation to such balance is obtained, however, by means of transpositions whereby the unbalances existing in one section are neutralized by those in a nearby section of line. Thus, transposition errors are a source of mutual unbalances.

In any practicable method of determining unbalances, the current or voltage measured in the circuit, or the impedance of a compensating unbalance which would reduce the voltage and current to zero, are complicated functions of the location, character and magnitude of the self and mutual unbalances, and the mode of energization and terminal impedances of the test circuit and neighboring conductors. The effects of mutual unbalances are particularly dependent upon the energization and impedance conditions of the neighboring conductors.

Methods of Energizing Circuits

If unbalances are measured by energizing a superposed circuit consisting of the two sides of the circuit in parallel to ground, leaving other conductors free, and observing the unbalanced current, the effect of transposition errors will usually be small as compared to even small self unbalances which might be present. The current due to the mutual unbalances is, in this case, a secondary effect, because of the relatively small amount of power in each of the neighboring conductors. The current and voltage in the energized superposed circuit induce current and voltage in circuits composed of the neighboring conductors and ground, since transpositions are not effective in reducing induction between grounded



Figs. 1, 2 and 3

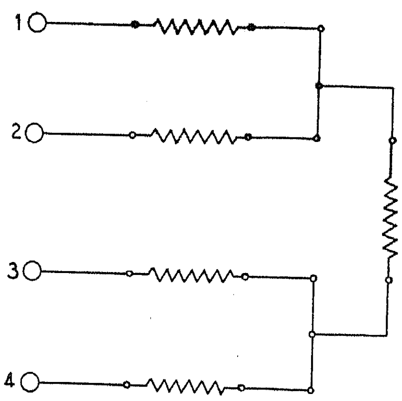


Fig. 4—Simple Network for Terminating a Phantom Group

circuits. These induced currents and voltages in the individual neighboring conductors are much smaller than the original current and voltage in the conductors of the superposed circuit. The induced currents and voltages in turn react by induction upon the current and voltage in the superposed circuit, and because of the transposition error this reaction is unsymmetrical, resulting in a current and voltage in the metallic circuit which forms one side of the superposed circuit.

If unbalances are measured by applying with respect to ground, the same voltage to all of the conductors on the line, the current set up in the test circuit due to a transposition error will no longer be a secondary effect, since the voltages and currents in each of the neighboring conductors will then be of approximately the same magnitudes as those in the conductors of the circuit whose unbalances are being determined. In this case the effect of a transposition error is increased relatively to that in the first case and becomes comparable in importance to the effect of a self unbalance.

If, however, the unbalances are measured by determining the crosstalk between the circuit in question and its phantom, with certain exceptions noted below the transposition error is also a primary effect. The current and voltage in closely adjacent conductors (those of the other side of the phantom) are of the same magnitude, but opposite in phase, as those in the conductors of the circuit whose unbalances are being measured. In this case the effects of a transposition error are probably more important as compared to those of self unbalances, than if all conductors of the line were energized simultaneously at the same voltage.

In an inductive exposure the voltages to ground and longitudinal currents of all conductors of the various circuits on the line are of the same order of magnitude. Consequently, the effects of transposition errors in the unexposed sections of line on the resultant unbalanced current in a circuit are more important than if this circuit alone were energized.

In maintaining telephone circuits in a condition of satisfactory balance from the standpoint of noise it is necessary that the methods employed to detect and locate unbalances be such as to cover all types of dissymmetry which contribute to noise. In order that all unbalances might be given proper weight in the measurement, it would be desirable, of course, to energize the test and the neighboring conductors in the same manner, relatively, as when the energization is due to inductive exposures. This is, however, impracticable.

If the methods used are sensitive enough to detect all sources of unbalance on any part of the circuit, large enough to contribute to noise or crosstalk, it is not essential that they cause the same relative effects on the measurements as on the noise or crosstalk. As brought out in the previous discussion, the important sources of unbalance contributing to noise are defects in line and connected apparatus (self unbalances) and transposition errors (mutual unbalances). With the exception of vertical pole-pair phantom groups all such unbalances in side circuits contribute to crosstalk in the phantom and this crosstalk may be used therefore as a measure of unbalance of side circuits. Transposition errors in vertical pole-pair phantom groups have very little effect on phantom-to-side crosstalk. Unbalances may exist in any phantom group which will not contribute to crosstalk but which will contribute to noise in the phantom circuit. Similarly, a non-phantom pair may be unbalanced for noise, but these unbalances may not cause appreciable crosstalk to any existing circuit. However, in all these cases where crosstalk is not caused in any existing circuits, special superposed circuits may be set up for test purposes in which these unbalances will cause crosstalk.

A simple arrangement for such a purpose is a superposed circuit having for one side the conductors in parallel of the circuit under test, and ground for the other. As already stated, however, transposition errors cause relatively small effect on crosstalk in this case. If, instead of the ground, the conductors in parallel of another phantom or non-phantomed pair, preferably on the same crossarm, be used to form the superposed circuit, transposition errors will be indicated as well as the self unbalances, in the same manner as in the case of crosstalk between a horizontal phantom and its side.

The methods which are in use in maintaining balanced telephone circuits in the plant of the Bell System utilize the crosstalk to metallic superposed circuits made up as above described as a criterion of unbalance. These methods provide means for indicating the unbalanced condition of a circuit by measurements of crosstalk made at its terminals; for obtaining an approximate location of sources of unbalance by measurements from one end of the circuit; and for securing a definite location of the unbalance by local measurements along the circuit, these latter measurements not involving opening the circuit.

In general, the magnitude of the crosstalk measured between two circuits will depend upon the frequency of the testing current used. When the source of the crosstalk is a capacitance or inductance unbalance, this is due to the increase in crosstalk with frequency. Also, when more than one unbalance is present, the crosstalk currents caused by the different unbalances add at some frequencies and subtract at others. Thus, a single-frequency crosstalk measurement is not a reliable indication of the general condition of the circuit as regards unbalances. Crosstalk measurements must, therefore, be made at a number of single frequencies or with a source of energy of complex wave form.

In practise both schemes are used, a complex wave being employed to obtain an indication of the general condition of the circuit and a number of single frequencies being employed successively to determine the location of unbalances.

Termination of Circuits

It is desirable in making unbalance or crosstalk tests that the measurements be made in such a way that the effect of any given unbalance on the crosstalk or equivalent unbalance of the circuit be independent of the length of circuit tested. In order to obtain this result, reflected waves from the distant ends of the circuits must be minimized. This is accomplished by terminating the circuits in networks whose impedances approximately equal the impedances of infinite lengths of the circuit. The precision with which it is necessary for these network impedances to simulate the characteristic line impedances depends upon the attenuation losses in the section of line under test. If the section be long enough terminations are unnecessary. Three types of network have been developed for use in these measurements which simulate the line impedances to different degrees of precision.

The first type of network, which is shown by Fig. 4, consists of a group of carefully balanced resistances. It is for use principally on cable circuits but may also be

used on open-wire circuits for the crosstalk tests to indicate the general condition of a circuit as to unbalances. More accurate terminations are required for the measurements to obtain unbalance locations unless the section tested is long.

A second type of network is shown by Fig. 5 which is more particularly adapted to tests on open-wire circuits. In this case the circuit under test and the superposed circuit used as the reference are terminated by a

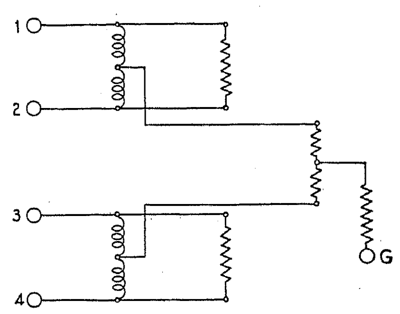


Fig. 5—Network for Terminating Simultaneously All Circuits of a Phantom Group. Binding Post G may be Connected to Four Wires in Parallel of a Nearby Phantom Group

carefully balanced retardation coil and resistances, rather than by resistances alone. Through the use of keys it is possible to arrange for terminating circuits of various types including both non-loaded and loaded circuits.

The third type of network shown by Fig. 6 is used when a more accurate termination than is possible with the other types is required. This network is usually

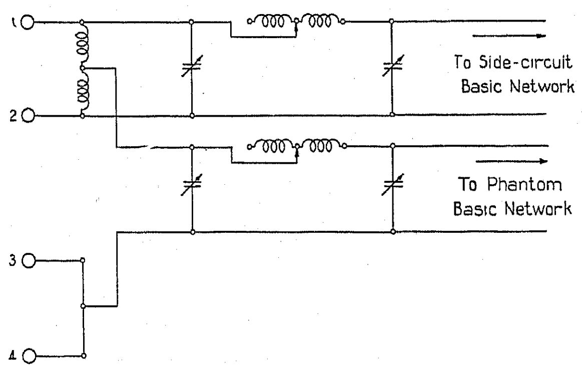


Fig. 6—Adjustable Building-Out Section. Schematic Circuit when used for Building-Out Simultaneously Phantom and Side Circuit

required only in connection with measurements made at one of the circuit terminals to locate unbalances. By the use of this adjustable "building-out section" in conjunction with an adjustable "basic network," it is possible to terminate open-wire circuits of any type and in the case of loaded circuits, at any point in the loading section, with a minimum of departure from the correct impedance. The networks of this third type are similar to those used in securing two-way repeater operation.

Crosstalk Set

The crosstalk set shown by Fig. 7 is used for measuring crosstalk between telephone circuits at frequencies within the voice range. This figure shows the schematic circuit arrangement when measuring "near-end" phantom-to-side crosstalk. "Near-end" crosstalk occurs when the talking and listening are done at the same end of the line. In the left-hand position of the switch, a source of testing current is connected to a crosstalk meter, which is a simple form of potentiometer. In the right-hand position the switch is connected to transformer A, the impedance ratio of which is adjustable, so that the impedance of any standard type of telephone circuit as measured through the transformer can be made approximately equal to that of the crosstalk meter. With this arrangement the power into the transformer is approximately equal to that into the crosstalk meter, particularly when the impedance of the source of current is about equal to that of the crosstalk meter.

Transformer B, also an adjustable impedance ratio

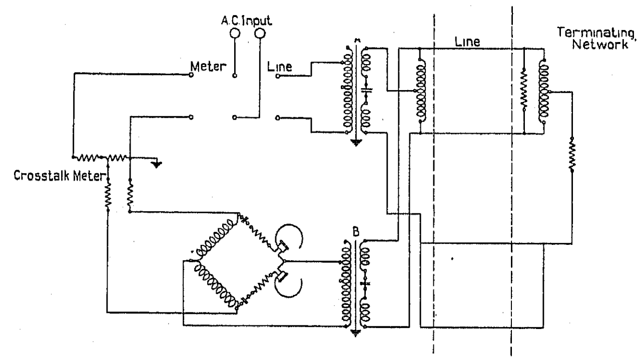


Fig. 7—Crosstalk Set. Schematic Circuit when Measuring Near-End Phantom-to-Side Crosstalk

transformer, is designed to permit a disturbed circuit of any standard type to deliver about the same power to the crosstalk set as it would deliver to a long circuit of its own type. Both transformer B and the crosstalk meter are connected to a network of resistances, condensers, receivers and a retardation coil, arranged in the same general manner as the four arms of a Wheatstone bridge. The branches to which the crosstalk meter and the disturbed circuit are connected have practically no effect upon each other and the disturbed circuit may therefore remain connected to the receivers while the source of disturbing current is switched to the crosstalk meter, thereby allowing the noise from the disturbed circuit to remain constant. This arrangement greatly facilitates accurate crosstalk comparisons on noisy circuits.

Measurements are made by listening to the crosstalk on the disturbed circuit and then to the tone produced by the current from the crosstalk meter, adjusting the latter until the two are equal. The reading of the scale on the crosstalk meter then represents the crosstalk

between the circuits under test expressed in crosstalk units. The crosstalk unit is chosen to indicate a convenient relation between power sent into the disturbing circuit and that received from the disturbed circuit.

Bridge for Locating Unbalances

When the measurement of crosstalk indicates that the unbalances of the circuit are greater than the allowable limit, measurements may be made with an impedance unbalance bridge to obtain an approximate location of the unbalances. The method is adapted from a similar one for locating irregularities in the impedances of metallic telephone circuits which interfere with repeater operation. It may be usefully applied only to circuits of considerable length, approximating 50 miles or more of open wire. Shorter lengths may be tested in cables. The bridge is shown in schematic form in Fig. 8 connected, with a phantom circuit as a superposed reference, to locate the unbalances of a side circuit.

The bridge is made up of a pair of equal ratio arms with a fixed inductance and resistance which are con-

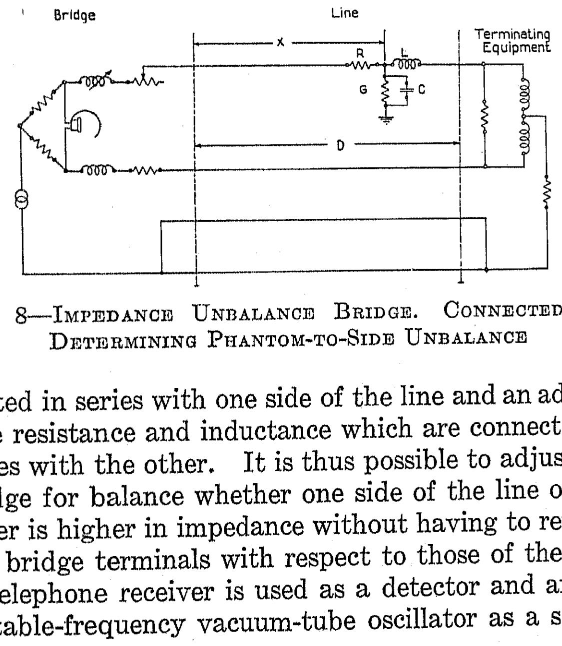


Fig. 8—Impedance Unbalance Bridge. Connected for Determining Phantom-to-Side Unbalance

nected in series with one side of the line and an adjustable resistance and inductance which are connected in series with the other. It is thus possible to adjust the bridge for balance whether one side of the line or the other is higher in impedance without having to reverse the bridge terminals with respect to those of the line. A telephone receiver is used as a detector and an adjustable-frequency vacuum-tube oscillator as a source of energy.

The method consists in balancing the bridge at a number of frequencies at definite intervals in the range from 200 to 2000 cycles and determining the magnitude and sign of the resistance and inductance unbalances which must be inserted in the bridge in order to compensate for those in the line. The values of equivalent resistance and inductance unbalance thus obtained are plotted as functions of the frequency. From these curves a location may be determined for the unbalance or unbalances if their number is not too great.

In principle this method is based upon the finite velocity of phase propagation of electric waves along the reference circuit to the unbalance and back along the circuit under test to the bridge. Thereby, the wave of unbalanced current received at the bridge from the distant unbalance is retarded in phase with respect to the current sent into the reference circuit, in addition to any localized phase shift produced by this unbalance. Only a localized phase shift occurs in the crosstalk or unbalance current set up by the compensating unbalance which is inserted in the bridge. Therefore. the compensating unbalance must be of such a character that its phase shift differs from that produced by the distant unbalance by an amount equal to the phase retardation caused by the finite velocity of phase propagation along the circuits. This phase retardation for a given location of unbalance depends on the frequency of the test current, since the distance to the unbalance measured in wave lengths varies inversely with the frequency. By measuring over a range of frequencies, the localized phase shift may be eliminated and that frequency ascertained for which the total distance of propagation to and from the unbalance is equal to one wave length. Thus its approximate location may be determined if the velocities of phase propagation on the two circuits be known. The method is based upon the assumption that the velocities of phase propagation and localized phase shifts are substantially independent of the frequency. These assumptions hold sufficiently well over the necessary working range of frequencies, and for the types of unbalance usually encountered. A mathematical discussion of the method is given in an appendix.

As an example of the method, consider a non-loaded open-wire phantom group in which the velocities of phase propagation are 180,000 mi. per sec. Let it be assumed that a series resistance unbalance is present 100 miles from the bridge. This is a wave length at 1800 cycles. It is various fractions of a wave length at other frequencies as indicated by the table below. This table indicates also the phase of the current in the reference (phantom) circuit at the position of the unbalance and that of the crosstalk current when propagated back over the circuit under test to the bridge, with respect to the current sent into the reference circuit at the bridge.

f	Fraction of Wave Length	Phase of Current in Reference Circuit at Unbalance	Phase of Crosstalk Current at Bridge Due to Unbalance at $L = 100$
225	1/8	−45 deg.	−90 deg.
450	1/4	−90 deg.	−180 deg.
675	3/8	−135 deg.	−270 deg.
900	1/2	−180 deg.	0 deg.
1125	5/8	−225 deg.	−90 deg.
1350	3/4	−270 deg.	−180 deg.
1575	7/8	−315 deg.	−270 deg.
1800	1	0 deg.	0 deg.

If unbalance be inserted at the bridge to compensate for that in the line, it is evident that at 900 cycles and at 1800 cycles this compensating unbalance must be of the same character (resistance in this case) as that out on the line and on the opposite side of the circuit.

At 450 and 1350 cycles the compensating unbalance will also be resistance but in the same side of the line. At 225 and 1125 cycles the compensating unbalance must be an inductive reactance in the same side of the line and at 675 and 1575 cycles an inductive reactance in the opposite side of the line. The values of equivalent unbalance are plotted on Fig. 9, changes in attenuation with frequency being neglected. It will be seen that periodic curves are obtained having a frequency interval of 900 cycles. This is the frequency for which the distance out to the unbalance and back corresponds to a full wave length. The rule for cases in which the velocity of phase propagation is the same in the test and reference circuits, is expressed by the formula:

$$X = \frac{V}{2 f_i}$$

where X is the distance to the V is the unbalance, velocity of phase propagation and f_i is the frequency interval of the curves.

When more than one unbalance is present, the curves obtained are the resultants of the simple periodic curves

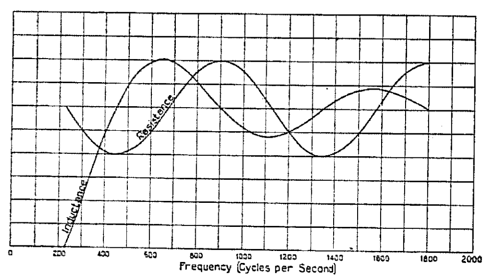


FIG. 9—EQUIVALENT IMPEDANCE UNBALANCE. NON-LOADED OPEN-WIRE SIDE CIRCUIT

which each of the individual unbalances would give if present alone. It is evident that when there are a large number of unbalances present the curves become very difficult of analysis. As a rule, the periods corresponding to the several unbalances present have no common integral multiple within the range of frequencies tested. Analysis by the methods used for analyzing periodic waves is therefore not possible. While the curves are thus made up of periodic waves they are not in themselves periodic.

When the several unbalances differ considerably in size it is often possible to make an analysis by inspection locating initially the largest unbalance. After this one unbalance has been located and cleared a second set of curves may be made which will now contain one less unbalance than before and will be correspondingly simpler to analyze. Where the unbalances are of approximately the same size but at a considerable distance apart they may often be separated by arranging to terminate the circuit at some intermediate point which will include one or two of the unbalances and eliminate the others. When the nearer unbalance has been located and cleared the longer section may be tested and the more distant ones located.

Fig. 10 shows curves taken on a cable circuit before and after removing a series resistance unbalance. Figs. 11 and 12 show curves taken on open-wire circuits, the former on a phantom circuit, before and after removing an inductance unbalance caused by the use of the wrong type of loading coil on one of the sides, and

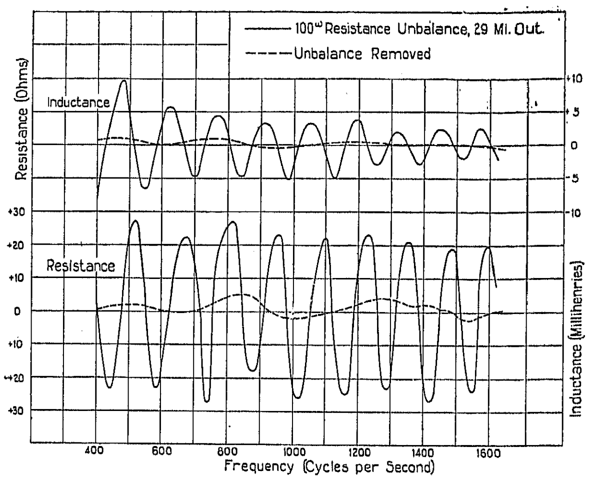


FIG. 10—EQUIVALENT IMPEDANCE UNBALANCE. HEAVY-LOADED NO. 19-GAGE CABLE SIDE CIRCUIT

the latter on a side circuit, before and after correcting a transposition error. It will be noted that equivalent resistance and inductance unbalances are plotted rather than equivalent resistance and reactance unbalances. This is done both because of greater convenience and because the former curves are more easily analyzed. The reactance in the bridge is introduced by an inductometer from which inductance unbalance may be read directly. If reactance were desired it would have to be calculated from the inductance setting. When the unbalance is a resistance, its effect is approximately independent of frequency and an equivalent resistance curve is obtained which is periodic while the inductance curve is the product of a periodic curve and of a curve

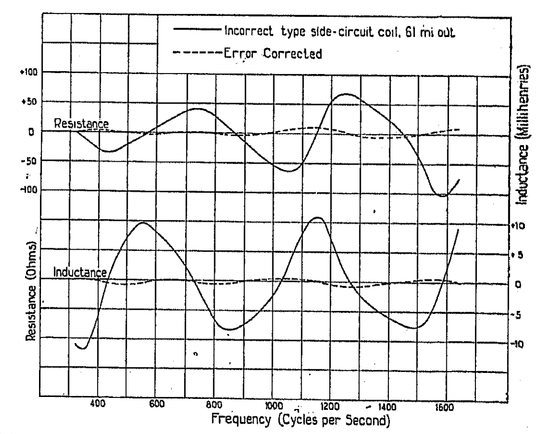


FIG. 11—EQUIVALENT IMPEDANCE UNBALANCE. LOADED NO. 8 B. W. G. OPEN-WIRE PHANTOM CIRCUIT

whose ordinates are inversely proportional to frequency. When the unbalance is series inductance or shunt capacitance, the effect increases with frequency and an equivalent inductance curve is obtained which is periodic while the resistance curve is the product of a periodic curve and of a curve whose ordinates are directly proportional to frequency. Thus, for both types of unbalance one periodic curve is obtained, while if reactance were plotted, only resistance unbalances would yield such curves. These curves are, of course,

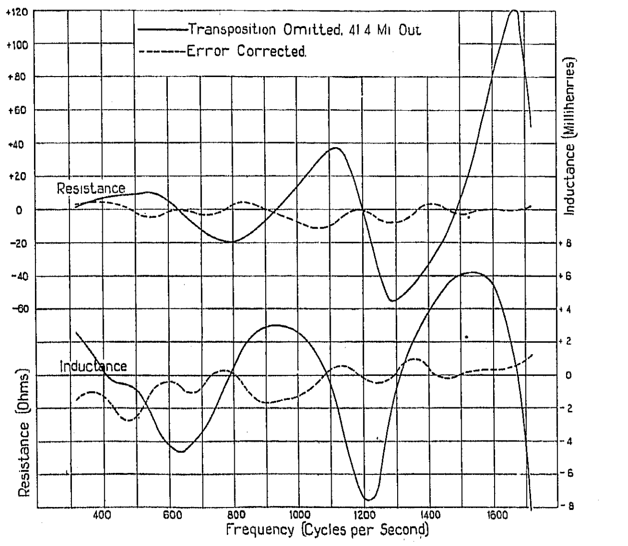


Fig. 12—Equivalent Impedance Unbalance. Loaded No. 12 N. B. S. G. Open-Wire Side Circuit

modified somewhat by the variation of attenuation with frequency but usually this effect is too small to introduce much difficulty into the analysis.

From the information given by these curves it is usually practicable to locate a single unbalance or one considerably larger than any other existing on the circuit, within one or two 8-mile transposition sections. Series unbalances in loading coils or immediately adjacent thereto may usually be located at the correct load point. To locate definitely transposition errors and resistance in joints, in either loaded or non-loaded lines, it will often be necessary to cover two 8-mi. transposition sections with the unbalance detector as described below.

Unbalance Detector

This unbalance detector consists of two parts, known as the sending and receiving sets. These parts are bridged to a circuit at points on either side of the suspected point of unbalance as indicated by the bridge, it being unnecessary to open the line wires. A test with the apparatus shows definitely if the circuit is unbalanced between the points at which the two parts are connected. It does not show the location of the unbalance. If an unbalance is found to exist in the section of line between the sets, they are moved closer together and the test repeated, this process being continued until the unbalance is reached. Fig. 13 shows a schematic circuit of the two parts of the set as arranged for testing side circuits of a phantom group. The entire equipment can be operated by one person.

The sending set is shown as a single-frequency generator connected across the phantom or other reference circuit, the sides being short-circuited. The receiving set consists essentially of two phantom coils, two potentiometers and a receiver. The single-frequency generator causes a current in the phantom circuit in both directions from the point of connection. If the four wires are perfectly balanced, the currents in them will be equal in magnitude and phase if the potentiometers are set so that the resistances on the two sides of the contacts are equal. In this case no sound will be heard in a receiver when connected to either of the phantom coils.

If a resistance unbalance X is present in wire 1, the impedances of wires 1 and 2 will be unequal and the currents in them, therefore, unequal. In this case a tone will be heard in the receiver. By adjusting the potentiometer contact until the unbalance of the potentiometer offsets the effect of X, the currents in the line windings of the coil will be approximately equal and the currents in the receiver consequently small. The reading of the potentiometer can, therefore, be used to indicate the condition of the circuit between the sending and receiving sets. When a transposition error is present the unbalance caused is indicated by current in the receiver which it is not possible to balance out by adjusting the potentiometer.

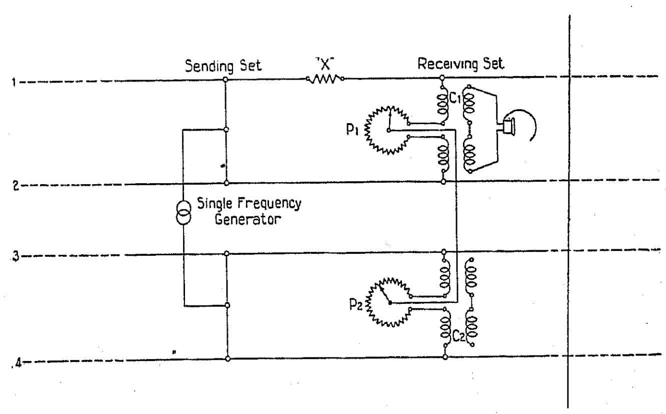


Fig. 13—Unbalance Detector. Schematic Circuit when Testing Side Circuits of Phantom Group

The actual circuit of the apparatus differs somewhat from that shown in the schematic circuit. The oscillator is not permanently connected across the circuit to be tested, but is arranged so that it can be connected to it by relays controlled by a key in the receiving set. When the key is not operated, the oscillator is not in operation and the sending set is connected to the circuit in such a manner as not to interfere with the use of the circuit for telephone purposes.

In addition to its use for finally locating an unbalance

whose approximate location has been determined by the impedance unbalance bridge, this unbalance detector may be used also when the length of line is too short to justify the use of the bridge method and when the number of the unbalances makes the complexity of the curves too great for even an approximate location with the bridge. In these latter cases the entire length of circuit may be covered with the detector.

Office Unbalances

The normal equipment of a toll telephone circuit includes a transformer or repeating coil which isolates the telephone apparatus from the line and from the telegraph apparatus. This transformer interrupts the transmission of longitudinal currents through the telephone office apparatus and prevents impressing noise voltage to ground on the unbalances of such apparatus. Apparatus used for superposing direct-current telegraph and carrier-frequency channels on the telephone

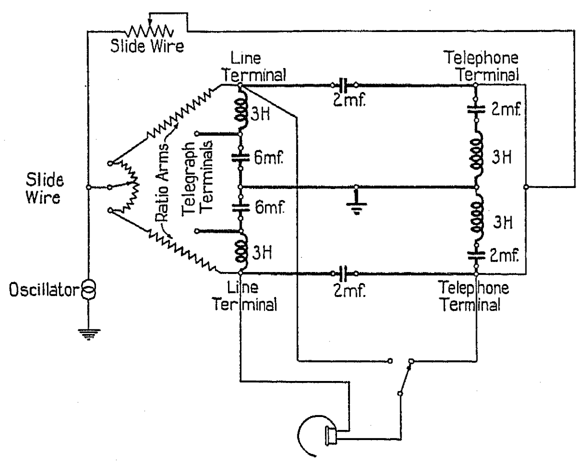


FIG. 14—COMPOSITE SET BRIDGE. SCHEMATIC CIRCUIT WHEN MEASURING UNBALANCE OF TELEGRAPH BRANCHES OF COMPOSITE SET. HEAVY LINES SHOW COMPOSITE SET

circuit is on the line side of this transformer and is therefore subjected to the action of longitudinal currents and voltages to ground. The line-filter set used for separating the carrier channels from the voice-frequency and d-c. telegraph channels has series and shunt branches in the two side circuits of a phantom group but no branches to ground. Unbalances in this set have a more important effect on crosstalk than on noise. Unbalances in the line-filter set may be detected by the use of the crosstalk set, connecting it to the office side of the filter with a terminating network on the line side.

The composite set used for superposing the d-c. telegraph channels on the telephone circuit has both series branches and branches connected from each wire to ground. For this reason, and because it is more generally used in the toll telephone plant than the line-filter set, special methods and apparatus have been developed for conveniently determining its condition from the standpoint of balance and for locating and determining the magnitude of such unbalances as may exist.

Some of the parts of the composite set can be tested with a simple slide wire bridge. Other parts, particularly the telegraph branches, cannot be so tested. By means of a special arrangement in the composite set bridge, the device which has been designed for rapid testing of the parts of a composite set as well as the whole set, the telegraph branches can be accurately measured without disconnecting any of the parts. A schematic circuit of the bridge, as arranged for testing these branches, is shown in Fig. 14. A simple slide wire bridge is connected across the line terminals of the composite set. A source of testing current is connected between the moving contact on the slide wire and ground. In order to prevent unbalances in the other branches of the composite set from affecting the measurements, the source of testing current is connected through an adjustable resistance to the telephone terminals. By simultaneous adjustment of this resistance and the slide wire of the bridge, it is possible to bring the line and telephone terminals to approximately the same potential. If this be done, the effect of the remainder of the composite set is eliminated and a measurement with the receiver connected across the ratio arms in the usual manner gives the unbalance of the telegraph branches alone. A receiver connected across either of the series condensers, as indicated, is used to determine when the line and telephone terminals are at the same potential, no sound being heard when this condition obtains.

Conclusion

While the effect of unbalances of a telephone circuit in contributing to noise depends much upon how and where the circuit is energized and on the energization and condition of other nearby telephone circuits, the test methods which have been devised permit the detection of all important unbalances which may contribute to noise. All of the apparatus for applying these methods has been standardized and made up in portable form for convenience in field testing. Much of it has been in use for several years.

The development of this apparatus, and of the methods outlined, provides the operating telephone companies with effective means for detecting faults of balance, and for maintaining circuits in a high state of efficiency from this standpoint. These methods are particularly applicable to toll circuits but some of them can also be applied to local circuits where necessary. This is important in minimizing crosstalk among telephone circuits and noise interference when such circuits are raised above the potential of the earth by induction from neighboring power circuits.

Further investigations of unbalances are in progress to determine more definitely the relationship between the magnitude and location of unbalances and their resultant effect at the circuit terminals when the circuits are energized inductively.

ACKNOWLEDGMENT

Credit is due chiefly to F. H. Best and P. W. Blye for carrying out the work leading to the development in practical form of the apparatus described. The authors wish to express their appreciation to them and also to H. M. Trueblood, A. G. Chapman and other colleagues for many helpful suggestions and criticisms received during the preparation of this paper.

Appendix

Theory of Bridge Method for Locating Unbalances

Fig. 8 shows, schematically, an unbalanced two-wire side-circuit connected to an unbalance bridge with the phantom used as a reference. In what follows, the circuit is assumed to be uniform (non-loaded) and the unbalances are assumed to be lumped and so small that they do not alter appreciably the total impedance of the circuit. Both the test and reference circuits are assumed to be terminated in characteristic impedance at the distant end.

Let:

γ_s = propagation constant of the side-circuit

γ_r = " " " " reference circuit

$\gamma = \gamma_s + \gamma_r = \alpha + j\,\beta$

Z_r = characteristic impedance of the reference circuit

Z_s = " " " " side-circuit

Z'_s = impedance of side-circuit looking toward bridge at distance x from bridge

Z_b = impedance of bridge looked at from line

I_r = current in the reference circuit at point of connection to bridge

I_r' = current in the reference circuit at distance x from bridge

i_s = side-circuit current at the bridge due to unbalances inserted in the bridge

i_s' = side-circuit current at the bridge due to unbalances in the line

i_s'' = side-circuit current due to unbalances in the line at distance x from bridge, looking toward the bridge.

Series unbalances R and L and shunt unbalances to ground G and C are assumed at a distance x from bridge. As regards the side-circuit current at the bridge, the same effect might be produced by values of equivalent series resistance and inductance unbalances R_e and L_e at the bridge. The bridge may, therefore, be balanced by inserting series unbalances $-R_e$ and $-L_e$.

The reference circuit current in one wire at a distance x from the bridge is

$$\frac{1}{2}\,I_r' = \frac{1}{2}\,I_r\,e^{-\gamma_r x} \qquad (1)$$

The reference circuit current in traversing the series unbalance sets up a voltage in the side-circuit which is

$$v_s' = \frac{1}{2}\,I_r'\,(R + j\,\omega\,L) \qquad (2)$$

This voltage acts through an impedance equal to the sum of the impedances of the side-circuit in two directions from the unbalance and sets up a current in the side-circuit which is

$$i_s''' = \frac{1}{2} I_r'\,\frac{R + j\,\omega\,L}{Z_s + Z_s'} \qquad (3)$$

The voltage to ground (one-half of the reference circuit voltage) at the distance x from the bridge is

$$V_g' = \frac{1}{2}\,I_r'\,Z_r \qquad (4)$$

This voltage sets up a current through the admittance unbalance in series with the admittance to ground of the whole circuit which, on the assumption that $G + j\,\omega\,C$ is small, is determined by the equation

$$i_g' = \frac{1}{2}\,I_r'\,Z_r\,(G + j\,\omega\,C) \qquad (5)$$

In obtaining the current in the side-circuit due to this current i_g' to ground, it is convenient to think of another path to ground on the other side of the circuit of zero admittance and, therefore, of zero total current to ground. The zero current in the path of zero admittance on the one wire and the current i_g' in the "unbalance" of finite admittance to ground on the other, may be said to be resolved into two equal components each equal to $i_g'/2$. Those in the zero current path are opposite in phase and those in the unbalance in the same phase, thus

$$\frac{1}{2}\,i_g' - \frac{1}{2}\,i_g' = 0 \qquad (6)$$

$$\frac{1}{2}\,i_g' + \frac{1}{2}\,i_g' = i_g' \qquad (7)$$

It will thus be seen that this is equivalent to supplying two equal sets of currents, one of which is a current in the reference circuit and the other in the side-circuit. As $G + j\,\omega\,C$ is assumed small, i_g' will be small compared to I_r' and the change in reference circuit current may, therefore, be neglected.

The current in the side-circuit will divide between the sections of circuit in the two directions from the unbalance in inverse proportion to the side-circuit impedances and the current propagated toward the bridge will be

$$i_s^{iv} = -\,\frac{1}{4}\,\frac{G + j\,\omega\,C}{Z_s + Z_s'}\,Z_s\,Z_r\,I_r' \qquad (8)$$

The total unbalanced current propagated toward the bridge is

$$i_s'' = i_s''' + i_s^{iv} \qquad (9)$$

or, adding equations (3) and (8), and substituting from (1)

$$i_s'' = \left[\frac{1}{2}\frac{R + j\,\omega\,L}{Z_s + Z_s'} - \frac{1}{4}\,Z_r\,Z_s\,\frac{G + j\,\omega\,C}{Z_s + Z_s'}\right] I_r e^{-\gamma_r x} \tag{10}$$

and, combining terms,

$$i_s'' = \frac{1}{2}\,\frac{R + j\,\omega\,L - \frac{1}{2}\,Z_r Z_s\,(G + j\,\omega\,C)}{Z_s + Z_s'}\,I_r e^{-\gamma_r x} \tag{11}$$

In terms of the side-circuit current at the unbalance, the side-circuit current at the bridge, caused by the unbalance, is

$$i_s' = i_s''\,\frac{Z_s}{Z_s \cosh\,\gamma_s x + Z_b \sinh\,\gamma_s x} \tag{12}$$

and the impedance looking from the unbalance toward the bridge is

$$Z_s' = Z_s\,\frac{Z_b \cosh\,\gamma_s x + Z_s \sinh\,\gamma_s x}{Z_s \cosh\,\gamma_s x + Z_b \sinh\,\gamma_s x} \tag{13}$$

Substituting equation (13) in (11) and the result in (12),

$$i_s' = \frac{1}{2}\,\frac{R + j\,\omega\,L - \frac{1}{2}\,Z_r Z_s\,(G + j\,\omega\,C)}{(Z_b + Z_s)\,(\cosh\,\gamma_s x + \sinh\,\gamma_s x)}\,I_r\,e^{-\gamma_r x} \tag{14}$$

$$i_s' = \frac{1}{2}\,\frac{R + j\,\omega\,L - \frac{1}{2}\,Z_r Z_s\,(G + j\,\omega\,C)}{Z_b + Z_s}\,I_r\,e^{(-\gamma_r - \gamma_s)x} \tag{15}$$

The side-circuit current set up by the action of I_r on the compensating unbalances $-\,R_e$ and $-\,L_e$ is

$$i_s = -\,\frac{1}{2}\,\frac{R_e + j\,\omega\,L_e}{Z_b + Z_s}\,I_r \tag{16}$$

If the bridge is balanced, there is no voltage across the receiver, therefore

$$i_s + i_s' = 0 \tag{17}$$

Substituting the values of i_s and i_s' from (16) and (15) in (17)

$$R_e + j\,\omega\,L_e = \left[R + j\,\omega\,L - \frac{1}{2}\,Z_r Z_s\,(G + j\,\omega\,C)\right] e^{-\gamma x} \tag{18}$$

or

$$R_e + j\,\omega\,L_e = \left[R + j\,\omega\,L - \frac{1}{2}\,Z_r Z_s\,(G + j\,\omega\,C)\right] \cdot (\cos\beta\,x - j\sin\beta\,x)\,e^{-\alpha x} \tag{19}$$

Resolving equation (19) into its real and imaginary parts, on the assumption that Z_r and Z_s are pure resistances[1],

$$R_e = e^{-\alpha x}\left[\left(R - \frac{1}{2}\,Z_s Z_r\,G\right)\cos\beta\,x + \left(\omega\,L - \frac{1}{2}\,Z_r Z_s\,\omega\,C\right)\sin\beta\,x\right] \tag{20}$$

$$\omega\,L_e = e^{-\alpha x}\left[\left(\omega\,L - \frac{1}{2}\,Z_r Z_s\,\omega\,C\right)\cos\beta\,x - \left(R - \frac{1}{2}\,Z_r Z_s\,G\right)\sin\beta\,x\right] \tag{21}$$

or

$$R_e = e^{-\alpha x}\sqrt{\left(R - \frac{1}{2}\,Z_r Z_s G\right)^2 + \omega^2\left(L - \frac{1}{2}\,Z_r Z_s C\right)^2}\;\cos\left[\beta\,x - \tan^{-1}\frac{\omega\left(L - \frac{1}{2}\,Z_r Z_s C\right)}{R - \frac{1}{2}\,Z_r Z_s G}\right] \tag{22}$$

$$\omega\,L_e = -\,e^{-\alpha x}\sqrt{\left(R - \frac{1}{2}\,Z_r Z_s G\right)^2 + \omega^2\left(L - \frac{1}{2}\,Z_r Z_s C\right)^2}\;\sin\left[\beta\,x - \tan^{-1}\frac{\omega\left(L - \frac{1}{2}\,Z_r Z_s C\right)}{R - \frac{1}{2}\,Z_r Z_s G}\right] \tag{23}$$

The value of β in these formulas may be expressed in terms of frequency as

$\beta = \dfrac{2\,\pi\,f}{V}$, V = a function of V_s and V_r, the velocities of phase propagation on the side and reference circuits, respectively. Since

$$\beta = \beta_s + \beta_r \text{ and } \beta_s = \frac{2\,\pi\,f}{V_s},\ \beta_r = \frac{2\,\pi\,f}{V_r},$$

$$\beta = 2\,\pi\,f\,\frac{(V_s + V_r)}{V_s V_r}$$

Substituting $\dfrac{2\,\pi\,f}{V}$ for β, formulas (22) and (23) may be rewritten as follows:

$$R_e = K\cos\left(\frac{2\,\pi\,x}{V}\,f - \phi\right) \tag{24}$$

$$\omega\,L_e = -\,K\sin\left(\frac{2\,\pi\,x}{V}\,f - \phi\right) \tag{25}$$

If R_e and $\omega\,L_e$ were plotted against frequency the curves would be perfect sine and cosine curves provided the values K, V and ϕ were constant with frequency. If this were so, the resistance curve would pass through a maximum when $\dfrac{2\,\pi\,x}{V}\,f - \phi = 0,\ 2\,\pi,\ 4\,\pi,$

1. This assumption holds fairly well for non-loaded circuits at frequencies above 400 cycles. The magnitude of the reactance term is smaller on circuits of the larger gage conductors.

etc. Let f_0, f_2, f_4, etc., indicate the frequencies at which the successive maxima occur, then

$$\frac{2\,\pi\,x}{V}\,f_0 - \phi = 0,\ f_0 = \frac{\phi\,V}{2\,\pi\,x}$$

$$\frac{2\,\pi\,x}{V}\,f_2 - \phi = 2\,\pi,\ f_2 = \frac{V}{x} + \frac{\phi\,V}{2\,\pi\,x}$$

$$\frac{2\,\pi\,x}{V}\,f_4 - \phi = 4\,\pi,\ f_4 = \frac{2\,V}{x} + \frac{\phi\,V}{2\,\pi\,x}$$

The frequency interval between positive peaks would then be

$$f_i = f_4 - f_2 = f_2 - f_0 = \frac{V}{x} \tag{26}$$

and conversely the distance from the bridge out to the unbalance would be equal to

$$x = \left(\frac{V_s\,V_r}{V_s + V_r}\right)\frac{1}{f_i} \tag{27}$$

If inductance and capacitance unbalances are absent the lag angle ϕ becomes 0 and is constant with frequency. If resistance and leakage unbalances are absent the angle ϕ becomes constant and has a value of 90 deg. For other cases the tangent of the angle ϕ is proportional to frequency, assuming R, L, G, C, Z_r and Z_s to be constants. The types of unbalance ordinarily encountered are such as to give values of ϕ nearly equal to 0 or to 90 deg. for all frequencies within the working range. Examples of such are: resistances in joints, for which ϕ is 0; and transposition errors and inductance unbalances as in loading coils, for which ϕ is practically 90 deg.

Inspection of equations (**24**) and (**25**) shows that variations in V and ϕ with frequency will result in variations in the *period* of the curves. In order to eliminate as far as practicable the effect of changes in period due to either of these factors, it is advisable to use the average period over as large a range of frequencies as possible.

Obviously, the factor

$$K = e^{-\alpha x}\sqrt{\left(R - \frac{1}{2}\,Z_r Z_s G\right)^2 + \omega^2\left(L - \frac{1}{2}\,Z_r Z_s C\right)^2}$$

cannot be independent of frequency. Variations in this factor cause variations in the amplitude of the curves but do not cause much change in the intervals between successive maxima. The actual changes in this factor with frequency depend upon the type of circuit involved and the particular type of unbalance existing on that circuit. In general terms, the variation of amplitude as frequency increases is as follows:

Series Resistance Unbalance. (R)

R_e. Amplitude decreases slowly due to increase in the attenuation constant, α, with frequency.

L_e. Since L_e is inversely proportional to ω its amplitude decreases directly as frequency increases. A further decrease in amplitude is caused by the increase in α with frequency.

In this case usually the resistance curve is of greater usefulness in determining the frequency interval.

Series Inductance Unbalance. (L)

R_e. In this case R_e is directly proportional to ω which tends to make the amplitude of R_e increase directly with frequency. This increase in amplitude is opposed by the increase in the attenuation constant α with frequency. This latter effect is, however, small compared to the former.

L_e. Amplitude decreases slowly due to increase in α with frequency.

In this case the inductance curve is of greater usefulness in determining the frequency interval.

Shunt Leakage Unbalance. (G)

Assuming Z_s and Z_r to be resistances and constant with frequency the amplitudes of the R_e and L_e curves behave as in the case of a series resistance unbalance. This assumption holds fairly well for non-loaded circuits at frequencies above 400 cycles.

Shunt Capacitance Unbalance. (C)

Assuming Z_s and Z_r to be resistances and constant with frequency the amplitudes of the R_e and L_e curves behave as in the case of a series inductance unbalance. Shunt capacitance unbalances are usually due to transposition errors which introduce simultaneously inductance unbalance. Such unbalances are not lumped but distributed along the circuit and, therefore, the curves obtained become somewhat distorted in the upper part of the frequency range.

In many cases two or more unbalances are found on a circuit. In such a case the curve might be represented by an equation made up by adding the equivalent resistance and inductance corresponding to each unbalance. The equation would consist of two or more terms similar to equation (**19**).

It should be noted that only very rarely would the individual curves resulting from the two or more unbalances have periods with a common multiple corresponding to a frequency interval included in the range tested. The composite curves are not, therefore, susceptible of analysis by the methods used in analyzing periodic waves.

While the theory given above is based upon circuits with uniformly distributed constants, it may also be applied to loaded circuits, with the assumption that the capacitance in the successive sections of line is lumped. The end section of the line adjacent to the bridge is assumed to be a full section, although 0, one-half coil or full coil loading may be present in the office adjacent to the bridge. All series unbalances are assumed to occur in or immediately adjacent to the loading coils. If such unbalances occur within the section or if less than full section termination is used at the end adjacent to the bridge, distortion in the curves will result.

Shunt capacitance unbalance is generally introduced by transposition errors and is accompanied by series

inductance unbalance. These unbalances are distributed along the sections. The characteristic impedance of loaded circuits varies at different points along the section, and except for mid-section the reactance term is important. The magnitude of the impedance varies considerably with frequency, although the phase-angle changes much less rapidly.

The velocity of phase propagation changes with frequency on loaded circuits much more rapidly than on non-loaded circuits, especially as the cut-off frequency is approached.

On account of these various factors tending to distort the curves obtained, it is not generally practicable to locate unbalances in loaded circuits closer than within a loading section on either side of the correct point unless the unbalance is a series unbalance in a loading coil or immediately adjacent thereto. If the proper end section be used at the bridge or if the circuit be built out to give this condition, it will usually be possible to locate such a series unbalance at the correct load point. To locate definitely a shunt unbalance, it will often be necessary to cover two successive sections (16 miles of open-wire line) with the unbalance detector in order to obtain the final location.

Termination of Circuits under Test

In making measurements of the unbalance of a circuit, it is necessary to terminate both the circuit under test and also the reference circuit. If the circuit were balanced with respect to the reference, the measurements would reveal no unbalance irrespective of the termination at the distant end but in the case of an unbalanced circuit, lack of terminations would result in unbalances at a number of apparent locations at and beyond the distant end of the circuit.

In examining this problem it is convenient to assume that the bridge impedances as looked at from the terminals of either the reference or the test circuits are equal to the characteristic impedances of these circuits. This assumption is valid, as far as obtaining the relative values of the line and compensating unbalances is concerned, because any reflection from the bridge end of the circuits affects the line and compensating unbalances relatively in the same manner as the original voltages and currents.

Referring again to Fig. 8, let both circuits be open at the distance D from the bridge and suppose that the unbalance at x is a pure resistance R. Then the current in the reference circuit at any point will consist of two terms, one being the original wave and the second the wave reflected from the open end of the circuit. Thus, if $2\,E_r$ be the open-circuit voltage of the oscillator, the current in the reference circuit at any point x will be

$$I_r' = \frac{E_r}{Z_r}\left[e^{-\gamma_r x} - e^{-\gamma_r (2D - x)}\right] \qquad (28)$$

Each of these two components of current in the reference circuit acting on the unbalance R, will cause two components of current in the side-circuit, one the current propagated directly from the unbalance to the bridge, and the other propagated to the open end and reflected back to the bridge. At the bridge this current will be, using the methods of the first part of this appendix,

$$I_s' = \frac{E_r}{Z_r}\cdot\frac{R}{4\,Z_s}\left[e^{-(\gamma_r+\gamma_s)x} - e^{-\gamma_r(2D-x)-\gamma_s x}\right]$$

$$-\frac{E_r}{Z_r}\cdot\frac{R}{4\,Z_s}\left[e^{-\gamma_r x-\gamma_s(2D-x)} - e^{-(\gamma_r+\gamma_s)(2D-x)}\right] \qquad (29)$$

Let the compensating unbalance $-R_e - j\,\omega\,L_e$ equal $-Z_e$. This unbalance will cause, similarly, four more components of unbalance current in the side-circuit. This current at the bridge is

$$I_s = -\frac{E_r}{Z_r}\,\frac{Z_e}{4\,Z_s}\left[1 - e^{-\gamma_r 2D} - e^{-\gamma_s 2D} + e^{-(\gamma_r+\gamma_s)2D}\right] \qquad (30)$$

These four components in each case are respectively propagation (1) to the unbalance along the reference circuit and back over the side-circuit, (2) along the reference circuit to the open end, reflection in opposite phase back to the unbalance on the reference circuit and to the bridge over the side circuit, (3) along the reference circuit to the unbalance, along the side circuit to the open end, reflection in opposite phase and back to the bridge along the side current, and (4) along the reference circuit to the open end and back to the unbalance in opposite phase, along the side-circuit to the open end, reflection again in opposite phase and back along the side-circuit to the bridge.

Z_e may be evaluated in terms of R, the propagation constants of the circuits and the distances involved, by equating the sum of the right-hand sides of equations (29) and (30) to zero. Thus,

$$Z_e = R\left[\frac{e^{-(\gamma_r+\gamma_s)x} - e^{-\gamma_r(2D-x)-\gamma_s x}}{1 - e^{-\gamma_r 2D} - e^{-\gamma_s 2D} + e^{-(\gamma_r+\gamma_s)2D}}\right]$$

$$+ R\left[\frac{-e^{-\gamma_r x-\gamma_s(2D-x)} + e^{-(\gamma_r+\gamma_s)(2D-x)}}{1 - e^{-\gamma_r 2D} - e^{-\gamma_s 2D} + e^{-(\gamma_r+\gamma_s)2D}}\right] \qquad (31)$$

As an example of the interpretation of this expression in terms of the equivalent resistance and inductance unbalance, consider the case of phantom-side unbalance measurements on a fully loaded or non-loaded group, for which the velocities of phase propagation on the two circuits are substantially equal. It will also be assumed that the difference in the attenuation constants of the two circuits is small as compared to their sum and that a wave propagated over lengths equal to or greater than twice the total length of both circuits becomes negligibly small. Then equation (31) becomes, calling $\gamma = \gamma_r + \gamma_s$

$$Z_e = R\,\frac{e^{-\gamma x} - 2\,e^{-\gamma D} + e^{-\gamma(2D-x)}}{1 - 2\,e^{-\gamma D}} \qquad (32)$$

Dividing and rejecting terms as small or smaller than $e^{-2\gamma D}$ as before.

$$Z_e = R\left[e^{-\gamma x} - 2\, e^{-\gamma D} + e^{-\gamma(2D-x)} + 2\, e^{-\gamma(D+x)} \right] \quad (33)$$

Substituting $\alpha + j\,\dfrac{2\,\pi\,f}{V}$ for γ and equating real and imaginary terms as in the first part of this appendix,

$$R_e = R\left[e^{-\alpha x} \cos \frac{2\,\pi\,x}{V} f - 2\, e^{-\alpha D} \cos \frac{2\,\pi\,D}{V} f \right]$$

$$+ R\left[e^{-\alpha(2D-x)} \cos \frac{2\,\pi\,(2\,D - x)}{V} \right.$$

$$\left. + 2\, e^{-\alpha(D+x)} \cos \frac{2\,\pi\,(D + x)\,f}{V} \right] \quad (34)$$

Examination of this equation shows that the curve will be composed of four periodic components, the first corresponding to the actual unbalance at x, the second corresponding to a smaller unbalance apparently located at the distant end, and the third and fourth apparently located beyond the distant end. Other smaller and apparently more distant unbalances have been neglected, in rejecting terms of the order $e^{-2\gamma D}$ and smaller.

The inductance curve, the expression for which has not been written out, would be similarly complex.

If the distant ends of the circuits were short-circuited instead of open these apparent locations would still be indicated, the signs of some of the terms merely being reversed. If one of the circuits were open or shorted and the other were properly terminated, the magnitudes of the unbalances apparently located at D and $D + x$ would be reduced and that located at $D - x$ would vanish.

Tests to locate unbalances on non-phantomed pairs, phantom circuits or sides of vertical phantoms, employ as a reference a superposed circuit having for its other side the conductors in parallel of some nearby non-phantomed pair or phantom group. Such a reference circuit is not transposed and when energized from the bridge, voltages and currents will be induced in other conductors of the line. These voltages and currents will react by induction on the voltages and currents of both the reference and test circuits and on account of reflections from the distant ends, unbalances will be indicated apparently located at and beyond the distant end. Experience shows that these reflection effects are not large and that the complication in the curves is not important, provided the test and reference circuits themselves are properly terminated.

Discussion

PRACTICES IN TELEPHONE TRANSMISSION MAINTENANCE WORK

(Harden)

TELEPHONE CIRCUIT UNBALANCES

(Ferris and McCurdy)

Pasadena, Cal., October 17, 1924

D. I. Cone: To the engineers and maintenance people of the operating telephone systems these advances in the means for maintaining the circuits are very gratifying. There are two major reasons for their need: First, in order to employ economically the communication plant, as many types of service as possible are put on the wires. Balancing the circuits is one means employed to separate one channel of communication from another. Second: The growth of power circuits and their inductive fields has been so great that communication circuits, which originally had the field very much to themselves, are now forced to exist in the presence of large inductive and conductive fields. Since the same people want both power and communication service it is impossible altogether to prevent that. The original telephone circuits would be wholly inoperative under present-day conditions but for the advances made, on the one hand, by the method of balancing the communication circuits, and on the other hand, by measures taken in the supply circuits to limit their fields of influence.

The Pacific Telephone & Telegraph System has about 3000 toll circuits, which it is our duty to maintain in efficient operating condition. Mr. Harden has described to you the testing technique employed for transmission maintenance. As a part of this we have, for several years, been making annual noise tests on these toll circuits, in addition to special tests made at times when changes either in the telephone circuit or in the neighboring power circuit are made, and acceptance tests of new circuits. When the crosstalk meter, which Mr. Ferris described, became available, measurements with it to determine the condition of the circuits were incorporated and are now regularly made as a part of our routine.

Since the impedance-unbalance bridge became available some years ago, we have made much use of it in locating unbalances of the types mentioned, particularly resistance unbalances and transposition irregularities. In this Pacific Coast toll network we have several hundred thousand circuit transpositions whose maintenance is a large problem. It would be possible to present to you many curves obtained on our circuits, similar to those shown in Figs. 9, 10, 11 and 12 of the paper, but these illustrate so well the characteristics obtained and the ability of this apparatus to show the nature of unbalances that it seems unnecessary to add more examples.

Referring to Fig. 10, where an artificial unbalance was inserted at a known distance, it is easily possible, by the formula given, to calculate the velocity of travel of the waves in that circuit. This was a loaded cable circuit and I find the velocity to be 7900 mi. per second. That is very different from the velocity in the open wire lines and we find it desirable to take account of this difference of velocity in cables and open wires when we are testing circuits which enter the office through long toll entering cables. The setup given in Fig. 10 affords a means of obtaining a "calibration" of the velocity factor.

To illustrate the necessity for obtaining balance, not only where there are inductive exposures, but throughout the line, suppose a line into which a branch line connects, the branch line being exposed to a source of noise. The induced energy will flow into the main section of the line and if there are unbalances on this main line, we will have trouble from the exposure, even though perfect balance were obtained on the branch circuit.

It might be asked, what is the relation of these more recent methods to the previously available working methods used? The advance seems to me to come in the form of a very definite

improvement of technique. The impedance-unbalance bridge represents the application of alternating current at varying frequencies to the old familiar Varley loop test much used for location of faults with the Wheatstone Bridge. Using the same idea, but a variety of alternating-current frequencies, it detects effects which the direct-current bridge cannot discover.

We often make use of the induced noise voltage to ground, comparing it with the induced voltage between wires. That has served as a rough method of indicating balance, which had some usefulness, employing the source of energy which happened to be there in the form of the disturbing circuits. On the other hand, in the methods here described, we have a definitely controlled and applied source of energy, with its manifest advantages.

By the pursuit of the methods indicated in the paper in providing and maintaining a high grade of balance in the telephone circuits important reductions in noise and in crosstalk have been realized in our plant. To offset this we have been confronted with the rapid expansion of the power circuits. That it has been possible to improve the conditions has been due to the faithful efforts of a large group of men. Having devoted their energies to the solution of this problem, they have enabled us to make remarkable advances in this direction. I feel moved to pay tribute to the work of these men because of the exceptional hours at which they often have to work in order that the normal service of the communication and power circuits may not be interrupted.

H. W. Hitchcock: In regard to the value of adequate transmission maintenance, such as is described in Mr. Harden's paper, I think it quite obvious that in the case of a toll circuit which involves an investment of possibly several thousand dollars, it is extremely important that the circuit be kept in a high degree of efficiency, as we can hardly afford to lose a considerable portion of its volume efficiency by allowing small troubles to creep in. As this is quite evident, it needs no further elaboration and I shall confine my further remarks to a discussion of the advantages to be derived from proper transmission maintenance in the local exchange plant where its value perhaps is not so apparent.

Unless one has given the matter close consideration, he may have the idea that a telephone plant grows in a more or less haphazard way. In fact, we set up quite a definite transmission standard of efficiency, which applies to the entire plant. That is, we attempt to make it possible for every subscriber to talk to every other subscriber with approximately the same degree of efficiency. This standard of efficiency has been arrived at from a study of extensive laboratory tests and from trying out certain standards in actual practice. In the latter case, particularly, the result is a combined reaction from a large number of people having all kinds of ears and voices and personal temperaments.

As a result of years of experience and trial, we have established a standard which we call a twenty-mile standard. This means that the telephone circuit to give good transmission can have the same loss as that produced by 20 miles of standard 19 gage cable. This standard is possible because the modern telephone transmitter produces in the circuit, telephone currents representing an amplification of several hundred times over the acoustic power directly by the voice against the transmitter diaphragm and therefore for satisfactory telephone conversations the telephone power delivered to the receiver at the far end of the circuit need be only a fraction of 1 per cent of the input power at the transmitting end. The fact that the overall electrical efficiency of the circuit itself is low is not of importance as we are dealing in the ultimate with the overall efficiency from the voice of the speaker to the ear of the listener. Furthermore, a very small amount of acoustic power at the receiving end is sufficient to give a very clear conversation and when you are talking with a person over the telephone you are anxious to make him understand what you have to say; you are not trying to warm up his ear.

Disregarding, for the present, the matter of cost, the proper volume of efficiency might be arrived at somewhat as follows:

Two telephone instruments might be connected together with a very short circuit and speech exchanged over them. In such a case, it would be found that the volume of tone would be unnecessarily loud, in fact it would be something like trying to talk to another person if he insisted upon shouting in your ear. The circuit between the telephones might then be extended until the speech became very weak and made it necessary to listen very closely and possibly ask that the speaker shout or repeat frequently in order that the speech might be understood. Obviously, a circuit having some intermediate efficiency would be preferable to either of these, and the one which, on the whole, would give the best results could be determined by repeated trial.

In establishing a standard, however, the economic factor cannot be ignored. The more power which you transmit over a circuit of a given length, the more copper you must place in it, with a corresponding increase in cost. From an economic consideration, then we would use the smallest wire which it is possible to employ and still transmit understandable speech.

As a result of these somewhat conflicting requirements, we select a standard which we believe will give good speech transmission, enabling one person to talk to another without unnecessary repetition and without having to shout, and at the same time provide the service at a price which will make it attractive for him to use freely.

Having established such a standard, it is obviously poor economy to depart from it in either direction. To do so tends to limit the service, to discourage its use, or make it unsatisfactory to the public. This will result on the one hand, from too great a cost if too high a degree of efficiency is attempted, and on the other, from poor and unsatisfactory transmission if a circuit of insufficient efficiency is provided.

In the design of a telephone plant, we usually assume approximately one hundred per cent efficiency for all the parts which go to make it up. Of course, it is impossible to expect that such parts will always be. and remain indefinitely, at this high degree of efficiency. Troubles are bound to creep in, just as they creep into any plant, so that we must face the fact that our plant tends to deteriorate, or that we will occasionally find defective instruments, such as transmitters, receivers or other parts. There are in general two ways by which this deterioration can be cared for. One is by providing a margin of safety in our plant design sufficient to make up for these occasional defects. The other is to detect these troubles as they arise and remove them. I believe it is obvious to all of you that the first method is not a very satisfactory one. It means that the entire plant must be much more expensively constructed than would otherwise be necessary. Moreover it is not satisfactory from the subscriber's point of view. For example, a subscriber's instrument may, for some reason, have deteriorated until it only is ten per cent as as good as it should be. He reports it and the company sends an adjuster to discuss the matter with him. The company's representative may say to the subscriber: "We appreciate that your set may be in very poor shape; in fact, it may only be ten per cent as good as it should be. To offset this, however we have designed your circuit, and those of your ten neighbors, lines, ten per cent better than we think is necessary, so that the average efficiency of our plant is satisfactory: nine instruments are ten per cent better than is necessary as against one instrument which is only ten per cent as good as it should be." The reaction of such an individual is obvious. He would reply that this is very nice for his neighbors perhaps but is of very little benefit to him.

In order to make such a method effective, it would be necessary to have all of the instruments several times as good as would normally be required and this would be prohibitive in expense. When we provide only a small margin of safety to take care of

isolated cases of trouble, we fail to accomplish the thing which we set out to do. The only way, then, is to detect these troubles as they appear and clear them promptly. Unfortunately, for the telephone company, they do not always announce themselves. Some of them, of course, do, such as "opens," "grounds," "crosses," or troubles of that kind. In these cases the supervisory signals fail to function or it is impossible to talk at all or the circuit is very noisy. Such troubles quickly call attention to themselves and are promptly remedied.

Another class of trouble is one which may produce only a moderate reduction in efficiency such as a few short-circuited turns on a coil or a partially defective condenser or a high-resistance joint. Such a circuit will usually give the proper supervisory response and can be talked over with a certain degree of facility. However, should such troubles be allowed to remain they will cause a gradual deterioration in the plant with a corresponding increase in the number of unsatisfactory connections. The only way to detect such trouble is by a series of systematic tests conducted periodically which cover the entire plant. In this way we find the troubles which are hidden away and which will not appear on ordinary inspection or as a result of the usual maintenance tests. To make such tests it is necessary that special instruments and routines be developed and followed and it is these tools, these instruments, these methods, which Mr. Harden has described and which we now have available for this kind of work. These new facilities are of tremendous value to us as you can well appreciate since they make it possible to give a good and uniform standard of service with a plant designed with the maximum degree of economy.

Farley Osgood: I might just say a word on the matter of inductive interference. I think it is the duty of the power engineer to make known any contemplated plan he may have for construction to his telephone friends as soon as it is practicable for him to do so. A number of years ago the lines were built without due consideration to any neighbors and the problem of interference got to be more and more a disturbing one, until its seriousness was recognized and committees formed to deal with it. These committees, the National Electric Light Committee and the Joint Overall Committee, with men from both groups, worked out a most friendly relationship all over the country and the basis of it all is to confide in each other any plans of construction, of course, with the understanding that there will be no propaganda about the plans. The engineers of the two properties should know each other's plans at the start in order that disturbances may be avoided by injecting into the construction program the present known methods of elimination.

V. D. Elliot: I think, that the men who are employed by the larger power companies, particularly those which have extensive private telephone systems, could, with advantage to themselves and to their companies, become more familiar with the subjects treated in these papers. I know that a number of these instruments, if not all of them, could, with certain modifications, be used for testing work on the private telephone systems of the large power companies.

I wish to say, by way of appreciation, that the telephone men of Southern California have shown a cooperative spirit to the Edison Company, by giving helpful suggestions, loaning apparatus and offering their help in any way possible to aid us in following out these ideas on our system.

It may be of interest to some of the telephone men and, perhaps, to the power men to know of a case of telephone line balance, which occurred on the telephone system of the Edison Company.

The telephone lead, which was built for the operation of the Big Creek System of the Southern California Edison Company, as originally constructed, had four wires, carried on a pole line separated by varying distances from the 150,000-volt power circuits. It was never intended to be other than a two-circuit line as the crossarms were of such size as to accommodate only the four wires, and the transposition system was only designed to take care of crosstalk balance between the pairs.

When we started construction work on raising the Big Creek Dams there was need for an additional circuit from our Los Angeles office to Big Creek. I was asked to see if a phantom circuit could be installed without great expense. We put on phantom coils at the two ends and made a trial and found that we could operate without undue crosstalk, without any phantom transpositions. The line has been rebuilt in certain sections and phantom transpositions have there been installed and provisions have been made to accommodate any number of future wires and also to take care of any outside induction that may be present. Exclusive of these places there are 100 to 150 miles of the lead which have no phantom transpositions, but the crosstalk and noise on that phantom circuit is tolerable from our standpoint and we can carry on a conversation and without interfering with the talking on the side circuits.

This I think is a different experience from that which the telephone men have had and I thought it might be of interest to them and also to the power men who have telephone circuits and are contemplating the use of phantom circuits.

Four years ago we superimposed a simplex telegraph circuit on top of the others and it is operating successfully without interference with the other channels.

Mr. Ashbrook, of our company, has tried out recently, in an experimental way, apparatus for stopping, or minimizing acoustic shocks which occur on these telephone circuits due to flashovers on the power lines. The power line is in general proximity to this line for its entire length of 241 mi. At some places the lines are 8 mi. apart and in other cases they run parallel at 1000 ft. separation, but in general the average horizontal separation is about one-half mile. At times of flashover currents as high as 900 amperes going from the power circuit with a ground return, create a large magnetic loop which induces considerable voltage between ground and the telephone line. This voltage has a very steep wave front and part of it appears as a voltage between wires due to unavoidable telephone line unbalances resulting in an acoustic shock or "bat in the ear."

Mr. Ashbrook has a method to overcome this effect, but we do not know as yet whether or not it will be applicable as a regular adjunct to the telephone system.

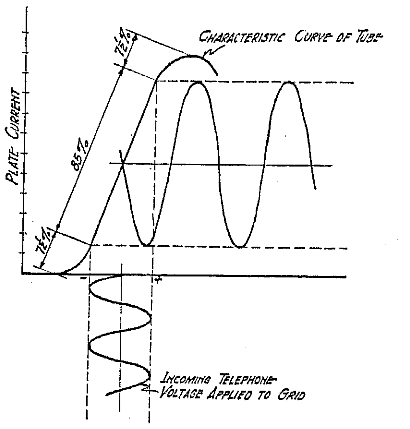
R. B. Ashbrook: The vacuum tube is utilized as a voltage-limiting device by operating the vacuum tube at such a point on its static characteristic curve that the voltage produced by normal telephone communication and applied to the grid causes a change of plate current over approximately 85 per cent of its characteristic curve. It is obvious that 7½ per cent increase in negative voltage will result in the plate current being reduced to zero and further increase will have no effect. If the increase is in the positive direction the plate current will increase until the saturation current corresponding to the existing filament temperature is reached. Further positive increase in the grid potential cannot result in further increase in the plate current, in fact as it becomes more positive it attracts a greater number of electrons to itself and causes a corresponding decrease in plate current.

A recent demonstration has proven beyond a doubt its usefulness to power companies operating telephone lines subjected to between-wire voltages occasioned by energizing or de-energizing parallel transmission lines.

Two operators, one a student wearing an operator's head set equipped with the acoustic-shock-suppression apparatus, the other with the usual operator's telephone set were connected with a telephone line at the time that a transmission line failed. The operator received a severe acoustic shock which was very painful and necessitated doctor's care for several days before resuming her duties while the student operator was not positive whether she heard it at all.

J. E. Woodbridge: The work of the Department of Develop-

ment and Research of the American Telephone and Telegraph Company on the lines described in this paper is of vital importance to the settlement of the problems of inductive interference which confront two divisions of the electrical industry. Such work is essential to the establishment of precise rules for legislative enactment in this controversy where such legislation is found necessary.



This is for the following reason:

A telephone circuit balanced in all respects, including balance to other circuits, is immune to inductive interference.

The real difficulty is, of course, to get such a balanced telephone circuit. An equally real difficulty in the past negotiations between the two interests has been to get either a criterion of unbalance, or a means of measuring the unbalance of communication circuits.

At the time when the Joint Committee on Inductive Interference, appointed by the California State Railroad Commission, was struggling with its problems, namely, from 1913 to 1917, the Committee members recognized the importance of the balance of telephone circuits, and that the legislation based on the Committee's findings ought to include rules on such balance so that undue diligence for the mitigation of interference should not be put upon the power systems. The difficulty, however, of measuring unbalances or of defining practicable balance made it impossible to include in the code any quantitative rules for limiting unbalances of telephone circuits.

This difficulty has now been partially removed by the work outlined in this paper. It is to be hoped that the work will be carried further to determine and make it possible to define reasonable or practical units of unbalance that can be specified as limits or tolerances in commerical construction and operation.

R. G. McCurdy: Mr. Cone refers to a method of measuring unbalances by determining the ratio of the noise measured in the metallic telephone circuit to the noise voltage measured to ground. While this test is valuable as preliminary indication it is generally not conclusive, because as mentioned in the paper, the noise measured in the metallic circuit includes two effects; one dependent upon unbalances of the circuit in its relations to ground and other telephone circuits, and the other dependent upon the dissymmetry in its relations to parallel power circuits. If this ratio indicates a high voltage to ground and a low noise in the metallic circuit, we are justified in concluding that the circuit is well balanced. If, however, the ratio is in the opposite direction, it may be concluded either that the circuit is unbalanced or that relatively large voltages are induced in the metallic circuit because of dissymmetry to parallel power circuits. Thus, in order to obtain any definite indication of the existence of unbalances of the circuit to ground or to other telephone circuits, tests must be made under controlled conditions, as described in the paper.

Mr. Elliott brings up the case of a line with a single phantom group, which was in a satisfactory condition from the standpoints of crosstalk and noise on both the side and phantom circuits with no transpositions in the phantom circuit. It should be observed that when no other circuits are nearby, complete balance may be secured between the two side circuits and between each side circuit and the phantom, provided each side circuit is made up of wires of the same size and with transpositions placed only in the side circuits. If any more telephone circuits are added to the same line, or if power circuits are constructed in proximity, it then becomes necessary to consider the balance of the phantom circuit. Under these conditions, all four wires of the phantom group must be of the same size and transpositions must be placed in the phantom circuit. I take it, in the case discussed by Mr. Elliott, no close exposures existed to other circuits.

L. P. Ferris: The use of the vacuum tube as an energy-limiting device referred to by Mr. Ashbrook has long been recognized by telephone engineers and has been proposed specifically for this purpose (H. D. Arnold, U. S. Patent No. 1168270, January 18, 1916). The telephone repeater in common use in many of our long telephone circuits employs vacuum tubes, and of course possesses this energy-limiting characteristic. Application of this method was considered a number of years ago in a case of parallelism between a telephone cable and a power circuit, which at times of short circuit caused acoustic shock to telephone operators. The large number of telephone circuits to be protected made the cost of vacuum-tube terminal equipment practically prohibitive. Other methods were found effective and more economical. It is, however, a different matter where only one or two circuits are involved as on a power company's private dispatch line.

Mr Woodbridge has stated that "a telephone circuit balanced in all respects, including balance to other circuits, is immune to inductive interference." This is theoretically true if the induced voltages to ground are not large enough to endanger personnel, break down insulation, or operate protective devices. Voltages high enough to interrupt service may be induced, even though the circuit be balanced in the manner Mr. Woodbridge suggests. Without exception, however, it might be said that a power circuit balanced in all respects, including balance to other circuits and its mode of energization, would cause no inductive interference. Both conditions are ideal but unrealizable in practise and, therefore, largely of academic interest. Balance of both systems, power and telephone within themselves, and with respect to each other, is important in preventing interference. Practically useful definitions of unbalance must, however, recognize these three distinct problems—balance of each system within itself, and mutual balance in exposures. Aside from the undesirable technical complications involved in making the unbalance of one system dependent upon the condition of the other, the importance is apparent from an administrative standpoint, of placing upon each party the responsibility for maintenance of its system in balance, only so far as conditions are under its control. This reduces the zone of joint responsibility to its minimum dimensions; *i. e.*, to the section where the two systems are in relatively close proximity, the exposure, which in general, involves but a small part of each system. Here responsibility for balance is inherently joint or divided, a situation not to be unnecessarily extended. It was with a background of such practical considerations in mind that our definition of a balanced telephone circuit was formulated, and in the opening paragraphs of the paper it is pointed out that it deals only with unbalances of the telephone circuits themselves, independent of their relation to power circuits. Thus, the problem of mutual unbalance within exposures, largely a problem of coordinated transposition design, and the problem of power-circuit balance, were purposely excluded from

the scope of the present paper. This, of course, does not mean we feel them any less important than the problem considered.

Mr. Woodbridge referring to the work of the California Joint Committee on Inductive Interference, mentions the difficulty then recognized of getting a criterion and measure of unbalance, and of setting any quantitative limits. The authors were associated with Mr. Woodbridge in this early joint work and as an outgrowth of the need then developed, the investigation reported in this paper was undertaken. That the results have been put into effective use on the Pacific Coast, it is gratifying to note from Mr. Cone's discussion. We recognize and have so stated in the conclusion of the paper that not all has been done along this line and with the joint development and research work now under way with power engineers, we hope as does Mr. Woodbridge that it may be possible to specify practical tolerances in construction and operation.

Report of the Board of Directors

FOR THE FISCAL YEAR ENDING APRIL 30, 1924

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Fortieth Annual Report, for the fiscal year ending April 30, 1924. A general balance sheet showing the condition of the Institute's finances on April 30, 1924, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year; more detailed information has been published from month to month in the Institute JOURNAL.

Directors' Meetings.—The Board of Directors held seven meetings during the year; five were held in New York and one each in Swampscott, Mass., and Philadelphia, Pa. The Executive Committee functioned when necessary in the intervals between regular board meetings.

Information regarding the more important activities of the Institute which have been under consideration of the Board of Directors, the committees and the various officers, is published each month in the section of the JOURNAL devoted to "Institute Activities."

Meetings.—The policy of holding in addition to the Annual business meeting four general meetings of the Institute each year was continued. The meetings held were as follows: Annual, Pacific Coast, Midwinter and Spring Conventions.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 18, 1923. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1923 was presented. The Tellers Committee made its report upon the election of officers for the administrative year beginning August 1, 1923.

In the evening the members of the Board of Directors, past-presidents of the Institute and the president-elect met at dinner and informally discussed the affairs of the Institute.

Annual Convention.—The Thirty-ninth Annual Convention was held at Swampscott, Mass. on June 25 to June 29, 1923. Six technical sessions were held including parallel sessions, two mornings. A total of twenty-eight papers were presented. A large number of entertainment features were provided including lectures, musicals, moving pictures, dancing, tennis, golf, etc. The total registration of 1616 surpassed by several hundred, all previous records. An event of interest was the past-president's luncheon attended by sixteen of the twenty-four living past-presidents. The annual conferences of the Sections Committee were held on the Monday preceding the official opening of the convention, forty Sections were represented. Results of the conferences were published in pamphlet form.

Pacific Coast Convention.—The Twelfth Pacific Coast Convention was held at Del Monte, Cal., on October 2nd to 5th, 1923. Four technical sessions were held at which twenty-eight papers were presented. On the evening of October 4th the Edison Medal was presented to Dr. Robt. A. Millikan. The total attendance at the convention was 252.

Midwinter Convention. The Twelfth Midwinter Convention and 40th Anniversary Celebration was held in Philadelphia, Pa. February 4th to 8th, 1924. This was the largest convention in Institute history with a total attendance of 1738 registered members and guests, and the first midwinter meeting held outside of New York. Forty technical papers were presented at nine technical sessions. Among events of particular interest were the 40th Anniversary ceremonies, the Transportation Sessions addressed by men prominent in the railway field, the dedication of the Moore School of Electrical Engineering, conference on Engineering Education and the presentation of the Edison Medal to John W. Lieb. A variety of inspection trips and entertainment features was provided.

Spring Convention.—The Third Spring Convention was held in Birmingham, Alabama, April 7th to 11th, 1924. Seven technical sessions were held and twenty-nine papers presented in addition to three addresses on water-power development.

Abstracts of the reports of the chairman of many of the Institute committees and delegations are included herein under various headings.

Meetings and Papers Committee.—During the past year this Committee has prepared the programs for the four national conventions in conjunction with local convention committees. In addition to the technical sessions an attempt was made to stage certain feature sessions at each convention which would aid the industry or the membership in that locality and would prove interesting and attractive to all attending the meetings. At the Midwinter Convention the feature sessions were devoted to the reminiscences of electrical pioneers and to the broad aspects of railroad transportation. At the Birmingham Convention operating problems were discussed and feature meetings were held on hydroelectric developments. At the Edgewater Beach Convention the problems connected with metro-

politan transmission and distribution will be treated in technical sessions and in addition the work of the Standards Committee will be outlined. Another departure at this meeting is a complete session devoted to the reports of the technical committees. At the Pasadena Convention the technical aspects of energy utilization will be featured and in addition the value of interconnection will be outlined, while Dr. Millikan and his associates will divulge some notable researches.

At each of the conventions every effort was made to secure the hearty cooperation of public officials, utility executives and other business men in the convention activities so that the Institute would become better known and receive greater consideration. The results of this policy have been good and an appreciable stride has been made in securing a better understanding of the work of the Institute among those more directly connected with the business, legal and financial aspects of the electrical industry.

The committee has codified and coordinated its work so that definite methods of procedure and permanent policies are now on record. A complete guide for handling Institute conventions has been produced to be given to local convention committees.

Another positive contribution has been the revision of the pamphlet for authors of Institute papers.

The Committee has been particularly active in coordinating the work of the technical committees and has formulated very definitely the scope and type of technical paper suitable for Institute presentation and has established a uniform method of procedure for securing, examining and scheduling papers for convention and Section presentation.

It has given considerable thought to Section and Geographical District activities and advocates the development of the Geographical meetings as a permanent plan for stimulating Institute activity in each section of the country. These regional meetings also will give an outlet for the presentation of many papers submitted to the committee and are capable of being developed so that only two national conventions each year will ultimately be advisable, the Midwinter and June meetings.

A definite plan for cooperating with Sections and Geographical Districts in arranging programs has been made.

The committee has met at least once each month during the year and there has been a great stimulation of interest in the Institute and its work which is reflected in the very large number of papers presented during the year.

Publication Committee.—Regular meetings of the committee are now held monthly, and the chairman maintains continuous contact with the secretary and with the editor of the JOURNAL so that there will be no delay in disposing of routine matters which require cooperative consideration.

As distinguished from the period covered by the War years when there was a dearth of material submitted for publication and when it was difficult to procure a sufficient amount of matter for the JOURNAL, the present experience is that the number of technical papers submitted for presentation at meetings and for publication has increased so that the task of properly disposing of these many contributions is greatly enlarged.

The present plan of holding four national meetings each year, for each of which from thirty to forty technical papers are submitted, brings for publication a mass of material which far exceeds that brought in under the old order. The problem as it has enlarged, has required certain alterations in publication policies such as those presented fully in the October, 1923, JOURNAL. Editorial analysis of the papers received during the past year discloses that engineer authors, in many instances, follow the plan of building their stories around blueprints and photographs available in the files instead of writing clear stories and drawing upon illustration for the purpose only of illustrating features which cannot clearly be described in words. The former is perhaps the easy way for the author, but the publication costs thus imposed on the Institute are mounting seriously.

The committee hopes that improvement in this respect can be effected by revision of the pamphlet of suggestions to authors. This undertaking is now in hand.

Sections and Branches.—The various Sections of the Institute throughout the country continued their activities during the past year upon about the same scope as heretofore as indicated by the brief reports published each month in the JOURNAL. The Mexico Section authorized by the Directors during the previous year was established.

The activity among the Student Branches has been particularly marked during the past year. Many of the colleges have held a larger number of meetings than heretofore as shown in the following table. New Branches were authorized at the Catholic University of America, University of Florida, Rhode Island State College, University of South Dakota, University of Tennessee and University of Utah.

	For Fiscal Year Ending						
	May 1 1918	May 1 1919	May 1 1920	May 1 1921	May 1 1922	May 1 1923	May 1 1924
SECTION							
Number of Sections	34	34	36	42	45	46	47
Number of Section meetings held	245	217	262	303	373	344	381
Total Attendance	34,614	25,837	30,741	37,823	54,378	46,672	58,945
BRANCHES							
Number of Branches	59	61	62	65	67	68	77
Number of Branch meetings held	268	156	360	443	439	503	530
Attendance	10,683	6,441	16,827	21,629	25,358	26,893	25,674

Standards Committee.—The main activities of the Standards Committee throughout the year have been concentrated on the general revision of the Institute Standards, the start of which was reported last year. In brief the Standards Committee can report that the work of revision has been proceeding rapidly and effectively and that in all the subject matter considered a very few points have been encountered in which there is great difficulty in reaching agreement.

The revision involves the separation of the Institute Standards, present and proposed, into a large number of pamphlets dealing with different classes of apparatus, there now being over 40 in the complete list. This involves careful work in the organization of the material so that the subdivisions shall be as useful as possible, to avoid overlapping of scope and to produce uniformity in treatment in the different groups of standards. This work has been carried on very ably by a Working Committee under the direction of Mr. Hobart, which cooperates with the other Working Committees and in most cases has prepared initial drafts for the use of the other committees.

The Standards Committee now has 22 Working Committees with a membership of about 200, not excluding duplications. These committees are working actively on 19 sections of the Institute Standards. Five more Working Committees are in the process of formation. One section of the standards on Industrial Control Apparatus has been approved as Institute Standards by the Committee and is waiting approval by the Directors. Four other sections have been accepted as proposed standards and are in process of publication.

The translation of the 1922 Standards into Spanish, which was undertaken a year ago under the direction of one of the Working Committees has been completed and it is understood that the Department of Commerce will print this translation in the immediate future.

The Institute is now sole sponsor for three projects under the A. E. S. C. procedure, joint sponsor with others for five other projects, and is represented in about twenty other sectional committees.

There has been a continuation of active cooperation with other organizations interested in electrical standardization, both in connection with the revision of the Institute rules and the other matters now before the Standards Committee. There have been conferences looking forward to the united action of these organizations in connection with the adoption as American Standards of many of the standards of the various cooperating organizations. The Board of Directors have instructed the Standards Committee that standards prepared by the Institute should be adopted as Institute standards before being placed under A. E. S. C. procedure, and it is expected that many of the revised sections of the Institute standards when so adopted will, together with similar material which has been adopted by the other organizations, form the basis for cooperation in the preparation of American Electrical Standards.

Closer relations have been brought about during the year with the U. S. National Committee of the International Electrotechnical Commission.

Arrangements have been made with the Meetings and Papers Committee for a session on Standards at the Annual Convention in June.

American Engineering Standards Committee.—The American Engineering Standards Committee is the national clearing house for industrial standardization—a federation of the principal national organizations and governmental departments that are active in such work. It serves as the official channel of cooperation in international standardization activities, and it also serves as a bureau of information on standardization activities both in this country and abroad.

Through its methods and procedure all organizations concerned participate in deciding whether a given piece of work shall be undertaken, in developing the standard, and in recommending its approval. In this way the results of the work of hundreds of bodies are being broadened and unified into a truly national system of standards.

There now exists the most widespread interest and activity in industrial standardization that there has ever been. The Great War and the reconstruction problems arising from it have given an enormous impetus to the movement.

The growth of the work of the American Engineering Standards Committee is indicative of the growth of the movement as a whole. The following statistical summary, giving the figures shown in the A. E. S. C. 1923 and 1924 Year Books, shows the advance which has been made in the work of the Committee:

	1923	1924
Member-Bodies—organizations or groups of organizations whose representatives form the A. E. S. C.	23	23
National organizations included in the Member-Bodies	33	34
Representatives forming the Main Committee	55	57
Standards approved by A. E. S. C.	29	53
Standards up for approval by A. E. S. C.	45	35
Projects having official status (already approved or on which work is under way)	121	152
Projects for which sponsorship has been accepted	76	113
Organizations acting as sponsors for projects	48	57
Trade, technical or governmental bodies cooperating through representatives on special or sectional committees	275	313
Individuals on sectional committees	917	1081

The sustaining-membership plan announced in last year's report has been sufficiently successful to permit considerable increased activities on the part of the American Engineering Standards Committee, one of these being the addition to the staff of an engineer-translator, making possible more satisfactory cooperative work with foreign standardizing bodies. An attempt is made to keep a complete file of foreign standards at the offices of the A. E. S. C., and information in regard to any of these may be obtained by communicating with the Secretary.

Close cooperative work has been maintained with the Federal Specifications Board, whereby the specifications which are being considered by the Board are reviewed by those groups cooperating in A. E. S. C. work which may be interested in the specifications.

During the year certain changes have been made in the procedure of the American Engineering Standards Committee, which permit of more freedom in the acceptance of specifications and the revision of specifications previously approved. This is resulting in the submission of an increased number of engineering specifications for adoption as "American Standards."

U. S. National Committee of I. E. C.—At the meeting of the Council of the International Electrotechnical Commission, held in Paris December 3rd last, Dr. C. O. Mailloux, who had been President of the Commission since 1919 retired, and was succeeded by Mr. Guido Semenza of Milan. Subsequently, in view of his distinguished services with the Commission, Dr. Mailloux was elected Honorary President of the Commission. An election was held also of the other general officers of the Commission. The Commission is represented in this country by the United States National Committee of the International Electrotechnical Commission.

The U. S. National Committee held its first meeting on March 12th. Dr. C. H. Sharp was elected President of the Committee, and immediately thereafter Dr. C. O. Mailloux, its former President, was elected Honorary President of the Committee.

The Institute of Radio Engineers was invited to become affiliated with the U. S. National Committee and appoint delegates to that Committee.

It was announced that meetings would be held in London on July 15th, 16th and 17th of the Advisory Committees on Rating of Electrical Machinery, on Nomenclature and on Graphical Symbols of the I. E. C., and the Executive Council was authorized to nominate delegates to these meetings.

U. S. National Committee International Commission on Illumination.—It was noted in the report of last year that at the plenary meeting of this Commission held in Paris in 1921, it was determined to hold the next plenary meeting of the Commission in the United States in 1924. Consequently, during the earlier part of the year the matter of holding such meeting in this country was the chief topic of discussion of the U. S. National Committee. The matter of the advisability of attempting to hold this meeting in the face of adverse European conditions was gone into very carefully, the advice of prominent English and European members of the Commission being obtained informally as to this point. It had been hoped that conditions would show a sufficient improvement so as to offer a reasonable prospect of carrying out the plan. However, political and economic developments were not of such a character as to justify this hope, and in order that the U. S. National Committee should not be put in the position of obstructing in any way the best interests of the Commission, a resolution was offered and carried at a meeting of the U. S. National Committee held on June 7th, 1923 waiving the right to hold the next plenary meeting in the United States, but expressing the hope that in case the 1924 meeting is not held in the United States the next plenary meeting will be held here.

The U. S. National Committee having thus, for the best interests of the I. C. I., waived its right to hold the meeting in this country, the officers of the I. C. I. determined to fix upon Geneva, Switzerland, as the place for the meeting. This meeting will be held on July 21st to 24th, next. The U. S. National Committee has appointed a committee to take care of the matter of American participation in this meeting. A suitable number of papers have been promised for this purpose and the attendance of a representative delegation from this country seems assured.

Committee on Safety Codes.—The Committee on Safety Codes is made up of representatives of the Institute assigned to cooperate with various standardizing committees in matters relating to different kinds of safety codes. In connection with the revision of the National Electrical Safety Code of the Bureau of Standards, the Institute is represented on the Sectional Committees covering the various subdivisions. Some progress has been made.

The Institute is represented on the Sectional Committee on Safety Codes for Elevators. This committee has partly completed its work.

The Institute is represented on the Sectional Committee for Safety Codes for Floor Openings and Railings. There has been little activity on this committee during the past year.

The Institute is represented on the Sectional Committee to consider electrical regulations in coal mines. The work of this Sectional Committee has been organized and allotted to various subcommittees and is progressing satisfactorily.

The Institute is represented on the Electrical Committee of the National Fire Protection Association. This committee has issued a regular biennial revision of the electrical code during the past year.

Technical Committees.—Reports of Technical Committees embracing an outline of the year's work

and a summary of progress in the industry will be presented at the Annual Convention and printed in the JOURNAL.

Membership.—The results of the Membership Committee's efforts this year are shown in the following table:

	Honorary Member	Fellow	Member	Associate	Total
Membership, April 30, 1923...	6	578	2,264	12,450	15,298
Additions:					
Transferred		20	79		
New Members Qualified		4	96	2,040	
Reinstated		4	9	53	
Deductions:					
Died		6	11	51	
Resigned		5	15	222	
Transferred			14	85	
Dropped		1	49	689	
Membership, April 30, 1924...	6	594	2,359	13,496	16,455

Net increase in Membership during the year 1,157

Deaths.—The following deaths have occured during the year.

Fellows: Robert F. Hayward, Henry St. Clair Putman, William M. Riggs, Charles P. Steinmetz, Arthur G. Webster, Charles Wirt.

Members: Frederick C. Bates, Louis Bell, John A. Britton, J. Stanford Brown, George A. Cellar, Berrywick S. Craig, Ralph E. Gilman, Wallace P. Hurley, Frank E. Kinsman, John Ely Moore, Herman J. Strobel.

Associates: Chester H. Arnold, E. W. Babcock, Otto C. Beck, Joseph A. Brennan, Arthur H. Burnett, Wilfred W. Chandler, Micheal F. D'Andre, Delwyn Dessar, John W. Ellard, William N. Fashbaugh, Frank E. Fielding, M. L. Gilmore, Frank R. Grady, John Grant, John A. Grimmons, George M. Haesler, Yagenda Hayashi, L. W. Hendricks, A. M. Hennefer, Henry J. Herman, John S. Holliday, William Hoopes, Edward J. Hunt, P. N. Jones, William E. Kampf, Mohachi Komatsu, Joseph A. Leahy, A. R. Ledoux, Earle S. Libby, Burton E. Lucas, Arthur Lund, Louis F. Matty, R. H. McKibben, John Mustard, Sam L. Naphtaly, L. L. Phillips, Herbert S. Pope, Louis L. Proctor, Yasuo Riko, Henry C. Russell, Karl E. Schrieber, C. P. Seidler, George A. Speer, Joshua W. Taylor, Chuzaburo Tsukamoto, Clinton H. Turner, Ralph A. Whiteman, Allan H. Whiting, Henry M. Whitney, Henry C. Wilson, Montraville Wood.

Total deaths, 68.

Board of Examiners.—The Board of Examiners during the year held eleven meetings, averaging about two hours and 45 minutes each. It considered and referred to the Board of Directors a total of 4036 applications for admission or transfer to the higher grades. This is very close to last year's figure.

APPLICATIONS FOR ADMISSION

Recommended for grade of Associate......	2137	
Not recommended........................	1	2138
Recommended for grade of Member.......	88	
Not recommended for admission to this grade................................	70	158
Recommended for grade of Fellow.........	2	
Not recommended for admission to this grade................................	5	7
Recommended for enrolment as Students..	1594	1594

APPLICATIONS FOR TRANSFER

Recommended for grade of Member.......	82	
Not recommended for transfer to this grade.	19	101
Recommended for grade of Fellow.........	24	
Not recommended for transfer to this grade.	4	28
Total number of applications considered....		4026

American Committee on Electrolysis.—The American Committee on Electrolysis has been practically inactive during the past year, pending the decision by the various organizations represented on the Committee as to the continuance of the work. Present indications are that the work of the Committee will receive the necessary financial support from these organizations, and that the active work will go forward during the present year.

Scholarships.—The governing bodies of Columbia University have placed at the disposal of the Institute two scholarships in electrical engineering in addition to the one previously granted. In consequence, the Institute is now authorized to award a scholarship each year so that there may be one man in each class holding a scholarship on the nomination of the Institute; these scholarships will continue until further notice. Each scholarship pays $350 toward the annual tuition, and reappointment for completion of course is conditioned upon the maintenance of good standing.

The first scholarship for the academic year 1923-24 was awarded by the Institute to Dudley P. South; this leaves the two other scholarships available for students who plan to graduate from the electrical engineering course in 1926 and 1927 respectively.

Institute Prizes.—At the meeting of the Board of Directors of the Institute of April 16, 1921, recommendations were approved establishing two Institute prizes to be awarded yearly to authors of worthy papers. The prize consists each of $100 in cash with suitable certificate.

"The Transmission Prize for 1922 was awarded R. J. C. Wood for his paper entitled '220-kv. Transmission system, Southern California Edison Company and Some 220-kv. Researches.' Honorable mention should be given H. B. Dwight for his paper entitled 'Electric Characteristics of Transmission Lines.'

"The First-Paper Prize for 1922 was awarded to C. H. Van Asperen for his paper entitled 'Mechanical Forces on Busbars Under Short-Circuit Conditions.' Honorable mention should be given to E. B. Shand for his paper entitled 'An Analytical Investigation of the Causes of Flashing of Synchronous Converters.' "

Edison Medal.—The Edison Medal for 1923 was awarded to John W. Lieb, New York, N. Y., "for the development and operation of electric central stations for illumination and power." The presentation ceremonies took place Monday evening, February 4th, 1924 at the Midwinter Convention, Philadelphia, Pa.

John Fritz Medal.—The John Fritz Medal Board of Award, which is composed of representatives of the national societies of Civil, Mining, Mechanical and Electrical Engineers, awarded the 20th medal to Dr. Ambrose Swasey in recognition of his achievements "as a designer and manufacturer of instruments and machines of precision, a builder of great telescopes, a benefactor of education and the founder of Engineering Foundation." The medal was presented at a gathering of engineers at the Engineering Societies Building, New York on the evening of April 23, 1924.

Kelvin Medal Award.—On December 14, 1923 the Committee of Award of the Kelvin Medal Trust named Dr. Elihu Thomson of Swampscott, Mass., as the recipient of the second triennial award. The Trust Fund represents the surplus obtained in connection with a call for subscriptions to erect a memorial window to Lord Kelvin in Westminster Abbey. The award is made by a committee composed of the Presidents of the principal representative British Engineering Institutions, as a mark of distinction to a person who has achieved eminence as an engineer or investigator in the kind of work applicable to the advancement of engineering with which Lord Kelvin was especially identified. The presentation of the medal will take place at the Kelvin Centenary Celebration, London, England on the afternoon of Thursday, July 10, 1924.

The Joseph Henry Bust.—In accordance with the custom of inviting civic, patriotic, scientific and other organizations and individuals to contribute bronze busts of those who have been elected to the Hall of Fame by the Senate of New York University, an invitation was received asking the Institute to act as sponsor in providing a bust of Joseph Henry. The necessary funds were raised by general subscription and by appropriation for the presentation of the bust and the unveiling ceremonies will take place on May 13, 1924

1922 Commission of Washington Award.—The Washington Award for 1922 was voted to Captain Robert W. Hunt and the presentation was made at the annual meeting of the Western Society of Engineers, held June 18, 1923. This 1922 award was made "For preeminent service in promoting the public welfare, for his pioneer work in the development of the steel industry in the United States and for a life devoted to the advancement of the engineering profession."

The award is made annually by a committee composed of nine representatives of the Western Society of Engineers and two each from the A. S. C. E., the A. I. M. E., the A. S. M. E. and the A. I. E. E. The award of the medal was established in 1917 by Past President J. W. Alvord of the Western Society "to be annually presented to an engineer whose work in some special instance, or whose services in general have been noteworthy for their merit in promoting the public good."

Employment Service.—The employment service which the Institute has maintained for many years is now conducted as a cooperative bureau in conjunction with a similar service maintained by the national societies of civil, mining and mechanical engineers under the title "Engineering Societies Employment Service."

The bureau was placed on a cooperative basis September 1, 1923 as the result of a report rendered in March 1923 by a joint committee of four major societies appointed to make a thorough investigation of all factors concerned in the maintenance and operation of an employment service; it having been generally recognized for some time that the scope of the service and its value to the individual member and employer was evidently falling far short of what might be possible under a more comprehensive plan of operation, were larger funds available.

With the committee's report as a basis the Secretaries of the A. I. E. E., A. S. M. E., A. S. C. E. and A. I. M. E., under whose supervision the service is conducted, drew up a plan for the maintenance of the bureau through the joint contributions of the societies and those individual members who are benefited, free service to be continued only to a limited extent principally by the publication from month to month without charge of announcements of available men in the respective journals of the societies concerned. An employment bulletin is issued weekly but only to those regularly registered as available. It is hoped that sufficient surplus funds may become available to permit the extension of the service to other cities, this plan to be continued from year to year until the bureau is on a truly national basis.

American Engineering Council.—During the past year American Engineering Council of the Federated American Engineering Societies has continued to function. The Annual Meeting was held in Washington

on January 11th and 12th, 1923, and was well attended. One of several Constitutional amendments endorsed by the Council at its Annual Meeting and which has since become effective eliminates from the name "Federated American Engineering Societies" so that the organization will be hereafter entitled "American Engineering Council."

One day of the annual session was set aside for considering Federal Government Reorganization by the establishment of a Department of Public Works, and a large number of engineering organizations, many of them not members of American Engineering Council, sent delegates.

During the year the Administrative Board held the following meetings:

January 10th	at Washington,
March 23-24	at Cincinnati,
June 6-7	at St. Paul,
Oct. 12-13	at Rochester.

The Institute has been represented on the Administrative Board, the Executive Committee and on most of the standing and special Committees and the representatives have been diligent in endeavoring to maintain and increase the prestige of the engineering profession through the work of the Council and its Committees, having always in mind the interests of our organization.

It is satisfying to be able to report that a very cordial cooperative attitude towards the Institute has been uniformly manifested by the delegates representing other organizations, and it is undoubtedly a fact that the frequent points of contact afforded by Council's activities have helped to strengthen the pleasant relationships between the Institute and such other organizations.

It is the belief of your representatives that an organization having the functions allocated to American Engineering Council is needed. The detail activities of American Engineering Council have been chronicled from month to month in the Institute JOURNAL.

World Power Conference.—The first World Power Conference will be held in London, June 30 to July 12, 1924. It is to be held at the British Empire Exhibition, Wembly, London, and is promoted by the Council of the British Electrical and Allied Manufacturer's Association, in cooperation with Technical and Scientific Institutions in Great Britain and other countries.

The Conference has been called for the purpose of discussing the technical and economic problems of power development, transmission and utilization and for promoting general interest in power development. It will consider how the industrial and scientific sources of power may be adjusted nationally and internationally.

The A. I. E. E. is officially cooperating in the World Power Conference and is represented on the organizing committee for American participation by three of its members. Several other members will present technical papers. The plan of the Program Committee was to explain the character and extent of American power development from both construction and operation standpoints, and give opportunity for setting forth conditions under which American capital has been able to undertake this development. The sections into which the program has been divided are as follows: Review of power resources; Water power and fuels; Steam and electric power; Industrial phases; Economic phases.

United Engineering Society.—This Society performs for the national societies of Civil, Mining, Mechanical and Electrical Engineers, certain specific acts which are governed by contracts; the primary function of the United Society being to hold in trust and to administer for these societies the Engineering Societies Building, in which the headquarters of the national societies are located.

Extracts from the annual financial report of the United Engineering Society were published in the March 1924 JOURNAL.

Engineering Societies Library.—The library of the Institute is combined with the libraries of the national societies of Civil, Mining and Mechanical Engineers, administered as the "Engineering Societies Library" under the direction of the Library Board of the United Engineering Society; this board is composed of representatives of each of the four societies referred to above.

In order to place the facilities of the library at the disposal of persons residing at a distance from New York, a Library Service Bureau has been established, and a staff of expert searchers and translators is employed to cover almost any engineering topic, in the following manner: abstracting, translating, bibliographing, statistical searches and reports, searches for patent purposes, copying, preparing reference cards, etc. During the year arrangements have been made for a lending department.

An abstract of the annual report of the Engineering Societies Library covering the calendar year 1923 was published in the March 1924 JOURNAL.

Engineering Foundation.—Engineering Foundation is a trust fund established in 1914 by Ambrose Swasey, of Cleveland, Ohio, by gifts to United Engineering Society as a nucleus of a large endowment "for the furtherance of research in science and in engineering, or for the advancement in any other manner of the profession of engineering and the good of mankind." It is administered by the Engineering Foundation Board upon which the Institute and other national engineering societies are represented. The Board is a Department of United Engineering Society.

The Foundation has made appropriations for various research projects and has cooperated in others.

An abstract of the annual report for the year 1923 was published in the March 1924 JOURNAL.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and has appointed representatives upon a number of new Sectional Committees of American Engineering Standards Committee and upon the Apparatus Makers and Users Committee, and the Board of Investigation and Coordination of the Society for the Promotion of Engineering Education. A complete list of representatives is published frequently in the JOURNAL.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report follows:

Respectfully submitted for the Board of Directors.

F. L. HUTCHINSON, *Secretary.*

New York, May 16, 1924.

ATLANTA
BALTIMORE
BIRMINGHAM
BOSTON
BUFFALO
CHICAGO
CINCINNATI
CLEVELAND
DALLAS
DENVER
DETROIT
KANSAS CITY
LOS ANGELES
MINNEAPOLIS
NEWARK
NEW ORLEANS

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CERTIFIED PUBLIC ACCOUNTANTS
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NEW YORK

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PHILADELPHIA
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PORTLAND
PROVIDENCE
SAINT LOUIS
SALT LAKE CITY
SAN DIEGO
SAN FRANCISCO
SEATTLE
TULSA
WATERTOWN
HAVANA
LONDON
PARIS
SHANGHAI

May 12, 1924

American Institute of Electrical Engineers,
33 West 39th Street,
New York.

Dear Sirs:

Pursuant to engagement, we have audited your books and accounts for the year ended April 30, 1924, and submit herewith our certificate and the following described exhibits and schedule:

Exhibit "A"—General Balance Sheet, April 30, 1924.

Schedule No. 1—Reserve Capital Fund—Securities.

Exhibit "B"—Summary of Income and Profit & Loss for the Year ended April 30, 1924.

Yours truly,

HASKINS & SELLS

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE OF AUDIT

We have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1924, and

WE HEREBY CERTIFY that the accompanying General Balance Sheet properly exhibits the financial condition of the Institute at April 30, 1924, that the Summary of Income and Profit & Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS

NEW YORK,
May 12, 1924.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

EXHIBIT A.

GENERAL BALANCE SHEET, APRIL 30, 1924

ASSETS

REAL ESTATE:			
One-Fourth Interest in United Engineering Society's Land, Building, and Building Equipment, 25 to 33 West 39th Street (Depreciation carried on Books of United Engineering Society).			$491,642.36
EQUIPMENT:			
Library—Volumes and Fixtures		$40,477.27	
Works of Art, Paintings, etc		3,001.35	
Office Furniture and Fixtures	$15,949.72		
Less Reserve for Depreciation (including $3,000.00 funded)	11,121.13	4,828.59	
Total Equipment			48,307.21
WORKING ASSETS:			
"Transactions, etc"		$10,451.63	
Paper and Cover Paper		1,577.93	
Badges		2,486.46	
Total Working Assets			14,516.02
CURRENT ASSETS:			
Cash		$21,510.55	
Accounts Receivable:			
Members—For Dues		14,667.22	
Advertisers		2,640.90	
Miscellaneous		789.07	
Accrued Interest on Investments		323.55	
Accrued Interest on Bank Balances		206.48	
Total Current Assets			40,137.77
FUNDS:			
Reserve Capital Fund—Securities—Schedule No. 1.		$44,271.35	
Life Membership Fund:			
Cash	$1,638.67		
Chicago, Burlington & Quincy Railroad Company 4% Bonds, 1958, Par Value $5,000.00	4,868.75		
Accrued Interest	33.33	6,540.75	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$494.03		
New York City 4½% Bonds, 1957, Par Value $2,000.00	2,210.43		
New York Telephone Company 4½% Bond, 1939, Par Value $1,000.00	878.75		
Accrued Interest	67.50	3,650.71	
Mailloux Fund:			
Cash	$5.17		
New York Telephone Company 4½% Bond, 1939, Par Value $1,000.00	1,000.00		
Accrued Interest	22.50	1,027.67	
Midwinter Convention Fund—Cash		149.56	
Depreciation of Furniture and Fixtures Fund—Cash		3,000.00	
Total Funds			58,640.04
Total			$653,243.40

LIABILITIES

CURRENT LIABILITIES:		
Accounts Payable	$7,585.40	
Dues Received in Advance	3,402.96	
Entrance Fees and Dues advanced by Applicants for Membership	498.50	
Subscriptions for "Transactions" received in Advance	85.00	
Total Current Liabilities		$11,571.86
FUND RESERVES (NOT INCLUDING DEPRECIATION RESERVE):		
Reserve Capital Fund	$44,271.35	
Life Membership Fund	6,540.75	
International Electrical Congress of St. Louis—Library Fund	3,650.71	
Mailloux Fund	1,027.67	
Midwinter Convention Fund	149.56	
Total Fund Reserves (not Including Depreciation Reserve)		55,640.04
SURPLUS, Per Exhibit "B"		586,031.50
Total		653,243.40

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Summary of Income and Profit & Loss for the Year Ended April 30, 1924

Exhibit B.

Income:			
Entrance Fees		$11,133.80	
Dues		*174,318.01	
Students' Dues		11,792.50	
Transfer Fees		955.00	
Advertising		63,172.99	
Journal Subscriptions		6,700.98	
Transactions Subscriptions		9,720.00	
Miscellaneous Sales		4,261.63	
Badges Sold	$5,559.75		
Less Cost	4,332.02	1,227.73	
Interest on Securities in Reserve Capital Fund		1,643.37	
Interest on Bank Balances		1,181.37	
Total			$286,107.38
Expenses:			
Publications:			
Journal	$94,713.56		
Transactions	12,126.63		
Year Book	7,321.89	$114,162.08	
Meetings		14,373.77	
Administrative Expenses		47,303.92	
Sections Committee		25,219.31	
Membership Committee		7,501.78	
Standards Committee		1,376.14	
Finance Committee		238.61	
Headquarters Committee		855.54	
Code Committee		60.00	
Edison Medal Committee		361.16	
Geographical District Executive Committees		623.47	
American Engineering Standards Committee		1,500.00	
International Electrotechnical Commission		773.33	
United States National Committee of International Commission on Illumination		600.00	
American Committee on Electrolysis		100.00	
President's Special Appropriation		354.65	
Board of Directors—Mileage		1,558.30	
Honorary Secretary		4,000.00	
John Fritz Medal Award		64.50	
Bust of Joseph Henry		513.60	
First Paper Prize, 1922		100.00	
Transmission Prize, 1922		100.00	
Engineering Societies Library:			
Maintenance	$5,500.00		
Recataloging	2,500.00	8,000.00	
United Engineering Society Assessment		4,860.00	
Federated American Engineering Society		14,892.00	
Engineering Societies Employment Service		2,626.25	
International Annual Tables		100.00	
Total			252,218.41
Net Income (Forward)			$33,888.97

*Includes $76,490.00 allocated to subscriptions for the Journal.

Net Income (Forward)		$33,888.97
Profit & Loss Credits:		
Adjustment of Institute's One-Fourth Interest in United Engineering Society's Real Estate	$1,857.19	
Adjustment of Inventory of Library Volumes and Fixtures	94.21	
Total		1,951.40
Gross Surplus for the Year		$35,840.37
Profit & Loss Charges:		
Uncollectable Dues Written off	$6,720.50	
Provision for Depreciation of Furniture and Fixtures	580.70	
Adjustment of Inventory of Furniture and Fixtures, April 30, 1924	202.50	
Adjustment of Inventory of Transactions, April 30, 1924	547.50	
Total		8,051.20
Surplus for the Year		$27,789.17
Surplus, May 1, 1923	$571,954.83	
Less Transferred to Capital Fund Reserve in Accordance with Resolution of Board of Directors	13,712.50	558,242.33
Surplus, April 30, 1924		$586,031.50

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

Reserve Capital Fund—Securities, April 30, 1924

Exhibit A.

Schedule No. 1.

	Par Value	Book Value
The Detroit Edison Company 1st and Refunding, 6% Series "B", Gold Bonds, due 1940	$5,000.00	$5,190.00
The New York Central Railroad Company Registered, 5%, Refunding and Improvement Mortgage Bonds, due 2013, Series "C"	6,000.00	5,742.50
Chicago, Burlington & Quincy Railroad Company 5%, 1st and Refunding Mortgage, Registered, Gold Bond, Series "A", due 1971	1,000.00	1,010.00
Great Northern Railroad Company 5½%, General Mortgage, Registered, Gold Bonds, Series "B", due 1952	6,000.00	5,827.50
Southern Railway Company 5%, 1st Consolidated Mortgage, Registered, Gold Bond, due 1994	1,000.00	980.00
City of Wilmington, Delaware, 4½% Bonds, due 1934	15,000.00	15,521.35
United States Third Liberty Loan 4¼% Bonds, due 1928	10,000.00	10,000.00
Total	$44,000.00	$44,271.35

Officers A. I. E. E. 1923-1924

PRESIDENT
(Term expires July 31, 1924)
HARRIS J. RYAN

JUNIOR PAST-PRESIDENTS

(Term expires July 31, 1924)
WILLIAM McCLELLAN

(Term expires July 31, 1925)
FRANK B. JEWETT

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(Terms expire July 31, 1924)
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R. F. SCHUCHARDT (District No. 5)
H. T. PLUMB (District No. 9)
G. FACCIOLI (District No. 1)
H. W. EALES (District No. 7)

(Terms expire July 31, 1925)
J. E. MACDONALD (District No. 8)
HERBERT S. SANDS (District No. 6)
S.E.M. HENDERSON (District No.10)
H. E. BUSSEY (District No. 4)
WILLIAM F. JAMES (District No. 2)

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(Terms expire July 31, 1924)
HAROLD B. SMITH
JAMES F. LINCOLN
E. B. CRAFT

(Terms expire July 31, 1925)
R. B. WILLIAMSON
A. G. PIERCE
HARLAN A. PRATT

(Terms expire July 31, 1926)
H. M. HOBART
ERNEST LUNN
G. L. KNIGHT

(Terms expire July 31, 1927)
WILLIAM M. McCONAHEY
W. K. VANDERPOEL
H. P. CHARLESWORTH

TREASURER / **SECRETARY**
(Terms expire July 31, 1924)
GEORGE A. HAMILTON / F. L. HUTCHINSON

HONORARY SECRETARY
RALPH W. POPE

GENERAL COUNSEL
PARKER & AARON
30 Broad Street, New York.

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*Norvin Green, 1884-5-6.
*Franklin L. Pope, 1886-7.
*T. Commerford Martin, 1887-8.
Edward Weston, 1888-9.
Elihu Thomson, 1889-90.
*William A. Anthony, 1890-91.
*Alexander Graham Bell, 1891-2.
Frank Julian Sprague, 1892-3.
*Edwin J. Houston, 1893-4-5.
*Louis Duncan, 1895-6-7.
*Francis Bacon Crocker, 1897-8.
A. E. Kennelly, 1898-1900.
Carl Hering, 1900-1.
*Charles P. Steinmetz, 1901-2.
Charles F. Scott, 1902-3.
Bion J. Arnold, 1903-4.
John W. Lieb, 1904-5.
*Schuyler Skaats Wheeler, 1905-6.
*Samuel Sheldon, 1906-7.
*Henry G. Stott, 1907-8.
Louis A. Ferguson, 1908-9.
Lewis B. Stillwell, 1909-10.
Dugald C. Jackson, 1910-11.
Gano Dunn, 1911-12.
Ralph D. Mershon, 1912-13.
C. O. Mailloux, 1913-14.
Paul M. Lincoln, 1914-15.
John J. Carty, 1915-16.
H. W. Buck, 1916-17.
E. W. Rice, Jr., 1917-18.
Comfort A. Adams, 1918-19.
Calvert Townley, 1919-20.
A. W. Berresford, 1920-21.
William McClellan, 1921-22.
Frank B. Jewett, 1922-23.

*Deceased

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A. S. Garfield, 45 Bd. Beausejour, Paris 16 E, France.
H. P. Gibbs, Tata Sons Ltd., Navsari Building, Fort Bombay, India.
Guido Semenza, 39 Via Monte Napoleone, Milan, Italy.
Eiji Aoyagi, Kyoto Imperial University, Kyoto, Japan.
Lawrence Birks, Public Works Department, Wellington, New Zealand.
Axel F. Enstrom, 24a Grefturegatan, Stockholm, Sweden.
W. Elsdon-Dew, P. O. Box 4563, Johannesburg, Transvaal, Africa.

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Chairmen of Technical Committees.
Chairmen of Sections.

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(Term expires July 31, 1925)
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(Term expires July 31, 1926)
B. A. Behrend, John H. Finney, C. S. Ruffner.
(Term expires July 31, 1927)
Gano Dunn, F. A. Scheffler, W. R. Whitney.
(Term expires July 31, 1928)
C. C. Chesney, Robert A. Millikan, M. I. Pupin.

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(Term expires July 31, 1924)
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(Term expires July 31, 1925)
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William M. McConahey,

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Graham Bright, R. L. Kingsland, C. D. Woodward.
A. B. Kiser,

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J. T. Barron, R. B. Howland, C. P. Osborne,
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H. W. Eales, H. A. Kidder, F. A. Scheffler,
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John Harisberger, H. P. Liversidge, R. B. Williamson.
D. C. McClure,

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L. B. Chubbuck, B. G. Jamieson, C. H. Sanderson,
W. S. Edsall, M. G. Lloyd, E. R. Stauffacher,
Clarence T. Evans, A. A. Meyer, E. C. Stone,
F. C. Hanker, W. H. Millan, A. H. Sweetnam,
S. E. M. Henderson, J. M. Oliver, John B. Taylor.
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F. S. Dellenbaugh, A. E. Kennelly, Clayton H. Sharp,
W. A. Del Mar, M. G. Lloyd, C. E. Skinner,
W. P. Dobson, R. W. Sorensen.

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L. W. Chubb, F. H. Kroger, Edgar Russel,
E. H. Colpitts, N. M. Lash, H. A. Shepard,
R. N. Conwell, Ray H. Manson, I. J. Thorp,
C. E. Davies, G. W. McRae, F. A. Wolff,
H. W. Drake, R. D. Parker, C. A. Wright.
L. F. Fuller, F. A. Raymond,

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Dugald C. Jackson, W. B. Potter.

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M. O. Bolser, C. H. Kraft, C. C. Robinson,
Wallace S. Clark, W. S. Lee, D. W. Roper,
W. H. Cole, W. M. McConahey, C. E. Schwenger,
R. N. Conwell, E. B. Meyer, Carroll H. Shaw,
W. A. Del Mar, G. J. Newton, Arthur Simon,
L . L. Elden, F. W. Peek, Jr., W. K. Vanderpoel,
F. M. Farmer, A. M. Perry, C. T. Wilkinson,
J. P. Jollyman, R. J. C. Wood.

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ON AMERICAN BUREAU OF WELDING

H. M. Hobart.

ON AMERICAN COMMITTEE ON ELECTROLYSIS

B. J. Arnold, N. A. Carle, F. N. Waterman.

ON AMERICAN ENGINEERING COUNCIL

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C. G. Adsit, William McClellan, *C. E. Skinner,
H. W. Eales, *L. F. Morehouse, L. B. Stillwell,
*John H. Finney, E. W. Rice, Jr., Calvert Townley,
F. L. Hutchinson, R. B. Williamson.
*D. C. Jackson,

*Members of Administrative Board.

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L. T. Robinson (Alternate)

ON APPARATUS MAKERS AND USERS COMMITTEE

C. E. Skinner.

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H. H. Barnes, Jr., Bancroft Gherardi, H. A. Lardner.

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Harris J. Ryan

ON ENGINEERING FOUNDATION BOARD

Frank B. Jewett, L. B. Stillwell.

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Frank B. Jewett.

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[6]S. G. Rhodes, Secretary, 124 E. 15th St., New York, N. Y.

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Gano Dunn,	A. S. McAllister,	[2]C. L. Warwick,
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[2]Representative of American Society for Testing Materials.
[3]Representative of War Department.
[4]Representative of Navy Department.
[5]Representative of Bureau of Foreign and Domestic Commerce.
[6]Representative of National Electric Light Association.
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[8]Representative of Associated Manufacturers of Electrical Supplies.

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